Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.4.2-a+b-sec-^m-d-sec-^n-A+B-sec+C-sec^2-

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1373]. This is test number [125].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1373)	% 0. (0)
Mathematica	% 97.82 (1343)	% 2.18 (30)
Maple	% 91.99 (1263)	% 8.01 (110)
Maxima	% 33.43 (459)	% 66.57 (914)
Fricas	% 53.02 (728)	% 46.98 (645)
Sympy	% 0.8 (11)	% 99.2 (1362)
Giac	% 38.67 (531)	% 61.33 (842)

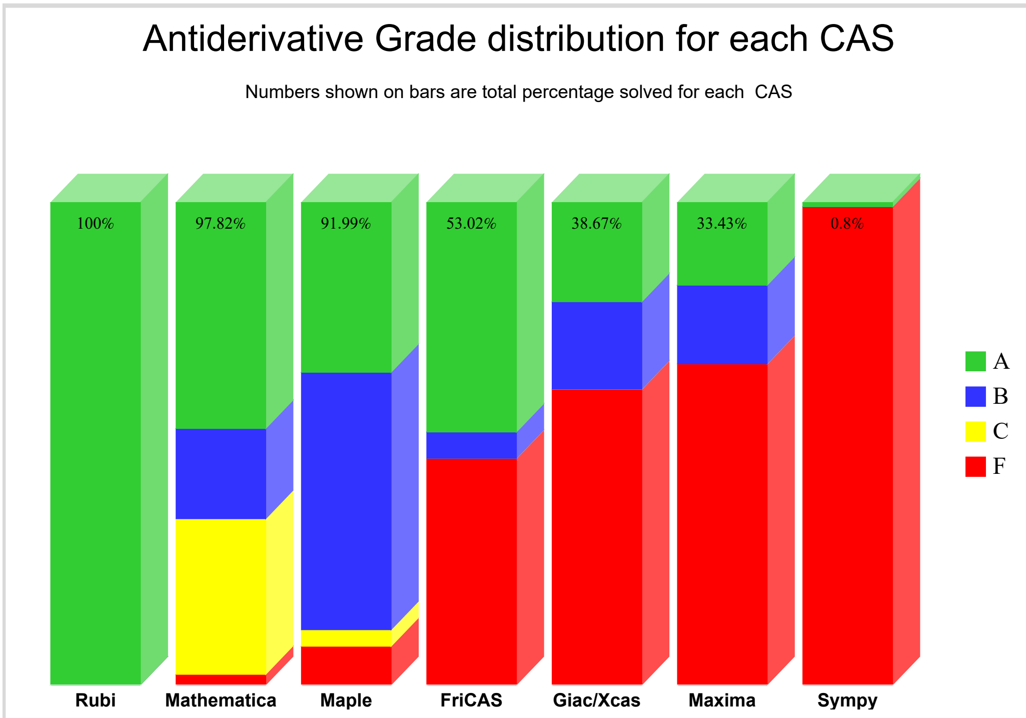
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

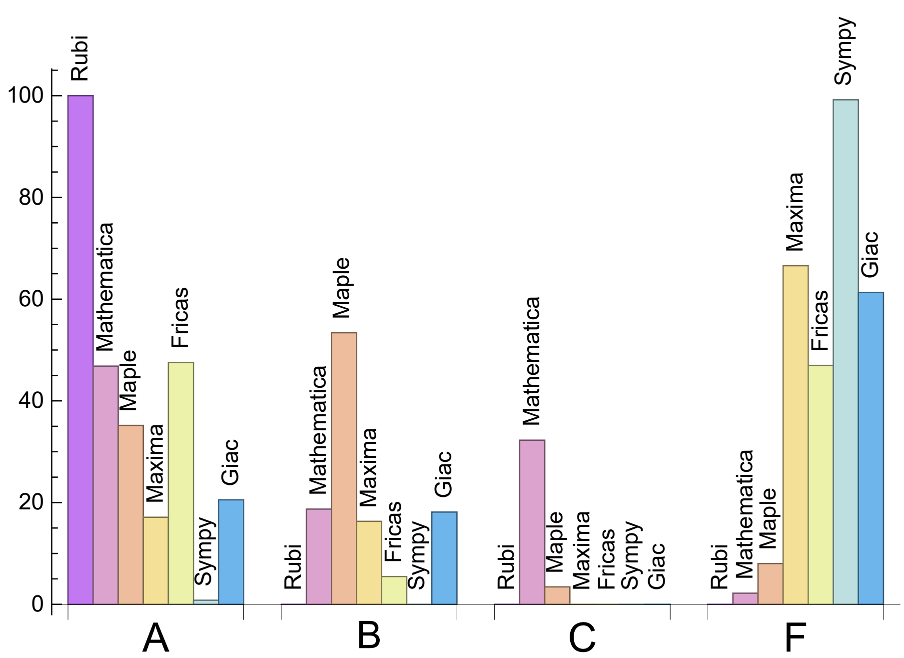
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	46.83	18.72	32.27	2.18
Maple	35.18	53.39	3.42	8.01
Maxima	17.12	16.31	0.	66.57
Fricas	47.56	5.46	0.	46.98
Sympy	0.8	0.	0.	99.2
Giac	20.54	18.14	0.	61.33

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.63	233.36	0.99	208.	1.
Mathematica	6.59	2839.5	8.42	311.	1.58
Maple	1.54	1097.44	3.59	476.	2.59
Maxima	1.85	1283.95	6.69	400.	2.47
Fricas	3.28	939.42	5.02	700.	4.64
Sympy	0.61	10.55	0.5	0.	0.
Giac	2.91	539.41	2.94	386.	2.4

1.4 list of integrals that has no closed form antiderivative

{761, 762, 763, 764, 1070, 1071, 1072, 1073, 1074}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {5, 10, 15, 21, 22, 23, 27, 28, 29, 30, 32, 33, 34, 35, 36, 39, 45, 46, 51, 52, 65, 66, 67, 68, 69, 72, 73, 74, 76, 77, 78, 79, 80, 81, 122, 130, 131, 138, 146, 147, 189, 198, 206, 216, 218, 222, 246, 247, 248, 249, 250, 251, 252, 254, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 277, 278, 279, 280, 282, 283, 284, 285, 289, 290, 390, 399, 406, 407, 510, 511, 512, 513, 516, 519, 520, 521, 522, 523, 525, 527, 528, 529, 530, 531, 533, 541, 549, 550, 558, 559, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 625, 626, 627, 628, 629, 630, 631, 632, 636, 684, 686, 687, 691, 692, 693, 694, 695, 696, 698, 699, 701, 702, 703, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 735, 736, 737, 738, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 758, 801, 808, 814, 815, 816, 817, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 842, 843, 844, 845, 846, 848, 849, 850, 851, 852, 853, 854, 857, 858, 859, 860, 908, 909, 910, 911, 912, 917, 918, 919, 920, 921, 924, 926, 927, 928, 935, 936, 937, 938, 939, 940, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 970, 971, 972, 973, 974, 975, 976, 979, 981, 982, 983, 989, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1043, 1044, 1045, 1046, 1047, 1048, 1050, 1051, 1052, 1055, 1056, 1057, 1059, 1061, 1062, 1063, 1064, 1066, 1067, 1068, 1069, 1078, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1182, 1183, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1271, 1294, 1295, 1296, 1300, 1301, 1302, 1303, 1306, 1307, 1308, 1309, 1310, 1311, 1313, 1314, 1315, 1316, 1317, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
```

```
        return expr
    else:
        return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

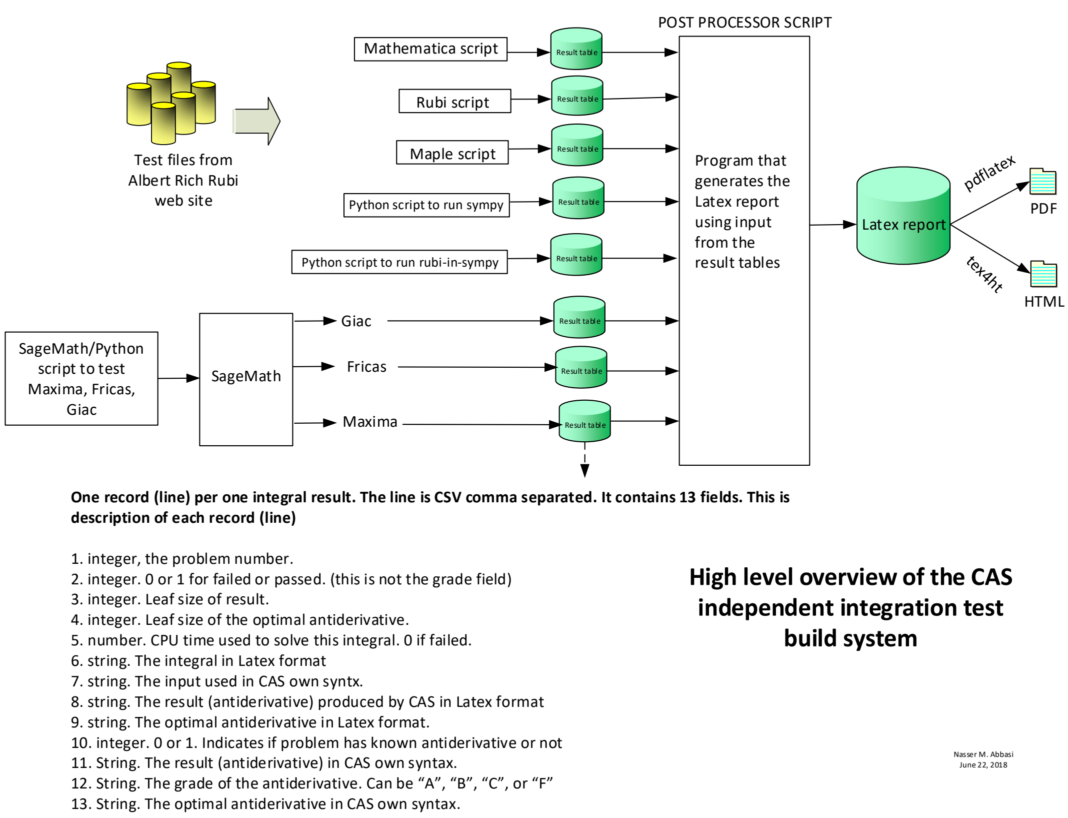
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106,
107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127,
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B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 4, 5, 10, 15, 19, 20, 31, 32, 33, 40, 44, 45, 46, 51, 52, 75, 76, 77, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 98, 99, 100, 101, 102, 108, 109, 110, 111, 117, 118, 119, 120, 127, 128, 129, 140, 141, 148, 149, 150, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 190, 191, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 286, 287, 288, 291, 292, 293, 294, 305, 308, 309, 310, 311, 312, 313, 316, 318, 319, 320, 321, 322, 324, 326, 328, 329, 330, 331, 337, 342, 343, 349, 350, 351, 352, 356, 357, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 370, 371, 374, 376, 377, 378, 379, 380, 381, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 408, 409, 412, 413, 414, 415, 416, 417, 423, 424, 425, 426, 427, 428, 434, 435, 436, 437, 439, 445, 446, 447, 448, 454, 455, 469, 476, 477, 478, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 514, 515, 517, 518, 524, 526, 532, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 672, 674, 675, 680, 681, 682, 683, 684, 688, 689, 690, 691, 692, 697, 698, 700, 705, 706, 707, 708, 710, 711, 724, 727, 737, 739, 744, 749, 753, 756, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 790, 791, 792, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 816, 817, 824, 839, 840, 841, 845, 846, 852, 855, 856, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 874, 875, 876, 877, 878, 879, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 904, 905, 906, 907, 908, 909, 913, 914, 915, 917, 922, 925, 930, 931, 932, 933, 937, 941, 942, 949, 950, 964, 970, 978, 980, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1024, 1025, 1026, 1030, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1081, 1082, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1184, 1185, 1186, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1297, 1298, 1299, 1304, 1305, 1312, 1318, 1319, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333 }

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C grade: { 1, 2, 3, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 34, 35, 36, 37, 38, 39, 41, 42, 43, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 162, 163, 189, 198, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 363, 364, 372, 373, 382, 383, 390, 399, 400, 405, 406, 407, 489, 490, 510, 511, 512, 513, 516, 519, 520, 521, 522, 523, 525, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 676, 677, 678, 679, 686, 687, 693, 694, 695, 696, 699, 701, 702, 703, 712, 714, 722, 734, 738, 740, 747, 755, 757, 758, 818, 819, 827, 842, 847, 853, 900, 901, 902, 903, 911, 912, 918, 919, 920, 921, 924, 926, 927, 928, 929, 940, 957, 963, 969, 977, 979, 981, 982, 983, 984, 985, 986, 987, 988, 989, 1031, 1032, 1033, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1043, 1044, 1045, 1046, 1047, 1048, 1050, 1051, 1052, 1053, 1055, 1056, 1057, 1058, 1059, 1061, 1062, 1063, 1064, 1066, 1067, 1068, 1069, 1078, 1079, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1182, 1183, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1294, 1295, 1296, 1300, 1301, 1302, 1303, 1306, 1307, 1308, 1309, 1310, 1311, 1313, 1314, 1315, 1316, 1317, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373 }

F grade: { 82, 83, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 633, 634, 759, 760, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1034, 1042, 1049, 1054, 1060, 1065, 1320}

2.1.3 Maple

A grade: { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 164, 165, 166, 168, 173, 174, 175, 211, 212, 213, 218, 219, 220, 221, 228, 229, 230, 233, 234, 235, 236, 240, 241, 242, 245, 246, 247, 248, 249, 255, 256, 257, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 276, 281, 282, 283, 287, 288, 294, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 335, 336, 337, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 365, 366, 367, 368, 374, 375, 376, 377, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 447, 454, 462, 463, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 491, 492, 493, 500, 501, 502, 546, 547, 548, 555, 556, 557, 560, 561, 562, 563, 564, 569, 570, 577, 583, 584, 585, 591, 592, 593, 594, 601, 602, 603, 604, 609, 610, 611, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 679, 680, 694, 700, 701, 705, 706, 756, 759, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 796, 797, 804, 810, 811, 841, 855, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 919, 925, 926, 930, 931, 978, 1013, 1014, 1015, 1070, 1071, 1072, 1073, 1074, 1090, 1091, 1092, 1098, 1099, 1100, 1107, 1108, 1109, 1110, 1111, 1114, 1120, 1121, 1127, 1128, 1129, 1130, 1131, 1135, 1136, 1137, 1138, 1139, 1144, 1145, 1146, 1147, 1148, 1149, 1153, 1154, 1155, 1156, 1157, 1158, 1161, 1162, 1163, 1164, 1165, 1167, 1168, 1169, 1173, 1176, 1177, 1183, 1194, 1202, 1203, 1209, 1210, 1217, 1218, 1219, 1220, 1224, 1225, 1245, 1246, 1247, 1248, 1253, 1254, 1255, 1256, 1262, 1263, 1264, 1265, 1266, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1320 }

B grade: { 121, 122, 123, 124, 128, 129, 130, 137, 159, 161, 162, 163, 167, 169, 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 214, 215, 216, 217, 222, 223, 224, 225, 226, 227, 231, 232, 237, 238, 239, 243, 244, 250, 251, 252, 253, 254, 258, 259, 260, 261, 267, 268, 269, 270, 277, 278, 279, 280, 284, 285, 286, 289, 290, 291, 292, 293, 332, 333, 334, 338, 339, 340, 347, 361, 362, 363, 364, 369, 370, 371, 372, 373, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 446, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 464, 465, 466, 473, 474, 486, 487, 488, 489, 490, 494, 495, 496, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 549, 550, 551, 552, 553, 554, 558, 559, 565, 566, 567, 568, 571, 572, 573, 574, 575, 576, 578, 579, 580, 581, 582, 586, 587, 588, 589, 590, 595, 596, 597, 598, 599, 600, 605, 606, 607, 608, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 676, 677, 678, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 695, 696, 697, 698, 699, 702, 703, 704, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 757, 758, 793, 794, 795, 798, 799, 800, 801, 802, 803, 805, 806, 807, 808, 809, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 888, 889, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 920, 921, 922, 923, 924, 927, 928, 929, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1043, 1044, 1050, 1051, 1055, 1056, 1057, 1061, 1062, 1063, 1066, 1067, 1068, 1069, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1093, 1094, 1095, 1096, 1097, 1101, 1102, 1103, 1104, 1105, 1106, 1112, 1113, 1115, 1116, 1117, 1118, 1119, 1122, 1123, 1124, 1125, 1126, 1132, 1133, 1134, 1140, 1141, 1142, 1143, 1150, 1151, 1152, 1159, 1160, 1166, 1170, 1171, 1172, 1174, 1175, 1178, 1179, 1180, 1181, 1182, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1204, 1205, 1206, 1207, 1208, 1211, 1212, 1213, 1214, 1215, 1216, 1221, 1222, 1223, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1249, 1250, 1251, 1252, 1257, 1258, 1259, 1260, 1261, 1267, 1268, 1269, 1270, 1271, 1276, 1277, 1278, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1341, 1342, 1348, 1349, 1356, 1357, 1358, 1363, 1364, 1365, 1368, 1369, 1370, 1371 }

C grade: { 760, 856, 1031, 1032, 1033, 1034, 1038, 1039, 1040, 1041, 1042, 1045, 1046, 1047, 1048, 1049, 1052, 1053, 1054, 1058, 1059, 1060, 1064, 1065, 1337, 1338, 1339, 1340, 1343, 1344, 1345, 1346, 1347, 1350, 1351, 1352, 1353, 1354, 1355, 1359, 1360, 1361, 1362, 1366, 1367, 1372, 1373 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 857, 858, 859, 860 }

2.1.4 Maxima

A grade: { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 127, 134, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 331, 342, 343, 344, 348, 349, 350, 351, 352, 353, 354, 355, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 423, 424, 425, 426, 431, 432, 433, 434, 435, 436, 437, 442, 443, 444, 445, 447, 468, 469, 470, 476, 477, 478, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 761, 762, 763, 764, 765, 766, 767, 768, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 1070, 1071, 1072, 1073, 1074

B grade: { 102, 111, 112, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 135, 137, 160, 161, 162, 163, 168, 177, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 290, 305, 309, 323, 324, 325, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 345, 346, 347, 361, 362, 363, 364, 369, 370, 378, 379, 417, 418, 419, 427, 428, 429, 430, 438, 439, 440, 441, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 473, 474, 475, 479, 480, 481, 487, 488, 489, 495, 504, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 635, 769, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1164, 1170, 1171, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1262, 1263, 1264, 1265, 1266, 1267, 1272, 1273, 1274, 1275, 1277, 1278, 1279, 1283, 1289 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 155, 156, 157, 158, 159, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 270, 272, 284, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 356, 357, 358, 359, 360, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 482, 483, 484, 485, 486, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 586, 595, 596, 597, 598, 599, 600, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1150, 1158, 1161, 1162, 1163, 1165, 1166, 1167, 1168, 1169, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1261, 1268, 1269, 1270, 1271, 1276, 1280, 1281, 1282, 1284, 1285, 1286, 1287, 1288, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373 }

2.1.5 FriCAS

A grade: { 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 201, 202, 203, 205, 206, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 291, 292, 293, 294, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 399, 400, 401, 402, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 524, 525, 526, 527, 528, 529, 530, 532, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 616, 617, 618, 619, 621, 622, 623, 624, 635, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 688, 689, 690, 706, 765, 766, 767, 768, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 796, 797, 798, 799, 800, 804, 807, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 903, 904, 905, 906, 907, 914, 915, 930, 931, 1074, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291 }

B grade: { 88, 196, 204, 285, 290, 309, 334, 397, 398, 406, 452, 523, 531, 612, 615, 620, 641, 677, 678, 684, 685, 686, 687, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 707, 708, 769, 793, 794, 795, 801, 802, 803, 805, 806, 808, 809, 810, 811, 812, 813, 901, 902, 909, 910, 911, 912, 913, 918, 919, 920, 921, 922, 925, 926, 927, 928, 929, 932, 933, 934 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 908, 916, 917, 923, 924, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373 }

2.1.6 Sympy

A grade: { 705, 761, 762, 763, 764, 930, 1070, 1071, 1072, 1073, 1074 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 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C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 159, 167, 168, 176, 197, 198, 199, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 361, 369, 378, 399, 400, 407, 486, 494, 503, 524, 525, 526, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 675, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	189	0	0	0	0	0
normalized size	1	1.	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	2.091	0.135	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	185	0	0	0	0	0
normalized size	1	1.	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	1.507	0.134	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.178	0.119	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	93	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.138	0.199	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	89	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.102	0.311	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	236	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	1.741	0.136	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	192	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	2.511	0.136	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	182	0	0	0	0	0
normalized size	1	1.	2.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	1.036	0.119	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	163	0	0	0	0	0
normalized size	1	1.	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	1.05	0.23	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.13	0.313	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	207	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	2.725	0.141	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	168	0	0	0	0	0
normalized size	1	1.	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	1.253	0.132	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	127	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.724	0.115	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	121	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.552	0.184	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	89	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.107	0.295	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	165	0	0	0	0	0
normalized size	1	1.	1.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.958	0.14	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	130	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.591	0.162	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	124	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.434	0.121	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.109	0.197	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.671	0.305	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	303	0	0	0	0	0
normalized size	1	1.	2.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	3.403	0.167	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	303	0	0	0	0	0
normalized size	1	1.	2.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	2.584	0.156	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	303	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	2.588	0.174	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	311	0	0	0	0	0
normalized size	1	1.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	6.829	0.151	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	311	0	0	0	0	0
normalized size	1	1.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	8.244	0.149	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	340	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	3.511	0.155	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	289	0	0	0	0	0
normalized size	1	1.	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	7.792	1.011	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	274	0	0	0	0	0
normalized size	1	1.	2.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	6.9	0.533	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	282	0	0	0	0	0
normalized size	1	1.	2.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	6.587	0.712	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	273	0	0	0	0	0
normalized size	1	1.	2.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	5.61	0.687	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	119	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.232	0.752	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	107	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.185	0.948	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	118	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.249	1.328	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	341	0	0	0	0	0
normalized size	1	1.	2.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	3.163	0.213	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	303	0	0	0	0	0
normalized size	1	1.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	2.273	0.234	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	303	0	0	0	0	0
normalized size	1	1.	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	2.331	0.222	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	311	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	4.565	0.234	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	343	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	2.859	0.222	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	338	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	3.671	0.22	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	129	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.237	1.178	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	346	0	0	0	0	0
normalized size	1	1.	2.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	4.85	0.153	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	265	0	0	0	0	0
normalized size	1	1.	1.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	5.635	0.149	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	311	0	0	0	0	0
normalized size	1	1.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	2.052	0.138	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	120	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	0.204	0.247	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	116	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	0.168	0.36	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	118	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.225	0.638	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	444	0	0	0	0	0
normalized size	1	1.	2.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	6.486	0.177	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	465	0	0	0	0	0
normalized size	1	1.	3.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	6.981	0.155	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	290	0	0	0	0	0
normalized size	1	1.	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	2.397	0.139	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	303	0	0	0	0	0
normalized size	1	1.	2.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	2.351	0.24	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	117	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.206	0.36	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	118	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.173	0.64	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	304	0	0	0	0	0
normalized size	1	1.	1.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	2.979	0.161	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	305	0	0	0	0	0
normalized size	1	1.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	1.905	0.15	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	173	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	1.831	0.131	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	305	0	0	0	0	0
normalized size	1	1.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	1.463	0.006	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	304	0	0	0	0	0
normalized size	1	1.	1.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	2.899	0.006	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	333	0	0	0	0	0
normalized size	1	1.	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	2.441	0.169	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	299	0	0	0	0	0
normalized size	1	1.	1.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	1.667	0.151	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	175	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	1.921	0.154	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	298	0	0	0	0	0
normalized size	1	1.	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	1.931	0.143	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	175	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	1.196	0.008	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	299	0	0	0	0	0
normalized size	1	1.	1.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	1.676	0.006	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	699	0	0	0	0	0
normalized size	1	1.	4.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	6.342	0.164	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	484	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	7.95	0.194	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	547	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	6.706	0.18	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	225	225	494	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	7.231	0.184	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	228	228	548	0	0	0	0	0
normalized size	1	1.	2.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	11.555	0.179	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	545	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	10.529	0.178	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	492	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	8.8	0.184	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	436	0	0	0	0	0
normalized size	1	1.	1.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	6.471	1.165	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	462	0	0	0	0	0
normalized size	1	1.	2.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	5.514	0.679	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	460	0	0	0	0	0
normalized size	1	1.	2.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	5.171	0.871	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	401	0	0	0	0	0
normalized size	1	1.	2.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	4.83	0.612	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	161	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.34	0.822	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	155	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	0.273	1.056	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	168	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	0.452	1.424	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	493	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	7.877	0.236	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	487	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.188	7.225	0.234	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	492	0	0	0	0	0
normalized size	1	1.	2.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	7.962	0.242	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	548	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	8.534	0.247	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	180.001	0.255	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	180.001	0.24	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	93	192	236	389	0	294
normalized size	1	1.	0.66	1.37	1.69	2.78	0.	2.1
time (sec)	N/A	0.185	0.718	0.048	0.942	0.517	0.	1.24

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	149	205	335	0	254
normalized size	1	1.	0.64	1.27	1.75	2.86	0.	2.17
time (sec)	N/A	0.17	0.428	0.043	0.941	0.509	0.	1.257

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	56	108	135	285	0	211
normalized size	1	1.	0.65	1.26	1.57	3.31	0.	2.45
time (sec)	N/A	0.108	0.27	0.046	0.931	0.508	0.	1.184

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	85	119	267	0	142
normalized size	1	1.	1.16	1.47	2.05	4.6	0.	2.45
time (sec)	N/A	0.054	0.027	0.041	0.926	0.516	0.	1.212

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	57	80	232	0	161
normalized size	1	1.	1.29	1.36	1.9	5.52	0.	3.83
time (sec)	N/A	0.101	0.027	0.071	0.929	0.53	0.	1.271

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	77	95	167	0	134
normalized size	1	1.	0.9	1.33	1.64	2.88	0.	2.31
time (sec)	N/A	0.127	0.081	0.081	0.93	0.52	0.	1.299

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	68	90	140	0	169
normalized size	1	1.	0.77	0.88	1.17	1.82	0.	2.19
time (sec)	N/A	0.145	0.128	0.082	0.929	0.485	0.	1.254

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	96	122	189	0	211
normalized size	1	1.	0.81	1.01	1.28	1.99	0.	2.22
time (sec)	N/A	0.177	0.231	0.089	0.929	0.496	0.	1.155

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	86	117	153	242	0	251
normalized size	1	1.	0.66	0.89	1.17	1.85	0.	1.92
time (sec)	N/A	0.187	0.278	0.107	0.933	0.501	0.	1.19

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	321	210	294	409	0	332
normalized size	1	1.	1.87	1.22	1.71	2.38	0.	1.93
time (sec)	N/A	0.385	1.79	0.119	0.944	0.52	0.	1.269

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	291	166	306	356	0	286
normalized size	1	1.	2.2	1.26	2.32	2.7	0.	2.17
time (sec)	N/A	0.212	1.408	0.049	0.944	0.512	0.	1.244

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	1090	134	177	331	0	252
normalized size	1	1.	11.35	1.4	1.84	3.45	0.	2.62
time (sec)	N/A	0.14	6.512	0.051	0.931	0.517	0.	1.203

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	330	114	192	320	0	205
normalized size	1	1.	2.95	1.02	1.71	2.86	0.	1.83
time (sec)	N/A	0.19	2.612	0.086	0.939	0.526	0.	1.231

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	292	107	136	296	0	193
normalized size	1	1.	2.45	0.9	1.14	2.49	0.	1.62
time (sec)	N/A	0.29	1.06	0.079	0.94	0.525	0.	1.228

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	109	128	154	236	0	242
normalized size	1	1.	0.99	1.16	1.4	2.15	0.	2.2
time (sec)	N/A	0.268	0.212	0.091	0.94	0.527	0.	1.223

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	73	142	178	207	0	238
normalized size	1	1.	0.54	1.04	1.31	1.52	0.	1.75
time (sec)	N/A	0.307	0.23	0.097	0.943	0.491	0.	1.2

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	97	160	211	258	0	284
normalized size	1	1.	0.57	0.95	1.25	1.53	0.	1.68
time (sec)	N/A	0.387	0.397	0.111	0.942	0.499	0.	1.189

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	123	211	275	315	0	329
normalized size	1	1.	0.63	1.09	1.42	1.62	0.	1.7
time (sec)	N/A	0.412	0.633	0.111	0.945	0.512	0.	1.208

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	387	257	516	474	0	378
normalized size	1	1.	1.96	1.3	2.62	2.41	0.	1.92
time (sec)	N/A	0.417	2.787	0.06	0.961	0.531	0.	1.247

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	323	212	385	419	0	332
normalized size	1	1.	2.06	1.35	2.45	2.67	0.	2.11
time (sec)	N/A	0.256	1.993	0.066	0.952	0.52	0.	1.252

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	363	180	338	386	0	300
normalized size	1	1.	2.47	1.22	2.3	2.63	0.	2.04
time (sec)	N/A	0.219	1.945	0.054	0.944	0.534	0.	1.258

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	1250	152	239	379	0	296
normalized size	1	1.	8.62	1.05	1.65	2.61	0.	2.04
time (sec)	N/A	0.25	6.395	0.098	0.942	0.53	0.	1.243

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	364	151	236	365	0	311
normalized size	1	1.	2.25	0.93	1.46	2.25	0.	1.92
time (sec)	N/A	0.414	4.245	0.093	0.945	0.531	0.	1.303

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	1014	146	185	350	0	284
normalized size	1	1.	6.5	0.94	1.19	2.24	0.	1.82
time (sec)	N/A	0.397	6.175	0.103	0.945	0.53	0.	1.255

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	124	175	231	284	0	288
normalized size	1	1.	0.73	1.04	1.37	1.68	0.	1.7
time (sec)	N/A	0.41	0.305	0.095	0.95	0.532	0.	1.292

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	97	197	257	266	0	284
normalized size	1	1.	0.6	1.22	1.6	1.65	0.	1.76
time (sec)	N/A	0.33	0.321	0.101	0.945	0.504	0.	1.235

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	123	245	323	321	0	329
normalized size	1	1.	0.57	1.13	1.5	1.49	0.	1.52
time (sec)	N/A	0.551	0.456	0.134	0.956	0.515	0.	1.261

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	419	303	624	531	0	424
normalized size	1	1.	1.84	1.33	2.74	2.33	0.	1.86
time (sec)	N/A	0.481	4.948	0.065	0.966	0.542	0.	1.267

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	387	258	606	471	0	378
normalized size	1	1.	2.06	1.37	3.22	2.51	0.	2.01
time (sec)	N/A	0.303	3.456	0.072	0.964	0.533	0.	1.238

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	418	226	416	452	0	347
normalized size	1	1.	2.36	1.28	2.35	2.55	0.	1.96
time (sec)	N/A	0.294	2.918	0.066	0.956	0.539	0.	1.254

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	379	197	400	437	0	342
normalized size	1	1.	2.09	1.09	2.21	2.41	0.	1.89
time (sec)	N/A	0.342	2.459	0.112	0.956	0.547	0.	1.311

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	1420	189	285	425	0	335
normalized size	1	1.	7.4	0.98	1.48	2.21	0.	1.74
time (sec)	N/A	0.528	6.412	0.102	0.953	0.543	0.	1.261

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	1250	190	285	432	0	335
normalized size	1	1.	6.31	0.96	1.44	2.18	0.	1.69
time (sec)	N/A	0.546	6.218	0.108	0.953	0.547	0.	1.245

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	375	191	262	408	0	329
normalized size	1	1.	1.88	0.96	1.31	2.04	0.	1.64
time (sec)	N/A	0.574	2.283	0.095	0.95	0.548	0.	1.275

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	147	221	309	351	0	335
normalized size	1	1.	0.71	1.07	1.49	1.7	0.	1.62
time (sec)	N/A	0.552	0.423	0.132	0.955	0.551	0.	1.327

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	119	284	369	319	0	329
normalized size	1	1.	0.62	1.48	1.92	1.66	0.	1.71
time (sec)	N/A	0.366	0.344	0.118	0.957	0.512	0.	1.242

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	145	322	431	377	0	375
normalized size	1	1.	0.57	1.27	1.7	1.48	0.	1.48
time (sec)	N/A	0.719	0.638	0.119	0.956	0.525	0.	1.255

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	792	386	551	477	0	288
normalized size	1	1.	4.8	2.34	3.34	2.89	0.	1.75
time (sec)	N/A	0.202	6.333	0.068	0.957	0.517	0.	1.211

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	1090	294	439	429	0	250
normalized size	1	1.	8.2	2.21	3.3	3.23	0.	1.88
time (sec)	N/A	0.179	6.518	0.061	0.95	0.519	0.	1.224

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	316	209	323	377	0	176
normalized size	1	1.	2.95	1.95	3.02	3.52	0.	1.64
time (sec)	N/A	0.167	3.011	0.059	0.944	0.504	0.	1.221

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	227	121	194	288	0	136
normalized size	1	1.	3.98	2.12	3.4	5.05	0.	2.39
time (sec)	N/A	0.152	1.792	0.051	0.94	0.497	0.	1.209

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	143	98	169	242	0	108
normalized size	1	1.	2.92	2.	3.45	4.94	0.	2.2
time (sec)	N/A	0.108	0.435	0.06	1.407	0.507	0.	1.22

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	108	88	158	132	0	100
normalized size	1	1.	2.08	1.69	3.04	2.54	0.	1.92
time (sec)	N/A	0.109	0.279	0.08	1.425	0.478	0.	1.172

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	159	144	248	192	0	130
normalized size	1	1.	1.66	1.5	2.58	2.	0.	1.35
time (sec)	N/A	0.153	0.351	0.099	1.416	0.484	0.	1.168

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	225	280	363	243	0	205
normalized size	1	1.	1.81	2.26	2.93	1.96	0.	1.65
time (sec)	N/A	0.169	0.795	0.092	1.426	0.49	0.	1.173

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	283	352	474	290	0	243
normalized size	1	1.	1.81	2.26	3.04	1.86	0.	1.56
time (sec)	N/A	0.187	0.696	0.1	1.435	0.498	0.	1.166

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	623	338	512	591	0	304
normalized size	1	1.	3.62	1.97	2.98	3.44	0.	1.77
time (sec)	N/A	0.329	2.94	0.071	0.966	0.526	0.	1.182

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	513	249	389	554	0	231
normalized size	1	1.	3.42	1.66	2.59	3.69	0.	1.54
time (sec)	N/A	0.305	2.115	0.069	0.957	0.517	0.	1.252

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	280	164	258	429	0	192
normalized size	1	1.	2.83	1.66	2.61	4.33	0.	1.94
time (sec)	N/A	0.252	1.475	0.056	0.953	0.505	0.	1.196

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	81	377	119	197	338	0	151
normalized size	1	1.08	5.03	1.59	2.63	4.51	0.	2.01
time (sec)	N/A	0.162	0.831	0.059	0.945	0.498	0.	1.216

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	141	97	161	228	0	113
normalized size	1	1.	2.07	1.43	2.37	3.35	0.	1.66
time (sec)	N/A	0.121	0.496	0.057	1.416	0.469	0.	1.194

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	195	130	223	261	0	154
normalized size	1	1.	2.38	1.59	2.72	3.18	0.	1.88
time (sec)	N/A	0.218	0.765	0.088	1.429	0.483	0.	1.219

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	281	184	319	332	0	185
normalized size	1	1.	2.05	1.34	2.33	2.42	0.	1.35
time (sec)	N/A	0.31	1.141	0.094	1.43	0.491	0.	1.217

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	349	322	439	369	0	258
normalized size	1	1.	2.14	1.98	2.69	2.26	0.	1.58
time (sec)	N/A	0.325	0.911	0.095	1.434	0.504	0.	1.235

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	632	289	446	740	0	279
normalized size	1	1.	3.19	1.46	2.25	3.74	0.	1.41
time (sec)	N/A	0.49	2.867	0.074	0.963	0.523	0.	1.232

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	457	204	315	576	0	240
normalized size	1	1.	3.15	1.41	2.17	3.97	0.	1.66
time (sec)	N/A	0.427	3.012	0.062	0.959	0.513	0.	1.246

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	236	139	225	481	0	177
normalized size	1	1.	1.92	1.13	1.83	3.91	0.	1.44
time (sec)	N/A	0.33	1.589	0.059	0.957	0.503	0.	1.255

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	121	88	181	221	0	120
normalized size	1	1.	1.16	0.85	1.74	2.12	0.	1.15
time (sec)	N/A	0.187	0.541	0.059	0.952	0.464	0.	1.227

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	227	117	189	351	0	140
normalized size	1	1.	2.14	1.1	1.78	3.31	0.	1.32
time (sec)	N/A	0.18	0.844	0.068	1.424	0.476	0.	1.216

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	283	170	277	386	0	204
normalized size	1	1.	2.36	1.42	2.31	3.22	0.	1.7
time (sec)	N/A	0.355	1.82	0.092	1.442	0.49	0.	1.239

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	385	224	373	478	0	235
normalized size	1	1.	2.1	1.22	2.04	2.61	0.	1.28
time (sec)	N/A	0.466	1.416	0.107	1.434	0.507	0.	1.196

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	455	362	493	525	0	308
normalized size	1	1.	2.11	1.68	2.28	2.43	0.	1.43
time (sec)	N/A	0.497	1.817	0.105	1.441	0.511	0.	1.209

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	746	329	502	923	0	325
normalized size	1	1.	3.22	1.42	2.16	3.98	0.	1.4
time (sec)	N/A	0.649	4.229	0.08	0.974	0.534	0.	1.233

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	544	244	370	722	0	286
normalized size	1	1.	2.97	1.33	2.02	3.95	0.	1.56
time (sec)	N/A	0.589	2.604	0.067	0.97	0.52	0.	1.217

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	283	199	308	624	0	246
normalized size	1	1.	1.76	1.24	1.91	3.88	0.	1.53
time (sec)	N/A	0.484	2.193	0.068	0.961	0.512	0.	1.246

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	151	88	236	308	0	158
normalized size	1	1.	1.09	0.64	1.71	2.23	0.	1.14
time (sec)	N/A	0.374	0.641	0.067	0.974	0.463	0.	1.232

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	171	90	236	309	0	158
normalized size	1	1.	1.2	0.63	1.66	2.18	0.	1.11
time (sec)	N/A	0.247	0.718	0.064	0.965	0.467	0.	1.212

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	315	177	271	477	0	208
normalized size	1	1.	2.32	1.3	1.99	3.51	0.	1.53
time (sec)	N/A	0.262	1.05	0.071	1.438	0.489	0.	1.215

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	371	210	332	516	0	248
normalized size	1	1.	2.44	1.38	2.18	3.39	0.	1.63
time (sec)	N/A	0.499	1.228	0.104	1.448	0.5	0.	1.204

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	505	264	429	632	0	279
normalized size	1	1.	2.35	1.23	2.	2.94	0.	1.3
time (sec)	N/A	0.627	2.317	0.117	1.448	0.517	0.	1.191

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	575	402	547	676	0	352
normalized size	1	1.	2.32	1.62	2.21	2.73	0.	1.42
time (sec)	N/A	0.687	2.756	0.111	1.456	0.53	0.	1.197

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	143	151	0	352	0	424
normalized size	1	1.	0.64	0.68	0.	1.58	0.	1.9
time (sec)	N/A	0.516	1.065	0.406	0.	0.506	0.	4.71

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	122	129	0	302	0	362
normalized size	1	1.	0.68	0.72	0.	1.68	0.	2.01
time (sec)	N/A	0.445	0.986	0.352	0.	0.496	0.	4.62

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	99	107	0	255	0	300
normalized size	1	1.	0.72	0.78	0.	1.86	0.	2.19
time (sec)	N/A	0.391	0.784	0.331	0.	0.486	0.	4.56

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	71	85	0	205	0	238
normalized size	1	1.	0.75	0.89	0.	2.16	0.	2.51
time (sec)	N/A	0.198	0.987	0.309	0.	0.483	0.	4.587

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	216	0	771	0	0
normalized size	1	1.	1.	2.25	0.	8.03	0.	0.
time (sec)	N/A	0.146	0.638	0.327	0.	0.547	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	138	1069	679	0	485
normalized size	1	1.	0.89	1.47	11.37	7.22	0.	5.16
time (sec)	N/A	0.201	0.328	0.348	1.819	0.544	0.	6.338

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	108	376	1629	779	0	602
normalized size	1	1.	0.98	3.42	14.81	7.08	0.	5.47
time (sec)	N/A	0.251	0.414	0.376	2.086	0.638	0.	6.441

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	117	569	3663	873	0	1156
normalized size	1	1.	0.76	3.72	23.94	5.71	0.	7.56
time (sec)	N/A	0.351	0.36	0.444	2.704	0.649	0.	6.697

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	70	751	10394	984	0	1458
normalized size	1	1.	0.36	3.83	53.03	5.02	0.	7.44
time (sec)	N/A	0.42	0.191	0.375	3.507	0.737	0.	6.814

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	144	152	0	375	0	424
normalized size	1	1.	0.64	0.68	0.	1.67	0.	1.88
time (sec)	N/A	0.655	1.365	0.327	0.	0.516	0.	4.828

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	121	130	0	319	0	362
normalized size	1	1.	0.7	0.75	0.	1.83	0.	2.08
time (sec)	N/A	0.477	1.24	0.29	0.	0.504	0.	4.735

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	100	108	0	266	0	300
normalized size	1	1.	0.76	0.82	0.	2.02	0.	2.27
time (sec)	N/A	0.263	1.172	0.286	0.	0.499	0.	4.7

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	122	330	0	887	0	0
normalized size	1	1.	0.92	2.48	0.	6.67	0.	0.
time (sec)	N/A	0.223	1.198	0.295	0.	0.567	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	113	239	1085	863	0	0
normalized size	1	1.	0.83	1.76	7.98	6.35	0.	0.
time (sec)	N/A	0.289	1.195	0.332	1.862	0.566	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	109	397	0	828	0	695
normalized size	1	1.	0.72	2.63	0.	5.48	0.	4.6
time (sec)	N/A	0.428	0.754	0.356	0.	0.651	0.	6.649

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	118	570	0	919	0	1166
normalized size	1	1.	0.76	3.68	0.	5.93	0.	7.52
time (sec)	N/A	0.462	1.201	0.393	0.	0.652	0.	6.934

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	140	752	0	1018	0	1467
normalized size	1	1.	0.7	3.76	0.	5.09	0.	7.34
time (sec)	N/A	0.571	1.402	0.329	0.	0.744	0.	7.437

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	159	934	0	1157	0	1771
normalized size	1	1.	0.65	3.81	0.	4.72	0.	7.23
time (sec)	N/A	0.639	2.154	0.366	0.	0.763	0.	7.638

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	169	176	0	470	0	486
normalized size	1	1.	0.62	0.64	0.	1.72	0.	1.78
time (sec)	N/A	0.865	1.924	0.359	0.	0.527	0.	5.317

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	147	154	0	382	0	424
normalized size	1	1.	0.7	0.73	0.	1.81	0.	2.01
time (sec)	N/A	0.533	1.494	0.301	0.	0.517	0.	5.134

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	125	132	0	333	0	362
normalized size	1	1.	0.74	0.78	0.	1.97	0.	2.14
time (sec)	N/A	0.316	1.397	0.287	0.	0.507	0.	5.182

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	151	434	0	1030	0	0
normalized size	1	1.	0.89	2.55	0.	6.06	0.	0.
time (sec)	N/A	0.308	1.836	0.314	0.	0.583	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	145	343	1868	1008	0	649
normalized size	1	1.	0.84	1.98	10.8	5.83	0.	3.75
time (sec)	N/A	0.369	1.783	0.339	1.948	0.582	0.	6.882

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	137	402	0	1022	0	748
normalized size	1	1.	0.73	2.14	0.	5.44	0.	3.98
time (sec)	N/A	0.598	0.886	0.361	0.	0.672	0.	7.031

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	132	583	0	968	0	1261
normalized size	1	1.	0.69	3.04	0.	5.04	0.	6.57
time (sec)	N/A	0.619	1.481	0.406	0.	0.67	0.	7.348

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	143	754	0	1076	0	1480
normalized size	1	1.	0.72	3.77	0.	5.38	0.	7.4
time (sec)	N/A	0.655	1.6	0.29	0.	0.751	0.	7.726

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	160	936	0	1197	0	1782
normalized size	1	1.	0.65	3.82	0.	4.89	0.	7.27
time (sec)	N/A	0.761	2.478	0.329	0.	0.766	0.	8.166

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	182	1118	0	1311	0	2084
normalized size	1	1.	0.63	3.86	0.	4.52	0.	7.19
time (sec)	N/A	0.858	2.228	0.371	0.	0.802	0.	8.535

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	474	966	0	1189	0	556
normalized size	1	1.	2.01	4.09	0.	5.04	0.	2.36
time (sec)	N/A	0.803	6.67	0.429	0.	0.648	0.	9.485

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	173	776	0	1098	0	333
normalized size	1	1.	0.9	4.02	0.	5.69	0.	1.73
time (sec)	N/A	0.594	6.119	0.366	0.	0.611	0.	9.31

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	160	586	0	998	0	394
normalized size	1	1.	1.05	3.86	0.	6.57	0.	2.59
time (sec)	N/A	0.417	4.339	0.346	0.	0.605	0.	8.929

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	125	385	0	895	0	193
normalized size	1	1.	1.15	3.53	0.	8.21	0.	1.77
time (sec)	N/A	0.206	1.519	0.326	0.	0.6	0.	9.11

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	126	271	0	1150	0	405
normalized size	1	1.	1.1	2.36	0.	10.	0.	3.52
time (sec)	N/A	0.162	0.923	0.289	0.	2.602	0.	11.005

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	282	0	1177	0	531
normalized size	1	1.	1.	2.5	0.	10.42	0.	4.7
time (sec)	N/A	0.224	0.773	0.339	0.	2.584	0.	11.608

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	10837	695	0	1299	0	680
normalized size	1	1.	68.16	4.37	0.	8.17	0.	4.28
time (sec)	N/A	0.369	26.409	0.37	0.	5.886	0.	11.746

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	145	1056	0	1399	0	1153
normalized size	1	1.	0.72	5.28	0.	7.	0.	5.76
time (sec)	N/A	0.563	0.556	0.332	0.	5.901	0.	12.093

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	160	1406	0	1523	0	1418
normalized size	1	1.	0.66	5.79	0.	6.27	0.	5.84
time (sec)	N/A	0.729	0.764	0.379	0.	8.696	0.	14.307

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	527	974	0	1392	0	593
normalized size	1	1.	2.03	3.76	0.	5.37	0.	2.29
time (sec)	N/A	0.845	6.802	0.438	0.	0.65	0.	9.31

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	189	784	0	1268	0	420
normalized size	1	1.	0.88	3.66	0.	5.93	0.	1.96
time (sec)	N/A	0.627	4.906	0.339	0.	0.627	0.	9.494

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	162	594	0	1173	0	398
normalized size	1	1.	0.96	3.51	0.	6.94	0.	2.36
time (sec)	N/A	0.448	3.645	0.311	0.	0.62	0.	9.102

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	127	405	0	987	0	251
normalized size	1	1.	1.01	3.21	0.	7.83	0.	1.99
time (sec)	N/A	0.23	1.813	0.275	0.	0.608	0.	9.059

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	154	554	0	1432	0	416
normalized size	1	1.	1.23	4.43	0.	11.46	0.	3.33
time (sec)	N/A	0.19	1.388	0.232	0.	7.925	0.	11.228

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	167	561	0	1486	0	0
normalized size	1	1.	1.06	3.55	0.	9.41	0.	0.
time (sec)	N/A	0.386	1.577	0.333	0.	7.982	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	12015	1064	0	1646	0	0
normalized size	1	1.	55.37	4.9	0.	7.59	0.	0.
time (sec)	N/A	0.588	26.764	0.369	0.	15.148	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	204	1414	0	1756	0	0
normalized size	1	1.	0.77	5.32	0.	6.6	0.	0.
time (sec)	N/A	0.776	2.921	0.325	0.	15.046	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	220	976	0	1567	0	591
normalized size	1	1.	0.85	3.77	0.	6.05	0.	2.28
time (sec)	N/A	0.837	3.373	0.356	0.	0.662	0.	9.373

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	196	786	0	1447	0	419
normalized size	1	1.	0.92	3.71	0.	6.83	0.	1.98
time (sec)	N/A	0.659	2.336	0.365	0.	0.62	0.	9.619

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	136	597	0	1238	0	386
normalized size	1	1.	0.82	3.62	0.	7.5	0.	2.34
time (sec)	N/A	0.456	3.203	0.301	0.	0.602	0.	9.088

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	120	602	0	1238	0	257
normalized size	1	1.	0.92	4.63	0.	9.52	0.	1.98
time (sec)	N/A	0.261	2.274	0.278	0.	0.612	0.	9.272

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	153	824	0	1748	0	470
normalized size	1	1.	0.94	5.09	0.	10.79	0.	2.9
time (sec)	N/A	0.261	3.54	0.242	0.	16.525	0.	11.129

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	166	835	0	1829	0	0
normalized size	1	1.	0.83	4.2	0.	9.19	0.	0.
time (sec)	N/A	0.558	3.857	0.366	0.	16.696	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	12059	1416	0	2013	0	0
normalized size	1	1.	46.03	5.4	0.	7.68	0.	0.
time (sec)	N/A	0.803	27.027	0.429	0.	28.733	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	409	838	0	0	0	0
normalized size	1	1.	2.	4.09	0.	0.	0.	0.
time (sec)	N/A	0.222	4.634	7.056	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	286	729	0	0	0	0
normalized size	1	1.	1.66	4.24	0.	0.	0.	0.
time (sec)	N/A	0.202	2.304	5.874	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	168	437	0	0	0	0
normalized size	1	1.	1.24	3.24	0.	0.	0.	0.
time (sec)	N/A	0.183	1.317	4.953	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	169	458	0	0	0	0
normalized size	1	1.	1.25	3.39	0.	0.	0.	0.
time (sec)	N/A	0.185	1.354	2.349	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	169	345	0	0	0	0
normalized size	1	1.	1.2	2.45	0.	0.	0.	0.
time (sec)	N/A	0.188	1.709	2.079	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	188	378	0	0	0	0
normalized size	1	1.	1.08	2.17	0.	0.	0.	0.
time (sec)	N/A	0.204	2.221	2.322	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	204	406	0	0	0	0
normalized size	1	1.	1.	1.98	0.	0.	0.	0.
time (sec)	N/A	0.232	2.838	2.351	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	821	1168	0	0	0	0
normalized size	1	1.	3.04	4.33	0.	0.	0.	0.
time (sec)	N/A	0.439	6.802	9.129	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	436	919	0	0	0	0
normalized size	1	1.	1.84	3.88	0.	0.	0.	0.
time (sec)	N/A	0.414	5.885	7.912	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	312	756	0	0	0	0
normalized size	1	1.	1.59	3.86	0.	0.	0.	0.
time (sec)	N/A	0.389	5.883	6.369	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	191	651	0	0	0	0
normalized size	1	1.	0.96	3.29	0.	0.	0.	0.
time (sec)	N/A	0.402	2.006	5.308	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	318	440	0	0	0	0
normalized size	1	1.	1.62	2.24	0.	0.	0.	0.
time (sec)	N/A	0.392	4.641	2.163	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	189	380	0	0	0	0
normalized size	1	1.	0.93	1.86	0.	0.	0.	0.
time (sec)	N/A	0.421	2.38	1.952	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	206	408	0	0	0	0
normalized size	1	1.	0.87	1.72	0.	0.	0.	0.
time (sec)	N/A	0.436	2.911	1.928	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	228	436	0	0	0	0
normalized size	1	1.	0.84	1.61	0.	0.	0.	0.
time (sec)	N/A	0.473	3.223	2.097	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	863	1409	0	0	0	0
normalized size	1	1.	2.71	4.42	0.	0.	0.	0.
time (sec)	N/A	0.618	7.001	10.123	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	818	1247	0	0	0	0
normalized size	1	1.	2.86	4.36	0.	0.	0.	0.
time (sec)	N/A	0.59	6.845	9.044	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	280	1014	0	0	0	0
normalized size	1	1.	1.11	4.01	0.	0.	0.	0.
time (sec)	N/A	0.563	3.497	7.518	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	255	939	0	0	0	0
normalized size	1	1.	0.98	3.63	0.	0.	0.	0.
time (sec)	N/A	0.565	2.86	6.838	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	221	704	0	0	0	0
normalized size	1	1.	0.87	2.78	0.	0.	0.	0.
time (sec)	N/A	0.57	2.271	2.469	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	218	569	0	0	0	0
normalized size	1	1.	0.86	2.25	0.	0.	0.	0.
time (sec)	N/A	0.573	2.231	2.406	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	206	408	0	0	0	0
normalized size	1	1.	0.81	1.61	0.	0.	0.	0.
time (sec)	N/A	0.58	2.879	1.934	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	228	436	0	0	0	0
normalized size	1	1.	0.8	1.52	0.	0.	0.	0.
time (sec)	N/A	0.605	3.294	2.077	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	250	464	0	0	0	0
normalized size	1	1.	0.78	1.45	0.	0.	0.	0.
time (sec)	N/A	0.655	4.209	1.992	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	342	803	0	0	0	0
normalized size	1	1.	1.47	3.46	0.	0.	0.	0.
time (sec)	N/A	0.236	6.245	7.168	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	324	486	0	0	0	0
normalized size	1	1.	1.71	2.56	0.	0.	0.	0.
time (sec)	N/A	0.217	4.307	5.797	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	776	316	0	0	0	0
normalized size	1	1.	5.11	2.08	0.	0.	0.	0.
time (sec)	N/A	0.189	6.636	4.051	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	795	245	0	0	0	0
normalized size	1	1.	6.41	1.98	0.	0.	0.	0.
time (sec)	N/A	0.171	6.42	2.075	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	232	262	0	0	0	0
normalized size	1	1.	1.43	1.62	0.	0.	0.	0.
time (sec)	N/A	0.2	2.722	2.204	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	248	277	0	0	0	0
normalized size	1	1.	1.25	1.39	0.	0.	0.	0.
time (sec)	N/A	0.214	3.234	2.206	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	884	738	0	0	0	0
normalized size	1	1.	3.86	3.22	0.	0.	0.	0.
time (sec)	N/A	0.379	7.447	7.02	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	293	450	0	0	0	0
normalized size	1	1.	1.53	2.36	0.	0.	0.	0.
time (sec)	N/A	0.336	4.611	2.339	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	859	423	0	0	0	0
normalized size	1	1.	5.21	2.56	0.	0.	0.	0.
time (sec)	N/A	0.316	6.662	2.28	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	298	352	0	0	0	0
normalized size	1	1.	1.75	2.07	0.	0.	0.	0.
time (sec)	N/A	0.317	3.497	2.348	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	912	437	0	0	0	0
normalized size	1	1.	4.54	2.17	0.	0.	0.	0.
time (sec)	N/A	0.362	6.782	2.401	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	301	451	0	0	0	0
normalized size	1	1.	1.28	1.91	0.	0.	0.	0.
time (sec)	N/A	0.376	6.343	2.474	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	984	876	0	0	0	0
normalized size	1	1.	3.49	3.11	0.	0.	0.	0.
time (sec)	N/A	0.537	7.778	3.141	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	953	679	0	0	0	0
normalized size	1	1.	3.83	2.73	0.	0.	0.	0.
time (sec)	N/A	0.524	7.049	2.905	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	952	451	0	0	0	0
normalized size	1	1.	4.33	2.05	0.	0.	0.	0.
time (sec)	N/A	0.51	6.89	2.594	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	954	451	0	0	0	0
normalized size	1	1.	4.3	2.03	0.	0.	0.	0.
time (sec)	N/A	0.492	6.946	2.405	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	975	451	0	0	0	0
normalized size	1	1.	4.31	2.	0.	0.	0.	0.
time (sec)	N/A	0.506	6.955	2.395	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	1008	465	0	0	0	0
normalized size	1	1.	4.05	1.87	0.	0.	0.	0.
time (sec)	N/A	0.533	7.111	2.18	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	1052	479	0	0	0	0
normalized size	1	1.	3.63	1.65	0.	0.	0.	0.
time (sec)	N/A	0.57	7.267	2.701	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	238	449	5963	1238	0	0
normalized size	1	1.	1.11	2.1	27.86	5.79	0.	0.
time (sec)	N/A	0.46	2.16	0.387	3.377	1.025	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	211	385	3699	1127	0	0
normalized size	1	1.	1.25	2.28	21.89	6.67	0.	0.
time (sec)	N/A	0.388	1.668	0.369	2.552	0.786	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	202	325	2034	1023	0	0
normalized size	1	1.	1.63	2.62	16.4	8.25	0.	0.
time (sec)	N/A	0.306	2.323	0.403	2.219	0.774	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	177	210	923	882	0	0
normalized size	1	1.	1.54	1.83	8.03	7.67	0.	0.
time (sec)	N/A	0.304	2.54	0.403	2.044	0.587	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	179	198	479	927	0	0
normalized size	1	1.	1.54	1.71	4.13	7.99	0.	0.
time (sec)	N/A	0.291	1.601	0.382	2.023	0.596	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	68	87	302	231	0	0
normalized size	1	1.	0.56	0.71	2.48	1.89	0.	0.
time (sec)	N/A	0.315	0.532	0.371	1.944	0.486	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	84	107	551	281	0	0
normalized size	1	1.	0.5	0.64	3.28	1.67	0.	0.
time (sec)	N/A	0.388	0.775	0.417	2.032	0.483	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	102	129	792	328	0	0
normalized size	1	1.	0.48	0.61	3.72	1.54	0.	0.
time (sec)	N/A	0.459	1.184	0.394	2.088	0.491	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	273	512	9767	1405	0	0
normalized size	1	1.	1.03	1.93	36.86	5.3	0.	0.
time (sec)	N/A	0.672	3.275	0.359	5.86	1.056	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	251	448	7777	1268	0	0
normalized size	1	1.	1.15	2.06	35.67	5.82	0.	0.
time (sec)	N/A	0.601	2.902	0.349	3.664	1.041	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	223	388	4733	1168	0	0
normalized size	1	1.	1.3	2.27	27.68	6.83	0.	0.
time (sec)	N/A	0.505	2.239	0.346	2.615	0.786	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	209	375	3402	1107	0	0
normalized size	1	1.	1.22	2.19	19.89	6.47	0.	0.
time (sec)	N/A	0.49	4.624	0.378	2.408	0.792	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	382	229	1597	980	0	0
normalized size	1	1.	2.26	1.36	9.45	5.8	0.	0.
time (sec)	N/A	0.467	6.427	0.362	2.058	0.608	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	428	222	655	1034	0	0
normalized size	1	1.	2.63	1.36	4.02	6.34	0.	0.
time (sec)	N/A	0.464	6.291	0.592	2.107	0.604	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	85	108	462	292	0	0
normalized size	1	1.	0.5	0.64	2.73	1.73	0.	0.
time (sec)	N/A	0.414	1.075	0.329	2.005	0.487	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	103	130	819	344	0	0
normalized size	1	1.	0.47	0.59	3.74	1.57	0.	0.
time (sec)	N/A	0.586	1.372	0.375	2.101	0.494	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	125	152	1072	401	0	0
normalized size	1	1.	0.47	0.57	4.03	1.51	0.	0.
time (sec)	N/A	0.677	1.905	0.365	2.152	0.5	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	295	576	14959	1559	0	0
normalized size	1	1.	0.95	1.85	47.95	5.	0.	0.
time (sec)	N/A	0.897	4.076	0.394	11.056	1.07	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	273	512	11950	1446	0	0
normalized size	1	1.	1.03	1.93	45.09	5.46	0.	0.
time (sec)	N/A	0.783	3.792	0.391	6.27	1.051	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	250	452	9027	1324	0	0
normalized size	1	1.	1.15	2.07	41.41	6.07	0.	0.
time (sec)	N/A	0.677	3.382	0.445	22.036	1.043	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	411	399	0	1257	0	0
normalized size	1	1.	1.89	1.83	0.	5.77	0.	0.
time (sec)	N/A	0.658	6.707	0.375	0.	0.805	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	416	380	4618	1219	0	0
normalized size	1	1.	1.86	1.7	20.62	5.44	0.	0.
time (sec)	N/A	0.674	6.899	0.444	20.847	0.818	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	428	255	0	1114	0	0
normalized size	1	1.	2.04	1.21	0.	5.3	0.	0.
time (sec)	N/A	0.657	6.58	0.413	0.	0.637	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	474	246	1238	1176	0	0
normalized size	1	1.	2.26	1.17	5.9	5.6	0.	0.
time (sec)	N/A	0.64	6.417	0.409	2.241	0.631	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	105	132	652	359	0	0
normalized size	1	1.	0.49	0.61	3.02	1.66	0.	0.
time (sec)	N/A	0.5	1.563	0.436	2.035	0.488	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	127	154	1141	408	0	0
normalized size	1	1.	0.48	0.58	4.29	1.53	0.	0.
time (sec)	N/A	0.797	2.077	0.384	2.16	0.506	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	148	176	1408	495	0	0
normalized size	1	1.	0.47	0.56	4.5	1.58	0.	0.
time (sec)	N/A	0.865	2.175	0.394	2.213	0.512	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	368	448	4809	1712	0	0
normalized size	1	1.	1.63	1.98	21.28	7.58	0.	0.
time (sec)	N/A	0.718	4.92	0.395	2.772	0.902	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	730	388	2867	1596	0	0
normalized size	1	1.	3.99	2.12	15.67	8.72	0.	0.
time (sec)	N/A	0.546	6.664	0.402	2.384	0.881	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	717	252	1307	1364	0	0
normalized size	1	1.	5.39	1.89	9.83	10.26	0.	0.
time (sec)	N/A	0.368	6.665	0.375	2.123	0.66	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	504	273	783	1351	0	0
normalized size	1	1.	3.73	2.02	5.8	10.01	0.	0.
time (sec)	N/A	0.364	2.972	0.35	2.108	0.661	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	273	171	504	917	0	0
normalized size	1	1.	2.01	1.26	3.71	6.74	0.	0.
time (sec)	N/A	0.337	3.882	0.378	2.019	0.532	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	528	194	624	1010	0	0
normalized size	1	1.	2.92	1.07	3.45	5.58	0.	0.
time (sec)	N/A	0.486	6.393	0.394	2.085	0.536	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	573	216	986	1108	0	0
normalized size	1	1.	2.56	0.96	4.4	4.95	0.	0.
time (sec)	N/A	0.675	6.48	0.431	2.188	0.547	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	800	370	0	1658	0	0
normalized size	1	1.	4.26	1.97	0.	8.82	0.	0.
time (sec)	N/A	0.562	7.068	0.388	0.	0.703	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	795	314	4257	1613	0	0
normalized size	1	1.	5.48	2.17	29.36	11.12	0.	0.
time (sec)	N/A	0.392	7.243	0.364	2.327	0.687	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	303	285	0	1098	0	0
normalized size	1	1.	1.99	1.88	0.	7.22	0.	0.
time (sec)	N/A	0.351	3.114	0.347	0.	0.543	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	316	306	0	1197	0	0
normalized size	1	1.	1.57	1.52	0.	5.96	0.	0.
time (sec)	N/A	0.515	3.007	0.381	0.	0.539	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	331	328	0	1280	0	0
normalized size	1	1.	1.33	1.32	0.	5.16	0.	0.
time (sec)	N/A	0.702	5.239	0.391	0.	0.556	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	903	615	0	2001	0	0
normalized size	1	1.	3.81	2.59	0.	8.44	0.	0.
time (sec)	N/A	0.768	7.309	0.375	0.	0.749	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	445	550	10615	1967	0	0
normalized size	1	1.	2.32	2.86	55.29	10.24	0.	0.
time (sec)	N/A	0.561	6.655	0.352	4.434	0.736	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	308	348	0	1308	0	0
normalized size	1	1.	2.	2.26	0.	8.49	0.	0.
time (sec)	N/A	0.369	2.84	0.364	0.	0.54	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	317	419	0	1349	0	0
normalized size	1	1.	1.59	2.11	0.	6.78	0.	0.
time (sec)	N/A	0.546	2.849	0.35	0.	0.551	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	331	438	0	1472	0	0
normalized size	1	1.	1.35	1.78	0.	5.98	0.	0.
time (sec)	N/A	0.703	3.226	0.345	0.	0.561	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	349	460	0	1580	0	0
normalized size	1	1.	1.18	1.56	0.	5.36	0.	0.
time (sec)	N/A	0.909	4.068	0.385	0.	0.579	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	434	434	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.704	4.525	0.178	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.434	2.803	0.161	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.458	3.134	0.161	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	457	457	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.504	3.864	0.163	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	815	815	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.016	27.529	0.173	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	774	774	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.827	14.999	0.159	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	791	791	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.879	17.881	0.176	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	841	841	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.927	9.657	0.16	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.533	18.196	1.099	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.522	24.651	0.376	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	419	142	0	0
normalized size	1	1.	1.	0.	11.03	3.74	0.	0.
time (sec)	N/A	0.928	0.162	1.27	11.131	0.909	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	337	171	220	339	0	254
normalized size	1	1.	3.18	1.61	2.08	3.2	0.	2.4
time (sec)	N/A	0.176	0.628	0.043	0.945	0.521	0.	1.145

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	181	128	171	288	0	208
normalized size	1	1.	2.1	1.49	1.99	3.35	0.	2.42
time (sec)	N/A	0.145	0.518	0.039	0.942	0.541	0.	1.173

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	119	239	0	167
normalized size	1	1.	1.34	1.54	2.12	4.27	0.	2.98
time (sec)	N/A	0.059	0.038	0.035	0.931	0.501	0.	1.154

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	65	99	220	0	113
normalized size	1	1.	1.34	2.03	3.09	6.88	0.	3.53
time (sec)	N/A	0.072	0.014	0.065	0.94	0.525	0.	1.136

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	46	56	78	139	0	107
normalized size	1	1.	1.44	1.75	2.44	4.34	0.	3.34
time (sec)	N/A	0.097	0.024	0.069	0.937	0.509	0.	1.175

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	74	99	0	126
normalized size	1	1.	0.94	1.21	1.57	2.11	0.	2.68
time (sec)	N/A	0.135	0.094	0.077	0.936	0.479	0.	1.148

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	107	146	0	167
normalized size	1	1.	0.84	1.1	1.39	1.9	0.	2.17
time (sec)	N/A	0.157	0.171	0.082	0.938	0.484	0.	1.128

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	136	193	0	211
normalized size	1	1.	0.77	1.1	1.4	1.99	0.	2.18
time (sec)	N/A	0.169	0.23	0.091	0.937	0.493	0.	1.122

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	391	235	375	421	0	332
normalized size	1	1.	2.31	1.39	2.22	2.49	0.	1.96
time (sec)	N/A	0.323	0.769	0.05	0.952	0.532	0.	1.177

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	339	187	311	362	0	286
normalized size	1	1.	2.46	1.36	2.25	2.62	0.	2.07
time (sec)	N/A	0.268	0.617	0.044	0.947	0.539	0.	1.197

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	63	141	225	315	0	240
normalized size	1	1.	0.61	1.37	2.18	3.06	0.	2.33
time (sec)	N/A	0.107	0.335	0.041	0.942	0.509	0.	1.189

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	277	113	192	297	0	208
normalized size	1	1.	3.38	1.38	2.34	3.62	0.	2.54
time (sec)	N/A	0.146	1.186	0.074	0.946	0.517	0.	1.173

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	143	107	142	278	0	212
normalized size	1	1.	1.96	1.47	1.95	3.81	0.	2.9
time (sec)	N/A	0.206	0.318	0.072	0.941	0.516	0.	1.163

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	96	108	136	194	0	196
normalized size	1	1.	1.09	1.23	1.55	2.2	0.	2.23
time (sec)	N/A	0.22	0.152	0.084	0.942	0.516	0.	1.177

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	61	116	149	165	0	192
normalized size	1	1.	0.6	1.14	1.46	1.62	0.	1.88
time (sec)	N/A	0.23	0.169	0.084	0.94	0.483	0.	1.178

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	86	154	194	213	0	238
normalized size	1	1.	0.64	1.14	1.44	1.58	0.	1.76
time (sec)	N/A	0.307	0.36	0.091	0.946	0.494	0.	1.155

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	240	271	0	284
normalized size	1	1.	0.68	1.16	1.5	1.69	0.	1.78
time (sec)	N/A	0.329	0.374	0.099	0.948	0.526	0.	1.203

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	391	234	455	431	0	332
normalized size	1	1.	2.4	1.44	2.79	2.64	0.	2.04
time (sec)	N/A	0.313	0.808	0.049	0.958	0.523	0.	1.192

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	81	188	354	366	0	286
normalized size	1	1.	0.65	1.5	2.83	2.93	0.	2.29
time (sec)	N/A	0.139	0.45	0.051	0.954	0.524	0.	1.193

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	772	158	286	356	0	255
normalized size	1	1.	6.95	1.42	2.58	3.21	0.	2.3
time (sec)	N/A	0.205	6.401	0.084	0.956	0.526	0.	1.202

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	208	144	223	342	0	259
normalized size	1	1.	1.93	1.33	2.06	3.17	0.	2.4
time (sec)	N/A	0.312	1.938	0.085	0.945	0.569	0.	1.219

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	272	145	189	323	0	259
normalized size	1	1.	2.32	1.24	1.62	2.76	0.	2.21
time (sec)	N/A	0.335	1.682	0.079	0.945	0.559	0.	1.218

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	113	153	200	254	0	243
normalized size	1	1.	0.9	1.22	1.6	2.03	0.	1.94
time (sec)	N/A	0.339	0.249	0.088	0.949	0.53	0.	1.238

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	86	176	225	216	0	238
normalized size	1	1.	0.69	1.42	1.81	1.74	0.	1.92
time (sec)	N/A	0.252	0.274	0.089	0.949	0.504	0.	1.207

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	108	223	288	278	0	284
normalized size	1	1.	0.61	1.27	1.64	1.58	0.	1.61
time (sec)	N/A	0.447	0.43	0.104	0.943	0.501	0.	1.209

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	354	332	0	329
normalized size	1	1.	0.67	1.32	1.76	1.65	0.	1.64
time (sec)	N/A	0.479	0.481	0.105	0.953	0.513	0.	1.227

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	550	340	497	417	0	246
normalized size	1	1.	4.2	2.6	3.79	3.18	0.	1.88
time (sec)	N/A	0.253	1.119	0.062	0.954	0.519	0.	1.185

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	383	252	381	386	0	211
normalized size	1	1.	3.55	2.33	3.53	3.57	0.	1.95
time (sec)	N/A	0.24	0.665	0.054	0.949	0.514	0.	1.155

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	234	163	265	319	0	147
normalized size	1	1.	3.77	2.63	4.27	5.15	0.	2.37
time (sec)	N/A	0.166	0.482	0.046	0.942	0.506	0.	1.15

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	106	78	134	197	0	95
normalized size	1	1.	2.41	1.77	3.05	4.48	0.	2.16
time (sec)	N/A	0.073	0.185	0.044	0.932	0.495	0.	1.148

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	72	56	99	105	0	59
normalized size	1	1.	2.06	1.6	2.83	3.	0.	1.69
time (sec)	N/A	0.132	0.131	0.069	1.43	0.47	0.	1.149

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	76	108	193	149	0	107
normalized size	1	1.	1.27	1.8	3.22	2.48	0.	1.78
time (sec)	N/A	0.196	0.343	0.083	1.417	0.476	0.	1.121

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	197	211	304	203	0	166
normalized size	1	1.	2.01	2.15	3.1	2.07	0.	1.69
time (sec)	N/A	0.232	0.41	0.093	1.434	0.484	0.	1.131

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	249	281	419	243	0	204
normalized size	1	1.	2.04	2.3	3.43	1.99	0.	1.67
time (sec)	N/A	0.241	0.604	0.096	1.433	0.493	0.	1.153

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	379	294	454	566	0	267
normalized size	1	1.	2.43	1.88	2.91	3.63	0.	1.71
time (sec)	N/A	0.381	1.488	0.066	0.963	0.518	0.	1.22

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	245	205	329	501	0	204
normalized size	1	1.	2.27	1.9	3.05	4.64	0.	1.89
time (sec)	N/A	0.336	1.049	0.053	0.953	0.504	0.	1.18

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	106	119	196	338	0	151
normalized size	1	1.	1.34	1.51	2.48	4.28	0.	1.91
time (sec)	N/A	0.233	0.758	0.049	0.958	0.502	0.	1.169

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	60	126	144	0	81
normalized size	1	1.	0.74	0.97	2.03	2.32	0.	1.31
time (sec)	N/A	0.074	0.287	0.052	0.948	0.467	0.	1.126

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	153	97	162	228	0	115
normalized size	1	1.	2.19	1.39	2.31	3.26	0.	1.64
time (sec)	N/A	0.178	0.394	0.079	1.435	0.481	0.	1.122

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	245	149	258	296	0	163
normalized size	1	1.	2.5	1.52	2.63	3.02	0.	1.66
time (sec)	N/A	0.315	0.609	0.086	1.43	0.521	0.	1.156

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	315	252	382	342	0	221
normalized size	1	1.	2.2	1.76	2.67	2.39	0.	1.55
time (sec)	N/A	0.378	0.754	0.091	1.434	0.496	0.	1.137

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	369	322	502	389	0	259
normalized size	1	1.	2.17	1.89	2.95	2.29	0.	1.52
time (sec)	N/A	0.402	0.709	0.089	1.456	0.503	0.	1.141

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	428	334	509	757	0	315
normalized size	1	1.	2.12	1.65	2.52	3.75	0.	1.56
time (sec)	N/A	0.553	1.972	0.069	0.976	0.526	0.	1.192

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	294	245	386	668	0	251
normalized size	1	1.	1.88	1.57	2.47	4.28	0.	1.61
time (sec)	N/A	0.498	1.645	0.057	0.969	0.521	0.	1.212

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	136	159	252	481	0	198
normalized size	1	1.	1.09	1.27	2.02	3.85	0.	1.58
time (sec)	N/A	0.403	0.66	0.054	0.953	0.502	0.	1.169

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	64	155	227	0	101
normalized size	1	1.	0.69	0.63	1.52	2.23	0.	0.99
time (sec)	N/A	0.251	0.172	0.053	0.959	0.461	0.	1.178

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	64	155	227	0	101
normalized size	1	1.	0.69	0.63	1.52	2.23	0.	0.99
time (sec)	N/A	0.11	0.359	0.056	0.958	0.462	0.	1.164

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	241	137	216	351	0	163
normalized size	1	1.	2.23	1.27	2.	3.25	0.	1.51
time (sec)	N/A	0.252	0.571	0.088	1.456	0.481	0.	1.157

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	365	189	312	431	0	212
normalized size	1	1.	2.68	1.39	2.29	3.17	0.	1.56
time (sec)	N/A	0.444	1.01	0.109	1.444	0.493	0.	1.179

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	435	292	435	495	0	270
normalized size	1	1.	2.33	1.56	2.33	2.65	0.	1.44
time (sec)	N/A	0.545	0.719	0.102	1.458	0.521	0.	1.159

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	115	160	0	373	0	424
normalized size	1	1.	0.5	0.7	0.	1.62	0.	1.84
time (sec)	N/A	0.485	5.886	0.403	0.	0.506	0.	4.627

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	98	138	0	308	0	362
normalized size	1	1.	0.52	0.74	0.	1.65	0.	1.94
time (sec)	N/A	0.419	0.537	0.398	0.	0.506	0.	4.562

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	81	116	0	265	0	300
normalized size	1	1.	0.56	0.81	0.	1.84	0.	2.08
time (sec)	N/A	0.359	0.281	0.317	0.	0.494	0.	4.453

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	80	94	0	217	0	238
normalized size	1	1.	0.79	0.93	0.	2.15	0.	2.36
time (sec)	N/A	0.277	0.306	0.285	0.	0.493	0.	4.412

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	43	70	0	169	0	174
normalized size	1	1.	0.69	1.13	0.	2.73	0.	2.81
time (sec)	N/A	0.084	0.16	0.275	0.	0.496	0.	4.313

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	198	620	0	0
normalized size	1	1.	1.15	1.79	3.	9.39	0.	0.
time (sec)	N/A	0.171	0.276	0.255	1.639	0.554	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	93	198	1268	694	0	450
normalized size	1	1.	1.37	2.91	18.65	10.21	0.	6.62
time (sec)	N/A	0.21	0.242	0.316	1.971	0.636	0.	6.321

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	398	2499	801	0	851
normalized size	1	1.	1.	3.4	21.36	6.85	0.	7.27
time (sec)	N/A	0.281	0.367	0.379	2.257	0.646	0.	6.616

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	70	580	4024	898	0	1156
normalized size	1	1.	0.44	3.62	25.15	5.61	0.	7.22
time (sec)	N/A	0.336	0.188	0.42	2.845	0.652	0.	6.707

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	113	161	0	394	0	424
normalized size	1	1.	0.48	0.69	0.	1.68	0.	1.81
time (sec)	N/A	0.638	6.104	0.463	0.	0.519	0.	4.91

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	100	139	0	329	0	362
normalized size	1	1.	0.53	0.74	0.	1.74	0.	1.92
time (sec)	N/A	0.564	0.687	0.287	0.	0.538	0.	4.775

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	82	117	0	279	0	300
normalized size	1	1.	0.59	0.85	0.	2.02	0.	2.17
time (sec)	N/A	0.353	0.373	0.265	0.	0.511	0.	4.676

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	62	95	0	225	0	238
normalized size	1	1.	0.61	0.94	0.	2.23	0.	2.36
time (sec)	N/A	0.129	0.254	0.253	0.	0.499	0.	4.526

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	1347	814	0	0
normalized size	1	1.	0.97	2.26	12.83	7.75	0.	0.
time (sec)	N/A	0.243	0.483	0.267	1.849	0.557	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	97	212	2431	755	0	544
normalized size	1	1.	0.94	2.06	23.6	7.33	0.	5.28
time (sec)	N/A	0.357	0.427	0.289	2.186	0.653	0.	6.384

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	101	399	0	833	0	863
normalized size	1	1.	0.85	3.35	0.	7.	0.	7.25
time (sec)	N/A	0.378	0.711	0.3	0.	0.693	0.	6.688

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	740	581	0	949	0	1166
normalized size	1	1.	4.51	3.54	0.	5.79	0.	7.11
time (sec)	N/A	0.47	11.387	0.381	0.	0.679	0.	7.014

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	1031	763	0	1049	0	1469
normalized size	1	1.	4.93	3.65	0.	5.02	0.	7.03
time (sec)	N/A	0.558	11.991	0.332	0.	0.767	0.	7.268

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	131	185	0	479	0	486
normalized size	1	1.	0.46	0.66	0.	1.7	0.	1.72
time (sec)	N/A	0.84	0.479	0.365	0.	0.538	0.	5.253

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	487	163	0	409	0	424
normalized size	1	1.	2.05	0.69	0.	1.73	0.	1.79
time (sec)	N/A	0.759	6.166	0.382	0.	0.562	0.	5.104

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	96	141	0	346	0	362
normalized size	1	1.	0.55	0.81	0.	1.98	0.	2.07
time (sec)	N/A	0.41	0.647	0.295	0.	0.511	0.	5.028

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	79	119	0	292	0	300
normalized size	1	1.	0.57	0.86	0.	2.12	0.	2.17
time (sec)	N/A	0.175	0.351	0.271	0.	0.495	0.	4.793

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1885	954	0	0
normalized size	1	1.	0.9	2.4	13.27	6.72	0.	0.
time (sec)	N/A	0.317	0.858	0.281	1.935	0.573	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	126	256	3753	968	0	644
normalized size	1	1.	0.88	1.79	26.24	6.77	0.	4.5
time (sec)	N/A	0.515	0.806	0.364	2.37	0.659	0.	6.689

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	116	410	0	890	0	957
normalized size	1	1.	0.75	2.66	0.	5.78	0.	6.21
time (sec)	N/A	0.529	0.824	0.388	0.	0.656	0.	7.092

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	121	583	0	976	0	1177
normalized size	1	1.	0.74	3.55	0.	5.95	0.	7.18
time (sec)	N/A	0.567	1.298	0.567	0.	0.655	0.	7.608

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	366	765	0	1103	0	1480
normalized size	1	1.	1.75	3.66	0.	5.28	0.	7.08
time (sec)	N/A	0.667	1.291	0.42	0.	0.749	0.	7.964

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	416	947	0	1227	0	1782
normalized size	1	1.	1.64	3.73	0.	4.83	0.	7.02
time (sec)	N/A	0.749	1.747	0.366	0.	0.765	0.	8.223

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	183	975	0	1211	0	528
normalized size	1	1.	0.75	4.01	0.	4.98	0.	2.17
time (sec)	N/A	0.876	1.17	0.43	0.	0.662	0.	9.223

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	140	785	0	1116	0	387
normalized size	1	1.	0.69	3.89	0.	5.52	0.	1.92
time (sec)	N/A	0.694	0.527	0.371	0.	0.623	0.	9.109

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	123	595	0	1019	0	366
normalized size	1	1.	0.77	3.74	0.	6.41	0.	2.3
time (sec)	N/A	0.514	0.377	0.332	0.	0.652	0.	8.94

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	106	405	0	917	0	251
normalized size	1	1.	0.9	3.43	0.	7.77	0.	2.13
time (sec)	N/A	0.304	0.296	0.303	0.	0.642	0.	8.89

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	88	200	0	751	0	194
normalized size	1	1.	1.13	2.56	0.	9.63	0.	2.49
time (sec)	N/A	0.095	0.172	0.252	0.	0.587	0.	8.822

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	814	0	302
normalized size	1	1.	1.01	2.13	0.	8.95	0.	3.32
time (sec)	N/A	0.186	0.3	0.249	0.	2.618	0.	10.862

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	10104	353	0	1214	0	531
normalized size	1	1.	84.91	2.97	0.	10.2	0.	4.46
time (sec)	N/A	0.32	26.438	0.345	0.	3.241	0.	11.376

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	135	717	0	1323	0	876
normalized size	1	1.	0.82	4.35	0.	8.02	0.	5.31
time (sec)	N/A	0.471	0.411	0.388	0.	5.95	0.	11.557

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	150	1067	0	1426	0	1142
normalized size	1	1.	0.73	5.18	0.	6.92	0.	5.54
time (sec)	N/A	0.645	0.763	0.349	0.	5.971	0.	11.726

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	204	983	0	1419	0	593
normalized size	1	1.	0.78	3.77	0.	5.44	0.	2.27
time (sec)	N/A	0.919	1.23	0.37	0.	0.672	0.	9.288

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	160	793	0	1315	0	421
normalized size	1	1.	0.74	3.67	0.	6.09	0.	1.95
time (sec)	N/A	0.725	2.349	0.335	0.	0.65	0.	9.14

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	141	603	0	1189	0	400
normalized size	1	1.	0.82	3.53	0.	6.95	0.	2.34
time (sec)	N/A	0.556	1.399	0.295	0.	0.618	0.	9.03

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	125	405	0	1003	0	257
normalized size	1	1.	1.06	3.43	0.	8.5	0.	2.18
time (sec)	N/A	0.319	0.727	0.26	0.	0.597	0.	8.796

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	127	402	0	957	0	208
normalized size	1	1.	1.46	4.62	0.	11.	0.	2.39
time (sec)	N/A	0.106	0.784	0.24	0.	0.605	0.	8.649

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	147	552	0	1416	0	417
normalized size	1	1.	1.16	4.35	0.	11.15	0.	3.28
time (sec)	N/A	0.27	1.576	0.234	0.	7.923	0.	11.05

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	10898	713	0	1575	0	0
normalized size	1	1.	64.11	4.19	0.	9.26	0.	0.
time (sec)	N/A	0.494	27.014	0.291	0.	10.519	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	395	1075	0	1673	0	0
normalized size	1	1.	1.79	4.86	0.	7.57	0.	0.
time (sec)	N/A	0.685	2.329	0.351	0.	15.29	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	177	985	0	1597	0	593
normalized size	1	1.	0.68	3.77	0.	6.12	0.	2.27
time (sec)	N/A	0.928	2.603	0.325	0.	0.687	0.	10.03

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	161	795	0	1474	0	420
normalized size	1	1.	0.75	3.68	0.	6.82	0.	1.94
time (sec)	N/A	0.748	2.598	0.306	0.	0.653	0.	9.785

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	144	597	0	1273	0	390
normalized size	1	1.	0.85	3.53	0.	7.53	0.	2.31
time (sec)	N/A	0.564	1.471	0.281	0.	0.627	0.	9.553

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	131	602	0	1233	0	258
normalized size	1	1.	1.04	4.78	0.	9.79	0.	2.05
time (sec)	N/A	0.334	1.433	0.261	0.	0.617	0.	9.294

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	206	594	0	1219	0	258
normalized size	1	1.	1.63	4.71	0.	9.67	0.	2.05
time (sec)	N/A	0.152	1.488	0.207	0.	0.616	0.	8.976

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	10133	824	0	1754	0	471
normalized size	1	1.	61.79	5.02	0.	10.7	0.	2.87
time (sec)	N/A	0.346	26.663	0.24	0.	16.806	0.	11.751

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	10956	1065	0	1948	0	0
normalized size	1	1.	52.93	5.14	0.	9.41	0.	0.
time (sec)	N/A	0.672	26.915	0.348	0.	22.253	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	101	287	359	437	0	404
normalized size	1	1.	0.66	1.89	2.36	2.88	0.	2.66
time (sec)	N/A	0.213	1.006	0.056	0.953	0.532	0.	1.302

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	84	223	294	375	0	343
normalized size	1	1.	0.66	1.76	2.31	2.95	0.	2.7
time (sec)	N/A	0.191	0.623	0.049	0.961	0.514	0.	1.299

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	485	160	209	312	0	277
normalized size	1	1.	5.27	1.74	2.27	3.39	0.	3.01
time (sec)	N/A	0.12	4.351	0.048	0.948	0.517	0.	1.245

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	305	117	157	292	0	190
normalized size	1	1.	4.84	1.86	2.49	4.63	0.	3.02
time (sec)	N/A	0.064	1.763	0.048	0.938	0.534	0.	1.3

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	71	88	124	257	0	181
normalized size	1	1.	1.54	1.91	2.7	5.59	0.	3.93
time (sec)	N/A	0.116	0.042	0.076	0.948	0.523	0.	1.189

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	59	100	120	184	0	177
normalized size	1	1.	0.95	1.61	1.94	2.97	0.	2.85
time (sec)	N/A	0.15	0.141	0.09	0.94	0.52	0.	1.249

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	64	102	132	162	0	231
normalized size	1	1.	0.78	1.24	1.61	1.98	0.	2.82
time (sec)	N/A	0.176	0.229	0.087	0.937	0.498	0.	1.249

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	97	141	178	221	0	294
normalized size	1	1.	0.95	1.38	1.75	2.17	0.	2.88
time (sec)	N/A	0.211	0.403	0.109	0.937	0.504	0.	1.243

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	94	173	224	282	0	355
normalized size	1	1.	0.67	1.23	1.59	2.	0.	2.52
time (sec)	N/A	0.225	0.442	0.105	0.948	0.513	0.	1.28

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	359	386	644	535	0	529
normalized size	1	1.	1.62	1.74	2.9	2.41	0.	2.38
time (sec)	N/A	0.43	3.298	0.068	0.977	0.555	0.	1.367

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	417	315	486	463	0	460
normalized size	1	1.	2.19	1.66	2.56	2.44	0.	2.42
time (sec)	N/A	0.41	3.066	0.061	0.967	0.536	0.	1.312

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	386	246	417	397	0	392
normalized size	1	1.	2.63	1.67	2.84	2.7	0.	2.67
time (sec)	N/A	0.235	2.414	0.057	0.959	0.519	0.	1.269

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	542	193	284	379	0	338
normalized size	1	1.	4.52	1.61	2.37	3.16	0.	2.82
time (sec)	N/A	0.157	5.685	0.056	0.952	0.526	0.	1.293

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	365	166	259	358	0	275
normalized size	1	1.	3.02	1.37	2.14	2.96	0.	2.27
time (sec)	N/A	0.217	3.57	0.103	0.955	0.53	0.	1.235

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	329	160	204	331	0	267
normalized size	1	1.	2.57	1.25	1.59	2.59	0.	2.09
time (sec)	N/A	0.287	3.654	0.091	0.959	0.529	0.	1.295

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	181	216	269	0	317
normalized size	1	1.	0.9	1.35	1.61	2.01	0.	2.37
time (sec)	N/A	0.292	0.29	0.103	0.947	0.528	0.	1.28

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	95	203	257	239	0	335
normalized size	1	1.	0.64	1.36	1.72	1.6	0.	2.25
time (sec)	N/A	0.328	0.331	0.102	0.951	0.504	0.	1.265

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	132	247	319	305	0	404
normalized size	1	1.	0.71	1.32	1.71	1.63	0.	2.16
time (sec)	N/A	0.415	0.603	0.11	0.958	0.51	0.	1.243

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	170	304	400	373	0	473
normalized size	1	1.	0.8	1.43	1.88	1.75	0.	2.22
time (sec)	N/A	0.441	0.966	0.122	0.96	0.523	0.	1.303

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	402	455	876	617	0	598
normalized size	1	1.	1.47	1.66	3.2	2.25	0.	2.18
time (sec)	N/A	0.598	6.162	0.074	0.989	0.557	0.	1.351

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	359	385	755	540	0	529
normalized size	1	1.	1.66	1.78	3.5	2.5	0.	2.45
time (sec)	N/A	0.455	4.241	0.067	0.982	0.544	0.	1.369

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	431	316	593	477	0	460
normalized size	1	1.	2.46	1.81	3.39	2.73	0.	2.63
time (sec)	N/A	0.277	3.608	0.069	0.975	0.528	0.	1.341

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	464	262	475	447	0	406
normalized size	1	1.	2.86	1.62	2.93	2.76	0.	2.51
time (sec)	N/A	0.237	3.093	0.065	0.96	0.558	0.	1.328

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	1503	226	370	425	0	389
normalized size	1	1.	9.63	1.45	2.37	2.72	0.	2.49
time (sec)	N/A	0.284	6.455	0.107	0.965	0.558	0.	1.369

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	406	219	320	410	0	378
normalized size	1	1.	2.37	1.28	1.87	2.4	0.	2.21
time (sec)	N/A	0.425	5.891	0.108	0.964	0.554	0.	1.25

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	379	221	284	398	0	379
normalized size	1	1.	2.24	1.31	1.68	2.36	0.	2.24
time (sec)	N/A	0.442	2.185	0.099	0.963	0.537	0.	1.3

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	147	251	324	336	0	386
normalized size	1	1.	0.8	1.37	1.77	1.84	0.	2.11
time (sec)	N/A	0.441	0.42	0.116	0.955	0.538	0.	1.336

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	130	295	381	315	0	404
normalized size	1	1.	0.73	1.65	2.13	1.76	0.	2.26
time (sec)	N/A	0.365	0.43	0.109	0.961	0.53	0.	1.294

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	170	364	478	378	0	473
normalized size	1	1.	0.72	1.55	2.03	1.61	0.	2.01
time (sec)	N/A	0.591	0.753	0.125	0.972	0.584	0.	1.327

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	204	427	574	454	0	541
normalized size	1	1.	0.77	1.61	2.17	1.71	0.	2.04
time (sec)	N/A	0.611	1.391	0.132	0.983	0.579	0.	1.333

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	1087	454	987	617	0	598
normalized size	1	1.	4.31	1.8	3.92	2.45	0.	2.37
time (sec)	N/A	0.52	6.456	0.077	1.003	0.556	0.	1.305

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	359	385	861	539	0	529
normalized size	1	1.	1.72	1.84	4.12	2.58	0.	2.53
time (sec)	N/A	0.328	5.923	0.08	0.984	0.544	0.	1.33

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	538	331	651	521	0	475
normalized size	1	1.	2.76	1.7	3.34	2.67	0.	2.44
time (sec)	N/A	0.304	5.091	0.074	0.967	0.561	0.	1.294

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	530	294	562	493	0	458
normalized size	1	1.	2.7	1.5	2.87	2.52	0.	2.34
time (sec)	N/A	0.382	4.656	0.117	0.979	0.566	0.	1.326

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	524	279	432	487	0	468
normalized size	1	1.	2.51	1.33	2.07	2.33	0.	2.24
time (sec)	N/A	0.566	4.627	0.125	0.966	0.56	0.	1.318

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	1518	280	400	486	0	468
normalized size	1	1.	7.	1.29	1.84	2.24	0.	2.16
time (sec)	N/A	0.602	6.312	0.122	0.965	0.565	0.	1.306

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	1436	289	392	466	0	448
normalized size	1	1.	6.62	1.33	1.81	2.15	0.	2.06
time (sec)	N/A	0.622	6.229	0.11	0.969	0.553	0.	1.295

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	182	320	448	408	0	455
normalized size	1	1.	0.81	1.42	1.99	1.81	0.	2.02
time (sec)	N/A	0.583	0.617	0.125	0.968	0.561	0.	1.343

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	163	416	540	378	0	473
normalized size	1	1.	0.77	1.95	2.54	1.77	0.	2.22
time (sec)	N/A	0.409	0.497	0.13	0.973	0.529	0.	1.336

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	204	490	652	454	0	541
normalized size	1	1.	0.73	1.76	2.35	1.63	0.	1.95
time (sec)	N/A	0.763	1.007	0.141	0.985	0.53	0.	1.325

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	237	577	782	536	0	610
normalized size	1	1.	0.78	1.9	2.58	1.77	0.	2.01
time (sec)	N/A	0.794	1.934	0.221	0.993	0.547	0.	1.318

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	1099	576	825	545	0	385
normalized size	1	1.	6.01	3.15	4.51	2.98	0.	2.1
time (sec)	N/A	0.218	6.393	0.074	0.979	0.543	0.	1.315

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	898	442	655	489	0	328
normalized size	1	1.	6.07	2.99	4.43	3.3	0.	2.22
time (sec)	N/A	0.198	6.314	0.073	0.965	0.531	0.	1.259

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	392	311	481	428	0	234
normalized size	1	1.	3.29	2.61	4.04	3.6	0.	1.97
time (sec)	N/A	0.192	4.289	0.064	0.957	0.51	0.	1.244

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	255	180	294	324	0	161
normalized size	1	1.	4.05	2.86	4.67	5.14	0.	2.56
time (sec)	N/A	0.168	1.396	0.059	0.943	0.513	0.	1.283

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	163	115	197	247	0	124
normalized size	1	1.	3.13	2.21	3.79	4.75	0.	2.38
time (sec)	N/A	0.116	0.493	0.063	1.416	0.508	0.	1.208

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	77	125	223	154	0	122
normalized size	1	1.	1.24	2.02	3.6	2.48	0.	1.97
time (sec)	N/A	0.13	0.402	0.093	1.429	0.48	0.	1.189

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	213	248	369	227	0	185
normalized size	1	1.	1.97	2.3	3.42	2.1	0.	1.71
time (sec)	N/A	0.182	0.505	0.099	1.433	0.486	0.	1.187

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	307	420	540	288	0	279
normalized size	1	1.	2.21	3.02	3.88	2.07	0.	2.01
time (sec)	N/A	0.197	1.025	0.102	1.437	0.496	0.	1.244

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	393	526	709	344	0	336
normalized size	1	1.	2.26	3.02	4.07	1.98	0.	1.93
time (sec)	N/A	0.212	1.01	0.111	1.452	0.509	0.	1.279

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	1069	506	765	689	0	409
normalized size	1	1.	5.51	2.61	3.94	3.55	0.	2.11
time (sec)	N/A	0.363	6.422	0.079	0.981	0.529	0.	1.271

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	901	373	582	630	0	317
normalized size	1	1.	5.33	2.21	3.44	3.73	0.	1.88
time (sec)	N/A	0.337	6.326	0.074	0.972	0.526	0.	1.269

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	312	243	387	513	0	244
normalized size	1	1.	2.79	2.17	3.46	4.58	0.	2.18
time (sec)	N/A	0.283	2.076	0.063	0.965	0.522	0.	1.224

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	87	219	157	257	351	0	194
normalized size	1	1.07	2.7	1.94	3.17	4.33	0.	2.4
time (sec)	N/A	0.172	0.828	0.066	0.96	0.527	0.	1.293

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	175	135	221	242	0	157
normalized size	1	1.	2.36	1.82	2.99	3.27	0.	2.12
time (sec)	N/A	0.129	0.505	0.069	1.428	0.473	0.	1.286

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	279	187	317	309	0	205
normalized size	1	1.	2.79	1.87	3.17	3.09	0.	2.05
time (sec)	N/A	0.259	0.831	0.098	1.448	0.487	0.	1.265

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	377	309	475	383	0	267
normalized size	1	1.	2.42	1.98	3.04	2.46	0.	1.71
time (sec)	N/A	0.335	1.518	0.106	1.442	0.497	0.	1.243

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	473	482	657	437	0	359
normalized size	1	1.	2.56	2.61	3.55	2.36	0.	1.94
time (sec)	N/A	0.363	1.819	0.112	1.456	0.512	0.	1.251

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	1081	433	666	849	0	389
normalized size	1	1.	5.	2.	3.08	3.93	0.	1.8
time (sec)	N/A	0.516	6.453	0.082	0.989	0.533	0.	1.259

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	839	303	473	690	0	316
normalized size	1	1.	5.21	1.88	2.94	4.29	0.	1.96
time (sec)	N/A	0.45	6.367	0.072	0.982	0.522	0.	1.28

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	277	197	313	505	0	243
normalized size	1	1.	2.1	1.49	2.37	3.83	0.	1.84
time (sec)	N/A	0.351	1.614	0.069	0.967	0.511	0.	1.311

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	156	113	242	251	0	155
normalized size	1	1.	1.42	1.03	2.2	2.28	0.	1.41
time (sec)	N/A	0.207	0.566	0.068	0.977	0.46	0.	1.278

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	289	175	277	375	0	207
normalized size	1	1.	2.51	1.52	2.41	3.26	0.	1.8
time (sec)	N/A	0.196	0.908	0.079	1.439	0.485	0.	1.307

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	419	247	398	455	0	278
normalized size	1	1.	2.97	1.75	2.82	3.23	0.	1.97
time (sec)	N/A	0.404	1.636	0.11	1.459	0.498	0.	1.23

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	557	369	555	556	0	340
normalized size	1	1.	2.77	1.84	2.76	2.77	0.	1.69
time (sec)	N/A	0.517	1.585	0.128	1.465	0.511	0.	1.229

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	655	542	738	613	0	432
normalized size	1	1.	2.76	2.29	3.11	2.59	0.	1.82
time (sec)	N/A	0.545	2.648	0.122	1.465	0.52	0.	1.194

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	1322	493	751	1062	0	458
normalized size	1	1.	5.2	1.94	2.96	4.18	0.	1.8
time (sec)	N/A	0.69	6.476	0.086	1.003	0.547	0.	1.325

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	1208	363	555	872	0	385
normalized size	1	1.	5.92	1.78	2.72	4.27	0.	1.89
time (sec)	N/A	0.629	6.394	0.076	0.994	0.531	0.	1.362

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	335	277	423	660	0	335
normalized size	1	1.	1.94	1.6	2.45	3.82	0.	1.94
time (sec)	N/A	0.497	2.814	0.077	0.991	0.537	0.	1.235

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	200	106	350	343	0	231
normalized size	1	1.	1.35	0.72	2.36	2.32	0.	1.56
time (sec)	N/A	0.406	0.684	0.075	0.986	0.472	0.	1.211

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	231	108	350	343	0	231
normalized size	1	1.	1.5	0.7	2.27	2.23	0.	1.5
time (sec)	N/A	0.26	0.762	0.078	0.99	0.474	0.	1.206

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	405	255	386	512	0	297
normalized size	1	1.	2.74	1.72	2.61	3.46	0.	2.01
time (sec)	N/A	0.284	1.362	0.085	1.458	0.498	0.	1.179

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	567	307	481	608	0	346
normalized size	1	1.	3.22	1.74	2.73	3.45	0.	1.97
time (sec)	N/A	0.559	1.928	0.122	1.468	0.512	0.	1.16

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	345	429	640	730	0	408
normalized size	1	1.	1.44	1.79	2.68	3.05	0.	1.71
time (sec)	N/A	0.699	5.253	0.128	1.474	0.525	0.	1.166

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	185	204	0	405	0	554
normalized size	1	1.	0.77	0.85	0.	1.69	0.	2.32
time (sec)	N/A	0.558	1.73	0.433	0.	0.518	0.	4.981

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	153	171	0	343	0	470
normalized size	1	1.	0.79	0.89	0.	1.78	0.	2.44
time (sec)	N/A	0.473	1.847	0.37	0.	0.506	0.	4.838

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	119	138	0	286	0	386
normalized size	1	1.	0.81	0.94	0.	1.95	0.	2.63
time (sec)	N/A	0.428	1.317	0.334	0.	0.495	0.	4.65

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	83	105	0	225	0	302
normalized size	1	1.	0.8	1.01	0.	2.16	0.	2.9
time (sec)	N/A	0.209	0.786	0.319	0.	0.495	0.	4.542

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	101	236	0	792	0	0
normalized size	1	1.	1.01	2.36	0.	7.92	0.	0.
time (sec)	N/A	0.152	0.688	0.321	0.	0.551	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	94	210	1268	717	0	531
normalized size	1	1.	0.96	2.14	12.94	7.32	0.	5.42
time (sec)	N/A	0.211	0.396	0.351	1.97	0.667	0.	6.387

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	113	548	2695	833	0	891
normalized size	1	1.	0.97	4.68	23.03	7.12	0.	7.62
time (sec)	N/A	0.277	0.489	0.383	2.407	0.913	0.	6.677

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	152	832	5090	946	0	1596
normalized size	1	1.	0.93	5.1	31.23	5.8	0.	9.79
time (sec)	N/A	0.37	0.482	0.362	3.143	0.913	0.	6.983

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	90	1105	0	1084	0	2049
normalized size	1	1.	0.43	5.29	0.	5.19	0.	9.8
time (sec)	N/A	0.454	0.239	0.398	0.	1.292	0.	7.167

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	185	205	0	435	0	554
normalized size	1	1.	0.76	0.84	0.	1.79	0.	2.28
time (sec)	N/A	0.695	2.04	0.36	0.	0.523	0.	5.191

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	152	172	0	363	0	470
normalized size	1	1.	0.81	0.92	0.	1.94	0.	2.51
time (sec)	N/A	0.516	2.148	0.315	0.	0.509	0.	5.033

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	120	139	0	300	0	386
normalized size	1	1.	0.83	0.97	0.	2.08	0.	2.68
time (sec)	N/A	0.283	1.633	0.296	0.	0.501	0.	4.892

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	132	361	0	944	0	0
normalized size	1	1.	0.93	2.54	0.	6.65	0.	0.
time (sec)	N/A	0.236	1.353	0.322	0.	0.573	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	115	409	2431	941	0	633
normalized size	1	1.	0.8	2.84	16.88	6.53	0.	4.4
time (sec)	N/A	0.31	1.949	0.371	2.193	0.654	0.	6.542

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	117	569	0	887	0	990
normalized size	1	1.	0.75	3.62	0.	5.65	0.	6.31
time (sec)	N/A	0.451	0.761	0.395	0.	0.924	0.	6.764

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	124	833	0	1003	0	1612
normalized size	1	1.	0.75	5.05	0.	6.08	0.	9.77
time (sec)	N/A	0.481	1.671	0.297	0.	0.916	0.	7.285

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	157	1106	0	1133	0	2066
normalized size	1	1.	0.73	5.14	0.	5.27	0.	9.61
time (sec)	N/A	0.586	1.749	0.342	0.	1.304	0.	7.695

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	182	1379	0	1295	0	2519
normalized size	1	1.	0.69	5.24	0.	4.92	0.	9.58
time (sec)	N/A	0.673	2.67	0.365	0.	1.319	0.	8.007

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	222	240	0	541	0	637
normalized size	1	1.	0.76	0.82	0.	1.84	0.	2.17
time (sec)	N/A	0.905	2.143	0.37	0.	0.571	0.	5.707

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	188	207	0	456	0	554
normalized size	1	1.	0.82	0.9	0.	1.99	0.	2.42
time (sec)	N/A	0.587	1.669	0.33	0.	0.544	0.	5.526

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	156	174	0	374	0	470
normalized size	1	1.	0.85	0.95	0.	2.03	0.	2.55
time (sec)	N/A	0.345	2.45	0.29	0.	0.534	0.	5.363

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	170	476	0	1111	0	0
normalized size	1	1.	0.93	2.62	0.	6.1	0.	0.
time (sec)	N/A	0.324	2.436	0.355	0.	0.595	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	209	604	3753	1108	0	771
normalized size	1	1.	1.14	3.28	20.4	6.02	0.	4.19
time (sec)	N/A	0.404	1.754	0.368	2.403	0.682	0.	6.865

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	152	583	0	1111	0	1095
normalized size	1	1.	0.77	2.96	0.	5.64	0.	5.56
time (sec)	N/A	0.631	1.191	0.388	0.	0.956	0.	7.082

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	140	846	0	1060	0	1712
normalized size	1	1.	0.68	4.09	0.	5.12	0.	8.27
time (sec)	N/A	0.658	1.704	0.323	0.	0.927	0.	7.799

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	156	1108	0	1187	0	2082
normalized size	1	1.	0.73	5.15	0.	5.52	0.	9.68
time (sec)	N/A	0.699	2.335	0.309	0.	1.295	0.	8.354

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	183	1381	0	1332	0	2535
normalized size	1	1.	0.7	5.29	0.	5.1	0.	9.71
time (sec)	N/A	0.81	2.391	0.368	0.	1.327	0.	8.927

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	217	1654	0	1507	0	2989
normalized size	1	1.	0.7	5.32	0.	4.85	0.	9.61
time (sec)	N/A	0.895	3.421	0.404	0.	1.336	0.	9.335

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	7186	1429	0	1289	0	689
normalized size	1	1.	28.29	5.63	0.	5.07	0.	2.71
time (sec)	N/A	0.857	29.625	0.45	0.	0.649	0.	9.53

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	7134	1144	0	1176	0	412
normalized size	1	1.	34.3	5.5	0.	5.65	0.	1.98
time (sec)	N/A	0.637	29.546	0.39	0.	0.634	0.	9.265

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	1666	859	0	1057	0	464
normalized size	1	1.	10.16	5.24	0.	6.45	0.	2.83
time (sec)	N/A	0.452	10.657	0.353	0.	0.619	0.	9.157

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	628	563	0	937	0	252
normalized size	1	1.	5.32	4.77	0.	7.94	0.	2.14
time (sec)	N/A	0.225	7.132	0.339	0.	0.616	0.	8.955

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	123	347	0	1172	0	396
normalized size	1	1.	1.04	2.94	0.	9.93	0.	3.36
time (sec)	N/A	0.172	1.184	0.299	0.	12.625	0.	10.992

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	430	0	1237	0	532
normalized size	1	1.	1.	3.58	0.	10.31	0.	4.43
time (sec)	N/A	0.239	1.147	0.352	0.	16.585	0.	11.119

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	16865	1025	0	1374	0	886
normalized size	1	1.	99.79	6.07	0.	8.13	0.	5.24
time (sec)	N/A	0.397	27.668	0.33	0.	43.098	0.	11.804

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	161	1561	0	1496	0	1490
normalized size	1	1.	0.76	7.33	0.	7.02	0.	7.
time (sec)	N/A	0.589	0.81	0.361	0.	42.795	0.	11.857

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	178	2086	0	1639	0	1887
normalized size	1	1.	0.69	8.05	0.	6.33	0.	7.29
time (sec)	N/A	0.777	0.969	0.413	0.	64.537	0.	12.302

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	2746	1437	0	1535	0	756
normalized size	1	1.	9.91	5.19	0.	5.54	0.	2.73
time (sec)	N/A	0.877	11.774	0.407	0.	0.695	0.	9.753

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	2025	1152	0	1399	0	456
normalized size	1	1.	8.84	5.03	0.	6.11	0.	1.99
time (sec)	N/A	0.683	8.847	0.376	0.	0.653	0.	9.618

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	7119	867	0	1257	0	491
normalized size	1	1.	39.33	4.79	0.	6.94	0.	2.71
time (sec)	N/A	0.476	25.268	0.321	0.	0.637	0.	9.286

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	135	748	583	0	1046	0	271
normalized size	1	1.12	6.23	4.86	0.	8.72	0.	2.26
time (sec)	N/A	0.246	6.481	0.306	0.	0.612	0.	8.983

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	16094	732	0	1625	0	448
normalized size	1	1.	122.85	5.59	0.	12.4	0.	3.42
time (sec)	N/A	0.195	28.184	0.249	0.	24.714	0.	10.926

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	179	889	0	1619	0	0
normalized size	1	1.	1.03	5.14	0.	9.36	0.	0.
time (sec)	N/A	0.417	2.374	0.352	0.	60.345	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	17669	1569	0	1789	0	0
normalized size	1	1.	76.16	6.76	0.	7.71	0.	0.
time (sec)	N/A	0.623	28.2	0.319	0.	118.304	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	221	2094	0	1935	0	0
normalized size	1	1.	0.78	7.37	0.	6.81	0.	0.
time (sec)	N/A	0.834	2.97	0.357	0.	118.417	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	7237	1439	0	1735	0	755
normalized size	1	1.	26.13	5.19	0.	6.26	0.	2.73
time (sec)	N/A	0.902	25.175	0.421	0.	0.698	0.	10.461

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	7197	1154	0	1590	0	455
normalized size	1	1.	31.7	5.08	0.	7.	0.	2.
time (sec)	N/A	0.695	25.693	0.356	0.	0.651	0.	10.032

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	7172	870	0	1365	0	487
normalized size	1	1.	40.07	4.86	0.	7.63	0.	2.72
time (sec)	N/A	0.491	25.03	0.319	0.	0.626	0.	9.764

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	7163	875	0	1330	0	277
normalized size	1	1.	52.28	6.39	0.	9.71	0.	2.02
time (sec)	N/A	0.273	24.931	0.292	0.	0.624	0.	9.414

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	16181	1097	0	1845	0	490
normalized size	1	1.	94.63	6.42	0.	10.79	0.	2.87
time (sec)	N/A	0.278	28.272	0.244	0.	96.835	0.	11.259

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	181	1338	0	2043	0	0
normalized size	1	1.	0.83	6.17	0.	9.41	0.	0.
time (sec)	N/A	0.61	5.283	0.39	0.	129.546	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	17747	2096	0	0	0	0
normalized size	1	1.	63.38	7.49	0.	0.	0.	0.
time (sec)	N/A	0.869	28.428	0.367	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	527	850	0	0	0	0
normalized size	1	1.	2.43	3.92	0.	0.	0.	0.
time (sec)	N/A	0.256	6.696	8.038	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	366	741	0	0	0	0
normalized size	1	1.	2.02	4.09	0.	0.	0.	0.
time (sec)	N/A	0.221	6.26	6.857	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	208	516	0	0	0	0
normalized size	1	1.	1.45	3.61	0.	0.	0.	0.
time (sec)	N/A	0.219	1.847	5.522	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	183	380	0	0	0	0
normalized size	1	1.	1.33	2.75	0.	0.	0.	0.
time (sec)	N/A	0.209	1.873	2.306	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	177	447	0	0	0	0
normalized size	1	1.	1.21	3.06	0.	0.	0.	0.
time (sec)	N/A	0.217	1.801	2.182	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	201	481	0	0	0	0
normalized size	1	1.	1.1	2.64	0.	0.	0.	0.
time (sec)	N/A	0.244	2.202	2.415	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	231	512	0	0	0	0
normalized size	1	1.	1.07	2.38	0.	0.	0.	0.
time (sec)	N/A	0.282	2.511	2.228	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	1270	1183	0	0	0	0
normalized size	1	1.	4.36	4.07	0.	0.	0.	0.
time (sec)	N/A	0.495	7.149	10.199	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	1216	934	0	0	0	0
normalized size	1	1.	4.77	3.66	0.	0.	0.	0.
time (sec)	N/A	0.506	7.008	9.	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	265	908	0	0	0	0
normalized size	1	1.	1.24	4.24	0.	0.	0.	0.
time (sec)	N/A	0.453	3.508	6.886	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	209	801	0	0	0	0
normalized size	1	1.	1.	3.85	0.	0.	0.	0.
time (sec)	N/A	0.446	2.451	6.223	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	187	595	0	0	0	0
normalized size	1	1.	0.87	2.78	0.	0.	0.	0.
time (sec)	N/A	0.493	2.063	2.448	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	189	483	0	0	0	0
normalized size	1	1.	0.86	2.21	0.	0.	0.	0.
time (sec)	N/A	0.497	2.255	2.378	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	234	514	0	0	0	0
normalized size	1	1.	0.92	2.02	0.	0.	0.	0.
time (sec)	N/A	0.502	3.564	2.476	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	270	545	0	0	0	0
normalized size	1	1.	0.93	1.87	0.	0.	0.	0.
time (sec)	N/A	0.524	4.546	2.298	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	1324	1427	0	0	0	0
normalized size	1	1.	3.86	4.16	0.	0.	0.	0.
time (sec)	N/A	0.699	7.354	12.427	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	1267	1265	0	0	0	0
normalized size	1	1.	4.13	4.12	0.	0.	0.	0.
time (sec)	N/A	0.646	7.195	11.043	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	359	1099	0	0	0	0
normalized size	1	1.	1.32	4.06	0.	0.	0.	0.
time (sec)	N/A	0.631	5.47	9.23	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	316	1328	0	0	0	0
normalized size	1	1.	1.17	4.9	0.	0.	0.	0.
time (sec)	N/A	0.631	4.721	8.219	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	275	950	0	0	0	0
normalized size	1	1.	1.02	3.52	0.	0.	0.	0.
time (sec)	N/A	0.658	3.068	7.704	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	266	727	0	0	0	0
normalized size	1	1.	0.98	2.68	0.	0.	0.	0.
time (sec)	N/A	0.633	3.089	2.711	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	214	514	0	0	0	0
normalized size	1	1.	0.79	1.9	0.	0.	0.	0.
time (sec)	N/A	0.647	3.048	2.337	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	246	545	0	0	0	0
normalized size	1	1.	0.8	1.78	0.	0.	0.	0.
time (sec)	N/A	0.675	5.181	2.188	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	300	576	0	0	0	0
normalized size	1	1.	0.87	1.68	0.	0.	0.	0.
time (sec)	N/A	0.718	6.501	2.474	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	1307	812	0	0	0	0
normalized size	1	1.	5.23	3.25	0.	0.	0.	0.
time (sec)	N/A	0.276	7.872	8.685	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	1261	494	0	0	0	0
normalized size	1	1.	6.15	2.41	0.	0.	0.	0.
time (sec)	N/A	0.248	7.435	6.934	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	1224	353	0	0	0	0
normalized size	1	1.	7.56	2.18	0.	0.	0.	0.
time (sec)	N/A	0.214	6.808	4.813	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	1243	281	0	0	0	0
normalized size	1	1.	9.35	2.11	0.	0.	0.	0.
time (sec)	N/A	0.209	6.543	1.951	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	1287	300	0	0	0	0
normalized size	1	1.	7.4	1.72	0.	0.	0.	0.
time (sec)	N/A	0.234	6.684	2.449	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	1350	320	0	0	0	0
normalized size	1	1.	6.31	1.5	0.	0.	0.	0.
time (sec)	N/A	0.254	6.772	2.247	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	1406	341	0	0	0	0
normalized size	1	1.	5.62	1.36	0.	0.	0.	0.
time (sec)	N/A	0.282	6.904	2.282	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1347	751	0	0	0	0
normalized size	1	1.	5.37	2.99	0.	0.	0.	0.
time (sec)	N/A	0.412	7.84	8.239	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	567	559	0	0	0	0
normalized size	1	1.	2.74	2.7	0.	0.	0.	0.
time (sec)	N/A	0.381	4.449	5.62	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	1097	509	0	0	0	0
normalized size	1	1.	6.34	2.94	0.	0.	0.	0.
time (sec)	N/A	0.363	6.822	2.318	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	1114	509	0	0	0	0
normalized size	1	1.	6.05	2.77	0.	0.	0.	0.
time (sec)	N/A	0.364	6.893	2.51	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	762	472	0	0	0	0
normalized size	1	1.	3.46	2.15	0.	0.	0.	0.
time (sec)	N/A	0.403	6.744	3.004	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	1442	491	0	0	0	0
normalized size	1	1.	5.68	1.93	0.	0.	0.	0.
time (sec)	N/A	0.425	7.229	2.749	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	1462	1040	0	0	0	0
normalized size	1	1.	4.75	3.38	0.	0.	0.	0.
time (sec)	N/A	0.621	8.608	10.019	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	1430	789	0	0	0	0
normalized size	1	1.	5.32	2.93	0.	0.	0.	0.
time (sec)	N/A	0.584	7.404	3.349	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1425	624	0	0	0	0
normalized size	1	1.	6.17	2.7	0.	0.	0.	0.
time (sec)	N/A	0.549	7.104	2.55	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1431	624	0	0	0	0
normalized size	1	1.	6.19	2.7	0.	0.	0.	0.
time (sec)	N/A	0.555	7.206	2.505	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	241	241	1449	624	0	0	0	0
normalized size	1	1.	6.01	2.59	0.	0.	0.	0.
time (sec)	N/A	0.562	7.533	2.616	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	1497	638	0	0	0	0
normalized size	1	1.	5.46	2.33	0.	0.	0.	0.
time (sec)	N/A	0.6	7.552	2.704	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	1555	666	0	0	0	0
normalized size	1	1.	4.97	2.13	0.	0.	0.	0.
time (sec)	N/A	0.639	7.743	2.47	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	179	638	8764	1338	0	0
normalized size	1	1.	0.79	2.81	38.61	5.89	0.	0.
time (sec)	N/A	0.518	2.105	0.443	4.196	2.4	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	141	543	5403	1200	0	0
normalized size	1	1.	0.79	3.03	30.18	6.7	0.	0.
time (sec)	N/A	0.423	1.19	0.433	3.026	1.493	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	452	2925	1077	0	0
normalized size	1	1.	0.83	3.45	22.33	8.22	0.	0.
time (sec)	N/A	0.338	0.839	0.47	2.605	1.486	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	94	344	1246	919	0	0
normalized size	1	1.	0.79	2.89	10.47	7.72	0.	0.
time (sec)	N/A	0.331	0.583	0.445	2.353	0.783	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	94	210	504	949	0	0
normalized size	1	1.	0.78	1.75	4.2	7.91	0.	0.
time (sec)	N/A	0.321	0.723	0.431	2.233	0.597	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	77	99	452	251	0	0
normalized size	1	1.	0.6	0.77	3.5	1.95	0.	0.
time (sec)	N/A	0.354	0.5	0.426	2.226	0.483	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	99	130	822	312	0	0
normalized size	1	1.	0.56	0.73	4.62	1.75	0.	0.
time (sec)	N/A	0.426	0.866	0.422	2.348	0.489	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	121	163	1185	369	0	0
normalized size	1	1.	0.54	0.72	5.24	1.63	0.	0.
time (sec)	N/A	0.51	1.352	0.441	2.37	0.495	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	211	732	0	1543	0	0
normalized size	1	1.	0.75	2.59	0.	5.45	0.	0.
time (sec)	N/A	0.748	3.899	0.417	0.	2.415	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	177	637	10963	1381	0	0
normalized size	1	1.	0.76	2.73	47.05	5.93	0.	0.
time (sec)	N/A	0.646	2.441	0.394	4.528	2.389	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	142	546	7760	1251	0	0
normalized size	1	1.	0.78	3.02	42.87	6.91	0.	0.
time (sec)	N/A	0.548	1.48	0.379	3.273	1.502	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	129	533	4942	1166	0	0
normalized size	1	1.	0.7	2.91	27.01	6.37	0.	0.
time (sec)	N/A	0.525	1.236	0.388	2.801	1.496	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	383	1964	1058	0	0
normalized size	1	1.	0.69	2.16	11.1	5.98	0.	0.
time (sec)	N/A	0.523	1.074	0.405	2.44	0.799	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	162	245	705	1089	0	0
normalized size	1	1.	0.94	1.42	4.1	6.33	0.	0.
time (sec)	N/A	0.512	1.849	0.411	2.319	0.627	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	100	131	743	325	0	0
normalized size	1	1.	0.55	0.72	4.1	1.8	0.	0.
time (sec)	N/A	0.466	0.965	0.361	2.316	0.489	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	123	164	1226	389	0	0
normalized size	1	1.	0.53	0.71	5.28	1.68	0.	0.
time (sec)	N/A	0.653	1.558	0.402	2.452	0.499	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	158	197	1604	460	0	0
normalized size	1	1.	0.56	0.69	5.65	1.62	0.	0.
time (sec)	N/A	0.739	2.334	0.403	2.514	0.507	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	245	827	0	1756	0	0
normalized size	1	1.	0.74	2.48	0.	5.27	0.	0.
time (sec)	N/A	0.949	4.687	0.451	0.	2.405	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	213	732	0	1581	0	0
normalized size	1	1.	0.76	2.6	0.	5.63	0.	0.
time (sec)	N/A	0.858	3.217	0.402	0.	2.435	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	179	641	0	1435	0	0
normalized size	1	1.	0.77	2.75	0.	6.16	0.	0.
time (sec)	N/A	0.752	2.159	0.375	0.	2.378	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	158	568	0	1349	0	0
normalized size	1	1.	0.68	2.44	0.	5.79	0.	0.
time (sec)	N/A	0.726	1.755	0.399	0.	1.539	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	155	549	0	1308	0	0
normalized size	1	1.	0.67	2.36	0.	5.61	0.	0.
time (sec)	N/A	0.741	1.273	0.351	0.	1.501	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	149	420	0	1214	0	0
normalized size	1	1.	0.67	1.88	0.	5.44	0.	0.
time (sec)	N/A	0.719	1.136	0.36	0.	0.819	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	194	280	1318	1257	0	0
normalized size	1	1.	0.87	1.26	5.94	5.66	0.	0.
time (sec)	N/A	0.697	6.353	0.436	2.522	0.643	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	124	166	1085	400	0	0
normalized size	1	1.	0.54	0.72	4.7	1.73	0.	0.
time (sec)	N/A	0.555	1.64	0.371	2.41	0.497	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	157	199	1709	482	0	0
normalized size	1	1.	0.55	0.7	6.02	1.7	0.	0.
time (sec)	N/A	0.863	1.528	0.387	2.545	0.509	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	190	232	2109	567	0	0
normalized size	1	1.	0.57	0.69	6.31	1.7	0.	0.
time (sec)	N/A	0.941	1.613	0.418	2.636	0.518	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	198	640	7028	1809	0	0
normalized size	1	1.	0.82	2.66	29.16	7.51	0.	0.
time (sec)	N/A	0.804	1.426	0.417	3.364	1.802	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	174	549	4047	1671	0	0
normalized size	1	1.	0.89	2.82	20.75	8.57	0.	0.
time (sec)	N/A	0.599	0.836	0.39	2.877	1.765	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	107	378	1947	1424	0	0
normalized size	1	1.	0.76	2.68	13.81	10.1	0.	0.
time (sec)	N/A	0.418	0.624	0.409	2.576	0.89	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	96	319	902	1373	0	0
normalized size	1	1.	0.7	2.31	6.54	9.95	0.	0.
time (sec)	N/A	0.388	0.502	0.351	2.408	0.678	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	88	229	641	960	0	0
normalized size	1	1.	0.62	1.6	4.48	6.71	0.	0.
time (sec)	N/A	0.361	0.586	0.401	2.311	0.541	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	155	263	1002	1069	0	0
normalized size	1	1.	0.81	1.38	5.25	5.6	0.	0.
time (sec)	N/A	0.571	0.885	0.427	2.457	0.545	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	175	296	1465	1187	0	0
normalized size	1	1.	0.74	1.25	6.18	5.01	0.	0.
time (sec)	N/A	0.758	1.563	0.377	2.585	0.566	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	118	515	2589	1563	0	0
normalized size	1	1.	0.78	3.39	17.03	10.28	0.	0.
time (sec)	N/A	0.472	0.515	0.419	2.994	1.834	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	239	741	0	2103	0	0
normalized size	1	1.	0.92	2.85	0.	8.09	0.	0.
time (sec)	N/A	0.906	2.275	0.363	0.	2.237	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	177	561	0	1793	0	0
normalized size	1	1.	0.88	2.78	0.	8.88	0.	0.
time (sec)	N/A	0.604	2.185	0.389	0.	1.038	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	175	384	0	1656	0	0
normalized size	1	1.	1.17	2.58	0.	11.11	0.	0.
time (sec)	N/A	0.406	1.304	0.371	0.	0.715	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	147	397	0	1157	0	0
normalized size	1	1.	0.91	2.47	0.	7.19	0.	0.
time (sec)	N/A	0.383	1.119	0.366	0.	0.546	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	126	427	0	1281	0	0
normalized size	1	1.	0.59	2.	0.	6.01	0.	0.
time (sec)	N/A	0.575	1.526	0.379	0.	0.558	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	148	460	0	1411	0	0
normalized size	1	1.	0.56	1.75	0.	5.37	0.	0.
time (sec)	N/A	0.753	2.08	0.402	0.	0.569	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	222	982	0	2217	0	0
normalized size	1	1.	0.87	3.87	0.	8.73	0.	0.
time (sec)	N/A	0.825	3.547	0.415	0.	1.184	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	684	0	2064	0	0
normalized size	1	1.	1.01	3.4	0.	10.27	0.	0.
time (sec)	N/A	0.596	1.609	0.381	0.	0.767	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	119	482	0	1400	0	0
normalized size	1	1.	0.73	2.96	0.	8.59	0.	0.
time (sec)	N/A	0.408	1.9	0.374	0.	0.544	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	128	594	0	1476	0	0
normalized size	1	1.	0.61	2.82	0.	7.	0.	0.
time (sec)	N/A	0.59	1.538	0.378	0.	0.557	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	146	624	0	1615	0	0
normalized size	1	1.	0.56	2.39	0.	6.19	0.	0.
time (sec)	N/A	0.802	2.088	0.391	0.	0.573	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	221	657	0	1747	0	0
normalized size	1	1.	0.71	2.1	0.	5.58	0.	0.
time (sec)	N/A	0.99	3.296	0.423	0.	0.586	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	446	446	5449	0	0	0	0	0
normalized size	1	1.	12.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.723	21.062	0.182	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	2931	0	0	0	0	0
normalized size	1	1.	7.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.452	19.595	0.178	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	3029	0	0	0	0	0
normalized size	1	1.	7.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.469	19.533	0.185	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	466	466	3111	0	0	0	0	0
normalized size	1	1.	6.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.518	19.677	0.187	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	839	839	4995	0	0	0	0	0
normalized size	1	1.	5.95	0.	0.	0.	0.	0.
time (sec)	N/A	1.068	20.309	0.19	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	786	786	4191	0	0	0	0	0
normalized size	1	1.	5.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.845	19.749	0.191	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	803	803	4253	0	0	0	0	0
normalized size	1	1.	5.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.888	19.681	0.185	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	856	856	4383	0	0	0	0	0
normalized size	1	1.	5.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.974	19.685	0.192	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.612	4.19	1.272	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	258	258	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.555	2.492	0.4	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	419	142	0	0
normalized size	1	1.	1.	0.	11.03	3.74	0.	0.
time (sec)	N/A	0.981	0.155	1.303	19.496	0.595	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	2582	0	0	0	0	0
normalized size	1	1.	15.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.264	16.977	0.487	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	96	192	236	389	0	451
normalized size	1	1.	0.69	1.37	1.69	2.78	0.	3.22
time (sec)	N/A	0.175	0.855	0.039	0.991	0.577	0.	1.235

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	80	149	205	335	0	410
normalized size	1	1.	0.68	1.27	1.75	2.86	0.	3.5
time (sec)	N/A	0.163	0.49	0.035	0.962	0.563	0.	1.223

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	59	108	135	285	0	248
normalized size	1	1.	0.69	1.26	1.57	3.31	0.	2.88
time (sec)	N/A	0.103	0.306	0.035	0.974	0.571	0.	1.236

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	85	119	267	0	181
normalized size	1	1.	1.16	1.47	2.05	4.6	0.	3.12
time (sec)	N/A	0.054	0.02	0.035	0.965	0.573	0.	1.154

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	57	80	232	0	161
normalized size	1	1.	1.29	1.36	1.9	5.52	0.	3.83
time (sec)	N/A	0.096	0.02	0.047	0.951	0.541	0.	1.147

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	77	95	167	0	171
normalized size	1	1.	1.26	1.33	1.64	2.88	0.	2.95
time (sec)	N/A	0.123	0.132	0.061	0.983	0.518	0.	1.242

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	64	68	90	140	0	207
normalized size	1	1.	0.83	0.88	1.17	1.82	0.	2.69
time (sec)	N/A	0.141	0.118	0.057	0.956	0.487	0.	1.144

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	84	96	122	189	0	367
normalized size	1	1.	0.88	1.01	1.28	1.99	0.	3.86
time (sec)	N/A	0.173	0.209	0.066	0.957	0.494	0.	1.203

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	89	117	153	242	0	408
normalized size	1	1.	0.68	0.89	1.17	1.85	0.	3.11
time (sec)	N/A	0.173	0.31	0.067	0.979	0.507	0.	1.165

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	275	257	292	455	0	718
normalized size	1	1.	1.22	1.14	1.29	2.01	0.	3.18
time (sec)	N/A	0.503	2.298	0.044	1.001	0.539	0.	1.21

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	1123	229	304	424	0	575
normalized size	1	1.	6.61	1.35	1.79	2.49	0.	3.38
time (sec)	N/A	0.308	6.319	0.044	0.989	0.537	0.	1.219

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	242	145	174	347	0	354
normalized size	1	1.	2.35	1.41	1.69	3.37	0.	3.44
time (sec)	N/A	0.139	1.241	0.045	0.989	0.532	0.	1.247

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	352	133	189	347	0	258
normalized size	1	1.	3.23	1.22	1.73	3.18	0.	2.37
time (sec)	N/A	0.165	0.878	0.063	0.99	0.537	0.	1.176

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	130	120	134	309	0	236
normalized size	1	1.	1.26	1.17	1.3	3.	0.	2.29
time (sec)	N/A	0.29	0.748	0.059	1.027	0.53	0.	1.194

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	144	137	151	250	0	346
normalized size	1	1.	1.29	1.22	1.35	2.23	0.	3.09
time (sec)	N/A	0.291	0.252	0.067	0.964	0.537	0.	1.217

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	104	140	176	248	0	510
normalized size	1	1.	0.72	0.97	1.21	1.71	0.	3.52
time (sec)	N/A	0.382	0.392	0.071	0.996	0.51	0.	1.167

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	126	158	208	300	0	672
normalized size	1	1.	0.78	0.98	1.29	1.86	0.	4.17
time (sec)	N/A	0.4	0.452	0.077	0.99	0.514	0.	1.194

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	407	430	521	636	0	1258
normalized size	1	1.	1.33	1.41	1.7	2.08	0.	4.11
time (sec)	N/A	0.719	3.617	0.051	1.021	0.576	0.	1.274

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	324	338	390	552	0	886
normalized size	1	1.	1.38	1.44	1.67	2.36	0.	3.79
time (sec)	N/A	0.488	1.881	0.056	1.004	0.554	0.	1.247

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	1241	267	343	485	0	710
normalized size	1	1.	7.43	1.6	2.05	2.9	0.	4.25
time (sec)	N/A	0.313	6.414	0.049	0.964	0.557	0.	1.25

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	325	195	244	440	0	435
normalized size	1	1.	1.95	1.17	1.46	2.63	0.	2.6
time (sec)	N/A	0.312	1.613	0.069	0.97	0.556	0.	1.248

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	287	196	242	419	0	522
normalized size	1	1.	1.71	1.17	1.44	2.49	0.	3.11
time (sec)	N/A	0.393	1.917	0.07	1.017	0.559	0.	1.245

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	184	183	190	394	0	413
normalized size	1	1.	1.13	1.12	1.17	2.42	0.	2.53
time (sec)	N/A	0.524	0.912	0.069	0.977	0.55	0.	1.237

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	177	252	235	354	0	679
normalized size	1	1.	0.97	1.38	1.29	1.95	0.	3.73
time (sec)	N/A	0.559	0.554	0.073	0.994	0.554	0.	1.269

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	155	201	262	367	0	818
normalized size	1	1.	0.71	0.92	1.2	1.68	0.	3.75
time (sec)	N/A	0.651	0.645	0.073	1.009	0.528	0.	1.233

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	253	249	328	450	0	1191
normalized size	1	1.	0.98	0.97	1.28	1.75	0.	4.63
time (sec)	N/A	0.752	1.127	0.084	1.022	0.553	0.	1.265

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	371	591	637	783	0	1728
normalized size	1	1.	0.97	1.55	1.67	2.06	0.	4.54
time (sec)	N/A	0.983	2.728	0.061	1.006	0.609	0.	1.281

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	460	511	620	716	0	1485
normalized size	1	1.	1.48	1.65	2.	2.31	0.	4.79
time (sec)	N/A	0.705	2.905	0.059	1.023	0.592	0.	1.236

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	503	377	429	610	0	1050
normalized size	1	1.	2.22	1.66	1.89	2.69	0.	4.63
time (sec)	N/A	0.478	2.573	0.056	0.979	0.582	0.	1.225

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	1357	316	413	575	0	797
normalized size	1	1.	5.93	1.38	1.8	2.51	0.	3.48
time (sec)	N/A	0.493	6.574	0.082	0.992	0.59	0.	1.264

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	416	258	298	509	0	536
normalized size	1	1.	1.9	1.18	1.36	2.32	0.	2.45
time (sec)	N/A	0.605	6.115	0.085	0.988	0.585	0.	1.216

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	324	259	298	516	0	537
normalized size	1	1.	1.29	1.03	1.19	2.06	0.	2.14
time (sec)	N/A	0.754	3.273	0.076	1.016	0.581	0.	1.247

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	270	296	275	504	0	753
normalized size	1	1.	1.1	1.2	1.12	2.05	0.	3.06
time (sec)	N/A	0.847	1.398	0.071	1.013	0.585	0.	1.288

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	223	364	323	473	0	1017
normalized size	1	1.	0.89	1.46	1.29	1.89	0.	4.07
time (sec)	N/A	0.884	0.833	0.085	0.988	0.58	0.	1.29

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	302	294	382	504	0	1396
normalized size	1	1.	1.01	0.99	1.28	1.69	0.	4.68
time (sec)	N/A	1.037	0.906	0.084	1.02	0.568	0.	1.277

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	351	332	444	595	0	1658
normalized size	1	1.	1.04	0.98	1.31	1.76	0.	4.89
time (sec)	N/A	1.151	0.859	0.1	0.992	0.582	0.	1.256

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	1299	205	259	428	0	512
normalized size	1	1.	8.22	1.3	1.64	2.71	0.	3.24
time (sec)	N/A	0.286	6.405	0.05	0.975	0.546	0.	1.268

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	86	118	142	323	0	227
normalized size	1	1.	0.81	1.11	1.34	3.05	0.	2.14
time (sec)	N/A	0.172	0.238	0.043	0.994	0.528	0.	1.179

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	94	126	281	0	0
normalized size	1	1.	1.	1.25	1.68	3.75	0.	0.
time (sec)	N/A	0.087	0.02	0.039	0.971	0.516	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	657	554	0	1486	0	502
normalized size	1	1.	3.53	2.98	0.	7.99	0.	2.7
time (sec)	N/A	0.645	3.71	0.087	0.	7.081	0.	1.46

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	428	362	0	1254	0	327
normalized size	1	1.	3.12	2.64	0.	9.15	0.	2.39
time (sec)	N/A	0.381	1.967	0.076	0.	7.02	0.	1.337

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	331	183	0	967	0	220
normalized size	1	1.	3.48	1.93	0.	10.18	0.	2.32
time (sec)	N/A	0.193	2.302	0.072	0.	1.989	0.	1.39

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	239	158	0	802	0	194
normalized size	1	1.	2.72	1.8	0.	9.11	0.	2.2
time (sec)	N/A	0.149	0.409	0.084	0.	1.996	0.	1.284

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	149	0	656	0	184
normalized size	1	1.	0.95	1.73	0.	7.63	0.	2.14
time (sec)	N/A	0.177	0.234	0.103	0.	0.55	0.	1.255

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	126	115	296	0	848	0	269
normalized size	1	0.98	0.9	2.31	0.	6.62	0.	2.1
time (sec)	N/A	0.387	0.358	0.114	0.	0.569	0.	1.25

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	149	551	0	1067	0	440
normalized size	1	0.99	0.85	3.15	0.	6.1	0.	2.51
time (sec)	N/A	0.605	0.5	0.118	0.	0.596	0.	1.214

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	191	1060	0	1328	0	775
normalized size	1	1.	0.82	4.57	0.	5.72	0.	3.34
time (sec)	N/A	0.927	0.658	0.125	0.	0.663	0.	1.39

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	461	646	0	2553	0	483
normalized size	1	1.	1.7	2.38	0.	9.42	0.	1.78
time (sec)	N/A	0.861	3.797	0.104	0.	29.97	0.	1.4

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	336	402	0	1904	0	516
normalized size	1	1.	2.2	2.63	0.	12.44	0.	3.37
time (sec)	N/A	0.485	2.678	0.084	0.	7.496	0.	1.288

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	331	350	0	1513	0	312
normalized size	1	1.	2.45	2.59	0.	11.21	0.	2.31
time (sec)	N/A	0.272	2.342	0.089	0.	5.483	0.	1.272

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	270	328	0	1188	0	277
normalized size	1	1.	2.16	2.62	0.	9.5	0.	2.22
time (sec)	N/A	0.227	1.978	0.094	0.	0.6	0.	1.196

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	137	367	0	1404	0	479
normalized size	1	1.	0.8	2.15	0.	8.21	0.	2.8
time (sec)	N/A	0.434	0.953	0.123	0.	0.63	0.	1.185

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	176	577	0	1820	0	425
normalized size	1	1.	0.69	2.25	0.	7.11	0.	1.66
time (sec)	N/A	0.86	0.947	0.128	0.	0.715	0.	1.259

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	212	836	0	2187	0	595
normalized size	1	1.	0.65	2.56	0.	6.71	0.	1.83
time (sec)	N/A	1.255	1.163	0.128	0.	0.785	0.	1.254

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	559	1547	0	4635	0	1602
normalized size	1	1.	1.47	4.06	0.	12.17	0.	4.2
time (sec)	N/A	1.62	4.15	0.109	0.	78.179	0.	1.349

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	421	1167	0	3416	0	703
normalized size	1	1.	1.55	4.31	0.	12.61	0.	2.59
time (sec)	N/A	1.022	2.263	0.098	0.	28.108	0.	1.318

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	445	1165	0	2866	0	687
normalized size	1	1.	2.1	5.5	0.	13.52	0.	3.24
time (sec)	N/A	0.594	5.177	0.097	0.	17.103	0.	1.299

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	342	230	0	1577	0	501
normalized size	1	1.	1.93	1.3	0.	8.91	0.	2.83
time (sec)	N/A	0.321	3.395	0.093	0.	0.648	0.	1.287

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	642	1143	0	2295	0	653
normalized size	1	1.	3.18	5.66	0.	11.36	0.	3.23
time (sec)	N/A	0.445	4.729	0.101	0.	0.712	0.	1.245

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	902	1132	0	2654	0	663
normalized size	1	1.	3.39	4.26	0.	9.98	0.	2.49
time (sec)	N/A	0.963	6.581	0.138	0.	0.787	0.	1.294

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	256	1478	0	3407	0	1551
normalized size	1	1.	0.69	4.01	0.	9.23	0.	4.2
time (sec)	N/A	1.615	2.439	0.149	0.	0.944	0.	1.308

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	564	2318	0	5434	0	1185
normalized size	1	1.	1.49	6.13	0.	14.38	0.	3.13
time (sec)	N/A	1.811	4.407	0.11	0.	65.167	0.	1.411

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	1092	2428	0	4779	0	1183
normalized size	1	1.	3.49	7.76	0.	15.27	0.	3.78
time (sec)	N/A	1.251	7.17	0.113	0.	51.219	0.	1.384

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	221	374	0	2475	0	936
normalized size	1	1.	0.85	1.43	0.	9.48	0.	3.59
time (sec)	N/A	0.673	1.273	0.095	0.	0.755	0.	1.295

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	438	373	0	2473	0	936
normalized size	1	1.	1.74	1.48	0.	9.81	0.	3.71
time (sec)	N/A	0.553	6.164	0.102	0.	0.751	0.	1.312

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	995	2407	0	3868	0	1141
normalized size	1	1.	3.41	8.24	0.	13.25	0.	3.91
time (sec)	N/A	0.937	7.212	0.113	0.	0.922	0.	1.317

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	367	367	1089	2283	0	4319	0	1143
normalized size	1	1.	2.97	6.22	0.	11.77	0.	3.11
time (sec)	N/A	1.824	7.422	0.152	0.	1.063	0.	1.364

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	1314	3023	0	5536	0	1392
normalized size	1	1.	2.56	5.89	0.	10.79	0.	2.71
time (sec)	N/A	2.345	5.524	0.161	0.	1.444	0.	1.359

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	31	0	95	41	58
normalized size	1	1.	1.	1.82	0.	5.59	2.41	3.41
time (sec)	N/A	0.044	0.007	0.043	0.	0.498	3.25	1.193

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	56	61	0	491	0	113
normalized size	1	1.	1.08	1.17	0.	9.44	0.	2.17
time (sec)	N/A	0.131	0.099	0.079	0.	0.528	0.	1.181

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	139	202	0	1050	0	236
normalized size	1	1.	1.3	1.89	0.	9.81	0.	2.21
time (sec)	N/A	0.206	0.46	0.098	0.	0.616	0.	1.262

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	223	659	0	1925	0	428
normalized size	1	1.	1.38	4.07	0.	11.88	0.	2.64
time (sec)	N/A	0.34	0.776	0.102	0.	0.684	0.	1.306

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	467	467	3518	4131	0	0	0	0
normalized size	1	1.	7.53	8.85	0.	0.	0.	0.
time (sec)	N/A	1.305	23.727	1.671	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	375	375	560	2784	0	0	0	0
normalized size	1	1.	1.49	7.42	0.	0.	0.	0.
time (sec)	N/A	0.772	19.958	1.	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	507	2453	0	0	0	0
normalized size	1	1.	1.65	7.96	0.	0.	0.	0.
time (sec)	N/A	0.505	18.011	0.754	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	570	1510	0	0	0	0
normalized size	1	1.	1.61	4.25	0.	0.	0.	0.
time (sec)	N/A	0.367	11.181	0.499	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	727	1602	0	0	0	0
normalized size	1	1.	2.07	4.55	0.	0.	0.	0.
time (sec)	N/A	0.379	18.251	0.508	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	1417	1834	0	0	0	0
normalized size	1	1.	3.45	4.46	0.	0.	0.	0.
time (sec)	N/A	0.681	19.17	0.427	0.	0.	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	1347	2535	0	0	0	0
normalized size	1	1.	2.68	5.05	0.	0.	0.	0.
time (sec)	N/A	1.061	19.33	0.485	0.	0.	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	587	587	1845	3606	0	0	0	0
normalized size	1	1.	3.14	6.14	0.	0.	0.	0.
time (sec)	N/A	1.575	15.378	0.619	0.	0.	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	550	550	3988	4696	0	0	0	0
normalized size	1	1.	7.25	8.54	0.	0.	0.	0.
time (sec)	N/A	1.919	26.097	2.226	0.	0.	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	454	454	3537	4115	0	0	0	0
normalized size	1	1.	7.79	9.06	0.	0.	0.	0.
time (sec)	N/A	1.05	24.128	1.669	0.	0.	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	3214	2986	0	0	0	0
normalized size	1	1.	8.59	7.98	0.	0.	0.	0.
time (sec)	N/A	0.761	23.782	0.986	0.	0.	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	415	415	6143	2834	0	0	0	0
normalized size	1	1.	14.8	6.83	0.	0.	0.	0.
time (sec)	N/A	0.576	25.114	0.773	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	4024	2147	0	0	0	0
normalized size	1	1.	9.86	5.26	0.	0.	0.	0.
time (sec)	N/A	0.575	24.43	0.638	0.	0.	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	414	414	1618	2617	0	0	0	0
normalized size	1	1.	3.91	6.32	0.	0.	0.	0.
time (sec)	N/A	0.675	19.701	0.648	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	504	504	1393	2723	0	0	0	0
normalized size	1	1.	2.76	5.4	0.	0.	0.	0.
time (sec)	N/A	1.178	18.711	0.477	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	583	583	651	3798	0	0	0	0
normalized size	1	1.	1.12	6.51	0.	0.	0.	0.
time (sec)	N/A	1.54	14.871	0.615	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	650	650	4418	6077	0	0	0	0
normalized size	1	1.	6.8	9.35	0.	0.	0.	0.
time (sec)	N/A	2.77	26.41	3.911	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	3989	4695	0	0	0	0
normalized size	1	1.	7.47	8.79	0.	0.	0.	0.
time (sec)	N/A	1.49	26.682	2.224	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	454	454	710	4333	0	0	0	0
normalized size	1	1.	1.56	9.54	0.	0.	0.	0.
time (sec)	N/A	1.032	22.241	1.669	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	481	481	4087	3384	0	0	0	0
normalized size	1	1.	8.5	7.04	0.	0.	0.	0.
time (sec)	N/A	0.831	25.511	1.008	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	478	478	6811	3498	0	0	0	0
normalized size	1	1.	14.25	7.32	0.	0.	0.	0.
time (sec)	N/A	0.81	25.742	0.977	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	463	463	4903	3206	0	0	0	0
normalized size	1	1.	10.59	6.92	0.	0.	0.	0.
time (sec)	N/A	0.913	25.989	0.772	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	507	507	1513	3512	0	0	0	0
normalized size	1	1.	2.98	6.93	0.	0.	0.	0.
time (sec)	N/A	1.202	19.731	0.886	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	587	587	5006	3986	0	0	0	0
normalized size	1	1.	8.53	6.79	0.	0.	0.	0.
time (sec)	N/A	1.633	24.139	0.644	0.	0.	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	960	2169	0	0	0	0
normalized size	1	1.	2.38	5.38	0.	0.	0.	0.
time (sec)	N/A	0.57	15.3	0.694	0.	0.	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	598	1508	0	0	0	0
normalized size	1	1.	1.69	4.27	0.	0.	0.	0.
time (sec)	N/A	0.403	12.708	0.505	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	3255	2784	0	0	0	0
normalized size	1	1.	8.28	7.08	0.	0.	0.	0.
time (sec)	N/A	0.908	23.64	1.015	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	2993	2256	0	0	0	0
normalized size	1	1.	9.35	7.05	0.	0.	0.	0.
time (sec)	N/A	0.572	22.828	0.78	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	409	1125	0	0	0	0
normalized size	1	1.	1.62	4.45	0.	0.	0.	0.
time (sec)	N/A	0.322	14.799	0.5	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	914	1011	0	0	0	0
normalized size	1	1.	2.92	3.23	0.	0.	0.	0.
time (sec)	N/A	0.234	16.414	0.467	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	386	841	0	0	0	0
normalized size	1	1.	1.1	2.39	0.	0.	0.	0.
time (sec)	N/A	0.395	15.852	0.462	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	1475	1652	0	0	0	0
normalized size	1	1.	3.59	4.02	0.	0.	0.	0.
time (sec)	N/A	0.64	15.051	0.414	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	506	506	1363	2347	0	0	0	0
normalized size	1	1.	2.69	4.64	0.	0.	0.	0.
time (sec)	N/A	1.028	19.044	0.495	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	460	460	3853	4055	0	0	0	0
normalized size	1	1.	8.38	8.82	0.	0.	0.	0.
time (sec)	N/A	1.081	25.859	1.272	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	3312	2672	0	0	0	0
normalized size	1	1.	10.13	8.17	0.	0.	0.	0.
time (sec)	N/A	0.675	23.576	0.629	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	541	2272	0	0	0	0
normalized size	1	1.	1.94	8.14	0.	0.	0.	0.
time (sec)	N/A	0.391	18.757	0.504	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	1127	2043	0	0	0	0
normalized size	1	1.	2.96	5.36	0.	0.	0.	0.
time (sec)	N/A	0.421	18.052	0.442	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	1259	2489	0	0	0	0
normalized size	1	1.	2.92	5.77	0.	0.	0.	0.
time (sec)	N/A	0.659	18.946	0.405	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	501	501	2500	3529	0	0	0	0
normalized size	1	1.	4.99	7.04	0.	0.	0.	0.
time (sec)	N/A	1.012	17.46	0.51	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	488	488	4050	7051	0	0	0	0
normalized size	1	1.	8.3	14.45	0.	0.	0.	0.
time (sec)	N/A	1.301	26.402	1.344	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	702	6135	0	0	0	0
normalized size	1	1.	1.72	15.04	0.	0.	0.	0.
time (sec)	N/A	0.821	22.567	0.805	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	3369	4550	0	0	0	0
normalized size	1	1.	8.91	12.04	0.	0.	0.	0.
time (sec)	N/A	0.694	23.804	0.44	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	517	517	1727	6380	0	0	0	0
normalized size	1	1.	3.34	12.34	0.	0.	0.	0.
time (sec)	N/A	0.773	20.758	0.477	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	559	559	1714	6418	0	0	0	0
normalized size	1	1.	3.07	11.48	0.	0.	0.	0.
time (sec)	N/A	1.19	20.494	0.683	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	645	645	801	9631	0	0	0	0
normalized size	1	1.	1.24	14.93	0.	0.	0.	0.
time (sec)	N/A	1.559	14.478	0.955	0.	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	626	626	2204	11805	0	0	0	0
normalized size	1	1.	3.52	18.86	0.	0.	0.	0.
time (sec)	N/A	1.276	22.085	0.721	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	939	1020	0	0	0	0
normalized size	1	1.	3.1	3.37	0.	0.	0.	0.
time (sec)	N/A	0.289	15.64	0.463	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	145	214	0	0	0	0
normalized size	1	1.	0.72	1.07	0.	0.	0.	0.
time (sec)	N/A	0.164	2.082	0.379	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	616	1392	0	0	0	0
normalized size	1	1.	1.82	4.12	0.	0.	0.	0.
time (sec)	N/A	0.404	14.009	0.407	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	445	445	1849	3887	0	0	0	0
normalized size	1	1.	4.16	8.73	0.	0.	0.	0.
time (sec)	N/A	0.626	14.544	0.433	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	0	259	0	0	0	0
normalized size	1	1.	0.	1.79	0.	0.	0.	0.
time (sec)	N/A	0.251	31.299	2.295	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	0	1160	0	0	0	0
normalized size	1	1.	0.	5.45	0.	0.	0.	0.
time (sec)	N/A	0.566	9.074	0.483	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	242	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	56.904	0.172	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	242	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.292	46.17	0.179	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	239	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.295	76.261	0.163	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	239	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.295	58.79	0.173	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	106	213	270	397	0	446
normalized size	1	1.	0.73	1.47	1.86	2.74	0.	3.08
time (sec)	N/A	0.203	0.88	0.036	0.969	0.959	0.	1.275

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	85	171	220	352	0	410
normalized size	1	1.	0.75	1.5	1.93	3.09	0.	3.6
time (sec)	N/A	0.185	0.627	0.036	0.976	1.012	0.	1.216

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	67	128	171	298	0	284
normalized size	1	1.	0.72	1.38	1.84	3.2	0.	3.05
time (sec)	N/A	0.153	0.29	0.034	0.987	0.683	0.	1.188

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	119	247	0	207
normalized size	1	1.	1.23	1.41	1.95	4.05	0.	3.39
time (sec)	N/A	0.066	0.027	0.03	0.962	0.522	0.	1.186

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	65	99	225	0	113
normalized size	1	1.	1.23	1.86	2.83	6.43	0.	3.23
time (sec)	N/A	0.074	0.014	0.046	0.972	0.511	0.	1.159

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	56	78	142	0	107
normalized size	1	1.	1.31	1.6	2.23	4.06	0.	3.06
time (sec)	N/A	0.102	0.027	0.055	0.97	0.51	0.	1.211

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	74	104	0	163
normalized size	1	1.	0.98	1.1	1.42	2.	0.	3.13
time (sec)	N/A	0.142	0.087	0.053	0.955	0.483	0.	1.179

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	107	149	0	243
normalized size	1	1.	0.89	1.01	1.27	1.77	0.	2.89
time (sec)	N/A	0.166	0.161	0.069	0.957	0.489	0.	1.175

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	136	205	0	367
normalized size	1	1.	0.87	1.02	1.3	1.95	0.	3.5
time (sec)	N/A	0.178	0.275	0.069	0.983	0.495	0.	1.185

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	88	128	167	248	0	405
normalized size	1	1.	0.65	0.94	1.23	1.82	0.	2.98
time (sec)	N/A	0.2	0.24	0.067	0.971	0.508	0.	1.18

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	150	312	373	521	0	713
normalized size	1	1.	0.76	1.58	1.88	2.63	0.	3.6
time (sec)	N/A	0.352	1.515	0.046	1.011	0.544	0.	1.54

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	120	241	308	443	0	645
normalized size	1	1.	0.67	1.35	1.72	2.47	0.	3.6
time (sec)	N/A	0.348	0.761	0.037	0.977	0.535	0.	1.428

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	92	174	223	371	0	397
normalized size	1	1.	0.79	1.5	1.92	3.2	0.	3.42
time (sec)	N/A	0.146	0.465	0.036	0.96	0.518	0.	1.318

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	133	189	335	0	259
normalized size	1	1.	0.78	1.55	2.2	3.9	0.	3.01
time (sec)	N/A	0.142	0.269	0.056	0.963	0.525	0.	1.239

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	109	104	139	294	0	208
normalized size	1	1.	1.82	1.73	2.32	4.9	0.	3.47
time (sec)	N/A	0.176	0.493	0.054	0.972	0.522	0.	1.226

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	120	120	134	213	0	240
normalized size	1	1.	1.5	1.5	1.68	2.66	0.	3.
time (sec)	N/A	0.252	0.221	0.058	0.96	0.522	0.	1.21

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	146	201	0	343
normalized size	1	1.	0.84	1.07	1.36	1.88	0.	3.21
time (sec)	N/A	0.289	0.244	0.063	0.963	0.494	0.	1.186

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	118	152	192	274	0	590
normalized size	1	1.	0.87	1.12	1.41	2.01	0.	4.34
time (sec)	N/A	0.321	0.513	0.068	0.965	0.507	0.	1.224

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	146	184	238	350	0	657
normalized size	1	1.	0.81	1.02	1.32	1.94	0.	3.65
time (sec)	N/A	0.338	0.473	0.073	0.968	0.529	0.	1.228

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	214	478	552	706	0	1258
normalized size	1	1.	0.77	1.72	1.99	2.54	0.	4.53
time (sec)	N/A	0.609	2.613	0.052	0.991	0.58	0.	1.296

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	181	382	460	612	0	975
normalized size	1	1.	0.72	1.52	1.83	2.43	0.	3.87
time (sec)	N/A	0.499	3.291	0.043	0.985	0.561	0.	1.289

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	140	290	359	510	0	791
normalized size	1	1.	0.78	1.61	1.99	2.83	0.	4.39
time (sec)	N/A	0.262	0.828	0.043	0.987	0.543	0.	1.26

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	108	223	292	458	0	454
normalized size	1	1.	0.79	1.63	2.13	3.34	0.	3.31
time (sec)	N/A	0.24	0.586	0.066	0.984	0.548	0.	1.238

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	277	172	228	401	0	325
normalized size	1	1.	2.11	1.31	1.74	3.06	0.	2.48
time (sec)	N/A	0.291	2.15	0.065	0.995	0.545	0.	1.242

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	217	168	194	369	0	316
normalized size	1	1.	1.75	1.35	1.56	2.98	0.	2.55
time (sec)	N/A	0.404	0.672	0.059	0.997	0.55	0.	1.22

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	159	207	205	317	0	424
normalized size	1	1.	1.1	1.43	1.41	2.19	0.	2.92
time (sec)	N/A	0.415	0.369	0.064	0.973	0.544	0.	1.27

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	140	180	231	321	0	724
normalized size	1	1.	0.78	1.01	1.29	1.79	0.	4.04
time (sec)	N/A	0.491	0.415	0.067	0.988	0.521	0.	1.241

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	176	227	293	423	0	907
normalized size	1	1.	0.8	1.03	1.33	1.91	0.	4.1
time (sec)	N/A	0.548	0.668	0.08	0.974	0.537	0.	1.271

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	422	688	0	1650	0	556
normalized size	1	1.	2.26	3.68	0.	8.82	0.	2.97
time (sec)	N/A	0.717	2.348	0.095	0.	2.29	0.	1.23

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	300	410	0	1353	0	363
normalized size	1	1.	2.1	2.87	0.	9.46	0.	2.54
time (sec)	N/A	0.45	1.736	0.08	0.	11.399	0.	1.217

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	130	228	0	1065	0	236
normalized size	1	1.	1.33	2.33	0.	10.87	0.	2.41
time (sec)	N/A	0.266	0.579	0.064	0.	0.889	0.	1.237

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	707	0	173
normalized size	1	1.	1.47	1.78	0.	9.3	0.	2.28
time (sec)	N/A	0.12	0.168	0.069	0.	1.965	0.	1.224

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	540	0	136
normalized size	1	1.	1.01	1.69	0.	8.06	0.	2.03
time (sec)	N/A	0.164	0.116	0.093	0.	0.536	0.	1.271

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	172	0	702	0	190
normalized size	1	1.	0.94	1.91	0.	7.8	0.	2.11
time (sec)	N/A	0.226	0.203	0.114	0.	0.556	0.	1.199

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	367	0	934	0	306
normalized size	1	1.	0.9	2.74	0.	6.97	0.	2.28
time (sec)	N/A	0.479	0.323	0.116	0.	0.574	0.	1.225

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	152	641	0	1177	0	486
normalized size	1	1.	0.85	3.6	0.	6.61	0.	2.73
time (sec)	N/A	0.704	0.491	0.12	0.	0.637	0.	1.224

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	438	698	0	2969	0	518
normalized size	1	1.	1.61	2.57	0.	10.92	0.	1.9
time (sec)	N/A	0.884	6.273	0.11	0.	50.219	0.	1.257

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	240	510	0	2433	0	545
normalized size	1	1.	1.46	3.11	0.	14.84	0.	3.32
time (sec)	N/A	0.623	2.05	0.095	0.	31.158	0.	1.239

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	191	350	0	1551	0	309
normalized size	1	1.	1.46	2.67	0.	11.84	0.	2.36
time (sec)	N/A	0.319	0.661	0.087	0.	9.642	0.	1.233

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	861	0	235
normalized size	1	1.	0.97	1.32	0.	8.61	0.	2.35
time (sec)	N/A	0.126	0.339	0.085	0.	0.543	0.	1.179

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	119	328	0	1226	0	271
normalized size	1	1.	0.96	2.65	0.	9.89	0.	2.19
time (sec)	N/A	0.274	0.55	0.111	0.	0.601	0.	1.212

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	147	453	0	1715	0	505
normalized size	1	1.	0.82	2.52	0.	9.53	0.	2.81
time (sec)	N/A	0.633	0.779	0.121	0.	0.681	0.	1.225

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	184	651	0	2136	0	459
normalized size	1	1.	0.7	2.49	0.	8.18	0.	1.76
time (sec)	N/A	0.931	1.019	0.121	0.	0.756	0.	1.196

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	418	1406	0	4591	0	784
normalized size	1	1.	1.45	4.87	0.	15.89	0.	2.71
time (sec)	N/A	1.425	6.45	0.101	0.	114.551	0.	1.371

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	270	1085	0	3051	0	656
normalized size	1	1.	1.23	4.93	0.	13.87	0.	2.98
time (sec)	N/A	0.751	1.881	0.098	0.	43.407	0.	1.462

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	157	238	0	1631	0	540
normalized size	1	1.	0.87	1.32	0.	9.06	0.	3.
time (sec)	N/A	0.371	0.665	0.087	0.	0.649	0.	1.41

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	172	236	0	1631	0	539
normalized size	1	1.	1.05	1.44	0.	9.95	0.	3.29
time (sec)	N/A	0.265	0.84	0.091	0.	0.651	0.	1.422

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	203	1063	0	2479	0	617
normalized size	1	1.	0.99	5.19	0.	12.09	0.	3.01
time (sec)	N/A	0.581	1.382	0.125	0.	0.752	0.	1.615

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	232	1349	0	3394	0	737
normalized size	1	1.	0.8	4.65	0.	11.7	0.	2.54
time (sec)	N/A	1.533	2.043	0.138	0.	0.897	0.	1.401

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	3734	4395	0	0	0	0
normalized size	1	1.	7.7	9.06	0.	0.	0.	0.
time (sec)	N/A	1.454	25.551	1.737	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	397	3330	3439	0	0	0	0
normalized size	1	1.	8.39	8.66	0.	0.	0.	0.
time (sec)	N/A	1.003	24.462	1.141	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	434	2498	0	0	0	0
normalized size	1	1.	1.38	7.96	0.	0.	0.	0.
time (sec)	N/A	0.602	18.46	0.781	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	358	1752	0	0	0	0
normalized size	1	1.	1.4	6.84	0.	0.	0.	0.
time (sec)	N/A	0.289	14.925	0.524	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	863	1372	0	0	0	0
normalized size	1	1.	2.7	4.29	0.	0.	0.	0.
time (sec)	N/A	0.361	17.532	0.45	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	1107	1386	0	0	0	0
normalized size	1	1.	3.22	4.03	0.	0.	0.	0.
time (sec)	N/A	0.464	18.014	0.423	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	1161	2065	0	0	0	0
normalized size	1	1.	2.71	4.81	0.	0.	0.	0.
time (sec)	N/A	0.805	18.65	0.403	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	573	573	4220	5368	0	0	0	0
normalized size	1	1.	7.36	9.37	0.	0.	0.	0.
time (sec)	N/A	1.981	26.506	2.543	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	475	475	3766	4395	0	0	0	0
normalized size	1	1.	7.93	9.25	0.	0.	0.	0.
time (sec)	N/A	1.224	25.914	1.678	0.	0.	0.	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	3342	3424	0	0	0	0
normalized size	1	1.	8.64	8.85	0.	0.	0.	0.
time (sec)	N/A	0.853	24.34	1.112	0.	0.	0.	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	456	2683	0	0	0	0
normalized size	1	1.	1.46	8.6	0.	0.	0.	0.
time (sec)	N/A	0.468	18.625	0.732	0.	0.	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	6047	2340	0	0	0	0
normalized size	1	1.	15.91	6.16	0.	0.	0.	0.
time (sec)	N/A	0.529	24.093	0.509	0.	0.	0.	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	979	2199	0	0	0	0
normalized size	1	1.	2.71	6.09	0.	0.	0.	0.
time (sec)	N/A	0.548	18.326	0.492	0.	0.	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	428	428	1580	2439	0	0	0	0
normalized size	1	1.	3.69	5.7	0.	0.	0.	0.
time (sec)	N/A	0.874	19.285	0.416	0.	0.	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	520	520	1532	3142	0	0	0	0
normalized size	1	1.	2.95	6.04	0.	0.	0.	0.
time (sec)	N/A	1.325	18.963	0.459	0.	0.	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	565	565	4227	5368	0	0	0	0
normalized size	1	1.	7.48	9.5	0.	0.	0.	0.
time (sec)	N/A	1.811	26.529	2.526	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	3781	4395	0	0	0	0
normalized size	1	1.	8.06	9.37	0.	0.	0.	0.
time (sec)	N/A	1.196	26.013	1.703	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	2913	3637	0	0	0	0
normalized size	1	1.	7.59	9.47	0.	0.	0.	0.
time (sec)	N/A	0.678	22.937	1.117	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	442	442	7124	3285	0	0	0	0
normalized size	1	1.	16.12	7.43	0.	0.	0.	0.
time (sec)	N/A	0.715	25.039	0.781	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	1146	3215	0	0	0	0
normalized size	1	1.	2.65	7.42	0.	0.	0.	0.
time (sec)	N/A	0.777	19.203	0.658	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	1338	3271	0	0	0	0
normalized size	1	1.	2.97	7.27	0.	0.	0.	0.
time (sec)	N/A	0.9	19.218	0.652	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	1546	3511	0	0	0	0
normalized size	1	1.	2.98	6.78	0.	0.	0.	0.
time (sec)	N/A	1.274	19.34	0.496	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	617	617	5186	4231	0	0	0	0
normalized size	1	1.	8.41	6.86	0.	0.	0.	0.
time (sec)	N/A	1.828	24.052	0.582	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	3426	3439	0	0	0	0
normalized size	1	1.	8.34	8.37	0.	0.	0.	0.
time (sec)	N/A	1.045	24.631	1.115	0.	0.	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	3000	2499	0	0	0	0
normalized size	1	1.	9.12	7.6	0.	0.	0.	0.
time (sec)	N/A	0.685	22.667	0.761	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	372	1563	0	0	0	0
normalized size	1	1.	1.43	5.99	0.	0.	0.	0.
time (sec)	N/A	0.437	16.03	0.509	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	312	829	0	0	0	0
normalized size	1	1.	1.49	3.95	0.	0.	0.	0.
time (sec)	N/A	0.174	14.285	0.418	0.	0.	0.	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	147	215	0	0	0	0
normalized size	1	1.	0.71	1.03	0.	0.	0.	0.
time (sec)	N/A	0.209	2.191	0.365	0.	0.	0.	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	348	348	1027	1025	0	0	0	0
normalized size	1	1.	2.95	2.95	0.	0.	0.	0.
time (sec)	N/A	0.499	16.745	0.385	0.	0.	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	471	471	3953	4320	0	0	0	0
normalized size	1	1.	8.39	9.17	0.	0.	0.	0.
time (sec)	N/A	1.273	25.818	1.248	0.	0.	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	3460	3333	0	0	0	0
normalized size	1	1.	10.52	10.13	0.	0.	0.	0.
time (sec)	N/A	0.81	24.569	0.732	0.	0.	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	466	2275	0	0	0	0
normalized size	1	1.	1.69	8.27	0.	0.	0.	0.
time (sec)	N/A	0.505	18.494	0.48	0.	0.	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	426	1633	0	0	0	0
normalized size	1	1.	1.68	6.43	0.	0.	0.	0.
time (sec)	N/A	0.304	15.345	0.392	0.	0.	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	1445	2009	0	0	0	0
normalized size	1	1.	3.84	5.34	0.	0.	0.	0.
time (sec)	N/A	0.519	14.494	0.392	0.	0.	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	1613	2871	0	0	0	0
normalized size	1	1.	3.78	6.72	0.	0.	0.	0.
time (sec)	N/A	0.793	19.411	0.388	0.	0.	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	509	509	4342	8046	0	0	0	0
normalized size	1	1.	8.53	15.81	0.	0.	0.	0.
time (sec)	N/A	1.575	26.461	1.589	0.	0.	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	3920	6455	0	0	0	0
normalized size	1	1.	9.4	15.48	0.	0.	0.	0.
time (sec)	N/A	1.005	26.099	0.802	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	3514	5170	0	0	0	0
normalized size	1	1.	9.08	13.36	0.	0.	0.	0.
time (sec)	N/A	0.735	24.456	0.444	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	559	4213	0	0	0	0
normalized size	1	1.	1.58	11.93	0.	0.	0.	0.
time (sec)	N/A	0.521	19.088	0.383	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	495	495	2039	5710	0	0	0	0
normalized size	1	1.	4.12	11.54	0.	0.	0.	0.
time (sec)	N/A	0.857	16.213	0.426	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	446	446	3729	7695	0	0	0	0
normalized size	1	1.	8.36	17.25	0.	0.	0.	0.
time (sec)	N/A	0.828	24.731	0.577	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	217	0	0	0	0
normalized size	1	1.	0.77	2.15	0.	0.	0.	0.
time (sec)	N/A	0.284	0.669	2.124	0.	0.	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	91	277	0	0	0	0
normalized size	1	1.	0.66	2.01	0.	0.	0.	0.
time (sec)	N/A	0.514	0.267	0.394	0.	0.	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	21744	0	0	0	0	0
normalized size	1	1.	94.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.267	26.87	0.148	0.	0.	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	21684	0	0	0	0	0
normalized size	1	1.	94.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	26.625	0.151	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	12792	0	0	0	0	0
normalized size	1	1.	56.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	27.003	0.158	0.	0.	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	12774	0	0	0	0	0
normalized size	1	1.	56.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	27.049	0.151	0.	0.	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	124	287	359	467	0	639
normalized size	1	1.	0.75	1.74	2.18	2.83	0.	3.87
time (sec)	N/A	0.232	1.219	0.043	1.059	0.555	0.	1.439

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	100	223	294	400	0	578
normalized size	1	1.	0.73	1.63	2.15	2.92	0.	4.22
time (sec)	N/A	0.204	0.664	0.041	1.025	0.535	0.	1.319

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	160	209	331	0	352
normalized size	1	1.	0.74	1.58	2.07	3.28	0.	3.49
time (sec)	N/A	0.138	0.37	0.038	1.05	0.598	0.	1.256

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	92	117	157	305	0	230
normalized size	1	1.	1.33	1.7	2.28	4.42	0.	3.33
time (sec)	N/A	0.071	0.021	0.038	1.153	0.592	0.	1.231

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	71	88	124	265	0	181
normalized size	1	1.	1.37	1.69	2.38	5.1	0.	3.48
time (sec)	N/A	0.122	0.022	0.057	0.985	0.551	0.	1.177

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	100	120	192	0	215
normalized size	1	1.	0.99	1.45	1.74	2.78	0.	3.12
time (sec)	N/A	0.156	0.122	0.064	1.03	0.541	0.	1.193

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	85	102	132	173	0	306
normalized size	1	1.	0.92	1.11	1.43	1.88	0.	3.33
time (sec)	N/A	0.18	0.183	0.061	1.022	0.509	0.	1.177

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	117	141	178	239	0	529
normalized size	1	1.	1.01	1.22	1.53	2.06	0.	4.56
time (sec)	N/A	0.215	0.316	0.073	1.034	0.518	0.	1.166

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	117	173	224	305	0	590
normalized size	1	1.	0.75	1.11	1.44	1.96	0.	3.78
time (sec)	N/A	0.235	0.456	0.073	1.04	0.527	0.	1.149

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	281	371	404	482	598	0	1034
normalized size	1	1.21	1.59	1.73	2.07	2.57	0.	4.44
time (sec)	N/A	0.592	2.134	0.05	1.069	0.561	0.	1.259

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	300	321	413	510	0	851
normalized size	1	1.	1.5	1.6	2.06	2.55	0.	4.26
time (sec)	N/A	0.348	1.556	0.05	1.029	0.56	0.	1.217

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	322	225	279	444	0	491
normalized size	1	1.	2.4	1.68	2.08	3.31	0.	3.66
time (sec)	N/A	0.172	1.74	0.047	1.043	0.552	0.	1.231

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	453	184	255	406	0	325
normalized size	1	1.	3.6	1.46	2.02	3.22	0.	2.58
time (sec)	N/A	0.203	1.41	0.069	1.073	0.556	0.	1.225

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	153	171	200	366	0	309
normalized size	1	1.	1.3	1.45	1.69	3.1	0.	2.62
time (sec)	N/A	0.317	1.164	0.065	1.063	0.548	0.	1.256

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	157	204	212	313	0	467
normalized size	1	1.	1.11	1.45	1.5	2.22	0.	3.31
time (sec)	N/A	0.365	0.521	0.077	1.02	0.549	0.	1.208

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	134	200	252	320	0	779
normalized size	1	1.	0.77	1.14	1.44	1.83	0.	4.45
time (sec)	N/A	0.448	0.731	0.072	1.012	0.519	0.	1.217

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	169	244	315	414	0	972
normalized size	1	1.	0.79	1.13	1.47	1.93	0.	4.52
time (sec)	N/A	0.514	0.802	0.081	1.033	0.536	0.	1.209

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	384	644	763	824	0	1850
normalized size	1	1.	1.01	1.69	2.	2.16	0.	4.86
time (sec)	N/A	0.87	3.175	0.062	1.102	0.613	0.	1.263

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	451	504	601	711	0	1335
normalized size	1	1.	1.58	1.76	2.1	2.49	0.	4.67
time (sec)	N/A	0.587	2.934	0.061	1.055	0.59	0.	1.269

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	525	389	483	624	0	1025
normalized size	1	1.	2.54	1.88	2.33	3.01	0.	4.95
time (sec)	N/A	0.34	5.52	0.057	1.148	0.595	0.	1.344

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	1335	294	378	543	0	591
normalized size	1	1.	6.95	1.53	1.97	2.83	0.	3.08
time (sec)	N/A	0.371	6.586	0.078	1.031	0.579	0.	1.373

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	320	267	328	500	0	729
normalized size	1	1.	1.57	1.31	1.61	2.45	0.	3.57
time (sec)	N/A	0.456	3.039	0.081	1.055	0.576	0.	1.336

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	263	278	292	486	0	564
normalized size	1	1.	1.34	1.42	1.49	2.48	0.	2.88
time (sec)	N/A	0.601	1.354	0.076	1.033	0.573	0.	1.336

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	215	362	332	463	0	976
normalized size	1	1.	0.96	1.62	1.49	2.08	0.	4.38
time (sec)	N/A	0.657	0.937	0.08	0.992	0.587	0.	1.356

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	288	301	389	498	0	1250
normalized size	1	1.	1.07	1.12	1.45	1.85	0.	4.65
time (sec)	N/A	0.751	1.061	0.08	1.068	0.555	0.	1.39

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	369	370	486	610	0	1764
normalized size	1	1.	1.15	1.16	1.52	1.91	0.	5.51
time (sec)	N/A	0.895	1.188	0.093	1.054	0.577	0.	1.334

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	491	486	905	1007	1085	0	2549
normalized size	1	1.	0.99	1.84	2.05	2.21	0.	5.19
time (sec)	N/A	1.234	4.61	0.077	1.079	0.677	0.	1.467

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	424	745	882	941	0	2238
normalized size	1	1.	1.1	1.94	2.3	2.45	0.	5.83
time (sec)	N/A	0.876	3.687	0.069	1.059	0.659	0.	1.404

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	690	572	671	824	0	1539
normalized size	1	1.	2.38	1.97	2.31	2.84	0.	5.31
time (sec)	N/A	0.543	4.104	0.066	1.051	0.625	0.	1.411

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	813	457	582	725	0	1134
normalized size	1	1.	2.98	1.67	2.13	2.66	0.	4.15
time (sec)	N/A	0.583	6.893	0.09	1.088	0.63	0.	1.455

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	348	377	452	644	0	743
normalized size	1	1.	1.27	1.38	1.65	2.35	0.	2.71
time (sec)	N/A	0.677	2.452	0.091	1.054	0.615	0.	1.403

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	370	374	420	628	0	733
normalized size	1	1.	1.22	1.23	1.39	2.07	0.	2.42
time (sec)	N/A	0.85	5.208	0.09	1.034	0.617	0.	1.391

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	382	434	412	636	0	1083
normalized size	1	1.	1.3	1.48	1.41	2.17	0.	3.7
time (sec)	N/A	0.972	3.973	0.082	1.044	0.616	0.	1.357

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	382	543	468	640	0	1477
normalized size	1	1.	1.22	1.73	1.49	2.04	0.	4.7
time (sec)	N/A	1.049	1.248	0.092	1.062	0.613	0.	1.415

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	432	431	560	698	0	2130
normalized size	1	1.	1.16	1.16	1.51	1.88	0.	5.73
time (sec)	N/A	1.208	1.65	0.09	1.109	0.595	0.	1.462

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	528	505	672	852	0	2450
normalized size	1	1.	1.21	1.15	1.53	1.95	0.	5.59
time (sec)	N/A	1.384	1.596	0.097	1.078	0.644	0.	1.509

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	170	360	432	633	0	888
normalized size	1	1.	0.79	1.68	2.02	2.96	0.	4.15
time (sec)	N/A	0.476	1.274	0.059	1.109	0.592	0.	1.372

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	114	228	275	470	0	406
normalized size	1	1.	0.77	1.53	1.85	3.15	0.	2.72
time (sec)	N/A	0.309	0.919	0.052	1.029	0.556	0.	1.262

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	77	157	192	363	0	288
normalized size	1	1.	0.79	1.62	1.98	3.74	0.	2.97
time (sec)	N/A	0.169	0.562	0.042	1.027	0.552	0.	1.213

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	512	825	0	1810	0	652
normalized size	1	1.	2.38	3.84	0.	8.42	0.	3.03
time (sec)	N/A	0.741	3.589	0.091	0.	54.432	0.	1.319

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	472	499	0	1465	0	387
normalized size	1	1.	3.08	3.26	0.	9.58	0.	2.53
time (sec)	N/A	0.454	2.596	0.083	0.	37.088	0.	1.285

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	365	272	0	1085	0	243
normalized size	1	1.	3.44	2.57	0.	10.24	0.	2.29
time (sec)	N/A	0.224	2.439	0.075	0.	11.357	0.	1.33

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	261	202	0	824	0	201
normalized size	1	1.	2.78	2.15	0.	8.77	0.	2.14
time (sec)	N/A	0.172	0.52	0.081	0.	6.41	0.	1.332

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	216	0	724	0	197
normalized size	1	1.	0.94	2.2	0.	7.39	0.	2.01
time (sec)	N/A	0.216	0.232	0.11	0.	0.568	0.	1.244

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	131	434	0	1004	0	323
normalized size	1	1.	0.9	2.99	0.	6.92	0.	2.23
time (sec)	N/A	0.452	0.437	0.123	0.	0.591	0.	1.229

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	178	814	0	1301	0	572
normalized size	1	1.	0.87	3.97	0.	6.35	0.	2.79
time (sec)	N/A	0.74	0.602	0.137	0.	0.625	0.	1.299

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	235	1580	0	1690	0	1081
normalized size	1	1.	0.85	5.72	0.	6.12	0.	3.92
time (sec)	N/A	1.104	0.832	0.134	0.	0.704	0.	1.315

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	407	407	605	1254	0	0	0	846
normalized size	1	1.	1.49	3.08	0.	0.	0.	2.08
time (sec)	N/A	1.74	3.819	0.117	0.	0.	0.	1.441

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	519	926	0	3341	0	578
normalized size	1	1.	1.66	2.97	0.	10.71	0.	1.85
time (sec)	N/A	1.233	2.854	0.105	0.	160.142	0.	1.384

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	382	630	0	2514	0	598
normalized size	1	1.	2.16	3.56	0.	14.2	0.	3.38
time (sec)	N/A	0.647	3.001	0.092	0.	63.785	0.	1.384

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	356	470	0	1621	0	338
normalized size	1	1.	2.41	3.18	0.	10.95	0.	2.28
time (sec)	N/A	0.304	3.089	0.095	0.	18.089	0.	1.296

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	299	448	0	1296	0	300
normalized size	1	1.	2.17	3.25	0.	9.39	0.	2.17
time (sec)	N/A	0.252	2.209	0.097	0.	0.626	0.	1.243

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	160	573	0	1796	0	551
normalized size	1	1.	0.79	2.84	0.	8.89	0.	2.73
time (sec)	N/A	0.644	0.956	0.131	0.	0.726	0.	1.295

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	206	857	0	2398	0	513
normalized size	1	1.	0.69	2.88	0.	8.05	0.	1.72
time (sec)	N/A	1.235	1.409	0.138	0.	0.819	0.	1.277

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	255	1241	0	3000	0	761
normalized size	1	1.	0.64	3.13	0.	7.58	0.	1.92
time (sec)	N/A	1.759	1.749	0.16	0.	0.931	0.	1.298

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	1124	2275	0	0	0	2349
normalized size	1	1.	2.42	4.89	0.	0.	0.	5.05
time (sec)	N/A	4.745	6.479	0.123	0.	0.	0.	1.473

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	492	1813	0	0	0	952
normalized size	1	1.	1.52	5.61	0.	0.	0.	2.95
time (sec)	N/A	2.987	2.91	0.106	0.	0.	0.	1.539

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	514	1572	0	3240	0	853
normalized size	1	1.	2.12	6.5	0.	13.39	0.	3.52
time (sec)	N/A	0.918	6.002	0.11	0.	78.26	0.	1.422

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	410	268	0	1852	0	689
normalized size	1	1.	2.03	1.33	0.	9.17	0.	3.41
time (sec)	N/A	0.421	4.253	0.092	0.	0.686	0.	1.38

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	793	1550	0	2668	0	818
normalized size	1	1.	3.46	6.77	0.	11.65	0.	3.57
time (sec)	N/A	0.759	6.196	0.107	0.	0.785	0.	1.391

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	1015	1756	0	3634	0	900
normalized size	1	1.	3.08	5.32	0.	11.01	0.	2.73
time (sec)	N/A	3.149	7.124	0.139	0.	0.962	0.	1.522

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	881	2206	0	4668	0	2291
normalized size	1	1.	1.94	4.87	0.	10.3	0.	5.06
time (sec)	N/A	4.824	5.224	0.164	0.	1.18	0.	1.521

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	1197	3764	0	0	0	1706
normalized size	1	1.	2.55	8.01	0.	0.	0.	3.63
time (sec)	N/A	9.913	6.487	0.117	0.	0.	0.	1.535

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	1302	3244	0	0	0	1532
normalized size	1	1.	3.64	9.06	0.	0.	0.	4.28
time (sec)	N/A	2.509	7.346	0.125	0.	0.	0.	1.579

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	299	453	0	3109	0	1310
normalized size	1	1.	0.95	1.44	0.	9.9	0.	4.17
time (sec)	N/A	1.039	1.472	0.101	0.	1.409	0.	1.481

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	1069	452	0	3109	0	1307
normalized size	1	1.	3.58	1.51	0.	10.4	0.	4.37
time (sec)	N/A	0.856	7.559	0.103	0.	1.31	0.	1.512

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	1230	3223	0	4535	0	1493
normalized size	1	1.	3.66	9.59	0.	13.5	0.	4.44
time (sec)	N/A	2.137	7.904	0.125	0.	1.785	0.	1.445

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	471	471	1367	3707	0	6179	0	1654
normalized size	1	1.	2.9	7.87	0.	13.12	0.	3.51
time (sec)	N/A	10.101	8.219	0.159	0.	1.4	0.	1.495

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	648	648	658	4523	0	7804	0	1941
normalized size	1	1.	1.02	6.98	0.	12.04	0.	3.
time (sec)	N/A	12.514	7.	0.162	0.	1.804	0.	1.529

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	46	0	115	75	72
normalized size	1	1.	0.96	1.92	0.	4.79	3.12	3.
time (sec)	N/A	0.023	0.011	0.046	0.	0.498	3.512	1.255

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	76	133	0	613	0	153
normalized size	1	1.	1.01	1.77	0.	8.17	0.	2.04
time (sec)	N/A	0.151	0.206	0.089	0.	0.533	0.	1.29

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	211	415	0	1526	0	301
normalized size	1	1.	1.51	2.96	0.	10.9	0.	2.15
time (sec)	N/A	0.39	0.828	0.105	0.	0.642	0.	1.357

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	302	1308	0	3089	0	695
normalized size	1	1.	1.31	5.66	0.	13.37	0.	3.01
time (sec)	N/A	1.1	1.854	0.114	0.	0.806	0.	1.424

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	1097	2853	0	5277	0	1161
normalized size	1	1.	3.26	8.49	0.	15.71	0.	3.46
time (sec)	N/A	4.671	5.594	0.127	0.	1.123	0.	1.614

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	517	517	4780	5961	0	0	0	0
normalized size	1	1.	9.25	11.53	0.	0.	0.	0.
time (sec)	N/A	1.557	27.739	2.016	0.	0.	0.	0.

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	413	413	3706	4339	0	0	0	0
normalized size	1	1.	8.97	10.51	0.	0.	0.	0.
time (sec)	N/A	0.92	26.173	1.204	0.	0.	0.	0.

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	579	3344	0	0	0	0
normalized size	1	1.	1.79	10.32	0.	0.	0.	0.
time (sec)	N/A	0.592	20.502	0.821	0.	0.	0.	0.

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	366	366	5313	2334	0	0	0	0
normalized size	1	1.	14.52	6.38	0.	0.	0.	0.
time (sec)	N/A	0.406	24.454	0.51	0.	0.	0.	0.

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	930	2153	0	0	0	0
normalized size	1	1.	2.57	5.95	0.	0.	0.	0.
time (sec)	N/A	0.412	18.701	0.49	0.	0.	0.	0.

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	435	435	1842	2626	0	0	0	0
normalized size	1	1.	4.23	6.04	0.	0.	0.	0.
time (sec)	N/A	0.773	20.025	0.417	0.	0.	0.	0.

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	538	538	544	3761	0	0	0	0
normalized size	1	1.	1.01	6.99	0.	0.	0.	0.
time (sec)	N/A	1.231	14.475	0.478	0.	0.	0.	0.

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	628	628	1087	7208	0	0	0	0
normalized size	1	1.	1.73	11.48	0.	0.	0.	0.
time (sec)	N/A	2.627	21.623	3.062	0.	0.	0.	0.

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	4186	5945	0	0	0	0
normalized size	1	1.	8.29	11.77	0.	0.	0.	0.
time (sec)	N/A	1.321	26.485	2.063	0.	0.	0.	0.

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	3724	4527	0	0	0	0
normalized size	1	1.	9.17	11.15	0.	0.	0.	0.
time (sec)	N/A	0.83	26.018	1.217	0.	0.	0.	0.

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	443	443	6972	3927	0	0	0	0
normalized size	1	1.	15.74	8.86	0.	0.	0.	0.
time (sec)	N/A	0.661	26.172	0.812	0.	0.	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	426	426	7722	3361	0	0	0	0
normalized size	1	1.	18.13	7.89	0.	0.	0.	0.
time (sec)	N/A	0.664	26.136	0.644	0.	0.	0.	0.

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	442	442	4520	3595	0	0	0	0
normalized size	1	1.	10.23	8.13	0.	0.	0.	0.
time (sec)	N/A	0.811	23.967	0.651	0.	0.	0.	0.

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	540	540	5054	4138	0	0	0	0
normalized size	1	1.	9.36	7.66	0.	0.	0.	0.
time (sec)	N/A	1.313	24.684	0.501	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	650	650	761	5474	0	0	0	0
normalized size	1	1.	1.17	8.42	0.	0.	0.	0.
time (sec)	N/A	1.906	16.978	0.738	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	610	610	1090	7208	0	0	0	0
normalized size	1	1.	1.79	11.82	0.	0.	0.	0.
time (sec)	N/A	2.11	21.861	3.07	0.	0.	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	4220	6163	0	0	0	0
normalized size	1	1.	8.41	12.28	0.	0.	0.	0.
time (sec)	N/A	1.256	26.677	2.056	0.	0.	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	1405	5138	0	0	0	0
normalized size	1	1.	2.7	9.86	0.	0.	0.	0.
time (sec)	N/A	0.971	21.276	1.259	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	1498	4981	0	0	0	0
normalized size	1	1.	2.97	9.86	0.	0.	0.	0.
time (sec)	N/A	0.965	20.961	1.099	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	507	507	4902	4884	0	0	0	0
normalized size	1	1.	9.67	9.63	0.	0.	0.	0.
time (sec)	N/A	1.061	25.408	0.94	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	549	549	5361	5113	0	0	0	0
normalized size	1	1.	9.77	9.31	0.	0.	0.	0.
time (sec)	N/A	1.279	25.824	0.967	0.	0.	0.	0.

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	652	652	5681	5850	0	0	0	0
normalized size	1	1.	8.71	8.97	0.	0.	0.	0.
time (sec)	N/A	2.02	26.106	0.706	0.	0.	0.	0.

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	774	774	800	7029	0	0	0	0
normalized size	1	1.	1.03	9.08	0.	0.	0.	0.
time (sec)	N/A	3.203	20.655	0.938	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	3811	4340	0	0	0	0
normalized size	1	1.	8.88	10.12	0.	0.	0.	0.
time (sec)	N/A	1.057	25.958	1.293	0.	0.	0.	0.

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	3332	3147	0	0	0	0
normalized size	1	1.	9.74	9.2	0.	0.	0.	0.
time (sec)	N/A	0.664	24.896	0.871	0.	0.	0.	0.

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	2741	1757	0	0	0	0
normalized size	1	1.	10.27	6.58	0.	0.	0.	0.
time (sec)	N/A	0.365	21.834	0.508	0.	0.	0.	0.

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	762	1193	0	0	0	0
normalized size	1	1.	2.4	3.76	0.	0.	0.	0.
time (sec)	N/A	0.249	18.338	0.446	0.	0.	0.	0.

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	861	1210	0	0	0	0
normalized size	1	1.	2.41	3.38	0.	0.	0.	0.
time (sec)	N/A	0.424	18.012	0.423	0.	0.	0.	0.

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	439	439	1905	2259	0	0	0	0
normalized size	1	1.	4.34	5.15	0.	0.	0.	0.
time (sec)	N/A	0.749	16.145	0.408	0.	0.	0.	0.

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	510	874	5857	0	0	0	0
normalized size	1	1.	1.71	11.48	0.	0.	0.	0.
time (sec)	N/A	1.345	20.839	1.534	0.	0.	0.	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	3856	4183	0	0	0	0
normalized size	1	1.	10.95	11.88	0.	0.	0.	0.
time (sec)	N/A	0.801	26.062	0.745	0.	0.	0.	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	603	3071	0	0	0	0
normalized size	1	1.	2.06	10.48	0.	0.	0.	0.
time (sec)	N/A	0.464	20.941	0.507	0.	0.	0.	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	395	395	1275	2844	0	0	0	0
normalized size	1	1.	3.23	7.2	0.	0.	0.	0.
time (sec)	N/A	0.489	18.822	0.358	0.	0.	0.	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	451	451	1814	3673	0	0	0	0
normalized size	1	1.	4.02	8.14	0.	0.	0.	0.
time (sec)	N/A	0.751	20.885	0.408	0.	0.	0.	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	490	5176	0	0	0	0
normalized size	1	1.	0.89	9.38	0.	0.	0.	0.
time (sec)	N/A	1.246	17.045	0.56	0.	0.	0.	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	549	549	989	10856	0	0	0	0
normalized size	1	1.	1.8	19.77	0.	0.	0.	0.
time (sec)	N/A	1.85	21.58	1.746	0.	0.	0.	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	4504	8858	0	0	0	0
normalized size	1	1.	10.03	19.73	0.	0.	0.	0.
time (sec)	N/A	1.044	27.601	0.845	0.	0.	0.	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	3980	6953	0	0	0	0
normalized size	1	1.	9.57	16.71	0.	0.	0.	0.
time (sec)	N/A	0.794	26.043	0.483	0.	0.	0.	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	541	541	11444	8177	0	0	0	0
normalized size	1	1.	21.15	15.11	0.	0.	0.	0.
time (sec)	N/A	0.91	28.012	0.465	0.	0.	0.	0.

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	618	618	20207	10319	0	0	0	0
normalized size	1	1.	32.7	16.7	0.	0.	0.	0.
time (sec)	N/A	1.481	28.869	0.654	0.	0.	0.	0.

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	448	448	4778	3700	0	0	0	0
normalized size	1	1.	10.67	8.26	0.	0.	0.	0.
time (sec)	N/A	0.893	23.636	0.799	0.	0.	0.	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	382	382	1139	2761	0	0	0	0
normalized size	1	1.	2.98	7.23	0.	0.	0.	0.
time (sec)	N/A	0.623	18.534	0.529	0.	0.	0.	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	1232	1588	0	0	0	0
normalized size	1	1.	3.9	5.03	0.	0.	0.	0.
time (sec)	N/A	0.41	18.093	0.447	0.	0.	0.	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	160	289	0	0	0	0
normalized size	1	1.	0.75	1.36	0.	0.	0.	0.
time (sec)	N/A	0.164	2.856	0.359	0.	0.	0.	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	2090	2613	0	0	0	0
normalized size	1	1.	5.51	6.89	0.	0.	0.	0.
time (sec)	N/A	0.537	15.222	0.413	0.	0.	0.	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	814	7862	0	0	0	0
normalized size	1	1.	1.57	15.15	0.	0.	0.	0.
time (sec)	N/A	1.	14.971	0.466	0.	0.	0.	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	1262	1020	0	0	0	0
normalized size	1	1.	4.74	3.83	0.	0.	0.	0.
time (sec)	N/A	0.301	7.309	8.952	0.	0.	0.	0.

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	1202	851	0	0	0	0
normalized size	1	1.	5.23	3.7	0.	0.	0.	0.
time (sec)	N/A	0.276	7.122	8.213	0.	0.	0.	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	1140	742	0	0	0	0
normalized size	1	1.	5.94	3.86	0.	0.	0.	0.
time (sec)	N/A	0.235	7.023	6.992	0.	0.	0.	0.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	223	666	0	0	0	0
normalized size	1	1.	1.47	4.38	0.	0.	0.	0.
time (sec)	N/A	0.223	2.51	5.243	0.	0.	0.	0.

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	197	388	0	0	0	0
normalized size	1	1.	1.35	2.66	0.	0.	0.	0.
time (sec)	N/A	0.227	2.77	2.235	0.	0.	0.	0.

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	194	465	0	0	0	0
normalized size	1	1.	1.24	2.98	0.	0.	0.	0.
time (sec)	N/A	0.227	2.661	2.278	0.	0.	0.	0.

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	219	515	0	0	0	0
normalized size	1	1.	1.13	2.65	0.	0.	0.	0.
time (sec)	N/A	0.253	3.502	2.316	0.	0.	0.	0.

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	249	565	0	0	0	0
normalized size	1	1.	1.08	2.46	0.	0.	0.	0.
time (sec)	N/A	0.281	4.681	2.036	0.	0.	0.	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	1371	611	0	0	0	0
normalized size	1	1.	5.15	2.3	0.	0.	0.	0.
time (sec)	N/A	0.308	6.917	2.638	0.	0.	0.	0.

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	507	1196	0	0	0	0
normalized size	1	1.	1.48	3.49	0.	0.	0.	0.
time (sec)	N/A	0.588	6.741	11.647	0.	0.	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	333	947	0	0	0	0
normalized size	1	1.	1.15	3.28	0.	0.	0.	0.
time (sec)	N/A	0.529	2.414	9.167	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	271	1000	0	0	0	0
normalized size	1	1.	1.12	4.15	0.	0.	0.	0.
time (sec)	N/A	0.513	2.088	7.509	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	227	1301	0	0	0	0
normalized size	1	1.	1.01	5.81	0.	0.	0.	0.
time (sec)	N/A	0.5	2.986	6.732	0.	0.	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	234	932	0	0	0	0
normalized size	1	1.	1.04	4.14	0.	0.	0.	0.
time (sec)	N/A	0.518	4.714	2.896	0.	0.	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	251	706	0	0	0	0
normalized size	1	1.	1.04	2.92	0.	0.	0.	0.
time (sec)	N/A	0.522	6.576	2.546	0.	0.	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	286	784	0	0	0	0
normalized size	1	1.	0.99	2.7	0.	0.	0.	0.
time (sec)	N/A	0.552	3.655	2.181	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	566	1292	0	0	0	0
normalized size	1	1.	1.43	3.25	0.	0.	0.	0.
time (sec)	N/A	0.867	7.08	12.11	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	377	1205	0	0	0	0
normalized size	1	1.	1.13	3.61	0.	0.	0.	0.
time (sec)	N/A	0.786	3.778	9.369	0.	0.	0.	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	311	1419	0	0	0	0
normalized size	1	1.	0.97	4.45	0.	0.	0.	0.
time (sec)	N/A	0.83	3.534	8.263	0.	0.	0.	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	295	1837	0	0	0	0
normalized size	1	1.	0.94	5.87	0.	0.	0.	0.
time (sec)	N/A	0.834	2.697	7.42	0.	0.	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	234	1278	0	0	0	0
normalized size	1	1.	0.74	4.03	0.	0.	0.	0.
time (sec)	N/A	0.832	5.063	2.813	0.	0.	0.	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	323	975	0	0	0	0
normalized size	1	1.	0.96	2.9	0.	0.	0.	0.
time (sec)	N/A	0.856	6.135	2.434	0.	0.	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	538	1082	0	0	0	0
normalized size	1	1.	1.34	2.7	0.	0.	0.	0.
time (sec)	N/A	0.915	6.836	2.287	0.	0.	0.	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	713	1550	0	0	0	0
normalized size	1	1.	1.38	3.01	0.	0.	0.	0.
time (sec)	N/A	1.306	7.505	14.859	0.	0.	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	609	1550	0	0	0	0
normalized size	1	1.	1.38	3.51	0.	0.	0.	0.
time (sec)	N/A	1.239	7.384	12.626	0.	0.	0.	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	530	1624	0	0	0	0
normalized size	1	1.	1.26	3.88	0.	0.	0.	0.
time (sec)	N/A	1.226	7.368	10.528	0.	0.	0.	0.

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	485	1884	0	0	0	0
normalized size	1	1.	1.19	4.61	0.	0.	0.	0.
time (sec)	N/A	1.247	7.365	10.602	0.	0.	0.	0.

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	394	2507	0	0	0	0
normalized size	1	1.	0.92	5.84	0.	0.	0.	0.
time (sec)	N/A	1.309	6.668	9.3	0.	0.	0.	0.

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	517	1652	0	0	0	0
normalized size	1	1.	1.21	3.88	0.	0.	0.	0.
time (sec)	N/A	1.313	7.338	3.615	0.	0.	0.	0.

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	580	1273	0	0	0	0
normalized size	1	1.	1.31	2.87	0.	0.	0.	0.
time (sec)	N/A	1.316	7.065	2.626	0.	0.	0.	0.

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	516	658	1407	0	0	0	0
normalized size	1	1.	1.28	2.73	0.	0.	0.	0.
time (sec)	N/A	1.401	7.14	2.689	0.	0.	0.	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	0	800	0	0	0	0
normalized size	1	1.	0.	2.7	0.	0.	0.	0.
time (sec)	N/A	1.11	80.329	9.906	0.	0.	0.	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	0	472	0	0	0	0
normalized size	1	1.	0.	2.17	0.	0.	0.	0.
time (sec)	N/A	0.773	58.265	7.304	0.	0.	0.	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	0	409	0	0	0	0
normalized size	1	1.	0.	2.3	0.	0.	0.	0.
time (sec)	N/A	0.477	44.585	4.575	0.	0.	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	0	323	0	0	0	0
normalized size	1	1.	0.	2.06	0.	0.	0.	0.
time (sec)	N/A	0.288	13.291	2.664	0.	0.	0.	0.

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	0	945	0	0	0	0
normalized size	1	1.	0.	4.57	0.	0.	0.	0.
time (sec)	N/A	0.531	59.833	2.712	0.	0.	0.	0.

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	0	801	0	0	0	0
normalized size	1	1.	0.	2.98	0.	0.	0.	0.
time (sec)	N/A	0.855	55.298	5.605	0.	0.	0.	0.

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	342	342	0	1095	0	0	0	0
normalized size	1	1.	0.	3.2	0.	0.	0.	0.
time (sec)	N/A	1.222	68.768	7.123	0.	0.	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	931	1031	0	0	0	0
normalized size	1	1.	2.08	2.31	0.	0.	0.	0.
time (sec)	N/A	1.37	7.422	11.463	0.	0.	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	865	897	0	0	0	0
normalized size	1	1.	2.38	2.47	0.	0.	0.	0.
time (sec)	N/A	0.959	7.186	7.709	0.	0.	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	829	809	0	0	0	0
normalized size	1	1.	2.77	2.71	0.	0.	0.	0.
time (sec)	N/A	0.673	7.044	5.817	0.	0.	0.	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	835	856	0	0	0	0
normalized size	1	1.	2.63	2.7	0.	0.	0.	0.
time (sec)	N/A	0.679	7.131	7.458	0.	0.	0.	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	887	1123	0	0	0	0
normalized size	1	1.	2.18	2.77	0.	0.	0.	0.
time (sec)	N/A	1.03	7.273	9.103	0.	0.	0.	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	507	507	976	1377	0	0	0	0
normalized size	1	1.	1.93	2.72	0.	0.	0.	0.
time (sec)	N/A	1.516	7.596	8.982	0.	0.	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	667	667	1161	2185	0	0	0	0
normalized size	1	1.	1.74	3.28	0.	0.	0.	0.
time (sec)	N/A	2.248	7.781	20.032	0.	0.	0.	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	556	556	1092	2049	0	0	0	0
normalized size	1	1.	1.96	3.69	0.	0.	0.	0.
time (sec)	N/A	1.692	7.482	12.217	0.	0.	0.	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	1051	1879	0	0	0	0
normalized size	1	1.	2.24	4.01	0.	0.	0.	0.
time (sec)	N/A	1.18	7.239	10.419	0.	0.	0.	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	478	478	1051	1972	0	0	0	0
normalized size	1	1.	2.2	4.13	0.	0.	0.	0.
time (sec)	N/A	1.208	7.199	10.687	0.	0.	0.	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	486	486	1064	2022	0	0	0	0
normalized size	1	1.	2.19	4.16	0.	0.	0.	0.
time (sec)	N/A	1.223	7.37	12.197	0.	0.	0.	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	598	598	1121	2289	0	0	0	0
normalized size	1	1.	1.87	3.83	0.	0.	0.	0.
time (sec)	N/A	1.805	7.6	13.84	0.	0.	0.	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	782	4821	0	0	0	0
normalized size	1	1.	1.75	10.79	0.	0.	0.	0.
time (sec)	N/A	1.625	6.887	0.628	0.	0.	0.	0.

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	346	346	478	3182	0	0	0	0
normalized size	1	1.	1.38	9.2	0.	0.	0.	0.
time (sec)	N/A	1.186	6.256	0.461	0.	0.	0.	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	438	2345	0	0	0	0
normalized size	1	1.	1.7	9.09	0.	0.	0.	0.
time (sec)	N/A	0.832	3.723	0.456	0.	0.	0.	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	0	2548	0	0	0	0
normalized size	1	1.	0.	9.2	0.	0.	0.	0.
time (sec)	N/A	0.904	31.686	0.431	0.	0.	0.	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	3426	3639	0	0	0	0
normalized size	1	1.	12.55	13.33	0.	0.	0.	0.
time (sec)	N/A	0.807	6.608	0.505	0.	0.	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	4441	4764	0	0	0	0
normalized size	1	1.	12.34	13.23	0.	0.	0.	0.
time (sec)	N/A	1.173	6.767	0.686	0.	0.	0.	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	5993	6551	0	0	0	0
normalized size	1	1.	13.11	14.33	0.	0.	0.	0.
time (sec)	N/A	1.644	6.978	0.9	0.	0.	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	551	551	916	7134	0	0	0	0
normalized size	1	1.	1.66	12.95	0.	0.	0.	0.
time (sec)	N/A	2.162	7.058	0.855	0.	0.	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	446	446	800	5245	0	0	0	0
normalized size	1	1.	1.79	11.76	0.	0.	0.	0.
time (sec)	N/A	1.654	6.955	0.583	0.	0.	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	709	4335	0	0	0	0
normalized size	1	1.	2.01	12.28	0.	0.	0.	0.
time (sec)	N/A	1.182	6.938	0.53	0.	0.	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	340	685	3823	0	0	0	0
normalized size	1	1.	2.01	11.24	0.	0.	0.	0.
time (sec)	N/A	1.231	6.897	0.465	0.	0.	0.	0.

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	0	4247	0	0	0	0
normalized size	1	1.	0.	11.93	0.	0.	0.	0.
time (sec)	N/A	1.255	39.345	0.501	0.	0.	0.	0.

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	4862	4944	0	0	0	0
normalized size	1	1.	13.54	13.77	0.	0.	0.	0.
time (sec)	N/A	1.236	6.824	0.622	0.	0.	0.	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	455	5997	6526	0	0	0	0
normalized size	1	1.	13.18	14.34	0.	0.	0.	0.
time (sec)	N/A	1.709	7.031	0.868	0.	0.	0.	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	550	550	925	7346	0	0	0	0
normalized size	1	1.	1.68	13.36	0.	0.	0.	0.
time (sec)	N/A	2.189	7.22	0.816	0.	0.	0.	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	453	453	817	6194	0	0	0	0
normalized size	1	1.	1.8	13.67	0.	0.	0.	0.
time (sec)	N/A	1.681	7.301	0.672	0.	0.	0.	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	766	5629	0	0	0	0
normalized size	1	1.	1.79	13.18	0.	0.	0.	0.
time (sec)	N/A	1.662	7.053	0.642	0.	0.	0.	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	419	419	755	5634	0	0	0	0
normalized size	1	1.	1.8	13.45	0.	0.	0.	0.
time (sec)	N/A	1.645	7.052	0.646	0.	0.	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	441	441	0	5602	0	0	0	0
normalized size	1	1.	0.	12.7	0.	0.	0.	0.
time (sec)	N/A	1.671	51.153	0.731	0.	0.	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	452	452	6410	6758	0	0	0	0
normalized size	1	1.	14.18	14.95	0.	0.	0.	0.
time (sec)	N/A	1.746	7.036	0.892	0.	0.	0.	0.

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	565	565	7479	7971	0	0	0	0
normalized size	1	1.	13.24	14.11	0.	0.	0.	0.
time (sec)	N/A	2.299	7.362	1.17	0.	0.	0.	0.

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	350	350	503	3178	0	0	0	0
normalized size	1	1.	1.44	9.08	0.	0.	0.	0.
time (sec)	N/A	1.145	4.966	0.462	0.	0.	0.	0.

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	427	1638	0	0	0	0
normalized size	1	1.	1.64	6.3	0.	0.	0.	0.
time (sec)	N/A	0.84	6.485	0.389	0.	0.	0.	0.

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	219	219	0	1358	0	0	0	0
normalized size	1	1.	0.	6.2	0.	0.	0.	0.
time (sec)	N/A	0.618	16.525	0.431	0.	0.	0.	0.

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	1959	1931	0	0	0	0
normalized size	1	1.	9.07	8.94	0.	0.	0.	0.
time (sec)	N/A	0.521	6.538	0.406	0.	0.	0.	0.

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	3039	3439	0	0	0	0
normalized size	1	1.	10.44	11.82	0.	0.	0.	0.
time (sec)	N/A	0.817	6.658	0.532	0.	0.	0.	0.

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	4470	4764	0	0	0	0
normalized size	1	1.	11.76	12.54	0.	0.	0.	0.
time (sec)	N/A	1.178	6.929	0.685	0.	0.	0.	0.

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	377	1431	0	0	0	0
normalized size	1	1.	1.49	5.66	0.	0.	0.	0.
time (sec)	N/A	1.079	4.353	0.468	0.	0.	0.	0.

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	774	3121	0	0	0	0
normalized size	1	1.	1.97	7.94	0.	0.	0.	0.
time (sec)	N/A	1.354	7.039	0.41	0.	0.	0.	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	311	311	0	2053	0	0	0	0
normalized size	1	1.	0.	6.6	0.	0.	0.	0.
time (sec)	N/A	0.977	33.507	0.467	0.	0.	0.	0.

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	3541	1889	0	0	0	0
normalized size	1	1.	14.22	7.59	0.	0.	0.	0.
time (sec)	N/A	0.605	7.048	0.457	0.	0.	0.	0.

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	350	350	4557	2733	0	0	0	0
normalized size	1	1.	13.02	7.81	0.	0.	0.	0.
time (sec)	N/A	0.943	7.566	0.404	0.	0.	0.	0.

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	461	461	6134	4114	0	0	0	0
normalized size	1	1.	13.31	8.92	0.	0.	0.	0.
time (sec)	N/A	1.373	8.06	0.52	0.	0.	0.	0.

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	563	563	938	9944	0	0	0	0
normalized size	1	1.	1.67	17.66	0.	0.	0.	0.
time (sec)	N/A	1.947	7.343	0.615	0.	0.	0.	0.

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	0	7030	0	0	0	0
normalized size	1	1.	0.	15.73	0.	0.	0.	0.
time (sec)	N/A	1.449	52.962	0.478	0.	0.	0.	0.

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	5040	5169	0	0	0	0
normalized size	1	1.	13.33	13.67	0.	0.	0.	0.
time (sec)	N/A	1.037	7.61	0.463	0.	0.	0.	0.

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	401	401	6142	6945	0	0	0	0
normalized size	1	1.	15.32	17.32	0.	0.	0.	0.
time (sec)	N/A	1.034	8.309	0.51	0.	0.	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	7608	8777	0	0	0	0
normalized size	1	1.	14.6	16.85	0.	0.	0.	0.
time (sec)	N/A	1.602	9.236	0.599	0.	0.	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	663	663	9192	11337	0	0	0	0
normalized size	1	1.	13.86	17.1	0.	0.	0.	0.
time (sec)	N/A	2.227	10.392	0.879	0.	0.	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	247	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.323	52.546	0.168	0.	0.	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	247	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.305	27.286	0.17	0.	0.	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	244	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.303	49.644	0.164	0.	0.	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	244	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.312	26.523	0.176	0.	0.	0.	0.

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	136	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	8.989	0.422	0.	0.	0.	0.

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	65	313	0	0	0	0
normalized size	1	1.	0.81	3.91	0.	0.	0.	0.
time (sec)	N/A	0.079	0.33	1.864	0.	0.	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	285	0	0	0	0
normalized size	1	1.	0.79	3.56	0.	0.	0.	0.
time (sec)	N/A	0.074	0.304	2.288	0.	0.	0.	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	252	0	0	0	0
normalized size	1	1.	0.96	5.04	0.	0.	0.	0.
time (sec)	N/A	0.059	0.108	2.193	0.	0.	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	124	228	0	0	0	0
normalized size	1	1.	2.58	4.75	0.	0.	0.	0.
time (sec)	N/A	0.06	0.94	1.782	0.	0.	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	289	149	0	0	0	0
normalized size	1	1.	6.57	3.39	0.	0.	0.	0.
time (sec)	N/A	0.062	1.613	2.312	0.	0.	0.	0.

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	266	0	0	0	0
normalized size	1	1.	0.9	5.54	0.	0.	0.	0.
time (sec)	N/A	0.06	0.184	2.162	0.	0.	0.	0.

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	73	593	0	0	0	0
normalized size	1	1.	0.91	7.41	0.	0.	0.	0.
time (sec)	N/A	0.075	0.364	5.565	0.	0.	0.	0.

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	73	376	0	0	0	0
normalized size	1	1.	0.91	4.7	0.	0.	0.	0.
time (sec)	N/A	0.076	0.574	4.45	0.	0.	0.	0.

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	918	406	0	0	0	0
normalized size	1	1.	5.56	2.46	0.	0.	0.	0.
time (sec)	N/A	0.241	6.292	3.306	0.	0.	0.	0.

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	872	378	0	0	0	0
normalized size	1	1.	6.51	2.82	0.	0.	0.	0.
time (sec)	N/A	0.219	6.239	2.119	0.	0.	0.	0.

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	824	345	0	0	0	0
normalized size	1	1.	8.16	3.42	0.	0.	0.	0.
time (sec)	N/A	0.203	6.296	2.169	0.	0.	0.	0.

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	813	458	0	0	0	0
normalized size	1	1.	8.56	4.82	0.	0.	0.	0.
time (sec)	N/A	0.204	6.351	2.524	0.	0.	0.	0.

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	817	437	0	0	0	0
normalized size	1	1.	8.6	4.6	0.	0.	0.	0.
time (sec)	N/A	0.207	6.37	4.892	0.	0.	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	851	729	0	0	0	0
normalized size	1	1.	6.45	5.52	0.	0.	0.	0.
time (sec)	N/A	0.223	6.444	6.617	0.	0.	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	895	838	0	0	0	0
normalized size	1	1.	5.42	5.08	0.	0.	0.	0.
time (sec)	N/A	0.245	6.515	7.763	0.	0.	0.	0.

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	976	436	0	0	0	0
normalized size	1	1.	4.24	1.9	0.	0.	0.	0.
time (sec)	N/A	0.518	6.315	2.239	0.	0.	0.	0.

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	1118	408	0	0	0	0
normalized size	1	1.	5.68	2.07	0.	0.	0.	0.
time (sec)	N/A	0.486	6.304	2.286	0.	0.	0.	0.

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	1070	380	0	0	0	0
normalized size	1	1.	6.52	2.32	0.	0.	0.	0.
time (sec)	N/A	0.466	6.379	2.148	0.	0.	0.	0.

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	799	440	0	0	0	0
normalized size	1	1.	5.06	2.78	0.	0.	0.	0.
time (sec)	N/A	0.464	6.449	2.156	0.	0.	0.	0.

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	1040	651	0	0	0	0
normalized size	1	1.	6.75	4.23	0.	0.	0.	0.
time (sec)	N/A	0.468	6.467	5.49	0.	0.	0.	0.

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	800	756	0	0	0	0
normalized size	1	1.	5.13	4.85	0.	0.	0.	0.
time (sec)	N/A	0.482	6.542	6.708	0.	0.	0.	0.

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	1092	918	0	0	0	0
normalized size	1	1.	5.54	4.66	0.	0.	0.	0.
time (sec)	N/A	0.512	6.614	8.003	0.	0.	0.	0.

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	1137	1168	0	0	0	0
normalized size	1	1.	4.94	5.08	0.	0.	0.	0.
time (sec)	N/A	0.542	6.671	9.685	0.	0.	0.	0.

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	1022	464	0	0	0	0
normalized size	1	1.	3.66	1.66	0.	0.	0.	0.
time (sec)	N/A	0.685	6.373	2.321	0.	0.	0.	0.

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	976	436	0	0	0	0
normalized size	1	1.	3.97	1.77	0.	0.	0.	0.
time (sec)	N/A	0.645	6.326	2.155	0.	0.	0.	0.

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	1116	408	0	0	0	0
normalized size	1	1.	5.24	1.92	0.	0.	0.	0.
time (sec)	N/A	0.623	6.439	2.142	0.	0.	0.	0.

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	1108	569	0	0	0	0
normalized size	1	1.	5.15	2.65	0.	0.	0.	0.
time (sec)	N/A	0.628	6.562	2.736	0.	0.	0.	0.

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	1089	704	0	0	0	0
normalized size	1	1.	5.16	3.34	0.	0.	0.	0.
time (sec)	N/A	0.629	6.586	2.693	0.	0.	0.	0.

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	1085	939	0	0	0	0
normalized size	1	1.	5.09	4.41	0.	0.	0.	0.
time (sec)	N/A	0.634	6.646	7.404	0.	0.	0.	0.

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	1102	1012	0	0	0	0
normalized size	1	1.	5.17	4.75	0.	0.	0.	0.
time (sec)	N/A	0.652	6.717	8.409	0.	0.	0.	0.

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	1135	1246	0	0	0	0
normalized size	1	1.	4.61	5.07	0.	0.	0.	0.
time (sec)	N/A	0.678	6.76	9.904	0.	0.	0.	0.

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	1179	1408	0	0	0	0
normalized size	1	1.	4.23	5.05	0.	0.	0.	0.
time (sec)	N/A	0.713	6.872	10.798	0.	0.	0.	0.

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	1393	295	0	0	0	0
normalized size	1	1.	7.26	1.54	0.	0.	0.	0.
time (sec)	N/A	0.287	6.725	2.47	0.	0.	0.	0.

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	1345	277	0	0	0	0
normalized size	1	1.	8.46	1.74	0.	0.	0.	0.
time (sec)	N/A	0.265	6.642	2.252	0.	0.	0.	0.

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	1300	262	0	0	0	0
normalized size	1	1.	10.66	2.15	0.	0.	0.	0.
time (sec)	N/A	0.241	6.57	2.121	0.	0.	0.	0.

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	1270	245	0	0	0	0
normalized size	1	1.	15.12	2.92	0.	0.	0.	0.
time (sec)	N/A	0.231	6.524	2.263	0.	0.	0.	0.

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	1304	316	0	0	0	0
normalized size	1	1.	11.64	2.82	0.	0.	0.	0.
time (sec)	N/A	0.244	6.622	4.679	0.	0.	0.	0.

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	1337	486	0	0	0	0
normalized size	1	1.	8.91	3.24	0.	0.	0.	0.
time (sec)	N/A	0.263	6.959	6.177	0.	0.	0.	0.

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	1382	803	0	0	0	0
normalized size	1	1.	7.2	4.18	0.	0.	0.	0.
time (sec)	N/A	0.273	7.243	8.174	0.	0.	0.	0.

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	1398	451	0	0	0	0
normalized size	1	1.	7.13	2.3	0.	0.	0.	0.
time (sec)	N/A	0.405	6.804	2.513	0.	0.	0.	0.

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	1355	437	0	0	0	0
normalized size	1	1.	8.42	2.71	0.	0.	0.	0.
time (sec)	N/A	0.381	6.732	2.59	0.	0.	0.	0.

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	934	352	0	0	0	0
normalized size	1	1.	7.18	2.71	0.	0.	0.	0.
time (sec)	N/A	0.349	6.58	2.494	0.	0.	0.	0.

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	1322	423	0	0	0	0
normalized size	1	1.	10.58	3.38	0.	0.	0.	0.
time (sec)	N/A	0.358	6.648	2.459	0.	0.	0.	0.

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	954	450	0	0	0	0
normalized size	1	1.	6.32	2.98	0.	0.	0.	0.
time (sec)	N/A	0.376	6.685	2.887	0.	0.	0.	0.

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	1391	738	0	0	0	0
normalized size	1	1.	7.36	3.9	0.	0.	0.	0.
time (sec)	N/A	0.414	7.255	7.296	0.	0.	0.	0.

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	1507	479	0	0	0	0
normalized size	1	1.	6.03	1.92	0.	0.	0.	0.
time (sec)	N/A	0.588	7.111	2.33	0.	0.	0.	0.

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	1470	465	0	0	0	0
normalized size	1	1.	7.03	2.22	0.	0.	0.	0.
time (sec)	N/A	0.542	6.947	2.751	0.	0.	0.	0.

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	1446	451	0	0	0	0
normalized size	1	1.	7.77	2.42	0.	0.	0.	0.
time (sec)	N/A	0.541	6.835	2.496	0.	0.	0.	0.

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	1439	451	0	0	0	0
normalized size	1	1.	7.82	2.45	0.	0.	0.	0.
time (sec)	N/A	0.531	6.775	2.484	0.	0.	0.	0.

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	1436	451	0	0	0	0
normalized size	1	1.	7.98	2.51	0.	0.	0.	0.
time (sec)	N/A	0.523	6.763	2.521	0.	0.	0.	0.

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	1473	679	0	0	0	0
normalized size	1	1.	7.05	3.25	0.	0.	0.	0.
time (sec)	N/A	0.565	6.976	3.061	0.	0.	0.	0.

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	1505	876	0	0	0	0
normalized size	1	1.	6.22	3.62	0.	0.	0.	0.
time (sec)	N/A	0.567	7.544	3.187	0.	0.	0.	0.

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	109	119	684	302	0	0
normalized size	1	1.	0.51	0.56	3.21	1.42	0.	0.
time (sec)	N/A	0.57	0.307	0.358	2.178	0.511	0.	0.

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	90	97	522	252	0	0
normalized size	1	1.	0.54	0.58	3.11	1.5	0.	0.
time (sec)	N/A	0.493	0.289	0.338	2.113	0.487	0.	0.

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	68	77	312	205	0	0
normalized size	1	1.	0.56	0.63	2.56	1.68	0.	0.
time (sec)	N/A	0.423	0.244	0.327	2.02	0.501	0.	0.

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	92	199	479	867	0	0
normalized size	1	1.	0.68	1.46	3.52	6.38	0.	0.
time (sec)	N/A	0.399	0.568	0.352	2.083	0.568	0.	0.

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	90	210	987	942	0	0
normalized size	1	1.	0.67	1.56	7.31	6.98	0.	0.
time (sec)	N/A	0.409	0.632	0.358	2.23	0.576	0.	0.

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	105	313	2034	1014	0	0
normalized size	1	1.	0.73	2.17	14.12	7.04	0.	0.
time (sec)	N/A	0.406	0.571	0.349	2.318	0.689	0.	0.

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	125	375	3699	1107	0	0
normalized size	1	1.	0.66	1.98	19.57	5.86	0.	0.
time (sec)	N/A	0.485	0.982	0.337	2.69	0.7	0.	0.

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	152	437	5963	1218	0	0
normalized size	1	1.	0.65	1.87	25.48	5.21	0.	0.
time (sec)	N/A	0.573	1.632	0.352	3.504	0.837	0.	0.

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	125	142	880	378	0	0
normalized size	1	1.	0.47	0.53	3.31	1.42	0.	0.
time (sec)	N/A	0.807	1.996	0.351	2.248	0.5	0.	0.

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	103	120	734	321	0	0
normalized size	1	1.	0.47	0.55	3.35	1.47	0.	0.
time (sec)	N/A	0.723	1.309	0.332	2.198	0.499	0.	0.

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	85	98	497	269	0	0
normalized size	1	1.	0.5	0.58	2.94	1.59	0.	0.
time (sec)	N/A	0.545	0.888	0.293	2.127	0.491	0.	0.

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	105	212	937	973	0	0
normalized size	1	1.	0.57	1.16	5.12	5.32	0.	0.
time (sec)	N/A	0.596	0.867	0.336	2.206	0.577	0.	0.

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	110	243	1828	1041	0	0
normalized size	1	1.	0.58	1.29	9.67	5.51	0.	0.
time (sec)	N/A	0.6	0.924	0.346	2.126	0.585	0.	0.

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	120	345	3402	1098	0	0
normalized size	1	1.	0.63	1.81	17.81	5.75	0.	0.
time (sec)	N/A	0.613	1.429	0.365	2.455	0.704	0.	0.

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	126	376	4733	1148	0	0
normalized size	1	1.	0.66	1.97	24.78	6.01	0.	0.
time (sec)	N/A	0.62	1.873	0.319	2.674	0.701	0.	0.

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	154	438	7777	1247	0	0
normalized size	1	1.	0.65	1.84	32.68	5.24	0.	0.
time (sec)	N/A	0.731	3.073	0.322	3.721	0.839	0.	0.

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	176	500	9767	1385	0	0
normalized size	1	1.	0.62	1.75	34.27	4.86	0.	0.
time (sec)	N/A	0.808	4.622	0.317	5.97	0.863	0.	0.

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	148	166	1150	473	0	0
normalized size	1	1.	0.47	0.53	3.67	1.51	0.	0.
time (sec)	N/A	1.016	3.512	0.368	2.267	0.51	0.	0.

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	127	144	938	385	0	0
normalized size	1	1.	0.48	0.54	3.53	1.45	0.	0.
time (sec)	N/A	0.937	2.424	0.345	2.192	0.517	0.	0.

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	105	122	783	336	0	0
normalized size	1	1.	0.49	0.56	3.62	1.56	0.	0.
time (sec)	N/A	0.632	1.582	0.305	2.176	0.495	0.	0.

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	125	236	1146	1115	0	0
normalized size	1	1.	0.54	1.03	4.98	4.85	0.	0.
time (sec)	N/A	0.781	1.501	0.272	2.228	0.588	0.	0.

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	131	245	11036	1175	0	0
normalized size	1	1.	0.57	1.07	47.98	5.11	0.	0.
time (sec)	N/A	0.798	1.576	0.271	3.305	0.594	0.	0.

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	139	378	4618	1210	0	0
normalized size	1	1.	0.57	1.55	18.93	4.96	0.	0.
time (sec)	N/A	0.813	1.712	0.316	21.455	0.71	0.	0.

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	144	409	0	1237	0	0
normalized size	1	1.	0.61	1.72	0.	5.2	0.	0.
time (sec)	N/A	0.793	2.385	0.309	0.	0.716	0.	0.

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	155	440	9027	1304	0	0
normalized size	1	1.	0.65	1.85	37.93	5.48	0.	0.
time (sec)	N/A	0.834	3.628	0.328	22.414	0.841	0.	0.

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	178	502	11950	1426	0	0
normalized size	1	1.	0.62	1.76	41.93	5.	0.	0.
time (sec)	N/A	0.933	3.576	0.346	6.461	0.856	0.	0.

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	200	564	14959	1539	0	0
normalized size	1	1.	0.6	1.7	45.06	4.64	0.	0.
time (sec)	N/A	1.032	4.769	0.373	11.31	0.864	0.	0.

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	166	206	906	1062	0	0
normalized size	1	1.	0.68	0.84	3.71	4.35	0.	0.
time (sec)	N/A	0.82	1.736	0.296	2.25	0.544	0.	0.

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	153	184	747	964	0	0
normalized size	1	1.	0.76	0.92	3.72	4.8	0.	0.
time (sec)	N/A	0.625	0.43	0.393	2.224	0.536	0.	0.

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	73	171	504	876	0	0
normalized size	1	1.	0.47	1.1	3.23	5.62	0.	0.
time (sec)	N/A	0.452	0.599	0.335	2.148	0.531	0.	0.

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	93	224	961	1331	0	0
normalized size	1	1.	0.53	1.28	5.49	7.61	0.	0.
time (sec)	N/A	0.486	0.524	0.366	2.216	0.616	0.	0.

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	105	248	0	1465	0	0
normalized size	1	1.	0.61	1.43	0.	8.47	0.	0.
time (sec)	N/A	0.484	0.445	0.355	0.	0.624	0.	0.

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	130	384	3110	1593	0	0
normalized size	1	1.	0.58	1.72	13.95	7.14	0.	0.
time (sec)	N/A	0.678	0.939	0.326	2.54	0.784	0.	0.

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	149	446	5084	1692	0	0
normalized size	1	1.	0.56	1.68	19.11	6.36	0.	0.
time (sec)	N/A	0.848	1.498	0.362	2.878	0.784	0.	0.

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	118	318	0	1234	0	0
normalized size	1	1.	0.44	1.19	0.	4.6	0.	0.
time (sec)	N/A	0.865	2.188	0.375	0.	0.558	0.	0.

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	104	262	0	1152	0	0
normalized size	1	1.	0.47	1.19	0.	5.21	0.	0.
time (sec)	N/A	0.668	1.645	0.355	0.	0.545	0.	0.

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	114	235	0	1052	0	0
normalized size	1	1.	0.66	1.37	0.	6.12	0.	0.
time (sec)	N/A	0.491	1.858	0.332	0.	0.536	0.	0.

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	114	304	4257	1593	0	0
normalized size	1	1.	0.62	1.64	23.01	8.61	0.	0.
time (sec)	N/A	0.528	1.609	0.321	2.437	0.666	0.	0.

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	169	362	0	1764	0	0
normalized size	1	1.	0.74	1.59	0.	7.74	0.	0.
time (sec)	N/A	0.704	2.239	0.341	0.	0.681	0.	0.

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	213	508	0	1956	0	0
normalized size	1	1.	0.75	1.78	0.	6.86	0.	0.
time (sec)	N/A	0.911	3.767	0.3	0.	0.869	0.	0.

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	150	450	0	1534	0	0
normalized size	1	1.	0.48	1.43	0.	4.87	0.	0.
time (sec)	N/A	1.07	3.971	0.39	0.	0.581	0.	0.

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	132	390	0	1426	0	0
normalized size	1	1.	0.5	1.47	0.	5.36	0.	0.
time (sec)	N/A	0.859	3.184	0.387	0.	0.563	0.	0.

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	118	365	0	1303	0	0
normalized size	1	1.	0.54	1.67	0.	5.95	0.	0.
time (sec)	N/A	0.688	2.68	0.354	0.	0.554	0.	0.

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	110	340	7466	1262	0	0
normalized size	1	1.	0.63	1.95	42.91	7.25	0.	0.
time (sec)	N/A	0.508	1.951	0.323	4.876	0.548	0.	0.

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	144	539	10615	1901	0	0
normalized size	1	1.	0.62	2.32	45.75	8.19	0.	0.
time (sec)	N/A	0.711	3.537	0.29	4.668	0.699	0.	0.

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	187	605	0	2107	0	0
normalized size	1	1.	0.68	2.18	0.	7.61	0.	0.
time (sec)	N/A	0.919	3.577	0.305	0.	0.717	0.	0.

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	0	0	0
normalized size	1	1.	0.69	2.61	0.	0.	0.	0.
time (sec)	N/A	0.095	0.496	2.125	0.	0.	0.	0.

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.083	0.23	2.036	0.	0.	0.	0.

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	228	0	0	0	0
normalized size	1	1.	0.87	3.74	0.	0.	0.	0.
time (sec)	N/A	0.071	0.109	2.279	0.	0.	0.	0.

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	152	0	0	0	0
normalized size	1	1.	1.	4.34	0.	0.	0.	0.
time (sec)	N/A	0.061	0.064	1.767	0.	0.	0.	0.

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	148	0	0	0	0
normalized size	1	1.	0.89	2.6	0.	0.	0.	0.
time (sec)	N/A	0.07	0.14	2.441	0.	0.	0.	0.

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	0	0	0
normalized size	1	1.	0.78	4.78	0.	0.	0.	0.
time (sec)	N/A	0.079	0.402	4.829	0.	0.	0.	0.

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	0	0	0
normalized size	1	1.	0.86	4.52	0.	0.	0.	0.
time (sec)	N/A	0.094	0.301	5.675	0.	0.	0.	0.

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	342	0	0	0	0
normalized size	1	1.	0.7	2.78	0.	0.	0.	0.
time (sec)	N/A	0.13	0.593	2.33	0.	0.	0.	0.

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	72	308	0	0	0	0
normalized size	1	1.	0.77	3.31	0.	0.	0.	0.
time (sec)	N/A	0.116	0.288	2.305	0.	0.	0.	0.

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	682	274	0	0	0	0
normalized size	1	1.	10.49	4.22	0.	0.	0.	0.
time (sec)	N/A	0.104	6.205	2.191	0.	0.	0.	0.

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	759	195	0	0	0	0
normalized size	1	1.	12.44	3.2	0.	0.	0.	0.
time (sec)	N/A	0.107	6.254	2.326	0.	0.	0.	0.

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	69	500	0	0	0	0
normalized size	1	1.	0.79	5.75	0.	0.	0.	0.
time (sec)	N/A	0.121	0.585	5.282	0.	0.	0.	0.

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	112	799	0	0	0	0
normalized size	1	1.	0.91	6.5	0.	0.	0.	0.
time (sec)	N/A	0.134	0.517	6.925	0.	0.	0.	0.

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	129	684	0	0	0	0
normalized size	1	1.	0.88	4.65	0.	0.	0.	0.
time (sec)	N/A	0.152	0.561	7.17	0.	0.	0.	0.

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	1292	512	0	0	0	0
normalized size	1	1.	7.38	2.93	0.	0.	0.	0.
time (sec)	N/A	0.308	6.362	2.536	0.	0.	0.	0.

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	1240	481	0	0	0	0
normalized size	1	1.	8.73	3.39	0.	0.	0.	0.
time (sec)	N/A	0.279	6.316	2.479	0.	0.	0.	0.

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	1186	447	0	0	0	0
normalized size	1	1.	11.19	4.22	0.	0.	0.	0.
time (sec)	N/A	0.244	6.378	2.425	0.	0.	0.	0.

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	1173	380	0	0	0	0
normalized size	1	1.	11.97	3.88	0.	0.	0.	0.
time (sec)	N/A	0.24	6.447	2.57	0.	0.	0.	0.

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	1180	515	0	0	0	0
normalized size	1	1.	11.46	5.	0.	0.	0.	0.
time (sec)	N/A	0.249	6.505	5.812	0.	0.	0.	0.

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	1228	739	0	0	0	0
normalized size	1	1.	8.71	5.24	0.	0.	0.	0.
time (sec)	N/A	0.267	6.577	7.517	0.	0.	0.	0.

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	1284	849	0	0	0	0
normalized size	1	1.	7.25	4.8	0.	0.	0.	0.
time (sec)	N/A	0.302	6.651	8.987	0.	0.	0.	0.

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1364	545	0	0	0	0
normalized size	1	1.	5.43	2.17	0.	0.	0.	0.
time (sec)	N/A	0.597	6.455	2.334	0.	0.	0.	0.

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	1699	514	0	0	0	0
normalized size	1	1.	7.9	2.39	0.	0.	0.	0.
time (sec)	N/A	0.554	6.416	2.522	0.	0.	0.	0.

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	2001	483	0	0	0	0
normalized size	1	1.	11.18	2.7	0.	0.	0.	0.
time (sec)	N/A	0.546	6.727	2.266	0.	0.	0.	0.

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	1356	595	0	0	0	0
normalized size	1	1.	7.98	3.5	0.	0.	0.	0.
time (sec)	N/A	0.529	6.638	2.99	0.	0.	0.	0.

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	1583	800	0	0	0	0
normalized size	1	1.	9.31	4.71	0.	0.	0.	0.
time (sec)	N/A	0.538	6.841	6.76	0.	0.	0.	0.

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	1599	906	0	0	0	0
normalized size	1	1.	9.19	5.21	0.	0.	0.	0.
time (sec)	N/A	0.542	6.946	7.461	0.	0.	0.	0.

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	2041	932	0	0	0	0
normalized size	1	1.	9.49	4.33	0.	0.	0.	0.
time (sec)	N/A	0.573	7.05	8.954	0.	0.	0.	0.

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1741	1181	0	0	0	0
normalized size	1	1.	6.94	4.71	0.	0.	0.	0.
time (sec)	N/A	0.599	6.987	10.903	0.	0.	0.	0.

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	1364	545	0	0	0	0
normalized size	1	1.	5.11	2.04	0.	0.	0.	0.
time (sec)	N/A	0.732	6.464	2.18	0.	0.	0.	0.

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1697	514	0	0	0	0
normalized size	1	1.	7.35	2.23	0.	0.	0.	0.
time (sec)	N/A	0.717	6.583	2.302	0.	0.	0.	0.

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	1688	727	0	0	0	0
normalized size	1	1.	7.44	3.2	0.	0.	0.	0.
time (sec)	N/A	0.705	6.74	2.993	0.	0.	0.	0.

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	1672	950	0	0	0	0
normalized size	1	1.	7.4	4.2	0.	0.	0.	0.
time (sec)	N/A	0.734	6.814	7.306	0.	0.	0.	0.

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1673	1328	0	0	0	0
normalized size	1	1.	7.24	5.75	0.	0.	0.	0.
time (sec)	N/A	0.709	6.923	8.903	0.	0.	0.	0.

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	1692	1097	0	0	0	0
normalized size	1	1.	7.32	4.75	0.	0.	0.	0.
time (sec)	N/A	0.727	6.962	9.569	0.	0.	0.	0.

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	1739	1262	0	0	0	0
normalized size	1	1.	6.51	4.73	0.	0.	0.	0.
time (sec)	N/A	0.751	7.065	11.067	0.	0.	0.	0.

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	1416	576	0	0	0	0
normalized size	1	1.	4.57	1.86	0.	0.	0.	0.
time (sec)	N/A	0.909	6.512	2.398	0.	0.	0.	0.

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	1751	545	0	0	0	0
normalized size	1	1.	6.39	1.99	0.	0.	0.	0.
time (sec)	N/A	0.883	6.713	2.405	0.	0.	0.	0.

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	1742	786	0	0	0	0
normalized size	1	1.	6.45	2.91	0.	0.	0.	0.
time (sec)	N/A	0.888	6.854	3.204	0.	0.	0.	0.

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	1451	864	0	0	0	0
normalized size	1	1.	5.39	3.21	0.	0.	0.	0.
time (sec)	N/A	0.879	6.954	3.245	0.	0.	0.	0.

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	1449	1214	0	0	0	0
normalized size	1	1.	5.43	4.55	0.	0.	0.	0.
time (sec)	N/A	0.875	7.077	9.332	0.	0.	0.	0.

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	1454	1535	0	0	0	0
normalized size	1	1.	5.37	5.66	0.	0.	0.	0.
time (sec)	N/A	0.871	7.228	10.01	0.	0.	0.	0.

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	1748	1427	0	0	0	0
normalized size	1	1.	6.38	5.21	0.	0.	0.	0.
time (sec)	N/A	0.894	7.353	11.766	0.	0.	0.	0.

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	1795	1505	0	0	0	0
normalized size	1	1.	5.79	4.85	0.	0.	0.	0.
time (sec)	N/A	0.925	7.413	12.971	0.	0.	0.	0.

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	2117	341	0	0	0	0
normalized size	1	1.	10.08	1.62	0.	0.	0.	0.
time (sec)	N/A	0.33	6.893	2.439	0.	0.	0.	0.

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	2063	320	0	0	0	0
normalized size	1	1.	11.86	1.84	0.	0.	0.	0.
time (sec)	N/A	0.309	6.841	2.26	0.	0.	0.	0.

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	2008	300	0	0	0	0
normalized size	1	1.	14.99	2.24	0.	0.	0.	0.
time (sec)	N/A	0.289	6.703	2.276	0.	0.	0.	0.

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	1973	281	0	0	0	0
normalized size	1	1.	21.22	3.02	0.	0.	0.	0.
time (sec)	N/A	0.266	6.655	2.697	0.	0.	0.	0.

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	2009	353	0	0	0	0
normalized size	1	1.	16.47	2.89	0.	0.	0.	0.
time (sec)	N/A	0.285	6.764	4.773	0.	0.	0.	0.

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	2052	494	0	0	0	0
normalized size	1	1.	12.44	2.99	0.	0.	0.	0.
time (sec)	N/A	0.306	7.188	6.898	0.	0.	0.	0.

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	2111	812	0	0	0	0
normalized size	1	1.	10.05	3.87	0.	0.	0.	0.
time (sec)	N/A	0.323	7.556	8.668	0.	0.	0.	0.

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	2174	513	0	0	0	0
normalized size	1	1.	8.43	1.99	0.	0.	0.	0.
time (sec)	N/A	0.499	7.289	2.915	0.	0.	0.	0.

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	2120	491	0	0	0	0
normalized size	1	1.	9.91	2.29	0.	0.	0.	0.
time (sec)	N/A	0.47	7.101	2.776	0.	0.	0.	0.

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	2064	472	0	0	0	0
normalized size	1	1.	11.47	2.62	0.	0.	0.	0.
time (sec)	N/A	0.451	6.943	2.996	0.	0.	0.	0.

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	1628	509	0	0	0	0
normalized size	1	1.	11.31	3.53	0.	0.	0.	0.
time (sec)	N/A	0.41	6.771	2.353	0.	0.	0.	0.

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	1620	509	0	0	0	0
normalized size	1	1.	12.18	3.83	0.	0.	0.	0.
time (sec)	N/A	0.414	6.735	2.559	0.	0.	0.	0.

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	1660	559	0	0	0	0
normalized size	1	1.	9.94	3.35	0.	0.	0.	0.
time (sec)	N/A	0.439	6.894	6.183	0.	0.	0.	0.

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	2107	751	0	0	0	0
normalized size	1	1.	9.99	3.56	0.	0.	0.	0.
time (sec)	N/A	0.468	7.498	9.138	0.	0.	0.	0.

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	2164	1072	0	0	0	0
normalized size	1	1.	8.66	4.29	0.	0.	0.	0.
time (sec)	N/A	0.49	8.153	10.952	0.	0.	0.	0.

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	2257	666	0	0	0	0
normalized size	1	1.	8.27	2.44	0.	0.	0.	0.
time (sec)	N/A	0.66	7.404	3.01	0.	0.	0.	0.

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	2206	638	0	0	0	0
normalized size	1	1.	9.43	2.73	0.	0.	0.	0.
time (sec)	N/A	0.631	7.203	2.836	0.	0.	0.	0.

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	2175	624	0	0	0	0
normalized size	1	1.	10.82	3.1	0.	0.	0.	0.
time (sec)	N/A	0.61	7.103	2.704	0.	0.	0.	0.

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	2167	624	0	0	0	0
normalized size	1	1.	11.23	3.23	0.	0.	0.	0.
time (sec)	N/A	0.593	6.955	2.575	0.	0.	0.	0.

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	2164	624	0	0	0	0
normalized size	1	1.	11.33	3.27	0.	0.	0.	0.
time (sec)	N/A	0.591	6.94	2.629	0.	0.	0.	0.

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	2205	789	0	0	0	0
normalized size	1	1.	9.63	3.45	0.	0.	0.	0.
time (sec)	N/A	0.633	7.233	3.202	0.	0.	0.	0.

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	2248	1040	0	0	0	0
normalized size	1	1.	8.39	3.88	0.	0.	0.	0.
time (sec)	N/A	0.665	8.021	10.748	0.	0.	0.	0.

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	2319	680	0	0	0	0
normalized size	1	1.	8.34	2.45	0.	0.	0.	0.
time (sec)	N/A	0.833	7.599	3.224	0.	0.	0.	0.

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	2286	666	0	0	0	0
normalized size	1	1.	9.37	2.73	0.	0.	0.	0.
time (sec)	N/A	0.79	7.39	3.206	0.	0.	0.	0.

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	1862	595	0	0	0	0
normalized size	1	1.	8.03	2.56	0.	0.	0.	0.
time (sec)	N/A	0.774	7.207	2.987	0.	0.	0.	0.

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	1862	595	0	0	0	0
normalized size	1	1.	8.13	2.6	0.	0.	0.	0.
time (sec)	N/A	0.765	7.068	3.024	0.	0.	0.	0.

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	1862	595	0	0	0	0
normalized size	1	1.	7.96	2.54	0.	0.	0.	0.
time (sec)	N/A	0.763	7.09	3.019	0.	0.	0.	0.

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	2316	1017	0	0	0	0
normalized size	1	1.	8.39	3.68	0.	0.	0.	0.
time (sec)	N/A	0.826	7.446	4.102	0.	0.	0.	0.

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	127	153	902	344	0	0
normalized size	1	1.	0.56	0.68	3.99	1.52	0.	0.
time (sec)	N/A	0.629	0.567	0.401	2.426	0.496	0.	0.

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	105	120	686	284	0	0
normalized size	1	1.	0.59	0.67	3.85	1.6	0.	0.
time (sec)	N/A	0.551	0.38	0.373	2.318	0.489	0.	0.

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	82	89	433	225	0	0
normalized size	1	1.	0.64	0.69	3.36	1.74	0.	0.
time (sec)	N/A	0.468	0.188	0.351	2.235	0.481	0.	0.

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	94	222	513	883	0	0
normalized size	1	1.	0.67	1.59	3.66	6.31	0.	0.
time (sec)	N/A	0.447	0.611	0.38	2.279	0.572	0.	0.

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	94	304	1310	986	0	0
normalized size	1	1.	0.68	2.19	9.42	7.09	0.	0.
time (sec)	N/A	0.45	0.747	0.408	2.325	0.693	0.	0.

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	109	440	2925	1068	0	0
normalized size	1	1.	0.72	2.91	19.37	7.07	0.	0.
time (sec)	N/A	0.456	0.688	0.356	2.591	1.092	0.	0.

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	140	533	5403	1180	0	0
normalized size	1	1.	0.7	2.68	27.15	5.93	0.	0.
time (sec)	N/A	0.548	1.361	0.378	3.013	1.106	0.	0.

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	178	626	8764	1318	0	0
normalized size	1	1.	0.72	2.53	35.48	5.34	0.	0.
time (sec)	N/A	0.633	2.162	0.381	4.139	1.66	0.	0.

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	158	187	1164	437	0	0
normalized size	1	1.	0.56	0.66	4.1	1.54	0.	0.
time (sec)	N/A	0.885	2.106	0.284	2.493	0.511	0.	0.

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	123	154	949	366	0	0
normalized size	1	1.	0.53	0.66	4.09	1.58	0.	0.
time (sec)	N/A	0.788	1.424	0.352	2.434	0.5	0.	0.

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	100	121	693	302	0	0
normalized size	1	1.	0.55	0.67	3.83	1.67	0.	0.
time (sec)	N/A	0.601	0.984	0.319	2.372	0.493	0.	0.

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	115	235	1022	1029	0	0
normalized size	1	1.	0.6	1.22	5.32	5.36	0.	0.
time (sec)	N/A	0.639	1.113	0.401	2.412	0.582	0.	0.

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	122	366	2558	1119	0	0
normalized size	1	1.	0.62	1.86	12.98	5.68	0.	0.
time (sec)	N/A	0.649	1.341	0.383	2.451	0.702	0.	0.

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	129	472	4942	1157	0	0
normalized size	1	1.	0.64	2.33	24.34	5.7	0.	0.
time (sec)	N/A	0.661	1.469	0.326	2.808	1.121	0.	0.

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	141	534	7760	1231	0	0
normalized size	1	1.	0.7	2.66	38.61	6.12	0.	0.
time (sec)	N/A	0.678	2.294	0.342	3.256	1.115	0.	0.

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	176	627	10963	1361	0	0
normalized size	1	1.	0.7	2.48	43.33	5.38	0.	0.
time (sec)	N/A	0.79	3.759	0.382	4.495	1.632	0.	0.

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	210	720	0	1523	0	0
normalized size	1	1.	0.69	2.38	0.	5.03	0.	0.
time (sec)	N/A	0.869	5.929	0.382	0.	1.65	0.	0.

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	190	222	1532	544	0	0
normalized size	1	1.	0.57	0.66	4.59	1.63	0.	0.
time (sec)	N/A	1.098	2.628	0.288	2.596	0.516	0.	0.

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	157	189	1249	459	0	0
normalized size	1	1.	0.55	0.67	4.4	1.62	0.	0.
time (sec)	N/A	1.014	2.321	0.28	2.506	0.509	0.	0.

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	124	156	1014	377	0	0
normalized size	1	1.	0.54	0.68	4.39	1.63	0.	0.
time (sec)	N/A	0.706	1.574	0.342	2.443	0.496	0.	0.

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	137	270	1357	1196	0	0
normalized size	1	1.	0.57	1.12	5.61	4.94	0.	0.
time (sec)	N/A	0.836	1.841	0.279	2.487	0.598	0.	0.

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	149	410	11426	1274	0	0
normalized size	1	1.	0.61	1.69	47.02	5.24	0.	0.
time (sec)	N/A	0.875	1.483	0.398	3.649	0.719	0.	0.

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	155	536	7309	1299	0	0
normalized size	1	1.	0.61	2.12	28.89	5.13	0.	0.
time (sec)	N/A	0.881	1.91	0.347	21.964	1.14	0.	0.

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	157	567	0	1328	0	0
normalized size	1	1.	0.62	2.24	0.	5.25	0.	0.
time (sec)	N/A	0.873	2.592	0.384	0.	1.134	0.	0.

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	178	629	0	1415	0	0
normalized size	1	1.	0.7	2.49	0.	5.59	0.	0.
time (sec)	N/A	0.891	4.074	0.348	0.	1.647	0.	0.

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	212	722	0	1561	0	0
normalized size	1	1.	0.7	2.4	0.	5.19	0.	0.
time (sec)	N/A	1.006	6.328	0.375	0.	1.677	0.	0.

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	947	815	0	1736	0	0
normalized size	1	1.	2.68	2.31	0.	4.92	0.	0.
time (sec)	N/A	1.104	6.587	0.414	0.	1.662	0.	0.

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	178	286	1310	1141	0	0
normalized size	1	1.	0.69	1.11	5.1	4.44	0.	0.
time (sec)	N/A	0.877	0.913	0.316	2.576	0.554	0.	0.

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	163	253	1045	1023	0	0
normalized size	1	1.	0.77	1.2	4.95	4.85	0.	0.
time (sec)	N/A	0.667	0.568	0.404	2.469	0.56	0.	0.

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	88	220	764	914	0	0
normalized size	1	1.	0.54	1.35	4.69	5.61	0.	0.
time (sec)	N/A	0.486	0.7	0.364	2.416	0.552	0.	0.

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	96	250	1080	1353	0	0
normalized size	1	1.	0.54	1.4	6.07	7.6	0.	0.
time (sec)	N/A	0.52	0.607	0.369	2.412	0.623	0.	0.

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	113	374	0	1530	0	0
normalized size	1	1.	0.62	2.07	0.	8.45	0.	0.
time (sec)	N/A	0.526	0.571	0.323	0.	0.788	0.	0.

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	127	545	4501	1667	0	0
normalized size	1	1.	0.54	2.32	19.15	7.09	0.	0.
time (sec)	N/A	0.735	1.089	0.349	2.918	1.31	0.	0.

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	154	638	7547	1789	0	0
normalized size	1	1.	0.55	2.27	26.86	6.37	0.	0.
time (sec)	N/A	0.927	1.113	0.367	3.412	1.323	0.	0.

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	137	282	1202	1439	0	0
normalized size	1	1.	0.74	1.53	6.53	7.82	0.	0.
time (sec)	N/A	0.602	0.417	0.389	2.698	0.805	0.	0.

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	135	450	0	1365	0	0
normalized size	1	1.	0.48	1.59	0.	4.82	0.	0.
time (sec)	N/A	0.918	3.065	0.29	0.	0.566	0.	0.

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	113	359	0	1235	0	0
normalized size	1	1.	0.48	1.54	0.	5.3	0.	0.
time (sec)	N/A	0.725	2.066	0.375	0.	0.553	0.	0.

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	96	306	0	1111	0	0
normalized size	1	1.	0.53	1.69	0.	6.14	0.	0.
time (sec)	N/A	0.525	1.602	0.362	0.	0.542	0.	0.

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	118	374	5785	1636	0	0
normalized size	1	1.	0.62	1.98	30.61	8.66	0.	0.
time (sec)	N/A	0.557	1.722	0.313	2.769	0.684	0.	0.

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	198	551	0	1905	0	0
normalized size	1	1.	0.82	2.28	0.	7.87	0.	0.
time (sec)	N/A	0.754	1.564	0.298	0.	0.87	0.	0.

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	239	731	0	2099	0	0
normalized size	1	1.	0.8	2.44	0.	7.	0.	0.
time (sec)	N/A	0.985	2.29	0.331	0.	1.588	0.	0.

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	173	647	0	1701	0	0
normalized size	1	1.	0.52	1.94	0.	5.11	0.	0.
time (sec)	N/A	1.153	4.283	0.302	0.	0.585	0.	0.

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	146	550	0	1569	0	0
normalized size	1	1.	0.52	1.96	0.	5.58	0.	0.
time (sec)	N/A	0.956	3.361	0.305	0.	0.574	0.	0.

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	128	500	0	1430	0	0
normalized size	1	1.	0.55	2.16	0.	6.19	0.	0.
time (sec)	N/A	0.739	2.833	0.307	0.	0.56	0.	0.

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	119	474	11343	1354	0	0
normalized size	1	1.	0.65	2.59	61.98	7.4	0.	0.
time (sec)	N/A	0.559	2.023	0.309	6.193	0.546	0.	0.

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	675	0	1998	0	0
normalized size	1	1.	0.63	2.8	0.	8.29	0.	0.
time (sec)	N/A	0.749	3.291	0.3	0.	0.711	0.	0.

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	222	972	0	2329	0	0
normalized size	1	1.	0.76	3.31	0.	7.92	0.	0.
time (sec)	N/A	0.997	3.965	0.322	0.	0.963	0.	0.

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	143	565	0	0	0	0
normalized size	1	1.	0.75	2.97	0.	0.	0.	0.
time (sec)	N/A	0.297	1.013	2.467	0.	0.	0.	0.

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	117	515	0	0	0	0
normalized size	1	1.	0.76	3.34	0.	0.	0.	0.
time (sec)	N/A	0.269	0.9	2.419	0.	0.	0.	0.

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	1569	465	0	0	0	0
normalized size	1	1.	13.53	4.01	0.	0.	0.	0.
time (sec)	N/A	0.252	6.491	2.481	0.	0.	0.	0.

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	1904	388	0	0	0	0
normalized size	1	1.	17.96	3.66	0.	0.	0.	0.
time (sec)	N/A	0.261	6.916	2.518	0.	0.	0.	0.

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	1909	666	0	0	0	0
normalized size	1	1.	17.04	5.95	0.	0.	0.	0.
time (sec)	N/A	0.278	6.991	5.604	0.	0.	0.	0.

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	136	742	0	0	0	0
normalized size	1	1.	0.89	4.88	0.	0.	0.	0.
time (sec)	N/A	0.305	1.552	7.888	0.	0.	0.	0.

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	173	851	0	0	0	0
normalized size	1	1.	0.91	4.48	0.	0.	0.	0.
time (sec)	N/A	0.319	4.278	9.324	0.	0.	0.	0.

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	194	784	0	0	0	0
normalized size	1	1.	0.78	3.14	0.	0.	0.	0.
time (sec)	N/A	0.603	1.336	2.587	0.	0.	0.	0.

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	2361	706	0	0	0	0
normalized size	1	1.	11.69	3.5	0.	0.	0.	0.
time (sec)	N/A	0.57	6.853	2.666	0.	0.	0.	0.

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	3011	932	0	0	0	0
normalized size	1	1.	16.19	5.01	0.	0.	0.	0.
time (sec)	N/A	0.552	7.462	2.903	0.	0.	0.	0.

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	2779	1301	0	0	0	0
normalized size	1	1.	15.44	7.23	0.	0.	0.	0.
time (sec)	N/A	0.545	7.545	6.998	0.	0.	0.	0.

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	3017	1000	0	0	0	0
normalized size	1	1.	15.01	4.98	0.	0.	0.	0.
time (sec)	N/A	0.571	7.648	8.06	0.	0.	0.	0.

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	218	947	0	0	0	0
normalized size	1	1.	0.88	3.8	0.	0.	0.	0.
time (sec)	N/A	0.605	4.595	10.196	0.	0.	0.	0.

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	286	1082	0	0	0	0
normalized size	1	1.	0.79	3.	0.	0.	0.	0.
time (sec)	N/A	0.955	2.026	2.615	0.	0.	0.	0.

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	296	3237	975	0	0	0	0
normalized size	1	1.	10.94	3.29	0.	0.	0.	0.
time (sec)	N/A	0.91	7.274	2.827	0.	0.	0.	0.

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	3915	1278	0	0	0	0
normalized size	1	1.	14.13	4.61	0.	0.	0.	0.
time (sec)	N/A	0.906	8.076	3.283	0.	0.	0.	0.

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	3868	1837	0	0	0	0
normalized size	1	1.	14.49	6.88	0.	0.	0.	0.
time (sec)	N/A	0.879	8.294	8.522	0.	0.	0.	0.

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	3871	1419	0	0	0	0
normalized size	1	1.	14.13	5.18	0.	0.	0.	0.
time (sec)	N/A	0.871	8.402	9.362	0.	0.	0.	0.

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	3933	1205	0	0	0	0
normalized size	1	1.	13.38	4.1	0.	0.	0.	0.
time (sec)	N/A	0.896	8.49	10.504	0.	0.	0.	0.

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	357	357	3345	1292	0	0	0	0
normalized size	1	1.	9.37	3.62	0.	0.	0.	0.
time (sec)	N/A	0.974	8.291	13.236	0.	0.	0.	0.

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	320	1273	0	0	0	0
normalized size	1	1.	0.79	3.15	0.	0.	0.	0.
time (sec)	N/A	1.32	2.516	2.933	0.	0.	0.	0.

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	377	377	4114	1652	0	0	0	0
normalized size	1	1.	10.91	4.38	0.	0.	0.	0.
time (sec)	N/A	1.305	8.414	3.588	0.	0.	0.	0.

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	4776	2507	0	0	0	0
normalized size	1	1.	12.87	6.76	0.	0.	0.	0.
time (sec)	N/A	1.295	9.058	10.084	0.	0.	0.	0.

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	388	388	4960	1884	0	0	0	0
normalized size	1	1.	12.78	4.86	0.	0.	0.	0.
time (sec)	N/A	1.303	9.209	11.548	0.	0.	0.	0.

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	4791	1624	0	0	0	0
normalized size	1	1.	12.48	4.23	0.	0.	0.	0.
time (sec)	N/A	1.316	9.447	12.151	0.	0.	0.	0.

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	401	401	4150	1550	0	0	0	0
normalized size	1	1.	10.35	3.87	0.	0.	0.	0.
time (sec)	N/A	1.331	9.091	15.119	0.	0.	0.	0.

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	274	801	0	0	0	0
normalized size	1	1.	1.31	3.83	0.	0.	0.	0.
time (sec)	N/A	0.965	2.393	6.517	0.	0.	0.	0.

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	218	945	0	0	0	0
normalized size	1	1.	1.48	6.43	0.	0.	0.	0.
time (sec)	N/A	0.668	1.345	3.227	0.	0.	0.	0.

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	0	323	0	0	0	0
normalized size	1	1.	0.	3.33	0.	0.	0.	0.
time (sec)	N/A	0.392	52.372	2.645	0.	0.	0.	0.

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	206	409	0	0	0	0
normalized size	1	1.	1.75	3.47	0.	0.	0.	0.
time (sec)	N/A	0.6	2.797	5.244	0.	0.	0.	0.

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	269	472	0	0	0	0
normalized size	1	1.	1.7	2.99	0.	0.	0.	0.
time (sec)	N/A	0.935	2.482	7.112	0.	0.	0.	0.

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	334	800	0	0	0	0
normalized size	1	1.	1.42	3.39	0.	0.	0.	0.
time (sec)	N/A	1.278	4.825	10.045	0.	0.	0.	0.

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	339	1123	0	0	0	0
normalized size	1	1.	0.98	3.25	0.	0.	0.	0.
time (sec)	N/A	1.215	3.686	9.059	0.	0.	0.	0.

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	301	856	0	0	0	0
normalized size	1	1.	1.17	3.33	0.	0.	0.	0.
time (sec)	N/A	0.801	3.287	8.351	0.	0.	0.	0.

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	301	809	0	0	0	0
normalized size	1	1.	1.26	3.38	0.	0.	0.	0.
time (sec)	N/A	0.788	4.202	6.981	0.	0.	0.	0.

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	340	897	0	0	0	0
normalized size	1	1.	1.11	2.92	0.	0.	0.	0.
time (sec)	N/A	1.102	5.021	9.099	0.	0.	0.	0.

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	474	1031	0	0	0	0
normalized size	1	1.	1.22	2.66	0.	0.	0.	0.
time (sec)	N/A	1.496	7.176	12.576	0.	0.	0.	0.

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	538	538	604	2289	0	0	0	0
normalized size	1	1.	1.12	4.25	0.	0.	0.	0.
time (sec)	N/A	2.043	7.421	14.63	0.	0.	0.	0.

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	441	2022	0	0	0	0
normalized size	1	1.	1.04	4.75	0.	0.	0.	0.
time (sec)	N/A	1.438	6.353	13.492	0.	0.	0.	0.

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	428	1972	0	0	0	0
normalized size	1	1.	1.01	4.66	0.	0.	0.	0.
time (sec)	N/A	1.359	5.813	11.946	0.	0.	0.	0.

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	444	1879	0	0	0	0
normalized size	1	1.	1.09	4.59	0.	0.	0.	0.
time (sec)	N/A	1.354	6.265	11.249	0.	0.	0.	0.

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	594	2049	0	0	0	0
normalized size	1	1.	1.2	4.13	0.	0.	0.	0.
time (sec)	N/A	1.881	7.3	14.485	0.	0.	0.	0.

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	3595	4075	0	0	0	0
normalized size	1	1.	7.87	8.92	0.	0.	0.	0.
time (sec)	N/A	1.798	24.337	1.109	0.	0.	0.	0.

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	3071	2829	0	0	0	0
normalized size	1	1.	8.53	7.86	0.	0.	0.	0.
time (sec)	N/A	1.299	22.935	0.837	0.	0.	0.	0.

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	404	1966	0	0	0	0
normalized size	1	1.	1.48	7.2	0.	0.	0.	0.
time (sec)	N/A	0.938	17.767	0.552	0.	0.	0.	0.

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	43023	1256	0	0	0	0
normalized size	1	1.	155.32	4.53	0.	0.	0.	0.
time (sec)	N/A	1.029	33.262	0.588	0.	0.	0.	0.

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	64644	1114	0	0	0	0
normalized size	1	1.	250.56	4.32	0.	0.	0.	0.
time (sec)	N/A	0.964	32.795	0.529	0.	0.	0.	0.

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	346	346	100266	1579	0	0	0	0
normalized size	1	1.	289.79	4.56	0.	0.	0.	0.
time (sec)	N/A	1.327	33.459	0.51	0.	0.	0.	0.

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	131249	2548	0	0	0	0
normalized size	1	1.	293.62	5.7	0.	0.	0.	0.
time (sec)	N/A	1.772	34.17	0.663	0.	0.	0.	0.

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	455	3703	4075	0	0	0	0
normalized size	1	1.	8.14	8.96	0.	0.	0.	0.
time (sec)	N/A	1.857	24.486	1.086	0.	0.	0.	0.

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	3261	2911	0	0	0	0
normalized size	1	1.	9.08	8.11	0.	0.	0.	0.
time (sec)	N/A	1.383	23.203	0.691	0.	0.	0.	0.

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	56321	2220	0	0	0	0
normalized size	1	1.	158.21	6.24	0.	0.	0.	0.
time (sec)	N/A	1.399	35.183	0.52	0.	0.	0.	0.

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	340	340	79958	1865	0	0	0	0
normalized size	1	1.	235.17	5.49	0.	0.	0.	0.
time (sec)	N/A	1.385	33.683	0.643	0.	0.	0.	0.

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	120732	2099	0	0	0	0
normalized size	1	1.	342.02	5.95	0.	0.	0.	0.
time (sec)	N/A	1.341	34.786	0.599	0.	0.	0.	0.

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	446	446	132839	2725	0	0	0	0
normalized size	1	1.	297.85	6.11	0.	0.	0.	0.
time (sec)	N/A	1.811	34.525	0.609	0.	0.	0.	0.

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	551	551	179293	3943	0	0	0	0
normalized size	1	1.	325.4	7.16	0.	0.	0.	0.
time (sec)	N/A	2.348	36.	0.862	0.	0.	0.	0.

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	565	565	4170	5307	0	0	0	0
normalized size	1	1.	7.38	9.39	0.	0.	0.	0.
time (sec)	N/A	2.485	25.73	1.532	0.	0.	0.	0.

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	452	452	3785	4157	0	0	0	0
normalized size	1	1.	8.37	9.2	0.	0.	0.	0.
time (sec)	N/A	1.91	24.836	1.032	0.	0.	0.	0.

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	441	441	64878	3164	0	0	0	0
normalized size	1	1.	147.12	7.17	0.	0.	0.	0.
time (sec)	N/A	1.823	35.285	0.75	0.	0.	0.	0.

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	419	419	86542	2893	0	0	0	0
normalized size	1	1.	206.54	6.9	0.	0.	0.	0.
time (sec)	N/A	1.828	34.893	0.706	0.	0.	0.	0.

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	129353	2792	0	0	0	0
normalized size	1	1.	302.93	6.54	0.	0.	0.	0.
time (sec)	N/A	1.828	35.332	0.955	0.	0.	0.	0.

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	453	453	157926	3162	0	0	0	0
normalized size	1	1.	348.62	6.98	0.	0.	0.	0.
time (sec)	N/A	1.876	36.586	1.01	0.	0.	0.	0.

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	550	550	180789	4031	0	0	0	0
normalized size	1	1.	328.71	7.33	0.	0.	0.	0.
time (sec)	N/A	2.386	35.967	0.953	0.	0.	0.	0.

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	674	674	211844	5292	0	0	0	0
normalized size	1	1.	314.31	7.85	0.	0.	0.	0.
time (sec)	N/A	2.974	37.518	1.381	0.	0.	0.	0.

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	492	2829	0	0	0	0
normalized size	1	1.	1.29	7.44	0.	0.	0.	0.
time (sec)	N/A	1.35	19.326	0.771	0.	0.	0.	0.

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	379	1885	0	0	0	0
normalized size	1	1.	1.3	6.48	0.	0.	0.	0.
time (sec)	N/A	0.975	18.23	0.529	0.	0.	0.	0.

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	216	216	359	1012	0	0	0	0
normalized size	1	1.	1.66	4.69	0.	0.	0.	0.
time (sec)	N/A	0.665	12.467	0.57	0.	0.	0.	0.

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	36160	2005	0	0	0	0
normalized size	1	1.	165.11	9.16	0.	0.	0.	0.
time (sec)	N/A	0.756	33.851	0.514	0.	0.	0.	0.

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	52620	866	0	0	0	0
normalized size	1	1.	202.38	3.33	0.	0.	0.	0.
time (sec)	N/A	0.98	32.557	0.462	0.	0.	0.	0.

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	350	350	98830	3191	0	0	0	0
normalized size	1	1.	282.37	9.12	0.	0.	0.	0.
time (sec)	N/A	1.285	33.45	0.49	0.	0.	0.	0.

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	25325	2301	0	0	0	0
normalized size	1	1.	121.75	11.06	0.	0.	0.	0.
time (sec)	N/A	0.898	6.292	0.512	0.	0.	0.	0.

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	461	461	3870	2418	0	0	0	0
normalized size	1	1.	8.39	5.25	0.	0.	0.	0.
time (sec)	N/A	1.567	24.839	0.613	0.	0.	0.	0.

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	350	350	3283	1518	0	0	0	0
normalized size	1	1.	9.38	4.34	0.	0.	0.	0.
time (sec)	N/A	1.114	23.121	0.669	0.	0.	0.	0.

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	517	966	0	0	0	0
normalized size	1	1.	2.08	3.88	0.	0.	0.	0.
time (sec)	N/A	0.755	17.612	0.546	0.	0.	0.	0.

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	311	311	63246	950	0	0	0	0
normalized size	1	1.	203.36	3.05	0.	0.	0.	0.
time (sec)	N/A	1.167	35.031	0.456	0.	0.	0.	0.

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	393	393	111509	1503	0	0	0	0
normalized size	1	1.	283.74	3.82	0.	0.	0.	0.
time (sec)	N/A	1.492	34.766	0.488	0.	0.	0.	0.

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	663	663	4917	6912	0	0	0	0
normalized size	1	1.	7.42	10.43	0.	0.	0.	0.
time (sec)	N/A	2.46	27.611	1.135	0.	0.	0.	0.

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	4327	5097	0	0	0	0
normalized size	1	1.	8.31	9.78	0.	0.	0.	0.
time (sec)	N/A	1.813	25.974	1.163	0.	0.	0.	0.

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	401	401	3834	3773	0	0	0	0
normalized size	1	1.	9.56	9.41	0.	0.	0.	0.
time (sec)	N/A	1.229	24.853	1.005	0.	0.	0.	0.

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	673	2767	0	0	0	0
normalized size	1	1.	1.78	7.32	0.	0.	0.	0.
time (sec)	N/A	1.229	19.36	0.785	0.	0.	0.	0.

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	119861	3739	0	0	0	0
normalized size	1	1.	268.15	8.36	0.	0.	0.	0.
time (sec)	N/A	1.648	36.981	0.718	0.	0.	0.	0.

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	563	563	215866	5561	0	0	0	0
normalized size	1	1.	383.42	9.88	0.	0.	0.	0.
time (sec)	N/A	2.163	38.055	0.839	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [299] had the largest ratio of [0.4444]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.	33	0.121
2	A	4	4	1.	31	0.129
3	A	3	3	1.	25	0.12
4	A	4	4	1.	31	0.129
5	A	4	4	1.	33	0.121
6	A	4	4	1.	33	0.121
7	A	4	4	1.	31	0.129
8	A	3	3	1.	25	0.12
9	A	4	4	1.	31	0.129
10	A	4	4	1.	33	0.121
11	A	4	4	1.	33	0.121
12	A	4	4	1.	31	0.129
13	A	3	3	1.	25	0.12
14	A	4	4	1.	31	0.129
15	A	4	4	1.	33	0.121
16	A	4	4	1.	33	0.121
17	A	4	4	1.	31	0.129
18	A	3	3	1.	25	0.12
19	A	4	4	1.	31	0.129
20	A	4	4	1.	33	0.121
21	A	4	4	1.	33	0.121
22	A	4	4	1.	33	0.121
23	A	4	4	1.	33	0.121
24	A	4	4	1.	33	0.121
25	A	4	4	1.	33	0.121
26	A	4	4	1.	33	0.121
27	A	4	4	1.	31	0.129
28	A	4	4	1.	31	0.129
29	A	4	4	1.	29	0.138
30	A	3	3	1.	23	0.13
31	A	4	4	1.	29	0.138
32	A	4	4	1.	31	0.129
33	A	4	4	1.	31	0.129
34	A	4	4	1.	33	0.121
35	A	4	4	1.	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	4	1.	33	0.121
37	A	4	4	1.	33	0.121
38	A	4	4	1.	33	0.121
39	A	4	4	1.	33	0.121
40	A	7	5	1.	38	0.132
41	A	7	5	1.	41	0.122
42	A	7	5	1.	39	0.128
43	A	6	4	1.	33	0.121
44	A	7	5	1.	39	0.128
45	A	7	5	1.	41	0.122
46	A	7	5	1.	41	0.122
47	A	7	5	1.	41	0.122
48	A	7	5	1.	39	0.128
49	A	6	4	1.	33	0.121
50	A	7	5	1.	39	0.128
51	A	7	5	1.	41	0.122
52	A	7	5	1.	41	0.122
53	A	7	5	1.	41	0.122
54	A	7	5	1.	39	0.128
55	A	6	4	1.	33	0.121
56	A	7	5	1.	39	0.128
57	A	7	5	1.	41	0.122
58	A	7	5	1.	41	0.122
59	A	7	5	1.	41	0.122
60	A	7	5	1.	39	0.128
61	A	6	4	1.	33	0.121
62	A	7	5	1.	39	0.128
63	A	7	5	1.	41	0.122
64	A	7	5	1.	41	0.122
65	A	7	5	1.	41	0.122
66	A	7	5	1.	41	0.122
67	A	7	5	1.	41	0.122
68	A	7	5	1.	41	0.122
69	A	7	5	1.	41	0.122
70	A	7	5	1.	41	0.122
71	A	7	5	1.	39	0.128
72	A	7	5	1.	39	0.128
73	A	7	5	1.	37	0.135
74	A	6	4	1.	31	0.129
75	A	7	5	1.	37	0.135
76	A	7	5	1.	39	0.128
77	A	7	5	1.	39	0.128
78	A	7	5	1.	41	0.122
79	A	7	5	1.	41	0.122

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	7	5	1.	41	0.122
81	A	7	5	1.	41	0.122
82	A	7	5	1.	41	0.122
83	A	7	5	1.	41	0.122
84	A	7	6	1.	31	0.194
85	A	7	7	1.	31	0.226
86	A	6	6	1.	29	0.207
87	A	5	4	1.	23	0.174
88	A	5	5	1.	29	0.172
89	A	5	5	1.	31	0.161
90	A	5	5	1.	31	0.161
91	A	7	6	1.	31	0.194
92	A	7	6	1.	31	0.194
93	A	8	8	1.	33	0.242
94	A	7	7	1.	31	0.226
95	A	6	6	1.	25	0.24
96	A	6	6	1.	31	0.194
97	A	5	4	1.	33	0.121
98	A	5	4	1.	33	0.121
99	A	6	6	1.	33	0.182
100	A	7	7	1.	33	0.212
101	A	8	7	1.	33	0.212
102	A	12	8	1.	33	0.242
103	A	11	7	1.	31	0.226
104	A	7	6	1.	25	0.24
105	A	7	6	1.	31	0.194
106	A	6	4	1.	33	0.121
107	A	6	5	1.	33	0.152
108	A	6	4	1.	33	0.121
109	A	9	7	1.	33	0.212
110	A	8	7	1.	33	0.212
111	A	15	8	1.	33	0.242
112	A	14	7	1.	31	0.226
113	A	8	6	1.	25	0.24
114	A	8	6	1.	31	0.194
115	A	7	4	1.	33	0.121
116	A	7	5	1.	33	0.152
117	A	7	5	1.	33	0.152
118	A	7	4	1.	33	0.121
119	A	12	7	1.	33	0.212
120	A	9	7	1.	33	0.212
121	A	7	5	1.	33	0.152
122	A	6	5	1.	33	0.152
123	A	6	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	4	4	1.	31	0.129
125	A	4	4	1.	25	0.16
126	A	4	4	1.	31	0.129
127	A	5	5	1.	33	0.152
128	A	6	5	1.	33	0.152
129	A	7	5	1.	33	0.152
130	A	7	6	1.	33	0.182
131	A	7	7	1.	33	0.212
132	A	6	6	1.	33	0.182
133	A	4	4	1.08	31	0.129
134	A	3	3	1.	25	0.12
135	A	5	5	1.	31	0.161
136	A	6	6	1.	33	0.182
137	A	7	6	1.	33	0.182
138	A	8	7	1.	33	0.212
139	A	7	7	1.	33	0.212
140	A	5	5	1.	33	0.152
141	A	3	3	1.	31	0.097
142	A	4	4	1.	25	0.16
143	A	6	5	1.	31	0.161
144	A	7	6	1.	33	0.182
145	A	8	6	1.	33	0.182
146	A	9	7	1.	33	0.212
147	A	8	7	1.	33	0.212
148	A	6	6	1.	33	0.182
149	A	4	4	1.	33	0.121
150	A	4	4	1.	31	0.129
151	A	5	4	1.	25	0.16
152	A	7	5	1.	31	0.161
153	A	8	6	1.	33	0.182
154	A	9	6	1.	33	0.182
155	A	6	6	1.	35	0.171
156	A	5	5	1.	35	0.143
157	A	4	4	1.	35	0.114
158	A	3	3	1.	33	0.091
159	A	5	5	1.	27	0.185
160	A	5	5	1.	33	0.152
161	A	4	4	1.	35	0.114
162	A	5	5	1.	35	0.143
163	A	6	5	1.	35	0.143
164	A	6	6	1.	35	0.171
165	A	5	5	1.	35	0.143
166	A	4	4	1.	33	0.121
167	A	6	6	1.	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
168	A	6	6	1.	33	0.182
169	A	5	5	1.	35	0.143
170	A	5	5	1.	35	0.143
171	A	6	6	1.	35	0.171
172	A	7	6	1.	35	0.171
173	A	7	6	1.	35	0.171
174	A	6	5	1.	35	0.143
175	A	5	4	1.	33	0.121
176	A	7	6	1.	27	0.222
177	A	7	6	1.	33	0.182
178	A	6	5	1.	35	0.143
179	A	6	6	1.	35	0.171
180	A	6	5	1.	35	0.143
181	A	7	6	1.	35	0.171
182	A	8	6	1.	35	0.171
183	A	7	6	1.	35	0.171
184	A	6	6	1.	35	0.171
185	A	5	5	1.	35	0.143
186	A	4	4	1.	33	0.121
187	A	6	5	1.	27	0.185
188	A	6	5	1.	33	0.152
189	A	7	6	1.	35	0.171
190	A	8	6	1.	35	0.171
191	A	9	6	1.	35	0.171
192	A	7	6	1.	35	0.171
193	A	6	6	1.	35	0.171
194	A	5	5	1.	35	0.143
195	A	4	4	1.	33	0.121
196	A	6	5	1.	27	0.185
197	A	7	6	1.	33	0.182
198	A	8	6	1.	35	0.171
199	A	9	6	1.	35	0.171
200	A	7	7	1.	35	0.2
201	A	6	6	1.	35	0.171
202	A	5	5	1.	35	0.143
203	A	4	4	1.	33	0.121
204	A	7	6	1.	27	0.222
205	A	8	7	1.	33	0.212
206	A	9	7	1.	35	0.2
207	A	9	7	1.	33	0.212
208	A	8	7	1.	33	0.212
209	A	7	6	1.	33	0.182
210	A	7	6	1.	33	0.182
211	A	7	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	8	7	1.	33	0.212
213	A	9	7	1.	33	0.212
214	A	10	8	1.	35	0.229
215	A	9	8	1.	35	0.229
216	A	8	7	1.	35	0.2
217	A	8	7	1.	35	0.2
218	A	8	7	1.	35	0.2
219	A	8	7	1.	35	0.2
220	A	9	8	1.	35	0.229
221	A	10	8	1.	35	0.229
222	A	11	8	1.	35	0.229
223	A	10	8	1.	35	0.229
224	A	9	7	1.	35	0.2
225	A	9	7	1.	35	0.2
226	A	9	8	1.	35	0.229
227	A	9	7	1.	35	0.2
228	A	9	7	1.	35	0.2
229	A	10	8	1.	35	0.229
230	A	11	8	1.	35	0.229
231	A	9	6	1.	35	0.171
232	A	8	6	1.	35	0.171
233	A	7	6	1.	35	0.171
234	A	6	5	1.	35	0.143
235	A	7	6	1.	35	0.171
236	A	8	6	1.	35	0.171
237	A	9	7	1.	35	0.2
238	A	8	7	1.	35	0.2
239	A	7	6	1.	35	0.171
240	A	7	6	1.	35	0.171
241	A	8	7	1.	35	0.2
242	A	9	7	1.	35	0.2
243	A	10	7	1.	35	0.2
244	A	9	7	1.	35	0.2
245	A	8	6	1.	35	0.171
246	A	8	7	1.	35	0.2
247	A	8	6	1.	35	0.171
248	A	9	7	1.	35	0.2
249	A	10	7	1.	35	0.2
250	A	6	5	1.	37	0.135
251	A	5	5	1.	37	0.135
252	A	4	4	1.	37	0.108
253	A	4	4	1.	37	0.108
254	A	4	4	1.	37	0.108
255	A	3	3	1.	37	0.081

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
256	A	4	4	1.	37	0.108
257	A	5	4	1.	37	0.108
258	A	7	6	1.	37	0.162
259	A	6	6	1.	37	0.162
260	A	5	5	1.	37	0.135
261	A	5	5	1.	37	0.135
262	A	5	5	1.	37	0.135
263	A	5	5	1.	37	0.135
264	A	4	4	1.	37	0.108
265	A	5	5	1.	37	0.135
266	A	6	5	1.	37	0.135
267	A	8	6	1.	37	0.162
268	A	7	6	1.	37	0.162
269	A	6	5	1.	37	0.135
270	A	6	5	1.	37	0.135
271	A	6	5	1.	37	0.135
272	A	6	6	1.	37	0.162
273	A	6	5	1.	37	0.135
274	A	5	4	1.	37	0.108
275	A	6	5	1.	37	0.135
276	A	7	5	1.	37	0.135
277	A	8	7	1.	37	0.189
278	A	7	7	1.	37	0.189
279	A	6	6	1.	37	0.162
280	A	6	6	1.	37	0.162
281	A	4	4	1.	37	0.108
282	A	5	5	1.	37	0.135
283	A	6	5	1.	37	0.135
284	A	7	7	1.	37	0.189
285	A	6	6	1.	37	0.162
286	A	4	4	1.	37	0.108
287	A	5	5	1.	37	0.135
288	A	6	5	1.	37	0.135
289	A	8	8	1.	37	0.216
290	A	7	7	1.	37	0.189
291	A	4	4	1.	37	0.108
292	A	5	5	1.	37	0.135
293	A	6	6	1.	37	0.162
294	A	7	6	1.	37	0.162
295	A	10	10	1.	27	0.37
296	A	9	9	1.	27	0.333
297	A	9	9	1.	27	0.333
298	A	10	10	1.	27	0.37
299	A	12	12	1.	27	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
300	A	11	11	1.	27	0.407
301	A	11	11	1.	27	0.407
302	A	12	12	1.	27	0.444
303	A	8	5	1.	33	0.152
304	A	8	6	1.	37	0.162
305	A	16	6	1.	88	0.068
306	A	7	6	1.	38	0.158
307	A	7	7	1.	36	0.194
308	A	5	4	1.	30	0.133
309	A	5	5	1.	36	0.139
310	A	4	3	1.	38	0.079
311	A	5	5	1.	38	0.132
312	A	6	6	1.	38	0.158
313	A	7	6	1.	38	0.158
314	A	8	7	1.	40	0.175
315	A	8	8	1.	38	0.21
316	A	7	7	1.	32	0.219
317	A	6	6	1.	38	0.158
318	A	5	4	1.	40	0.1
319	A	5	4	1.	40	0.1
320	A	6	6	1.	40	0.15
321	A	7	7	1.	40	0.175
322	A	8	7	1.	40	0.175
323	A	12	8	1.	38	0.21
324	A	11	7	1.	32	0.219
325	A	7	6	1.	38	0.158
326	A	6	4	1.	40	0.1
327	A	6	5	1.	40	0.125
328	A	6	4	1.	40	0.1
329	A	9	7	1.	40	0.175
330	A	8	7	1.	40	0.175
331	A	9	7	1.	40	0.175
332	A	7	6	1.	40	0.15
333	A	7	7	1.	40	0.175
334	A	6	6	1.	38	0.158
335	A	4	4	1.	32	0.125
336	A	3	3	1.	38	0.079
337	A	5	5	1.	40	0.125
338	A	6	6	1.	40	0.15
339	A	7	6	1.	40	0.15
340	A	8	7	1.	40	0.175
341	A	7	7	1.	40	0.175
342	A	5	5	1.	38	0.132
343	A	3	3	1.	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
344	A	4	4	1.	38	0.105
345	A	6	5	1.	40	0.125
346	A	7	6	1.	40	0.15
347	A	8	6	1.	40	0.15
348	A	9	7	1.	40	0.175
349	A	8	7	1.	40	0.175
350	A	6	6	1.	40	0.15
351	A	4	4	1.	38	0.105
352	A	4	4	1.	32	0.125
353	A	5	4	1.	38	0.105
354	A	7	5	1.	40	0.125
355	A	8	6	1.	40	0.15
356	A	7	6	1.	42	0.143
357	A	6	6	1.	42	0.143
358	A	5	5	1.	42	0.119
359	A	4	4	1.	40	0.1
360	A	3	3	1.	34	0.088
361	A	5	5	1.	40	0.125
362	A	4	4	1.	42	0.095
363	A	5	5	1.	42	0.119
364	A	6	5	1.	42	0.119
365	A	7	7	1.	42	0.167
366	A	6	6	1.	42	0.143
367	A	5	5	1.	40	0.125
368	A	4	4	1.	34	0.118
369	A	6	6	1.	40	0.15
370	A	5	5	1.	42	0.119
371	A	5	5	1.	42	0.119
372	A	6	6	1.	42	0.143
373	A	7	6	1.	42	0.143
374	A	8	7	1.	42	0.167
375	A	7	6	1.	42	0.143
376	A	6	5	1.	40	0.125
377	A	5	4	1.	34	0.118
378	A	7	6	1.	40	0.15
379	A	6	5	1.	42	0.119
380	A	6	6	1.	42	0.143
381	A	6	5	1.	42	0.119
382	A	7	6	1.	42	0.143
383	A	8	6	1.	42	0.143
384	A	8	6	1.	42	0.143
385	A	7	6	1.	42	0.143
386	A	6	6	1.	42	0.143
387	A	5	5	1.	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
388	A	4	4	1.	34	0.118
389	A	6	5	1.	40	0.125
390	A	7	6	1.	42	0.143
391	A	8	6	1.	42	0.143
392	A	9	6	1.	42	0.143
393	A	8	7	1.	42	0.167
394	A	7	7	1.	42	0.167
395	A	6	6	1.	42	0.143
396	A	5	5	1.	40	0.125
397	A	4	4	1.	34	0.118
398	A	7	6	1.	40	0.15
399	A	8	7	1.	42	0.167
400	A	9	7	1.	42	0.167
401	A	8	7	1.	42	0.167
402	A	7	6	1.	42	0.143
403	A	6	6	1.	42	0.143
404	A	5	5	1.	40	0.125
405	A	5	5	1.	34	0.147
406	A	8	6	1.	40	0.15
407	A	9	7	1.	42	0.167
408	A	7	6	1.	39	0.154
409	A	7	7	1.	39	0.18
410	A	6	6	1.	37	0.162
411	A	5	4	1.	31	0.129
412	A	5	5	1.	37	0.135
413	A	5	5	1.	39	0.128
414	A	5	5	1.	39	0.128
415	A	7	6	1.	39	0.154
416	A	7	6	1.	39	0.154
417	A	8	7	1.	41	0.171
418	A	8	8	1.	41	0.195
419	A	7	7	1.	39	0.18
420	A	6	6	1.	33	0.182
421	A	6	6	1.	39	0.154
422	A	5	4	1.	41	0.098
423	A	5	4	1.	41	0.098
424	A	6	6	1.	41	0.146
425	A	7	7	1.	41	0.171
426	A	8	7	1.	41	0.171
427	A	9	7	1.	41	0.171
428	A	12	8	1.	41	0.195
429	A	11	7	1.	39	0.18
430	A	7	6	1.	33	0.182
431	A	7	6	1.	39	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	6	4	1.	41	0.098
433	A	6	5	1.	41	0.122
434	A	6	4	1.	41	0.098
435	A	9	7	1.	41	0.171
436	A	8	7	1.	41	0.171
437	A	9	7	1.	41	0.171
438	A	15	8	1.	41	0.195
439	A	14	7	1.	39	0.18
440	A	8	6	1.	33	0.182
441	A	8	6	1.	39	0.154
442	A	7	4	1.	41	0.098
443	A	7	5	1.	41	0.122
444	A	7	5	1.	41	0.122
445	A	7	4	1.	41	0.098
446	A	12	7	1.	41	0.171
447	A	9	7	1.	41	0.171
448	A	10	7	1.	41	0.171
449	A	7	5	1.	41	0.122
450	A	6	5	1.	41	0.122
451	A	6	6	1.	41	0.146
452	A	4	4	1.	39	0.103
453	A	4	4	1.	33	0.121
454	A	4	4	1.	39	0.103
455	A	5	5	1.	41	0.122
456	A	6	5	1.	41	0.122
457	A	7	5	1.	41	0.122
458	A	7	6	1.	41	0.146
459	A	7	7	1.	41	0.171
460	A	6	6	1.	41	0.146
461	A	4	4	1.07	39	0.103
462	A	3	3	1.	33	0.091
463	A	5	5	1.	39	0.128
464	A	6	6	1.	41	0.146
465	A	7	6	1.	41	0.146
466	A	8	7	1.	41	0.171
467	A	7	7	1.	41	0.171
468	A	5	5	1.	41	0.122
469	A	3	3	1.	39	0.077
470	A	4	4	1.	33	0.121
471	A	6	5	1.	39	0.128
472	A	7	6	1.	41	0.146
473	A	8	6	1.	41	0.146
474	A	9	7	1.	41	0.171
475	A	8	7	1.	41	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
476	A	6	6	1.	41	0.146
477	A	4	4	1.	41	0.098
478	A	4	4	1.	39	0.103
479	A	5	4	1.	33	0.121
480	A	7	5	1.	39	0.128
481	A	8	6	1.	41	0.146
482	A	6	6	1.	43	0.14
483	A	5	5	1.	43	0.116
484	A	4	4	1.	43	0.093
485	A	3	3	1.	41	0.073
486	A	5	5	1.	35	0.143
487	A	5	5	1.	41	0.122
488	A	4	4	1.	43	0.093
489	A	5	5	1.	43	0.116
490	A	6	5	1.	43	0.116
491	A	6	6	1.	43	0.14
492	A	5	5	1.	43	0.116
493	A	4	4	1.	41	0.098
494	A	6	6	1.	35	0.171
495	A	6	6	1.	41	0.146
496	A	5	5	1.	43	0.116
497	A	5	5	1.	43	0.116
498	A	6	6	1.	43	0.14
499	A	7	6	1.	43	0.14
500	A	7	6	1.	43	0.14
501	A	6	5	1.	43	0.116
502	A	5	4	1.	41	0.098
503	A	7	6	1.	35	0.171
504	A	7	6	1.	41	0.146
505	A	6	5	1.	43	0.116
506	A	6	6	1.	43	0.14
507	A	6	5	1.	43	0.116
508	A	7	6	1.	43	0.14
509	A	8	6	1.	43	0.14
510	A	7	6	1.	43	0.14
511	A	6	6	1.	43	0.14
512	A	5	5	1.	43	0.116
513	A	4	4	1.	41	0.098
514	A	6	5	1.	35	0.143
515	A	6	5	1.	41	0.122
516	A	7	6	1.	43	0.14
517	A	8	6	1.	43	0.14
518	A	9	6	1.	43	0.14
519	A	7	6	1.	43	0.14

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
520	A	6	6	1.	43	0.14
521	A	5	5	1.	43	0.116
522	A	4	4	1.12	41	0.098
523	A	6	5	1.	35	0.143
524	A	7	6	1.	41	0.146
525	A	8	6	1.	43	0.14
526	A	9	6	1.	43	0.14
527	A	7	7	1.	43	0.163
528	A	6	6	1.	43	0.14
529	A	5	5	1.	43	0.116
530	A	4	4	1.	41	0.098
531	A	7	6	1.	35	0.171
532	A	8	7	1.	41	0.171
533	A	9	7	1.	43	0.163
534	A	9	7	1.	41	0.171
535	A	8	7	1.	41	0.171
536	A	7	6	1.	41	0.146
537	A	7	6	1.	41	0.146
538	A	7	6	1.	41	0.146
539	A	8	7	1.	41	0.171
540	A	9	7	1.	41	0.171
541	A	10	8	1.	43	0.186
542	A	9	8	1.	43	0.186
543	A	8	7	1.	43	0.163
544	A	8	7	1.	43	0.163
545	A	8	7	1.	43	0.163
546	A	8	7	1.	43	0.163
547	A	9	8	1.	43	0.186
548	A	10	8	1.	43	0.186
549	A	11	8	1.	43	0.186
550	A	10	8	1.	43	0.186
551	A	9	7	1.	43	0.163
552	A	9	7	1.	43	0.163
553	A	9	8	1.	43	0.186
554	A	9	7	1.	43	0.163
555	A	9	7	1.	43	0.163
556	A	10	8	1.	43	0.186
557	A	11	8	1.	43	0.186
558	A	9	6	1.	43	0.14
559	A	8	6	1.	43	0.14
560	A	7	6	1.	43	0.14
561	A	6	5	1.	43	0.116
562	A	7	6	1.	43	0.14
563	A	8	6	1.	43	0.14

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
564	A	9	6	1.	43	0.14
565	A	9	7	1.	43	0.163
566	A	8	7	1.	43	0.163
567	A	7	6	1.	43	0.14
568	A	7	6	1.	43	0.14
569	A	8	7	1.	43	0.163
570	A	9	7	1.	43	0.163
571	A	10	7	1.	43	0.163
572	A	9	7	1.	43	0.163
573	A	8	6	1.	43	0.14
574	A	8	7	1.	43	0.163
575	A	8	6	1.	43	0.14
576	A	9	7	1.	43	0.163
577	A	10	7	1.	43	0.163
578	A	6	5	1.	45	0.111
579	A	5	5	1.	45	0.111
580	A	4	4	1.	45	0.089
581	A	4	4	1.	45	0.089
582	A	4	4	1.	45	0.089
583	A	3	3	1.	45	0.067
584	A	4	4	1.	45	0.089
585	A	5	4	1.	45	0.089
586	A	7	6	1.	45	0.133
587	A	6	6	1.	45	0.133
588	A	5	5	1.	45	0.111
589	A	5	5	1.	45	0.111
590	A	5	5	1.	45	0.111
591	A	5	5	1.	45	0.111
592	A	4	4	1.	45	0.089
593	A	5	5	1.	45	0.111
594	A	6	5	1.	45	0.111
595	A	8	6	1.	45	0.133
596	A	7	6	1.	45	0.133
597	A	6	5	1.	45	0.111
598	A	6	5	1.	45	0.111
599	A	6	5	1.	45	0.111
600	A	6	6	1.	45	0.133
601	A	6	5	1.	45	0.111
602	A	5	4	1.	45	0.089
603	A	6	5	1.	45	0.111
604	A	7	5	1.	45	0.111
605	A	8	7	1.	45	0.156
606	A	7	7	1.	45	0.156
607	A	6	6	1.	45	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
608	A	6	6	1.	45	0.133
609	A	4	4	1.	45	0.089
610	A	5	5	1.	45	0.111
611	A	6	5	1.	45	0.111
612	A	6	6	1.	54	0.111
613	A	8	7	1.	45	0.156
614	A	7	7	1.	45	0.156
615	A	6	6	1.	45	0.133
616	A	4	4	1.	45	0.089
617	A	5	5	1.	45	0.111
618	A	6	5	1.	45	0.111
619	A	8	8	1.	45	0.178
620	A	7	7	1.	45	0.156
621	A	4	4	1.	45	0.089
622	A	5	5	1.	45	0.111
623	A	6	6	1.	45	0.133
624	A	7	6	1.	45	0.133
625	A	10	10	1.	35	0.286
626	A	9	9	1.	35	0.257
627	A	9	9	1.	35	0.257
628	A	10	10	1.	35	0.286
629	A	12	12	1.	35	0.343
630	A	11	11	1.	35	0.314
631	A	11	11	1.	35	0.314
632	A	12	12	1.	35	0.343
633	A	8	5	1.	41	0.122
634	A	8	6	1.	45	0.133
635	A	16	6	1.	102	0.059
636	A	8	8	1.	36	0.222
637	A	7	6	1.	31	0.194
638	A	7	7	1.	31	0.226
639	A	6	6	1.	29	0.207
640	A	5	4	1.	23	0.174
641	A	5	5	1.	29	0.172
642	A	5	5	1.	31	0.161
643	A	5	5	1.	31	0.161
644	A	7	6	1.	31	0.194
645	A	7	6	1.	31	0.194
646	A	8	8	1.	33	0.242
647	A	7	7	1.	31	0.226
648	A	6	5	1.	25	0.2
649	A	6	5	1.	31	0.161
650	A	6	6	1.	33	0.182
651	A	6	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
652	A	6	6	1.	33	0.182
653	A	8	7	1.	33	0.212
654	A	9	8	1.	33	0.242
655	A	8	7	1.	31	0.226
656	A	7	6	1.	25	0.24
657	A	7	6	1.	31	0.194
658	A	7	6	1.	33	0.182
659	A	7	7	1.	33	0.212
660	A	7	7	1.	33	0.212
661	A	7	7	1.	33	0.212
662	A	9	8	1.	33	0.242
663	A	10	8	1.	33	0.242
664	A	9	7	1.	31	0.226
665	A	8	6	1.	25	0.24
666	A	8	6	1.	31	0.194
667	A	8	7	1.	33	0.212
668	A	8	6	1.	33	0.182
669	A	8	7	1.	33	0.212
670	A	8	7	1.	33	0.212
671	A	8	7	1.	33	0.212
672	A	10	8	1.	33	0.242
673	A	8	7	1.	30	0.233
674	A	7	6	1.	30	0.2
675	A	6	5	1.	28	0.179
676	A	8	8	1.	33	0.242
677	A	7	7	1.	33	0.212
678	A	6	6	1.	31	0.194
679	A	6	6	1.	25	0.24
680	A	5	5	1.	31	0.161
681	A	6	6	0.98	33	0.182
682	A	7	6	0.99	33	0.182
683	A	8	6	1.	33	0.182
684	A	8	8	1.	33	0.242
685	A	7	7	1.	33	0.212
686	A	6	6	1.	31	0.194
687	A	5	5	1.	25	0.2
688	A	6	6	1.	31	0.194
689	A	7	6	1.	33	0.182
690	A	8	6	1.	33	0.182
691	A	9	9	1.	33	0.273
692	A	8	8	1.	33	0.242
693	A	7	7	1.	33	0.212
694	A	6	6	1.	31	0.194
695	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
696	A	7	7	1.	31	0.226
697	A	8	7	1.	33	0.212
698	A	9	9	1.	33	0.273
699	A	8	8	1.	33	0.242
700	A	7	7	1.	33	0.212
701	A	7	6	1.	31	0.194
702	A	7	6	1.	25	0.24
703	A	8	7	1.	31	0.226
704	A	9	7	1.	33	0.212
705	A	3	2	1.	30	0.067
706	A	5	5	1.	30	0.167
707	A	6	6	1.	30	0.2
708	A	7	7	1.	30	0.233
709	A	7	7	1.	35	0.2
710	A	6	6	1.	35	0.171
711	A	5	5	1.	33	0.152
712	A	6	6	1.	27	0.222
713	A	6	6	1.	33	0.182
714	A	7	7	1.	35	0.2
715	A	8	7	1.	35	0.2
716	A	9	7	1.	35	0.2
717	A	8	8	1.	35	0.229
718	A	7	6	1.	35	0.171
719	A	6	5	1.	33	0.152
720	A	7	7	1.	27	0.259
721	A	7	7	1.	33	0.212
722	A	7	7	1.	35	0.2
723	A	8	8	1.	35	0.229
724	A	9	8	1.	35	0.229
725	A	9	8	1.	35	0.229
726	A	8	6	1.	35	0.171
727	A	7	5	1.	33	0.152
728	A	8	7	1.	27	0.259
729	A	8	7	1.	33	0.212
730	A	8	8	1.	35	0.229
731	A	8	7	1.	35	0.2
732	A	9	8	1.	35	0.229
733	A	8	8	1.	32	0.25
734	A	7	7	1.	32	0.219
735	A	6	6	1.	35	0.171
736	A	5	5	1.	35	0.143
737	A	4	4	1.	33	0.121
738	A	5	5	1.	27	0.185
739	A	6	6	1.	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
740	A	7	7	1.	35	0.2
741	A	8	7	1.	35	0.2
742	A	6	6	1.	35	0.171
743	A	5	5	1.	35	0.143
744	A	4	4	1.	33	0.121
745	A	6	6	1.	27	0.222
746	A	7	7	1.	33	0.212
747	A	8	8	1.	35	0.229
748	A	6	6	1.	35	0.171
749	A	5	5	1.	35	0.143
750	A	5	5	1.	33	0.152
751	A	7	7	1.	27	0.259
752	A	8	7	1.	33	0.212
753	A	9	8	1.	35	0.229
754	A	8	7	1.	27	0.259
755	A	7	7	1.	32	0.219
756	A	4	4	1.	32	0.125
757	A	7	7	1.	32	0.219
758	A	8	8	1.	32	0.25
759	A	8	7	1.	35	0.2
760	A	11	11	1.	37	0.297
761	A	0	0	0.	0	0.
762	A	0	0	0.	0	0.
763	A	0	0	0.	0	0.
764	A	0	0	0.	0	0.
765	A	8	6	1.	38	0.158
766	A	7	6	1.	38	0.158
767	A	7	7	1.	36	0.194
768	A	5	4	1.	30	0.133
769	A	5	5	1.	36	0.139
770	A	4	3	1.	38	0.079
771	A	5	5	1.	38	0.132
772	A	6	6	1.	38	0.158
773	A	7	6	1.	38	0.158
774	A	8	6	1.	38	0.158
775	A	8	7	1.	40	0.175
776	A	8	8	1.	38	0.21
777	A	6	5	1.	32	0.156
778	A	6	5	1.	38	0.132
779	A	6	5	1.	40	0.125
780	A	6	6	1.	40	0.15
781	A	6	6	1.	40	0.15
782	A	8	7	1.	40	0.175
783	A	8	7	1.	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
784	A	9	8	1.	40	0.2
785	A	9	8	1.	38	0.21
786	A	7	5	1.	32	0.156
787	A	7	6	1.	38	0.158
788	A	7	6	1.	40	0.15
789	A	7	7	1.	40	0.175
790	A	7	7	1.	40	0.175
791	A	7	7	1.	40	0.175
792	A	9	8	1.	40	0.2
793	A	9	9	1.	40	0.225
794	A	8	8	1.	40	0.2
795	A	8	8	1.	38	0.21
796	A	6	6	1.	32	0.188
797	A	5	5	1.	38	0.132
798	A	6	6	1.	40	0.15
799	A	7	7	1.	40	0.175
800	A	8	7	1.	40	0.175
801	A	9	9	1.	40	0.225
802	A	8	8	1.	40	0.2
803	A	7	7	1.	38	0.184
804	A	5	5	1.	32	0.156
805	A	6	6	1.	38	0.158
806	A	7	7	1.	40	0.175
807	A	8	7	1.	40	0.175
808	A	9	9	1.	40	0.225
809	A	8	8	1.	40	0.2
810	A	7	7	1.	38	0.184
811	A	6	5	1.	32	0.156
812	A	7	7	1.	38	0.184
813	A	8	8	1.	40	0.2
814	A	8	8	1.	42	0.19
815	A	7	7	1.	42	0.167
816	A	6	6	1.	40	0.15
817	A	5	5	1.	34	0.147
818	A	6	6	1.	40	0.15
819	A	7	7	1.	42	0.167
820	A	8	8	1.	42	0.19
821	A	9	8	1.	42	0.19
822	A	8	7	1.	42	0.167
823	A	7	6	1.	40	0.15
824	A	6	5	1.	34	0.147
825	A	7	7	1.	40	0.175
826	A	7	7	1.	42	0.167
827	A	8	8	1.	42	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
828	A	9	8	1.	42	0.19
829	A	9	7	1.	42	0.167
830	A	8	6	1.	40	0.15
831	A	7	5	1.	34	0.147
832	A	8	8	1.	40	0.2
833	A	8	8	1.	42	0.19
834	A	8	8	1.	42	0.19
835	A	9	9	1.	42	0.214
836	A	10	9	1.	42	0.214
837	A	7	7	1.	42	0.167
838	A	6	6	1.	42	0.143
839	A	5	5	1.	40	0.125
840	A	4	4	1.	34	0.118
841	A	4	4	1.	40	0.1
842	A	7	7	1.	42	0.167
843	A	7	7	1.	42	0.167
844	A	6	6	1.	42	0.143
845	A	5	5	1.	40	0.125
846	A	5	5	1.	34	0.147
847	A	7	7	1.	40	0.175
848	A	8	8	1.	42	0.19
849	A	7	7	1.	42	0.167
850	A	6	6	1.	42	0.143
851	A	6	6	1.	40	0.15
852	A	6	5	1.	34	0.147
853	A	8	8	1.	40	0.2
854	A	7	5	1.	34	0.147
855	A	6	6	1.	42	0.143
856	A	8	8	1.	44	0.182
857	A	8	5	1.	34	0.147
858	A	8	5	1.	34	0.147
859	A	8	5	1.	34	0.147
860	A	8	5	1.	34	0.147
861	A	7	6	1.	39	0.154
862	A	7	7	1.	39	0.18
863	A	6	6	1.	37	0.162
864	A	5	4	1.	31	0.129
865	A	5	5	1.	37	0.135
866	A	5	5	1.	39	0.128
867	A	5	5	1.	39	0.128
868	A	7	6	1.	39	0.154
869	A	7	6	1.	39	0.154
870	A	8	8	1.21	41	0.195
871	A	7	7	1.	39	0.18

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
872	A	6	5	1.	33	0.152
873	A	6	5	1.	39	0.128
874	A	6	6	1.	41	0.146
875	A	6	6	1.	41	0.146
876	A	6	6	1.	41	0.146
877	A	8	7	1.	41	0.171
878	A	9	8	1.	41	0.195
879	A	8	7	1.	39	0.18
880	A	7	5	1.	33	0.152
881	A	7	6	1.	39	0.154
882	A	7	5	1.	41	0.122
883	A	7	6	1.	41	0.146
884	A	7	6	1.	41	0.146
885	A	7	6	1.	41	0.146
886	A	9	7	1.	41	0.171
887	A	10	8	1.	41	0.195
888	A	9	7	1.	39	0.18
889	A	8	5	1.	33	0.152
890	A	8	6	1.	39	0.154
891	A	8	6	1.	41	0.146
892	A	8	5	1.	41	0.122
893	A	8	6	1.	41	0.146
894	A	8	6	1.	41	0.146
895	A	8	6	1.	41	0.146
896	A	10	7	1.	41	0.171
897	A	8	7	1.	48	0.146
898	A	7	6	1.	48	0.125
899	A	6	5	1.	46	0.109
900	A	8	8	1.	41	0.195
901	A	7	7	1.	41	0.171
902	A	6	6	1.	39	0.154
903	A	6	6	1.	33	0.182
904	A	5	5	1.	39	0.128
905	A	6	5	1.	41	0.122
906	A	7	5	1.	41	0.122
907	A	8	5	1.	41	0.122
908	A	9	9	1.	41	0.22
909	A	8	8	1.	41	0.195
910	A	7	7	1.	41	0.171
911	A	6	6	1.	39	0.154
912	A	5	5	1.	33	0.152
913	A	6	6	1.	39	0.154
914	A	7	6	1.	41	0.146
915	A	8	6	1.	41	0.146

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
916	A	9	8	1.	41	0.195
917	A	8	8	1.	41	0.195
918	A	7	7	1.	41	0.171
919	A	6	6	1.	39	0.154
920	A	6	5	1.	33	0.152
921	A	7	6	1.	39	0.154
922	A	8	6	1.	41	0.146
923	A	9	8	1.	41	0.195
924	A	8	8	1.	41	0.195
925	A	7	7	1.	41	0.171
926	A	7	6	1.	39	0.154
927	A	7	5	1.	33	0.152
928	A	8	6	1.	39	0.154
929	A	9	6	1.	41	0.146
930	A	3	2	1.	48	0.042
931	A	5	5	1.	48	0.104
932	A	6	6	1.	48	0.125
933	A	7	7	1.	48	0.146
934	A	8	7	1.	48	0.146
935	A	7	7	1.	43	0.163
936	A	6	6	1.	43	0.14
937	A	5	5	1.	41	0.122
938	A	6	6	1.	35	0.171
939	A	6	6	1.	41	0.146
940	A	7	7	1.	43	0.163
941	A	8	7	1.	43	0.163
942	A	8	7	1.	43	0.163
943	A	7	6	1.	43	0.14
944	A	6	5	1.	41	0.122
945	A	7	6	1.	35	0.171
946	A	7	7	1.	41	0.171
947	A	7	6	1.	43	0.14
948	A	8	7	1.	43	0.163
949	A	9	7	1.	43	0.163
950	A	8	6	1.	43	0.14
951	A	7	5	1.	41	0.122
952	A	8	6	1.	35	0.171
953	A	8	7	1.	41	0.171
954	A	8	7	1.	43	0.163
955	A	8	6	1.	43	0.14
956	A	9	7	1.	43	0.163
957	A	10	7	1.	43	0.163
958	A	6	6	1.	43	0.14
959	A	5	5	1.	43	0.116

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
960	A	4	4	1.	41	0.098
961	A	5	5	1.	35	0.143
962	A	6	6	1.	41	0.146
963	A	7	6	1.	43	0.14
964	A	6	6	1.	43	0.14
965	A	5	5	1.	43	0.116
966	A	4	4	1.	41	0.098
967	A	6	6	1.	35	0.171
968	A	7	7	1.	41	0.171
969	A	8	7	1.	43	0.163
970	A	6	6	1.	43	0.14
971	A	5	5	1.	43	0.116
972	A	5	5	1.	41	0.122
973	A	7	6	1.	35	0.171
974	A	8	7	1.	41	0.171
975	A	8	8	1.	50	0.16
976	A	7	7	1.	50	0.14
977	A	6	6	1.	50	0.12
978	A	4	4	1.	50	0.08
979	A	7	7	1.	50	0.14
980	A	8	8	1.	50	0.16
981	A	10	7	1.	41	0.171
982	A	9	7	1.	41	0.171
983	A	8	7	1.	41	0.171
984	A	7	6	1.	41	0.146
985	A	7	6	1.	41	0.146
986	A	7	6	1.	41	0.146
987	A	8	7	1.	41	0.171
988	A	9	7	1.	41	0.171
989	A	10	7	1.	41	0.171
990	A	10	8	1.	43	0.186
991	A	9	8	1.	43	0.186
992	A	8	7	1.	43	0.163
993	A	8	7	1.	43	0.163
994	A	8	7	1.	43	0.163
995	A	8	7	1.	43	0.163
996	A	9	8	1.	43	0.186
997	A	10	8	1.	43	0.186
998	A	9	7	1.	43	0.163
999	A	9	8	1.	43	0.186
1000	A	9	7	1.	43	0.163
1001	A	9	7	1.	43	0.163
1002	A	9	7	1.	43	0.163
1003	A	10	8	1.	43	0.186

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1004	A	11	8	1.	43	0.186
1005	A	10	7	1.	43	0.163
1006	A	10	8	1.	43	0.186
1007	A	10	8	1.	43	0.186
1008	A	10	7	1.	43	0.163
1009	A	10	7	1.	43	0.163
1010	A	10	7	1.	43	0.163
1011	A	11	8	1.	43	0.186
1012	A	11	8	1.	43	0.186
1013	A	10	8	1.	43	0.186
1014	A	9	8	1.	43	0.186
1015	A	8	7	1.	43	0.163
1016	A	9	8	1.	43	0.186
1017	A	10	8	1.	43	0.186
1018	A	11	8	1.	43	0.186
1019	A	11	9	1.	43	0.209
1020	A	10	9	1.	43	0.209
1021	A	9	8	1.	43	0.186
1022	A	9	8	1.	43	0.186
1023	A	10	9	1.	43	0.209
1024	A	11	9	1.	43	0.209
1025	A	12	9	1.	43	0.209
1026	A	11	9	1.	43	0.209
1027	A	10	8	1.	43	0.186
1028	A	10	9	1.	43	0.209
1029	A	10	8	1.	43	0.186
1030	A	11	9	1.	43	0.209
1031	A	14	13	1.	45	0.289
1032	A	13	13	1.	45	0.289
1033	A	12	12	1.	45	0.267
1034	A	12	12	1.	45	0.267
1035	A	9	9	1.	45	0.2
1036	A	10	9	1.	45	0.2
1037	A	11	9	1.	45	0.2
1038	A	15	13	1.	45	0.289
1039	A	14	13	1.	45	0.289
1040	A	13	12	1.	45	0.267
1041	A	13	13	1.	45	0.289
1042	A	13	12	1.	45	0.267
1043	A	10	9	1.	45	0.2
1044	A	11	9	1.	45	0.2
1045	A	15	13	1.	45	0.289
1046	A	14	12	1.	45	0.267
1047	A	14	13	1.	45	0.289

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1048	A	14	13	1.	45	0.289
1049	A	14	12	1.	45	0.267
1050	A	11	9	1.	45	0.2
1051	A	12	9	1.	45	0.2
1052	A	13	12	1.	45	0.267
1053	A	12	12	1.	45	0.267
1054	A	11	11	1.	45	0.244
1055	A	8	8	1.	45	0.178
1056	A	9	8	1.	45	0.178
1057	A	10	8	1.	45	0.178
1058	A	13	13	1.	54	0.241
1059	A	13	13	1.	45	0.289
1060	A	12	12	1.	45	0.267
1061	A	8	8	1.	45	0.178
1062	A	9	9	1.	45	0.2
1063	A	10	9	1.	45	0.2
1064	A	14	13	1.	45	0.289
1065	A	13	12	1.	45	0.267
1066	A	9	9	1.	45	0.2
1067	A	9	8	1.	45	0.178
1068	A	10	9	1.	45	0.2
1069	A	11	9	1.	45	0.2
1070	A	0	0	0.	0	0.
1071	A	0	0	0.	0	0.
1072	A	0	0	0.	0	0.
1073	A	0	0	0.	0	0.
1074	A	0	0	0.	0	0.
1075	A	4	4	1.	23	0.174
1076	A	4	4	1.	23	0.174
1077	A	3	3	1.	23	0.13
1078	A	3	3	1.	23	0.13
1079	A	3	3	1.	23	0.13
1080	A	3	3	1.	23	0.13
1081	A	4	4	1.	23	0.174
1082	A	4	4	1.	23	0.174
1083	A	8	7	1.	33	0.212
1084	A	7	7	1.	33	0.212
1085	A	6	6	1.	33	0.182
1086	A	6	6	1.	33	0.182
1087	A	6	6	1.	33	0.182
1088	A	7	7	1.	33	0.212
1089	A	8	7	1.	33	0.212
1090	A	10	9	1.	35	0.257
1091	A	9	9	1.	35	0.257

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1092	A	8	8	1.	35	0.229
1093	A	8	8	1.	35	0.229
1094	A	8	8	1.	35	0.229
1095	A	8	8	1.	35	0.229
1096	A	9	9	1.	35	0.257
1097	A	10	9	1.	35	0.257
1098	A	11	9	1.	35	0.257
1099	A	10	9	1.	35	0.257
1100	A	9	8	1.	35	0.229
1101	A	9	8	1.	35	0.229
1102	A	9	9	1.	35	0.257
1103	A	9	8	1.	35	0.229
1104	A	9	8	1.	35	0.229
1105	A	10	9	1.	35	0.257
1106	A	11	9	1.	35	0.257
1107	A	8	6	1.	35	0.171
1108	A	7	6	1.	35	0.171
1109	A	6	6	1.	35	0.171
1110	A	5	5	1.	35	0.143
1111	A	6	6	1.	35	0.171
1112	A	7	6	1.	35	0.171
1113	A	8	6	1.	35	0.171
1114	A	8	7	1.	35	0.2
1115	A	7	7	1.	35	0.2
1116	A	6	6	1.	35	0.171
1117	A	6	6	1.	35	0.171
1118	A	7	7	1.	35	0.2
1119	A	8	7	1.	35	0.2
1120	A	9	7	1.	35	0.2
1121	A	8	7	1.	35	0.2
1122	A	7	6	1.	35	0.171
1123	A	7	7	1.	35	0.2
1124	A	7	6	1.	35	0.171
1125	A	8	7	1.	35	0.2
1126	A	9	7	1.	35	0.2
1127	A	6	5	1.	37	0.135
1128	A	5	5	1.	37	0.135
1129	A	4	4	1.	37	0.108
1130	A	5	5	1.	37	0.135
1131	A	5	5	1.	37	0.135
1132	A	5	5	1.	37	0.135
1133	A	6	6	1.	37	0.162
1134	A	7	6	1.	37	0.162
1135	A	7	6	1.	37	0.162

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1136	A	6	6	1.	37	0.162
1137	A	5	5	1.	37	0.135
1138	A	6	6	1.	37	0.162
1139	A	6	6	1.	37	0.162
1140	A	6	6	1.	37	0.162
1141	A	6	6	1.	37	0.162
1142	A	7	7	1.	37	0.189
1143	A	8	7	1.	37	0.189
1144	A	8	6	1.	37	0.162
1145	A	7	6	1.	37	0.162
1146	A	6	5	1.	37	0.135
1147	A	7	6	1.	37	0.162
1148	A	7	7	1.	37	0.189
1149	A	7	6	1.	37	0.162
1150	A	7	6	1.	37	0.162
1151	A	7	6	1.	37	0.162
1152	A	8	7	1.	37	0.189
1153	A	9	7	1.	37	0.189
1154	A	7	6	1.	37	0.162
1155	A	6	6	1.	37	0.162
1156	A	5	5	1.	37	0.135
1157	A	7	7	1.	37	0.189
1158	A	7	7	1.	37	0.189
1159	A	8	8	1.	37	0.216
1160	A	9	8	1.	37	0.216
1161	A	7	6	1.	37	0.162
1162	A	6	6	1.	37	0.162
1163	A	5	5	1.	37	0.135
1164	A	7	7	1.	37	0.189
1165	A	8	8	1.	37	0.216
1166	A	9	8	1.	37	0.216
1167	A	8	7	1.	37	0.189
1168	A	7	7	1.	37	0.189
1169	A	6	6	1.	37	0.162
1170	A	5	5	1.	37	0.135
1171	A	8	8	1.	37	0.216
1172	A	9	9	1.	37	0.243
1173	A	7	5	1.	30	0.167
1174	A	6	5	1.	30	0.167
1175	A	5	5	1.	30	0.167
1176	A	4	4	1.	30	0.133
1177	A	5	5	1.	30	0.167
1178	A	6	5	1.	30	0.167
1179	A	7	5	1.	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1180	A	7	6	1.	31	0.194
1181	A	6	6	1.	31	0.194
1182	A	5	5	1.	31	0.161
1183	A	5	5	1.	31	0.161
1184	A	6	6	1.	31	0.194
1185	A	7	6	1.	31	0.194
1186	A	8	6	1.	31	0.194
1187	A	8	7	1.	41	0.171
1188	A	7	7	1.	41	0.171
1189	A	6	6	1.	41	0.146
1190	A	6	6	1.	41	0.146
1191	A	6	6	1.	41	0.146
1192	A	7	7	1.	41	0.171
1193	A	8	7	1.	41	0.171
1194	A	10	9	1.	43	0.209
1195	A	9	9	1.	43	0.209
1196	A	8	8	1.	43	0.186
1197	A	8	8	1.	43	0.186
1198	A	8	8	1.	43	0.186
1199	A	8	8	1.	43	0.186
1200	A	9	9	1.	43	0.209
1201	A	10	9	1.	43	0.209
1202	A	10	9	1.	43	0.209
1203	A	9	8	1.	43	0.186
1204	A	9	8	1.	43	0.186
1205	A	9	9	1.	43	0.209
1206	A	9	8	1.	43	0.186
1207	A	9	8	1.	43	0.186
1208	A	10	9	1.	43	0.209
1209	A	11	9	1.	43	0.209
1210	A	10	8	1.	43	0.186
1211	A	10	8	1.	43	0.186
1212	A	10	9	1.	43	0.209
1213	A	10	9	1.	43	0.209
1214	A	10	8	1.	43	0.186
1215	A	10	8	1.	43	0.186
1216	A	11	9	1.	43	0.209
1217	A	8	6	1.	43	0.14
1218	A	7	6	1.	43	0.14
1219	A	6	6	1.	43	0.14
1220	A	5	5	1.	43	0.116
1221	A	6	6	1.	43	0.14
1222	A	7	6	1.	43	0.14
1223	A	8	6	1.	43	0.14

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1224	A	9	7	1.	43	0.163
1225	A	8	7	1.	43	0.163
1226	A	7	7	1.	43	0.163
1227	A	6	6	1.	43	0.14
1228	A	6	6	1.	43	0.14
1229	A	7	7	1.	43	0.163
1230	A	8	7	1.	43	0.163
1231	A	9	7	1.	43	0.163
1232	A	9	7	1.	43	0.163
1233	A	8	7	1.	43	0.163
1234	A	7	6	1.	43	0.14
1235	A	7	7	1.	43	0.163
1236	A	7	6	1.	43	0.14
1237	A	8	7	1.	43	0.163
1238	A	9	7	1.	43	0.163
1239	A	9	7	1.	43	0.163
1240	A	8	6	1.	43	0.14
1241	A	8	7	1.	43	0.163
1242	A	8	7	1.	43	0.163
1243	A	8	6	1.	43	0.14
1244	A	9	7	1.	43	0.163
1245	A	6	5	1.	45	0.111
1246	A	5	5	1.	45	0.111
1247	A	4	4	1.	45	0.089
1248	A	5	5	1.	45	0.111
1249	A	5	5	1.	45	0.111
1250	A	5	5	1.	45	0.111
1251	A	6	6	1.	45	0.133
1252	A	7	6	1.	45	0.133
1253	A	7	6	1.	45	0.133
1254	A	6	6	1.	45	0.133
1255	A	5	5	1.	45	0.111
1256	A	6	6	1.	45	0.133
1257	A	6	6	1.	45	0.133
1258	A	6	6	1.	45	0.133
1259	A	6	6	1.	45	0.133
1260	A	7	7	1.	45	0.156
1261	A	8	7	1.	45	0.156
1262	A	8	6	1.	45	0.133
1263	A	7	6	1.	45	0.133
1264	A	6	5	1.	45	0.111
1265	A	7	6	1.	45	0.133
1266	A	7	7	1.	45	0.156
1267	A	7	6	1.	45	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1268	A	7	6	1.	45	0.133
1269	A	7	6	1.	45	0.133
1270	A	8	7	1.	45	0.156
1271	A	9	7	1.	45	0.156
1272	A	7	6	1.	45	0.133
1273	A	6	6	1.	45	0.133
1274	A	5	5	1.	45	0.111
1275	A	7	7	1.	45	0.156
1276	A	7	7	1.	45	0.156
1277	A	8	8	1.	45	0.178
1278	A	9	8	1.	45	0.178
1279	A	7	7	1.	54	0.13
1280	A	7	6	1.	45	0.133
1281	A	6	6	1.	45	0.133
1282	A	5	5	1.	45	0.111
1283	A	7	7	1.	45	0.156
1284	A	8	8	1.	45	0.178
1285	A	9	8	1.	45	0.178
1286	A	8	7	1.	45	0.156
1287	A	7	7	1.	45	0.156
1288	A	6	6	1.	45	0.133
1289	A	5	5	1.	45	0.111
1290	A	8	8	1.	45	0.178
1291	A	9	9	1.	45	0.2
1292	A	8	7	1.	41	0.171
1293	A	7	7	1.	41	0.171
1294	A	6	6	1.	41	0.146
1295	A	6	6	1.	41	0.146
1296	A	6	6	1.	41	0.146
1297	A	7	7	1.	41	0.171
1298	A	8	7	1.	41	0.171
1299	A	8	8	1.	43	0.186
1300	A	7	7	1.	43	0.163
1301	A	7	7	1.	43	0.163
1302	A	7	7	1.	43	0.163
1303	A	7	7	1.	43	0.163
1304	A	8	8	1.	43	0.186
1305	A	9	8	1.	43	0.186
1306	A	8	7	1.	43	0.163
1307	A	8	8	1.	43	0.186
1308	A	8	7	1.	43	0.163
1309	A	8	7	1.	43	0.163
1310	A	8	7	1.	43	0.163
1311	A	9	8	1.	43	0.186

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1312	A	9	7	1.	43	0.163
1313	A	9	8	1.	43	0.186
1314	A	9	8	1.	43	0.186
1315	A	9	7	1.	43	0.163
1316	A	9	7	1.	43	0.163
1317	A	9	7	1.	43	0.163
1318	A	8	7	1.	43	0.163
1319	A	7	7	1.	43	0.163
1320	A	6	6	1.	43	0.14
1321	A	7	7	1.	43	0.163
1322	A	8	7	1.	43	0.163
1323	A	9	7	1.	43	0.163
1324	A	8	8	1.	43	0.186
1325	A	7	7	1.	43	0.163
1326	A	7	7	1.	43	0.163
1327	A	8	7	1.	43	0.163
1328	A	9	7	1.	43	0.163
1329	A	9	8	1.	43	0.186
1330	A	8	7	1.	43	0.163
1331	A	8	8	1.	43	0.186
1332	A	8	7	1.	43	0.163
1333	A	9	7	1.	43	0.163
1334	A	12	10	1.	45	0.222
1335	A	11	10	1.	45	0.222
1336	A	10	10	1.	45	0.222
1337	A	13	13	1.	45	0.289
1338	A	13	13	1.	45	0.289
1339	A	14	14	1.	45	0.311
1340	A	15	14	1.	45	0.311
1341	A	12	10	1.	45	0.222
1342	A	11	10	1.	45	0.222
1343	A	14	13	1.	45	0.289
1344	A	14	14	1.	45	0.311
1345	A	14	13	1.	45	0.289
1346	A	15	14	1.	45	0.311
1347	A	16	14	1.	45	0.311
1348	A	13	10	1.	45	0.222
1349	A	12	10	1.	45	0.222
1350	A	15	13	1.	45	0.289
1351	A	15	14	1.	45	0.311
1352	A	15	14	1.	45	0.311
1353	A	15	13	1.	45	0.289
1354	A	16	14	1.	45	0.311
1355	A	17	14	1.	45	0.311

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1356	A	11	9	1.	45	0.2
1357	A	10	9	1.	45	0.2
1358	A	9	9	1.	45	0.2
1359	A	12	12	1.	45	0.267
1360	A	13	13	1.	45	0.289
1361	A	14	13	1.	45	0.289
1362	A	13	13	1.	54	0.241
1363	A	11	10	1.	45	0.222
1364	A	10	10	1.	45	0.222
1365	A	9	9	1.	45	0.2
1366	A	13	13	1.	45	0.289
1367	A	14	14	1.	45	0.311
1368	A	12	10	1.	45	0.222
1369	A	11	10	1.	45	0.222
1370	A	10	9	1.	45	0.2
1371	A	10	10	1.	45	0.222
1372	A	14	13	1.	45	0.289
1373	A	15	14	1.	45	0.311

Chapter 3

Listing of integrals

3.1 $\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+ \ldots}{10b^2d}$$

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c +
 d*x])^(7/3)*Tan[c + d*x])/(10*b^2*d)
```

Rubi [A] time = 0.0801958, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4046, 3772, 2643}

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{7/3}}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c +
 d*x])^(7/3)*Tan[c + d*x])/(10*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{7/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$

$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10b^2d} + \frac{(10A+7C)\int (b\sec(c+dx)}{10b^2}$$

$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10b^2d} + \frac{\left((10A+7C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\,\mathrm{s}}\right.}{1(}$$

$$= \frac{3(10A+7C)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(}{40bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.09117, size = 189, normalized size = 1.99

$$\frac{3\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(\sin(c+dx)\sec^{\frac{10}{3}}(c+dx)((10A+7C)\cos(2(c+dx))+5(2A+3C))-2i\sqrt[3]{2}(10A\right.}{40d\sec^{\frac{7}{3}}(c+dx)(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2)*((-2*I)*2^(1/3)*(10*A + 7*
C)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(1 + E^((2*I)*(c + d*x
)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c + d*x))] + (5*(2*A
+ 3*C) + (10*A + 7*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^(10/3)*Sin[c + d*x]))/
(40*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/3))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\sqrt[3]{b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec\left(c+dx\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*(A + C*sec(c + d*x)**2)*sec(c + d*x)**2, x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.2 $\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=92

$$\frac{3(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{4/3}}{7bd}$$

```
[Out] (3*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x
])^(4/3)*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.0801193, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{3(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{4/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x
])^(4/3)*Tan[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx}{b}$$
$$= \frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7bd} + \frac{(7A+4C)\int (b\sec(c+dx}{7b}$$
$$= \frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7bd} + \frac{\left((7A+4C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b}\right.}{}$$
$$= \frac{3(7A+4C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c}{7d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.50716, size = 185, normalized size = 2.01

$$\frac{3ie^{i(c+dx)}\cos^3(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)\left((7A+4C)\left(1+e^{2i(c+dx)}\right)^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\right.\right.}{7bd\left(1+e^{2i(c+dx)}\right)^2(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (((3*I)/7)*E^(I*(c + d*x))*Cos[c + d*x]^3*(-14*A*(1 + E^((2*I)*(c + d*x)))^
2 - 4*C*(1 + 5*E^((2*I)*(c + d*x)) + 2*E^((4*I)*(c + d*x))) + (7*A + 4*C)*(
1 + E^((2*I)*(c + d*x)))^(7/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(
c + d*x))])*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2))/(b*d*(1 + E^((2*
I)*(c + d*x)))^2*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int \sec(dx+c)\sqrt[3]{b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[3]{b\sec\left(c+dx\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*(A + C*sec(c + d*x)**2)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.3 $\int \sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=88

$$\frac{3C\tan(c+dx)\sqrt[3]{b\sec(c+dx)}}{4d}-\frac{3b(4A+C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*b*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*
x])/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x
])^(1/3)*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0641493, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4046, 3772, 2643}

$$\frac{3C\tan(c+dx)\sqrt[3]{b\sec(c+dx)}}{4d}-\frac{3b(4A+C)\sin(c+dx)\,_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-3*b*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*
x])/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x
])^(1/3)*Tan[c + d*x])/(4*d)
```


Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{3C\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4d}+\frac{1}{4}(4A+C)\int \sqrt[3]{b\sec(c+dx)}\,dx$$
$$= \frac{3C\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4d}+\frac{1}{4}\left((4A+C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right.$$
$$= -\frac{3(4A+C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+d}{8d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.17849, size = 162, normalized size = 1.84

$$\frac{3\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(C\sin(c+dx)\sec^{\frac{4}{3}}(c+dx)-i\sqrt[3]{2}(4A+C)\sqrt[3]{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt[3]{1+e^{2i(c+dx)}}\text{Hypergeome}\right.}{2d\sec^{\frac{7}{3}}(c+dx)(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2)*((-I)*2^(1/3)*(4*A + C)*(E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(1 + E^((2*I)*(c + d*x)))^(
1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c + d*x))] + C*Sec[c + d*x
]^(4/3)*Sin[c + d*x]))/(2*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/
3))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[3]{b\sec\left(c+dx\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*(A + C*sec(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3), x)
```

3.4 $\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=89

$$\frac{3bC\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}-\frac{3b^2(A-2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

```
[Out] (-3*b^2*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*C*Tan[c + d
*x])/(d*(b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.0874223, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{3bC\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}-\frac{3b^2(A-2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*b^2*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*C*Tan[c + d
*x])/(d*(b*Sec[c + d*x])^(2/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = b\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}dx$$
$$= \frac{3bC\tan(c+dx)}{d(b\sec(c+dx))^{2/3}} + (b(A-2C))\int \frac{1}{(b\sec(c+dx))^{2/3}}dx$$
$$= \frac{3bC\tan(c+dx)}{d(b\sec(c+dx))^{2/3}} + \left(b(A-2C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+d\ldots}\right.$$
$$= -\frac{3(A-2C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(\ldots}}{5d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.137783, size = 93, normalized size = 1.04

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{4/3}\left(2A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)-C\ldots\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x]*(2*A*Cos[c + d*x]^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[
c + d*x]^2] - C*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \cos(dx+c)\sqrt[3]{b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)\sec(dx+c)^2+A\cos(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*(b*sec(d*x + c))^
(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec\left(dx+c\right)^{2} + A\right) \left(b \sec\left(dx+c\right)\right)^{\frac{1}{3}} \cos\left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.5 $\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=93

$$\frac{3Ab^2\tan(c+dx)}{5d(b\sec(c+dx))^{5/3}}-\frac{3b(2A+5C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*b*(2*A + 5*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(10*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3))
```

Rubi [A] time = 0.104483, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4045, 3772, 2643}

$$\frac{3Ab^2\tan(c+dx)}{5d(b\sec(c+dx))^{5/3}}-\frac{3b(2A+5C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*b*(2*A + 5*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(10*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx &= b^2\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/3}}\,dx \\
&= \frac{3Ab^2\tan(c+dx)}{5d(b\sec(c+dx))^{5/3}}+\frac{1}{5}(2A+5C)\int \sqrt[3]{b\sec(c+dx)}\,dx \\
&= \frac{3Ab^2\tan(c+dx)}{5d(b\sec(c+dx))^{5/3}}+\frac{1}{5}\left((2A+5C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+\right. \\
&= -\frac{3(2A+5C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+}{10d\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.10182, size = 89, normalized size = 0.96

$$
-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(c+dx)\right)-5C\text{Hype}\right.}{5d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x]*(A*Cos[c + d*x]^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[c
+ d*x]^2] - 5*C*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(5*d)
```

Maple [F] time = 0.311, size = 0, normalized size = 0.

$$
\int (\cos(dx+c))^2\sqrt[3]{b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+A\cos\left(dx+c\right)^2\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*(b*sec(d*x +
c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.6 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{3(13A+10C)\sin(c+dx)(b\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{91bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+d}{13b^2d}$$

```
[Out] (3*(13*A + 10*C)*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(7/3)*Sin[c + d*x])/(91*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c
 + d*x])^(10/3)*Tan[c + d*x])/(13*b^2*d)
```

Rubi [A] time = 0.0857016, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4046, 3772, 2643}

$$\frac{3(13A+10C)\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{91bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{10/3}}{13b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(13*A + 10*C)*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(7/3)*Sin[c + d*x])/(91*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c
 + d*x])^(10/3)*Tan[c + d*x])/(13*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{10/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$
$$= \frac{3C(b\sec(c+dx))^{10/3}\tan(c+dx)}{13b^2d} + \frac{(13A+10C)\int(b\sec(}{13b^2}$$
$$= \frac{3C(b\sec(c+dx))^{10/3}\tan(c+dx)}{13b^2d} + \frac{\left((13A+10C)\sqrt[3]{\frac{\cos(c+}{b}}\right.}{}$$
$$= \frac{3C(b\sec(c+dx))^{10/3}\tan(c+dx)}{13b^2d} + \frac{3b(13A+10C)\,{}_2F_1\left(-\right.}{}$$

Mathematica [C] time = 1.74144, size = 236, normalized size = 2.48

$$\frac{12ie^{i(c+dx)}\cos^3(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)\left((13A+10C)\left(1+e^{2i(c+dx)}\right)^{13/3}\text{Hypergeometric2F1}\left(\right.\right.}{91d\left(1+e^{2i(c+dx)}\right)^4\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (((12*I)/91)*E^(I*(c + d*x))*Cos[c + d*x]^3*(-13*A*(1 + E^((2*I)*(c + d*x))
)^2*(1 + 5*E^((2*I)*(c + d*x)) + 2*E^((4*I)*(c + d*x))) - 2*C*(5 + 21*E^((2
*I)*(c + d*x)) + 79*E^((4*I)*(c + d*x)) + 45*E^((6*I)*(c + d*x)) + 10*E^((8
*I)*(c + d*x))) + (13*A + 10*C)*(1 + E^((2*I)*(c + d*x)))^(13/3)*Hypergeome
tric2F1[1/3, 2/3, 5/3, -E^((2*I)*(c + d*x))])*(b*Sec[c + d*x])^(4/3)*(A + C
*Sec[c + d*x]^2))/(d*(1 + E^((2*I)*(c + d*x)))^4*(A + 2*C + A*Cos[2*(c + d*
x)]))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\sec(dx+c))^{\frac{4}{3}} \left(A + C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{5}+Ab\sec\left(dx+c\right)^{3}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^5 + A*b*sec(d*x + c)^3)*(b*sec(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.7 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=92

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{40d\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+c}{10bd}$$

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d
*x])^(7/3)*Tan[c + d*x])/(10*b*d)
```

Rubi [A] time = 0.0804184, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{40d\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{7/3}}{10bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d
*x])^(7/3)*Tan[c + d*x])/(10*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{7/3}\left(A+C\sec^2(c+dx)\right)dx}{b}$$
$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10bd} + \frac{(10A+7C)\int (b\sec(c+dx$$
$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10bd} + \frac{\left((10A+7C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b}\right.}{}$$
$$= \frac{3(10A+7C)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin}{40d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.5108, size = 192, normalized size = 2.09

$$\frac{3(b\sec(c+dx))^{7/3}\left(A+C\sec^2(c+dx)\right)\left(\sin(c+dx)\sec^{\frac{10}{3}}(c+dx)((10A+7C)\cos(2(c+dx))+5(2A+3C))-2i\sqrt[3]{2}(10A\right.}{40bd\sec^{\frac{13}{3}}(c+dx)(A\cos(2(c+dx))+A+2C}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(7/3)*(A + C*Sec[c + d*x]^2)*((-2*I)*2^(1/3)*(10*A + 7*
C)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(1 + E^((2*I)*(c + d*x
)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c + d*x))] + (5*(2*A
+ 3*C) + (10*A + 7*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^(10/3)*Sin[c + d*x]))/
(40*b*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(13/3))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{4}+Ab\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^4 + A*b*sec(d*x + c)^2)*(b*sec(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.8 $\int (b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=90

$$\frac{3b(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{4/3}}{7d}$$

```
[Out] (3*b*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c
 + d*x])^(1/3)*Sin[c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d
*x])^(4/3)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.0838983, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4046, 3772, 2643}

$$\frac{3b(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{4/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*b*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c
 + d*x])^(1/3)*Sin[c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d
*x])^(4/3)*Tan[c + d*x])/(7*d)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7d}+\frac{1}{7}(7A+4C)\int (b\sec(c+dx))^{4/3}dx$$
$$= \frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7d}+\frac{1}{7}\left((7A+4C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec}\right.$$
$$= \frac{3b(7A+4C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{7d\sqrt{\sin^2(c+dx)}}+$$

Mathematica [C] time = 1.03609, size = 182, normalized size = 2.02

$$\frac{3ie^{i(c+dx)}\cos^3(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)\left((7A+4C)\left(1+e^{2i(c+dx)}\right)^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\right.\right.}{7d\left(1+e^{2i(c+dx)}\right)^2(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (((3*I)/7)*E^(I*(c + d*x))*Cos[c + d*x]^3*(-14*A*(1 + E^((2*I)*(c + d*x)))^
2 - 4*C*(1 + 5*E^((2*I)*(c + d*x)) + 2*E^((4*I)*(c + d*x))) + (7*A + 4*C)*(
1 + E^((2*I)*(c + d*x)))^(7/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(
c + d*x))])*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2))/(d*(1 + E^((2*I)
*(c + d*x)))^2*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec(dx+c)^3+Ab\sec(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3), x)
```

3.9 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=91

$$\frac{3bC\tan(c+dx)\sqrt[3]{b\sec(c+dx)}}{4d}-\frac{3b^2(4A+C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*b^2*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*C*(b*Sec[c +
 d*x])^(1/3)*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0944594, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{3bC\tan(c+dx)\sqrt[3]{b\sec(c+dx)}}{4d}-\frac{3b^2(4A+C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*b^2*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*C*(b*Sec[c +
 d*x])^(1/3)*Tan[c + d*x])/(4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = b\int \sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$$
$$= \frac{3bC\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4d} + \frac{1}{4}(b(4A+C))\int \sqrt[3]{b\sec(c+$$
$$= \frac{3bC\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4d} + \frac{1}{4}\left(b(4A+C)\sqrt[3]{\frac{\cos(c+dx}{b}}\right.$$
$$= -\frac{3b(4A+C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+}}{8d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.05036, size = 163, normalized size = 1.79

$$\frac{3b\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(C\sin(c+dx)\sec^{\frac{4}{3}}(c+dx)-i\sqrt[3]{2}(4A+C)\sqrt[3]{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt[3]{1+e^{2i(c+dx)}}\text{Hypergeom}\right.}{2d\sec^{\frac{7}{3}}(c+dx)(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*b*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2)*((-I)*2^(1/3)*(4*A + C)*
(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(1 + E^((2*I)*(c + d*x)))
^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c + d*x))] + C*Sec[c + d
*x]^(4/3)*Sin[c + d*x]))/(2*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(
7/3))
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b\sec(dx+c))^{\frac{4}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+Ab\cos\left(dx+c\right)\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*b*cos(d*x + c)*sec(d*x + c)^3 + A*b*cos(d*x + c)*sec(d*x + c))*
(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.10 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=91

$$\frac{3b^2C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}-\frac{3b^3(A-2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

```
[Out] (-3*b^3*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b^2*C*Tan[c +
 d*x])/(d*(b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.115196, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4046, 3772, 2643}

$$\frac{3b^2C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}-\frac{3b^3(A-2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*b^3*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b^2*C*Tan[c +
 d*x])/(d*(b*Sec[c + d*x])^(2/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = b^2\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}dx$$
$$= \frac{3b^2C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}} + \left(b^2(A-2C)\right)\int \frac{1}{(b\sec(c+dx))^{2/3}}d$$
$$= \frac{3b^2C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}} + \left(b^2(A-2C)\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c\right.$$
$$= -\frac{3b(A-2C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sqrt[3]{b}}{5d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.130123, size = 90, normalized size = 0.99

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{4/3}\left(2A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)-C\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x]*(2*A*Cos[c + d*x]^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[
c + d*x]^2] - C*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^{\frac{4}{3}} \left(A+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^2\sec(dx+c)^3+Ab\cos(dx+c)^2\sec(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + A*b*cos(d*x + c)^2*sec(d*x +
c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx + c\right)^{2} + A\right) \left(b \sec \left(dx + c\right)\right)^{\frac{4}{3}} \cos \left(dx + c\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.11 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{3(8A+5C)\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{16bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+d}{8b^2d}$$

```
[Out] (3*(8*A + 5*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sin[c + d*x])/(16*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c +
d*x])^(5/3)*Tan[c + d*x])/(8*b^2*d)
```

Rubi [A] time = 0.0868797, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4046, 3772, 2643}

$$\frac{3(8A+5C)\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{16bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan(c+dx)(b\sec(c+dx))^{5/3}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(8*A + 5*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sin[c + d*x])/(16*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c +
d*x])^(5/3)*Tan[c + d*x])/(8*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{5/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$
$$= \frac{3C(b\sec(c+dx))^{5/3}\tan(c+dx)}{8b^2d} + \frac{(8A+5C)\int (b\sec(c+dx))^{5/3}\,dx}{8b^2}$$
$$= \frac{3C(b\sec(c+dx))^{5/3}\tan(c+dx)}{8b^2d} + \frac{\left((8A+5C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right.}{8b^2}$$
$$= \frac{3(8A+5C)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{16bd\sqrt{\sin^2(c+dx)}} + \frac{3C(b}{}$$

Mathematica [C] time = 2.72516, size = 207, normalized size = 2.18

$$\frac{3ie^{-i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{8/3}\left(A+C\sec^2(c+dx)\right)\left((8A+5C)\left(1+e^{2i(c+dx)}\right)^{8/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(c+dx)}\right)-5\right.}{20\sqrt[3]{2}d\sec^{\frac{5}{3}}(c+dx)\sqrt[3]{b\sec(c+dx)}(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (((3*I)/20)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(8/3)*(-5*(8*A*(1 +
 E^((2*I)*(c + d*x)))^2 + C*(1 + 14*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d
*x)))) + (8*A + 5*C)*(1 + E^((2*I)*(c + d*x)))^(8/3)*Hypergeometric2F1[2/3,
 5/6, 11/6, -E^((2*I)*(c + d*x))])*(A + C*Sec[c + d*x]^2))/(2^(1/3)*d*E^(I*
(c + d*x))*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/3)*(b*Sec[c + d*x
])^(1/3))
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\left(A+C(\sec(dx+c))^2\right)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*(b*sec(d*x + c))^(2/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(b*sec(c + d*x))**(1/3), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

3.12 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{3C\tan(c+dx)(b\sec(c+dx))^{2/3}}{5bd}-\frac{3(5A+2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.0835011, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{3C\tan(c+dx)(b\sec(c+dx))^{2/3}}{5bd}-\frac{3(5A+2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(5*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx}{b}$$

$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{(5A+2C)\int (b\sec(c+dx))^{2/3}\,dx}{5b}$$

$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left((5A+2C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))\right.}{5b}$$

$$= -\frac{3(5A+2C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.25291, size = 168, normalized size = 1.83

$$\frac{3(b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)\left(2C\sin(c+dx)\sec^{\frac{5}{3}}(c+dx)-i2^{2/3}(5A+2C)\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(1+e^{2i(c+dx)}\right)^{2/3}\mathrm{I}\right.}{5bd\sec^{\frac{8}{3}}(c+dx)(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2)*((-I)*2^(2/3)*(5*A + 2*C)*
(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E^((2*I)*(c + d*x)))
^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)*(c + d*x))] + 2*C*Sec[c +
 d*x]^(5/3)*Sin[c + d*x]))/(5*b*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*
x]^(8/3))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \sec(dx+c)\left(A+C(\sec(dx+c))^2\right)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sec(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

3.13 $\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=90

$$\frac{3C\tan(c+dx)}{2d\sqrt[3]{b\sec(c+dx)}}-\frac{3b(2A-C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*b*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*
x])/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/
(2*d*(b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.0739019, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4046, 3772, 2643}

$$\frac{3C\tan(c+dx)}{2d\sqrt[3]{b\sec(c+dx)}}-\frac{3b(2A-C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*
x])/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/
(2*d*(b*Sec[c + d*x])^(1/3))
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{3C\tan(c+dx)}{2d\sqrt[3]{b\sec(c+dx)}} + \frac{1}{2}(2A-C)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx$$
$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{b\sec(c+dx)}} + \frac{1}{2}\left((2A-C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx$$
$$= -\frac{3(2A-C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}} + \frac{3C\tan(}{2d\sqrt[3]{b\sec}}$$

Mathematica [C] time = 0.724211, size = 127, normalized size = 1.41

$$-\frac{3i\left((2A-C)e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(c+dx)}\right)-5\left(Ae^{2i(c+dx)}+A-Ce^{2i(c+dx)}\right)\right)}{5d\left(1+e^{2i(c+dx)}\right)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (((-3*I)/5)*(-5*(A + A*E^((2*I)*(c + d*x)) - C*E^((2*I)*(c + d*x))) + (2*A
- C)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[
2/3, 5/6, 11/6, -E^((2*I)*(c + d*x))]))/(d*(1 + E^((2*I)*(c + d*x)))*(b*Sec
[c + d*x])^(1/3))
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (A+C(\sec(dx+c))^2)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+C\sec^{2}\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(1/3), x)
```

3.14 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=88

$$\frac{3Ab\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*(A + 4*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x]
)/(4*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b*Tan[c + d*x])/
(4*d*(b*Sec[c + d*x])^(4/3))
```

Rubi [A] time = 0.0968436, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4045, 3772, 2643}

$$\frac{3Ab\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*(A + 4*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x]
)/(4*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b*Tan[c + d*x])/
(4*d*(b*Sec[c + d*x])^(4/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx = b\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$$
$$= \frac{3Ab\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} + \frac{(A+4C)\int (b\sec(c+dx))^{2/3}\,dx}{4b}$$
$$= \frac{3Ab\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} + \frac{\left((A+4C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right.}}{4b}$$
$$= -\frac{3(A+4C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+}{4bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 0.551716, size = 121, normalized size = 1.38

$$-\frac{3ie^{-i(c+dx)}\left(2(A+4C)e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-e^{2i(c+dx)}\right)+A\left(-1+e^{4i(c+dx)}\right)\right)}{8d\left(1+e^{2i(c+dx)}\right)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (((-3*I)/8)*(A*(-1 + E^((4*I)*(c + d*x))) + 2*(A + 4*C)*E^((2*I)*(c + d*x))
*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)
*(c + d*x))]))/(d*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*(b*Sec[c + d*x]
)^(1/3))
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(A+C(\sec(dx+c))^2\right)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\cos(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*(b*sec(d*x + c))^
(2/3)/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\cos\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

3.15 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{3Ab^2\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}}-\frac{3b(4A+7C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*b*(4*A + 7*C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(28*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c +
 d*x])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Rubi [A] time = 0.109311, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4045, 3772, 2643}

$$\frac{3Ab^2\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}}-\frac{3b(4A+7C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*(4*A + 7*C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(28*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c +
 d*x])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx = b^2\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/3}}\,dx$$
$$= \frac{3Ab^2\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}} + \frac{1}{7}(4A+7C)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx$$
$$= \frac{3Ab^2\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}} + \frac{1}{7}\left((4A+7C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{}$$
$$= -\frac{3(4A+7C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+}{28bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.107267, size = 89, normalized size = 0.96

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)+7C\text{Hypergeometric2}\right.}{7d\sqrt[3]{b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(A*Cos[c + d*x]^2*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c
 + d*x]^2] + 7*C*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-T
an[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2\left(A+C(\sec(dx+c))^2\right)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*(b*sec(d*x +
c))^(2/3)/(b*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

3.16 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3C\tan(c+dx)(b\sec(c+dx))^{2/3}}{5b^2d}-\frac{3(5A+2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d
*x])^(2/3)*Tan[c + d*x])/(5*b^2*d)
```

Rubi [A] time = 0.079406, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4046, 3772, 2643}

$$\frac{3C\tan(c+dx)(b\sec(c+dx))^{2/3}}{5b^2d}-\frac{3(5A+2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d
*x])^(2/3)*Tan[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}dx = \frac{\int (b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$
$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5b^2d} + \frac{(5A+2C)\int (b\sec(c+dx))^{2/3}dx}{5b^2}$$
$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5b^2d} + \frac{\left((5A+2C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx)\right.}{5b^2}$$
$$= -\frac{3(5A+2C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 0.958264, size = 165, normalized size = 1.74

$$\frac{3\left(A+C\sec^2(c+dx)\right)\left(2C\sin(c+dx)\sec^{\frac{5}{3}}(c+dx)-i2^{2/3}(5A+2C)\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometri}\right.}{5d\sec^{\frac{2}{3}}(c+dx)(b\sec(c+dx))^{4/3}(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*(A + C*Sec[c + d*x]^2)*((-I)*2^(2/3)*(5*A + 2*C)*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(2/3)*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F
1[1/3, 2/3, 4/3, -E^((2*I)*(c + d*x))] + 2*C*Sec[c + d*x]^(5/3)*Sin[c + d*x
]))/(5*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(2/3)*(b*Sec[c + d*x])
^(4/3))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\left(A+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

3.17 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=92

$$\frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}}-\frac{3(2A-C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(2
*b*d*(b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.0750487, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}}-\frac{3(2A-C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```


```
[Out] (-3*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(2
*b*d*(b*Sec[c + d*x])^(1/3))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$= \frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}} + \frac{(2A-C)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{2b}$$

$$= \frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}} + \frac{\left((2A-C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,d}{2b}$$

$$= -\frac{3(2A-C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+d}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 0.591473, size = 130, normalized size = 1.41

$$-\frac{3i\left((2A-C)e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(c+dx)}\right)-5\left(Ae^{2i(c+dx)}+A-Ce^{2i(c+dx)}\right)\right)}{5bd\left(1+e^{2i(c+dx)}\right)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (((-3*I)/5)*(-5*(A + A*E^((2*I)*(c + d*x)) - C*E^((2*I)*(c + d*x))) + (2*A
- C)*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[
2/3, 5/6, 11/6, -E^((2*I)*(c + d*x))]))/(b*d*(1 + E^((2*I)*(c + d*x)))*(b*S
ec[c + d*x])^(1/3))
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int \sec(dx+c)\left(A+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

3.18 $\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3A\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} - \frac{3(A+4C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*(A + 4*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x]
)/(4*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*Tan[c + d*x])/
(4*d*(b*Sec[c + d*x])^(4/3))
```

Rubi [A] time = 0.0707745, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4045, 3772, 2643}

$$\frac{3A\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} - \frac{3(A+4C)\sin(c+dx)\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*(A + 4*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x]
)/(4*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*Tan[c + d*x])/
(4*d*(b*Sec[c + d*x])^(4/3))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{3A\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} + \frac{(A+4C)\int (b\sec(c+dx))^{2/3}\,dx}{4b^2}$$

$$= \frac{3A\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} + \frac{\left((A+4C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{4b^2}$$

$$= -\frac{3(A+4C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} + \frac{3A\,\mathrm{t}}{4d(b\,\mathrm{se}}$$

Mathematica [C] time = 0.433661, size = 124, normalized size = 1.38

$$-\frac{3ie^{-i(c+dx)}\left(2(A+4C)e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-e^{2i(c+dx)}\right)+A\left(-1+e^{4i(c+dx)}\right)\right)}{8bd\left(1+e^{2i(c+dx)}\right)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (((-3*I)/8)*(A*(-1 + E^((4*I)*(c + d*x))) + 2*(A + 4*C)*E^((2*I)*(c + d*x))
*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)
*(c + d*x))]))/(b*d*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*(b*Sec[c + d*
x])^(1/3))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (A+C(\sec(dx+c))^2)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+C\sec^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c))^(4/3), x)
```

3.19 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3Ab\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}}-\frac{3(4A+7C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*(4*A + 7*C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*
x])/(28*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b*Tan[c + d*x
])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Rubi [A] time = 0.0901464, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4045, 3772, 2643}

$$\frac{3Ab\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}}-\frac{3(4A+7C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*(4*A + 7*C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*
x])/(28*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b*Tan[c + d*x
])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = b\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/3}}\,dx$$
$$= \frac{3Ab\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}} + \frac{(4A+7C)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{7b}$$
$$= \frac{3Ab\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}} + \frac{\left((4A+7C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx}{b}}}{7b}$$
$$= -\frac{3(4A+7C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+}{28b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.109125, size = 92, normalized size = 1.02

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)+7C\text{Hypergeometric2}\right.}{7bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(A*Cos[c + d*x]^2*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c
 + d*x]^2] + 7*C*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-T
an[c + d*x]^2])/(7*b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(A+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\cos(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*(b*sec(d*x + c))^
(2/3)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

3.20 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3Ab^2\tan(c+dx)}{10d(b\sec(c+dx))^{10/3}}-\frac{3b(7A+10C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

```
[Out] (-3*b*(7*A + 10*C)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c
 + d*x])/(10*d*(b*Sec[c + d*x])^(10/3))
```

Rubi [A] time = 0.109126, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 4045, 3772, 2643}

$$\frac{3Ab^2\tan(c+dx)}{10d(b\sec(c+dx))^{10/3}}-\frac{3b(7A+10C)\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{70d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*(7*A + 10*C)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c
 + d*x])/(10*d*(b*Sec[c + d*x])^(10/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = b^2\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{10/3}}\,dx$$
$$= \frac{3Ab^2\tan(c+dx)}{10d(b\sec(c+dx))^{10/3}} + \frac{1}{10}(7A+10C)\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx$$
$$= \frac{3Ab^2\tan(c+dx)}{10d(b\sec(c+dx))^{10/3}} + \frac{1}{10}\left((7A+10C)\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/\cdots}\right.$$
$$= -\frac{3(7A+10C)\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\,\mathrm{si}\cdots}{70b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.67149, size = 96, normalized size = 1.03

$$\frac{\tan(c+dx)\left((7A+10C)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(c+dx)\right)+3\sqrt[6]{\cos^2(c+dx)}(2A\cos(2(c+dx))+9A+10\cdots\right.}{40d\sqrt[6]{\cos^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] ((3*(Cos[c + d*x]^2)^(1/6)*(9*A + 10*C + 2*A*Cos[2*(c + d*x)]) + (7*A + 10*
C)*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[c + d*x]^2])*Tan[c + d*x])/(40*d*(C
os[c + d*x]^2)^(1/6)*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2\left(A+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*(b*sec(d*x +
c))^(2/3)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

3.21 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=146

$$\frac{3b(A(3m+7)+C(3m+4))\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-1),\frac{1}{6}(5-3m),\right.}{d(3m+1)(3m+7)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*b*C*Sec[c + d*x]^(2 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*
m)) + (3*b*(C*(4 + 3*m) + A*(7 + 3*m))*Hypergeometric2F1[1/2, (-1 - 3*m)/6,
 (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(1 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.116383, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{3b(A(3m+7)+C(3m+4))\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)(3m+7)\sqrt{\sin^2(c+dx)}}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*b*C*Sec[c + d*x]^(2 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*
m)) + (3*b*(C*(4 + 3*m) + A*(7 + 3*m))*Hypergeometric2F1[1/2, (-1 - 3*m)/6,
 (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(1 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{3bC\sec^{2+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d(7+3m)} + \frac{\left(b\left(C\left(\frac{4}{3}+\right.\right.\right.}{}$$

$$= \frac{3bC\sec^{2+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d(7+3m)} + \frac{\left(b\left(C\left(\frac{4}{3}+\right.\right.\right.}{}$$

$$= \frac{3bC\sec^{2+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d(7+3m)} + \frac{3b(C(4+3}{}$$

Mathematica [C] time = 3.40345, size = 303, normalized size = 2.08

$$-\frac{3i2^{m+\frac{7}{3}}e^{-\frac{1}{3}i(3m+7)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{7}{3}}(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)\left(\frac{2(A+2C)e^{\frac{1}{3}i(3m+10)(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{1}{6}(-3\right.}{3m+10}\right.}{d\sec^{\frac{10}{3}}(c+dx)(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-3*I)*2^(7/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(7/3 + m)*(
(2*(A + 2*C)*E^((I/3)*(10 + 3*m)*(c + d*x))*Hypergeometric2F1[1, (-4 - 3*m)
/6, 8/3 + m/2, -E^((2*I)*(c + d*x))])/(10 + 3*m) + (A*E^((I/3)*(16 + 3*m)*(
c + d*x))*Hypergeometric2F1[1, (2 - 3*m)/6, (22 + 3*m)/6, -E^((2*I)*(c + d*
x))])/(16 + 3*m) + (A*E^((I/3)*(4 + 3*m)*(c + d*x))*Hypergeometric2F1[1, -5
/3 - m/2, 5/3 + m/2, -E^((2*I)*(c + d*x))])/(4 + 3*m))*(b*Sec[c + d*x])^(4/
3)*(A + C*Sec[c + d*x]^2))/(d*E^((I/3)*(7 + 3*m)*(c + d*x))*(A + 2*C + A*Co
s[2*c + 2*d*x])*Sec[c + d*x]^(10/3))
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{\frac{4}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{3}+Ab\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3)*sec
(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

3.22 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=146

$$\frac{3C\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m+1}(c+dx)}{d(3m+5)}-\frac{3(A(3m+5)+C(3m+2))\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)}{d(1-3m)(3m+5)\sqrt{}}$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(5 + 3*m)
) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 -
 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c
 + d*x])/(d*(1 - 3*m)*(5 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.117809, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{3C\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m+1}(c+dx)}{d(3m+5)}-\frac{3(A(3m+5)+C(3m+2))\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)}{d(1-3m)(3m+5)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(5 + 3*m)
) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 -
 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c
 + d*x])/(d*(1 - 3*m)*(5 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\sec^{1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c+dx)}{d(5+3m)} + \frac{\left(\left(C\left(\frac{2}{3}\right.\right.\right.}{}$$

$$= \frac{3C\sec^{1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c+dx)}{d(5+3m)} + \frac{\left(\left(C\left(\frac{2}{3}\right.\right.\right.}{}$$

$$= \frac{3C\sec^{1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c+dx)}{d(5+3m)} - \frac{3(C(2}{}$$

Mathematica [C] time = 2.58443, size = 303, normalized size = 2.08

$$-\frac{3i2^{m+\frac{5}{3}}e^{-\frac{1}{3}i(3m+5)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{5}{3}}(b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)\left(\frac{2(A+2C)e^{\frac{1}{3}i(3m+8)(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{1}{6}\right.}{3m+8}\right.}{d\sec^{\frac{8}{3}}(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-3*I)*2^(5/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(5/3 + m)*(
(A*E^((I/3)*(2 + 3*m)*(c + d*x))*Hypergeometric2F1[1, (-8 - 3*m)/6, (8 + 3*
m)/6, -E^((2*I)*(c + d*x))])/(2 + 3*m) + (2*(A + 2*C)*E^((I/3)*(8 + 3*m)*(c
 + d*x))*Hypergeometric2F1[1, (-2 - 3*m)/6, 7/3 + m/2, -E^((2*I)*(c + d*x))
])/(8 + 3*m) + (A*E^((I/3)*(14 + 3*m)*(c + d*x))*Hypergeometric2F1[1, (4 -
3*m)/6, (20 + 3*m)/6, -E^((2*I)*(c + d*x))])/(14 + 3*m))*(b*Sec[c + d*x])^(
2/3)*(A + C*Sec[c + d*x]^2))/(d*E^((I/3)*(5 + 3*m)*(c + d*x))*(A + 2*C + A*
Cos[2*c + 2*d*x])*Sec[c + d*x]^(8/3))
```

Maple [F] time = 0.156, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{\frac{2}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

3.23 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=144

$$\frac{3C\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m+1}(c+dx)}{d(3m+4)}-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\text{H}}{d(2-3m)(3m+4)\sqrt{}}$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3
*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(2 - 3*m)*(4 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.117013, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{3C\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m+1}(c+dx)}{d(3m+4)}-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\,{}_2}{d(2-3m)(3m+4)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3
*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(2 - 3*m)*(4 + 3*m)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d(4+3m)} + \frac{\left(\left(C\left(\frac{1}{3}+m\right)+\right.\right.}{}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d(4+3m)} + \frac{\left(\left(C\left(\frac{1}{3}+m\right)+\right.\right.}{}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d(4+3m)} - \frac{3(C+3Cm+}{}$$

Mathematica [C] time = 2.58827, size = 303, normalized size = 2.1

$$-\frac{3i2^{m+\frac{4}{3}}e^{-\frac{1}{3}i(3m+4)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{4}{3}}\sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(\frac{2(A+2C)e^{\frac{1}{3}i(3m+7)(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{1}{6}(-3m-\right.}{3m+7}\right.}{d\sec^{\frac{7}{3}}(c+d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-3*I)*2^(4/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(4/3 + m)*(
(A*E^((I/3)*(1 + 3*m)*(c + d*x))*Hypergeometric2F1[1, (-7 - 3*m)/6, (7 + 3*
m)/6, -E^((2*I)*(c + d*x))])/(1 + 3*m) + (2*(A + 2*C)*E^((I/3)*(7 + 3*m)*(c
 + d*x))*Hypergeometric2F1[1, (-1 - 3*m)/6, (13 + 3*m)/6, -E^((2*I)*(c + d*
x))])/(7 + 3*m) + (A*E^((I/3)*(13 + 3*m)*(c + d*x))*Hypergeometric2F1[1, (5
 - 3*m)/6, (19 + 3*m)/6, -E^((2*I)*(c + d*x))])/(13 + 3*m))*(b*Sec[c + d*x]
)^(1/3)*(A + C*Sec[c + d*x]^2))/(d*E^((I/3)*(4 + 3*m)*(c + d*x))*(A + 2*C +
 A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/3))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m \sqrt[3]{b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

3.24 $\int \frac{\sec^m(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=147

$$\frac{3(C(1-3m)-A(3m+2))\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4-3m),\frac{1}{6}(10-3m),\cos^2(c+dx)\right)}{d(4-3m)(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + 3*m)*(b*Sec[c + d*x])^(1/3)
) + (3*(C*(1 - 3*m) - A*(2 + 3*m))*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10
- 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*
(2 + 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.119844, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{3(C(1-3m)-A(3m+2))\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{d(4-3m)(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3C\sin(c+dx)\sec^{m}}{d(3m+2)\sqrt[3]{b\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + 3*m)*(b*Sec[c + d*x])^(1/3)
) + (3*(C*(1 - 3*m) - A*(2 + 3*m))*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10
- 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*
(2 + 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\sec(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{3}+m\right)+A\left(\frac{2}{3}+m\right)\right)\sqrt[3]{\sec(c+d}\right.}{\left(\frac{2}{3}+m\right)\sqrt[3]{b\sec(c+}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\sec(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{3}+m\right)+A\left(\frac{2}{3}+m\right)\right)\cos^{\frac{2}{3}+m}(c+\right.}{\left(\frac{2}{3}+m\right)\sqrt[3]{b}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\sec(c+dx)}} + \frac{3(C(1-3m)-A(2+3m))\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-\right.}{d(4-3m)(2+}$$

Mathematica [C] time = 6.82919, size = 311, normalized size = 2.12

$$-\frac{3i2^{m+\frac{2}{3}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m-\frac{1}{3}}\left(1+e^{2i(c+dx)}\right)^{m-\frac{1}{3}}\left(A+C\sec^2(c+dx)\right)\left((3m-1)e^{2i(c+dx)}\left(2(3m+11)(A+2C)\text{Hypergeomet}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((-3*I)*2^(2/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/3 + m)*
(1 + E^((2*I)*(c + d*x)))^(-1/3 + m)*(A*(55 + 48*m + 9*m^2)*Hypergeometric2
F1[5/3 + m, (-1 + 3*m)/6, (5 + 3*m)/6, -E^((2*I)*(c + d*x))] + E^((2*I)*(c
+ d*x))*(-1 + 3*m)*(2*(A + 2*C)*(11 + 3*m)*Hypergeometric2F1[5/3 + m, (5 +
3*m)/6, (11 + 3*m)/6, -E^((2*I)*(c + d*x))] + A*E^((2*I)*(c + d*x))*(5 + 3*
m)*Hypergeometric2F1[5/3 + m, (11 + 3*m)/6, (17 + 3*m)/6, -E^((2*I)*(c + d*
x))]))*(A + C*Sec[c + d*x]^2))/(d*(-1 + 3*m)*(5 + 3*m)*(11 + 3*m)*(A + 2*C
+ A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/3)*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m\left(A+C(\sec(dx+c))^2\right)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{m}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*se
c(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{m}\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{m}}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

3.25 $\int \frac{\sec^m(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=145

$$\frac{3C\sin(c+dx)\sec^{m+1}(c+dx)}{d(3m+1)(b\sec(c+dx))^{2/3}}-\frac{3(3Am+A-C(2-3m))\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5-\right.}{d(5-3m)(3m+1)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + 3*m)*(b*Sec[c + d*x])^(2/3)
) - (3*(A - C*(2 - 3*m) + 3*A*m)*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 -
3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(1
 + 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.126757, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{3C\sin(c+dx)\sec^{m+1}(c+dx)}{d(3m+1)(b\sec(c+dx))^{2/3}}-\frac{3(3Am+A-C(2-3m))\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);c\right.}{d(5-3m)(3m+1)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + 3*m)*(b*Sec[c + d*x])^(2/3)
) - (3*(A - C*(2 - 3*m) + 3*A*m)*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 -
3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(1
 + 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{(b\sec(c+dx))^{2/3}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+3m)(b\sec(c+dx))^{2/3}} + \frac{\left(\left(C\left(-\frac{2}{3}+m\right)+A\left(\frac{1}{3}+m\right)\right)\sec^{\frac{2}{3}}(c+dx)\right)}{\left(\frac{1}{3}+m\right)(b\sec(c+dx))^{2}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+3m)(b\sec(c+dx))^{2/3}} + \frac{\left(\left(C\left(-\frac{2}{3}+m\right)+A\left(\frac{1}{3}+m\right)\right)\cos^{\frac{1}{3}+m}(c+dx\right.}{\left(\frac{1}{3}+m\right)(b\sec(}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+3m)(b\sec(c+dx))^{2/3}} - \frac{3(A-C(2-3m)+3Am)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m),\right.}{d(5-3m)(1+3m)(}$$

Mathematica [C] time = 8.24401, size = 311, normalized size = 2.14

$$-\frac{3i2^{m+\frac{1}{3}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m-\frac{2}{3}}\left(1+e^{2i(c+dx)}\right)^{m-\frac{2}{3}}\left(A+C\sec^2(c+dx)\right)\left((3m+10)\left(2(3m-2)(A+2C)e^{2i(c+dx)}\text{Hypergeometric}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((-3*I)*2^(1/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-2/3 + m)*
(1 + E^((2*I)*(c + d*x)))^(-2/3 + m)*(A*E^((4*I)*(c + d*x))*(-8 + 6*m + 9*m
^2)*Hypergeometric2F1[5/3 + m/2, 4/3 + m, 8/3 + m/2, -E^((2*I)*(c + d*x))]
+ (10 + 3*m)*(A*(4 + 3*m)*Hypergeometric2F1[4/3 + m, (-2 + 3*m)/6, (4 + 3*m
)/6, -E^((2*I)*(c + d*x))] + 2*(A + 2*C)*E^((2*I)*(c + d*x))*(-2 + 3*m)*Hyp
ergeometric2F1[4/3 + m, (4 + 3*m)/6, 5/3 + m/2, -E^((2*I)*(c + d*x))]))*(A
+ C*Sec[c + d*x]^2))/(d*(-2 + 3*m)*(4 + 3*m)*(10 + 3*m)*(A + 2*C + A*Cos[2*
c + 2*d*x])*Sec[c + d*x]^(4/3)*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m \left(A+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{m}}{b\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*se
c(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{m}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{m}}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

3.26 $\int \frac{\sec^m(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=148

$$-\frac{3(-3Am+A+C(4-3m))\sin(c+dx)\sec^{m-2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7-3m),\frac{1}{6}(13-3m),\cos^2(c+dx)\right)}{bd(1-3m)(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}} -$$

```
[Out] (-3*C*Sec[c + d*x]^m*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)) -
 (3*(A + C*(4 - 3*m) - 3*A*m)*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m
)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(1 - 3*m)*(7
- 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.134481, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$-\frac{3(-3Am+A+C(4-3m))\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)}{bd(1-3m)(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3C\sin(c+dx)\,\text{se}}{bd(1-3m)\sqrt[3]{b\,\text{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```


```
[Out] (-3*C*Sec[c + d*x]^m*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)) -
 (3*(A + C*(4 - 3*m) - 3*A*m)*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m
)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(1 - 3*m)*(7
- 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)\left(A+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx=\frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{b\sqrt[3]{b\sec(c+dx)}}$$

$$=-\frac{3C\sec^m(c+dx)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\sec(c+dx)}}+\frac{\left(\left(C\left(-\frac{4}{3}+m\right)+A\left(-\frac{1}{3}+m\right)\right)\sqrt[3]{\sec(c+d}}{b\left(-\frac{1}{3}+m\right)\sqrt[3]{b\sec(c}}$$

$$=-\frac{3C\sec^m(c+dx)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\sec(c+dx)}}+\frac{\left(\left(C\left(-\frac{4}{3}+m\right)+A\left(-\frac{1}{3}+m\right)\right)\cos^{\frac{2}{3}+m}(c-}{b\left(-\frac{1}{3}+m\right)\sqrt[3]{}}$$

$$=-\frac{3C\sec^m(c+dx)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\sec(c+dx)}}-\frac{3(A(1-3m)+C(4-3m))\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3}{bd(1-3m)(7-3}$$

Mathematica [C] time = 3.51081, size = 340, normalized size = 2.3

$$-\frac{3i2^{m-\frac{1}{3}}e^{-\frac{1}{3}i(6c+d(3m+2)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{2}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{2}{3}}\left(A+C\sec^2(c+dx)\right)\left(\frac{e^{\frac{1}{3}i(6c+d(3m+2)x)}\left(2(3m+8)(A+2C)\text{Hypergeometr}\right.}{}\right.}{d\sec^{\frac{2}{3}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] ((-3*I)*2^(-1/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3 + m)*
(1 + E^((2*I)*(c + d*x)))^(2/3 + m)*((A*E^((I/3)*d*(-4 + 3*m)*x)*Hypergeome
tric2F1[2/3 + m, (-4 + 3*m)/6, (2 + 3*m)/6, -E^((2*I)*(c + d*x))])/(-4 + 3*
m) + (E^((I/3)*(6*c + d*(2 + 3*m)*x))*(2*(A + 2*C)*(8 + 3*m)*Hypergeometric
2F1[2/3 + m, (2 + 3*m)/6, (8 + 3*m)/6, -E^((2*I)*(c + d*x))] + A*E^((2*I)*(
c + d*x))*(2 + 3*m)*Hypergeometric2F1[2/3 + m, (8 + 3*m)/6, 7/3 + m/2, -E^(
(2*I)*(c + d*x))]))/((2 + 3*m)*(8 + 3*m)))*(A + C*Sec[c + d*x]^2))/(d*E^((I
/3)*(6*c + d*(2 + 3*m)*x))*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(2/3
)*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m\left(A+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^m}{b^2\sec\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*
sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

3.27 $\int \sec^m(c+dx)(b\sec(c+dx))^n \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=145

$$\frac{C\sin(c+dx)\sec^{m+1}(c+dx)(b\sec(c+dx))^n}{d(m+n+1)} - \frac{(A(m+n+1)+C(m+n))\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n}{d(-m-n+1)(m+}$$

```
[Out] (C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)) -
((C*(m + n) + A*(1 + m + n))*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m -
 n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x
])/(d*(1 - m - n)*(1 + m + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.110575, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {20, 4046, 3772, 2643}

$$\frac{C\sin(c+dx)\sec^{m+1}(c+dx)(b\sec(c+dx))^n}{d(m+n+1)} - \frac{(A(m+n+1)+C(m+n))\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n}{d(-m-n+1)(m+n+1)\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)) -
((C*(m + n) + A*(1 + m + n))*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m -
 n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x
])/(d*(1 - m - n)*(1 + m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{m+n}(c+dx)\left(A+C\sec^2\right.$$

$$= \frac{C\sec^{1+m}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \left(\left(A+\frac{C(m+}{1+m}\right.\right.$$

$$= \frac{C\sec^{1+m}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \left(\left(A+\frac{C(m+}{1+m}\right.\right.$$

$$= \frac{C\sec^{1+m}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} - \frac{\left(A+\frac{C(m+n)}{1+m+n}\right)}{}$$

Mathematica [C] time = 7.79207, size = 289, normalized size = 1.99

$$-\frac{i2^{m+n+1}e^{-i(m+n+1)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n+1}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{2(A+2C)e^{i(m+n+2)(c+dx)}\text{Hyperg}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(1 + m + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1 + m + n)
*((A*E^(I*(m + n)*(c + d*x))*Hypergeometric2F1[1, (-2 - m - n)/2, (2 + m +
n)/2, -E^((2*I)*(c + d*x))])/(m + n) + (2*(A + 2*C)*E^(I*(2 + m + n)*(c + d
*x))*Hypergeometric2F1[1, (-m - n)/2, (4 + m + n)/2, -E^((2*I)*(c + d*x))])
/(2 + m + n) + (A*E^(I*(4 + m + n)*(c + d*x))*Hypergeometric2F1[1, (2 - m -
 n)/2, (6 + m + n)/2, -E^((2*I)*(c + d*x))])/(4 + m + n))*Sec[c + d*x]^(-2
- n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*E^(I*(1 + m + n)*(c + d*
x))*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 1.011, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m(b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^n\sec\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(c+dx\right)\right)^n\left(A+C\sec^2\left(c+dx\right)\right)\sec^m\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + C*sec(c + d*x)**2)*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^n\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

3.28 $\int \sec^2(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=120

$$\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-n-1),\frac{1-n}{2},\cos^2(c+dx)\right)}{bd(n+1)(n+3)\sqrt{\sin^2(c+dx)}}+\frac{C\tan(c+$$

```
[Out] ((C*(2 + n) + A*(3 + n))*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[
c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*(3 + n)*Sqr
t[Sin[c + d*x]^2]) + (C*(b*Sec[c + d*x])^(2 + n)*Tan[c + d*x])/(b^2*d*(3 +
n))
```

Rubi [A] time = 0.111613, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)(n+3)\sqrt{\sin^2(c+dx)}}+\frac{C\tan(c+dx)(b\sec(c+dx))^n}{b^2d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((C*(2 + n) + A*(3 + n))*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[
c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*(3 + n)*Sqr
t[Sin[c + d*x]^2]) + (C*(b*Sec[c + d*x])^(2 + n)*Tan[c + d*x])/(b^2*d*(3 +
n))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{2+n}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$
$$= \frac{C(b\sec(c+dx))^{2+n}\tan(c+dx)}{b^2d(3+n)} + \frac{\left(A+\frac{C(2+n)}{3+n}\right)\int (b\sec(c$$
$$= \frac{C(b\sec(c+dx))^{2+n}\tan(c+dx)}{b^2d(3+n)} + \frac{\left(\left(A+\frac{C(2+n)}{3+n}\right)\left(\frac{\cos(c+dx)}{b}\right.\right.}{}$$
$$= \frac{\left(A+\frac{C(2+n)}{3+n}\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 6.89962, size = 274, normalized size = 2.28

$$-\frac{i2^{n+3}e^{-in(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{2(A+2C)e^{i(n+4)(c+dx)}\text{Hypergeometric2F1}(1,}{n+4}\right.}{d\left(1+e^{2i(c+dx)}\right)^3(A\cos(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(3 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*((A*E^(I*(2 +
 n)*(c + d*x))*Hypergeometric2F1[1, -2 - n/2, (4 + n)/2, -E^((2*I)*(c + d*x
))])/(2 + n) + (2*(A + 2*C)*E^(I*(4 + n)*(c + d*x))*Hypergeometric2F1[1, -1
 - n/2, (6 + n)/2, -E^((2*I)*(c + d*x))])/(4 + n) + (A*E^(I*(6 + n)*(c + d*
x))*Hypergeometric2F1[1, -n/2, (8 + n)/2, -E^((2*I)*(c + d*x))])/(6 + n))*S
ec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*E^(I*n*(
c + d*x))*(1 + E^((2*I)*(c + d*x)))^3*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.533, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2(b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(c+dx\right)\right)^{n}\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + C*sec(c + d*x)**2)*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

3.29 $\int \sec(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=109

$$\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)}{dn(n+2)\sqrt{\sin^2(c+dx)}}+\frac{C\tan(c+dx)(b}{bd(n}$$

```
[Out] ((C*(1 + n) + A*(2 + n))*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(2 + n)*Sqrt[Sin[c + d*x]^2]) +
 (C*(b*Sec[c + d*x])^(1 + n)*Tan[c + d*x])/(b*d*(2 + n))
```

Rubi [A] time = 0.102033, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 4046, 3772, 2643}

$$\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn(n+2)\sqrt{\sin^2(c+dx)}}+\frac{C\tan(c+dx)(b\sec(c+dx))^{n+1}}{bd(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((C*(1 + n) + A*(2 + n))*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(2 + n)*Sqrt[Sin[c + d*x]^2]) +
 (C*(b*Sec[c + d*x])^(1 + n)*Tan[c + d*x])/(b*d*(2 + n))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = \frac{\int (b\sec(c+dx))^{1+n}\left(A+C\sec^2(c+dx)\right) dx}{b}$$
$$= \frac{C(b\sec(c+dx))^{1+n}\tan(c+dx)}{bd(2+n)} + \frac{\left(A+\frac{C(1+n)}{2+n}\right)\int (b\sec(c+dx$$
$$= \frac{C(b\sec(c+dx))^{1+n}\tan(c+dx)}{bd(2+n)} + \frac{\left(\left(A+\frac{C(1+n)}{2+n}\right)\left(\frac{\cos(c+dx)}{b}\right)^n (b$$
$$= \frac{\left(A+\frac{C(1+n)}{2+n}\right) {}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n \sin}{dn\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 6.58729, size = 282, normalized size = 2.59

$$-\frac{i2^{n+2}e^{-in(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n \sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{2(A+2C)e^{i(n+3)(c+dx)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-\right.}{n+3}\right.}{d\left(1+e^{2i(c+dx)}\right)^2(A\cos(2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*((A*E^(I*(1 +
 n)*(c + d*x))*Hypergeometric2F1[1, (-3 - n)/2, (3 + n)/2, -E^((2*I)*(c + d
*x))])/(1 + n) + (2*(A + 2*C)*E^(I*(3 + n)*(c + d*x))*Hypergeometric2F1[1,
(-1 - n)/2, (5 + n)/2, -E^((2*I)*(c + d*x))])/(3 + n) + (A*E^(I*(5 + n)*(c
+ d*x))*Hypergeometric2F1[1, (1 - n)/2, (7 + n)/2, -E^((2*I)*(c + d*x))])/(
5 + n))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d
*E^(I*n*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^2*(A + 2*C + A*Cos[2*c + 2*d*x
]))
```

Maple [F] time = 0.712, size = 0, normalized size = 0.

$$\int \sec(dx+c)(b\sec(dx+c))^n \left(A+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n \sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(c+dx\right)\right)^{n}\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + C*sec(c + d*x)**2)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)
```

3.30 $\int (b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=113

$$\frac{C\tan(c+dx)(b\sec(c+dx))^n}{d(n+1)}-\frac{b(An+A+Cn)\sin(c+dx)(b\sec(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(c\right.}{d(1-n)(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((b*(A + A*n + C*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d
*x]^2]*(b*Sec[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*(1 + n)*Sqrt[Sin[
c + d*x]^2])) + (C*(b*Sec[c + d*x])^n*Tan[c + d*x])/(d*(1 + n))
```

Rubi [A] time = 0.0831302, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4046, 3772, 2643}

$$\frac{C\tan(c+dx)(b\sec(c+dx))^n}{d(n+1)}-\frac{b(An+A+Cn)\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)}{d(1-n)(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] -((b*(A + A*n + C*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d
*x]^2]*(b*Sec[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*(1 + n)*Sqrt[Sin[
c + d*x]^2])) + (C*(b*Sec[c + d*x])^n*Tan[c + d*x])/(d*(1 + n))
```


Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = \frac{C(b\sec(c+dx))^n \tan(c+dx)}{d(1+n)} + \frac{(A+An+Cn)\int (b\sec(c+dx))^n dx}{1+n}$$

$$= \frac{C(b\sec(c+dx))^n \tan(c+dx)}{d(1+n)} + \frac{\left((A+An+Cn)\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+\right.}{1+n}$$

$$= -\frac{(A+An+Cn)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+d}{d\left(1-n^2\right)\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 5.61037, size = 273, normalized size = 2.42

$$-\frac{i2^{n+1}e^{-i(n+1)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+1}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(ne^{i(n+2)(c+dx)}\left(2(n+4)(A+2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(1 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1 + n)*(A*E^(I
*n*(c + d*x))*(8 + 6*n + n^2)*Hypergeometric2F1[1, -1 - n/2, (2 + n)/2, -E^
((2*I)*(c + d*x))] + E^(I*(2 + n)*(c + d*x))*n*(A*E^((2*I)*(c + d*x))*(2 +
n)*Hypergeometric2F1[1, 1 - n/2, (6 + n)/2, -E^((2*I)*(c + d*x))] + 2*(A +
2*C)*(4 + n)*Hypergeometric2F1[1, -n/2, (4 + n)/2, -E^((2*I)*(c + d*x))]))*
Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*E^(I*(1
 + n)*(c + d*x))*n*(2 + n)*(4 + n)*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.687, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \left(A+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + C*sec(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n, x)
```

3.31 $\int \cos(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=117

$$\frac{b^2(C(1-n)-An)\sin(c+dx)(b\sec(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(c+dx)\right)}{d(2-n)n\sqrt{\sin^2(c+dx)}}+\frac{bC\tan(c+dx)(b}{dn}$$

```
[Out] (b^2*(C*(1 - n) - A*n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c +
 d*x]^2]*(b*Sec[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*n*Sqrt[Sin[c +
d*x]^2]) + (b*C*(b*Sec[c + d*x])^(-1 + n)*Tan[c + d*x])/(d*n)
```

Rubi [A] time = 0.11834, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {16, 4046, 3772, 2643}

$$\frac{b^2(C(1-n)-An)\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)n\sqrt{\sin^2(c+dx)}}+\frac{bC\tan(c+dx)(b\sec(c+dx))^{n-1}}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (b^2*(C*(1 - n) - A*n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c +
 d*x]^2]*(b*Sec[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*n*Sqrt[Sin[c +
d*x]^2]) + (b*C*(b*Sec[c + d*x])^(-1 + n)*Tan[c + d*x])/(d*n)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = b\int (b\sec(c+dx))^{-1+n} \left(A+C\sec^2(c+dx)\right) dx$$

$$= \frac{bC(b\sec(c+dx))^{-1+n}\tan(c+dx)}{dn} + \frac{(b(C(-1+n)+An))\int(b\ldots}{n}$$

$$= \frac{bC(b\sec(c+dx))^{-1+n}\tan(c+dx)}{dn} + \frac{\left(b(C(-1+n)+An)\left(\frac{\cos(\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= \frac{(C(1-n)-An)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\ldots}{d(2-n)n\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.231657, size = 119, normalized size = 1.02

$$\frac{\sqrt{-\tan^2(c+dx)}(b\sec(c+dx))^n \left(A(n+1)\cos(c+dx)\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\sec^2(c+dx)\right)+C\ldots\right.}{d(n-1)(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((A*(1 + n)*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1
 + n)/2, Sec[c + d*x]^2] + C*(-1 + n)*Csc[c + d*x]*Hypergeometric2F1[1/2, (
1 + n)/2, (3 + n)/2, Sec[c + d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]
^2])/(d*(-1 + n)*(1 + n))
```

Maple [F] time = 0.752, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b\sec(dx+c))^n \left(A+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*(b*sec(d*x + c))^
n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)
```

3.32 $\int \cos^2(c+dx)(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=132

$$-\frac{b^3(A(1-n)+C(2-n))\sin(c+dx)(b\sec(c+dx))^{n-3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-n}{2},\frac{5-n}{2},\cos^2(c+dx)\right)}{d(1-n)(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2C\tan(c+d}{d}$$

```
[Out] -((b^3*(A*(1 - n) + C*(2 - n))*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(1 - n)*(3 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^2*C*(b*Sec[c + d*x])^(-2 + n)*Tan[c + d*x])/(d
*(1 - n))
```

Rubi [A] time = 0.145624, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$-\frac{b^3(A(1-n)+C(2-n))\sin(c+dx)(b\sec(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(1-n)(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2C\tan(c+dx)(b\sec(c+dx))^{n-}}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -((b^3*(A*(1 - n) + C*(2 - n))*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(1 - n)*(3 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^2*C*(b*Sec[c + d*x])^(-2 + n)*Tan[c + d*x])/(d
*(1 - n))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = b^2\int (b\sec(c+dx))^{-2+n}\left(A+C\sec^2(c+dx)\right) dx$$
$$= -\frac{b^2C(b\sec(c+dx))^{-2+n}\tan(c+dx)}{d(1-n)} + \left(b^2\left(A+\frac{C(2-n)}{1-n}\right.\right.$$
$$= -\frac{b^2C(b\sec(c+dx))^{-2+n}\tan(c+dx)}{d(1-n)} + \left(b^2\left(A+\frac{C(2-n)}{1-n}\right.\right.$$
$$= -\frac{\left(A+\frac{C(2-n)}{1-n}\right)\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)(}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.185405, size = 107, normalized size = 0.81

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^n\left(An\cos^2(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\sec^2(c+dx)\right)+C(n-\right.}{d(n-2)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cot[c + d*x]*(A*n*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, S
ec[c + d*x]^2] + C*(-2 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c +
d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-2 + n)*n)
```

Maple [F] time = 0.948, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2(b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*(b*sec(d*x +
c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

3.33 $\int \cos^3(c+dx)(b\sec(c+dx))^n \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=132

$$-\frac{b^4(A(2-n)+C(3-n))\sin(c+dx)(b\sec(c+dx))^{n-4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-n}{2},\frac{6-n}{2},\cos^2(c+dx)\right)}{d(2-n)(4-n)\sqrt{\sin^2(c+dx)}}-\frac{b^3C\tan(c$$

```
[Out] -((b^4*(A*(2 - n) + C*(3 - n))*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(2 - n)*(4 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^3*C*(b*Sec[c + d*x])^(-3 + n)*Tan[c + d*x])/(d
*(2 - n))
```

Rubi [A] time = 0.143475, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 4046, 3772, 2643}

$$-\frac{b^4(A(2-n)+C(3-n))\sin(c+dx)(b\sec(c+dx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)}{d(2-n)(4-n)\sqrt{\sin^2(c+dx)}}-\frac{b^3C\tan(c+dx)(b\sec(c+dx}{d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] -((b^4*(A*(2 - n) + C*(3 - n))*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(2 - n)*(4 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^3*C*(b*Sec[c + d*x])^(-3 + n)*Tan[c + d*x])/(d
*(2 - n))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = b^3 \int (b\sec(c+dx))^{-3+n} \left(A+C\sec^2(c+dx)\right) dx$$
$$= -\frac{b^3C(b\sec(c+dx))^{-3+n}\tan(c+dx)}{d(2-n)} + \left(b^3\left(A+\frac{C(3-n)}{2-n}\right)\right)$$
$$= -\frac{b^3C(b\sec(c+dx))^{-3+n}\tan(c+dx)}{d(2-n)} + \left(b^3\left(A+\frac{C(3-n)}{2-n}\right)\right)\Big($$
$$= -\frac{\left(A+\frac{C(3-n)}{2-n}\right)\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)(b\,s}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.24919, size = 118, normalized size = 0.89

$$\frac{b\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{n-1}\left(A(n-1)\cos^2(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\sec^2(c+dx)\right)\right.}{d(n-3)(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (b*Cot[c + d*x]*(A*(-1 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-3 + n)/
2, (-1 + n)/2, Sec[c + d*x]^2] + C*(-3 + n)*Hypergeometric2F1[1/2, (-1 + n)
/2, (1 + n)/2, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x
]^2])/(d*(-3 + n)*(-1 + n))
```

Maple [F] time = 1.328, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3 (b\sec(dx+c))^n \left(A+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{3}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + A*cos(d*x + c)^3)*(b*sec(d*x +
c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

3.34 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=142

$$\frac{2(A(2n+7)+C(2n+5))\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-3),\frac{1}{4}(1-2n),\cos^2(\right.}{d(2n+3)(2n+7)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (2*C*Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) + (2
*(C*(5 + 2*n) + A*(7 + 2*n))*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)
/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(3 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.123714, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{2(A(2n+7)+C(2n+5))\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-3);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)}{d(2n+3)(2n+7)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*C*Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) + (2
*(C*(5 + 2*n) + A*(7 + 2*n))*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)
/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(3 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{5}{2}+n}(c+dx)\left(A+C\sec\right.$$

$$= \frac{2C\sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(7+2n)} + \frac{\left(\left(C\left(\frac{5}{2}+n\right)\right.\right.}{}$$

$$= \frac{2C\sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(7+2n)} + \frac{\left(\left(C\left(\frac{5}{2}+n\right)\right.\right.}{}$$

$$= \frac{2C\sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(7+2n)} + \frac{2(C(5+2n)}{}$$

Mathematica [C] time = 3.16323, size = 341, normalized size = 2.4

$$-\frac{i2^{n+\frac{9}{2}}e^{2ic-\frac{1}{2}id(2n+1)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{1}{2}}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{e^{\frac{1}{2}i(4c+d(2n+9}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(9/2 + n)*E^((2*I)*c - (I/2)*d*(1 + 2*n)*x)*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*((A*E^((
I/2)*d*(5 + 2*n)*x)*Hypergeometric2F1[9/2 + n, (5 + 2*n)/4, (9 + 2*n)/4, -E
^((2*I)*(c + d*x))])/(5 + 2*n) + (E^((I/2)*(4*c + d*(9 + 2*n)*x))*(2*(A + 2
*C)*(13 + 2*n)*Hypergeometric2F1[9/2 + n, (9 + 2*n)/4, (13 + 2*n)/4, -E^((2
*I)*(c + d*x))] + A*E^((2*I)*(c + d*x))*(9 + 2*n)*Hypergeometric2F1[9/2 + n
, (13 + 2*n)/4, (17 + 2*n)/4, -E^((2*I)*(c + d*x))]))/((9 + 2*n)*(13 + 2*n)
))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*(A +
 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{5}{2}}(b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{n}\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^n*sqrt(sec(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

3.35 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=142

$$\frac{2(A(2n+5)+C(2n+3))\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-1),\frac{1}{4}(3-2n),\text{co}\right.}{d(2n+1)(2n+5)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (2*C*Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) + (2
*(C*(3 + 2*n) + A*(5 + 2*n))*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)
/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(1 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.126935, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{2(A(2n+5)+C(2n+3))\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)(2n+5)\sqrt{\sin^2(c+dx)}}+\frac{2C\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*C*Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) + (2
*(C*(3 + 2*n) + A*(5 + 2*n))*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)
/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(1 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{3}{2}+n}(c+dx)\left(A+C\sec^2\right.$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(5+2n)} + \frac{\left(\left(C\left(\frac{3}{2}+n\right)+\right.\right.}{}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(5+2n)} + \frac{\left(\left(C\left(\frac{3}{2}+n\right)+\right.\right.}{}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(5+2n)} + \frac{2(C(3+2n)+}{}$$

Mathematica [C] time = 2.2733, size = 303, normalized size = 2.13

$$-\frac{i2^{n+\frac{7}{2}}e^{-\frac{1}{2}i(2n+5)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{5}{2}}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{2(A+2C)e^{\frac{1}{2}i(2n+7)(c+dx)}\text{Hypergeomet}}{2}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(7/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(5/2 + n)*((A
*E^((I/2)*(3 + 2*n)*(c + d*x))*Hypergeometric2F1[1, (-7 - 2*n)/4, (7 + 2*n)
/4, -E^((2*I)*(c + d*x))])/(3 + 2*n) + (2*(A + 2*C)*E^((I/2)*(7 + 2*n)*(c +
 d*x))*Hypergeometric2F1[1, (-3 - 2*n)/4, (11 + 2*n)/4, -E^((2*I)*(c + d*x)
)])/(7 + 2*n) + (A*E^((I/2)*(11 + 2*n)*(c + d*x))*Hypergeometric2F1[1, (1 -
 2*n)/4, (15 + 2*n)/4, -E^((2*I)*(c + d*x))])/(11 + 2*n))*Sec[c + d*x]^(-2
- n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*E^((I/2)*(5 + 2*n)*(c +
d*x))*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{3}{2}}(b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n}\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*(b*sec(d*x + c))^n*sqrt(sec(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

3.36 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=140

$$\frac{2C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n}{d(2n+3)}-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}}{d(1-2n)(2n+3)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c}}$$

```
[Out] (2*C*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 + 2*n)*Sq
rt[Sec[c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.119767, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{2C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n}{d(2n+3)}-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5\right.}{d(1-2n)(2n+3)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*C*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 + 2*n)*Sq
rt[Sec[c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right) \int \sec^{\frac{1}{2}+n}(c+dx) \left(A+C\mathrm{s}\right.$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(3+2n)} + \frac{\left(\left(C\left(\frac{1}{2}+n\right.\right.\right.}{}$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(3+2n)} + \frac{\left(\left(C\left(\frac{1}{2}+n\right.\right.\right.}{}$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(3+2n)} - \frac{2(C+2Cn}{}$$

Mathematica [C] time = 2.33077, size = 303, normalized size = 2.16

$$-\frac{i2^{n+\frac{5}{2}}e^{-\frac{1}{2}i(2n+3)(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{3}{2}}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{2(A+2C)e^{\frac{1}{2}i(2n+5)(c+dx)}\text{Hypergeo}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(5/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(3/2 + n)*((A
*E^((I/2)*(1 + 2*n)*(c + d*x))*Hypergeometric2F1[1, (-5 - 2*n)/4, (5 + 2*n)
/4, -E^((2*I)*(c + d*x))])/(1 + 2*n) + (2*(A + 2*C)*E^((I/2)*(5 + 2*n)*(c +
 d*x))*Hypergeometric2F1[1, (-1 - 2*n)/4, (9 + 2*n)/4, -E^((2*I)*(c + d*x))
])/(5 + 2*n) + (A*E^((I/2)*(9 + 2*n)*(c + d*x))*Hypergeometric2F1[1, (3 - 2
*n)/4, (13 + 2*n)/4, -E^((2*I)*(c + d*x))])/(9 + 2*n))*Sec[c + d*x]^(-2 - n
)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*E^((I/2)*(3 + 2*n)*(c + d*x
))*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \left(A+C(\sec(dx+c))^2\right)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \sec\left(dx+c\right)^{2}+A\right)\left(b \sec\left(dx+c\right)\right)^{n} \sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec\left(dx+c\right)^{2}+A\right)\left(b \sec\left(dx+c\right)\right)^{n} \sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

3.37 $\int \frac{(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2C\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n}{d(2n+1)}-\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2}}{d(3-2n)(2n+1)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}}$$

```
[Out] (2*C*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)) - (2
*(A - C*(1 - 2*n) + 2*A*n)*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(1 + 2*n)*Se
c[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.113877, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$\frac{2C\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n}{d(2n+1)}-\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}\right.}{d(3-2n)(2n+1)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*C*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)) - (2
*(A - C*(1 - 2*n) + 2*A*n)*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(1 + 2*n)*Se
c[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)\left(A+C\sec^2(c+dx)\right)\,d$$

$$= \frac{2C\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)} + \frac{\left(\left(C\left(-\frac{1}{2}+n\right)+A\left(\frac{1}{2}+n\right)\right)\right.}{}$$

$$= \frac{2C\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)} + \frac{\left(\left(C\left(-\frac{1}{2}+n\right)+A\left(\frac{1}{2}+n\right)\right)\right.}{}$$

$$= \frac{2C\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)} - \frac{2(A-C(1-2n)+2An)\,{}_2F}{d(}$$

Mathematica [C] time = 4.56532, size = 311, normalized size = 2.21

$$-\frac{i2^{n+\frac{3}{2}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n-\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n-\frac{1}{2}}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left((2n-1)e^{2i(c+dx)}\left(2(2n+7)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + n)*(1
 + E^((2*I)*(c + d*x)))^(-1/2 + n)*(A*(21 + 20*n + 4*n^2)*Hypergeometric2F1
[3/2 + n, (-1 + 2*n)/4, (3 + 2*n)/4, -E^((2*I)*(c + d*x))] + E^((2*I)*(c +
d*x))*(-1 + 2*n)*(2*(A + 2*C)*(7 + 2*n)*Hypergeometric2F1[3/2 + n, (3 + 2*n
)/4, (7 + 2*n)/4, -E^((2*I)*(c + d*x))] + A*E^((2*I)*(c + d*x))*(3 + 2*n)*H
ypergeometric2F1[3/2 + n, (7 + 2*n)/4, (11 + 2*n)/4, -E^((2*I)*(c + d*x))])
)*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*(-1 +
 2*n)*(3 + 2*n)*(7 + 2*n)*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)\frac{1}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)\right)^{n}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

3.38 $\int \frac{(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{2(-2An+A+C(3-2n))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5-2n),\frac{1}{4}(9-2n),\cos^2(c+dx)\right)}{d(1-2n)(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}-2$$

```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]) - (
2*(A + C*(3 - 2*n) - 2*A*n)*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4
, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(5 - 2*n)*S
ec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.125937, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$-\frac{2(-2An+A+C(3-2n))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)}{d(1-2n)(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b\sec}{d(1-2n)\sqrt{\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```



```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]) - (
2*(A + C*(3 - 2*n) - 2*A*n)*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4
, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(5 - 2*n)*S
ec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)\left(A+C\sec^2(c+dx)\right)$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}} + \frac{\left(\left(C\left(-\frac{3}{2}+n\right)+A\left(-\frac{1}{2}+n\right)\right)\sec^{-n}(\right.}{}$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}} + \frac{\left(\left(C\left(-\frac{3}{2}+n\right)+A\left(-\frac{1}{2}+n\right)\right)\cos^{\frac{1}{2}+n}\right.}{}$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}} - \frac{2(A(1-2n)+C(3-2n))\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5\right.}{d(1-2n)(5}$$

Mathematica [C] time = 2.85872, size = 343, normalized size = 2.45

$$-\frac{i2^{n+\frac{1}{2}}e^{-\frac{1}{2}i(4c+d(2n+1)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{1}{2}}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{e^{\frac{1}{2}i(4c+d(2n}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((-I)*2^(1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(1
+ E^((2*I)*(c + d*x)))^(1/2 + n)*((A*E^((I/2)*d*(-3 + 2*n)*x)*Hypergeometri
c2F1[1/2 + n, (-3 + 2*n)/4, (1 + 2*n)/4, -E^((2*I)*(c + d*x))])/(d*(-3 + 2*
n)) + (E^((I/2)*(4*c + d*(1 + 2*n)*x))*(2*(A + 2*C)*(5 + 2*n)*Hypergeometri
c2F1[1/2 + n, (1 + 2*n)/4, (5 + 2*n)/4, -E^((2*I)*(c + d*x))] + A*E^((2*I)*
(c + d*x))*(1 + 2*n)*Hypergeometric2F1[1/2 + n, (5 + 2*n)/4, (9 + 2*n)/4, -
E^((2*I)*(c + d*x))]))/(d*(1 + 2*n)*(5 + 2*n)))*Sec[c + d*x]^(-2 - n)*(b*Se
c[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(E^((I/2)*(4*c + d*(1 + 2*n)*x))*(A +
 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)(\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+A\right) \left(b \sec \left(dx+c\right)\right)^{n}}{\sec \left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec \left(dx+c\right)^{2}+A\right) \left(b \sec \left(dx+c\right)\right)^{n}}{\sec \left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sec \left(c+dx\right)\right)^{n} \left(A+C \sec^{2}\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + C*sec(c + d*x)**2)/sec(c + d*x)**(3/2), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+A\right) \left(b \sec \left(dx+c\right)\right)^{n}}{\sec \left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

3.39 $\int \frac{(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{2(A(3-2n)+C(5-2n))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7-2n),\frac{1}{4}(11-2n),\cos^2(c+dx)\right)}{d(3-2n)(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}$$

```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)) - (
2*(A*(3 - 2*n) + C*(5 - 2*n))*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n
)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(7 - 2*n
)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.128632, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {20, 4046, 3772, 2643}

$$-\frac{2(A(3-2n)+C(5-2n))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)}{d(3-2n)(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b}{d(3-2n)\sec}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)) - (
2*(A*(3 - 2*n) + C*(5 - 2*n))*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n
)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(7 - 2*n
)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{5}{2}+n}(c+dx)\left(A+C\sec^2(c+dx)\right)dx$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\left(C\left(-\frac{5}{2}+n\right)+A\left(-\frac{3}{2}+n\right)\right)\sec^{-n}(c+$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\left(C\left(-\frac{5}{2}+n\right)+A\left(-\frac{3}{2}+n\right)\right)\cos^{\frac{1}{2}+n}(c$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)} - \frac{2(A(3-2n)+C(5-2n))\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-\right.}{d(3-2n)(7-}$$

Mathematica [C] time = 3.6713, size = 338, normalized size = 2.38

$$-\frac{i2^{n-\frac{1}{2}}e^{-\frac{1}{2}i(4c+d(2n-1)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n-\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n-\frac{1}{2}}\sec^{-n-2}(c+dx)\left(A+C\sec^2(c+dx)\right)(b\sec(c+dx))^n\left(\frac{e^{\frac{1}{2}i(4c+d(2n-1)x)}}{\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((-I)*2^(-1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(-1/2 + n)*((A*E^((I/2)*d*(-5 + 2*n)*x)*Hypergeome
tric2F1[-1/2 + n, (-5 + 2*n)/4, (-1 + 2*n)/4, -E^((2*I)*(c + d*x))])/(-5 +
2*n) + (E^((I/2)*(4*c + d*(-1 + 2*n)*x))*(2*(A + 2*C)*(3 + 2*n)*Hypergeomet
ric2F1[-1/2 + n, (-1 + 2*n)/4, (3 + 2*n)/4, -E^((2*I)*(c + d*x))] + A*E^((2
*I)*(c + d*x))*(-1 + 2*n)*Hypergeometric2F1[-1/2 + n, (3 + 2*n)/4, (7 + 2*n
)/4, -E^((2*I)*(c + d*x))]))/(-3 + 4*n + 4*n^2))*Sec[c + d*x]^(-2 - n)*(b*S
ec[c + d*x])^n*(A + C*Sec[c + d*x]^2))/(d*E^((I/2)*(4*c + d*(-1 + 2*n)*x))*
(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)(\sec(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

3.40 $\int \sec^m(c+dx)(b\sec(c+dx))^n \left(B\sec(c+dx)+C\sec^2(c+dx)\right.$

Optimal. Leaf size=167

$$\frac{B\sin(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-n),\frac{1}{2}(-m-n+2),\cos^2(c+dx)\right)}{d(m+n)\sqrt{\sin^2(c+dx)}}+\frac{C\sin(c\cdots}{}$$

```
[Out] (B*Hypergeometric2F1[1/2, (-m - n)/2, (2 - m - n)/2, Cos[c + d*x]^2]*Sec[c
+ d*x]^m*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(m + n)*Sqrt[Sin[c + d*x]^2])
+ (C*Hypergeometric2F1[1/2, (-1 - m - n)/2, (1 - m - n)/2, Cos[c + d*x]^2]*
Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)*Sqrt[S
in[c + d*x]^2])
```

Rubi [A] time = 0.125212, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {20, 4047, 3772, 2643, 12}

$$\frac{B\sin(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n);\frac{1}{2}(-m-n+2);\cos^2(c+dx)\right)}{d(m+n)\sqrt{\sin^2(c+dx)}}+\frac{C\sin(c+dx)\sec^{m+1}(c+d\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x
]
```

```
[Out] (B*Hypergeometric2F1[1/2, (-m - n)/2, (2 - m - n)/2, Cos[c + d*x]^2]*Sec[c
+ d*x]^m*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(m + n)*Sqrt[Sin[c + d*x]^2])
+ (C*Hypergeometric2F1[1/2, (-1 - m - n)/2, (1 - m - n)/2, Cos[c + d*x]^2]*
Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)*Sqrt[S
in[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n) \int \sec^{m+n}(c+d$$

$$= (\sec^{-n}(c+dx)(b\sec(c+dx))^n) \int C\sec^{2+m+n}(c$$

$$= (B\cos^{m+n}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n$$

$$= \frac{B\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n);\frac{1}{2}(2-m-n);\cos^2(c+dx\right.}{d(m+n)\sqrt{\sin}}$$

$$= \frac{B\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n);\frac{1}{2}(2-m-n);\cos^2(c+dx\right.}{d(m+n)\sqrt{\sin}}$$

Mathematica [A] time = 0.237104, size = 129, normalized size = 0.77

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n \left(B(m+n+2)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m\right.\right.}{d(m+n+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(B*(2 + m + n)*Hypergeometr
ic2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sec[c + d*x]^2] + C*(1 + m + n)*Hy
pergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Sec[c + d*x]^2]*Sec[c +
d*x])*Sqrt[-Tan[c + d*x]^2])/(d*(1 + m + n)*(2 + m + n))
```

Maple [F] time = 1.178, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^n \left(B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c))^n*sec(d*x +
c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c))^n*sec(d*x + c
)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c))^n*sec(d*x +
c)^m, x)
```

3.41 $\int \sec^2(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=154

$$\frac{3(11A+8C)\sin(c+dx)(b\sec(c+dx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)}{55bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+d}{}$$

```
[Out] (3*(11*A + 8*C)*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(5/3)*Sin[c + d*x])/(55*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeome
tric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(8/3)*Sin[c + d*x
])/(8*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(8/3)*Tan[c + d*x
])/(11*b^2*d)
```

Rubi [A] time = 0.159226, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(11A+8C)\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{55bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{8/3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\right.}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (3*(11*A + 8*C)*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(5/3)*Sin[c + d*x])/(55*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeome
tric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(8/3)*Sin[c + d*x
])/(8*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(8/3)*Tan[c + d*x
])/(11*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{8/3}\left(A+B\sec(c+dx)+C\,\mathrm{se}\right.}{b^2}$$

$$= \frac{\int (b\sec(c+dx))^{8/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$

$$= \frac{3C(b\sec(c+dx))^{8/3}\tan(c+dx)}{11b^2d} + \frac{(11A+8}{}$$

$$= \frac{3C(b\sec(c+dx))^{8/3}\tan(c+dx)}{11b^2d} + \frac{3B\,{}_2F_1\left(-\right.}{}$$

$$= \frac{3(11A+8C)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\,\mathrm{s}}{55bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 4.84952, size = 346, normalized size = 2.25

$$\frac{3(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\sec^{\frac{2}{3}}(c+dx)\left(2\tan(c+dx)\sec^2(c+dx)(4(11A+8C)\cos(2(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-I)*2^
(2/3)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(275*B*(1 + E^((2*I
)*(c + d*x))) + 275*B*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hy
pergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(c + d*x))] + 16*(11*A + 8*C)*E^(
I*(c + d*x))*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*
I)*c))) + Sec[c + d*x]^(2/3)*(275*B*Cos[d*x]*Csc[c] + 2*(44*A + 72*C + 55*B
*Cos[c + d*x] + 4*(11*A + 8*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x
])))/(440*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^
(8/3))
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2(b\sec(dx+c))^{\frac{2}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*
x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*se
c(d*x + c)^2, x)
```

3.42 $\int \sec(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=151

$$\frac{3(8A+5C)\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{16d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx)}{}$$

```
[Out] (3*(8*A + 5*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sin[c + d*x])/(16*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometri
c2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x])/(
5*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*
b*d)
```

Rubi [A] time = 0.160207, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(8A+5C)\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{16d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\right.}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```



```
[Out] (3*(8*A + 5*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sin[c + d*x])/(16*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometri
c2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x])/(
5*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*
b*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\int(b\sec(c+dx))^{5/3}\left(A+B\sec(c+dx)+C\sec\right.}{b}$$

$$= \frac{\int(b\sec(c+dx))^{5/3}\left(A+C\sec^2(c+dx)\right)dx}{b} +$$

$$= \frac{3C(b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + \frac{(8A+5C)}{}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))}{5bd\sqrt{\sin^2(c+dx)}}$$

$$= \frac{3(8A+5C)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec}{16d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 5.63532, size = 265, normalized size = 1.75

$$3i\left(\frac{be^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left((8A+5C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{8/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(c+dx)}\right)-16B\left(1+e^{2i(c+dx)}\right)^{8/3}\text{Hyp}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (((3*I)/40)*((b*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(16*B - 4
0*A*E^(I*(c + d*x)) - 5*C*E^(I*(c + d*x)) - 80*A*E^((3*I)*(c + d*x)) - 70*C
*E^((3*I)*(c + d*x)) - 16*B*E^((4*I)*(c + d*x)) - 40*A*E^((5*I)*(c + d*x))
- 25*C*E^((5*I)*(c + d*x)) - 16*B*(1 + E^((2*I)*(c + d*x)))^(8/3)*Hypergeom
etric2F1[1/3, 2/3, 4/3, -E^((2*I)*(c + d*x))] + (8*A + 5*C)*E^(I*(c + d*x))
*(1 + E^((2*I)*(c + d*x)))^(8/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I
)*(c + d*x))]))/(2^(1/3)*d*(1 + E^((2*I)*(c + d*x)))^2)
```

Maple [F] time = 0.149, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{2}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*se
c(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x
+ c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*se
c(d*x + c), x)
```

3.43 $\int (b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=146

$$-\frac{3b(5A+2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeor}}{2d\sqrt{\sin^2(c+}}$$

```
[Out] (-3*b*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(5*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/
(2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d
)
```

Rubi [A] time = 0.139643, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4047, 3772, 2643, 4046}

$$-\frac{3b(5A+2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (-3*b*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(5*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/
(2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d
)
```


Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b} + \int (b\sec(c+dx))^{2/3}\left(A+C\sec\right.$$

$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{1}{5}(5A+2C)\int (b\sec($$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+d}{2d\sqrt{\sin^2(c+dx)}}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+d}{2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.05197, size = 311, normalized size = 2.13

$$\frac{(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{3\cos(c+dx)(5B\csc(c)\cos(dx)\cos(c+dx)+2C\sin(c+dx))}{d} - \frac{3i2^{2/3}e^{-i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right.}{}\right.}{5(A\cos(2(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-3*I)*2^
(2/3)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(5*B*(1 + E^((2*I)*
(c + d*x))) + 5*B*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hyperg
eometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(c + d*x))] + (5*A + 2*C)*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1
/3, 2/3, 4/3, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))
*Sec[c + d*x]^(8/3)) + (3*Cos[c + d*x]*(5*B*Cos[d*x]*Cos[c + d*x]*Csc[c] +
2*C*Sin[c + d*x]))/d))/(5*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])
)
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{2}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x
)
```

3.44 $\int \cos(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=148

$$-\frac{3b^2(2A-C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3bB\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\right.}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*b^2*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Hypergeome
tric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(1
/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*C*Tan[c + d*x])/(2*d*(b*Sec[c + d*x])^(1/3
))
```

Rubi [A] time = 0.163401, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3b^2(2A-C)\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3bB\sin(c+dx)\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3bC\tan(c+dx)}{2d\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (-3*b^2*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Hypergeome
tric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(1
/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*C*Tan[c + d*x])/(2*d*(b*Sec[c + d*x])^(1/3
))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = b\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

$$= b\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx + B\int (b\sec(c$$

$$= \frac{3bC\tan(c+dx)}{2d\sqrt[3]{b\sec(c+dx)}} + \frac{1}{2}(b(2A-C))\int \frac{}{\sqrt[3]{b\,\mathrm{s}}}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+d}}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+d}}$$

Mathematica [A] time = 0.204417, size = 120, normalized size = 0.81

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{5/3}\left(10A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)-5\right.}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x]*(10*A*Cos[c + d*x]^2*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec
[c + d*x]^2] - 5*B*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*
x]^2] - 2*C*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c + d
*x])^(5/3)*Sqrt[-Tan[c + d*x]^2])/(10*b*d)
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{2}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*co
s(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*co
s(d*x + c), x)
```

3.45 $\int \cos^2(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=150

$$-\frac{3b(A+4C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3b^2B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3}\right.}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx)}$$

```
[Out] (-3*b^2*B*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4
*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*(A + 4*C)*Hypergeome
tric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^
(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c + d*x])/(4*d*(b*Sec[c + d*x])^
(4/3))
```

Rubi [A] time = 0.195291, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4045}

$$\frac{3Ab^2\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}}-\frac{3b(A+4C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (-3*b^2*B*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4
*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*(A + 4*C)*Hypergeome
tric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^
(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^2*Tan[c + d*x])/(4*d*(b*Sec[c + d*x])^
(4/3))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = b^2\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$$
$$= b^2\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx + (bB)\int \frac{}{\sqrt[3]{b\sec}}$$
$$= \frac{3Ab^2\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} + \frac{1}{4}(A+4C)\int (b\sec(c$$
$$= -\frac{3B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(}{4d\sqrt{\sin^2(c+dx}}$$
$$= -\frac{3(A+4C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+\right.}{4d\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.168129, size = 116, normalized size = 0.77

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{2/3}\left(A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)+4B\cos\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x]*(A*Cos[c + d*x]^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c
+ d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]
^2] - 2*C*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*(b*Sec[c + d*x]
)^(2/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^{\frac{2}{3}} \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2+B \sec (dx+c)+A\right) (b \sec (dx+c))^{\frac{2}{3}} \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*co
s(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos (dx+c)^2 \sec (dx+c)^2+B \cos (dx+c)^2 \sec (dx+c)+A \cos (dx+c)^2\right) (b \sec (dx+c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2+B \sec (dx+c)+A\right) (b \sec (dx+c))^{\frac{2}{3}} \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*co
s(d*x + c)^2, x)
```

3.46 $\int \cos^3(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=154

$$-\frac{3b^2(4A+7C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3b^3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\right.}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))}$$

```
[Out] (-3*b^3*B*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
7*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*(4*A + 7*C)*Hyper
geometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*d*(b*Sec[c +
d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^3*Tan[c + d*x])/(7*d*(b*Sec[c +
d*x])^(7/3))
```

Rubi [A] time = 0.192032, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4045}

$$-\frac{3b^2(4A+7C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}+\frac{3Ab^3\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}}-\frac{3b^3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx\right.}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (-3*b^3*B*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
7*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*(4*A + 7*C)*Hyper
geometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*d*(b*Sec[c +
d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^3*Tan[c + d*x])/(7*d*(b*Sec[c +
d*x])^(7/3))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = b^3\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/3}}$$

$$= b^3\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{7/3}}\,dx + \left(b^2B\right)\int \frac{}{(b}$$

$$= \frac{3Ab^3\tan(c+dx)}{7d(b\sec(c+dx))^{7/3}} + \frac{1}{7}(b(4A+7C))\int$$

$$= -\frac{3B\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+d\right.}{7d\sqrt{\sin^2(c+}}$$

$$= -\frac{3(4A+7C)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos\right.}{28d\sqrt{\sin}}$$

Mathematica [A] time = 0.224726, size = 118, normalized size = 0.77

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(4A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)+7B\cos(c+dx)\text{H}\right.}{28d\sqrt[3]{b\sec(c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (-3*b*Cot[c + d*x]*(4*A*Cos[c + d*x]^2*Hypergeometric2F1[-7/6, 1/2, -1/6, S
ec[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c +
 d*x]^2] + 28*C*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Ta
n[c + d*x]^2])/(28*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.638, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3(b\sec(dx+c))^{\frac{2}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^3*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*co
s(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2 + B\cos\left(dx+c\right)^3\sec\left(dx+c\right) + A\cos\left(dx+c\right)^3\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*co
s(d*x + c)^3, x)
```

3.47 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=154

$$\frac{3(13A+10C)\sin(c+dx)(b\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{91bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+}{}$$

```
[Out] (3*(13*A + 10*C)*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(7/3)*Sin[c + d*x])/(91*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeo
metric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(10/3)*Sin[c +
d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(10/3)*Tan[c
+ d*x])/(13*b^2*d)
```

Rubi [A] time = 0.153347, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(13A+10C)\sin(c+dx)(b\sec(c+dx))^{7/3}{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{91bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{10/3}{}_2F_1\left(-\frac{5}{3},\frac{1}{2}\right.}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (3*(13*A + 10*C)*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(7/3)*Sin[c + d*x])/(91*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeo
metric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(10/3)*Sin[c +
d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(10/3)*Tan[c
+ d*x])/(13*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{10/3}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.}{b^2}$$

$$= \frac{\int (b\sec(c+dx))^{10/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2}$$

$$= \frac{3C(b\sec(c+dx))^{10/3}\tan(c+dx)}{13b^2d} + \frac{(13A+}{}$$

$$= \frac{3C(b\sec(c+dx))^{10/3}\tan(c+dx)}{13b^2d} + \frac{3bB\,{}_2F_1}{}$$

$$= \frac{3C(b\sec(c+dx))^{10/3}\tan(c+dx)}{13b^2d} + \frac{3b(13A}{}$$

Mathematica [C] time = 6.48562, size = 444, normalized size = 2.88

$$\frac{3b\csc(c)e^{-idx}\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(40\sqrt[3]{2}\left(-1+e^{2ic}\right)(13A+10C)e^{2idx}\sqrt[3]{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt[3]{1+e^{2i}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (3*b*Csc[c]*(40*2^(1/3)*(13*A + 10*C)*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*(E^(
I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(1 + E^((2*I)*(c + d*x)))^(1/
3)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(c + d*x))] - ((-1 + E^((2*I)
*c))*(91*B*(-7 - 30*E^((2*I)*(c + d*x)) + 30*E^((6*I)*(c + d*x)) + 7*E^((8*
I)*(c + d*x))) + 80*E^(I*(c + d*x))*(13*A*(1 + E^((2*I)*(c + d*x)))^2*(1 +
5*E^((2*I)*(c + d*x)) + 2*E^((4*I)*(c + d*x))) + 2*C*(5 + 21*E^((2*I)*(c +
d*x)) + 79*E^((4*I)*(c + d*x)) + 45*E^((6*I)*(c + d*x)) + 10*E^((8*I)*(c +
d*x)))) + 637*B*(1 + E^((2*I)*(c + d*x)))^(13/3)*Hypergeometric2F1[1/6, 1/3
, 7/6, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^(1/3))/(2*E^(I*(c - d*x))*(1 + E
^((2*I)*(c + d*x)))^4))*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2))/(1820*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d
*x)])*Sec[c + d*x]^(7/3))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^2 (b \sec (dx + c))^{\frac{4}{3}} \left(A + B \sec (dx + c) + C (\sec (dx + c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \sec (dx + c)^5 + Bb \sec (dx + c)^4 + Ab \sec (dx + c)^3\right) (b \sec (dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^5 + B*b*sec(d*x + c)^4 + A*b*sec(d*x + c)^3)*(b*
sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^2 + B \sec (dx + c) + A\right) (b \sec (dx + c))^{\frac{4}{3}} \sec (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*se
c(d*x + c)^2, x)
```

3.48 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=151

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{40d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+d}{}$$

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d*x])
/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(7/3)*Tan[c + d*x])/(
10*b*d)
```

Rubi [A] time = 0.154035, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{40d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\right.}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d*x])
/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(7/3)*Tan[c + d*x])/(
10*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\int (b\sec(c+dx))^{7/3}\left(A+B\sec(c+dx)+C\sec\ldots\right.}{b}$$

$$= \frac{\int (b\sec(c+dx))^{7/3}\left(A+C\sec^2(c+dx)\right)dx}{b} + \ldots$$

$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10bd} + \frac{(10A+7C\ldots}{\ldots}$$

$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10bd} + \frac{3bB\,{}_2F_1\left(-\ldots\right.}{\ldots}$$

$$= \frac{3(10A+7C)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec\ldots}{40d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 6.9811, size = 465, normalized size = 3.08

$$\frac{\frac{\cos^4(c+dx)(b\sec(c+dx))^{7/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{3\sec(c)\sec(c+dx)(70A\sin(dx)+40B\sin(c)+49C\sin(dx))}{140d}+\frac{3(10A+7C)\tan(c)}{20d}+\frac{3\sec(c)\sec^2(c+dx)(10B\sin(dx)+7C\ldots}{35d}\right.}{A\cos(2c+2dx)+A+2B\cos(c+dx)+2C}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] ((((-3*I)/70)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(160*B*(1 +
 E^((2*I)*(c + d*x))) + 160*B*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^
(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, -E^((2*I)*(c + d*x))] + 7*(10*A + 7
*C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hype
rgeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c + d*x))])*(b*Sec[c + d*x])^(7/3)*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(2^(2/3)*d*E^(I*(c + d*x))*(-1 + E
^((2*I)*c))*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^
(13/3)) + (Cos[c + d*x]^4*(b*Sec[c + d*x])^(7/3)*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((24*B*Cos[d*x]*Csc[c])/(7*d) + (3*C*Sec[c]*Sec[c + d*x]^3*Si
n[d*x])/(5*d) + (3*Sec[c]*Sec[c + d*x]^2*(7*C*Sin[c] + 10*B*Sin[d*x]))/(35*
d) + (3*Sec[c]*Sec[c + d*x]*(40*B*Sin[c] + 70*A*Sin[d*x] + 49*C*Sin[d*x]))/
(140*d) + (3*(10*A + 7*C)*Tan[c])/(20*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x]))/b
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\sec(dx+c)^4+Bb\sec(dx+c)^3+Ab\sec(dx+c)^2\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^4 + B*b*sec(d*x + c)^3 + A*b*sec(d*x + c)^2)*(b*
sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*se
c(d*x + c), x)
```

3.49 $\int (b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=146

$$\frac{3b(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx)}{}$$

```
[Out] (3*b*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c
 + d*x])^(1/3)*Sin[c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/
(4*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(4/3)*Tan[c + d*x])/(7*d
)
```

Rubi [A] time = 0.137347, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4047, 3772, 2643, 4046}

$$\frac{3b(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\text{c}\right.}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (3*b*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c
 + d*x])^(1/3)*Sin[c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/
(4*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(4/3)*Tan[c + d*x])/(7*d
)
```


Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b}+\int (b\sec(c+dx))^{4/3}\left(A+C\sec\right.$$

$$=\frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7d}+\frac{1}{7}(7A+4C)\int (b\sec(c$$

$$=\frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+d}{4d\sqrt{\sin^2(c+dx)}}$$

$$=\frac{3b(7A+4C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,s}{7d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.39704, size = 290, normalized size = 1.99

$$-\frac{3ib\sqrt[3]{b\sec(c+dx)}\left(-14Ae^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-e^{2i(c+dx)}\right)+7B\left(1+e^{2i(c+dx)}\right)^{7/3}\text{Hyper}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (((-3*I)/28)*b*(-7*B + 28*A*E^(I*(c + d*x)) + 8*C*E^(I*(c + d*x)) + 56*A*E^
((3*I)*(c + d*x)) + 40*C*E^((3*I)*(c + d*x)) + 7*B*E^((4*I)*(c + d*x)) + 28
*A*E^((5*I)*(c + d*x)) + 16*C*E^((5*I)*(c + d*x)) + 7*B*(1 + E^((2*I)*(c +
d*x)))^(7/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c + d*x))] - 14*A*
E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(7/3)*Hypergeometric2F1[1/3, 2/3,
 5/3, -E^((2*I)*(c + d*x))] - 8*C*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))
^(7/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(c + d*x))])*(b*Sec[c + d
*x])^(1/3))/(d*(1 + E^((2*I)*(c + d*x)))^2)
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec(dx+c)^3+Bb\sec(dx+c)^2+Ab\sec(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*se
c(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x
)
```

3.50 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=146

$$-\frac{3b^2(4A+C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometr}}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*b^2*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeome
tric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x]
)/(d*Sqrt[Sin[c + d*x]^2]) + (3*b*C*(b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4
*d)
```

Rubi [A] time = 0.158827, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3b^2(4A+C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}+\frac{3bC}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (-3*b^2*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeome
tric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x]
)/(d*Sqrt[Sin[c + d*x]^2]) + (3*b*C*(b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4
*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = b\int \sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\right.$$

$$= b\int \sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,d$$

$$= \frac{3bC\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4d} + \frac{1}{4}(b(4A$$

$$= \frac{3bB\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c-}}{d\sqrt{\sin^2(c+dx)}}$$

$$= \frac{3bB\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c-}}{d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.35104, size = 303, normalized size = 2.08

$$\frac{3b\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\sqrt[3]{\sec(c+dx)}(4B\csc(c)\cos(dx)+C\tan(c+dx)) - \frac{i\sqrt[3]{2}e^{-i(c+dx)}}{\cdots}\right.}{2d\sec^{\frac{7}{3}}(c+dx)(A\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (3*b*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-I)*
2^(1/3)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(4*B*(1 + E^((2*I
)*(c + d*x))) + 4*B*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hype
rgeometric2F1[-1/3, 1/3, 2/3, -E^((2*I)*(c + d*x))] + (4*A + C)*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[1
/6, 1/3, 7/6, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c)))
+ Sec[c + d*x]^(1/3)*(4*B*Cos[d*x]*Csc[c] + C*Tan[c + d*x])))/(2*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/3))
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^{2}+B\sec \left(dx+c\right)+A\right)\left(b\sec \left(dx+c\right)\right)^{\frac{4}{3}}\cos \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*co
s(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos \left(dx+c\right)\sec \left(dx+c\right)^{3}+Bb\cos \left(dx+c\right)\sec \left(dx+c\right)^{2}+Ab\cos \left(dx+c\right)\sec \left(dx+c\right)\right)\left(b\sec \left(dx+c\right)\right)^{\frac{1}{3}},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)*sec(d*x + c)^3 + B*b*cos(d*x + c)*sec(d*x + c)^2
 + A*b*cos(d*x + c)*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^{2}+B\sec \left(dx+c\right)+A\right)\left(b\sec \left(dx+c\right)\right)^{\frac{4}{3}}\cos \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*co
s(d*x + c), x)
```

3.51 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=150

$$-\frac{3b^3(A-2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3b^2B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\right.}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^2}$$

```
[Out] (-3*b^3*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hyperge
ometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x
])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b^2*C*Tan[c + d*x])/(d*(b*Sec[c + d*x])
^(2/3))
```

Rubi [A] time = 0.185851, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3b^3(A-2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3b^2C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (-3*b^3*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hyperge
ometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x
])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b^2*C*Tan[c + d*x])/(d*(b*Sec[c + d*x])
^(2/3))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = b^2\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}$$

$$= b^2\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}dx + (bB)\int \sqrt[3]{b}$$

$$= \frac{3b^2C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}} + \left(b^2(A-2C)\right)\int \frac{}{(b s}$$

$$= -\frac{3bB\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx\right.}{2d\sqrt{\sin^2(c+}}$$

$$= -\frac{3bB\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx\right.}{2d\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.205939, size = 117, normalized size = 0.78

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{4/3}\left(2A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)-4B\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x]*(2*A*Cos[c + d*x]^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[
c + d*x]^2] - 4*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x
]^2] - C*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*x])
^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^{\frac{4}{3}} \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx+c)^2 + B\sec (dx+c) + A\right) (b\sec (dx+c))^{\frac{4}{3}} \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*co
s(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos (dx+c)^2 \sec (dx+c)^3 + Bb\cos (dx+c)^2 \sec (dx+c)^2 + Ab\cos (dx+c)^2 \sec (dx+c)\right) (b\sec (dx+c))^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + B*b*cos(d*x + c)^2*sec(d*x +
c)^2 + A*b*cos(d*x + c)^2*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx+c)^2 + B\sec (dx+c) + A\right) (b\sec (dx+c))^{\frac{4}{3}} \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*co
s(d*x + c)^2, x)
```

3.52 $\int \cos^3(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=154

$$-\frac{3b^2(2A+5C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}-\frac{3b^3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+d}$$

```
[Out] (-3*b^3*B*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*(2*A + 5*C)*Hyper
geometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*(b*Sec[c +
d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^3*Tan[c + d*x])/(5*d*(b*Sec[c +
d*x])^(5/3))
```

Rubi [A] time = 0.191297, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4045}

$$-\frac{3b^2(2A+5C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3Ab^3\tan(c+dx)}{5d(b\sec(c+dx))^{5/3}}-\frac{3b^3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+\right.}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (-3*b^3*B*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*(2*A + 5*C)*Hyper
geometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*(b*Sec[c +
d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*A*b^3*Tan[c + d*x])/(5*d*(b*Sec[c +
d*x])^(5/3))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = b^3\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/3}}\,dx$$
$$= b^3\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{5/3}}\,dx + \left(b^2B\right)\int \frac{}{(b\,\mathrm{se}}$$
$$= \frac{3Ab^3\tan(c+dx)}{5d(b\sec(c+dx))^{5/3}} + \frac{1}{5}(b(2A+5C))\int \sqrt[3]{b}$$
$$= -\frac{3bB\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx}}$$
$$= -\frac{3b(2A+5C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(\right.}{10d\sqrt{\sin^2(c}}$$

Mathematica [A] time = 0.172995, size = 118, normalized size = 0.77

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\sqrt[3]{b\sec(c+dx)}\left(2A\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(c+dx)\right)+5B\cos\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (-3*b*Cot[c + d*x]*(2*A*Cos[c + d*x]^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Se
c[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c +
d*x]^2] - 10*C*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(10*d)
```

Maple [F] time = 0.64, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3 (b\sec(dx+c))^{\frac{4}{3}} \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^3*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^2 + B \sec (dx + c) + A\right) (b \sec (dx + c))^{\frac{4}{3}} \cos (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*co
s(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos (dx + c)^3 \sec (dx + c)^3 + Bb \cos (dx + c)^3 \sec (dx + c)^2 + Ab \cos (dx + c)^3 \sec (dx + c)\right) (b \sec (dx + c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3*sec(d*x + c)^3 + B*b*cos(d*x + c)^3*sec(d*x +
c)^2 + A*b*cos(d*x + c)^3*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^2 + B \sec (dx + c) + A\right) (b \sec (dx + c))^{\frac{4}{3}} \cos (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*co
s(d*x + c)^3, x)
```

3.53 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\text{H}}{4b^2}$$

```
[Out] (3*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/
(4*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(4/3)*Tan[c + d*x])/
(7*b^2*d)
```

Rubi [A] time = 0.151808, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c\right.}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(2/3),x]
```


```
[Out] (3*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/
(4*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(4/3)*Tan[c + d*x])/
(7*b^2*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,d}{b^2}$$

$$= \frac{\int(b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)\,dx}{b^2} + \frac{B\int(b\sec(c}{b}$$

$$= \frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7b^2d} + \frac{(7A+4C)\int(b\sec(c+}{7b^2}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx}{4b^2d\sqrt{\sin^2(c+dx)}}$$

$$= \frac{3(7A+4C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin}{7bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.97887, size = 304, normalized size = 1.97

$$\frac{3be^{-ic}\left(-1+e^{2ic}\right)\csc(c)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2(7A+4C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{7/3}\text{Hypergeometric2}\right.}{28d\,(1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(2/3),x]
```

```
[Out] (3*b*(-1 + E^((2*I)*c))*Csc[c]*(7*B - 28*A*E^(I*(c + d*x)) - 8*C*E^(I*(c +
d*x)) - 56*A*E^((3*I)*(c + d*x)) - 40*C*E^((3*I)*(c + d*x)) - 7*B*E^((4*I)*
(c + d*x)) - 28*A*E^((5*I)*(c + d*x)) - 16*C*E^((5*I)*(c + d*x)) - 7*B*(1 +
 E^((2*I)*(c + d*x)))^(7/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c +
 d*x))] + 2*(7*A + 4*C)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(7/3)*Hyp
ergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(c + d*x))])*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2))/(28*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(5/3))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^3+B\sec\left(dx+c\right)^2+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x
+ c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(b*sec(c
+ d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(2/3), x)
```

3.54 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=147

$$-\frac{3(4A+C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2}}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F
1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d
*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.146348, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3(4A+C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3C\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])
^(2/3),x]
```

```
[Out] (-3*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F
1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d
*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx}{b}$$

$$= \frac{\int \sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx}{b} + \frac{B\int(b\sec(c+}{b^2}$$

$$= \frac{3C\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4bd} + \frac{(4A+C)\int\sqrt[3]{b\sec(c+dx}}{4b}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} +$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} -$$

Mathematica [C] time = 1.90498, size = 305, normalized size = 2.07

$$\frac{3\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\sqrt[3]{\sec(c+dx)}(4B\csc(c)\cos(dx)+C\tan(c+dx)) - \frac{i\sqrt[3]{2}e^{-i(c+dx)}\sqrt[3]{}}{}\right.}{2bd\sec^{\frac{7}{3}}(c+dx)(A\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c +
 d*x])^(2/3),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-I)*2^
(1/3)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(4*B*(1 + E^((2*I)*
(c + d*x))) + 4*B*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hyperg
eometric2F1[-1/3, 1/3, 2/3, -E^((2*I)*(c + d*x))] + (4*A + C)*E^(I*(c + d*x
))*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[1/6
, 1/3, 7/6, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) +
Sec[c + d*x]^(1/3)*(4*B*Cos[d*x]*Csc[c] + C*Tan[c + d*x])))/(2*b*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/3))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int \sec(dx+c)\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b,
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c+dx\right) + C\sec^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2/3)
,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(b*sec(c + d
*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(2/3), x)
```

3.55 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=142

$$-\frac{3b(A-2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},c\right.}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*b*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d
*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometri
c2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/
3)*Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(d*(b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.135153, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4047, 3772, 2643, 4046}

$$-\frac{3b(A-2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(2/3),x]
```



```
[Out] (-3*b*(A - 2*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d
*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometri
c2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/
3)*Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(d*(b*Sec[c + d*x])^(2/3))
```


Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
```

```
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx=\frac{B\int \sqrt[3]{b\sec(c+dx)}\,dx}{b}+\int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$$

$$=\frac{3C\tan(c+dx)}{d(b\sec(c+dx))^{2/3}}+(A-2C)\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx+\frac{\left(B\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\ldots}\right.}{\ldots}$$

$$=-\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}+\ldots$$

$$=-\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}-\ldots$$

Mathematica [C] time = 1.83104, size = 173, normalized size = 1.22

$$\frac{3e^{-idx}(\sin(dx)-i\cos(dx))\sqrt[3]{b\sec(c+dx)}\left((A-2C)e^{i(c+dx)}\sqrt[3]{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-e^{2i(c+dx)}\right)+4\ldots\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(1/3)*((-I)*Cos[d*x] + Sin[d*x])*(-2*A*Cos[c + d*x] + 4
*C*Cos[c + d*x] + 4*B*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[1/6
, 1/3, 7/6, -E^((2*I)*(c + d*x))] + (A - 2*C)*E^(I*(c + d*x))*(1 + E^((2*I)
*(c + d*x)))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(c + d*x))] +
 (4*I)*C*Sin[c + d*x]))/(4*b*d*E^(I*d*x))
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int (A+B\sec(dx+c)+C(\sec(dx+c))^2)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+B\sec(dx+c)+A}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/(b*
sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(2/3),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x
)
```

3.56 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=147

$$-\frac{3(4A+C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometr}}{bd\sqrt{\sin^2(c+dx}}$$

```
[Out] (-3*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F
1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d
*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.14016, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3(4A+C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}+\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3C\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])
^(2/3),x]
```


```
[Out] (-3*(4*A + C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F
1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d
*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx}{b}$$

$$= \frac{\int \sqrt[3]{b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx}{b} + \frac{B\int(b\sec(c+dx)}{b^2}$$

$$= \frac{3C\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{4bd} + \frac{(4A+C)\int\sqrt[3]{b\sec(c+dx)}\,dx}{4b}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} + 3($$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - 3($$

Mathematica [C] time = 1.46259, size = 305, normalized size = 2.07

$$\frac{3\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\sqrt[3]{\sec(c+dx)}(4B\csc(c)\cos(dx)+C\tan(c+dx)) - \frac{i\sqrt[3]{2}e^{-i(c+dx)}\sqrt[3]{\frac{e^{i}}{1+e}}}{}\right.}{2bd\sec^{\frac{7}{3}}(c+dx)(A\cos(2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c +
 d*x])^(2/3),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-I)*2^
(1/3)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3)*(4*B*(1 + E^((2*I)*
(c + d*x))) + 4*B*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hyperg
eometric2F1[-1/3, 1/3, 2/3, -E^((2*I)*(c + d*x))] + (4*A + C)*E^(I*(c + d*x
))*(-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[1/6
, 1/3, 7/6, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) +
Sec[c + d*x]^(1/3)*(4*B*Cos[d*x]*Csc[c] + C*Tan[c + d*x])))/(2*b*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/3))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \sec(dx+c)\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b,
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2/3)
,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(b*sec(c + d
*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(2/3), x)
```

3.57 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4}}{}$$

```
[Out] (3*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/
(4*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(4/3)*Tan[c + d*x])/
(7*b^2*d)
```

Rubi [A] time = 0.147516, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(7A+4C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\text{co}\right.}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(2/3),x]
```

```
[Out] (3*(7*A + 4*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/
(4*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(4/3)*Tan[c + d*x])/
(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int (b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx}{b^2}$$

$$= \frac{\int (b\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2} + \frac{B\int (b\sec(c+d}{b^3}$$

$$= \frac{3C(b\sec(c+dx))^{4/3}\tan(c+dx)}{7b^2d} + \frac{(7A+4C)\int (b\sec(c+dx}{7b^2}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} +$$

$$= \frac{3(7A+4C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c}{7bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.89875, size = 304, normalized size = 1.97

$$\frac{3be^{-ic}\left(-1+e^{2ic}\right)\csc(c)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2(7A+4C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{7/3}\text{Hypergeometric2F1}\right.}{28d\left(1+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(2/3),x]
```

```
[Out] (3*b*(-1 + E^((2*I)*c))*Csc[c]*(7*B - 28*A*E^(I*(c + d*x)) - 8*C*E^(I*(c +
d*x)) - 56*A*E^((3*I)*(c + d*x)) - 40*C*E^((3*I)*(c + d*x)) - 7*B*E^((4*I)*
(c + d*x)) - 28*A*E^((5*I)*(c + d*x)) - 16*C*E^((5*I)*(c + d*x)) - 7*B*(1 +
 E^((2*I)*(c + d*x)))^(7/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(c +
 d*x))] + 2*(7*A + 4*C)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(7/3)*Hyp
ergeometric2F1[1/3, 2/3, 5/3, -E^((2*I)*(c + d*x))])*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2))/(28*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(5/3))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^3+B\sec\left(dx+c\right)^2+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x
+ c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(b*sec(c
+ d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(2/3), x)
```

3.58 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+d}{}$$

```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeo
metric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d
*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(7/3)*Tan[c + d
*x])/(10*b^3*d)
```

Rubi [A] time = 0.157399, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(10A+7C)\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\right.}{7b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(2/3),x]
```


```
[Out] (3*(10*A + 7*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeo
metric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d
*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(7/3)*Tan[c + d
*x])/(10*b^3*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int (b\sec(c+dx))^{7/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx}{b^3}$$
$$= \frac{\int (b\sec(c+dx))^{7/3}\left(A+C\sec^2(c+dx)\right)dx}{b^3} + \frac{B\int (b\sec(c+ d}{b^4}$$
$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10b^3d} + \frac{(10A+7C)\int (b\sec(c+dx}{10b^3}$$
$$= \frac{3C(b\sec(c+dx))^{7/3}\tan(c+dx)}{10b^3d} + \frac{3B\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c-\right.}{}$$
$$= \frac{3(10A+7C)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin}{40b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 2.4415, size = 333, normalized size = 2.16

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{3(7(10A+7C)\sin(c+dx)+4\tan(c+dx)(10B+7C\sec(c+dx))+160B\csc(c)\cos(dx)\cos(c+dx))}{d}-\frac{3i\sqrt[3]{2}e^{-i(c+dx)}}{}\right.}{140(b\sec(c+dx))^{2/3}(A}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(2/3),x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-3*I)*2^(1/3)*(E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x))))^(1/3)*(160*B*(1 + E^((2*I)*(c + d*x))) + 160*B*(
-1 + E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[-1/3, 1
/3, 2/3, -E^((2*I)*(c + d*x))] + 7*(10*A + 7*C)*E^(I*(c + d*x))*(-1 + E^((2
*I)*c))*(1 + E^((2*I)*(c + d*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E
^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sec[c + d*x]^(4
/3)) + (3*(160*B*Cos[d*x]*Cos[c + d*x]*Csc[c] + 7*(10*A + 7*C)*Sin[c + d*x]
 + 4*(10*B + 7*C*Sec[c + d*x])*Tan[c + d*x]))/d))/(140*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x
 + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*
x + c))^(1/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(b*sec(c
+ d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec \left(dx+c\right)^2+B\sec \left(dx+c\right)+A\right)\sec \left(dx+c\right)^3}{\left(b\sec \left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x
 + c))^(2/3), x)
```

3.59 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=154

$$-\frac{3(5A+2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeom}}{2b^2d\sqrt{\sin^2(c+}}$$

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/
(2*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/
(5*b^2*d)
```

Rubi [A] time = 0.150623, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3(5A+2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(4/3),x]
```


```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/
(2*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/
(5*b^2*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx}{b^2}$$

$$= \frac{\int (b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2} + \frac{B\int (b\sec(c+$$

$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5b^2d} + \frac{(5A+2C)\int (b\sec(c+dx)}{5b^2}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} +$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} -$$

Mathematica [C] time = 1.66668, size = 299, normalized size = 1.94

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{3(5B\csc(c)\cos(dx)+2C\tan(c+dx))}{d} - \frac{3i2^{2/3}e^{-i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(\left(-1+e^{2ic}\right)(5A+2C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right.}{}\right.}{5(b\sec(c+dx))^{4/3}(A\cos(2(c+dx))+A+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(4/3),x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-3*I)*2^(2/3)*(E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x))))^(2/3)*(5*B*(1 + E^((2*I)*(c + d*x))) + 5*B*(-1 +
 E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3,
5/6, -E^((2*I)*(c + d*x))] + (5*A + 2*C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))
*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)
*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sec[c + d*x]^(2/3)) +
(3*(5*B*Cos[d*x]*Csc[c] + 2*C*Tan[c + d*x]))/d))/(5*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(b*sec(c
+ d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(4/3), x)
```

3.60 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3(2A-C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\right.}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F
1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*
Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.144028, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3(2A-C)\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])
^(4/3),x]
```



```
[Out] (-3*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F
1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*
Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$= \frac{\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$

$$= \frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}} + \frac{(2A-C)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{2b} + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right.\right.}{}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin}{b^2d\sqrt{\sin^2(c+dx)}}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin}{b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.92147, size = 175, normalized size = 1.17

$$\frac{3e^{-idx}(\sin(dx)-i\cos(dx))(b\sec(c+dx))^{2/3}\left((2A-C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(c+dx)}\right)-\right.}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c +
 d*x])^(4/3),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(2/3)*((-I)*Cos[d*x] + Sin[d*x])*(-10*A*Cos[c + d*x] +
5*C*Cos[c + d*x] + 5*B*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/
3, 2/3, 4/3, -E^((2*I)*(c + d*x))] + (2*A - C)*E^(I*(c + d*x))*(1 + E^((2*I
)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(c + d*x))]
 + (5*I)*C*Sin[c + d*x]))/(10*b^2*d*E^(I*d*x))
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \sec(dx+c)\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx + c\right)^2 + B \sec \left(dx + c\right) + A\right) \sec \left(dx + c\right)}{\left(b \sec \left(dx + c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec \left(dx + c\right)^2 + B \sec \left(dx + c\right) + A\right) \left(b \sec \left(dx + c\right)\right)^{\frac{2}{3}}}{b^2 \sec \left(dx + c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^
2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B \sec \left(c + dx\right) + C \sec^2 \left(c + dx\right)\right) \sec \left(c + dx\right)}{\left(b \sec \left(c + dx\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3)
,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(b*sec(c + d
*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx + c\right)^2 + B \sec \left(dx + c\right) + A\right) \sec \left(dx + c\right)}{\left(b \sec \left(dx + c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(4/3), x)
```

3.61 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=146

$$-\frac{3(A+4C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\right.}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/}}$$

```
[Out] (-3*B*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*Hypergeometric2F
1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*(b*Sec[c + d*x])^(1/3
)*Sqrt[Sin[c + d*x]^2]) + (3*A*Tan[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Rubi [A] time = 0.138117, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4047, 3772, 2643, 4045}

$$-\frac{3(A+4C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3A\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(4/3),x]
```


```
[Out] (-3*B*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*Hypergeometric2F
1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*(b*Sec[c + d*x])^(1/3
)*Sqrt[Sin[c + d*x]^2]) + (3*A*Tan[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3))
```


Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{B\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b} + \int \frac{A+C\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$$

$$= \frac{3A\tan(c+dx)}{4d(b\sec(c+dx))^{4/3}} + \frac{(A+4C)\int (b\sec(c+dx))^{2/3}\,dx}{4b^2} + \frac{\left(B\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\right.}{}$$

$$= -\frac{3B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} + \cdots$$

$$= -\frac{3(A+4C)\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+\cdots}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.93147, size = 298, normalized size = 2.04

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-\frac{30\cos(c+dx)(4B\cot(c)-A\sin(c+dx))}{d}+\frac{3i2^{2/3}e^{-idx}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(1+e^{2i(c+dx)}\right)^{2/3}\left(e^{idx}\left(8Be^{i(c+dx)}\text{Hyperg}\cdots\right.\right.}{\cdots}\right.}{20(b\sec(c+dx))^{4/3}(A\cos(2(c+dx))\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((3*I)*2^(2/3)*(E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E^((2*I)*(c + d*x)))^(2/3)*(40*B*E^(I*
c)*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(c + d*x))] + E^(I*d*x)*(-5*
(A + 4*C)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)*(c
+ d*x))] + 8*B*E^(I*(c + d*x))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*
(c + d*x))])))/(d*E^(I*d*x)*(-1 + E^((2*I)*c))*Sec[c + d*x]^(2/3)) - (30*Co
s[c + d*x]*(4*B*Cot[c] - A*Sin[c + d*x]))/d))/(20*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + B \sec (dx+c) + A}{(b \sec (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec (dx+c)^2 + B \sec (dx+c) + A\right) (b \sec (dx+c))^{\frac{2}{3}}}{b^2 \sec (dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^
2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \sec (c+dx) + C \sec^2 (c+dx)}{(b \sec (c+dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(b*sec(c + d*x))**(4/3),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + B \sec (dx+c) + A}{(b \sec (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x
)
```

3.62 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3(2A-C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos\right.}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F
1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*
Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.141136, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3(2A-C)\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x])
^(4/3),x]
```


```
[Out] (-3*(2*A - C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(8*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F
1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*
Sqrt[Sin[c + d*x]^2]) + (3*C*Tan[c + d*x])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx=\frac{\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$=\frac{\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}+\frac{B\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$

$$=\frac{3C\tan(c+dx)}{2bd\sqrt[3]{b\sec(c+dx)}}+\frac{(2A-C)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{2b}+\frac{\Big(B\Big(\frac{\cos(c\cdots}{b\cdots}$$

$$=-\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\cdots}{b^2d\sqrt{\sin^2(c+dx)}\cdots}$$

$$=-\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\cdots}{b^2d\sqrt{\sin^2(c+dx)}\cdots}$$

Mathematica [C] time = 1.19564, size = 175, normalized size = 1.17

$$\frac{3e^{-idx}(\sin(dx)-i\cos(dx))(b\sec(c+dx))^{2/3}\left((2A-C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(c+dx\cdots}\right.\right.}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c +
 d*x])^(4/3),x]
```

```
[Out] (3*(b*Sec[c + d*x])^(2/3)*((-I)*Cos[d*x] + Sin[d*x])*(-10*A*Cos[c + d*x] +
5*C*Cos[c + d*x] + 5*B*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/
3, 2/3, 4/3, -E^((2*I)*(c + d*x))] + (2*A - C)*E^(I*(c + d*x))*(1 + E^((2*I
)*(c + d*x)))^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(c + d*x))]
 + (5*I)*C*Sin[c + d*x]))/(10*b^2*d*E^(I*d*x))
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \sec(dx+c)\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^
2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4/3)
,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(b*sec(c + d
*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c))^(4/3), x)
```

3.63 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=154

$$-\frac{3(5A+2C)\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometri}}{2b^2d\sqrt{\sin^2(c+dx}}$$

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/
(2*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/
(5*b^2*d)
```

Rubi [A] time = 0.147556, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{3(5A+2C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}+\frac{3C}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(4/3),x]
```

```
[Out] (-3*(5*A + 2*C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(5*b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/
(2*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/
(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) d}{b^2}$$

$$= \frac{\int (b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx}{b^2} + \frac{B\int (b\sec(c}{l}$$

$$= \frac{3C(b\sec(c+dx))^{2/3}\tan(c+dx)}{5b^2d} + \frac{(5A+2C)\int (b\sec(c+}{5b^2}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx}{2b^2d\sqrt{\sin^2(c+dx)}}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 1.67554, size = 299, normalized size = 1.94

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{3(5B\csc(c)\cos(dx)+2C\tan(c+dx))}{d} - \frac{3i2^{2/3}e^{-i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(\left(-1+e^{2ic}\right)(5A+2C)e^{i(c+dx)}\left(1+e^{2i(c+}\right.\right.}{}\right.}{5(b\sec(c+dx))^{4/3}(A\cos(2(c+dx))+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(4/3),x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-3*I)*2^(2/3)*(E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x))))^(2/3)*(5*B*(1 + E^((2*I)*(c + d*x))) + 5*B*(-1 +
 E^((2*I)*c))*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3,
5/6, -E^((2*I)*(c + d*x))] + (5*A + 2*C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))
*(1 + E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)
*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sec[c + d*x]^(2/3)) +
(3*(5*B*Cos[d*x]*Csc[c] + 2*C*Tan[c + d*x]))/d))/(5*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(b*sec(c
+ d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c))^(4/3), x)
```

3.64 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3(8A+5C)\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{16b^2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{5/}}{5}$$

```
[Out] (3*(8*A + 5*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sin[c + d*x])/(16*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeom
etric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x
])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(5/3)*Tan[c + d*x
])/(8*b^3*d)
```

Rubi [A] time = 0.150722, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{3(8A+5C)\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{16b^2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos\right.}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(4/3),x]
```

```
[Out] (3*(8*A + 5*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sin[c + d*x])/(16*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeom
etric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x
])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]) + (3*C*(b*Sec[c + d*x])^(5/3)*Tan[c + d*x
])/(8*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{5/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) d}{b^3}$$

$$= \frac{\int (b\sec(c+dx))^{5/3}\left(A+C\sec^2(c+dx)\right) dx}{b^3} + \frac{B\int (b\sec(c}{b}$$

$$= \frac{3C(b\sec(c+dx))^{5/3}\tan(c+dx)}{8b^3d} + \frac{(8A+5C)\int (b\sec(c+}{8b^3}$$

$$= \frac{3B\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{5/3}\sin(c+dx}{5b^3d\sqrt{\sin^2(c+dx)}}$$

$$= \frac{3(8A+5C)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\,s}{16b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 6.34181, size = 699, normalized size = 4.54

$$-\frac{6i2^{2/3}B\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(1+e^{2i(c+dx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-e^{2i(c+dx)}\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{5d\sec^{\frac{2}{3}}(c+dx)(b\sec(c+dx))^{4/3}(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(4/3),x]
```

```
[Out] (((-6*I)/5)*2^(2/3)*B*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1
+ E^((2*I)*(c + d*x)))^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^((2*I)*(c
+ d*x))]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(2/3)*(b*Sec[c + d*x])^(4/3)) + (3
*A*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E^((2*I)*(c + d*x
)))^(2/3)*Csc[c]*(-5*(1 + E^((2*I)*(c + d*x)))^(1/3) + E^((2*I)*d*x)*(-1 +
E^((2*I)*c))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(c + d*x))])*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*2^(1/3)*d*E^(I*d*x)*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(2/3)*(b*Sec[c + d*x])^(4/3)
) + (3*C*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E^((2*I)*(c
 + d*x)))^(2/3)*Csc[c]*(-5*(1 + E^((2*I)*(c + d*x)))^(1/3) + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(c + d*x))])
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(8*2^(1/3)*d*E^(I*d*x)*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(2/3)*(b*Sec[c + d*x])
^(4/3)) + ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((3*(8*A + 5*C)*Cos[d*x]
*Csc[c])/(8*d) + (3*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(4*d) + (3*Sec[c]*Sec
```

```
[c + d*x]*(5*C*Sin[c] + 8*B*Sin[d*x]))/(20*d) + (6*B*Tan[c])/(5*d)))/((A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{3} \left(A + B\sec (dx+c) + C (\sec (dx+c))^{2}\right) (b\sec (dx+c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec (dx+c)^{2} + B\sec (dx+c) + A\right)\sec (dx+c)^{3}}{(b\sec (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x
 + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec (dx+c)^{3} + B\sec (dx+c)^{2} + A\sec (dx+c)\right)(b\sec (dx+c))^{\frac{2}{3}}}{b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x
+ c))^(2/3)/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec (c+dx) + C\sec^{2}(c+dx)\right)\sec^{3}(c+dx)}{(b\sec (c+dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(b*sec(c
+ d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x
 + c))^(4/3), x)
```

3.65 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=230

$$\frac{3b(A(3m+7)+C(3m+4))\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-1),\frac{1}{6}(5-3m),\text{co}\right.}{d(3m+1)(3m+7)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*b*C*Sec[c + d*x]^(2 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*
m)) + (3*b*(C*(4 + 3*m) + A*(7 + 3*m))*Hypergeometric2F1[1/2, (-1 - 3*m)/6,
 (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(1 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric
2F1[1/2, (-4 - 3*m)/6, (2 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(1 + m)*(b
*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.188354, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$\frac{3b(A(3m+7)+C(3m+4))\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)(3m+7)\sqrt{\sin^2(c+dx)}}+\frac{3bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (3*b*C*Sec[c + d*x]^(2 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*
m)) + (3*b*(C*(4 + 3*m) + A*(7 + 3*m))*Hypergeometric2F1[1/2, (-1 - 3*m)/6,
 (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(1 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric
2F1[1/2, (-4 - 3*m)/6, (2 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(1 + m)*(b
*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\left(A+\right.}{\sqrt[3]{\sec(c+}}$$

$$= \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\left(A+\right.}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{3bC\sec^{2+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c}{d(7+3m)}$$

$$= \frac{3bC\sec^{2+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c}{d(7+3m)}$$

$$= \frac{3bC\sec^{2+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c}{d(7+3m)}$$

Mathematica [C] time = 7.9503, size = 484, normalized size = 2.1

$$-\frac{3i2^{m+\frac{7}{3}}e^{-\frac{1}{3}id(3m+4)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{4}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{4}{3}}(b\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2(A+2C)e}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-3*I)*2^(7/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(4/3 + m)*(
1 + E^((2*I)*(c + d*x)))^(4/3 + m)*((2*(A + 2*C)*E^((I/3)*(6*c + d*(10 + 3*
m)*x))*Hypergeometric2F1[5/3 + m/2, 10/3 + m, 8/3 + m/2, -E^((2*I)*(c + d*x
))])/(10 + 3*m) + (A*E^((4*I)*c + (I/3)*d*(16 + 3*m)*x)*Hypergeometric2F1[8
/3 + m/2, 10/3 + m, (22 + 3*m)/6, -E^((2*I)*(c + d*x))])/(16 + 3*m) + (A*E^
((I/3)*d*(4 + 3*m)*x)*Hypergeometric2F1[10/3 + m, (4 + 3*m)/6, 5/3 + m/2, -
E^((2*I)*(c + d*x))])/(4 + 3*m) + (2*B*E^((I/3)*(3*c + d*(7 + 3*m)*x))*Hype
rgeometric2F1[10/3 + m, (7 + 3*m)/6, (13 + 3*m)/6, -E^((2*I)*(c + d*x))])/(
7 + 3*m) + (2*B*E^((I/3)*(9*c + d*(13 + 3*m)*x))*Hypergeometric2F1[10/3 + m
, (13 + 3*m)/6, (19 + 3*m)/6, -E^((2*I)*(c + d*x))])/(13 + 3*m))*(b*Sec[c +
 d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^((I/3)*d*(4 + 3*
m)*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(10/3)
```

)

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{m} (b \sec (dx+c))^{\frac{4}{3}} \left(A+B \sec (dx+c)+C (\sec (dx+c))^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \sec (dx+c)^{3} + Bb \sec (dx+c)^{2} + Ab \sec (dx+c)\right) (b \sec (dx+c))^{\frac{1}{3}} \sec (dx+c)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*se
c(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*se
c(d*x + c)^m, x)
```

3.66 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=227

$$-\frac{3(A(3m+5)+C(3m+2))\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1-3m),\frac{1}{6}(7-3m),\right.}{d(1-3m)(3m+5)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(5 + 3*m)
) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 -
 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c
 + d*x])/(d*(1 - 3*m)*(5 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric
2F1[1/2, (-2 - 3*m)/6, (4 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c
 + d*x])^(2/3)*Sin[c + d*x])/(d*(2 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.190569, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{3(A(3m+5)+C(3m+2))\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)}{d(1-3m)(3m+5)\sqrt{\sin^2(c+dx)}}+-\;3$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(5 + 3*m)
) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 -
 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c
 + d*x])/(d*(1 - 3*m)*(5 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric
2F1[1/2, (-2 - 3*m)/6, (4 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c
 + d*x])^(2/3)*Sin[c + d*x])/(d*(2 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)\left(A+\right.}{\sec^{\frac{2}{3}}(c+}$$

$$= \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)\left(A+\right.}{\sec^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\sec^{1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c}{d(5+3m)}$$

$$= \frac{3C\sec^{1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c}{d(5+3m)}$$

$$= \frac{3C\sec^{1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c}{d(5+3m)}$$

Mathematica [C] time = 6.70575, size = 547, normalized size = 2.41

$$-\frac{3i2^{m+\frac{5}{3}}e^{-\frac{1}{3}id(3m+2)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{2}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{2}{3}}(b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{Ae^{4ic+\frac{1}{3}id(}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-3*I)*2^(5/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3 + m)*(
1 + E^((2*I)*(c + d*x)))^(2/3 + m)*((A*E^((4*I)*c + (I/3)*d*(14 + 3*m)*x)*H
ypergeometric2F1[7/3 + m/2, 8/3 + m, (20 + 3*m)/6, -E^((2*I)*(c + d*x))])/(
14 + 3*m) + (A*E^((I/3)*d*(2 + 3*m)*x)*Hypergeometric2F1[8/3 + m, (2 + 3*m)
/6, (8 + 3*m)/6, -E^((2*I)*(c + d*x))])/(2 + 3*m) + (2*B*E^((I/3)*(3*c + d*
(5 + 3*m)*x))*Hypergeometric2F1[8/3 + m, (5 + 3*m)/6, (11 + 3*m)/6, -E^((2*
I)*(c + d*x))])/(5 + 3*m) + (2*A*E^((I/3)*(6*c + d*(8 + 3*m)*x))*Hypergeome
tric2F1[8/3 + m, (8 + 3*m)/6, 7/3 + m/2, -E^((2*I)*(c + d*x))])/(8 + 3*m) +
 (4*C*E^((I/3)*(6*c + d*(8 + 3*m)*x))*Hypergeometric2F1[8/3 + m, (8 + 3*m)/
6, 7/3 + m/2, -E^((2*I)*(c + d*x))])/(8 + 3*m) + (2*B*E^((I/3)*(9*c + d*(11
 + 3*m)*x))*Hypergeometric2F1[8/3 + m, (11 + 3*m)/6, (17 + 3*m)/6, -E^((2*I
)*(c + d*x))])/(11 + 3*m))*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*S
```

```
ec[c + d*x]^2))/(d*E^((I/3)*d*(2 + 3*m)*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sec[c + d*x]^(8/3))
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{m} (b \sec (dx+c))^{\frac{2}{3}} \left(A+B \sec (dx+c)+C (\sec (dx+c))^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \sec (dx+c)^{2}+B \sec (dx+c)+A\right) (b \sec (dx+c))^{\frac{2}{3}} \sec (dx+c)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec
(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right) (b \sec(dx+c))^{\frac{2}{3}} \sec(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*se
c(d*x + c)^m, x)
```

3.67 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=225

$$-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2-3m),\frac{1}{6}(8-3m),\text{co}\right.}{d(2-3m)(3m+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3
*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(2 - 3*m)*(4 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F
1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.188582, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)}{d(2-3m)(3m+4)\sqrt{\sin^2(c+dx)}}+\frac{3B\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 + 3*m)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3
*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c +
 d*x])/(d*(2 - 3*m)*(4 + 3*m)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F
1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c +
 d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)\left(A+B\sec\right.}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)\left(A+C\sec\right.}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+d}{d(4+3m)}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+d}{d(4+3m)}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c+d}{d(4+3m)}$$

Mathematica [C] time = 7.23119, size = 494, normalized size = 2.2

$$-\frac{3i2^{m+\frac{4}{3}}e^{-\frac{1}{3}id(3m+1)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{1}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{1}{3}}\sqrt[3]{b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{e^{ic}\left(e^{\frac{1}{3}i(3c+d(3}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-3*I)*2^(4/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3 + m)*(
1 + E^((2*I)*(c + d*x)))^(1/3 + m)*((2*B*E^((I/3)*(9*c + d*(10 + 3*m)*x))*H
ypergeometric2F1[5/3 + m/2, 7/3 + m, 8/3 + m/2, -E^((2*I)*(c + d*x))])/(d*(
10 + 3*m)) + (A*E^((I/3)*(d + 3*d*m)*x)*Hypergeometric2F1[7/3 + m, (1 + 3*m
)/6, (7 + 3*m)/6, -E^((2*I)*(c + d*x))])/(d + 3*d*m) + (E^(I*c)*((2*B*E^((I
/3)*d*(4 + 3*m)*x)*Hypergeometric2F1[7/3 + m, (4 + 3*m)/6, 5/3 + m/2, -E^((
2*I)*(c + d*x))])/(4 + 3*m) + (E^((I/3)*(3*c + d*(7 + 3*m)*x))*(2*(A + 2*C)
*(13 + 3*m)*Hypergeometric2F1[7/3 + m, (7 + 3*m)/6, (13 + 3*m)/6, -E^((2*I)
*(c + d*x))] + A*E^((2*I)*(c + d*x))*(7 + 3*m)*Hypergeometric2F1[7/3 + m, (
13 + 3*m)/6, (19 + 3*m)/6, -E^((2*I)*(c + d*x))]))/((7 + 3*m)*(13 + 3*m)))
```

```
/d)*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(E^((I/
3)*d*(1 + 3*m)*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c +
 d*x]^(7/3))
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{m} \sqrt[3]{b \sec (dx+c)} \left(A+B \sec (dx+c)+C (\sec (dx+c))^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \sec (dx+c)^{2}+B \sec (dx+c)+A\right) (b \sec (dx+c))^{\frac{1}{3}} \sec (dx+c)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec
(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*se
c(d*x + c)^m, x)
```

3.68 $\int \frac{\sec^m(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=228

$$\frac{3(C(1-3m)-A(3m+2))\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4-3m),\frac{1}{6}(10-3m),\cos^2(c+dx)\right)}{d(4-3m)(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\cdots$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + 3*m)*(b*Sec[c + d*x])^(1/3)
) + (3*(C*(1 - 3*m) - A*(2 + 3*m))*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10
- 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*
(2 + 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometri
c2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c +
d*x])/(d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.190819, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$\frac{3(C(1-3m)-A(3m+2))\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{d(4-3m)(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^{m}\cdots}{d(1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(1/3),x]
```


```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + 3*m)*(b*Sec[c + d*x])^(1/3)
) + (3*(C*(1 - 3*m) - A*(2 + 3*m))*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10
- 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*
(2 + 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometri
c2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c +
d*x])/(d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.}{\sqrt[3]{b\sec(c+dx)}}$$

$$= \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\sec(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{3}+m\right)+A\left(\frac{2}{3}+\right.\right.\right.}{\left(\frac{2}{3}\right.}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\sec(c+dx)}} - \frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7\right.}{d(1-3m}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\sec(c+dx)}} + \frac{3(C(1-3m)-A(2+3m}{}$$

Mathematica [C] time = 11.5551, size = 548, normalized size = 2.4

$$-\frac{3i2^{m+\frac{2}{3}}e^{-\frac{1}{3}i(3c+d(3m+2)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{2}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{2}{3}}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(e^{ic}(3m-1)\left((3m+2)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(1/3),x]
```

```
[Out] ((-3*I)*2^(2/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3 + m)*(
1 + E^((2*I)*(c + d*x)))^(2/3 + m)*(A*E^((I/3)*d*(-1 + 3*m)*x)*(880 + 2418*
m + 2079*m^2 + 702*m^3 + 81*m^4)*Hypergeometric2F1[5/3 + m, (-1 + 3*m)/6, (
5 + 3*m)/6, -E^((2*I)*(c + d*x))] + E^(I*c)*(-1 + 3*m)*(2*B*E^((I/3)*d*(2 +
 3*m)*x)*(440 + 549*m + 216*m^2 + 27*m^3)*Hypergeometric2F1[5/3 + m, (2 + 3
*m)/6, (8 + 3*m)/6, -E^((2*I)*(c + d*x))] + E^((I/3)*(3*c + d*(5 + 3*m)*x))
*(2 + 3*m)*(2*(A + 2*C)*(88 + 57*m + 9*m^2)*Hypergeometric2F1[5/3 + m, (5 +
 3*m)/6, (11 + 3*m)/6, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(5 + 3*m)*(2
*B*(11 + 3*m)*Hypergeometric2F1[5/3 + m, (8 + 3*m)/6, 7/3 + m/2, -E^((2*I)*
(c + d*x))] + A*E^(I*(c + d*x))*(8 + 3*m)*Hypergeometric2F1[5/3 + m, (11 +
3*m)/6, (17 + 3*m)/6, -E^((2*I)*(c + d*x))]))))*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2))/(d*E^((I/3)*(3*c + d*(2 + 3*m)*x))*(-1 + 3*m)*(2 + 3*m)*(5 +
```

```
3*m)*(8 + 3*m)*(11 + 3*m)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sec[c + d*x]^(5/3)*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{m} \left(A+B\sec (dx+c)+C (\sec (dx+c))^{2}\right) \frac{1}{\sqrt[3]{b\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec (dx+c)^{2}+B\sec (dx+c)+A\right)\sec (dx+c)^{m}}{(b\sec (dx+c))^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x
 + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec (dx+c)^{2}+B\sec (dx+c)+A\right)(b\sec (dx+c))^{\frac{2}{3}}\sec (dx+c)^{m}}{b\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec
(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec (c+dx)+C\sec^{2}(c+dx)\right)\sec^{m}(c+dx)}{\sqrt[3]{b\sec (c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(1
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**m/(b*sec(c
+ d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{m}}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x
 + c))^(1/3), x)
```

3.69 $\int \frac{\sec^m(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=226

$$-\frac{3(3Am+A-C(2-3m))\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5-3m),\frac{1}{6}(11-3m),\cos^2(c+dx)\right)}{d(5-3m)(3m+1)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}-$$

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + 3*m)*(b*Sec[c + d*x])^(2/3)
) - (3*(A - C*(2 - 3*m) + 3*A*m)*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 -
3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(1
 + 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2
F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*
x])/(d*(2 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.191646, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{3(3Am+A-C(2-3m))\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)}{d(5-3m)(3m+1)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sec^{m}}{d(2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(2/3),x]
```

```
[Out] (3*C*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + 3*m)*(b*Sec[c + d*x])^(2/3)
) - (3*(A - C*(2 - 3*m) + 3*A*m)*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 -
3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(1
 + 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2
F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*
x])/(d*(2 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)\left(A+B\sec(c+dx)+C\sec\right.}{(b\sec(c+dx))^{2/3}}$$

$$= \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{(b\sec(c+dx))^{2/3}}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+3m)(b\sec(c+dx))^{2/3}} + \frac{\left(\left(C\left(-\frac{2}{3}+m\right)+A\left(\frac{1}{3}+\right.\right.\right.}{\left(\frac{1}{3}+\right.}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+3m)(b\sec(c+dx))^{2/3}} - \frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8\right.}{d(2-3m)}$$

$$= \frac{3C\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+3m)(b\sec(c+dx))^{2/3}} - \frac{3(A-C(2-3m)+3Am}{}$$

Mathematica [C] time = 10.5294, size = 545, normalized size = 2.41

$$-\frac{3i2^{m+\frac{1}{3}}e^{-\frac{1}{3}i(3c+d(3m+1)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{1}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{1}{3}}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left((3m+10)\left(2(3m-2)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(2/3),x]
```

```
[Out] ((-3*I)*2^(1/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/3 + m)*(
1 + E^((2*I)*(c + d*x)))^(1/3 + m)*(A*E^((4*I)*c + (I/3)*d*(10 + 3*m)*x)*(-
56 - 150*m + 135*m^2 + 270*m^3 + 81*m^4)*Hypergeometric2F1[5/3 + m/2, 4/3 +
 m, 8/3 + m/2, -E^((2*I)*(c + d*x))] + (10 + 3*m)*(A*E^((I/3)*d*(-2 + 3*m)*
x)*(28 + 117*m + 108*m^2 + 27*m^3)*Hypergeometric2F1[4/3 + m, (-2 + 3*m)/6,
 (4 + 3*m)/6, -E^((2*I)*(c + d*x))] + 2*E^((I/3)*(3*c + d*(1 + 3*m)*x))*(-2
 + 3*m)*(B*(28 + 33*m + 9*m^2)*Hypergeometric2F1[4/3 + m, (1 + 3*m)/6, (7 +
 3*m)/6, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(1 + 3*m)*((A + 2*C)*(7 +
3*m)*Hypergeometric2F1[4/3 + m, (4 + 3*m)/6, 5/3 + m/2, -E^((2*I)*(c + d*x)
)] + B*E^(I*(c + d*x))*(4 + 3*m)*Hypergeometric2F1[4/3 + m, (7 + 3*m)/6, (1
3 + 3*m)/6, -E^((2*I)*(c + d*x))]))))*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/(d*E^((I/3)*(3*c + d*(1 + 3*m)*x))*(-2 + 3*m)*(1 + 3*m)*(4 + 3*m)*(7 +
```

```
3*m)*(10 + 3*m)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d
*x]^(4/3)*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^m \left(A+B\sec (dx+c)+C(\sec (dx+c))^2\right)(b\sec (dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec (dx+c)^2+B\sec (dx+c)+A\right)\sec (dx+c)^m}{(b\sec (dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x
 + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec (dx+c)^2+B\sec (dx+c)+A\right)(b\sec (dx+c))^{\frac{1}{3}}\sec (dx+c)^m}{b\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec
(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec (c+dx)+C\sec^2 (c+dx)\right)\sec^m (c+dx)}{(b\sec (c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(2
/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**m/(b*sec(c
+ d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{m}}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(2/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x
 + c))^(2/3), x)
```

3.70 $\int \frac{\sec^m(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=234

$$-\frac{3(-3Am+A+C(4-3m))\sin(c+dx)\sec^{m-2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7-3m),\frac{1}{6}(13-3m),\cos^2(c+dx)\right)}{bd(1-3m)(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}} -$$

```
[Out] (-3*C*Sec[c + d*x]^m*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)) -
 (3*(A + C*(4 - 3*m) - 3*A*m)*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m
)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(1 - 3*m)*(7
- 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F
1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin
[c + d*x])/(b*d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.204798, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{3(-3Am+A+C(4-3m))\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)}{bd(1-3m)(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}} - \frac{3B\sin(c+dx)\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c + d*x
])^(4/3),x]
```

```
[Out] (-3*C*Sec[c + d*x]^m*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Sec[c + d*x])^(1/3)) -
 (3*(A + C*(4 - 3*m) - 3*A*m)*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m
)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(1 - 3*m)*(7
- 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F
1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin
[c + d*x])/(b*d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.}{b\sqrt[3]{b\sec(c+dx)}}$$

$$= \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)\left(A+C\sec^2(c+dx)\right)dx}{b\sqrt[3]{b\sec(c+dx)}}$$

$$= -\frac{3C\sec^m(c+dx)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\sec(c+dx)}} + \frac{\left(\left(C\left(-\frac{4}{3}+m\right)+A\left(-\frac{1}{3}+\right.\right.\right.}{b\left(-\frac{1}{3}\right.}$$

$$= -\frac{3C\sec^m(c+dx)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\sec(c+dx)}} - \frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(1\right.}{bd(4-}$$

$$= -\frac{3C\sec^m(c+dx)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\sec(c+dx)}} - \frac{3(A(1-3m)+C(4-3m)}{}$$

Mathematica [C] time = 8.79981, size = 492, normalized size = 2.1

$$-\frac{3i2^{m-\frac{1}{3}}e^{-\frac{1}{3}i(6c+d(3m+2)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{2}{3}}\left(1+e^{2i(c+dx)}\right)^{m+\frac{2}{3}}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(e^{2ic}\left(\frac{2(A+2C)e^{\frac{1}{3}id(3m+2)x}}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b*Sec[c
 + d*x])^(4/3),x]
```

```
[Out] ((-3*I)*2^(-1/3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(2/3 + m)*
(1 + E^((2*I)*(c + d*x)))^(2/3 + m)*((A*E^((I/3)*d*(-4 + 3*m)*x)*Hypergeome
tric2F1[2/3 + m, (-4 + 3*m)/6, (2 + 3*m)/6, -E^((2*I)*(c + d*x))])/(-4 + 3*
m) + (2*B*E^((I/3)*(3*c + d*(-1 + 3*m)*x))*Hypergeometric2F1[2/3 + m, (-1 +
 3*m)/6, (5 + 3*m)/6, -E^((2*I)*(c + d*x))])/(-1 + 3*m) + E^((2*I)*c)*((2*(
A + 2*C)*E^((I/3)*d*(2 + 3*m)*x)*Hypergeometric2F1[2/3 + m, (2 + 3*m)/6, (8
 + 3*m)/6, -E^((2*I)*(c + d*x))])/(2 + 3*m) + (2*B*E^((I/3)*(3*c + d*(5 + 3
*m)*x))*Hypergeometric2F1[2/3 + m, (5 + 3*m)/6, (11 + 3*m)/6, -E^((2*I)*(c
+ d*x))])/(5 + 3*m) + (A*E^((I/3)*(6*c + d*(8 + 3*m)*x))*Hypergeometric2F1[
2/3 + m, (8 + 3*m)/6, 7/3 + m/2, -E^((2*I)*(c + d*x))])/(8 + 3*m)))*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^((I/3)*(6*c + d*(2 + 3*m)*x))*(A + 2
```

```
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(2/3)*(b*Sec[c + d
*x])^(4/3))
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^m \left(A+B\sec (dx+c)+C(\sec (dx+c))^2\right)(b\sec (dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec (dx+c)^2+B\sec (dx+c)+A\right)\sec (dx+c)^m}{(b\sec (dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x
 + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec (dx+c)^2+B\sec (dx+c)+A\right)(b\sec (dx+c))^{\frac{2}{3}}\sec (dx+c)^m}{b^2\sec (dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec
(d*x + c)^m/(b^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(b*sec(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{m}}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(b*sec(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x
 + c))^(4/3), x)
```

3.71 $\int \sec^m(c+dx)(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=226

$$-\frac{(A(m+n+1)+C(m+n))\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-n+1),\frac{1}{2}(-m-\right.}{d(-m-n+1)(m+n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)) -
((C*(m + n) + A*(1 + m + n))*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m -
 n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x
])/(d*(1 - m - n)*(1 + m + n)*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[
1/2, (-m - n)/2, (2 - m - n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d
*x])^n*Sin[c + d*x])/(d*(m + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.180782, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{(A(m+n+1)+C(m+n))\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n+1);\frac{1}{2}(-m-n+3);\cos^2(c+d\right.}{d(-m-n+1)(m+n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (C*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)) -
((C*(m + n) + A*(1 + m + n))*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m -
 n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x
])/(d*(1 - m - n)*(1 + m + n)*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[
1/2, (-m - n)/2, (2 - m - n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d
*x])^n*Sin[c + d*x])/(d*(m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{m+n}(\ldots$$

$$= (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{m+n}(\ldots$$

$$= \frac{C\sec^{1+m}(c+dx)(b\sec(c+dx))^n\sin(c+d\ldots}{d(1+m+n)}$$

$$= \frac{C\sec^{1+m}(c+dx)(b\sec(c+dx))^n\sin(c+d\ldots}{d(1+m+n)}$$

$$= \frac{C\sec^{1+m}(c+dx)(b\sec(c+dx))^n\sin(c+d\ldots}{d(1+m+n)}$$

Mathematica [C] time = 6.47147, size = 436, normalized size = 1.93

$$-\frac{i2^{m+n+1}e^{-idx(m+n)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n}\left(1+e^{2i(c+dx)}\right)^{m+n}\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+d\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] ((-I)*2^(1 + m + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(m + n)*(1
+ E^((2*I)*(c + d*x)))^(m + n)*((A*E^(I*d*(m + n)*x)*Hypergeometric2F1[(m +
 n)/2, 2 + m + n, (2 + m + n)/2, -E^((2*I)*(c + d*x))])/(m + n) + (2*B*E^(I
*(c + d*(1 + m + n)*x))*Hypergeometric2F1[(1 + m + n)/2, 2 + m + n, (3 + m
+ n)/2, -E^((2*I)*(c + d*x))])/(1 + m + n) + E^((2*I)*c)*((2*(A + 2*C)*E^(I
*d*(2 + m + n)*x)*Hypergeometric2F1[(2 + m + n)/2, 2 + m + n, (4 + m + n)/2
, -E^((2*I)*(c + d*x))])/(2 + m + n) + (2*B*E^(I*(c + d*(3 + m + n)*x))*Hyp
ergeometric2F1[2 + m + n, (3 + m + n)/2, (5 + m + n)/2, -E^((2*I)*(c + d*x)
)])/(3 + m + n) + (A*E^(I*(2*c + d*(4 + m + n)*x))*Hypergeometric2F1[2 + m
+ n, (4 + m + n)/2, (6 + m + n)/2, -E^((2*I)*(c + d*x))])/(4 + m + n)))*Sec
[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/(d*E^(I*d*(m + n)*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 1.165, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x
 + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(c+dx\right)\right)^{n}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c
 + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c)^m, x)
```

3.72 $\int \sec^2(c+dx)(b\sec(c+dx))^n \left(A + B\sec(c+dx) + C\sec^2(c+\right.$

Optimal. Leaf size=189

$$\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-n-1),\frac{1-n}{2},\cos^2(c+dx)\right)}{bd(n+1)(n+3)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+}{}$$

```
[Out] ((C*(2 + n) + A*(3 + n))*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[
c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*(3 + n)*Sqr
t[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (-2 - n)/2, -n/2, Cos[c + d*
x]^2]*(b*Sec[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*
x]^2]) + (C*(b*Sec[c + d*x])^(2 + n)*Tan[c + d*x])/(b^2*d*(3 + n))
```

Rubi [A] time = 0.195178, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)(n+3)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n}}{b^2d(n+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] ((C*(2 + n) + A*(3 + n))*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[
c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*(3 + n)*Sqr
t[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (-2 - n)/2, -n/2, Cos[c + d*
x]^2]*(b*Sec[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*
x]^2]) + (C*(b*Sec[c + d*x])^(2 + n)*Tan[c + d*x])/(b^2*d*(3 + n))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{\int(b\sec(c+dx))^{2+n}\left(A+B\sec(c+dx)+C\right.}{b^2}$$

$$=\frac{\int(b\sec(c+dx))^{2+n}\left(A+C\sec^2(c+dx)\right)d}{b^2}$$

$$=\frac{C(b\sec(c+dx))^{2+n}\tan(c+dx)}{b^2d(3+n)}+\frac{\left(A+\frac{C(}{3}\right.}{}$$

$$=\frac{C(b\sec(c+dx))^{2+n}\tan(c+dx)}{b^2d(3+n)}+\frac{B\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

$$=\frac{\left(A+\frac{C(2+n)}{3+n}\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c\right.}{bd(1+n)\sqrt{\sin}}$$

Mathematica [C] time = 5.51426, size = 462, normalized size = 2.44

$$-\frac{i2^{n+3}e^{2ic-idnx}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^n\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{Ae^{id(n}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] ((-I)*2^(3 + n)*E^((2*I)*c - I*d*n*x)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*((A*E^(I*d*(2 + n)*x)*Hypergeometric2F
1[(2 + n)/2, 4 + n, (4 + n)/2, -E^((2*I)*(c + d*x))])/(2 + n) + (2*B*E^(I*(
c + d*(3 + n)*x))*Hypergeometric2F1[(3 + n)/2, 4 + n, (5 + n)/2, -E^((2*I)*
(c + d*x))])/(3 + n) + (2*A*E^(I*(2*c + d*(4 + n)*x))*Hypergeometric2F1[(4
+ n)/2, 4 + n, (6 + n)/2, -E^((2*I)*(c + d*x))])/(4 + n) + (4*C*E^(I*(2*c +
 d*(4 + n)*x))*Hypergeometric2F1[(4 + n)/2, 4 + n, (6 + n)/2, -E^((2*I)*(c
+ d*x))])/(4 + n) + (2*B*E^(I*(3*c + d*(5 + n)*x))*Hypergeometric2F1[4 + n,
 (5 + n)/2, (7 + n)/2, -E^((2*I)*(c + d*x))])/(5 + n) + (A*E^(I*(4*c + d*(6
 + n)*x))*Hypergeometric2F1[4 + n, (6 + n)/2, (8 + n)/2, -E^((2*I)*(c + d*x
))])/(6 + n))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.679, size = 0, normalized size = 0.

$$\int(\sec(dx+c))^2(b\sec(dx+c))^n\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*
x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(c+dx\right)\right)^{n}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c
 + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c)^2, x)
```

3.73 $\int \sec(c+dx)(b\sec(c+dx))^n \left(A + B\sec(c + dx) + C\sec^2(c +$

Optimal. Leaf size=182

$$\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)}{dn(n+2)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec($$

```
[Out] ((C*(1 + n) + A*(2 + n))*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(2 + n)*Sqrt[Sin[c + d*x]^2]) +
 (B*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]) + (C*(b*Se
c[c + d*x])^(1 + n)*Tan[c + d*x])/(b*d*(2 + n))
```

Rubi [A] time = 0.185408, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn(n+2)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\right.}{bd(n+1)\sqrt{\sin^2(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] ((C*(1 + n) + A*(2 + n))*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(2 + n)*Sqrt[Sin[c + d*x]^2]) +
 (B*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]) + (C*(b*Se
c[c + d*x])^(1 + n)*Tan[c + d*x])/(b*d*(2 + n))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{\int (b\sec(c+dx))^{1+n} \left(A+B\sec(c+dx)+C\right.}{b}$$

$$= \frac{\int (b\sec(c+dx))^{1+n} \left(A+C\sec^2(c+dx)\right) dx}{b}$$

$$= \frac{C(b\sec(c+dx))^{1+n}\tan(c+dx)}{bd(2+n)} + \frac{\left(A+\frac{C(1}{2+}\right.}{}$$

$$= \frac{B\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec}{bd(1+n)\sqrt{\sin^2(c+d}}$$

$$= \frac{\left(A+\frac{C(1+n)}{2+n}\right){}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 5.17132, size = 460, normalized size = 2.53

$$-\frac{i2^{n+2}e^{i(c-dnx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^n\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{Ae^{id(n+}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((-I)*2^(2 + n)*E^(I*(c - d*n*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
))^n*(1 + E^((2*I)*(c + d*x)))^n*((A*E^(I*d*(1 + n)*x)*Hypergeometric2F1[(1
 + n)/2, 3 + n, (3 + n)/2, -E^((2*I)*(c + d*x))])/(1 + n) + (2*B*E^(I*(c +
d*(2 + n)*x))*Hypergeometric2F1[(2 + n)/2, 3 + n, (4 + n)/2, -E^((2*I)*(c +
 d*x))])/(2 + n) + (2*A*E^(I*(2*c + d*(3 + n)*x))*Hypergeometric2F1[(3 + n)
/2, 3 + n, (5 + n)/2, -E^((2*I)*(c + d*x))])/(3 + n) + (4*C*E^(I*(2*c + d*(
3 + n)*x))*Hypergeometric2F1[(3 + n)/2, 3 + n, (5 + n)/2, -E^((2*I)*(c + d*
x))])/(3 + n) + (2*B*E^(I*(3*c + d*(4 + n)*x))*Hypergeometric2F1[3 + n, (4
+ n)/2, (6 + n)/2, -E^((2*I)*(c + d*x))])/(4 + n) + (A*E^(I*(4*c + d*(5 + n
)*x))*Hypergeometric2F1[3 + n, (5 + n)/2, (7 + n)/2, -E^((2*I)*(c + d*x))])
/(5 + n))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.871, size = 0, normalized size = 0.

$$\int \sec(dx+c)(b\sec(dx+c))^n\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^3+B\sec\left(dx+c\right)^2+A\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x
+ c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(c+dx\right)\right)^n\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c
 + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c), x)
```

3.74 $\int (b\sec(c+dx))^n \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=175

$$-\frac{b(An + A + Cn)\sin(c+dx)(b\sec(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos^2(c+dx)\right)}{d(1-n)(n+1)\sqrt{\sin^2(c+dx)}} + \frac{B\sin(c+dx)(b\sec(c$$

```
[Out] -((b*(A + A*n + C*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d
*x]^2]*(b*Sec[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*(1 + n)*Sqrt[Sin[
c + d*x]^2])) + (B*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*
(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]) + (C*(b*Sec[c +
 d*x])^n*Tan[c + d*x])/(d*(1 + n))
```

Rubi [A] time = 0.142766, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4047, 3772, 2643, 4046}

$$-\frac{b(An + A + Cn)\sin(c+dx)(b\sec(c+dx))^{n-1}{}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c+dx)\right)}{d(1-n)(n+1)\sqrt{\sin^2(c+dx)}} + \frac{B\sin(c+dx)(b\sec(c+dx))^n{}_2F_1\left(\frac{1}{2}, -\frac{n}{2}\right.}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] -((b*(A + A*n + C*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d
*x]^2]*(b*Sec[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*(1 + n)*Sqrt[Sin[
c + d*x]^2])) + (B*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*
(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]) + (C*(b*Sec[c +
 d*x])^n*Tan[c + d*x])/(d*(1 + n))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{B\int (b\sec(c+dx))^{1+n}\,dx}{b} + \int (b\sec(c+dx))^n \left(A+C\,\mathrm{se}\right.$$

$$= \frac{C(b\sec(c+dx))^n\tan(c+dx)}{d(1+n)} + \frac{(A+An+Cn)\int (b\,\mathrm{se}}{1+n}$$

$$= \frac{B\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+}{dn\sqrt{\sin^2(c+dx)}}$$

$$= -\frac{(A+An+Cn)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+\right.}{d\left(1-n^2\right)\sqrt{\sin^2(c+dx}}$$

Mathematica [C] time = 4.8301, size = 401, normalized size = 2.29

$$-\frac{i2^{n+1}e^{-idnx}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^n\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(e^{2ic}\left(\frac{2(A}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(1 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(1 + E^((2*I)
*(c + d*x)))^n*((A*E^(I*d*n*x)*Hypergeometric2F1[n/2, 2 + n, (2 + n)/2, -E^
((2*I)*(c + d*x))])/n + (2*B*E^(I*(c + d*(1 + n)*x))*Hypergeometric2F1[(1 +
 n)/2, 2 + n, (3 + n)/2, -E^((2*I)*(c + d*x))])/(1 + n) + E^((2*I)*c)*((2*(
A + 2*C)*E^(I*d*(2 + n)*x)*Hypergeometric2F1[(2 + n)/2, 2 + n, (4 + n)/2, -
E^((2*I)*(c + d*x))])/(2 + n) + (2*B*E^(I*(c + d*(3 + n)*x))*Hypergeometric
2F1[2 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(c + d*x))])/(3 + n) + (A*E^(I*(
2*c + d*(4 + n)*x))*Hypergeometric2F1[2 + n, (4 + n)/2, (6 + n)/2, -E^((2*I
)*(c + d*x))])/(4 + n)))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*n*x)*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.612, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x) + C*sec(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

3.75 $\int \cos(c+dx)(b\sec(c+dx))^n \left(A + B\sec(c + dx) + C\sec^2(c +$

Optimal. Leaf size=191

$$\frac{b^2(C(1-n)-An)\sin(c+dx)(b\sec(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(c+dx)\right)}{d(2-n)n\sqrt{\sin^2(c+dx)}}-\frac{bB\sin(c+dx)(b\,\mathrm{s}}{}$$

```
[Out] (b^2*(C*(1 - n) - A*n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c +
 d*x]^2]*(b*Sec[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*n*Sqrt[Sin[c +
d*x]^2]) - (b*B*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2
]*(b*Sec[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
+ (b*C*(b*Sec[c + d*x])^(-1 + n)*Tan[c + d*x])/(d*n)
```

Rubi [A] time = 0.193549, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {16, 4047, 3772, 2643, 4046}

$$\frac{b^2(C(1-n)-An)\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)n\sqrt{\sin^2(c+dx)}}-\frac{bB\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1}{d(1-n)\sqrt{\sin^2(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (b^2*(C*(1 - n) - A*n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c +
 d*x]^2]*(b*Sec[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*n*Sqrt[Sin[c +
d*x]^2]) - (b*B*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2
]*(b*Sec[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
+ (b*C*(b*Sec[c + d*x])^(-1 + n)*Tan[c + d*x])/(d*n)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = b\int (b\sec(c+dx))^{-1+n} \left(A+B\sec(c+dx)+C\right.$$

$$= b\int (b\sec(c+dx))^{-1+n} \left(A+C\sec^2(c+dx)\right) d$$

$$= \frac{bC(b\sec(c+dx))^{-1+n}\tan(c+dx)}{dn} + \frac{(b(C(-1+}{}$$

$$= -\frac{B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(}{d(1-n)\sqrt{\sin^2(c+d}}$$

$$= -\frac{B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(}{d(1-n)\sqrt{\sin^2(c+d}}$$

Mathematica [A] time = 0.340157, size = 161, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(c+dx)}(b\sec(c+dx))^n \left(An(n+1)\cos(c+dx)\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\sec^2(c+dx)\right)+(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((A*n*(1 + n)*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2,
(1 + n)/2, Sec[c + d*x]^2] + (-1 + n)*Csc[c + d*x]*(B*(1 + n)*Cos[c + d*x]*
Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d*x]^2] + C*n*Hypergeometric
2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d*x]^2]))*(b*Sec[c + d*x])^n*Sqrt[-T
an[c + d*x]^2])/(d*(-1 + n)*n*(1 + n))
```

Maple [F] time = 0.822, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*
x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*(b*sec(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*
x + c), x)
```

3.76 $\int \cos^2(c+dx)(b\sec(c+dx))^n \left(A + B\sec(c + dx) + C\sec^2(c +\right.$

Optimal. Leaf size=208

$$-\frac{b^3(A(1-n)+C(2-n))\sin(c+dx)(b\sec(c+dx))^{n-3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-n}{2},\frac{5-n}{2},\cos^2(c+dx)\right)}{d(1-n)(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2B\sin(c+d}{}$$

```
[Out] -((b^3*(A*(1 - n) + C*(2 - n))*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(1 - n)*(3 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^2*B*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/
2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[
Sin[c + d*x]^2]) - (b^2*C*(b*Sec[c + d*x])^(-2 + n)*Tan[c + d*x])/(d*(1 - n
))
```

Rubi [A] time = 0.216861, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{b^3(A(1-n)+C(2-n))\sin(c+dx)(b\sec(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(1-n)(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2B\sin(c+dx)(b\sec(c+dx))^{n-2}}{d(2-n)\sqrt{\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] -((b^3*(A*(1 - n) + C*(2 - n))*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(1 - n)*(3 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^2*B*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/
2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[
Sin[c + d*x]^2]) - (b^2*C*(b*Sec[c + d*x])^(-2 + n)*Tan[c + d*x])/(d*(1 - n
))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = b^2\int (b\sec(c+dx))^{-2+n}\left(A+B\sec(c+dx\right.$$

$$= b^2\int (b\sec(c+dx))^{-2+n}\left(A+C\sec^2(c+d\right.$$

$$= -\frac{b^2C(b\sec(c+dx))^{-2+n}\tan(c+dx)}{d(1-n)} + \left(b^2\right.$$

$$= -\frac{B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+\right.}{d(2-n)\sqrt{\sin^2(}}$$

$$= -\frac{B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+\right.}{d(2-n)\sqrt{\sin^2(}}$$

Mathematica [A] time = 0.272728, size = 155, normalized size = 0.75

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^n\left(A(n-1)n\cos^2(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\sec^2(c+dx)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (Cot[c + d*x]*(A*(-1 + n)*n*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-2 + n)/
2, n/2, Sec[c + d*x]^2] + (-2 + n)*(B*n*Cos[c + d*x]*Hypergeometric2F1[1/2,
 (-1 + n)/2, (1 + n)/2, Sec[c + d*x]^2] + C*(-1 + n)*Hypergeometric2F1[1/2,
 n/2, (2 + n)/2, Sec[c + d*x]^2]))*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2]
)/(d*(-2 + n)*(-1 + n)*n)
```

Maple [F] time = 1.056, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*
x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*(b*sec(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*
x + c)^2, x)
```

3.77 $\int \cos^3(c+dx)(b\sec(c+dx))^n \left(A + B\sec(c+dx) + C\sec^2(c\right.$

Optimal. Leaf size=208

$$-\frac{b^4(A(2-n)+C(3-n))\sin(c+dx)(b\sec(c+dx))^{n-4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-n}{2},\frac{6-n}{2},\cos^2(c+dx)\right)}{d(2-n)(4-n)\sqrt{\sin^2(c+dx)}}-\frac{b^3B\sin(c\cdot}{}$$

```
[Out] -((b^4*(A*(2 - n) + C*(3 - n))*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(2 - n)*(4 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^3*B*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/
2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[
Sin[c + d*x]^2]) - (b^3*C*(b*Sec[c + d*x])^(-3 + n)*Tan[c + d*x])/(d*(2 - n
))
```

Rubi [A] time = 0.210411, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {16, 4047, 3772, 2643, 4046}

$$-\frac{b^4(A(2-n)+C(3-n))\sin(c+dx)(b\sec(c+dx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)}{d(2-n)(4-n)\sqrt{\sin^2(c+dx)}}-\frac{b^3B\sin(c+dx)(b\sec(c+dx)}{d(3-n)\cdot}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] -((b^4*(A*(2 - n) + C*(3 - n))*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(2 - n)*(4 - n)
*Sqrt[Sin[c + d*x]^2])) - (b^3*B*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/
2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[
Sin[c + d*x]^2]) - (b^3*C*(b*Sec[c + d*x])^(-3 + n)*Tan[c + d*x])/(d*(2 - n
))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = b^3 \int (b\sec(c+dx))^{-3+n} \left(A+B\sec(c+dx)+\right.$$

$$= b^3 \int (b\sec(c+dx))^{-3+n} \left(A+C\sec^2(c+dx)\right)$$

$$= -\frac{b^3C(b\sec(c+dx))^{-3+n}\tan(c+dx)}{d(2-n)} + \left(b^3\left(A\right.\right.$$

$$= -\frac{B\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+}}$$

$$= -\frac{B\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.4523, size = 168, normalized size = 0.81

$$\frac{b\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{n-1}\left(A\left(n^2-3n+2\right)\cos^2(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\sec^2(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (b*Cot[c + d*x]*(A*(2 - 3*n + n^2)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-
3 + n)/2, (-1 + n)/2, Sec[c + d*x]^2] + (-3 + n)*(B*(-1 + n)*Cos[c + d*x]*H
ypergeometric2F1[1/2, (-2 + n)/2, n/2, Sec[c + d*x]^2] + C*(-2 + n)*Hyperge
ometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[c + d*x]^2]))*(b*Sec[c + d*x])^(
-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + n)*(-2 + n)*(-1 + n))
```

Maple [F] time = 1.424, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3 (b\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*
x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{3}\right)\left(b\sec\left(dx+c\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*(b*sec(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*
x + c)^3, x)
```

3.78 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n \left(A + B\sec(c+dx) + C\sec^2(c+\right.$

Optimal. Leaf size=223

$$\frac{2(A(2n+7)+C(2n+5))\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-3),\frac{1}{4}(1-2n),\cos^2(\right.}{d(2n+3)(2n+7)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (2*C*Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) + (2
*(C*(5 + 2*n) + A*(7 + 2*n))*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)
/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(3 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-5
 - 2*n)/4, (-1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c + d*x]^(5/2)*(b*Sec[c + d*x]
)^n*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.195954, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$\frac{2(A(2n+7)+C(2n+5))\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-3);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)}{d(2n+3)(2n+7)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (2*C*Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) + (2
*(C*(5 + 2*n) + A*(7 + 2*n))*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)
/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(3 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-5
 - 2*n)/4, (-1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c + d*x]^(5/2)*(b*Sec[c + d*x]
)^n*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{\frac{5}{2}+n}(c$$

$$= (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{\frac{5}{2}+n}(c$$

$$= \frac{2C\sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx}{d(7+2n)}$$

$$= \frac{2C\sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx}{d(7+2n)}$$

$$= \frac{2C\sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx}{d(7+2n)}$$

Mathematica [C] time = 7.87678, size = 493, normalized size = 2.21

$$-\frac{i2^{n+\frac{9}{2}}e^{2ic-\frac{1}{2}id(2n+1)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{1}{2}}\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(9/2 + n)*E^((2*I)*c - (I/2)*d*(1 + 2*n)*x)*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*((A*E^((
I/2)*d*(5 + 2*n)*x)*Hypergeometric2F1[9/2 + n, (5 + 2*n)/4, (9 + 2*n)/4, -E
^((2*I)*(c + d*x))])/(5 + 2*n) + (2*B*E^((I/2)*(2*c + d*(7 + 2*n)*x))*Hyper
geometric2F1[9/2 + n, (7 + 2*n)/4, (11 + 2*n)/4, -E^((2*I)*(c + d*x))])/(7
+ 2*n) + E^((2*I)*c)*((2*(A + 2*C)*E^((I/2)*d*(9 + 2*n)*x)*Hypergeometric2F
1[9/2 + n, (9 + 2*n)/4, (13 + 2*n)/4, -E^((2*I)*(c + d*x))])/(9 + 2*n) + (2
*B*E^((I/2)*(2*c + d*(11 + 2*n)*x))*Hypergeometric2F1[9/2 + n, (11 + 2*n)/4
, (15 + 2*n)/4, -E^((2*I)*(c + d*x))])/(11 + 2*n) + (A*E^((I/2)*(4*c + d*(1
3 + 2*n)*x))*Hypergeometric2F1[9/2 + n, (13 + 2*n)/4, (17 + 2*n)/4, -E^((2*
I)*(c + d*x))])/(13 + 2*n)))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x]))
```

Maple [F] time = 0.236, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{5}{2}} (b\sec (dx+c))^n \left(A+B\sec (dx+c)+C(\sec (dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec (dx+c)^4+B\sec (dx+c)^3+A\sec (dx+c)^2\right)(b\sec (dx+c))^n\sqrt{\sec (dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*
x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx+c)^2+B\sec (dx+c)+A\right)(b\sec (dx+c))^n\sec (dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c)^(5/2), x)
```

3.79 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=223

$$\frac{2(A(2n+5)+C(2n+3))\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-1),\frac{1}{4}(3-2n),\cos^2(\right.}{d(2n+1)(2n+5)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (2*C*Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) + (2
*(C*(3 + 2*n) + A*(5 + 2*n))*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)
/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(1 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-3
 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])
^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.187871, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$\frac{2(A(2n+5)+C(2n+3))\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)(2n+5)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (2*C*Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) + (2
*(C*(3 + 2*n) + A*(5 + 2*n))*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)
/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*
(1 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-3
 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])
^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{3}{2}+n}(c$$

$$= \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{3}{2}+n}(c$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx}{d(5+2n)}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx}{d(5+2n)}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx}{d(5+2n)}$$

Mathematica [C] time = 7.22507, size = 487, normalized size = 2.18

$$-\frac{i2^{n+\frac{7}{2}}e^{-\frac{1}{2}id(2n+3)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{3}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{3}{2}}\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(7/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(3/2 + n)*(1
+ E^((2*I)*(c + d*x)))^(3/2 + n)*((A*E^((I/2)*d*(3 + 2*n)*x)*Hypergeometric
2F1[7/2 + n, (3 + 2*n)/4, (7 + 2*n)/4, -E^((2*I)*(c + d*x))])/(3 + 2*n) + (
2*B*E^((I/2)*(2*c + d*(5 + 2*n)*x))*Hypergeometric2F1[7/2 + n, (5 + 2*n)/4,
 (9 + 2*n)/4, -E^((2*I)*(c + d*x))])/(5 + 2*n) + E^((2*I)*c)*((2*(A + 2*C)*
E^((I/2)*d*(7 + 2*n)*x)*Hypergeometric2F1[7/2 + n, (7 + 2*n)/4, (11 + 2*n)/
4, -E^((2*I)*(c + d*x))])/(7 + 2*n) + (2*B*E^((I/2)*(2*c + d*(9 + 2*n)*x))*
Hypergeometric2F1[7/2 + n, (9 + 2*n)/4, (13 + 2*n)/4, -E^((2*I)*(c + d*x))]
)/(9 + 2*n) + (A*E^((I/2)*(4*c + d*(11 + 2*n)*x))*Hypergeometric2F1[7/2 + n
, (11 + 2*n)/4, (15 + 2*n)/4, -E^((2*I)*(c + d*x))])/(11 + 2*n)))*Sec[c + d
*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d
*E^((I/2)*d*(3 + 2*n)*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{3}{2}} (b\sec (dx+c))^n \left(A+B\sec (dx+c)+C(\sec (dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec (dx+c)^3+B\sec (dx+c)^2+A\sec (dx+c)\right)(b\sec (dx+c))^n\sqrt{\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x
+ c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx+c)^2+B\sec (dx+c)+A\right)(b\sec (dx+c))^n\sec (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*
x + c)^(3/2), x)
```

3.80 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=221

$$-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1-2n),\frac{1}{4}(5-2n),\cos^2(c+dx)\right)}{d(1-2n)(2n+3)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}+\frac{2B}{}$$

```
[Out] (2*C*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 + 2*n)*Sq
rt[Sec[c + d*x]]*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-1 -
2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*
Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.17899, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)}{d(1-2n)(2n+3)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{\sec(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (2*C*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 + 2*n)*Sq
rt[Sec[c + d*x]]*Sqrt[Sin[c + d*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-1 -
2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*
Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(\cdots$$

$$= \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(\cdots$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+d\cdots}{d(3+2n)}$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+d\cdots}{d(3+2n)}$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+d\cdots}{d(3+2n)}$$

Mathematica [C] time = 7.96238, size = 492, normalized size = 2.23

$$i2^{n+\frac{5}{2}}e^{-\frac{1}{2}id(2n+1)x}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{1}{2}}\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+d\cdots\right.$$

$$-\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-I)*2^(5/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(1
+ E^((2*I)*(c + d*x)))^(1/2 + n)*((A*E^((I/2)*(d + 2*d*n)*x)*Hypergeometric
2F1[5/2 + n, (1 + 2*n)/4, (5 + 2*n)/4, -E^((2*I)*(c + d*x))])/(d + 2*d*n) +
 (E^(I*c)*((2*B*E^((I/2)*d*(3 + 2*n)*x)*Hypergeometric2F1[5/2 + n, (3 + 2*n
)/4, (7 + 2*n)/4, -E^((2*I)*(c + d*x))])/(3 + 2*n) + E^(I*c)*((2*(A + 2*C)*
E^((I/2)*d*(5 + 2*n)*x)*Hypergeometric2F1[5/2 + n, (5 + 2*n)/4, (9 + 2*n)/4
, -E^((2*I)*(c + d*x))])/(5 + 2*n) + (2*B*E^((I/2)*(2*c + d*(7 + 2*n)*x))*H
ypergeometric2F1[5/2 + n, (7 + 2*n)/4, (11 + 2*n)/4, -E^((2*I)*(c + d*x))])
/(7 + 2*n) + (A*E^((I/2)*(4*c + d*(9 + 2*n)*x))*Hypergeometric2F1[5/2 + n,
(9 + 2*n)/4, (13 + 2*n)/4, -E^((2*I)*(c + d*x))])/(9 + 2*n))))/d)*Sec[c + d
*x]^(-2 - n)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(E
^((I/2)*d*(1 + 2*n)*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.242, size = 0, normalized size = 0.

$$\int (b \sec (dx + c))^{n} \left(A + B \sec (dx + c) + C (\sec (dx + c))^{2}\right) \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \sec (dx + c)^{2} + B \sec (dx + c) + A\right) (b \sec (dx + c))^{n} \sqrt{\sec (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(se
c(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^{2} + B \sec (dx + c) + A\right) (b \sec (dx + c))^{n} \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(s
ec(d*x + c)), x)
```

3.81 $\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=222

$$-\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3-2n),\frac{1}{4}(7-2n),\cos^2(c+dx)\right)}{d(3-2n)(2n+1)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}-\frac{2B}{}$$

```
[Out] (2*C*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)) - (2
*(A - C*(1 - 2*n) + 2*A*n)*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(1 + 2*n)*Se
c[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) - (2*B*Hypergeometric2F1[1/2, (1 - 2
*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1
- 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.180655, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)}{d(3-2n)(2n+1)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(c}{d(1}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[Sec[c
 + d*x]],x]
```


```
[Out] (2*C*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)) - (2
*(A - C*(1 - 2*n) + 2*A*n)*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4,
 Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(1 + 2*n)*Se
c[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) - (2*B*Hypergeometric2F1[1/2, (1 - 2
*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1
- 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)\left(A+\right.$$

$$= \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)\left(A+\right.$$

$$= \frac{2C\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)} + \left(B\cos^{\frac{1}{2}}\right.$$

$$= \frac{2C\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)} - \frac{2B\,{}_2F_1\Big(}{}$$

$$= \frac{2C\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)} - \frac{2(A-C}{}$$

Mathematica [C] time = 8.53417, size = 548, normalized size = 2.47

$$-\frac{i2^{n+\frac{3}{2}}e^{-\frac{1}{2}i(2c+d(2n+1)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{1}{2}}\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(1
+ E^((2*I)*(c + d*x)))^(1/2 + n)*(A*E^((I/2)*d*(-1 + 2*n)*x)*(105 + 352*n +
 344*n^2 + 128*n^3 + 16*n^4)*Hypergeometric2F1[3/2 + n, (-1 + 2*n)/4, (3 +
2*n)/4, -E^((2*I)*(c + d*x))] + E^(I*c)*(-1 + 2*n)*(2*B*E^((I/2)*d*(1 + 2*n
)*x)*(105 + 142*n + 60*n^2 + 8*n^3)*Hypergeometric2F1[3/2 + n, (1 + 2*n)/4,
 (5 + 2*n)/4, -E^((2*I)*(c + d*x))] + E^((I/2)*(2*c + d*(3 + 2*n)*x))*(1 +
2*n)*(2*(A + 2*C)*(35 + 24*n + 4*n^2)*Hypergeometric2F1[3/2 + n, (3 + 2*n)/
4, (7 + 2*n)/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(3 + 2*n)*(2*B*(7 +
 2*n)*Hypergeometric2F1[3/2 + n, (5 + 2*n)/4, (9 + 2*n)/4, -E^((2*I)*(c + d
*x))] + A*E^(I*(c + d*x))*(5 + 2*n)*Hypergeometric2F1[3/2 + n, (7 + 2*n)/4,
 (11 + 2*n)/4, -E^((2*I)*(c + d*x))]))))*Sec[c + d*x]^(-2 - n)*(b*Sec[c + d
*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^((I/2)*(2*c + d*(1 + 2
*n)*x))*(-1 + 2*n)*(1 + 2*n)*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*(A + 2*C + 2*B*C
```

```
os[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{n}\left(A+B\sec(dx+c)+C(\sec(dx+c))^{2}\right)\frac{1}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^{2}+B\sec(dx+c)+A\right)(b\sec(dx+c))^{n}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(s
ec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec(dx+c)^{2}+B\sec(dx+c)+A\right)(b\sec(dx+c))^{n}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(se
c(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(s
ec(d*x + c)), x)
```

3.82 $\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$-\frac{2(-2An+A+C(3-2n))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5-2n),\frac{1}{4}(9-2n),\cos^2(c+dx)\right)}{d(1-2n)(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{}$$

```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]) - (
2*(A + C*(3 - 2*n) - 2*A*n)*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4
, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(5 - 2*n)*S
ec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2]) - (2*B*Hypergeometric2F1[1/2, (3 -
2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.199144, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{2(-2An+A+C(3-2n))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)}{d(1-2n)(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec}{d(}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c + d*
x]^(3/2),x]
```


```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]) - (
2*(A + C*(3 - 2*n) - 2*A*n)*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4
, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(5 - 2*n)*S
ec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2]) - (2*B*Hypergeometric2F1[1/2, (3 -
2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)\left(A+\right.$$

$$= \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)\left(A+\right.$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}} + \left(B\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\right.$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}} - \frac{2B\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n)\right.}{d(}$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}} - \frac{2(A(1-2n)+C(3-}{}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*
x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x
 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(c+dx\right)\right)^n\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3
/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sec(c
 + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*
x + c)^(3/2), x)
```

3.83 $\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{2(A(3-2n)+C(5-2n))\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7-2n),\frac{1}{4}(11-2n),\cos^2(c+dx)\right)}{d(3-2n)(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}-\cdots$$

```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)) - (
2*(A*(3 - 2*n) + C*(5 - 2*n))*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n
)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(7 - 2*n
)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*x]^2]) - (2*B*Hypergeometric2F1[1/2, (5
 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d
*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.201044, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {20, 4047, 3772, 2643, 4046}

$$-\frac{2(A(3-2n)+C(5-2n))\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)}{d(3-2n)(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec\cdots}{d(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c + d*
x]^(5/2),x]
```

```
[Out] (-2*C*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)) - (
2*(A*(3 - 2*n) + C*(5 - 2*n))*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n
)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*(7 - 2*n
)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*x]^2]) - (2*B*Hypergeometric2F1[1/2, (5
 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d
*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{5}{2}+n}(c+dx)\left(A+\right.$$

$$= \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{5}{2}+n}(c+dx)\left(A+\right.$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)} + \left(B\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\right.$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)} - \frac{2B\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n)\right.}{d(}$$

$$= -\frac{2C(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)} - \frac{2(A(3-2n)+C(5-}{}$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(5/2),x]

[Out] $Aborted
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(\sec(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)

[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*
x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x
 + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)\right)^n}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*
x + c)^(5/2), x)
```

3.84 $\int \sec^3(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=140

$$\frac{a(5A+4C)\tan^3(c+dx)}{15d}+\frac{a(5A+4C)\tan(c+dx)}{5d}+\frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\tan(c+dx)\,\text{se}}{8d}$$

```
[Out] (a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 4*C)*Tan[c + d*x])/
(5*d) + (a*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^3*Tan[c + d*x])/(4*d) + (a*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(5*A
+ 4*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.185496, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4077, 4047, 3767, 4046, 3768, 3770}

$$\frac{a(5A+4C)\tan^3(c+dx)}{15d}+\frac{a(5A+4C)\tan(c+dx)}{5d}+\frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\tan(c+dx)\,\text{se}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 4*C)*Tan[c + d*x])/
(5*d) + (a*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^3*Tan[c + d*x])/(4*d) + (a*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(5*A
+ 4*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] && !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3(c+dx)\left(5aA+a(\right.$$
$$= \frac{aC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3(c+dx)\left(5aA+5a\right.$$
$$= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{aC\sec^4(c+dx)\tan(c+dx)}{5d}$$
$$= \frac{a(5A+4C)\tan(c+dx)}{5d}+\frac{a(4A+3C)\sec(c+dx)\tan(c+}{8d}$$
$$= \frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(5A+4C)\tan(c+dx}{5d}$$

Mathematica [A] time = 0.717998, size = 93, normalized size = 0.66

$$\frac{a\left(15(4A+3C)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5(A+2C)\tan^2(c+dx)+15(A+C)+3C\tan^4(c+dx)\right)+15(4A\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(15*(4*A + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(4*A + 3*C)*Sec
[c + d*x] + 30*C*Sec[c + d*x]^3 + 8*(15*(A + C) + 5*(A + 2*C)*Tan[c + d*x]^
2 + 3*C*Tan[c + d*x]^4))))/(120*d)
```

Maple [A] time = 0.048, size = 192, normalized size = 1.4

$$\frac{Aa\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aC(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3aC\sec(dx+c)t}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/4*a*C
*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec
(d*x+c)+tan(d*x+c))+2/3/d*A*a*tan(d*x+c)+1/3/d*A*a*tan(d*x+c)*sec(d*x+c)^2+
8/15*a*C*tan(d*x+c)/d+1/5*a*C*sec(d*x+c)^4*tan(d*x+c)/d+4/15*a*C*sec(d*x+c)
^2*tan(d*x+c)/d
```

Maxima [A] time = 0.941858, size = 236, normalized size = 1.69

$$\frac{80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ca-15\,Ca\left(\frac{2\left(3\sin(\cdots}{\sin(dx+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a - 15*C*a*(2*(3*sin(d*x + c)^3 - 5*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) - 60*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.51743, size = 389, normalized size = 2.78

$$\frac{15(4A+3C)a\cos(dx+c)^5\log(\sin(dx+c)+1)-15(4A+3C)a\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(16(5A\cdots\right.}{240\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*A + 3*C)*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*A + 3*
C)*a*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*A + 4*C)*a*cos(d*x +
c)^4 + 15*(4*A + 3*C)*a*cos(d*x + c)^3 + 8*(5*A + 4*C)*a*cos(d*x + c)^2 + 3
0*C*a*cos(d*x + c) + 24*C*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int C\sec^5(c+dx)\,dx+\int C\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integr
al(C*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.23969, size = 294, normalized size = 2.1

$$\frac{15(4Aa+3Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15(4Aa+3Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(60\,Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+45\,C\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(4*A*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a +
3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a*tan(1/2*d*x + 1/2*c)^
9 + 45*C*a*tan(1/2*d*x + 1/2*c)^9 - 200*A*a*tan(1/2*d*x + 1/2*c)^7 - 130*C*
a*tan(1/2*d*x + 1/2*c)^7 + 400*A*a*tan(1/2*d*x + 1/2*c)^5 + 464*C*a*tan(1/2
*d*x + 1/2*c)^5 - 440*A*a*tan(1/2*d*x + 1/2*c)^3 - 190*C*a*tan(1/2*d*x + 1/
2*c)^3 + 180*A*a*tan(1/2*d*x + 1/2*c) + 195*C*a*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.85 $\int \sec^2(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=117

$$\frac{a(3A+2C)\tan(c+dx)}{3d}+\frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aC\tan(c+dx)\,\text{s}}{4d}$$

```
[Out] (a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(3*A + 2*C)*Tan[c + d*x])/
(3*d) + (a*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (a*C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.170044, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4077, 4047, 3768, 3770, 4046, 3767, 8}

$$\frac{a(3A+2C)\tan(c+dx)}{3d}+\frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aC\tan(c+dx)\,\text{s}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(3*A + 2*C)*Tan[c + d*x])/
(3*d) + (a*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (a*C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx &= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^2(c+dx)\left(4aA+a(\right.\\ &= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^2(c+dx)\left(4aA+4a\right.\\ &= \frac{a(4A+3C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aC\sec^2(c+dx)\tan}{3d}\\ &= \frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\sec(c+dx)}{8d}\\ &= \frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+2C)\tan(c+dx}{3d}\end{aligned}$$

Mathematica [A] time = 0.428452, size = 75, normalized size = 0.64

$$\frac{a\left(3(4A+3C)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(4A+3C)\sec(c+dx)+24(A+C)+8C\tan^2(c+dx)+6C\sec^3(c+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(3*(4*A + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(A + C) + 3*(4*A
 + 3*C)*Sec[c + d*x] + 6*C*Sec[c + d*x]^3 + 8*C*Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.043, size = 149, normalized size = 1.3

$$\frac{Aa\tan(dx+c)}{d}+\frac{2aC\tan(dx+c)}{3d}+\frac{aC(\sec(dx+c))^2\tan(dx+c)}{3d}+\frac{Aa\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa\ln(\sec(dx+}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*tan(d*x+c)+2/3*a*C*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d+1
/2/d*A*a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/4*a*C*
sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec(
d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.941002, size = 205, normalized size = 1.75

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca-3\,Ca\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-\ldots\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a - 3*C*a*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a*tan(d*x +
 c))/d
```

Fricas [A] time = 0.509292, size = 335, normalized size = 2.86

$$\frac{3\,(4\,A+3\,C)a\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(4\,A+3\,C)a\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8\,(3\,A+2\ldots\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(4*A + 3*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A + 3*C)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(3*A + 2*C)*a*cos(d*x + c)^3
 + 3*(4*A + 3*C)*a*cos(d*x + c)^2 + 8*C*a*cos(d*x + c) + 6*C*a)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^2(c+dx)\,dx+\int A\sec^3(c+dx)\,dx+\int C\sec^4(c+dx)\,dx+\int C\sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integr
al(C*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**5, x))
```

Giac [A] time = 1.25697, size = 254, normalized size = 2.17

$$3\,(4\,Aa+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,Aa+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+9\,Ca\tan\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(4*A*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*C
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*A*a*tan(1/2*d*x + 1/2*c)^7 +
 9*C*a*tan(1/2*d*x + 1/2*c)^7 - 60*A*a*tan(1/2*d*x + 1/2*c)^5 - 49*C*a*tan(
1/2*d*x + 1/2*c)^5 + 84*A*a*tan(1/2*d*x + 1/2*c)^3 + 31*C*a*tan(1/2*d*x + 1
/2*c)^3 - 36*A*a*tan(1/2*d*x + 1/2*c) - 39*C*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.86 $\int \sec(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=86

$$\frac{a(3A+2C)\tan(c+dx)}{3d}+\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{aC\tan(c+dx)\sec(c+d}{2d}$$

```
[Out] (a*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 2*C)*Tan[c + d*x])/(3
*d) + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*C*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 0.107609, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4077, 4047, 3767, 8, 4046, 3770}

$$\frac{a(3A+2C)\tan(c+dx)}{3d}+\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{aC\tan(c+dx)\sec(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 2*C)*Tan[c + d*x])/(3
*d) + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*C*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
```

```
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx = \frac{aC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)\left(3aA+a(3A\right.$$
$$=\frac{aC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)\left(3aA+3aC\right.$$
$$=\frac{aC\sec(c+dx)\tan(c+dx)}{2d}+\frac{aC\sec^2(c+dx)\tan(c+dx)}{3d}+$$
$$=\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(3A+2C)\tan(c+dx)}{3d}+$$

Mathematica [A] time = 0.270418, size = 56, normalized size = 0.65

$$\frac{a\left(3(2A+C)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6(A+C)+2C\tan^2(c+dx)+3C\sec(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(3*(2*A + C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + C) + 3*C*Sec[c
 + d*x] + 2*C*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.046, size = 108, normalized size = 1.3

$$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\sec(dx+c)\tan(dx+c)}{2d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Aa\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*C
*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*tan(d*x+c)+2/3*a*C*tan(d*x+c)/d+1/3*a*C*
sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.931099, size = 135, normalized size = 1.57

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca-3Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12Aa\log(\sec}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a - 3*C*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*A*a*l
og(sec(d*x + c) + tan(d*x + c)) + 12*A*a*tan(d*x + c))/d
```

Fricas [A] time = 0.507627, size = 285, normalized size = 3.31

$$\frac{3\,(2\,A+C)a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+C)a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(3\,A+2\,C)\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*A + C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + C)*a*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A + 2*C)*a*cos(d*x + c)^2 + 3
*C*a*cos(d*x + c) + 2*C*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\sec^{2}\left(c+dx\right)dx+\int C\sec^{3}\left(c+dx\right)dx+\int C\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sec(c + d*x)**2, x) + Integral(
C*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**4, x))
```

Giac [B] time = 1.18443, size = 211, normalized size = 2.45

$$\frac{3\,(2\,Aa+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,Aa+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ca\tan\left(\frac{1}{2}\,d\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*A*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a + C*a)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*C*a
*tan(1/2*d*x + 1/2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 6*A*a*tan(1/2*d*x + 1/2*c) + 9*C*a*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.87 $\int (a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=58

$$\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+aAx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] a*A*x + (a*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*Tan[c + d*x])/d +
(a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0543391, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4049, 3770, 3767, 8}

$$\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+aAx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (a*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*Tan[c + d*x])/d +
(a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4049

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x])/(2*f), x] + D
ist[1/2, Int[Simp[2*A*a + b*(2*A + C)*Csc[e + f*x] + 2*a*C*Csc[e + f*x]^2,
x], x], x] /; FreeQ[{a, b, e, f, A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx &= \frac{aC\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2aA+a(2A+C)\sec(c+dx)+2aC\right.\\ &= aAx+\frac{aC\sec(c+dx)\tan(c+dx)}{2d}+(aC)\int \sec^2(c+dx)\,dx+\frac{1}{2}(a(2A\\ &= aAx+\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\sec(c+dx)\tan(c+dx)}{2d}-\\ &= aAx+\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)}{d}+\frac{aC\sec(c+\end{aligned}$$

Mathematica [A] time = 0.0272733, size = 67, normalized size = 1.16

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])/(2*d) +
 (a*C*Tan[c + d*x])/d + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.041, size = 85, normalized size = 1.5

$$aAx+\frac{Aac}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\sec(dx+c)\tan(dx+c)}{2d}+\frac{aC\ln(\sec(dx+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a*A*x+1/d*A*a*c+a*C*tan(d*x+c)/d+1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*C*
sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.925916, size = 119, normalized size = 2.05

$$\frac{4(dx+c)Aa-Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Aa\log(\sec(dx+c)+\tan(dx+c)\cdots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a - C*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*A*a*log(sec(d*x + c) + tan(d*x +
c)) + 4*C*a*tan(d*x + c))/d
```

Fricas [A] time = 0.51587, size = 267, normalized size = 4.6

$$\frac{4Aadx\cos(dx+c)^2+(2A+C)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(2A+C)a\cos(dx+c)^2\log(-\sin(dx+c)+1\cdots}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*A*a*d*x*cos(d*x + c)^2 + (2*A + C)*a*cos(d*x + c)^2*log(sin(d*x + c)
 + 1) - (2*A + C)*a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*C*a*cos(d*
x + c) + C*a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\,dx + \int A\sec\left(c + dx\right)dx + \int C\sec^{2}\left(c + dx\right)dx + \int C\sec^{3}\left(c + dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A, x) + Integral(A*sec(c + d*x), x) + Integral(C*sec(c + d*x)**
2, x) + Integral(C*sec(c + d*x)**3, x))
```

Giac [A] time = 1.21176, size = 142, normalized size = 2.45

$$\frac{2\,(dx + c)Aa + (2\,Aa + Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (2\,Aa + Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 3\,C\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + (2*A*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
2*A*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(C*a*tan(1/2*d*x + 1/2*
c)^3 - 3*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.88 $\int \cos(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=42

$$\frac{aA\sin(c+dx)}{d}+aAx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*A*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d + (a*C*Tan[c +
 d*x])/d
```

Rubi [A] time = 0.100562, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4077, 4047, 8, 4045, 3770}

$$\frac{aA\sin(c+dx)}{d}+aAx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d + (a*C*Tan[c +
 d*x])/d
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aC\tan(c+dx)}{d}+\int \cos(c+dx)\left(aA+aA\sec(c+dx)+aC\right.$$
$$= \frac{aC\tan(c+dx)}{d}+(aA)\int 1\,dx+\int \cos(c+dx)\left(aA+aC\,\mathrm{se}\right.$$
$$= aAx+\frac{aA\sin(c+dx)}{d}+\frac{aC\tan(c+dx)}{d}+(aC)\int \sec(c+a$$
$$= aAx+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)}{d}+\frac{aC\tan(}{d}$$

Mathematica [A] time = 0.027497, size = 54, normalized size = 1.29

$$\frac{aA\sin(c)\cos(dx)}{d}+\frac{aA\cos(c)\sin(dx)}{d}+aAx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (a*A*Cos[
c]*Sin[d*x])/d + (a*C*Tan[c + d*x])/d
```

Maple [A] time = 0.071, size = 57, normalized size = 1.4

$$aAx+\frac{Aa\sin(dx+c)}{d}+\frac{Aac}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a*A*x+a*A*sin(d*x+c)/d+1/d*A*a*c+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+a*C*tan(
d*x+c)/d
```

Maxima [A] time = 0.928543, size = 80, normalized size = 1.9

$$\frac{2(dx+c)Aa+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2Aa\sin(dx+c)+2Ca\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*A*a*sin(d*x + c) + 2*C*a*tan(d*x + c))/d
```

Fricas [B] time = 0.529703, size = 232, normalized size = 5.52

$$\frac{2Aadx\cos(dx+c)+Ca\cos(dx+c)\log(\sin(dx+c)+1)-Ca\cos(dx+c)\log(-\sin(dx+c)+1)+2(Aa\cos(dx+c)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + C*a*cos(d*x + c)*log(sin(d*x + c) + 1) - C*a*
cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a*cos(d*x + c) + C*a)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\cos(c+dx)\,dx+\int A\cos(c+dx)\sec(c+dx)\,dx+\int C\cos(c+dx)\sec^2(c+dx)\,dx+\int C\cos(c+dx)\sec^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A*cos(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(C*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(C*cos(c + d*x)*sec(
c + d*x)**3, x))
```

Giac [B] time = 1.27122, size = 161, normalized size = 3.83

$$\frac{(dx+c)Aa+Ca\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-Ca\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Aa\,t}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] ((d*x + c)*A*a + C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*a*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - C*a*tan(1/2*d*x + 1
/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^4 - 1))/d
```

3.89 $\int \cos^2(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=58

$$\frac{aA\sin(c+dx)}{d}+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+2C)+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*(A + 2*C)*x)/2 + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d +
(a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.126799, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4075, 4047, 8, 4045, 3770}

$$\frac{aA\sin(c+dx)}{d}+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+2C)+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(A + 2*C)*x)/2 + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d +
(a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)\left(-2aA-\right.$$
$$= \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)\left(-2aA-\right.$$
$$= \frac{1}{2}a(A+2C)x + \frac{aA\sin(c+dx)}{d} + \frac{aA\cos(c+dx)\sin(c\cdot}{2d}$$
$$= \frac{1}{2}a(A+2C)x + \frac{aC\tanh^{-1}(\sin(c+dx))}{d} + \frac{aA\sin(c+d}{d}$$

Mathematica [A] time = 0.0808933, size = 52, normalized size = 0.9

$$\frac{a\left(4A\sin(c+dx)+A\sin(2(c+dx))+2Ac+2Adx+4C\tanh^{-1}(\sin(c+dx))+4Cdx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(2*A*c + 2*A*d*x + 4*C*d*x + 4*C*ArcTanh[Sin[c + d*x]] + 4*A*Sin[c + d*x
] + A*Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.081, size = 77, normalized size = 1.3

$$\frac{Aa\cos(dx+c)\sin(dx+c)}{2d} + \frac{aAx}{2} + \frac{Aac}{2d} + aCx + \frac{Cac}{d} + \frac{Aa\sin(dx+c)}{d} + \frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2*a*A*cos(d*x+c)*sin(d*x+c)/d+1/2*a*A*x+1/2/d*A*a*c+a*C*x+1/d*C*a*c+a*A*s
in(d*x+c)/d+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.930492, size = 95, normalized size = 1.64

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa+4\,(dx+c)Ca+2\,Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4\,Aa\sin(dx+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*C*a + 2*C*a*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a*sin(d*x + c))/d
```

Fricas [A] time = 0.519841, size = 167, normalized size = 2.88

$$\frac{(A+2\,C)adx+Ca\log(\sin(dx+c)+1)-Ca\log(-\sin(dx+c)+1)+(Aa\cos(dx+c)+2\,Aa)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/2*((A + 2*C)*a*d*x + C*a*log(sin(d*x + c) + 1) - C*a*log(-sin(d*x + c) +
1) + (A*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29866, size = 134, normalized size = 2.31

$$\frac{2\,Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Aa+2\,Ca)(dx+c)+\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,Aa\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*C*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (A*a + 2*C*a)*(d*x + c) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 3
*A*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.90 $\int \cos^3(c+dx)(a+a\sec(c+dx)) \left(A + C\sec^2(c + dx)\right) dx$

Optimal. Leaf size=77

$$\frac{a(2A + 3C)\sin(c + dx)}{3d} + \frac{aA\sin(c + dx)\cos^2(c + dx)}{3d} + \frac{aA\sin(c + dx)\cos(c + dx)}{2d} + \frac{1}{2}ax(A + 2C)$$

```
[Out] (a*(A + 2*C)*x)/2 + (a*(2*A + 3*C)*Sin[c + d*x])/(3*d) + (a*A*Cos[c + d*x]*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.145037, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4075, 4047, 2637, 4045, 8}

$$\frac{a(2A + 3C)\sin(c + dx)}{3d} + \frac{aA\sin(c + dx)\cos^2(c + dx)}{3d} + \frac{aA\sin(c + dx)\cos(c + dx)}{2d} + \frac{1}{2}ax(A + 2C)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(A + 2*C)*x)/2 + (a*(2*A + 3*C)*Sin[c + d*x])/(3*d) + (a*A*Cos[c + d*x]*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)\left(-3aA-a\right.$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)\left(-3aA-3\right.$$
$$= \frac{a(2A+3C)\sin(c+dx)}{3d} + \frac{aA\cos(c+dx)\sin(c+dx)}{2d} + \frac{aA}{}$$
$$= \frac{1}{2}a(A+2C)x + \frac{a(2A+3C)\sin(c+dx)}{3d} + \frac{aA\cos(c+dx)\sin}{2d}$$

Mathematica [A] time = 0.127982, size = 59, normalized size = 0.77

$$\frac{a(3(3A+4C)\sin(c+dx)+3A\sin(2(c+dx))+A\sin(3(c+dx))+6Ac+6Adx+12Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(6*A*c + 6*A*d*x + 12*C*d*x + 3*(3*A + 4*C)*Sin[c + d*x] + 3*A*Sin[2*(c
+ d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.082, size = 68, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aC\sin(dx+c) + aC(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*sin(d*x+c)+a*C*(d*x+c))
```

Maxima [A] time = 0.928591, size = 90, normalized size = 1.17

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-3(2dx+2c+\sin(2dx+2c))Aa-12(dx+c)Ca-12Ca\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*A*a - 12*(d*x + c)*C*a - 12*C*a*sin(d*x + c))/d
```

Fricas [A] time = 0.484788, size = 140, normalized size = 1.82

$$\frac{3(A+2C)adx + \left(2Aa\cos(dx+c)^2 + 3Aa\cos(dx+c) + 2(2A+3C)a\right)\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(A + 2*C)*a*d*x + (2*A*a*cos(d*x + c)^2 + 3*A*a*cos(d*x + c) + 2*(2*
A + 3*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25374, size = 169, normalized size = 2.19

$$\frac{3\,(Aa+2\,Ca)(dx+c)+\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Ca\,\mathrm{ta}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(A*a + 2*C*a)*(d*x + c) + 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*ta
n(1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*C*a*tan(1/2*d*x +
1/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 6*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.91 $\int \cos^4(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{a(A+C)\sin(c+dx)}{d}+\frac{a(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}-\frac{aA\sin^3(c+dx)}{3d}+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{8}ax(3A$$

```
[Out] (a*(3*A + 4*C)*x)/8 + (a*(A + C)*Sin[c + d*x])/d + (a*(3*A + 4*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*A*Si
n[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.177301, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4075, 4047, 2635, 8, 4044, 3013}

$$\frac{a(A+C)\sin(c+dx)}{d}+\frac{a(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}-\frac{aA\sin^3(c+dx)}{3d}+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{8}ax(3A$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(3*A + 4*C)*x)/8 + (a*(A + C)*Sin[c + d*x])/d + (a*(3*A + 4*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*A*Si
n[c + d*x]^3)/(3*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
```

```
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)\left(-4aA\right.$$

$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)\left(-4aA\right.$$

$$= \frac{a(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA\cos^3(c+dx)\text{s}}{4d}$$

$$= \frac{1}{8}a(3A+4C)x + \frac{a(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} +$$

$$= \frac{1}{8}a(3A+4C)x + \frac{a(A+C)\sin(c+dx)}{d} + \frac{a(3A+4C)\text{cc}}{}$$

Mathematica [A] time = 0.230785, size = 77, normalized size = 0.81

$$\frac{a(24(3A+4C)\sin(c+dx)+24(A+C)\sin(2(c+dx))+8A\sin(3(c+dx))+3A\sin(4(c+dx))+36Ac+36Adx+48}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(36*A*c + 48*c*C + 36*A*d*x + 48*C*d*x + 24*(3*A + 4*C)*Sin[c + d*x] + 2
4*(A + C)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/
(96*d)
```

Maple [A] time = 0.089, size = 96, normalized size = 1.

$$\frac{1}{d}\left(Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+aC\left(\frac{\text{cc}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+a*C*sin(d*x+c))
```

Maxima [A] time = 0.929224, size = 122, normalized size = 1.28

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa-24(2dx+2c+\text{si}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
C*a - 96*C*a*sin(d*x + c))/d
```

Fricas [A] time = 0.496195, size = 189, normalized size = 1.99

$$\frac{3\,(3\,A+4\,C)adx+\left(6\,Aa\cos\left(dx+c\right)^{3}+8\,Aa\cos\left(dx+c\right)^{2}+3\,(3\,A+4\,C)a\cos\left(dx+c\right)+8\,(2\,A+3\,C)a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(3*A + 4*C)*a*d*x + (6*A*a*cos(d*x + c)^3 + 8*A*a*cos(d*x + c)^2 +
3*(3*A + 4*C)*a*cos(d*x + c) + 8*(2*A + 3*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15484, size = 211, normalized size = 2.22

$$\frac{3\,(3\,Aa+4\,Ca)(dx+c)+\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+49\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+60\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+84\,C\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a + 4*C*a)*(d*x + c) + 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 + 12*C*
a*tan(1/2*d*x + 1/2*c)^7 + 49*A*a*tan(1/2*d*x + 1/2*c)^5 + 60*C*a*tan(1/2*d
*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 84*C*a*tan(1/2*d*x + 1/2*c)
^3 + 39*A*a*tan(1/2*d*x + 1/2*c) + 36*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

3.92 $\int \cos^5(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=131

$$-\frac{a(4A+5C)\sin^3(c+dx)}{15d}+\frac{a(4A+5C)\sin(c+dx)}{5d}+\frac{a(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{aA\sin(c+dx)\cos^4(c+}{5d}$$

```
[Out] (a*(3*A + 4*C)*x)/8 + (a*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (a*(3*A + 4*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*C)*Sin[c + d*x]^3)/(
15*d)
```

Rubi [A] time = 0.186607, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4075, 4047, 2633, 4045, 2635, 8}

$$-\frac{a(4A+5C)\sin^3(c+dx)}{15d}+\frac{a(4A+5C)\sin(c+dx)}{5d}+\frac{a(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{aA\sin(c+dx)\cos^4(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(3*A + 4*C)*x)/8 + (a*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (a*(3*A + 4*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*C)*Sin[c + d*x]^3)/(
15*d)
```


Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)\left(-5aA-a\right.$$
$$= \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)\left(-5aA-5\right.$$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d}$$
$$= \frac{a(4A+5C)\sin(c+dx)}{5d} + \frac{a(3A+4C)\cos(c+dx)\sin(c+}{8d}$$
$$= \frac{1}{8}a(3A+4C)x + \frac{a(4A+5C)\sin(c+dx)}{5d} + \frac{a(3A+4C)\text{co}}{}$$

Mathematica [A] time = 0.278168, size = 86, normalized size = 0.66

$$\frac{a\left(-160(2A+C)\sin^3(c+dx)+480(A+C)\sin(c+dx)+15(4(3A+4C)(c+dx)+8(A+C)\sin(2(c+dx))+A\sin(4(c\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(480*(A + C)*Sin[c + d*x] - 160*(2*A + C)*Sin[c + d*x]^3 + 96*A*Sin[c +
d*x]^5 + 15*(4*(3*A + 4*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + A*Sin[4
*(c + d*x)])))/(480*d)
```

Maple [A] time = 0.107, size = 117, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Aa\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*C*(2+cos(d*x+c)^2)*
sin(d*x+c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.933367, size = 153, normalized size = 1.17

$$\frac{32\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Aa}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 160*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*C*a + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a)/d
```

Fricas [A] time = 0.500726, size = 242, normalized size = 1.85

$$\frac{15\left(3A+4C\right)adx+\left(24\,Aa\cos\left(dx+c\right)^4+30\,Aa\cos\left(dx+c\right)^3+8\left(4A+5C\right)a\cos\left(dx+c\right)^2+15\left(3A+4C\right)a\cos\left(dx+c\right)\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*A + 4*C)*a*d*x + (24*A*a*cos(d*x + c)^4 + 30*A*a*cos(d*x + c)^
3 + 8*(4*A + 5*C)*a*cos(d*x + c)^2 + 15*(3*A + 4*C)*a*cos(d*x + c) + 16*(4*
A + 5*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1905, size = 251, normalized size = 1.92

$$\frac{15\left(3\,Aa+4\,Ca\right)\left(dx+c\right)+\frac{2\left(45\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+130\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+200\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+464\,Aa\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*A*a + 4*C*a)*(d*x + c) + 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 60
*C*a*tan(1/2*d*x + 1/2*c)^9 + 130*A*a*tan(1/2*d*x + 1/2*c)^7 + 200*C*a*tan(
```

```
1/2*d*x + 1/2*c)^7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*C*a*tan(1/2*d*x +
 1/2*c)^5 + 190*A*a*tan(1/2*d*x + 1/2*c)^3 + 440*C*a*tan(1/2*d*x + 1/2*c)^3
 + 195*A*a*tan(1/2*d*x + 1/2*c) + 180*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^5)/d
```

3.93 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=172

$$\frac{a^2(4A+3C)\tan(c+dx)}{3d}+\frac{a^2(4A+3C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2(4A+3C)\tan(c+dx)\sec(c+dx)}{12d}+\frac{(10A+3C)\,\mathrm{t}}{}$$

```
[Out] (a^2*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^2*(4*A + 3*C)*Tan[c + d*
x])/(3*d) + (a^2*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(12*d) + ((10*A + 3
*C)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(30*d) + (C*Sec[c + d*x]^2*(a + a*
Sec[c + d*x])^2*Tan[c + d*x])/(5*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c + d*x
])/(10*a*d)
```

Rubi [A] time = 0.385351, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4089, 4010, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(4A+3C)\tan(c+dx)}{3d}+\frac{a^2(4A+3C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2(4A+3C)\tan(c+dx)\sec(c+dx)}{12d}+\frac{(10A+3C)\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^2*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^2*(4*A + 3*C)*Tan[c + d*
x])/(3*d) + (a^2*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(12*d) + ((10*A + 3
*C)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(30*d) + (C*Sec[c + d*x]^2*(a + a*
Sec[c + d*x])^2*Tan[c + d*x])/(5*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c + d*x
])/(10*a*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^2\tan(c+dx)}{5d} + \frac{\int \sec^2(c}{}$$

$$= \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^2\tan(c+dx)}{5d} + \frac{C(a+a\,s}{}$$

$$= \frac{(10A+3C)(a+a\sec(c+dx))^2\tan(c+dx)}{30d} + \frac{C\sec^2(c+}{}$$

$$= \frac{(10A+3C)(a+a\sec(c+dx))^2\tan(c+dx)}{30d} + \frac{C\sec^2(c+}{}$$

$$= \frac{a^2(4A+3C)\sec(c+dx)\tan(c+dx)}{12d} + \frac{(10A+3C)(a+a}{}$$

$$= \frac{a^2(4A+3C)\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a^2(4A+3C)\tan(c+}{3d}$$

Mathematica [A] time = 1.78983, size = 321, normalized size = 1.87

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(A\cos^2(c+dx)+C\right)\left(240(4A+3C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^4*Sec[c
+ d*x]^5*(240*(4*A + 3*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(40*(16*A + 15
*C)*Sin[d*x] - 120*(3*A + C)*Sin[2*c + d*x] + 120*A*Sin[c + 2*d*x] + 210*C*
Sin[c + 2*d*x] + 120*A*Sin[3*c + 2*d*x] + 210*C*Sin[3*c + 2*d*x] + 440*A*Si
n[2*c + 3*d*x] + 360*C*Sin[2*c + 3*d*x] - 60*A*Sin[4*c + 3*d*x] + 60*A*Sin[
3*c + 4*d*x] + 45*C*Sin[3*c + 4*d*x] + 60*A*Sin[5*c + 4*d*x] + 45*C*Sin[5*c
 + 4*d*x] + 100*A*Sin[4*c + 5*d*x] + 72*C*Sin[4*c + 5*d*x])))/(1920*d*(A +
2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.119, size = 210, normalized size = 1.2

$$\frac{5\,a^2A\tan(dx+c)}{3d}+\frac{6\,a^2C\tan(dx+c)}{5d}+\frac{3\,a^2C\tan(dx+c)(\sec(dx+c))^2}{5d}+\frac{a^2A\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2A\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 5/3/d*a^2*A*tan(d*x+c)+6/5/d*a^2*C*tan(d*x+c)+3/5/d*a^2*C*tan(d*x+c)*sec(d*
x+c)^2+1/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+
1/2/d*a^2*C*tan(d*x+c)*sec(d*x+c)^3+3/4/d*a^2*C*sec(d*x+c)*tan(d*x+c)+3/4/d
*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*a^2*A*tan(d*x+c)*sec(d*x+c)^2+1/5/d*
a^2*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 0.944061, size = 294, normalized size = 1.71

$$\frac{40\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2+8\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Ca^2+40\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 8*(3*tan(d*x + c)^5 + 1
0*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^2 + 40*(tan(d*x + c)^3 + 3*tan(d*x
+ c))*C*a^2 - 15*C*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 60*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)) + 120*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.520133, size = 409, normalized size = 2.38

$$\frac{15\,(4\,A+3\,C)a^2\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(4\,A+3\,C)a^2\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(4\,(25\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(4*A + 3*C)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*A +
3*C)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(4*(25*A + 18*C)*a^2*cos
(d*x + c)^4 + 15*(4*A + 3*C)*a^2*cos(d*x + c)^3 + 4*(5*A + 9*C)*a^2*cos(d*x
 + c)^2 + 30*C*a^2*cos(d*x + c) + 12*C*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\sec^2(c+dx)\,dx+\int 2A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int C\sec^4(c+dx)\,dx+\int 2C\sec^5(c+dx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*sec(c + d*x)**3, x) + I
ntegral(A*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**4, x) + Integral(2
*C*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.26917, size = 332, normalized size = 1.93

$$\frac{15\left(4Aa^2+3Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4Aa^2+3Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(60Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+4\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/60*(15*(4*A*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a
^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a^2*tan(1/2*d*x
+ 1/2*c)^9 + 45*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 280*A*a^2*tan(1/2*d*x + 1/2*
c)^7 - 210*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 560*A*a^2*tan(1/2*d*x + 1/2*c)^5
+ 432*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 520*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 270
*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 180*A*a^2*tan(1/2*d*x + 1/2*c) + 195*C*a^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.94 $\int \sec(c+dx)(a+a\sec(c+dx))^2 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=132

$$\frac{a^2(12A+7C)\tan(c+dx)}{6d} + \frac{a^2(12A+7C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(12A+7C)\tan(c+dx)\sec(c+dx)}{24d} + \frac{C\tan(c+$$

```
[Out] (a^2*(12*A + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(12*A + 7*C)*Tan[c +
d*x])/(6*d) + (a^2*(12*A + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (C*(a +
 a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c +
 d*x])/(4*a*d)
```

Rubi [A] time = 0.212292, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4083, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(12A+7C)\tan(c+dx)}{6d} + \frac{a^2(12A+7C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(12A+7C)\tan(c+dx)\sec(c+dx)}{24d} + \frac{C\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^2*(12*A + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(12*A + 7*C)*Tan[c +
d*x])/(6*d) + (a^2*(12*A + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (C*(a +
 a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c +
 d*x])/(4*a*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4ad}+\frac{\int \sec(c+dx)(a+a\sec}{}$$

$$= -\frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{C(a+a\sec(c+dx))^3}{4ad}$$

$$= -\frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{C(a+a\sec(c+dx))^3}{4ad}$$

$$= \frac{a^2(12A+7C)\sec(c+dx)\tan(c+dx)}{24d}-\frac{C(a+a\sec(c+dx}{12d}$$

$$= \frac{a^2(12A+7C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(12A+7C)\tan(c+}{6d}$$

Mathematica [B] time = 1.40833, size = 291, normalized size = 2.2

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(A\cos^2(c+dx)+C\right)\left(24(12A+7C)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^4*Sec[c
+ d*x]^4*(24*(12*A + 7*C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-48*(3*A + 2*
C)*Sin[c] + 3*(4*A + 15*C)*Sin[d*x] + 12*A*Sin[2*c + d*x] + 45*C*Sin[2*c +
d*x] + 144*A*Sin[c + 2*d*x] + 128*C*Sin[c + 2*d*x] - 48*A*Sin[3*c + 2*d*x]
+ 12*A*Sin[2*c + 3*d*x] + 21*C*Sin[2*c + 3*d*x] + 12*A*Sin[4*c + 3*d*x] + 2
1*C*Sin[4*c + 3*d*x] + 48*A*Sin[3*c + 4*d*x] + 32*C*Sin[3*c + 4*d*x])))/(38
4*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.049, size = 166, normalized size = 1.3

$$\frac{3a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{7a^2C\sec(dx+c)\tan(dx+c)}{8d}+\frac{7a^2C\ln(\sec(dx+c)+\tan(dx+c))}{8d}+2\frac{a^2A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 3/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+7/8/d*a^2*C*sec(d*x+c)*tan(d*x+c)+7/8
/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*A*tan(d*x+c)+4/3/d*a^2*C*tan(d*x
+c)+2/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1
/4/d*a^2*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 0.943957, size = 306, normalized size = 2.32

$$32\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ca^2-3\,Ca^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 - 3*C*a^2*(2*(3*sin(d*x +
c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*C*a^2*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 48*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 96*A*a^2*tan(d*x +
c))/d
```

Fricas [A] time = 0.51219, size = 356, normalized size = 2.7

$$\frac{3\,(12\,A+7\,C)a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(12\,A+7\,C)a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(3\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(12*A + 7*C)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(12*A + 7
*C)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(3*A + 2*C)*a^2*cos(d
*x + c)^3 + 3*(4*A + 7*C)*a^2*cos(d*x + c)^2 + 16*C*a^2*cos(d*x + c) + 6*C*
a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\sec\left(c+dx\right)dx+\int 2A\sec^2\left(c+dx\right)dx+\int A\sec^3\left(c+dx\right)dx+\int C\sec^3\left(c+dx\right)dx+\int 2C\sec^4\left(c+dx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sec(c + d*x)**2, x) + Inte
gral(A*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**3, x) + Integral(2*C*
sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**5, x))
```

Giac [A] time = 1.24423, size = 286, normalized size = 2.17

$$3\left(12\,Aa^{2}+7\,Ca^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(12\,Aa^{2}+7\,Ca^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(36\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(12*A*a^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(12*A*a
^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(36*A*a^2*tan(1/2*d*x
+ 1/2*c)^7 + 21*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 132*A*a^2*tan(1/2*d*x + 1/2*
c)^5 - 77*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 156*A*a^2*tan(1/2*d*x + 1/2*c)^3 +
 83*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 60*A*a^2*tan(1/2*d*x + 1/2*c) - 75*C*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.95 $\int (a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=96

$$\frac{a^2(A+C)\tan(c+dx)}{d}+\frac{a^2(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+a^2Ax+\frac{C\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d}+\frac{C\tan(c+$$

```
[Out] a^2*A*x + (a^2*(2*A + C)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Tan[c + d*
x])/d + (C*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (C*(a^2 + a^2*Sec[c
 + d*x])*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.140093, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4055, 3917, 3914, 3767, 8, 3770}

$$\frac{a^2(A+C)\tan(c+dx)}{d}+\frac{a^2(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+a^2Ax+\frac{C\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d}+\frac{C\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a^2*A*x + (a^2*(2*A + C)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Tan[c + d*
x])/d + (C*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (C*(a^2 + a^2*Sec[c
 + d*x])*Tan[c + d*x])/(3*d)
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx=\frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{\int (a+a\sec(c+dx))^2(3aA+2aC}{3a}$$
$$=\frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{C\left(a^2+a^2\sec(c+dx)\right)\tan(c+dx}{3d}$$
$$=a^2Ax+\frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{C\left(a^2+a^2\sec(c+dx)\right)\text{ta}}{3d}$$
$$=a^2Ax+\frac{a^2(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{C(a+a\sec(c+dx))^2\tan(c}{3d}$$
$$=a^2Ax+\frac{a^2(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(A+C)\tan(c+dx)}{d}+\frac{C}{}$$

Mathematica [B] time = 6.51247, size = 1090, normalized size = 11.35

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (A*x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2))/(2*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((-2*A - C)*Cos[c + d*x]^
4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*
Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/(2*d*(A + 2*C + A*Cos[2*c + 2*d*x])
) + ((2*A + C)*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/(2*d*(A
 + 2*C + A*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(12*d*(A + 2*C + A*
Cos[2*c + 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*
x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A
+ C*Sec[c + d*x]^2)*(7*C*Cos[c/2] - 5*C*Sin[c/2]))/(24*d*(A + 2*C + A*Cos[2
*c + 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)^2) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*S
ec[c + d*x]^2)*(3*A*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2]))/(6*d*(A + 2*C + A*Cos
[2*c + 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])) + (C*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C
*Sec[c + d*x]^2)*Sin[(d*x)/2])/(12*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/
2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]
^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*(-7*C
*Cos[c/2] - 5*C*Sin[c/2]))/(24*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^4*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*(3*A*Sin[
(d*x)/2] + 5*C*Sin[(d*x)/2]))/(6*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2]
 + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.051, size = 134, normalized size = 1.4

$$a^2Ax+\frac{Aa^2c}{d}+\frac{5\,a^2C\tan(dx+c)}{3d}+2\,\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2C\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2C\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^2*A*x+1/d*A*a^2*c+5/3/d*a^2*C*tan(d*x+c)+2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+
c))+1/d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/d
*a^2*A*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.931493, size = 177, normalized size = 1.84

$$\frac{6\,(dx+c)Aa^2+2\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^2-3\,Ca^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^2 + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 - 3*C*a^
2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 6*A*a^2*tan(d*x
+ c) + 6*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.517296, size = 331, normalized size = 3.45

$$\frac{6\,Aa^2dx\cos(dx+c)^3+3\,(2\,A+C)a^2\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(2\,A+C)a^2\cos(dx+c)^3\log(-\sin(dx+}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*A*a^2*d*x*cos(d*x + c)^3 + 3*(2*A + C)*a^2*cos(d*x + c)^3*log(sin(d*
x + c) + 1) - 3*(2*A + C)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*((3
*A + 5*C)*a^2*cos(d*x + c)^2 + 3*C*a^2*cos(d*x + c) + C*a^2)*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\,dx+\int 2A\sec(c+dx)\,dx+\int A\sec^2(c+dx)\,dx+\int C\sec^2(c+dx)\,dx+\int 2C\sec^3(c+dx)\,dx+\int C\sec\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**2*(Integral(A, x) + Integral(2*A*sec(c + d*x), x) + Integral(A*sec(c + d
*x)**2, x) + Integral(C*sec(c + d*x)**2, x) + Integral(2*C*sec(c + d*x)**3,
```

```
x) + Integral(C*sec(c + d*x)**4, x))
```

Giac [B] time = 1.20263, size = 252, normalized size = 2.62

$$\frac{3\,(dx+c)Aa^2+3\left(2\,Aa^2+Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^2+Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa^2\tan\left(\right.\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*A*a^2 + 3*(2*A*a^2 + C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 3*(2*A*a^2 + C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*
tan(1/2*d*x + 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*tan(1/2*d
*x + 1/2*c)^3 - 8*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^2*tan(1/2*d*x + 1/2*
c) + 9*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.96 $\int \cos(c+dx)(a+a\sec(c+dx))^2 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=112

$$-\frac{a^2(2A-3C)\tan(c+dx)}{2d}+\frac{a^2(2A+3C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(2A-C)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+2a^2A\text{:}$$

```
[Out] 2*a^2*A*x + (a^2*(2*A + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*(a + a*Sec[c
 + d*x])^2*Sin[c + d*x])/d - (a^2*(2*A - 3*C)*Tan[c + d*x])/(2*d) - ((2*A -
 C)*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.189976, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4087, 3917, 3914, 3767, 8, 3770}

$$-\frac{a^2(2A-3C)\tan(c+dx)}{2d}+\frac{a^2(2A+3C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(2A-C)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+2a^2A\text{:}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] 2*a^2*A*x + (a^2*(2*A + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*(a + a*Sec[c
 + d*x])^2*Sin[c + d*x])/d - (a^2*(2*A - 3*C)*Tan[c + d*x])/(2*d) - ((2*A -
 C)*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^2\sin(c+dx)}{d} + \frac{\int(a+a\sec(c+dx))^2(2}{}$$

$$= \frac{A(a+a\sec(c+dx))^2\sin(c+dx)}{d} - \frac{(2A-C)\left(a^2+a^2\sec(\right.}{2d}$$

$$= 2a^2Ax + \frac{A(a+a\sec(c+dx))^2\sin(c+dx)}{d} - \frac{(2A-C)\left(a^2\right.}{}$$

$$= 2a^2Ax + \frac{a^2(2A+3C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{A(a+a\sec(}{}$$

$$= 2a^2Ax + \frac{a^2(2A+3C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{A(a+a\sec(}{}$$

Mathematica [B] time = 2.61197, size = 330, normalized size = 2.95

$$a^2\cos^4(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(A+C\sec^2(c+dx)\right)\left(-\frac{2(2A+3C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(2A+3C)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + C*Sec[c +
d*x]^2)*(8*A*x - (2*(2*A + 3*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d
 + (2*(2*A + 3*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*A*Cos[d*
x]*Sin[c])/d + (4*A*Cos[c]*Sin[d*x])/d + C/(d*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2) + (8*C*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])) - C/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (8*
C*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))))/(8*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.086, size = 114, normalized size = 1.

$$\frac{a^2A\sin(dx+c)}{d} + \frac{3a^2C\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 2a^2Ax + 2\frac{Aa^2c}{d} + 2\frac{a^2C\tan(dx+c)}{d} + \frac{a^2A\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*sin(d*x+c)+3/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*A*x+2/d*A*
a^2*c+2/d*a^2*C*tan(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*C*
```

```
sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.938584, size = 192, normalized size = 1.71

$$\frac{8\,(dx+c)Aa^2 - Ca^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Aa^2(\log\left(\sin\left(dx+c\right)+1\right) - \log(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/4*(8*(d*x + c)*A*a^2 - C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*A*a^2*sin(d*x + c) + 8*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.525572, size = 320, normalized size = 2.86

$$\frac{8\,Aa^2dx\cos\left(dx+c\right)^2 + \left(2\,A+3\,C\right)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(2\,A+3\,C\right)a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(8*A*a^2*d*x*cos(d*x + c)^2 + (2*A + 3*C)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (2*A + 3*C)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
A*a^2*cos(d*x + c)^2 + 4*C*a^2*cos(d*x + c) + C*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23061, size = 205, normalized size = 1.83

$$\frac{4\,(dx+c)Aa^2 + \frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + \left(2\,Aa^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(2\,Aa^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(d*x + c)*A*a^2 + 4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)
^2 + 1) + (2*A*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A*a^2
 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*C*a^2*tan(1/2*d*x + 1
/2*c)^3 - 5*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.97 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=119

$$\frac{a^2(3A-2C)\sin(c+dx)}{2d}-\frac{(A-2C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{1}{2}a^2x(3A+2C)+\frac{2a^2C\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a^2*(3*A + 2*C)*x)/2 + (2*a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*A - 2*C
)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((A - 2*C)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.289675, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4087, 4018, 3996, 3770}

$$\frac{a^2(3A-2C)\sin(c+dx)}{2d}-\frac{(A-2C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{1}{2}a^2x(3A+2C)+\frac{2a^2C\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(3*A + 2*C)*x)/2 + (2*a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*A - 2*C
)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((A - 2*C)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{\int \cos(c+$$
$$= \frac{A\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{(A-2C)}{}$$
$$= \frac{a^2(3A-2C)\sin(c+dx)}{2d} + \frac{A\cos(c+dx)(a+a\sec(c+dx}{2d}$$
$$= \frac{1}{2}a^2(3A+2C)x + \frac{a^2(3A-2C)\sin(c+dx)}{2d} + \frac{A\cos(c+d}{}$$
$$= \frac{1}{2}a^2(3A+2C)x + \frac{2a^2C\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2(3A-2}{}$$

Mathematica [B] time = 1.06006, size = 292, normalized size = 2.45

$$-\frac{a^2\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4\cos(dx)\left(3Adx-4C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4C\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^2*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*(3*A*d*x + 2*C*d*x - 4*C*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + 4*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 + 4*Cos[2*c + d*x]*(3*A*d*x + 2*C*d*x - 4*C*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 4*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + A*Sin[d*x] + 1
6*C*Sin[d*x] + A*Sin[2*c + d*x] + 8*A*Sin[c + 2*d*x] + 8*A*Sin[3*c + 2*d*x]
 + A*Sin[2*c + 3*d*x] + A*Sin[4*c + 3*d*x]))/(16*d*(Cos[c/2] - Sin[c/2])*(C
os[c/2] + Sin[c/2])*(-1 + Tan[(c + d*x)/2])*(1 + Tan[(c + d*x)/2]))
```

Maple [A] time = 0.079, size = 107, normalized size = 0.9

$$\frac{a^2A\cos(dx+c)\sin(dx+c)}{2d} + \frac{3a^2Ax}{2} + \frac{3a^2Ac}{2d} + a^2Cx + \frac{Ca^2c}{d} + 2\frac{a^2A\sin(dx+c)}{d} + 2\frac{a^2C\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*a^2*A*cos(d*x+c)*sin(d*x+c)+3/2*a^2*A*x+3/2/d*A*a^2*c+a^2*C*x+1/d*C*a
^2*c+2/d*a^2*A*sin(d*x+c)+2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*tan
(d*x+c)
```

Maxima [A] time = 0.93985, size = 136, normalized size = 1.14

$$\frac{(2dx+2c+\sin(2dx+2c))Aa^2+4(dx+c)Aa^2+4(dx+c)Ca^2+4Ca^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*A*a^2 + 4*(d*x +
c)*C*a^2 + 4*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*A*a^
2*sin(d*x + c) + 4*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.525422, size = 296, normalized size = 2.49

$$\frac{(3A + 2C)a^2dx\cos(dx + c) + 2Ca^2\cos(dx + c)\log(\sin(dx + c) + 1) - 2Ca^2\cos(dx + c)\log(-\sin(dx + c) + 1) + (}{2d\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/2*((3*A + 2*C)*a^2*d*x*cos(d*x + c) + 2*C*a^2*cos(d*x + c)*log(sin(d*x +
c) + 1) - 2*C*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (A*a^2*cos(d*x + c)
^2 + 4*A*a^2*cos(d*x + c) + 2*C*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22831, size = 193, normalized size = 1.62

$$\frac{4Ca^2\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 4Ca^2\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{4Ca^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1} + \left(3Aa^2 + 2Ca^2\right)(dx + c) +}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/2*(4*C*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*C*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*C*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (3*A*a^2 + 2*C*a^2)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*
A*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.98 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=110

$$\frac{a^2(A+C)\sin(c+dx)}{d}+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d}+a^2x(A+2C)+\frac{a^2C\tanh^{-1}(\sin(c+dx))}{d}+\frac{A}{}$$

```
[Out] a^2*(A + 2*C)*x + (a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Sin[c + d*
x])/d + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d) + (A*C
os[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.267877, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4087, 4017, 3996, 3770}

$$\frac{a^2(A+C)\sin(c+dx)}{d}+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d}+a^2x(A+2C)+\frac{a^2C\tanh^{-1}(\sin(c+dx))}{d}+\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a^2*(A + 2*C)*x + (a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(A + C)*Sin[c + d*
x])/d + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d) + (A*C
os[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d} + \frac{\int \cos$$

$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d} + \frac{A\cos}{}$$

$$= \frac{a^2(A+C)\sin(c+dx)}{d} + \frac{A\cos^2(c+dx)(a+a\sec(c+}{3d}$$

$$= a^2(A+2C)x + \frac{a^2(A+C)\sin(c+dx)}{d} + \frac{A\cos^2(c+dx}{}$$

$$= a^2(A+2C)x + \frac{a^2C\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2(A+C)\mathrm{s}}{d}$$

Mathematica [A] time = 0.212069, size = 109, normalized size = 0.99

$$\frac{a^2\left(3(7A+4C)\sin(c+dx)+6A\sin(2(c+dx))+A\sin(3(c+dx))+12Adx-12C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(12*A*d*x + 24*C*d*x - 12*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(7*A + 4*C)*Sin[c + d*x]
 + 6*A*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.091, size = 128, normalized size = 1.2

$$\frac{A(\cos(dx+c))^2\sin(dx+c)a^2}{3d} + \frac{5a^2A\sin(dx+c)}{3d} + \frac{a^2C\sin(dx+c)}{d} + \frac{a^2A\cos(dx+c)\sin(dx+c)}{d} + a^2Ax + \frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^2+5/3/d*a^2*A*sin(d*x+c)+1/d*a^2*C*sin(d*
x+c)+1/d*a^2*A*cos(d*x+c)*sin(d*x+c)+a^2*A*x+1/d*A*a^2*c+2*a^2*C*x+2/d*C*a^
2*c+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.940133, size = 154, normalized size = 1.4

$$-\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-3(2dx+2c+\sin(2dx+2c))Aa^2-12(dx+c)Ca^2-3Ca^2(\log(\sin(dx+c}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a^2 - 12*(d*x + c)*C*a^2 - 3*C*a^2*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) - 6*A*a^2*sin(d*x + c) - 6*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.527179, size = 236, normalized size = 2.15

$$\frac{6(A+2C)a^2dx + 3Ca^2\log(\sin(dx+c)+1) - 3Ca^2\log(-\sin(dx+c)+1) + 2\left(Aa^2\cos(dx+c)^2 + 3Aa^2\cos(dx+c)\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(6*(A + 2*C)*a^2*d*x + 3*C*a^2*log(sin(d*x + c) + 1) - 3*C*a^2*log(-sin
(d*x + c) + 1) + 2*(A*a^2*cos(d*x + c)^2 + 3*A*a^2*cos(d*x + c) + (5*A + 3*
C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22258, size = 242, normalized size = 2.2

$$\frac{3Ca^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 3Ca^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + 3\left(Aa^2+2Ca^2\right)(dx+c) + \frac{2\left(3Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3\right.}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/3*(3*C*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*C*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(A*a^2 + 2*C*a^2)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x
+ 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 8*A*a^2*tan(1/2*d*x + 1/2*c)^
3 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 9*A*a^2*tan(1/2*d*x + 1/2*c) + 3*C*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.99 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=136

$$\frac{a^2(7A+12C)\sin(c+dx)}{6d}+\frac{a^2(7A+12C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(7A+12C)+\frac{A\sin(c+dx)\cos^3(c+dx)(a}{4d}$$

```
[Out] (a^2*(7*A + 12*C)*x)/8 + (a^2*(7*A + 12*C)*Sin[c + d*x])/(6*d) + (a^2*(7*A
+ 12*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c
+ d*x])^2*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(4*d)
```

Rubi [A] time = 0.307353, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4087, 4013, 3788, 2637, 4045, 8}

$$\frac{a^2(7A+12C)\sin(c+dx)}{6d}+\frac{a^2(7A+12C)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(7A+12C)+\frac{A\sin(c+dx)\cos^3(c+dx)(a}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(7*A + 12*C)*x)/8 + (a^2*(7*A + 12*C)*Sin[c + d*x])/(6*d) + (a^2*(7*A
+ 12*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c
+ d*x])^2*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(4*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx &= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{4d}+\frac{\int \cos^3(c\\ &= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d}+\frac{A\cos^3(c\\ &= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d}+\frac{A\cos^3(c\\ &= \frac{a^2(7A+12C)\sin(c+dx)}{6d}+\frac{a^2(7A+12C)\cos(c+dx)\sin\\ &= \frac{1}{8}a^2(7A+12C)x+\frac{a^2(7A+12C)\sin(c+dx)}{6d}+\frac{a^2(7A+1\end{aligned}$$

Mathematica [A] time = 0.230261, size = 73, normalized size = 0.54

$$\frac{a^2(48(3A+4C)\sin(c+dx)+24(2A+C)\sin(2(c+dx))+16A\sin(3(c+dx))+3A\sin(4(c+dx))+84Adx+144Cdx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(84*A*d*x + 144*C*d*x + 48*(3*A + 4*C)*Sin[c + d*x] + 24*(2*A + C)*Sin
[2*(c + d*x)] + 16*A*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.097, size = 142, normalized size = 1.

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2A\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^2A\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3
*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*C*sin(d*x+c)+
a^2*C*(d*x+c))
```

Maxima [A] time = 0.943066, size = 178, normalized size = 1.31

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{2}-24\left(2\,dx+2\,c+\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*
c))*A*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 - 96*(d*x + c)*C*a^2
- 192*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.490619, size = 207, normalized size = 1.52

$$\frac{3\left(7\,A+12\,C\right)a^{2}dx+\left(6\,Aa^{2}\cos\left(dx+c\right)^{3}+16\,Aa^{2}\cos\left(dx+c\right)^{2}+3\left(7\,A+4\,C\right)a^{2}\cos\left(dx+c\right)+16\left(2\,A+3\,C\right)a^{2}\right)\sin}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/24*(3*(7*A + 12*C)*a^2*d*x + (6*A*a^2*cos(d*x + c)^3 + 16*A*a^2*cos(d*x +
 c)^2 + 3*(7*A + 4*C)*a^2*cos(d*x + c) + 16*(2*A + 3*C)*a^2)*sin(d*x + c))/
d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20009, size = 238, normalized size = 1.75

$$\frac{3\left(7\,Aa^{2}+12\,Ca^{2}\right)\left(dx+c\right)+\frac{2\left(21\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+36\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+77\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+132\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+83\,Aa^{2}\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 12*C*a^2)*(d*x + c) + 2*(21*A*a^2*tan(1/2*d*x + 1/2*c)^7
 + 36*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 132*
C*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 156*C*a^2*
tan(1/2*d*x + 1/2*c)^3 + 75*A*a^2*tan(1/2*d*x + 1/2*c) + 60*C*a^2*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.100 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=169

$$\frac{a^2(18A+25C)\sin(c+dx)}{15d}+\frac{a^2(9A+10C)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{a^2(3A+4C)\sin(c+dx)\cos(c+dx)}{4d}+\frac{A\sin(}{}$$

```
[Out] (a^2*(3*A + 4*C)*x)/4 + (a^2*(18*A + 25*C)*Sin[c + d*x])/(15*d) + (a^2*(3*A
 + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^2*(9*A + 10*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(30*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin[c + d*
x])/(5*d) + (A*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(10*d)
```

Rubi [A] time = 0.387358, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4087, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^2(18A+25C)\sin(c+dx)}{15d}+\frac{a^2(9A+10C)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{a^2(3A+4C)\sin(c+dx)\cos(c+dx)}{4d}+\frac{A\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^2*(3*A + 4*C)*x)/4 + (a^2*(18*A + 25*C)*Sin[c + d*x])/(15*d) + (a^2*(3*A
 + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^2*(9*A + 10*C)*Cos[c + d*x]^2
*Sin[c + d*x])/(30*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin[c + d*
x])/(5*d) + (A*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(10*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{\int \cos^4(c \ldots}{\ldots}$$
$$= \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{A\cos^3(c \ldots}{\ldots}$$
$$= \frac{a^2(9A+10C)\cos^2(c+dx)\sin(c+dx)}{30d} + \frac{A\cos^4(c+dx)(\ldots}{\ldots}$$
$$= \frac{a^2(9A+10C)\cos^2(c+dx)\sin(c+dx)}{30d} + \frac{A\cos^4(c+dx)(\ldots}{\ldots}$$
$$= \frac{a^2(18A+25C)\sin(c+dx)}{15d} + \frac{a^2(3A+4C)\cos(c+dx)\sin \ldots}{4d}$$
$$= \frac{1}{4}a^2(3A+4C)x + \frac{a^2(18A+25C)\sin(c+dx)}{15d} + \frac{a^2(3A+ \ldots}{\ldots}$$

Mathematica [A] time = 0.396537, size = 97, normalized size = 0.57

$$\frac{a^2(30(11A+14C)\sin(c+dx)+120(A+C)\sin(2(c+dx))+45A\sin(3(c+dx))+15A\sin(4(c+dx))+3A\sin(5(c+dx \ldots}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(120*A*c + 180*A*d*x + 240*C*d*x + 30*(11*A + 14*C)*Sin[c + d*x] + 120
*(A + C)*Sin[2*(c + d*x)] + 45*A*Sin[3*(c + d*x)] + 20*C*Sin[3*(c + d*x)] +
 15*A*Sin[4*(c + d*x)] + 3*A*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.111, size = 160, normalized size = 1.

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+\frac{a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+2a^2A\left(1/4\left((\cos \ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/3*a^2*C*(2+
cos(d*x+c)^2)*sin(d*x+c)+2*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)+3/8*d*x+3/8*c)+2*a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^
2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*C*sin(d*x+c))
```

Maxima [A] time = 0.941552, size = 211, normalized size = 1.25

$$16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-80\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2+15\,(12\,dx+12\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 15*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 80*(sin(d*x + c)^3 - 3*sin(d*x + c))*C
*a^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 240*C*a^2*sin(d*x + c))
/d
```

Fricas [A] time = 0.498604, size = 258, normalized size = 1.53

$$\frac{15\,(3\,A+4\,C)a^2dx+\left(12\,Aa^2\cos(dx+c)^4+30\,Aa^2\cos(dx+c)^3+4\,(9\,A+5\,C)a^2\cos(dx+c)^2+15\,(3\,A+4\,C)a^2\,c}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/60*(15*(3*A + 4*C)*a^2*d*x + (12*A*a^2*cos(d*x + c)^4 + 30*A*a^2*cos(d*x
+ c)^3 + 4*(9*A + 5*C)*a^2*cos(d*x + c)^2 + 15*(3*A + 4*C)*a^2*cos(d*x + c)
 + 4*(18*A + 25*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18922, size = 284, normalized size = 1.68

$$\frac{15\left(3\,Aa^{2}+4\,Ca^{2}\right)(dx+c)+\frac{2\left(45\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+60\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+210\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+280\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+432\,Aa^{2}\tan\left(\frac{1}{2}\right.\cdots}{\left(\cdots\right.}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/60*(15*(3*A*a^2 + 4*C*a^2)*(d*x + c) + 2*(45*A*a^2*tan(1/2*d*x + 1/2*c)^9
 + 60*C*a^2*tan(1/2*d*x + 1/2*c)^9 + 210*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 280
*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 432*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 560*C*a^
2*tan(1/2*d*x + 1/2*c)^5 + 270*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 520*C*a^2*tan
(1/2*d*x + 1/2*c)^3 + 195*A*a^2*tan(1/2*d*x + 1/2*c) + 180*C*a^2*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.101 $\int \cos^6(c+dx)(a+a\sec(c+dx))^2 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=194

$$-\frac{2a^2(4A+5C)\sin^3(c+dx)}{15d}+\frac{2a^2(4A+5C)\sin(c+dx)}{5d}+\frac{a^2(9A+10C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^2(11A+14C)\text{s}}{}$$

```
[Out] (a^2*(11*A + 14*C)*x)/16 + (2*a^2*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (a^2*(1
1*A + 14*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(9*A + 10*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Sin[c
 + d*x])/(6*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(
15*d) - (2*a^2*(4*A + 5*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.412487, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4087, 4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{2a^2(4A+5C)\sin^3(c+dx)}{15d}+\frac{2a^2(4A+5C)\sin(c+dx)}{5d}+\frac{a^2(9A+10C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^2(11A+14C)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^2*(11*A + 14*C)*x)/16 + (2*a^2*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (a^2*(1
1*A + 14*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(9*A + 10*C)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Sin[c
 + d*x])/(6*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(
15*d) - (2*a^2*(4*A + 5*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d} + \frac{\int \cos^5(c ...}{}$$
$$= \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d} + \frac{A\cos^4(c ...}{}$$
$$= \frac{a^2(9A+10C)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{A\cos^5(c+dx)(a ...}{}$$
$$= \frac{a^2(9A+10C)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{A\cos^5(c+dx)(a ...}{}$$
$$= \frac{a^2(11A+14C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^2(9A+10C)\cos ...}{}$$
$$= \frac{1}{16}a^2(11A+14C)x + \frac{2a^2(4A+5C)\sin(c+dx)}{5d} + \frac{a^2(11A ...}{}$$

Mathematica [A] time = 0.633017, size = 123, normalized size = 0.63

$$\frac{a^2(240(5A+6C)\sin(c+dx)+15(31A+32C)\sin(2(c+dx))+200A\sin(3(c+dx))+75A\sin(4(c+dx))+24A\sin(5(c ...}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(240*A*c + 660*A*d*x + 840*C*d*x + 240*(5*A + 6*C)*Sin[c + d*x] + 15*(
31*A + 32*C)*Sin[2*(c + d*x)] + 200*A*Sin[3*(c + d*x)] + 160*C*Sin[3*(c + d
*x)] + 75*A*Sin[4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 24*A*Sin[5*(c + d*x)
] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.111, size = 211, normalized size = 1.1

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+a^2C\left(\frac{\sin(dx+c)}{4}\left((\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+a^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*
x+3/8*c)+2/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2/3*a^2*C
*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.94507, size = 275, normalized size = 1.42

$$\frac{128\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/960*(128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*A*a^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 30*(12*d*x + 12*c +
 sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 + 240*(2*d*x + 2*c + sin(2*d*
x + 2*c))*C*a^2)/d
```

Fricas [A] time = 0.512383, size = 315, normalized size = 1.62

$$\frac{15(11A+14C)a^2dx+\left(40Aa^2\cos(dx+c)^5+96Aa^2\cos(dx+c)^4+10(11A+6C)a^2\cos(dx+c)^3+32(4A+5C\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(11*A + 14*C)*a^2*d*x + (40*A*a^2*cos(d*x + c)^5 + 96*A*a^2*cos(d
*x + c)^4 + 10*(11*A + 6*C)*a^2*cos(d*x + c)^3 + 32*(4*A + 5*C)*a^2*cos(d*x
 + c)^2 + 15*(11*A + 14*C)*a^2*cos(d*x + c) + 64*(4*A + 5*C)*a^2)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20802, size = 329, normalized size = 1.7

$$15\left(11\,Aa^2+14\,Ca^2\right)(dx+c)+\frac{2\left(165\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+210\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+935\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1190\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1986\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(15*(11*A*a^2 + 14*C*a^2)*(d*x + c) + 2*(165*A*a^2*tan(1/2*d*x + 1/2*
c)^11 + 210*C*a^2*tan(1/2*d*x + 1/2*c)^11 + 935*A*a^2*tan(1/2*d*x + 1/2*c)^
9 + 1190*C*a^2*tan(1/2*d*x + 1/2*c)^9 + 1986*A*a^2*tan(1/2*d*x + 1/2*c)^7 +
 2580*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 3006*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 31
80*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 1305*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 2330*
C*a^2*tan(1/2*d*x + 1/2*c)^3 + 795*A*a^2*tan(1/2*d*x + 1/2*c) + 750*C*a^2*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.102 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=197

$$\frac{a^3(30A+23C)\tan^3(c+dx)}{120d}+\frac{a^3(30A+23C)\tan(c+dx)}{10d}+\frac{a^3(30A+23C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3a^3(30A+23C)}{}$$

```
[Out] (a^3*(30*A + 23*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(30*A + 23*C)*Tan[c
 + d*x])/(10*d) + (3*a^3*(30*A + 23*C)*Sec[c + d*x]*Tan[c + d*x])/(80*d) +
((30*A + 7*C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(120*d) + (C*Sec[c + d*x
]^2*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(6*d) + (C*(a + a*Sec[c + d*x])^4*
Tan[c + d*x])/(10*a*d) + (a^3*(30*A + 23*C)*Tan[c + d*x]^3)/(120*d)
```

Rubi [A] time = 0.416659, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4089, 4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(30A+23C)\tan^3(c+dx)}{120d}+\frac{a^3(30A+23C)\tan(c+dx)}{10d}+\frac{a^3(30A+23C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3a^3(30A+23C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^3*(30*A + 23*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(30*A + 23*C)*Tan[c
 + d*x])/(10*d) + (3*a^3*(30*A + 23*C)*Sec[c + d*x]*Tan[c + d*x])/(80*d) +
((30*A + 7*C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(120*d) + (C*Sec[c + d*x
]^2*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(6*d) + (C*(a + a*Sec[c + d*x])^4*
Tan[c + d*x])/(10*a*d) + (a^3*(30*A + 23*C)*Tan[c + d*x]^3)/(120*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^3\tan(c+dx)}{6d} + \frac{\int \sec^2(c ...}{...}$$
$$= \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^3\tan(c+dx)}{6d} + \frac{C(a+a ...}{...}$$
$$= \frac{(30A+7C)(a+a\sec(c+dx))^3\tan(c+dx)}{120d} + \frac{C\sec^2(c+ ...}{...}$$
$$= \frac{(30A+7C)(a+a\sec(c+dx))^3\tan(c+dx)}{120d} + \frac{C\sec^2(c+ ...}{...}$$
$$= \frac{(30A+7C)(a+a\sec(c+dx))^3\tan(c+dx)}{120d} + \frac{C\sec^2(c+ ...}{...}$$
$$= \frac{a^3(30A+23C)\tanh^{-1}(\sin(c+dx))}{40d} + \frac{3a^3(30A+23C)\mathrm{se} ...}{8 ...}$$
$$= \frac{a^3(30A+23C)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^3(30A+23C)\mathrm{tan} ...}{10d}$$

Mathematica [A] time = 2.78739, size = 387, normalized size = 1.96

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(A\cos^2(c+dx)+C\right)\left(480(30A+23C)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+ ...}{...}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^6*Sec[c
+ d*x]^6*(480*(30*A + 23*C)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(45*A
+ 34*C)*Sin[c] + 30*(38*A + 75*C)*Sin[d*x] + 1140*A*Sin[2*c + d*x] + 2250*C
*Sin[2*c + d*x] + 8160*A*Sin[c + 2*d*x] + 7680*C*Sin[c + 2*d*x] - 2640*A*Si
n[3*c + 2*d*x] - 480*C*Sin[3*c + 2*d*x] + 1590*A*Sin[2*c + 3*d*x] + 1955*C*
Sin[2*c + 3*d*x] + 1590*A*Sin[4*c + 3*d*x] + 1955*C*Sin[4*c + 3*d*x] + 4080
*A*Sin[3*c + 4*d*x] + 3264*C*Sin[3*c + 4*d*x] - 240*A*Sin[5*c + 4*d*x] + 45
0*A*Sin[4*c + 5*d*x] + 345*C*Sin[4*c + 5*d*x] + 450*A*Sin[6*c + 5*d*x] + 34
5*C*Sin[6*c + 5*d*x] + 720*A*Sin[5*c + 6*d*x] + 544*C*Sin[5*c + 6*d*x])))/(
30720*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.06, size = 257, normalized size = 1.3

$$3\frac{Aa^3\tan(dx+c)}{d}+\frac{34\,a^3C\tan(dx+c)}{15\,d}+\frac{17\,a^3C\tan(dx+c)(\sec(dx+c))^2}{15\,d}+\frac{15\,Aa^3\sec(dx+c)\tan(dx+c)}{8\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 3/d*A*a^3*tan(d*x+c)+34/15*a^3*C*tan(d*x+c)/d+17/15/d*a^3*C*tan(d*x+c)*sec(
d*x+c)^2+15/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*A*a^3*ln(sec(d*x+c)+tan(
d*x+c))+23/24/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+23/16/d*a^3*C*sec(d*x+c)*tan(
d*x+c)+23/16/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^3*tan(d*x+c)*sec(d*x
+c)^2+3/5/d*a^3*C*tan(d*x+c)*sec(d*x+c)^4+1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)
^3+1/6/d*a^3*C*tan(d*x+c)*sec(d*x+c)^5
```

Maxima [B] time = 0.960638, size = 516, normalized size = 2.62

$$480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+96\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ca^3+160\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/480*(480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 96*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^3 + 160*(tan(d*x + c)^3 + 3*tan(d
*x + c))*C*a^3 - 5*C*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin
(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 30*A*a^3*(2*(3*sin(d*x
+ c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 90*C*a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) - 360*A*a^3*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*A*a^3*tan(
d*x + c))/d
```

Fricas [A] time = 0.531258, size = 474, normalized size = 2.41

$$\frac{15\,(30\,A+23\,C)a^3\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\,(30\,A+23\,C)a^3\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(16\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/480*(15*(30*A + 23*C)*a^3*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(30*A
 + 23*C)*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(45*A + 34*C)*a^
3*cos(d*x + c)^5 + 15*(30*A + 23*C)*a^3*cos(d*x + c)^4 + 16*(15*A + 17*C)*a
^3*cos(d*x + c)^3 + 10*(6*A + 23*C)*a^3*cos(d*x + c)^2 + 144*C*a^3*cos(d*x
+ c) + 40*C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\sec^2\left(c+dx\right)dx+\int 3A\sec^3\left(c+dx\right)dx+\int 3A\sec^4\left(c+dx\right)dx+\int A\sec^5\left(c+dx\right)dx+\int C\sec^4\left(c+dx\right)dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sec(c + d*x)**3, x) + I
ntegral(3*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral
(C*sec(c + d*x)**4, x) + Integral(3*C*sec(c + d*x)**5, x) + Integral(3*C*se
c(c + d*x)**6, x) + Integral(C*sec(c + d*x)**7, x))
```

Giac [A] time = 1.24728, size = 378, normalized size = 1.92

$$\frac{15\left(30\,Aa^3+23\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(30\,Aa^3+23\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(450\,Aa^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(15*(30*A*a^3 + 23*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(30
*A*a^3 + 23*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(450*A*a^3*tan(1/
2*d*x + 1/2*c)^11 + 345*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 2550*A*a^3*tan(1/2*
d*x + 1/2*c)^9 - 1955*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 5940*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 + 4554*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 7500*A*a^3*tan(1/2*d*x +
1/2*c)^5 - 5814*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 5130*A*a^3*tan(1/2*d*x + 1/2
*c)^3 + 3165*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1470*A*a^3*tan(1/2*d*x + 1/2*c)
 - 1575*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.103 $\int \sec(c+dx)(a+a\sec(c+dx))^3 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=157

$$\frac{a^3(20A+13C)\tan^3(c+dx)}{60d} + \frac{a^3(20A+13C)\tan(c+dx)}{5d} + \frac{a^3(20A+13C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3a^3(20A+13C)}{}$$

```
[Out] (a^3*(20*A + 13*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(20*A + 13*C)*Tan[c
+ d*x])/(5*d) + (3*a^3*(20*A + 13*C)*Sec[c + d*x]*Tan[c + d*x])/(40*d) - (C
*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (C*(a + a*Sec[c + d*x])^4*Ta
n[c + d*x])/(5*a*d) + (a^3*(20*A + 13*C)*Tan[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.255788, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4083, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(20A+13C)\tan^3(c+dx)}{60d} + \frac{a^3(20A+13C)\tan(c+dx)}{5d} + \frac{a^3(20A+13C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3a^3(20A+13C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^3*(20*A + 13*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(20*A + 13*C)*Tan[c
+ d*x])/(5*d) + (3*a^3*(20*A + 13*C)*Sec[c + d*x]*Tan[c + d*x])/(40*d) - (C
*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (C*(a + a*Sec[c + d*x])^4*Ta
n[c + d*x])/(5*a*d) + (a^3*(20*A + 13*C)*Tan[c + d*x]^3)/(60*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{5ad} + \frac{\int \sec(c+dx)(a+a\sec}{}$$
$$= -\frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a+a\sec(c+dx))^4}{5ad}$$
$$= -\frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a+a\sec(c+dx))^4}{5ad}$$
$$= -\frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a+a\sec(c+dx))^4}{5ad}$$
$$= \frac{a^3(20A+13C)\tanh^{-1}(\sin(c+dx))}{20d} + \frac{3a^3(20A+13C)\sec}{40}$$
$$= \frac{a^3(20A+13C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(20A+13C)\tan(}{5d}$$

Mathematica [B] time = 1.99294, size = 323, normalized size = 2.06

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(A\cos^2(c+dx)+C\right)\left(240(20A+13C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^6*Sec[c
+ d*x]^5*(240*(20*A + 13*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(34*A +
29*C)*Sin[d*x] - 240*(7*A + 3*C)*Sin[2*c + d*x] + 360*A*Sin[c + 2*d*x] + 75
0*C*Sin[c + 2*d*x] + 360*A*Sin[3*c + 2*d*x] + 750*C*Sin[3*c + 2*d*x] + 1840
*A*Sin[2*c + 3*d*x] + 1520*C*Sin[2*c + 3*d*x] - 360*A*Sin[4*c + 3*d*x] + 18
0*A*Sin[3*c + 4*d*x] + 195*C*Sin[3*c + 4*d*x] + 180*A*Sin[5*c + 4*d*x] + 19
5*C*Sin[5*c + 4*d*x] + 440*A*Sin[4*c + 5*d*x] + 304*C*Sin[4*c + 5*d*x])))/(
7680*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.066, size = 212, normalized size = 1.4

$$\frac{5\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + \frac{13\,a^3C\sec(dx+c)\tan(dx+c)}{8\,d} + \frac{13\,a^3C\ln(\sec(dx+c)+\tan(dx+c))}{8\,d} + \frac{11\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+13/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+13
/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*A*a^3*tan(d*x+c)+38/15*a^3*C*ta
n(d*x+c)/d+19/15/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+3/2/d*A*a^3*sec(d*x+c)*tan
(d*x+c)+3/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+
c)^2+1/5/d*a^3*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 0.951773, size = 385, normalized size = 2.45

$$80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ca^3+240\left(\tan(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(3*tan(d*x + c)^5 +
10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d*
x + c))*C*a^3 - 45*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 180*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) - 60*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 -
1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*log(sec(d*x
 + c) + tan(d*x + c)) + 720*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.520395, size = 419, normalized size = 2.67

$$15\,(20\,A+13\,C)a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(20\,A+13\,C)a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\,\left(8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(20*A + 13*C)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(20*A
 + 13*C)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(55*A + 38*C)*a^3
*cos(d*x + c)^4 + 15*(12*A + 13*C)*a^3*cos(d*x + c)^3 + 8*(5*A + 19*C)*a^3*
cos(d*x + c)^2 + 90*C*a^3*cos(d*x + c) + 24*C*a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\sec(c+dx)\,dx+\int 3A\sec^2(c+dx)\,dx+\int 3A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int C\sec^3(c+dx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sec(c + d*x)**2, x) + Inte
gral(3*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integral(C*
sec(c + d*x)**3, x) + Integral(3*C*sec(c + d*x)**4, x) + Integral(3*C*sec(c
 + d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.25224, size = 332, normalized size = 2.11

$$\frac{15\left(20\,Aa^3+13\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(20\,Aa^3+13\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(300\,Aa^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(20*A*a^3 + 13*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(20
*A*a^3 + 13*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(300*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 195*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 1400*A*a^3*tan(1/2*d*
x + 1/2*c)^7 - 910*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 2560*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 1664*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 2120*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1330*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 660*A*a^3*tan(1/2*d*x + 1/2*c)
+ 765*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.104 $\int (a+a\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=147

$$\frac{5a^3(4A+3C)\tan(c+dx)}{8d}+\frac{a^3(28A+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4A+5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{8d}+a^3$$

```
[Out] a^3*A*x + (a^3*(28*A + 15*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(4*A + 3
*C)*Tan[c + d*x])/(8*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + (
C*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(4*a*d) + ((4*A + 5*C)*(a^3 + a^
3*Sec[c + d*x])*Tan[c + d*x])/(8*d)
```

Rubi [A] time = 0.218977, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4055, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^3(4A+3C)\tan(c+dx)}{8d}+\frac{a^3(28A+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4A+5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{8d}+a^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a^3*A*x + (a^3*(28*A + 15*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(4*A + 3
*C)*Tan[c + d*x])/(8*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + (
C*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(4*a*d) + ((4*A + 5*C)*(a^3 + a^
3*Sec[c + d*x])*Tan[c + d*x])/(8*d)
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{\int (a+a\sec(c+dx))^3(4aA+3aC\ldots}{4a}$$

$$=\frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{C\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+d\ldots}{4ad}$$

$$=\frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{C\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+d\ldots}{4ad}$$

$$=a^3Ax+\frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{C\left(a^2+a^2\sec(c+dx)\right)^2\ldots}{4ad}$$

$$=a^3Ax+\frac{a^3(28A+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{C(a+a\sec(c+dx))^3\ldots}{4d}$$

$$=a^3Ax+\frac{a^3(28A+15C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^3(4A+3C)\tan(c+\ldots}{8d}$$

Mathematica [B] time = 1.94526, size = 363, normalized size = 2.47

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(A\cos^2(c+dx)+C\right)\left(\sec(c)(4A\sin(2c+dx)+72A\sin(c+2dx)-24\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^6*Sec[c +
 d*x]^4*(-8*(28*A + 15*C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(24*A*d*x*Cos[
c] + 16*A*d*x*Cos[c + 2*d*x] + 16*A*d*x*Cos[3*c + 2*d*x] + 4*A*d*x*Cos[3*c
+ 4*d*x] + 4*A*d*x*Cos[5*c + 4*d*x] - 72*A*Sin[c] - 72*C*Sin[c] + 4*A*Sin[d
*x] + 23*C*Sin[d*x] + 4*A*Sin[2*c + d*x] + 23*C*Sin[2*c + d*x] + 72*A*Sin[c
 + 2*d*x] + 88*C*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] - 8*C*Sin[3*c + 2*d
*x] + 4*A*Sin[2*c + 3*d*x] + 15*C*Sin[2*c + 3*d*x] + 4*A*Sin[4*c + 3*d*x] +
 15*C*Sin[4*c + 3*d*x] + 24*A*Sin[3*c + 4*d*x] + 24*C*Sin[3*c + 4*d*x])))/(
256*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.054, size = 180, normalized size = 1.2

$$a^3Ax+\frac{Aa^3c}{d}+3\,\frac{a^3C\tan(dx+c)}{d}+\frac{7\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{15\,a^3C\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{15\,a^3C\,\mathrm{l}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*x+1/d*A*a^3*c+3*a^3*C*tan(d*x+c)/d+7/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+
c))+15/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c
))+3/d*A*a^3*tan(d*x+c)+1/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*A*a^3*sec(d
*x+c)*tan(d*x+c)+1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 0.94449, size = 338, normalized size = 2.3

$$16\,(dx+c)Aa^3+16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^3-Ca^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*(d*x + c)*A*a^3 + 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - C*a
^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^
2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 4*A*a^3*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) - 12*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) + 48*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 4
8*A*a^3*tan(d*x + c) + 16*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.533892, size = 386, normalized size = 2.63

$$\frac{16\,Aa^3dx\cos(dx+c)^4+(28\,A+15\,C)a^3\cos(dx+c)^4\log(\sin(dx+c)+1)-(28\,A+15\,C)a^3\cos(dx+c)^4\log(-\sin}{16\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(16*A*a^3*d*x*cos(d*x + c)^4 + (28*A + 15*C)*a^3*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - (28*A + 15*C)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) +
 2*(24*(A + C)*a^3*cos(d*x + c)^3 + (4*A + 15*C)*a^3*cos(d*x + c)^2 + 8*C*a
^3*cos(d*x + c) + 2*C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\,dx+\int 3A\sec(c+dx)\,dx+\int 3A\sec^2(c+dx)\,dx+\int A\sec^3(c+dx)\,dx+\int C\sec^2(c+dx)\,dx+\int 3C\,\mathrm{se}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**3*(Integral(A, x) + Integral(3*A*sec(c + d*x), x) + Integral(3*A*sec(c +
 d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**2,
```

```
x) + Integral(3*C*sec(c + d*x)**3, x) + Integral(3*C*sec(c + d*x)**4, x) +
Integral(C*sec(c + d*x)**5, x))
```

Giac [A] time = 1.25785, size = 300, normalized size = 2.04

$$8(dx+c)Aa^3 + \left(28\,Aa^3 + 15\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(28\,Aa^3 + 15\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(20\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)*A*a^3 + (28*A*a^3 + 15*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - (28*A*a^3 + 15*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(20*A
*a^3*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 68*A*a^3*ta
n(1/2*d*x + 1/2*c)^5 - 55*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 76*A*a^3*tan(1/2*d
*x + 1/2*c)^3 + 73*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a^3*tan(1/2*d*x + 1/
2*c) - 49*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.105 $\int \cos(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=145

$$\frac{a^3(6A+5C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A-5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}-\frac{(3A-C)\tan(c+dx)\left(a^2\sec(c+a\right.}{3ad}$$

```
[Out] 3*a^3*A*x + (a^3*(6*A + 5*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*(a + a*Sec[c
 + d*x])^3*Sin[c + d*x])/d + (5*a^3*C*Tan[c + d*x])/(2*d) - ((3*A - C)*(a^2
 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*a*d) - ((6*A - 5*C)*(a^3 + a^3*Sec[
c + d*x])*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.250234, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4087, 3917, 3914, 3767, 8, 3770}

$$\frac{a^3(6A+5C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A-5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}-\frac{(3A-C)\tan(c+dx)\left(a^2\sec(c+a\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] 3*a^3*A*x + (a^3*(6*A + 5*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*(a + a*Sec[c
 + d*x])^3*Sin[c + d*x])/d + (5*a^3*C*Tan[c + d*x])/(2*d) - ((3*A - C)*(a^2
 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*a*d) - ((6*A - 5*C)*(a^3 + a^3*Sec[
c + d*x])*Tan[c + d*x])/(6*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} + \frac{\int (a+a\sec(c+dx))^3(3}{}$$

$$= \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} - \frac{(3A-C)\left(a^2+a^2\sec(\right.}{3a}$$

$$= \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} - \frac{(3A-C)\left(a^2+a^2\sec(\right.}{3a}$$

$$= 3a^3Ax + \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} - \frac{(3A-C)\left(a^2\right.}{}$$

$$= 3a^3Ax + \frac{a^3(6A+5C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{A(a+a\sec(}{}$$

$$= 3a^3Ax + \frac{a^3(6A+5C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{A(a+a\sec(}{}$$

Mathematica [B] time = 6.39511, size = 1250, normalized size = 8.62

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*A*x*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Se
c[c + d*x]^2))/(4*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((-6*A - 5*C)*Cos[c + d
*x]^5*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(8*d*(A + 2*C + A*Cos[2*c + 2*d
*x])) + ((6*A + 5*C)*Cos[c + d*x]^5*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x
)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(
8*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[d*x]*Cos[c + d*x]^5*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sin[c])/(4*d*(A +
2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[c]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*
(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sin[d*x])/(4*d*(A + 2*C + A*C
os[2*c + 2*d*x])) + (C*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d
*x])^3*(A + C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(24*d*(A + 2*C + A*Cos[2*c + 2*
d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) +
(Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c +
d*x]^2)*(5*C*Cos[c/2] - 4*C*Sin[c/2]))/(24*d*(A + 2*C + A*Cos[2*c + 2*d*x])
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[
c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^
2)*(3*A*Sin[(d*x)/2] + 11*C*Sin[(d*x)/2]))/(12*d*(A + 2*C + A*Cos[2*c + 2*d
*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (C*
Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d
```

```
*x]^2)*Sin[(d*x)/2])/(24*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c
/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^5*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*(-5*C*Cos[c/2]
- 4*C*Sin[c/2]))/(24*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c/2])
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^5*Sec[c/2 + (
d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*(3*A*Sin[(d*x)/2] +
 11*C*Sin[(d*x)/2]))/(12*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c
/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.098, size = 152, normalized size = 1.1

$$\frac{Aa^3\sin(dx+c)}{d}+\frac{5a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3a^3Ax+3\frac{Aa^3c}{d}+\frac{11a^3C\tan(dx+c)}{3d}+3\frac{Aa^3\ln(\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*sin(d*x+c)/d+5/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*A*x+3/d*A*a^
3*c+11/3*a^3*C*tan(d*x+c)/d+3/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3*C
*sec(d*x+c)*tan(d*x+c)+1/d*A*a^3*tan(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+
c)^2
```

Maxima [A] time = 0.941689, size = 239, normalized size = 1.65

$$\frac{36(dx+c)Aa^3+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^3-9Ca^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(36*(d*x + c)*A*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 9*C*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 18*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^3*sin(d*x
 + c) + 12*A*a^3*tan(d*x + c) + 36*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.530178, size = 379, normalized size = 2.61

$$\frac{36Aa^3dx\cos(dx+c)^3+3(6A+5C)a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-3(6A+5C)a^3\cos(dx+c)^3\log(-\sin(}{12d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(36*A*a^3*d*x*cos(d*x + c)^3 + 3*(6*A + 5*C)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(6*A + 5*C)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
```

```
 2*(6*A*a^3*cos(d*x + c)^3 + 2*(3*A + 11*C)*a^3*cos(d*x + c)^2 + 9*C*a^3*co
s(d*x + c) + 2*C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24342, size = 296, normalized size = 2.04

$$18\,(dx+c)Aa^3 + \frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 3\left(6\,Aa^3+5\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\left(6\,Aa^3+5\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/6*(18*(d*x + c)*A*a^3 + 12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*
c)^2 + 1) + 3*(6*A*a^3 + 5*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(6
*A*a^3 + 5*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*tan(1/2*d*x + 1/
2*c)^3 - 40*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3
3*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.106 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=162

$$\frac{5a^3(A-C)\sin(c+dx)}{2d}+\frac{a^3(2A+7C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(A-4C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}-\frac{(A-C)\mathrm{s}}{}$$

```
[Out] (a^3*(7*A + 2*C)*x)/2 + (a^3*(2*A + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*
a^3*(A - C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x])^3*Si
n[c + d*x])/(2*d) - ((A - C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*a*
d) - ((A - 4*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.413954, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4087, 4018, 3996, 3770}

$$\frac{5a^3(A-C)\sin(c+dx)}{2d}+\frac{a^3(2A+7C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(A-4C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}-\frac{(A-C)\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(7*A + 2*C)*x)/2 + (a^3*(2*A + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*
a^3*(A - C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x])^3*Si
n[c + d*x])/(2*d) - ((A - C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*a*
d) - ((A - 4*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d} + \frac{\int \cos(c+$$
$$= \frac{A\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d} - \frac{(A-C)(a$$
$$= \frac{A\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d} - \frac{(A-C)(a$$
$$= \frac{5a^3(A-C)\sin(c+dx)}{2d} + \frac{A\cos(c+dx)(a+a\sec(c+dx)}{2d}$$
$$= \frac{1}{2}a^3(7A+2C)x + \frac{5a^3(A-C)\sin(c+dx)}{2d} + \frac{A\cos(c+dx}{}$$
$$= \frac{1}{2}a^3(7A+2C)x + \frac{a^3(2A+7C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a}{}$$

Mathematica [B] time = 4.24472, size = 364, normalized size = 2.25

$$\frac{a^3\cos^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(A+C\sec^2(c+dx)\right)\left(-\frac{2(2A+7C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(2A+7C)}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*Cos[c + d*x]^5*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + C*Sec[c +
d*x]^2)*(2*(7*A + 2*C)*x - (2*(2*A + 7*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]])/d + (2*(2*A + 7*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (
12*A*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])/d + (12*A*Cos[c]*Sin[d*x]
)/d + (A*Cos[2*c]*Sin[2*d*x])/d + C/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^2) + (12*C*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])) - C/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (12*C*Sin
[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
)/(16*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.093, size = 151, normalized size = 0.9

$$\frac{Aa^3\sin(dx+c)\cos(dx+c)}{2d} + \frac{7a^3Ax}{2} + \frac{7Aa^3c}{2d} + a^3Cx + \frac{Ca^3c}{d} + 3\frac{Aa^3\sin(dx+c)}{d} + \frac{7a^3C\ln(\sec(dx+c)+\tan(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^3*sin(d*x+c)*cos(d*x+c)+7/2*a^3*A*x+7/2/d*A*a^3*c+a^3*C*x+1/d*C*a
^3*c+3*a^3*A*sin(d*x+c)/d+7/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*tan
```

```
(d*x+c)/d+1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^3*C*sec(d*x+c)*tan(d*
x+c)
```

Maxima [A] time = 0.945354, size = 236, normalized size = 1.46

$$(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^3 + 12\,(dx + c)Aa^3 + 4\,(dx + c)Ca^3 - Ca^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*A*a^3 + 4*(d*x +
 c)*C*a^3 - C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) + 2*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*
A*a^3*sin(d*x + c) + 12*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.53135, size = 365, normalized size = 2.25

$$\frac{2\,(7\,A + 2\,C)a^3dx\cos(dx + c)^2 + (2\,A + 7\,C)a^3\cos(dx + c)^2\log(\sin(dx + c) + 1) - (2\,A + 7\,C)a^3\cos(dx + c)^2\log(-}{4\,d\cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*(7*A + 2*C)*a^3*d*x*cos(d*x + c)^2 + (2*A + 7*C)*a^3*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (2*A + 7*C)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) +
1) + 2*(A*a^3*cos(d*x + c)^3 + 6*A*a^3*cos(d*x + c)^2 + 6*C*a^3*cos(d*x + c
) + C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30287, size = 311, normalized size = 1.92

$$\left(7\,Aa^3 + 2\,Ca^3\right)(dx + c) + \left(2\,Aa^3 + 7\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(2\,Aa^3 + 7\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/2*((7*A*a^3 + 2*C*a^3)*(d*x + c) + (2*A*a^3 + 7*C*a^3)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) - (2*A*a^3 + 7*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 3*A*a
^3*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^3*tan(1/
2*d*x + 1/2*c)^3 + 9*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^3*tan(1/2*d*x + 1
/2*c) + 7*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.107 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=156

$$-\frac{(5A-6C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{5a^3A\sin(c+dx)}{2d}+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2ad}-$$

```
[Out] (a^3*(5*A + 6*C)*x)/2 + (3*a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*A*Sin[c
+ d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(3*d
) + (A*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*a*d) - ((5*
A - 6*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.396585, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4087, 4017, 4018, 3996, 3770}

$$-\frac{(5A-6C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{5a^3A\sin(c+dx)}{2d}+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2ad}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(5*A + 6*C)*x)/2 + (3*a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*A*Sin[c
+ d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(3*d
) + (A*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*a*d) - ((5*
A - 6*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(6*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{\int \cos^2(c}{}$$

$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{A\cos(c}{}$$

$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{A\cos(c}{}$$

$$= \frac{5a^3A\sin(c+dx)}{2d} + \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin}{3d}$$

$$= \frac{1}{2}a^3(5A+6C)x + \frac{5a^3A\sin(c+dx)}{2d} + \frac{A\cos^2(c+dx)(a+}{}$$

$$= \frac{1}{2}a^3(5A+6C)x + \frac{3a^3C\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3A\sin(c}{2d}$$

Mathematica [B] time = 6.17475, size = 1014, normalized size = 6.5

$$a^3\left(-\frac{3C\cos^2(c+dx)(\cos(c+dx)+1)^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(C\sec^2(c+dx)+A\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d(\cos(2c+2dx)A+A+2C)} + \frac{3C\cos^2(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a^3*(((5*A + 6*C)*x*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^
6*(A + C*Sec[c + d*x]^2))/(8*(A + 2*C + A*Cos[2*c + 2*d*x])) - (3*C*Cos[c +
 d*x]^2*(1 + Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*S
ec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2))/(4*d*(A + 2*C + A*Cos[2*c + 2*d
*x])) + (3*C*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2))/(4*d*(A + 2
*C + A*Cos[2*c + 2*d*x])) + ((15*A + 4*C)*Cos[d*x]*Cos[c + d*x]^2*(1 + Cos[
c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[c])/(16*d*(A +
2*C + A*Cos[2*c + 2*d*x])) + (3*A*Cos[2*d*x]*Cos[c + d*x]^2*(1 + Cos[c + d*
x])^3*Sec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[2*c])/(16*d*(A + 2*C
+ A*Cos[2*c + 2*d*x])) + (A*Cos[3*d*x]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*
Sec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[3*c])/(48*d*(A + 2*C + A*Co
s[2*c + 2*d*x])) + ((15*A + 4*C)*Cos[c]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3
```

```
*Sec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[d*x])/(16*d*(A + 2*C + A*C
os[2*c + 2*d*x])) + (3*A*Cos[2*c]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*Sec[c
/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[2*d*x])/(16*d*(A + 2*C + A*Cos[2
*c + 2*d*x])) + (A*Cos[3*c]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*Sec[c/2 + (
d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[3*d*x])/(48*d*(A + 2*C + A*Cos[2*c + 2
*d*x])) + (C*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(A +
C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(4*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/
2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (C*Cos[c + d*x]
^2*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*Sin[(d*
x)/2])/(4*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])))
```

Maple [A] time = 0.103, size = 146, normalized size = 0.9

$$\frac{A(\cos(dx+c))^2\sin(dx+c)a^3}{3d}+\frac{11Aa^3\sin(dx+c)}{3d}+\frac{a^3C\sin(dx+c)}{d}+\frac{3Aa^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{5a^3Ax}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3+11/3*a^3*A*sin(d*x+c)/d+a^3*C*sin(d*x+c
)/d+3/2/d*A*a^3*sin(d*x+c)*cos(d*x+c)+5/2*a^3*A*x+5/2/d*A*a^3*c+3*a^3*C*x+3
/d*C*a^3*c+3/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+a^3*C*tan(d*x+c)/d
```

Maxima [A] time = 0.944634, size = 185, normalized size = 1.19

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-9(2dx+2c+\sin(2dx+2c))Aa^3-12(dx+c)Aa^3-36(dx+c)Ca^3-18C}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 - 36*(d*x + c)*C*a^3 - 18*C*a^3*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 36*A*a^3*sin(d*x + c) - 12*C*a
^3*sin(d*x + c) - 12*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.530032, size = 350, normalized size = 2.24

$$\frac{3(5A+6C)a^3dx\cos(dx+c)+9Ca^3\cos(dx+c)\log(\sin(dx+c)+1)-9Ca^3\cos(dx+c)\log(-\sin(dx+c)+1)+}{6d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(5*A + 6*C)*a^3*d*x*cos(d*x + c) + 9*C*a^3*cos(d*x + c)*log(sin(d*x
+ c) + 1) - 9*C*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*A*a^3*cos(d*x
```

```
+ c)^3 + 9*A*a^3*cos(d*x + c)^2 + 2*(11*A + 3*C)*a^3*cos(d*x + c) + 6*C*a^3
)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25528, size = 284, normalized size = 1.82

$$\frac{18\,Ca^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,Ca^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+3\left(5\,Aa^3+6\,Ca^3\right)(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(18*C*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*C*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 12*C*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 - 1) + 3*(5*A*a^3 + 6*C*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^
5 + 6*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*C
*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d*x + 1/2*c) + 6*C*a^3*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.108 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=169

$$\frac{5a^3(3A+4C)\sin(c+dx)}{8d}+\frac{(5A+4C)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{8d}+\frac{A\sin(c+dx)\cos^2(c+dx)\left(a^2\right.}{4ad}$$

```
[Out] (a^3*(15*A + 28*C)*x)/8 + (a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(3*A + 4
*C)*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c +
d*x])/(4*d) + (A*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(4
*a*d) + ((5*A + 4*C)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(8
*d)
```

Rubi [A] time = 0.410138, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4087, 4017, 3996, 3770}

$$\frac{5a^3(3A+4C)\sin(c+dx)}{8d}+\frac{(5A+4C)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{8d}+\frac{A\sin(c+dx)\cos^2(c+dx)\left(a^2\right.}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^3*(15*A + 28*C)*x)/8 + (a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(3*A + 4
*C)*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c +
d*x])/(4*d) + (A*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(4
*a*d) + ((5*A + 4*C)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(8
*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{\int \cos^3(c$$

$$= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{A\cos^2(c$$

$$= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d} + \frac{A\cos^2(c$$

$$= \frac{5a^3(3A+4C)\sin(c+dx)}{8d} + \frac{A\cos^3(c+dx)(a+a\sec(c+}{4d}$$

$$= \frac{1}{8}a^3(15A+28C)x + \frac{5a^3(3A+4C)\sin(c+dx)}{8d} + \frac{A\cos^3(c$$

$$= \frac{1}{8}a^3(15A+28C)x + \frac{a^3C\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3(3A+$$

Mathematica [A] time = 0.304805, size = 124, normalized size = 0.73

$$\frac{a^3\left(8(13A+12C)\sin(c+dx)+8(4A+C)\sin(2(c+dx))+8A\sin(3(c+dx))+A\sin(4(c+dx))+60Adx-32C\log\left(\text{co}\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(60*A*d*x + 112*C*d*x - 32*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 32*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*(13*A + 12*C)*Sin[c + d
*x] + 8*(4*A + C)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + A*Sin[4*(c + d*
x)]))/(32*d)
```

Maple [A] time = 0.095, size = 175, normalized size = 1.

$$\frac{Aa^3\sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{15Aa^3\sin(dx+c)\cos(dx+c)}{8d} + \frac{15a^3Ax}{8} + \frac{15Aa^3c}{8d} + \frac{a^3C\sin(dx+c)\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/4/d*A*a^3*sin(d*x+c)*cos(d*x+c)^3+15/8/d*A*a^3*sin(d*x+c)*cos(d*x+c)+15/8
*a^3*A*x+15/8/d*A*a^3*c+1/2/d*a^3*C*sin(d*x+c)*cos(d*x+c)+7/2*a^3*C*x+7/2/d
*C*a^3*c+1/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3+3*a^3*A*sin(d*x+c)/d+3*a^3*C*sin
(d*x+c)/d+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.950342, size = 231, normalized size = 1.37

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-24\left(2\,dx+2\,c+\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - (12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*A*a^3 - 8*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 - 96*(d*x + c)*C*a^3 - 1
6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 32*A*a^3*sin(d*x
+ c) - 96*C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.532458, size = 284, normalized size = 1.68

$$\frac{(15\,A+28\,C)a^{3}dx+4\,Ca^{3}\log\left(\sin\left(dx+c\right)+1\right)-4\,Ca^{3}\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Aa^{3}\cos\left(dx+c\right)^{3}+8\,Aa^{3}\cos\left(d\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/8*((15*A + 28*C)*a^3*d*x + 4*C*a^3*log(sin(d*x + c) + 1) - 4*C*a^3*log(-s
in(d*x + c) + 1) + (2*A*a^3*cos(d*x + c)^3 + 8*A*a^3*cos(d*x + c)^2 + (15*A
 + 4*C)*a^3*cos(d*x + c) + 24*(A + C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29159, size = 288, normalized size = 1.7

$$8\,Ca^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,Ca^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(15\,Aa^{3}+28\,Ca^{3}\right)(dx+c)+\frac{2\left(15\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/8*(8*C*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*C*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (15*A*a^3 + 28*C*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d
*x + 1/2*c)^7 + 20*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 55*A*a^3*tan(1/2*d*x + 1/
2*c)^5 + 68*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 73*A*a^3*tan(1/2*d*x + 1/2*c)^3
+ 76*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*A*a^3*tan(1/2*d*x + 1/2*c) + 28*C*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.109 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=161

$$-\frac{a^3(13A+20C)\sin^3(c+dx)}{60d}+\frac{a^3(13A+20C)\sin(c+dx)}{5d}+\frac{3a^3(13A+20C)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(13A+$$

```
[Out] (a^3*(13*A + 20*C)*x)/8 + (a^3*(13*A + 20*C)*Sin[c + d*x])/(5*d) + (3*a^3*(
13*A + 20*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (3*A*Cos[c + d*x]^3*(a + a
*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(5*d) - (a^3*(13*A + 20*C)*Sin[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.329812, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4087, 4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{a^3(13A+20C)\sin^3(c+dx)}{60d}+\frac{a^3(13A+20C)\sin(c+dx)}{5d}+\frac{3a^3(13A+20C)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(13A+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(13*A + 20*C)*x)/8 + (a^3*(13*A + 20*C)*Sin[c + d*x])/(5*d) + (3*a^3*(
13*A + 20*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (3*A*Cos[c + d*x]^3*(a + a
*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(5*d) - (a^3*(13*A + 20*C)*Sin[c + d*x]^3)/(60*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx &= \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{5d}+\frac{\int\cos^4(c\\ &= \frac{3A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{20d}+\frac{A\cos^4(\\ &= \frac{3A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{20d}+\frac{A\cos^4(\\ &= \frac{1}{20}a^3(13A+20C)x+\frac{3A\cos^3(c+dx)(a+a\sec(c+dx))^3}{20d}\\ &= \frac{1}{20}a^3(13A+20C)x+\frac{3a^3(13A+20C)\sin(c+dx)}{20d}+\frac{3a^3(\\ &= \frac{1}{8}a^3(13A+20C)x+\frac{a^3(13A+20C)\sin(c+dx)}{5d}+\frac{3a^3(13\end{aligned}$$

Mathematica [A] time = 0.3213, size = 97, normalized size = 0.6

$$\frac{a^3(60(23A+30C)\sin(c+dx)+120(4A+3C)\sin(2(c+dx))+170A\sin(3(c+dx))+45A\sin(4(c+dx))+6A\sin(5(c+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(780*A*d*x + 1200*C*d*x + 60*(23*A + 30*C)*Sin[c + d*x] + 120*(4*A + 3
*C)*Sin[2*(c + d*x)] + 170*A*Sin[3*(c + d*x)] + 40*C*Sin[3*(c + d*x)] + 45*
A*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.101, size = 197, normalized size = 1.2

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3Aa^3\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^3*(2+cos(d*x+c)
^2)*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(1/2*cos(d*x+c)*
sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
+3*a^3*C*sin(d*x+c)+a^3*C*(d*x+c))
```

Maxima [A] time = 0.944548, size = 257, normalized size = 1.6

$$32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{3}-480\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}+45\,(12\,dx+12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 45*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 360*(2*d*x + 2*c + si
n(2*d*x + 2*c))*C*a^3 + 480*(d*x + c)*C*a^3 + 1440*C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.503565, size = 266, normalized size = 1.65

$$\frac{15\,(13\,A+20\,C)a^{3}dx+\left(24\,Aa^{3}\cos\left(dx+c\right)^{4}+90\,Aa^{3}\cos\left(dx+c\right)^{3}+8\,(19\,A+5\,C)a^{3}\cos\left(dx+c\right)^{2}+15\,(13\,A+12}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(13*A + 20*C)*a^3*d*x + (24*A*a^3*cos(d*x + c)^4 + 90*A*a^3*cos(d
*x + c)^3 + 8*(19*A + 5*C)*a^3*cos(d*x + c)^2 + 15*(13*A + 12*C)*a^3*cos(d*
x + c) + 8*(38*A + 55*C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2349, size = 284, normalized size = 1.76

$$\frac{15\left(13\,Aa^3+20\,Ca^3\right)(dx+c)+\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+300\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1400\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,Aa^3\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 20*C*a^3)*(d*x + c) + 2*(195*A*a^3*tan(1/2*d*x + 1/2*
c)^9 + 300*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 1400*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 2
560*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2120
*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c) + 660*C*a^3*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.110 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=216

$$\frac{a^3(34A+45C)\sin(c+dx)}{15d}+\frac{a^3(73A+90C)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{a^3(23A+30C)\sin(c+dx)\cos(c+dx)}{16d}+\frac{(3}{}$$

```
[Out] (a^3*(23*A + 30*C)*x)/16 + (a^3*(34*A + 45*C)*Sin[c + d*x])/(15*d) + (a^3*(
23*A + 30*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(73*A + 90*C)*Cos[c +
 d*x]^2*Sin[c + d*x])/(120*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*Si
n[c + d*x])/(6*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*
x])/(10*a*d) + ((31*A + 30*C)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin[c
 + d*x])/(120*d)
```

Rubi [A] time = 0.550766, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4087, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^3(34A+45C)\sin(c+dx)}{15d}+\frac{a^3(73A+90C)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{a^3(23A+30C)\sin(c+dx)\cos(c+dx)}{16d}+\frac{(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(23*A + 30*C)*x)/16 + (a^3*(34*A + 45*C)*Sin[c + d*x])/(15*d) + (a^3*(
23*A + 30*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(73*A + 90*C)*Cos[c +
 d*x]^2*Sin[c + d*x])/(120*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*Si
n[c + d*x])/(6*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*
x])/(10*a*d) + ((31*A + 30*C)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin[c
 + d*x])/(120*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{6d} + \frac{\int \cos^5(c$$

$$= \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{6d} + \frac{A\cos^4(c$$

$$= \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{6d} + \frac{A\cos^4(c$$

$$= \frac{a^3(73A+90C)\cos^2(c+dx)\sin(c+dx)}{120d} + \frac{A\cos^5(c+dx)$$

$$= \frac{a^3(73A+90C)\cos^2(c+dx)\sin(c+dx)}{120d} + \frac{A\cos^5(c+dx)$$

$$= \frac{a^3(34A+45C)\sin(c+dx)}{15d} + \frac{a^3(23A+30C)\cos(c+dx)}{16d}$$

$$= \frac{1}{16}a^3(23A+30C)x + \frac{a^3(34A+45C)\sin(c+dx)}{15d} + \frac{a^3(23$$

Mathematica [A] time = 0.456372, size = 123, normalized size = 0.57

$$\frac{a^3(120(21A+26C)\sin(c+dx)+15(63A+64C)\sin(2(c+dx))+380A\sin(3(c+dx))+135A\sin(4(c+dx))+36A\sin(}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(900*A*c + 1380*A*d*x + 1800*C*d*x + 120*(21*A + 26*C)*Sin[c + d*x] +
15*(63*A + 64*C)*Sin[2*(c + d*x)] + 380*A*Sin[3*(c + d*x)] + 240*C*Sin[3*(c
```

```
+ d*x)] + 135*A*Sin[4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 36*A*Sin[5*(c +
d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.134, size = 245, normalized size = 1.1

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+a^3C\left(\frac{\sin(dx+c)}{4}\left((\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*
x+3/8*c)+3/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(2+
cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)+3/8*d*x+3/8*c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*A*
a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*C*sin(d*x+c))
```

Maxima [A] time = 0.956301, size = 323, normalized size = 1.5

$$192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 960*(sin(d*x + c)^3
 - 3*sin(d*x + c))*C*a^3 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*C*a^3 + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 960*C*a^3*s
in(d*x + c))/d
```

Fricas [A] time = 0.514645, size = 321, normalized size = 1.49

$$\frac{15(23A+30C)a^3dx+\left(40Aa^3\cos(dx+c)^5+144Aa^3\cos(dx+c)^4+10(23A+6C)a^3\cos(dx+c)^3+16(17A+1\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*(23*A + 30*C)*a^3*d*x + (40*A*a^3*cos(d*x + c)^5 + 144*A*a^3*cos(
d*x + c)^4 + 10*(23*A + 6*C)*a^3*cos(d*x + c)^3 + 16*(17*A + 15*C)*a^3*cos(
d*x + c)^2 + 15*(23*A + 30*C)*a^3*cos(d*x + c) + 16*(34*A + 45*C)*a^3)*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26057, size = 329, normalized size = 1.52

$$15\left(23\,Aa^3+30\,Ca^3\right)(dx+c)+\frac{2\left(345\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+450\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2550\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4554\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(15*(23*A*a^3 + 30*C*a^3)*(d*x + c) + 2*(345*A*a^3*tan(1/2*d*x + 1/2*
c)^11 + 450*C*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*A*a^3*tan(1/2*d*x + 1/2*c)
^9 + 2550*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 5940*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 7
500*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 5130
*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^3*tan(1/2*d*x + 1/2*c) + 1470*C*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.111 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=228

$$\frac{8a^4(14A+11C)\tan^3(c+dx)}{105d}+\frac{16a^4(14A+11C)\tan(c+dx)}{35d}+\frac{a^4(14A+11C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(14A+11C$$

```
[Out] (a^4*(14*A + 11*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (16*a^4*(14*A + 11*C)*Tan
[c + d*x])/(35*d) + (27*a^4*(14*A + 11*C)*Sec[c + d*x]*Tan[c + d*x])/(140*d
) + (a^4*(14*A + 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(70*d) + ((21*A + 4*C)*
(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(105*d) + (C*Sec[c + d*x]^2*(a + a*Sec
[c + d*x])^4*Tan[c + d*x])/(7*d) + (2*C*(a + a*Sec[c + d*x])^5*Tan[c + d*x]
)/(21*a*d) + (8*a^4*(14*A + 11*C)*Tan[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.481239, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4089, 4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{8a^4(14A+11C)\tan^3(c+dx)}{105d}+\frac{16a^4(14A+11C)\tan(c+dx)}{35d}+\frac{a^4(14A+11C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(14A+11C$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^4*(14*A + 11*C)*ArcTanh[Sin[c + d*x]])/(4*d) + (16*a^4*(14*A + 11*C)*Tan
[c + d*x])/(35*d) + (27*a^4*(14*A + 11*C)*Sec[c + d*x]*Tan[c + d*x])/(140*d
) + (a^4*(14*A + 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(70*d) + ((21*A + 4*C)*
(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(105*d) + (C*Sec[c + d*x]^2*(a + a*Sec
[c + d*x])^4*Tan[c + d*x])/(7*d) + (2*C*(a + a*Sec[c + d*x])^5*Tan[c + d*x]
)/(21*a*d) + (8*a^4*(14*A + 11*C)*Tan[c + d*x]^3)/(105*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
```

```
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^4\tan(c+dx)}{7d}+\frac{\int \sec^2(c}{}$$

$$= \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^4\tan(c+dx)}{7d}+\frac{2C(a+a}{}$$

$$= \frac{(21A+4C)(a+a\sec(c+dx))^4\tan(c+dx)}{105d}+\frac{C\sec^2(c+}{}$$

$$= \frac{(21A+4C)(a+a\sec(c+dx))^4\tan(c+dx)}{105d}+\frac{C\sec^2(c+}{}$$

$$= \frac{(21A+4C)(a+a\sec(c+dx))^4\tan(c+dx)}{105d}+\frac{C\sec^2(c+}{}$$

$$= \frac{2a^4(14A+11C)\tanh^{-1}(\sin(c+dx))}{35d}+\frac{6a^4(14A+11C)\mathrm{se}}{}$$

$$= \frac{8a^4(14A+11C)\tanh^{-1}(\sin(c+dx))}{35d}+\frac{16a^4(14A+11C)\mathrm{t}}{35d}$$

$$= \frac{a^4(14A+11C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{16a^4(14A+11C)\mathrm{ta}}{35d}$$

Mathematica [A] time = 4.9479, size = 419, normalized size = 1.84

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(A\cos^2(c+dx)+C\right)\left(6720(14A+11C)\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^8*Sec[c
+ d*x]^7*(6720*(14*A + 11*C)*Cos[c + d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(560*(91*A
+ 83*C)*Sin[d*x] - 140*(217*A + 122*C)*Sin[2*c + d*x] + 10710*A*Sin[c + 2*d
*x] + 16415*C*Sin[c + 2*d*x] + 10710*A*Sin[3*c + 2*d*x] + 16415*C*Sin[3*c +
 2*d*x] + 41244*A*Sin[2*c + 3*d*x] + 37296*C*Sin[2*c + 3*d*x] - 7560*A*Sin[
4*c + 3*d*x] - 840*C*Sin[4*c + 3*d*x] + 7560*A*Sin[3*c + 4*d*x] + 7700*C*Si
n[3*c + 4*d*x] + 7560*A*Sin[5*c + 4*d*x] + 7700*C*Sin[5*c + 4*d*x] + 15848*
A*Sin[4*c + 5*d*x] + 12712*C*Sin[4*c + 5*d*x] - 420*A*Sin[6*c + 5*d*x] + 14
70*A*Sin[5*c + 6*d*x] + 1155*C*Sin[5*c + 6*d*x] + 1470*A*Sin[7*c + 6*d*x] +
 1155*C*Sin[7*c + 6*d*x] + 2324*A*Sin[6*c + 7*d*x] + 1816*C*Sin[6*c + 7*d*x
])))/(215040*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.065, size = 303, normalized size = 1.3

$$\frac{83\,Aa^4\tan(dx+c)}{15\,d}+\frac{454\,a^4C\tan(dx+c)}{105\,d}+\frac{227\,a^4C\tan(dx+c)(\sec(dx+c))^2}{105\,d}+\frac{7\,Aa^4\sec(dx+c)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 83/15/d*A*a^4*tan(d*x+c)+454/105/d*a^4*C*tan(d*x+c)+227/105/d*a^4*C*tan(d*x
+c)*sec(d*x+c)^2+7/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+7/2/d*A*a^4*ln(sec(d*x+c
)+tan(d*x+c))+11/6/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+11/4/d*a^4*C*sec(d*x+c)*
tan(d*x+c)+11/4/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+34/15/d*A*a^4*tan(d*x+c)*
sec(d*x+c)^2+48/35/d*a^4*C*tan(d*x+c)*sec(d*x+c)^4+1/d*A*a^4*tan(d*x+c)*sec
(d*x+c)^3+2/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^5+1/5/d*A*a^4*tan(d*x+c)*sec(d*
x+c)^4+1/7/d*a^4*C*tan(d*x+c)*sec(d*x+c)^6
```

Maxima [B] time = 0.965796, size = 624, normalized size = 2.74

$$56\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+1680\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+24\left(5\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/840*(56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
1680*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 24*(5*tan(d*x + c)^7 + 21*ta
n(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*C*a^4 + 336*(3*tan(d*x
+ c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^4 + 280*(tan(d*x + c)^3 +
 3*tan(d*x + c))*C*a^4 - 35*C*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3
 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 -
 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 210*A*a^4*(2*(
3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1)
- 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 210*C*a^4*(2*(3*sin(
```

```
d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*A*a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 840
*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.542161, size = 531, normalized size = 2.33

$$\frac{105\,(14\,A+11\,C)a^4\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\,(14\,A+11\,C)a^4\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/840*(105*(14*A + 11*C)*a^4*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(14
*A + 11*C)*a^4*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 2*(4*(581*A + 454*C)
*a^4*cos(d*x + c)^6 + 105*(14*A + 11*C)*a^4*cos(d*x + c)^5 + 4*(238*A + 227
*C)*a^4*cos(d*x + c)^4 + 70*(6*A + 11*C)*a^4*cos(d*x + c)^3 + 12*(7*A + 48*
C)*a^4*cos(d*x + c)^2 + 280*C*a^4*cos(d*x + c) + 60*C*a^4)*sin(d*x + c))/(d
*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int A\sec^2\left(c+dx\right)dx+\int 4A\sec^3\left(c+dx\right)dx+\int 6A\sec^4\left(c+dx\right)dx+\int 4A\sec^5\left(c+dx\right)dx+\int A\sec^6\left(c+dx\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + I
ntegral(6*A*sec(c + d*x)**4, x) + Integral(4*A*sec(c + d*x)**5, x) + Integr
al(A*sec(c + d*x)**6, x) + Integral(C*sec(c + d*x)**4, x) + Integral(4*C*se
c(c + d*x)**5, x) + Integral(6*C*sec(c + d*x)**6, x) + Integral(4*C*sec(c +
 d*x)**7, x) + Integral(C*sec(c + d*x)**8, x))
```

Giac [A] time = 1.26657, size = 424, normalized size = 1.86

$$\frac{105\left(14\,Aa^4+11\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(14\,Aa^4+11\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(1470\,Aa^4\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/420*(105*(14*A*a^4 + 11*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(
14*A*a^4 + 11*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(1470*A*a^4*tan
(1/2*d*x + 1/2*c)^13 + 1155*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 9800*A*a^4*tan(
```

```
1/2*d*x + 1/2*c)^11 - 7700*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 27734*A*a^4*tan(
1/2*d*x + 1/2*c)^9 + 21791*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 43008*A*a^4*tan(1
/2*d*x + 1/2*c)^7 - 33792*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 39914*A*a^4*tan(1/
2*d*x + 1/2*c)^5 + 31521*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 21560*A*a^4*tan(1/2
*d*x + 1/2*c)^3 - 14700*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 5250*A*a^4*tan(1/2*d
*x + 1/2*c) + 5565*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^7)/d
```

3.112 $\int \sec(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=188

$$\frac{2a^4(10A+7C)\tan^3(c+dx)}{15d}+\frac{4a^4(10A+7C)\tan(c+dx)}{5d}+\frac{7a^4(10A+7C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(10A+7C)\tan(c}{40}$$

```
[Out] (7*a^4*(10*A + 7*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(10*A + 7*C)*Tan
[c + d*x])/(5*d) + (27*a^4*(10*A + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(80*d) +
 (a^4*(10*A + 7*C)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) - (C*(a + a*Sec[c +
d*x])^4*Tan[c + d*x])/(30*d) + (C*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/(6*a
*d) + (2*a^4*(10*A + 7*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.302775, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4083, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{2a^4(10A+7C)\tan^3(c+dx)}{15d}+\frac{4a^4(10A+7C)\tan(c+dx)}{5d}+\frac{7a^4(10A+7C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(10A+7C)\tan(c}{40}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] (7*a^4*(10*A + 7*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(10*A + 7*C)*Tan
[c + d*x])/(5*d) + (27*a^4*(10*A + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(80*d) +
 (a^4*(10*A + 7*C)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) - (C*(a + a*Sec[c +
d*x])^4*Tan[c + d*x])/(30*d) + (C*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/(6*a
*d) + (2*a^4*(10*A + 7*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx &= \frac{C(a+a\sec(c+dx))^5\tan(c+dx)}{6ad}+\frac{\int \sec(c+dx)(a+a\\ &= -\frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{30d}+\frac{C(a+a\sec(c+d}{6a}\\ &= -\frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{30d}+\frac{C(a+a\sec(c+d}{6a}\\ &= -\frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{30d}+\frac{C(a+a\sec(c+d}{6a}\\ &= \frac{a^4(10A+7C)\tanh^{-1}(\sin(c+dx))}{10d}+\frac{3a^4(10A+7C)\sec}{1}\\ &= \frac{2a^4(10A+7C)\tanh^{-1}(\sin(c+dx))}{5d}+\frac{4a^4(10A+7C)\,t}{5d}\\ &= \frac{7a^4(10A+7C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^4(10A+7C)\,t}{5d}\end{aligned}$$

Mathematica [B] time = 3.45619, size = 387, normalized size = 2.06

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(A\cos^2(c+dx)+C\right)\left(3360(10A+7C)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^8*Sec[c
+ d*x]^6*(3360*(10*A + 7*C)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-640*(25*A
+ 18*C)*Sin[c] + 30*(62*A + 125*C)*Sin[d*x] + 1860*A*Sin[2*c + d*x] + 3750*
C*Sin[2*c + d*x] + 17280*A*Sin[c + 2*d*x] + 15360*C*Sin[c + 2*d*x] - 6720*A
*Sin[3*c + 2*d*x] - 1920*C*Sin[3*c + 2*d*x] + 2670*A*Sin[2*c + 3*d*x] + 384
5*C*Sin[2*c + 3*d*x] + 2670*A*Sin[4*c + 3*d*x] + 3845*C*Sin[4*c + 3*d*x] +
8640*A*Sin[3*c + 4*d*x] + 6912*C*Sin[3*c + 4*d*x] - 960*A*Sin[5*c + 4*d*x]
+ 810*A*Sin[4*c + 5*d*x] + 735*C*Sin[4*c + 5*d*x] + 810*A*Sin[6*c + 5*d*x]
```

```
+ 735*C*Sin[6*c + 5*d*x] + 1600*A*Sin[5*c + 6*d*x] + 1152*C*Sin[5*c + 6*d*x
])))/(61440*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.072, size = 258, normalized size = 1.4

$$\frac{35\,Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{49\,a^4C\sec(dx+c)\tan(dx+c)}{16d}+\frac{49\,a^4C\ln(\sec(dx+c)+\tan(dx+c))}{16d}+\frac{20}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 35/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+49/16/d*a^4*C*sec(d*x+c)*tan(d*x+c)+
49/16/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*A*a^4*tan(d*x+c)+24/5/d*a^4*
C*tan(d*x+c)+12/5/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+27/8/d*A*a^4*sec(d*x+c)*t
an(d*x+c)+41/24/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+4/3/d*A*a^4*tan(d*x+c)*sec(
d*x+c)^2+4/5/d*a^4*C*tan(d*x+c)*sec(d*x+c)^4+1/4/d*A*a^4*tan(d*x+c)*sec(d*x
+c)^3+1/6/d*a^4*C*tan(d*x+c)*sec(d*x+c)^5
```

Maxima [B] time = 0.964257, size = 606, normalized size = 3.22

$$\frac{640\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+128\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Ca^4+640\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 128*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^4 + 640*(tan(d*x + c)^3 + 3*tan(
d*x + c))*C*a^4 - 5*C*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*si
n(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15
*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 30*A*a^4*(2*(3*sin(d*x
 + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(s
in(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*C*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 720*A*a^4*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*C*a^4*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 480*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 1920*A*a^4*tan(d*x
+ c))/d
```

Fricas [A] time = 0.533052, size = 471, normalized size = 2.51

$$\frac{105\,(10\,A+7\,C)a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-105\,(10\,A+7\,C)a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\left(64\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*(10*A + 7*C)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(10*
A + 7*C)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(64*(25*A + 18*C)*a^
4*cos(d*x + c)^5 + 15*(54*A + 49*C)*a^4*cos(d*x + c)^4 + 64*(5*A + 9*C)*a^4
*cos(d*x + c)^3 + 10*(6*A + 41*C)*a^4*cos(d*x + c)^2 + 192*C*a^4*cos(d*x +
c) + 40*C*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int A\sec(c+dx)\,dx+\int 4A\sec^2(c+dx)\,dx+\int 6A\sec^3(c+dx)\,dx+\int 4A\sec^4(c+dx)\,dx+\int A\sec^5(c+dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*sec(c + d*x)**2, x) + Inte
gral(6*A*sec(c + d*x)**3, x) + Integral(4*A*sec(c + d*x)**4, x) + Integral(
A*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**3, x) + Integral(4*C*sec(c
 + d*x)**4, x) + Integral(6*C*sec(c + d*x)**5, x) + Integral(4*C*sec(c + d*
x)**6, x) + Integral(C*sec(c + d*x)**7, x))
```

Giac [A] time = 1.23798, size = 378, normalized size = 2.01

$$\frac{105\left(10\,Aa^4+7\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(10\,Aa^4+7\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(1050\,Aa^4\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(10*A*a^4 + 7*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(1
0*A*a^4 + 7*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(1050*A*a^4*tan(1
/2*d*x + 1/2*c)^11 + 735*C*a^4*tan(1/2*d*x + 1/2*c)^11 - 5950*A*a^4*tan(1/2
*d*x + 1/2*c)^9 - 4165*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 13860*A*a^4*tan(1/2*d
*x + 1/2*c)^7 + 9702*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 16860*A*a^4*tan(1/2*d*x
 + 1/2*c)^5 - 11802*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 10690*A*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 7355*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 2790*A*a^4*tan(1/2*d*x + 1
/2*c) - 3105*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.113 $\int (a+a\sec(c+dx))^4 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=177

$$\frac{a^4(10A+7C)\tan(c+dx)}{2d}+\frac{a^4(12A+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+7C)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{15d}+\frac{(8A+$$

```
[Out] a^4*A*x + (a^4*(12*A + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*(10*A + 7*C
)*Tan[c + d*x])/(2*d) + (a*C*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + (
C*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(5*d) + ((5*A + 7*C)*(a^2 + a^2*Sec[
c + d*x])^2*Tan[c + d*x])/(15*d) + ((8*A + 7*C)*(a^4 + a^4*Sec[c + d*x])*Ta
n[c + d*x])/(6*d)
```

Rubi [A] time = 0.294305, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4055, 3917, 3914, 3767, 8, 3770}

$$\frac{a^4(10A+7C)\tan(c+dx)}{2d}+\frac{a^4(12A+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+7C)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{15d}+\frac{(8A+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] a^4*A*x + (a^4*(12*A + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*(10*A + 7*C
)*Tan[c + d*x])/(2*d) + (a*C*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + (
C*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(5*d) + ((5*A + 7*C)*(a^2 + a^2*Sec[
c + d*x])^2*Tan[c + d*x])/(15*d) + ((8*A + 7*C)*(a^4 + a^4*Sec[c + d*x])*Ta
n[c + d*x])/(6*d)
```


Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx=\frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{\int (a+a\sec(c+dx))^4(5aA+4a}{5a}$$

$$=\frac{aC(a+a\sec(c+dx))^3\tan(c+dx)}{5d}+\frac{C(a+a\sec(c+dx))^4\tan(c+}{5d}$$

$$=\frac{aC(a+a\sec(c+dx))^3\tan(c+dx)}{5d}+\frac{C(a+a\sec(c+dx))^4\tan(c+}{5d}$$

$$=\frac{aC(a+a\sec(c+dx))^3\tan(c+dx)}{5d}+\frac{C(a+a\sec(c+dx))^4\tan(c+}{5d}$$

$$=a^4Ax+\frac{aC(a+a\sec(c+dx))^3\tan(c+dx)}{5d}+\frac{C(a+a\sec(c+dx))^4}{5d}$$

$$=a^4Ax+\frac{a^4(12A+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC(a+a\sec(c+dx))}{5d}$$

$$=a^4Ax+\frac{a^4(12A+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4(10A+7C)\tan(c+}{2d}$$

Mathematica [B] time = 2.9184, size = 418, normalized size = 2.36

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(A\cos^2(c+dx)+C\right)\left(\sec(c)(-780A\sin(2c+dx)+120A\sin(c+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^8*Sec[c +
 d*x]^5*(-240*(12*A + 7*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(150*A*d*x*Co
s[d*x] + 150*A*d*x*Cos[2*c + d*x] + 75*A*d*x*Cos[2*c + 3*d*x] + 75*A*d*x*Co
s[4*c + 3*d*x] + 15*A*d*x*Cos[4*c + 5*d*x] + 15*A*d*x*Cos[6*c + 5*d*x] + 12
20*A*Sin[d*x] + 1180*C*Sin[d*x] - 780*A*Sin[2*c + d*x] - 480*C*Sin[2*c + d*
x] + 120*A*Sin[c + 2*d*x] + 330*C*Sin[c + 2*d*x] + 120*A*Sin[3*c + 2*d*x] +
 330*C*Sin[3*c + 2*d*x] + 820*A*Sin[2*c + 3*d*x] + 800*C*Sin[2*c + 3*d*x] -
 180*A*Sin[4*c + 3*d*x] - 30*C*Sin[4*c + 3*d*x] + 60*A*Sin[3*c + 4*d*x] + 1
05*C*Sin[3*c + 4*d*x] + 60*A*Sin[5*c + 4*d*x] + 105*C*Sin[5*c + 4*d*x] + 20
0*A*Sin[4*c + 5*d*x] + 166*C*Sin[4*c + 5*d*x])))/(3840*d*(A + 2*C + A*Cos[2
*(c + d*x)]))
```

Maple [A] time = 0.066, size = 226, normalized size = 1.3

$$a^4Ax+\frac{Aa^4c}{d}+\frac{83\,a^4C\tan(dx+c)}{15\,d}+6\,\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{7\,a^4C\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{7\,a^4C\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^4*A*x+1/d*A*a^4*c+83/15/d*a^4*C*tan(d*x+c)+6/d*A*a^4*ln(sec(d*x+c)+tan(d*
x+c))+7/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+7/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c
))+20/3/d*A*a^4*tan(d*x+c)+34/15/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+2/d*A*a^4*
sec(d*x+c)*tan(d*x+c)+1/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+1/3/d*A*a^4*tan(d*x
+c)*sec(d*x+c)^2+1/5/d*a^4*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 0.955568, size = 416, normalized size = 2.35

$$\frac{20\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+60\,(dx+c)Aa^4+4\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Ca^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(20*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 60*(d*x + c)*A*a^4 + 4*(
3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^4 + 120*(tan(d*
x + c)^3 + 3*tan(d*x + c))*C*a^4 - 15*C*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*
x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) +
 3*log(sin(d*x + c) - 1)) - 60*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
 log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*C*a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240
*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 360*A*a^4*tan(d*x + c) + 60*C*a^4
*tan(d*x + c))/d
```

Fricas [A] time = 0.539463, size = 452, normalized size = 2.55

$$\frac{60\,Aa^4dx\cos(dx+c)^5+15\,(12\,A+7\,C)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(12\,A+7\,C)a^4\cos(dx+c)^5\log(-\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(60*A*a^4*d*x*cos(d*x + c)^5 + 15*(12*A + 7*C)*a^4*cos(d*x + c)^5*log(
sin(d*x + c) + 1) - 15*(12*A + 7*C)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) +
1) + 2*(2*(100*A + 83*C)*a^4*cos(d*x + c)^4 + 15*(4*A + 7*C)*a^4*cos(d*x +
c)^3 + 2*(5*A + 34*C)*a^4*cos(d*x + c)^2 + 30*C*a^4*cos(d*x + c) + 6*C*a^4)
*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int A\,dx+\int 4A\sec(c+dx)\,dx+\int 6A\sec^2(c+dx)\,dx+\int 4A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int C\sec^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] a**4*(Integral(A, x) + Integral(4*A*sec(c + d*x), x) + Integral(6*A*sec(c +
 d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**
4, x) + Integral(C*sec(c + d*x)**2, x) + Integral(4*C*sec(c + d*x)**3, x) +
 Integral(6*C*sec(c + d*x)**4, x) + Integral(4*C*sec(c + d*x)**5, x) + Inte
gral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.25417, size = 347, normalized size = 1.96

$$\frac{30\,(dx+c)Aa^4+15\left(12\,Aa^4+7\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(12\,Aa^4+7\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/30*(30*(d*x + c)*A*a^4 + 15*(12*A*a^4 + 7*C*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 15*(12*A*a^4 + 7*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(150*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 105*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 680
*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 490*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 1180*A*a
^4*tan(1/2*d*x + 1/2*c)^5 + 896*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 920*A*a^4*ta
n(1/2*d*x + 1/2*c)^3 - 790*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 270*A*a^4*tan(1/2
*d*x + 1/2*c) + 375*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^5)/d
```

3.114 $\int \cos(c+dx)(a+a\sec(c+dx))^4 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=181

$$\frac{5a^4(4A+7C)\tan(c+dx)}{8d} + \frac{a^4(52A+35C)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(12A-7C)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d} - \frac{(12$$

```
[Out] 4*a^4*A*x + (a^4*(52*A + 35*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (A*(a + a*Sec
[c + d*x])^4*Sin[c + d*x])/d + (5*a^4*(4*A + 7*C)*Tan[c + d*x])/(8*d) - (a*
(4*A - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) - ((12*A - 7*C)*(a^2 +
 a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - ((12*A - 35*C)*(a^4 + a^4*Sec[c
 + d*x])*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.341742, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4087, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^4(4A+7C)\tan(c+dx)}{8d} + \frac{a^4(52A+35C)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(12A-7C)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d} - \frac{(12$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] 4*a^4*A*x + (a^4*(52*A + 35*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (A*(a + a*Sec
[c + d*x])^4*Sin[c + d*x])/d + (5*a^4*(4*A + 7*C)*Tan[c + d*x])/(8*d) - (a*
(4*A - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) - ((12*A - 7*C)*(a^2 +
 a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - ((12*A - 35*C)*(a^4 + a^4*Sec[c
 + d*x])*Tan[c + d*x])/(24*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d}+\frac{\int(a+a\sec(c+dx)}{}$$
$$=\frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d}-\frac{a(4A-C)(a+a\sec}{}$$
$$=\frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d}-\frac{a(4A-C)(a+a\sec}{}$$
$$=\frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d}-\frac{a(4A-C)(a+a\sec}{}$$
$$=4a^4Ax+\frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d}-\frac{a(4A-C)}{}$$
$$=4a^4Ax+\frac{a^4(52A+35C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{A(a+}{}$$
$$=4a^4Ax+\frac{a^4(52A+35C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{A(a+}{}$$

Mathematica [B] time = 2.45913, size = 379, normalized size = 2.09

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(A\cos^2(c+dx)+C\right)\left(\sec(c)(24A\sin(2c+dx)+288A\sin(c+2dx)-96A\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*(C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(5
2*A + 35*C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(288*A*d*x*Cos[c] + 192*A*d*
x*Cos[c + 2*d*x] + 192*A*d*x*Cos[3*c + 2*d*x] + 48*A*d*x*Cos[3*c + 4*d*x] +
 48*A*d*x*Cos[5*c + 4*d*x] - 288*A*Sin[c] - 480*C*Sin[c] + 24*A*Sin[d*x] +
105*C*Sin[d*x] + 24*A*Sin[2*c + d*x] + 105*C*Sin[2*c + d*x] + 288*A*Sin[c +
 2*d*x] + 544*C*Sin[c + 2*d*x] - 96*A*Sin[3*c + 2*d*x] - 96*C*Sin[3*c + 2*d
*x] + 30*A*Sin[2*c + 3*d*x] + 81*C*Sin[2*c + 3*d*x] + 30*A*Sin[4*c + 3*d*x]
 + 81*C*Sin[4*c + 3*d*x] + 96*A*Sin[3*c + 4*d*x] + 160*C*Sin[3*c + 4*d*x] +
 6*A*Sin[4*c + 5*d*x] + 6*A*Sin[6*c + 5*d*x])))/(1536*d*(A + 2*C + A*Cos[2*
(c + d*x)]))
```

Maple [A] time = 0.112, size = 197, normalized size = 1.1

$$\frac{Aa^4\sin(dx+c)}{d}+\frac{35\,a^4C\ln(\sec(dx+c)+\tan(dx+c))}{8d}+4\,a^4Ax+4\,\frac{Aa^4c}{d}+\frac{20\,a^4C\tan(dx+c)}{3d}+\frac{13\,Aa^4\ln(\sec(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+35/8/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*A*x+4/d*A
*a^4*c+20/3/d*a^4*C*tan(d*x+c)+13/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+27/8/
d*a^4*C*sec(d*x+c)*tan(d*x+c)+4/d*A*a^4*tan(d*x+c)+4/3/d*a^4*C*tan(d*x+c)*s
ec(d*x+c)^2+1/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+1/4/d*a^4*C*tan(d*x+c)*sec(d*
x+c)^3
```

Maxima [A] time = 0.955576, size = 400, normalized size = 2.21

$$\frac{192\,(dx+c)Aa^4+64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^4-3\,Ca^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(192*(d*x + c)*A*a^4 + 64*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 - 3*
C*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a^4*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 72*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) + 144*A*a^4*(log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 24*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 48*A*a^4*sin(d*x + c) + 192*A*a^4*tan(d*x + c) + 192*C*a^4*tan(d*x + c))
/d
```

Fricas [A] time = 0.546793, size = 437, normalized size = 2.41

$$\frac{192\,Aa^4dx\cos(dx+c)^4+3\,(52\,A+35\,C)a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(52\,A+35\,C)a^4\cos(dx+c)^4\log(-\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(192*A*a^4*d*x*cos(d*x + c)^4 + 3*(52*A + 35*C)*a^4*cos(d*x + c)^4*log
(sin(d*x + c) + 1) - 3*(52*A + 35*C)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
 1) + 2*(24*A*a^4*cos(d*x + c)^4 + 32*(3*A + 5*C)*a^4*cos(d*x + c)^3 + 3*(4
*A + 27*C)*a^4*cos(d*x + c)^2 + 32*C*a^4*cos(d*x + c) + 6*C*a^4)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31122, size = 342, normalized size = 1.89

$$96\,(dx+c)Aa^4+\frac{48\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+3\left(52\,Aa^4+35\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(52\,Aa^4+35\,Ca^4\right)\log\left(\left|\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(96*(d*x + c)*A*a^4 + 48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 + 1) + 3*(52*A*a^4 + 35*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(52*A*a^4 + 35*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(84*A*a^4*tan
(1/2*d*x + 1/2*c)^7 + 105*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 276*A*a^4*tan(1/2*
d*x + 1/2*c)^5 - 385*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 300*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 511*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 108*A*a^4*tan(1/2*d*x + 1/2*
c) - 279*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.115 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=192

$$\frac{5a^4(A-2C)\sin(c+dx)}{2d}+\frac{2a^4(2A+3C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(A-2C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{(3A+22$$

```
[Out] (a^4*(13*A + 2*C)*x)/2 + (2*a^4*(2*A + 3*C)*ArcTanh[Sin[c + d*x]])/d + (5*a
^4*(A - 2*C)*Sin[c + d*x])/(2*d) - (a*(3*A - 2*C)*(a + a*Sec[c + d*x])^3*Si
n[c + d*x])/(6*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(2
*d) - ((A - 2*C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((3*A + 2
2*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.527648, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4087, 4018, 3996, 3770}

$$\frac{5a^4(A-2C)\sin(c+dx)}{2d}+\frac{2a^4(2A+3C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(A-2C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{(3A+22$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^4*(13*A + 2*C)*x)/2 + (2*a^4*(2*A + 3*C)*ArcTanh[Sin[c + d*x]])/d + (5*a
^4*(A - 2*C)*Sin[c + d*x])/(2*d) - (a*(3*A - 2*C)*(a + a*Sec[c + d*x])^3*Si
n[c + d*x])/(6*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(2
*d) - ((A - 2*C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((3*A + 2
2*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{2d} + \frac{\int \cos(}{}$$

$$= -\frac{a(3A-2C)(a+a\sec(c+dx))^3\sin(c+dx)}{6d} + \frac{A\cos(}{}$$

$$= -\frac{a(3A-2C)(a+a\sec(c+dx))^3\sin(c+dx)}{6d} + \frac{A\cos(}{}$$

$$= -\frac{a(3A-2C)(a+a\sec(c+dx))^3\sin(c+dx)}{6d} + \frac{A\cos(}{}$$

$$= \frac{5a^4(A-2C)\sin(c+dx)}{2d} - \frac{a(3A-2C)(a+a\sec(c+d}{6d}$$

$$= \frac{1}{2}a^4(13A+2C)x + \frac{5a^4(A-2C)\sin(c+dx)}{2d} - \frac{a(3A-}{}$$

$$= \frac{1}{2}a^4(13A+2C)x + \frac{2a^4(2A+3C)\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [B] time = 6.41151, size = 1420, normalized size = 7.4

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((13*A + 2*C)*x*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*
(A + C*Sec[c + d*x]^2))/(16*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((-2*A - 3*C)
*Cos[c + d*x]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x
)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(4*d*(A + 2*C + A*Cos
[2*c + 2*d*x])) + ((2*A + 3*C)*Cos[c + d*x]^6*Log[Cos[c/2 + (d*x)/2] + Sin[
c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c +
d*x]^2))/(4*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[d*x]*Cos[c + d*x]^6*
Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[c])/
(2*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[2*d*x]*Cos[c + d*x]^6*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[2*c])/(32*d
*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[c]*Cos[c + d*x]^6*Sec[c/2 + (d*x)
/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[d*x])/(2*d*(A + 2*C
 + A*Cos[2*c + 2*d*x])) + (A*Cos[2*c]*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(
a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[2*d*x])/(32*d*(A + 2*C + A
*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c +
 d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(48*d*(A + 2*C + A*Cos[2*c +
2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3)
+ (Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c
+ d*x]^2)*(13*C*Cos[c/2] - 11*C*Sin[c/2]))/(96*d*(A + 2*C + A*Cos[2*c + 2*d
*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (
Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d
```

```
*x]^2)*(3*A*Sin[(d*x)/2] + 20*C*Sin[(d*x)/2]))/(24*d*(A + 2*C + A*Cos[2*c +
 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) +
 (C*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c
 + d*x]^2)*Sin[(d*x)/2])/(48*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + S
in[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^6*Sec
[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*(-13*C*Cos[
c/2] - 11*C*Sin[c/2]))/(96*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin
[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^6*Sec[c
/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*(3*A*Sin[(d*x
)/2] + 20*C*Sin[(d*x)/2]))/(24*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.102, size = 189, normalized size = 1.

$$\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13\,a^4Ax}{2}+\frac{13\,Aa^4c}{2d}+a^4Cx+\frac{Ca^4c}{d}+4\,\frac{Aa^4\sin(dx+c)}{d}+6\,\frac{a^4C\ln(\sec(dx+c)+\tan(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*a^4*A*x+13/2/d*A*a^4*c+a^4*C*x+1/d*C
*a^4*c+4/d*A*a^4*sin(d*x+c)+6/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*a^4*
C*tan(d*x+c)+4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^4*C*sec(d*x+c)*tan(d
*x+c)+1/d*A*a^4*tan(d*x+c)+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.953046, size = 285, normalized size = 1.48

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+72\,(dx+c)Aa^4+4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^4+12\,(dx+c)Ca^4-12\,Ca^4\big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 72*(d*x + c)*A*a^4 + 4*(ta
n(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 12*(d*x + c)*C*a^4 - 12*C*a^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*C*a^4*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^4*sin(d*x + c) + 1
2*A*a^4*tan(d*x + c) + 72*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.543052, size = 425, normalized size = 2.21

$$\frac{3\,(13\,A+2\,C)a^4dx\cos(dx+c)^3+6\,(2\,A+3\,C)a^4\cos(dx+c)^3\log(\sin(dx+c)+1)-6\,(2\,A+3\,C)a^4\cos(dx+c)^3\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(13*A + 2*C)*a^4*d*x*cos(d*x + c)^3 + 6*(2*A + 3*C)*a^4*cos(d*x + c)
^3*log(sin(d*x + c) + 1) - 6*(2*A + 3*C)*a^4*cos(d*x + c)^3*log(-sin(d*x +
c) + 1) + (3*A*a^4*cos(d*x + c)^4 + 24*A*a^4*cos(d*x + c)^3 + 2*(3*A + 20*C
)*a^4*cos(d*x + c)^2 + 12*C*a^4*cos(d*x + c) + 2*C*a^4)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26086, size = 335, normalized size = 1.74

$$\frac{3\left(13\,Aa^4+2\,Ca^4\right)(dx+c)+12\left(2\,Aa^4+3\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\left(2\,Aa^4+3\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(3*(13*A*a^4 + 2*C*a^4)*(d*x + c) + 12*(2*A*a^4 + 3*C*a^4)*log(abs(tan(
1/2*d*x + 1/2*c) + 1)) - 12*(2*A*a^4 + 3*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) + 6*(7*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 4*(3*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 15*C*
a^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 38*C*a^4*tan(
1/2*d*x + 1/2*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) + 27*C*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.116 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=198

$$\frac{5a^4(2A-C)\sin(c+dx)}{2d}+\frac{a^4(2A+13C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(2A-C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}-\frac{(4A-9C}{}$$

```
[Out] 2*a^4*(3*A + 2*C)*x + (a^4*(2*A + 13*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a
^4*(2*A - C)*Sin[c + d*x])/(2*d) + (2*a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])
^3*Sin[c + d*x])/(3*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*Sin[c + d
*x])/(3*d) - ((2*A - C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) - ((
4*A - 9*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.546137, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4087, 4017, 4018, 3996, 3770}

$$\frac{5a^4(2A-C)\sin(c+dx)}{2d}+\frac{a^4(2A+13C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(2A-C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}-\frac{(4A-9C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] 2*a^4*(3*A + 2*C)*x + (a^4*(2*A + 13*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a
^4*(2*A - C)*Sin[c + d*x])/(2*d) + (2*a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])
^3*Sin[c + d*x])/(3*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*Sin[c + d
*x])/(3*d) - ((2*A - C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) - ((
4*A - 9*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
```

```
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{3d}+\frac{\int \cos\ldots}{\ldots}$$

$$= \frac{2aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d}+\frac{A\,c\ldots}{\ldots}$$

$$= \frac{2aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d}+\frac{A\,c\ldots}{\ldots}$$

$$= \frac{2aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d}+\frac{A\,c\ldots}{\ldots}$$

$$= \frac{5a^4(2A-C)\sin(c+dx)}{2d}+\frac{2aA\cos(c+dx)(a+a\sec(\ldots}{3d}$$

$$= 2a^4(3A+2C)x+\frac{5a^4(2A-C)\sin(c+dx)}{2d}+\frac{2aA\cos(\ldots}{\ldots}$$

$$= 2a^4(3A+2C)x+\frac{a^4(2A+13C)\tanh^{-1}(\sin(c+dx))}{2d}+\ldots$$

Mathematica [B] time = 6.21813, size = 1250, normalized size = 6.31

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((3*A + 2*C)*x*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(
A + C*Sec[c + d*x]^2))/(4*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((-2*A - 13*C)*
Cos[c + d*x]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)
/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(16*d*(A + 2*C + A*Cos
[2*c + 2*d*x])) + ((2*A + 13*C)*Cos[c + d*x]^6*Log[Cos[c/2 + (d*x)/2] + Sin
[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c +
 d*x]^2))/(16*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((27*A + 4*C)*Cos[d*x]*Co
s[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x
]^2)*Sin[c])/(32*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[2*d*x]*Cos[c +
d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*S
in[2*c])/(8*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[3*d*x]*Cos[c + d*x]^
```

```
6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[3*
c])/(96*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((27*A + 4*C)*Cos[c]*Cos[c + d*
x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin
[d*x])/(32*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[2*c]*Cos[c + d*x]^6*S
ec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[2*d*x
])/(8*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Cos[3*c]*Cos[c + d*x]^6*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[3*d*x])/(9
6*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^
8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(32*d*(A + 2*C + A*Cos[2*c
 + 2*d*x])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (C*Cos[c + d*x]^6
*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[(d*
x)/2])/(2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])) - (C*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(
a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(32*d*(A + 2*C + A*Cos[2*c +
2*d*x])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (C*Cos[c + d*x]^6*Se
c[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[(d*x)/
2])/(2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.108, size = 190, normalized size = 1.

$$\frac{A(\cos(dx+c))^2\sin(dx+c)a^4}{3d}+\frac{20Aa^4\sin(dx+c)}{3d}+\frac{a^4C\sin(dx+c)}{d}+2\frac{Aa^4\cos(dx+c)\sin(dx+c)}{d}+6a^4Ax+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^4+20/3/d*A*a^4*sin(d*x+c)+1/d*a^4*C*sin(d
*x+c)+2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+6*a^4*A*x+6/d*A*a^4*c+4*a^4*C*x+4/d*C
*a^4*c+13/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^4*C*tan(d*x+c)+1/d*A*a^
4*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.952799, size = 285, normalized size = 1.44

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-12(2dx+2c+\sin(2dx+2c))Aa^4-48(dx+c)Aa^4-48(dx+c)Ca^4+3Ca^4\big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 12*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a^4 - 48*(d*x + c)*A*a^4 - 48*(d*x + c)*C*a^4 + 3*C*a^4*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) - 6*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 36*C*a^4*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 72*A*a^4*sin(d*x + c) - 1
2*C*a^4*sin(d*x + c) - 48*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.547472, size = 432, normalized size = 2.18

$$\frac{24(3A+2C)a^4dx\cos(dx+c)^2+3(2A+13C)a^4\cos(dx+c)^2\log(\sin(dx+c)+1)-3(2A+13C)a^4\cos(dx+c)^2\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/12*(24*(3*A + 2*C)*a^4*d*x*cos(d*x + c)^2 + 3*(2*A + 13*C)*a^4*cos(d*x +
c)^2*log(sin(d*x + c) + 1) - 3*(2*A + 13*C)*a^4*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*A*a^4*cos(d*x + c)^4 + 12*A*a^4*cos(d*x + c)^3 + 2*(20*A
+ 3*C)*a^4*cos(d*x + c)^2 + 24*C*a^4*cos(d*x + c) + 3*C*a^4)*sin(d*x + c))/
(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24505, size = 335, normalized size = 1.69

$$\frac{12\left(3\,Aa^4+2\,Ca^4\right)(dx+c)+3\left(2\,Aa^4+13\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^4+13\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(12*(3*A*a^4 + 2*C*a^4)*(d*x + c) + 3*(2*A*a^4 + 13*C*a^4)*log(abs(tan(
1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^4 + 13*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - 6*(7*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*C*a^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 4*(15*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 3*C*
a^4*tan(1/2*d*x + 1/2*c)^5 + 38*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^4*tan(
1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.117 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=200

$$\frac{5a^4(7A+4C)\sin(c+dx)}{8d} - \frac{(35A-12C)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{24d} + \frac{(7A+4C)\sin(c+dx)\cos(c+dx)\left(a^2\sec(c\right.}{8d}$$

```
[Out] (a^4*(35*A + 52*C)*x)/8 + (4*a^4*C*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(7*A +
 4*C)*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*Sin[
c + d*x])/(3*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(4
*d) + ((7*A + 4*C)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(8
*d) - ((35*A - 12*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.573874, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4087, 4017, 4018, 3996, 3770}

$$\frac{5a^4(7A+4C)\sin(c+dx)}{8d} - \frac{(35A-12C)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{24d} + \frac{(7A+4C)\sin(c+dx)\cos(c+dx)\left(a^2\sec(c\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^4*(35*A + 52*C)*x)/8 + (4*a^4*C*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(7*A +
 4*C)*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*Sin[
c + d*x])/(3*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(4
*d) + ((7*A + 4*C)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(8
*d) - ((35*A - 12*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(24*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
```

```
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{4d} + \frac{\int \cos \ldots}{}$$

$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{A\cos \ldots}{}$$

$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{A\cos \ldots}{}$$

$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{A\cos \ldots}{}$$

$$= \frac{5a^4(7A+4C)\sin(c+dx)}{8d} + \frac{aA\cos^2(c+dx)(a+a\sec \ldots}{3d}$$

$$= \frac{1}{8}a^4(35A+52C)x + \frac{5a^4(7A+4C)\sin(c+dx)}{8d} + \frac{aA\,c \ldots}{}$$

$$= \frac{1}{8}a^4(35A+52C)x + \frac{4a^4C\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^4(\ldots}{}$$

Mathematica [A] time = 2.28263, size = 375, normalized size = 1.88

$$\frac{a^4\cos^2(c+dx)(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(A+C\sec^2(c+dx)\right)\left(\frac{96(7A+4C)\sin(c)\cos(dx)}{d}+\frac{24(7A+C)\sin(2c)\cos(2dx)}{d} \ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(A + C*Sec[c +
d*x]^2)*(12*(35*A + 52*C)*x - (384*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]])/d + (384*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (96*(7*A + 4*C
)*Cos[d*x]*Sin[c])/d + (24*(7*A + C)*Cos[2*d*x]*Sin[2*c])/d + (32*A*Cos[3*d
*x]*Sin[3*c])/d + (3*A*Cos[4*d*x]*Sin[4*c])/d + (96*(7*A + 4*C)*Cos[c]*Sin[
d*x])/d + (24*(7*A + C)*Cos[2*c]*Sin[2*d*x])/d + (32*A*Cos[3*c]*Sin[3*d*x])
/d + (3*A*Cos[4*c]*Sin[4*d*x])/d + (96*C*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c
/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (96*C*Sin[(d*x)/2])/(d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(768*(A + 2*C + A*
```

```
Cos[2*(c + d*x)]))
```

Maple [A] time = 0.095, size = 191, normalized size = 1.

$$\frac{Aa^4 \sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{27\,Aa^4 \cos(dx+c)\sin(dx+c)}{8d} + \frac{35\,a^4 Ax}{8} + \frac{35\,Aa^4 c}{8d} + \frac{a^4 C \sin(dx+c)\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+27/8/d*A*a^4*cos(d*x+c)*sin(d*x+c)+35/8
*a^4*A*x+35/8/d*A*a^4*c+1/2/d*a^4*C*sin(d*x+c)*cos(d*x+c)+13/2*a^4*C*x+13/2
/d*C*a^4*c+4/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^4+20/3/d*A*a^4*sin(d*x+c)+4/d*
a^4*C*sin(d*x+c)+4/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^4*C*tan(d*x+c)
```

Maxima [A] time = 0.949653, size = 262, normalized size = 1.31

$$-\frac{128\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Aa^4 - 3\,(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\sin(2\,dx+2\,c))Aa^4 - 144\,(2\,dx + 2\,c + s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 144*(2*d*x + 2*c + sin(2*d*x +
2*c))*A*a^4 - 96*(d*x + c)*A*a^4 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^
4 - 576*(d*x + c)*C*a^4 - 192*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) - 384*A*a^4*sin(d*x + c) - 384*C*a^4*sin(d*x + c) - 96*C*a^4*tan(d
*x + c))/d
```

Fricas [A] time = 0.548459, size = 408, normalized size = 2.04

$$\frac{3\,(35\,A + 52\,C)a^4 dx\cos(dx+c) + 48\,Ca^4\cos(dx+c)\log(\sin(dx+c)+1) - 48\,Ca^4\cos(dx+c)\log(-\sin(dx+c)+1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/24*(3*(35*A + 52*C)*a^4*d*x*cos(d*x + c) + 48*C*a^4*cos(d*x + c)*log(sin(
d*x + c) + 1) - 48*C*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*A*a^4*cos
(d*x + c)^4 + 32*A*a^4*cos(d*x + c)^3 + 3*(27*A + 4*C)*a^4*cos(d*x + c)^2 +
 32*(5*A + 3*C)*a^4*cos(d*x + c) + 24*C*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27484, size = 329, normalized size = 1.64

$$96\,Ca^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-96\,Ca^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{48\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+3\left(35\,Aa^4+52\,Ca^4\right)(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/24*(96*C*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 96*C*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 48*C*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 - 1) + 3*(35*A*a^4 + 52*C*a^4)*(d*x + c) + 2*(105*A*a^4*tan(1/2*d*x + 1/2
*c)^7 + 84*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*A*a^4*tan(1/2*d*x + 1/2*c)^5
+ 276*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 300
*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*A*a^4*tan(1/2*d*x + 1/2*c) + 108*C*a^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.118 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=207

$$\frac{a^4(7A+10C)\sin(c+dx)}{2d} + \frac{(7A+5C)\sin(c+dx)\cos^2(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{15d} + \frac{(7A+8C)\sin(c+dx)\cos(c+d}{6d}$$

```
[Out] (a^4*(7*A + 12*C)*x)/2 + (a^4*C*ArcTanh[Sin[c + d*x]])/d + (a^4*(7*A + 10*C
)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c +
d*x])/(5*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(5*d)
+ ((7*A + 5*C)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(15*
d) + ((7*A + 8*C)*Cos[c + d*x]*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.551686, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4087, 4017, 3996, 3770}

$$\frac{a^4(7A+10C)\sin(c+dx)}{2d} + \frac{(7A+5C)\sin(c+dx)\cos^2(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{15d} + \frac{(7A+8C)\sin(c+dx)\cos(c+d}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^4*(7*A + 12*C)*x)/2 + (a^4*C*ArcTanh[Sin[c + d*x]])/d + (a^4*(7*A + 10*C
)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c +
d*x])/(5*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(5*d)
+ ((7*A + 5*C)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(15*
d) + ((7*A + 8*C)*Cos[c + d*x]*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{5d} + \int \cos$$

$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{5d} + \frac{A\cos}{}$$

$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{5d} + \frac{A\cos}{}$$

$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{5d} + \frac{A\cos}{}$$

$$= \frac{a^4(7A+10C)\sin(c+dx)}{2d} + \frac{aA\cos^3(c+dx)(a+a\sec}{5d}$$

$$= \frac{1}{2}a^4(7A+12C)x + \frac{a^4(7A+10C)\sin(c+dx)}{2d} + \frac{aA\cos}{}$$

$$= \frac{1}{2}a^4(7A+12C)x + \frac{a^4C\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^4(7A}{}$$

Mathematica [A] time = 0.422612, size = 147, normalized size = 0.71

$$\frac{a^4\Big(30(49A+54C)\sin(c+dx)+240(2A+C)\sin(2(c+dx))+145A\sin(3(c+dx))+30A\sin(4(c+dx))+3A\sin(5(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*(840*A*d*x + 1440*C*d*x - 240*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 240*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 30*(49*A + 54*C)*Sin[
c + d*x] + 240*(2*A + C)*Sin[2*(c + d*x)] + 145*A*Sin[3*(c + d*x)] + 20*C*S
in[3*(c + d*x)] + 30*A*Sin[4*(c + d*x)] + 3*A*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.132, size = 221, normalized size = 1.1

$$\frac{83\,Aa^4\sin(dx+c)}{15\,d} + \frac{Aa^4\sin(dx+c)(\cos(dx+c))^4}{5\,d} + \frac{34\,A(\cos(dx+c))^2\sin(dx+c)\,a^4}{15\,d} + \frac{C(\cos(dx+c))^2\sin(d}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 83/15/d*A*a^4*sin(d*x+c)+1/5/d*A*a^4*sin(d*x+c)*cos(d*x+c)^4+34/15/d*A*cos(
d*x+c)^2*sin(d*x+c)*a^4+1/3/d*C*cos(d*x+c)^2*sin(d*x+c)*a^4+20/3/d*a^4*C*si
n(d*x+c)+1/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+7/2/d*A*a^4*cos(d*x+c)*sin(d*x+c
```

```
)+7/2*a^4*A*x+7/2/d*A*a^4*c+2/d*a^4*C*sin(d*x+c)*cos(d*x+c)+6*a^4*C*x+6/d*C
*a^4*c+1/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.954818, size = 309, normalized size = 1.49

$$\frac{8\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{4}-240\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}+15\left(12\,dx+12\,c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 - 2
40*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 15*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A
*a^4 - 40*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 120*(2*d*x + 2*c + sin(
2*d*x + 2*c))*C*a^4 + 480*(d*x + c)*C*a^4 + 60*C*a^4*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 120*A*a^4*sin(d*x + c) + 720*C*a^4*sin(d*x + c)
)/d
```

Fricas [A] time = 0.551146, size = 351, normalized size = 1.7

$$\frac{15\left(7\,A+12\,C\right)a^{4}dx+15\,Ca^{4}\log\left(\sin\left(dx+c\right)+1\right)-15\,Ca^{4}\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Aa^{4}\cos\left(dx+c\right)^{4}+30\,Aa^{4}\cos\left(\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(7*A + 12*C)*a^4*d*x + 15*C*a^4*log(sin(d*x + c) + 1) - 15*C*a^4*l
og(-sin(d*x + c) + 1) + (6*A*a^4*cos(d*x + c)^4 + 30*A*a^4*cos(d*x + c)^3 +
 2*(34*A + 5*C)*a^4*cos(d*x + c)^2 + 15*(7*A + 4*C)*a^4*cos(d*x + c) + 2*(8
3*A + 100*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32693, size = 335, normalized size = 1.62

$$\frac{30\,Ca^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-30\,Ca^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\left(7\,Aa^{4}+12\,Ca^{4}\right)\left(dx+c\right)+\frac{2\left(105\,Aa^{4}\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/30*(30*C*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 30*C*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 15*(7*A*a^4 + 12*C*a^4)*(d*x + c) + 2*(105*A*a^4*tan
(1/2*d*x + 1/2*c)^9 + 150*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 490*A*a^4*tan(1/2*
d*x + 1/2*c)^7 + 680*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 896*A*a^4*tan(1/2*d*x +
 1/2*c)^5 + 1180*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 790*A*a^4*tan(1/2*d*x + 1/2
*c)^3 + 920*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 375*A*a^4*tan(1/2*d*x + 1/2*c) +
 270*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.119 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=192

$$-\frac{2a^4(7A+10C)\sin^3(c+dx)}{15d}+\frac{4a^4(7A+10C)\sin(c+dx)}{5d}+\frac{a^4(7A+10C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(7A+10C)}{}$$

```
[Out] (7*a^4*(7*A + 10*C)*x)/16 + (4*a^4*(7*A + 10*C)*Sin[c + d*x])/(5*d) + (27*a
^4*(7*A + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(7*A + 10*C)*Cos[c
 + d*x]^3*Sin[c + d*x])/(40*d) + (2*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4
*Sin[c + d*x])/(15*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*Sin[c + d*
x])/(6*d) - (2*a^4*(7*A + 10*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.366457, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4087, 4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{2a^4(7A+10C)\sin^3(c+dx)}{15d}+\frac{4a^4(7A+10C)\sin(c+dx)}{5d}+\frac{a^4(7A+10C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(7A+10C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (7*a^4*(7*A + 10*C)*x)/16 + (4*a^4*(7*A + 10*C)*Sin[c + d*x])/(5*d) + (27*a
^4*(7*A + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(7*A + 10*C)*Cos[c
 + d*x]^3*Sin[c + d*x])/(40*d) + (2*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4
*Sin[c + d*x])/(15*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*Sin[c + d*
x])/(6*d) - (2*a^4*(7*A + 10*C)*Sin[c + d*x]^3)/(15*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{6d} + \int \cos$$
$$= \frac{2A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{15d} + \frac{A\,\text{co}}{}$$
$$= \frac{2A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{15d} + \frac{A\,\text{co}}{}$$
$$= \frac{1}{10}a^4(7A+10C)x + \frac{2A\cos^4(c+dx)(a+a\sec(c+dx)}{15d}$$
$$= \frac{1}{10}a^4(7A+10C)x + \frac{2a^4(7A+10C)\sin(c+dx)}{5d} + \frac{3a^4(}{}$$
$$= \frac{2}{5}a^4(7A+10C)x + \frac{4a^4(7A+10C)\sin(c+dx)}{5d} + \frac{27a^4(}{}$$
$$= \frac{7}{16}a^4(7A+10C)x + \frac{4a^4(7A+10C)\sin(c+dx)}{5d} + \frac{27a^4}{}$$

Mathematica [A] time = 0.344172, size = 119, normalized size = 0.62

$$\frac{a^4(480(11A+14C)\sin(c+dx)+15(127A+112C)\sin(2(c+dx))+720A\sin(3(c+dx))+225A\sin(4(c+dx))+48A}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*(2940*A*d*x + 4200*C*d*x + 480*(11*A + 14*C)*Sin[c + d*x] + 15*(127*A
+ 112*C)*Sin[2*(c + d*x)] + 720*A*Sin[3*(c + d*x)] + 320*C*Sin[3*(c + d*x)]
 + 225*A*Sin[4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 48*A*Sin[5*(c + d*x)] +
 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.118, size = 284, normalized size = 1.5

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4Aa^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6
*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^4*C*(
1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c)+4/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/2*cos
(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*
x+1/2*c)+4*a^4*C*sin(d*x+c)+a^4*C*(d*x+c))
```

Maxima [A] time = 0.957015, size = 369, normalized size = 1.92

$$\frac{256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 180*(12*d*
x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 240*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*A*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 +
30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4 + 1440*(2*
d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 960*(d*x + c)*C*a^4 + 3840*C*a^4*sin(
d*x + c))/d
```

Fricas [A] time = 0.511888, size = 319, normalized size = 1.66

$$\frac{105(7A+10C)a^4dx+\left(40Aa^4\cos(dx+c)^5+192Aa^4\cos(dx+c)^4+10(41A+6C)a^4\cos(dx+c)^3+64(9A+5C)a\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(105*(7*A + 10*C)*a^4*d*x + (40*A*a^4*cos(d*x + c)^5 + 192*A*a^4*cos(
d*x + c)^4 + 10*(41*A + 6*C)*a^4*cos(d*x + c)^3 + 64*(9*A + 5*C)*a^4*cos(d*
x + c)^2 + 15*(49*A + 54*C)*a^4*cos(d*x + c) + 64*(18*A + 25*C)*a^4)*sin(d*
x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24157, size = 329, normalized size = 1.71

$$105\left(7\,Aa^4+10\,Ca^4\right)(dx+c)+\frac{2\left(735\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1050\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5950\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 10*C*a^4)*(d*x + c) + 2*(735*A*a^4*tan(1/2*d*x + 1/2*
c)^11 + 1050*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*A*a^4*tan(1/2*d*x + 1/2*c
)^9 + 5950*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x + 1/2*c)^7
 + 13860*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*A*a^4*tan(1/2*d*x + 1/2*c)^5
+ 16860*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/2*c)^3 +
10690*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*A*a^4*tan(1/2*d*x + 1/2*c) + 2790
*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.120 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=254

$$\frac{a^4(454A+581C)\sin(c+dx)}{105d}+\frac{a^4(247A+308C)\sin(c+dx)\cos^2(c+dx)}{210d}+\frac{a^4(11A+14C)\sin(c+dx)\cos(c+dx)}{4d}+$$

```
[Out] (a^4*(11*A + 14*C)*x)/4 + (a^4*(454*A + 581*C)*Sin[c + d*x])/(105*d) + (a^4
*(11*A + 14*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^4*(247*A + 308*C)*Cos[
c + d*x]^2*Sin[c + d*x])/(210*d) + (2*a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x
])^3*Sin[c + d*x])/(21*d) + (A*Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*Sin[c
+ d*x])/(7*d) + ((8*A + 7*C)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(35*d) + ((109*A + 126*C)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c + d*x])
*Sin[c + d*x])/(210*d)
```

Rubi [A] time = 0.718647, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4087, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^4(454A+581C)\sin(c+dx)}{105d}+\frac{a^4(247A+308C)\sin(c+dx)\cos^2(c+dx)}{210d}+\frac{a^4(11A+14C)\sin(c+dx)\cos(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^4*(11*A + 14*C)*x)/4 + (a^4*(454*A + 581*C)*Sin[c + d*x])/(105*d) + (a^4
*(11*A + 14*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^4*(247*A + 308*C)*Cos[
c + d*x]^2*Sin[c + d*x])/(210*d) + (2*a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x
])^3*Sin[c + d*x])/(21*d) + (A*Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*Sin[c
+ d*x])/(7*d) + ((8*A + 7*C)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(35*d) + ((109*A + 126*C)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c + d*x])
*Sin[c + d*x])/(210*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^6(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{7d} + \frac{\int \cos\ldots}{}$$

$$= \frac{2aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{21d} + \frac{A\,\mathrm{c}\ldots}{}$$

$$= \frac{2aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{21d} + \frac{A\,\mathrm{c}\ldots}{}$$

$$= \frac{2aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{21d} + \frac{A\,\mathrm{c}\ldots}{}$$

$$= \frac{a^4(247A+308C)\cos^2(c+dx)\sin(c+dx)}{210d} + \frac{2aA\cos^5\ldots}{}$$

$$= \frac{a^4(247A+308C)\cos^2(c+dx)\sin(c+dx)}{210d} + \frac{2aA\cos^5\ldots}{}$$

$$= \frac{a^4(454A+581C)\sin(c+dx)}{105d} + \frac{a^4(11A+14C)\cos(c+\ldots}{4d}$$

$$= \frac{1}{4}a^4(11A+14C)x + \frac{a^4(454A+581C)\sin(c+dx)}{105d} + \frac{a^4\ldots}{}$$

Mathematica [A] time = 0.637744, size = 145, normalized size = 0.57

$$\frac{a^4(105(323A+392C)\sin(c+dx)+420(31A+32C)\sin(2(c+dx))+5495A\sin(3(c+dx))+2100A\sin(4(c+dx))+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*(11760*A*c + 18480*A*d*x + 23520*C*d*x + 105*(323*A + 392*C)*Sin[c + d
*x] + 420*(31*A + 32*C)*Sin[2*(c + d*x)] + 5495*A*Sin[3*(c + d*x)] + 4060*C
*Sin[3*(c + d*x)] + 2100*A*Sin[4*(c + d*x)] + 840*C*Sin[4*(c + d*x)] + 651*
A*Sin[5*(c + d*x)] + 84*C*Sin[5*(c + d*x)] + 140*A*Sin[6*(c + d*x)] + 15*A*
Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.119, size = 322, normalized size = 1.3

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)+\frac{a^4C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/7*A*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)+1/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/6*
(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)
+4*a^4*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*A
*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^4*C*(2+cos(d*x+c)^2
)*sin(d*x+c)+4*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+4*a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*A*a^4*(2+cos(d
*x+c)^2)*sin(d*x+c)+a^4*C*sin(d*x+c))
```

Maxima [A] time = 0.956173, size = 431, normalized size = 1.7

$$-\frac{48\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)Aa^4-672\left(3\sin(dx+c)^5-10\sin(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*
sin(d*x + c))*A*a^4 - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*
x + c))*A*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*
c) - 48*sin(2*d*x + 2*c))*A*a^4 + 560*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a
^4 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 11
2*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^4 + 3360*(si
n(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c
) + 8*sin(2*d*x + 2*c))*C*a^4 - 1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4
 - 1680*C*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.525151, size = 377, normalized size = 1.48

$$\frac{105(11A+14C)a^4dx+\left(60Aa^4\cos(dx+c)^6+280Aa^4\cos(dx+c)^5+12(48A+7C)a^4\cos(dx+c)^4+70(11A+6C\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/420*(105*(11*A + 14*C)*a^4*d*x + (60*A*a^4*cos(d*x + c)^6 + 280*A*a^4*cos
(d*x + c)^5 + 12*(48*A + 7*C)*a^4*cos(d*x + c)^4 + 70*(11*A + 6*C)*a^4*cos(
d*x + c)^3 + 4*(227*A + 238*C)*a^4*cos(d*x + c)^2 + 105*(11*A + 14*C)*a^4*c
os(d*x + c) + 4*(454*A + 581*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25544, size = 375, normalized size = 1.48

$$105\left(11\,Aa^4+14\,Ca^4\right)(dx+c)+\frac{2\left(1155\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+1470\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+7700\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+9800\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/420*(105*(11*A*a^4 + 14*C*a^4)*(d*x + c) + 2*(1155*A*a^4*tan(1/2*d*x + 1/
2*c)^13 + 1470*C*a^4*tan(1/2*d*x + 1/2*c)^13 + 7700*A*a^4*tan(1/2*d*x + 1/2
*c)^11 + 9800*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 21791*A*a^4*tan(1/2*d*x + 1/2
*c)^9 + 27734*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 33792*A*a^4*tan(1/2*d*x + 1/2*
c)^7 + 43008*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 31521*A*a^4*tan(1/2*d*x + 1/2*c
)^5 + 39914*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 14700*A*a^4*tan(1/2*d*x + 1/2*c)
^3 + 21560*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 5565*A*a^4*tan(1/2*d*x + 1/2*c) +
 5250*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.121 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=165

$$-\frac{(3A+4C)\tan^3(c+dx)}{3ad}-\frac{(3A+4C)\tan(c+dx)}{ad}+\frac{3(4A+5C)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{(A+C)\tan(c+dx)\sec^4(c+d}{d(a\sec(c+dx)+a)}$$

```
[Out] (3*(4*A + 5*C)*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((3*A + 4*C)*Tan[c + d*x])/
(a*d) + (3*(4*A + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + ((4*A + 5*C)*Se
c[c + d*x]^3*Tan[c + d*x])/(4*a*d) - ((A + C)*Sec[c + d*x]^4*Tan[c + d*x])/
(d*(a + a*Sec[c + d*x])) - ((3*A + 4*C)*Tan[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.201955, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4085, 3787, 3767, 3768, 3770}

$$-\frac{(3A+4C)\tan^3(c+dx)}{3ad}-\frac{(3A+4C)\tan(c+dx)}{ad}+\frac{3(4A+5C)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{(A+C)\tan(c+dx)\sec^4(c+d}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*(4*A + 5*C)*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((3*A + 4*C)*Tan[c + d*x])/
(a*d) + (3*(4*A + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d) + ((4*A + 5*C)*Se
c[c + d*x]^3*Tan[c + d*x])/(4*a*d) - ((A + C)*Sec[c + d*x]^4*Tan[c + d*x])/
(d*(a + a*Sec[c + d*x])) - ((3*A + 4*C)*Tan[c + d*x]^3)/(3*a*d)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^4(c+dx)(a(3A+4C)-a(4A+}{a^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A+4C)\int \sec^4(c+dx)\,dx}{a} + \frac{(4A}{}$$

$$= \frac{(4A+5C)\sec^3(c+dx)\tan(c+dx)}{4ad} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} +$$

$$= -\frac{(3A+4C)\tan(c+dx)}{ad} + \frac{3(4A+5C)\sec(c+dx)\tan(c+dx)}{8ad} + \frac{(4A+5}{}$$

$$= \frac{3(4A+5C)\tanh^{-1}(\sin(c+dx))}{8ad} - \frac{(3A+4C)\tan(c+dx)}{ad} + \frac{3(4A+5C)\,\mathrm{s}}{}$$

Mathematica [B] time = 6.3334, size = 792, normalized size = 4.8

$$\frac{\sec\left(\frac{c}{2}\right)\sec(c)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\left(204A\sin\left(c-\frac{dx}{2}\right)-60A\sin\left(c+\frac{dx}{2}\right)+84A\sin\left(2c+\frac{dx}{2}\right)+36A\sin\left(c+\frac{3dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*(4*A + 5*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Log[Cos[c/2 + (d*x)/2] -
Sin[c/2 + (d*x)/2]]*(A + C*Sec[c + d*x]^2))/(2*d*(A + 2*C + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])) + (3*(4*A + 5*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*
x]*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(A + C*Sec[c + d*x]^2))/(2*
d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d*x)/2
]*Sec[c/2]*Sec[c]*Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2)*(-60*A*Sin[(d*x)/2]
 - 75*C*Sin[(d*x)/2] - 60*A*Sin[(3*d*x)/2] - 91*C*Sin[(3*d*x)/2] + 204*A*Si
n[c - (d*x)/2] + 219*C*Sin[c - (d*x)/2] - 60*A*Sin[c + (d*x)/2] + 21*C*Sin[
c + (d*x)/2] + 84*A*Sin[2*c + (d*x)/2] + 165*C*Sin[2*c + (d*x)/2] + 36*A*Si
n[c + (3*d*x)/2] + 5*C*Sin[c + (3*d*x)/2] + 36*A*Sin[2*c + (3*d*x)/2] + 69*
C*Sin[2*c + (3*d*x)/2] + 132*A*Sin[3*c + (3*d*x)/2] + 165*C*Sin[3*c + (3*d*
x)/2] - 156*A*Sin[c + (5*d*x)/2] - 211*C*Sin[c + (5*d*x)/2] - 60*A*Sin[2*c
+ (5*d*x)/2] - 115*C*Sin[2*c + (5*d*x)/2] - 60*A*Sin[3*c + (5*d*x)/2] - 51*
C*Sin[3*c + (5*d*x)/2] + 36*A*Sin[4*c + (5*d*x)/2] + 45*C*Sin[4*c + (5*d*x)
/2] - 12*A*Sin[2*c + (7*d*x)/2] - 19*C*Sin[2*c + (7*d*x)/2] + 12*A*Sin[3*c
+ (7*d*x)/2] + 5*C*Sin[3*c + (7*d*x)/2] + 12*A*Sin[4*c + (7*d*x)/2] + 21*C*
Sin[4*c + (7*d*x)/2] + 36*A*Sin[5*c + (7*d*x)/2] + 45*C*Sin[5*c + (7*d*x)/2
] - 48*A*Sin[3*c + (9*d*x)/2] - 64*C*Sin[3*c + (9*d*x)/2] - 24*A*Sin[4*c +
(9*d*x)/2] - 40*C*Sin[4*c + (9*d*x)/2] - 24*A*Sin[5*c + (9*d*x)/2] - 24*C*S
in[5*c + (9*d*x)/2]))/(192*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
d*x]))
```

Maple [B] time = 0.068, size = 386, normalized size = 2.3

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{4\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-4}+\frac{5\,C}{6\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{15\,C}{8\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-1/4/a/d*C/(tan(1/2*d
*x+1/2*c)+1)^4+5/6/a/d*C/(tan(1/2*d*x+1/2*c)+1)^3-15/8/a/d/(tan(1/2*d*x+1/2
*c)+1)^2*C-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*A+15/8/a/d*ln(tan(1/2*d*x+1/2*c
)+1)*C+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A+25/8/a/d/(tan(1/2*d*x+1/2*c)+1)*C
+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*A+1/4/a/d*C/(tan(1/2*d*x+1/2*c)-1)^4+5/6/a/
d*C/(tan(1/2*d*x+1/2*c)-1)^3-15/8/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-3/2/a/d*ln
(tan(1/2*d*x+1/2*c)-1)*A+15/8/a/d/(tan(1/2*d*x+1/2*c)-1)^2*C+1/2/a/d/(tan(1
/2*d*x+1/2*c)-1)^2*A+25/8/a/d/(tan(1/2*d*x+1/2*c)-1)*C+3/2/a/d/(tan(1/2*d*x
+1/2*c)-1)*A
```

Maxima [B] time = 0.956555, size = 551, normalized size = 3.34

$$-\frac{C\left(\frac{2\left(\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{45\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{24\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+12\,A\left(\frac{2}{a-}\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/24*(C*(2*(21*sin(d*x + c)/(cos(d*x + c) + 1) - 109*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 75*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a + 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 24*sin(d
*x + c)/(a*(cos(d*x + c) + 1))) + 12*A*(2*(sin(d*x + c)/(cos(d*x + c) + 1)
- 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(c
os(d*x + c) + 1) + 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*
sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.517481, size = 477, normalized size = 2.89

$$\frac{9\left((4\,A+5\,C)\cos(dx+c)^5+(4\,A+5\,C)\cos(dx+c)^4\right)\log(\sin(dx+c)+1)-9\left((4\,A+5\,C)\cos(dx+c)^5+(4\,A+5\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(9*((4*A + 5*C)*cos(d*x + c)^5 + (4*A + 5*C)*cos(d*x + c)^4)*log(sin(d
*x + c) + 1) - 9*((4*A + 5*C)*cos(d*x + c)^5 + (4*A + 5*C)*cos(d*x + c)^4)*
log(-sin(d*x + c) + 1) - 2*(16*(3*A + 4*C)*cos(d*x + c)^4 + (12*A + 19*C)*c
os(d*x + c)^3 - (12*A + 13*C)*cos(d*x + c)^2 + 2*C*cos(d*x + c) - 6*C)*sin(
d*x + c))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{C\sec^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x
)**6/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.21083, size = 288, normalized size = 1.75

$$\frac{\frac{9(4A+5C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{9(4A+5C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{24\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} + \frac{2\left(36A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+75\right.}{}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(9*(4*A + 5*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(4*A + 5*C)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 24*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2
*d*x + 1/2*c))/a + 2*(36*A*tan(1/2*d*x + 1/2*c)^7 + 75*C*tan(1/2*d*x + 1/2*
c)^7 - 84*A*tan(1/2*d*x + 1/2*c)^5 - 115*C*tan(1/2*d*x + 1/2*c)^5 + 60*A*ta
n(1/2*d*x + 1/2*c)^3 + 109*C*tan(1/2*d*x + 1/2*c)^3 - 12*A*tan(1/2*d*x + 1/
2*c) - 21*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a))/d
```

3.122 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=133

$$\frac{(3A+4C)\tan^3(c+dx)}{3ad}+\frac{(3A+4C)\tan(c+dx)}{ad}-\frac{(2A+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] -((2*A + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((3*A + 4*C)*Tan[c + d*x])/(
a*d) - ((2*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d
*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((3*A + 4*C)*Tan[c + d*x]^3)
/(3*a*d)
```

Rubi [A] time = 0.178874, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4085, 3787, 3768, 3770, 3767}

$$\frac{(3A+4C)\tan^3(c+dx)}{3ad}+\frac{(3A+4C)\tan(c+dx)}{ad}-\frac{(2A+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((2*A + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((3*A + 4*C)*Tan[c + d*x])/(
a*d) - ((2*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d
*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((3*A + 4*C)*Tan[c + d*x]^3)
/(3*a*d)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^3(c+dx)(a(2A+3C)-a(3A+}{a^2}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(2A+3C)\int \sec^3(c+dx)\,dx}{a} + \frac{(3A}{}$$

$$= -\frac{(2A+3C)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} -$$

$$= -\frac{(2A+3C)\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(3A+4C)\tan(c+dx)}{ad} - \frac{(2A+3C)\,\mathrm{se}}{}$$

Mathematica [B] time = 6.51847, size = 1090, normalized size = 8.2

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(2*A + 3*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Log[Cos[c/2 + (d*x)/2] - S
in[c/2 + (d*x)/2]]*(A + C*Sec[c + d*x]^2))/(d*(A + 2*C + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])) - (2*(2*A + 3*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*
Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(A + C*Sec[c + d*x]^2))/(d*(A
+ 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (4*Cos[c/2 + (d*x)/2]*C
os[c + d*x]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2
]))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*C*Cos[c/2
+ (d*x)/2]^2*Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(3*d*(A + 2*
C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2
 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) - (2*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]
*(A + C*Sec[c + d*x]^2)*(C*Cos[c/2] - 2*C*Sin[c/2]))/(3*d*(A + 2*C + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2])^2) + (4*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*(A + C*Se
c[c + d*x]^2)*(3*A*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2]))/(3*d*(A + 2*C + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2])) + (2*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*(A + C*Se
c[c + d*x]^2)*Sin[(d*x)/2])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[
c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3
) + (2*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*(C*Cos[c/2]
 + 2*C*Sin[c/2]))/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])*
(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (4*Cos
[c/2 + (d*x)/2]^2*Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*(3*A*Sin[(d*x)/2] + 5
*C*Sin[(d*x)/2]))/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])*
(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.061, size = 294, normalized size = 2.2

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{3ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{3C}{2ad}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-1/3/a/d*C/(tan(1/2*d*
x+1/2*c)+1)^3+1/a/d/(tan(1/2*d*x+1/2*c)+1)^2*C-3/2/a/d*ln(tan(1/2*d*x+1/2*c
)+1)*C-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A-5/2/a/d/(tan(1/2*d*x+1/2*c)+1)*C-1/
a/d/(tan(1/2*d*x+1/2*c)+1)*A-1/3/a/d*C/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d/(tan(
1/2*d*x+1/2*c)-1)^2*C+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C+1/a/d*ln(tan(1/2*d
*x+1/2*c)-1)*A-5/2/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/a/d/(tan(1/2*d*x+1/2*c)-1
)*A
```

Maxima [B] time = 0.949656, size = 439, normalized size = 3.3

$$\frac{C\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-6A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6*(C*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 6*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin
(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c)
+ 1))))/d
```

Fricas [A] time = 0.519058, size = 429, normalized size = 3.23

$$-\frac{3\left((2A+3C)\cos(dx+c)^4+(2A+3C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((2A+3C)\cos(dx+c)^4+(2A+3\right.}{12\left(ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((2*A + 3*C)*cos(d*x + c)^4 + (2*A + 3*C)*cos(d*x + c)^3)*log(sin(
d*x + c) + 1) - 3*((2*A + 3*C)*cos(d*x + c)^4 + (2*A + 3*C)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) - 2*(4*(3*A + 4*C)*cos(d*x + c)^3 + (6*A + 7*C)*cos
(d*x + c)^2 - C*cos(d*x + c) + 2*C)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d
*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x
)**5/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.2242, size = 250, normalized size = 1.88

$$-\frac{\frac{3(2A+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{3(2A+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} + \frac{2\left(6A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15C\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(2*A + 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*(2*A + 3*C)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*
d*x + 1/2*c))/a + 2*(6*A*tan(1/2*d*x + 1/2*c)^5 + 15*C*tan(1/2*d*x + 1/2*c)
^5 - 12*A*tan(1/2*d*x + 1/2*c)^3 - 16*C*tan(1/2*d*x + 1/2*c)^3 + 6*A*tan(1/
2*d*x + 1/2*c) + 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*
a))/d
```

3.123 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{(A+2C)\tan(c+dx)}{ad}+\frac{(2A+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(2A+3C)\tan(c+dx)\mathrm{s}}{2ad}$$

```
[Out] ((2*A + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((A + 2*C)*Tan[c + d*x])/(a*d
) + ((2*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d*x]
^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.166728, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 3787, 3767, 8, 3768, 3770}

$$-\frac{(A+2C)\tan(c+dx)}{ad}+\frac{(2A+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(2A+3C)\tan(c+dx)\mathrm{s}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((2*A + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((A + 2*C)*Tan[c + d*x])/(a*d
) + ((2*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A + C)*Sec[c + d*x]
^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^2(c+dx)(a(A+2C)-a(2A+}{a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A+2C)\int \sec^2(c+dx)\,dx}{a} + \frac{(2A+}{}$$

$$= \frac{(2A+3C)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + $$

$$= \frac{(2A+3C)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(A+2C)\tan(c+dx)}{ad} + \frac{(2A+3C)\sec(}{}$$

Mathematica [B] time = 3.01149, size = 316, normalized size = 2.95

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-2(2A+3C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*(-4*(A + C)*Sec[c/2]*
Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-2*(2*A + 3*C)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 4*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 6*C*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + C/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^
2 - C/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (4*C*Sin[d*x])/((Cos[c/2] -
 Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d*(A + 2*C + A*Cos[2*(c + d*x)])*(
1 + Sec[c + d*x]))
```

Maple [B] time = 0.059, size = 209, normalized size = 2.

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3\,C}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{3\,C}{2\,ad}\ln\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-1/2/a/d/(tan(1/2*d*x
+1/2*c)+1)^2*C+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*C+3/2/a/d*ln(tan(1/2*d*x+1/2*
c)+1)*C+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*C
+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*C-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-1/a/d*
```

```
ln(tan(1/2*d*x+1/2*c)-1)*A
```

Maxima [B] time = 0.94444, size = 323, normalized size = 3.02

$$-\frac{C\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-2A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2*(C*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) - 2*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.503715, size = 377, normalized size = 3.52

$$\frac{\left((2A+3C)\cos(dx+c)^3+(2A+3C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((2A+3C)\cos(dx+c)^3+(2A+3C)\,\text{co}\right.}{4\left(ad\cos(dx+c)^3+ad\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*(((2*A + 3*C)*cos(d*x + c)^3 + (2*A + 3*C)*cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - ((2*A + 3*C)*cos(d*x + c)^3 + (2*A + 3*C)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(2*(A + 2*C)*cos(d*x + c)^2 + C*cos(d*x + c) - C)*sin
(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{C\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x
)**4/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.22082, size = 176, normalized size = 1.64

$$\frac{\frac{(2A+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(2A+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-C\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((2*A + 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (2*A + 3*C)*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x +
 1/2*c))/a + 2*(3*C*tan(1/2*d*x + 1/2*c)^3 - C*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.124 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{(A+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{C\tan(c+dx)}{ad}-\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] -((C*ArcTanh[Sin[c + d*x]])/(a*d)) + (C*Tan[c + d*x])/(a*d) + ((A + C)*Tan[
c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Rubi [A] time = 0.151981, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4083, 3998, 3770, 3794}

$$\frac{(A+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{C\tan(c+dx)}{ad}-\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((C*ArcTanh[Sin[c + d*x]])/(a*d)) + (C*Tan[c + d*x])/(a*d) + ((A + C)*Tan[
c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \frac{C\tan(c+dx)}{ad} + \frac{\int \frac{\sec(c+dx)(aA-aC\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{a}$$
$$= \frac{C\tan(c+dx)}{ad} - \frac{C\int \sec(c+dx)\,dx}{a} + (A+C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{C\tanh^{-1}(\sin(c+dx))}{ad} + \frac{C\tan(c+dx)}{ad} + \frac{(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 1.79207, size = 227, normalized size = 3.98

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\left(A+C\sec^2(c+dx)\right)\left((A+C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+C\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)\right.}\right.\right.}{ad(\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*((A + C)*Sec[c/2]*S
in[(d*x)/2] + C*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2]
)*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])))))/(a*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c
 + d*x]))
```

Maple [B] time = 0.051, size = 121, normalized size = 2.1

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-1/a/d/(tan(1/2*d*x+1/
2*c)+1)*C-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C-1/a/d/(tan(1/2*d*x+1/2*c)-1)*C+1
/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.940495, size = 194, normalized size = 3.4

$$-\frac{C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - A*sin(d*x
 + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.496646, size = 288, normalized size = 5.05

$$-\frac{\left(C\cos\left(dx+c\right)^{2}+C\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(C\cos\left(dx+c\right)^{2}+C\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2}{2\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*((C*cos(d*x + c)^2 + C*cos(d*x + c))*log(sin(d*x + c) + 1) - (C*cos(d*
x + c)^2 + C*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*((A + 2*C)*cos(d*x +
c) + C)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^{3}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**
3/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.20918, size = 136, normalized size = 2.39

$$-\frac{\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -(C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - C*log(abs(tan(1/2*d*x + 1/2*c) -
 1))/a - (A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*C*tan(1/2*
d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.125 $\int \frac{A+C\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{(A+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(a*d) - ((A + C)*Tan[c + d*x])/(a*d*(1
+ Sec[c + d*x]))
```

Rubi [A] time = 0.107575, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4051, 3770, 3919, 3794}

$$-\frac{(A+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(a*d) - ((A + C)*Tan[c + d*x])/(a*d*(1
+ Sec[c + d*x]))
```


Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{aA-aC\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{a} + \frac{C\int \sec(c+dx)\,dx}{a}$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{ad} + (-A-C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.435218, size = 143, normalized size = 2.92

$$-\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A\cos^2(c+dx)+C\right)\left((A+C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-\cos\left(\frac{1}{2}(c+dx)\right)\left(Adx-C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.\right.}{ad(\cos(c+dx)+1)(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(C + A*Cos[c + d*x]^2)*(-(Cos[(c + d*x)/2]*(A*d*x - C*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + C*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]])) + (A + C)*Sec[c/2]*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x])*(A +
 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.06, size = 98, normalized size = 2.

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{A\arctan(\tan(1/2\,dx+c/2))}{ad}-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+2/a/d*A*arctan(tan(1/2*d*x+1/2*c))-1/a/d*C*tan(
1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C+1/a/d*ln(tan(1/2*d*x+1/2*c)
+1)*C
```

Maxima [B] time = 1.40678, size = 169, normalized size = 3.45

$$\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.506627, size = 242, normalized size = 4.94

$$\frac{2\,Adx\cos\left(dx+c\right)+2\,Adx+\left(C\cos\left(dx+c\right)+C\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(C\cos\left(dx+c\right)+C\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*d*x*cos(d*x + c) + 2*A*d*x + (C*cos(d*x + c) + C)*log(sin(d*x + c)
 + 1) - (C*cos(d*x + c) + C)*log(-sin(d*x + c) + 1) - 2*(A + C)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^{2}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*
x) + 1), x))/a
```

Giac [A] time = 1.21956, size = 108, normalized size = 2.2

$$\frac{\frac{(dx+c)A}{a}+\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A/a + C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a
)/d
```

3.126 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{(2A+C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{Ax}{a}$$

```
[Out] -((A*x)/a) + ((2*A + C)*Sin[c + d*x])/(a*d) - ((A + C)*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x]))
```

Rubi [A] time = 0.108917, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4085, 3787, 2637, 8}

$$\frac{(2A+C)\sin(c+dx)}{ad}-\frac{(A+C)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{Ax}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((A*x)/a) + ((2*A + C)*Sin[c + d*x])/(a*d) - ((A + C)*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos(c+dx)(-a(2A+C)+aA\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{(A+C)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{A\int 1\,dx}{a} + \frac{(2A+C)\int \cos(c+dx)\,dx}{a}$$
$$= -\frac{Ax}{a} + \frac{(2A+C)\sin(c+dx)}{ad} - \frac{(A+C)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.279259, size = 108, normalized size = 2.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(A\sin\left(c+\frac{dx}{2}\right)+A\sin\left(c+\frac{3dx}{2}\right)+A\sin\left(2c+\frac{3dx}{2}\right)-2Adx\cos\left(c+\frac{dx}{2}\right)+5A\sin\left(\frac{dx}{2}\right)-2Ad\right.}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*A*d*x*Cos[(d*x)/2] - 2*A*d*x*Cos[c + (d*x)/2
] + 5*A*Sin[(d*x)/2] + 4*C*Sin[(d*x)/2] + A*Sin[c + (d*x)/2] + A*Sin[c + (3
*d*x)/2] + A*Sin[2*c + (3*d*x)/2]))/(4*a*d)
```

Maple [A] time = 0.08, size = 88, normalized size = 1.7

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{A\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{A\arctan(\tan(1/2\,dx+c/2))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d*A*tan(1/2*d*x+1
/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.42461, size = 158, normalized size = 3.04

$$-\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-\frac{C\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - C*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.47804, size = 132, normalized size = 2.54

$$-\frac{Adx\cos\left(dx+c\right)+Adx-\left(A\cos\left(dx+c\right)+2\,A+C\right)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -(A*d*x*cos(d*x + c) + A*d*x - (A*cos(d*x + c) + 2*A + C)*sin(d*x + c))/(a*
d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int\frac{C\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*cos(c + d*x)*s
ec(c + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.17157, size = 100, normalized size = 1.92

$$-\frac{\frac{(dx+c)A}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((d*x + c)*A/a - (A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*A
*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.127 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{(2A+C)\sin(c+dx)}{ad}+\frac{(3A+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A+C)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A+2C)}{2a}$$

```
[Out] ((3*A + 2*C)*x)/(2*a) - ((2*A + C)*Sin[c + d*x])/(a*d) + ((3*A + 2*C)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rubi [A] time = 0.152574, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4085, 3787, 2635, 8, 2637}

$$-\frac{(2A+C)\sin(c+dx)}{ad}+\frac{(3A+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A+C)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A+2C)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((3*A + 2*C)*x)/(2*a) - ((2*A + C)*Sin[c + d*x])/(a*d) + ((3*A + 2*C)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^2(c+dx)(-a(3A+2C)+a(2A+C$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(2A+C)\int\cos(c+dx)\,dx}{a} + \frac{(3A+2C)}{}$$

$$= -\frac{(2A+C)\sin(c+dx)}{ad} + \frac{(3A+2C)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(A+C)\cos(c}{d(a+a\,s}$$

$$= \frac{(3A+2C)x}{2a} - \frac{(2A+C)\sin(c+dx)}{ad} + \frac{(3A+2C)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(}{}$$

Mathematica [A] time = 0.350722, size = 159, normalized size = 1.66

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4dx(3A+2C)\cos\left(c+\frac{dx}{2}\right)-4A\sin\left(c+\frac{dx}{2}\right)-3A\sin\left(c+\frac{3dx}{2}\right)-3A\sin\left(2c+\frac{3dx}{2}\right)+A\sin\left(2c\right.\right.}{8ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(3*A + 2*C)*d*x*Cos[(d*x)/2] + 4*(3*A + 2*C)*
d*x*Cos[c + (d*x)/2] - 20*A*Sin[(d*x)/2] - 16*C*Sin[(d*x)/2] - 4*A*Sin[c +
(d*x)/2] - 3*A*Sin[c + (3*d*x)/2] - 3*A*Sin[2*c + (3*d*x)/2] + A*Sin[2*c +
(5*d*x)/2] + A*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.099, size = 144, normalized size = 1.5

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\frac{(\tan(1/2\,dx+c/2))^3A}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(
1/2*d*x+1/2*c)+3/a/d*A*arctan(tan(1/2*d*x+1/2*c))+2/a/d*arctan(tan(1/2*d*x+
1/2*c))*C
```

Maxima [A] time = 1.4161, size = 248, normalized size = 2.58

$$-\frac{A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -(A*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) - C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.483527, size = 192, normalized size = 2.

$$\frac{(3A+2C)dx\cos(dx+c)+(3A+2C)dx+\left(A\cos(dx+c)^2-A\cos(dx+c)-4A-2C\right)\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((3*A + 2*C)*d*x*cos(d*x + c) + (3*A + 2*C)*d*x + (A*cos(d*x + c)^2 - A
*cos(d*x + c) - 4*A - 2*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16776, size = 130, normalized size = 1.35

$$\frac{\frac{(dx+c)(3A+2C)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((d*x + c)*(3*A + 2*C)/a - 2*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x +
1/2*c))/a - 2*(3*A*tan(1/2*d*x + 1/2*c)^3 + A*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.128 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{(4A+3C)\sin^3(c+dx)}{3ad}+\frac{(4A+3C)\sin(c+dx)}{ad}-\frac{(3A+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A+C)\sin(c+dx)\cos^2(c+d}{d(a\sec(c+dx)+a)}$$

```
[Out] -((3*A + 2*C)*x)/(2*a) + ((4*A + 3*C)*Sin[c + d*x])/(a*d) - ((3*A + 2*C)*Co
s[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(d
*(a + a*Sec[c + d*x])) - ((4*A + 3*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.168757, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4085, 3787, 2633, 2635, 8}

$$-\frac{(4A+3C)\sin^3(c+dx)}{3ad}+\frac{(4A+3C)\sin(c+dx)}{ad}-\frac{(3A+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A+C)\sin(c+dx)\cos^2(c+d}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((3*A + 2*C)*x)/(2*a) + ((4*A + 3*C)*Sin[c + d*x])/(a*d) - ((3*A + 2*C)*Co
s[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])/(d
*(a + a*Sec[c + d*x])) - ((4*A + 3*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^3(c+dx)(-a(4A+3C)+a(3A}{a^2}$$
$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A+2C)\int \cos^2(c+dx)\,dx}{a} + \frac{(4A}{}$$
$$= -\frac{(3A+2C)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} -$$
$$= -\frac{(3A+2C)x}{2a} + \frac{(4A+3C)\sin(c+dx)}{ad} - \frac{(3A+2C)\cos(c+dx)\sin(c+d}{2ad}$$

Mathematica [A] time = 0.794644, size = 225, normalized size = 1.81

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-12dx(3A+2C)\cos\left(c+\frac{dx}{2}\right)+21A\sin\left(c+\frac{dx}{2}\right)+18A\sin\left(c+\frac{3dx}{2}\right)+18A\sin\left(2c+\frac{3dx}{2}\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-12*(3*A + 2*C)*d*x*Cos[(d*x)/2] - 12*(3*A + 2*
C)*d*x*Cos[c + (d*x)/2] + 69*A*Sin[(d*x)/2] + 60*C*Sin[(d*x)/2] + 21*A*Sin[
c + (d*x)/2] + 12*C*Sin[c + (d*x)/2] + 18*A*Sin[c + (3*d*x)/2] + 12*C*Sin[c
 + (3*d*x)/2] + 18*A*Sin[2*c + (3*d*x)/2] + 12*C*Sin[2*c + (3*d*x)/2] - 2*A
*Sin[2*c + (5*d*x)/2] - 2*A*Sin[3*c + (5*d*x)/2] + A*Sin[3*c + (7*d*x)/2] +
 A*Sin[4*c + (7*d*x)/2]))/(24*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.092, size = 280, normalized size = 2.3

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\frac{(\tan(1/2\,dx+c/2))^5 A}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+2\frac{(\tan(1/2\,dx+c/2))^5 C}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{16A}{3ad}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)+5/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)^5*C+16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+
4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*C+3/a/d/(1+tan(1/2*d*
x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1
/2*d*x+1/2*c)-3/a/d*A*arctan(tan(1/2*d*x+1/2*c))-2/a/d*arctan(tan(1/2*d*x+1
/2*c))*C
```

Maxima [B] time = 1.42638, size = 363, normalized size = 2.93

$$\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(c}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*C*(2*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.48975, size = 243, normalized size = 1.96

$$-\frac{3(3A+2C)dx\cos(dx+c)+3(3A+2C)dx-\left(2A\cos(dx+c)^3-A\cos(dx+c)^2+(7A+6C)\cos(dx+c)+16A\right.}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(3*A + 2*C)*d*x*cos(d*x + c) + 3*(3*A + 2*C)*d*x - (2*A*cos(d*x + c
)^3 - A*cos(d*x + c)^2 + (7*A + 6*C)*cos(d*x + c) + 16*A + 12*C)*sin(d*x +
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17271, size = 205, normalized size = 1.65

$$-\frac{\frac{3(dx+c)(3A+2C)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+16A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(3*A + 2*C)/a - 6*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x
 + 1/2*c))/a - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5
+ 16*A*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d
*x + 1/2*c) + 6*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))
/d
```

3.129 $\int \frac{\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{(4A+3C)\sin^3(c+dx)}{3ad}-\frac{(4A+3C)\sin(c+dx)}{ad}+\frac{(5A+4C)\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3(5A+4C)\sin(c+dx)\cos(c+}{8ad}$$

```
[Out] (3*(5*A + 4*C)*x)/(8*a) - ((4*A + 3*C)*Sin[c + d*x])/(a*d) + (3*(5*A + 4*C)
*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((5*A + 4*C)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
)) + ((4*A + 3*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.186714, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4085, 3787, 2635, 8, 2633}

$$\frac{(4A+3C)\sin^3(c+dx)}{3ad}-\frac{(4A+3C)\sin(c+dx)}{ad}+\frac{(5A+4C)\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3(5A+4C)\sin(c+dx)\cos(c+}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```


```
[Out] (3*(5*A + 4*C)*x)/(8*a) - ((4*A + 3*C)*Sin[c + d*x])/(a*d) + (3*(5*A + 4*C)
*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((5*A + 4*C)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a*d) - ((A + C)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
)) + ((4*A + 3*C)*Sin[c + d*x]^3)/(3*a*d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^4(c+dx)(-a(5A+4C)+a(4A}{a^2}$$

$$= -\frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(4A+3C)\int\cos^3(c+dx)\,dx}{a} + \frac{(5A}{}$$

$$= \frac{(5A+4C)\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{(A+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} +$$

$$= -\frac{(4A+3C)\sin(c+dx)}{ad} + \frac{3(5A+4C)\cos(c+dx)\sin(c+dx)}{8ad} + \frac{(5A+4C}{}$$

$$= \frac{3(5A+4C)x}{8a} - \frac{(4A+3C)\sin(c+dx)}{ad} + \frac{3(5A+4C)\cos(c+dx)\sin(c+d}{8ad}$$

Mathematica [A] time = 0.695898, size = 283, normalized size = 1.81

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(72dx(5A+4C)\cos\left(c+\frac{dx}{2}\right)-168A\sin\left(c+\frac{dx}{2}\right)-120A\sin\left(c+\frac{3dx}{2}\right)-120A\sin\left(2c+\frac{3dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(72*(5*A + 4*C)*d*x*Cos[(d*x)/2] + 72*(5*A + 4*C
)*d*x*Cos[c + (d*x)/2] - 552*A*Sin[(d*x)/2] - 480*C*Sin[(d*x)/2] - 168*A*Si
n[c + (d*x)/2] - 96*C*Sin[c + (d*x)/2] - 120*A*Sin[c + (3*d*x)/2] - 72*C*Si
n[c + (3*d*x)/2] - 120*A*Sin[2*c + (3*d*x)/2] - 72*C*Sin[2*c + (3*d*x)/2] +
 40*A*Sin[2*c + (5*d*x)/2] + 24*C*Sin[2*c + (5*d*x)/2] + 40*A*Sin[3*c + (5*
d*x)/2] + 24*C*Sin[3*c + (5*d*x)/2] - 5*A*Sin[3*c + (7*d*x)/2] - 5*A*Sin[4*
c + (7*d*x)/2] + 3*A*Sin[4*c + (9*d*x)/2] + 3*A*Sin[5*c + (9*d*x)/2]))/(192
*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.1, size = 352, normalized size = 2.3

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{25A}{4ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-3\frac{(\tan(1/2\,dx+c/2))}{ad\left(1+(\tan(1/2\,dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-25/4/a/d/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A-3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^7*C-115/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c
)^5*A-7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*C-109/12/a/d/(1
+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A-5/a/d/(1+tan(1/2*d*x+1/2*c)
```

```
^2)^4*tan(1/2*d*x+1/2*c)^3*C-7/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*A*tan(1/2*d
*x+1/2*c)-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*C*tan(1/2*d*x+1/2*c)+15/4/a/d*A*
arctan(tan(1/2*d*x+1/2*c))+3/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.43518, size = 474, normalized size = 3.04

$$-\frac{A\left(\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+12C\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)}{(\cos(dx+c)+1}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx}{(\cos(dx+c)}}\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(A*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) + 12*C*((sin(d*x +
c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) -
 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c
) + 1))))/d
```

Fricas [A] time = 0.497758, size = 290, normalized size = 1.86

$$\frac{9(5A+4C)dx\cos(dx+c)+9(5A+4C)dx+\left(6A\cos(dx+c)^4-2A\cos(dx+c)^3+(13A+12C)\cos(dx+c)^2-(1\right.}{24(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(9*(5*A + 4*C)*d*x*cos(d*x + c) + 9*(5*A + 4*C)*d*x + (6*A*cos(d*x + c
)^4 - 2*A*cos(d*x + c)^3 + (13*A + 12*C)*cos(d*x + c)^2 - (19*A + 12*C)*cos
(d*x + c) - 64*A - 48*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16628, size = 243, normalized size = 1.56

$$\frac{\frac{9\,(dx+c)(5\,A+4\,C)}{a}-\frac{24\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(75\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+36\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+115\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+84\,C\tan\left(\frac{1}{2}\,dx+\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(9*(d*x + c)*(5*A + 4*C)/a - 24*(A*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*
x + 1/2*c))/a - 2*(75*A*tan(1/2*d*x + 1/2*c)^7 + 36*C*tan(1/2*d*x + 1/2*c)^
7 + 115*A*tan(1/2*d*x + 1/2*c)^5 + 84*C*tan(1/2*d*x + 1/2*c)^5 + 109*A*tan(
1/2*d*x + 1/2*c)^3 + 60*C*tan(1/2*d*x + 1/2*c)^3 + 21*A*tan(1/2*d*x + 1/2*c
) + 12*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.130 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=172

$$\frac{(5A+12C)\tan^3(c+dx)}{3a^2d}+\frac{(5A+12C)\tan(c+dx)}{a^2d}-\frac{(2A+5C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2(2A+5C)\tan(c+dx)\sec^3(c}{3a^2d(\sec(c+dx)+1)}$$

```
[Out] -(((2*A + 5*C)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + ((5*A + 12*C)*Tan[c + d*x]
)/(a^2*d) - ((2*A + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(a^2*d) - (2*(2*A + 5*C
)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[
c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((5*A + 12*C)*Tan[c
 + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.329126, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4019, 3787, 3768, 3770, 3767}

$$\frac{(5A+12C)\tan^3(c+dx)}{3a^2d}+\frac{(5A+12C)\tan(c+dx)}{a^2d}-\frac{(2A+5C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2(2A+5C)\tan(c+dx)\sec^3(c}{3a^2d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((2*A + 5*C)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + ((5*A + 12*C)*Tan[c + d*x]
)/(a^2*d) - ((2*A + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(a^2*d) - (2*(2*A + 5*C
)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[
c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((5*A + 12*C)*Tan[c
 + d*x]^3)/(3*a^2*d)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^4(c+dx)(a(A+4C)-3a(A+2C)\sec(c+dx))}{a+a\sec(c+dx)}\,d}{3a^2}$$

$$= -\frac{2(2A+5C)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{2(2A+5C)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(2A+5C)\sec(c+dx)\tan(c+dx)}{a^2d} - \frac{2(2A+5C)\sec^3(c+dx)\tan(c+d}{3a^2d(1+\sec(c+dx))}$$

$$= -\frac{(2A+5C)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{(5A+12C)\tan(c+dx)}{a^2d} - \frac{(2A+5C)\,s}{}$$

Mathematica [B] time = 2.94003, size = 623, normalized size = 3.62

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left(-60A\sin\left(c-\frac{dx}{2}\right)+24A\sin\left(c+\frac{dx}{2}\right)-60A\sin\left(2c\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*(A + C*Sec[c + d*x]^2)*(192*(2*A + 5*C)*Cos[(c + d*x)/2]^
3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^3*(-3*(8*A + C)*Sin[(d*x)/2] +
(66*A + 155*C)*Sin[(3*d*x)/2] - 60*A*Sin[c - (d*x)/2] - 153*C*Sin[c - (d*x)
/2] + 24*A*Sin[c + (d*x)/2] + 21*C*Sin[c + (d*x)/2] - 60*A*Sin[2*c + (d*x)/
2] - 135*C*Sin[2*c + (d*x)/2] - 4*A*Sin[c + (3*d*x)/2] + 25*C*Sin[c + (3*d*
x)/2] + 36*A*Sin[2*c + (3*d*x)/2] + 45*C*Sin[2*c + (3*d*x)/2] - 34*A*Sin[3*
c + (3*d*x)/2] - 85*C*Sin[3*c + (3*d*x)/2] + 42*A*Sin[c + (5*d*x)/2] + 99*C
*Sin[c + (5*d*x)/2] + 21*C*Sin[2*c + (5*d*x)/2] + 24*A*Sin[3*c + (5*d*x)/2]
 + 33*C*Sin[3*c + (5*d*x)/2] - 18*A*Sin[4*c + (5*d*x)/2] - 45*C*Sin[4*c + (
5*d*x)/2] + 24*A*Sin[2*c + (7*d*x)/2] + 57*C*Sin[2*c + (7*d*x)/2] + 3*A*Sin
```

```
[3*c + (7*d*x)/2] + 18*C*Sin[3*c + (7*d*x)/2] + 15*A*Sin[4*c + (7*d*x)/2] +
 24*C*Sin[4*c + (7*d*x)/2] - 6*A*Sin[5*c + (7*d*x)/2] - 15*C*Sin[5*c + (7*d
*x)/2] + 10*A*Sin[3*c + (9*d*x)/2] + 24*C*Sin[3*c + (9*d*x)/2] + 3*A*Sin[4*
c + (9*d*x)/2] + 11*C*Sin[4*c + (9*d*x)/2] + 7*A*Sin[5*c + (9*d*x)/2] + 13*
C*Sin[5*c + (9*d*x)/2])))/(24*a^2*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec
[c + d*x])^2)
```

Maple [B] time = 0.071, size = 338, normalized size = 2.

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{9\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)\right.}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+9/2/d/a^2*C*tan(1/2*d*x+1/2*c)-2/d/a^2*ln(tan(1/2*d*x
+1/2*c)+1)*A-5/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-5/d/a^2/(tan(1/2*d*x+1/2*c)
+1)*C-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A-1/3/d/a^2*C/(tan(1/2*d*x+1/2*c)+1)^3
+3/2/d/a^2*C/(tan(1/2*d*x+1/2*c)+1)^2-5/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C-1/d/
a^2/(tan(1/2*d*x+1/2*c)-1)*A+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A+5/d/a^2*ln(
tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^2*C/(tan(1/2*d*x+1/2*c)-1)^3-3/2/d/a^2*C/(t
an(1/2*d*x+1/2*c)-1)^2
```

Maxima [B] time = 0.965534, size = 512, normalized size = 2.98

$$\frac{C\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)+A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] 1/6*(C*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) + A*
((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.526268, size = 591, normalized size = 3.44

$$-\frac{3\left((2\,A+5\,C)\cos\left(dx+c\right)^5+2\,(2\,A+5\,C)\cos\left(dx+c\right)^4+(2\,A+5\,C)\cos\left(dx+c\right)^3\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left((2\,A+5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] -1/6*(3*((2*A + 5*C)*cos(d*x + c)^5 + 2*(2*A + 5*C)*cos(d*x + c)^4 + (2*A +
 5*C)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((2*A + 5*C)*cos(d*x + c)^5
 + 2*(2*A + 5*C)*cos(d*x + c)^4 + (2*A + 5*C)*cos(d*x + c)^3)*log(-sin(d*x
+ c) + 1) - 2*(2*(5*A + 12*C)*cos(d*x + c)^4 + (14*A + 33*C)*cos(d*x + c)^3
 + 3*(A + 2*C)*cos(d*x + c)^2 - C*cos(d*x + c) + C)*sin(d*x + c))/(a^2*d*co
s(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(C*sec(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.18158, size = 304, normalized size = 1.77

$$-\frac{\frac{6(2A+5C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{6(2A+5C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{4\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-20\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cdots\right.}}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/6*(6*(2*A + 5*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*A + 5*C)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(3*A*tan(1/2*d*x + 1/2*c)^5 + 15
*C*tan(1/2*d*x + 1/2*c)^5 - 6*A*tan(1/2*d*x + 1/2*c)^3 - 20*C*tan(1/2*d*x +
 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) + 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*
d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*
d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) + 27*C*a^4*tan(1/2*d*x + 1/2
*c))/a^6)/d
```

3.131 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{4(A+4C)\tan(c+dx)}{3a^2d}+\frac{(2A+7C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2(A+4C)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(2A+7C)\tan(c+d}{2a^2d}$$

```
[Out] ((2*A + 7*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(A + 4*C)*Tan[c + d*x])/
(3*a^2*d) + ((2*A + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - (2*(A + 4*C
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[
c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.305003, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4085, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{4(A+4C)\tan(c+dx)}{3a^2d}+\frac{(2A+7C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2(A+4C)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(2A+7C)\tan(c+d}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((2*A + 7*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(A + 4*C)*Tan[c + d*x])/
(3*a^2*d) + ((2*A + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - (2*(A + 4*C
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[
c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)(3aC-a(2A+5C)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{2(A+4C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{2(A+4C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(2A+7C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{2(A+4C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}$$

$$= \frac{(2A+7C)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{4(A+4C)\tan(c+dx)}{3a^2d} + \frac{(2A+7C)\sec}{}$$

Mathematica [B] time = 2.11466, size = 513, normalized size = 3.42

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(-36A\sin\left(c-\frac{dx}{2}\right)+36A\sin\left(c+\frac{dx}{2}\right)-20A\sin\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]*(A + C*Sec[c + d*x]^2)*(96*(2*A + 7*C)*Cos[(c + d*x)/2]^
3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-2*(10*A + 7*C)*Sin[(d*x)/2]
 + (22*A + 97*C)*Sin[(3*d*x)/2] - 36*A*Sin[c - (d*x)/2] - 126*C*Sin[c - (d*
x)/2] + 36*A*Sin[c + (d*x)/2] + 42*C*Sin[c + (d*x)/2] - 20*A*Sin[2*c + (d*x
)/2] - 98*C*Sin[2*c + (d*x)/2] - 18*A*Sin[c + (3*d*x)/2] - 3*C*Sin[c + (3*d
*x)/2] + 22*A*Sin[2*c + (3*d*x)/2] + 37*C*Sin[2*c + (3*d*x)/2] - 18*A*Sin[3
*c + (3*d*x)/2] - 63*C*Sin[3*c + (3*d*x)/2] + 18*A*Sin[c + (5*d*x)/2] + 75*
C*Sin[c + (5*d*x)/2] - 6*A*Sin[2*c + (5*d*x)/2] + 15*C*Sin[2*c + (5*d*x)/2]
 + 18*A*Sin[3*c + (5*d*x)/2] + 39*C*Sin[3*c + (5*d*x)/2] - 6*A*Sin[4*c + (5
```

```
*d*x)/2] - 21*C*Sin[4*c + (5*d*x)/2] + 8*A*Sin[2*c + (7*d*x)/2] + 32*C*Sin[
2*c + (7*d*x)/2] + 12*C*Sin[3*c + (7*d*x)/2] + 8*A*Sin[4*c + (7*d*x)/2] + 2
0*C*Sin[4*c + (7*d*x)/2])))/(24*a^2*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + S
ec[c + d*x])^2)
```

Maple [A] time = 0.069, size = 249, normalized size = 1.7

$$-\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{A}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)+1/d/a^2*ln(tan(1/2*d*
x+1/2*c)+1)*A+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^2*C/(tan(1/2*d*x
+1/2*c)+1)^2+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^2*ln(tan(1/2*d*x+1/2*
c)-1)*A-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^2*C/(tan(1/2*d*x+1/2*c
)-1)^2+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.95685, size = 389, normalized size = 2.59

$$-\frac{C\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)+A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)}{(\cos(dx+c)+}}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/6*(C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) + A*((9*sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a^2))/d
```

Fricas [A] time = 0.516834, size = 554, normalized size = 3.69

$$\frac{3\left((2A+7C)\cos(dx+c)^4+2(2A+7C)\cos(dx+c)^3+(2A+7C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((2A+7C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/12*(3*((2*A + 7*C)*cos(d*x + c)^4 + 2*(2*A + 7*C)*cos(d*x + c)^3 + (2*A +
 7*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((2*A + 7*C)*cos(d*x + c)^4
 + 2*(2*A + 7*C)*cos(d*x + c)^3 + (2*A + 7*C)*cos(d*x + c)^2)*log(-sin(d*x
+ c) + 1) - 2*(8*(A + 4*C)*cos(d*x + c)^3 + (10*A + 43*C)*cos(d*x + c)^2 +
6*C*cos(d*x + c) - 3*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d
*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(C*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.25169, size = 231, normalized size = 1.54

$$\frac{\frac{3(2A+7C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{3(2A+7C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{6\left(5C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2} - \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ca^4 \cdots}{\cdots}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/6*(3*(2*A + 7*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(2*A + 7*C)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*C*tan(1/2*d*x + 1/2*c)^3 - 3*C
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/
2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2
*c) + 21*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.132 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=99

$$\frac{(A+4C)\tan(c+dx)}{3a^2d}-\frac{2C\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2C\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (-2*C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((A + 4*C)*Tan[c + d*x])/(3*a^2*d) +
 (2*C*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.252496, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4008, 3787, 3770, 3767, 8}

$$\frac{(A+4C)\tan(c+dx)}{3a^2d}-\frac{2C\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2C\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((A + 4*C)*Tan[c + d*x])/(3*a^2*d) +
 (2*C*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(-a(A-2C)-a(A+4C)\sec(c+dx))}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{2C\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)(}{}$$

$$= \frac{2C\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(2C)\int \sec(c+}{a^2}$$

$$= -\frac{2C\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{2C\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^2(c+dx)\,t}{3d(a+a\sec(c+}$$

$$= -\frac{2C\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{(A+4C)\tan(c+dx)}{3a^2d} + \frac{2C\tan(c+dx)}{a^2d(1+\sec(c+dx))}$$

Mathematica [B] time = 1.47486, size = 280, normalized size = 2.83

$$4\cos\left(\frac{1}{2}(c+dx)\right)\left(A+C\sec^2(c+dx)\right)\left((A+C)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A+C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(A+7C)\sec\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(A + C*Sec[c + d*x]^2)*((A + C)*Sec[c/2]*Sin[(d*x)/2] +
 2*(A + 7*C)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*C*Cos[(c + d*x)/2
]^3*(2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
) + (A + C)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(A + 2*C + A*Cos[2*(c + d*
x)])*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.056, size = 164, normalized size = 1.7

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\,C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2/(tan(1/2*d*x+1
```

```
/2*c)+1)*C-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^2/(tan(1/2*d*x+1/2*c)-1
)*C+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.953288, size = 258, normalized size = 2.61

$$\frac{C\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)+\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)}\right.}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] 1/6*(C*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) + A*(3*sin(d*x + c)/(cos(
d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.504845, size = 429, normalized size = 4.33

$$-\frac{3\left(C\cos(dx+c)^3+2C\cos(dx+c)^2+C\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left(C\cos(dx+c)^3+2C\cos(dx+c)^2+\right.}{3\left(a^2d\cos(dx+c)^3+2a^2d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] -1/3*(3*(C*cos(d*x + c)^3 + 2*C*cos(d*x + c)^2 + C*cos(d*x + c))*log(sin(d*
x + c) + 1) - 3*(C*cos(d*x + c)^3 + 2*C*cos(d*x + c)^2 + C*cos(d*x + c))*lo
g(-sin(d*x + c) + 1) - ((A + 10*C)*cos(d*x + c)^2 + 2*(A + 7*C)*cos(d*x + c
) + 3*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{C\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(C*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.19624, size = 192, normalized size = 1.94

$$-\frac{\frac{12\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{12\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Aa^4\tan\left(\frac{1}{2}\,d\right.}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/6*(12*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*C*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^2 + 12*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 3
*A*a^4*tan(1/2*d*x + 1/2*c) + 15*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.133 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{(A-5C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{(A+C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((A - 5*C)*Tan[c + d*x])/(3*a^2*d*(1 +
Sec[c + d*x])) + ((A + C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.162227, antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4079, 3998, 3770, 3794}

$$\frac{2(A-2C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^2*d) + (2*(A - 2*C)*Tan[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rule 4079

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A + C)*Cot[e +
 f*x]*Csc[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] - Dist[1/(a*b*
(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[-(b*C) - 2*A
*b*(m + 1) + a*(A*(m + 2) - C*(m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(a(2A-C)+3aC\sec(c+dx))}{a+a\sec(c+dx)}dx}{3a^2}$$
$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2(A-2C))\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}dx}{3a} + \frac{C\int \mathrm{se}}{}$$
$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{2(A-2C)\tan}{3d\left(a^2+a^2\sec\right.}$$

Mathematica [B] time = 0.831484, size = 377, normalized size = 5.03

$$-\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6A\sin\left(c+\frac{dx}{2}\right)-4A\sin\left(c+\frac{3dx}{2}\right)-6A\sin\left(\frac{dx}{2}\right)-6C\sin\left(c+\frac{dx}{2}\right)+8C\sin\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c/2]*Sec[c + d*x]^2*(3*C*Cos[c + (3*d*x)/2]*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*C*Cos[2*c + (3*d*x)/2]*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] + 9*C*Cos[(d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 9*C*Cos[c + (d*x)
/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) - 3*C*Cos[c + (3*d*x)/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 3*C*Cos[2*c + (3*d*x)/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
 6*A*Sin[(d*x)/2] + 18*C*Sin[(d*x)/2] + 6*A*Sin[c + (d*x)/2] - 6*C*Sin[c +
(d*x)/2] - 4*A*Sin[c + (3*d*x)/2] + 8*C*Sin[c + (3*d*x)/2]))/(6*a^2*d*(1 +
Sec[c + d*x])^2)
```

Maple [A] time = 0.059, size = 119, normalized size = 1.6

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan(1/2*d*
x+1/2*c)-1)*C+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [B] time = 0.945317, size = 197, normalized size = 2.63

$$-\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*(3*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.497714, size = 338, normalized size = 4.51

$$\frac{3\left(C\cos\left(dx+c\right)^{2}+2\,C\cos\left(dx+c\right)+C\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(C\cos\left(dx+c\right)^{2}+2\,C\cos\left(dx+c\right)+C\right)\log\left(-\sin\left(dx\right.\right.}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*log(sin(d*x + c) + 1) - 3*
(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*log(-sin(d*x + c) + 1) + 2*(2*(A
- 2*C)*cos(d*x + c) + A - 5*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(C*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.21579, size = 151, normalized size = 2.01

$$\frac{\frac{6\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{6\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}-\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*C*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2
*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) + 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.134 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$-\frac{2(2A-C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A+C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (A*x)/a^2 - (2*(2*A - C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A +
 C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.120653, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4053, 3919, 3794}

$$-\frac{2(2A-C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A+C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (A*x)/a^2 - (2*(2*A - C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A +
 C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{(A+C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{-3aA+a(A-2C)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{Ax}{a^2}-\frac{(A+C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(2(2A-C))\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\
&= \frac{Ax}{a^2}-\frac{(A+C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{2(2A-C)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}
\end{aligned}$$

Mathematica [B] time = 0.495736, size = 141, normalized size = 2.07

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(12A\sin\left(c+\frac{dx}{2}\right)-10A\sin\left(c+\frac{3dx}{2}\right)+9Adx\cos\left(c+\frac{dx}{2}\right)+3Adx\cos\left(c+\frac{3dx}{2}\right)+3Adx\cos\left(2c\right.\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*A*d*x*Cos[(d*x)/2] + 9*A*d*x*Cos[c + (d*x)/
2] + 3*A*d*x*Cos[c + (3*d*x)/2] + 3*A*d*x*Cos[2*c + (3*d*x)/2] - 18*A*Sin[(
d*x)/2] + 6*C*Sin[(d*x)/2] + 12*A*Sin[c + (d*x)/2] - 10*A*Sin[c + (3*d*x)/2
] + 2*C*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.057, size = 97, normalized size = 1.4

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{A\arctan(\tan(1/2\,dx}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*A*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.41635, size = 161, normalized size = 2.37

$$-\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{C\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - C*(3*sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.468711, size = 228, normalized size = 3.35

$$\frac{3\,Adx\cos(dx+c)^2+6\,Adx\cos(dx+c)+3\,Adx-((5\,A-C)\cos(dx+c)+4\,A-2\,C)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*d*x*cos(d*x + c)^2 + 6*A*d*x*cos(d*x + c) + 3*A*d*x - ((5*A - C)*c
os(d*x + c) + 4*A - 2*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(C*sec(c +
 d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.19422, size = 113, normalized size = 1.66

$$\frac{\frac{6\,(dx+c)A}{a^2} + \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 3\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.135 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{(10A+C)\sin(c+dx)}{3a^2d}-\frac{2A\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{2Ax}{a^2}-\frac{(A+C)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (-2*A*x)/a^2 + ((10*A + C)*Sin[c + d*x])/(3*a^2*d) - (2*A*Sin[c + d*x])/(a^
2*d*(1 + Sec[c + d*x])) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^
2)
```

Rubi [A] time = 0.217872, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4085, 4020, 3787, 2637, 8}

$$\frac{(10A+C)\sin(c+dx)}{3a^2d}-\frac{2A\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{2Ax}{a^2}-\frac{(A+C)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*A*x)/a^2 + ((10*A + C)*Sin[c + d*x])/(3*a^2*d) - (2*A*Sin[c + d*x])/(a^
2*d*(1 + Sec[c + d*x])) - ((A + C)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^
2)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}dx = -\frac{(A+C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)(-a(4A+C)+a(2A-C)\sec(c+dx))}{a+a\sec(c+dx)}dx}{3a^2}$$
$$= -\frac{2A\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos(c+dx)\left(-a^2(10A\right.}{}$$
$$= -\frac{2A\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(2A)\int 1\,dx}{a^2} + \frac{(10A+C)}{}$$
$$= -\frac{2Ax}{a^2} + \frac{(10A+C)\sin(c+dx)}{3a^2d} - \frac{2A\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sin(c+}{3d(a+a\sec(c+}$$

Mathematica [B] time = 0.764618, size = 195, normalized size = 2.38

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-30A\sin\left(c+\frac{dx}{2}\right)+41A\sin\left(c+\frac{3dx}{2}\right)+9A\sin\left(2c+\frac{3dx}{2}\right)+3A\sin\left(2c+\frac{5dx}{2}\right)+3A\sin\left(3c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-36*A*d*x*Cos[(d*x)/2] - 36*A*d*x*Cos[c + (d*
x)/2] - 12*A*d*x*Cos[c + (3*d*x)/2] - 12*A*d*x*Cos[2*c + (3*d*x)/2] + 66*A*
Sin[(d*x)/2] + 12*C*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2] - 12*C*Sin[c + (d*
x)/2] + 41*A*Sin[c + (3*d*x)/2] + 8*C*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3
*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2]))/(48*a^2*d)
```

Maple [A] time = 0.088, size = 130, normalized size = 1.6

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{5A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{A\tan(1/2\,d}{da^2\left(1+(\tan(1/\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*A*tan(1/2*d*x
+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.42876, size = 223, normalized size = 2.72

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right) + \frac{C\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) +
 C*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/a^2)/d
```

Fricas [A] time = 0.483188, size = 261, normalized size = 3.18

$$-\frac{6\,Adx\cos\left(dx+c\right)^{2}+12\,Adx\cos\left(dx+c\right)+6\,Adx-\left(3\,A\cos\left(dx+c\right)^{2}+2\,(7\,A+C)\cos\left(dx+c\right)+10\,A+C\right)\sin\left(dx+\right.}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*A*d*x*cos(d*x + c)^2 + 12*A*d*x*cos(d*x + c) + 6*A*d*x - (3*A*cos(d
*x + c)^2 + 2*(7*A + C)*cos(d*x + c) + 10*A + C)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{C\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(C*cos(c + d*x)*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1),
x))/a**2
```

Giac [A] time = 1.21883, size = 154, normalized size = 1.88

$$-\frac{\frac{12\,(dx+c)A}{a^{2}}-\frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}+\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(d*x + c)*A/a^2 - 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3
 - 15*A*a^4*tan(1/2*d*x + 1/2*c) - 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.136 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{4(4A+C)\sin(c+dx)}{3a^2d}+\frac{(7A+2C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2(4A+C)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7A+2C)}{2a^2}-\frac{(A+$$

```
[Out] ((7*A + 2*C)*x)/(2*a^2) - (4*(4*A + C)*Sin[c + d*x])/(3*a^2*d) + ((7*A + 2*
C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - (2*(4*A + C)*Cos[c + d*x]*Sin[c +
 d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x]*Sin[c + d*x])/(
3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.310433, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4020, 3787, 2635, 8, 2637}

$$-\frac{4(4A+C)\sin(c+dx)}{3a^2d}+\frac{(7A+2C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2(4A+C)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7A+2C)}{2a^2}-\frac{(A+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((7*A + 2*C)*x)/(2*a^2) - (4*(4*A + C)*Sin[c + d*x])/(3*a^2*d) + ((7*A + 2*
C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - (2*(4*A + C)*Cos[c + d*x]*Sin[c +
 d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x]*Sin[c + d*x])/(
3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(-a(5A+2C)+3aA\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{2(4A+C)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \ldots$$

$$= -\frac{2(4A+C)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - (\ldots$$

$$= -\frac{4(4A+C)\sin(c+dx)}{3a^2d} + \frac{(7A+2C)\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{2(4A+C)\ldots}{3a^2\ldots}$$

$$= \frac{(7A+2C)x}{2a^2} - \frac{4(4A+C)\sin(c+dx)}{3a^2d} + \frac{(7A+2C)\cos(c+dx)\sin(c+dx)}{2a^2d}\ldots$$

Mathematica [B] time = 1.14137, size = 281, normalized size = 2.05

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(36dx(7A+2C)\cos\left(c+\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-239A\sin\left(c+\frac{3dx}{2}\right)-63A\sin\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*Sec[c + d*x]^2*(36*(7*A + 2*C)*d*x*Cos[(d*x)/2]
+ 36*(7*A + 2*C)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] + 24*C*
d*x*Cos[c + (3*d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] + 24*C*d*x*Cos[2*c +
 (3*d*x)/2] - 381*A*Sin[(d*x)/2] - 144*C*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)
/2] + 96*C*Sin[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] - 80*C*Sin[c + (3*d*
x)/2] - 63*A*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*
c + (5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] + 3*A*Sin[4*c + (7*d*x)/2]))/(48*
a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.094, size = 184, normalized size = 1.3

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\frac{(\tan(1/2dx+\ldots}{da^2\left(1+(\tan(1/2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*A*ta
n(1/2*d*x+1/2*c)+7/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^2*arctan(tan(1/
2*d*x+1/2*c))*C
```

Maxima [A] time = 1.42979, size = 319, normalized size = 2.33

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)+C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) + C*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 0.490655, size = 332, normalized size = 2.42

$$\frac{3\,(7\,A+2\,C)dx\cos{(dx+c)}^2+6\,(7\,A+2\,C)dx\cos{(dx+c)}+3\,(7\,A+2\,C)dx+\left(3\,A\cos{(dx+c)}^3-6\,A\cos{(dx+c)}^2-\right.}{6\left(a^2d\cos{(dx+c)}^2+2\,a^2d\cos{(dx+c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(7*A + 2*C)*d*x*cos(d*x + c)^2 + 6*(7*A + 2*C)*d*x*cos(d*x + c) + 3*
(7*A + 2*C)*d*x + (3*A*cos(d*x + c)^3 - 6*A*cos(d*x + c)^2 - (43*A + 10*C)*
cos(d*x + c) - 32*A - 8*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

[Out] Timed out

Giac [A] time = 1.21661, size = 185, normalized size = 1.35

$$\frac{\frac{3\,(dx+c)(7\,A+2\,C)}{a^2}-\frac{6\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A + 2*C)/a^2 - 6*(5*A*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x
 + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c)
- 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.137 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(12A+5C)\sin^3(c+dx)}{3a^2d}+\frac{(12A+5C)\sin(c+dx)}{a^2d}-\frac{(5A+2C)\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{2(5A+2C)\sin(c+dx)\cos^2}{3a^2d(\sec(c+dx)+1}$$

```
[Out] -(((5*A + 2*C)*x)/a^2) + ((12*A + 5*C)*Sin[c + d*x])/(a^2*d) - ((5*A + 2*C)
*Cos[c + d*x]*Sin[c + d*x])/(a^2*d) - (2*(5*A + 2*C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2) - ((12*A + 5*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.324637, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4020, 3787, 2633, 2635, 8}

$$-\frac{(12A+5C)\sin^3(c+dx)}{3a^2d}+\frac{(12A+5C)\sin(c+dx)}{a^2d}-\frac{(5A+2C)\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{2(5A+2C)\sin(c+dx)\cos^2}{3a^2d(\sec(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((5*A + 2*C)*x)/a^2) + ((12*A + 5*C)*Sin[c + d*x])/(a^2*d) - ((5*A + 2*C)
*Cos[c + d*x]*Sin[c + d*x])/(a^2*d) - (2*(5*A + 2*C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x]^2*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2) - ((12*A + 5*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(-3a(2A+C)+a(4A+C)\sec(c+dx))}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{2(5A+2C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{2(5A+2C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(5A+2C)\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{2(5A+2C)\cos^2(c+dx)\sin(c+dx}{3a^2d(1+\sec(c+dx))}$$

$$= -\frac{(5A+2C)x}{a^2} + \frac{(12A+5C)\sin(c+dx)}{a^2d} - \frac{(5A+2C)\cos(c+dx)\sin(c+}{a^2d}$$

Mathematica [B] time = 0.911198, size = 349, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-72dx(5A+2C)\cos\left(c+\frac{dx}{2}\right)-156A\sin\left(c+\frac{dx}{2}\right)+342A\sin\left(c+\frac{3dx}{2}\right)+118A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*Sec[c + d*x]^2*(-72*(5*A + 2*C)*d*x*Cos[(d*x)/2]
 - 72*(5*A + 2*C)*d*x*Cos[c + (d*x)/2] - 120*A*d*x*Cos[c + (3*d*x)/2] - 48*
C*d*x*Cos[c + (3*d*x)/2] - 120*A*d*x*Cos[2*c + (3*d*x)/2] - 48*C*d*x*Cos[2*
c + (3*d*x)/2] + 516*A*Sin[(d*x)/2] + 264*C*Sin[(d*x)/2] - 156*A*Sin[c + (d
*x)/2] - 120*C*Sin[c + (d*x)/2] + 342*A*Sin[c + (3*d*x)/2] + 164*C*Sin[c +
(3*d*x)/2] + 118*A*Sin[2*c + (3*d*x)/2] + 36*C*Sin[2*c + (3*d*x)/2] + 30*A*
Sin[2*c + (5*d*x)/2] + 12*C*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2
] + 12*C*Sin[3*c + (5*d*x)/2] - 3*A*Sin[3*c + (7*d*x)/2] - 3*A*Sin[4*c + (7
*d*x)/2] + A*Sin[4*c + (9*d*x)/2] + A*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 +
 Sec[c + d*x])^2)
```

Maple [B] time = 0.095, size = 322, normalized size = 2.

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{9A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 10\frac{(\tan(1/2\,d}{da^2\left(1+(\tan(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+9/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+10/d/a^2/(1+tan(1/2*d
*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^5*C+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*
c)^3*A+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)^3+6/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+2/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*C*tan(1/2*d*x+1/2*c)-10/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))-4/d/a^2*ar
ctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.43361, size = 439, normalized size = 2.69

$$\frac{A\left(\frac{4\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)+C\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\mathrm{a}}{}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) + C*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)
^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^2 + 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(
d*x + c) + 1))))/d
```

Fricas [A] time = 0.503547, size = 369, normalized size = 2.26

$$-\frac{3\,(5\,A+2\,C)dx\cos(dx+c)^2+6\,(5\,A+2\,C)dx\cos(dx+c)+3\,(5\,A+2\,C)dx-\left(A\cos(dx+c)^4-A\cos(dx+c)^3+3\,(\right.}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] -1/3*(3*(5*A + 2*C)*d*x*cos(d*x + c)^2 + 6*(5*A + 2*C)*d*x*cos(d*x + c) + 3
*(5*A + 2*C)*d*x - (A*cos(d*x + c)^4 - A*cos(d*x + c)^3 + 3*(2*A + C)*cos(d
*x + c)^2 + (33*A + 14*C)*cos(d*x + c) + 24*A + 10*C)*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23518, size = 258, normalized size = 1.58

$$-\frac{\frac{6\,(dx+c)(5\,A+2\,C)}{a^2} - \frac{4\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/6*(6*(d*x + c)*(5*A + 2*C)/a^2 - 4*(15*A*tan(1/2*d*x + 1/2*c)^5 + 3*C*ta
n(1/2*d*x + 1/2*c)^5 + 20*A*tan(1/2*d*x + 1/2*c)^3 + 6*C*tan(1/2*d*x + 1/2*
c)^3 + 9*A*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x +
 1/2*c)^3 - 27*A*a^4*tan(1/2*d*x + 1/2*c) - 15*C*a^4*tan(1/2*d*x + 1/2*c))/
a^6)/d
```

3.138 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=198

$$-\frac{2(11A+76C)\tan(c+dx)}{15a^3d}+\frac{(2A+13C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(11A+76C)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2A+13C)\,\text{t}}{}$$

```
[Out] ((2*A + 13*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(11*A + 76*C)*Tan[c + d
*x])/(15*a^3*d) + ((2*A + 13*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A
+ C)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((A + 11*C
)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((11*A + 7
6*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.489821, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4085, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{2(11A+76C)\tan(c+dx)}{15a^3d}+\frac{(2A+13C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(11A+76C)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2A+13C)\,\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((2*A + 13*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(11*A + 76*C)*Tan[c + d
*x])/(15*a^3*d) + ((2*A + 13*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A
+ C)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((A + 11*C
)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((11*A + 7
6*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)(-a(A-4C)-a(2A+7C)\sec(c+dx))}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+11C)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+11C)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+11C)\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(2A+13C)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} -$$

$$= \frac{(2A+13C)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{2(11A+76C)\tan(c+dx)}{15a^3d} + \frac{(2A+13}{}$$

Mathematica [B] time = 2.86662, size = 632, normalized size = 3.19

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(-654A\sin\left(c-\frac{dx}{2}\right)+654A\sin\left(c+\frac{d}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*(1920*(2*A + 13*C)*C
os[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-5*(98*A + 2
```

```
47*C)*Sin[(d*x)/2] + 5*(106*A + 761*C)*Sin[(3*d*x)/2] - 654*A*Sin[c - (d*x)
/2] - 4329*C*Sin[c - (d*x)/2] + 654*A*Sin[c + (d*x)/2] + 1989*C*Sin[c + (d*
x)/2] - 490*A*Sin[2*c + (d*x)/2] - 3575*C*Sin[2*c + (d*x)/2] - 350*A*Sin[c
+ (3*d*x)/2] - 475*C*Sin[c + (3*d*x)/2] + 530*A*Sin[2*c + (3*d*x)/2] + 2005
*C*Sin[2*c + (3*d*x)/2] - 350*A*Sin[3*c + (3*d*x)/2] - 2275*C*Sin[3*c + (3*
d*x)/2] + 378*A*Sin[c + (5*d*x)/2] + 2673*C*Sin[c + (5*d*x)/2] - 150*A*Sin[
2*c + (5*d*x)/2] + 105*C*Sin[2*c + (5*d*x)/2] + 378*A*Sin[3*c + (5*d*x)/2]
+ 1593*C*Sin[3*c + (5*d*x)/2] - 150*A*Sin[4*c + (5*d*x)/2] - 975*C*Sin[4*c
+ (5*d*x)/2] + 190*A*Sin[2*c + (7*d*x)/2] + 1325*C*Sin[2*c + (7*d*x)/2] - 3
0*A*Sin[3*c + (7*d*x)/2] + 255*C*Sin[3*c + (7*d*x)/2] + 190*A*Sin[4*c + (7*
d*x)/2] + 875*C*Sin[4*c + (7*d*x)/2] - 30*A*Sin[5*c + (7*d*x)/2] - 195*C*Si
n[5*c + (7*d*x)/2] + 44*A*Sin[3*c + (9*d*x)/2] + 304*C*Sin[3*c + (9*d*x)/2]
 + 90*C*Sin[4*c + (9*d*x)/2] + 44*A*Sin[5*c + (9*d*x)/2] + 214*C*Sin[5*c +
(9*d*x)/2])))/(240*a^3*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^
3)
```

Maple [A] time = 0.074, size = 289, normalized size = 1.5

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{2\,C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-2/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*C*tan(1/2*d*x+1/2*c)+1/d/a^3*ln(tan(1/2*d*x+1/2*
c)+1)*A+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^3*C/(tan(1/2*d*x+1/2*
c)+1)^2+7/2/d/a^3*C/(tan(1/2*d*x+1/2*c)+1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)
*A-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^3*C/(tan(1/2*d*x+1/2*c)-1)
^2+7/2/d/a^3*C/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 0.962566, size = 446, normalized size = 2.25

$$-\frac{C\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)+A\left(\frac{105\,\mathrm{s}}{\cos(}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] -1/60*(C*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) + A*((105*sin(d*x + c)/(cos(d*x + c) + 1)
 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 0.523106, size = 740, normalized size = 3.74

$$\frac{15\left((2\,A+13\,C)\cos\left(dx+c\right)^{5}+3\,(2\,A+13\,C)\cos\left(dx+c\right)^{4}+3\,(2\,A+13\,C)\cos\left(dx+c\right)^{3}+(2\,A+13\,C)\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/60*(15*((2*A + 13*C)*cos(d*x + c)^5 + 3*(2*A + 13*C)*cos(d*x + c)^4 + 3*(
2*A + 13*C)*cos(d*x + c)^3 + (2*A + 13*C)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - 15*((2*A + 13*C)*cos(d*x + c)^5 + 3*(2*A + 13*C)*cos(d*x + c)^4 + 3*
(2*A + 13*C)*cos(d*x + c)^3 + (2*A + 13*C)*cos(d*x + c)^2)*log(-sin(d*x + c
) + 1) - 2*(4*(11*A + 76*C)*cos(d*x + c)^4 + 3*(34*A + 239*C)*cos(d*x + c)^
3 + (64*A + 479*C)*cos(d*x + c)^2 + 45*C*cos(d*x + c) - 15*C)*sin(d*x + c))
/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{4}(c+dx)}{\sec^{3}(c+dx)+3\sec^{2}(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^{6}(c+dx)}{\sec^{3}(c+dx)+3\sec^{2}(c+dx)+3\sec(c+dx)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.23234, size = 279, normalized size = 1.41

$$\frac{\frac{30\,(2\,A+13\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{30\,(2\,A+13\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{60\left(7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(30*(2*A + 13*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(2*A + 13
*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*C*tan(1/2*d*x + 1/2*c)^3
 - 5*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^
12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*A*a^12*tan
(1/2*d*x + 1/2*c)^3 + 40*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2
*d*x + 1/2*c) + 465*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.139 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{(2A+27C)\tan(c+dx)}{15a^3d}-\frac{3C\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3C\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(A-$$

```
[Out] (-3*C*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((2*A + 27*C)*Tan[c + d*x])/(15*a^3*
d) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + (
(A - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (3
*C*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.427468, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4085, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{(2A+27C)\tan(c+dx)}{15a^3d}-\frac{3C\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3C\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(A-$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-3*C*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((2*A + 27*C)*Tan[c + d*x])/(15*a^3*
d) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + (
(A - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (3
*C*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
```

```
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)(-a(2A-3C)-a(A+6C)\sec(c+dx))}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-9C)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-9C)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-9C)\sec^2(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{3C\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-9C)}{15ad}$$

$$= -\frac{3C\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(2A+27C)\tan(c+dx)}{15a^3d} - \frac{(A+C)\sec^3(c+d}{5d(a+a\sec(}$$

Mathematica [B] time = 3.01238, size = 457, normalized size = 3.15

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec(c+dx)\left(-10A\sin\left(c-\frac{dx}{2}\right)+10A\sin\left(c+\frac{dx}{2}\right)-2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*(2880*C*Cos[(c + d*x)
/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]*(-5*(4*A + 51*C)*Sin[(d*x)/
```

```
2] + (22*A + 567*C)*Sin[(3*d*x)/2] - 10*A*Sin[c - (d*x)/2] - 600*C*Sin[c -
(d*x)/2] + 10*A*Sin[c + (d*x)/2] + 375*C*Sin[c + (d*x)/2] - 20*A*Sin[2*c +
(d*x)/2] - 480*C*Sin[2*c + (d*x)/2] - 60*C*Sin[c + (3*d*x)/2] + 22*A*Sin[2*
c + (3*d*x)/2] + 402*C*Sin[2*c + (3*d*x)/2] - 225*C*Sin[3*c + (3*d*x)/2] +
10*A*Sin[c + (5*d*x)/2] + 315*C*Sin[c + (5*d*x)/2] + 30*C*Sin[2*c + (5*d*x)
/2] + 10*A*Sin[3*c + (5*d*x)/2] + 240*C*Sin[3*c + (5*d*x)/2] - 45*C*Sin[4*c
 + (5*d*x)/2] + 2*A*Sin[2*c + (7*d*x)/2] + 72*C*Sin[2*c + (7*d*x)/2] + 15*C
*Sin[3*c + (7*d*x)/2] + 2*A*Sin[4*c + (7*d*x)/2] + 57*C*Sin[4*c + (7*d*x)/2
]))/(60*a^3*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.062, size = 204, normalized size = 1.4

$$\frac{A}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{6\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{2\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{A}{4\,d a^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(
1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)-1/d/a^3*C/(tan(1/2*d*x+1/2*c
)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^3*C/(tan(1/2*d*x+1/2*c)-1)+3/
d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [A] time = 0.958932, size = 315, normalized size = 2.17

$$\frac{3\,C\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)+A\left(\frac{15\,\sin(d}{\cos(dx+c}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(3*C*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) + A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.513473, size = 576, normalized size = 3.97

$$-\frac{45\left(C\cos(dx+c)^4+3\,C\cos(dx+c)^3+3\,C\cos(dx+c)^2+C\cos(dx+c)\right)\log(\sin(dx+c)+1)-45\left(C\cos(dx+c)^4\right.}{30\left(a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] -1/30*(45*(C*cos(d*x + c)^4 + 3*C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + C*c
os(d*x + c))*log(sin(d*x + c) + 1) - 45*(C*cos(d*x + c)^4 + 3*C*cos(d*x + c
)^3 + 3*C*cos(d*x + c)^2 + C*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(A
 + 36*C)*cos(d*x + c)^3 + 3*(2*A + 57*C)*cos(d*x + c)^2 + (7*A + 117*C)*cos
(d*x + c) + 15*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.24574, size = 240, normalized size = 1.66

$$-\frac{\frac{180\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{180\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{120\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,Aa}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/60*(180*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 180*C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^3 + 120*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/
2*c)^5 + 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*C*a^12*tan(1/2*d*x + 1/2*c)^
3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) + 255*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.140 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{(6A-29C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(3A-7C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((3*A - 7*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((6*A - 29*C)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.330017, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4085, 4008, 3998, 3770, 3794}

$$\frac{(6A-29C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(3A-7C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((3*A - 7*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((6*A - 29*C)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)(-a(3A-2C)-5aC\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(3A-7C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \sec(c+d\ldots}{\ldots}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(3A-7C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(6A-29\ldots}{\ldots}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(3A-7C)\,\mathrm{t}\ldots}{15ad(a+a\,\mathrm{s}\ldots}$$

Mathematica [A] time = 1.58916, size = 236, normalized size = 1.92

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\left(15(A-5C)\sin\left(c+\frac{dx}{2}\right)-15A\sin\left(c+\frac{3dx}{2}\right)-3A\sin\left(2c\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*(240*C*Cos[(c + d*x)
/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + Sec[c/2]*(-5*(3*A - 29*C)*Sin[(d*x)/2] + 15*(A - 5*C)*Si
n[c + (d*x)/2] - 15*A*Sin[c + (3*d*x)/2] + 95*C*Sin[c + (3*d*x)/2] - 15*C*S
in[2*c + (3*d*x)/2] - 3*A*Sin[2*c + (5*d*x)/2] + 22*C*Sin[2*c + (5*d*x)/2])
))/(15*a^3*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.059, size = 139, normalized size = 1.1

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{A}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{7\,C}{4\,da^3}\tan\left(\frac{d\ldots}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/3/d/
a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1
/2*d*x+1/2*c)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C+1/d/a^3*ln(tan(1/2*d*x+1/2
*c)+1)*C
```

Maxima [A] time = 0.957302, size = 225, normalized size = 1.83

$$-\frac{C\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] -1/60*(C*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.502743, size = 481, normalized size = 3.91

$$\frac{15\left(C\cos(dx+c)^3+3\,C\cos(dx+c)^2+3\,C\cos(dx+c)+C\right)\log(\sin(dx+c)+1)-15\left(C\cos(dx+c)^3+3\,C\cos(dx+\right.}{30\left(a^3d\cos(dx+c)^3+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*log
(sin(d*x + c) + 1) - 15*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*
x + c) + C)*log(-sin(d*x + c) + 1) + 2*((3*A - 22*C)*cos(d*x + c)^2 + 3*(3*
A - 17*C)*cos(d*x + c) + 3*A - 32*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int\frac{C\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.25467, size = 177, normalized size = 1.44

$$\frac{\frac{60\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^{12}\tan\left(\frac{1}{2}\,d\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(60*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*C*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*
d*x + 1/2*c)^5 + 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^12*tan(1/2*d*x +
 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.141 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{(2A+7C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A-C)\tan(c+dx)}{3ad(a\sec(c+dx)+a)^2}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -((A + C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((A - C
)*Tan[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 7*C)*Tan[c + d*x])
/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.187035, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4079, 4000, 3794}

$$\frac{(2A+7C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A-C)\tan(c+dx)}{3ad(a\sec(c+dx)+a)^2}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((A + C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((A - C
)*Tan[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 7*C)*Tan[c + d*x])
/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4079

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A + C)*Cot[e +
 f*x]*Csc[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] - Dist[1/(a*b*
(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[-(b*C) - 2*A
*b*(m + 1) + a*(A*(m + 2) - C*(m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(a(4A-C)-a(A-4C)\sec(c+dx))}{(a+a\sec(c+dx))^2}dx}{5a^2}$$
$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-C)\tan(c+dx)}{3ad(a+a\sec(c+dx))^2} + \frac{(2A+7C)\int}{1}$$
$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-C)\tan(c+dx)}{3ad(a+a\sec(c+dx))^2} + \frac{(2A+7C)}{15d\left(a^3+a^3\right.}$$

Mathematica [A] time = 0.540961, size = 121, normalized size = 1.16

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-30A\sin\left(c+\frac{dx}{2}\right)+20A\sin\left(c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{3dx}{2}\right)+7A\sin\left(2c+\frac{5dx}{2}\right)+20(2A+\right.}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(20*(2*A + C)*Sin[(d*x)/2] - 30*A*Sin[c + (d*x
)/2] + 20*A*Sin[c + (3*d*x)/2] + 10*C*Sin[c + (3*d*x)/2] - 15*A*Sin[2*c + (
3*d*x)/2] + 7*A*Sin[2*c + (5*d*x)/2] + 2*C*Sin[2*c + (5*d*x)/2]))/(240*a^3*
d)
```

Maple [A] time = 0.059, size = 88, normalized size = 0.9

$$\frac{1}{4da^3}\left(\frac{A}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{2C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5*A+1/5*C*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/
2*d*x+1/2*c)^3*A+2/3*C*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+C*tan(1/2*
d*x+1/2*c))
```

Maxima [A] time = 0.95209, size = 181, normalized size = 1.74

$$\frac{\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(C*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + A*(15*sin(d*x + c)
/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x +
```

```
c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.46378, size = 221, normalized size = 2.12

$$\frac{\left((7\,A+2\,C)\cos\left(dx+c\right)^2+6\,(A+C)\cos\left(dx+c\right)+2\,A+7\,C\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((7*A + 2*C)*cos(d*x + c)^2 + 6*(A + C)*cos(d*x + c) + 2*A + 7*C)*sin(
d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int\frac{C\sec^3\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(C*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.22729, size = 120, normalized size = 1.15

$$\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 + 3*C*tan(1/2*d*x + 1/2*c)^5 - 10*A*tan(1/
2*d*x + 1/2*c)^3 + 10*C*tan(1/2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c)
+ 15*C*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

3.142 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$-\frac{(22A-3C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-3C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A+C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] (A*x)/a^3 - ((A + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A - 3
*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((22*A - 3*C)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.180472, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4053, 3922, 3919, 3794}

$$-\frac{(22A-3C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-3C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A+C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (A*x)/a^3 - ((A + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A - 3
*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((22*A - 3*C)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5aA+a(2A-3C)\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$
$$= -\frac{(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2A-a^2(7A-3C)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^4}$$
$$= \frac{Ax}{a^3} - \frac{(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(22A-3C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}$$
$$= \frac{Ax}{a^3} - \frac{(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(22A-3C)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

Mathematica [B] time = 0.844389, size = 227, normalized size = 2.14

$$\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(270A\sin\left(c+\frac{dx}{2}\right)-230A\sin\left(c+\frac{3dx}{2}\right)+90A\sin\left(2c+\frac{3dx}{2}\right)-64A\sin\left(2c+\frac{5dx}{2}\right)+150Adx\,\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*A*d*x*Cos[(d*x)/2] + 150*A*d*x*Cos[c + (d
*x)/2] + 75*A*d*x*Cos[c + (3*d*x)/2] + 75*A*d*x*Cos[2*c + (3*d*x)/2] + 15*A
*d*x*Cos[2*c + (5*d*x)/2] + 15*A*d*x*Cos[3*c + (5*d*x)/2] - 370*A*Sin[(d*x)
/2] + 30*C*Sin[(d*x)/2] + 270*A*Sin[c + (d*x)/2] - 30*C*Sin[c + (d*x)/2] -
230*A*Sin[c + (3*d*x)/2] + 30*C*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c + (3*d*x)
/2] - 64*A*Sin[2*c + (5*d*x)/2] + 6*C*Sin[2*c + (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.068, size = 117, normalized size = 1.1

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,A}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{4\,da^3}\tan\left(\frac{dx}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-7/4/d/a^3*A*tan(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1
/2*d*x+1/2*c)+2/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.42385, size = 189, normalized size = 1.78

$$-\frac{A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{3C\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*C*(5*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.476095, size = 351, normalized size = 3.31

$$\frac{15\,Adx\cos(dx+c)^3+45\,Adx\cos(dx+c)^2+45\,Adx\cos(dx+c)+15\,Adx-\left((32\,A-3\,C)\cos(dx+c)^2+3\,(17\,A-\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*A*d*x*cos(d*x + c)^3 + 45*A*d*x*cos(d*x + c)^2 + 45*A*d*x*cos(d*x
+ c) + 15*A*d*x - ((32*A - 3*C)*cos(d*x + c)^2 + 3*(17*A - 3*C)*cos(d*x + c
) + 22*A - 3*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)
+ Integral(C*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x))/a**3
```

Giac [A] time = 1.21603, size = 140, normalized size = 1.32

$$\frac{\frac{60\,(dx+c)A}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*A/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(
1/2*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*
d*x + 1/2*c) - 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.143 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=120

$$\frac{2(36A+C)\sin(c+dx)}{15a^3d}-\frac{3A\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3Ax}{a^3}-\frac{(9A-C)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A+C)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] (-3*A*x)/a^3 + (2*(36*A + C)*Sin[c + d*x])/(15*a^3*d) - ((A + C)*Sin[c + d*
x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - C)*Sin[c + d*x])/(15*a*d*(a + a*
Sec[c + d*x])^2) - (3*A*Sin[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.354935, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4085, 4020, 3787, 2637, 8}

$$\frac{2(36A+C)\sin(c+dx)}{15a^3d}-\frac{3A\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3Ax}{a^3}-\frac{(9A-C)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A+C)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-3*A*x)/a^3 + (2*(36*A + C)*Sin[c + d*x])/(15*a^3*d) - ((A + C)*Sin[c + d*
x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - C)*Sin[c + d*x])/(15*a*d*(a + a*
Sec[c + d*x])^2) - (3*A*Sin[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(-a(6A+C)+a(3A-2C)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-a^2(27A+2C\right.}{a+a\,\mathrm{se}}}{15}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{3A\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx\right.}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{3A\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx\right.}$$

$$= -\frac{3Ax}{a^3} + \frac{2(36A+C)\sin(c+dx)}{15a^3d} - \frac{(A+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-C)\sin}{15ad(a+a\sec}$$

Mathematica [B] time = 1.82001, size = 283, normalized size = 2.36

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(1125A\sin\left(c+\frac{dx}{2}\right)-1215A\sin\left(c+\frac{3dx}{2}\right)+225A\sin\left(2c+\frac{3dx}{2}\right)-363A\sin\left(2c+\frac{5dx}{2}\right)-7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^5*(900*A*d*x*Cos[(d*x)/2] + 900*A*d*x*Cos[c + (
d*x)/2] + 450*A*d*x*Cos[c + (3*d*x)/2] + 450*A*d*x*Cos[2*c + (3*d*x)/2] + 9
0*A*d*x*Cos[2*c + (5*d*x)/2] + 90*A*d*x*Cos[3*c + (5*d*x)/2] - 1755*A*Sin[(
d*x)/2] - 160*C*Sin[(d*x)/2] + 1125*A*Sin[c + (d*x)/2] + 120*C*Sin[c + (d*x
)/2] - 1215*A*Sin[c + (3*d*x)/2] - 80*C*Sin[c + (3*d*x)/2] + 225*A*Sin[2*c
+ (3*d*x)/2] + 60*C*Sin[2*c + (3*d*x)/2] - 363*A*Sin[2*c + (5*d*x)/2] - 28*
C*Sin[2*c + (5*d*x)/2] - 75*A*Sin[3*c + (5*d*x)/2] - 15*A*Sin[3*c + (7*d*x)
/2] - 15*A*Sin[4*c + (7*d*x)/2]))/(960*a^3*d)
```

Maple [A] time = 0.092, size = 170, normalized size = 1.4

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,A}{4\,da^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
```

```
(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*A*tan(1/2*d*x+1/2*c)
/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.44244, size = 277, normalized size = 2.31

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{C\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\mathrm{s}}{(\mathrm{cos}}\right.}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + C*(15*sin(d*x + c)/(cos(d*x +
 c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.489513, size = 386, normalized size = 3.22

$$-\frac{45\,Adx\cos(dx+c)^3+135\,Adx\cos(dx+c)^2+135\,Adx\cos(dx+c)+45\,Adx-\left(15\,A\cos(dx+c)^3+(117\,A+7\,C)\,\mathrm{co}\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/15*(45*A*d*x*cos(d*x + c)^3 + 135*A*d*x*cos(d*x + c)^2 + 135*A*d*x*cos(d
*x + c) + 45*A*d*x - (15*A*cos(d*x + c)^3 + (117*A + 7*C)*cos(d*x + c)^2 +
3*(57*A + 2*C)*cos(d*x + c) + 72*A + 2*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\cos(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\cos(c+dx)\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(C*cos(c + d*x)*sec(c + d*x)**2/(sec(c + d*x)**3 + 3
*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.23877, size = 204, normalized size = 1.7

$$-\frac{\frac{180\,(dx+c)A}{a^3}-\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+255\,Aa^{1}}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(180*(d*x + c)*A/a^3 - 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x +
 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*C*a^12*tan(1/2*d*x + 1/2*
c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) + 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^
15)/d
```

3.144 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=183

$$-\frac{2(76A+11C)\sin(c+dx)}{15a^3d}+\frac{(13A+2C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(76A+11C)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13A+2C)}{2a^3}$$

```
[Out] ((13*A + 2*C)*x)/(2*a^3) - (2*(76*A + 11*C)*Sin[c + d*x])/(15*a^3*d) + ((13
*A + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]*Sin[
c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A + C)*Cos[c + d*x]*Sin[c + d
*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((76*A + 11*C)*Cos[c + d*x]*Sin[c +
d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.466027, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4020, 3787, 2635, 8, 2637}

$$-\frac{2(76A+11C)\sin(c+dx)}{15a^3d}+\frac{(13A+2C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(76A+11C)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13A+2C)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] ((13*A + 2*C)*x)/(2*a^3) - (2*(76*A + 11*C)*Sin[c + d*x])/(15*a^3*d) + ((13
*A + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]*Sin[
c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A + C)*Cos[c + d*x]*Sin[c + d
*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((76*A + 11*C)*Cos[c + d*x]*Sin[c +
d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-a(7A+2C)+a(4A-C)\sec(c+dx))}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A+C)\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \int$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A+C)\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - ($$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A+C)\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - ($$

$$= -\frac{2(76A+11C)\sin(c+dx)}{15a^3d} + \frac{(13A+2C)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A+}{5}$$

$$= \frac{(13A+2C)x}{2a^3} - \frac{2(76A+11C)\sin(c+dx)}{15a^3d} + \frac{(13A+2C)\cos(c+dx)\sin(c}{2a^3d}$$

Mathematica [B] time = 1.41611, size = 385, normalized size = 2.1

$$\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(600dx(13A+2C)\cos\left(c+\frac{dx}{2}\right)+7560A\sin\left(c+\frac{dx}{2}\right)-9230A\sin\left(c+\frac{3dx}{2}\right)+930A\sin\left(2c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(600*(13*A + 2*C)*d*x*Cos[(d*x)/2] + 600*(13*A
 + 2*C)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] + 600*C*d*x*Co
s[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] + 600*C*d*x*Cos[2*c + (3
*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] + 120*C*d*x*Cos[2*c + (5*d*x)/2]
+ 780*A*d*x*Cos[3*c + (5*d*x)/2] + 120*C*d*x*Cos[3*c + (5*d*x)/2] - 12760*A
*Sin[(d*x)/2] - 2960*C*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] + 2160*C*Sin[
c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] - 1840*C*Sin[c + (3*d*x)/2] + 930*
A*Sin[2*c + (3*d*x)/2] + 720*C*Sin[2*c + (3*d*x)/2] - 2782*A*Sin[2*c + (5*d
*x)/2] - 512*C*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] - 105*A*Si
n[3*c + (7*d*x)/2] - 105*A*Sin[4*c + (7*d*x)/2] + 15*A*Sin[4*c + (9*d*x)/2]
 + 15*A*Sin[5*c + (9*d*x)/2]))/(3840*a^3*d)
```

Maple [A] time = 0.107, size = 224, normalized size = 1.2

$$-\frac{A}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,A}{3\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{3\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31\,A}{4\,d a^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A+1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*ta
n(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2*
c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2
*d*x+1/2*c)+13/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arctan(tan(1/2*d*
x+1/2*c))*C
```

Maxima [A] time = 1.43358, size = 373, normalized size = 2.04

$$-\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+C\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3}{(c}}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + C*((105*sin(d*x
+ c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^3))/d
```

Fricas [A] time = 0.506768, size = 478, normalized size = 2.61

$$\frac{15\,(13\,A+2\,C)dx\cos\left(dx+c\right)^3+45\,(13\,A+2\,C)dx\cos\left(dx+c\right)^2+45\,(13\,A+2\,C)dx\cos\left(dx+c\right)+15\,(13\,A+2\,C)dx+}{30\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(13*A + 2*C)*d*x*cos(d*x + c)^3 + 45*(13*A + 2*C)*d*x*cos(d*x + c)
^2 + 45*(13*A + 2*C)*d*x*cos(d*x + c) + 15*(13*A + 2*C)*d*x + (15*A*cos(d*x
 + c)^4 - 45*A*cos(d*x + c)^3 - (479*A + 64*C)*cos(d*x + c)^2 - 3*(239*A +
34*C)*cos(d*x + c) - 304*A - 44*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*
```

```
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19616, size = 235, normalized size = 1.28

$$\frac{\frac{30\,(dx+c)(13\,A+2\,C)}{a^3}-\frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,Ca}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*A + 2*C)/a^3 - 60*(7*A*tan(1/2*d*x + 1/2*c)^3 + 5*A*
tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(
1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*
x + 1/2*c)^3 - 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x +
1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.145 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$-\frac{4(34A+9C)\sin^3(c+dx)}{15a^3d}+\frac{4(34A+9C)\sin(c+dx)}{5a^3d}-\frac{(23A+6C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(23A+6C)\sin(c+dx)\cos}{3d\left(a^3\sec(c+dx)\right.}$$

```
[Out] -((23*A + 6*C)*x)/(2*a^3) + (4*(34*A + 9*C)*Sin[c + d*x])/(5*a^3*d) - ((23*
A + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]^2*Sin
[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A + 3*C)*Cos[c + d*x]^2*Sin[
c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((23*A + 6*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d*x])) - (4*(34*A + 9*C)*Sin[c + d*x]^3
)/(15*a^3*d)
```

Rubi [A] time = 0.497261, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4020, 3787, 2633, 2635, 8}

$$-\frac{4(34A+9C)\sin^3(c+dx)}{15a^3d}+\frac{4(34A+9C)\sin(c+dx)}{5a^3d}-\frac{(23A+6C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(23A+6C)\sin(c+dx)\cos}{3d\left(a^3\sec(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -((23*A + 6*C)*x)/(2*a^3) + (4*(34*A + 9*C)*Sin[c + d*x])/(5*a^3*d) - ((23*
A + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A + C)*Cos[c + d*x]^2*Sin
[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A + 3*C)*Cos[c + d*x]^2*Sin[
c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((23*A + 6*C)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d*x])) - (4*(34*A + 9*C)*Sin[c + d*x]^3
)/(15*a^3*d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)(-a(8A+3C)+5aA\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A+3C)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A+3C)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A+3C)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(23A+6C)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} -$$

$$= -\frac{(23A+6C)x}{2a^3} + \frac{4(34A+9C)\sin(c+dx)}{5a^3d} - \frac{(23A+6C)\cos(c+dx)\sin(c}{2a^3d}$$

Mathematica [B] time = 1.81681, size = 455, normalized size = 2.11

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(600dx(23A+6C)\cos\left(c+\frac{dx}{2}\right)+11110A\sin\left(c+\frac{dx}{2}\right)-15380A\sin\left(c+\frac{3dx}{2}\right)+380A\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^5*(600*(23*A + 6*C)*d*x*Cos[(d*x)/2] + 600*(23*
A + 6*C)*d*x*Cos[c + (d*x)/2] + 6900*A*d*x*Cos[c + (3*d*x)/2] + 1800*C*d*x*
Cos[c + (3*d*x)/2] + 6900*A*d*x*Cos[2*c + (3*d*x)/2] + 1800*C*d*x*Cos[2*c +
 (3*d*x)/2] + 1380*A*d*x*Cos[2*c + (5*d*x)/2] + 360*C*d*x*Cos[2*c + (5*d*x)
/2] + 1380*A*d*x*Cos[3*c + (5*d*x)/2] + 360*C*d*x*Cos[3*c + (5*d*x)/2] - 20
410*A*Sin[(d*x)/2] - 7020*C*Sin[(d*x)/2] + 11110*A*Sin[c + (d*x)/2] + 4500*
C*Sin[c + (d*x)/2] - 15380*A*Sin[c + (3*d*x)/2] - 4860*C*Sin[c + (3*d*x)/2]
 + 380*A*Sin[2*c + (3*d*x)/2] + 900*C*Sin[2*c + (3*d*x)/2] - 4777*A*Sin[2*c
```

```
 + (5*d*x)/2] - 1452*C*Sin[2*c + (5*d*x)/2] - 1625*A*Sin[3*c + (5*d*x)/2] -
 300*C*Sin[3*c + (5*d*x)/2] - 230*A*Sin[3*c + (7*d*x)/2] - 60*C*Sin[3*c + (
7*d*x)/2] - 230*A*Sin[4*c + (7*d*x)/2] - 60*C*Sin[4*c + (7*d*x)/2] + 20*A*S
in[4*c + (9*d*x)/2] + 20*A*Sin[5*c + (9*d*x)/2] - 5*A*Sin[5*c + (11*d*x)/2]
 - 5*A*Sin[6*c + (11*d*x)/2]))/(3840*a^3*d)
```

Maple [A] time = 0.105, size = 362, normalized size = 1.7

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{5\,A}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{49\,A}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-5/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*A-1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+49/4/d/a^3*A*tan
(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+17/d/a^3/(1+tan(1/2*d*x+1/2
*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/
2*d*x+1/2*c)^5+76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A
+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)^3+11/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3
*C*tan(1/2*d*x+1/2*c)-23/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))-6/d/a^3*arctan(
tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.44132, size = 493, normalized size = 2.28

$$\frac{A\left(\frac{20\left(\frac{33\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3+\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1380\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+3\,C\left(\frac{40\,\mathrm{s}}{\left(a^3+\frac{a^3\sin(dx+}{(\cos(dx+c)}\right.}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] 1/60*(A*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 3*C*
(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x
+ c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.510645, size = 525, normalized size = 2.43

$$-\frac{15\,(23\,A+6\,C)dx\cos(dx+c)^3+45\,(23\,A+6\,C)dx\cos(dx+c)^2+45\,(23\,A+6\,C)dx\cos(dx+c)+15\,(23\,A+6\,C)dx}{30\left(a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] -1/30*(15*(23*A + 6*C)*d*x*cos(d*x + c)^3 + 45*(23*A + 6*C)*d*x*cos(d*x + c
)^2 + 45*(23*A + 6*C)*d*x*cos(d*x + c) + 15*(23*A + 6*C)*d*x - (10*A*cos(d*
x + c)^5 - 15*A*cos(d*x + c)^4 + 5*(19*A + 6*C)*cos(d*x + c)^3 + (869*A + 2
34*C)*cos(d*x + c)^2 + 9*(143*A + 38*C)*cos(d*x + c) + 544*A + 144*C)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20886, size = 308, normalized size = 1.43

$$-\frac{\frac{30\,(dx+c)(23\,A+6\,C)}{a^3} - \frac{20\left(51\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+76\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,C\tan\left(\frac{1}{2}\,dx \cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^3} \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/60*(30*(d*x + c)*(23*A + 6*C)/a^3 - 20*(51*A*tan(1/2*d*x + 1/2*c)^5 + 6*
C*tan(1/2*d*x + 1/2*c)^5 + 76*A*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1/2*d*x +
 1/2*c)^3 + 33*A*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*t
an(1/2*d*x + 1/2*c)^5 - 50*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*C*a^12*tan(1/
2*d*x + 1/2*c)^3 + 735*A*a^12*tan(1/2*d*x + 1/2*c) + 255*C*a^12*tan(1/2*d*x
 + 1/2*c))/a^15)/d
```

3.146 $\int \frac{\sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$-\frac{32(5A+54C)\tan(c+dx)}{105a^4d}+\frac{(2A+21C)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{(10A+129C)\tan(c+dx)\sec^3(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{16(5A+54C}{105a}$$

```
[Out] ((2*A + 21*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (32*(5*A + 54*C)*Tan[c + d
*x])/(105*a^4*d) + ((2*A + 21*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((1
0*A + 129*C)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2)
- (16*(5*A + 54*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x
])) - ((A + C)*Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) -
(2*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.64895, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4085, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{32(5A+54C)\tan(c+dx)}{105a^4d}+\frac{(2A+21C)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{(10A+129C)\tan(c+dx)\sec^3(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{16(5A+54C}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] ((2*A + 21*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (32*(5*A + 54*C)*Tan[c + d
*x])/(105*a^4*d) + ((2*A + 21*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((1
0*A + 129*C)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2)
- (16*(5*A + 54*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x
])) - ((A + C)*Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) -
(2*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^5(c+dx)(-a(2A-5C)-a(2A+9C)\sec(c+dx)}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= -\frac{(A+C)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2C\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \sec^4}{}$$

$$= -\frac{(10A+129C)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^5(c+dx)\tan(c+d}{7d(a+a\sec(c+dx))^4}$$

$$= -\frac{(10A+129C)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^5(c+dx)\tan(c+d}{7d(a+a\sec(c+dx))^4}$$

$$= -\frac{(10A+129C)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^5(c+dx)\tan(c+d}{7d(a+a\sec(c+dx))^4}$$

$$= \frac{(2A+21C)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(10A+129C)\sec^3(c+dx)\tan(c+}{105a^4d(1+\sec(c+dx))^2}$$

$$= \frac{(2A+21C)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{32(5A+54C)\tan(c+dx)}{105a^4d} + \frac{(2A+21}{}$$

Mathematica [B] time = 4.22883, size = 746, normalized size = 3.22

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(-17220A\sin\left(c-\frac{dx}{2}\right)+17220A\sin\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^5*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(53760*(2*A + 21*C
)*Cos[(c + d*x)/2]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-14*(1010
*A + 5229*C)*Sin[(d*x)/2] + 4*(3790*A + 41667*C)*Sin[(3*d*x)/2] - 17220*A*S
in[c - (d*x)/2] - 183162*C*Sin[c - (d*x)/2] + 17220*A*Sin[c + (d*x)/2] + 10
0842*C*Sin[c + (d*x)/2] - 14140*A*Sin[2*c + (d*x)/2] - 155526*C*Sin[2*c + (
d*x)/2] - 9800*A*Sin[c + (3*d*x)/2] - 37380*C*Sin[c + (3*d*x)/2] + 15160*A*
Sin[2*c + (3*d*x)/2] + 101148*C*Sin[2*c + (3*d*x)/2] - 9800*A*Sin[3*c + (3*
d*x)/2] - 102900*C*Sin[3*c + (3*d*x)/2] + 10920*A*Sin[c + (5*d*x)/2] + 1193
64*C*Sin[c + (5*d*x)/2] - 4760*A*Sin[2*c + (5*d*x)/2] - 8820*C*Sin[2*c + (5
*d*x)/2] + 10920*A*Sin[3*c + (5*d*x)/2] + 78204*C*Sin[3*c + (5*d*x)/2] - 47
60*A*Sin[4*c + (5*d*x)/2] - 49980*C*Sin[4*c + (5*d*x)/2] + 5890*A*Sin[2*c +
 (7*d*x)/2] + 64053*C*Sin[2*c + (7*d*x)/2] - 1470*A*Sin[3*c + (7*d*x)/2] +
3885*C*Sin[3*c + (7*d*x)/2] + 5890*A*Sin[4*c + (7*d*x)/2] + 44733*C*Sin[4*c
 + (7*d*x)/2] - 1470*A*Sin[5*c + (7*d*x)/2] - 15435*C*Sin[5*c + (7*d*x)/2]
+ 2030*A*Sin[3*c + (9*d*x)/2] + 21987*C*Sin[3*c + (9*d*x)/2] - 210*A*Sin[4*
c + (9*d*x)/2] + 3675*C*Sin[4*c + (9*d*x)/2] + 2030*A*Sin[5*c + (9*d*x)/2]
+ 16107*C*Sin[5*c + (9*d*x)/2] - 210*A*Sin[6*c + (9*d*x)/2] - 2205*C*Sin[6*
c + (9*d*x)/2] + 320*A*Sin[4*c + (11*d*x)/2] + 3456*C*Sin[4*c + (11*d*x)/2]
 + 840*C*Sin[5*c + (11*d*x)/2] + 320*A*Sin[6*c + (11*d*x)/2] + 2616*C*Sin[6
*c + (11*d*x)/2])))/(3360*a^4*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c +
 d*x])^4)
```

Maple [A] time = 0.08, size = 329, normalized size = 1.4

$$-\frac{A}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{9\,C}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{11\,A}{24\,d a^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^7-1/8/d/
a^4*tan(1/2*d*x+1/2*c)^5*A-9/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-11/24/d/a^4*A*
tan(1/2*d*x+1/2*c)^3-13/8/d/a^4*C*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2
*d*x+1/2*c)-111/8/d/a^4*C*tan(1/2*d*x+1/2*c)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+
1)*A+21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^4*C/(tan(1/2*d*x+1/2*c)+
1)^2+9/2/d/a^4*C/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A-
21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^4*C/(tan(1/2*d*x+1/2*c)-1)^2+
9/2/d/a^4*C/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 0.97384, size = 502, normalized size = 2.16

$$-\frac{3\,C\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c}\right.}{a^4}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(3*C*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
```

```
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) + 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [A] time = 0.53375, size = 923, normalized size = 3.98

$$\frac{105\left((2A+21C)\cos(dx+c)^6+4(2A+21C)\cos(dx+c)^5+6(2A+21C)\cos(dx+c)^4+4(2A+21C)\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/420*(105*((2*A + 21*C)*cos(d*x + c)^6 + 4*(2*A + 21*C)*cos(d*x + c)^5 + 6
*(2*A + 21*C)*cos(d*x + c)^4 + 4*(2*A + 21*C)*cos(d*x + c)^3 + (2*A + 21*C)
*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((2*A + 21*C)*cos(d*x + c)^6 +
 4*(2*A + 21*C)*cos(d*x + c)^5 + 6*(2*A + 21*C)*cos(d*x + c)^4 + 4*(2*A + 2
1*C)*cos(d*x + c)^3 + (2*A + 21*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) -
 2*(64*(5*A + 54*C)*cos(d*x + c)^5 + (1070*A + 11619*C)*cos(d*x + c)^4 + 4*
(310*A + 3411*C)*cos(d*x + c)^3 + 4*(130*A + 1509*C)*cos(d*x + c)^2 + 420*C
*cos(d*x + c) - 105*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*
x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x
+ c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{C\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**7/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.23292, size = 325, normalized size = 1.4

$$\frac{\frac{420(2A+21C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{420(2A+21C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{840\left(9C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}-\frac{15Aa^{24}\tan\left(\frac{1}{2}c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(420*(2*A + 21*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(2*A +
 21*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*C*tan(1/2*d*x + 1/2*
c)^3 - 7*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15
*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 105*A*a
^24*tan(1/2*d*x + 1/2*c)^5 + 189*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24
*tan(1/2*d*x + 1/2*c)^3 + 1365*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^24*
tan(1/2*d*x + 1/2*c) + 11655*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.147 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=183

$$\frac{2(3A+122C)\tan(c+dx)}{105a^4d}+\frac{(3A-88C)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4C\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{4C\tan(c+dx)}{a^4d(\sec(c+dx)+1)}$$

```
[Out] (-4*C*ArcTanh[Sin[c + d*x]])/(a^4*d) + (2*(3*A + 122*C)*Tan[c + d*x])/(105*
a^4*d) + ((3*A - 88*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c +
 d*x])^2) + (4*C*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[c
+ d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (2*(A - 6*C)*Sec[c +
d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.589016, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4085, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{2(3A+122C)\tan(c+dx)}{105a^4d}+\frac{(3A-88C)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4C\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{4C\tan(c+dx)}{a^4d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (-4*C*ArcTanh[Sin[c + d*x]])/(a^4*d) + (2*(3*A + 122*C)*Tan[c + d*x])/(105*
a^4*d) + ((3*A - 88*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c +
 d*x])^2) + (4*C*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[c
+ d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (2*(A - 6*C)*Sec[c +
d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)(-a(3A-4C)-a(A+8C)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(A-6C)\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \int$$

$$= \frac{(3A-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + 2$$

$$= \frac{(3A-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + 2$$

$$= \frac{(3A-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + 2$$

$$= -\frac{4C\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(3A-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)s}{7d(}$$

$$= -\frac{4C\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{2(3A+122C)\tan(c+dx)}{105a^4d} + \frac{(3A-88C)\sec^2(c}{105a^4d(1+se}$$

Mathematica [B] time = 2.60388, size = 544, normalized size = 2.97

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec(c+dx)\left(-126A\sin\left(c-\frac{dx}{2}\right)+126A\sin\left(c+\frac{dx}{2}\right)-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(107520*C*Cos[(c +
d*x)/2]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]*(-70*(3*A + 154*C)*Sin[
(d*x)/2] + 28*(9*A + 671*C)*Sin[(3*d*x)/2] - 126*A*Sin[c - (d*x)/2] - 20524
*C*Sin[c - (d*x)/2] + 126*A*Sin[c + (d*x)/2] + 14644*C*Sin[c + (d*x)/2] - 2
10*A*Sin[2*c + (d*x)/2] - 16660*C*Sin[2*c + (d*x)/2] - 4690*C*Sin[c + (3*d*
x)/2] + 252*A*Sin[2*c + (3*d*x)/2] + 14378*C*Sin[2*c + (3*d*x)/2] - 9100*C*
Sin[3*c + (3*d*x)/2] + 132*A*Sin[c + (5*d*x)/2] + 11668*C*Sin[c + (5*d*x)/2
] - 630*C*Sin[2*c + (5*d*x)/2] + 132*A*Sin[3*c + (5*d*x)/2] + 9358*C*Sin[3*
c + (5*d*x)/2] - 2940*C*Sin[4*c + (5*d*x)/2] + 42*A*Sin[2*c + (7*d*x)/2] +
4228*C*Sin[2*c + (7*d*x)/2] + 315*C*Sin[3*c + (7*d*x)/2] + 42*A*Sin[4*c + (
7*d*x)/2] + 3493*C*Sin[4*c + (7*d*x)/2] - 420*C*Sin[5*c + (7*d*x)/2] + 6*A*
Sin[3*c + (9*d*x)/2] + 664*C*Sin[3*c + (9*d*x)/2] + 105*C*Sin[4*c + (9*d*x)
/2] + 6*A*Sin[5*c + (9*d*x)/2] + 559*C*Sin[5*c + (9*d*x)/2])))/(840*a^4*d*(
A + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^4)
```

Maple [A] time = 0.067, size = 244, normalized size = 1.3

$$\frac{A}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,C}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{8\,d a^4}\left(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^7+3/40/d/
a^4*tan(1/2*d*x+1/2*c)^5*A+7/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+1/8/d/a^4*A*ta
n(1/2*d*x+1/2*c)^3+23/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*A*tan(1/2*d
*x+1/2*c)+49/8/d/a^4*C*tan(1/2*d*x+1/2*c)-1/d/a^4*C/(tan(1/2*d*x+1/2*c)+1)-
4/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^4*C/(tan(1/2*d*x+1/2*c)-1)+4/d/a^4
*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [A] time = 0.96958, size = 370, normalized size = 2.02

$$\frac{C\left(\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^4}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(C*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) + 3
*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.520267, size = 722, normalized size = 3.95

$$-\frac{210\left(C\cos\left(dx+c\right)^{5}+4\,C\cos\left(dx+c\right)^{4}+6\,C\cos\left(dx+c\right)^{3}+4\,C\cos\left(dx+c\right)^{2}+C\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] -1/105*(210*(C*cos(d*x + c)^5 + 4*C*cos(d*x + c)^4 + 6*C*cos(d*x + c)^3 + 4
*C*cos(d*x + c)^2 + C*cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(C*cos(d*x
+ c)^5 + 4*C*cos(d*x + c)^4 + 6*C*cos(d*x + c)^3 + 4*C*cos(d*x + c)^2 + C*c
os(d*x + c))*log(-sin(d*x + c) + 1) - (2*(3*A + 332*C)*cos(d*x + c)^4 + 4*(
6*A + 559*C)*cos(d*x + c)^3 + (39*A + 2636*C)*cos(d*x + c)^2 + 4*(9*A + 296
*C)*cos(d*x + c) + 105*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos
(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d
*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{4}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^{6}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**6/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.21663, size = 286, normalized size = 1.56

$$-\frac{\frac{3360\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{3360\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{4}}+\frac{1680\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{4}}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,Aa}{}}{840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/840*(3360*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*C*log(abs(tan(
1/2*d*x + 1/2*c) - 1))/a^4 + 1680*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*
x + 1/2*c)^7 + 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*C*a^24*tan(1/2*d*x +
1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 805*C*a^24*tan(1/2*d*x + 1/2
*c)^3 + 105*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*C*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

3.148 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=161

$$\frac{(16A-215C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(8A-55C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((8*A - 55*C)*Tan[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) + ((16*A - 215*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c
 + d*x])) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + (2*(2*A - 5*C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*
x])^3)
```

Rubi [A] time = 0.484186, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4019, 4008, 3998, 3770, 3794}

$$\frac{(16A-215C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(8A-55C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((8*A - 55*C)*Tan[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) + ((16*A - 215*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c
 + d*x])) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + (2*(2*A - 5*C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*
x])^3)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^3(c+dx)(-a(4A-3C)-7aC\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(2A-5C)\sec^2(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - $$

$$= -\frac{(8A-55C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(2A-5C)}{35ad(}$$

$$= -\frac{(8A-55C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(2A-5C)}{35ad(}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(8A-55C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^3(c+dx)\tan}{7d(a+a\sec(c+d}$$

Mathematica [A] time = 2.19314, size = 283, normalized size = 1.76

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\left(6720C\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\text{lo}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(6720*C*Cos[(c + d
*x)/2]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) - Sec[c/2]*(70*(2*A - 49*C)*Sin[(d*x)/2] - 70*(2*A - 31
*C)*Sin[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] - 2625*C*Sin[c + (3*d*x)/2]
 + 735*C*Sin[2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] - 1015*C*Sin[2*c
+ (5*d*x)/2] + 105*C*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] - 160*
C*Sin[3*c + (7*d*x)/2])))/(210*a^4*d*(A + 2*C + A*Cos[2*(c + d*x)])*(1 + Se
```

```
c[c + d*x])^4)
```

Maple [A] time = 0.068, size = 199, normalized size = 1.2

$$\frac{A}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{15\,C}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{A}{56\,da^4}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/24/d/a^4*A*tan(1/2*d*x+1/2*c)^3-1/40/d/a^4*tan(1/2*d*x+1/2*c)^5*A-1/8/d/a
^4*C*tan(1/2*d*x+1/2*c)^5-15/8/d/a^4*C*tan(1/2*d*x+1/2*c)-1/56/d/a^4*tan(1/
2*d*x+1/2*c)^7*A+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)-1/d/a^4*ln(tan(1/2*d*x+1/2*
c)-1)*C-11/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3-1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^
7+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [A] time = 0.961218, size = 308, normalized size = 1.91

$$-\frac{5\,C\left(\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - A*(105*sin(d*x + c
)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

Fricas [A] time = 0.51241, size = 624, normalized size = 3.88

$$\frac{105\left(C\cos(dx+c)^4+4\,C\cos(dx+c)^3+6\,C\cos(dx+c)^2+4\,C\cos(dx+c)+C\right)\log(\sin(dx+c)+1)-105\left(C\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/210*(105*(C*cos(d*x + c)^4 + 4*C*cos(d*x + c)^3 + 6*C*cos(d*x + c)^2 + 4*
C*cos(d*x + c) + C)*log(sin(d*x + c) + 1) - 105*(C*cos(d*x + c)^4 + 4*C*cos
(d*x + c)^3 + 6*C*cos(d*x + c)^2 + 4*C*cos(d*x + c) + C)*log(-sin(d*x + c)
+ 1) + 2*(8*(A - 20*C)*cos(d*x + c)^3 + (32*A - 535*C)*cos(d*x + c)^2 + 4*(
```

```
13*A - 155*C)*cos(d*x + c) + 13*A - 260*C)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{C\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**5/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.24557, size = 246, normalized size = 1.53

$$\frac{\frac{840\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Ca^{24}\,\mathrm{ta}}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(840*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan
(1/2*d*x + 1/2*c)^7 + 21*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*C*a^24*tan(1/2
*d*x + 1/2*c)^5 - 35*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 385*C*a^24*tan(1/2*d*x
 + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*C*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

3.149 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{4(2A+9C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}+\frac{(23A-54C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2(3A-4C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

```
[Out] ((23*A - 54*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + (4*(2*A + 9
*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[c + d*x]^2*
Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*(3*A - 4*C)*Tan[c + d*x])/(
35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.374343, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4085, 4008, 4000, 3794}

$$\frac{4(2A+9C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}+\frac{(23A-54C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2(3A-4C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ((23*A - 54*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + (4*(2*A + 9
*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Sec[c + d*x]^2*
Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*(3*A - 4*C)*Tan[c + d*x])/(
35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^2(c+dx)(-a(5A-2C)+a(A-6C)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(3A-4C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \int \frac{\sec(c+dx)(6a}{}$$

$$= \frac{(23A-54C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(3A-4C)\,\mathrm{t}}{35ad(a+a\,\mathrm{se}}$$

$$= \frac{(23A-54C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(3A-4C)\,\mathrm{t}}{35ad(a+a\,\mathrm{se}}$$

Mathematica [A] time = 0.641402, size = 151, normalized size = 1.09

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-175A\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(2c+\frac{3dx}{2}\right)+91A\sin\left(2c+\frac{5dx}{2}\right)+13A\sin\right.}{6720a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(70*(4*A + 3*C)*Sin[(d*x)/2] - 175*A*Sin[c + (
d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 126*C*Sin[c + (3*d*x)/2] - 105*A*Sin[2
*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*x)/2] + 42*C*Sin[2*c + (5*d*x)/2] + 1
3*A*Sin[3*c + (7*d*x)/2] + 6*C*Sin[3*c + (7*d*x)/2]))/(6720*a^4*d)
```

Maple [A] time = 0.067, size = 88, normalized size = 0.6

$$\frac{1}{8\,da^4}\left(\frac{A+C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A+3\,C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-A+3\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A+C)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*C)*tan(1/2*d*x+1/2*c)^5
+1/3*(-A+3*C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 0.974169, size = 236, normalized size = 1.71

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,C\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*C*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.462843, size = 308, normalized size = 2.23

$$\frac{\left((13\,A+6\,C)\cos\left(dx+c\right)^{3}+4\,(13\,A+6\,C)\cos\left(dx+c\right)^{2}+(32\,A+39\,C)\cos\left(dx+c\right)+8\,A+36\,C\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/105*((13*A + 6*C)*cos(d*x + c)^3 + 4*(13*A + 6*C)*cos(d*x + c)^2 + (32*A
+ 39*C)*cos(d*x + c) + 8*A + 36*C)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^{2}(c+dx)}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{C\sec^{4}(c+dx)}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**4/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A] time = 1.23181, size = 158, normalized size = 1.14

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+63\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,dx+\right.}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 + 15*C*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan
(1/2*d*x + 1/2*c)^5 + 63*C*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*
c)^3 + 105*C*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*C*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

3.150 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=142

$$\frac{(6A+13C)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{(6A+13C)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{2(4A-3C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A+C)\tan(c+dx)\sec(c+d}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] -((A + C)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (2*(4*A
 - 3*C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((6*A + 13*C)*Tan[c
 + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((6*A + 13*C)*Tan[c + d*x])/(
105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.246962, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4079, 4000, 3796, 3794}

$$\frac{(6A+13C)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{(6A+13C)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{2(4A-3C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A+C)\tan(c+dx)\sec(c+d}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -((A + C)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (2*(4*A
 - 3*C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((6*A + 13*C)*Tan[c
 + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((6*A + 13*C)*Tan[c + d*x])/(
105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 4079

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A + C)*Cot[e +
 f*x]*Csc[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] - Dist[1/(a*b*
(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[-(b*C) - 2*A
*b*(m + 1) + a*(A*(m + 2) - C*(m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)(a(6A-C)-a(2A-5C)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(4A-3C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(6A+13C)\int}{3}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(4A-3C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(6A+13C)}{105d\left(a^2+a^2\right.}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{2(4A-3C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(6A+13C)}{105d\left(a^2+a^2\right.}$$

Mathematica [A] time = 0.718021, size = 171, normalized size = 1.2

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-70(9A+2C)\sin\left(c+\frac{dx}{2}\right)+441A\sin\left(c+\frac{3dx}{2}\right)-315A\sin\left(2c+\frac{3dx}{2}\right)+147A\sin\left(2c+\frac{5dx}{2}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(70*(9*A + 2*C)*Sin[(d*x)/2] - 70*(9*A + 2*C)*
Sin[c + (d*x)/2] + 441*A*Sin[c + (3*d*x)/2] + 168*C*Sin[c + (3*d*x)/2] - 31
5*A*Sin[2*c + (3*d*x)/2] + 147*A*Sin[2*c + (5*d*x)/2] + 56*C*Sin[2*c + (5*d
*x)/2] - 105*A*Sin[3*c + (5*d*x)/2] + 36*A*Sin[3*c + (7*d*x)/2] + 8*C*Sin[3
*c + (7*d*x)/2]))/(6720*a^4*d)
```

Maple [A] time = 0.064, size = 90, normalized size = 0.6

$$\frac{1}{8da^4}\left(\frac{-A-C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3A-C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-3A+C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A-C)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-C)*tan(1/2*d*x+1/2*c)^5
+1/3*(-3*A+C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c
))
```

Maxima [A] time = 0.965073, size = 236, normalized size = 1.66

$$\frac{\frac{C\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(C*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.467458, size = 309, normalized size = 2.18

$$\frac{\left(4\,(9\,A+2\,C)\cos\left(dx+c\right)^3+(39\,A+32\,C)\cos\left(dx+c\right)^2+4\,(6\,A+13\,C)\cos\left(dx+c\right)+6\,A+13\,C\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(4*(9*A + 2*C)*cos(d*x + c)^3 + (39*A + 32*C)*cos(d*x + c)^2 + 4*(6*A
 + 13*C)*cos(d*x + c) + 6*A + 13*C)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^3\left(c+dx\right)}{\sec^4\left(c+dx\right)+4\sec^3\left(c+dx\right)+6\sec^2\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**3/(sec(c + d*x)
**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.21229, size = 158, normalized size = 1.11

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,A\tan\left(\frac{1}{2}\,dx\right.}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 + 15*C*tan(1/2*d*x + 1/2*c)^7 - 63*A*ta
n(1/2*d*x + 1/2*c)^5 + 21*C*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/
```

```
2*c)^3 - 35*C*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*C*t
an(1/2*d*x + 1/2*c))/(a^4*d)
```

3.151 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=136

$$-\frac{8(20A-C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-8C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{2(5A-2C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A+C)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] (A*x)/a^4 - ((55*A - 8*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) -
(8*(20*A - C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Tan[c
 + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*(5*A - 2*C)*Tan[c + d*x])/(35*a*
d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.262356, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4053, 3922, 3919, 3794}

$$-\frac{8(20A-C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-8C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{2(5A-2C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A+C)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (A*x)/a^4 - ((55*A - 8*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) -
(8*(20*A - C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Tan[c
 + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*(5*A - 2*C)*Tan[c + d*x])/(35*a*
d*(a + a*Sec[c + d*x])^3)
```

Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{-7aA+a(3A-4C)\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$
$$= -\frac{(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(5A-2C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{\int \frac{35a^2A-4a^2(5A-2C)\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$
$$= -\frac{(55A-8C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(5A-2C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{-105a^3A}{}}{}$$
$$= \frac{Ax}{a^4} - \frac{(55A-8C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(5A-2C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{(8(2}{}$$
$$= \frac{Ax}{a^4} - \frac{(55A-8C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(5A-2C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8(2}{105a}$$

Mathematica [B] time = 1.04983, size = 315, normalized size = 2.32

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(8260A\sin\left(c+\frac{dx}{2}\right)-7140A\sin\left(c+\frac{3dx}{2}\right)+3780A\sin\left(2c+\frac{3dx}{2}\right)-2800A\sin\left(2c+\frac{5dx}{2}\right)+84\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*A*d*x*Cos[(d*x)/2] + 3675*A*d*x*Cos[c +
(d*x)/2] + 2205*A*d*x*Cos[c + (3*d*x)/2] + 2205*A*d*x*Cos[2*c + (3*d*x)/2]
+ 735*A*d*x*Cos[2*c + (5*d*x)/2] + 735*A*d*x*Cos[3*c + (5*d*x)/2] + 105*A*d
*x*Cos[3*c + (7*d*x)/2] + 105*A*d*x*Cos[4*c + (7*d*x)/2] - 9940*A*Sin[(d*x)
/2] + 560*C*Sin[(d*x)/2] + 8260*A*Sin[c + (d*x)/2] - 350*C*Sin[c + (d*x)/2]
 - 7140*A*Sin[c + (3*d*x)/2] + 336*C*Sin[c + (3*d*x)/2] + 3780*A*Sin[2*c +
(3*d*x)/2] - 210*C*Sin[2*c + (3*d*x)/2] - 2800*A*Sin[2*c + (5*d*x)/2] + 182
*C*Sin[2*c + (5*d*x)/2] + 840*A*Sin[3*c + (5*d*x)/2] - 520*A*Sin[3*c + (7*d
*x)/2] + 26*C*Sin[3*c + (7*d*x)/2]))/(13440*a^4*d)
```

Maple [A] time = 0.071, size = 177, normalized size = 1.3

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{11\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^7-1/8/d/a
^4*tan(1/2*d*x+1/2*c)^5*A-1/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*A*t
an(1/2*d*x+1/2*c)^3-1/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2*
d*x+1/2*c)+1/8/d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*A*arctan(tan(1/2*d*x+1/2*
c))
```

Maxima [A] time = 1.43845, size = 271, normalized size = 1.99

$$-\frac{5\,A\left(\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{C\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - C*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.489385, size = 477, normalized size = 3.51

$$\frac{105\,Adx\cos(dx+c)^4+420\,Adx\cos(dx+c)^3+630\,Adx\cos(dx+c)^2+420\,Adx\cos(dx+c)+105\,Adx-\left(13\,(20\,A\ldots\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*A*d*x*cos(d*x + c)^4 + 420*A*d*x*cos(d*x + c)^3 + 630*A*d*x*cos(
d*x + c)^2 + 420*A*d*x*cos(d*x + c) + 105*A*d*x - (13*(20*A - C)*cos(d*x +
c)^3 + 4*(155*A - 13*C)*cos(d*x + c)^2 + (535*A - 32*C)*cos(d*x + c) + 160*
A - 8*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a
^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{C\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*se
c(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c
 + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.2146, size = 208, normalized size = 1.53

$$\frac{\frac{840\,(dx+c)A}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,Ca^{2}\ldots}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)*A/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*
tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 21*C*a^24*tan(
1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 35*C*a^24*tan(1/2*
d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 105*C*a^24*tan(1/2*d*x
+ 1/2*c))/a^28)/d
```

3.152 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=152

$$\frac{2(332A+3C)\sin(c+dx)}{105a^4d}-\frac{(88A-3C)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4A\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{4Ax}{a^4}-\frac{2(6A-C)\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(}{7a}$$

```
[Out] (-4*A*x)/a^4 + (2*(332*A + 3*C)*Sin[c + d*x])/(105*a^4*d) - ((88*A - 3*C)*S
in[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*A*Sin[c + d*x])/(a^4*d*(
1 + Sec[c + d*x])) - ((A + C)*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) -
(2*(6*A - C)*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.49945, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4085, 4020, 3787, 2637, 8}

$$\frac{2(332A+3C)\sin(c+dx)}{105a^4d}-\frac{(88A-3C)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4A\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{4Ax}{a^4}-\frac{2(6A-C)\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-4*A*x)/a^4 + (2*(332*A + 3*C)*Sin[c + d*x])/(105*a^4*d) - ((88*A - 3*C)*S
in[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*A*Sin[c + d*x])/(a^4*d*(
1 + Sec[c + d*x])) - ((A + C)*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) -
(2*(6*A - C)*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(-a(8A+C)+a(4A-3C)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(6A-C)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-a^2(52A+3C)+6\right.}{(a+a\sec(c+}}{35a^4}$$

$$= -\frac{(88A-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(6A-C)\sin(c+dx)}{35ad(a+a\sec(c+dx))}$$

$$= -\frac{(88A-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(6A-C)\sin(c+dx)}{35ad(a+a\sec(c+dx))}$$

$$= -\frac{(88A-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(6A-C)\sin(c+dx)}{35ad(a+a\sec(c+dx))}$$

$$= -\frac{4Ax}{a^4} + \frac{2(332A+3C)\sin(c+dx)}{105a^4d} - \frac{(88A-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\sin}{7d(a+a\sec}$$

Mathematica [B] time = 1.22754, size = 371, normalized size = 2.44

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(46130A\sin\left(c+\frac{dx}{2}\right)-46116A\sin\left(c+\frac{3dx}{2}\right)+18060A\sin\left(2c+\frac{3dx}{2}\right)-19292A\sin\left(2c+\frac{5dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*A*d*x*Cos[(d*x)/2] + 29400*A*d*x*Cos[c
 + (d*x)/2] + 17640*A*d*x*Cos[c + (3*d*x)/2] + 17640*A*d*x*Cos[2*c + (3*d*x
)/2] + 5880*A*d*x*Cos[2*c + (5*d*x)/2] + 5880*A*d*x*Cos[3*c + (5*d*x)/2] +
840*A*d*x*Cos[3*c + (7*d*x)/2] + 840*A*d*x*Cos[4*c + (7*d*x)/2] - 60830*A*S
in[(d*x)/2] - 2520*C*Sin[(d*x)/2] + 46130*A*Sin[c + (d*x)/2] + 2520*C*Sin[c
 + (d*x)/2] - 46116*A*Sin[c + (3*d*x)/2] - 1764*C*Sin[c + (3*d*x)/2] + 1806
0*A*Sin[2*c + (3*d*x)/2] + 1260*C*Sin[2*c + (3*d*x)/2] - 19292*A*Sin[2*c +
(5*d*x)/2] - 588*C*Sin[2*c + (5*d*x)/2] + 2100*A*Sin[3*c + (5*d*x)/2] + 420
*C*Sin[3*c + (5*d*x)/2] - 3791*A*Sin[3*c + (7*d*x)/2] - 144*C*Sin[3*c + (7*
d*x)/2] - 735*A*Sin[4*c + (7*d*x)/2] - 105*A*Sin[4*c + (9*d*x)/2] - 105*A*S
in[5*c + (9*d*x)/2]))/(26880*a^4*d)
```

Maple [A] time = 0.104, size = 210, normalized size = 1.4

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{23\,A}{24\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^7+7/40/d
/a^4*tan(1/2*d*x+1/2*c)^5*A+3/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-23/24/d/a^4*A
*tan(1/2*d*x+1/2*c)^3-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/2
*d*x+1/2*c)+1/8/d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*A*tan(1/2*d*x+1/2*c)/(1+
tan(1/2*d*x+1/2*c)^2)-8/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.4476, size = 332, normalized size = 2.18

$$\frac{A\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)+3C\left(\frac{35\sin(dx+c)}{\cos(dx+c)+\cdots}\right.\cdots}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) + 3*C*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.499998, size = 516, normalized size = 3.39

$$-\frac{420\,Adx\cos(dx+c)^4+1680\,Adx\cos(dx+c)^3+2520\,Adx\cos(dx+c)^2+1680\,Adx\cos(dx+c)+420\,Adx-\left(105\cdots\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/105*(420*A*d*x*cos(d*x + c)^4 + 1680*A*d*x*cos(d*x + c)^3 + 2520*A*d*x*c
os(d*x + c)^2 + 1680*A*d*x*cos(d*x + c) + 420*A*d*x - (105*A*cos(d*x + c)^4
 + 4*(296*A + 9*C)*cos(d*x + c)^3 + (2636*A + 39*C)*cos(d*x + c)^2 + 4*(559
*A + 6*C)*cos(d*x + c) + 664*A + 6*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 +
 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a
^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20368, size = 248, normalized size = 1.63

$$-\frac{\frac{3360\,(dx+c)A}{a^4} - \frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-63\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,Aa}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(3360*(d*x + c)*A/a^4 - 1680*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*
d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 63*C*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 105*C*a^24*tan(1/2*d*x + 1
/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c) - 105*C*a^24*tan(1/2*d*x + 1/2*c
))/a^28)/d
```

3.153 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=215

$$-\frac{32(54A+5C)\sin(c+dx)}{105a^4d}+\frac{(21A+2C)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{16(54A+5C)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(129A+}{10}$$

```
[Out] ((21*A + 2*C)*x)/(2*a^4) - (32*(54*A + 5*C)*Sin[c + d*x])/(105*a^4*d) + ((2
1*A + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129*A + 10*C)*Cos[c + d
*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (16*(54*A + 5*C)*Cos[c
 + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x
]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*A*Cos[c + d*x]*Sin[c + d*
x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.627105, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4020, 3787, 2635, 8, 2637}

$$-\frac{32(54A+5C)\sin(c+dx)}{105a^4d}+\frac{(21A+2C)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{16(54A+5C)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(129A+}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] ((21*A + 2*C)*x)/(2*a^4) - (32*(54*A + 5*C)*Sin[c + d*x])/(105*a^4*d) + ((2
1*A + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129*A + 10*C)*Cos[c + d
*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (16*(54*A + 5*C)*Cos[c
 + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x
]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*A*Cos[c + d*x]*Sin[c + d*
x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(-a(9A+2C)+a(5A-2C)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2A\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(}{}}{}$$

$$= -\frac{(129A+10C)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - 2$$

$$= -\frac{(129A+10C)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - 2$$

$$= -\frac{(129A+10C)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - 2$$

$$= -\frac{32(54A+5C)\sin(c+dx)}{105a^4d} + \frac{(21A+2C)\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{(129A+}{10}$$

$$= \frac{(21A+2C)x}{2a^4} - \frac{32(54A+5C)\sin(c+dx)}{105a^4d} + \frac{(21A+2C)\cos(c+dx)\sin(c+}{2a^4d}$$

Mathematica [B] time = 2.31692, size = 505, normalized size = 2.35

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(14700dx(21A+2C)\cos\left(c+\frac{dx}{2}\right)+386190A\sin\left(c+\frac{dx}{2}\right)-422478A\sin\left(c+\frac{3dx}{2}\right)+132930A\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(14700*(21*A + 2*C)*d*x*Cos[(d*x)/2] + 14700*(
21*A + 2*C)*d*x*Cos[c + (d*x)/2] + 185220*A*d*x*Cos[c + (3*d*x)/2] + 17640*
C*d*x*Cos[c + (3*d*x)/2] + 185220*A*d*x*Cos[2*c + (3*d*x)/2] + 17640*C*d*x*
Cos[2*c + (3*d*x)/2] + 61740*A*d*x*Cos[2*c + (5*d*x)/2] + 5880*C*d*x*Cos[2*
c + (5*d*x)/2] + 61740*A*d*x*Cos[3*c + (5*d*x)/2] + 5880*C*d*x*Cos[3*c + (5
*d*x)/2] + 8820*A*d*x*Cos[3*c + (7*d*x)/2] + 840*C*d*x*Cos[3*c + (7*d*x)/2]
 + 8820*A*d*x*Cos[4*c + (7*d*x)/2] + 840*C*d*x*Cos[4*c + (7*d*x)/2] - 53949
```

```
0*A*Sin[(d*x)/2] - 79520*C*Sin[(d*x)/2] + 386190*A*Sin[c + (d*x)/2] + 66080
*C*Sin[c + (d*x)/2] - 422478*A*Sin[c + (3*d*x)/2] - 57120*C*Sin[c + (3*d*x)
/2] + 132930*A*Sin[2*c + (3*d*x)/2] + 30240*C*Sin[2*c + (3*d*x)/2] - 181461
*A*Sin[2*c + (5*d*x)/2] - 22400*C*Sin[2*c + (5*d*x)/2] + 3675*A*Sin[3*c + (
5*d*x)/2] + 6720*C*Sin[3*c + (5*d*x)/2] - 36003*A*Sin[3*c + (7*d*x)/2] - 41
60*C*Sin[3*c + (7*d*x)/2] - 9555*A*Sin[4*c + (7*d*x)/2] - 945*A*Sin[4*c + (
9*d*x)/2] - 945*A*Sin[5*c + (9*d*x)/2] + 105*A*Sin[5*c + (11*d*x)/2] + 105*
A*Sin[6*c + (11*d*x)/2]))/(107520*a^4*d)
```

Maple [A] time = 0.117, size = 264, normalized size = 1.2

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{13\,A}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^7-9/40/d/
a^4*tan(1/2*d*x+1/2*c)^5*A-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*A*ta
n(1/2*d*x+1/2*c)^3+11/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*A*tan(1/2
*d*x+1/2*c)-15/8/d/a^4*C*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)^3*A-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x
+1/2*c)+21/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^4*arctan(tan(1/2*d*x+1/
2*c))*C
```

Maxima [A] time = 1.44788, size = 429, normalized size = 2.

$$-\frac{3\,A\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)+5\,C\left(\frac{315\sin}{\cos(d}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) + 5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) -
 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c
)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 0.51676, size = 632, normalized size = 2.94

$$105\,(21\,A+2\,C)dx\cos(dx+c)^4+420\,(21\,A+2\,C)dx\cos(dx+c)^3+630\,(21\,A+2\,C)dx\cos(dx+c)^2+420\,(21\,A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/210*(105*(21*A + 2*C)*d*x*cos(d*x + c)^4 + 420*(21*A + 2*C)*d*x*cos(d*x +
 c)^3 + 630*(21*A + 2*C)*d*x*cos(d*x + c)^2 + 420*(21*A + 2*C)*d*x*cos(d*x
+ c) + 105*(21*A + 2*C)*d*x + (105*A*cos(d*x + c)^5 - 420*A*cos(d*x + c)^4
- 4*(1509*A + 130*C)*cos(d*x + c)^3 - 4*(3411*A + 310*C)*cos(d*x + c)^2 - (
11619*A + 1070*C)*cos(d*x + c) - 3456*A - 320*C)*sin(d*x + c))/(a^4*d*cos(d
*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d
*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19131, size = 279, normalized size = 1.3

$$\frac{\frac{420\,(dx+c)(21\,A+2\,C)}{a^4} - \frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(420*(d*x + c)*(21*A + 2*C)/a^4 - 840*(9*A*tan(1/2*d*x + 1/2*c)^3 + 7
*A*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*
tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*A*a^24*tan(
1/2*d*x + 1/2*c)^5 - 105*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/
2*d*x + 1/2*c)^3 + 385*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 11655*A*a^24*tan(1/2
*d*x + 1/2*c) - 1575*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.154 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=248

$$-\frac{4(454A+83C)\sin^3(c+dx)}{105a^4d}+\frac{4(454A+83C)\sin(c+dx)}{35a^4d}-\frac{2(11A+2C)\sin(c+dx)\cos(c+dx)}{a^4d}-\frac{4(11A+2C)\text{si}}{3a^4d(\text{se}}$$

```
[Out] (-2*(11*A + 2*C)*x)/a^4 + (4*(454*A + 83*C)*Sin[c + d*x])/(35*a^4*d) - (2*(
11*A + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(a^4*d) - ((178*A + 31*C)*Cos[c + d*
x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*(11*A + 2*C)*Cos[c
 + d*x]^2*Sin[c + d*x])/(3*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x
]^2*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*(8*A + C)*Cos[c + d*x]^
2*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (4*(454*A + 83*C)*Sin[c +
 d*x]^3)/(105*a^4*d)
```

Rubi [A] time = 0.687439, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4020, 3787, 2633, 2635, 8}

$$-\frac{4(454A+83C)\sin^3(c+dx)}{105a^4d}+\frac{4(454A+83C)\sin(c+dx)}{35a^4d}-\frac{2(11A+2C)\sin(c+dx)\cos(c+dx)}{a^4d}-\frac{4(11A+2C)\text{si}}{3a^4d(\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```



```
[Out] (-2*(11*A + 2*C)*x)/a^4 + (4*(454*A + 83*C)*Sin[c + d*x])/(35*a^4*d) - (2*(
11*A + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(a^4*d) - ((178*A + 31*C)*Cos[c + d*
x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*(11*A + 2*C)*Cos[c
 + d*x]^2*Sin[c + d*x])/(3*a^4*d*(1 + Sec[c + d*x])) - ((A + C)*Cos[c + d*x
]^2*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*(8*A + C)*Cos[c + d*x]^
2*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (4*(454*A + 83*C)*Sin[c +
 d*x]^3)/(105*a^4*d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^3(c+dx)(-a(10A+3C)+a(6A-C)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2(8A+C)\cos^2(c+dx)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \int$$

$$= -\frac{(178A+31C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} -$$

$$= -\frac{(178A+31C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} -$$

$$= -\frac{(178A+31C)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} -$$

$$= -\frac{2(11A+2C)\cos(c+dx)\sin(c+dx)}{a^4d} - \frac{(178A+31C)\cos^2(c+dx)\sin(c+d}{105a^4d(1+\sec(c+dx))^2}$$

$$= -\frac{2(11A+2C)x}{a^4} + \frac{4(454A+83C)\sin(c+dx)}{35a^4d} - \frac{2(11A+2C)\cos(c+dx)\sin}{a^4d}$$

Mathematica [B] time = 2.75575, size = 575, normalized size = 2.32

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(58800dx(11A+2C)\cos\left(c+\frac{dx}{2}\right)+687260A\sin\left(c+\frac{dx}{2}\right)-814107A\sin\left(c+\frac{3dx}{2}\right)+204645A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^7*(58800*(11*A + 2*C)*d*x*Cos[(d*x)/2] + 58800*
(11*A + 2*C)*d*x*Cos[c + (d*x)/2] + 388080*A*d*x*Cos[c + (3*d*x)/2] + 70560
*C*d*x*Cos[c + (3*d*x)/2] + 388080*A*d*x*Cos[2*c + (3*d*x)/2] + 70560*C*d*x
```

```
*Cos[2*c + (3*d*x)/2] + 129360*A*d*x*Cos[2*c + (5*d*x)/2] + 23520*C*d*x*Cos
[2*c + (5*d*x)/2] + 129360*A*d*x*Cos[3*c + (5*d*x)/2] + 23520*C*d*x*Cos[3*c
 + (5*d*x)/2] + 18480*A*d*x*Cos[3*c + (7*d*x)/2] + 3360*C*d*x*Cos[3*c + (7*
d*x)/2] + 18480*A*d*x*Cos[4*c + (7*d*x)/2] + 3360*C*d*x*Cos[4*c + (7*d*x)/2
] - 1010660*A*Sin[(d*x)/2] - 243320*C*Sin[(d*x)/2] + 687260*A*Sin[c + (d*x)
/2] + 184520*C*Sin[c + (d*x)/2] - 814107*A*Sin[c + (3*d*x)/2] - 184464*C*Si
n[c + (3*d*x)/2] + 204645*A*Sin[2*c + (3*d*x)/2] + 72240*C*Sin[2*c + (3*d*x
)/2] - 357609*A*Sin[2*c + (5*d*x)/2] - 77168*C*Sin[2*c + (5*d*x)/2] - 18025
*A*Sin[3*c + (5*d*x)/2] + 8400*C*Sin[3*c + (5*d*x)/2] - 72522*A*Sin[3*c + (
7*d*x)/2] - 15164*C*Sin[3*c + (7*d*x)/2] - 24010*A*Sin[4*c + (7*d*x)/2] - 2
940*C*Sin[4*c + (7*d*x)/2] - 2310*A*Sin[4*c + (9*d*x)/2] - 420*C*Sin[4*c +
(9*d*x)/2] - 2310*A*Sin[5*c + (9*d*x)/2] - 420*C*Sin[5*c + (9*d*x)/2] + 175
*A*Sin[5*c + (11*d*x)/2] + 175*A*Sin[6*c + (11*d*x)/2] - 35*A*Sin[6*c + (13
*d*x)/2] - 35*A*Sin[7*c + (13*d*x)/2]))/(107520*a^4*d)
```

Maple [A] time = 0.111, size = 402, normalized size = 1.6

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{11\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,C}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{59\,A}{24\,da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*C*tan(1/2*d*x+1/2*c)^7+11/40/
d/a^4*tan(1/2*d*x+1/2*c)^5*A+7/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-59/24/d/a^4*
A*tan(1/2*d*x+1/2*c)^3-23/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3+209/8/d/a^4*A*tan
(1/2*d*x+1/2*c)+49/8/d/a^4*C*tan(1/2*d*x+1/2*c)+26/d/a^4/(1+tan(1/2*d*x+1/2
*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/
2*d*x+1/2*c)^5+124/3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*
A+4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)^3+18/d/a^4/(1+tan
(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^
3*C*tan(1/2*d*x+1/2*c)-44/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))-8/d/a^4*arctan
(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.45614, size = 547, normalized size = 2.21

$$\frac{A\left(\frac{560\left(\frac{27\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{62\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{39\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4+\frac{3\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{21945\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2065\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{231\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{36960\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] 1/840*(A*(560*(27*sin(d*x + c)/(cos(d*x + c) + 1) + 62*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 39*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4 + 3*a^4*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (21945*sin(d*x + c)/(cos(d*x
+ c) + 1) - 2065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 231*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 36960*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4) + C*(1680*sin(d*x + c)/((a^4 +
```

```
a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*
x + c)/(cos(d*x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 0.529699, size = 676, normalized size = 2.73

$$-\frac{210\,(11\,A+2\,C)dx\cos\left(dx+c\right)^4+840\,(11\,A+2\,C)dx\cos\left(dx+c\right)^3+1260\,(11\,A+2\,C)dx\cos\left(dx+c\right)^2+840\,(11\,A+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] -1/105*(210*(11*A + 2*C)*d*x*cos(d*x + c)^4 + 840*(11*A + 2*C)*d*x*cos(d*x
+ c)^3 + 1260*(11*A + 2*C)*d*x*cos(d*x + c)^2 + 840*(11*A + 2*C)*d*x*cos(d*
x + c) + 210*(11*A + 2*C)*d*x - (35*A*cos(d*x + c)^6 - 70*A*cos(d*x + c)^5
+ 35*(14*A + 3*C)*cos(d*x + c)^4 + 8*(799*A + 148*C)*cos(d*x + c)^3 + 4*(35
92*A + 659*C)*cos(d*x + c)^2 + 2*(6109*A + 1118*C)*cos(d*x + c) + 3632*A +
664*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4
*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19713, size = 352, normalized size = 1.42

$$-\frac{\frac{1680\,(dx+c)(11\,A+2\,C)}{a^4}-\frac{560\left(39\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+62\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,C\tan\left(\frac{1}{2}\,dx+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/840*(1680*(d*x + c)*(11*A + 2*C)/a^4 - 560*(39*A*tan(1/2*d*x + 1/2*c)^5
+ 3*C*tan(1/2*d*x + 1/2*c)^5 + 62*A*tan(1/2*d*x + 1/2*c)^3 + 6*C*tan(1/2*d*
x + 1/2*c)^3 + 27*A*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^3*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a
^24*tan(1/2*d*x + 1/2*c)^7 - 231*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*C*a^24
*tan(1/2*d*x + 1/2*c)^5 + 2065*A*a^24*tan(1/2*d*x + 1/2*c)^3 + 805*C*a^24*t
```

```
an(1/2*d*x + 1/2*c)^3 - 21945*A*a^24*tan(1/2*d*x + 1/2*c) - 5145*C*a^24*tan
(1/2*d*x + 1/2*c))/a^28)/d
```

3.155 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=223

$$\frac{2a(99A+80C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4(99A+80C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+80C)\tan(c+dx)}{3465d}$$

```
[Out] (4*a*(99*A + 80*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9
9*A + 80*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) +
 (2*a*C*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(
99*A + 80*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3465*d) + (2*C*Sec[c +
 d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(11*d) + (4*(99*A + 80*C)*(a
 + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1155*a*d)
```

Rubi [A] time = 0.515776, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4089, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a(99A+80C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4(99A+80C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+80C)\tan(c+dx)}{3465d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a*(99*A + 80*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9
9*A + 80*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) +
 (2*a*C*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(
99*A + 80*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3465*d) + (2*C*Sec[c +
 d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(11*d) + (4*(99*A + 80*C)*(a
 + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1155*a*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
```

```
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx &= \frac{2C\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{11d}+\frac{2\int \mathrm{se}
\\
&= \frac{2aC\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sec^4(c+dx)\sqrt{a+a}
\\
&= \frac{2a(99A+80C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2aC\sec^4(c}{99d\sqrt{a+}}
\\
&= \frac{2a(99A+80C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2aC\sec^4(c}{99d\sqrt{a+}}
\\
&= \frac{2a(99A+80C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2aC\sec^4(c}{99d\sqrt{a+}}
\\
&= \frac{4a(99A+80C)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}}+\frac{2a(99A+80C)\sec^3(c+}{693d\sqrt{a+a\sec(}}
\end{aligned}
$$

Mathematica [A] time = 1.06486, size = 143, normalized size = 0.64

$$
\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}((2871A+3020C)\cos(c+dx)+13(99A+80C)\cos(2(c+dx))+128}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((1089*A + 1510*C + (2871*A + 3020*C)*Cos[c + d*x] + 13*(99*A + 80*C)*Cos[2
*(c + d*x)] + 1287*A*Cos[3*(c + d*x)] + 1040*C*Cos[3*(c + d*x)] + 198*A*Cos
```

```
[4*(c + d*x)] + 160*C*Cos[4*(c + d*x)] + 198*A*Cos[5*(c + d*x)] + 160*C*Cos
[5*(c + d*x)])*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/
(3465*d)
```

Maple [A] time = 0.406, size = 151, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(1584A(\cos(dx+c))^5+1280C(\cos(dx+c))^5+792A(\cos(dx+c))^4+640C(\cos(dx+c))^4+\right.}{3465d(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(1584*A*cos(d*x+c)^5+1280*C*cos(d*x+c)^5+792*A*co
s(d*x+c)^4+640*C*cos(d*x+c)^4+594*A*cos(d*x+c)^3+480*C*cos(d*x+c)^3+495*A*c
os(d*x+c)^2+400*C*cos(d*x+c)^2+350*C*cos(d*x+c)+315*C)*(a*(cos(d*x+c)+1)/co
s(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.505868, size = 352, normalized size = 1.58

$$\frac{2\left(16(99A+80C)\cos(dx+c)^5+8(99A+80C)\cos(dx+c)^4+6(99A+80C)\cos(dx+c)^3+5(99A+80C)\cos(d\right.}{3465\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2/3465*(16*(99*A + 80*C)*cos(d*x + c)^5 + 8*(99*A + 80*C)*cos(d*x + c)^4 +
6*(99*A + 80*C)*cos(d*x + c)^3 + 5*(99*A + 80*C)*cos(d*x + c)^2 + 350*C*cos
(d*x + c) + 315*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*
cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.7097, size = 424, normalized size = 1.9

$$-\frac{2\left(3465\sqrt{2}Aa^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+3465\sqrt{2}Ca^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(10395\sqrt{2}Aa^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+5775\sqrt{2}Ca^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -2/3465*(3465*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 3465*sqrt(2)*C*a^6*sgn(cos(
d*x + c)) - (10395*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 5775*sqrt(2)*C*a^6*sgn
(cos(d*x + c)) - (15246*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 16170*sqrt(2)*C*a
^6*sgn(cos(d*x + c)) - (14058*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 8910*sqrt(2
)*C*a^6*sgn(cos(d*x + c)) - (6633*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 5885*sq
rt(2)*C*a^6*sgn(cos(d*x + c)) - (891*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 755*
sqrt(2)*C*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^
2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*d)
```

3.156 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=180

$$\frac{2(21A+16C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2a(21A+16C)\tan(c}{45d\sqrt{a\sec(c+dx)}}$$

```
[Out] (2*a*(21*A + 16*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*S
ec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(21*A + 16
*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*C*Sec[c + d*x]^3*Sq
rt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(21*A + 16*C)*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.445481, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4089, 4016, 3800, 4001, 3792}

$$\frac{2(21A+16C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2a(21A+16C)\tan(c}{45d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a*(21*A + 16*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*S
ec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(21*A + 16
*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*C*Sec[c + d*x]^3*Sq
rt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(21*A + 16*C)*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{9d} + \frac{2\int \mathrm{se}}{}$$
$$= \frac{2aC\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx)\sqrt{a+a}}{9}$$
$$= \frac{2aC\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx)\sqrt{a+a}}{9}$$
$$= \frac{2aC\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} - \frac{4(21A+16C)\sqrt{a+a\,\mathrm{se}}}{315}$$
$$= \frac{2a(21A+16C)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sec^3(c+dx)\tan(c+}{63d\sqrt{a+a\sec(c+dx}}$$

Mathematica [A] time = 0.98639, size = 122, normalized size = 0.68

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}(2(63A+88C)\cos(c+dx)+11(21A+16C)\cos(2(c+dx))+42A\,\mathrm{co}}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((189*A + 214*C + 2*(63*A + 88*C)*Cos[c + d*x] + 11*(21*A + 16*C)*Cos[2*(c
+ d*x)] + 42*A*Cos[3*(c + d*x)] + 32*C*Cos[3*(c + d*x)] + 42*A*Cos[4*(c + d
*x)] + 32*C*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*Tan
[(c + d*x)/2])/(315*d)
```

Maple [A] time = 0.352, size = 129, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(168A(\cos(dx+c))^4+128C(\cos(dx+c))^4+84A(\cos(dx+c))^3+64C(\cos(dx+c))^3+6}{315d(\cos(dx+c))^4\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(168*A*cos(d*x+c)^4+128*C*cos(d*x+c)^4+84*A*cos(d*
x+c)^3+64*C*cos(d*x+c)^3+63*A*cos(d*x+c)^2+48*C*cos(d*x+c)^2+40*C*cos(d*x+c
)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.496488, size = 302, normalized size = 1.68

$$\frac{2\left(8(21A+16C)\cos(dx+c)^4+4(21A+16C)\cos(dx+c)^3+3(21A+16C)\cos(dx+c)^2+40C\cos(dx+c)+35C\right.}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2/315*(8*(21*A + 16*C)*cos(d*x + c)^4 + 4*(21*A + 16*C)*cos(d*x + c)^3 + 3*
(21*A + 16*C)*cos(d*x + c)^2 + 40*C*cos(d*x + c) + 35*C)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\sec(c+dx)+1)}\left(A+C\sec^2(c+dx)\right)\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + C*sec(c + d*x)**2)*sec(c + d*x)**3
, x)
```

Giac [A] time = 4.62029, size = 362, normalized size = 2.01

$$\frac{2\left(315\sqrt{2}Aa^5\mathrm{sgn}(\cos(dx+c))+315\sqrt{2}Ca^5\mathrm{sgn}(\cos(dx+c))-\left(840\sqrt{2}Aa^5\mathrm{sgn}(\cos(dx+c))+420\sqrt{2}Ca^5\mathrm{sgn}(\cos(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 2/315*(315*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 315*sqrt(2)*C*a^5*sgn(cos(d*x
+ c)) - (840*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 420*sqrt(2)*C*a^5*sgn(cos(d*
x + c)) - (882*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 882*sqrt(2)*C*a^5*sgn(cos(
d*x + c)) - (504*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 324*sqrt(2)*C*a^5*sgn(co
s(d*x + c)) - (147*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 107*sqrt(2)*C*a^5*sgn(
cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.157 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=137

$$\frac{2(35A+18C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(35A+27C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sec^2(c+dx)\sqrt{a\sec(c+dx)+a}}{7d}$$

```
[Out] (2*a*(35*A + 27*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(35*
A + 18*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*C*Sec[c + d*x
]^2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(7*d) + (2*C*(a + a*Sec[c + d*x]
)^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.3905, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4089, 4010, 4001, 3792}

$$\frac{2(35A+18C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(35A+27C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sec^2(c+dx)\sqrt{a\sec(c+dx)+a}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(35*A + 27*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(35*
A + 18*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*C*Sec[c + d*x
]^2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(7*d) + (2*C*(a + a*Sec[c + d*x]
)^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{7d}+\frac{2\int \text{se}}{}$$

$$= \frac{2C\sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{7d}+\frac{2C(a}{}$$

$$= \frac{2(35A+18C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d}+\frac{2C\sec^2}{}$$

$$= \frac{2a(35A+27C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2(35A+18C)\sqrt{a+a\sec}}{105d}$$

Mathematica [A] time = 0.783647, size = 99, normalized size = 0.72

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}(3(35A+36C)\cos(c+dx)+(35A+24C)\cos(2(c+dx))+35A\cos(}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((35*A + 54*C + 3*(35*A + 36*C)*Cos[c + d*x] + (35*A + 24*C)*Cos[2*(c + d*x
)] + 35*A*Cos[3*(c + d*x)] + 24*C*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Sqrt[a*(
1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(105*d)
```

Maple [A] time = 0.331, size = 107, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(70A(\cos(dx+c))^3+48C(\cos(dx+c))^3+35A(\cos(dx+c))^2+24C(\cos(dx+c))^2+18C}{105d(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(70*A*cos(d*x+c)^3+48*C*cos(d*x+c)^3+35*A*cos(d*x+
c)^2+24*C*cos(d*x+c)^2+18*C*cos(d*x+c)+15*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.486188, size = 255, normalized size = 1.86

$$\frac{2\left(2\,(35\,A+24\,C)\cos\left(dx+c\right)^{3}+(35\,A+24\,C)\cos\left(dx+c\right)^{2}+18\,C\cos\left(dx+c\right)+15\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2/105*(2*(35*A + 24*C)*cos(d*x + c)^3 + (35*A + 24*C)*cos(d*x + c)^2 + 18*C
*cos(d*x + c) + 15*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + C*sec(c + d*x)**2)*sec(c + d*x)**2
, x)
```

Giac [A] time = 4.56029, size = 300, normalized size = 2.19

$$-\frac{2\left(105\,\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\,\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(245\,\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\,\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -2/105*(105*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^4*sgn(cos(d*x
 + c)) - (245*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^4*sgn(cos(d
*x + c)) - (175*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 147*sqrt(2)*C*a^4*sgn(cos
(d*x + c)) - (35*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 27*sqrt(2)*C*a^4*sgn(cos
(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.158 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{2a(15A+7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}$$

```
[Out] (2*a*(15*A + 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (4*C*Sqrt
[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec[c + d*x])^(3/2)
*Tan[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.198207, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4083, 4001, 3792}

$$\frac{2a(15A+7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(15*A + 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (4*C*Sqrt
[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec[c + d*x])^(3/2)
*Tan[c + d*x])/(5*a*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{2\int \sec(c+dx)\sqrt{a+}}{}$$
$$= -\frac{4C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2C(a+a\sec(c+dx)}{5ad}$$
$$= \frac{2a(15A+7C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{4C\sqrt{a+a\sec(c+dx)}\tan(c+}{15d}$$

Mathematica [A] time = 0.986517, size = 71, normalized size = 0.75

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}((15A+8C)\cos(2(c+dx))+15A+8C\cos(c+dx)+14C)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((15*A + 14*C + 8*C*Cos[c + d*x] + (15*A + 8*C)*Cos[2*(c + d*x)])*Sec[c + d
*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.309, size = 85, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^2+8C(\cos(dx+c))^2+4C\cos(dx+c)+3C\right)}{15d(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^2+8*C*cos(d*x+c)^2+4*C*cos(d*x+c)+
3*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.483187, size = 205, normalized size = 2.16

$$\frac{2\left((15A+8C)\cos(dx+c)^2+4C\cos(dx+c)+3C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/15*((15*A + 8*C)*cos(d*x + c)^2 + 4*C*cos(d*x + c) + 3*C)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + C*sec(c + d*x)**2)*sec(c + d*x), x
)
```

Giac [B] time = 4.58747, size = 238, normalized size = 2.51

$$\frac{2\left(15\sqrt{2}Aa^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\sqrt{2}Ca^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(30\sqrt{2}Aa^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+10\sqrt{2}Ca^{3}\mathrm{sgn}\left(\cos\left(dx\right.\right.\right.\right.}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 2/15*(15*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 15*sqrt(2)*C*a^3*sgn(cos(d*x + c
)) - (30*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 10*sqrt(2)*C*a^3*sgn(cos(d*x + c
)) - (15*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 7*sqrt(2)*C*a^3*sgn(cos(d*x + c)
))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a
*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.159 $\int \sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=96

$$\frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2aC\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*C*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sqrt[a + a*Sec[c
+ d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.145721, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4055, 3915, 3774, 203, 3792}

$$\frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2aC\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*C*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sqrt[a + a*Sec[c
+ d*x]]*Tan[c + d*x])/(3*d)
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2\int \sqrt{a+a\sec(c+dx)}\left(\frac{3aA}{2}+\right.}{3a}$$

$$= \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + A\int \sqrt{a+a\sec(c+dx)}\,dx + \frac{1}{3}$$

$$= \frac{2aC\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} - \frac{(2aA)}{}$$

$$= \frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aC\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sqrt{a+a\,\mathrm{se}}}{}$$

Mathematica [A] time = 0.63819, size = 96, normalized size = 1.

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(3A\cos(c+dx)\tan^{-1}\left(\sqrt{\sec(c+dx)-1}\right)+C(2\cos(c+dx)+1)\sqrt{\mathrm{se}}\right.}{3d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[c + d*x] + C*(1 + 2*Cos[c + d*x
])*Sqrt[-1 + Sec[c + d*x]])*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
+ d*x)/2])/(3*d*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.327, size = 216, normalized size = 2.3

$$\frac{1}{6d\sin(dx+c)\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+3*A*2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*C*cos(d*x+c)^2+4*C*cos(d*x+c)+4*C)/sin(
d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.546649, size = 771, normalized size = 8.03

$$\left[\frac{3\left(A\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,C\cos\left(dx\right.\right.}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^
2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*C*cos(d*x + c) + C)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d
*cos(d*x + c)), -2/3*(3*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(a)*arctan(
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))
) - (2*C*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + C*sec(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.160 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=94

$$-\frac{a(A-2C)\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (A*
Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d - (a*(A - 2*C)*Tan[c + d*x])/(d*Sq
rt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.200783, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4087, 3915, 3774, 203, 3792}

$$-\frac{a(A-2C)\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (A*
Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d - (a*(A - 2*C)*Tan[c + d*x])/(d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{\int \sqrt{a+a\sec(c+dx)}\left(\frac{a}{2}\right.}{}$$
$$= \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}A\int \sqrt{a+a\sec(c+d}$$
$$= \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} - \frac{a(A-2C)\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} -$$
$$= \frac{\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+a}{d}$$

Mathematica [A] time = 0.328001, size = 84, normalized size = 0.89

$$\frac{a\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}(A\cos(c+dx)+2C)+A\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (2*C + A*Cos[c + d*x])*Sqrt[1 - Sec
[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])])
```

Maple [A] time = 0.348, size = 138, normalized size = 1.5

$$-\frac{1}{2d\sin(dx+c)}\left(A\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+2A(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*
cos(d*x+c)^2-2*A*cos(d*x+c)+4*C*cos(d*x+c)-4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.81915, size = 1069, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A/d
```

Fricas [A] time = 0.543701, size = 679, normalized size = 7.22

$$\left[\frac{(A\cos(dx+c)+A)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(A\cos(dx+c)+2\,C)\sqrt{}}{2\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/2*((A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
 + c) - a)/(cos(d*x + c) + 1)) + 2*(A*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -((A*cos(d*x + c
) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - (A*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.338, size = 485, normalized size = 5.16

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}Ca\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+A\sqrt{-a}\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a*sgn(cos(d*x + c))*t
an(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + A*sqrt(-a)*log(abs((sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2
*sqrt(2) + 3)))*sgn(cos(d*x + c)) - A*sqrt(-a)*log(abs((sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*s
gn(cos(d*x + c)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a*sgn(cos(d*x + c)) - A*sqrt(-a)*a^
2*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.161 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=110

$$\frac{\sqrt{a}(3A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (Sqrt[a]*(3*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*A*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d
*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.250998, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4087, 4015, 3774, 203}

$$\frac{\sqrt{a}(3A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aA\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(3*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*A*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d
*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d} + \frac{\int \cos(c+d}{}$$
$$= \frac{aA\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\mathrm{s}}{2d}$$
$$= \frac{aA\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\mathrm{s}}{2d}$$
$$= \frac{\sqrt{a}(3A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{aA\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx}}$$

Mathematica [A] time = 0.413832, size = 108, normalized size = 0.98

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(3A+8C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\mathrm{si}\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(3
*A + 8*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sqrt[Cos[c + d*x]]*(2*Sin[
(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)
```

Maple [B] time = 0.376, size = 376, normalized size = 3.4

$$-\frac{1}{16\,d\cos(dx+c)\sin(dx+c)}\left(-3\,A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\left(-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d*(-3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(3/2)-8*C*cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*sin(d*x+c)-3*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c
)-8*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+8*A*cos(
d*x+c)^4+4*A*cos(d*x+c)^3-12*A*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 2.08601, size = 1629, normalized size = 14.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/16*(16*C*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1)))*A)/d
```

Fricas [A] time = 0.637843, size = 779, normalized size = 7.08

$$\left[\frac{((3A+8C)\cos(dx+c)+3A+8C)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2A\,\mathrm{co}\right.}{8\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/8*(((3*A + 8*C)*cos(d*x + c) + 3*A + 8*C)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^2 +
3*A*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c) + d), -1/4*(((3*A + 8*C)*cos(d*x + c) + 3*A + 8*C)*sqrt(a)*arc
tan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) - (2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.441, size = 602, normalized size = 5.47

$$-\frac{\left(3\,A\sqrt{-a}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+8\,C\sqrt{-a}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/8*((3*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*sgn(cos(d*x + c)))*log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*sg
n(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a*sgn(co
s(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^4*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 17*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^3*sgn(cos(d*x
 + c)) + A*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.162 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=153

$$\frac{a(5A+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{A\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{aA\text{s}}{12}$$

```
[Out] (Sqrt[a]*(5*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*A + 8*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c +
d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.351455, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4087, 4015, 3805, 3774, 203}

$$\frac{a(5A+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{A\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{aA\text{s}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (Sqrt[a]*(5*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*A + 8*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c +
d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\int \cos^2(c+}{}$$

$$= \frac{aA\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sqrt{a+a\sec(c-}{3d}$$

$$= \frac{a(5A+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A}{}$$

$$= \frac{a(5A+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A}{}$$

$$= \frac{\sqrt{a}(5A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a(5A+8C)\sin(c+}{8d\sqrt{a+a\sec(c+d}}$$

Mathematica [C] time = 0.360273, size = 117, normalized size = 0.76

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2A\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)+C\left(\cos(c+d\right.\right.}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((C*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])
 + 2*A*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]]
)
```

Maple [B] time = 0.444, size = 569, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d*(15*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)
*cos(d*x+c)^2+24*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
```

```
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*cos(d*x+c)^2+30*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(5/2)*cos(d*x+c)+48*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*cos(d*x+c)+15*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*sin(d*x+c)+24*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x
+c)+64*A*cos(d*x+c)^6+16*A*cos(d*x+c)^5+40*A*cos(d*x+c)^4+192*C*cos(d*x+c)^
4-120*A*cos(d*x+c)^3-192*C*cos(d*x+c)^3)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2
)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [B] time = 2.70446, size = 3663, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/96*((4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
```

```
 + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arct
an2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + 24*(2*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*
(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
 c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + s
in(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1)
- arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1)
 - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1)))*C)/d
```

Fricas [A] time = 0.648524, size = 873, normalized size = 5.71

$$\left[\frac{3\left((5A+8C)\cos(dx+c)+5A+8C\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8A\right.}{48\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/48*(3*((5*A + 8*C)*cos(d*x + c) + 5*A + 8*C)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d*x + c)^3
 + 10*A*cos(d*x + c)^2 + 3*(5*A + 8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(3*((5*A + 8*C)
*cos(d*x + c) + 5*A + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^3 + 10*A*co
s(d*x + c)^2 + 3*(5*A + 8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.69748, size = 1156, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/48*(3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*sgn(cos(d*x + c)))*
log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*C*sqrt(-
a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(63*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a
*sgn(cos(d*x + c)) + 72*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a*sgn(cos(d*x + c)) - 369*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^2*sgn(
cos(d*x + c)) - 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1638*(sqrt(-a)*tan(1/2*
```

```
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(co
s(d*x + c)) + 3024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^6*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 1074*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos
(d*x + c)) - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^4*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 171*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d
*x + c)) + 360*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 13*A*sqrt(-a)*a^6*sgn(cos(d*
x + c)) - 24*C*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.163 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=196

$$\frac{a(35A+48C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(35A+48C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a(35A+48C)\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(}{}$$

```
[Out] (Sqrt[a]*(35*A + 48*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a*(35*A + 48*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]
) + (a*(35*A + 48*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) +
(A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.419558, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4087, 4015, 3805, 3774, 203}

$$\frac{a(35A+48C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(35A+48C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a(35A+48C)\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(35*A + 48*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a*(35*A + 48*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]
) + (a*(35*A + 48*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) +
(A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{\int \cos^3(c+$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^3(c+dx)\sqrt{a+a\sec(c}{4d}$$
$$= \frac{a(35A+48C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos^2(c+dx)\,\mathrm{s}}{24d\sqrt{a+a\sec(}}$$
$$= \frac{a(35A+48C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a(35A+48C)\cos(c+dx)\sin}{96d\sqrt{a+a\sec(c+dx}}$$
$$= \frac{a(35A+48C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a(35A+48C)\cos(c+dx)\sin}{96d\sqrt{a+a\sec(c+dx}}$$
$$= \frac{\sqrt{a}(35A+48C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a(35A+48C)\sin}{64d\sqrt{a+a\sec(c}}$$

Mathematica [C] time = 0.190977, size = 70, normalized size = 0.36

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(A\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)+C\text{Hypergeometric2F1}\left(\frac{1}{2},3\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(C*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F
1[1/2, 5, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/
2])/d
```

Maple [B] time = 0.375, size = 751, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3072/d*(105*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(7/2)*2^(1/2)+144*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(7/2)*2^(1/2)+315*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*2^(1/2)+432*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)+315*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+432*C*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+105*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)*sin(d*x+c)+144*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-128*A*cos(d*x+c)^7-224*A*cos(d*x+c)^
6-1536*C*cos(d*x+c)^6-560*A*cos(d*x+c)^5-768*C*cos(d*x+c)^5+1680*A*cos(d*x+
c)^4+2304*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/co
s(d*x+c)^3
```

<u>Maxima</u> [B] time = 3.50748, size = 10394, normalized size = 53.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/768*(48*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
```

```
+ 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - 1)))*C - (2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^
3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(
4*d*x + 4*c) + 9*sin(4*d*x + 4*c)^3 + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x +
 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x
+ 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2
*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)
^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c)
- 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4
*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*
x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 9*cos(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) - 36*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*c
os(4*d*x + 4*c)^3 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(
4*d*x + 4*c)^2 - 10*cos(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) + 8)*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (9*cos(4*d*x + 4*c) + 8)*sin
(4*d*x + 4*c)^2 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*
d*x + 4*c)^2 + 26*cos(4*d*x + 4*c)^2 + 25*cos(4*d*x + 4*c) + 8)*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 - (32*(c
os(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin
(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 1
6*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*co
s(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(9*cos(4*d*x + 4*c)^
3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c)^2 - 8*co
s(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*
(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + sin(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 4*(4*(9*cos(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c))*sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((
64*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 20*(sin(4*d*x + 4*c)^3
+ (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) + 8*(cos(4
```

```
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 5*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 5*sin(4*d*x +
 4*c)^3 + 4*(5*sin(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c)^2 + 10*cos(4*d*x +
4*c) - 11)*sin(4*d*x + 4*c) - 64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))*sin(4*d*x + 4*c) + 40*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 +
 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 10*(2*sin(4*d*
x + 4*c)^3 + 2*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + c
os(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (16*
cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 17*cos(4*d*x + 4*c) + 1)*sin(1
/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 5*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + 2*(32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
- 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 8*cos(4*d*x + 4*c)^2 + 8*(4*cos(4*d*x + 4*c)^2 - sin(4*d*x + 4*c)^2
 - 40*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - 4*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - 5*(cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) - 2*sin(4*d*x + 4*c)^2 - 85*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 5*(8*cos(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c)
)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1)) - (64*(cos(4*d*x + 4*c)^2 + sin(4*d*x +
4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^3 + 5*cos(4*d*x + 4*c)^3 + 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*
x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 18*cos(4*d*x + 4*c)^2 + 8*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 37*cos(4*d*x + 4*c) - 24)*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*
x + 4*c)^2 + 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x
+ 4*c)^2 - 14*cos(4*d*x + 4*c)^2 + 16*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c
)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 8*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) +
 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 43*cos(4*d*x + 4
*c) - 24)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 24*cos(4
*d*x + 4*c)^2 + 2*(10*cos(4*d*x + 4*c)^3 + 10*(cos(4*d*x + 4*c) - 4)*sin(4*
d*x + 4*c)^2 - 50*cos(4*d*x + 4*c)^2 + (16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*
x + 4*c)^2 - 21*cos(4*d*x + 4*c) + 5)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) - 5*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 48*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + (8*cos(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)^2 - 5*cos(4*d*
x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(128*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 8*(5
*(cos(4*d*x + 4*c) - 4)*sin(4*d*x + 4*c) + 8*cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 2*(5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c) + 21*cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 5*(cos(
4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*sin(4*d*x + 4*c)*sin(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 105*((4*(cos(4*d*x + 4*c)^2 + sin(4
*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*
d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + co
s(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x +
 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4
```

```
*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
 + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2
(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(co
s(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + si
n(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4
*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(
cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1))) - 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos
(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x +
 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)) + 1) + (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4
*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(c
```

```
os(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4
*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1)) - 1))*sqrt(a))*A/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^
2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4
*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

Fricas [A] time = 0.737268, size = 984, normalized size = 5.02

$$\left[\frac{3\left((35\,A+48\,C)\cos\left(dx+c\right)+35\,A+48\,C\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,(}{384\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/384*(3*((35*A + 48*C)*cos(d*x + c) + 35*A + 48*C)*sqrt(-a)*log((2*a*cos(
d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*cos(d*x
+ c)^4 + 56*A*cos(d*x + c)^3 + 2*(35*A + 48*C)*cos(d*x + c)^2 + 3*(35*A + 4
8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d), -1/192*(3*((35*A + 48*C)*cos(d*x + c) + 35*A + 48*C)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - (48*A*cos(d*x + c)^4 + 56*A*cos(d*x + c)^3 + 2*(35*A + 48*
C)*cos(d*x + c)^2 + 3*(35*A + 48*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.8139, size = 1458, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/384*(3*(35*A*sqrt(-a)*sgn(cos(d*x + c)) + 48*C*sqrt(-a)*sgn(cos(d*x + c)
))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3))) - 3*(35*A*sqrt(-a)*sgn(cos(d*x + c)) + 48*C*s
qrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(279*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt
(-a)*a*sgn(cos(d*x + c)) + 240*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a*sgn(cos(d*x + c)) + 285*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*
a^2*sgn(cos(d*x + c)) - 1968*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 4605*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)
*a^3*sgn(cos(d*x + c)) - 2640*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 37281*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a
)*a^4*sgn(cos(d*x + c)) + 41616*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 35643*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-
a)*a^5*sgn(cos(d*x + c)) - 42288*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 9175*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-
a)*a^6*sgn(cos(d*x + c)) + 12528*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 1311*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-
a)*a^7*sgn(cos(d*x + c)) - 1392*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 43*A*sqrt(-
a)*a^8*sgn(cos(d*x + c)) + 48*C*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/
d
```

3.164 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=225

$$\frac{2a^2(33A+28C)\tan(c+dx)\sec^3(c+dx)}{231d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(143A+112C)\tan(c+dx)}{165d\sqrt{a\sec(c+dx)+a}}+\frac{2(143A+112C)\tan(c+dx)(a\sec(c+}{385d}$$

```
[Out] (2*a^2*(143*A + 112*C)*Tan[c + d*x])/(165*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(33*A + 28*C)*Sec[c + d*x]^3*Tan[c + d*x])/(231*d*Sqrt[a + a*Sec[c + d*
x]]) - (4*a*(143*A + 112*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(1155*d)
 + (2*a*C*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(33*d) + (2
*(143*A + 112*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(385*d) + (2*C*Se
c[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(11*d)
```

Rubi [A] time = 0.654957, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4089, 4018, 4016, 3800, 4001, 3792}

$$\frac{2a^2(33A+28C)\tan(c+dx)\sec^3(c+dx)}{231d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(143A+112C)\tan(c+dx)}{165d\sqrt{a\sec(c+dx)+a}}+\frac{2(143A+112C)\tan(c+dx)(a\sec(c+}{385d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a^2*(143*A + 112*C)*Tan[c + d*x])/(165*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(33*A + 28*C)*Sec[c + d*x]^3*Tan[c + d*x])/(231*d*Sqrt[a + a*Sec[c + d*
x]]) - (4*a*(143*A + 112*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(1155*d)
 + (2*a*C*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(33*d) + (2
*(143*A + 112*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(385*d) + (2*C*Se
c[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(11*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{11d} + \frac{2\int \text{s}\ldots}{\ldots}$$

$$= \frac{2aC\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{33d} + \frac{2C\,\text{se}\ldots}{\ldots}$$

$$= \frac{2a^2(33A+28C)\sec^3(c+dx)\tan(c+dx)}{231d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sec^3(c\ldots}{\ldots}$$

$$= \frac{2a^2(33A+28C)\sec^3(c+dx)\tan(c+dx)}{231d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sec^3(c\ldots}{\ldots}$$

$$= \frac{2a^2(33A+28C)\sec^3(c+dx)\tan(c+dx)}{231d\sqrt{a+a\sec(c+dx)}} - \frac{4a(143A+1\ldots}{\ldots}$$

$$= \frac{2a^2(143A+112C)\tan(c+dx)}{165d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(33A+28C)\sec^3(\ldots}{231d\sqrt{a+a\,\text{se}\ldots}}$$

Mathematica [A] time = 1.365, size = 144, normalized size = 0.64

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}((4147A+4228C)\cos(c+dx)+2(737A+728C)\cos(2(c+dx))+185\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a*(1188*A + 1652*C + (4147*A + 4228*C)*Cos[c + d*x] + 2*(737*A + 728*C)*Co
s[2*(c + d*x)] + 1859*A*Cos[3*(c + d*x)] + 1456*C*Cos[3*(c + d*x)] + 286*A*
Cos[4*(c + d*x)] + 224*C*Cos[4*(c + d*x)] + 286*A*Cos[5*(c + d*x)] + 224*C*
Cos[5*(c + d*x)])*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2
])/(2310*d)
```

Maple [A] time = 0.327, size = 152, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(1144A(\cos(dx+c))^5+896C(\cos(dx+c))^5+572A(\cos(dx+c))^4+448C(\cos(dx+c))^4\right.}{1155d(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/1155/d*a*(-1+cos(d*x+c))*(1144*A*cos(d*x+c)^5+896*C*cos(d*x+c)^5+572*A*c
os(d*x+c)^4+448*C*cos(d*x+c)^4+429*A*cos(d*x+c)^3+336*C*cos(d*x+c)^3+165*A*
cos(d*x+c)^2+280*C*cos(d*x+c)^2+245*C*cos(d*x+c)+105*C)*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.515857, size = 375, normalized size = 1.67

$$\frac{2\left(8(143A+112C)a\cos(dx+c)^5+4(143A+112C)a\cos(dx+c)^4+3(143A+112C)a\cos(dx+c)^3+5(33A+\right.}{1155\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/1155*(8*(143*A + 112*C)*a*cos(d*x + c)^5 + 4*(143*A + 112*C)*a*cos(d*x +
c)^4 + 3*(143*A + 112*C)*a*cos(d*x + c)^3 + 5*(33*A + 56*C)*a*cos(d*x + c)^
2 + 245*C*a*cos(d*x + c) + 105*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.82788, size = 424, normalized size = 1.88

$$-\frac{4\left(1155\sqrt{2}Aa^7\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+1155\sqrt{2}Ca^7\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-\left(3850\sqrt{2}Aa^7\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+2310\sqrt{2}Ca^7\operatorname{sgn}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -4/1155*(1155*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 1155*sqrt(2)*C*a^7*sgn(cos(
d*x + c)) - (3850*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 2310*sqrt(2)*C*a^7*sgn(
cos(d*x + c)) - (5698*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 5082*sqrt(2)*C*a^7*
sgn(cos(d*x + c)) - (4884*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 3696*sqrt(2)*C*
a^7*sgn(cos(d*x + c)) - (2299*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 1771*sqrt(2
)*C*a^7*sgn(cos(d*x + c)) - 2*(209*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 161*sq
rt(2)*C*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)
*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)*d)
```

3.165 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=174

$$\frac{8a^2(63A+47C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(63A+22C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{315d}+\frac{2a(63A+47C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}$$

```
[Out] (8*a^2*(63*A + 47*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(63*A + 47*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*(63*A + 2
2*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*d) + (2*C*Sec[c + d*x]^2
*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(9*d) + (2*C*(a + a*Sec[c + d*x])
^(5/2)*Tan[c + d*x])/(21*a*d)
```

Rubi [A] time = 0.476666, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4089, 4010, 4001, 3793, 3792}

$$\frac{8a^2(63A+47C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(63A+22C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{315d}+\frac{2a(63A+47C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (8*a^2*(63*A + 47*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(63*A + 47*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*(63*A + 2
2*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*d) + (2*C*Sec[c + d*x]^2
*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(9*d) + (2*C*(a + a*Sec[c + d*x])
^(5/2)*Tan[c + d*x])/(21*a*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{9d} + \frac{2\int \mathrm{s}}{}$$

$$= \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{9d} + \frac{2C(a}{}$$

$$= \frac{2(63A+22C)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{315d} + \frac{2C\,\mathrm{se}}{}$$

$$= \frac{2a(63A+47C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{2(63A}{}$$

$$= \frac{8a^2(63A+47C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a(63A+47C)\sqrt{a+a\,\mathrm{se}}}{315}$$

Mathematica [A] time = 1.24025, size = 121, normalized size = 0.7

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}((567A+748C)\cos(c+dx)+(882A+748C)\cos(2(c+dx))+189A\,\mathrm{c}}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a*(693*A + 752*C + (567*A + 748*C)*Cos[c + d*x] + (882*A + 748*C)*Cos[2*(c
 + d*x)] + 189*A*Cos[3*(c + d*x)] + 136*C*Cos[3*(c + d*x)] + 189*A*Cos[4*(c
 + d*x)] + 136*C*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])
]*Tan[(c + d*x)/2])/(630*d)
```

Maple [A] time = 0.29, size = 130, normalized size = 0.8

$$-\frac{2a(-1+\cos(dx+c))\left(378A(\cos(dx+c))^4+272C(\cos(dx+c))^4+189A(\cos(dx+c))^3+136C(\cos(dx+c))^3+6\right.}{315d(\cos(dx+c))^4\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(378*A*cos(d*x+c)^4+272*C*cos(d*x+c)^4+189*A*cos
(d*x+c)^3+136*C*cos(d*x+c)^3+63*A*cos(d*x+c)^2+102*C*cos(d*x+c)^2+85*C*cos(
d*x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.503773, size = 319, normalized size = 1.83

$$\frac{2\left(2\,(189\,A+136\,C)a\cos\left(dx+c\right)^{4}+(189\,A+136\,C)a\cos\left(dx+c\right)^{3}+3\,(21\,A+34\,C)a\cos\left(dx+c\right)^{2}+85\,Ca\cos\left(dx\right.\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/315*(2*(189*A + 136*C)*a*cos(d*x + c)^4 + (189*A + 136*C)*a*cos(d*x + c)^
3 + 3*(21*A + 34*C)*a*cos(d*x + c)^2 + 85*C*a*cos(d*x + c) + 35*C*a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*
x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.73506, size = 362, normalized size = 2.08

$$4\left(315\,\sqrt{2}Aa^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+315\,\sqrt{2}Ca^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(945\,\sqrt{2}Aa^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+525\,\sqrt{2}Ca^{6}\mathrm{sgn}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] 4/315*(315*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 315*sqrt(2)*C*a^6*sgn(cos(d*x
+ c)) - (945*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 525*sqrt(2)*C*a^6*sgn(cos(d*
x + c)) - (1071*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 819*sqrt(2)*C*a^6*sgn(cos
(d*x + c)) - (567*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 423*sqrt(2)*C*a^6*sgn(c
os(d*x + c)) - 2*(63*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 47*sqrt(2)*C*a^6*sgn
(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
 + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.166 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=132

$$\frac{8a^2(35A+19C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(35A+19C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

```
[Out] (8*a^2*(35*A + 19*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(35*A + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) - (4*C*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + a*Sec[c + d*x])^(5/2)*
Tan[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.263148, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4083, 4001, 3793, 3792}

$$\frac{8a^2(35A+19C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(35A+19C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (8*a^2*(35*A + 19*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(35*A + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) - (4*C*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + a*Sec[c + d*x])^(5/2)*
Tan[c + d*x])/(7*a*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
```

```
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{2\int \sec(c+dx)(a\,\cdots}{\cdots}$$
$$= -\frac{4C(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2C(a+a\sec(c+\cdots}{\cdots}$$
$$= \frac{2a(35A+19C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} - \frac{4C(a+\cdots}{\cdots}$$
$$= \frac{8a^2(35A+19C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(35A+19C)\sqrt{a+a\sec\cdots}}{105d}$$

Mathematica [A] time = 1.17178, size = 100, normalized size = 0.76

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}((525A+468C)\cos(c+dx)+2(35A+52C)\cos(2(c+dx))+175A\,\text{co}\cdots}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(70*A + 164*C + (525*A + 468*C)*Cos[c + d*x] + 2*(35*A + 52*C)*Cos[2*(c
+ d*x)] + 175*A*Cos[3*(c + d*x)] + 104*C*Cos[3*(c + d*x)])*Sec[c + d*x]^3*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.286, size = 108, normalized size = 0.8

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(175\,A\,(\cos(dx+c))^3+104\,C\,(\cos(dx+c))^3+35\,A\,(\cos(dx+c))^2+52\,C\,(\cos(dx+c))^2+39\,\text{C}\cdots}{105\,d\,(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(175*A*cos(d*x+c)^3+104*C*cos(d*x+c)^3+35*A*cos(
d*x+c)^2+52*C*cos(d*x+c)^2+39*C*cos(d*x+c)+15*C)*(a*(cos(d*x+c)+1)/cos(d*x+
c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

[Out] Timed out

Fricas [A] time = 0.49882, size = 266, normalized size = 2.02

$$\frac{2\left((175\,A+104\,C)a\cos\left(dx+c\right)^{3}+(35\,A+52\,C)a\cos\left(dx+c\right)^{2}+39\,Ca\cos\left(dx+c\right)+15\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+\right.}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/105*((175*A + 104*C)*a*cos(d*x + c)^3 + (35*A + 52*C)*a*cos(d*x + c)^2 +
39*C*a*cos(d*x + c) + 15*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.70037, size = 300, normalized size = 2.27

$$-\frac{4\left(105\sqrt{2}Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ca^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(280\sqrt{2}Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+140\sqrt{2}Ca^{5}\mathrm{sgn}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] -4/105*(105*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^5*sgn(cos(d*x
 + c)) - (280*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 140*sqrt(2)*C*a^5*sgn(cos(d
*x + c)) - (245*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 133*sqrt(2)*C*a^5*sgn(cos
(d*x + c)) - 2*(35*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 19*sqrt(2)*C*a^5*sgn(c
os(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.167 $\int (a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=133

$$\frac{2a^2(5A+4C)\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2C\tan(c+dx)(a\sec}{5d}$$

```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(5*A + 4*C)*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(5*d) + (2*C*(a + a*Sec[c + d*x])^(3/2)
*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.222669, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4055, 3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(5A+4C)\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2C\tan(c+dx)(a\sec}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(5*A + 4*C)*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(5*d) + (2*C*(a + a*Sec[c + d*x])^(3/2)
*Tan[c + d*x])/(5*d)
```


Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2\int(a+a\sec(c+dx))^{3/2}\left(\frac{5}{\cdots}\right.}{5\cdots}$$
$$= \frac{2aC\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2C(a+a\sec(c+dx))^{3/2}\tan\cdots}{5d}$$
$$= \frac{2aC\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2C(a+a\sec(c+dx))^{3/2}\tan\cdots}{5d}$$
$$= \frac{2a^2(5A+4C)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d} \cdots$$
$$= \frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2(5A+4C)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\cdots}{\cdots}$$

Mathematica [A] time = 1.1978, size = 122, normalized size = 0.92

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}((5A+6C)\cos(2(c+dx))+5A+6C\cos(c+dx\cdots\right.}{5d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(10*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[c + d*x]^2 + (5*A + 8*C + 6*C*
Cos[c + d*x] + (5*A + 6*C)*Cos[2*(c + d*x)])*Sqrt[-1 + Sec[c + d*x]])*Sec[c
 + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(5*d*Sqrt[-1 + Sec[c
 + d*x]])
```

Maple [B] time = 0.295, size = 330, normalized size = 2.5

$$-\frac{a}{20\,d\,(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(5\,A\sin(dx+c)\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(\cdots}{\cos(dx\cdots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/20/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*A*sin(d*x+c)*2^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2+10*A*sin(d*x+c)*2^(1/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)+5*A*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+40*A*cos(d*x+c)^3+48*C*cos(d*x+c)
^3-40*A*cos(d*x+c)^2-24*C*cos(d*x+c)^2-16*C*cos(d*x+c)-8*C)/cos(d*x+c)^2/si
n(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.566698, size = 887, normalized size = 6.67

$$\left[\frac{5\left(Aa\cos\left(dx+c\right)^{3}+Aa\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left((5\,A+6\right.}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/5*(5*(A*a*cos(d*x + c)^3 + A*a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((5*A + 6*C)*a*co
s(d*x + c)^2 + 3*C*a*cos(d*x + c) + C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), -2/5*(5*(A*a*cos
(d*x + c)^3 + A*a*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((5*A + 6*C)*a*cos(d*x
 + c)^2 + 3*C*a*cos(d*x + c) + C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + C*sec(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.168 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=136

$$-\frac{a^2(3A-8C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+A\sin(c+dx)$$

```
[Out] (3*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (a^2*(3*A - 8*C)*Tan[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (a*(3*A - 2*C)*Sqrt[a + a*Sec[c + d*x]]
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.288896, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4087, 3917, 3915, 3774, 203, 3792}

$$-\frac{a^2(3A-8C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+A\sin(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (3*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (a^2*(3*A - 8*C)*Tan[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (a*(3*A - 2*C)*Sqrt[a + a*Sec[c + d*x]]
*Tan[c + d*x])/(3*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} + \frac{\int(a+a\sec(c+\ldots}{\ldots}$$

$$= \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{a(3A-2C)\sqrt{a+\ldots}}{\ldots}$$

$$= \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{a(3A-2C)\sqrt{a+\ldots}}{\ldots}$$

$$= \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{a^2(3A-8C)\tan\ldots}{3d\sqrt{a+a\sec(\ldots}}$$

$$= \frac{3a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{A(a+a\sec(c+dx))^{3/2}\ldots}{d}$$

Mathematica [A] time = 1.19531, size = 113, normalized size = 0.83

$$\frac{a\tan(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}(3A\cos(2(c+dx))+3A+20C\cos(c+dx)+4C)+18A\cos(c+\ldots\right.}{6d(\cos(c+dx)+1)\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(18*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[c + d*x] + (3*A + 4*C + 20*C*C
os[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[-1 + Sec[c + d*x]])*Sqrt[a*(1 + Se
c[c + d*x])]*Tan[c + d*x])/(6*d*(1 + Cos[c + d*x])*Sqrt[-1 + Sec[c + d*x]])
```

Maple [A] time = 0.332, size = 239, normalized size = 1.8

$$-\frac{a}{12\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-9\,A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/12/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-9*A*2^(1/2)*sin(d*x+c)*cos(
d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-9*A*2^(1/2)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+12*A*cos(d*x+c)^3-12*A*cos(d*x+c)^2+
40*C*cos(d*x+c)^2-32*C*cos(d*x+c)-8*C)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 1.8616, size = 1085, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A/d
```

Fricas [A] time = 0.565599, size = 863, normalized size = 6.35

$$\left[\frac{9\left(Aa\cos\left(dx+c\right)^{2} + Aa\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2} - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx+c) - a}{\cos(dx+c)+1}\right) + 2\left(3\,Aa\cos\right.}{6\left(d\cos\left(dx+c\right)^{2} + d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] [1/6*(9*(A*a*cos(d*x + c)^2 + A*a*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(3*A*a*cos(d*x + c)
^2 + 10*C*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), -1/3*(9*(A*a*cos(d*x + c)
^2 + A*a*cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (3*A*a*cos(d*x + c)^2 + 10*C*a*co
s(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.169 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=151

$$\frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}-\frac{a(A-4C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{A\sin(c+}{}$$

```
[Out] (a^(3/2)*(7*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a^2*(5*A - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) -
(a*(A - 4*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.427823, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4087, 4018, 4015, 3774, 203}

$$\frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}-\frac{a(A-4C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{A\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(7*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a^2*(5*A - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) -
(a*(A - 4*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d} + \frac{\int c}{}$$

$$= -\frac{a(A-4C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d} + \frac{A\cos(c}{}$$

$$= \frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a(A-4C)\sqrt{a+a\sec(c+}}{2d}$$

$$= \frac{a^2(5A-8C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a(A-4C)\sqrt{a+a\sec(c+}}{2d}$$

$$= \frac{a^{3/2}(7A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^2(5A-8C)\,\mathrm{s}}{4d\sqrt{a+a\,\mathrm{sec}}}$$

Mathematica [A] time = 0.753918, size = 109, normalized size = 0.72

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(7A+8C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(7A}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(7*A + 8*C)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(A + 8*C + 7*A*Cos[c + d*x
] + A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.356, size = 397, normalized size = 2.6

$$-\frac{a}{16\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-7A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/16/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-7*A*2^(1/2)*sin(d*x+c)*cos(
d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-8*C*cos(d*x+c)*2^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-7*A*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)*sin(d*x+c)+8*A*cos(d*x+c)^4+20*A*cos(d*x+c)^3-28*A*cos(d
*x+c)^2+32*C*cos(d*x+c)^2-32*C*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.651124, size = 828, normalized size = 5.48

$$\left[\frac{((7A+8C)a\cos(dx+c)+(7A+8C)a)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2Aa\right.}{8(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/8*(((7*A + 8*C)*a*cos(d*x + c) + (7*A + 8*C)*a)*sqrt(-a)*log((2*a*cos(d*
x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*
sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a*cos(d*x +
 c)^2 + 7*A*a*cos(d*x + c) + 8*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((7*A + 8*C)*a*cos(d*x + c) + (7
*A + 8*C)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a*cos(d*x + c)^2 + 7*A*a*cos(d*x + c)
+ 8*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.6494, size = 695, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/8*(16*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a^2*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (7*A*sqrt(-a)*a*sgn
(cos(d*x + c)) + 8*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)
)) - (7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*a*sgn(cos(d*x + c)))*
log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 9
5*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A
*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 53*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 5*A*sq
rt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.170 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=155

$$\frac{a^2(19A+24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11A+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{A\sin(c+dx)\cos^2(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+$$

```
[Out] (a^(3/2)*(11*A + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(19*A + 24*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a*A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (A*Co
s[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.462273, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4087, 4017, 4015, 3774, 203}

$$\frac{a^2(19A+24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11A+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{A\sin(c+dx)\cos^2(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(11*A + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(19*A + 24*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a*A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (A*Co
s[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{\int ...}{...}$$

$$= \frac{aA\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{A\,\mathrm{co}...}{...}$$

$$= \frac{a^2(19A+24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos(c+dx)\sqrt{a+a\,\mathrm{s}...}}{4...}$$

$$= \frac{a^2(19A+24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos(c+dx)\sqrt{a+a\,\mathrm{s}...}}{4...}$$

$$= \frac{a^{3/2}(11A+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a^2(19A+2...}{24d\sqrt{a+...}}$$

Mathematica [A] time = 1.20122, size = 118, normalized size = 0.76

$$\frac{a\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\cos(c+dx)\sqrt{\sec(c+dx)-1}(22A\cos(c+dx)+4A\cos(2(c+dx))+37A+24C)+...\right.}{24d(\cos(c+dx)+1)\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a*((33*A + 72*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]] + Cos[c + d*x]*(37*A + 24
*C + 22*A*Cos[c + d*x] + 4*A*Cos[2*(c + d*x)])*Sqrt[-1 + Sec[c + d*x]])*Sqr
t[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(24*d*(1 + Cos[c + d*x])*Sqrt[-1 + Se
c[c + d*x]])
```

Maple [B] time = 0.393, size = 570, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d*a*(33*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*cos(d*x+c)^2+72*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(5/2)*cos(d*x+c)^2+66*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*cos(d*x+c)+144*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(5/2)*cos(d*x+c)+33*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*sin(d*x+c)+72*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(
d*x+c)+64*A*cos(d*x+c)^6+112*A*cos(d*x+c)^5+88*A*cos(d*x+c)^4+192*C*cos(d*x
+c)^4-264*A*cos(d*x+c)^3-192*C*cos(d*x+c)^3)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.652477, size = 919, normalized size = 5.93

$$\left[\frac{3\,((11\,A+24\,C)a\cos{(dx+c)}+(11\,A+24\,C)a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2}{48\,(d\cos{(dx+c)}+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/48*(3*((11*A + 24*C)*a*cos(d*x + c) + (11*A + 24*C)*a)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*a*co
s(d*x + c)^3 + 22*A*a*cos(d*x + c)^2 + 3*(11*A + 8*C)*a*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/2
4*(3*((11*A + 24*C)*a*cos(d*x + c) + (11*A + 24*C)*a)*sqrt(a)*arctan(sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8
*A*a*cos(d*x + c)^3 + 22*A*a*cos(d*x + c)^2 + 3*(11*A + 8*C)*a*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.9336, size = 1166, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/48*(3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 24*C*sqrt(-a)*a*sgn(cos(d*x +
 c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2
4*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3
3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*
A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 72*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 303*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*s
qrt(-a)*a^3*sgn(cos(d*x + c)) - 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqr
t(-a)*a^4*sgn(cos(d*x + c)) + 3024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt
(-a)*a^5*sgn(cos(d*x + c)) - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 309*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-
a)*a^6*sgn(cos(d*x + c)) + 360*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*A*sqrt(-a
)*a^7*sgn(cos(d*x + c)) - 24*C*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.171 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=200

$$\frac{a^2(75A+112C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75A+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(13A+16C)\sin(c+dx)\cos(c+dx)}{32d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin}{}$$

```
[Out] (a^(3/2)*(75*A + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a^2*(75*A + 112*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d
*x]]) + (a^2*(13*A + 16*C)*Cos[c + d*x]*Sin[c + d*x])/(32*d*Sqrt[a + a*Sec[
c + d*x]]) + (a*A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(8*
d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.57143, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4017, 4015, 3805, 3774, 203}

$$\frac{a^2(75A+112C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75A+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(13A+16C)\sin(c+dx)\cos(c+dx)}{32d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(75*A + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a^2*(75*A + 112*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d
*x]]) + (a^2*(13*A + 16*C)*Cos[c + d*x]*Sin[c + d*x])/(32*d*Sqrt[a + a*Sec[
c + d*x]]) + (a*A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(8*
d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d}+\frac{\int c}{}$$

$$=\frac{aA\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{8d}+\frac{A\,c}{}$$

$$=\frac{a^2(13A+16C)\cos(c+dx)\sin(c+dx)}{32d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos^2(c+}{}$$

$$=\frac{a^2(75A+112C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(13A+16C)\cos(c+}{32d\sqrt{a+a\sec}}$$

$$=\frac{a^2(75A+112C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(13A+16C)\cos(c+}{32d\sqrt{a+a\sec}}$$

$$=\frac{a^{3/2}(75A+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{a^2(75A+1}{64d\sqrt{a-}}$$

Mathematica [A] time = 1.40235, size = 140, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(75A+112C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+dx)\right)-\right.\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(75*A + 112*C)*ArcS
in[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (95*A + 112*C + (62*A + 3
2*C)*Cos[c + d*x] + 20*A*Cos[2*(c + d*x)] + 4*A*Cos[3*(c + d*x)])*(-Sin[(c
```

```
+ d*x)/2] + Sin[(3*(c + d*x))/2])))/(128*d)
```

Maple [B] time = 0.329, size = 752, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/1024/d*a*(75*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(7/2)*2^(1/2)+112*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(7/2)*2^(1/2)+225*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(7/2)*2^(1/2)+336*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)*2^(1/2)+225*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+336*C*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+75*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)*sin(d*x+c)+112*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*sin(d*x+c)-256*A*cos(d*x+c)^8-384*A*cos(d*x+c)^7-160*A*cos(d*x+c)^
6-512*C*cos(d*x+c)^6-400*A*cos(d*x+c)^5-1280*C*cos(d*x+c)^5+1200*A*cos(d*x+
c)^4+1792*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/
sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.743615, size = 1018, normalized size = 5.09

$$\left[\frac{((75\,A + 112\,C)a\cos\left(dx + c\right) + (75\,A + 112\,C)a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx+c) - a}{\cos(dx+c)+1}\right) + 2}{128\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/128*(((75*A + 112*C)*a*cos(d*x + c) + (75*A + 112*C)*a)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(16*A*a*
cos(d*x + c)^4 + 40*A*a*cos(d*x + c)^3 + 2*(25*A + 16*C)*a*cos(d*x + c)^2 +
 (75*A + 112*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c) + d), -1/64*(((75*A + 112*C)*a*cos(d*x + c) + (7
5*A + 112*C)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(sqrt(a)*sin(d*x + c))) - (16*A*a*cos(d*x + c)^4 + 40*A*a*cos(d*x
+ c)^3 + 2*(25*A + 16*C)*a*cos(d*x + c)^2 + (75*A + 112*C)*a*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.4372, size = 1467, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/128*((75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 112*C*sqrt(-a)*a*sgn(cos(d*x +
 c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3))) - (75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 112
*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(75
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A
*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 112*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 2087
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A
*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 2864*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 119
75*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10
*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 23344*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) -
42483*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^8*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 69360*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) +
 33889*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^6*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 51536*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^6*sgn(cos(d*x + c))
- 8693*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
```

```
)^4*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 14736*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^7*sgn(cos(d*x + c))
+ 1101*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 1808*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) -
 49*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 80*C*sqrt(-a)*a^9*sgn(cos(d*x + c)))
/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 -
 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*
a + a^2)^4)/d
```

3.172 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=245

$$\frac{a^2(133A+176C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(133A+176C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(67A+80C)\sin(c+dx)\cos^2(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (a^(3/2)*(133*A + 176*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^2*(133*A + 176*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^2*(133*A + 176*C)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^2*(67*A + 80*C)*Cos[c + d*x]^2*Sin[c + d*x])/(240*d*
Sqrt[a + a*Sec[c + d*x]]) + (3*a*A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(40*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*Sin[c +
 d*x])/(5*d)
```

Rubi [A] time = 0.639246, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4017, 4015, 3805, 3774, 203}

$$\frac{a^2(133A+176C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(133A+176C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(67A+80C)\sin(c+dx)\cos^2(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(133*A + 176*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^2*(133*A + 176*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^2*(133*A + 176*C)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^2*(67*A + 80*C)*Cos[c + d*x]^2*Sin[c + d*x])/(240*d*
Sqrt[a + a*Sec[c + d*x]]) + (3*a*A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(40*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*Sin[c +
 d*x])/(5*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\int \cos \ldots}{\ldots}$$

$$= \frac{3aA\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{40d} + \frac{A\cos \ldots}{\ldots}$$

$$= \frac{a^2(67A+80C)\cos^2(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} + \frac{3aA\cos^3(c+\ldots}{\ldots}$$

$$= \frac{a^2(133A+176C)\cos(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(67A+80\ldots}{240d\ldots}$$

$$= \frac{a^2(133A+176C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(133A+176C)\cos(c\ldots}{192d\sqrt{a+a\sec\ldots}}$$

$$= \frac{a^2(133A+176C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(133A+176C)\cos(c\ldots}{192d\sqrt{a+a\sec\ldots}}$$

$$= \frac{a^{3/2}(133A+176C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^2(133A+1\ldots}{128d\sqrt{a+\ldots}}$$

Mathematica [A] time = 2.15396, size = 159, normalized size = 0.65

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(15\sqrt{2}(133A+176C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+dx)\right)-\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[2]*(133*A + 176*C)*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (2671*A + 2960*C + 2*
(1007*A + 880*C)*Cos[c + d*x] + 4*(181*A + 80*C)*Cos[2*(c + d*x)] + 228*A*C
os[3*(c + d*x)] + 48*A*Cos[4*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d
*x))/2])))/(3840*d)
```

Maple [B] time = 0.366, size = 934, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/61440/d*a*(1995*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(
d*x+c)^4*sin(d*x+c)+2640*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2
)*cos(d*x+c)^4*sin(d*x+c)+7980*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*
2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+10560*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(9/2)*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+11970*A*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15840*C*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+7980*A*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)+10560*C*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)+1995*A*2^(1/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+2640*C*2^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+12288*A*cos(d*x+c)^10+16896
*A*cos(d*x+c)^9+4864*A*cos(d*x+c)^8+20480*C*cos(d*x+c)^8+8512*A*cos(d*x+c)^
7+35840*C*cos(d*x+c)^7+21280*A*cos(d*x+c)^6+28160*C*cos(d*x+c)^6-63840*A*co
s(d*x+c)^5-84480*C*cos(d*x+c)^5)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*
x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.763288, size = 1157, normalized size = 4.72

$$\left[\frac{15\,((133\,A+176\,C)a\cos{(dx+c)}+(133\,A+176\,C)a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/3840*(15*((133*A + 176*C)*a*cos(d*x + c) + (133*A + 176*C)*a)*sqrt(-a)*l
og((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(3
84*A*a*cos(d*x + c)^5 + 912*A*a*cos(d*x + c)^4 + 8*(133*A + 80*C)*a*cos(d*x
 + c)^3 + 10*(133*A + 176*C)*a*cos(d*x + c)^2 + 15*(133*A + 176*C)*a*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c) + d), -1/1920*(15*((133*A + 176*C)*a*cos(d*x + c) + (133*A + 176*C)*a)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) - (384*A*a*cos(d*x + c)^5 + 912*A*a*cos(d*x + c)^4 + 8*(133
*A + 80*C)*a*cos(d*x + c)^3 + 10*(133*A + 176*C)*a*cos(d*x + c)^2 + 15*(133
*A + 176*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.63812, size = 1771, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/3840*(15*(133*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 176*C*sqrt(-a)*a*sgn(cos(
d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(133*A*sqrt(-a)*a*sgn(cos(d*x +
c)) + 176*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sq
rt(2)*(1995*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
```

```
 + a))^18*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2640*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*sqrt(-a)*a^2*sgn(cos(d*x
+ c)) - 38505*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^16*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 55920*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*sqrt(-a)*a^3*sgn(cos(d
*x + c)) + 561660*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^14*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 582720*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^4*sgn(
cos(d*x + c)) - 2684100*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 3395520*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a
^5*sgn(cos(d*x + c)) + 7371738*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 9329760*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqr
t(-a)*a^6*sgn(cos(d*x + c)) - 6407470*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 81108
80*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*
C*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 2176620*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) +
2882880*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^6*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 399860*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^9*sgn(cos(d*x + c)
) - 498880*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 34035*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^10*sgn(cos(d*x +
 c)) + 42960*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*C*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 1201*A*sqrt(-a)*a^11*sgn(cos(
d*x + c)) - 1520*C*sqrt(-a)*a^11*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^5)/d
```

3.173 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=273

$$\frac{2a^3(2717A+2224C)\tan(c+dx)\sec^3(c+dx)}{9009d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(143A+136C)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{1287d}+\frac{2a^3(1\cdots}{6\cdots}$$

```
[Out] (2*a^3*(10439*A + 8368*C)*Tan[c + d*x])/(6435*d*Sqrt[a + a*Sec[c + d*x]]) +
 (2*a^3*(2717*A + 2224*C)*Sec[c + d*x]^3*Tan[c + d*x])/(9009*d*Sqrt[a + a*S
ec[c + d*x]]) - (4*a^2*(10439*A + 8368*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
d*x])/(45045*d) + (2*a^2*(143*A + 136*C)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(1287*d) + (2*a*(10439*A + 8368*C)*(a + a*Sec[c + d*x])
^(3/2)*Tan[c + d*x])/(15015*d) + (10*a*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x]
)^(3/2)*Tan[c + d*x])/(143*d) + (2*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5
/2)*Tan[c + d*x])/(13*d)
```

Rubi [A] time = 0.864525, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4089, 4018, 4016, 3800, 4001, 3792}

$$\frac{2a^3(2717A+2224C)\tan(c+dx)\sec^3(c+dx)}{9009d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(143A+136C)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{1287d}+\frac{2a^3(1\cdots}{6\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] (2*a^3*(10439*A + 8368*C)*Tan[c + d*x])/(6435*d*Sqrt[a + a*Sec[c + d*x]]) +
 (2*a^3*(2717*A + 2224*C)*Sec[c + d*x]^3*Tan[c + d*x])/(9009*d*Sqrt[a + a*S
ec[c + d*x]]) - (4*a^2*(10439*A + 8368*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
d*x])/(45045*d) + (2*a^2*(143*A + 136*C)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(1287*d) + (2*a*(10439*A + 8368*C)*(a + a*Sec[c + d*x])
^(3/2)*Tan[c + d*x])/(15015*d) + (10*a*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x]
)^(3/2)*Tan[c + d*x])/(143*d) + (2*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5
/2)*Tan[c + d*x])/(13*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{13d} + \frac{2}{}$$

$$= \frac{10aC\sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{143d} +$$

$$= \frac{2a^2(143A+136C)\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan}{1287d}$$

$$= \frac{2a^3(2717A+2224C)\sec^3(c+dx)\tan(c+dx)}{9009d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^3(2717A+2224C)\sec^3(c+dx)\tan(c+dx)}{9009d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^3(2717A+2224C)\sec^3(c+dx)\tan(c+dx)}{9009d\sqrt{a+a\sec(c+dx)}} - \frac{4a^2(}{}$$

$$= \frac{2a^3(10439A+8368C)\tan(c+dx)}{6435d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(2717A+2}{9009}$$

Mathematica [A] time = 1.92376, size = 169, normalized size = 0.62

$$\frac{a^2 \tan\left(\frac{1}{2}(c+dx)\right) \sec^6(c+dx)\sqrt{a(\sec(c+dx)+1)}(1120(286A+347C)\cos(c+dx)+14(32747A+30334C)\cos(2(c+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*(322751*A + 343612*C + 1120*(286*A + 347*C)*Cos[c + d*x] + 14*(32747*A
 + 30334*C)*Cos[2*(c + d*x)] + 141570*A*Cos[3*(c + d*x)] + 125520*C*Cos[3*(
c + d*x)] + 156585*A*Cos[4*(c + d*x)] + 125520*C*Cos[4*(c + d*x)] + 20878*A
*Cos[5*(c + d*x)] + 16736*C*Cos[5*(c + d*x)] + 20878*A*Cos[6*(c + d*x)] + 1
6736*C*Cos[6*(c + d*x)])*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c +
 d*x)/2])/(180180*d)
```

Maple [A] time = 0.359, size = 176, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(83512A(\cos(dx+c))^6+66944C(\cos(dx+c))^6+41756A(\cos(dx+c))^5+33472C(\cos(d\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(83512*A*cos(d*x+c)^6+66944*C*cos(d*x+c)^6+4
1756*A*cos(d*x+c)^5+33472*C*cos(d*x+c)^5+31317*A*cos(d*x+c)^4+25104*C*cos(d
*x+c)^4+18590*A*cos(d*x+c)^3+20920*C*cos(d*x+c)^3+5005*A*cos(d*x+c)^2+18305
*C*cos(d*x+c)^2+11970*C*cos(d*x+c)+3465*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)/cos(d*x+c)^6/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.527034, size = 470, normalized size = 1.72

$$\frac{2\left(8(10439A+8368C)a^2\cos(dx+c)^6+4(10439A+8368C)a^2\cos(dx+c)^5+3(10439A+8368C)a^2\cos(dx+c)^4\ldots}{45045\left(d\cos\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/45045*(8*(10439*A + 8368*C)*a^2*cos(d*x + c)^6 + 4*(10439*A + 8368*C)*a^2
*cos(d*x + c)^5 + 3*(10439*A + 8368*C)*a^2*cos(d*x + c)^4 + 10*(1859*A + 20
92*C)*a^2*cos(d*x + c)^3 + 35*(143*A + 523*C)*a^2*cos(d*x + c)^2 + 11970*C*
a^2*cos(d*x + c) + 3465*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.31685, size = 486, normalized size = 1.78

$$\frac{8\left(45045\,\sqrt{2}Aa^9\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+45045\,\sqrt{2}Ca^9\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(180180\,\sqrt{2}Aa^9\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+120120\,\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] 8/45045*(45045*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 45045*sqrt(2)*C*a^9*sgn(co
s(d*x + c)) - (180180*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 120120*sqrt(2)*C*a^
9*sgn(cos(d*x + c)) - (342342*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 294294*sqrt
(2)*C*a^9*sgn(cos(d*x + c)) - (391248*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 310
596*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - (265837*sqrt(2)*A*a^9*sgn(cos(d*x + c
)) + 212069*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - 4*(24167*sqrt(2)*A*a^9*sgn(co
s(d*x + c)) + 19279*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - 2*(1859*sqrt(2)*A*a^9
*sgn(cos(d*x + c)) + 1483*sqrt(2)*C*a^9*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c
)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(
(a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.174 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=211

$$\frac{16a^2(33A+25C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{693d}+\frac{64a^3(33A+25C)\tan(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2(99A+26C)\tan(c+dx)(a\sec(c+$$

```
[Out] (64*a^3*(33*A + 25*C)*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(33*A + 25*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(693*d) + (2*a*(33
*A + 25*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(231*d) + (2*(99*A + 26
*C)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(693*d) + (2*C*Sec[c + d*x]^2*
(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(11*d) + (10*C*(a + a*Sec[c + d*x]
)^(7/2)*Tan[c + d*x])/(99*a*d)
```

Rubi [A] time = 0.532672, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4089, 4010, 4001, 3793, 3792}

$$\frac{16a^2(33A+25C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{693d}+\frac{64a^3(33A+25C)\tan(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2(99A+26C)\tan(c+dx)(a\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (64*a^3*(33*A + 25*C)*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(33*A + 25*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(693*d) + (2*a*(33
*A + 25*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(231*d) + (2*(99*A + 26
*C)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(693*d) + (2*C*Sec[c + d*x]^2*
(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(11*d) + (10*C*(a + a*Sec[c + d*x]
)^(7/2)*Tan[c + d*x])/(99*a*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{11d} + \frac{2}{}$$

$$= \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{11d} + \frac{1}{}$$

$$= \frac{2(99A+26C)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{693d} + \frac{2C}{}$$

$$= \frac{2a(33A+25C)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{231d} + \frac{2}{}$$

$$= \frac{16a^2(33A+25C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{693d} + \frac{2}{}$$

$$= \frac{64a^3(33A+25C)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(33A+25C)\sqrt{a}}{}$$

Mathematica [A] time = 1.49389, size = 147, normalized size = 0.7

$$a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}(2(4983A+5014C)\cos(c+dx)+52(66A+71C)\cos(2(c+dx))+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*(2673*A + 3628*C + 2*(4983*A + 5014*C)*Cos[c + d*x] + 52*(66*A + 71*C)
*Cos[2*(c + d*x)] + 4587*A*Cos[3*(c + d*x)] + 3692*C*Cos[3*(c + d*x)] + 759
*A*Cos[4*(c + d*x)] + 568*C*Cos[4*(c + d*x)] + 759*A*Cos[5*(c + d*x)] + 568
*C*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x
)/2])/(2772*d)
```

Maple [A] time = 0.301, size = 154, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(1518A(\cos(dx+c))^5+1136C(\cos(dx+c))^5+759A(\cos(dx+c))^4+568C(\cos(dx+c}{693d(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/693/d*a^2*(-1+cos(d*x+c))*(1518*A*cos(d*x+c)^5+1136*C*cos(d*x+c)^5+759*A
*cos(d*x+c)^4+568*C*cos(d*x+c)^4+396*A*cos(d*x+c)^3+426*C*cos(d*x+c)^3+99*A
*cos(d*x+c)^2+355*C*cos(d*x+c)^2+224*C*cos(d*x+c)+63*C)*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.516659, size = 382, normalized size = 1.81

$$\frac{2\left(2\,(759\,A + 568\,C)a^2\cos\left(dx + c\right)^5 + (759\,A + 568\,C)a^2\cos\left(dx + c\right)^4 + 6\,(66\,A + 71\,C)a^2\cos\left(dx + c\right)^3 + (99\,A + 355\,C \cdots}{693\left(d\cos\left(dx + c\right)^6 + d\cos\left(dx + c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] 2/693*(2*(759*A + 568*C)*a^2*cos(d*x + c)^5 + (759*A + 568*C)*a^2*cos(d*x +
 c)^4 + 6*(66*A + 71*C)*a^2*cos(d*x + c)^3 + (99*A + 355*C)*a^2*cos(d*x + c
)^2 + 224*C*a^2*cos(d*x + c) + 63*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.13425, size = 424, normalized size = 2.01

$$-\frac{8\left(693\sqrt{2}Aa^{8}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+693\sqrt{2}Ca^{8}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-\left(2541\sqrt{2}Aa^{8}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+1617\sqrt{2}Ca^{8}\operatorname{sgn}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -8/693*(693*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 693*sqrt(2)*C*a^8*sgn(cos(d*x
 + c)) - (2541*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 1617*sqrt(2)*C*a^8*sgn(cos
(d*x + c)) - (3927*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 3003*sqrt(2)*C*a^8*sgn
(cos(d*x + c)) - (3267*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 2475*sqrt(2)*C*a^8
*sgn(cos(d*x + c)) - 4*(363*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 275*sqrt(2)*C
*a^8*sgn(cos(d*x + c)) - 2*(33*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 25*sqrt(2)
*C*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*t
an(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(
1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)*d)
```

3.175 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=169

$$\frac{16a^2(21A+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(21A+13C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a(21A+13C)\tan(c+dx)(a\sec(c-}{105d}$$

```
[Out] (64*a^3*(21*A + 13*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(21*A + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(21
*A + 13*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) - (4*C*(a + a*S
ec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*C*(a + a*Sec[c + d*x])^(7/2)*T
an[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.316348, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4083, 4001, 3793, 3792}

$$\frac{16a^2(21A+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(21A+13C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a(21A+13C)\tan(c+dx)(a\sec(c-}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (64*a^3*(21*A + 13*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(21*A + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(21
*A + 13*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) - (4*C*(a + a*S
ec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*C*(a + a*Sec[c + d*x])^(7/2)*T
an[c + d*x])/(9*a*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad} + \frac{2\int \sec(c+dx)}{}$$
$$= -\frac{4C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d} + \frac{2C(a+a\sec(}{}$$
$$= \frac{2a(21A+13C)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d} - \frac{4C}{}$$
$$= \frac{16a^2(21A+13C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{2a(}{}$$
$$= \frac{64a^3(21A+13C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(21A+13C)\sqrt{a}}{}$$

Mathematica [A] time = 1.39697, size = 125, normalized size = 0.74

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}(4(441A+698C)\cos(c+dx)+4(966A+803C)\cos(2(c+dx))+}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(2961*A + 2908*C + 4*(441*A + 698*C)*Cos[c + d*x] + 4*(966*A + 803*C)*
Cos[2*(c + d*x)] + 588*A*Cos[3*(c + d*x)] + 584*C*Cos[3*(c + d*x)] + 903*A*
Cos[4*(c + d*x)] + 584*C*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c
 + d*x])]*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.287, size = 132, normalized size = 0.8

$$-\frac{2a^2(-1+\cos(dx+c))\left(903A(\cos(dx+c))^4+584C(\cos(dx+c))^4+294A(\cos(dx+c))^3+292C(\cos(dx+c))^3}{315d(\cos(dx+c))^4\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(903*A*cos(d*x+c)^4+584*C*cos(d*x+c)^4+294*A*c
os(d*x+c)^3+292*C*cos(d*x+c)^3+63*A*cos(d*x+c)^2+219*C*cos(d*x+c)^2+130*C*c
os(d*x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.507498, size = 333, normalized size = 1.97

$$\frac{2\left((903\,A+584\,C)a^2\cos(dx+c)^4+2\,(147\,A+146\,C)a^2\cos(dx+c)^3+3\,(21\,A+73\,C)a^2\cos(dx+c)^2+130\,Ca^2\cos(d\right.}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/315*((903*A + 584*C)*a^2*cos(d*x + c)^4 + 2*(147*A + 146*C)*a^2*cos(d*x +
 c)^3 + 3*(21*A + 73*C)*a^2*cos(d*x + c)^2 + 130*C*a^2*cos(d*x + c) + 35*C*
a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5
 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.18204, size = 362, normalized size = 2.14

$$\frac{8\left(315\sqrt{2}Aa^7\mathrm{sgn}(\cos(dx+c))+315\sqrt{2}Ca^7\mathrm{sgn}(\cos(dx+c))-\left(1050\sqrt{2}Aa^7\mathrm{sgn}(\cos(dx+c))+630\sqrt{2}Ca^7\mathrm{sgn}(\cos(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] 8/315*(315*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 315*sqrt(2)*C*a^7*sgn(cos(d*x
+ c)) - (1050*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 630*sqrt(2)*C*a^7*sgn(cos(d
*x + c)) - (1323*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 819*sqrt(2)*C*a^7*sgn(co
s(d*x + c)) - 4*(189*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 117*sqrt(2)*C*a^7*sg
n(cos(d*x + c)) - 2*(21*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 13*sqrt(2)*C*a^7*
sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*
d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.176 $\int (a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=170

$$\frac{2a^3(49A+32C)\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(7A+8C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\,\text{t}}{}$$

```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(49*A + 32*C)*Tan[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(7*A + 8*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*C*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(7*d) + (2*C*(a + a*Sec[c + d*x])^(5/2)*Ta
n[c + d*x])/(7*d)
```

Rubi [A] time = 0.308362, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4055, 3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(49A+32C)\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(7A+8C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\,\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(49*A + 32*C)*Tan[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(7*A + 8*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*C*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(7*d) + (2*C*(a + a*Sec[c + d*x])^(5/2)*Ta
n[c + d*x])/(7*d)
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{2\int (a+a\sec(c+dx))^{5/2}\left(\frac{7aA}{2}\right.}{7a}$$

$$= \frac{2aC(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d} + \frac{2C(a+a\sec(c+dx))^{5/2}\tan(}{7d}$$

$$= \frac{2a^2(7A+8C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{21d} + \frac{2aC(a+a\sec(c+dx}{7d}$$

$$= \frac{2a^2(7A+8C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{21d} + \frac{2aC(a+a\sec(c+dx}{7d}$$

$$= \frac{2a^3(49A+32C)\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(7A+8C)\sqrt{a+a\sec(c+dx)}\tan(}{21d}$$

$$= \frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3(49A+32C)\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + 2a^2(7$$

Mathematica [A] time = 1.83551, size = 151, normalized size = 0.89

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}((84A+93C)\cos(c+dx)+(7A+23C)\cos(2(c+\right.}{21d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(42*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[c + d*x]^3 + (7*A + 29*C + (
84*A + 93*C)*Cos[c + d*x] + (7*A + 23*C)*Cos[2*(c + d*x)] + 28*A*Cos[3*(c +
 d*x)] + 23*C*Cos[3*(c + d*x)])*Sqrt[-1 + Sec[c + d*x]])*Sec[c + d*x]^3*Sqr
t[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(21*d*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.314, size = 434, normalized size = 2.6

$$\frac{a^2}{168\,d\,(\cos(dx+c))^3\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(21\,A\sin(dx+c)(\cos(dx+c))^3\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/168/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(21*A*sin(d*x+c)*cos(d*x+c)
^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+63*A*sin(d*x+c)*cos(d*
x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+63*A*sin(d*x+c)*co
s(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+21*A*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)-896*A*cos(d*x+c)^4-736*C*
cos(d*x+c)^4+784*A*cos(d*x+c)^3+368*C*cos(d*x+c)^3+112*A*cos(d*x+c)^2+176*C
*cos(d*x+c)^2+144*C*cos(d*x+c)+48*C)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.582656, size = 1030, normalized size = 6.06

$$\left[\frac{21\left(Aa^2\cos\left(dx+c\right)^4+Aa^2\cos\left(dx+c\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\right.}{21\left(d\cos\left(dx+c\right)^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/21*(21*(A*a^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*(28*A +
23*C)*a^2*cos(d*x + c)^3 + (7*A + 23*C)*a^2*cos(d*x + c)^2 + 12*C*a^2*cos(d
*x + c) + 3*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/21*(21*(A*a^2*cos(d*x + c)^4 + A*a^
2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*(28*A + 23*C)*a^2*cos(d*x + c)^3 +
(7*A + 23*C)*a^2*cos(d*x + c)^2 + 12*C*a^2*cos(d*x + c) + 3*C*a^2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x
 + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.177 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=173

$$\frac{a^3(15A+64C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(15A-16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{5a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(5A-$$

```
[Out] (5*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d + (a^3*(15*A + 64*C)*Tan[c + d
*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(15*A - 16*C)*Sqrt[a + a*Sec[c
+ d*x]]*Tan[c + d*x])/(15*d) - (a*(5*A - 2*C)*(a + a*Sec[c + d*x])^(3/2)*Ta
n[c + d*x])/(5*d)
```

Rubi [A] time = 0.369443, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4087, 3917, 3915, 3774, 203, 3792}

$$\frac{a^3(15A+64C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(15A-16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{5a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(5A-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (5*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d + (a^3*(15*A + 64*C)*Tan[c + d
*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(15*A - 16*C)*Sqrt[a + a*Sec[c
+ d*x]]*Tan[c + d*x])/(15*d) - (a*(5*A - 2*C)*(a + a*Sec[c + d*x])^(3/2)*Ta
n[c + d*x])/(5*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} + \frac{\int(a+a\sec(c+dx)}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{a(5A-2C)(a+a\,\mathrm{se}}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{a^2(15A-16C)\sqrt{a+}}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{a^2(15A-16C)\sqrt{a+}}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} + \frac{a^3(15A+64C)\tan(}{15d\sqrt{a+a\sec(c}}$$

$$= \frac{5a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{A(a+a\sec(c+dx))^{5/2}\sin}{d}$$

Mathematica [A] time = 1.78311, size = 145, normalized size = 0.84

$$\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}((45A+112C)\cos(c+dx)+4(15A+43C)\cos(2(c+dx}{60d(\cos(c+dx)+1)\sqrt{\sec(c+dx)-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(300*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[c + d*x]^2 + (60*A + 196*C
+ (45*A + 112*C)*Cos[c + d*x] + 4*(15*A + 43*C)*Cos[2*(c + d*x)] + 15*A*Cos
[3*(c + d*x)])*Sqrt[-1 + Sec[c + d*x]])*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[c + d*x])/(60*d*(1 + Cos[c + d*x])*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.339, size = 343, normalized size = 2.

$$-\frac{a^2}{120\,d\,(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(75\,A\sin(dx+c)\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(d}{\cos(dx}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/120/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(75*A*sin(d*x+c)*2^(1/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2+150*A*sin(d*x+c)*2^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)+75*A*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+120*A*cos(d*x+c)^4+120*A*co
s(d*x+c)^3+688*C*cos(d*x+c)^3-240*A*cos(d*x+c)^2-464*C*cos(d*x+c)^2-176*C*c
os(d*x+c)-48*C)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [B] time = 1.9482, size = 1868, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/4*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
```

```
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d
)
```

Fricas [A] time = 0.58181, size = 1008, normalized size = 5.83

$$\left[\frac{75\left(Aa^{2}\cos\left(dx+c\right)^{3}+Aa^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(15\,A\right.}{30\left(d\cos\left(dx+c\right)^{3}+d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] [1/30*(75*(A*a^2*cos(d*x + c)^3 + A*a^2*cos(d*x + c)^2)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(15*A*a^2*c
os(d*x + c)^3 + 2*(15*A + 43*C)*a^2*cos(d*x + c)^2 + 28*C*a^2*cos(d*x + c)
+ 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3 + d*cos(d*x + c)^2), -1/15*(75*(A*a^2*cos(d*x + c)^3 + A*a^2*cos(d*
x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c))) - (15*A*a^2*cos(d*x + c)^3 + 2*(15*A + 43*C)*a^2
*cos(d*x + c)^2 + 28*C*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.88202, size = 649, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] -1/30*(75*A*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*
A*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
```

```
 + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 60*sqrt(2)*(3
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*
sqrt(-a)*a^3*sgn(cos(d*x + c)) - A*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)
- 4*(15*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 60*sqrt(2)*C*a^5*sgn(cos(d*x + c)
) - (30*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 80*sqrt(2)*C*a^5*sgn(cos(d*x + c)
) - (15*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 32*sqrt(2)*C*a^5*sgn(cos(d*x + c)
))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a
*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.178 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=188

$$\frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}-\frac{a(3A$$

```
[Out] (a^(5/2)*(19*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^3*(27*A - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]
) - (a^2*(A - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) - (a*(3*A -
 4*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]*(a +
 a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.598209, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4087, 4018, 4015, 3774, 203}

$$\frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}-\frac{a(3A$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(5/2)*(19*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^3*(27*A - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]
) - (a^2*(A - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) - (a*(3*A -
 4*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]*(a +
 a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{2d} + \frac{\int c\ldots}{\ldots}$$

$$= -\frac{a(3A-4C)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{6d} + \frac{A\cos\ldots}{\ldots}$$

$$= -\frac{a^2(A-8C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d} - \frac{a(3A-\ldots}{\ldots}$$

$$= \frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(A-8C)\sqrt{a+a\sec(\ldots}}{2d}$$

$$= \frac{a^3(27A-56C)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(A-8C)\sqrt{a+a\sec(\ldots}}{2d}$$

$$= \frac{a^{5/2}(19A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^3(27A-56\ldots}{12d\sqrt{a+a\ldots}}$$

Mathematica [A] time = 0.885886, size = 137, normalized size = 0.73

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(6\sqrt{2}(19A+8C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\sin\left(\frac{1}{2}\ldots\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(1
9*A + 8*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(33*A +
16*C + (9*A + 128*C)*Cos[c + d*x] + 33*A*Cos[2*(c + d*x)] + 3*A*Cos[3*(c +
d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.361, size = 402, normalized size = 2.1

$$-\frac{a^2}{48\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-57\,A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-57*A*2^(1/2)*sin(d*x+c)*c
os(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-24*C*cos(d*x+c)*2^(1/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-57*A*2^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-24*C*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+24*A*cos(d*x+c)^4+108*A*cos(d*x+c)^3-13
2*A*cos(d*x+c)^2+256*C*cos(d*x+c)^2-224*C*cos(d*x+c)-32*C)/cos(d*x+c)/sin(d
*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.671674, size = 1022, normalized size = 5.44

$$\left[\frac{3\left((19\,A+8\,C)a^2\cos\left(dx+c\right)^2+(19\,A+8\,C)a^2\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos}{\cos(dx+c)+1}\right.}{24\left(d\cos\left(dx+c\right)^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/24*(3*((19*A + 8*C)*a^2*cos(d*x + c)^2 + (19*A + 8*C)*a^2*cos(d*x + c))*
sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
1)) + 2*(6*A*a^2*cos(d*x + c)^3 + 33*A*a^2*cos(d*x + c)^2 + 64*C*a^2*cos(d*
x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2 + d*cos(d*x + c)), -1/12*(3*((19*A + 8*C)*a^2*cos(d*x + c)^2
 + (19*A + 8*C)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (6*A*a^2*cos(d*x + c)^
3 + 33*A*a^2*cos(d*x + c)^2 + 64*C*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.03085, size = 748, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/24*(3*(19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*a^2*sgn(cos(d*
x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)
) + 8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 16*(7*
sqrt(2)*C*a^4*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*C*a^4*sg
n(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)) + 12*sqrt(2)*(19*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(d*x +
c)) - 171*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 89*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c))
- 9*A*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.179 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=192

$$\frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(3A-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{5a^{5/2}(5A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{A\,\mathrm{si}}{}$$

```
[Out] (5*a^(5/2)*(5*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*A - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) - (a^2*(3*A - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (5*a*A
*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c +
d*x]^2*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.619128, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4017, 4018, 4015, 3774, 203}

$$\frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(3A-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{5a^{5/2}(5A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{A\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (5*a^(5/2)*(5*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*A - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) - (a^2*(3*A - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (5*a*A
*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c +
d*x]^2*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
```

```
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \frac{\int \cdots}{\cdots}$$

$$= \frac{5aA\cos(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{12d} + \cdots$$

$$= -\frac{a^2(3A-8C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{5aA\cdots}{\cdots}$$

$$= \frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(3A-8C)\sqrt{a+a\sec\cdots}}{4d}$$

$$= \frac{a^3(49A-24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(3A-8C)\sqrt{a+a\sec\cdots}}{4d}$$

$$= \frac{5a^{5/2}(5A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a^3(49A-24\cdots}{24d\sqrt{a+a\cdots}}$$

Mathematica [A] time = 1.48149, size = 132, normalized size = 0.69

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}(3(27A+8C)\cos(c+dx)+17A\cos(2(c+dx))+2A\cos(3(c+d\cdots\right.}{24d(\cos(c+dx)+1)\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*(15*(5*A + 8*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]] + (17*A + 48*C + 3*(27
*A + 8*C)*Cos[c + d*x] + 17*A*Cos[2*(c + d*x)] + 2*A*Cos[3*(c + d*x)])*Sqrt
[-1 + Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(24*d*(1 + Co
s[c + d*x])*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.406, size = 583, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(75*A*sin(d*x+c)*2^(1/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2+120*C*sin(d*x+c)*2^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2+150*A*sin(d*x+c)
*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)+240*C*sin(d*x
+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)+75*A*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+120*C*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+64*A*cos(d*x+c)^6+208*A*cos
(d*x+c)^5+328*A*cos(d*x+c)^4+192*C*cos(d*x+c)^4-600*A*cos(d*x+c)^3+192*C*co
s(d*x+c)^3-384*C*cos(d*x+c)^2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.67021, size = 968, normalized size = 5.04

$$\left[\frac{15\left((5A+8C)a^2\cos(dx+c)+(5A+8C)a^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2(\cdots}{48(d\cos(dx+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/48*(15*((5*A + 8*C)*a^2*cos(d*x + c) + (5*A + 8*C)*a^2)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*a^2
*cos(d*x + c)^3 + 34*A*a^2*cos(d*x + c)^2 + 3*(25*A + 8*C)*a^2*cos(d*x + c)
 + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c) + d), -1/24*(15*((5*A + 8*C)*a^2*cos(d*x + c) + (5*A + 8*C)*a^2)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - (8*A*a^2*cos(d*x + c)^3 + 34*A*a^2*cos(d*x + c)^2 + 3*(25*
A + 8*C)*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.3483, size = 1261, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/48*(96*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a^3*sgn(cos(d*x + c
))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 15*(5*A*sqrt(-a)*a
^2*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt
(2) + 3))) - 15*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*a^2*sgn(
cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(75*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^3*sgn(
cos(d*x + c)) + 72*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^10*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 1125*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^4*sgn(co
s(d*x + c)) - 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^8*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 6174*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^5*sgn(cos(
d*x + c)) + 3024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 4314*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^6*sgn(cos(d
*x + c)) - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 807*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^7*sgn(cos(d*x
 + c)) + 360*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
```

```
2 + a))^2*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 49*A*sqrt(-a)*a^8*sgn(cos(d*x
+ c)) - 24*C*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.180 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=200

$$\frac{a^3(299A+432C)\sin(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163A+304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+16C)\sin(c+dx)\cos(c+dx)\sqrt{a}}{32d}$$

```
[Out] (a^(5/2)*(163*A + 304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^3*(299*A + 432*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(17*A + 16*C)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
 d*x])/(32*d) + (5*a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(24*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*
d)
```

Rubi [A] time = 0.654743, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4087, 4017, 4015, 3774, 203}

$$\frac{a^3(299A+432C)\sin(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163A+304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+16C)\sin(c+dx)\cos(c+dx)\sqrt{a}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(163*A + 304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^3*(299*A + 432*C)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(17*A + 16*C)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
 d*x])/(32*d) + (5*a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(24*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*
d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{4d} + \frac{\int \cos}{}$$

$$= \frac{5aA\cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{24d} + \frac{A\,c}{}$$

$$= \frac{a^2(17A+16C)\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx}{32d}$$

$$= \frac{a^3(299A+432C)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(17A+16C)\cos(c+}{}$$

$$= \frac{a^3(299A+432C)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(17A+16C)\cos(c+}{}$$

$$= \frac{a^{5/2}(163A+304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^3(299A+4}{192d\sqrt{a+}}$$

Mathematica [A] time = 1.59983, size = 143, normalized size = 0.72

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(163A+304C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+dx)\right)-\right.\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(163*A + 304*C)
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (581*A + 528*C + (36
2*A + 96*C)*Cos[c + d*x] + 92*A*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)])*(
-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384*d)
```

Maple [B] time = 0.29, size = 754, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3072/d*a^2*(489*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(7/2)*2^(1/2)+912*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)*2^(1/2)+1467*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)+2736*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1467*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+2736*C*sin(d*x+c)*cos(d*x+c)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+489*A*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+912*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-2176*A*cos(d*x+c)^7-2272*A*c
os(d*x+c)^6-1536*C*cos(d*x+c)^6-2608*A*cos(d*x+c)^5-6912*C*cos(d*x+c)^5+782
4*A*cos(d*x+c)^4+8448*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/c
os(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.751069, size = 1076, normalized size = 5.38

$$\left[\frac{3\left((163\,A+304\,C)a^2\cos\left(dx+c\right)+(163\,A+304\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(d}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/384*(3*((163*A + 304*C)*a^2*cos(d*x + c) + (163*A + 304*C)*a^2)*sqrt(-a)
*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*
(48*A*a^2*cos(d*x + c)^4 + 184*A*a^2*cos(d*x + c)^3 + 2*(163*A + 48*C)*a^2*
```

```
cos(d*x + c)^2 + 3*(163*A + 176*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((163*A + 3
04*C)*a^2*cos(d*x + c) + (163*A + 304*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*a^2*
cos(d*x + c)^4 + 184*A*a^2*cos(d*x + c)^3 + 2*(163*A + 48*C)*a^2*cos(d*x +
c)^2 + 3*(163*A + 176*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)

[Out] Timed out
```

Giac [B] time = 7.72581, size = 1480, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")

[Out] -1/384*(3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 304*C*sqrt(-a)*a^2*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x
 + c)) + 304*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) +
 4*sqrt(2)*(489*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^14*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 912*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^3*sgn(cos(d
*x + c)) - 10269*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^12*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 19152*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^4*sgn(co
s(d*x + c)) + 69885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^10*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 137424*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^5*sg
n(cos(d*x + c)) - 259233*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 374544*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^6
*sgn(cos(d*x + c)) + 209979*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 266928*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*
a^7*sgn(cos(d*x + c)) - 55511*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 75888*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)
*a^8*sgn(cos(d*x + c)) + 6687*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 9456*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*
a^9*sgn(cos(d*x + c)) - 299*A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 432*C*sqrt(
```

```
-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.181 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=245

$$\frac{a^3(283A+400C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283A+400C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(79A+80C)\sin(c+dx)\cos^2(c+dx)\sqrt{a\,\mathrm{s}}}{240d}$$

```
[Out] (a^(5/2)*(283*A + 400*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*A + 400*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(787*A + 1040*C)*Cos[c + d*x]*Sin[c + d*x])/(960*d*Sqrt[a
+ a*Sec[c + d*x]]) + (a^2*(79*A + 80*C)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d
*x]]*Sin[c + d*x])/(240*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*Sin[c +
 d*x])/(5*d)
```

Rubi [A] time = 0.761214, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4017, 4015, 3805, 3774, 203}

$$\frac{a^3(283A+400C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283A+400C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(79A+80C)\sin(c+dx)\cos^2(c+dx)\sqrt{a\,\mathrm{s}}}{240d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(283*A + 400*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*A + 400*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(787*A + 1040*C)*Cos[c + d*x]*Sin[c + d*x])/(960*d*Sqrt[a
+ a*Sec[c + d*x]]) + (a^2*(79*A + 80*C)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d
*x]]*Sin[c + d*x])/(240*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*Sin[c +
 d*x])/(5*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{\int \cdots}{\cdots}$$

$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{8d} + \frac{A \cdots}{\cdots}$$

$$= \frac{a^2(79A+80C)\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c \cdots}{240d}$$

$$= \frac{a^3(787A+1040C)\cos(c+dx)\sin(c+dx)}{960d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(79A \cdots}{\cdots}$$

$$= \frac{a^3(283A+400C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(787A+1040C)\cos \cdots}{960d\sqrt{a+a \cdots}}$$

$$= \frac{a^3(283A+400C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(787A+1040C)\cos \cdots}{960d\sqrt{a+a \cdots}}$$

$$= \frac{a^{5/2}(283A+400C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^3(283A \cdots}{128d\sqrt{\cdots}}$$

Mathematica [A] time = 2.47798, size = 160, normalized size = 0.65

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(15\sqrt{2}(283A+400C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+d \cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[2]*(283*A + 400*C
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (5521*A + 6320*C +
(3874*A + 2720*C)*Cos[c + d*x] + 4*(331*A + 80*C)*Cos[2*(c + d*x)] + 348*A*
Cos[3*(c + d*x)] + 48*A*Cos[4*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c +
d*x))/2])))/(3840*d)
```

Maple [B] time = 0.329, size = 936, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/61440/d*a^2*(4245*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*co
s(d*x+c)^4*sin(d*x+c)+6000*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1
/2)*cos(d*x+c)^4*sin(d*x+c)+16980*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/
2)*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+24000*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(9/2)*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+25470*A*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+36000*C*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+16980*A*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)+24000*C*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)+4245*A*2^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+6000*C*2^(1/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+12288*A*cos(d*x+c)^10+3
2256*A*cos(d*x+c)^9+27904*A*cos(d*x+c)^8+20480*C*cos(d*x+c)^8+18112*A*cos(d
*x+c)^7+66560*C*cos(d*x+c)^7+45280*A*cos(d*x+c)^6+104960*C*cos(d*x+c)^6-135
840*A*cos(d*x+c)^5-192000*C*cos(d*x+c)^5)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.766085, size = 1197, normalized size = 4.89

$$\left[\frac{15\left((283\,A+400\,C)a^2\cos\left(dx+c\right)+(283\,A+400\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(\right.}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/3840*(15*((283*A + 400*C)*a^2*cos(d*x + c) + (283*A + 400*C)*a^2)*sqrt(-
a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) +
2*(384*A*a^2*cos(d*x + c)^5 + 1392*A*a^2*cos(d*x + c)^4 + 8*(283*A + 80*C)*
a^2*cos(d*x + c)^3 + 10*(283*A + 272*C)*a^2*cos(d*x + c)^2 + 15*(283*A + 40
0*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d), -1/1920*(15*((283*A + 400*C)*a^2*cos(d*x + c) + (28
3*A + 400*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)/(sqrt(a)*sin(d*x + c))) - (384*A*a^2*cos(d*x + c)^5 + 1392*A*a^2
*cos(d*x + c)^4 + 8*(283*A + 80*C)*a^2*cos(d*x + c)^3 + 10*(283*A + 272*C)*
a^2*cos(d*x + c)^2 + 15*(283*A + 400*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.16618, size = 1782, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/3840*(15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 400*C*sqrt(-a)*a^2*sgn(
cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(283*A*sqrt(-a)*a^2*sgn(cos(
d*x + c)) + 400*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(4245*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
```

```
1/2*c)^2 + a))^18*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 6000*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*sqrt(-a)*a^3*sgn(
cos(d*x + c)) - 114615*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^16*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 162000*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*sqrt(-a)*a^4
*sgn(cos(d*x + c)) + 1298820*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 1801920*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(
-a)*a^5*sgn(cos(d*x + c)) - 6176700*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 976416
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*
C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 16394598*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^7*sgn(cos(d*x + c))
+ 24060960*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^10*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 14042770*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^8*sgn(cos(d*
x + c)) - 19910240*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 4791060*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^9*sgn(
cos(d*x + c)) + 7135680*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 860300*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^10
*sgn(cos(d*x + c)) - 1268800*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^10*sgn(cos(d*x + c)) + 75885*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)
*a^11*sgn(cos(d*x + c)) + 111600*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^11*sgn(cos(d*x + c)) - 2671*A*sq
rt(-a)*a^12*sgn(cos(d*x + c)) - 3920*C*sqrt(-a)*a^12*sgn(cos(d*x + c)))/((s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
a^2)^5)/d
```

3.182 $\int \cos^6(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=290

$$\frac{a^3(1015A+1304C)\sin(c+dx)}{512d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(1015A+1304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{512d}+\frac{a^2(23A+24C)\sin(c+dx)\cos^3(c+}{96d}$$

```
[Out] (a^(5/2)*(1015*A + 1304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
 d*x]]])/(512*d) + (a^3*(1015*A + 1304*C)*Sin[c + d*x])/(512*d*Sqrt[a + a*S
ec[c + d*x]]) + (a^3*(1015*A + 1304*C)*Cos[c + d*x]*Sin[c + d*x])/(768*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^3*(109*A + 136*C)*Cos[c + d*x]^2*Sin[c + d*x])
/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(23*A + 24*C)*Cos[c + d*x]^3*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(96*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x]
)^(5/2)*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.85847, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4017, 4015, 3805, 3774, 203}

$$\frac{a^3(1015A+1304C)\sin(c+dx)}{512d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(1015A+1304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{512d}+\frac{a^2(23A+24C)\sin(c+dx)\cos^3(c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(5/2)*(1015*A + 1304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
 d*x]]])/(512*d) + (a^3*(1015*A + 1304*C)*Sin[c + d*x])/(512*d*Sqrt[a + a*S
ec[c + d*x]]) + (a^3*(1015*A + 1304*C)*Cos[c + d*x]*Sin[c + d*x])/(768*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^3*(109*A + 136*C)*Cos[c + d*x]^2*Sin[c + d*x])
/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(23*A + 24*C)*Cos[c + d*x]^3*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(96*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x]
)^(5/2)*Sin[c + d*x])/(6*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{6d} + \frac{\int \cos}{}$$

$$= \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{12d} + \frac{A\cos}{}$$

$$= \frac{a^2(23A+24C)\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+d}{96d}$$

$$= \frac{a^3(109A+136C)\cos^2(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(23A+2}{}$$

$$= \frac{a^3(1015A+1304C)\cos(c+dx)\sin(c+dx)}{768d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(109A+}{1}$$

$$= \frac{a^3(1015A+1304C)\sin(c+dx)}{512d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1015A+1304C)\cos}{768d\sqrt{a+a}}$$

$$= \frac{a^3(1015A+1304C)\sin(c+dx)}{512d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1015A+1304C)\cos}{768d\sqrt{a+a}}$$

$$= \frac{a^{5/2}(1015A+1304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{512d} + \frac{a^3(1015A}{512d\sqrt{}}$$

Mathematica [A] time = 2.22812, size = 182, normalized size = 0.63

$$a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(1015A+1304C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+dx)\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(1015*A + 1304*
C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (4193*A + 4648*C +
 (3234*A + 2896*C)*Cos[c + d*x] + 4*(315*A + 184*C)*Cos[2*(c + d*x)] + 428*
A*Cos[3*(c + d*x)] + 96*C*Cos[3*(c + d*x)] + 112*A*Cos[4*(c + d*x)] + 16*A*
Cos[5*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(3072*d)
```

Maple [B] time = 0.371, size = 1118, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/98304/d*a^2*(-3045*A*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))-3912*C*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)/cos(d*x+c))-15225*A*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-19560*C*cos(d*x+c)^4*sin(d*x+c)*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-30450*A*cos(d*x+c)^3*sin(d*x
+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-39120*C*cos(d*x+c)^3
*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-30450*A*cos(
d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-3912
0*C*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))-15225*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))-19560*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))-3045*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*s
in(d*x+c)-3912*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x
+c)+16384*A*cos(d*x+c)^12+40960*A*cos(d*x+c)^11+31744*A*cos(d*x+c)^10+24576
*C*cos(d*x+c)^10+14848*A*cos(d*x+c)^9+69632*C*cos(d*x+c)^9+25984*A*cos(d*x+
c)^8+72704*C*cos(d*x+c)^8+64960*A*cos(d*x+c)^7+83456*C*cos(d*x+c)^7-194880*
A*cos(d*x+c)^6-250368*C*cos(d*x+c)^6)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/c
os(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.801721, size = 1311, normalized size = 4.52

$$\left[\frac{3\left(\left(1015\,A+1304\,C\right)a^2\cos\left(dx+c\right)+\left(1015\,A+1304\,C\right)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(\right.}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/3072*(3*((1015*A + 1304*C)*a^2*cos(d*x + c) + (1015*A + 1304*C)*a^2)*sqr
t(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
 + 2*(256*A*a^2*cos(d*x + c)^6 + 896*A*a^2*cos(d*x + c)^5 + 48*(29*A + 8*C)
*a^2*cos(d*x + c)^4 + 8*(203*A + 184*C)*a^2*cos(d*x + c)^3 + 2*(1015*A + 13
04*C)*a^2*cos(d*x + c)^2 + 3*(1015*A + 1304*C)*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/1536*(
3*((1015*A + 1304*C)*a^2*cos(d*x + c) + (1015*A + 1304*C)*a^2)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c))) - (256*A*a^2*cos(d*x + c)^6 + 896*A*a^2*cos(d*x + c)^5 + 48*(29*A + 8*
C)*a^2*cos(d*x + c)^4 + 8*(203*A + 184*C)*a^2*cos(d*x + c)^3 + 2*(1015*A +
1304*C)*a^2*cos(d*x + c)^2 + 3*(1015*A + 1304*C)*a^2*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.53483, size = 2084, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] -1/3072*(3*(1015*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1304*C*sqrt(-a)*a^2*sgn
(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(1015*A*sqrt(-a)*a^2*sgn(cos
(d*x + c)) + 1304*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3
))) + 4*sqrt(2)*(3045*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^22*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 3912*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^22*C*sqrt(-a)*a^3*sg
n(cos(d*x + c)) - 100485*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^20*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 129096*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^20*C*sqrt(-a)*a
^4*sgn(cos(d*x + c)) + 1303699*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^18*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 1693560*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*sqr
t(-a)*a^5*sgn(cos(d*x + c)) - 9936699*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 1195
1544*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
16*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 38257266*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^7*sgn(cos(d*x + c
)) + 48800976*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^14*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 83779026*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^8*sgn(co
s(d*x + c)) - 106200016*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 74917446*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*
a^9*sgn(cos(d*x + c)) + 94661616*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 30850806*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*s
qrt(-a)*a^10*sgn(cos(d*x + c)) - 39751536*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^10*sgn(cos(d*x + c)) +
7187801*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^6*A*sqrt(-a)*a^11*sgn(cos(d*x + c)) + 9070440*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^11*sgn(cos(d*x +
 c)) - 929817*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*A*sqrt(-a)*a^12*sgn(cos(d*x + c)) - 1176936*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^12*sgn(cos
(d*x + c)) + 64887*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*A*sqrt(-a)*a^13*sgn(cos(d*x + c)) + 82200*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^13*sgn(
cos(d*x + c)) - 1887*A*sqrt(-a)*a^14*sgn(cos(d*x + c)) - 2392*C*sqrt(-a)*a^
14*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*a + a^2)^6)/d
```

3.183 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=236

$$\frac{2(21A+19C)\tan(c+dx)\sec^2(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(21A+29C)\tan(c+dx)\sqrt{a\sec(c+dx}}{315ad}$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(147*A + 143*C)*Tan[c + d*x])/(315*d*Sqrt[a +
a*Sec[c + d*x]]) + (2*(21*A + 19*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) - (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(21*A + 29*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*a*d)
```

Rubi [A] time = 0.802611, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4089, 4021, 4010, 4001, 3795, 203}

$$\frac{2(21A+19C)\tan(c+dx)\sec^2(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(21A+29C)\tan(c+dx)\sqrt{a\sec(c+dx}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(147*A + 143*C)*Tan[c + d*x])/(315*d*Sqrt[a +
a*Sec[c + d*x]]) + (2*(21*A + 19*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) - (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(21*A + 29*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*a*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^4(c+dx)\left(\frac{1}{2}a(9A+8C)-\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{9a}$$

$$= -\frac{2C\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \cdots$$

$$= \frac{2(21A+19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \cdots$$

$$= \frac{2(21A+19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \cdots$$

$$= \frac{4(147A+143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2(21A+19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \cdots$$

$$= \frac{4(147A+143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2(21A+19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \cdots$$

$$= -\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{4(147A+143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \cdots$$

Mathematica [B] time = 6.67025, size = 474, normalized size = 2.01

$$\cos^2(c+dx)\sqrt{\sec(c+dx)+1}\sqrt{(\cos(c+dx)+1)\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(-\frac{4\sec(c)\sec^2(c+dx)(-63A\sin(dx)+40C\sin(c)-9}{315d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]
]*(A + C*Sec[c + d*x]^2)*((8*(-84*A - 126*C + 273*A*Cos[c] + 257*C*Cos[c])*
Sin[c/2])/(315*d*(Cos[c/2] + Cos[(3*c)/2])) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2
]*(357*A*Sin[(d*x)/2] + 383*C*Sin[(d*x)/2]))/(315*d) + (4*C*Sec[c]*Sec[c +
d*x]^4*Sin[d*x])/(9*d) + (4*Sec[c]*Sec[c + d*x]*(63*A*Sin[c] + 97*C*Sin[c]
- 84*A*Sin[d*x] - 126*C*Sin[d*x]))/(315*d) - (4*Sec[c]*Sec[c + d*x]^2*(40*C
*Sin[c] - 63*A*Sin[d*x] - 97*C*Sin[d*x]))/(315*d) + (4*Sec[c]*Sec[c + d*x]^
3*(7*C*Sin[c] - 8*C*Sin[d*x]))/(63*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sqr
t[a*(1 + Sec[c + d*x])]) - (2*Sqrt[2]*(A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]
]/Sqrt[2]]*Cos[c + d*x]^4*Sqrt[-1 + Sec[c + d*x]]*(1 + Sec[c + d*x])^2*(A +
 C*Sec[c + d*x]^2)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])*Sqrt[1 - Cos[c + d*x
]^2]*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Sqrt[Cos[c +
 d*x]^2*(-1 + Sec[c + d*x])*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.429, size = 966, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/5040/d/a*(315*A*cos(d*x+c)^4*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*s
in(d*x+c)+315*C*cos(d*x+c)^4*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(
d*x+c)+1260*A*cos(d*x+c)^3*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*
x+c)+1260*C*cos(d*x+c)^3*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+
c)+1890*A*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)
+1890*C*cos(d*x+c)^2*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+1
260*A*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+1260*
C*cos(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+315*A*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+315*C*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+8736*A*cos(d*x+c)^5+8224*C*cos(d*x+c)^5-9
408*A*cos(d*x+c)^4-9152*C*cos(d*x+c)^4+2688*A*cos(d*x+c)^3+2752*C*cos(d*x+c
)^3-2016*A*cos(d*x+c)^2-1984*C*cos(d*x+c)^2+1280*C*cos(d*x+c)-1120*C)*(a*(c
```

```
os(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.647738, size = 1189, normalized size = 5.04

$$\left[\frac{315\sqrt{2}\left((A+C)a\cos(dx+c)^5+(A+C)a\cos(dx+c)^4\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/630*(315*sqrt(2)*((A + C)*a*cos(d*x + c)^5 + (A + C)*a*cos(d*x + c)^4)*s
qrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)
*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((273*A + 257*C)*cos(d*x + c)^4 - (21*A
 + 29*C)*cos(d*x + c)^3 + 3*(21*A + 19*C)*cos(d*x + c)^2 - 5*C*cos(d*x + c)
 + 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x
 + c)^5 + a*d*cos(d*x + c)^4), 1/315*(2*((273*A + 257*C)*cos(d*x + c)^4 - (
21*A + 29*C)*cos(d*x + c)^3 + 3*(21*A + 19*C)*cos(d*x + c)^2 - 5*C*cos(d*x
+ c) + 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 315*sqr
t(2)*((A + C)*a*cos(d*x + c)^5 + (A + C)*a*cos(d*x + c)^4)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
/sqrt(a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2(c+dx)\right)\sec^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1))
, x)
```

Giac [A] time = 9.48507, size = 556, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/315*(315*(sqrt(2)*A + sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c)
+ sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)) + 2*(315*sqrt(2)*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 315*sqrt(2
)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (1050*sqrt(2)*A*a^4*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1) + 840*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (1
512*sqrt(2)*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 1638*sqrt(2)*C*a^4*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1) - (1134*sqrt(2)*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1) + 936*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (357*sqrt(2)*A
*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 383*sqrt(2)*C*a^4*sgn(tan(1/2*d*x +
1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.184 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(35A+31C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}-\frac{4(35A+37C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+2$$

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(35*A + 37*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec
[c + d*x]]) - (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (
2*(35*A + 31*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.593652, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4089, 4021, 4010, 4001, 3795, 203}

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(35A+31C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}-\frac{4(35A+37C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+2$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```



```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(35*A + 37*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec
[c + d*x]]) - (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (
2*(35*A + 31*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
```

```
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^3(c+dx)\left(\frac{1}{2}a(7A+6C)-\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{7a}$$

$$= -\frac{2C\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{4\int \frac{\sec^2(c+dx)(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2C\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2(35A+31C)\cdots}{\cdots}$$

$$= -\frac{4(35A+37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx\cdots}{7d\sqrt{a+a\,\mathrm{se}\cdots}}$$

$$= -\frac{4(35A+37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx\cdots}{7d\sqrt{a+a\,\mathrm{se}\cdots}}$$

$$= \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(35A+37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sec^2(c\cdots}{35d\sqrt{a\cdots}}$$

Mathematica [A] time = 6.11893, size = 173, normalized size = 0.9

$$\frac{2\cos^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(A+C\sec^2(c+dx)\right)\left(105\sqrt{2}(A+C)\cot(c+dx)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)\cdots\right.}{105ad(A\cos(2(c+d\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] (2*Cos[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(A + C*Sec[c + d*x]^2)*(105*Sq
rt[2]*(A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cot[c + d*x]*Sqrt[-1
+ Sec[c + d*x]] + 2*(35*A + 73*C + 24*C*Cos[c + d*x] + (35*A + 43*C)*Cos[2*
(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a*d*(
A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.366, size = 776, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/840/d/a*(105*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(
d*x+c)^3+105*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x
+c)^3+315*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)
^2+315*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+
315*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+315*C
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+105*A*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+105*C*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-560*A*cos(d*x+c)^4-688*C*cos(d*x+c)^4+1120
*A*cos(d*x+c)^3+1184*C*cos(d*x+c)^3-560*A*cos(d*x+c)^2-544*C*cos(d*x+c)^2+2
88*C*cos(d*x+c)-240*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin
(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.611454, size = 1098, normalized size = 5.69

$$\left[\frac{105\sqrt{2}\left((A+C)a\cos(dx+c)^4+(A+C)a\cos(dx+c)^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{210\left(ad\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/210*(105*sqrt(2)*((A + C)*a*cos(d*x + c)^4 + (A + C)*a*cos(d*x + c)^3)*s
qrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a
)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((35*A + 43*C)*cos(d*x + c)^3 - (35*A
+ 31*C)*cos(d*x + c)^2 + 3*C*cos(d*x + c) - 15*C)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3), -1/
105*(2*((35*A + 43*C)*cos(d*x + c)^3 - (35*A + 31*C)*cos(d*x + c)^2 + 3*C*c
os(d*x + c) - 15*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) +
105*sqrt(2)*((A + C)*a*cos(d*x + c)^4 + (A + C)*a*cos(d*x + c)^3)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1))
, x)
```

Giac [A] time = 9.30975, size = 333, normalized size = 1.73

$$\frac{\frac{105\sqrt{2}(A+C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{4\left(\left(\frac{\sqrt{2}(35Aa^3+46Ca^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{14\sqrt{2}(5Aa^3+4Ca^3)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/105*(105*sqrt(2)*(A + C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) +
 4*((sqrt(2)*(35*A*a^3 + 46*C*a^3)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x +
 1/2*c)^2 - 1) - 14*sqrt(2)*(5*A*a^3 + 4*C*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2
- 1))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(A*a^3 + 2*C*a^3)/sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.185 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(15A+14C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2C\tan(c+dx)\sqrt{\ }}{15\ }$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(15*A + 14*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]]) + (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d
*x]]) - (2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.416758, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4089, 4010, 4001, 3795, 203}

$$-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(15A+14C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2C\tan(c+dx)\sqrt{\ }}{15\ }$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(15*A + 14*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]]) + (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d
*x]]) - (2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
+ 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^2(c+dx)\left(\frac{1}{2}a(5A+4C)-\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad} + \frac{4\int \frac{\sec(c+}{}}{}$$

$$= \frac{2(15A+14C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sqrt{a+a\sec(c}}{15}$$

$$= \frac{2(15A+14C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2C\sqrt{a+a\sec(c}}{15}$$

$$= -\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(15A+14C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(}{5d\sqrt{}}$$

Mathematica [A] time = 4.33884, size = 160, normalized size = 1.05

$$\frac{2\cos^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)((15A+13C)\cos(2(c+dx))+15A}{15ad(A\cos(2(c+dx))+A+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] (2*Cos[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(A + C*Sec[c + d*x]^2)*(-15*Sq
rt[2]*(A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cot[c + d*x]*Sqrt[-1
+ Sec[c + d*x]] + (15*A + 19*C - 2*C*Cos[c + d*x] + (15*A + 13*C)*Cos[2*(c
+ d*x)])*Sec[c + d*x]^2*Tan[(c + d*x)/2]))/(15*a*d*(A + 2*C + A*Cos[2*(c +
d*x)]))
```

Maple [B] time = 0.346, size = 586, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/60/d/a*(15*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*
x+c)^2+15*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)
^2+30*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)+30*
C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)+15*A*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+15*C*ln(-(-(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(5/2)*sin(d*x+c)+120*A*cos(d*x+c)^3+104*C*cos(d*x+c)^3-120*A
*cos(d*x+c)^2-112*C*cos(d*x+c)^2+32*C*cos(d*x+c)-24*C)*(a*(cos(d*x+c)+1)/co
s(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.605205, size = 998, normalized size = 6.57

$$\left[\frac{15\sqrt{2}\left((A+C)a\cos\left(dx+c\right)^{3}+(A+C)a\cos\left(dx+c\right)^{2}\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^{2}+2\cos\ldots}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{30\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(d\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*((A + C)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*sqr
t(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*c
os(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 4*((15*A + 13*C)*cos(d*x + c)^2 - C*cos(d*x
 + c) + 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos
(d*x + c)^3 + a*d*cos(d*x + c)^2), 1/15*(2*((15*A + 13*C)*cos(d*x + c)^2 -
C*cos(d*x + c) + 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
+ 15*sqrt(2)*((A + C)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1))
, x)
```

Giac [B] time = 8.92879, size = 394, normalized size = 2.59

$$-\frac{\frac{15\left(\sqrt{2}A+\sqrt{2}C\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{2\left(15\sqrt{2}Aa^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)+15\sqrt{2}Ca^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)-\left(30\sqrt{2}\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/15*(15*(sqrt(2)*A + sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) +
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)) + 2*(15*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 15*sqrt(2)*C*
a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (30*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1) + 20*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (15*sqrt(
2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 17*sqrt(2)*C*a^2*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d
*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)))/d
```

3.186 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4C\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*C*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.205821, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4083, 4001, 3795, 203}

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4C\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*C*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx = \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}+\frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}a(3A+C)-aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{3a}$$

$$= -\frac{4C\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}+(A+C)\int \frac{}{\sqrt{a}}$$

$$= -\frac{4C\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}-\frac{(2(A+C))\,\text{Sub}}{}$$

$$= \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{4C\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sqrt{a+a\sec(c}{3}$$

Mathematica [A] time = 1.5188, size = 125, normalized size = 1.15

$$-\frac{2\cos\left(\frac{c}{2}\right)\cos(c)\sin(c+dx)\left(3\sqrt{2}(A+C)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)+8C\sin^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\right)}{3d\left(\cos\left(\frac{c}{2}\right)+\cos\left(\frac{3c}{2}\right)\right)(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Cos[c/2]*Cos[c]*(3*Sqrt[2]*(A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[
2]]*Sqrt[-1 + Sec[c + d*x]] + 8*C*Sec[c + d*x]^2*Sin[(c + d*x)/2]^4)*Sin[c
+ d*x])/(3*d*(Cos[c/2] + Cos[(3*c)/2])*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [B] time = 0.326, size = 385, normalized size = 3.5

$$-\frac{1}{6\,ad\sin(dx+c)\cos(dx+c)}\left(3\,A\sin(dx+c)\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+\cos(dx+c)-1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d/a*(3*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+
c)+3*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)+3*A*
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+3*C*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-4*C*cos(d*x+c)^2+8*C*cos(d*x+c)-4*C)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.600128, size = 895, normalized size = 8.21

$$\left[\frac{3\sqrt{2}\left((A+C)a\cos\left(dx+c\right)^2+(A+C)a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{6\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos(d*x + c))*sqrt(-1
/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(
d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)) - 4*(C*cos(d*x + c) - C)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/
3*(2*(C*cos(d*x + c) - C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) + 3*sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos(d*x + c))*arctan(
sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 9.10977, size = 193, normalized size = 1.77

$$\frac{\frac{4\sqrt{2}Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{3\sqrt{2}(A+C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/3*(4*sqrt(2)*C*a*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*sqr
t(2)*(A + C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.187 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d) + (2*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.16219, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4055, 3920, 3774, 203, 3795}

$$-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d) + (2*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2\int \frac{\frac{aA}{2}-\frac{1}{2}aC\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}\\
&= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{a}+(-A-C)\int\frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2A)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{(2(A+C))\,\text{Subst}\left(\int\frac{1}{2a+\dots}\right.}{d}\\
&= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.923437, size = 126, normalized size = 1.1

$$\frac{\tan(c+dx)\left(\sqrt{2}(A+C)\cos(c+dx)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)-2A\cos(c+dx)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\sqrt{\sec\dots}\right.\right.}{d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*C*(-1 + Cos[c + d*x]) - 2*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[c + d*x
]*Sqrt[-1 + Sec[c + d*x]] + Sqrt[2]*(A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/
Sqrt[2]]*Cos[c + d*x]*Sqrt[-1 + Sec[c + d*x]])*Tan[c + d*x])/(d*(-1 + Cos[c
 + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.289, size = 271, normalized size = 2.4

$$-\frac{1}{ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(\dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(A*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*sin(d*x+c)+C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+2*C*cos(
d*x+c)-2*C)/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60155, size = 1150, normalized size = 10.

$$\left[\frac{\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-2(A\cdots}{2(ad\cos(d\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*log((2*sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x +
 c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 2*(A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*
cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*C*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -(2*(A*cos(d*x + c) + A)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) - 2*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
- sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*arctan(sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a
*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 11.0052, size = 405, normalized size = 3.52

$$\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\left(A\sqrt{-a}+C\sqrt{-a}\right)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.\right.}{\sqrt{-a}\operatorname{sgn}(}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*tan(1/2*d*x + 1/2*c)/(
(a*tan(1/2*d*x + 1/2*c)^2 - a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(
A*sqrt(-a) + C*sqrt(-a))*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*A*log(ab
s((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
 a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*A*log(a
bs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.188 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d))
+ (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.224349, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4087, 3920, 3774, 203, 3795}

$$\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d))
+ (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-\frac{aA}{2}+\frac{1}{2}a(A+2C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{2a} + (A+C)\int\frac{\sec(c+dx}{\sqrt{a+a\sec(c+}}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{A\,\mathrm{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{(2(A+C))\,\mathrm{Su}}{}$$

$$= -\frac{A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{A\sin(c}{d\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.772834, size = 113, normalized size = 1.

$$\frac{\sin(c+dx)\left(-\sqrt{2}(A+C)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)+A(\cos(c+dx)-1)+A\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\sqrt{\sec(c\right.\right.}{d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((A*(-1 + Cos[c + d*x]) + A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Sqrt[-1 + Sec[c
 + d*x]] - Sqrt[2]*(A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Sqrt[-1
+ Sec[c + d*x]])*Sin[c + d*x])/(d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [B] time = 0.339, size = 282, normalized size = 2.5

$$-\frac{1}{2\,ad\sin(dx+c)}\left(-A\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-2\,A\sqrt{-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d/a*(-A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2
*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-2*C*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+2*A*cos(d*x+c)^2-2*A*cos(d*x+c))*(a*(
```

```
cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+A\right) \cos \left(dx+c\right)}{\sqrt{a \sec \left(dx+c\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.58359, size = 1177, normalized size = 10.42

$$\left[\frac{2 A \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos\left(dx+c\right) \sin\left(dx+c\right) + \sqrt{2}\left((A+C)a \cos\left(dx+c\right)+(A+C)a\right)\sqrt{-\frac{1}{a}} \log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos \cdots}{\cos \cdots}\right.}{2\left(ad \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*log(-(2*sqrt(2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c
) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)) - (A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(
d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d*cos(d*x + c) + a*d), (A*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + (A*cos(d*x + c) +
A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))) - sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C \sec^{2}{\left(c + dx \right)}\right) \cos{\left(c + dx \right)}}{\sqrt{a \left(\sec{\left(c + dx \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [B] time = 11.608, size = 531, normalized size = 4.7

$$-\frac{\frac{\sqrt{2}\left(A\sqrt{-a}+C\sqrt{-a}\right)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}+\frac{A\log\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/2*(sqrt(2)*(A*sqrt(-a) + C*sqrt(-a))*log((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)) - A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) + A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
 - 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2*A*sqrt(-a) - A*sqrt(-a)*a)/((((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))/d
```

3.189 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{(7A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{A\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c}{2d\sqrt{a\sec(c+dx)}}$$

```
[Out] ((7*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) - (A*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c +
d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.369351, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4022, 3920, 3774, 203, 3795}

$$\frac{(7A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{A\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c}{2d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((7*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) - (A*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c +
d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{aA}{2}+\frac{1}{2}a(3A+4C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= -\frac{A\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\frac{1}{4}a^2(7A+8C)-\frac{1}{4}a^2A\sec(c+\cdots}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{A\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + (-A-C)\int \frac{\sec(c\cdots}{\sqrt{a+a\,\mathrm{se}\cdots}}$$

$$= -\frac{A\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(2(A+C))\,\mathrm{Subst}\left(\int \frac{\cdots}{2\cdots}\right.}{\cdots}$$

$$= \frac{(7A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\cdots}{4d\sqrt{\cdots}}$$

Mathematica [C] time = 26.4094, size = 10837, normalized size = 68.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.37, size = 695, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d/a*(-7*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(3/2)-8*C*cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/
2)*sin(d*x+c)-8*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(
d*x+c)-7*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*C
*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)-8*C*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*A*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(3/2)*sin(d*x+c)+8*A*cos(d*x+c)^4-12*A*cos(d*x+c)^3+4*A*cos(d*x+c)^2)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x
)
```

Fricas [A] time = 5.88597, size = 1299, normalized size = 8.17

$$\left[\frac{4\sqrt{2}((A+C)a\cos\left(dx+c\right)+(A+C)a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-((7\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*log((2*sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
 + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - ((7*A + 8*C)*cos(d*x + c) + 7*A + 8*C)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x +
c)^2 - A*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
```

```
)/(a*d*cos(d*x + c) + a*d), -1/4*(((7*A + 8*C)*cos(d*x + c) + 7*A + 8*C)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - (2*A*cos(d*x + c)^2 - A*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*((A + C)*a*cos(d*x + c) + (A +
 C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.7457, size = 680, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*(A*sqrt(-a) + C*sqrt(-a))*log((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)) - (7*A + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1)) + (7*A + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)) - 4*sqrt(2)*(17*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a) - 57*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a + 19*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a
^2 - 3*A*sqrt(-a)*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*a + a^2)^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.190 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{(7A+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos^2}{3d\sqrt{a\sec(c+d}}$$

```
[Out] -((9*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*S
qrt[a]*d) + (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A + 8*C)*Sin[c + d*x])/(8*d*Sqrt[a +
 a*Sec[c + d*x]]) - (A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c +
d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.56293, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4022, 3920, 3774, 203, 3795}

$$\frac{(7A+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)\cos^2}{3d\sqrt{a\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((9*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*S
qrt[a]*d) + (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A + 8*C)*Sin[c + d*x])/(8*d*Sqrt[a +
 a*Sec[c + d*x]]) - (A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c +
d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(-\frac{aA}{2}+\frac{1}{2}a(5A+6C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(\frac{3}{4}a^2(7A+\cdots\right.}{\sqrt{a+a\,\mathrm{s}\cdots}}}{\cdots}$$

$$= \frac{(7A+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+\cdots}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(7A+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+\cdots}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(7A+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+\cdots}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(9A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{(7\cdots}{8\cdots}$$

Mathematica [A] time = 0.555509, size = 145, normalized size = 0.72

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left(8A\cos^2(c+dx)-2A\cos(c+dx)+21A+24C\right)-3(9A+8C)\tanh^{-1}\left(\sqrt{1-\cdots}\right.\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] ((-3*(9*A + 8*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 24*Sqrt[2]*(A + C)*ArcTa
nh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(21*A + 24*C - 2*A*Cos[c
+ d*x] + 8*A*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(24*d*Sq
```

```
rt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.332, size = 1056, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/192/d/a*(27*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*cos(d*x+c)^2+24*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)*cos(d*x+c)^2+54*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)*cos(d*x+c)+48*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x
+c)*cos(d*x+c)^2+48*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*cos(d*x+c)+48*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c
)*cos(d*x+c)^2+27*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d
*x+c)+96*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)+
24*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+96*C*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)+48*A*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+48*C*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)*sin(d*x+c)-64*A*cos(d*x+c)^6+80*A*cos(d*x+c)^5-184*A*cos(d*x
+c)^4-192*C*cos(d*x+c)^4+168*A*cos(d*x+c)^3+192*C*cos(d*x+c)^3)*(a*(cos(d*x
+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{3}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x
)
```

Fricas [A] time = 5.90081, size = 1399, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/48*(24*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*log(-(2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 3*((9*A + 8*C)*cos(d*x + c) + 9*A + 8*C)*sqrt(-a)*log((2*a*
cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d
*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*(7*A + 8*C)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), 1/24*(3*(
(9*A + 8*C)*cos(d*x + c) + 9*A + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (8*A*cos(d*x + c)^
3 - 2*A*cos(d*x + c)^2 + 3*(7*A + 8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c) - 24*sqrt(2)*((A + C)*a*cos(d*x + c) + (A +
C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 12.0926, size = 1153, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/48*(24*sqrt(2)*(A*sqrt(-a) + C*sqrt(-a))*log((sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)) - 3*(9*A + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + 3*(9*A + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)) - 4*sqrt(2)*(165*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a) + 72*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a) - 1323
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*
sqrt(-a)*a - 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^8*C*sqrt(-a)*a + 3906*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2 + 3024*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^2 - 2118*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt
```

```
(-a)*a^3 - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*C*sqrt(-a)*a^3 + 393*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4 + 360*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^4 - 31*A*sqrt
(-a)*a^5 - 24*C*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/
d
```

3.191 $\int \frac{\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=243

$$-\frac{(21A+16C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{(107A+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(43A+48C)}{96d\sqrt{a}}$$

```
[Out] ((107*A + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(
64*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((21*A + 16*C)*Sin[c + d*x])/(64*d*S
qrt[a + a*Sec[c + d*x]]) + ((43*A + 48*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*
Sqrt[a + a*Sec[c + d*x]]) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a +
a*Sec[c + d*x]]) + (A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c +
d*x]])
```

Rubi [A] time = 0.728525, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4087, 4022, 3920, 3774, 203, 3795}

$$-\frac{(21A+16C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{(107A+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(43A+48C)}{96d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] ((107*A + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(
64*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((21*A + 16*C)*Sin[c + d*x])/(64*d*S
qrt[a + a*Sec[c + d*x]]) + ((43*A + 48*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*
Sqrt[a + a*Sec[c + d*x]]) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a +
a*Sec[c + d*x]]) + (A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c +
d*x]])
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^3(c+dx)\left(-\frac{aA}{2}+\frac{1}{2}a(7A+8C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(\frac{1}{4}\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{(43A+48C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos\cdots}{4d\cdots}$$

$$= -\frac{(21A+16C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{(43A+48C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos\cdots}{24d\cdots}$$

$$= -\frac{(21A+16C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{(43A+48C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos\cdots}{24d\cdots}$$

$$= -\frac{(21A+16C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{(43A+48C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} - \frac{A\cos\cdots}{24d\cdots}$$

$$= \frac{(107A+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}\cdots$$

Mathematica [A] time = 0.764234, size = 160, normalized size = 0.66

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left((86A+96C)\cos(c+dx)+48A\cos^3(c+dx)-8A\cos^2(c+dx)-63A-4\cdots\right.\right.}{192d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] (((321*A + 336*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 192*Sqrt[2]*(A + C)*Arc
Tanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-63*A - 48*C + (86*A +
 96*C)*Cos[c + d*x] - 8*A*Cos[c + d*x]^2 + 48*A*Cos[c + d*x]^3)*Sqrt[1 - Se
c[c + d*x]])*Tan[c + d*x])/(192*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c
+ d*x])])
```

Maple [B] time = 0.379, size = 1406, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3072/d/a*(321*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)+336*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*2^(1/2)+384*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)^3+963*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)*2^(1/2)+384*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin
(d*x+c)*cos(d*x+c)^3+1008*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(7/2)*2^(1/2)+1152*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*sin(d*x+c)*cos(d*x+c)^2+963*A*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(7/2)*2^(1/2)+1152*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)*sin(d*x+c)*cos(d*x+c)^2+1008*C*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)+1152*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*sin(d*x+c)*cos(d*x+c)+321*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2
^(1/2)*sin(d*x+c)+1152*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+
c)*cos(d*x+c)+336*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d
*x+c)+384*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+384*C*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-768*A*cos(d*x+c)^8+896*A*c
os(d*x+c)^7-1504*A*cos(d*x+c)^6-1536*C*cos(d*x+c)^6+2384*A*cos(d*x+c)^5+230
4*C*cos(d*x+c)^5-1008*A*cos(d*x+c)^4-768*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/
cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{4}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x
)
```

Fricas [A] time = 8.69633, size = 1523, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/384*(192*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*log((2*
sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin
(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) - 3*((107*A + 112*C)*cos(d*x + c) + 107*A + 112*C)*sqrt(-a)*
log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(
48*A*cos(d*x + c)^4 - 8*A*cos(d*x + c)^3 + 2*(43*A + 48*C)*cos(d*x + c)^2 -
 3*(21*A + 16*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a*d*cos(d*x + c) + a*d), -1/192*(3*((107*A + 112*C)*cos(d*x + c)
 + 107*A + 112*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*cos(d*x + c)^4 - 8*A*cos(d*x + c
)^3 + 2*(43*A + 48*C)*cos(d*x + c)^2 - 3*(21*A + 16*C)*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 192*sqrt(2)*((A + C)*a*cos
(d*x + c) + (A + C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 14.3068, size = 1418, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] 1/384*(192*sqrt(2)*(A*sqrt(-a) + C*sqrt(-a))*log((sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(a*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)) - 3*(107*A + 112*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)) + 3*(107*A + 112*C)*log(abs((sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt
(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 4*sqrt(2)*(1599*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a) + 816*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt
(-a) - 18219*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^12*A*sqrt(-a)*a - 12528*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a + 91467*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^2 + 64752*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqr
t(-a)*a^2 - 177735*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*A*sqrt(-a)*a^3 - 124848*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^3 + 100413*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^4 +
70032*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^6*C*sqrt(-a)*a^4 - 26881*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^5 - 19152*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^5 + 3321*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^
6 + 2640*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*C*sqrt(-a)*a^6 - 205*A*sqrt(-a)*a^7 - 144*C*sqrt(-a)*a^7)/(((sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.192 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{(11A+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(245A+397C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{210a^2d}-\frac{(A+C)\tan(c+dx)\sec^4(c-}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((11*A + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^4*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((455*A + 799*C)*Tan[c + d*x])/(105*a*d*Sqrt[a
 + a*Sec[c + d*x]]) - ((35*A + 67*C)*Sec[c + d*x]^2*Tan[c + d*x])/(70*a*d*S
qrt[a + a*Sec[c + d*x]]) + ((7*A + 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(14*a
*d*Sqrt[a + a*Sec[c + d*x]]) + ((245*A + 397*C)*Sqrt[a + a*Sec[c + d*x]]*Ta
n[c + d*x])/(210*a^2*d)
```

Rubi [A] time = 0.845068, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4021, 4010, 4001, 3795, 203}

$$\frac{(11A+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(245A+397C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{210a^2d}-\frac{(A+C)\tan(c+dx)\sec^4(c-}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((11*A + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^4*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((455*A + 799*C)*Tan[c + d*x])/(105*a*d*Sqrt[a
 + a*Sec[c + d*x]]) - ((35*A + 67*C)*Sec[c + d*x]^2*Tan[c + d*x])/(70*a*d*S
qrt[a + a*Sec[c + d*x]]) + ((7*A + 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(14*a
*d*Sqrt[a + a*Sec[c + d*x]]) + ((245*A + 397*C)*Sqrt[a + a*Sec[c + d*x]]*Ta
n[c + d*x])/(210*a^2*d)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeqQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^4(c+dx)\left(2a(A+2C)-\frac{1}{2}a(7A+11C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(7A+11C)\sec^3(c+dx)\tan(c+dx)}{14ad\sqrt{a+a\sec(c+dx)}} -$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(35A+67C)\sec^2(c+dx)\tan(c+dx)}{70ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(35A+67C)\sec^2(c+dx)\tan(c+dx)}{70ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(455A+799C)\tan(c+dx)}{105ad\sqrt{a+a\sec(c+dx)}} - \frac{(35A+6}{70}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(455A+799C)\tan(c+dx)}{105ad\sqrt{a+a\sec(c+dx)}} - \frac{(35A+6}{70}$$

$$= \frac{(11A+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(4}{1}$$

Mathematica [B] time = 6.80202, size = 527, normalized size = 2.03

$$\cos^2(c+dx)(\sec(c+dx)+1)^{3/2}\sqrt{(\cos(c+dx)+1)\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+C\sin\left(\frac{dx}{2}\right.\right.}{2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(1 + Sec[c + d*x])^(3
/2)*(A + C*Sec[c + d*x]^2)*((-2*(-140*A - 448*C + 665*A*Cos[c] + 1201*C*Cos
[c])*Sin[c/2])/(105*d*(Cos[c/2] + Cos[(3*c)/2])) + (Sec[c/2]*Sec[c/2 + (d*x
)/2]^2*(A*Sin[c/2] + C*Sin[c/2]))/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(-80
5*A*Sin[(d*x)/2] - 1649*C*Sin[(d*x)/2]))/(105*d) + (Sec[c/2]*Sec[c/2 + (d*x
)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(2*d) + (4*C*Sec[c]*Sec[c + d*x]^
3*Sin[d*x])/(7*d) - (4*Sec[c]*Sec[c + d*x]*(39*C*Sin[c] - 35*A*Sin[d*x] - 1
12*C*Sin[d*x]))/(105*d) + (4*Sec[c]*Sec[c + d*x]^2*(5*C*Sin[c] - 13*C*Sin[d
*x]))/(35*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(3/2)
) + ((11*A + 19*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cos[c + d*x]^4*S
qrt[-1 + Sec[c + d*x]]*(1 + Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sin[c +
d*x])/(Sqrt[2]*d*(1 + Cos[c + d*x])*Sqrt[1 - Cos[c + d*x]^2]*(A + 2*C + A*C
os[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(3/2)*Sqrt[Cos[c + d*x]^2*(-1 + Sec
[c + d*x])*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.438, size = 974, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3360/d/a^2*(-1+cos(d*x+c))*(1155*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(7/2)*cos(d*x+c)^4+1995*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(7/2)*cos(d*x+c)^4+4620*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
sin(d*x+c)*cos(d*x+c)^3+7980*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*si
n(d*x+c)*cos(d*x+c)^3+6930*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(
d*x+c)*cos(d*x+c)^2+11970*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)^2+4620*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x
+c)*cos(d*x+c)+7980*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*
cos(d*x+c)+1155*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+1995
*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-10640*A*cos(d*x+c)^
5-19216*C*cos(d*x+c)^5+3920*A*cos(d*x+c)^4+6352*C*cos(d*x+c)^4+8960*A*cos(d
*x+c)^3+16000*C*cos(d*x+c)^3-2240*A*cos(d*x+c)^2-3712*C*cos(d*x+c)^2+1536*C
```

```
*cos(d*x+c)-960*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x
+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.649925, size = 1392, normalized size = 5.37

$$\left[-\frac{105\sqrt{2}\left((11A+19C)\cos(dx+c)^5+2(11A+19C)\cos(dx+c)^4+(11A+19C)\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos \ldots}{\cos \ldots}}\ldots}{\ldots}\right.\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/840*(105*sqrt(2)*((11*A + 19*C)*cos(d*x + c)^5 + 2*(11*A + 19*C)*cos(d*
x + c)^4 + (11*A + 19*C)*cos(d*x + c)^3)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(
d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
 4*((665*A + 1201*C)*cos(d*x + c)^4 + 12*(35*A + 67*C)*cos(d*x + c)^3 - 28*
(5*A + 7*C)*cos(d*x + c)^2 + 36*C*cos(d*x + c) - 60*C)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x +
 c)^4 + a^2*d*cos(d*x + c)^3), -1/420*(105*sqrt(2)*((11*A + 19*C)*cos(d*x +
 c)^5 + 2*(11*A + 19*C)*cos(d*x + c)^4 + (11*A + 19*C)*cos(d*x + c)^3)*sqrt
(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) + 2*((665*A + 1201*C)*cos(d*x + c)^4 + 12*(35*A + 67*C
)*cos(d*x + c)^3 - 28*(5*A + 7*C)*cos(d*x + c)^2 + 36*C*cos(d*x + c) - 60*C
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2(c+dx)\right)\sec^4(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3
/2), x)
```

Giac [A] time = 9.30967, size = 593, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] 1/420*(105*(11*sqrt(2)*A + 19*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/
2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)) - ((((105*(sqrt(2)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + s
qrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^3 -
4*(455*sqrt(2)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 877*sqrt(2)*C*a^5*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)*tan(1/2*d*x + 1/2*c)^2 + 14*(305*sqrt(2
)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 517*sqrt(2)*C*a^5*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1))/a^3)*tan(1/2*d*x + 1/2*c)^2 - 140*(25*sqrt(2)*A*a^5*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1) + 47*sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))/a^3)*tan(1/2*d*x + 1/2*c)^2 + 105*(9*sqrt(2)*A*a^5*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1) + 17*sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)*tan(1/
2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/d
```

3.193 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{(7A+15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(5A+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+$$

```
[Out] -((7*A + 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) + ((15*A + 31*C)*Tan[c + d*x])/(5*a*d*Sqrt[a + a
*Sec[c + d*x]]) + ((5*A + 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
+ a*Sec[c + d*x]]) - ((5*A + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(
10*a^2*d)
```

Rubi [A] time = 0.6272, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4021, 4010, 4001, 3795, 203}

$$-\frac{(7A+15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(5A+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{(A+C)\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] -((7*A + 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) + ((15*A + 31*C)*Tan[c + d*x])/(5*a*d*Sqrt[a + a
*Sec[c + d*x]]) + ((5*A + 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
+ a*Sec[c + d*x]]) - ((5*A + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(
10*a^2*d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^3(c+dx)\left(a(A+3C)-\frac{1}{2}a(5A+9C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A+9C)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A+9C)\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(15A+31C)\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{(5A+9}{10}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(15A+31C)\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{(5A+9}{10}$$

$$= -\frac{(7A+15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 4.90577, size = 189, normalized size = 0.88

$$\frac{\sin(c+dx)\cos(c+dx)\left(A+C\sec^2(c+dx)\right)\left(\sec^3(c+dx)((75A+131C)\cos(c+dx)+8(5A+9C)\cos(2(c+dx))+\right.}{20d(a(\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*(-10*Sqrt[2]*(7*A + 15*C)*ArcTan[Sqrt[
-1 + Sec[c + d*x]]/Sqrt[2]]*Cot[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] + (4
0*A + 88*C + (75*A + 131*C)*Cos[c + d*x] + 8*(5*A + 9*C)*Cos[2*(c + d*x)] +
 25*A*Cos[3*(c + d*x)] + 49*C*Cos[3*(c + d*x)])*Sec[c + d*x]^3)*Sin[c + d*x
])/(20*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.339, size = 784, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/80/d/a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)+75*C*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)+105*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x
+c)*cos(d*x+c)^2+225*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)
*cos(d*x+c)^2+105*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*co
s(d*x+c)+225*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x
+c)+35*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+75*C*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+200*A*cos(d*x+c)^4+392*C*cos(d
*x+c)^4-40*A*cos(d*x+c)^3-104*C*cos(d*x+c)^3-160*A*cos(d*x+c)^2-320*C*cos(d
*x+c)^2+64*C*cos(d*x+c)-32*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c
)^2/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.626887, size = 1268, normalized size = 5.93

$$\left[-\frac{5\sqrt{2}\left((7A+15C)\cos(dx+c)^4+2(7A+15C)\cos(dx+c)^3+(7A+15C)\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos}{\cos}}\cdots}{\cdots}\right.\cdots}{40\left(a^2d\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/40*(5*sqrt(2)*((7*A + 15*C)*cos(d*x + c)^4 + 2*(7*A + 15*C)*cos(d*x + c
)^3 + (7*A + 15*C)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x +
 c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((
25*A + 49*C)*cos(d*x + c)^3 + 4*(5*A + 9*C)*cos(d*x + c)^2 - 4*C*cos(d*x +
c) + 4*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(
d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), 1/20*(5*sqrt(2
)*((7*A + 15*C)*cos(d*x + c)^4 + 2*(7*A + 15*C)*cos(d*x + c)^3 + (7*A + 15*
C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((25*A + 49*C)*cos(d*x + c)
^3 + 4*(5*A + 9*C)*cos(d*x + c)^2 - 4*C*cos(d*x + c) + 4*C)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(
d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^2(c+dx)\right)\sec^3(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3
/2), x)
```

Giac [A] time = 9.49401, size = 420, normalized size = 1.96

$$-\frac{\frac{5\sqrt{2}(7A+15C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\left(\left(\left(\frac{5\sqrt{2}(Aa^3+Ca^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}(55Aa^3+127Ca^3)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\cdots\right.}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] -1/20*(5*sqrt(2)*(7*A + 15*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)) - (((5*sqrt(2)*(A*a^3 + C*a^3)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)) - sqrt(2)*(55*A*a^3 + 127*C*a^3)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*(19*A*a^3 + 35*C*a^3)/
(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*(
9*A*a^3 + 17*C*a^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/
2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
)/d
```

3.194 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(3A+11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A+7C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((3*A + 11*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a
 + a*Sec[c + d*x])^(3/2)) - ((3*A + 13*C)*Tan[c + d*x])/(3*a*d*Sqrt[a + a*S
ec[c + d*x]]) + ((3*A + 7*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2*
d)
```

Rubi [A] time = 0.448418, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4085, 4010, 4001, 3795, 203}

$$\frac{(3A+11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A+7C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((3*A + 11*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a
 + a*Sec[c + d*x])^(3/2)) - ((3*A + 13*C)*Tan[c + d*x])/(3*a*d*Sqrt[a + a*S
ec[c + d*x]]) + ((3*A + 7*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2*
d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2aC-\frac{1}{2}a(3A+7C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+7C)\sqrt{a+a\sec(c+dx)}\tan(c+d}{6a^2d}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A+13C)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{(3A+7C)\sqrt{}}{}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A+13C)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{(3A+7C)\sqrt{}}{}$$

$$= \frac{(3A+11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3}{3a}$$

Mathematica [A] time = 3.64517, size = 162, normalized size = 0.96

$$\frac{\sin(c+dx)\cos(c+dx)\left(A+C\sec^2(c+dx)\right)\left(3\sqrt{2}(3A+11C)\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)-\text{se}\right.}{6d(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+dx))+A+2C}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]*(A + C*Sec[c + d*x]^2)*(3*Sqrt[2]*(3*A + 11*C)*ArcTan[Sqrt[-1
 + Sec[c + d*x]]/Sqrt[2]]*Cot[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] - (3*A
 + 11*C + 24*C*Cos[c + d*x] + (3*A + 19*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^2
)*Sin[c + d*x])/(6*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^
(3/2))
```

Maple [B] time = 0.311, size = 594, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d/a^2*(-1+cos(d*x+c))*(9*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*s
in(d*x+c)*cos(d*x+c)^2+33*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d
*x+c)*cos(d*x+c)^2+18*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2
)*cos(d*x+c)+66*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(
d*x+c)+9*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+33*C*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-12*A*cos(d*x+c)^3-76*C*cos(d
*x+c)^3+12*A*cos(d*x+c)^2+28*C*cos(d*x+c)^2+64*C*cos(d*x+c)-16*C)*(a*(cos(d
*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.619937, size = 1173, normalized size = 6.94

$$\left[-\frac{3\sqrt{2}\left((3A+11C)\cos(dx+c)^3+2(3A+11C)\cos(dx+c)^2+(3A+11C)\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+}{\cos(dx}}}{}\right.}{24\left(a^2d\cos(dx+c)^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*((3*A + 11*C)*cos(d*x + c)^3 + 2*(3*A + 11*C)*cos(d*x + c
)^2 + (3*A + 11*C)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)
^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A
 + 19*C)*cos(d*x + c)^2 + 12*C*cos(d*x + c) - 4*C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^
2 + a^2*d*cos(d*x + c)), -1/12*(3*sqrt(2)*((3*A + 11*C)*cos(d*x + c)^3 + 2*
(3*A + 11*C)*cos(d*x + c)^2 + (3*A + 11*C)*cos(d*x + c))*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) + 2*((3*A + 19*C)*cos(d*x + c)^2 + 12*C*cos(d*x + c) - 4*C)*sqrt((a*
```

```
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2
*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3
/2), x)
```

Giac [B] time = 9.10165, size = 398, normalized size = 2.36

$$-\frac{\frac{\left(\left(\frac{3\left(\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)+\sqrt{2}Ca\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a}-\frac{2\left(3\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)+23\sqrt{2}Ca\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\cdots}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\cdots}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] -1/12*(((3*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a - 2*(3*sqrt(2)*A*a*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1) + 23*sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))/a)*tan(1/2*d*x + 1/2*c)^2 + 3*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1) + 9*sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a)*tan(1/2*d*x + 1/2*c
)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*
(3*sqrt(2)*A + 11*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)))/d
```

3.195 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(A-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A+5C)\tan(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((A - 7*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a + a*
Sec[c + d*x])^(3/2)) + ((A + 5*C)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rubi [A] time = 0.230168, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4079, 4001, 3795, 203}

$$\frac{(A-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A+5C)\tan(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((A - 7*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a + a*
Sec[c + d*x])^(3/2)) + ((A + 5*C)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rule 4079

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A + C)*Cot[e +
 f*x]*Csc[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] - Dist[1/(a*b*
(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[-(b*C) - 2*A
*b*(m + 1) + a*(A*(m + 2) - C*(m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)\left(a(A-C)+\frac{1}{2}a(A+5C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+5C)\tan(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(A-7C)\int \frac{\text{se}}{\sqrt{a+}}}{4a}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+5C)\tan(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(A-7C)\,\text{Subst}}{}$$

$$= \frac{(A-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+5}{2ad\sqrt{a}}$$

Mathematica [A] time = 1.81293, size = 127, normalized size = 1.01

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(2\sin^2\left(\frac{1}{2}(c+dx)\right)(A+4C\sec(c+dx)+5C)+\sqrt{2}(A-7C)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\right.\right.}{2ad(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] -((Sqrt[2]*(A - 7*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/
2]^2*Sqrt[-1 + Sec[c + d*x]] + 2*(A + 5*C + 4*C*Sec[c + d*x])*Sin[(c + d*x)
/2]^2)*Tan[(c + d*x)/2])/(2*a*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x
])]))
```

Maple [B] time = 0.275, size = 405, normalized size = 3.2

$$\frac{-1+\cos(dx+c)}{4\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-A\cos(dx+c)\sin(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ln\left(-\frac{1}{\sin(dx+c)}\left(-\sqrt{-2\,\frac{}{\text{c}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-A*cos(d*x+c
)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+7*C*cos(d*x+c)*sin(d
*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-A*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*sin(d*x+c)+7*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*s
in(d*x+c)+2*A*cos(d*x+c)^2+10*C*cos(d*x+c)^2-2*A*cos(d*x+c)-2*C*cos(d*x+c)-
```

```
8*C)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A] time = 0.607946, size = 987, normalized size = 7.83

$$\left[\frac{\sqrt{2}\left((A-7C)\cos\left(dx+c\right)^{2}+2\,(A-7C)\cos\left(dx+c\right)+A-7C\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3\,a\cos}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(sqrt(2)*((A - 7*C)*cos(d*x + c)^2 + 2*(A - 7*C)*cos(d*x + c) + A - 7*
C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A + 5*C)*cos(d*x + c) + 4*C)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((A - 7*C)*cos(d*x + c)^2 + 2*
(A - 7*C)*cos(d*x + c) + A - 7*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((A + 5*C)*c
os(d*x + c) + 4*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.0595, size = 251, normalized size = 1.99

$$\frac{\frac{\left(\frac{\sqrt{2}\left(Aa^2+Ca^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}\left(Aa^2+9Ca^2\right)}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{\sqrt{2}(A-7C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/4*((sqrt(2)*(A*a^2 + C*a^2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)) - sqrt(2)*(A*a^2 + 9*C*a^2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(2)*
(A - 7*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.196 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{(5A-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*
x])^(3/2))
```

Rubi [A] time = 0.190262, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4053, 3920, 3774, 203, 3795}

$$-\frac{(5A-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*
x])^(3/2))
```

Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2aA+\frac{1}{2}a(A-3C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{(5A-3C)\int\frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{(A+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} + \frac{(5A-3C)\,\text{Subst}}{}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 1.38849, size = 154, normalized size = 1.23

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left((A+C)(\cos(c+dx)-1)-\sqrt{2}(5A-3C)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)+8A\right.}{2ad(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -(((A + C)*(-1 + Cos[c + d*x]) + 8*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[(c
 + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] - Sqrt[2]*(5*A - 3*C)*ArcTan[Sqrt[-1 +
 Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]])*Tan[(c
+ d*x)/2])/(2*a*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.232, size = 554, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(4*A*sin(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)+5*A*cos(d*x+c)*sin(d*x+c)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+4*A*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))+5*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(
```

```
d*x+c)-3*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-2*A*cos(d*x
+c)^2-2*C*cos(d*x+c)^2+2*A*cos(d*x+c)+2*C*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*
x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 7.92495, size = 1432, normalized size = 11.46

$$\left[-\frac{4\,(A+C)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - \sqrt{2}\left((5\,A-3\,C)\cos(dx+c)^2 + 2\,(5\,A-3\,C)\cos(dx+c) + 5\,A -\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(A + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - sqrt(2)*((5*A - 3*C)*cos(d*x + c)^2 + 2*(5*A - 3*C)*cos(d*x + c)
+ 5*A - 3*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c
) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*(A*cos(d*x + c)^2 + 2*A*c
os(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)
/(cos(d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, -1/4*(2*(A + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) - sqrt(2)*((5*A - 3*C)*cos(d*x + c)^2 + 2*(5*A - 3*C)*cos(d*x + c)
 + 5*A - 3*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 8*(A*cos(d*x + c)^2 + 2*A*cos(d*x
+ c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 11.2279, size = 416, normalized size = 3.33

$$-\frac{\frac{\sqrt{2}(5A-3C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{8A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{8A\log\left(\left|\left(\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2)*(5*A - 3*C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) +
8*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
 - 8*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)) - 2*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x +
1/2*c)/a^3)/d
```

3.197 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{(9A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A+C)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((9*A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + ((3*A + C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.386058, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4085, 4022, 3920, 3774, 203, 3795}

$$\frac{(9A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A+C)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((9*A + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + ((3*A + C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-a(3A+C)+\frac{1}{2}a(3A-C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{3a^2A-\frac{1}{2}a^2(3A+C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}}{2a^3}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A)\int \sqrt{a+a\sec(c+d}}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(3A)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x\right.}{ad}$$

$$= -\frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sin}{2d(a+a\sec}$$

Mathematica [A] time = 1.57668, size = 167, normalized size = 1.06

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(2\sin^2\left(\frac{1}{2}(c+dx)\right)(2A\cos(c+dx)+3A+C)+\sqrt{2}(9A+C)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\right.\right.}{2ad(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] -((-12*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Sec[c
 + d*x]] + Sqrt[2]*(9*A + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cos[(c
 + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] + 2*(3*A + C + 2*A*Cos[c + d*x])*Sin[(
c + d*x)/2]^2)*Tan[(c + d*x)/2])/(2*a*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec
[c + d*x])]])
```

Maple [B] time = 0.333, size = 561, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(-1+cos(d*x+c))*(6*A*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*2^(1/2)*cos(d*x+c)+6*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)+9*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))+C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))+9*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-4*A*cos(d*x+c)^
3+C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-2*A*cos(d*x+c)^2-2
*C*cos(d*x+c)^2+6*A*cos(d*x+c)+2*C*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c)
)^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A] time = 7.98163, size = 1486, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((9*A + C)*cos(d*x + c)^2 + 2*(9*A + C)*cos(d*x + c) + 9*A +
 C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 12*(A*cos(d*x + c)^2 + 2*A*cos(d*x +
 c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*
x + c) + 1)) - 4*(2*A*cos(d*x + c)^2 + (3*A + C)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d), -1/4*(sqrt(2)*((9*A + C)*cos(d*x + c)^2 + 2*(9*A + C)
*cos(d*x + c) + 9*A + C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 12*(A*cos(d*x + c)^2 +
2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*A*cos(d*x + c)^2 + (3*A + C)*
```

```
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C \sec^{2}\left(c + dx\right)\right) \cos\left(c + dx\right)}{\left(a \left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.198 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(19A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A+2C)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A+C)\sin(c+}{2ad\sqrt{a\sec(}$$

```
[Out] ((19*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a
^(3/2)*d) - ((13*A + 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]*Sin[c + d*x
])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A + 2*C)*Sin[c + d*x])/(4*a*d*Sqr
t[a + a*Sec[c + d*x]]) + ((2*A + C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.587937, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4022, 3920, 3774, 203, 3795}

$$\frac{(19A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A+2C)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A+C)\sin(c+}{2ad\sqrt{a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((19*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a
^(3/2)*d) - ((13*A + 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]*Sin[c + d*x
])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A + 2*C)*Sin[c + d*x])/(4*a*d*Sqr
t[a + a*Sec[c + d*x]]) + ((2*A + C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[
a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)\left(-2a(2A+C)+\frac{1}{2}a(5A+C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(2A+C)\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \cos}{}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A+2C)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A+C)\cos(c}{2ad\sqrt{a+a}}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A+2C)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A+C)\cos(c}{2ad\sqrt{a+a}}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A+2C)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2A+C)\cos(c}{2ad\sqrt{a+a}}$$

$$= \frac{(19A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - (A$$

Mathematica [C] time = 26.7637, size = 12015, normalized size = 55.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.369, size = 1064, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/16/d/a^2*(-1+cos(d*x+c))*(-19*A*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(3/2)*2^(1/2)-8*C*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*2^(1/2)-26*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(3/2)*sin(d*x+c)*cos(d*x+c)^2-38*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-10*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*sin(d*x+c)*cos(d*x+c)^2-16*C*cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(3/2)*sin(d*x+c)-52*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(3/2)*cos(d*x+c)-19*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*sin(d*x+c)-20*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*
cos(d*x+c)-8*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)
+8*A*cos(d*x+c)^5-26*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)
-10*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-20*A*cos(d*x+c)^
4-16*A*cos(d*x+c)^3-8*C*cos(d*x+c)^3+28*A*cos(d*x+c)^2+8*C*cos(d*x+c)^2)*(a
*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2),
 x)
```

Fricas [A] time = 15.1479, size = 1646, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((13*A + 5*C)*cos(d*x + c)^2 + 2*(13*A + 5*C)*cos(d*x + c) +
 13*A + 5*C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x +
c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A + 8*C)*cos(d*x + c)
^2 + 2*(19*A + 8*C)*cos(d*x + c) + 19*A + 8*C)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*A*cos(d*x + c)^3
- 3*A*cos(d*x + c)^2 - (7*A + 2*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d), 1/4*(sqrt(2)*((13*A + 5*C)*cos(d*x + c)^2 + 2*(13*A + 5*C)*cos(d*x
+ c) + 13*A + 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((19*A + 8*C)*cos(d*x + c)^2
+ 2*(19*A + 8*C)*cos(d*x + c) + 19*A + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (2*A*cos(d*x
 + c)^3 - 3*A*cos(d*x + c)^2 - (7*A + 2*C)*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.199 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{(47A+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A+9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3(7A+4C)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}+\frac{(5A+3C)\,\mathrm{si}}{6ad\sqrt{a}}$$

```
[Out] -((47*A + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8
*a^(3/2)*d) + ((17*A + 9*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*(7*A + 4*C)*Sin[c + d*x])/(8*a
*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12
*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A + 3*C)*Cos[c + d*x]^2*Sin[c + d*x])/
(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.775701, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4022, 3920, 3774, 203, 3795}

$$-\frac{(47A+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A+9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3(7A+4C)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}+\frac{(5A+3C)\,\mathrm{si}}{6ad\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] -((47*A + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8
*a^(3/2)*d) + ((17*A + 9*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*(7*A + 4*C)*Sin[c + d*x])/(8*a
*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12
*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A + 3*C)*Cos[c + d*x]^2*Sin[c + d*x])/
(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos^3(c+dx)\left(-a(5A+3C)+\frac{1}{2}a(7A+3C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A+3C)\cos^2(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} - \int$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(13A+6C)\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a+a\sec(c+dx)}} + (5$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3(7A+4C)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A+6C)\,c}{12ad\sqrt{a}}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3(7A+4C)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A+6C)\,c}{12ad\sqrt{a}}$$

$$= -\frac{(A+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3(7A+4C)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \frac{(13A+6C)\,c}{12ad\sqrt{a}}$$

$$= -\frac{(47A+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A+9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} -$$

Mathematica [A] time = 2.92112, size = 204, normalized size = 0.77

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(\sin^2\left(\frac{1}{2}(c+dx)\right)((43A+24C)\cos(c+dx)-3A\cos(2(c+dx))+2A\cos(3(c+dx))+60A+36C)-3\right.}{12ad(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2
),x]
```

```
[Out] -((-3*(47*A + 24*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sqrt
[-1 + Sec[c + d*x]] + 6*Sqrt[2]*(17*A + 9*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]
/Sqrt[2]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] + (60*A + 36*C + (43*A
 + 24*C)*Cos[c + d*x] - 3*A*Cos[2*(c + d*x)] + 2*A*Cos[3*(c + d*x)])*Sin[(c
 + d*x)/2]^2)*Tan[(c + d*x)/2])/(12*a*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec
[c + d*x])]))
```

Maple [B] time = 0.325, size = 1414, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/192/d/a^2*(-1+cos(d*x+c))*(141*A*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+72*C*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+204*A*cos(d*x+c)^3*sin(d*x+c)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+423*A*sin(d*x+c)*2^(1/2)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2+108*C*cos(d*x+c)^3*sin(d*x+c)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+216*C*sin(d*x+c)*2^(1/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2+612*A*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)^2+423*A*sin(d*x+c)*2^(1/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)+324*C*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)^2+216*C*sin(d*x+c)*2^(1/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)+612*A*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)+141*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(5/2)*sin(d*x+c)+324*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*sin(d*x+c)*cos(d*x+c)+72*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*sin(d*x+c)+204*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-
64*A*cos(d*x+c)^7+108*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c
)+112*A*cos(d*x+c)^6-344*A*cos(d*x+c)^5-192*C*cos(d*x+c)^5-208*A*cos(d*x+c)
^4-96*C*cos(d*x+c)^4+504*A*cos(d*x+c)^3+288*C*cos(d*x+c)^3)*(a*(cos(d*x+c)+
1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2),
 x)
```

Fricas [A] time = 15.0457, size = 1756, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/48*(6*sqrt(2)*((17*A + 9*C)*cos(d*x + c)^2 + 2*(17*A + 9*C)*cos(d*x + c
) + 17*A + 9*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x
+ c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*((47*A + 24*C)*cos(d*x
 + c)^2 + 2*(47*A + 24*C)*cos(d*x + c) + 47*A + 24*C)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(8*A*cos(d*x
+ c)^4 - 6*A*cos(d*x + c)^3 + (37*A + 24*C)*cos(d*x + c)^2 + 9*(7*A + 4*C)*
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/24*(6*sqrt(2)*((17*A + 9
*C)*cos(d*x + c)^2 + 2*(17*A + 9*C)*cos(d*x + c) + 17*A + 9*C)*sqrt(a)*arct
an(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*si
n(d*x + c))) - 3*((47*A + 24*C)*cos(d*x + c)^2 + 2*(47*A + 24*C)*cos(d*x +
c) + 47*A + 24*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^4 - 6*A*cos(d*x + c)
^3 + (37*A + 24*C)*cos(d*x + c)^2 + 9*(7*A + 4*C)*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*
cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.200 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=259

$$\frac{(45A+157C)\tan(c+dx)\sec^2(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A+283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(195A+787C)\tan(c+dx)\sqrt{a\sec(c+dx)}}{240a^3d}$$

```
[Out] -((75*A + 283*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^4*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) - ((5*A + 21*C)*Sec[c + d*x]^3*Tan[c + d*x])/
(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((465*A + 1729*C)*Tan[c + d*x])/(120*
a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((45*A + 157*C)*Sec[c + d*x]^2*Tan[c + d*
x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]]) - ((195*A + 787*C)*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(240*a^3*d)
```

Rubi [A] time = 0.837182, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4019, 4021, 4010, 4001, 3795, 203}

$$\frac{(45A+157C)\tan(c+dx)\sec^2(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A+283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(195A+787C)\tan(c+dx)\sqrt{a\sec(c+dx)}}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((75*A + 283*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^4*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) - ((5*A + 21*C)*Sec[c + d*x]^3*Tan[c + d*x])/
(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((465*A + 1729*C)*Tan[c + d*x])/(120*
a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((45*A + 157*C)*Sec[c + d*x]^2*Tan[c + d*
x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]]) - ((195*A + 787*C)*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(240*a^3*d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^4(c+dx)\left(4aC-\frac{1}{2}a(5A+13C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A+21C)\sec^3(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A+21C)\sec^3(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A+21C)\sec^3(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A+21C)\sec^3(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A+21C)\sec^3(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{(75A+283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} -$$

Mathematica [A] time = 3.37294, size = 220, normalized size = 0.85

$$\frac{\tan(c+dx)\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)\left(50(153A+521C)\cos(c+dx)+108(45A+157C)\cos(2(c+dx))+\frac{60\sqrt{2}(7}{}\right.}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2
),x]
```

```
[Out] ((4125*A + 15053*C + 50*(153*A + 521*C)*Cos[c + d*x] + 108*(45*A + 157*C)*C
os[2*(c + d*x)] + 2550*A*Cos[3*(c + d*x)] + 9110*C*Cos[3*(c + d*x)] + 735*A
*Cos[4*(c + d*x)] + 2671*C*Cos[4*(c + d*x)] + (60*Sqrt[2]*(75*A + 283*C)*Ar
cTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cos[c + d*x]^3*(1 + Cos[c + d*x])^2*S
qrt[-1 + Sec[c + d*x]])/(-1 + Cos[c + d*x]))*Sec[c + d*x]^2*(A + C*Sec[c +
d*x]^2)*Tan[c + d*x])/(960*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c +
 d*x]))^(5/2))
```

Maple [B] time = 0.356, size = 976, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/1920/d/a^3*(-1+cos(d*x+c))^2*(1125*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos
```

```
(d*x+c)+1))^(5/2)*cos(d*x+c)^4+4245*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*cos(d*x+c)^4+4500*A*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(5/2)+16980*C*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)+6750*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*
sin(d*x+c)*cos(d*x+c)^2+25470*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*s
in(d*x+c)*cos(d*x+c)^2+4500*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin
(d*x+c)*cos(d*x+c)+16980*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*
x+c)*cos(d*x+c)+1125*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)
+4245*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+5880*A*cos(d*x
+c)^5+21368*C*cos(d*x+c)^5+4320*A*cos(d*x+c)^4+15072*C*cos(d*x+c)^4-6360*A*
cos(d*x+c)^3-23896*C*cos(d*x+c)^3-3840*A*cos(d*x+c)^2-13824*C*cos(d*x+c)^2+
2048*C*cos(d*x+c)-768*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/c
os(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.662063, size = 1567, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/960*(15*sqrt(2)*((75*A + 283*C)*cos(d*x + c)^5 + 3*(75*A + 283*C)*cos(d
*x + c)^4 + 3*(75*A + 283*C)*cos(d*x + c)^3 + (75*A + 283*C)*cos(d*x + c)^2
)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((735*A + 2671*C)*cos(d*x + c)^4 +
5*(255*A + 911*C)*cos(d*x + c)^3 + 32*(15*A + 49*C)*cos(d*x + c)^2 - 160*C*
cos(d*x + c) + 96*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a
^3*d*cos(d*x + c)^2), 1/480*(15*sqrt(2)*((75*A + 283*C)*cos(d*x + c)^5 + 3*
(75*A + 283*C)*cos(d*x + c)^4 + 3*(75*A + 283*C)*cos(d*x + c)^3 + (75*A + 2
83*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((735*A + 2671*C)*cos(d*
```

```
x + c)^4 + 5*(255*A + 911*C)*cos(d*x + c)^3 + 32*(15*A + 49*C)*cos(d*x + c)
^2 - 160*C*cos(d*x + c) + 96*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x
 + c)^3 + a^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 9.37325, size = 591, normalized size = 2.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] 1/480*((((15*(2*(sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*
a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^2 + (13*sqrt(
2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 29*sqrt(2)*C*a^2*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1))/a^2)*tan(1/2*d*x + 1/2*c)^2 - (1725*sqrt(2)*A*a^2*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1) + 6733*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))/a^2)*tan(1/2*d*x + 1/2*c)^2 + 5*(549*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1) + 1973*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^2)*tan(
1/2*d*x + 1/2*c)^2 - 15*(83*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) +
 291*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^2)*tan(1/2*d*x + 1/2*
c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) -
 15*(75*sqrt(2)*A + 283*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) +
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*
c)^2 - 1)))/d
```

3.201 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{(19A+163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5(3A+19C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{(21A+197C)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-$$

```
[Out] ((19*A + 163*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d
*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d
*(a + a*Sec[c + d*x])^(5/2)) - ((A + 17*C)*Sec[c + d*x]^2*Tan[c + d*x])/(16
*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((21*A + 197*C)*Tan[c + d*x])/(24*a^2*d*
Sqrt[a + a*Sec[c + d*x]]) + (5*(3*A + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c
+ d*x])/(48*a^3*d)
```

Rubi [A] time = 0.659367, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4019, 4010, 4001, 3795, 203}

$$\frac{(19A+163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5(3A+19C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{(21A+197C)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((19*A + 163*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d
*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d
*(a + a*Sec[c + d*x])^(5/2)) - ((A + 17*C)*Sec[c + d*x]^2*Tan[c + d*x])/(16
*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((21*A + 197*C)*Tan[c + d*x])/(24*a^2*d*
Sqrt[a + a*Sec[c + d*x]]) + (5*(3*A + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c
+ d*x])/(48*a^3*d)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^3(c+dx)\left(-a(A-3C)-\frac{1}{2}a(3A+11C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A+17C)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \int$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A+17C)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + 5$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A+17C)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2}{2}$$

$$= -\frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A+17C)\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2}{2}$$

$$= \frac{(19A+163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - ($$

Mathematica [A] time = 2.33556, size = 196, normalized size = 0.92

$$\frac{\tan(c+dx)\sec(c+dx)\left(A+C\sec^2(c+dx)\right)\left(-(81A+1537C)\cos(c+dx)-2(39A+503C)\cos(2(c+dx))-\frac{6\sqrt{2}(19A+16}{}\right.}{96d(a(\sec(c+dx)+1))^{5/2}(A\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2
),x]
```

```
[Out] ((-78*A - 878*C - (81*A + 1537*C)*Cos[c + d*x] - 2*(39*A + 503*C)*Cos[2*(c
+ d*x)] - 27*A*Cos[3*(c + d*x)] - 299*C*Cos[3*(c + d*x)] - (6*Sqrt[2]*(19*A
 + 163*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Cos[c + d*x]^2*(1 + Cos[c
 + d*x])^2*Sqrt[-1 + Sec[c + d*x]])/(-1 + Cos[c + d*x]))*Sec[c + d*x]*(A +
C*Sec[c + d*x]^2)*Tan[c + d*x])/(96*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1
+ Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.365, size = 786, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d/a^3*(-1+cos(d*x+c))^2*(57*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*sin(d*x+c)*cos(d*x+c)^3+489*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*sin(d*x+c)*cos(d*x+c)^3+171*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*si
n(d*x+c)*cos(d*x+c)^2+1467*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(
d*x+c)*cos(d*x+c)^2+171*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*cos(d*x+c)+1467*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*
cos(d*x+c)+57*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+489*C*
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-108*A*cos(d*x+c)^4-119
6*C*cos(d*x+c)^4-48*A*cos(d*x+c)^3-816*C*cos(d*x+c)^3+156*A*cos(d*x+c)^2+13
72*C*cos(d*x+c)^2+768*C*cos(d*x+c)-128*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.619588, size = 1447, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*((19*A + 163*C)*cos(d*x + c)^4 + 3*(19*A + 163*C)*cos(d*
x + c)^3 + 3*(19*A + 163*C)*cos(d*x + c)^2 + (19*A + 163*C)*cos(d*x + c))*s
qrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((27*A + 299*C)*cos(d*x + c)^3 + (39*A
+ 503*C)*cos(d*x + c)^2 + 160*C*cos(d*x + c) - 32*C)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), -1/96*(3*sqrt(2)*((19*A
 + 163*C)*cos(d*x + c)^4 + 3*(19*A + 163*C)*cos(d*x + c)^3 + 3*(19*A + 163*
C)*cos(d*x + c)^2 + (19*A + 163*C)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
 2*((27*A + 299*C)*cos(d*x + c)^3 + (39*A + 503*C)*cos(d*x + c)^2 + 160*C*c
os(d*x + c) - 32*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^
3*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^{2}\left(c + dx\right)\right)\sec^{3}\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5
/2), x)
```

Giac [A] time = 9.61947, size = 419, normalized size = 1.98

$$-\frac{\frac{\left(\left(\left(3\left(\frac{2\sqrt{2}\left(Aa^5+Ca^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\left(7Aa^5+23Ca^5\right)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}\left(15Aa^5+167Ca^5\right)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3\sqrt{2}\left(11Aa^5+155Ca^5\right)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx \cdots\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} \cdots}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^6*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(7*A*a^5 + 23*C*a^5)/(a^6*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(15*A*a^5 + 167*C*a^5
)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)
*(11*A*a^5 + 155*C*a^5)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)) - 3*sqrt(2)*(19*A + 163*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt
```

```
(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)))/d
```

3.202 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=165

$$\frac{5(A-15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+9C)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(3A-13C)\tan(c}{16ad(a\sec(c+dx)}$$

```
[Out] (5*(A - 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(
a + a*Sec[c + d*x])^(5/2)) - ((3*A - 13*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec
[c + d*x])^(3/2)) + ((A + 9*C)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*
x]])
```

Rubi [A] time = 0.456373, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4085, 4008, 4001, 3795, 203}

$$\frac{5(A-15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+9C)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(3A-13C)\tan(c}{16ad(a\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*(A - 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(
a + a*Sec[c + d*x])^(5/2)) - ((3*A - 13*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec
[c + d*x])^(3/2)) + ((A + 9*C)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*
x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^2(c+dx)\left(-2a(A-C)-\frac{1}{2}a(A+9C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(3A-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+}{}}{}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(3A-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(A+9}{4a^2d\sqrt{a}}$$

$$= -\frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(3A-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(A+9}{4a^2d\sqrt{a}}$$

$$= \frac{5(A-15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} -$$

Mathematica [A] time = 3.2032, size = 136, normalized size = 0.82

$$\frac{\tan^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(-(10(A+17C)\cos(c+dx)+(A+49C)\cos(2(c+dx))+A+113C))-5\sqrt{2}(A-15C)\mathrm{s}}{32a^2d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2
),x]
```

```
[Out] (-5*Sqrt[2]*(A - 15*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Sqrt[-1 + Se
c[c + d*x]]*Sin[c + d*x] - (A + 113*C + 10*(A + 17*C)*Cos[c + d*x] + (A + 4
9*C)*Cos[2*(c + d*x)])*Sec[c + d*x]*Tan[(c + d*x)/2]^3)/(32*a^2*d*(-1 + Cos
[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.301, size = 597, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(5*A*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-75*C*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+10*A*cos(d*x+c)*si
n(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-150*C*cos(d*x+c)*sin(d*x
+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+5*A*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*sin(d*x+c)-2*A*cos(d*x+c)^3-75*C*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*sin(d*x+c)-98*C*cos(d*x+c)^3-8*A*cos(d*x+c)^2-72*C*cos(d*x
+c)^2+10*A*cos(d*x+c)+106*C*cos(d*x+c)+64*C)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.602017, size = 1238, normalized size = 7.5

$$\left[\frac{5\sqrt{2}\left((A-15C)\cos(dx+c)^3+3(A-15C)\cos(dx+c)^2+3(A-15C)\cos(dx+c)+A-15C\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a}{\cdots}}}{\cdots}\right.}{64\left(a^3d\cos(dx+c)^3+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*((A - 15*C)*cos(d*x + c)^3 + 3*(A - 15*C)*cos(d*x + c)^2 +
 3*(A - 15*C)*cos(d*x + c) + A - 15*C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d
*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
4*((A + 49*C)*cos(d*x + c)^2 + 5*(A + 17*C)*cos(d*x + c) + 32*C)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*sqrt(2)*((A - 15*
C)*cos(d*x + c)^3 + 3*(A - 15*C)*cos(d*x + c)^2 + 3*(A - 15*C)*cos(d*x + c)
 + A - 15*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((A + 49*C)*cos(d*x + c)^2 + 5*(A
```

```
 + 17*C)*cos(d*x + c) + 32*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5
/2), x)
```

Giac [B] time = 9.0881, size = 386, normalized size = 2.34

$$\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Aa^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)+\sqrt{2}Ca^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}Aa^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)+17\,\sqrt{2}Ca^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] 1/32*(((2*(sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a^6*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^8 + (sqrt(2)*A*a^6*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 17*sqrt(2)*C*a^6*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1))/a^8)*tan(1/2*d*x + 1/2*c)^2 - (3*sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1) + 83*sqrt(2)*C*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*tan(1
/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) + 5*(sqrt(2)*A - 15*sqr
t(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.203 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=130

$$\frac{(3A+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(7A-9C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((3*A + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(4*d*(a
+ a*Sec[c + d*x])^(5/2)) + ((7*A - 9*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c
+ d*x])^(3/2))
```

Rubi [A] time = 0.260597, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4079, 4000, 3795, 203}

$$\frac{(3A+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(7A-9C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A+C)\tan(c+dx)\sec(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]*Tan[c + d*x])/(4*d*(a
+ a*Sec[c + d*x])^(5/2)) + ((7*A - 9*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c
+ d*x])^(3/2))
```

Rule 4079

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A + C)*Cot[e +
 f*x]*Csc[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] - Dist[1/(a*b*
(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[-(b*C) - 2*A
*b*(m + 1) + a*(A*(m + 2) - C*(m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec(c+dx)\left(a(3A-C)-\frac{1}{2}a(A-7C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-9C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(3A+19}{}$$

$$= -\frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-9C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(3A+19}{}$$

$$= \frac{(3A+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{}{1}$$

Mathematica [A] time = 2.274, size = 120, normalized size = 0.92

$$\frac{\tan^3\left(\frac{1}{2}(c+dx)\right)((9C-7A)\cos(c+dx)-3A+13C)-\frac{(3A+19C)\sin(c+dx)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)}{\sqrt{2}}}{16a^2d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] (-(((3*A + 19*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Sqrt[-1 + Sec[c +
d*x]]*Sin[c + d*x])/Sqrt[2]) + (-3*A + 13*C + (-7*A + 9*C)*Cos[c + d*x])*Ta
n[(c + d*x)/2]^3)/(16*a^2*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.278, size = 602, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*A*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+19*C*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+6*A*cos(d*x+c)*sin(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+38*C*cos(d*x+c)*sin(d*x+c)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+3*A*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*sin(d*x+c)-14*A*cos(d*x+c)^3+19*C*(-2*cos(d*x+c)/(cos(d*x+c)+
```

```
1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))*sin(d*x+c)+18*C*cos(d*x+c)^3+8*A*cos(d*x+c)^2+8*C*cos(d*x+c)
^2+6*A*cos(d*x+c)-26*C*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.611615, size = 1238, normalized size = 9.52

$$\left[-\frac{\sqrt{2}\left((3\,A+19\,C)\cos\left(dx+c\right)^3+3\,(3\,A+19\,C)\cos\left(dx+c\right)^2+3\,(3\,A+19\,C)\cos\left(dx+c\right)+3\,A+19\,C\right)\sqrt{-a}\log\left(\frac{2\,\sqrt{2}\sqrt{}}{}\right.}{64\left(a^3d\cos\left(dx+c\right)^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A + 19*C)*cos(d*x + c)^3 + 3*(3*A + 19*C)*cos(d*x + c)^
2 + 3*(3*A + 19*C)*cos(d*x + c) + 3*A + 19*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) - 4*((7*A - 9*C)*cos(d*x + c)^2 + (3*A - 13*C)*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A + 19*
C)*cos(d*x + c)^3 + 3*(3*A + 19*C)*cos(d*x + c)^2 + 3*(3*A + 19*C)*cos(d*x
+ c) + 3*A + 19*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 9*C)*cos(d*x + c)^2
 + (3*A - 13*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 9.27247, size = 257, normalized size = 1.98

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\left(Aa^{5}+Ca^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(5\,Aa^{5}-11\,Ca^{5}\right)}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(3\,A+19\,C)\log\left(\left|-\sqrt{-a}\right.\right.}{\sqrt{-a}a^{2}\mathrm{s}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 + C*a^5)*tan(1/
2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(5*A*a^5 -
 11*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + sq
rt(2)*(3*A + 19*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.204 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{(43A-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-5C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A+C)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*A - 5*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rubi [A] time = 0.260848, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4053, 3922, 3920, 3774, 203, 3795}

$$-\frac{(43A-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-5C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A+C)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*A - 5*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4aA+\frac{1}{2}a(3A-5C)\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2A-\frac{1}{4}a^2(11A-5C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\sec(c+dx)}\,dx}{a^3} - \cdots$$

$$= -\frac{(A+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\cdots}{\sqrt{a\cdots}}\right)}{a^2d}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\tan(c+dx)}{4d(a+a\sec(c+dx))\cdots}$$

Mathematica [A] time = 3.54011, size = 153, normalized size = 0.94

$$\frac{\tan^3\left(\frac{1}{2}(c+dx)\right)\left((((15A-C)\cos(c+dx)+11A-5C) + \frac{(43A-5C)\sin(c+dx)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)}{\sqrt{2}} - 32A\sin(c+dx\cdots\right)}{16a^2d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-32*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Sqrt[-1 + Sec[c + d*x]]*Sin[c + d*x]
 + ((43*A - 5*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Sqrt[-1 + Sec[c +
d*x]]*Sin[c + d*x])/Sqrt[2] + (11*A - 5*C + (15*A - C)*Cos[c + d*x])*Tan[(c
 + d*x)/2]^3)/(16*a^2*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.242, size = 824, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(32*A*sin(d*x+c)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^2*2^(1/2)+64*A*sin(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)+43*A*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-5*C*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+32*A*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+86*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))-10*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))+43*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin
(d*x+c)-30*A*cos(d*x+c)^3-5*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin
(d*x+c)+2*C*cos(d*x+c)^3+8*A*cos(d*x+c)^2+8*C*cos(d*x+c)^2+22*A*cos(d*x+c)-
10*C*cos(d*x+c))/(cos(d*x+c)+1)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 16.5249, size = 1748, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*A - 5*C)*cos(d*x + c)^3 + 3*(43*A - 5*C)*cos(d*x + c)^2
 + 3*(43*A - 5*C)*cos(d*x + c) + 43*A - 5*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*
cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt
(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
- 4*((15*A - C)*cos(d*x + c)^2 + (11*A - 5*C)*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*A - 5*C)*cos
(d*x + c)^3 + 3*(43*A - 5*C)*cos(d*x + c)^2 + 3*(43*A - 5*C)*cos(d*x + c) +
```

```
43*A - 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x
+ c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((15*A - C)*cos(d*x + c
)^2 + (11*A - 5*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*
x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [B] time = 11.129, size = 470, normalized size = 2.9

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a}\left(\left(\frac{2\sqrt{2}(Aa^5 + Ca^5)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)} - \frac{\sqrt{2}(13Aa^5 - 3Ca^5)}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{\sqrt{2}(43A - 5C)\log\left(\left(\sqrt{\cdots}\right.\right.}{\sqrt{-\cdots}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 + C*a^5)*tan
(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(13*A*a
^5 - 3*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) +
 sqrt(2)*(43*A - 5*C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 64
*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - 64*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))/d
```

3.205 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(35A+3C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(115A+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(15A-C)\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-5*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((115*A + 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*(a + a*Sec[c
+ d*x])^(5/2)) - ((15*A - C)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2)) + ((35*A + 3*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.557619, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4085, 4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(35A+3C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(115A+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(15A-C)\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-5*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((115*A + 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*(a + a*Sec[c
+ d*x])^(5/2)) - ((15*A - C)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2)) + ((35*A + 3*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)\left(-a(5A+C)+\frac{1}{2}a(5A-3C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{1}{2}a^2(35\ldots\right.}{\sqrt{\ldots}}}{\ldots}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A+3C)\sin(c\ldots}{16a^2d\sqrt{a+a\sec(\ldots}}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A+3C)\sin(c\ldots}{16a^2d\sqrt{a+a\sec(\ldots}}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A+3C)\sin(c\ldots}{16a^2d\sqrt{a+a\sec(\ldots}}$$

$$= -\frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A\ldots}{4d(a\ldots}$$

Mathematica [A] time = 3.85664, size = 166, normalized size = 0.83

$$\frac{\tan^3\left(\frac{1}{2}(c+dx)\right)(-((55A+7C)\cos(c+dx)+8A\cos(2(c+dx))+43A+3C)) - \frac{(115A+3C)\sin(c+dx)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\ldots\right.}{\sqrt{2}}}{16a^2d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] (80*A*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Sqrt[-1 + Sec[c + d*x]]*Sin[c + d*x]
- ((115*A + 3*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[2]]*Sqrt[-1 + Sec[c +
d*x]]*Sin[c + d*x])/Sqrt[2] - (43*A + 3*C + (55*A + 7*C)*Cos[c + d*x] + 8*A
*Cos[2*(c + d*x)])*Tan[(c + d*x)/2]^3)/(16*a^2*d*(-1 + Cos[c + d*x])*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.366, size = 835, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(80*A*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*cos(d*x+c)^2*2^(1/2)+115*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*sin(d*x+c)*cos(d*x+c)^2+160*A*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)+3*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))*sin(d*x+c)*cos(d*x+c)^2+230*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+co
s(d*x+c)-1)/sin(d*x+c))+80*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)-32*A*cos(d*x+c)^4+6*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
cos(d*x+c)-1)/sin(d*x+c))+115*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*s
in(d*x+c)-78*A*cos(d*x+c)^3+3*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*s
in(d*x+c)-14*C*cos(d*x+c)^3+40*A*cos(d*x+c)^2+8*C*cos(d*x+c)^2+70*A*cos(d*x
+c)+6*C*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 16.6962, size = 1829, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((115*A + 3*C)*cos(d*x + c)^3 + 3*(115*A + 3*C)*cos(d*x + c
)^2 + 3*(115*A + 3*C)*cos(d*x + c) + 115*A + 3*C)*sqrt(-a)*log((2*sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) + 160*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A
)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c)
+ 1)) - 4*(16*A*cos(d*x + c)^3 + (55*A + 7*C)*cos(d*x + c)^2 + (35*A + 3*C)
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -
1/32*(sqrt(2)*((115*A + 3*C)*cos(d*x + c)^3 + 3*(115*A + 3*C)*cos(d*x + c)^
2 + 3*(115*A + 3*C)*cos(d*x + c) + 115*A + 3*C)*sqrt(a)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
160*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*
arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) - 2*(16*A*cos(d*x + c)^3 + (55*A + 7*C)*cos(d*x + c)^2 + (35*A + 3
*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.206 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{(63A+11C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(39A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A+43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A+7C)\,\mathrm{si}}{16a^2d\sqrt{a}}$$

```
[Out] ((39*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a
^(5/2)*d) - ((219*A + 43*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]*Sin[c +
d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A + 3*C)*Cos[c + d*x]*Sin[c +
 d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((63*A + 11*C)*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*A + 7*C)*Cos[c + d*x]*Sin[c + d*x
])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.802955, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(63A+11C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(39A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A+43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31A+7C)\,\mathrm{si}}{16a^2d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((39*A + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a
^(5/2)*d) - ((219*A + 43*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]*Sin[c +
d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A + 3*C)*Cos[c + d*x]*Sin[c +
 d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((63*A + 11*C)*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*A + 7*C)*Cos[c + d*x]*Sin[c + d*x
])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\cos^2(c+dx)\left(-2a(3A+C)+\frac{1}{2}a(7A-C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A+3C)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \int$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A+3C)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(31}{}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A+3C)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(63}{16a}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A+3C)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(63}{16a}$$

$$= -\frac{(A+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A+3C)\cos(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(63}{16a}$$

$$= \frac{(39A+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A+43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} -$$

Mathematica [C] time = 27.0268, size = 12059, normalized size = 46.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.429, size = 1416, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/64/d/a^3*(-1+cos(d*x+c))^2*(156*A*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+32*C*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+219*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)*sin(d*x+c)*cos(d*x+c)^3+468*A*cos(d*x+c)^2*sin(d*x+c)*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*2^(1/2)+43*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(3/2)*sin(d*x+c)*cos(d*x+c)^3+96*C*cos(d*x+c)^2*sin(d*x+c)*arctanh(1
```

```
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*2^(1/2)+657*A*ln(-(-(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(3/2)*sin(d*x+c)*cos(d*x+c)^2+468*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+129*C*ln(-(-(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(3/2)*sin(d*x+c)*cos(d*x+c)^2+96*C*cos(d*x+c)*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+657*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)+156*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*sin(d*x+c)-32*A*cos(d*x+c)^6+129*C*sin(d*x+c)*ln(-(-(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)+32*C*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(3/2)*sin(d*x+c)+219*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*sin(d*x+c)+112*A*cos(d*x+c)^5+43*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*sin(d*x+c)+300*A*cos(d*x+c)^4+60*C*cos(d*x+c)^4-128*A*cos(d*x+c)^3-16
*C*cos(d*x+c)^3-252*A*cos(d*x+c)^2-44*C*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos
(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2),
 x)
```

Fricas [A] time = 28.7326, size = 2013, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((219*A + 43*C)*cos(d*x + c)^3 + 3*(219*A + 43*C)*cos(d*x +
 c)^2 + 3*(219*A + 43*C)*cos(d*x + c) + 219*A + 43*C)*sqrt(-a)*log(-(2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 8*((39*A + 8*C)*cos(d*x + c)^3 + 3*(39*A + 8*C)*cos(d*x + c)
^2 + 3*(39*A + 8*C)*cos(d*x + c) + 39*A + 8*C)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
```

```
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(8*A*cos(d*x + c)^4
- 20*A*cos(d*x + c)^3 - 5*(19*A + 3*C)*cos(d*x + c)^2 - (63*A + 11*C)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(s
qrt(2)*((219*A + 43*C)*cos(d*x + c)^3 + 3*(219*A + 43*C)*cos(d*x + c)^2 + 3
*(219*A + 43*C)*cos(d*x + c) + 219*A + 43*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 8*(
(39*A + 8*C)*cos(d*x + c)^3 + 3*(39*A + 8*C)*cos(d*x + c)^2 + 3*(39*A + 8*C
)*cos(d*x + c) + 39*A + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*(8*A*cos(d*x + c)^4 - 20*
A*cos(d*x + c)^3 - 5*(19*A + 3*C)*cos(d*x + c)^2 - (63*A + 11*C)*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="giac")

[Out] Exception raised: NotImplementedError
```

3.207 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=205

$$\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(7A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5A+3}{}$$

```
[Out] (-2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(5*d) + (2*a*(7*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (
2*a*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*C*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(7*d)
```

Rubi [A] time = 0.222209, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4077, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a(7A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(5*d) + (2*a*(7*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (
2*a*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*C*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(7*d)
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{2aC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{7aA}{2}+\right.$$

$$= \frac{2aC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{7aA}{2}+\right.$$

$$= \frac{2a(7A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{2aC\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2a(5A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a(7A+5C)\sec^{\frac{3}{2}}}{2}$$

$$= \frac{2a(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+$$

$$= -\frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 4.63431, size = 409, normalized size = 2.

$$2a\csc(c)e^{-idx}\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)(5A+3C)e^{2idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometr}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*Cos[c + d*x]^2*Csc[c]*(A + C*Sec[c + d*x]^2)*(7*Sqrt[2]*(5*A + 3*C)*E^
((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))] - ((-1 + E^((2*I)*c))*(35*A*(1 + E^((2*I)*(c + d*x)))^2*(-1 +
```

```
3*E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + 3*E^((3*I)*(c + d*x))) + C*(-25
+ 21*E^(I*(c + d*x)) - 85*E^((2*I)*(c + d*x)) + 189*E^((3*I)*(c + d*x)) + 8
5*E^((4*I)*(c + d*x)) + 231*E^((5*I)*(c + d*x)) + 25*E^((6*I)*(c + d*x)) +
63*E^((7*I)*(c + d*x))))*Sqrt[Sec[c + d*x]])/(E^(I*(c - d*x))*(1 + E^((2*I)
*(c + d*x)))^3) + 10*(7*A + 5*C)*E^(I*d*x)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]]*Sin[c]))/(105*d*E^(I*d*x)*(A + 2*C + A*Cos[
2*(c + d*x)]))
```

Maple [B] time = 7.056, size = 838, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\sec\left(dx+c\right)^{4}+Ca\sec\left(dx+c\right)^{3}+Aa\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^4 + C*a*sec(d*x + c)^3 + A*a*sec(d*x + c)^2 + A*
a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x
)
```

3.208 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=172

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(5A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2a(5A+3C}{}$$

```
[Out] (-2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*a*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*C*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.20153, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4077, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a(5A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+3C)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*a*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(5*d) + (2*a*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*C*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{2aC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\sec(c+dx)}\left(\frac{5aA}{2}+\right.$$

$$= \frac{2aC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\sec(c+dx)}\left(\frac{5aA}{2}+\right.$$

$$= \frac{2a(5A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2aC\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$= \frac{2a(5A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2aC\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$$= -\frac{2a(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [C] time = 2.30438, size = 286, normalized size = 1.66

$$\frac{2ae^{-ic}\left(-1+e^{2ic}\right)\csc(c)\left(A+C\sec^2(c+dx)\right)\left((5A+3C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right.\right.}{15d\,(1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(-1 + E^((2*I)*c))*Csc[c]*(5*C - 15*A*E^(I*(c + d*x)) - 3*C*E^(I*(c +
d*x)) - 30*A*E^((3*I)*(c + d*x)) - 24*C*E^((3*I)*(c + d*x)) - 5*C*E^((4*I)*
(c + d*x)) - 15*A*E^((5*I)*(c + d*x)) - 9*C*E^((5*I)*(c + d*x)) - (5*I)*(3*
A + C)*(1 + E^((2*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2] + (5*A + 3*C)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(A + C*Sec[c + d*x]^2))/(15
*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2*(A + 2*C + A*Cos[2*(c + d*x)])*Sec[c
 + d*x]^(3/2))
```

Maple [B] time = 5.874, size = 729, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*(
-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C/(8*sin(1/2*d*x+1/2*c)^6-12*si
n(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^{2}+A\right)\left(a \sec \left(dx+c\right)+a\right)\sqrt{\sec \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca \sec \left(dx+c\right)^{3}+Ca \sec \left(dx+c\right)^{2}+Aa \sec \left(dx+c\right)+Aa\right)\sqrt{\sec \left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x
)
```

3.209 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

```
[Out] (2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*C*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.183111, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4077, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aC}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*C*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \frac{2aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{1}{2}a(3A+C)\sec(c+dx)+\frac{3}{2}}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{3}{2}aC\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{1}{3}(a($$

$$= \frac{2aC\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+(a(A-$$

$$= \frac{2a(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2aC\sqrt{\sec(c+}}{}$$

$$= \frac{2a(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(3A+C)\sqrt{c}}{}$$

Mathematica [C] time = 1.31688, size = 168, normalized size = 1.24

$$\frac{ae^{-idx}\sec^{\frac{3}{2}}(c+dx)(\sin(dx)-i\cos(dx))\left((A-C)\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+2i(3A+C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*((-I)*Cos[d*x] + Sin[d*x])*(-3*A + 3*C - 3*A*Cos[2*(c
 + d*x)] + 3*C*Cos[2*(c + d*x)] + (2*I)*(3*A + C)*Cos[c + d*x]^(3/2)*Ellipt
icF[(c + d*x)/2, 2] + (A - C)*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (2*I)*C*Sin[c + d*x] + (3*I)*C*
Sin[2*(c + d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 4.953, size = 437, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))+2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*
c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^3+Ca\sec\left(dx+c\right)^2+Aa\sec\left(dx+c\right)+Aa}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x
)
```

3.210 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*C*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.184622, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4075, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a*C*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3aA}{2}-\frac{1}{2}a(A+3C)\sec(c+dx)-\frac{3}{2}aC\sec^2(c+\ldots}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3aA}{2}-\frac{3}{2}aC\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}(a(A+3C))\int \ldots$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2aC\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + (a(A-C))\int \frac{\ldots}{\sqrt{\sec(\ldots}}$$

$$= \frac{2a(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2aA\sin(c+\ldots}{3d\sqrt{\sec(c+\ldots}}$$

$$= \frac{2a(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a(A+3C)\sqrt{c\ldots}}{\ldots}$$

Mathematica [C] time = 1.35405, size = 169, normalized size = 1.25

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2i(A-C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+2(A\ldots\right.}{3d\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((6*I)*A*Cos[c + d*x] - (6*I)
*C*Cos[c + d*x] + 2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
- (2*I)*(A - C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 6*C*Sin[c + d*x] + A*Sin[2*(c +
 d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 2.349, size = 458, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*a*(4*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+3*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+Ca\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{A}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int C\sqrt{\sec\left(c+dx\right)}\,dx+\int C\sec^{\frac{3}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) +
 Integral(C*sqrt(sec(c + d*x)), x) + Integral(C*sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x
)
```

3.211 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{5d}$$

```
[Out] (2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.187561, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4075, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
 (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5aA}{2}-\frac{1}{2}a(3A+5C)\sec(c+dx)-\frac{5}{2}aC\sec^2(c}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5aA}{2}-\frac{5}{2}aC\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(a(3A+5C))$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(a(A+3C))\int\sqrt{\sec(c+dx)}\,dx$$

$$= \frac{2a(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2aA\sin(c+}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(A+3C)}{}$$

Mathematica [C] time = 1.7088, size = 169, normalized size = 1.2

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2i(3A+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+10\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(10*(A + 3*C)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] - (2*I)*(3*A + 5*C)*E^(I*(c + d*x))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
 + Cos[c + d*x]*((6*I)*(3*A + 5*C) + 10*A*Sin[c + d*x] + 3*A*Sin[2*(c + d*x
)])))/(15*d*E^(I*d*x))
```

Maple [A] time = 2.079, size = 345, normalized size = 2.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+44\,A\left(\sin(1/2\,dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+44*A*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*A*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^3+Ca\sec\left(dx+c\right)^2+Aa\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int \frac{A}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{C}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int C\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(A/sec(c + d*x)**(3/2), x)
+ Integral(C/sqrt(sec(c + d*x)), x) + Integral(C*sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+A\right)\left(a \sec \left(dx+c\right)+a\right)}{\sec \left(dx+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x
)
```

3.212 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(3A+5C)\sqrt{\cos(c+}}{}$$

```
[Out] (2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*C)*Sin[c
+ d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.204178, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4075, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*C)*Sin[c
+ d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int\frac{-\frac{7aA}{2}-\frac{1}{2}a(5A+7C)\sec(c+dx)-\frac{7}{2}aC\sec^2(c}{\sec^{\frac{5}{2}}(c+dx)}\\
&= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int\frac{-\frac{7aA}{2}-\frac{7}{2}aC\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{7}(a(5A+7C))\\
&= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{1}{5}(a(3\\
&= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{1}{5}\Big(a(\\
&= \frac{2a(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5A+7C}{}\end{aligned}$$

Mathematica [C] time = 2.22053, size = 188, normalized size = 1.08

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-28i(3A+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(5*A + 7*C)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] - (28*I)*(3*A + 5*C)*E^(I*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))] + Cos[c + d*x]*((84*I)*(3*A + 5*C) + 5*(23*A + 28*C)*Sin[c + d*x] + 42*
A*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Maple [A] time = 2.322, size = 378, normalized size = 2.2

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-528\,A\cos\left(1/2\,d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^6+(448*A+140*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-122*A-70*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+Ca\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x
)
```

3.213 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(15*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
)) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(7*A + 9*C)*Sin[c
 + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*C)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.231572, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4075, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*a*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(15*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
)) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(7*A + 9*C)*Sin[c
 + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*C)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]])
```


Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}-\frac{2}{9}\int \frac{-\frac{9aA}{2}-\frac{1}{2}a(7A+9C)\sec(c+dx)-\frac{9}{2}aC\sec^2(c\\
&= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}-\frac{2}{9}\int \frac{-\frac{9aA}{2}-\frac{9}{2}aC\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\frac{1}{9}(a(7A+9C))\\
&= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{7}(a(5\\
&= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A\\
&= \frac{2a(7A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2aA\sin(c+}{9d\sec^{\frac{7}{2}}(c+}\\
&= \frac{2a(7A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2a(5A+7C}{}\end{aligned}$$

Mathematica [C] time = 2.83771, size = 204, normalized size = 1.

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(7A+9C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(120*(5*A + 7*C)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(7*A + 9*C)*E^(I*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))] + Cos[c + d*x]*((1176*I)*A + (1512*I)*C + 30*(23*A + 28*C)*Sin[c + d*x
] + 14*(19*A + 18*C)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 35*A*Sin[4*
(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 2.351, size = 406, normalized size = 2.

$$-\frac{2a}{315d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-1120\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}+2960\,A\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2960*A*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+(-3152*A-504*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1792*A
+924*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-408*A-336*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+Ca\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(9/2), x
)
```

3.214 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=270

$$\frac{4a^2(7A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(21A+19C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(7$$

```
[Out] (-16*a^2*(3*A + 2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (16*a^2*(3*A + 2*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d) + (4*a^2*(7*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(21*d) + (2*a^2*(21*A + 19*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d) +
(2*C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d) + (8*C*S
ec[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(63*d)
```

Rubi [A] time = 0.438974, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4089, 4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a^2(21A+19C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(7A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{16a^2(3A+2C)\sin(c+dx)\sqrt{\sec}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-16*a^2*(3*A + 2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (16*a^2*(3*A + 2*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d) + (4*a^2*(7*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(21*d) + (2*a^2*(21*A + 19*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d) +
(2*C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d) + (8*C*S
ec[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(63*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{9d}+\frac{2\int \sec$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{9d}+\frac{8C\sec}{}$$

$$= \frac{2a^2(21A+19C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2C\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{2a^2(21A+19C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2C\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{16a^2(3A+2C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{4a^2(7A+5C)}{}$$

$$= \frac{16a^2(3A+2C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{4a^2(7A+5C)}{}$$

$$= -\frac{16a^2(3A+2C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{15d}$$

Mathematica [C] time = 6.80225, size = 821, normalized size = 3.04

$$\frac{4\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4(c+dx)\csc(c)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\right.}{15d(\cos(2c+2dx)A+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x
]^2))/(15*d*E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])) + (8*Sqrt[2]*C*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos
[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 +
E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/
2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/(45*d*E^(I*d*
x)*(A + 2*C + A*Cos[2*c + 2*d*x])) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]
^2))/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)) + (10*C*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
d*x])^2*(A + C*Sec[c + d*x]^2))/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c
+ d*x]^(7/2)) + (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c +
 d*x]^2)*((8*(3*A + 2*C)*Cos[d*x]*Csc[c])/(15*d) + (C*Sec[c]*Sec[c + d*x]^4
*Sin[d*x])/(9*d) + (Sec[c]*Sec[c + d*x]^3*(7*C*Sin[c] + 18*C*Sin[d*x]))/(63
*d) + (Sec[c]*Sec[c + d*x]^2*(90*C*Sin[c] + 63*A*Sin[d*x] + 112*C*Sin[d*x])
)/(315*d) + (Sec[c]*Sec[c + d*x]*(63*A*Sin[c] + 112*C*Sin[c] + 210*A*Sin[d*
x] + 150*C*Sin[d*x]))/(315*d) + (2*(7*A + 5*C)*Tan[c])/(21*d)))/((A + 2*C +
 A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 9.129, size = 1168, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*(-1/56*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/
144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+
```

```
4*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-8/5*(1/4*A+1/4*C)/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1
/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\sec(dx+c)^5 + 2\,Ca^2\sec(dx+c)^4 + (A+C)a^2\sec(dx+c)^3 + 2\,Aa^2\sec(dx+c)^2 + Aa^2\sec(dx+c)\right)\sqrt{\text{se}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^5 + 2*C*a^2*sec(d*x + c)^4 + (A + C)*a^2*sec(d
*x + c)^3 + 2*A*a^2*sec(d*x + c)^2 + A*a^2*sec(d*x + c))*sqrt(sec(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2),
 x)
```

3.215 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=237

$$\frac{8a^2(7A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(35A+33C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(5A\cdots}{}$$

```
[Out] (-4*a^2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (8*a^2*(7*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(5*A + 3*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (2*a^2*(35*A + 33*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d
) + (8*C*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.413934, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4089, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a^2(35A+33C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(5A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8a^2(7A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c\cdots}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-4*a^2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (8*a^2*(7*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(5*A + 3*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (2*a^2*(35*A + 33*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d
) + (8*C*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2\int}{}$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{8C}{}$$

$$= \frac{2a^2(35A+33C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2C\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{2a^2(35A+33C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2C\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{4a^2(5A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2(35A+}{}$$

$$= \frac{8a^2(7A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{21d}$$

$$= -\frac{4a^2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{5d}$$

Mathematica [C] time = 5.88483, size = 436, normalized size = 1.84

$$a^2\csc(c)e^{-idx}\cos^4(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(A+C\sec^2(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)(5A+3C)e^{2idx}\sqrt{\frac{e^{i(}}{1+e^{2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (a^2*Cos[c + d*x]^4*Csc[c]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + C*S
ec[c + d*x]^2)*(7*Sqrt[2]*(5*A + 3*C)*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt
[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - ((-1 + E^((2*I)*c))
*(35*A*(1 + E^((2*I)*(c + d*x)))^2*(-1 + 6*E^(I*(c + d*x)) + E^((2*I)*(c +
d*x)) + 6*E^((3*I)*(c + d*x))) + 6*C*(-10 + 7*E^(I*(c + d*x)) - 20*E^((2*I)
*(c + d*x)) + 63*E^((3*I)*(c + d*x)) + 20*E^((4*I)*(c + d*x)) + 77*E^((5*I)
*(c + d*x)) + 10*E^((6*I)*(c + d*x)) + 21*E^((7*I)*(c + d*x))))*Sqrt[Sec[c
+ d*x]])/(2*E^(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3) + 20*(7*A + 3*C)*E
^(I*d*x)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*Si
n[c]))/(105*d*E^(I*d*x)*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 7.912, size = 919, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+
8*(1/4*A+1/4*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*C/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+4*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1
```

```
/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\sec(dx+c)^4 + 2\,Ca^2\sec(dx+c)^3 + (A+C)a^2\sec(dx+c)^2 + 2\,Aa^2\sec(dx+c) + Aa^2\right)\sqrt{\sec(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)),
 x)
```

3.216 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=196

$$\frac{4a^2(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(15A+17C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{8C\sin(c+$$

```
[Out] (-16*a^2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (4*a^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a^2*(15*A + 17*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5
*d) + (8*C*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.389384, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4089, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(15A+17C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{8C\sin(c+dx)\sqrt{se}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (-16*a^2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (4*a^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a^2*(15*A + 17*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5
*d) + (8*C*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
```

```
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2\int \frac{(a+a\sec(c+dx))}{}}{}$$

$$= \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{8C\sqrt{\sec(c+dx)}}{}$$

$$= \frac{2a^2(15A+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2C\sqrt{\sec(c+dx)}(a+a}{}$$

$$= \frac{2a^2(15A+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2C\sqrt{\sec(c+dx)}(a+a}{}$$

$$= \frac{2a^2(15A+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2C\sqrt{\sec(c+dx)}(a+a}{}$$

$$= -\frac{16a^2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2(3A+C)}{}$$

Mathematica [C] time = 5.88315, size = 312, normalized size = 1.59

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(A+C\sec^2(c+dx)\right)\left(\frac{-3\csc(c)\cos(dx)(5A\cos(2c)-5A-16C)+30A\cos(c)\sin(dx)+2C\tan(c+dx)(}{2d\sec^{\frac{7}{2}}(c+dx)}\right.}{15(A}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*(((-2*I
)*Sqrt[2]*Cos[c + d*x]^4*(12*C*Sqrt[1 + E^((2*I)*(c + d*x))] + 12*C*(-1 + E
^((2*I)*c))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*(3*
A + C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))]))/(d*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (-3*(-5*A - 16*C + 5*A*
Cos[2*c])*Cos[d*x]*Csc[c] + 30*A*Cos[c]*Sin[d*x] + 2*C*(10 + 3*Sec[c + d*x]
)*Tan[c + d*x])/(2*d*Sec[c + d*x]^(7/2))))/(15*(A + 2*C + A*Cos[2*(c + d*x)
]))
```

Maple [B] time = 6.369, size = 756, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 4/15*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^4+48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-96*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+60*A*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2-48*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2+116*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-15*A*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+12*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-37*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4+2\,Ca^2\sec(dx+c)^3+(A+C)a^2\sec(dx+c)^2+2\,Aa^2\sec(dx+c)+Aa^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)),
 x)
```

3.217 $\int \frac{(a+a\sec(c+dx))^2(A+C\sec^2(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=198

$$\frac{8a^2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a^2(A-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}-\frac{2(A-C)\sin(c$$

```
[Out] (4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (8*a^2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) - (2*a^2*(A - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) - (
2*(A - C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.402225, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4087, 4018, 3997, 3787, 3771, 2639, 2641}

$$-\frac{2a^2(A-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}-\frac{2(A-C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2\sec(c+dx)+a^2\right)}{3d}+\frac{8a^2(A+C)\sqrt{\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```



```
[Out] (4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (8*a^2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) - (2*a^2*(A - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) - (
2*(A - C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec(c+dx))^2\left(2aA-\frac{3}{2}a(A-C)\right.}{\sqrt{\sec(c+dx)}}}{3a}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(A-C)\sqrt{\sec(c+dx)}\left(a^2+a\right.}{3d}$$

$$= -\frac{2a^2(A-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+a\sec(c+dx))^2\,\mathrm{si}}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2a^2(A-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+a\sec(c+dx))^2\,\mathrm{si}}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2a^2(A-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a+a\sec(c+dx))^2\,\mathrm{si}}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{4a^2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2(A+}{}$$

Mathematica [C] time = 2.00566, size = 191, normalized size = 0.96

$$\frac{a^2e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-4i(A-C)\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+16(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^2*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((12*I)*A - (12*I)*C + (12*
I)*A*Cos[2*(c + d*x)] - (12*I)*C*Cos[2*(c + d*x)] + 16*(A + C)*Cos[c + d*x]
^(3/2)*EllipticF[(c + d*x)/2, 2] - (4*I)*(A - C)*(1 + E^((2*I)*(c + d*x)))^
(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + A*Sin[c + d*
x] + 4*C*Sin[c + d*x] + 12*C*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(6*d*E
^(I*d*x))
```

Maple [B] time = 5.308, size = 651, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-4*A*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2
+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*C*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^4-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+A*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+7*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^{2} + A\right)\left(a \sec\left(dx+c\right) + a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4+2\,Ca^2\sec(dx+c)^3+(A+C)a^2\sec(dx+c)^2+2\,Aa^2\sec(dx+c)+Aa^2}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2),
 x)
```

3.218 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{4a^2(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a^2(7A-15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{8A\sin(c+$$

```
[Out] (16*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) - (2*a^2*(7*A - 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(15*d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)
) + (8*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.39229, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4087, 4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{2a^2(7A-15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{8A\sin(c+dx)\left(a^2\right.}{15d\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),x]
```


```
[Out] (16*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) - (2*a^2*(7*A - 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(15*d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)
) + (8*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^2\left(2aA-\frac{1}{2}a(A-5C\right.}{\sec^{\frac{3}{2}}(c+dx)}}{5a}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\sec(c+dx)\right)\sin(}{15d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2a^2(7A-15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+a}$$

$$= -\frac{2a^2(7A-15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+a}$$

$$= -\frac{2a^2(7A-15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+a}$$

$$= \frac{16a^2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2(A+3C)\sqrt{}}{}$$

Mathematica [C] time = 4.64055, size = 318, normalized size = 1.62

$$a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(A+C\sec^2(c+dx)\right)\left(-\frac{\csc(c)((99A+60C)\cos(2c+dx)-2A\sin(c)(20\sin(2(c+dx))+3\sin(3(c+dx)))}{8d\sec^{\frac{7}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*(((2*I)
*Sqrt[2]*Cos[c + d*x]^4*(12*A*Sqrt[1 + E^((2*I)*(c + d*x))] + 12*A*(-1 + E^
((2*I)*c))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*(A +
 3*C)*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -
E^((2*I)*(c + d*x))]))/(d*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) - (Csc[c]*((93*A - 60*C)*C
os[d*x] + (99*A + 60*C)*Cos[2*c + d*x] - 2*A*Sin[c]*(20*Sin[2*(c + d*x)] +
3*Sin[3*(c + d*x)])))/(8*d*Sec[c + d*x]^(7/2))))/(15*(A + 2*C + A*Cos[2*(c
+ d*x)]))
```

Maple [A] time = 2.163, size = 440, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*a^2*(-12*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+15*C)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2 \sec(dx+c)^4 + 2\,Ca^2 \sec(dx+c)^3 + (A+C)a^2 \sec(dx+c)^2 + 2\,Aa^2 \sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{2A}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{C}{\sqrt{\sec(c+dx)}}\,dx + \int 2C\sqrt{\sec(c+dx)}\,dx +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(2*A/sec(c + d*x)**(3/2)
, x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(C/sqrt(sec(c + d*x)), x
) + Integral(2*C*sqrt(sec(c + d*x)), x) + Integral(C*sec(c + d*x)**(3/2), x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c) + a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2),
 x)
```

3.219 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{8a^2(3A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(33A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^2(3A+5C)\sqrt{\cos(c}{}$$

```
[Out] (4*a^2*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (8*a^2*(3*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(33*A + 35*C)*Sin[c + d*x])/(105*
d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[
c + d*x]^(5/2)) + (8*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(35*d*Sec[c +
 d*x]^(3/2))
```

Rubi [A] time = 0.421424, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4087, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a^2(33A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{8a^2(3A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+5C)\sqrt{\cos(c+dx)}\sqrt{}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a^2*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (8*a^2*(3*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(33*A + 35*C)*Sin[c + d*x])/(105*
d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[
c + d*x]^(5/2)) + (8*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(35*d*Sec[c +
 d*x]^(3/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^2\left(2aA+\frac{1}{2}a(A+7C\right.}{\sec^{\frac{5}{2}}(c+dx)}}{7a}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\sec(c+dx)\right)\sin(c}{35d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(33A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(33A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(33A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^2(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2(3A}{}$$

Mathematica [C] time = 2.37989, size = 189, normalized size = 0.93

$$a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(3A+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(80*(3*A + 7*C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(3*A + 5*C)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((504*I)*A + (840*I)*C + 5*(51*A + 28*C)*Sin[c + d*x]
+ 84*A*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

Maple [A] time = 1.952, size = 380, normalized size = 1.9

$$-\frac{4a^2}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8-348\,A\cos\left(1/2\,dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-348*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^6+(378*A+70*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-117*A-35*C
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+70*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec\left(dx+c\right)^4+2\,Ca^2\sec\left(dx+c\right)^3+\left(A+C\right)a^2\sec\left(dx+c\right)^2+2\,Aa^2\sec\left(dx+c\right)+Aa^2}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2),
 x)
```

3.220 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{4a^2(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(19A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+7C)\sin(c+}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (16*a^2*(2*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(15*d) + (4*a^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(19*A + 21*C)*Sin[c + d*x])/(10
5*d*Sec[c + d*x]^(3/2)) + (4*a^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c
 + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/
2)) + (8*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.436035, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4087, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a^2(19A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(5A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (16*a^2*(2*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(15*d) + (4*a^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(19*A + 21*C)*Sin[c + d*x])/(10
5*d*Sec[c + d*x]^(3/2)) + (4*a^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c
 + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/
2)) + (8*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^2\left(2aA+\frac{3}{2}a(A+3C)\sec\right.}{\sec^{\frac{7}{2}}(c+dx)}}{9a}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\sec(c+dx)\right)\sin(c+}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(19A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(19A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8}{}$$

$$= \frac{2a^2(19A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\,s}{9}$$

$$= \frac{16a^2(2A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2a^2(19A}{105}$$

$$= \frac{16a^2(2A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^2(5A+}{}$$

Mathematica [C] time = 2.91122, size = 206, normalized size = 0.87

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-448i(2A+3C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(5*A + 7*C)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] - (448*I)*(2*A + 3*C)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((2688*I)*A + (4032*I)*C + 60*(23*A + 28*C)*Sin[c +
d*x] + 14*(37*A + 18*C)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 35*A*Si
n[4*(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 1.928, size = 408, normalized size = 1.7

$$-\frac{4a^2}{315d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-560A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^{10}+1840A\cos(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+1840*A*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-2368*A-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1568*
A+672*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-273*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4 + 2\,Ca^2\sec(dx+c)^3 + (A+C)a^2\sec(dx+c)^2 + 2\,Aa^2\sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c) + a)^2}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2),
 x)
```

3.221 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=270

$$\frac{8a^2(25A+33C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^2(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(89A+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] (4*a^2*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (8*a^2*(25*A + 33*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(89*A + 99*C)*Sin[c + d*x])/(
693*d*Sec[c + d*x]^(5/2)) + (4*a^2*(7*A + 9*C)*Sin[c + d*x])/(45*d*Sec[c +
d*x]^(3/2)) + (8*a^2*(25*A + 33*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
 + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8
*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.47287, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4087, 4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^2(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(89A+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2(25A+33C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{8a^2(25A+33C)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(11/2),x]
```


```
[Out] (4*a^2*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (8*a^2*(25*A + 33*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(89*A + 99*C)*Sin[c + d*x])/(
693*d*Sec[c + d*x]^(5/2)) + (4*a^2*(7*A + 9*C)*Sin[c + d*x])/(45*d*Sec[c +
d*x]^(3/2)) + (8*a^2*(25*A + 33*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
 + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8
*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^2\left(2aA+\frac{1}{2}a(5A+1\right.}{\sec^{\frac{9}{2}}(c+dx)}}{11a}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8A\left(a^2+a^2\sec(c+dx)\right)\sin(}{99d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(89A+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(89A+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(89A+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2(25}{23}$$

$$= \frac{2a^2(89A+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2(25}{23}$$

$$= \frac{4a^2(7A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{8a^2(25}{}$$

Mathematica [C] time = 3.22288, size = 228, normalized size = 0.84

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2464i(7A+9C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(960*(25*A + 33*C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(7*A + 9*C)*E^(I*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))] + Cos[c + d*x]*((51744*I)*A + (66528*I)*C + 30*(941*A + 1122*C)*
Sin[c + d*x] + 616*(19*A + 18*C)*Sin[2*(c + d*x)] + 4545*A*Sin[3*(c + d*x)]
 + 1980*C*Sin[3*(c + d*x)] + 1540*A*Sin[4*(c + d*x)] + 315*A*Sin[5*(c + d*x
)])))/(27720*d*E^(I*d*x))
```

Maple [A] time = 2.097, size = 436, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-37520*A*cos(1/2*d*x+1/2*c)*sin(1
```

```
/2*d*x+1/2*c)^10+(57040*A+3960*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-46192*A-11484*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(22022*A+12474*C)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4563*A-3861*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+750*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+990*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4 + 2\,Ca^2\sec(dx+c)^3 + (A+C)a^2\sec(dx+c)^2 + 2\,Aa^2\sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)\left(a\sec(dx+c) + a\right)^2}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(11/2)
, x)
```

3.222 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=319

$$\frac{4a^3(143A+105C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{8a^3(44A+35C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{385d}+\frac{4a^3(}{}$$

```
[Out] (-4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(143*A + 105*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(7*A + 5*C)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(5*d) + (4*a^3*(143*A + 105*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(231*d) + (8*a^3*(44*A + 35*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(385
*d) + (2*C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(11*d) +
 (4*C*Sec[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(33*a*d)
+ (2*(33*A + 35*C)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(231*d)
```

Rubi [A] time = 0.618146, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4089, 4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{8a^3(44A+35C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{385d}+\frac{4a^3(143A+105C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{231d}+\frac{2(33A+35C)\sin(c+dx)\sec}{23}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(143*A + 105*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(7*A + 5*C)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(5*d) + (4*a^3*(143*A + 105*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(231*d) + (8*a^3*(44*A + 35*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(385
*d) + (2*C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(11*d) +
 (4*C*Sec[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(33*a*d)
+ (2*(33*A + 35*C)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(231*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{11d} + \frac{2\int \sec}{}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{11d} + \frac{4C\sec}{}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{11d} + \frac{4C\sec}{}$$

$$= \frac{8a^3(44A+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{385d} + \frac{2C\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{8a^3(44A+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{385d} + \frac{2C\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{4a^3(7A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^3(143A+105}{}$$

$$= \frac{4a^3(7A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^3(143A+105}{}$$

$$= -\frac{4a^3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}$$

Mathematica [C] time = 7.00122, size = 863, normalized size = 2.71

$$\frac{7Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5(c+dx)\csc(c)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e}\right.}{15\sqrt{2}d(\cos(2c+2dx)A+A+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (7*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(1
5*Sqrt[2]*d*E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])) + (C*Sqrt[E^(I*(c + d
*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^
5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c
))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/
2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(3*Sqrt[2]*d*E^(I*d*x)*
(A + 2*C + A*Cos[2*c + 2*d*x])) + (13*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2
))/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2)) + (5*C*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*
x])^3*(A + C*Sec[c + d*x]^2))/(11*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c +
d*x]^(9/2)) + (Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d
*x]^2)*(((7*A + 5*C)*Cos[d*x]*Csc[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^5*Sin[
d*x])/(22*d) + (Sec[c]*Sec[c + d*x]^4*(3*C*Sin[c] + 11*C*Sin[d*x]))/(66*d)
+ (Sec[c]*Sec[c + d*x]^3*(77*C*Sin[c] + 33*A*Sin[d*x] + 126*C*Sin[d*x]))/(4
62*d) + (Sec[c]*Sec[c + d*x]^2*(165*A*Sin[c] + 630*C*Sin[c] + 693*A*Sin[d*x
] + 770*C*Sin[d*x]))/(2310*d) + (Sec[c]*Sec[c + d*x]*(693*A*Sin[c] + 770*C*
Sin[c] + 1430*A*Sin[d*x] + 1050*C*Sin[d*x]))/(2310*d) + ((143*A + 105*C)*Ta
```

```
n[c])/(231*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 10.123, size = 1409, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(1/8*A+3
/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))+6*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*C*(-1/144*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7
/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-16/5*(3/8*A+1/8*C
)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*
sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2
*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*C*(-1/352*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^6-9/61
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^4-15/154*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+15/77*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3 \sec(dx+c)^6 + 3\,Ca^3 \sec(dx+c)^5 + (A+3\,C)a^3 \sec(dx+c)^4 + (3\,A+C)a^3 \sec(dx+c)^3 + 3\,Aa^3 \sec(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^6 + 3*C*a^3*sec(d*x + c)^5 + (A + 3*C)*a^3*sec
(d*x + c)^4 + (3*A + C)*a^3*sec(d*x + c)^3 + 3*A*a^3*sec(d*x + c)^2 + A*a^3
*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c) + a)^3 \sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2),
 x)
```

3.223 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=286

$$\frac{4a^3(21A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{8a^3(21A+16C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(6}{}$$

```
[Out] (-4*a^3*(27*A + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(21*A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(27*A + 17*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (8*a^3*(21*A + 16*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*Sin[c + d*
x])/(9*d) + (4*C*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x]
)/(21*a*d) + (2*(63*A + 73*C)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*S
in[c + d*x])/(315*d)
```

Rubi [A] time = 0.589692, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4089, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{8a^3(21A+16C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(63A+73C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{315d}+\frac{4a^3(27A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-4*a^3*(27*A + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(21*A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(27*A + 17*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (8*a^3*(21*A + 16*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*Sin[c + d*
x])/(9*d) + (4*C*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x]
)/(21*a*d) + (2*(63*A + 73*C)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*S
in[c + d*x])/(315*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{9d} + \frac{2\int$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{9d} + \frac{4C$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{9d} + \frac{4C$$

$$= \frac{8a^3(21A+16C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2C\sec^{\frac{3}{2}}(c$$

$$= \frac{8a^3(21A+16C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2C\sec^{\frac{3}{2}}(c$$

$$= \frac{4a^3(27A+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{8a^3(21A$$

$$= \frac{4a^3(21A+11C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c$$

$$= -\frac{4a^3(27A+17C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec($$

Mathematica [C] time = 6.84525, size = 818, normalized size = 2.86

$$\frac{3Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5(c+dx)\csc(c)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1}\right.}{5\sqrt{2}d(\cos(2c+2dx)A+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (3*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(5
*Sqrt[2]*d*E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])) + (17*C*Sqrt[E^(I*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x
]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)
*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x
)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(45*Sqrt[2]*d*E^(I*d*
x)*(A + 2*C + A*Cos[2*c + 2*d*x])) + (A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2
))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2)) + (11*C*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x]
)^3*(A + C*Sec[c + d*x]^2))/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*
x]^(9/2)) + (Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x
]^2)*(((27*A + 17*C)*Cos[d*x]*Csc[c])/(15*d) + (C*Sec[c]*Sec[c + d*x]^4*Sin
[d*x])/(18*d) + (Sec[c]*Sec[c + d*x]^3*(7*C*Sin[c] + 27*C*Sin[d*x]))/(126*d
) + (Sec[c]*Sec[c + d*x]^2*(135*C*Sin[c] + 63*A*Sin[d*x] + 238*C*Sin[d*x]))
/(630*d) + (Sec[c]*Sec[c + d*x]*(63*A*Sin[c] + 238*C*Sin[c] + 315*A*Sin[d*x
] + 330*C*Sin[d*x]))/(630*d) + ((21*A + 22*C)*Tan[c])/(42*d)))/((A + 2*C +
A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 9.044, size = 1247, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+
16*(3/8*A+1/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/144*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)
^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-16/5*(1/8
*A+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\sec(dx+c)^5+3\,Ca^3\sec(dx+c)^4+(A+3\,C)a^3\sec(dx+c)^3+(3\,A+C)a^3\sec(dx+c)^2+3\,Aa^3\sec\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)*
sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)),
 x)
```

3.224 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{4a^3(35A+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{8a^3(70A+53C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(5A\cdots}{}$$

```
[Out] (-4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(35*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(70*A + 53*C)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*S
in[c + d*x])/(7*d) + (12*C*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])^2*Si
n[c + d*x])/(35*a*d) + (2*(5*A + 7*C)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c +
 d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.562782, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4089, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{8a^3(70A+53C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(5A+7C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(35A+13C\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (-4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(35*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(70*A + 53*C)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*S
in[c + d*x])/(7*d) + (12*C*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])^2*Si
n[c + d*x])/(35*a*d) + (2*(5*A + 7*C)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c +
 d*x])*Sin[c + d*x])/(15*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \frac{2\int \frac{(a+a\sec(c+dx))^3}{\cdots}}{\cdots}$$

$$= \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \frac{12C\sqrt{\sec(c+dx}\cdots}{\cdots}$$

$$= \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \frac{12C\sqrt{\sec(c+dx}\cdots}{\cdots}$$

$$= \frac{8a^3(70A+53C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\sec(c+dx)}(a+a\cdots}{\cdots}$$

$$= \frac{8a^3(70A+53C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\sec(c+dx)}(a+a\cdots}{\cdots}$$

$$= \frac{8a^3(70A+53C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} + \frac{2C\sqrt{\sec(c+dx)}(a+a\cdots}{\cdots}$$

$$= -\frac{4a^3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(3\cdots}{\cdots}$$

Mathematica [C] time = 3.49733, size = 280, normalized size = 1.11

$$a^3e^{-idx}\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(14i(5A+7C)e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{7/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^3*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-630*I)*A - (882*I)*C - (
840*I)*A*Cos[2*(c + d*x)] - (1176*I)*C*Cos[2*(c + d*x)] - (210*I)*A*Cos[4*(
c + d*x)] - (294*I)*C*Cos[4*(c + d*x)] + 80*(35*A + 13*C)*Cos[c + d*x]^(7/2
)*EllipticF[(c + d*x)/2, 2] + ((14*I)*(5*A + 7*C)*(1 + E^((2*I)*(c + d*x)))
^(7/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c
+ d*x)) + 70*A*Sin[c + d*x] + 380*C*Sin[c + d*x] + 630*A*Sin[2*(c + d*x)] +
 840*C*Sin[2*(c + d*x)] + 70*A*Sin[3*(c + d*x)] + 260*C*Sin[3*(c + d*x)] +
315*A*Sin[4*(c + d*x)] + 294*C*Sin[4*(c + d*x)]))/(420*d*E^(I*d*x))
```

Maple [B] time = 7.518, size = 1014, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*(-1/56*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+16*(1/8*A+3/8*C)*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x
+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+16*(3/8*A+
1/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca^3\sec(dx+c)^5+3\,Ca^3\sec(dx+c)^4+(A+3\,C)a^3\sec(dx+c)^3+(3\,A+C)a^3\sec(dx+c)^2+3\,Aa^3\sec(\right.$$
$$\sqrt{\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)),
 x)
```

3.225 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=259

$$\frac{4a^3(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(5A+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(5A-9C}{}$$

```
[Out] (4*a^3*(5*A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A + 21*C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(3*d*Sqrt[Se
c[c + d*x]]) - (2*(5*A - 3*C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])^2
*Sin[c + d*x])/(15*a*d) - (2*(5*A - 9*C)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[
c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.564843, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4087, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(5A+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(5A-9C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}-\frac{2(5A-3C)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (4*a^3*(5*A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A + 21*C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(3*d*Sqrt[Se
c[c + d*x]]) - (2*(5*A - 3*C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])^2
*Sin[c + d*x])/(15*a*d) - (2*(5*A - 9*C)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[
c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec(c+dx))^3\left(3aA-\frac{1}{2}a(5A-3C\cdots\right.}{\sqrt{\sec(c+dx)}}}{3a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(5A-3C)\sqrt{\sec(c+dx)}\left(a^2 \cdots\right.}{15\cdots}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(5A-3C)\sqrt{\sec(c+dx)}\left(a^2 \cdots\right.}{15\cdots}$$

$$= \frac{4a^3(5A+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^3 \mathrm{s}\cdots}{3d\sqrt{\sec(c+d\cdots}}$$

$$= \frac{4a^3(5A+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^3 \mathrm{s}\cdots}{3d\sqrt{\sec(c+d\cdots}}$$

$$= \frac{4a^3(5A+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^3 \mathrm{s}\cdots}{3d\sqrt{\sec(c+d\cdots}}$$

$$= \frac{4a^3(5A-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(5A\cdots}{\cdots}$$

Mathematica [C] time = 2.86039, size = 255, normalized size = 0.98

$$\frac{a^3 e^{-idx} \sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-4i(5A-9C)e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^3*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((180*I)*A*Cos[c + d*x] - (
324*I)*C*Cos[c + d*x] + (60*I)*A*Cos[3*(c + d*x)] - (108*I)*C*Cos[3*(c + d*
x)] + 80*(5*A + 3*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - ((4*I)*
(5*A - 9*C)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 30*A*Sin[c + d*x] + 132*C*Sin[c
+ d*x] + 10*A*Sin[2*(c + d*x)] + 60*C*Sin[2*(c + d*x)] + 30*A*Sin[3*(c + d*
x)] + 108*C*Sin[3*(c + d*x)] + 5*A*Sin[4*(c + d*x)]))/(60*d*E^(I*d*x))
```

Maple [B] time = 6.838, size = 939, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 4/15*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(40*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+100*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4-120*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+60*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^4-216*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+60*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2+90*A*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-60*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+246*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+25*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + 3\,Ca^3\sec(dx+c)^4 + (A+3\,C)a^3\sec(dx+c)^3 + (3\,A+C)a^3\sec(dx+c)^2 + 3\,Aa^3\sec(}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c) + a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2),
 x)
```

3.226 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=253

$$\frac{4a^3(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{8a^3(3A-10C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(9A-5C}{}$$

```
[Out] (4*a^3*(9*A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (8*a^3*(3*A - 10*C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(5*d*Sec[c +
 d*x]^(3/2)) + (4*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(5*a*d*Sqrt[Se
c[c + d*x]]) - (2*(9*A - 5*C)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*S
in[c + d*x])/(15*d)
```

Rubi [A] time = 0.569539, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4087, 4017, 4018, 3997, 3787, 3771, 2639, 2641}

$$-\frac{8a^3(3A-10C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(9A-5C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A+5C)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (4*a^3*(9*A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (8*a^3*(3*A - 10*C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(5*d*Sec[c +
 d*x]^(3/2)) + (4*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(5*a*d*Sqrt[Se
c[c + d*x]]) - (2*(9*A - 5*C)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*S
in[c + d*x])/(15*d)
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3\left(3aA-\frac{1}{2}a(3A-5C)\sec\right.}{\sec^{\frac{3}{2}}(c+dx)}}{5a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\sec(c+dx)\right)^2\sin(c+}{5ad\sqrt{\sec(c+dx)}}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\sec(c+dx)\right)^2\sin(c+}{5ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{8a^3(3A-10C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^3\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{8a^3(3A-10C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^3\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{8a^3(3A-10C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A(a+a\sec(c+dx))^3\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{4a^3(9A-5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(3A+}{}$$

Mathematica [C] time = 2.27082, size = 221, normalized size = 0.87

$$a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-8i(9A-5C)\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+80(3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((216*I)*A - (120*I)*C + (2
16*I)*A*Cos[2*(c + d*x)] - (120*I)*C*Cos[2*(c + d*x)] + 80*(3*A + 5*C)*Cos[
c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - (8*I)*(9*A - 5*C)*(1 + E^((2*I)*
(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] +
30*A*Sin[c + d*x] + 40*C*Sin[c + d*x] + 6*A*Sin[2*(c + d*x)] + 180*C*Sin[2*
(c + d*x)] + 30*A*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(60*d*E^(I*d*x)
)
```

Maple [B] time = 2.469, size = 704, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*(24*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+15*C)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A+25*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+15*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+15*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5+3\,Ca^3\sec(dx+c)^4+(A+3\,C)a^3\sec(dx+c)^3+(3\,A+C)a^3\sec(dx+c)^2+3\,Aa^3\sec(}{\sec(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2),
 x)
```

3.227 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=253

$$\frac{4a^3(13A+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^3(41A-35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7\ldots}{}$$

```
[Out] (4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(13*A + 35*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(41*A - 35*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(7*d*Se
c[c + d*x]^(5/2)) + (12*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(35*a*d*
Sec[c + d*x]^(3/2)) + (2*(7*A + 5*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.572871, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4087, 4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{4a^3(41A-35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7A+5C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+35C)\sqrt{\cos(c\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(13*A + 35*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(41*A - 35*C)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(7*d*Se
c[c + d*x]^(5/2)) + (12*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(35*a*d*
Sec[c + d*x]^(3/2)) + (2*(7*A + 5*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(15*d*Sqrt[Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3\left(3aA-\frac{1}{2}a(A-7C\right.}{\sec^{\frac{5}{2}}(c+dx)}}{7a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\sec(c+dx)\right)^2\sin}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\sec(c+dx)\right)^2\sin}{35ad\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(41A-35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} + \frac{2A(a+a\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+}$$

$$= -\frac{4a^3(41A-35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} + \frac{2A(a+a\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+}$$

$$= -\frac{4a^3(41A-35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} + \frac{2A(a+a\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{4a^3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(13}{}$$

Mathematica [C] time = 2.23135, size = 218, normalized size = 0.86

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(7A+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((2352*I)*A*Cos[c + d*x] +
(1680*I)*C*Cos[c + d*x] + 80*(13*A + 35*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(7*A + 5*C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 126*A*Sin[c
+ d*x] + 840*C*Sin[c + d*x] + 550*A*Sin[2*(c + d*x)] + 140*C*Sin[2*(c + d*x
)] + 126*A*Sin[3*(c + d*x)] + 15*A*Sin[4*(c + d*x)]))/(420*d*E^(I*d*x))
```

Maple [B] time = 2.406, size = 569, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*a^3*(120*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A+5*C)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)-4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(52
*A+35*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+175*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-
105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + 3\,Ca^3\sec(dx+c)^4 + (A+3\,C)a^3\sec(dx+c)^3 + (3\,A+C)a^3\sec(dx+c)^2 + 3\,Aa^3\sec(dx}{\sec(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2),
 x)
```

3.228 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=253

$$\frac{4a^3(11A+21C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(73A+63C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (4*a^3*(17*A + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(11*A + 21*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(16*A + 21*C)*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(9*d
*Sec[c + d*x]^(7/2)) + (4*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(21*a*
d*Sec[c + d*x]^(5/2)) + (2*(73*A + 63*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d
*x])/(315*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.579981, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4087, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(73A+63C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^3(16A+21C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(11A+21C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),x]
```


```
[Out] (4*a^3*(17*A + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(11*A + 21*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a^3*(16*A + 21*C)*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(9*d
*Sec[c + d*x]^(7/2)) + (4*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(21*a*
d*Sec[c + d*x]^(5/2)) + (2*(73*A + 63*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d
*x])/(315*d*Sec[c + d*x]^(3/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeqQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3\left(3aA+\frac{1}{2}a(A+9C)\sec\right.}{\sec^{\frac{7}{2}}(c+dx)}}{9a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\sec(c+dx)\right)^2\sin(c+}{21ad\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\sec(c+dx)\right)^2\sin(c+}{21ad\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8a^3(16A+21C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4.}{}$$

$$= \frac{8a^3(16A+21C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4.}{}$$

$$= \frac{8a^3(16A+21C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4.}{}$$

$$= \frac{4a^3(17A+27C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{4a^3(11A}{}$$

Mathematica [C] time = 2.87909, size = 206, normalized size = 0.81

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(17A+27C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(11*A + 21*C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(17*A + 27*C)*E^(I*(c + d*x)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))] + Cos[c + d*x]*((5712*I)*A + (9072*I)*C + 30*(97*A + 84*C)*Sin[
c + d*x] + 14*(73*A + 18*C)*Sin[2*(c + d*x)] + 270*A*Sin[3*(c + d*x)] + 35*
A*Sin[4*(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 1.934, size = 408, normalized size = 1.6

$$-\frac{4a^3}{315d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-560\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^{10}+2200\,A\cos(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2200*A*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-3412*A-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(2702*
A+882*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-738*A-378*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5+3\,Ca^3\sec(dx+c)^4+(A+3\,C)a^3\sec(dx+c)^3+(3\,A+C)a^3\sec(dx+c)^2+3\,Aa^3\sec(\right.}{\sec(dx+c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^2+A\right)\left(a \sec \left(dx+c\right)+a\right)^3}{\sec \left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2),
 x)
```

3.229 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{4a^3(105A+143C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{8a^3(35A+44C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(35A+33C)}{}$$

```
[Out] (4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(105*A + 143*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a^3*(35*A + 44*C)*Sin[c + d*x])/
(385*d*Sec[c + d*x]^(3/2)) + (4*a^3*(105*A + 143*C)*Sin[c + d*x])/(231*d*Sq
rt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(11*d*Sec[c +
 d*x]^(9/2)) + (4*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(33*a*d*Sec[c
+ d*x]^(7/2)) + (2*(35*A + 33*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(23
1*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.60518, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4087, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{8a^3(35A+44C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(35A+33C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(105A+143C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(11/2),x]
```


```
[Out] (4*a^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(105*A + 143*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (8*a^3*(35*A + 44*C)*Sin[c + d*x])/
(385*d*Sec[c + d*x]^(3/2)) + (4*a^3*(105*A + 143*C)*Sin[c + d*x])/(231*d*Sq
rt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(11*d*Sec[c +
 d*x]^(9/2)) + (4*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(33*a*d*Sec[c
+ d*x]^(7/2)) + (2*(35*A + 33*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(23
1*d*Sec[c + d*x]^(5/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3\left(3aA+\frac{1}{2}a(3A+1\right.}{\sec^{\frac{9}{2}}(c+dx)}}{11a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\sec(c+dx)\right)^2\sin}{33ad\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{4A\left(a^2+a^2\sec(c+dx)\right)^2\sin}{33ad\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{8a^3(35A+44C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{8a^3(35A+44C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{8a^3(35A+44C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(105A+143C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}} + 2A$$

$$= \frac{4a^3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^3(35}{38}$$

$$= \frac{4a^3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3(10}{}$$

Mathematica [C] time = 3.29351, size = 228, normalized size = 0.8

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2464i(5A+7C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(160*(105*A + 143*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(5*A + 7*C)*E^(I*(c + d*x
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)
*(c + d*x))] + Cos[c + d*x]*((36960*I)*A + (51744*I)*C + 10*(1953*A + 2354*
C)*Sin[c + d*x] + 308*(25*A + 18*C)*Sin[2*(c + d*x)] + 2835*A*Sin[3*(c + d*
x)] + 660*C*Sin[3*(c + d*x)] + 770*A*Sin[4*(c + d*x)] + 105*A*Sin[5*(c + d*
x)])))/(9240*d*E^(I*d*x))
```

Maple [A] time = 2.077, size = 436, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -4/1155*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(3360*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-14560*A*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^10+(25760*A+1320*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-
24080*A-4752*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13090*A+6622*C)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2940*A-2288*C)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+525*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1155*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+715*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + 3\,Ca^3\sec(dx+c)^4 + (A+3\,C)a^3\sec(dx+c)^3 + (3\,A+C)a^3\sec(dx+c)^2 + 3\,Aa^3\sec(dx}{\sec(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2)
, x)
```

3.230 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{4a^3(95A+121C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(175A+221C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{40a^3(118A+14}{9009d\sec}$$

```
[Out] (4*a^3*(175*A + 221*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(195*d) + (4*a^3*(95*A + 121*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (40*a^3*(118*A + 143*C)*Sin[c
+ d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (4*a^3*(175*A + 221*C)*Sin[c + d*x])/
(585*d*Sec[c + d*x]^(3/2)) + (4*a^3*(95*A + 121*C)*Sin[c + d*x])/(231*d*Sqr
t[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c +
d*x]^(11/2)) + (12*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(143*a*d*Sec[
c + d*x]^(9/2)) + (2*(145*A + 143*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(1287*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.655025, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4087, 4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^3(175A+221C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{40a^3(118A+143C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(145A+143C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{1287d\sec^{\frac{7}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(13/2),x]
```


```
[Out] (4*a^3*(175*A + 221*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(195*d) + (4*a^3*(95*A + 121*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (40*a^3*(118*A + 143*C)*Sin[c
+ d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (4*a^3*(175*A + 221*C)*Sin[c + d*x])/
(585*d*Sec[c + d*x]^(3/2)) + (4*a^3*(95*A + 121*C)*Sin[c + d*x])/(231*d*Sqr
t[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c +
d*x]^(11/2)) + (12*A*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(143*a*d*Sec[
c + d*x]^(9/2)) + (2*(145*A + 143*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(1287*d*Sec[c + d*x]^(7/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
```

```
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3\left(3aA+\frac{1}{2}a(5A+13C)s\right.}{\sec^{\frac{11}{2}}(c+dx)}}{13a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\sec(c+dx)\right)^2\sin(c}{143ad\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{12A\left(a^2+a^2\sec(c+dx)\right)^2\sin(c}{143ad\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{40a^3(118A+143C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)}$$

$$= \frac{40a^3(118A+143C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)}$$

$$= \frac{40a^3(118A+143C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(175A+221C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a}{}$$

$$= \frac{40a^3(118A+143C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(175A+221C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a}{}$$

$$= \frac{4a^3(175A+221C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{195d} + \frac{4a^3(9}{}$$

Mathematica [C] time = 4.20869, size = 250, normalized size = 0.78

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-4928i(175A+221C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(12480*(95*A + 121*C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (4928*I)*(175*A + 221*C)*E^(I*(c
+ d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))] + Cos[c + d*x]*((2587200*I)*A + (3267264*I)*C + 780*(1811
*A + 2134*C)*Sin[c + d*x] + 77*(7825*A + 7592*C)*Sin[2*(c + d*x)] + 251550*
A*Sin[3*(c + d*x)] + 154440*C*Sin[3*(c + d*x)] + 90860*A*Sin[4*(c + d*x)] +
 20020*C*Sin[4*(c + d*x)] + 24570*A*Sin[5*(c + d*x)] + 3465*A*Sin[6*(c + d*
x)])))/(720720*d*E^(I*d*x))
```

Maple [A] time = 1.992, size = 464, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+1058400*A*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^12+(-2122400*A-80080*C)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*
x+1/2*c)+(2331040*A+314600*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-153
5860*A-487916*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(633710*A+386386*C
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-121230*A-105534*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+18525*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-40425*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+23595*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-51051*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3 \sec(dx+c)^5 + 3\,Ca^3 \sec(dx+c)^4 + (A+3\,C)a^3 \sec(dx+c)^3 + (3\,A+C)a^3 \sec(dx+c)^2 + 3\,Aa^3 \sec(}{\sec(dx+c)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(13/2)
, x)
```

3.231 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=232

$$-\frac{(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A+C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A+7C)\sin(c\ldots}{}$$

```
[Out] (-3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) - ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a*d) + (3*(5*A + 7*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*a*d) - ((3*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((5*A +
 7*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.236459, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 3787, 3768, 3771, 2641, 2639}

$$-\frac{(A+C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A+7C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}-\frac{(3A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(5\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```


```
[Out] (-3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) - ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a*d) + (3*(5*A + 7*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*a*d) - ((3*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((5*A +
 7*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(3A+5C)-\frac{1}{2}a(5A\right.}{a^2}$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A+5C)\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a} + \frac{(5A+7}{}$$

$$= -\frac{(3A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{(5A+7C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} -$$

$$= \frac{3(5A+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad} - \frac{(3A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} +$$

$$= -\frac{(3A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad} + \frac{3(5A+7C)\sqrt{\sec(c}}{5a}$$

$$= -\frac{3(5A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(3A+5C)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 6.24486, size = 342, normalized size = 1.47

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-3i(5A+7C)e^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\left(1+e^{2i(c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(((-3*I)*(5*A + 7*C)*(1 + E^(I*(c + d
*x)))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 40*(3*A + 5*C)*Cos[(c + d*x)/2]*Co
s[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d
*x)/2]) + (2*I)*(30*A + 54*C + 2*(45*A + 56*C)*Cos[c + d*x] + 6*(5*A + 7*C)
*Cos[2*(c + d*x)] + 30*A*Cos[3*(c + d*x)] + 44*C*Cos[3*(c + d*x)] + (15*I)*
A*Sin[c + d*x] + (31*I)*C*Sin[c + d*x] - (4*I)*C*Sin[2*(c + d*x)] + (15*I)*
A*Sin[3*(c + d*x)] + (19*I)*C*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Si
```

```
n[(c + 3*d*x)/2]))/(60*a*d*E^(I*d*x)*(1 + Sec[c + d*x]))
```

Maple [B] time = 7.168, size = 803, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*C*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-A-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)-2/5*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*
sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*A+2*C)*(-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x
)
```

3.232 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=190

$$\frac{(3A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A+5C)\sin(c-}{3}$$

```
[Out] ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(3*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A + C)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.216725, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(A+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{(A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(3*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A + C)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(A+3C)-\frac{1}{2}a(3A+\right.}{a^2}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A+3C)\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{(3A+5C}{}$$

$$= -\frac{(A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(3A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} -$$

$$= -\frac{(A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{(3A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} -$$

$$= \frac{(A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A+5C)\sqrt{\cos(c+d}{}$$

Mathematica [C] time = 4.30655, size = 324, normalized size = 1.71

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-i(A+3C)e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}+e^{2i(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*(A + 3*C)*Sqrt[1 + E^((2*I)*(c
+ d*x))]*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 2
*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])*E
llipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*(Cos[c + d
*x] - I*Sin[c + d*x]) + (2*I)*(3*A + 5*C + 6*C*Cos[c + d*x] + (3*A + 7*C)*C
os[2*(c + d*x)] - (2*I)*C*Sin[c + d*x] + (2*I)*C*Sin[2*(c + d*x)]))*(Cos[(c
 + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a*d*E^(I*d*x)*(1 + Sec[c + d*x]))
```

Maple [B] time = 5.797, size = 486, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A+C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)-2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a*sec(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x
)
```

3.233 $\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(A+3C)\sin(c+dx}{ad}$$

```
[Out] -(((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
+ C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.189359, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```


```
[Out] -(((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((A + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
+ C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx &= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-C)-\frac{1}{2}a(A+\right.}{a^2} \\
&= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A-C)\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{(A+3C)}{} \\
&= \frac{(A+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A}{} \\
&= \frac{(A-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+3C)\sqrt{\sec(c+dx)}}{ad} \\
&= -\frac{(A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-C)\sqrt{\cos(c+d}}{}
\end{aligned}$$

Mathematica [C] time = 6.63623, size = 776, normalized size = 5.11

$$\frac{\sqrt{2}A\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos(c+dx)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{3d(a\sec(c+dx)+a)(A\cos(2c+2dx)+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*
d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Sqrt[2]*C*Sqrt
[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*C
os[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + A
*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Cos[c/2 + (d*x)/2]^2*Sqrt[C
os[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*(A + C*Sec[c + d*x
]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Se
c[c + d*x])) - (2*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*Ellipt
icF[(c + d*x)/2, 2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + A
*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d
*x)/2]^2*(A + C*Sec[c + d*x]^2)*((2*(A + 3*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d
```

```
 - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d - (4
*(A + C)*Tan[c/2])/d))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(
a + a*Sec[c + d*x]))
```

Maple [A] time = 4.051, size = 316, normalized size = 2.1

$$-\frac{1}{ad}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^4+(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+5*C)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx + c\right)^2 + A\right)\sqrt{\sec \left(dx + c\right)}}{a \sec \left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x
)
```

3.234 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=124

$$-\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(3A+C)\sqrt{\cos(c+$$

```
[Out] ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c +
 d*x]))
```

Rubi [A] time = 0.170953, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4085, 3787, 3771, 2639, 2641}

$$-\frac{(A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{se}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c +
 d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}dx = -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(3A+C)+\frac{1}{2}a(A-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-C)\int\sqrt{\sec(c+dx)}\,dx}{2a} + \frac{(3A+C)\int}{2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{c}}}{2a}$$

$$= \frac{(3A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-C)\sqrt{\cos(c+dx)}F\left(\right.}{}$$

Mathematica [C] time = 6.41976, size = 795, normalized size = 6.41

$$-\frac{\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos(c+dx)\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1}\right.}{d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),
x]
```

```
[Out] -((Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(d*E^(I*
d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x]))) - (Sqrt[2]*C*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C
+ A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*A*Cos[c/2 + (d*x)/2]^2*Sqr
t[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*(A + C*Sec[c +
d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a
*Sec[c + d*x])) + (2*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*Ell
ipticF[(c + d*x)/2, 2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C
+ A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (Cos[c/2 +
 (d*x)/2]^2*(A + C*Sec[c + d*x]^2)*((-2*(2*A + C + A*Cos[2*c])*Cos[d*x]*Csc
[c/2]*Sec[c/2])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[
(d*x)/2]))/d + (8*A*Cos[c]*Sin[d*x])/d + (4*(A + C)*Tan[c/2])/d))/((A + 2*C
 + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.075, size = 245, normalized size = 2.

$$\frac{1}{ad}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\left(A\text{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)+(2*A+2*C)*sin(1/2*d*x+1/2*c)^4+(-A-C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\sec(dx+c)}}{a\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^2 + a*se
c(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(C*sec
(c + d*x)**2/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

3.235 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=162

$$\frac{(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)}$$

```
[Out] -(((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*a*d) + ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x
]]) - ((A + C)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.200203, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 3787, 3769, 3771, 2641, 2639}

$$\frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}+\frac{(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] -(((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*a*d) + ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x
]]) - ((A + C)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(5A+3C)+\frac{1}{2}a(3A+C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{(3A+C)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{(5A+3C)\int -}{2a}$$

$$= \frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{(5A+3C)\int \sqrt{\sec}}{6a}$$

$$= -\frac{(3A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{(3A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A+3C)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 2.7222, size = 232, normalized size = 1.43

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(i(3A+C)e^{\frac{1}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*(2*(5*A + 3*C)
*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + I*(3*A + C
)*E^((I/2)*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2*Cos[c + d*x]*((-3
*I)*(3*A + C)*Cos[(c + d*x)/2] + (5*A + 3*C + 2*A*Cos[c + d*x])*Sin[(c + d*
x)/2])))/(3*a*d*E^(I*d*x)*(1 + Sec[c + d*x]))
```

Maple [A] time = 2.204, size = 262, normalized size = 1.6

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\left(5\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+3*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A+6*C)*sin(1/2*d*x+1/2*c)^4+(-7*A
-3*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{(a \sec(dx+c) + a) \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec(dx+c)^2 + A\right)\sqrt{\sec(dx+c)}}{a \sec(dx+c)^3 + a \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^3 + a*se
c(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)}\, dx + \int \frac{C \sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(C*se
c(c + d*x)**2/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)),
 x)
```

3.236 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=199

$$-\frac{(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A+5C)\sin(c}{5ad\sec^{\frac{3}{2}}(c+}$$

```
[Out] (3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a*d) + ((7*A + 5*C)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^
(3/2)) - ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - ((A + C)*S
in[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.21364, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(5A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a*d) + ((7*A + 5*C)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^
(3/2)) - ((5*A + 3*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - ((A + C)*S
in[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(7A+5C)+\frac{1}{2}a(5A+3C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(5A+3C)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(7A+5C)\int}{2a}$$

$$= \frac{(7A+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{(7A+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(5A+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{3(7A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(5A+3C)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 3.23445, size = 248, normalized size = 1.25

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-6i(7A+5C)e^{\frac{1}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometri}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*(-20*(5*A + 3*
C)*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (6*I)*(7
*A + 5*C)*E^((I/2)*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2*Cos[c + d
*x]*((18*I)*(7*A + 5*C)*Cos[(c + d*x)/2] - 2*(22*A + 15*C + 4*A*Cos[c + d*x
] - 3*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2])))/(30*a*d*E^(I*d*x)*(1 + Sec[c
+ d*x]))
```

Maple [A] time = 2.206, size = 277, normalized size = 1.4

$$-\frac{1}{15ad}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+15*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*C*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8-56*A*sin(1/2*d*x+1/2*c)^6+(-30
*A-30*C)*sin(1/2*d*x+1/2*c)^4+(23*A+15*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{4}+a\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c)^4 + a*se
c(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)),
 x)
```

3.237 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{2(A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(A+7C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{2(A+5C)\sin(c$$

```
[Out] ((A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + (2*(A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((A + 7*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + (2*(A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((A + 7*C)*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.379498, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(A+7C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{2(A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{(A+7C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(A$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] ((A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + (2*(A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((A + 7*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + (2*(A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((A + 7*C)*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}dx = -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(A-5C)-\frac{3}{2}a(A+3C)\sec(c+dx)\right)}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(A+7C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \mathrm{s}}{}$$

$$= -\frac{(A+7C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= -\frac{(A+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2(A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} -$$

$$= -\frac{(A+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2(A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} -$$

$$= \frac{(A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(A+5C)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 7.4472, size = 884, normalized size = 3.86

$$-\frac{2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right.}{3d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
2,x]
```

```
[Out] (-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A +
2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (14*Sqrt[2]*C*Sqrt[E^(I
*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/
2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(
-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*S
ec[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^2) + (8*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*C
sc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + C*Sec[c
+ d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
2) + (40*C*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c +
d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*Sin[c])/(3*d*
(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2
]^4*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((-4*(A + 7*C)*Cos[d*x]*Csc[c
/2]*Sec[c/2])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[
(d*x)/2]))/(3*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 4*C*Si
n[(d*x)/2]))/(3*d) + (16*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (16*(C + A
*Cos[c] + 5*C*Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (4*(A + C)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
2)
```

Maple [B] time = 7.02, size = 738, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A+
C)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c
)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+4
*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*C*(cos(1/2*d*x+1/2*c)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)-8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a^2*sec(
d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2,
 x)
```

3.238 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=191

$$\frac{(A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A-5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4C\sin(c+dx)\sqrt{s}}{a^2d}$$

```
[Out] (-4*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + ((A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + (4*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((A - 5*
C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.336472, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(A-5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4C\sin(c+dx)\sqrt{\sec(c+d}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-4*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + ((A - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + (4*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((A - 5*
C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A + C)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-C)-\frac{1}{2}a(A+7C)\sec(c+dx)\right)}{a+a\sec(c+dx)}\,d}{3a^2}$$

$$= \frac{(A-5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \int \sqrt{}$$

$$= \frac{(A-5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + (A-$$

$$= \frac{4C\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{(A-5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\,\mathrm{s}}{3d(}$$

$$= \frac{(A-5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4C\sqrt{\sec(c+dx)}\sin(c}{a^2d}$$

$$= -\frac{4C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A-5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{3a}$$

Mathematica [C] time = 4.61146, size = 293, normalized size = 1.53

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(8(A-5C)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\text{Ellip}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(I*A - (29*I)*C - (2*I)*(A + 25*C)*Cos
[c + d*x] + I*A*Cos[2*(c + d*x)] - (17*I)*C*Cos[2*(c + d*x)] + ((4*I)*C*(1
+ E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 8*(A - 5*C)*Cos[(c + d*x)
/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Si
n[(c + d*x)/2]) + 12*C*Sin[c + d*x] + A*Sin[2*(c + d*x)] + 7*C*Sin[2*(c + d
*x)])*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 +
Sec[c + d*x])^2)
```

Maple [B] time = 2.339, size = 450, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))*cos(1/2*d*x+1/2*c)-48*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(A+43*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A+37*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^
3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec\left(dx+c\right)^{3} + A \sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a^{2} \sec\left(dx+c\right)^{2} + 2\,a^{2} \sec\left(dx+c\right) + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a^2*sec(d*
x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2,
 x)
```

3.239 $\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$\frac{2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}-\frac{(A-C)\sqrt{\cos(c+d}}{}$$

```
[Out] -(((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d)) + (2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a^2*d) + ((A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(
1 + Sec[c + d*x])) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*
Sec[c + d*x])^2)
```

Rubi [A] time = 0.315646, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4019, 3787, 3771, 2639, 2641}

$$\frac{(A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -(((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d)) + (2*(A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a^2*d) + ((A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(
1 + Sec[c + d*x])) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*
Sec[c + d*x])^2)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}dx = -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(5A-C)+\frac{1}{2}a(A-5C)\sec(c+dx)\right)}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{(A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{3}{2}a^{}}{}$$

$$= \frac{(A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(A-}{}$$

$$= \frac{(A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{((A-}{}$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(A+C)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 6.66196, size = 859, normalized size = 5.21

$$\frac{2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)}{3d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
2,x]
```

```
[Out] (2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2
*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (2*Sqrt[2]*C*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
+ (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])
```

```
*(a + a*Sec[c + d*x])^2) + (8*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc
[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + C*Sec[c +
d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
 + (8*C*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x
)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A
+ 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4
*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((4*(A - C)*Cos[d*x]*Csc[c/2]*Se
c[c/2])/d - (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - C*Sin[(d*x)
/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x
)/2]))/(3*d) - (16*(2*A - C)*Tan[c/2])/(3*d) + (4*(A + C)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
2)
```

Maple [B] time = 2.28, size = 423, normalized size = 2.6

$$-\frac{1}{6\,a^2d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\,(\cos(1/2\,dx+c/2))^6+4\,A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^6+4*C*cos(1
/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+16*C*cos(1/2*d*x+1/2*c)^4+9*A*c
os(1/2*d*x+1/2*c)^2-3*C*cos(1/2*d*x+1/2*c)^2-A-C)/a^2/cos(1/2*d*x+1/2*c)^3/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2,
 x)
```

3.240 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{(5A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(5A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{4A\sqrt{\cos(c+dx)}}{}$$

```
[Out] (4*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) - ((5*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*a^2*d) - ((5*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
 Sec[c + d*x])) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec
[c + d*x])^2)
```

Rubi [A] time = 0.317311, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4020, 3787, 3771, 2639, 2641}

$$-\frac{(5A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{(5A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4A\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) - ((5*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*a^2*d) - ((5*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
 Sec[c + d*x])) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec
[c + d*x])^2)
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a(7A+C)+\frac{3}{2}a(A-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(5A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \int -$$

$$= -\frac{(5A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A}{}$$

$$= -\frac{(5A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A}{}$$

$$= \frac{4A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(5A-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{3a^2d}$$

Mathematica [C] time = 3.49726, size = 298, normalized size = 1.75

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(A+C\sec^2(c+dx)\right)\left(-\frac{8(5A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} - \frac{32iAe^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sqrt{\sec(c+dx)}\text{Hypergeo}}{d}\right.}{3a^2(\sec(c+dx)+1)^2(A\cos(2(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2
),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((8*I)*(1 + E^((2*I)*(c + d*x)))*(-(C*E^(I*(c + d*x))*
(-1 + E^(I*(c + d*x)))) + A*(3 + 16*E^(I*(c + d*x)) + 20*E^((2*I)*(c + d*x)
) + 9*E^((3*I)*(c + d*x))))*Sqrt[Sec[c + d*x]])/(d*E^(I*(c + d*x))*(1 + E^(
I*(c + d*x)))^3) - (8*(5*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d - ((32*I)*A*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sqrt[Sec[c +
d*x]])/d)*(A + C*Sec[c + d*x]^2))/(3*a^2*(A + 2*C + A*Cos[2*(c + d*x)])*(1
+ Sec[c + d*x])^2)
```

Maple [A] time = 2.348, size = 352, normalized size = 2.1

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+10\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,(\mathrm{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d
*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*cos(1/
2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4-2*C*cos(1/2*d*x+1/2*c)^4+15*A*co
s(1/2*d*x+1/2*c)^2+3*C*cos(1/2*d*x+1/2*c)^2-A-C)/a^2/cos(1/2*d*x+1/2*c)^3/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^3+2\,a^2\sec\left(dx+c\right)^2+a^2\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^3 + 2*
a^2*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^{\frac{5}{2}}\left(c+dx\right)+2\sec^{\frac{3}{2}}\left(c+dx\right)+\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{C\sec^2\left(c+dx\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)+2\sec^{\frac{3}{2}}\left(c+dx\right)+\sqrt{\sec\left(c+dx\right)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x
))), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)*
*(3/2) + sqrt(sec(c + d*x))), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))
), x)
```

3.241 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{2(5A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{2(5A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+d}$$

```
[Out] -(((7*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + (2*(5*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + (2*(5*A + C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c
 + d*x]]) - ((7*A + C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c
 + d*x])) - ((A + C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d
*x])^2)
```

Rubi [A] time = 0.362494, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{2(5A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}+\frac{2(5A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] -(((7*A + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + (2*(5*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + (2*(5*A + C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c
 + d*x]]) - ((7*A + C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c
 + d*x])) - ((A + C)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d
*x])^2)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{3}{2}a(3A+C)+\frac{1}{2}a(5A-C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(7A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - $$

$$= -\frac{(7A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} + $$

$$= \frac{2(5A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(7A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A+C)}{3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(7A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(5A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(7A+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(5A+C)\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 6.78187, size = 912, normalized size = 4.54

$$\frac{14\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)}{3d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2
),x]
```

```
[Out] (14*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A +
2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (2*Sqrt[2]*C*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*(A + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^2) + (40*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*C
sc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + C*Sec[c
+ d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
2) + (8*C*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(
A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]
^4*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((4*(5*A + C + 2*A*Cos[2*c])*C
os[d*x]*Csc[c/2]*Sec[c/2])/d + (8*A*Cos[2*d*x]*Sin[2*c])/(3*d) + (4*Sec[c/2
]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (16*Sec[c
/2]*Sec[c/2 + (d*x)/2]*(5*A*Sin[(d*x)/2] + 2*C*Sin[(d*x)/2]))/(3*d) - (32*A
*Cos[c]*Sin[d*x])/d + (8*A*Cos[2*c]*Sin[2*d*x])/(3*d) - (16*(5*A + 2*C)*Tan
[c/2])/(3*d) + (4*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C
+ A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 2.401, size = 437, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*cos(1/2*
d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+42*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*co
s(1/2*d*x+1/2*c)^6+4*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*C*c
os(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*A*cos(1/2*d*x+1/2*c)^4-20*
C*cos(1/2*d*x+1/2*c)^4+21*A*cos(1/2*d*x+1/2*c)^2+9*C*cos(1/2*d*x+1/2*c)^2-A
-C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{4}+2\,a^{2}\sec\left(dx+c\right)^{3}+a^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^4 + 2*
a^2*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)
), x)
```

3.242 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=236

$$-\frac{5(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(3A+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{4(14A+5C)\text{si}}{15a^2d\sec^{\frac{3}{2}}(c}$$

```
[Out] (4*(14*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a^2*d) - (5*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (4*(14*A + 5*C)*Sin[c + d*x])/(15*a^2*d*Se
c[c + d*x]^(3/2)) - (5*(3*A + C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
 - ((3*A + C)*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])) -
 ((A + C)*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.376041, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(3A+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{4(14A+5C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(3A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{5(3A+C)\sqrt{\cos(c+dx)}\sqrt{\text{s}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (4*(14*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a^2*d) - (5*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (4*(14*A + 5*C)*Sin[c + d*x])/(15*a^2*d*Se
c[c + d*x]^(3/2)) - (5*(3*A + C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
 - ((3*A + C)*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])) -
 ((A + C)*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A+C)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a(11A+5C)+\frac{1}{2}a(7A+C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(3A+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int}{}$$

$$= -\frac{(3A+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A+C)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{(5(}{}$$

$$= \frac{4(14A+5C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A+C)\sin(c+dx}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c}$$

$$= \frac{4(14A+5C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A+C)\sin(c+dx}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c}$$

$$= \frac{4(14A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5(3A+C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.34301, size = 301, normalized size = 1.28

$$-\frac{\sin(c)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(8i(14A+5C)e^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2
),x]
```

```
[Out] -(Cos[(c + d*x)/2]*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^(5/2)*Sin[c]*(Cos[d*x] +
I*Sin[d*x])*(400*(3*A + C)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2] + ((8*I)*(14*A + 5*C)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^
((I/2)*(c + d*x)) + 2*Cos[c + d*x]*((-72*I)*(14*A + 5*C)*Cos[(c + d*x)/2] -
 (24*I)*(14*A + 5*C)*Cos[(3*(c + d*x))/2] + 2*(158*A + 50*C + (179*A + 60*C
)*Cos[c + d*x] + 8*A*Cos[2*(c + d*x)] - 3*A*Cos[3*(c + d*x)])*Sin[(c + d*x)
/2])))/(120*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 2.474, size = 451, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*A*cos(1/2
*d*x+1/2*c)^10-352*A*cos(1/2*d*x+1/2*c)^8+120*A*cos(1/2*d*x+1/2*c)^6-150*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*A*cos(1/2*d*x+1/2*c)^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-120*C*cos(1/2*d*x+1/2*c)^6-50*C*cos(1/2*d*x+1/2*c)
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-120*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+266*A*cos(1/2*d*x+1/2*c)^4+190*C*cos(1/2*d*x+1/2*c)^4-135*A*cos(1/2*
d*x+1/2*c)^2-75*C*cos(1/2*d*x+1/2*c)^2+5*A+5*C)/a^2/cos(1/2*d*x+1/2*c)^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec\left(dx+c\right)^2 + A\right)\sqrt{\sec\left(dx+c\right)}}{a^2 \sec\left(dx+c\right)^5 + 2\,a^2 \sec\left(dx+c\right)^4 + a^2 \sec\left(dx+c\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^5 + 2*
a^2*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)
), x)
```

3.243 $\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$\frac{(A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{(9A+119C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+11C)\sin\dots}{}$$

```
[Out] ((9*A + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(2*a^3*d) - ((9*A + 119*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((A + 11*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) -
((A + C)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2
*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) - ((9*A
+ 119*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.536954, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(9A+119C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+11C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{(9A+119C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}\dots$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] ((9*A + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((A + 11*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(2*a^3*d) - ((9*A + 119*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((A + 11*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) -
((A + C)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2
*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) - ((9*A
+ 119*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(-\frac{1}{2}a(3A-7C)-\frac{1}{2}a(3A+13C)\sec(c+d\ldots}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2C\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \int \frac{\sec^{\frac{5}{2}}(c+\ldots}{}$$

$$= -\frac{(A+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2C\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{(9A+11\ldots}{30\ldots}$$

$$= -\frac{(A+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2C\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{(9A+11\ldots}{30\ldots}$$

$$= -\frac{(9A+119C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(A+11C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d}\ldots$$

$$= -\frac{(9A+119C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(A+11C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d}\ldots$$

$$= \frac{(9A+119C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+11C)\sqrt{\cos(c\ldots}{}$$

Mathematica [C] time = 7.778, size = 984, normalized size = 3.49

$$-\frac{6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+}\right.}{5d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(5*d*E^
(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (238*Sqrt[
2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*
(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (4*A*Cos[c/2 + (d*
x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[
c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])^3) + (44*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]
]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*Sec
[c + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])
^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((-4*
(9*A + 119*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*
x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d
*x)/2]^3*(3*A*Sin[(d*x)/2] + 13*C*Sin[(d*x)/2]))/(15*d) + (8*Sec[c/2]*Sec[c
/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + 29*C*Sin[(d*x)/2]))/(3*d) + (32*C*Sec[c]*
Sec[c + d*x]*Sin[d*x])/(3*d) + (8*(4*C + 3*A*Cos[c] + 33*C*Cos[c])*Sec[c]*T
an[c/2])/(3*d) + (8*(3*A + 13*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4
*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 3.141, size = 876, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+55*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-119*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-30*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+55*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+24*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+55*C*EllipticF(cos(1/2*d*x+1/
```

```
2*c),2^(1/2))-119*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+55*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(9*A+119*C)*sin(1/2*d*x+1/2*c)^10+24*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(29*A+389*C)*sin(1/2*d*x+1/2*c)^8-10*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(81*A+1111*C)*sin(1/2*d*x+1/2*c
)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(99*A+1414*C)*si
n(1/2*d*x+1/2*c)^4-3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
23*A+343*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1
/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{3}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + A*sec(d*x + c)^3)*sqrt(sec(d*x + c))/(a^3*sec(
d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3,
 x)
```

3.244 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=249

$$\frac{(A-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A-13C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-49C)\sin(c+}{10a^3}$$

```
[Out] ((A - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(6*a^3*d) - ((A - 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
10*a^3*d) - ((A + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d
*x])^3) + (2*(A - 4*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a + a*Sec[
c + d*x])^2) + ((A - 13*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a^3 + a^3
*Sec[c + d*x]))
```

Rubi [A] time = 0.523758, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(A-13C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] ((A - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(6*a^3*d) - ((A - 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
10*a^3*d) - ((A + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d
*x])^3) + (2*(A - 4*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a + a*Sec[
c + d*x])^2) + ((A - 13*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a^3 + a^3
*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{5}{2}a(A-C)-\frac{1}{2}a(A+11C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(A-4C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(A-4C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(A-4C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}$$

$$= \frac{(A-13C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{(A-49C)\sqrt{\sec(c}}{10a}$$

$$= \frac{(A-49C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A-13C)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 7.04886, size = 953, normalized size = 3.83

$$-\frac{2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right.}{15d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E
^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (98*Sqrt[
2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*
(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (4*A*Cos[c/2 + (d*
x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[
c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])^3) - (52*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*S
ec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d
*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*(
(-4*(A - 49*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d
*x)/2]*(A*Sin[(d*x)/2] - 13*C*Sin[(d*x)/2]))/(3*d) + (16*Sec[c/2]*Sec[c/2 +
 (d*x)/2]^3*(A*Sin[(d*x)/2] - 4*C*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c
/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (8*(-A + 13*C)*T
an[c/2])/(3*d) + (16*(A - 4*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(
A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.905, size = 679, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-65*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)
```

```
+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-49*C)*sin(1/2*d
*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A-81
7*C)*sin(1/2*d*x+1/2*c)^6+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(A-124*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(A-439*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*sqrt(sec(d*x + c))/(a^3*sec(
d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3,
 x)
```

3.245 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=220

$$\frac{(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A-9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-9C)\sqrt{\cos(c+\cdots}}{}$$

```
[Out] -((A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + (2*(2*A - 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
15*a*d*(a + a*Sec[c + d*x])^2) + ((A - 9*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.510215, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4019, 3787, 3771, 2639, 2641}

$$\frac{(A-9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-9C)\sqrt{\cos(c+dx)}\sqrt{s\cdots}}{10\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -((A - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + (2*(2*A - 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
15*a*d*(a + a*Sec[c + d*x])^2) + ((A - 9*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(10*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(7A-3C)+\frac{1}{2}a(A-9C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(2A-3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(2A-3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(2A-3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(2A-3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+3C)\sqrt{\cos(c+a}}{}$$

Mathematica [C] time = 6.89032, size = 952, normalized size = 4.33

$$\frac{2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)}{15d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E^
(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (6*Sqrt[2]
*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(
(2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A
+ 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (4*A*Cos[c/2 + (d*x)/
2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c +
 d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*
x])*(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*
Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*Sec[c
 + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3
) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((4*(A
- 9*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3
*(7*A*Sin[(d*x)/2] - 3*C*Sin[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x
)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*
x)/2]*(A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(3*d) + (8*(A + 3*C)*Tan[c/2])/(
3*d) - (8*(7*A - 3*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A + C)*Se
c[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a
*Sec[c + d*x])^3)
```

Maple [A] time = 2.594, size = 451, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8+30*C*c
os(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-54*C*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+138*C*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4-24*C*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)
^2-3*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^3+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*sqrt(sec(d*x + c))/(a^3*sec(d*
x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3,
 x)
```

3.246 $\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(3A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(9A-C)\sqrt{\cos(c\cdots}}{}$$

```
[Out] -((9*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + (2*(3*A - 2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.492153, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(3A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-C)\sqrt{\cos(c+dx)}\sqrt{s\cdots}}{10\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -((9*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + (2*(3*A - 2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(9A-C)+\frac{1}{2}a(3A-7C)\sec(c+dx)\right.}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(3A-2C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(3A-2C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(3A-2C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2(3A-2C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= -\frac{(9A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+C)\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 6.94646, size = 954, normalized size = 4.3

$$\frac{6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)}{5d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(5*d*E^(
I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (2*Sqrt[2]*
C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*
x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((
2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A
+ 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (4*A*Cos[c/2 + (d*x)/
2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c +
 d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Cs
c[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*Sec[c +
 d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3
) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((4*(9*
A - C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(
9*A*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2
]^3*(6*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(15*d) - (8*(9*A - C)*Tan[c/2])/(3
*d) + (16*(6*A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A + C)*Sec[
c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*S
ec[c + d*x])^3)
```

Maple [A] time = 2.405, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8+10*C*
cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+22*C*cos(1/2*d*x+1/2*c)^6
+114*A*cos(1/2*d*x+1/2*c)^4-6*C*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c
)^2-7*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*
a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3,
 x)
```

3.247 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=226

$$-\frac{(13A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(13A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-C)\sqrt{c\ldots}}{}$$

```
[Out] ((49*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) - ((13*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d
*(a + a*Sec[c + d*x])^3) - (2*(4*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.506204, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4085, 4020, 3787, 3771, 2639, 2641}

$$-\frac{(13A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-C)\sqrt{\cos(c+d\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] ((49*A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) - ((13*A - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(6*a^3*d) - ((A + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d
*(a + a*Sec[c + d*x])^3) - (2*(4*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{1}{2}a(11A+C)+\frac{5}{2}a(A-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int}{}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(13}{}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(13}{}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2(4A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(13}{}$$

$$= \frac{(49A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-C)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 6.95465, size = 975, normalized size = 4.31

$$-\frac{98\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx}}\right.}{15d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] (-98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
```

```
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*
E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (2*Sqrt[
2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*
(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (52*A*Cos[c/2 + (d
*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec
[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c +
2*d*x])*(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*S
ec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d
*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*(
(-4*(39*A - C + 10*A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (4*Sec[c
/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[
c/2]*Sec[c/2 + (d*x)/2]*(23*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (8*Se
c[c/2]*Sec[c/2 + (d*x)/2]^3*(17*A*Sin[(d*x)/2] + 7*C*Sin[(d*x)/2]))/(15*d)
+ (32*A*Cos[c]*Sin[d*x])/d + (8*(23*A + C)*Tan[c/2])/(3*d) - (8*(17*A + 7*C
)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A + C)*Sec[c/2 + (d*x)/2]^4*T
an[c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.395, size = 451, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos
(1/2*d*x+1/2*c)^8+130*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+294*
A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-
10*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+1/2*
c)^6+264*A*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+
1/2*c)^2-17*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{4}+3\,a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+a^{3}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^4 + 3*
a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))
), x)
```

3.248 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=249

$$\frac{(11A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{(11A+C)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{(119A+9C)\sin(c+}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+\right.}$$

```
[Out] -((119*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((11*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(2*a^3*d) + ((11*A + C)*Sin[c + d*x])/(2*a^3*d*Sqrt[Se
c[c + d*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
+ d*x])^3) - (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*
x])^2) - ((119*A + 9*C)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]*(a^3 + a^3*S
ec[c + d*x]))
```

Rubi [A] time = 0.533404, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(11A+C)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{(119A+9C)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{(11A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|2}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```



```
[Out] -((119*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((11*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(2*a^3*d) + ((11*A + C)*Sin[c + d*x])/(2*a^3*d*Sqrt[Se
c[c + d*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
+ d*x])^3) - (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*
x])^2) - ((119*A + 9*C)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]*(a^3 + a^3*S
ec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{1}{2}a(13A+3C)+\frac{1}{2}a(7A-3C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} -$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} -$$

$$= \frac{(11A+C)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2A\sin}{3ad\sqrt{\sec(c+dx}}$$

$$= -\frac{(119A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(11A+C)\sin(c+}{2a^3d\sqrt{\sec(c+dx}}$$

$$= -\frac{(119A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(11A+C)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 7.11061, size = 1008, normalized size = 4.05

$$\frac{238\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c}}\right.}{15d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] (238*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*
E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (6*Sqrt[
2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(
A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (44*A*Cos[c/2 + (d*
x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[
c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]
*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*Sec[
c + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((4*(8
9*A + 9*C + 30*A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (16*A*Cos[2*
d*x]*Sin[2*c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C
*Sin[(d*x)/2]))/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2
] + 6*C*Sin[(d*x)/2]))/(15*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(43*A*Sin[(d
*x)/2] + 9*C*Sin[(d*x)/2]))/(3*d) - (96*A*Cos[c]*Sin[d*x])/d + (16*A*Cos[2*
c]*Sin[2*d*x])/(3*d) - (8*(43*A + 9*C)*Tan[c/2])/(3*d) + (16*(11*A + 6*C)*S
ec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[
c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.18, size = 465, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*co
s(1/2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+108*C*cos(1/2*d*x+1/2*c)^8+30*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+54*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d
*x+1/2*c)^6-198*C*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4+114*C*cos
(1/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2-27*C*cos(1/2*d*x+1/2*c)^2+3*A+3
*C)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^5+3\,a^3\sec\left(dx+c\right)^4+3\,a^3\sec\left(dx+c\right)^3+a^3\sec\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^5 + 3*
a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)
), x)
```

3.249 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=290

$$-\frac{(63A+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(63A+13C)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33A+}{30a^3c}$$

```
[Out] (7*(33*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((63*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(33*A + 7*C)*Sin[c + d*x])/(30*a^3*d
*Sec[c + d*x]^(3/2)) - ((63*A + 13*C)*Sin[c + d*x])/(6*a^3*d*Sqrt[Sec[c + d
*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])
^3) - (2*(6*A + C)*Sin[c + d*x])/(15*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c +
d*x])^2) - ((63*A + 13*C)*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3
*Sec[c + d*x]))
```

Rubi [A] time = 0.570337, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4085, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(63A+13C)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33A+7C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(63A+13C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(63A+13C)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (7*(33*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((63*A + 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(33*A + 7*C)*Sin[c + d*x])/(30*a^3*d
*Sec[c + d*x]^(3/2)) - ((63*A + 13*C)*Sin[c + d*x])/(6*a^3*d*Sqrt[Sec[c + d
*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])
^3) - (2*(6*A + C)*Sin[c + d*x])/(15*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c +
d*x])^2) - ((63*A + 13*C)*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3
*Sec[c + d*x]))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
```

```
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{5}{2}a(3A+C)+\frac{1}{2}a(9A-C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2(6A+C)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2(6A+C)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2(6A+C)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= \frac{7(33A+7C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(63A+13C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(}{5d\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{7(33A+7C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(63A+13C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A+C)\sin(}{5d\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= \frac{7(33A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(63A+13C)\sqrt{\mathrm{co}}}{}$$

Mathematica **[C]** time = 7.26741, size = 1052, normalized size = 3.63

$$-\frac{154\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i}}\right.}{5d(\cos(2c+2dx)A+A+2C)(\sec(c+dx)a+a)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] (-154*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(5*d*
E^(I*d*x)*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (98*Sqrt
[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] +
E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)
*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)
*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (84*A*Cos[c/2 + (
d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Se
c[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])^3) - (52*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + C*S
ec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d
*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*(
(-2*(329*A + 78*C + 133*A*Cos[2*c] + 20*C*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c
/2])/(5*d) - (16*A*Cos[2*d*x]*Sin[2*c])/d + (8*A*Cos[3*d*x]*Sin[3*c])/(5*d)
```

```
+ (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d
) + (184*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(
3*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(27*A*Sin[(d*x)/2] + 17*C*Sin[(d*x)
/2]))/(15*d) + (8*(133*A + 20*C)*Cos[c]*Sin[d*x])/(5*d) - (16*A*Cos[2*c]*Si
n[2*d*x])/d + (8*A*Cos[3*c]*Sin[3*d*x])/(5*d) + (184*(3*A + C)*Tan[c/2])/(3
*d) - (8*(27*A + 17*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A + C)*S
ec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a +
a*Sec[c + d*x])^3)
```

Maple [A] time = 2.701, size = 479, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*co
s(1/2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10-228*A*cos(1/2*d*x+1/2*c)^8-
630*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1386*A*cos(1/2*d*x+1/2
*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*cos(1/2*d*
x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-294*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6+578*C*cos(1/2*d*x+1/2*c)^6-744*A
*cos(1/2*d*x+1/2*c)^4-264*C*cos(1/2*d*x+1/2*c)^4+57*A*cos(1/2*d*x+1/2*c)^2+
37*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{6}+3\,a^{3}\sec\left(dx+c\right)^{5}+3\,a^{3}\sec\left(dx+c\right)^{4}+a^{3}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^6 + 3*
a^3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)
), x)
```

3.250 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=214

$$\frac{a(48A+35C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+}}\right.}{64d}$$

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a*(48*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt
[a + a*Sec[c + d*x]]) + (a*(48*A + 35*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*
d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.45962, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4089, 4016, 3803, 3801, 215}

$$\frac{a(48A+35C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+}}\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a*(48*A + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt
[a + a*Sec[c + d*x]]) + (a*(48*A + 35*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*
d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(4*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{\int \sec^{\frac{5}{2}}}{}$$

$$= \frac{aC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}{4d}$$

$$= \frac{a(48A+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{aC\sec^{\frac{7}{2}}(c+}{24d\sqrt{a+a}}$$

$$= \frac{a(48A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+35C}{96d\sqrt{}}$$

$$= \frac{a(48A+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+35C}{96d\sqrt{}}$$

$$= \frac{\sqrt{a}(48A+35C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a(48A+35C}{64d\sqrt{}}$$

Mathematica [A] time = 2.1597, size = 238, normalized size = 1.11

$$\frac{\cos^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)((432A+539C)\cos(c+dx)+4(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Cos[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(A + C*Sec[c + d*x]^2)*((192*A +
 332*C + (432*A + 539*C)*Cos[c + d*x] + 4*(48*A + 35*C)*Cos[2*(c + d*x)] +
144*A*Cos[3*(c + d*x)] + 105*C*Cos[3*(c + d*x)])*Sec[c + d*x]^(9/2)*Tan[(c
+ d*x)/2] - (12*(48*A + 35*C)*Csc[c + d*x]*(Log[1 + Sec[c + d*x]] - Log[Sqr
t[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c +
```

```
d*x]^2]])*Sqrt[Tan[c + d*x]^2])/Sqrt[1 + Sec[c + d*x]]))/(384*d*(A + 2*C +
A*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.387, size = 449, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/768/d*(-144*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+144*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-105*C*cos(d*x+c)^
4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*
x+c)))+105*C*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1+sin(d*x+c)))+288*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c
)+1))^(1/2)+210*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+192*A*c
os(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+140*C*sin(d*x+c)*cos(d*x+c
)^2*(-2/(cos(d*x+c)+1))^(1/2)+112*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2)+96*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(5/2)*(a
*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/co
s(d*x+c)*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.37684, size = 5963, normalized size = 27.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/768*(48*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(
7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*
sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*
(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
```

```
 + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt
(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + s
in(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c
)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1) + (420*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d
*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(15
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 140*(sqrt(2)*sin(8*d*x + 8*c) + 4
*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*
x + 2*c))*cos(13/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1596*(sqrt(2)*sin
(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4
*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c))) +
500*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(
4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 500*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) +
 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 1596*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*si
n(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*c
os(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 140*(sqrt(2)*sin(8*d*x + 8*c)
 + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(
2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 420*(sqrt(2)*s
in(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) +
 4*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*
cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
 + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) +
 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2
*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)
^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2
*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) - 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
```

```
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c))) + 2) + 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 2) - 420*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c)
 + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 140*(sqrt(2)*cos(8*d*x + 8*c) +
4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*sin(13/2*arctan2(sin(d*x + c), cos(d*x + c))) - 1596*(
sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x
 + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 500*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x +
6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*s
in(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 500*(sqrt(2)*cos(8*d*x + 8*c)
 + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1596
*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d
*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 140*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x +
 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 420*(sqrt(2)*cos(8*d*x + 8*c
) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sq
rt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)
*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
+ 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*
x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c)
 + 1))/d
```

Fricas [A] time = 1.02463, size = 1238, normalized size = 5.79

$$\left[\frac{3\left((48\,A+35\,C)\cos\left(dx+c\right)^{4}+(48\,A+35\,C)\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx}{\cos(d}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{768\left(d\cos\left(dx+c\right)^{4}+d\cos\left(d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/768*(3*((48*A + 35*C)*cos(d*x + c)^4 + (48*A + 35*C)*cos(d*x + c)^3)*sqr
t(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos
(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(48*A + 35
*C)*cos(d*x + c)^3 + 2*(48*A + 35*C)*cos(d*x + c)^2 + 56*C*cos(d*x + c) + 4
8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/384*(3*((48*A + 35*C)*cos(d*x +
 c)^4 + (48*A + 35*C)*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(48*A + 35*C)*cos(d*x + c)^3 + 2*(48*A
 + 35*C)*cos(d*x + c)^2 + 56*C*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos
(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2
), x)
```

3.251 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=169

$$\frac{a(8A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{C\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)}}{3d}$$

```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*d) + (a*(8*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Se
c[c + d*x]]) + (C*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(3*d)
```

Rubi [A] time = 0.387715, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4089, 4016, 3803, 3801, 215}

$$\frac{a(8A+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{C\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*d) + (a*(8*A + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Se
c[c + d*x]]) + (C*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(3*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
```

```
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\int \sec^{\frac{3}{2}}}{}$$

$$= \frac{aC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}{3d}$$

$$= \frac{a(8A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{aC\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{a(8A+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{aC\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{\sqrt{a}(8A+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a(8A+5C)\sec}{8d\sqrt{a+}}$$

Mathematica [A] time = 1.66813, size = 211, normalized size = 1.25

$$\frac{\cos^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(3(8A+5C)\cos(2(c+dx))+24A\right.}{24d(A\cos(2(c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Cos[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(A + C*Sec[c + d*x]^2)*((24*A +
31*C + 20*C*Cos[c + d*x] + 3*(8*A + 5*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/
2)*Tan[(c + d*x)/2] - (6*(8*A + 5*C)*Csc[c + d*x]*(Log[1 + Sec[c + d*x]] -
Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[T
an[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2])/Sqrt[1 + Sec[c + d*x]]))/(24*d*(A +
2*C + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.369, size = 385, normalized size = 2.3

$$-\frac{-1+\cos(dx+c)}{48\,d\cos(dx+c)\,(\sin(dx+c))^{2}}\left(24\,A\,(\cos(dx+c))^{3}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(24*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-24*A*cos(d*x+c)^3*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+15*C
*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*2^(1/2)-15*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2)+30*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2
)+20*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*C*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(3/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(
1/2)/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)/sin(d*x+c)^2
```

Maxima [B] time = 2.55205, size = 3699, normalized size = 21.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*si
n(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)
*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c)
)*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*sin(6*d*x + 6
*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arct
an2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2
```

)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4
*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 15*(2*(3*
cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6
c)^2 + 6(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2
+ 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 15*(2*(3*co
s(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c
)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 +
9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(2*(3*cos(
4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^
2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*
cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 15*(2*(3*cos(4*
d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
+ 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*co
s(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sq
rt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 60*(sqrt(2)*cos(6
*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(6*d*x
+ 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*cos(6*d*x + 6
*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*si
n(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*cos(6*d*x + 6*c)
+ 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(6*d*x + 6*c) + 3*s
qrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2
)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(s
in(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 +
9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x
+ 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d

Fricas [A] time = 0.785601, size = 1127, normalized size = 6.67

$$\left[\frac{3\left((8A+5C)\cos(dx+c)^3+(8A+5C)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{s}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(3*((8*A + 5*C)*cos(d*x + c)^3 + (8*A + 5*C)*cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(8*A + 5*C)*cos
(d*x + c)^2 + 10*C*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2),
 1/48*(3*((8*A + 5*C)*cos(d*x + c)^3 + (8*A + 5*C)*cos(d*x + c)^2)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(8*A + 5*
C)*cos(d*x + c)^2 + 10*C*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x +
c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)\sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

3.252 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)$

Optimal. Leaf size=124

$$\frac{\sqrt{a}(8A+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{aC\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d
*x]]) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.305816, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4089, 4016, 3801, 215}

$$\frac{\sqrt{a}(8A+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{aC\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d
*x]]) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d} + \frac{\int\sqrt{\sec(c}{}$$

$$= \frac{aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(}{2d}$$

$$= \frac{aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(}{2d}$$

$$= \frac{\sqrt{a}(8A+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{aC\sec^{\frac{3}{2}}(c+dx)\sin}{4d\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 2.32312, size = 202, normalized size = 1.63

$$\frac{\cos^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(A+C\sec^2(c+dx)\right)\left(C\left(\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+a}{4d(A\cos(2(c+dx))+A+2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Cos[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(A + C*Sec[c + d*x]^2)*(C*Sec[(c
 + d*x)/2]*Sec[c + d*x]^(5/2)*(Sin[(c + d*x)/2] + 3*Sin[(3*(c + d*x))/2]) -
 (2*(8*A + 3*C)*Csc[c + d*x]*(Log[1 + Sec[c + d*x]] - Log[Sqrt[Sec[c + d*x]
] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt
[Tan[c + d*x]^2])/Sqrt[1 + Sec[c + d*x]]))/(4*d*(A + 2*C + A*Cos[2*(c + d*x
)]))
```

Maple [B] time = 0.403, size = 325, normalized size = 2.6

$$\frac{(\cos(dx+c))^2-1}{16\,d(\sin(dx+c))^2\cos(dx+c)}\left(8\,A\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))*cos(d*x+c)^2*2^(1/2)-8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)+3*C*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)-3
*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*
cos(d*x+c)^2*2^(1/2)+6*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*
```

```
C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)
+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(co
s(d*x+c)^2-1)
```

Maxima [B] time = 2.2186, size = 2034, normalized size = 16.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*(8*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2)) - (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(
7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*
sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*
(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt
(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + s
in(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c
)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
```

```
*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.774252, size = 1023, normalized size = 8.25

$$\left[\frac{\left((8\,A+3\,C)\cos\left(dx+c\right)^{2}+(8\,A+3\,C)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(\cdots}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\cdots\right.}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*(((8*A + 3*C)*cos(d*x + c)^2 + (8*A + 3*C)*cos(d*x + c))*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*C*cos(d*x + c) + 2*
C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((8*A + 3*C)*cos(d*x + c)^2 + (8
*A + 3*C)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d
*x + c) - 2*a)) + 2*(3*C*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x +
c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)
), x)
```

3.253 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{a(2A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
*(2*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) +
(C*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.303945, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4089, 4015, 3801, 215}

$$\frac{a(2A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
*(2*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) +
(C*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \frac{C\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{1}{2}a(2A-C\right.}{\sqrt{\sec(c+d}}}{a}$$

$$= \frac{a(2A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx}}{d}$$

$$= \frac{a(2A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx}}{d}$$

$$= \frac{\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a(2A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{C}{}$$

Mathematica [A] time = 2.54043, size = 177, normalized size = 1.54

$$-\frac{\cot(c+dx)\sqrt{a(\sec(c+dx)+1)}\left((C-2A)\sqrt{\sec(c+dx)+1}\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)+1}\sec^{\frac{3}{2}}(c+dx)(A\cos(2(c+\right.}{d\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] -((Cot[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((-2*A + C)*Sqrt[Sec[c + d*x]]*S
qrt[1 + Sec[c + d*x]] + (A - C + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)*Sqr
t[1 + Sec[c + d*x]] + C*(Log[1 + Sec[c + d*x]] - Log[Sqrt[Sec[c + d*x]] + S
ec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[
c + d*x]^2]))/(d*Sqrt[1 + Sec[c + d*x]]))
```

Maple [B] time = 0.403, size = 210, normalized size = 1.8

$$-\frac{1}{4d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(C\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(C*cos(d*x+c)*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1+sin(d*x+c)))*2^(1/2)-C*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1
/2)+8*A*cos(d*x+c)^2-8*A*cos(d*x+c)+4*C*cos(d*x+c)-4*C)*(1/cos(d*x+c))^(1/2
)/sin(d*x+c)
```

Maxima [B] time = 2.04424, size = 923, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*(8*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) - (4*sqrt(2)*cos(3/2*arctan2(
sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.586503, size = 882, normalized size = 7.67

$$\left[\frac{(C\cos(dx+c)+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4(2A\cos(dx+c)+C)\sqrt{\frac{a\cdots}{\cdots}}}{\sqrt{\cos(d\cdots}}}{4(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*((C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 4*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*((C*cos(d*x
 + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
```

```
)) + 2*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)
), x)
```

3.254 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.290942, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4087, 4015, 3801, 215}

$$\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{aA}{2}+\frac{3}{2}aC\sec(c+dx)\right)}{\sqrt{\sec(c+dx)}}}{3a}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + C$$

(2

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} -$$

$$= \frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + 2A\sqrt{a}$$

Mathematica [A] time = 1.60091, size = 179, normalized size = 1.54

$$-\frac{\csc(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2A\sqrt{\sec(c+dx)+1}+A(\cos(2(c+dx))-3)\sec(c+dx)\sqrt{\sec(c+dx)+1}+6C\sqrt{\tan^2}\right.}{3d\sec^{\frac{3}{2}}(c+dx)\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] -(Csc[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(2*A*Sqrt[1 + Sec[c + d*x]] + A*(
-3 + Cos[2*(c + d*x)])*Sec[c + d*x]*Sqrt[1 + Sec[c + d*x]] + 6*C*(Log[1 + S
ec[c + d*x]] - Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c
 + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]))/(
3*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]])
```

Maple [B] time = 0.382, size = 198, normalized size = 1.7

$$-\frac{(\cos(dx+c))^2}{6d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-3C\sqrt{-2(\cos(dx+c)+1)^{-1}}\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-3*C*(-2/(cos(d*x+c)+1))^(1/2)*
2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))*sin(d*x+c)+3*C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+4*A*cos(d*x+c
)^2+4*A*cos(d*x+c)-8*A)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.02298, size = 479, normalized size = 4.13

$$\sqrt{2}\left(3\,\cos\left(\frac{2}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\sin\left(\frac{2}{3}\,\arctan\left(\sin\left(\frac{3}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 3*C*sqrt(a)*(
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [A] time = 0.595523, size = 927, normalized size = 7.99

$$\left[\frac{3\,(C\cos\left(dx+c\right)+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(A\cos(dx+c)^2+2\,A\,c\right.}{}}{6\,(d\cos\left(dx+c\right)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*(3*(C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
, 1/3*(3*(C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a
*cos(d*x + c) - 2*a)) + 2*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c) + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\,(\sec\left(c+dx\right)+1)}\left(A+C\sec^2\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + C*sec(c + d*x)**2)/sec(c + d*x)**(
3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

3.255 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{2a(7A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a*(7*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/
2)) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.314592, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {4087, 4013, 3804}

$$\frac{2a(7A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(7*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/
2)) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{aA}{2}+\frac{1}{2}a(2A+5C)\sec(c\right.}{\sec^{\frac{3}{2}}(c+dx)}}{5a}$$

$$= \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a(7A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.531613, size = 68, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(8A\cos(c+dx)+3A\cos(2(c+dx))+19A+30C)}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] ((19*A + 30*C + 8*A*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c
+ d*x])]*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.371, size = 87, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(3A(\cos(dx+c))^2+4A\cos(dx+c)+8A+15C\right)(\cos(dx+c))^3}{15d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+8*A+15*C)*(a*(cos(
d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 1.94444, size = 302, normalized size = 2.48

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
```

```
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 + 120*sqrt(2)*C*sqrt(a)*sin(1/2*d*x + 1/2*c))/d
```

Fricas [A] time = 0.485701, size = 231, normalized size = 1.89

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{3}+4\,A\cos\left(dx+c\right)^{2}+\left(8\,A+15\,C\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^3 + 4*A*cos(d*x + c)^2 + (8*A + 15*C)*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*
sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2
), x)
```

3.256 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=168

$$\frac{4a(24A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(24A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{7d\sec^{\frac{5}{2}}(c+dx)}+$$

```
[Out] (2*a*A*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (4*a*(24*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec
[c + d*x]^(5/2))
```

Rubi [A] time = 0.387809, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4087, 4015, 3805, 3804}

$$\frac{4a(24A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(24A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{7d\sec^{\frac{5}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*a*A*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (4*a*(24*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec
[c + d*x]^(5/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
```

```
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{aA}{2}+\frac{1}{2}a(4A+7C)\sec\right.}{\sec^{\frac{5}{2}}(c+dx)}}{7a}$$

$$= \frac{2aA\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(24A+35C)\sin(c+}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{2aA\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(24A+35C)\sin(c+}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.775454, size = 84, normalized size = 0.5

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((141A+140C)\cos(c+dx)+36A\cos(2(c+dx))+15A\cos(3(c+dx))+228A+}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] ((228*A + 280*C + (141*A + 140*C)*Cos[c + d*x] + 36*A*Cos[2*(c + d*x)] + 15
*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*Sq
rt[Sec[c + d*x]])
```

Maple [A] time = 0.417, size = 107, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^3+18A(\cos(dx+c))^2+24A\cos(dx+c)+35C\cos(dx+c)+48A+70\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+24*A*cos(d*x+
c)+35*C*cos(d*x+c)+48*A+70*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c
)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.03238, size = 551, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) + 140*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x +
 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C
*sqrt(a))/d
```

Fricas [A] time = 0.483041, size = 281, normalized size = 1.67

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{4}+18\,A\cos\left(dx+c\right)^{3}+\left(24\,A+35\,C\right)\cos\left(dx+c\right)^{2}+2\left(24\,A+35\,C\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^4 + 18*A*cos(d*x + c)^3 + (24*A + 35*C)*cos(d*x +
c)^2 + 2*(24*A + 35*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2
), x)
```

3.257 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{2a(16A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+21C)\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*A*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*(16*A + 21*C)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
+ d*x]]) + (8*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) + (16*a*(16*A + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d
*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.458937, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4087, 4015, 3805, 3804}

$$\frac{2a(16A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+21C)\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),x]
```



```
[Out] (2*a*A*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*(16*A + 21*C)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
+ d*x]]) + (8*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) + (16*a*(16*A + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d
*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{aA}{2}+\frac{3}{2}a(2A+3C)\mathrm{s}\right.}{\sec^{\frac{7}{2}}(c+dx)}}{9a}$$

$$= \frac{2aA\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+21C)\sin(c+}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2aA\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+21C)\sin(c+}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2aA\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+21C)\sin(c+}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

Mathematica [A] time = 1.18383, size = 102, normalized size = 0.48

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(16(47A+42C)\cos(c+dx)+4(83A+63C)\cos(2(c+dx))+80A\cos(3(c+dx)}{1260d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] ((1321*A + 1596*C + 16*(47*A + 42*C)*Cos[c + d*x] + 4*(83*A + 63*C)*Cos[2*(
c + d*x)] + 80*A*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[
c + d*x])]*Tan[(c + d*x)/2])/(1260*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.394, size = 129, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(35A(\cos(dx+c))^4+40A(\cos(dx+c))^3+48A(\cos(dx+c))^2+63C(\cos(dx+c))^2+64\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+40*A*cos(d*x+c)^3+48*A*cos(d*x+
c)^2+63*C*cos(d*x+c)^2+64*A*cos(d*x+c)+84*C*cos(d*x+c)+128*A+168*C)*(a*(cos
```

```
(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.08835, size = 792, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(1890*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c)))*sin(9/2*d*x + 9/2*c) + 420*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 252*cos(4/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 45*cos(2/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 1890*c
os(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2
*c))) - 420*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) - 252*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) - 45*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*sin(9/2*d*x + 9/2*c) + 45*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) + 84*sqrt(2)*(30*cos(4/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) +
5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x
+ 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.491079, size = 328, normalized size = 1.54

$$\frac{2\left(35\,A\cos\left(dx+c\right)^{5}+40\,A\cos\left(dx+c\right)^{4}+3\,(16\,A+21\,C)\cos\left(dx+c\right)^{3}+4\,(16\,A+21\,C)\cos\left(dx+c\right)^{2}+8\,(16\,A+21\,C\cdots}{315\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/315*(35*A*cos(d*x + c)^5 + 40*A*cos(d*x + c)^4 + 3*(16*A + 21*C)*cos(d*x
+ c)^3 + 4*(16*A + 21*C)*cos(d*x + c)^2 + 8*(16*A + 21*C)*cos(d*x + c))*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqr
t(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+A\right) \sqrt{a \sec \left(dx+c\right)+a}}{\sec \left(dx+c\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(9/2
), x)
```

3.258 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=265

$$\frac{a^2(80A+67C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)\sec}{128d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^2*(176*A + 133*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(176*A + 133*C)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(80*A + 67*C)*Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (3*a*C*Sec[c + d*
x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (C*Sec[c + d*x]^(7
/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.672174, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(80A+67C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)\sec}{128d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^2*(176*A + 133*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(176*A + 133*C)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(80*A + 67*C)*Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (3*a*C*Sec[c + d*
x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (C*Sec[c + d*x]^(7
/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\int}{}$$

$$= \frac{3aC\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{40d} + \frac{C}{}$$

$$= \frac{a^2(80A+67C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} + \frac{3aC\sec^{\frac{7}{2}}(}{}$$

$$= \frac{a^2(176A+133C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(80A}{}$$

$$= \frac{a^2(176A+133C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A}{}$$

$$= \frac{a^2(176A+133C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A}{}$$

$$= \frac{a^{3/2}(176A+133C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^2(176A}{}$$

Mathematica [A] time = 3.27519, size = 273, normalized size = 1.03

$$\cos^3(c+dx)(a(\sec(c+dx)+1))^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{\sec(c+dx)+1}(12(880A+127$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(3/2)*(A + C*Sec[c + d*x]^2)*((10480
*A + 13313*C + 12*(880*A + 1273*C)*Cos[c + d*x] + 4*(3280*A + 3059*C)*Cos[2
*(c + d*x)] + 3520*A*Cos[3*(c + d*x)] + 2660*C*Cos[3*(c + d*x)] + 2640*A*Co
s[4*(c + d*x)] + 1995*C*Cos[4*(c + d*x)])*Sec[c + d*x]^(11/2)*Sqrt[1 + Sec[
c + d*x]]*Tan[(c + d*x)/2] - 120*(176*A + 133*C)*Csc[c + d*x]*(Log[1 + Sec[
c + d*x]] - Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c +
d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(7680*d*(A + 2*C + A*Co
s[2*(c + d*x)])*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.359, size = 512, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/7680/d*a*(2640*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^5-2640*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^5+1995*C*2^(1/2
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*c
os(d*x+c)^5-1995*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^5+5280*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)^4+3990*C*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4
+3520*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+2660*C*sin(d*x+c)
*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+1280*A*cos(d*x+c)^2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2)+2128*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1
/2)+1824*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+768*C*(-2/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+
c))^(5/2)*(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2
-1)
```

Maxima [B] time = 5.86039, size = 9767, normalized size = 36.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] -1/7680*(80*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d
```

```
*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d
*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c
) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) +
a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*
sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*c
os(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin
(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x +
 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c)
 + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 +
 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9
*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*
d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*
x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x +
2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*lo
g(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*
c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)
^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3
*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*c
os(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d
*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) +
 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*
d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt
(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6
*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)
*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*co
s(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c
) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(
sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos
(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*A*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x
```

+ 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 +
18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
 2*c) + 1) + (7980*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x +
8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2660*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) +
10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a
*sin(2*d*x + 2*c))*cos(17/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
38304*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqr
t(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2
*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12160*
(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a
*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x +
 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 45400*(sqrt(
2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6
*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))
*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 45400*(sqrt(2)*a*s
in(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x +
 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12160*(sqrt(2)*a*sin(10*d
*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) +
 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 38304*(sqrt(2)*a*sin(10*d*x + 10
*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqr
t(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2660*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*
sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*s
in(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 7980*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*
a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x
 + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8*c)^2
+ 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*x + 2*
c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6*d*x +
 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*x + 6*
c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 10*c) +
 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) +
 a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a
)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 10*a*
cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*a*sin(
4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6*d*x +
 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(
2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) + 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8*c)^
2 + 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*x +
2*c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6*d*x
 + 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*x +
6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 10*c)
 + 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c)
 + a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) +
 a)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 10*
a*cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*a*si

n(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6*d*x
 + 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100
*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) - 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8*c
)^2 + 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*x
+ 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6*d
*x + 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*x
+ 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 10*
c) + 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*
c) + a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c)
 + a)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 1
0*a*cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*a*
sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6*d
*x + 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 1
00*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) + 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8
*c)^2 + 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*
x + 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6
*d*x + 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*
x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 1
0*c) + 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x +
2*c) + a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*
c) + a)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 10*a*cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*
a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6
*d*x + 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
 100*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) - 7980*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*
a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x
 + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(19/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2660*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqr
t(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(
4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(17/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 38304*(sqrt(2)*a*cos(10*d*x + 10*c) +
 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*
a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(15/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12160*(sqrt(2)*a*cos(10*d*x + 1
0*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sq
rt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(13
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 45400*(sqrt(2)*a*cos(10*d
*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) +
 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 45400*(sqrt(2)*a*co
s(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x +
6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(
2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12160*(sqrt(2)
*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d
*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) +

```
sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 38304*(sq
rt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*co
s(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2660
*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*
a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
7980*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt
(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*
d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*C*sqrt(a)/(2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*d*x +
4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 +
10*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)
^2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2
*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x +
6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d
*x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.05582, size = 1405, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/7680*(15*((176*A + 133*C)*a*cos(d*x + c)^5 + (176*A + 133*C)*a*cos(d*x +
 c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)
^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(15
*(176*A + 133*C)*a*cos(d*x + c)^4 + 10*(176*A + 133*C)*a*cos(d*x + c)^3 + 8
*(80*A + 133*C)*a*cos(d*x + c)^2 + 912*C*a*cos(d*x + c) + 384*C*a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c)^5 + d*cos(d*x + c)^4), 1/3840*(15*((176*A + 133*C)*a*cos(d*x + c)^5 +
 (176*A + 133*C)*a*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 - a*cos(d*x + c) - 2*a)) + 2*(15*(176*A + 133*C)*a*cos(d*x + c)^4 + 10*(1
76*A + 133*C)*a*cos(d*x + c)^3 + 8*(80*A + 133*C)*a*cos(d*x + c)^2 + 912*C*
a*cos(d*x + c) + 384*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5
/2), x)
```

3.259 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=218

$$\frac{a^2(16A+13C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{32d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(112A+75C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{a\sec(c-}}\right.}{64d}$$

```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^2*(112*A + 75*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*
Sqrt[a + a*Sec[c + d*x]]) + (a^2*(16*A + 13*C)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(32*d*Sqrt[a + a*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.601322, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(16A+13C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{32d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(112A+75C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{a\sec(c-}}\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^2*(112*A + 75*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*
Sqrt[a + a*Sec[c + d*x]]) + (a^2*(16*A + 13*C)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(32*d*Sqrt[a + a*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d} + \frac{\int \cdots}{\cdots}$$

$$= \frac{aC\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{8d} + \frac{C\,\text{s}\cdots}{\cdots}$$

$$= \frac{a^2(16A+13C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{32d\sqrt{a+a\sec(c+dx)}} + \frac{aC\sec^{\frac{5}{2}}(c\cdots}{\cdots}$$

$$= \frac{a^2(112A+75C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A+\cdots}{\cdots}$$

$$= \frac{a^2(112A+75C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A+\cdots}{\cdots}$$

$$= \frac{a^{3/2}(112A+75C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^2(112A\cdots}{\cdots}$$

Mathematica [A] time = 2.90213, size = 251, normalized size = 1.15

$$\frac{\cos^3(c+dx)(a(\sec(c+dx)+1))^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{\sec(c+dx)+1}(7(48A+55C\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(3/2)*(A + C*Sec[c + d*x]^2)*((64*A
+ 164*C + 7*(48*A + 55*C)*Cos[c + d*x] + 4*(16*A + 25*C)*Cos[2*(c + d*x)] +
 112*A*Cos[3*(c + d*x)] + 75*C*Cos[3*(c + d*x)])*Sec[c + d*x]^(9/2)*Sqrt[1
+ Sec[c + d*x]]*Tan[(c + d*x)/2] - 4*(112*A + 75*C)*Csc[c + d*x]*(Log[1 + S
ec[c + d*x]] - Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c
 + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(128*d*(A + 2*C + A*
Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.349, size = 448, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/128/d*a*(-1+cos(d*x+c))*(112*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-112*A*cos(d*x+c)^4*2^(1
/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
+75*C*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))-75*C*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+224*A*sin(d*x+c)*cos(d*x+c)^
3*(-2/(cos(d*x+c)+1))^(1/2)+150*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1
))^(1/2)+64*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+100*C*sin(d
*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+80*C*sin(d*x+c)*cos(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2)+32*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d
*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(cos(d*x+c)+1))^(1/2)/c
os(d*x+c)^2/sin(d*x+c)^2
```

Maxima [B] time = 3.66433, size = 7777, normalized size = 35.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] -1/256*(16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
4*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*s
in(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*si
n(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*si
n(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos
```

```
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(
2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(
2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(2*(2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2
```

*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (300*(sqrt(2)*a*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) + 100(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*
sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(13/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*sin(8*d*x + 8*c) +
4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*s
in(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 22
8*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*
sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*
sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a
*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x +
4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 100*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x +
6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(sqrt(2)*a*sin(8*d*x +
8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt
(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4
*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6
*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) +
4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*
cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(
4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d
*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*
c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x +
 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x +
 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c)
+ 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*co
s(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4
*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x +
4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x
 + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x
 + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c
) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) +
4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*
cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin
(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x
+ 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(

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sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d
*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d
*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4
*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c)
+ 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a
)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*s
in(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*
x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
+ 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*a*cos(
8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c)
+ 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(13/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos
(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c)
 + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 228*(
sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos
(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*s
qrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(
2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*s
qrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*a*cos(8*d*x
+ 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sq
rt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x +
 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(
2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))))*C*sqrt(a)/(2*(4
*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x
+ 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(
6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.04114, size = 1268, normalized size = 5.82

$$\left[\frac{\left((112\,A+75\,C)a\cos\left(dx+c\right)^4+(112\,A+75\,C)a\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{}}{\sqrt{\cos(dx+}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{256\left(d\cos\left(dx+c\right)^4+d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/256*(((112*A + 75*C)*a*cos(d*x + c)^4 + (112*A + 75*C)*a*cos(d*x + c)^3)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((112*A +
 75*C)*a*cos(d*x + c)^3 + 2*(16*A + 25*C)*a*cos(d*x + c)^2 + 40*C*a*cos(d*x
 + c) + 16*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/128*(((112*A + 75*C)
*a*cos(d*x + c)^4 + (112*A + 75*C)*a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*((112*A + 75*C)*a*cos(d*x
+ c)^3 + 2*(16*A + 25*C)*a*cos(d*x + c)^2 + 40*C*a*cos(d*x + c) + 16*C*a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*
cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3
/2), x)
```

3.260 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=171

$$\frac{a^2(24A+19C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aC\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\,s}}{4d}$$

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^2*(24*A + 19*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(4*d) + (C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.504945, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4089, 4018, 4016, 3801, 215}

$$\frac{a^2(24A+19C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aC\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\,s}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^2*(24*A + 19*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a*C*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(4*d) + (C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(3*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
```

```
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx=\frac{C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{\int\sqrt{\text{s}}}{}$$

$$=\frac{aC\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}+\frac{C\sec}{}$$

$$=\frac{a^2(24A+19C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{aC\sec^{\frac{3}{2}}(c+}{}$$

$$=\frac{a^2(24A+19C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{aC\sec^{\frac{3}{2}}(c+}{}$$

$$=\frac{a^{3/2}(24A+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^2(24A+19C}{24d\sqrt{}}$$

Mathematica [A] time = 2.23935, size = 223, normalized size = 1.3

$$\frac{\cos^3(c+dx)(a(\sec(c+dx)+1))^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{\sec(c+dx)+1}(3(8A+11C)\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(3/2)*(A + C*Sec[c + d*x]^2)*((24*A
+ 49*C + 44*C*Cos[c + d*x] + 3*(8*A + 11*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^
(7/2)*Sqrt[1 + Sec[c + d*x]]*Tan[(c + d*x)/2] - 6*(24*A + 11*C)*Csc[c + d*x
]*(Log[1 + Sec[c + d*x]] - Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sq
rt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(24*d*(A
 + 2*C + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.346, size = 388, normalized size = 2.3

$$\frac{a\left((\cos(dx+c))^2-1\right)}{96\,d\,(\cos(dx+c))^2\,(\sin(dx+c))^2}\left(72\,A\,(\cos(dx+c))^3\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/96/d*a*(72*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*2^(1/2)-72*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+33*C*cos(d*x+c)^3*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1
/2)-33*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2)+66*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+44*C*sin(d*x+
c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.6148, size = 4733, normalized size = 27.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/96*(24*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d
*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
 + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
```

```
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin
(2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - (132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
 + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
 + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
 + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
 + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*
cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*
x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x +
2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) +
 a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2
+ a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(
2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x
 + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6
*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c
)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a
*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*c
os(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2
*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d
*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x
 + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2
 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6
*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*
sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a
*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*s
```

```
in(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*
d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(
2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*
x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2
)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*C*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos
(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*
c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(
2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.786487, size = 1168, normalized size = 6.83

$$\left[\frac{3\left((24\,A+11\,C)a\cos\left(dx+c\right)^{3}+(24\,A+11\,C)a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\cdots}}{\sqrt{\cos(dx+c\cdots}}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\cdots\right.}{96\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{\cdots}\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(3*((24*A + 11*C)*a*cos(d*x + c)^3 + (24*A + 11*C)*a*cos(d*x + c)^2)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(8*A +
11*C)*a*cos(d*x + c)^2 + 22*C*a*cos(d*x + c) + 8*C*a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*c
os(d*x + c)^2), 1/48*(3*((24*A + 11*C)*a*cos(d*x + c)^3 + (24*A + 11*C)*a*c
os(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
 2*a)) + 2*(3*(8*A + 11*C)*a*cos(d*x + c)^2 + 22*C*a*cos(d*x + c) + 8*C*a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
c)), x)
```

3.261 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{a^2(8A-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{3aC\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{4d}$$

```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(8*A - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Sec[c + d*x]]) + (3*a*C*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[
c + d*x])/(4*d) + (C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c +
d*x])/(2*d)
```

Rubi [A] time = 0.489834, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4089, 4018, 4015, 3801, 215}

$$\frac{a^2(8A-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{3aC\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(8*A - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Sec[c + d*x]]) + (3*a*C*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[
c + d*x])/(4*d) + (C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c +
d*x])/(2*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d} + \frac{\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{1}{2}\right.}{\sqrt{s}}}{}$$

$$= \frac{3aC\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{C\sqrt{\sec(c+dx)}(a}{}$$

$$= \frac{a^2(8A-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{3aC\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}{4d}$$

$$= \frac{a^2(8A-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{3aC\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}{4d}$$

$$= \frac{a^{3/2}(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^2(8A-5C)\sqrt{\sec(c+dx)}\sin}{4d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 4.62418, size = 209, normalized size = 1.22

$$\frac{(a(\sec(c+dx)+1))^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)(4A\cos(2(c+dx))+4A+7C\cos(c+dx)+2C)}{\sqrt{\frac{1}{\cos(c+dx)+1}}}-(8A+7C)\cos^2(c+dx)\sqrt{\tan^2}\right.}{2d(\sec(c+dx)+1)^{3/2}(A\cos}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c +
d*x]],x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(3/2)*(A + C*Sec[c + d*x]^2)*(((4*A + 2*C + 7*C*Cos
[c + d*x] + 4*A*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/Sqrt[(1 + Cos[c + d*x])
^(-1)] - (8*A + 7*C)*Cos[c + d*x]^2*Cot[c + d*x]*(Log[1 + Sec[c + d*x]] - L
og[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[Ta
n[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(2*d*(A + 2*C + A*Cos[2*(c + d*x)])*
(1 + Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.378, size = 375, normalized size = 2.2

$$-\frac{a}{16 d \sin(dx+c)\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(8 A\sqrt{2}\sin(dx+c)(\cos(dx+c))^{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/16/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*A*2^(1/2)*sin(d*x+c)*cos(d
*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c)))*(-2/(cos(d*x+c)+1))^(1/2)-8*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos
(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1-sin(d*x+c)))+7*C*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)-7
*C*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(
1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+32*A*cos(d*x+c)^3
-32*A*cos(d*x+c)^2+28*C*cos(d*x+c)^2-20*C*cos(d*x+c)-8*C)*(1/cos(d*x+c))^(1
/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [B] time = 2.40772, size = 3402, normalized size = 19.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*(4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A
*sqrt(a) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
4*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*s
in(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*si
n(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*si
n(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
```

```
3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(
2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(
2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sqrt(a)/(2*(2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*ar
```

```
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d
```

Fricas [A] time = 0.791901, size = 1107, normalized size = 6.47

$$\left[\frac{\left((8A+7C)a\cos(dx+c)^2+(8A+7C)a\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*(((8*A + 7*C)*a*cos(d*x + c)^2 + (8*A + 7*C)*a*cos(d*x + c))*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(8*A*a*cos(d*x + c
)^2 + 7*C*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*((
(8*A + 7*C)*a*cos(d*x + c)^2 + (8*A + 7*C)*a*cos(d*x + c))*sqrt(-a)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(8*A*a*cos(d*x + c)^
2 + 7*C*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x +
c)), x)
```

3.262 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=169

$$\frac{a^2(8A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+}}{3d}$$

```
[Out] (3*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a^2*(8*A - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]]) - (a*(2*A - 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(3*d) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]])
```

Rubi [A] time = 0.467308, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4018, 4015, 3801, 215}

$$\frac{a^2(8A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (3*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a^2*(8*A - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]]) - (a*(2*A - 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(3*d) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{3aA}{2}-\frac{1}{2}a(2\ldots\right.}{\sqrt{\sec(c+dx)}\ldots}}{3a}$$

$$= -\frac{a(2A-3C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A(a\ldots}{\ldots}$$

$$= \frac{a^2(8A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a(2A-3C)\sqrt{\sec(c+dx)}\ldots}{\ldots}$$

$$= \frac{a^2(8A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a(2A-3C)\sqrt{\sec(c+dx)}\ldots}{\ldots}$$

$$= \frac{3a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2(8A-3C)\sqrt{\sec(c+dx)}\sin(c+\ldots}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 6.42657, size = 382, normalized size = 2.26

$$\frac{6C\sin(c+dx)\cos^3(c+dx)\sqrt{\sec^2(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}\left(\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+\ldots}\right.\right.}{d\left(1-\cos^2(c+dx)\right)(\sec(c+dx)+1)^{3/2}(A\cos(2c+2dx)\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(3/2),x]
```

```
[Out] (6*C*Cos[c + d*x]^3*(-Log[1 + Sec[c + d*x]] + Log[Sqrt[Sec[c + d*x]] + Sec[
c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(a*(1 +
 Sec[c + d*x]))^(3/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin[
c + d*x])/(d*(1 - Cos[c + d*x]^2)*(A + 2*C + A*Cos[2*c + 2*d*x])*(1 + Sec[c
 + d*x])^(3/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(a*(1 + Sec[c + d*
```

```
x]))^(3/2)*(A + C*Sec[c + d*x]^2)*((16*A*Cos[d*x]*Sin[c])/(3*d) + (2*A*Cos[
2*d*x]*Sin[2*c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(8*A*Sin[(d*x)/2] -
 3*C*Sin[(d*x)/2]))/(3*d) + (16*A*Cos[c]*Sin[d*x])/(3*d) + (2*A*Cos[2*c]*Si
n[2*d*x])/(3*d) - (2*(8*A - 3*C)*Tan[c/2])/(3*d)))/((A + 2*C + A*Cos[2*c +
2*d*x])*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.362, size = 229, normalized size = 1.4

$$-\frac{a\cos(dx+c)}{12\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(9\,C\cos(dx+c)\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/12/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(9*C*cos(d*x+c)*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*2^(1/2)-9*C*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)
))*2^(1/2)+8*A*cos(d*x+c)^3+32*A*cos(d*x+c)^2-40*A*cos(d*x+c)+12*C*cos(d*x+
c)-12*C)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.05753, size = 1597, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))
*A*sqrt(a) + 3*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
- 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin
(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3
/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
```

```
- 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)
 + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c
))*sin(2*d*x + 2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.607756, size = 980, normalized size = 5.8

$$\left[\frac{9\,(Ca\cos(dx+c)+Ca)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(2\,Aa\cos(dx+c)^2 \cdots\right.}{\cdots}}{12\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/12*(9*(C*a*cos(d*x + c) + C*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(2*A*a*cos(d*x + c)^2 + 10*A*a*cos(d*x + c) + 3*C*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/
(d*cos(d*x + c) + d), 1/6*(9*(C*a*cos(d*x + c) + C*a)*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*a*cos(d*x + c)^2 + 1
0*A*a*cos(d*x + c) + 3*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3
/2), x)
```

3.263 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2a^2(4A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2aA$$

```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(4*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d
*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/
2))
```

Rubi [A] time = 0.464401, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4017, 4015, 3801, 215}

$$\frac{2a^2(4A+5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2aA$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),
x]
```


```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(4*A + 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d
*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/
2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{5}{2}aC\sec\right.}{\sec^{\frac{3}{2}}(c+dx)}}{5a}$$

$$= \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(4A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin}{5d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^2(4A+5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin}{5d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2(4A+5C)\sqrt{\sec(c+dx)}\sin(c+}{5d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 6.29136, size = 428, normalized size = 2.63

$$\frac{4C\sin(c+dx)\cos^3(c+dx)\sqrt{\sec^2(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}\left(\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+dx)}\right.\right.}{d\left(1-\cos^2(c+dx)\right)(\sec(c+dx)+1)^{3/2}(A\cos(2c+2dx)+A}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(5/2),x]
```

```
[Out] (4*C*Cos[c + d*x]^3*(-Log[1 + Sec[c + d*x]] + Log[Sqrt[Sec[c + d*x]] + Sec[
c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(a*(1 +
 Sec[c + d*x]))^(3/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin[
c + d*x])/(d*(1 - Cos[c + d*x]^2)*(A + 2*C + A*Cos[2*c + 2*d*x])*(1 + Sec[c
```

```
 + d*x])^(3/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(a*(1 + Sec[c + d*
x]))^(3/2)*(A + C*Sec[c + d*x]^2)*(((17*A + 20*C)*Cos[d*x]*Sin[c])/(5*d) +
(4*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (A*Cos[3*d*x]*Sin[3*c])/(5*d) - (4*Sec[c/
2]*Sec[c/2 + (d*x)/2]*(4*A*Sin[(d*x)/2] + 5*C*Sin[(d*x)/2]))/(5*d) + ((17*A
 + 20*C)*Cos[c]*Sin[d*x])/(5*d) + (4*A*Cos[2*c]*Sin[2*d*x])/(5*d) + (A*Cos[
3*c]*Sin[3*d*x])/(5*d) - (4*(4*A + 5*C)*Tan[c/2])/(5*d)))/((A + 2*C + A*Cos
[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.592, size = 222, normalized size = 1.4

$$-\frac{a\left(\cos\left(dx+c\right)\right)^{3}}{10\,d\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(5\,C\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/10/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*C*(-2/(cos(d*x+c)+1))^(1/2
)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))*sin(d*x+c)-5*C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)+4*A*cos(d*x
+c)^3+8*A*cos(d*x+c)^2+12*A*cos(d*x+c)+20*C*cos(d*x+c)-24*A-20*C)*cos(d*x+c
)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.1075, size = 655, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/20*(sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5
/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*si
n(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))))*A*sqrt(a) + 5*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) + sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x
+ 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 0.60433, size = 1034, normalized size = 6.34

$$\left[\frac{5\,(Ca\cos(dx+c)+Ca)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(Aa\cos(dx+c)^3+3\,Aa\right.}{}}{10\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/10*(5*(C*a*cos(d*x + c) + C*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(A*a*cos(d*x + c)^3 + 3*A*a*cos(d*x + c)^2 + (6*A +
 5*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/5*(5*(C*a*cos(d*x + c) + C*a)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(A*a*
cos(d*x + c)^3 + 3*A*a*cos(d*x + c)^2 + (6*A + 5*C)*a*cos(d*x + c))*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5
/2), x)
```

3.264 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=169

$$\frac{8a^2(19A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+35C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{6A\sin(c+dx)(a\sec(c+}{35d\sec^{\frac{3}{2}}(c+d}$$

```
[Out] (8*a^2*(19*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec
[c + d*x]]) + (2*a*(19*A + 35*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(5/2)) + (6*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d
*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.413674, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4087, 4013, 3809, 3804}

$$\frac{8a^2(19A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+35C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{6A\sin(c+dx)(a\sec(c+}{35d\sec^{\frac{3}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (8*a^2*(19*A + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec
[c + d*x]]) + (2*a*(19*A + 35*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(5/2)) + (6*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d
*Sec[c + d*x]^(3/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
```

```
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{1}{2}a(2A+\right.}{\sec^{\frac{5}{2}}(c+dx)}}{7a}$$

$$= \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6A(a+a\sec(c+dx))^{3/2}\sin(c}{35d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a(19A+35C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{8a^2(19A+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(19A+35C)\sqrt{a+a\,\mathrm{se}}}{105d\sqrt{\sec(}}$$

Mathematica [A] time = 1.07534, size = 85, normalized size = 0.5

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((253A+140C)\cos(c+dx)+78A\cos(2(c+dx))+15A\cos(3(c+dx))+494A+}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(7/2),x]
```

```
[Out] (a*(494*A + 700*C + (253*A + 140*C)*Cos[c + d*x] + 78*A*Cos[2*(c + d*x)] +
15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*
Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.329, size = 108, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+39A(\cos(dx+c))^2+52A\cos(dx+c)+35C\cos(dx+c)+104A+17}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+52*A*cos(d*
x+c)+35*C*cos(d*x+c)+104*A+175*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d
*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.00468, size = 462, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 280*(sqrt(2)*a*sin(3/2*d*x
 + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 0.486802, size = 292, normalized size = 1.73

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{4}+39\,Aa\cos\left(dx+c\right)^{3}+\left(52\,A+35\,C\right)a\cos\left(dx+c\right)^{2}+\left(104\,A+175\,C\right)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx}{\cos(dx}}}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^4 + 39*A*a*cos(d*x + c)^3 + (52*A + 35*C)*a*cos(
d*x + c)^2 + (104*A + 175*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7
/2), x)
```

3.265 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=219

$$\frac{2a^2(52A+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+189C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+189C)\sin(c+}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+d}}$$

```
[Out] (2*a^2*(52*A + 63*C)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]
]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(136*A + 189*C)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x
]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*A*(a + a*Sec[c + d*x])^(3/2
)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.586297, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4017, 4015, 3805, 3804}

$$\frac{2a^2(52A+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+189C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+189C)\sin(c+}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (2*a^2*(52*A + 63*C)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]
]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(136*A + 189*C)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x
]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*A*(a + a*Sec[c + d*x])^(3/2
)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{1}{2}a(4A+9}{\sec^{\frac{7}{2}}(c+dx)}}{9a}$$

$$= \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(52A+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c}{21d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(52A+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(136A+189C)\sin(c}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{2a^2(52A+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(136A+189C)\sin(c}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}$$

Mathematica [A] time = 1.37166, size = 103, normalized size = 0.47

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(799A+756C)\cos(c+dx)+4(137A+63C)\cos(2(c+dx))+170A\cos(3(c+d}{1260d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(9/2),x]
```

```
[Out] (a*(2689*A + 3276*C + 2*(799*A + 756*C)*Cos[c + d*x] + 4*(137*A + 63*C)*Cos
[2*(c + d*x)] + 170*A*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 +
 Sec[c + d*x])]*Tan[(c + d*x)/2])/(1260*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.375, size = 130, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(35A(\cos(dx+c))^4+85A(\cos(dx+c))^3+102A(\cos(dx+c))^2+63C(\cos(dx+c))^2+1\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+102*A*cos(d
*x+c)^2+63*C*cos(d*x+c)^2+136*A*cos(d*x+c)+189*C*cos(d*x+c)+272*A+378*C)*(a
*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x
+c)
```

Maxima [B] time = 2.10143, size = 819, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) + 252*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*s
in(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d
*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
2*a*sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.493636, size = 344, normalized size = 1.57

$$\frac{2\left(35\,Aa\cos(dx+c)^5+85\,Aa\cos(dx+c)^4+3(34\,A+21\,C)a\cos(dx+c)^3+(136\,A+189\,C)a\cos(dx+c)^2+2(13\right.}{315(d\cos(dx+c)+d)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^5 + 85*A*a*cos(d*x + c)^4 + 3*(34*A + 21*C)*a*co
s(d*x + c)^3 + (136*A + 189*C)*a*cos(d*x + c)^2 + 2*(136*A + 189*C)*a*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x +
 c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9
/2), x)
```

3.266 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{2a^2(112A+143C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(28A+33C)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(112A+143C)\sin(c+dx)\sqrt{\sec(}}{1155d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a^2*(28*A + 33*C)*Sin[c + d*x])/(231*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(112*A + 143*C)*Sin[c + d*x])/(385*d*Sec[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sin[c + d*x])/(1155*d*
Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(112*A + 143*C)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(33*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```

Rubi [A] time = 0.677311, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4017, 4015, 3805, 3804}

$$\frac{2a^2(112A+143C)\sin(c+dx)}{385d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(28A+33C)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(112A+143C)\sin(c+dx)\sqrt{\sec(}}{1155d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (2*a^2*(28*A + 33*C)*Sin[c + d*x])/(231*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(112*A + 143*C)*Sin[c + d*x])/(385*d*Sec[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sin[c + d*x])/(1155*d*
Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(112*A + 143*C)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(33*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{3/2}\left(\frac{3aA}{2}+\frac{1}{2}a(6A+\right.}{\sec^{\frac{9}{2}}(c+dx)}}{11a}$$

$$= \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{33d\sec^{\frac{7}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(c+dx)}\sin(c}{33d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(112A+143C)\sin(c}{385d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(112A+143C)\sin(c}{385d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2a^2(28A+33C)\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(112A+143C)\sin(c}{385d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

Mathematica [A] time = 1.9047, size = 125, normalized size = 0.47

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(5789A+5566C)\cos(c+dx)+8(581A+429C)\cos(2(c+dx))+1645A\cos(3(c}{9240d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(11/2),x]
```

```
[Out] (a*(18494*A + 21736*C + 2*(5789*A + 5566*C)*Cos[c + d*x] + 8*(581*A + 429*C
)*Cos[2*(c + d*x)] + 1645*A*Cos[3*(c + d*x)] + 660*C*Cos[3*(c + d*x)] + 490
*A*Cos[4*(c + d*x)] + 105*A*Cos[5*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Ta
n[(c + d*x)/2])/(9240*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.365, size = 152, normalized size = 0.6

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(105\,A\,(\cos(dx+c))^{5}+245\,A\,(\cos(dx+c))^{4}+280\,A\,(\cos(dx+c))^{3}+165\,C\,(\cos(dx+c))^{3}}{1155\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/1155/d*a*(-1+cos(d*x+c))*(105*A*cos(d*x+c)^5+245*A*cos(d*x+c)^4+280*A*co
s(d*x+c)^3+165*C*cos(d*x+c)^3+336*A*cos(d*x+c)^2+429*C*cos(d*x+c)^2+448*A*c
os(d*x+c)+572*C*cos(d*x+c)+896*A+1144*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2
)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [B] time = 2.15183, size = 1072, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 1/36960*(7*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11/2
*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*cos
(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x
 + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 11/
2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d*x
 + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)
)) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c),
cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/2*
c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55*a
*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d
*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11/2
*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/2*
d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/2*
d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) + 44*sqrt(2)*(735*a*cos(
6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*
c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin
(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 175*a*cos(7/2*d*x + 7/2*c)
*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 63*a*cos(7/
2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))
```

```
+ 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.499946, size = 401, normalized size = 1.51

$$\frac{2\left(105\,Aa\cos\left(dx+c\right)^{6}+245\,Aa\cos\left(dx+c\right)^{5}+5\,(56\,A+33\,C)a\cos\left(dx+c\right)^{4}+3\,(112\,A+143\,C)a\cos\left(dx+c\right)^{3}+4\,(1\right.}{1155\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/1155*(105*A*a*cos(d*x + c)^6 + 245*A*a*cos(d*x + c)^5 + 5*(56*A + 33*C)*a
*cos(d*x + c)^4 + 3*(112*A + 143*C)*a*cos(d*x + c)^3 + 4*(112*A + 143*C)*a*
cos(d*x + c)^2 + 8*(112*A + 143*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1
1/2), x)
```

3.267 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=312

$$\frac{a^2(24A+23C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{96d}+\frac{a^3(136A+109C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1304A}{}$$

```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(512*d) + (a^3*(1304*A + 1015*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(512*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1304*A + 1015*C)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(768*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(136*A + 109*C)*Se
c[c + d*x]^(7/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*
A + 23*C)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(96*d)
+ (a*C*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) +
 (C*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.897397, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(24A+23C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{96d}+\frac{a^3(136A+109C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1304A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(512*d) + (a^3*(1304*A + 1015*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(512*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1304*A + 1015*C)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(768*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(136*A + 109*C)*Se
c[c + d*x]^(7/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*
A + 23*C)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(96*d)
+ (a*C*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) +
 (C*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{6d} + \frac{\int\ }{}$$

$$= \frac{aC\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{12d} + \frac{C}{}$$

$$= \frac{a^2(24A+23C)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c}{96d}$$

$$= \frac{a^3(136A+109C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(24A}{}$$

$$= \frac{a^3(1304A+1015C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{768d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(13}{}$$

$$= \frac{a^3(1304A+1015C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{512d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(13}{}$$

$$= \frac{a^3(1304A+1015C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{512d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(13}{}$$

$$= \frac{a^{5/2}(1304A+1015C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{512d} + \frac{a^3(130}{}$$

Mathematica [A] time = 4.07644, size = 295, normalized size = 0.95

$$\frac{\cos^3(c+dx)(a(\sec(c+dx)+1))^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\sqrt{\sec(c+dx)+1}(14(4056A+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*(A + C*Sec[c + d*x]^2)*((18720
*A + 27412*C + 14*(4056*A + 4591*C)*Cos[c + d*x] + 16*(1496*A + 1711*C)*Cos
[2*(c + d*x)] + 25448*A*Cos[3*(c + d*x)] + 21721*C*Cos[3*(c + d*x)] + 5216*
A*Cos[4*(c + d*x)] + 4060*C*Cos[4*(c + d*x)] + 3912*A*Cos[5*(c + d*x)] + 30
45*C*Cos[5*(c + d*x)])*Sec[c + d*x]^(13/2)*Sqrt[1 + Sec[c + d*x]]*Tan[(c +
d*x)/2] - 48*(1304*A + 1015*C)*Csc[c + d*x]*(Log[1 + Sec[c + d*x]] - Log[Sq
rt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c +
 d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(12288*d*(A + 2*C + A*Cos[2*(c + d*x)])*(
1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.394, size = 576, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/6144/d*a^2*(3912*A*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-3912*A*cos(d*x+c)^6*2^(1/2)*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+3045*C*cos(
d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
+sin(d*x+c)))-3045*C*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+7824*A*cos(d*x+c)^5*(-2/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)+6090*C*cos(d*x+c)^5*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+5216*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+4060*C*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+2944*A*sin(d*x+c)*cos(d*x+c)^3*(-
2/(cos(d*x+c)+1))^(1/2)+3248*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^
(1/2)+768*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2784*C*sin(d*
x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+1792*C*sin(d*x+c)*cos(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)+512*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos
(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(cos(d*x+c)+1))^(1/2)
/cos(d*x+c)^3/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 11.0557, size = 14959, normalized size = 47.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] -1/6144*(8*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos
(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8
*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*
d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c
))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^
2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d
*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*
a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2
*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(s
qrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^
2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6
*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2
*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2
 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8
*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x +
 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x
 + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(
4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d
*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
```

```
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2
*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 +
16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c)
+ a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*
x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(
4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*co
s(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 +
16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c
)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6
*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*
d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(
6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^
2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*
a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c
)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x
 + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2
*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*co
s(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*
cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a
^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3
*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^
2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(
6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*si
n(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos
(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x +
4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sq
rt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2
*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x +
 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + s
qrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sq
rt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2
*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x +
8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*
```

cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*A*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c
) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*co
s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2
c)^2 + 4(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1) + (12180*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10
*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(23/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4060*(sqrt(2)*a^2*si
n(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*
d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(21/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 70644*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*si
n(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d
*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c
))*cos(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 22620*(sqrt(2)*a
^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*s
in(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*
x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(17/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 147592*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*
a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*s
in(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 37800*(sqr
t(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)
*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 37800*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqr
t(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*
a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 14759
2*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*s
qrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*
a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 22620*(sqrt(2)*a^2*sin(12*d*x + 12*c) +
6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqr
t(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2
*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 7
0644*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 1
5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(
2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4060*(sqrt(2)*a^2*sin(12*d*x + 12*c)
 + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*s
qrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a
^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 12180*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) +
 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqr
t(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3045*(a^2*cos(12*d*x + 12*c)^2 + 36
*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*cos(8*d*x + 8*c)^2 + 400*a^2*cos(6*d*x
 + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^2 + 36*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*x + 10*c)^2 + 225*a^2*sin(8*d*x + 8*c)^
2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225*a^2*sin(4*d*x + 4*c)^2 + 180*a^2*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^2*sin(2*d*x + 2*c)^2 + 12*a^2*cos(2*d*x
 + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x + 10*c) + 15*a^2*cos(8*d*x + 8*c) + 20*

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a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a
^2)*cos(12*d*x + 12*c) + 12*(15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6
*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x +
10*c) + 30*(20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2
*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 40*(15*a^2*cos(4*d*x + 4*c) + 6*a^2*c
os(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 30*(6*a^2*cos(2*d*x + 2*c) + a^2)
*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*d*x + 10*c) + 15*a^2*sin(8*d*x + 8*c) +
 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c)
)*sin(12*d*x + 12*c) + 12*(15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c
) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) +
30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + 120*(5*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3045*(a^2*cos(12*d*x +
 12*c)^2 + 36*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*cos(8*d*x + 8*c)^2 + 400*a
^2*cos(6*d*x + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^2 + 36*a^2*cos(2*d*x + 2*c
)^2 + a^2*sin(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*x + 10*c)^2 + 225*a^2*sin(
8*d*x + 8*c)^2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225*a^2*sin(4*d*x + 4*c)^2 +
180*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^2*sin(2*d*x + 2*c)^2 + 12*
a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x + 10*c) + 15*a^2*cos(8*d*x
 + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d
*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*cos(8*d*x + 8*c) + 20*a^2*
cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*
cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c)
+ 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 40*(15*a^2*cos(4*d*x + 4
*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 30*(6*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*d*x + 10*c) + 15*a^2*sin(8
*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin
(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*sin(8*d*x + 8*c) + 20*a^2*si
n(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(10*d
*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*
sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*sin(4*d*x + 4*c) + 2*a^2*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3045*(a^2
*cos(12*d*x + 12*c)^2 + 36*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*cos(8*d*x + 8
*c)^2 + 400*a^2*cos(6*d*x + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^2 + 36*a^2*co
s(2*d*x + 2*c)^2 + a^2*sin(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*x + 10*c)^2 +
 225*a^2*sin(8*d*x + 8*c)^2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225*a^2*sin(4*d*
x + 4*c)^2 + 180*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^2*sin(2*d*x +
 2*c)^2 + 12*a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x + 10*c) + 15*
a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) +
6*a^2*cos(2*d*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*cos(8*d*x + 8
*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x +
 2*c) + a^2)*cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(
4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 40*(15*a^2*
cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 30*(6*a
^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*d*x + 10*c) +
 15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c
) + 6*a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*sin(8*d*x + 8*c
) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2
*c))*sin(10*d*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x +
4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*sin(4*d*x + 4*
c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
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2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
) + 3045*(a^2*cos(12*d*x + 12*c)^2 + 36*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*
cos(8*d*x + 8*c)^2 + 400*a^2*cos(6*d*x + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^
2 + 36*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*
x + 10*c)^2 + 225*a^2*sin(8*d*x + 8*c)^2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225
*a^2*sin(4*d*x + 4*c)^2 + 180*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^
2*sin(2*d*x + 2*c)^2 + 12*a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x
+ 10*c) + 15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*
d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*
cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^
2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c)
+ 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c)
+ 40*(15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6
c) + 30(6*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*
d*x + 10*c) + 15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*si
n(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*si
n(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*
sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2
*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*s
in(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2) - 12180*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(
10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x
 + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(23/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4060
*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sq
rt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a
^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(21/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 70644*(sqrt(2)*a^2*cos(12*
d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x +
 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) +
 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(19/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 22620*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(
2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^
2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*
d*x + 2*c) + sqrt(2)*a^2)*sin(17/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 147592*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x +
10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) +
 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)
*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 37800*(sqrt(2
)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^
2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4
*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 37800*(sqrt(2)*a^2*cos(12*d*x + 12
*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) +
20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(
2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 147592*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*
cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6
*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
2620*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 1
5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(
2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin
(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 70644*(sqrt(2)*a^2*cos(
12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*
x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c

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) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4060*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt
(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a
^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2
*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 12180*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 1
0*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) +
15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(6*
cos(10*d*x + 10*c) + 15*cos(8*d*x + 8*c) + 20*cos(6*d*x + 6*c) + 15*cos(4*d
*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(12*d*x + 12*c) + cos(12*d*x + 12*c)
^2 + 12*(15*cos(8*d*x + 8*c) + 20*cos(6*d*x + 6*c) + 15*cos(4*d*x + 4*c) +
6*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + 36*cos(10*d*x + 10*c)^2 + 30*(
20*cos(6*d*x + 6*c) + 15*cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(8*d
*x + 8*c) + 225*cos(8*d*x + 8*c)^2 + 40*(15*cos(4*d*x + 4*c) + 6*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 400*cos(6*d*x + 6*c)^2 + 30*(6*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + 225*cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2
+ 2*(6*sin(10*d*x + 10*c) + 15*sin(8*d*x + 8*c) + 20*sin(6*d*x + 6*c) + 15*
sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + sin(12*d*x + 12
*c)^2 + 12*(15*sin(8*d*x + 8*c) + 20*sin(6*d*x + 6*c) + 15*sin(4*d*x + 4*c)
 + 6*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 36*sin(10*d*x + 10*c)^2 + 30*(2
0*sin(6*d*x + 6*c) + 15*sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + 225*sin(8*d*x + 8*c)^2 + 120*(5*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 400*sin(6*d*x + 6*c)^2 + 225*sin(4*d*x + 4*c)^2 + 180
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 12*cos(2*d*x +
 2*c) + 1))/d
```

Fricas [A] time = 1.06953, size = 1559, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/6144*(3*((1304*A + 1015*C)*a^2*cos(d*x + c)^6 + (1304*A + 1015*C)*a^2*co
s(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d
*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
+ 4*(3*(1304*A + 1015*C)*a^2*cos(d*x + c)^5 + 2*(1304*A + 1015*C)*a^2*cos(d
*x + c)^4 + 8*(184*A + 203*C)*a^2*cos(d*x + c)^3 + 48*(8*A + 29*C)*a^2*cos(
d*x + c)^2 + 896*C*a^2*cos(d*x + c) + 256*C*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^6 + d*cos(d*
x + c)^5), 1/3072*(3*((1304*A + 1015*C)*a^2*cos(d*x + c)^6 + (1304*A + 1015
*C)*a^2*cos(d*x + c)^5)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d
*x + c) - 2*a)) + 2*(3*(1304*A + 1015*C)*a^2*cos(d*x + c)^5 + 2*(1304*A + 1
015*C)*a^2*cos(d*x + c)^4 + 8*(184*A + 203*C)*a^2*cos(d*x + c)^3 + 48*(8*A
+ 29*C)*a^2*cos(d*x + c)^2 + 896*C*a^2*cos(d*x + c) + 256*C*a^2)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5
/2), x)
```

3.268 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=265

$$\frac{a^3(1040A+787C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{960d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(400A+283C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+d}{}$$

```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^3*(400*A + 283*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1040*A + 787*C)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(80*A + 79*C)*Sec[c + d
*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(240*d) + (a*C*Sec[c + d*x
]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d) + (C*Sec[c + d*x]^(5
/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.783079, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(1040A+787C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{960d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(400A+283C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^3*(400*A + 283*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1040*A + 787*C)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(80*A + 79*C)*Sec[c + d
*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(240*d) + (a*C*Sec[c + d*x
]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d) + (C*Sec[c + d*x]^(5
/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{\int \sec}{}$$

$$= \frac{aC\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{8d} + \frac{C\,\mathrm{se}}{}$$

$$= \frac{a^2(80A+79C)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{240d}$$

$$= \frac{a^3(1040A+787C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{960d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(80A+}{}$$

$$= \frac{a^3(400A+283C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1040A}{9}$$

$$= \frac{a^3(400A+283C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1040A}{9}$$

$$= \frac{a^{5/2}(400A+283C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{a^3(400A+}{12}$$

Mathematica [A] time = 3.79227, size = 273, normalized size = 1.03

$$\cos^3(c+dx)(a(\sec(c+dx)+1))^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{\sec(c+dx)+1}(12(1360A+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*(A + C*Sec[c + d*x]^2)*((20560
*A + 24863*C + 12*(1360*A + 2343*C)*Cos[c + d*x] + 4*(6640*A + 6509*C)*Cos[
2*(c + d*x)] + 5440*A*Cos[3*(c + d*x)] + 5660*C*Cos[3*(c + d*x)] + 6000*A*C
os[4*(c + d*x)] + 4245*C*Cos[4*(c + d*x)])*Sec[c + d*x]^(11/2)*Sqrt[1 + Sec
[c + d*x]]*Tan[(c + d*x)/2] - 120*(400*A + 283*C)*Csc[c + d*x]*(Log[1 + Sec
[c + d*x]] - Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c +
 d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(7680*d*(A + 2*C + A*C
os[2*(c + d*x)])*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.391, size = 512, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/3840/d*a^2*(-1+cos(d*x+c))*(6000*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^5-6000*A*2^(1/2)*arct
an(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x
+c)^5+4245*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1+sin(d*x+c)))*cos(d*x+c)^5-4245*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^5+12000*A*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+8490*C*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)^4+5440*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/
2)+5660*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+1280*A*cos(d*x+
c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4528*C*sin(d*x+c)*cos(d*x+c)^2*(-
2/(cos(d*x+c)+1))^(1/2)+2784*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2)+768*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)*(1/cos(d*x+c))^(3/2)/cos(d*x+c)^3/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))
^(1/2)
```

Maxima [B] time = 6.27029, size = 11950, normalized size = 45.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/7680*(80*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*s
```

qrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sq
rt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d
*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)
*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*si
n(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2
+ 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*si
n(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c
) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^
2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*co
s(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c
) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6
*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6
*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6
*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/

2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^
2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6
*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^
2*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c
) + 3*sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*
x + 3/2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*
x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) - 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(cos(6*d*x + 6*c)^2 +
6*(cos(6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 9*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(co
s(6*d*x + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
sin(6*d*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*cos(6*d*x + 6*c) + 1) - (16980*(sqrt(2)*a^2*sin(10*d*x + 10*c)
 + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(19/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*sin(10*d*x + 10
*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10
*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(17/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 81504*(sqrt(2)*a^2*sin(10*d*x
+ 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c)
+ 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/

4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8320*(sqrt(2)*a^2*sin(10*d
*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*
c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(
13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 86440*(sqrt(2)*a^2*sin(
10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*
cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*a^2*
sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*
d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^
2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x +
2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 81504*(sqrt(2)
*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*s
in(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 5660*(sqrt(
2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2
*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d
*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16980*(sq
rt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*
a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4245*(
a^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x +
6*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(1
0*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 +
100*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25
*a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*
x + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*
d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2
*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10
*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20
*(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c)
 + 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*
sin(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) +
a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*
sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a
^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6
*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10
*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 1
00*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*
a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x
 + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d
*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*
cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*
a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*
(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c)
+ 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*s
in(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a
^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*s
in(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 4245*(a^
2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*
c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*
d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 10

0*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a
^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x
+ 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*
x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*c
os(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a
^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(
5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) +
2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*si
n(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^
2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a^2
*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c
)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d
*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100
*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^
2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x +
8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*co
s(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^
2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5
*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) +
2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin
(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16980*(sqr
t(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a
^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2
*d*x + 2*c) + sqrt(2)*a^2)*sin(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) - 5660(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*
c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(17/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*
a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^2*cos(10*d*x + 1
0*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 1
0*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a
^2)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*
a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*co
s(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 86440*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) +
10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 8320*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*co
s(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x
+ 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) +
5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*cos(
10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x
+ 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) +

```
 sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16980*
(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(
2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*c
os(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))))*C*sqrt(a)/(2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4
*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*
c)^2 + 10*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) +
 1)*cos(8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*
cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x
 + 2*c)^2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c)
+ sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6
*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*
sin(8*d*x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.05128, size = 1446, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/7680*(15*((400*A + 283*C)*a^2*cos(d*x + c)^5 + (400*A + 283*C)*a^2*cos(d
*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
+ c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*(15*(400*A + 283*C)*a^2*cos(d*x + c)^4 + 10*(272*A + 283*C)*a^2*cos(d*x +
c)^3 + 8*(80*A + 283*C)*a^2*cos(d*x + c)^2 + 1392*C*a^2*cos(d*x + c) + 384*
C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/3840*(15*((400*A + 283*C)*a^2
*cos(d*x + c)^5 + (400*A + 283*C)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(15*(400*A + 283*C)*a^2*c
os(d*x + c)^4 + 10*(272*A + 283*C)*a^2*cos(d*x + c)^3 + 8*(80*A + 283*C)*a^
2*cos(d*x + c)^2 + 1392*C*a^2*cos(d*x + c) + 384*C*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 + d
*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^2 + A\right)(a \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3
/2), x)
```

3.269 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=218

$$\frac{a^3(432A+299C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+17C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{32d}+\frac{a^{5/2}(304A+\ldots}{}$$

```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(64*d) + (a^3*(432*A + 299*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(192
*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(16*A + 17*C)*Sec[c + d*x]^(3/2)*Sqrt[a
 + a*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (5*a*C*Sec[c + d*x]^(3/2)*(a + a*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sec[c + d*x]^(3/2)*(a + a*Sec
[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.677448, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4089, 4018, 4016, 3801, 215}

$$\frac{a^3(432A+299C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+17C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{32d}+\frac{a^{5/2}(304A+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(64*d) + (a^3*(432*A + 299*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(192
*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(16*A + 17*C)*Sec[c + d*x]^(3/2)*Sqrt[a
 + a*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (5*a*C*Sec[c + d*x]^(3/2)*(a + a*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sec[c + d*x]^(3/2)*(a + a*Sec
[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{4d} + \frac{\int}{}$$

$$= \frac{5aC\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{24d} +$$

$$= \frac{a^2(16A+17C)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c}{32d}$$

$$= \frac{a^3(432A+299C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A}{}$$

$$= \frac{a^3(432A+299C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A}{}$$

$$= \frac{a^{5/2}(304A+163C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^3(432A}{}$$

Mathematica [A] time = 3.38238, size = 250, normalized size = 1.15

$$\frac{\cos^3(c+dx)(a(\sec(c+dx)+1))^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{\sec(c+dx)+1}((1584A+220}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*(A + C*Sec[c + d*x]^2)*((192*A
 + 844*C + (1584*A + 2203*C)*Cos[c + d*x] + 4*(48*A + 163*C)*Cos[2*(c + d*x
)] + 528*A*Cos[3*(c + d*x)] + 489*C*Cos[3*(c + d*x)])*Sec[c + d*x]^(9/2)*Sq
rt[1 + Sec[c + d*x]]*Tan[(c + d*x)/2] - 12*(304*A + 163*C)*Csc[c + d*x]*(Lo
g[1 + Sec[c + d*x]] - Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1
+ Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(384*d*(A + 2
```

```
*C + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.445, size = 452, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/768/d*a^2*(-912*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+912*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-489*C*cos(d*x
+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))+489*C*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+1056*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(
d*x+c)+1))^(1/2)+978*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+19
2*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+652*C*sin(d*x+c)*cos(
d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+368*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2)+96*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/
cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)
^2/cos(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [B] time = 22.0357, size = 9027, normalized size = 41.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] -1/768*(48*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(
2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x +
 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^
2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
```

qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x +
 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/
2*c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)
 + 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2
*c) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2
c) - 19(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(
2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*
d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d
*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt
(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + (1956*(sqrt(2)*a^2*sin

(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6
*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^
2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d
*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqr
t(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*
sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(
2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*si
n(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*
(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*
a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*s
qrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^
2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x
+ 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(
6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(
4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) +
a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c)
+ a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c
) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8
*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(
2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^
2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*si
n(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c)
 + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c)
+ 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c
) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x
 + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c)
 + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*l
og(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d
*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*s
in(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 +
 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a
^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x +
 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*
x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*
d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x
 + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*c

```
os(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16
*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) +
a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*
x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*
d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin
(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
- 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1956*(sqrt(2)
*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(
4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*cos(8*d*x + 8*c)
 + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos
(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x +
2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*co
s(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x +
 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sq
rt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2
*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x +
6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sq
rt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqr
t(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*
cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(4*cos(6*d*x + 6*c
) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)
^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin
(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.04331, size = 1324, normalized size = 6.07

$$\left[\frac{3\left((304\,A+163\,C)a^2\cos(dx+c)^4+(304\,A+163\,C)a^2\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(d\cdots\right.}{\cdots}}{\cos(dx+c)^3+\cos(dx\cdots}\right.}{768\left(d\cos(dx+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/768*(3*((304*A + 163*C)*a^2*cos(d*x + c)^4 + (304*A + 163*C)*a^2*cos(d*x
 + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x +
c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(
3*(176*A + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 163*C)*a^2*cos(d*x + c)^2
+ 184*C*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1
/384*(3*((304*A + 163*C)*a^2*cos(d*x + c)^4 + (304*A + 163*C)*a^2*cos(d*x +
 c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) +
 2*(3*(176*A + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 163*C)*a^2*cos(d*x + c
)^2 + 184*C*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c) + a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
c)), x)
```

3.270 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=218

$$\frac{a^3(24A-49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+31C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{24d}+\frac{5a^{5/2}(8A+5$$

```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^3*(24*A - 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a^2*(24*A + 31*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]*Sin[c + d*x])/(24*d) + (5*a*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(12*d) + (C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d
*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.658138, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4089, 4018, 4015, 3801, 215}

$$\frac{a^3(24A-49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+31C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{24d}+\frac{5a^{5/2}(8A+5$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c + d*x]],
x]
```


```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^3*(24*A - 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a^2*(24*A + 31*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]*Sin[c + d*x])/(24*d) + (5*a*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(12*d) + (C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d
*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \frac{\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{1}{2}\right.}{\sqrt{s}}}{}$$

$$= \frac{5aC\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{12d} + \frac{C\sqrt{\sec(c+dx)}(}{}$$

$$= \frac{a^2(24A+31C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{24d} + \frac{5aC\sqrt{s}}{}$$

$$= \frac{a^3(24A-49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(24A+31C)\sqrt{\sec(c+a}}{}$$

$$= \frac{a^3(24A-49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(24A+31C)\sqrt{\sec(c+a}}{}$$

$$= \frac{5a^{5/2}(8A+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a^3(24A-49C)\sqrt{\sec(c+dx}}{24d\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 6.70738, size = 411, normalized size = 1.89

$$\frac{5(8A+5C)\sin(c+dx)\cos^3(c+dx)\sqrt{\sec^2(c+dx)-1}(a(\sec(c+dx)+1))^{5/2}\left(\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)+1}\sqrt{\sec}\right.\right.}{4d\left(1-\cos^2(c+dx)\right)(\sec(c+dx)+1)^{5/2}(A\cos(2c+2dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Sec[c +
d*x]],x]
```

```
[Out] (5*(8*A + 5*C)*Cos[c + d*x]^3*(-Log[1 + Sec[c + d*x]] + Log[Sqrt[Sec[c + d*
x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]
])*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*
```

```
x]^2)*Sin[c + d*x])/(4*d*(1 - Cos[c + d*x]^2)*(A + 2*C + A*Cos[2*c + 2*d*x]
)*(1 + Sec[c + d*x])^(5/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(a*(1
+ Sec[c + d*x]))^(5/2)*(A + C*Sec[c + d*x]^2)*((4*A*Cos[d*x]*Sin[c])/d + (S
ec[c/2]*Sec[c/2 + (d*x)/2]*(-24*A*Sin[(d*x)/2] + 49*C*Sin[(d*x)/2]))/(12*d)
 + (4*A*Cos[c]*Sin[d*x])/d + (2*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (
Sec[c]*Sec[c + d*x]*(4*C*Sin[c] + 13*C*Sin[d*x]))/(6*d) - ((-26*C + 24*A*Co
s[c] - 75*C*Cos[c])*Sec[c]*Tan[c/2])/(12*d)))/((A + 2*C + A*Cos[2*c + 2*d*x
])*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.375, size = 399, normalized size = 1.8

$$-\frac{a^2}{96\,d\sin(dx+c)(\cos(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(120\,A\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/96/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(120*A*(-2/(cos(d*x+c)+1))^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3-120*A*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)*
2^(1/2)*cos(d*x+c)^3+75*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)*2^(1/2)*cos(d*x
+c)^3-75*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3+192*A*cos
(d*x+c)^4-96*A*cos(d*x+c)^3+300*C*cos(d*x+c)^3-96*A*cos(d*x+c)^2-164*C*cos(
d*x+c)^2-104*C*cos(d*x+c)-32*C)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^
2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.804891, size = 1257, normalized size = 5.77

$$\left[\frac{15\left((8\,A+5\,C)a^2\cos(dx+c)^3+(8\,A+5\,C)a^2\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos}{\cos}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos(dx+c)^3+d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(15*((8*A + 5*C)*a^2*cos(d*x + c)^3 + (8*A + 5*C)*a^2*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(48*A*a^2
*cos(d*x + c)^3 + 3*(8*A + 25*C)*a^2*cos(d*x + c)^2 + 34*C*a^2*cos(d*x + c)
 + 8*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(15*((8*A + 5*C)*a^2*
cos(d*x + c)^3 + (8*A + 5*C)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(48*A*a^2*cos(d*x + c)^3 + 3*(
8*A + 25*C)*a^2*cos(d*x + c)^2 + 34*C*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
+ c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^2 + A\right)\left(a \sec\left(dx+c\right) + a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x +
c)), x)
```

3.271 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=224

$$\frac{a^3(56A-27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-21C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{12d}+\frac{a^{5/2}(8A+19C}{}$$

```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(56*A - 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt
[a + a*Sec[c + d*x]]) - (a^2*(8*A - 21*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(12*d) - (a*(4*A - 3*C)*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (2*A*(a + a*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.674266, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4018, 4015, 3801, 215}

$$\frac{a^3(56A-27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-21C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{12d}+\frac{a^{5/2}(8A+19C}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),
x]
```


```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(56*A - 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt
[a + a*Sec[c + d*x]]) - (a^2*(8*A - 21*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(12*d) - (a*(4*A - 3*C)*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (2*A*(a + a*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{5aA}{2}-\frac{1}{2}a(4A-3}{\sqrt{\sec(c+dx)}}}{3a}$$

$$= -\frac{a(4A-3C)\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{6d} + \frac{2A(a+}{}$$

$$= -\frac{a^2(8A-21C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{12d} - \frac{a(4A-}{}$$

$$= \frac{a^3(56A-27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(8A-21C)\sqrt{\sec(c+dx}}{}$$

$$= \frac{a^3(56A-27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(8A-21C)\sqrt{\sec(c+dx}}{}$$

$$= \frac{a^{5/2}(8A+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^3(56A-27C)\sqrt{\sec(c+dx}}{12d\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 6.89878, size = 416, normalized size = 1.86

$$\frac{(8A+19C)\sin(c+dx)\cos^3(c+dx)\sqrt{\sec^2(c+dx)-1}(a(\sec(c+dx)+1))^{5/2}\left(\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)+1}\sqrt{\sec}\right.\right.}{2d\left(1-\cos^2(c+dx)\right)(\sec(c+dx)+1)^{5/2}(A\cos(2c+2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(3/2),x]
```

```
[Out] ((8*A + 19*C)*Cos[c + d*x]^3*(-Log[1 + Sec[c + d*x]] + Log[Sqrt[Sec[c + d*x
]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]]
)*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x
]^2)*Sin[c + d*x])/(2*d*(1 - Cos[c + d*x]^2)*(A + 2*C + A*Cos[2*c + 2*d*x])
*(1 + Sec[c + d*x])^(5/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(a*(1 +
 Sec[c + d*x]))^(5/2)*(A + C*Sec[c + d*x]^2)*((28*A*Cos[d*x]*Sin[c])/(3*d)
+ (2*A*Cos[2*d*x]*Sin[2*c])/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(-56*A*Sin
[(d*x)/2] + 27*C*Sin[(d*x)/2]))/(6*d) + (28*A*Cos[c]*Sin[d*x])/(3*d) + (C*S
ec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*A*Cos[2*c]*Sin[2*d*x])/(3*d) - ((-6*C +
 56*A*Cos[c] - 33*C*Cos[c])*Sec[c]*Tan[c/2])/(6*d)))/((A + 2*C + A*Cos[2*c
+ 2*d*x])*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^(5/2))
```

Maple [A] time = 0.444, size = 380, normalized size = 1.7

$$\frac{a^2}{48\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(24\,A\sqrt{2}\sin(dx+c)(\cos(dx+c))^2\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\,\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(24*A*2^(1/2)*sin(d*x+c)*cos
(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-24*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arcta
n(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos
(d*x+c)+1))^(1/2)+57*C*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)
))-57*C*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)-32*A*cos(d*x
+c)^4-224*A*cos(d*x+c)^3+256*A*cos(d*x+c)^2-132*C*cos(d*x+c)^2+108*C*cos(d*
x+c)+24*C)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 20.8474, size = 4618, normalized size = 20.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/48*(4*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
```

arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*
d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*A*sqrt(a) - 3*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c
) - 56*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*s
in(3/2*d*x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4
*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))
*sin(4*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(
7/2*d*x + 7/2*c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin
(3/2*d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ 38*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +

```
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*
d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(
2*d*x + 2*c) + 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5
/2*d*x + 5/2*c) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2
)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a
^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c))*C*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.818258, size = 1219, normalized size = 5.44

$$\left[\frac{3\left((8A+19C)a^2\cos(dx+c)^2+(8A+19C)a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\ldots}{\ldots}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.\ldots}{48\left(d\cos(dx+c)^2+d\cos(dx\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/48*(3*((8*A + 19*C)*a^2*cos(d*x + c)^2 + (8*A + 19*C)*a^2*cos(d*x + c))*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(8*A*a^2*c
os(d*x + c)^3 + 64*A*a^2*cos(d*x + c)^2 + 33*C*a^2*cos(d*x + c) + 6*C*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^2 + d*cos(d*x + c)), 1/24*(3*((8*A + 19*C)*a^2*cos(d*x + c)^2
 + (8*A + 19*C)*a^2*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)) + 2*(8*A*a^2*cos(d*x + c)^3 + 64*A*a^2*cos(d*x +
c)^2 + 33*C*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
```

```
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3
/2), x)
```

3.272 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{a^3(64A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A-15C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{5a^{5/2}C\sinh^{-}}{}$$

```
[Out] (5*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a^3*(64*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) - (a^2*(16*A - 15*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(15*d) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Se
c[c + d*x]^(3/2))
```

Rubi [A] time = 0.656814, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4087, 4017, 4018, 4015, 3801, 215}

$$\frac{a^3(64A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A-15C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{5a^{5/2}C\sinh^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (5*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a^3*(64*A + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) - (a^2*(16*A - 15*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(15*d) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Se
c[c + d*x]^(3/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{5aA}{2}-\frac{1}{2}a(2A-5}{\sec^{\frac{3}{2}}(c+dx)}}{5a}$$

$$= \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+dx))^{5/2}\sin(}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{a^2(16A-15C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2aA(a}{}$$

$$= \frac{a^3(64A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(16A-15C)\sqrt{\sec(c+d}}{}$$

$$= \frac{a^3(64A+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(16A-15C)\sqrt{\sec(c+d}}{}$$

$$= \frac{5a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3(64A+15C)\sqrt{\sec(c+dx)}\sin(c+}{15d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 6.58031, size = 428, normalized size = 2.04

$$\frac{10C\sin(c+dx)\cos^3(c+dx)\sqrt{\sec^2(c+dx)-1}(a(\sec(c+dx)+1))^{5/2}\left(\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+\right.\right.}{d\left(1-\cos^2(c+dx)\right)(\sec(c+dx)+1)^{5/2}(A\cos(2c+2dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(5/2),x]
```

```
[Out] (10*C*Cos[c + d*x]^3*(-Log[1 + Sec[c + d*x]] + Log[Sqrt[Sec[c + d*x]] + Sec
[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(a*(1
+ Sec[c + d*x]))^(5/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin
[c + d*x])/(d*(1 - Cos[c + d*x]^2)*(A + 2*C + A*Cos[2*c + 2*d*x])*(1 + Sec[
c + d*x])^(5/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(a*(1 + Sec[c + d
*x]))^(5/2)*(A + C*Sec[c + d*x]^2)*(((131*A + 60*C)*Cos[d*x]*Sin[c])/(15*d)
 + (22*A*Cos[2*d*x]*Sin[2*c])/(15*d) + (A*Cos[3*d*x]*Sin[3*c])/(5*d) - (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]*(64*A*Sin[(d*x)/2] + 15*C*Sin[(d*x)/2]))/(15*d)
+ ((131*A + 60*C)*Cos[c]*Sin[d*x])/(15*d) + (22*A*Cos[2*c]*Sin[2*d*x])/(15*
d) + (A*Cos[3*c]*Sin[3*d*x])/(5*d) - (2*(64*A + 15*C)*Tan[c/2])/(15*d)))/((
A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^(5/2))
```

Maple [A] time = 0.413, size = 255, normalized size = 1.2

$$-\frac{a^2(\cos(dx+c))^2}{60\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(75\,C\cos(dx+c)\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/60/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(75*C*cos(d*x+c)*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))*2^(1/2)-75*C*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*
x+c)))*2^(1/2)+24*A*cos(d*x+c)^4+88*A*cos(d*x+c)^3+232*A*cos(d*x+c)^2+120*C
*cos(d*x+c)^2-344*A*cos(d*x+c)-60*C*cos(d*x+c)-60*C)*cos(d*x+c)^2*(1/cos(d*
x+c))^(5/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.637299, size = 1114, normalized size = 5.3

$$\left[\frac{75\left(Ca^2\cos(dx+c)+Ca^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(6Aa^2\cos(dx+c)\right.}{}}{60\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/60*(75*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(6*A*a^2*cos(d*x + c)^3 + 28*A*a^2*cos(d*x + c
)^2 + 2*(43*A + 15*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/30
*(75*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)) + 2*(6*A*a^2*cos(d*x + c)^3 + 28*A*a^2*cos(d*x +
c)^2 + 2*(43*A + 15*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\left(a\sec(dx+c)+a\right)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5
/2), x)
```

3.273 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{2a^3(32A+49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+7C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+d}{\sqrt{a\sec(c+dx}}\right.}{d}$$

```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(32*A + 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*
d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.640494, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4017, 4015, 3801, 215}

$$\frac{2a^3(32A+49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+7C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+d}{\sqrt{a\sec(c+dx}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(32*A + 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*
d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{7}{2}aC\sec\right.}{\sec^{\frac{5}{2}}(c+dx)}}{7a}$$

$$= \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^{5/2}\sin(}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(8A+7C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx}{7d\sec^{\frac{3}{2}}(c}$$

$$= \frac{2a^3(32A+49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+7C)\sqrt{a+a\sec}}{21d\sqrt{\sec(c}}$$

$$= \frac{2a^3(32A+49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+7C)\sqrt{a+a\sec}}{21d\sqrt{\sec(c}}$$

$$= \frac{2a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3(32A+49C)\sqrt{\sec(c+dx)}\sin(c}{21d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 6.41719, size = 474, normalized size = 2.26

$$\frac{4C\sin(c+dx)\cos^3(c+dx)\sqrt{\sec^2(c+dx)-1}(a(\sec(c+dx)+1))^{5/2}\left(\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+dx)}\right.\right.}{d\left(1-\cos^2(c+dx)\right)(\sec(c+dx)+1)^{5/2}(A\cos(2c+2dx)+A}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(7/2),x]
```

```
[Out] (4*C*Cos[c + d*x]^3*(-Log[1 + Sec[c + d*x]] + Log[Sqrt[Sec[c + d*x]] + Sec[
c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(a*(1 +
 Sec[c + d*x]))^(5/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin[
c + d*x])/(d*(1 - Cos[c + d*x]^2)*(A + 2*C + A*Cos[2*c + 2*d*x])*(1 + Sec[c
 + d*x])^(5/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(a*(1 + Sec[c + d*
x]))^(5/2)*(A + C*Sec[c + d*x]^2)*(((137*A + 196*C)*Cos[d*x]*Sin[c])/(21*d)
 + ((31*A + 14*C)*Cos[2*d*x]*Sin[2*c])/(21*d) + (3*A*Cos[3*d*x]*Sin[3*c])/(
7*d) + (A*Cos[4*d*x]*Sin[4*c])/(14*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(32*
A*Sin[(d*x)/2] + 49*C*Sin[(d*x)/2]))/(21*d) + ((137*A + 196*C)*Cos[c]*Sin[d
*x])/(21*d) + ((31*A + 14*C)*Cos[2*c]*Sin[2*d*x])/(21*d) + (3*A*Cos[3*c]*Si
n[3*d*x])/(7*d) + (A*Cos[4*c]*Sin[4*d*x])/(14*d) - (4*(32*A + 49*C)*Tan[c/2
])/(21*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*(1 + Sec[c +
 d*x])^(5/2))
```

Maple [A] time = 0.409, size = 246, normalized size = 1.2

$$-\frac{a^2 (\cos(dx+c))^4}{42\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(12\,A(\cos(dx+c))^4+21\,C\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/42/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(12*A*cos(d*x+c)^4+21*C*(-2
/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-21*C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/
2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*
sin(d*x+c)+36*A*cos(d*x+c)^3+44*A*cos(d*x+c)^2+28*C*cos(d*x+c)^2+92*A*cos(d
*x+c)+196*C*cos(d*x+c)-184*A-224*C)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d
*x+c)
```

Maxima [B] time = 2.24109, size = 1238, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/168*(sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*co
s(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), co
s(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*
sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3
/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 14*sqrt(2)
*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3
/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(sin(3
```

```
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x + 3/2*c) +
30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sqrt
(a))/d
```

Fricas [A] time = 0.631439, size = 1176, normalized size = 5.6

$$\left[\frac{21\left(Ca^2\cos(dx+c)+Ca^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3Aa^2\cos(dx+c)\right.}{}}{42\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, alg
orithm="fricas")
```

```
[Out] [1/42*(21*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(3*A*a^2*cos(d*x + c)^4 + 12*A*a^2*cos(d*x + c
)^3 + (23*A + 7*C)*a^2*cos(d*x + c)^2 + 2*(23*A + 28*C)*a^2*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*
cos(d*x + c) + d), 1/21*(21*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*A*a^2*cos(d*x + c)^
4 + 12*A*a^2*cos(d*x + c)^3 + (23*A + 7*C)*a^2*cos(d*x + c)^2 + 2*(23*A + 2
8*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7
/2), x)
```

3.274 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=216

$$\frac{64a^3(13A+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(13A+21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{315d\sqrt{\sec(c+dx)}}+\frac{2a(13A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}}$$

```
[Out] (64*a^3*(13*A + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Se
c[c + d*x]]) + (16*a^2*(13*A + 21*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(315*d*Sqrt[Sec[c + d*x]]) + (2*a*(13*A + 21*C)*(a + a*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)
*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (10*A*(a + a*Sec[c + d*x])^(5/2)*
Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.500494, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4087, 4013, 3809, 3804}

$$\frac{64a^3(13A+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(13A+21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{315d\sqrt{\sec(c+dx)}}+\frac{2a(13A+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (64*a^3*(13*A + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Se
c[c + d*x]]) + (16*a^2*(13*A + 21*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(315*d*Sqrt[Sec[c + d*x]]) + (2*a*(13*A + 21*C)*(a + a*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)
*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (10*A*(a + a*Sec[c + d*x])^(5/2)*
Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{1}{2}a(2}{\sec^{\frac{7}{2}}(c+dx}}{9a}$$
$$= \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{10A(a+a\sec(c+dx))^{5/2}\text{s}}{63d\sec^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2a(13A+21C)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c}{9d\,\text{se}}$$
$$= \frac{16a^2(13A+21C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{315d\sqrt{\sec(c+dx)}} + \frac{2a(13A+21C}{}$$
$$= \frac{64a^3(13A+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(13A+21C)\sqrt{a}}{315d}$$

Mathematica [A] time = 1.56315, size = 105, normalized size = 0.49

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(4(779A+588C)\cos(c+dx)+4(254A+63C)\cos(2(c+dx))+260A\cos(3(c}{1260d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(9/2),x]
```

```
[Out] (a^2*(5653*A + 7476*C + 4*(779*A + 588*C)*Cos[c + d*x] + 4*(254*A + 63*C)*C
os[2*(c + d*x)] + 260*A*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1
 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(1260*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.436, size = 132, normalized size = 0.6

$$-\frac{2\,a^2\,(-1+\cos(dx+c))\left(35\,A\,(\cos(dx+c))^4+130\,A\,(\cos(dx+c))^3+219\,A\,(\cos(dx+c))^2+63\,C\,(\cos(dx+c))^2+}{315\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+219*A*co
s(d*x+c)^2+63*C*cos(d*x+c)^2+292*A*cos(d*x+c)+294*C*cos(d*x+c)+584*A+903*C)
*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(
d*x+c)
```

Maxima [B] time = 2.03526, size = 652, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, alg
orithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) + 168*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) +
 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
)*C*sqrt(a))/d
```

Fricas [A] time = 0.487792, size = 359, normalized size = 1.66

$$\frac{2\left(35\,Aa^{2}\cos\left(dx+c\right)^{5}+130\,Aa^{2}\cos\left(dx+c\right)^{4}+3\,(73\,A+21\,C)a^{2}\cos\left(dx+c\right)^{3}+2\,(146\,A+147\,C)a^{2}\cos\left(dx+c\right)^{2}+(\ldots}{315\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, alg
orithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^5 + 130*A*a^2*cos(d*x + c)^4 + 3*(73*A + 21*C)
*a^2*cos(d*x + c)^3 + 2*(146*A + 147*C)*a^2*cos(d*x + c)^2 + (584*A + 903*C
)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((
d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9
/2), x)
```

3.275 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{2a^3(232A+297C)\sin(c+dx)}{693d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(32A+33C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(568A+759C)\sin(c+dx)}{693d\sqrt{a\sec(c+dx)}}$$

```
[Out] (2*a^3*(232*A + 297*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(568*A + 759*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(32*A + 33*C)*Sqrt[a
 + a*Sec[c + d*x]]*Sin[c + d*x])/(231*d*Sec[c + d*x]^(5/2)) + (10*a*A*(a +
a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a
*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```

Rubi [A] time = 0.797018, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4017, 4015, 3805, 3804}

$$\frac{2a^3(232A+297C)\sin(c+dx)}{693d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(32A+33C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(568A+759C)\sin(c+dx)}{693d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(11/2)
,x]
```


```
[Out] (2*a^3*(232*A + 297*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(568*A + 759*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(32*A + 33*C)*Sqrt[a
 + a*Sec[c + d*x]]*Sin[c + d*x])/(231*d*Sec[c + d*x]^(5/2)) + (10*a*A*(a +
a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a
*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{1}{2}a(4$$

$$= \frac{10aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^{5/2}}{11d\sec^{\frac{9}{2}}(c+d$$

$$= \frac{2a^2(32A+33C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{231d\sec^{\frac{5}{2}}(c+dx)} + \frac{10aA(a+a\sec}{99d\,s}$$

$$= \frac{2a^3(232A+297C)\sin(c+dx)}{693d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(32A+33C)\sqrt{a+a\,s}}{231d\sec^{\frac{5}{2}}(}$$

$$= \frac{2a^3(232A+297C)\sin(c+dx)}{693d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(568A+759C)\sin}{693d\sqrt{\sec(c+dx)}\sqrt{a+a}}$$

$$= \frac{2a^3(232A+297C)\sin(c+dx)}{693d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(568A+759C)\sin}{693d\sqrt{\sec(c+dx)}\sqrt{a+a}}$$

Mathematica [A] time = 2.07714, size = 127, normalized size = 0.48

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(6989A+6666C)\cos(c+dx)+16(325A+198C)\cos(2(c+dx))+1735A\,c}{5544d\sqrt{\sec(c+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(11/2),x]
```

```
[Out] (a^2*(22928*A + 27456*C + 2*(6989*A + 6666*C)*Cos[c + d*x] + 16*(325*A + 19
8*C)*Cos[2*(c + d*x)] + 1735*A*Cos[3*(c + d*x)] + 396*C*Cos[3*(c + d*x)] +
448*A*Cos[4*(c + d*x)] + 63*A*Cos[5*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*
Tan[(c + d*x)/2])/(5544*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.384, size = 154, normalized size = 0.6

$$-\frac{2 a^2 (-1+\cos(dx+c))\left(63 A(\cos(dx+c))^5+224 A(\cos(dx+c))^4+355 A(\cos(dx+c))^3+99 C(\cos(dx+c))^3+42\right.}{693 d \sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/693/d*a^2*(-1+cos(d*x+c))*(63*A*cos(d*x+c)^5+224*A*cos(d*x+c)^4+355*A*co
s(d*x+c)^3+99*C*cos(d*x+c)^3+426*A*cos(d*x+c)^2+396*C*cos(d*x+c)^2+568*A*co
s(d*x+c)+759*C*cos(d*x+c)+1136*A+1518*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2
)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [B] time = 2.15987, size = 1141, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, al
gorithm="maxima")
```

```
[Out] 1/22176*(sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(1
1/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(
11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*
a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(1
1/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(1
1/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*
a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*
c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287
*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c)
 + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))
) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c
))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2
*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11
/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
 11/2*c))))*A*sqrt(a) + 132*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^
2*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x
+ 7/2*c) - 315*a^2*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin
```

```
(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d
*x + 7/2*c) + 21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) + 77*a^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) + 315*a^2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*C
*sqrt(a))/d
```

Fricas [A] time = 0.505933, size = 408, normalized size = 1.53

$$\frac{2\left(63\,Aa^2\cos\left(dx+c\right)^6+224\,Aa^2\cos\left(dx+c\right)^5+\left(355\,A+99\,C\right)a^2\cos\left(dx+c\right)^4+6\left(71\,A+66\,C\right)a^2\cos\left(dx+c\right)^3+\left(\right.}{693\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, al
gorithm="fricas")
```

```
[Out] 2/693*(63*A*a^2*cos(d*x + c)^6 + 224*A*a^2*cos(d*x + c)^5 + (355*A + 99*C)*
a^2*cos(d*x + c)^4 + 6*(71*A + 66*C)*a^2*cos(d*x + c)^3 + (568*A + 759*C)*a
^2*cos(d*x + c)^2 + 2*(568*A + 759*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1
1/2), x)
```

3.276 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=313

$$\frac{2a^3(8368A+10439C)\sin(c+dx)}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+143C)\sin(c+dx)\sqrt{a\sec}}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

```
[Out] (2*a^3*(2224*A + 2717*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)*Sqrt[a +
a*Sec[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Sin[c + d*x])/(15015*d*Sec[c +
 d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sin[c + d
*x])/(45045*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^3*(8368*
A + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a + a*Sec[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1287
*d*Sec[c + d*x]^(7/2)) + (10*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
143*d*Sec[c + d*x]^(9/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(
13*d*Sec[c + d*x]^(11/2))
```

Rubi [A] time = 0.865055, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4017, 4015, 3805, 3804}

$$\frac{2a^3(8368A+10439C)\sin(c+dx)}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+143C)\sin(c+dx)\sqrt{a\sec}}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^(13/2)
,x]
```

```
[Out] (2*a^3*(2224*A + 2717*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)*Sqrt[a +
a*Sec[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Sin[c + d*x])/(15015*d*Sec[c +
 d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sin[c + d
*x])/(45045*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^3*(8368*
A + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a + a*Sec[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1287
*d*Sec[c + d*x]^(7/2)) + (10*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
143*d*Sec[c + d*x]^(9/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(
13*d*Sec[c + d*x]^(11/2))
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
```

```
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^{5/2}\left(\frac{5aA}{2}+\frac{1}{2}a(6}{\sec^{\frac{11}{2}}(c+d}}{13a}$$

$$= \frac{10aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{143d\sec^{\frac{9}{2}}(c+dx)} + \frac{2A(a+a\sec(c+dx))^{5/2}}{13d\sec^{\frac{11}{2}}(c+d}$$

$$= \frac{2a^2(136A+143C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{1287d\sec^{\frac{7}{2}}(c+dx)} + \frac{10aA(a+a\,\mathrm{se}}{143}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(136A+143C)\sqrt{a+}}{1287d\,\mathrm{sec}}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(8368A+10439C}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(8368A+10439C}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{2a^3(2224A+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(8368A+10439C}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

Mathematica [A] time = 2.17458, size = 148, normalized size = 0.47

$$\frac{a^2 \tan\left(\frac{1}{2}(c+dx)\right) \sqrt{a(\sec(c+dx)+1)}(8(226573A+222794C)\cos(c+dx)+(746519A+581152C)\cos(2(c+dx))+28$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sec[c + d*x]^
(13/2),x]
```

```
[Out] (a^2*(2798182*A + 3233516*C + 8*(226573*A + 222794*C)*Cos[c + d*x] + (74651
9*A + 581152*C)*Cos[2*(c + d*x)] + 287060*A*Cos[3*(c + d*x)] + 148720*C*Cos
[3*(c + d*x)] + 94010*A*Cos[4*(c + d*x)] + 20020*C*Cos[4*(c + d*x)] + 23940
*A*Cos[5*(c + d*x)] + 3465*A*Cos[6*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*T
an[(c + d*x)/2])/(720720*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.394, size = 176, normalized size = 0.6

$$-\frac{2a^2(-1+\cos(dx+c))\left(3465A(\cos(dx+c))^6+11970A(\cos(dx+c))^5+18305A(\cos(dx+c))^4+5005C(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(3465*A*cos(d*x+c)^6+11970*A*cos(d*x+c)^5+18
305*A*cos(d*x+c)^4+5005*C*cos(d*x+c)^4+20920*A*cos(d*x+c)^3+18590*C*cos(d*x
+c)^3+25104*A*cos(d*x+c)^2+31317*C*cos(d*x+c)^2+33472*A*cos(d*x+c)+41756*C*
cos(d*x+c)+66944*A+83512*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^
7*(1/cos(d*x+c))^(13/2)/sin(d*x+c)
```

Maxima [B] time = 2.21276, size = 1408, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, al
gorithm="maxima")
```

```
[Out] 1/2882880*(sqrt(2)*(3783780*a^2*cos(12/13*arctan2(sin(13/2*d*x + 13/2*c), c
os(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 1066065*a^2*cos(10/13*arct
an2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c)
 + 459459*a^2*cos(8/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*
c)))*sin(13/2*d*x + 13/2*c) + 193050*a^2*cos(6/13*arctan2(sin(13/2*d*x + 13
/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 70070*a^2*cos(4/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13
/2*c) + 20475*a^2*cos(2/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c)))*sin(13/2*d*x + 13/2*c) - 3783780*a^2*cos(13/2*d*x + 13/2*c)*sin(12
/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 1066065*a^2*
cos(13/2*d*x + 13/2*c)*sin(10/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d
*x + 13/2*c))) - 459459*a^2*cos(13/2*d*x + 13/2*c)*sin(8/13*arctan2(sin(13/
2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 193050*a^2*cos(13/2*d*x + 13/2*
```

```
c)*sin(6/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 7007
0*a^2*cos(13/2*d*x + 13/2*c)*sin(4/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) - 20475*a^2*cos(13/2*d*x + 13/2*c)*sin(2/13*arctan2(sin
(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 6930*a^2*sin(13/2*d*x + 13/
2*c) + 20475*a^2*sin(11/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c))) + 70070*a^2*sin(9/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x
+ 13/2*c))) + 193050*a^2*sin(7/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*
d*x + 13/2*c))) + 459459*a^2*sin(5/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) + 1066065*a^2*sin(3/13*arctan2(sin(13/2*d*x + 13/2*c),
cos(13/2*d*x + 13/2*c))) + 3783780*a^2*sin(1/13*arctan2(sin(13/2*d*x + 13/2
*c), cos(13/2*d*x + 13/2*c))))*A*sqrt(a) + 572*sqrt(2)*(8190*a^2*cos(8/9*ar
ctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2
100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/
2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9
/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a
^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) +
 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756
*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^
2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*s
in(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.512213, size = 495, normalized size = 1.58

$$\frac{2\left(3465\,Aa^2\cos\left(dx+c\right)^7+11970\,Aa^2\cos\left(dx+c\right)^6+35\,(523\,A+143\,C)a^2\cos\left(dx+c\right)^5+10\,(2092\,A+1859\,C)a^2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, al
gorithm="fricas")
```

```
[Out] 2/45045*(3465*A*a^2*cos(d*x + c)^7 + 11970*A*a^2*cos(d*x + c)^6 + 35*(523*A
 + 143*C)*a^2*cos(d*x + c)^5 + 10*(2092*A + 1859*C)*a^2*cos(d*x + c)^4 + 3*
(8368*A + 10439*C)*a^2*cos(d*x + c)^3 + 4*(8368*A + 10439*C)*a^2*cos(d*x +
c)^2 + 8*(8368*A + 10439*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1
3/2), x)
```

3.277 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=226

$$\frac{(8A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A+9C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+$$

```
[Out] -((8*A + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((8*A + 7*C)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) - (C*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.717771, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4089, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(8A+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A+9C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((8*A + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((8*A + 7*C)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) - (C*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(7/2
)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{C\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(6A+5C)-\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= -\frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3a^2 \ldots}{4}\right.}{\sqrt{a \ldots}}}{\ldots}$$

$$= \frac{(8A+7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{7}{2}}(c \ldots}{3d\sqrt{a \ldots}}$$

$$= \frac{(8A+7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{7}{2}}(c \ldots}{3d\sqrt{a \ldots}}$$

$$= \frac{(8A+7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{7}{2}}(c \ldots}{3d\sqrt{a \ldots}}$$

$$= -\frac{(8A+9C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx \ldots}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 4.91999, size = 368, normalized size = 1.63

$$\frac{\cos^2(c+dx)\sqrt{\sec(c+dx)+1}\left(A+C\sec^2(c+dx)\right)\left(\frac{6\tan(c+dx)\left((8A+9C)\log(\sec(c+dx)+1)-(8A+9C)\log\left(\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}+\sqrt{t\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((24*A + 37*C
 - 4*C*Cos[c + d*x] + 3*(8*A + 7*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Sq
rt[1 + Sec[c + d*x]]*Tan[(c + d*x)/2] + (6*((8*A + 9*C)*Log[1 + Sec[c + d*x
]] - (8*A + 9*C)*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec
[c + d*x]]*Sqrt[Tan[c + d*x]^2]] + 2*Sqrt[2]*(A + C)*(Log[1 - 2*Sec[c + d*x
] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*
Sqrt[Tan[c + d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[
2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]]))*Tan[c
+ d*x])/Sqrt[Tan[c + d*x]^2]))/(24*d*(A + 2*C + A*Cos[2*(c + d*x)])*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [B] time = 0.395, size = 448, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/48/d/a*(-1+cos(d*x+c))*(24*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-24*A*cos(d*x+c)^3*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+27*
C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+s
in(d*x+c)))*2^(1/2)-27*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-48*A*cos(d*x+c)^2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2)-96*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)*cos(d*x+c)^3-42*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-96*C*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+4*C*sin(d*x+c
)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/(-2/(cos(d*
x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.77223, size = 4809, normalized size = 21.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) + (84*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 100*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 312*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 312*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 100*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 84*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)

```
+ 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c)
+ 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2
*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 48*(sqrt(2)*cos(6*d*x + 6*c)
^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*
sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d
*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x
+ 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 1) + 48*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x
 + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*s
qrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 8
4*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) + 100*(
sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 312*(sqrt
(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 312*(sqrt(2)*
cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 100*(sqrt(2)*cos(
6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 84*(sqrt(2)*cos(6*d*x
 + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C/((2*(3*cos(4*d*x + 4*c) +
 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*
c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
 + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9
*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.901926, size = 1712, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(3*((8*A + 9*C)*cos(d*x + c)^3 + (8*A + 9*C)*cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x
```

```
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 48*sqrt(2)*((A + C)*
a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*log(-(cos(d*x + c)^2 - 2*sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a
) + 4*(3*(8*A + 7*C)*cos(d*x + c)^2 - 2*C*cos(d*x + c) + 8*C)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c)^3 + a*d*cos(d*x + c)^2), -1/48*(48*sqrt(2)*((A + C)*a*cos(d*x + c)^3 + (
A + C)*a*cos(d*x + c)^2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 3*((8*A + 9*C
)*cos(d*x + c)^3 + (8*A + 9*C)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*c
os(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*(3*(8*A + 7*C)*cos(d*x + c)^2 -
2*C*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a
), x)
```

3.278 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{ad}}+\frac{(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{ad}}+\frac{C\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{C\sin(\ldots}{4d\ldots}$$

```
[Out] ((8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - (C*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.545988, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4089, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{ad}}+\frac{(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{ad}}+\frac{C\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{C\sin(\ldots}{4d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - (C*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(4A+3C)-\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= -\frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \int \frac{\sqrt{\sec(c+dx)}\left(-\frac{a^2C}{4}\right.}{\sqrt{a+}}$$

$$= -\frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + (-A-C)\int \frac{}{\sqrt{a}}$$

$$= -\frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(2(A+C))\,\mathrm{Subs}}{}$$

$$= \frac{(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [B] time = 6.66378, size = 730, normalized size = 3.99

$$\frac{\sqrt{\sec(c+dx)+1}\sqrt{(\cos(c+dx)+1)\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(-\frac{3C\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}+\frac{C\sec(c)\sin(dx)\sec(c+dx)}{d}-\right.}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}(A\cos(2c+2dx)+A+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c
+ d*x]^2)*(-((C*(-2 + Cos[c])*Sin[c/2])/(d*(Cos[c/2] + Cos[(3*c)/2]))) - (3
*C*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(2*d) + (C*Sec[c]*Sec[c + d*x]
*Sin[d*x])/d))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a*(1
 + Sec[c + d*x])]) + (Cos[c + d*x]^2*Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c +
d*x]^2)*(-(C*Cos[c + d*x]^2*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*
Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]
] - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]
]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])*Sqr
t[-1 + Sec[c + d*x]^2]*Sin[c + d*x])/(2*d*(1 + Cos[c + d*x])*Sqrt[2 - 2*Cos
[c + d*x]^2]*Sqrt[1 - Cos[c + d*x]^2]) - ((-8*A - 7*C)*Cos[c + d*x]^2*(-8*L
og[1 + Sec[c + d*x]] + 8*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt
[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]] + Sqrt[2]*(-Log[1 - 2*Sec[c +
 d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*
x]]*Sqrt[-1 + Sec[c + d*x]^2]] + Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2
+ 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x
]^2]]))*(1 + Sec[c + d*x])*Sqrt[-1 + Sec[c + d*x]^2]*Sin[c + d*x])/(4*d*(1
+ Cos[c + d*x])*(1 - Cos[c + d*x]^2))))/(4*(A + 2*C + A*Cos[2*c + 2*d*x])*S
qrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.402, size = 388, normalized size = 2.1

$$\frac{(\cos(dx+c))^2-1}{16\,ad\,(\sin(dx+c))^2}\left(8\,A\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d/a*(8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*cos(d*x+c)^2*2^(1/2)-8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)+7*C*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)
-7*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)*cos(d*x+c)^2*2^(1/2)-16*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)*cos(d*x+c)^2-2*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-16*C*arc
tan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+4*C*(-2/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c)
)^(3/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.38408, size = 2867, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

[Out] -1/16*(8*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) - (4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c

```
), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos
(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))))*C/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
sqrt(a)))/d
```

Fricas [A] time = 0.881474, size = 1596, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*(((8*A + 7*C)*cos(d*x + c)^2 + (8*A + 7*C)*cos(d*x + c))*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 8*sqrt(2)*((A + C)*a*cos
(d*x + c)^2 + (A + C)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*(
C*cos(d*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), 1/8*(8*sqrt(2)
*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/s
in(d*x + c)) + ((8*A + 7*C)*cos(d*x + c)^2 + (8*A + 7*C)*cos(d*x + c))*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*(C*cos(d
*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a
), x)
```

3.279 $\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d))
 + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (C*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.367609, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4089, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d))
 + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (C*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(2A+C)-\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{C\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{2a} + (A+$$

$$= \frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{C\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} - \frac{(2(}{}$$

$$= -\frac{C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{C\,\text{s}}{}$$

Mathematica [B] time = 6.6647, size = 717, normalized size = 5.39

$$\frac{(2A+C)\sin(c+dx)\cos^4(c+dx)(\sec(c+dx)+1)^{3/2}\sqrt{\sec^2(c+dx)-1}\left(\log\left(-3\sec^2(c+dx)-2\sqrt{2}\sqrt{\sec(c+dx)+1}\sqrt{\text{se}}\right.\right.}{2d(\cos(c+dx)+1)\sqrt{2-2\,\text{cc}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] ((2*A + C)*Cos[c + d*x]^4*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sq
rt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]]
- Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*
Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])^(3/2)
*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin[c + d*x])/(2*d*(1 + C
os[c + d*x])*Sqrt[2 - 2*Cos[c + d*x]^2]*Sqrt[1 - Cos[c + d*x]^2]*(A + 2*C +
 A*Cos[2*c + 2*d*x])*Sqrt[a*(1 + Sec[c + d*x])]) - (C*Cos[c + d*x]^4*(-8*Lo
g[1 + Sec[c + d*x]] + 8*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[
1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]] + Sqrt[2]*(-Log[1 - 2*Sec[c +
d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x
]]*Sqrt[-1 + Sec[c + d*x]^2]] + Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 +
 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]
```

```
^2]]))*(1 + Sec[c + d*x])^(3/2)*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*
x]^2)*Sin[c + d*x])/(4*d*(1 + Cos[c + d*x])*(1 - Cos[c + d*x]^2)*(A + 2*C +
 A*Cos[2*c + 2*d*x])*Sqrt[a*(1 + Sec[c + d*x])]) + (Sqrt[(1 + Cos[c + d*x])
*Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((2*C*Sec[c/2]
*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (2*C*Tan[c/2])/d))/((A + 2*C + A*Cos[
2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.375, size = 252, normalized size = 1.9

$$\frac{(\cos(dx+c))^2-1}{4\,ad\,(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-C\cos(dx+c)\arctan\left(\frac{\sqrt{2}\,(\cos(dx+c)+1+\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-C*cos(d*
x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)*2^(1/2)+C*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))*2^(1/2)+4*A*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2))+4*C*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(
1/2))+2*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(-2/(cos(d*x+c)+1))^(1/2)/s
in(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.12321, size = 1307, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*(2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (4*sqrt(2)*cos(3/2*arc
tan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c
```

```
)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*
c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*l
og(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), c
os(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))))*C/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.659541, size = 1364, normalized size = 10.26

$$\left[\frac{(C\cos(dx+c)+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}((A+C)a\cos(dx+c)+(A+\cdots}{\cdots}}{4(ad\cos(dx+c)+ad)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*((C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 2*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1))/sqrt(a) + 4*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*((A + C)*a
*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + (C*cos(d*x
+ c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) - 2*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a
), x)
```

3.280 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.363927, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4087, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{aA}{2}+\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(-A-C)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx+\frac{C\int \sqrt{s}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}+$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2A}{}$$

Mathematica [B] time = 2.97222, size = 504, normalized size = 3.73

$$-\frac{\tan(c+dx)\left(\frac{8A}{\sqrt{\frac{1}{\cos(c+dx)+1}}}-8A\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}+\sqrt{2}A\sqrt{\tan^2(c+dx)}\log\left(-3\sec^2(c+dx)-2\sec(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] -(Tan[c + d*x]*((8*A)/Sqrt[(1 + Cos[c + d*x])^(-1)] - 8*A*Sqrt[Sec[c + d*x]
]*Sqrt[1 + Sec[c + d*x]] + 8*C*Log[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2] -
 8*C*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*S
qrt[Tan[c + d*x]^2]]*Sqrt[Tan[c + d*x]^2] + Sqrt[2]*A*Log[1 - 2*Sec[c + d*x
] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*
Sqrt[Tan[c + d*x]^2]]*Sqrt[Tan[c + d*x]^2] + Sqrt[2]*C*Log[1 - 2*Sec[c + d*
x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]
*Sqrt[Tan[c + d*x]^2]]*Sqrt[Tan[c + d*x]^2] - Sqrt[2]*A*Log[1 - 2*Sec[c + d
*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]
]*Sqrt[Tan[c + d*x]^2]]*Sqrt[Tan[c + d*x]^2] - Sqrt[2]*C*Log[1 - 2*Sec[c +
d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x
]]*Sqrt[Tan[c + d*x]^2]]*Sqrt[Tan[c + d*x]^2]))/(4*d*(-1 + Sec[c + d*x])*Sq
```

```
rt[1 + Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.35, size = 273, normalized size = 2.

$$-\frac{1}{2\,ad\sin\left(dx+c\right)}\left(-C\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\cos\left(dx+c\right)+1-\sin\left(dx+c\right)\right)}{4}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d/a*(-C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)+C*(-2/(cos(d*x+c)+1)
)^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))*sin(d*x+c)-2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*
(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-2*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*A*cos(d*x+c)-4*A)*(
a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/(1/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.10825, size = 783, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) + (sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2))*C/sqrt(a))/d
```

Fricas [A] time = 0.660838, size = 1351, normalized size = 10.01

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(C\cos(dx+c)+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\cdots}{\sqrt{\cos\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.\cdots}{2\,(ad\cos(dx+c)\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)) + sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*
x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A + C)*a*cos(d*x
+ c) + (A + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x + c) +
 C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*
d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c+dx)}{\sqrt{a\,(\sec(c+dx)+1)}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d
*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{\sqrt{a\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c
))), x)
```

3.281 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{ad}}-\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.337145, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4087, 4013, 3808, 206}

$$\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{ad}}-\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}a(2A+3C)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + (A+C)\int$$

$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{(2(A+C))}{}$$

$$= \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 3.88161, size = 273, normalized size = 2.01

$$\frac{\sqrt{\sec(c+dx)+1}\left(A+C\sec^2(c+dx)\right)\left(3\sqrt{2}(A+C)\cos^2(c+dx)\sqrt{\tan^2(c+dx)}\cot(c+dx)\left(\log\left(-3\sec^2(c+dx)-2\sec\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((-16*A*Sqrt[1 + Sec[c + d*x
]]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2])/Sec[c + d*x]^(5/2) + 3*Sqrt[2]*(A +
 C)*Cos[c + d*x]^2*Cot[c + d*x]*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2
- 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]]
 - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]
*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(6*d*
(A + 2*C + A*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.378, size = 171, normalized size = 1.3

$$-\frac{(\cos(dx+c))^2}{3ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+3*C*(-2/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)
+2*A*cos(d*x+c)^2-4*A*cos(d*x+c)+2*A)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin
```

```
(d*x+c)
```

Maxima [B] time = 2.01865, size = 504, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) - 3*(sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*C/sqrt(a))/d
```

Fricas [A] time = 0.5318, size = 917, normalized size = 6.74

$$\left[\frac{\frac{3\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(A\cos(dx+c)^2-A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2 -
 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
)/sqrt(a) + 4*(A*cos(d*x + c)^2 - A*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -
1/3*(3*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/si
n(d*x + c)) - 2*(A*cos(d*x + c)^2 - A*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}\sec^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**
(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{\sqrt{a\sec(dx + c) + a}\sec(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/
2)), x)
```

3.282 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{2(13A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)}}$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*A*Sin[c + d*x])/(15*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A + 15*C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.486149, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4022, 4013, 3808, 206}

$$\frac{2(13A+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*A*Sin[c + d*x])/(15*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A + 15*C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx = \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}a(4A+5C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(1}{}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(1}{}$$

$$= -\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 6.39342, size = 528, normalized size = 2.92

$$\frac{\sqrt{\sec(c+dx)+1}\sqrt{(\cos(c+dx)+1)\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(\frac{(71A+60C)\sin(c)\cos(dx)}{15d}+\frac{(71A+60C)\cos(c)\sin(dx)}{15d}-\frac{4\sec(}{}\right.}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] -(((A + C)*Cos[c + d*x]^4*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sq
rt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]]
- Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*
Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])^(3/2)
*Sqrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin[c + d*x])/(d*(1 + Cos
[c + d*x])*Sqrt[2 - 2*Cos[c + d*x]^2]*Sqrt[1 - Cos[c + d*x]^2]*(A + 2*C + A
```

```
*Cos[2*c + 2*d*x])*Sqrt[a*(1 + Sec[c + d*x])])) + (Sqrt[(1 + Cos[c + d*x])*
Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*(((71*A + 60*C)
*Cos[d*x]*Sin[c])/(15*d) - (8*A*Cos[2*d*x]*Sin[2*c])/(15*d) + (A*Cos[3*d*x]
*Sin[3*c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(17*A*Sin[(d*x)/2] + 15*C
*Sin[(d*x)/2]))/(15*d) + ((71*A + 60*C)*Cos[c]*Sin[d*x])/(15*d) - (8*A*Cos[
2*c]*Sin[2*d*x])/(15*d) + (A*Cos[3*c]*Sin[3*d*x])/(5*d) - (4*(17*A + 15*C)*
Tan[c/2])/(15*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[
a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.394, size = 194, normalized size = 1.1

$$\frac{(\cos(dx+c))^3}{15\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*arctan(1/2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-6*A*cos(d*x+c
)^3+15*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2))*sin(d*x+c)+8*A*cos(d*x+c)^2-28*A*cos(d*x+c)-30*C*cos(d*x+c)+26*A+3
0*C)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.08458, size = 624, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) - 30*(sqrt(2)*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*C/sqrt(a))/d
```

Fricas [A] time = 0.53628, size = 1010, normalized size = 5.58

$$\left[\frac{\frac{15\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(3A\cos(dx+c)^3-A\cos(dx+c)^2+(13A+15C)\cdots\right.}{\sqrt{\cos(dx+c)}}}{30\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2
 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))/sqrt(a) + 4*(3*A*cos(d*x + c)^3 - A*cos(d*x + c)^2 + (13*A + 15*C)*cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A + C)*a*cos(d*x + c)
+ (A + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x + c)^3 - A
*cos(d*x + c)^2 + (13*A + 15*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{\sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/
2)), x)
```

3.283 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=224

$$-\frac{2(43A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{se}}{\sqrt{2}\sqrt{a\sec(c+}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*A*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A + 35*C)*Sin[c + d*x])/(105*d*S
qrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A + 35*C)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.67462, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4087, 4022, 4013, 3808, 206}

$$-\frac{2(43A+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{se}}{\sqrt{2}\sqrt{a\sec(c+}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*A*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A + 35*C)*Sin[c + d*x])/(105*d*S
qrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A + 35*C)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{aA}{2}+\frac{1}{2}a(6A+7C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{105}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{105}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2A\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{105}$$

$$= \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 6.47997, size = 573, normalized size = 2.56

$$\frac{(A+C)\sin(c+dx)\cos^4(c+dx)(\sec(c+dx)+1)^{3/2}\sqrt{\sec^2(c+dx)-1}\left(\log\left(-3\sec^2(c+dx)-2\sqrt{2}\sqrt{\sec(c+dx)+1}\sqrt{\sec\right.\right.}{d(\cos(c+dx)+1)\sqrt{2-2\cos}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] ((A + C)*Cos[c + d*x]^4*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt
[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]] -
Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sq
rt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])^(3/2)*S
qrt[-1 + Sec[c + d*x]^2]*(A + C*Sec[c + d*x]^2)*Sin[c + d*x])/(d*(1 + Cos[c
 + d*x])*Sqrt[2 - 2*Cos[c + d*x]^2]*Sqrt[1 - Cos[c + d*x]^2]*(A + 2*C + A*C
os[2*c + 2*d*x])*Sqrt[a*(1 + Sec[c + d*x])]) + (Sqrt[(1 + Cos[c + d*x])*Sec
[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((-2*(193*A + 140*
C)*Cos[d*x]*Sin[c])/(105*d) + ((113*A + 70*C)*Cos[2*d*x]*Sin[2*c])/(105*d)
- (6*A*Cos[3*d*x]*Sin[3*c])/(35*d) + (A*Cos[4*d*x]*Sin[4*c])/(14*d) + (8*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(46*A*Sin[(d*x)/2] + 35*C*Sin[(d*x)/2]))/(105*d)
- (2*(193*A + 140*C)*Cos[c]*Sin[d*x])/(105*d) + ((113*A + 70*C)*Cos[2*c]*Si
n[2*d*x])/(105*d) - (6*A*Cos[3*c]*Sin[3*d*x])/(35*d) + (A*Cos[4*c]*Sin[4*d*
x])/(14*d) + (8*(46*A + 35*C)*Tan[c/2])/(105*d)))/((A + 2*C + A*Cos[2*c + 2
*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.431, size = 216, normalized size = 1.

$$-\frac{(\cos(dx+c))^4}{105\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\Bigg(30\,A(\cos(dx+c))^4-36\,A(\cos(dx+c))^3+105\,\arctan\Bigg(1/2\,\sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/105/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(30*A*cos(d*x+c)^4-36*A*cos(
d*x+c)^3+105*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+
c)+1))^(1/2)*A*sin(d*x+c)+105*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+68*A*cos(d*x+c)^2+70*C*cos(d*x+c
)^2-148*A*cos(d*x+c)-140*C*cos(d*x+c)+86*A+70*C)*cos(d*x+c)^4*(1/cos(d*x+c)
)^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.18826, size = 986, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
```

```
7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) + 140*
(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin
(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C/sqrt(a))/d
```

Fricas [A] time = 0.54735, size = 1108, normalized size = 4.95

$$\left[\frac{\frac{105\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(15A\cos(dx+c)^4-3A\cos(dx+c)^3+(31A+35\right.}{}}{210\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/210*(105*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a) + 4*(15*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 + (31*A + 35*C)
*cos(d*x + c)^2 - (43*A + 35*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/10
5*(105*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/si
n(d*x + c)) - 2*(15*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 + (31*A + 35*C)*c
os(d*x + c)^2 - (43*A + 35*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(7/
2)), x)
```

3.284 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{(A+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{(A+3C)\sin(}{2ad\sqrt{a\,s}}$$

```
[Out] (-3*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
 + ((A + 9*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((A + 3*C)*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.562369, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4085, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(A+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{(A+3C)\sin(}{2ad\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] (-3*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
 + ((A + 9*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((A + 3*C)*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(A-3C)-a(A+3C)\sec(c+dx\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} -$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} -$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+3C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{3C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(A+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - (A$$

Mathematica [B] time = 7.0676, size = 800, normalized size = 4.26

$$(\sec(c+dx)+1)^{3/2}\left(C\sec^2(c+dx)+A\right)\left(\frac{(A+3C)\cos^2(c+dx)\left(\log\left(-3\sec^2(c+dx)-2\sec(c+dx)-2\sqrt{2}\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+dx)-1}\sqrt{\sec(c+dx}\right.\right.}{2d(\cos(c\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(3/2),x]
```

```
[Out] (Cos[c + d*x]^2*(1 + Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*(((A + 3*C)
*Cos[c + d*x]^2*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt
[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]] - Log[1 -
2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + S
ec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])*Sqrt[-1 + Sec[c
 + d*x]^2]*Sin[c + d*x])/(2*d*(1 + Cos[c + d*x])*Sqrt[2 - 2*Cos[c + d*x]^2]
*Sqrt[1 - Cos[c + d*x]^2]) - (3*C*Cos[c + d*x]^2*(-8*Log[1 + Sec[c + d*x]]
+ 8*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sq
rt[-1 + Sec[c + d*x]^2]] + Sqrt[2]*(-Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x
]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c +
 d*x]^2]] + Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[
c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]]))*(1 + Sec[c +
d*x])*Sqrt[-1 + Sec[c + d*x]^2]*Sin[c + d*x])/(2*d*(1 + Cos[c + d*x])*(1 -
Cos[c + d*x]^2))))/(2*(A + 2*C + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))
^(3/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(1 + Sec[c + d*x])^(3/2)*(
A + C*Sec[c + d*x]^2)*((Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(-(A*Sin[c/2]) - C*Si
n[c/2]))/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) - C*Sin[
(d*x)/2]))/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 3*C*Sin[(
d*x)/2]))/d + ((A + 3*C)*Tan[c/2])/d))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sec[
c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.388, size = 370, normalized size = 2.

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\left(-3\,C\sin(dx+c)\sqrt{2}\cos(dx+c)\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(-3*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+3*C*sin(d*x+c)*2^(1/2)*cos(d*x+c
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+A
*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*c
os(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+9*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^
(1/2)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+C*cos(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2)+2*C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3*(co
s(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.702915, size = 1658, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((A + 9*C)*cos(d*x + c)^2 + 2*(A + 9*C)*cos(d*x + c) + A + 9*
C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 6*(C*cos(d*x + c)^2 + 2*C*cos(d*
x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x
 + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) +
4*((A + 3*C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d), -1/4*(sqrt(2)*((A + 9*C)*cos(d*x + c)^2 + 2*(A + 9*C)*cos(d*x +
c) + A + 9*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 6*(C*cos(d*x + c)^2 + 2*
C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)) - 2*((A + 3*C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx + c\right)^{2} + A\right) \sec\left(dx + c\right)^{\frac{3}{2}}}{\left(a \sec\left(dx + c\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3
/2), x)
```

3.285 $\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(3A-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((3*A - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.391786, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4085, 4023, 3808, 206, 3801, 215}

$$\frac{(3A-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((3*A - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(3A-C)-2aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{2a^2}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-5C)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a} + \frac{C\int}{}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A-5C)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{}}{}\right.}{2ad}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(3A-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(}{}$$

Mathematica [B] time = 7.24291, size = 795, normalized size = 5.48

$$\frac{(\sec(c+dx)+1)^{3/2}\left(C\sec^2(c+dx)+A\right)\left(\frac{(3A-C)\left(\log\left(-3\sec^2(c+dx)-2\sec(c+dx)-2\sqrt{2}\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+dx)-1}\sqrt{\sec(c+dx)}+1\right)-\log\left(-\right.}{2d(\cos(c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(3/2),x]
```

```
[Out] (Cos[c + d*x]^2*(1 + Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*(((3*A - C)
*Cos[c + d*x]^2*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt
[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]] - Log[1 -
2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + S
ec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])*Sqrt[-1 + Sec[c
 + d*x]^2]*Sin[c + d*x])/(2*d*(1 + Cos[c + d*x])*Sqrt[2 - 2*Cos[c + d*x]^2]
*Sqrt[1 - Cos[c + d*x]^2]) + (C*Cos[c + d*x]^2*(-8*Log[1 + Sec[c + d*x]] +
8*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]*Sqrt
[-1 + Sec[c + d*x]^2]] + Sqrt[2]*(-Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^
2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d
```

```
*x]^2]] + Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]]))*(1 + Sec[c + d*
x])*Sqrt[-1 + Sec[c + d*x]^2]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])*(1 - Cos[
c + d*x]^2))))/(2*(A + 2*C + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(3/
2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(1 + Sec[c + d*x])^(3/2)*(A +
C*Sec[c + d*x]^2)*((Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(A*Sin[c/2] + C*Sin[c/2])
)/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) - C*Sin[(d*x)/2])
)/d + (Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(2*
d) - ((A + C)*Tan[c/2])/d))/((A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3
/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.364, size = 314, normalized size = 2.2

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-2\,C\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+
c)*(-2*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))*sin(d*x+c)+2*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+3*A*arctan(1/2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*
C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+C*cos(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)-A*(-2/(cos(d*x+c)+1))^(1/2)-C*(-2/(cos(d*x+c)+1))
^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.32726, size = 4257, normalized size = 29.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
 + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin
(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2
*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log
```

```
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c
)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
+ 4*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*co
s(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*
sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2
*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a))
 + (4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*co
s(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x +
 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 +
 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*
x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2
*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(
```

```
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*C/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*
sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)
*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)))/d
```

Fricas [B] time = 0.687207, size = 1613, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((3*A - 5*C)*cos(d*x + c)^2 + 2*(3*A - 5*C)*cos(d*x + c) + 3
*A - 5*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(A + C)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(C*cos(d*x + c
)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*
(sqrt(2)*((3*A - 5*C)*cos(d*x + c)^2 + 2*(3*A - 5*C)*cos(d*x + c) + 3*A - 5
*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A + C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(C*cos(d*x + c)^2 + 2*C
*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
c) - 2*a)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3
/2), x)
```

3.286 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{(7A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -((7*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A + C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.350837, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4085, 4013, 3808, 206}

$$-\frac{(7A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),
x]
```

```
[Out] -((7*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A + C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(5A+C)+a(A-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} -$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(7A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+a}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 3.11372, size = 303, normalized size = 1.99

$$\frac{(\sec(c+dx)+1)^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{2\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)+1}(4A\cos(c+dx)+5A+C)}{\sec^{\frac{3}{2}}(c+dx)} - \sqrt{2}(7A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(
3/2)),x]
```

```
[Out] ((1 + Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((2*(5*A + C + 4*A*Cos[c +
 d*x])*Sec[(c + d*x)/2]^3*Sqrt[1 + Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(
3*(c + d*x))/2]))/Sec[c + d*x]^(3/2) - Sqrt[2]*(7*A - C)*Cos[c + d*x]^2*Cot
[c + d*x]*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c
 + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]] - Log[1 - 2*Sec[c + d
*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]
]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(8*d*(A + 2*C + A*Cos[2*(c
+ d*x)])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.347, size = 285, normalized size = 1.9

$$\frac{-1+\cos(dx+c)}{4\,da^2(\sin(dx+c))^3}\left(-7\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d/a^2*(-1+cos(d*x+c))*(-7*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+C*cos(d*x+c)*sin(d*x+
```

```
c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/
2)-7*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
1/2)*A*sin(d*x+c)+C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))*sin(d*x+c)+8*A*cos(d*x+c)^2+2*A*cos(d*x+c)+2*C*cos(d*x+
c)-10*A-2*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1/cos(d*x+c)
)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.542567, size = 1098, normalized size = 7.22

$$\left[-\frac{\sqrt{2}\left((7\,A-C)\cos\left(dx+c\right)^{2}+2\,(7\,A-C)\cos\left(dx+c\right)+7\,A-C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - C)*cos(d*x + c)^2 + 2*(7*A - C)*cos(d*x + c) + 7*A -
 C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)^2 + (5*A +
C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(
sqrt(2)*((7*A - C)*cos(d*x + c)^2 + 2*(7*A - C)*cos(d*x + c) + 7*A - C)*sqr
t(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c)^2 + (5*A + C)*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x +
 c))), x)
```

3.287 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(11A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(19A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((11*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2
*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A + 3*C)*Sin[c + d*
x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((19*A + 3*C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.514951, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4085, 4022, 4013, 3808, 206}

$$\frac{(11A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(19A+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),
x]
```


```
[Out] ((11*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(2
*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A + 3*C)*Sin[c + d*
x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((19*A + 3*C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(7A+3C)+2aA\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+a}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+a}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(7A+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+a}}$$

$$= \frac{(11A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+}$$

Mathematica [A] time = 3.00724, size = 316, normalized size = 1.57

$$\frac{(\sec(c+dx)+1)^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{2\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)+1}\sec^3\left(\frac{1}{2}(c+dx)\right)(12A\cos(c+dx)-2A\cos(2(c+dx))+17}{\sec^{\frac{3}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(
3/2)),x]
```

```
[Out] ((1 + Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((2*(17*A + 3*C + 12*A*Cos
[c + d*x] - 2*A*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^3*Sqrt[1 + Sec[c + d*x]]
*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/Sec[c + d*x]^(3/2) + 3*Sqrt[2]*
(11*A + 3*C)*Cos[c + d*x]^2*Cot[c + d*x]*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c
+ d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c +
 d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[
c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2
```

```
]))/(24*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.381, size = 306, normalized size = 1.5

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^2}{12\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(33\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(33*A*sin(d*
x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d
*x+c)+1))^(1/2)+9*C*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+33*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+8*A*cos(d*x+c)^3+
9*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*sin(d*x+c)-32*A*cos(d*x+c)^2-14*A*cos(d*x+c)-6*C*cos(d*x+c)+38*A+6*C)*c
os(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.538732, size = 1197, normalized size = 5.96

$$\left[\frac{3\sqrt{2}\left((11\,A+3\,C)\cos(dx+c)^2+2\,(11\,A+3\,C)\cos(dx+c)+11\,A+3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx}}{\cos(dx+c)^2+2\,\cos(d}\right.}{24\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((11*A + 3*C)*cos(d*x + c)^2 + 2*(11*A + 3*C)*cos(d*x + c)
 + 11*A + 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*A*cos(d*x + c)^
3 - 12*A*cos(d*x + c)^2 - (19*A + 3*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 +
```

```
2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A + 3*C)*cos(d*x + c)^
2 + 2*(11*A + 3*C)*cos(d*x + c) + 11*A + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x +
 c))) - 2*(4*A*cos(d*x + c)^3 - 12*A*cos(d*x + c)^2 - (19*A + 3*C)*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{(a \sec(dx+c) + a)^{\frac{3}{2}} \sec(dx+c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(
3/2)), x)
```

3.288 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{(15A+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3}}$$

```
[Out] -((15*A + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(
2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A + 5*C)*Sin[c + d
*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((13*A + 5*C)*S
in[c + d*x])/(10*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((49*A
+ 25*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.702256, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4085, 4022, 4013, 3808, 206}

$$-\frac{(15A+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),
x]
```


```
[Out] -((15*A + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(
2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A + 5*C)*Sin[c + d
*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((13*A + 5*C)*S
in[c + d*x])/(10*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((49*A
+ 25*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(9A+5C)+a(3A+C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= -\frac{(15A+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}$$

Mathematica [A] time = 5.23948, size = 331, normalized size = 1.33

$$\frac{(\sec(c+dx)+1)^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{2\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)+1}((39A+20C)\cos(c+dx)-2A\cos(2(c+}{\sec^{\frac{3}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(
3/2)),x]
```

```
[Out] ((1 + Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((2*(47*A + 25*C + (39*A +
 20*C)*Cos[c + d*x] - 2*A*Cos[2*(c + d*x)] + A*Cos[3*(c + d*x)])*Sec[(c + d
*x)/2]^3*Sqrt[1 + Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))
/Sec[c + d*x]^(3/2) - 5*Sqrt[2]*(15*A + 7*C)*Cos[c + d*x]^2*Cot[c + d*x]*(L
og[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqr
t[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*Sec[
c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c
 + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(40*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a
*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.391, size = 328, normalized size = 1.3

$$-\frac{(-1+\cos(dx+c))(\cos(dx+c))^3}{20\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(75\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/20/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(75*A*sin(d
*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(
d*x+c)+1))^(1/2)-8*A*cos(d*x+c)^4+35*C*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+75*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c
)+16*A*cos(d*x+c)^3+35*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-80*A*cos(d*x+c)^2-40*C*cos(d*x+c)^2-26*
A*cos(d*x+c)-10*C*cos(d*x+c)+98*A+50*C)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/s
in(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.555983, size = 1280, normalized size = 5.16

$$\left[\frac{5\sqrt{2}\left((15\,A+7\,C)\cos(dx+c)^2+2\,(15\,A+7\,C)\cos(dx+c)+15\,A+7\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(d}}{\cos(dx+c)^2+2\,\cos(d}\right.}{40\left(a^2d\cos(dx+c)^2+2\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/40*(5*sqrt(2)*((15*A + 7*C)*cos(d*x + c)^2 + 2*(15*A + 7*C)*cos(d*x + c)
 + 15*A + 7*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*A*cos(d*x + c)^
4 - 4*A*cos(d*x + c)^3 + 4*(9*A + 5*C)*cos(d*x + c)^2 + (49*A + 25*C)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(5*sqrt(
2)*((15*A + 7*C)*cos(d*x + c)^2 + 2*(15*A + 7*C)*cos(d*x + c) + 15*A + 7*C)
*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c)^4 - 4*A*cos(d*x +
 c)^3 + 4*(9*A + 5*C)*cos(d*x + c)^2 + (49*A + 25*C)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{(a \sec(dx+c) + a)^{\frac{3}{2}} \sec(dx+c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(
5/2)), x)
```

3.289 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{(3A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(3A+115C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+C}{4a}$$

```
[Out] (-5*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
 + ((3*A + 115*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x
]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((A - 15*C)*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((3*A + 35
*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.767542, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4085, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(3A+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(3A+115C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+C}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```


```
[Out] (-5*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
 + ((3*A + 115*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x
]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((A - 15*C)*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((3*A + 35
*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}dx = -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(3A-5C)-a(A+5C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \int$$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + ($$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + ($$

$$= -\frac{(A+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + ($$

$$= -\frac{5C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A+115C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - ($$

Mathematica [B] time = 7.30941, size = 903, normalized size = 3.81

$$\frac{\cos^2(c+dx)\left(C\sec^2(c+dx)+A\right)\left(\frac{(3A+35C)\cos^2(c+dx)\left(\log\left(-3\sec^2(c+dx)-2\sec(c+dx)-2\sqrt{2}\sqrt{\sec(c+dx)+1}\sqrt{\sec^2(c+dx)-1}\sqrt{\sec(c+dx)}+1\right)-\log(\right.}{2d(\cos(c+dx)+1)\sqrt{2-}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(1 + Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(((3*A + 35
*C)*Cos[c + d*x]^2*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*S
qrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]] - Log[1
 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1
+ Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]])*(1 + Sec[c + d*x])*Sqrt[-1 + Se
c[c + d*x]^2]*Sin[c + d*x])/(2*d*(1 + Cos[c + d*x])*Sqrt[2 - 2*Cos[c + d*x]
^2]*Sqrt[1 - Cos[c + d*x]^2]) - (20*C*Cos[c + d*x]^2*(-8*Log[1 + Sec[c + d*
x]] + 8*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]
]*Sqrt[-1 + Sec[c + d*x]^2]] + Sqrt[2]*(-Log[1 - 2*Sec[c + d*x] - 3*Sec[c +
 d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec
[c + d*x]^2]] + Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[
Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[-1 + Sec[c + d*x]^2]]))*(1 + Sec[
c + d*x])*Sqrt[-1 + Sec[c + d*x]^2]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])*(1
- Cos[c + d*x]^2))))/(16*(A + 2*C + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x
]))^(5/2)) + (Sqrt[(1 + Cos[c + d*x])*Sec[c + d*x]]*(1 + Sec[c + d*x])^(5/2
)*(A + C*Sec[c + d*x]^2)*((Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(A*Sin[c/2] - 15*C
*Sin[c/2]))/(16*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-(A*Sin[c/2]) - C*Sin[
c/2]))/(8*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - 15*C*Sin[(d
*x)/2]))/(16*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) - C*Sin
[(d*x)/2]))/(8*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + 35*C*S
in[(d*x)/2]))/(8*d) + ((3*A + 35*C)*Tan[c/2])/(8*d)))/((A + 2*C + A*Cos[2*c
```

```
+ 2*d*x])*Sec[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.375, size = 615, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)*(-1+cos(d*x+c))^2*(-40*C*2^(1/2)*sin(d*x+c)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+40*C*2^(1/2)*si
n(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x
+c)))*cos(d*x+c)^2+3*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)^2-3*A*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-40*C*sin(d*
x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))+40*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+115*C*sin(d*x+c)*arcta
n(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-35*C*(-2/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)^3+3*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(
-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+115*C*
sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-20*C
*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+7*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2)+39*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*C*(-2/(cos(d*x+c)+1))^(1
/2))/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.749329, size = 2001, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((3*A + 115*C)*cos(d*x + c)^3 + 3*(3*A + 115*C)*cos(d*x + c)
^2 + 3*(3*A + 115*C)*cos(d*x + c) + 3*A + 115*C)*sqrt(a)*log(-(a*cos(d*x +
c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1)) + 80*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c)
```

```
+ C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2
 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((3*A
 + 35*C)*cos(d*x + c)^2 + (7*A + 55*C)*cos(d*x + c) + 16*C)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(
2)*((3*A + 115*C)*cos(d*x + c)^3 + 3*(3*A + 115*C)*cos(d*x + c)^2 + 3*(3*A
+ 115*C)*cos(d*x + c) + 3*A + 115*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 8
0*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(-a)*a
rctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((3*A + 35*C)*
cos(d*x + c)^2 + (7*A + 55*C)*cos(d*x + c) + 16*C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5
/2), x)
```

3.290 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{(5A-43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5A-1\ldots}{16\ldots}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((5*A - 43*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A - 11*C)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.560581, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4085, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(5A-43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5A-1\ldots}{16\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((5*A - 43*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A - 11*C)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(5A-3C)-4aC\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-11C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \cdots$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-11C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-11C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \cdots$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(5A-43C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A \cdots}{}$$

Mathematica [B] time = 6.65486, size = 445, normalized size = 2.32

$$\left(A+C\sec^2(c+dx)\right)\left(8\tan\left(\frac{c}{2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{\sec(c+dx)+1}((5A-11C)\cos(c+dx)+A-15C)+\sin(\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(5/2),x]
```

```
[Out] ((A + C*Sec[c + d*x]^2)*(8*(A - 15*C + (5*A - 11*C)*Cos[c + d*x])*Sec[c/2]*
Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[1 + Sec[c + d*x]]*Sin[(d*x)/2]*Tan[(c +
d*x)/2]^2 + 8*(A - 15*C + (5*A - 11*C)*Cos[c + d*x])*Sec[c + d*x]*Sqrt[1 +
Sec[c + d*x]]*Tan[c/2]*Tan[(c + d*x)/2]^2 + (-256*C*Log[1 + Sec[c + d*x]] +
 256*C*Log[Sqrt[Sec[c + d*x]] + Sec[c + d*x]^(3/2) + Sqrt[1 + Sec[c + d*x]]
*Sqrt[Tan[c + d*x]^2]] + Sqrt[2]*(5*A - 43*C)*(Log[1 - 2*Sec[c + d*x] - 3*S
ec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Ta
n[c + d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt
[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]]))*Sqrt[Sec[c +
d*x]]*Sin[c + d*x]*Sqrt[Tan[c + d*x]^2]))/(64*d*(A + 2*C + A*Cos[2*(c + d*x
)])*(-1 + Sec[c + d*x])*(Sec[c + d*x]/(1 + Sec[c + d*x]))^(3/2)*(a*(1 + Sec
[c + d*x]))^(5/2))
```

Maple [B] time = 0.352, size = 550, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(-1+cos(d*x+c))^2*(16*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-16*C*sin(d*x
+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))+5*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-5*A*sin(d*x+
c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+16*C*2^(1/2)
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*si
n(d*x+c)-16*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1+sin(d*x+c)))*sin(d*x+c)-11*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+4
3*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-
4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*A*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))*sin(d*x+c)-4*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+43*C
*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-A*(-2/(cos(d*x
+c)+1))^(1/2)+15*C*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))
^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+
c)^5
```

Maxima [B] time = 4.43378, size = 10615, normalized size = 55.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/32*((4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
```

```
)))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*si
n(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x
+ 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*
c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x
+ 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*
cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*
d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8
*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3
*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c)
+ 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*
cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2
```

*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))*
A/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*
x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c)
 + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2
*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^
2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)) + (44*(sin(4*d*x + 4*c) + 6*sin(2
*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*sin(5/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 19*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 11*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x +
 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*s
in(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*
(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2
+ 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c
)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 1
6*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x +
4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*s
in(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2

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*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4
*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2
*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt
(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sq
rt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqr
t(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos
(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) +
 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c)
 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c)
+ 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*s
qrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x
+ 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*
d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos
(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x
+ 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
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x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2
*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 43*(2*(6*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^
2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(
cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(
2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 43*(2*(
6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*
x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
- 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(19*
cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*cos(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 176*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(cos(4*d*x + 4*c)
 + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 176*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((sqrt(2)*a^2*cos(4*d*x + 4*c)
^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt
(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)
^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 1
2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6
*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*
```

```
x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sq
rt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c)
 + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sqrt(a)))/d
```

Fricas [B] time = 0.736031, size = 1967, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*A - 43*C)*cos(d*x + c)^3 + 3*(5*A - 43*C)*cos(d*x + c)^
2 + 3*(5*A - 43*C)*cos(d*x + c) + 5*A - 43*C)*sqrt(a)*log(-(a*cos(d*x + c)^
2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 32*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C
)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 -
2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((5*A -
11*C)*cos(d*x + c)^2 + (A - 15*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A - 4
3*C)*cos(d*x + c)^3 + 3*(5*A - 43*C)*cos(d*x + c)^2 + 3*(5*A - 43*C)*cos(d*
x + c) + 5*A - 43*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 32*(C*cos(d*x + c
)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((5*A - 11*C)*cos(d*x + c)^2 +
 (A - 15*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*
a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5
/2), x)
```

3.291 $\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(19A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((19*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((9*A - 7*C)*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.369346, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4085, 4012, 3808, 206}

$$\frac{(19A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```


```
[Out] ((19*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((9*A - 7*C)*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}dx = -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(7A-C)+a(A-3C)\sec(c+dx\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(9A-7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(9A-7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{(19A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 2.84041, size = 308, normalized size = 2.

$$(\sec(c+dx)+1)^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)+1}\sec^5\left(\frac{1}{2}(c+dx)\right)((13A-3C)\cos(c+dx)+9A-7C)}{\sec^{\frac{3}{2}}(c+dx)} + \sqrt{2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(5/2),x]
```

```
[Out] ((1 + Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(((9*A - 7*C + (13*A - 3*C
)*Cos[c + d*x])*Sec[(c + d*x)/2]^5*Sqrt[1 + Sec[c + d*x]]*(Sin[(c + d*x)/2]
 - Sin[(3*(c + d*x))/2]))/Sec[c + d*x]^(3/2) + Sqrt[2]*(19*A + 3*C)*Cos[c +
 d*x]^2*Cot[c + d*x]*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]
*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]] - Log[1 -
2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + S
ec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(64*d*(A + 2*C +
 A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.364, size = 348, normalized size = 2.3

$$\frac{\cos(dx+c)(-1+\cos(dx+c))^2}{16da^3(\sin(dx+c))^5}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(19A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x
+c)*(-1+cos(d*x+c))^2*(19*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))+13*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+3*C*sin
(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*C*cos
(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+19*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))*sin(d*x+c)-4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*C*arct
an(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-4*C*cos(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2)-9*A*(-2/(cos(d*x+c)+1))^(1/2)+7*C*(-2/(cos(d*x+c)+1))
^(1/2))/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.539892, size = 1308, normalized size = 8.49

$$\left[\frac{\sqrt{2}\left((19\,A+3\,C)\cos\left(dx+c\right)^{3}+3\,(19\,A+3\,C)\cos\left(dx+c\right)^{2}+3\,(19\,A+3\,C)\cos\left(dx+c\right)+19\,A+3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx}{\quad}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 3*C)*cos(d*x + c)^3 + 3*(19*A + 3*C)*cos(d*x + c)^2
 + 3*(19*A + 3*C)*cos(d*x + c) + 19*A + 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((13*A - 3*C)*cos(d*x + c)^2 + (9*A - 7*C)*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/
32*(sqrt(2)*((19*A + 3*C)*cos(d*x + c)^3 + 3*(19*A + 3*C)*cos(d*x + c)^2 +
3*(19*A + 3*C)*cos(d*x + c) + 19*A + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))
) + 2*((13*A - 3*C)*cos(d*x + c)^2 + (9*A - 7*C)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5
/2), x)
```

3.292 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(49A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5(15A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-$$

```
[Out] (-5*(15*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 3*C)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((49*A + C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.545614, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4085, 4020, 4013, 3808, 206}

$$\frac{(49A+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5(15A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),
x]
```


```
[Out] (-5*(15*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 3*C)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((49*A + C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(9A+C)+2a(A-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{5(15A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sqrt{\sec(c+dx)}\sin(c-}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 2.8492, size = 317, normalized size = 1.59

$$\frac{(\sec(c+dx)+1)^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)+1}(5(17A+C)\cos(c+dx)+16A\cos(2(c+d}{\sec^{\frac{3}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(
5/2)),x]
```

```
[Out] ((1 + Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(((65*A + C + 5*(17*A + C)
*Cos[c + d*x] + 16*A*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*Sqrt[1 + Sec[c +
d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/Sec[c + d*x]^(3/2) - 5*Sq
rt[2]*(15*A - C)*Cos[c + d*x]^2*Cot[c + d*x]*(Log[1 - 2*Sec[c + d*x] - 3*Se
c[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan
[c + d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[
Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d*
x]^2]))/(64*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.35, size = 419, normalized size = 2.1

$$-\frac{(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\left(-75\,A\sin(dx+c)(\cos(dx+c))^2\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/32/d/a^3*(-1+cos(d*x+c))^2*(-75*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+5*C*cos(d*x+c)
^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-150*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+10*C*cos(d*x+c)*sin(d*x+c)*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+64*A*c
os(d*x+c)^3-75*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*
x+c)+1))^(1/2)*A*sin(d*x+c)+5*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+106*A*cos(d*x+c)^2+10*C*cos(d*x+
c)^2-72*A*cos(d*x+c)-8*C*cos(d*x+c)-98*A-2*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))
^(1/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.551441, size = 1349, normalized size = 6.78

$$\left[-\frac{5\sqrt{2}\left((15\,A-C)\cos(dx+c)^3+3\,(15\,A-C)\cos(dx+c)^2+3\,(15\,A-C)\cos(dx+c)+15\,A-C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2\cdots}{\cdots}\right.}{64\left(a^3d\cos(dx+c)^3+3\,a^3d\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/64*(5*sqrt(2)*((15*A - C)*cos(d*x + c)^3 + 3*(15*A - C)*cos(d*x + c)^2
+ 3*(15*A - C)*cos(d*x + c) + 15*A - C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*
sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 4*(32*A*cos(d*x + c)^3 + 5*(17*A + C)*cos(d*x + c)^2 + (49*A + C)*cos(
```

```
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d), 1/32*(5*sqrt(2)*((15*A - C)*cos(d*x + c)^3 + 3*(15*A - C)*cos(
d*x + c)^2 + 3*(15*A - C)*cos(d*x + c) + 15*A - C)*sqrt(-a)*arctan(sqrt(2)*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(
d*x + c))) + 2*(32*A*cos(d*x + c)^3 + 5*(17*A + C)*cos(d*x + c)^2 + (49*A +
 C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c) + a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x +
 c))), x)
```

3.293 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{(299A+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{5(19A+3C)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((163*A + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])
/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A + C)*Sin[c +
d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (5*(19*A + 3
*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
((299*A + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c
 + d*x]])
```

Rubi [A] time = 0.703322, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4085, 4020, 4022, 4013, 3808, 206}

$$-\frac{(299A+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{5(19A+3C)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),
x]
```

```
[Out] ((163*A + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])
/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A + C)*Sin[c +
d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (5*(19*A + 3
*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
((299*A + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c
 + d*x]])
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(11A+3C)+a(3A-C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= \frac{(163A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sin(c+dx}{4d\sqrt{\sec(c+dx)}(a+a\sec(c}$$

Mathematica [A] time = 3.22632, size = 331, normalized size = 1.35

$$(\sec(c+dx)+1)^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)+1}\sec^5\left(\frac{1}{2}(c+dx)\right)((479A+39C)\cos(c+dx)+80A\cos(2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(
5/2)),x]
```

```
[Out] ((1 + Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(((379*A + 27*C + (479*A +
 39*C)*Cos[c + d*x] + 80*A*Cos[2*(c + d*x)] - 8*A*Cos[3*(c + d*x)])*Sec[(c
+ d*x)/2]^5*Sqrt[1 + Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]
))/Sec[c + d*x]^(3/2) + 3*Sqrt[2]*(163*A + 19*C)*Cos[c + d*x]^2*Cot[c + d*x
]*(Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]
*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*
Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[T
an[c + d*x]^2]])*Sqrt[Tan[c + d*x]^2]))/(192*d*(A + 2*C + A*Cos[2*(c + d*x)
])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.345, size = 438, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*A*si
n(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)+57*C*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+
c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+978*A*sin(d*x+c)*cos(d*x
+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1
/2)+64*A*cos(d*x+c)^4+114*C*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+489*arctan(1/2*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-384*A*co
s(d*x+c)^3+57*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2))*sin(d*x+c)-686*A*cos(d*x+c)^2-78*C*cos(d*x+c)^2+408*A*cos(d
*x+c)+24*C*cos(d*x+c)+598*A+54*C)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x
+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.560931, size = 1472, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/192*(3*sqrt(2)*((163*A + 19*C)*cos(d*x + c)^3 + 3*(163*A + 19*C)*cos(d*x
 + c)^2 + 3*(163*A + 19*C)*cos(d*x + c) + 163*A + 19*C)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(32*A*cos(d*x + c)^4 - 160*A*cos(d*x + c)^3 - (503*A
 + 39*C)*cos(d*x + c)^2 - (299*A + 27*C)*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*
((163*A + 19*C)*cos(d*x + c)^3 + 3*(163*A + 19*C)*cos(d*x + c)^2 + 3*(163*A
 + 19*C)*cos(d*x + c) + 163*A + 19*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
2*(32*A*cos(d*x + c)^4 - 160*A*cos(d*x + c)^3 - (503*A + 39*C)*cos(d*x + c)
^2 - (299*A + 27*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(
3/2)), x)
```

3.294 $\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=295

$$\frac{(157A+45C)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A+735C)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A+195C)\sin(c+dx)}{240a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)}}$$

```
[Out] -((283*A + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x]
)/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A + 5*C)*Sin[c
 + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((157*A +
 45*C)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
 - ((787*A + 195*C)*Sin[c + d*x])/(240*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) + ((2671*A + 735*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a^
2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.908951, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4085, 4020, 4022, 4013, 3808, 206}

$$\frac{(157A+45C)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A+735C)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A+195C)\sin(c+dx)}{240a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),
x]
```


```
[Out] -((283*A + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x]
)/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A + 5*C)*Sin[c
 + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((157*A +
 45*C)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
 - ((787*A + 195*C)*Sin[c + d*x])/(240*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) + ((2671*A + 735*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a^
2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
```

```
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(13A+5C)+4aA\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(283A+75C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

Mathematica [A] time = 4.06811, size = 349, normalized size = 1.18

$$\frac{(\sec(c+dx)+1)^{5/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)+1}(5(887A+255C)\cos(c+dx)+16(52A+15C)}{\sec^{\frac{3}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(
5/2)),x]
```

```
[Out] ((1 + Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(((3491*A + 975*C + 5*(887
*A + 255*C)*Cos[c + d*x] + 16*(52*A + 15*C)*Cos[2*(c + d*x)] - 40*A*Cos[3*(
c + d*x)] + 12*A*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Sqrt[1 + Sec[c + d*x]
]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/Sec[c + d*x]^(3/2) - 15*Sqrt[
2]*(283*A + 75*C)*Cos[c + d*x]^2*Cot[c + d*x]*(Log[1 - 2*Sec[c + d*x] - 3*S
ec[c + d*x]^2 - 2*Sqrt[2]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Ta
n[c + d*x]^2]] - Log[1 - 2*Sec[c + d*x] - 3*Sec[c + d*x]^2 + 2*Sqrt[2]*Sqrt
[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*Sqrt[Tan[c + d*x]^2]])*Sqrt[Tan[c + d
*x]^2]))/(960*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(5/2)
)
```

Maple [A] time = 0.385, size = 460, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/480/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(192*A*c
os(d*x+c)^5-4245*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-1125*C*cos(d*x+c)^2*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5
12*A*cos(d*x+c)^4-8490*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-2250*C*cos(d*x+c)*sin(d*x+c)
*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)
+3456*A*cos(d*x+c)^3-4245*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*
(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+960*C*cos(d*x+c)^3-1125*C*(-2/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)
+5974*A*cos(d*x+c)^2+1590*C*cos(d*x+c)^2-3768*A*cos(d*x+c)-1080*C*cos(d*x+c
)-5342*A-1470*C)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.579132, size = 1580, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/960*(15*sqrt(2)*((283*A + 75*C)*cos(d*x + c)^3 + 3*(283*A + 75*C)*cos(d*
x + c)^2 + 3*(283*A + 75*C)*cos(d*x + c) + 283*A + 75*C)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(96*A*cos(d*x + c)^5 - 160*A*cos(d*x + c)^4 + 32*(4
9*A + 15*C)*cos(d*x + c)^3 + 5*(911*A + 255*C)*cos(d*x + c)^2 + (2671*A + 7
35*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sq
rt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*
cos(d*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A + 75*C)*cos(d*x + c)^3 + 3
*(283*A + 75*C)*cos(d*x + c)^2 + 3*(283*A + 75*C)*cos(d*x + c) + 283*A + 75
*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^5 - 160*A*cos
(d*x + c)^4 + 32*(49*A + 15*C)*cos(d*x + c)^3 + 5*(911*A + 255*C)*cos(d*x +
 c)^2 + (2671*A + 735*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + A}{(a \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(
5/2)), x)
```

3.295 $\int (a+a\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=434

$$-\frac{3^{3/4}C\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-1}\right.}{5\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*C*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*Sqrt[2]*A*AppellF1
[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]) + (3*C*(a + a*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d*(1 + Sec[c + d*x])) - (3^(3/4)*C*EllipticF
[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^
(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec
[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*d*(1 - Sec[c + d*x])*(1 +
Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x]
)^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.704333, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {4055, 3924, 3779, 3778, 136, 3828, 3827, 50, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}+\frac{3C\tan(c+dx)(a\sec(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (3*C*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*Sqrt[2]*A*AppellF1
[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]) + (3*C*(a + a*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d*(1 + Sec[c + d*x])) - (3^(3/4)*C*EllipticF
[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^
(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec
[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*d*(1 - Sec[c + d*x])*(1 +
Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x]
)^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\int(a+a\sec(c+dx))^{2/3}\left(\ldots\right.}{5\ldots}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + A\int(a+a\sec(c+dx))^{2/3}\ldots$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{\left(A(a+a\sec(c+dx))^{2/3}\right)\int\ldots}{(1+\sec(c+\ldots}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(A(a+a\sec(c+dx))^{2/3}\tan\ldots\right.}{d\sqrt{1-\sec\ldots}}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\ldots\right.}{\ldots}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\ldots\right.}{\ldots}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\ldots\right.}{\ldots}$$

Mathematica [F] time = 4.52548, size = 0, normalized size = 0.

$$\int (a+a\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2), x]
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^{\frac{2}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{2}{3}}\left(A+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*(A + C*sec(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(2/3), x)
```

3.296 $\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=384

$$\frac{3^{3/4}C\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (3*C*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sqrt[2]*A*AppellF1
[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d
*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*C*EllipticF[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c
+ d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[
c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[
c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-
(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) -
 (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.43397, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4055, 3924, 3779, 3778, 136, 3828, 3827, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3C\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}+\frac{3^{3/4}C\tan(c+dx)\left(\sqrt[3]{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(1/3),x]
```


```
[Out] (3*C*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sqrt[2]*A*AppellF1
[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d
*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*C*EllipticF[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c
+ d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[
c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[
c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-
(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) -
 (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```

```
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3\int \frac{\frac{2aA}{3}-\frac{1}{3}aC\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}+A\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx-\frac{1}{2}C\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}+\frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}-\frac{\left(C\sqrt[3]{1+\sec(c+dx)}\right)\int}{2\sqrt[3]{a+a\sec(c}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}-\frac{(A\tan(c+dx))\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{5/6}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}+\frac{(C\,t}{2d\sqrt{}}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}}+\frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [F] time = 2.80309, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(1/3), x]
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int (A+C(\sec(dx+c))^2)\frac{1}{\sqrt[3]{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2{(c + dx)}}{\sqrt[3]{a(\sec{(c + dx)} + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{(a\sec(dx + c) + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.297 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=396

$$\frac{3^{3/4}(A-4C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec}}\right)\right)}{5\sqrt[3]{2}ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (-3*(A + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^(4/3)) + (3*Sqrt[2]*A*A
ppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d
*x])/(a*d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*(A
- 4*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1
/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^
(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.45772, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4053, 3924, 3779, 3778, 136, 3828, 3827, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{ad\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}-\frac{3(A+C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^{4/3}}+\frac{3^{3/4}(A-4C)\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(4/3),x]
```


```
[Out] (-3*(A + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^(4/3)) + (3*Sqrt[2]*A*A
ppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d
*x])/(a*d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*(A
- 4*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1
/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^
(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
```

]

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx = -\frac{3(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} - \frac{3\int \frac{-\frac{5aA}{3}+\frac{1}{3}a(A-4C)\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{5a^2}$$

$$= -\frac{3(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{A\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{a} - \frac{(A-4C)\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= -\frac{3(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{a\sqrt[3]{a+a\sec(c+dx)}} - \frac{\left((A-4C)\sqrt[3]{1+s\right.}{5a\sqrt[3]{a}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} - \frac{(A\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}} +$$

$$= -\frac{3(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan}{ad\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan}{ad\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [F] time = 3.13432, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(4/3),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(4/3), x]
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \left(A+C(\sec(dx+c))^2\right)(a+a\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(a\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{(a\sec(dx + c) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(4/3), x)
```

3.298 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{7/3}}\,dx$

Optimal. Leaf size=457

$$\frac{3^{3/4}(4A-7C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\mathrm{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{s}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{s}}\right.\right.}{55\sqrt[3]{2}a^2d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (-3*(A + C)*Tan[c + d*x])/(11*d*(a + a*Sec[c + d*x])^(7/3)) - (3*(4*A - 7*C
)*Tan[c + d*x])/(55*a^2*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)) -
(3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(5*a^2*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a +
 a*Sec[c + d*x])^(1/3)) + (3^(3/4)*(4*A - 7*C)*EllipticF[ArcCos[(2^(1/3) -
(1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(55*2^(1
/3)*a^2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c
+ d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.504245, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {4053, 3924, 3779, 3778, 136, 3828, 3827, 51, 63, 225}

$$-\frac{3\sqrt{2}A\tan(c+dx)F_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5a^2d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}-\frac{3(4A-7C)\tan(c+dx)}{55a^2d(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(7/3),x]
```


```
[Out] (-3*(A + C)*Tan[c + d*x])/(11*d*(a + a*Sec[c + d*x])^(7/3)) - (3*(4*A - 7*C
)*Tan[c + d*x])/(55*a^2*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)) -
(3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(5*a^2*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a +
 a*Sec[c + d*x])^(1/3)) + (3^(3/4)*(4*A - 7*C)*EllipticF[ArcCos[(2^(1/3) -
(1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(55*2^(1
/3)*a^2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c
+ d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
```

```
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{7/3}}\,dx = -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3\int \frac{-\frac{11aA}{3}+\frac{1}{3}a(4A-7C)\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx}{11a^2}$$

$$= -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} + \frac{A\int \frac{1}{(a+a\sec(c+dx))^{4/3}}\,dx}{a} - \frac{(4A-7C)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx}{11a}$$

$$= -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} + \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{(1+\sec(c+dx))^{4/3}}\,dx}{a^2\sqrt[3]{a+a\sec(c+dx)}} - \frac{\left((4A-7C)\sqrt[3]{1}\cdots\right.}{11\cdots}$$

$$= -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{(A\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{11/6}}\,dx, x, \sec(c+dx)\right)}{a^2d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}\cdots$$

$$= -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3(4A-7C)\tan(c+dx)}{55a^2d(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}AF_1\left(-\frac{5}{6}\cdots\right.}{5a^2d\sqrt{1}\cdots}$$

$$= -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3(4A-7C)\tan(c+dx)}{55a^2d(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}AF_1\left(-\frac{5}{6}\cdots\right.}{5a^2d\sqrt{1}\cdots}$$

$$= -\frac{3(A+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3(4A-7C)\tan(c+dx)}{55a^2d(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}AF_1\left(-\frac{5}{6}\cdots\right.}{5a^2d\sqrt{1}\cdots}$$

Mathematica [F] time = 3.86418, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{7/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(7/3),x]

[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(7/3), x]
```

Maple [F] time = 0.163, size = 0, normalized size = 0.

$$\int (A+C(\sec(dx+c))^2)(a+a\sec(dx+c))^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + A}{(a \sec (dx+c) + a)^{\frac{7}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(7/3), x)
```

3.299 $\int (a+a\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=815

$$\frac{3C\tan(c+dx)(\sec(c+dx)a+a)^{4/3}}{7d}+\frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)(\sec(c+dx)+1)\,\mathrm{ta}}{11d\sqrt{1-\sec(c+dx)}}$$

```
[Out] (3*a*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*d) + (3*Sqrt[2]*a*A*Appe
llF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(1 + Sec[c
 + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 - Sec[c + d*
x]]) + (3*C*(a + a*Sec[c + d*x])^(4/3)*Tan[c + d*x])/(7*d) - (15*(1 + Sqrt[
3])*a*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*d*(1 + Sec[c + d*x])^(2
/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (15*2^(1/3)*3^(1/
4)*a*C*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a +
a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) +
2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*d*(1 - Sec[c + d*x
])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^
2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*C*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3
])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c
+ d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c
+ d*x])/(7*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1
 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 1.01616, antiderivative size = 815, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4055, 3924, 3779, 3778, 136, 3828, 3827, 50, 63, 308, 225, 1881}

$$\frac{3C\tan(c+dx)(\sec(c+dx)a+a)^{4/3}}{7d}+\frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)(\sec(c+dx)+1)\,\mathrm{ta}}{11d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*a*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*d) + (3*Sqrt[2]*a*A*Appe
llF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(1 + Sec[c
 + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 - Sec[c + d*
x]]) + (3*C*(a + a*Sec[c + d*x])^(4/3)*Tan[c + d*x])/(7*d) - (15*(1 + Sqrt[
3])*a*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*d*(1 + Sec[c + d*x])^(2
/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (15*2^(1/3)*3^(1/
4)*a*C*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a +
a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) +
```

```
2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*d*(1 - Sec[c + d*x
])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^
2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*C*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3
])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c
+ d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c
+ d*x])/(7*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1
 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} + \frac{3\int (a+a\sec(c+dx))^{4/3}\left(\frac{7aA}{3}\right.}{7a}$$

$$= \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} + A\int (a+a\sec(c+dx))^{4/3}\,dx +$$

$$= \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} + \frac{\left(aA\sqrt[3]{a+a\sec(c+dx)}\right)\int(1+}{\sqrt[3]{1+\sec(c+}}$$

$$= \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} - \frac{\left(aA\sqrt[3]{a+a\sec(c+dx)}\tan(c\right.}{d\sqrt{1-\sec(c}}$$

$$= \frac{3aC\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{7d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{se}\right.}{}$$

$$= \frac{3aC\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{7d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{se}\right.}{}$$

$$= \frac{3aC\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{7d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{se}\right.}{}$$

$$= \frac{3aC\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{7d} + \frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\mathrm{se}\right.}{}$$

Mathematica [F] time = 27.5295, size = 0, normalized size = 0.

$$\int (a+a\sec(c+dx))^{4/3}\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^(4/3)*(A + C*Sec[c + d*x]^2), x]
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^{\frac{4}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx + c\right)^{2} + A\right) \left(a \sec \left(dx + c\right) + a\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(4/3), x)
```

3.300 $\int \sqrt[3]{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=774

$$\frac{3^{3/4}\left(1-\sqrt{3}\right)C\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,\text{EllipticF}\left(\text{co}\right.}{4\,2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*A*AppellF1
[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqrt[3])*C
*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*(1 + Sec[c + d*x])^(2/3)*(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*3^(1/4)*C*EllipticE[A
rcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Se
c[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (3^
(3/4)*(1 - Sqrt[3])*C*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sq
rt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*S
qrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))
/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*2^(
2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x
])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1
+ Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.826751, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {4055, 3924, 3779, 3778, 136, 3828, 3827, 63, 308, 225, 1881}

$$\frac{3\sqrt{2}A\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5d\sqrt{1-\sec(c+dx)}}+\frac{3C\tan(c+dx)\sqrt[3]{a\sec(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] (3*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*A*AppellF1
[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqrt[3])*C
*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*(1 + Sec[c + d*x])^(2/3)*(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*3^(1/4)*C*EllipticE[A
rcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Se
```

```
c[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (3^
(3/4)*(1 - Sqrt[3])*C*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sq
rt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*S
qrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))
/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*2^(
2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x
])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1
+ Sec[c + d*x])^(1/3))^2)])
```

Rule 4055

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[(a + b*Csc[e + f*x])^m*Simp[A*b*
(m + 1) + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x]
 && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt[3]{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\int\sqrt[3]{a+a\sec(c+dx)}\left(\frac{4aA}{3}+\right.}{4a}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + A\int\sqrt[3]{a+a\sec(c+dx)}\,dx + \frac{1}{4}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\right)\int\sqrt[3]{1+}}{\sqrt[3]{1+\sec(c+dx}}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} - \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\tan(c+\right.}{d\sqrt{1-\sec(c}}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec\right.}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec\right.}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec\right.}{}$$

Mathematica [F] time = 14.9993, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[(a + a*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2), x]
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+a\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a\left(\sec\left(c+dx\right)+1\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(1/3)*(A + C*sec(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(1/3), x)
```

3.301 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=791

$$\frac{3^{3/4}\left(1-\sqrt{3}\right)(A+2C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\mathrm{El}}{2^{2/3}ad(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+d}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right.}}}$$

```
[Out] (-3*(A + C)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) + (3*Sqrt[2]*A*App
ellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec
[c + d*x])^(1/3)*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqr
t[3])*(A + 2*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(a*d*(1 + Sec[c +
d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^(1/3
)*3^(1/4)*(A + 2*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt
[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(a*d*(1
- Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(A + 2*C)*EllipticF[ArcCos[(2^(1
/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1
/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*a*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])
^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)
))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.878553, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {4053, 3924, 3779, 3778, 136, 3828, 3827, 63, 308, 225, 1881}

$$\frac{3\sqrt{2}A\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5ad\sqrt{1-\sec(c+dx)}}-\frac{3(A+C)\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(2/3),x]
```


```
[Out] (-3*(A + C)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) + (3*Sqrt[2]*A*App
ellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec
[c + d*x])^(1/3)*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqr
t[3])*(A + 2*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(a*d*(1 + Sec[c +
d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^(1/3
)*3^(1/4)*(A + 2*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt
[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(a*d*(1
```

```
- Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(A + 2*C)*EllipticF[ArcCos[(2^(1
/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1
/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*a*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])
^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)
))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx = -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\int \sqrt[3]{a+a\sec(c+dx)}\left(-\frac{aA}{3}-\frac{1}{3}a(A+2C)\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{A\int \sqrt[3]{a+a\sec(c+dx)}\,dx}{a} + \frac{(A+2C)\int \sec(c+dx)\sqrt[3]{a+a\sec}}{a}$$

$$= -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\right)\int \sqrt[3]{1+\sec(c+dx)}\,dx}{a\sqrt[3]{1+\sec(c+dx)}} + \frac{\left((A+2C)\sqrt[3]{a\right.}}{}$$

$$= -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}\sqrt[6]{1+x}}\,dx,x,\text{se}\right.}{ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{5/6}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a}}{5ad\sqrt{1-\sec(c+dx)}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a}}{5ad\sqrt{1-\sec(c+dx)}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a}}{5ad\sqrt{1-\sec(c+dx)}}$$

Mathematica [F] time = 17.8809, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(2/3), x]
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int \left(A+C(\sec(dx+c))^2\right)(a+a\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(a\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{(a\sec(dx + c) + a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(2/3), x)
```

3.302 $\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=841

$$\frac{3\sqrt[3]{2}\sqrt[4]{3}(2A-5C)E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)\cdots}{\left(\sqrt[3]{2}-(1\cdots\right.}}}{7ad(1-\sec(c+dx))(\sec(c+dx)a+a)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (-3*(A + C)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (3*(2*A - 5*C)
*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*AppellF1[-
1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(a*
d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) - (3*(1 + Sqrt[3])*(2*
A - 5*C)*(1 + Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])
^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^(1/3)*3^(
1/4)*(2*A - 5*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3]
)/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^
(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/
3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*(1 - S
ec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(2*A - 5*C)*EllipticF[ArcCos[(2^(
1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1
+ Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3)
 - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/
3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*
x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1
/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.926865, antiderivative size = 841, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4053, 3924, 3779, 3778, 136, 3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{3\sqrt[3]{2}\sqrt[4]{3}(2A-5C)E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)\cdots}{\left(\sqrt[3]{2}-(1\cdots\right.}}}{7ad(1-\sec(c+dx))(\sec(c+dx)a+a)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*(A + C)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (3*(2*A - 5*C)
*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*AppellF1[-
1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(a*
d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) - (3*(1 + Sqrt[3])*(2*
A - 5*C)*(1 + Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])
^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^(1/3)*3^(
1/4)*(2*A - 5*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3]
)/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^
```

```
(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/
3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*(1 - S
ec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(2*A - 5*C)*EllipticF[ArcCos[(2^(
1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1
+ Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3)
 - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/
3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*
x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1
/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 4053

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> -Simp[(a*(A + C)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*Simp[A*b*(2*m + 1) - a*(A*(m + 1) - C*m)*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3\int \frac{-\frac{7aA}{3}+\frac{1}{3}a(2A-5C)\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}dx}{7a^2}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{A\int \frac{1}{(a+a\sec(c+dx))^{2/3}}dx}{a} - \frac{(2A-5C)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}dx}{7a}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{\left(A(1+\sec(c+dx))^{2/3}\right)\int \frac{1}{(1+\sec(c+dx))^{2/3}}dx}{a(a+a\sec(c+dx))^{2/3}} - \frac{\left((2A-5C)(1\right.}{}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{\left(A\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{7/6}}dx,\right.}{ad\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec\right.}{ad\sqrt{1-\sec(c+d}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec\right.}{ad\sqrt{1-\sec(c+d}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec\right.}{ad\sqrt{1-\sec(c+d}}$$

$$= -\frac{3(A+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec\right.}{ad\sqrt{1-\sec(c+d}}$$

Mathematica [F] time = 9.65701, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/3), x]
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int \left(A+C(\sec(dx+c))^2\right)(a+a\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{(a\sec(dx + c) + a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.303 $\int \sec^m(c+dx)(a+a\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=244

$$\frac{2^{n+\frac{1}{2}}(A(m+n+1)+C(m-n))\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{1}{2};1-m,\frac{1}{2}-n;\frac{3}{2};1-\sec(c+\right.}{d(m+n+1)}$$

```
[Out] (C*Sec[c + d*x]^(1 + m)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)
) + (2^(3/2 + n)*C*n*AppellF1[1/2, 1 - m, -1/2 - n, 3/2, 1 - Sec[c + d*x],
(1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c + d*x])^n*
Tan[c + d*x])/(d*(1 + m + n)) + (2^(1/2 + n)*(C*(m - n) + A*(1 + m + n))*Ap
pellF1[1/2, 1 - m, 1/2 - n, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x])/2]*(1
 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x])/(d*(1 + m
+ n))
```

Rubi [A] time = 0.533392, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4089, 4023, 3828, 3825, 133}

$$\frac{2^{n+\frac{1}{2}}(A(m+n+1)+C(m-n))\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{1}{2};1-m,\frac{1}{2}-n;\frac{3}{2};1-\sec(c+\right.}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + a*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (C*Sec[c + d*x]^(1 + m)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)
) + (2^(3/2 + n)*C*n*AppellF1[1/2, 1 - m, -1/2 - n, 3/2, 1 - Sec[c + d*x],
(1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c + d*x])^n*
Tan[c + d*x])/(d*(1 + m + n)) + (2^(1/2 + n)*(C*(m - n) + A*(1 + m + n))*Ap
pellF1[1/2, 1 - m, 1/2 - n, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x])/2]*(1
 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x])/(d*(1 + m
+ n))
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
```

```
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^m(c+dx)(a+a\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \frac{\int \sec}{}$$

$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \frac{(Cn)}{}$$

$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \frac{(Cn(1}{}$$

$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \frac{\left(Cn(1\right.}{}$$

$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+m+n)} + \frac{2^{\frac{3}{2}+n}C}{}$$

Mathematica [F] time = 18.1964, size = 0, normalized size = 0.

$$\int \sec^m(c+dx)(a+a\sec(c+dx))^n\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^m*(a + a*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[Sec[c + d*x]^m*(a + a*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2), x]
```

Maple [F] time = 1.099, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m(a+a\sec(dx+c))^n\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^n\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^n\sec\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+a*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^n\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^m, x)
```

3.304 $\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=253

$$-\frac{(-An+Cn+C)\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(\frac{1}{2}-n,-n,1-n\right.}{dn(n+1)(\sec(c+dx)+1)}$$

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) - ((C -
A*n + C*n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c + d*x])/(1 - Sec
[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec[c + d*x]))^(1
/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1 + Sec[c + d*x]
)) + (2^(3/2 + n)*C*AppellF1[1/2, 1 + n, -1/2 - n, 3/2, 1 - Sec[c + d*x], (
1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c + d*x])^n*T
an[c + d*x])/d
```

Rubi [A] time = 0.521556, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4087, 4023, 3828, 3825, 132, 133}

$$-\frac{(-An+Cn+C)\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2}-n,-n;1-n;-\frac{2\sec(c+dx)}{1-\sec(c+dx)}\right)}{dn(n+1)(\sec(c+dx)+1)}+\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) - ((C -
A*n + C*n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c + d*x])/(1 - Sec
[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec[c + d*x]))^(1
/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1 + Sec[c + d*x]
)) + (2^(3/2 + n)*C*AppellF1[1/2, 1 + n, -1/2 - n, 3/2, 1 - Sec[c + d*x], (
1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c + d*x])^n*T
an[c + d*x])/d
```


Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx = \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \int$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{C}{}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + (C$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{(}{}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)} - \frac{(}{}$$

Mathematica [F] time = 24.6505, size = 0, normalized size = 0.

$$\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A+C\sec^2(c+dx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^
2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + C*Sec[c + d*x]^
2), x]
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-1-n} (a + a\sec (dx+c))^{n} \left(A + C (\sec (dx+c))^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx+c)^{2} + A\right)(a\sec (dx+c) + a)^{n} \sec (dx+c)^{-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algori
thm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n -
1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec (dx+c)^{2} + A\right)(a\sec (dx+c) + a)^{n} \sec (dx+c)^{-n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algori
thm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^{2}+A\right)\left(a\sec \left(dx+c\right)+a\right)^{n}\sec \left(dx+c\right)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algori
thm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n -
1), x)
```

3.305 $\int \left(\frac{\sec^{-n}(c+dx)(a+a\sec(c+dx))^n(-aAn-aC(1+n)\sec(c+dx))}{a(1+n)} + \sec^{-1-n}(c+\right.$

Optimal. Leaf size=38

$$\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a\sec(c+dx)+a)^n}{d(n+1)}$$

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Rubi [A] time = 0.927861, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 88, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {4023, 3828, 3825, 132, 133, 4087}

$$\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a\sec(c+dx)+a)^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^n*(-(a*A*n) - a*C*(1 + n)*Sec[c + d*x]))/(a*(1 +
n)*Sec[c + d*x]^n) + Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + C*Se
c[c + d*x]^2),x]
```

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] && !IntegerQ[n]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rubi steps

$$\int \left(\frac{\sec^{-n}(c+dx)(a+a\sec(c+dx))^n(-aAn-aC(1+n)\sec(c+dx))}{a(1+n)} + \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A+C\,\mathrm{se}\right.\right.$$

Mathematica [A] time = 0.162311, size = 38, normalized size = 1.

$$\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a(\sec(c+dx)+1))^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^n*(-(a*A*n) - a*C*(1 + n)*Sec[c + d*x]))/(a
*(1 + n)*Sec[c + d*x]^n) + Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A
+ C*Sec[c + d*x]^2),x]
```

```
[Out] (A*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Maple [F] time = 1.27, size = 0, normalized size = 0.

$$\int \frac{(a + a\sec(dx + c))^n (-aAn - aC(1 + n)\sec(dx + c))}{(1 + n)a(\sec(dx + c))^n} + (\sec(dx + c))^{-1-n} (a + a\sec(dx + c))^n \left(A + C(\sec(dx + c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(-a*A*n-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d*x+c)^n)
+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*(-a*A*n-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d*x+c)^n)
+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x)
```

Maxima [B] time = 11.1309, size = 419, normalized size = 11.03

$$\frac{\left(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1\right)^n Aa^n \cos(-(dn + d)x + 2n\arctan(\sin(dx + c), \cos(dx + c) + 1) - c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(-a*A*n-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d*x
+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] 1/2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(-(d
*n + d)*x + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1) - c)*sin(c*n) - (co
s(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(-(d*n - d)*
x + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1) + c)*sin(c*n) - (cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(c*n)*sin(-(d*n + d)
*x + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1) - c) + (cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(c*n)*sin(-(d*n - d)*x + 2*n
*arctan2(sin(d*x + c), cos(d*x + c) + 1) + c))/((d*n + d)*2^n*cos(c*n)^2 +
(d*n + d)*2^n*sin(c*n)^2)
```

Fricas [A] time = 0.908504, size = 142, normalized size = 3.74

$$\frac{A\left(\frac{a\cos(dx+c)+a}{\cos(dx+c)}\right)^n \frac{1}{\cos(dx+c)}^{-n-1} \sin(dx + c)}{(dn + d)\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(-a*A*n-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d*x
+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] A*((a*cos(d*x + c) + a)/cos(d*x + c))^n*(1/cos(d*x + c))^(-n - 1)*sin(d*x +
 c)/((d*n + d)*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(-a*A*n-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d*
x+c)**n)+sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^n\sec\left(dx+c\right)^{-n-1}-\frac{\left(Ca(n+1)\sec\left(dx+c\right)+Aan\right)\left(a\sec\left(dx+c\right)+a\right)^n}{a(n+1)\sec\left(dx+c\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(-a*A*n-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d*x
+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n -
1) - (C*a*(n + 1)*sec(d*x + c) + A*a*n)*(a*sec(d*x + c) + a)^n/(a*(n + 1)*s
ec(d*x + c)^n), x)
```

3.306 $\int \sec^2(c+dx)(a+a\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+d\right.$

Optimal. Leaf size=106

$$\frac{a(B+C)\tan^3(c+dx)}{3d}+\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4B+3C)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (a*(4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(B + C)*Tan[c + d*x])/d +
(a*(4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(B + C)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.17563, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3997, 3787, 3768, 3770, 3767}

$$\frac{a(B+C)\tan^3(c+dx)}{3d}+\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4B+3C)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (a*(4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(B + C)*Tan[c + d*x])/d +
(a*(4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(B + C)*Tan[c + d*x]^3)/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+a\sec(c+dx))(B+C\sec$$
$$= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^3(c+$$
$$= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+(a(B+C))\int$$
$$= \frac{a(4B+3C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aC\sec$$
$$= \frac{a(4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(B+C)}{}$$

Mathematica [B] time = 0.628324, size = 337, normalized size = 3.18

$$-\frac{a\sec^4(c+dx)\left(12(4B+3C)\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] -(a*Sec[c + d*x]^4*(36*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27*C*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*(4*B + 3*C)*Cos[2*(c + d*x)]*(L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]]) + 3*(4*B + 3*C)*Cos[4*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 36*B*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] - 27*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
 24*B*Sin[c + d*x] - 66*C*Sin[c + d*x] - 64*B*Sin[2*(c + d*x)] - 64*C*Sin[2
*(c + d*x)] - 24*B*Sin[3*(c + d*x)] - 18*C*Sin[3*(c + d*x)] - 16*B*Sin[4*(c
 + d*x)] - 16*C*Sin[4*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.043, size = 171, normalized size = 1.6

$$\frac{Ba\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2aC\tan(dx+c)}{3d}+\frac{aC(\sec(dx+c))^2\tan(dx+c}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*a*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*C
*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d+2/3/d*B*a*tan(d*x+c)+1/3/d*
B*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*C*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*C*sec(d
*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.944518, size = 220, normalized size = 2.08

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca-3\,Ca\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\mathrm{si}\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*C*a - 3*C*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.520872, size = 339, normalized size = 3.2

$$\frac{3\,(4\,B+3\,C)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,B+3\,C)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(B+C)a\,\mathrm{co}\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/48*(3*(4*B + 3*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*B + 3*C)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B + C)*a*cos(d*x + c)^3 +
3*(4*B + 3*C)*a*cos(d*x + c)^2 + 8*(B + C)*a*cos(d*x + c) + 6*C*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int B\sec^{3}(c+dx)\,dx+\int B\sec^{4}(c+dx)\,dx+\int C\sec^{4}(c+dx)\,dx+\int C\sec^{5}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x) + Integr
al(C*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**5, x))
```

Giac [A] time = 1.14518, size = 254, normalized size = 2.4

$$\frac{3\,(4\,Ba+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,Ba+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9\,Ca\tan\cdots\right.}{\cdots}}{24\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(4*B*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*B*a + 3*C
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*B*a*tan(1/2*d*x + 1/2*c)^7 +
 9*C*a*tan(1/2*d*x + 1/2*c)^7 - 28*B*a*tan(1/2*d*x + 1/2*c)^5 - 49*C*a*tan(
1/2*d*x + 1/2*c)^5 + 52*B*a*tan(1/2*d*x + 1/2*c)^3 + 31*C*a*tan(1/2*d*x + 1
/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c) - 39*C*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.307 $\int \sec(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx\right.$

Optimal. Leaf size=86

$$\frac{a(3B+2C)\tan(c+dx)}{3d}+\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(B+C)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tan(c+dx)\sec^2(c+}{3d}$$

```
[Out] (a*(B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*B + 2*C)*Tan[c + d*x])/(3*d
) + (a*(B + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*C*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.144769, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 3997, 3787, 3767, 8, 3768, 3770}

$$\frac{a(3B+2C)\tan(c+dx)}{3d}+\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(B+C)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tan(c+dx)\sec^2(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x
]
```

```
[Out] (a*(B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*B + 2*C)*Tan[c + d*x])/(3*d
) + (a*(B + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*C*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+a\sec(c+dx))(B+C\sec($$
$$= \frac{aC\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int \sec^2(c+$$
$$= \frac{aC\sec^2(c+dx)\tan(c+dx)}{3d} + (a(B+C))\int s$$
$$= \frac{a(B+C)\sec(c+dx)\tan(c+dx)}{2d} + \frac{aC\sec^2(c}{}$$
$$= \frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(3B+2C)\,t}{3a}$$

Mathematica [B] time = 0.517878, size = 181, normalized size = 2.1

$$-\frac{a\sec^3(c+dx)\left(-4\sin(c+dx)(3(B+C)\cos(c+dx)+(3B+2C)\cos(2(c+dx))+3B+4C)+9(B+C)\cos(c+dx)\right(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] -(a*Sec[c + d*x]^3*(9*(B + C)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*(B + C)*Cos[3*(c
+ d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) - 4*(3*B + 4*C + 3*(B + C)*Cos[c + d*x] + (3*B + 2*C)*Co
s[2*(c + d*x)])*Sin[c + d*x]))/(24*d)
```

Maple [A] time = 0.039, size = 128, normalized size = 1.5

$$\frac{Ba\tan(dx+c)}{d} + \frac{aC\sec(dx+c)\tan(dx+c)}{2d} + \frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{Ba\sec(dx+c)\tan(dx+c)}{2d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a*tan(d*x+c)+1/2*a*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*C*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*B*a*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x
+c))+2/3*a*C*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.942356, size = 171, normalized size = 1.99

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Ca-3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ca\left(\frac{2\sin(dx+\cdots}{\sin(dx+c)^{2}\cdots}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*C*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*a*tan(d*x + c))/d
```

Fricas [A] time = 0.540933, size = 288, normalized size = 3.35

$$\frac{3\left(B+C\right)a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(B+C\right)a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(3\,B+2\,C\right)a\cos\left(d\cdots\right.\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/12*(3*(B + C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B + C)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*B + 2*C)*a*cos(d*x + c)^2 + 3*(B
+ C)*a*cos(d*x + c) + 2*C*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int B\sec^{2}\left(c+dx\right)dx+\int B\sec^{3}\left(c+dx\right)dx+\int C\sec^{3}\left(c+dx\right)dx+\int C\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(B*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**3, x) + Integr
al(C*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**4, x))
```

Giac [A] time = 1.17324, size = 208, normalized size = 2.42

$$\frac{3\left(Ba+Ca\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ba+Ca\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] 1/6*(3*(B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + C*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*B*a*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*tan
(1/2*d*x + 1/2*c)^5 - 12*B*a*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*tan(1/2*d*x + 1
/2*c)^3 + 9*B*a*tan(1/2*d*x + 1/2*c) + 9*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

3.308 $\int (a+a\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=56

$$\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*(2*B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(B + C)*Tan[c + d*x])/d + (a
*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0586209, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4048, 3770, 3767, 8}

$$\frac{a(B+C)\tan(c+dx)}{d}+\frac{a(2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(2*B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(B + C)*Tan[c + d*x])/d + (a
*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aC\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(a(2B+C)\sec(c+dx\right.$$
$$= \frac{aC\sec(c+dx)\tan(c+dx)}{2d}+(a(B+C))\int \sec^2(c+dx)$$
$$= \frac{a(2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\sec(c+dx)\tan(c+}{2d}$$
$$= \frac{a(2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(B+C)\tan(c+dx)}{d}+$$

Mathematica [A] time = 0.0375233, size = 75, normalized size = 1.34

$$\frac{aB\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])/(2*d) + (a*B*Ta
n[c + d*x])/d + (a*C*Tan[c + d*x])/d + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.035, size = 86, normalized size = 1.5

$$\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Ba\tan(dx+c)}{d}+\frac{aC\sec(dx+c)\tan(dx+c)}{2d}+\frac{aC\ln(\sec(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+a*C*tan(d*x+c)/d+1/d*B*a*tan(d*x+c)+1/2*a
*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.930831, size = 119, normalized size = 2.12

$$-\frac{Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4Ba\log(\sec(dx+c)+\tan(dx+c))-4Ba\tan(d}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(C*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*B*a*log(sec(d*x + c) + tan(d*x + c)) - 4*B*a*tan(d
*x + c) - 4*C*a*tan(d*x + c))/d
```

Fricas [A] time = 0.500579, size = 239, normalized size = 4.27

$$\frac{(2B+C)a\cos(dx+c)^2\log(\sin(dx+c)+1)-(2B+C)a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2(B+C)a\cos(dx+}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/4*((2*B + C)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*B + C)*a*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(B + C)*a*cos(d*x + c) + C*a)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int B\sec(c+dx)\,dx+\int B\sec^2(c+dx)\,dx+\int C\sec^2(c+dx)\,dx+\int C\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(B*sec(c + d*x), x) + Integral(B*sec(c + d*x)**2, x) + Integral(
C*sec(c + d*x)**2, x) + Integral(C*sec(c + d*x)**3, x))
```

Giac [B] time = 1.15362, size = 167, normalized size = 2.98

$$\frac{(2Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*B*a + C*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 + C*a*tan(1
/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c) - 3*C*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.309 $\int \cos(c+dx)(a+a\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=32

$$\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+aBx+\frac{aC\tan(c+dx)}{d}$$

```
[Out] a*B*x + (a*(B + C)*ArcTanh[Sin[c + d*x]])/d + (a*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.0717028, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {4072, 3914, 3767, 8, 3770}

$$\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+aBx+\frac{aC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x
]
```

```
[Out] a*B*x + (a*(B + C)*ArcTanh[Sin[c + d*x]])/d + (a*C*Tan[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+a\sec(c+dx))(B+C\sec(c+dx))\,dx$$
$$= aBx + (aC)\int \sec^2(c+dx)\,dx + (a(B+C))\int \text{se}$$
$$= aBx + \frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d} - \frac{(aC)\,\text{Sub}}{}$$
$$= aBx + \frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d} + \frac{aC\tan(c}{d}$$

Mathematica [A] time = 0.0143111, size = 43, normalized size = 1.34

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{d} + aBx + \frac{aC\tan(c+dx)}{d} + \frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a*B*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*
C*Tan[c + d*x])/d
```

Maple [A] time = 0.065, size = 65, normalized size = 2.

$$aBx + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{Bac}{d} + \frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{aC\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a*B*x+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a*c+1/d*a*C*ln(sec(d*x+c)+tan
(d*x+c))+a*C*tan(d*x+c)/d
```

Maxima [B] time = 0.940278, size = 99, normalized size = 3.09

$$\frac{2(dx+c)Ba + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + Ca(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2C}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*C*a*tan(d*x + c))
/d
```

Fricas [B] time = 0.525431, size = 220, normalized size = 6.88

$$\frac{2Badx\cos(dx+c) + (B+C)a\cos(dx+c)\log(\sin(dx+c)+1) - (B+C)a\cos(dx+c)\log(-\sin(dx+c)+1) + 2Ca\,\text{si}}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/2*(2*B*a*d*x*cos(d*x + c) + (B + C)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (B + C)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*C*a*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int C\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int C\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c
 + d*x)**2, x) + Integral(C*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(C*c
os(c + d*x)*sec(c + d*x)**3, x))
```

Giac [B] time = 1.13647, size = 113, normalized size = 3.53

$$\frac{(dx+c)Ba+(Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] ((d*x + c)*B*a + (B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + C*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*C*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.310 $\int \cos^2(c+dx)(a+a\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=32

$$\frac{aB\sin(c+dx)}{d}+ax(B+C)+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*(B + C)*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rubi [A] time = 0.0969974, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {4072, 3996, 3770}

$$\frac{aB\sin(c+dx)}{d}+ax(B+C)+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a*(B + C)*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \int \cos(c+dx)(a+a\sec(c+dx))(B+C\sec(c+\\ &= \frac{aB\sin(c+dx)}{d}-\int(-a(B+C)-aC\sec(c+dx\\ &= a(B+C)x+\frac{aB\sin(c+dx)}{d}+(aC)\int\sec(c+d\\ &= a(B+C)x+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\sin(c}{d}\end{aligned}$$

Mathematica [A] time = 0.0237102, size = 46, normalized size = 1.44

$$\frac{aB\sin(c)\cos(dx)}{d}+\frac{aB\cos(c)\sin(dx)}{d}+aBx+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+aCx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] a*B*x + a*C*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Cos[d*x]*Sin[c])/d + (
a*B*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.069, size = 56, normalized size = 1.8

$$aBx+aCx+\frac{Ba\sin(dx+c)}{d}+\frac{Bac}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a*B*x+a*C*x+a*B*sin(d*x+c)/d+1/d*B*a*c+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+1/
d*C*a*c
```

Maxima [A] time = 0.936515, size = 78, normalized size = 2.44

$$\frac{2(dx+c)Ba+2(dx+c)Ca+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2Ba\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + 2*(d*x + c)*C*a + C*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*B*a*sin(d*x + c))/d
```

Fricas [A] time = 0.508625, size = 139, normalized size = 4.34

$$\frac{2(B+C)adx+Ca\log(\sin(dx+c)+1)-Ca\log(-\sin(dx+c)+1)+2Ba\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(B + C)*a*d*x + C*a*log(sin(d*x + c) + 1) - C*a*log(-sin(d*x + c) +
1) + 2*B*a*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17508, size = 107, normalized size = 3.34

$$\frac{Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ba+Ca)(dx+c)+\frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] (C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + C*a)*(d*x + c) + 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.311 $\int \cos^3(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=47

$$\frac{a(B+C)\sin(c+dx)}{d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(B+2C)$$

```
[Out] (a*(B + 2*C)*x)/2 + (a*(B + C)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rubi [A] time = 0.134582, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4072, 3996, 3787, 2637, 8}

$$\frac{a(B+C)\sin(c+dx)}{d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(B+2C)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (a*(B + 2*C)*x)/2 + (a*(B + C)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \int \cos^2(c+dx)(a+a\sec(c+dx))(B+C\sec(c+\\ &= \frac{aB\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}\int \cos(c+dx)(-\\ &= \frac{aB\cos(c+dx)\sin(c+dx)}{2d}+(a(B+C))\int \cos(\\ &= \frac{1}{2}a(B+2C)x+\frac{a(B+C)\sin(c+dx)}{d}+\frac{aB\cos(}{}\end{aligned}$$

Mathematica [A] time = 0.0940167, size = 44, normalized size = 0.94

$$\frac{a(4(B+C)\sin(c+dx)+B\sin(2(c+dx))+2Bc+2Bdx+4Cdx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a*(2*B*c + 2*B*d*x + 4*C*d*x + 4*(B + C)*Sin[c + d*x] + B*Sin[2*(c + d*x)]
))/(4*d)
```

Maple [A] time = 0.077, size = 57, normalized size = 1.2

$$\frac{1}{d}\left(Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\sin(dx+c)+aC\sin(dx+c)+aC(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c)+a*C*sin(d
*x+c)+a*C*(d*x+c))
```

Maxima [A] time = 0.935813, size = 74, normalized size = 1.57

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba+4\,(dx+c)Ca+4\,Ba\sin(dx+c)+4\,Ca\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 4*(d*x + c)*C*a + 4*B*a*sin(d*x
 + c) + 4*C*a*sin(d*x + c))/d
```

Fricas [A] time = 0.478606, size = 99, normalized size = 2.11

$$\frac{(B+2\,C)adx+(Ba\cos(dx+c)+2\,(B+C)a)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*((B + 2*C)*a*d*x + (B*a*cos(d*x + c) + 2*(B + C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14786, size = 126, normalized size = 2.68

$$\frac{(Ba+2\,Ca)(dx+c)+\frac{2\left(Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*((B*a + 2*C*a)*(d*x + c) + 2*(B*a*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*tan(1/
2*d*x + 1/2*c)^3 + 3*B*a*tan(1/2*d*x + 1/2*c) + 2*C*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.312 $\int \cos^4(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+$

Optimal. Leaf size=77

$$\frac{a(2B+3C)\sin(c+dx)}{3d}+\frac{a(B+C)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{1}{2}ax(B+C)$$

```
[Out] (a*(B + C)*x)/2 + (a*(2*B + 3*C)*Sin[c + d*x])/(3*d) + (a*(B + C)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.15688, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3996, 3787, 2635, 8, 2637}

$$\frac{a(2B+3C)\sin(c+dx)}{3d}+\frac{a(B+C)\sin(c+dx)\cos(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{1}{2}ax(B+C)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (a*(B + C)*x)/2 + (a*(2*B + 3*C)*Sin[c + d*x])/(3*d) + (a*(B + C)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx &= \int \cos^3(c+dx)(a+a\sec(c+dx))(B+C\sec\\ &= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d}-\frac{1}{3}\int \cos^2(c+\\ &= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d}+(a(B+C))\int\\ &= \frac{a(2B+3C)\sin(c+dx)}{3d}+\frac{a(B+C)\cos(c+}{2d}\\ &= \frac{1}{2}a(B+C)x+\frac{a(2B+3C)\sin(c+dx)}{3d}+\frac{a(B}{}\end{aligned}$$

Mathematica [A] time = 0.170921, size = 65, normalized size = 0.84

$$\frac{a(3(3B+4C)\sin(c+dx)+3(B+C)\sin(2(c+dx))+B\sin(3(c+dx))+6Bc+6Bdx+6cC+6Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a*(6*B*c + 6*c*C + 6*B*d*x + 6*C*d*x + 3*(3*B + 4*C)*Sin[c + d*x] + 3*(B +
 C)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.082, size = 85, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Ba\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*C*sin(d*x+c))
```

Maxima [A] time = 0.937938, size = 107, normalized size = 1.39

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba-3(2dx+2c+\sin(2dx+2c))Ba-3(2dx+2c+\sin(2dx+2c))Ca-12Ca}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a - 12*C*a*sin(d*x + c)
)/d
```

Fricas [A] time = 0.484199, size = 146, normalized size = 1.9

$$\frac{3\,(B+C)adx + \left(2\,Ba\cos\left(dx+c\right)^{2} + 3\,(B+C)a\cos\left(dx+c\right) + 2\,(2\,B+3\,C)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/6*(3*(B + C)*a*d*x + (2*B*a*cos(d*x + c)^2 + 3*(B + C)*a*cos(d*x + c) + 2
*(2*B + 3*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12818, size = 167, normalized size = 2.17

$$\frac{3\,(Ba+Ca)(dx+c) + \frac{2\left(3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + 12\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + 9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 9\,Ca\tan\left(\frac{1}{2}\,dx \cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(B*a + C*a)*(d*x + c) + 2*(3*B*a*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*tan(
1/2*d*x + 1/2*c)^5 + 4*B*a*tan(1/2*d*x + 1/2*c)^3 + 12*C*a*tan(1/2*d*x + 1/
2*c)^3 + 9*B*a*tan(1/2*d*x + 1/2*c) + 9*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3)/d
```

3.313 $\int \cos^5(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=97

$$-\frac{a(B+C)\sin^3(c+dx)}{3d}+\frac{a(B+C)\sin(c+dx)}{d}+\frac{a(3B+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}+$$

```
[Out] (a*(3*B + 4*C)*x)/8 + (a*(B + C)*Sin[c + d*x])/d + (a*(3*B + 4*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(B +
 C)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.169343, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3996, 3787, 2633, 2635, 8}

$$-\frac{a(B+C)\sin^3(c+dx)}{3d}+\frac{a(B+C)\sin(c+dx)}{d}+\frac{a(3B+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{aB\sin(c+dx)\cos^3(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (a*(3*B + 4*C)*x)/8 + (a*(B + C)*Sin[c + d*x])/d + (a*(3*B + 4*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(B +
 C)*Sin[c + d*x]^3)/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+a\sec(c+dx))(B+C\sec(c+$$
$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)$$
$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d} + (a(B+C))\int \cos$$
$$= \frac{a(3B+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aB\cos^3(c}{}$$
$$= \frac{1}{8}a(3B+4C)x + \frac{a(B+C)\sin(c+dx)}{d} + \frac{a(3B+}{}$$

Mathematica [A] time = 0.229937, size = 75, normalized size = 0.77

$$\frac{a\left(-32(B+C)\sin^3(c+dx)+96(B+C)\sin(c+dx)+24(B+C)\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bdx+48\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a*(36*B*c + 48*c*C + 36*B*d*x + 48*C*d*x + 96*(B + C)*Sin[c + d*x] - 32*(B
 + C)*Sin[c + d*x]^3 + 24*(B + C)*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))
/(96*d)
```

Maple [A] time = 0.091, size = 107, normalized size = 1.1

$$\frac{1}{d}\left(Ba\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ba\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aC\left(2+(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.93707, size = 136, normalized size = 1.4

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba-3\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba+32\left(\sin(dx+c)^3-3s\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))
*C*a - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a)/d
```

Fricas [A] time = 0.492934, size = 193, normalized size = 1.99

$$\frac{3\,(3\,B+4\,C)adx+\left(6\,Ba\cos\left(dx+c\right)^{3}+8\,(B+C)a\cos\left(dx+c\right)^{2}+3\,(3\,B+4\,C)a\cos\left(dx+c\right)+16\,(B+C)a\right)\sin\left(dx+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/24*(3*(3*B + 4*C)*a*d*x + (6*B*a*cos(d*x + c)^3 + 8*(B + C)*a*cos(d*x + c
)^2 + 3*(3*B + 4*C)*a*cos(d*x + c) + 16*(B + C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1221, size = 211, normalized size = 2.18

$$\frac{3\,(3\,Ba+4\,Ca)(dx+c)+\frac{2\left(9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+49\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+28\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a + 4*C*a)*(d*x + c) + 2*(9*B*a*tan(1/2*d*x + 1/2*c)^7 + 12*C*
a*tan(1/2*d*x + 1/2*c)^7 + 49*B*a*tan(1/2*d*x + 1/2*c)^5 + 28*C*a*tan(1/2*d
*x + 1/2*c)^5 + 31*B*a*tan(1/2*d*x + 1/2*c)^3 + 52*C*a*tan(1/2*d*x + 1/2*c)
^3 + 39*B*a*tan(1/2*d*x + 1/2*c) + 36*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

3.314 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=169

$$\frac{a^2(10B+9C)\tan^3(c+dx)}{15d}+\frac{a^2(10B+9C)\tan(c+dx)}{5d}+\frac{a^2(7B+6C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5B+6C)\tan(c+dx)\,s}{20d}$$

```
[Out] (a^2*(7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(10*B + 9*C)*Tan[c + d
*x])/(5*d) + (a^2*(7*B + 6*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*B
+ 6*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (C*Sec[c + d*x]^3*(a^2 + a^2*S
ec[c + d*x])*Tan[c + d*x])/(5*d) + (a^2*(10*B + 9*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.323242, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^2(10B+9C)\tan^3(c+dx)}{15d}+\frac{a^2(10B+9C)\tan(c+dx)}{5d}+\frac{a^2(7B+6C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5B+6C)\tan(c+dx)\,s}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (a^2*(7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(10*B + 9*C)*Tan[c + d
*x])/(5*d) + (a^2*(7*B + 6*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*B
+ 6*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (C*Sec[c + d*x]^3*(a^2 + a^2*S
ec[c + d*x])*Tan[c + d*x])/(5*d) + (a^2*(10*B + 9*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+a\sec(c+dx))^2(B+C\,\mathrm{se}$$

$$= \frac{C\sec^3(c+dx)\left(a^2+a^2\sec(c+dx)\right)\tan(c+}{5d}$$

$$= \frac{a^2(5B+6C)\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{C\,\mathrm{s}}{}$$

$$= \frac{a^2(5B+6C)\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{C\,\mathrm{s}}{}$$

$$= \frac{a^2(7B+6C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2(5}{}$$

$$= \frac{a^2(7B+6C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(10B}{}$$

Mathematica [B] time = 0.768951, size = 391, normalized size = 2.31

$$-\frac{a^2\sec^5(c+dx)\left(150(7B+6C)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] -(a^2*Sec[c + d*x]^5*(105*B*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 90*C*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 150*(7*B + 6*C)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 75*(7*B + 6*C)*Cos[3*(c + d*x)
]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]) - 105*B*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
```

```
2]] - 90*C*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 640*
B*Sin[c + d*x] - 960*C*Sin[c + d*x] - 660*B*Sin[2*(c + d*x)] - 840*C*Sin[2*
(c + d*x)] - 800*B*Sin[3*(c + d*x)] - 720*C*Sin[3*(c + d*x)] - 210*B*Sin[4*
(c + d*x)] - 180*C*Sin[4*(c + d*x)] - 160*B*Sin[5*(c + d*x)] - 144*C*Sin[5*
(c + d*x)]))/(1920*d)
```

Maple [A] time = 0.05, size = 235, normalized size = 1.4

$$\frac{7\,Ba^2\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{7\,Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{6\,a^2C\tan(dx+c)}{5\,d}+\frac{3\,a^2C\tan(dx+c)(\sec(d}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 7/8/d*B*a^2*sec(d*x+c)*tan(d*x+c)+7/8/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+6/5
/d*a^2*C*tan(d*x+c)+3/5/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+4/3/d*B*a^2*tan(d*x
+c)+2/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^2*C*tan(d*x+c)*sec(d*x+c)^3
+3/4/d*a^2*C*sec(d*x+c)*tan(d*x+c)+3/4/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/
4/d*B*a^2*tan(d*x+c)*sec(d*x+c)^3+1/5/d*a^2*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 0.952453, size = 375, normalized size = 2.22

$$\frac{160\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^2+16\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)Ca^2+80\left(\tan(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/240*(160*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^2 + 80*(tan(d*x + c)^3 + 3*tan(d*
x + c))*C*a^2 - 15*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 30*C*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.532336, size = 421, normalized size = 2.49

$$\frac{15\,(7\,B+6\,C)a^2\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(7\,B+6\,C)a^2\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(16\,(10\,B+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(7*B + 6*C)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(7*B +
6*C)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(10*B + 9*C)*a^2*cos
```

```
(d*x + c)^4 + 15*(7*B + 6*C)*a^2*cos(d*x + c)^3 + 8*(10*B + 9*C)*a^2*cos(d*
x + c)^2 + 30*(B + 2*C)*a^2*cos(d*x + c) + 24*C*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int B\sec^3(c+dx)\,dx+\int 2B\sec^4(c+dx)\,dx+\int B\sec^5(c+dx)\,dx+\int C\sec^4(c+dx)\,dx+\int 2C\sec^5(c+dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] a**2*(Integral(B*sec(c + d*x)**3, x) + Integral(2*B*sec(c + d*x)**4, x) + I
ntegral(B*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**4, x) + Integral(2
*C*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.17684, size = 332, normalized size = 1.96

$$15\left(7\,Ba^2+6\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(7\,Ba^2+6\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(7*B*a^2 + 6*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(7*B*
a^2 + 6*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(105*B*a^2*tan(1/2*d*
x + 1/2*c)^9 + 90*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 490*B*a^2*tan(1/2*d*x + 1/
2*c)^7 - 420*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 800*B*a^2*tan(1/2*d*x + 1/2*c)^
5 + 864*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 790*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 5
40*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 375*B*a^2*tan(1/2*d*x + 1/2*c) + 390*C*a^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.315 $\int \sec(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+d\right.$

Optimal. Leaf size=138

$$\frac{a^2(8B+7C)\tan(c+dx)}{6d}+\frac{a^2(8B+7C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8B+7C)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(4B-C)\tan(c+}{}$$

```
[Out] (a^2*(8*B + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*B + 7*C)*Tan[c + d*
x])/(6*d) + (a^2*(8*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*B - C)
*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a + a*Sec[c + d*x])^3*Ta
n[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.267637, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4072, 4010, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(8B+7C)\tan(c+dx)}{6d}+\frac{a^2(8B+7C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8B+7C)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(4B-C)\tan(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```


```
[Out] (a^2*(8*B + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*B + 7*C)*Tan[c + d*
x])/(6*d) + (a^2*(8*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*B - C)
*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a + a*Sec[c + d*x])^3*Ta
n[c + d*x])/(4*a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+a\sec(c+dx))^2(B+C\sec$$

$$= \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4ad} + \frac{\int \sec(c}{}$$

$$= \frac{(4B-C)(a+a\sec(c+dx))^2\tan(c+dx)}{12d} + $$

$$= \frac{(4B-C)(a+a\sec(c+dx))^2\tan(c+dx)}{12d} + $$

$$= \frac{a^2(8B+7C)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(4B}{}$$

$$= \frac{a^2(8B+7C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2(8B+}{}$$

Mathematica [B] time = 0.617076, size = 339, normalized size = 2.46

$$-\frac{a^2\sec^4(c+dx)\left(12(8B+7C)\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] -(a^2*Sec[c + d*x]^4*(72*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 63*C*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*(8*B + 7*C)*Cos[2*(c + d*x)]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
```

```
 d*x)/2]]) + 3*(8*B + 7*C)*Cos[4*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 72*B*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] - 63*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 - 48*B*Sin[c + d*x] - 90*C*Sin[c + d*x] - 112*B*Sin[2*(c + d*x)] - 128*C*S
in[2*(c + d*x)] - 48*B*Sin[3*(c + d*x)] - 42*C*Sin[3*(c + d*x)] - 40*B*Sin[
4*(c + d*x)] - 32*C*Sin[4*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.044, size = 187, normalized size = 1.4

$$\frac{5\,Ba^2\tan(dx+c)}{3\,d}+\frac{7\,a^2C\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{7\,a^2C\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{Ba^2\sec(dx+c)\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 5/3/d*B*a^2*tan(d*x+c)+7/8/d*a^2*C*sec(d*x+c)*tan(d*x+c)+7/8/d*a^2*C*ln(sec
(d*x+c)+tan(d*x+c))+1/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)
+tan(d*x+c))+4/3/d*a^2*C*tan(d*x+c)+2/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/3
/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^2*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 0.947203, size = 311, normalized size = 2.25

$$16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^2+32\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^2-3\,Ca^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a^2 - 3*C*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^2*tan(d*x +
 c))/d
```

Fricas [A] time = 0.539237, size = 362, normalized size = 2.62

$$\frac{3\,(8\,B+7\,C)a^2\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(8\,B+7\,C)a^2\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8\,(5\,B+4\,C\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/48*(3*(8*B + 7*C)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*B + 7*C
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(5*B + 4*C)*a^2*cos(d*x
```

```
+ c)^3 + 3*(8*B + 7*C)*a^2*cos(d*x + c)^2 + 8*(B + 2*C)*a^2*cos(d*x + c) +
6*C*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int B\sec^2(c+dx)\,dx+\int 2B\sec^3(c+dx)\,dx+\int B\sec^4(c+dx)\,dx+\int C\sec^3(c+dx)\,dx+\int 2C\sec^4(c+dx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**2*(Integral(B*sec(c + d*x)**2, x) + Integral(2*B*sec(c + d*x)**3, x) + I
ntegral(B*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**3, x) + Integral(2
*C*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**5, x))
```

Giac [A] time = 1.19679, size = 286, normalized size = 2.07

$$\frac{3\left(8Ba^2+7Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8Ba^2+7Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(24Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+2\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(8*B*a^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^2
 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*B*a^2*tan(1/2*d*x +
1/2*c)^7 + 21*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 88*B*a^2*tan(1/2*d*x + 1/2*c)^
5 - 77*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 136*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 83
*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*a^2*tan(1/2*d*x + 1/2*c) - 75*C*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.316 $\int (a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=103

$$\frac{2a^2(3B+2C)\tan(c+dx)}{3d}+\frac{a^2(3B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3B+2C)\tan(c+dx)\sec(c+dx)}{6d}+\frac{C\tan(c+dx)(a$$

```
[Out] (a^2*(3*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*(3*B + 2*C)*Tan[c +
d*x])/(3*d) + (a^2*(3*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (C*(a + a
*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.106857, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4054, 12, 3788, 3767, 8, 4046, 3770}

$$\frac{2a^2(3B+2C)\tan(c+dx)}{3d}+\frac{a^2(3B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3B+2C)\tan(c+dx)\sec(c+dx)}{6d}+\frac{C\tan(c+dx)(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(3*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*(3*B + 2*C)*Tan[c +
d*x])/(3*d) + (a^2*(3*B + 2*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (C*(a + a
*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(c+dx))^2 \left(B\sec(c+dx) + C\sec^2(c+dx)\right) dx = \frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{\int a(3B+2C)\sec(c}{}$$
$$= \frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}(3B+2C)\int \sec($$
$$= \frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}(3B+2C)\int \sec($$
$$= \frac{a^2(3B+2C)\sec(c+dx)\tan(c+dx)}{6d} + \frac{C(a+a\sec(c+}{3}$$
$$= \frac{a^2(3B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2(3B+2C)\tan(c}{3d}$$

Mathematica [A] time = 0.334829, size = 63, normalized size = 0.61

$$\frac{a^2\left((9B+6C)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(3(B+2C)\sec(c+dx) + 12(B+C) + 2C\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*((9*B + 6*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(12*(B + C) + 3*(B +
 2*C)*Sec[c + d*x] + 2*C*Tan[c + d*x]^2)))/(6*d)
```

Maple [A] time = 0.041, size = 141, normalized size = 1.4

$$\frac{3Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{5a^2C\tan(dx+c)}{3d} + 2\frac{Ba^2\tan(dx+c)}{d} + \frac{a^2C\sec(dx+c)\tan(dx+c)}{d} + \frac{a^2C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+5/3/d*a^2*C*tan(d*x+c)+2/d*B*a^2*tan(
d*x+c)+1/d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+
1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.941812, size = 225, normalized size = 2.18

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca^{2}-3\,Ba^{2}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,Ca^{2}\left(\frac{2\,\sin(\cdots}{\sin(dx\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 - 3*B*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*C*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 12*B*a^2*log(sec(d*x + c) + tan(d*x + c)) + 24*B*a^2*tan(d
*x + c) + 12*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.508882, size = 315, normalized size = 3.06

$$\frac{3\,(3\,B+2\,C)a^{2}\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,B+2\,C)a^{2}\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(6\,B+5\,C\cdots\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(3*B + 2*C)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*B + 2*C
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(6*B + 5*C)*a^2*cos(d*x
+ c)^2 + 3*(B + 2*C)*a^2*cos(d*x + c) + 2*C*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int B\sec\left(c+dx\right)dx+\int 2B\sec^{2}\left(c+dx\right)dx+\int B\sec^{3}\left(c+dx\right)dx+\int C\sec^{2}\left(c+dx\right)dx+\int 2C\sec^{3}\left(c+dx\right)dx+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**2*(Integral(B*sec(c + d*x), x) + Integral(2*B*sec(c + d*x)**2, x) + Inte
gral(B*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**2, x) + Integral(2*C*
sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**4, x))
```

Giac [A] time = 1.18855, size = 240, normalized size = 2.33

$$\frac{3\left(3\,Ba^{2}+2\,Ca^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Ba^{2}+2\,Ca^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ca^{2}\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(3*B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*B*a^2
+ 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*B*a^2*tan(1/2*d*x + 1/
2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 -
 16*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*B*a^2*tan(1/2*d*x + 1/2*c) + 18*C*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.317 $\int \cos(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=82

$$\frac{a^2(2B+3C)\tan(c+dx)}{2d}+\frac{a^2(4B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Bx+\frac{C\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}$$

```
[Out] a^2*B*x + (a^2*(4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*B + 3*C)*
Tan[c + d*x])/(2*d) + (C*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.146151, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3917, 3914, 3767, 8, 3770}

$$\frac{a^2(2B+3C)\tan(c+dx)}{2d}+\frac{a^2(4B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Bx+\frac{C\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^2*B*x + (a^2*(4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*B + 3*C)*
Tan[c + d*x])/(2*d) + (C*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+a\sec(c+dx))^2(B+C\sec(c+dx))\,dx$$
$$= \frac{C\left(a^2+a^2\sec(c+dx)\right)\tan(c+dx)}{2d}+\frac{1}{2}\int (a$$
$$= a^2Bx+\frac{C\left(a^2+a^2\sec(c+dx)\right)\tan(c+dx)}{2d}$$
$$= a^2Bx+\frac{a^2(4B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+$$
$$= a^2Bx+\frac{a^2(4B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+$$

Mathematica [B] time = 1.18582, size = 277, normalized size = 3.38

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(B+2C)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{d\left(\sin\left(\frac{c}{2}\right)+\cos\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(4*B*x - (2*(4*B + 3*C)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(4*B + 3*C)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + C/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
4*(B + 2*C)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - C/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(B + 2*
C)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))))/16
```

Maple [A] time = 0.074, size = 113, normalized size = 1.4

$$a^2Bx+\frac{Ba^2c}{d}+\frac{3\,a^2C\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+2\,\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\,\frac{a^2C\tan(dx+c)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^2*B*x+1/d*B*a^2*c+3/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a^2*ln(sec(
d*x+c)+tan(d*x+c))+2/d*a^2*C*tan(d*x+c)+1/d*B*a^2*tan(d*x+c)+1/2/d*a^2*C*se
c(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.945849, size = 192, normalized size = 2.34

$$\frac{4\,(dx+c)Ba^2 - Ca^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 4\,Ba^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 - C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*B*a^2*tan(d*x + c) + 8*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.517118, size = 297, normalized size = 3.62

$$\frac{4\,Ba^2dx\cos\left(dx+c\right)^2 + (4\,B+3\,C)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - (4\,B+3\,C)a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/4*(4*B*a^2*d*x*cos(d*x + c)^2 + (4*B + 3*C)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (4*B + 3*C)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(B + 2*C)*a^2*cos(d*x + c) + C*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17345, size = 208, normalized size = 2.54

$$\frac{2\,(dx+c)Ba^2 + \left(4\,Ba^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(4\,Ba^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(2\,Ba^2\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + (4*B*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (4*B*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a^2*
tan(1/2*d*x + 1/2*c)^3 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d
*x + 1/2*c) - 5*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

3.318 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=73

$$\frac{a^2(B-C)\sin(c+dx)}{d}+\frac{a^2(B+2C)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2B+C)+\frac{C\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{d}$$

```
[Out] a^2*(2*B + C)*x + (a^2*(B + 2*C)*ArcTanh[Sin[c + d*x]])/d + (a^2*(B - C)*Si
n[c + d*x])/d + (C*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/d
```

Rubi [A] time = 0.206269, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4072, 4018, 3996, 3770}

$$\frac{a^2(B-C)\sin(c+dx)}{d}+\frac{a^2(B+2C)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2B+C)+\frac{C\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] a^2*(2*B + C)*x + (a^2*(B + 2*C)*ArcTanh[Sin[c + d*x]])/d + (a^2*(B - C)*Si
n[c + d*x])/d + (C*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+a\sec(c+dx))^2(B+C\,\text{se}$$
$$= \frac{C\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{d} + \int \text{co}$$
$$= \frac{a^2(B-C)\sin(c+dx)}{d} + \frac{C\left(a^2+a^2\sec(c+\right.}{d}$$
$$= a^2(2B+C)x + \frac{a^2(B-C)\sin(c+dx)}{d} + \frac{C(}{}$$
$$= a^2(2B+C)x + \frac{a^2(B+2C)\tanh^{-1}(\sin(c+}{d}$$

Mathematica [A] time = 0.317977, size = 143, normalized size = 1.96

$$\frac{a^2\left(B\sin(c+dx)-B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+B\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+2Bc+2Bd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^2*(2*B*c + c*C + 2*B*d*x + C*d*x - B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - 2*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 2*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + B*S
in[c + d*x] + C*Tan[c + d*x]))/d
```

Maple [A] time = 0.072, size = 107, normalized size = 1.5

$$2a^2Bx + a^2Cx + \frac{Ba^2\sin(dx+c)}{d} + \frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\frac{Ba^2c}{d} + \frac{a^2C\tan(dx+c)}{d} + 2\frac{a^2C\ln(\sec}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2*a^2*B*x+a^2*C*x+a^2*B*sin(d*x+c)/d+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/
d*B*a^2*c+1/d*a^2*C*tan(d*x+c)+2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*a^
2*c
```

Maxima [A] time = 0.94135, size = 142, normalized size = 1.95

$$\frac{4(dx+c)Ba^2 + 2(dx+c)Ca^2 + Ba^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Ca^2(\log(\sin(dx+c)+1) - \text{log}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*B*a^2 + 2*(d*x + c)*C*a^2 + B*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*a^2*sin(d*x + c) + 2*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.516101, size = 278, normalized size = 3.81

$$\frac{2\,(2\,B+C)a^2dx\cos\left(dx+c\right)+(B+2\,C)a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(B+2\,C)a^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(2*(2*B + C)*a^2*d*x*cos(d*x + c) + (B + 2*C)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (B + 2*C)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^
2*cos(d*x + c) + C*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16266, size = 212, normalized size = 2.9

$$\frac{\left(2\,Ba^2+Ca^2\right)(dx+c)+\left(Ba^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(Ba^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Ba^2\,\mathrm{ta}\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] ((2*B*a^2 + C*a^2)*(d*x + c) + (B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(B*a^2*
tan(1/2*d*x + 1/2*c)^3 - C*a^2*tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x +
 1/2*c) - C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.319 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=88

$$\frac{a^2(3B+2C)\sin(c+dx)}{2d}+\frac{B\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{1}{2}a^2x(3B+4C)+\frac{a^2C\tanh^{-1}(\sin(c+dx$$

```
[Out] (a^2*(3*B + 4*C)*x)/2 + (a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*B + 2*C)*
Sin[c + d*x])/(2*d) + (B*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(2*d)
```

Rubi [A] time = 0.219644, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4072, 4017, 3996, 3770}

$$\frac{a^2(3B+2C)\sin(c+dx)}{2d}+\frac{B\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{1}{2}a^2x(3B+4C)+\frac{a^2C\tanh^{-1}(\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (a^2*(3*B + 4*C)*x)/2 + (a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*B + 2*C)*
Sin[c + d*x])/(2*d) + (B*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int \cos^2(c+dx)(a+a\sec(c+dx))^2(B+C\sec(\ldots$$
$$=\frac{B\cos(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{2d}\ldots$$
$$=\frac{a^2(3B+2C)\sin(c+dx)}{2d}+\frac{B\cos(c+dx)\left(a^2+\ldots\right.}{\ldots}$$
$$=\frac{1}{2}a^2(3B+4C)x+\frac{a^2(3B+2C)\sin(c+dx)}{2d}+\ldots$$
$$=\frac{1}{2}a^2(3B+4C)x+\frac{a^2C\tanh^{-1}(\sin(c+dx))}{d}+\ldots$$

Mathematica [A] time = 0.151612, size = 96, normalized size = 1.09

$$\frac{a^2\left(4(2B+C)\sin(c+dx)+B\sin(2(c+dx))+6Bdx-4C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4C\log\left(\sin\left(\frac{1}{2}(c+d\ldots\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^2*(6*B*d*x + 8*C*d*x - 4*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(2*B + C)*Sin[c + d*x] + B*S
in[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.084, size = 108, normalized size = 1.2

$$\frac{Ba^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{3a^2Bx}{2}+\frac{3Ba^2c}{2d}+\frac{a^2C\sin(dx+c)}{d}+2\frac{Ba^2\sin(dx+c)}{d}+2a^2Cx+2\frac{Ca^2c}{d}+\frac{a^2C\ln(\sec\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2*a^2*B*cos(d*x+c)*sin(d*x+c)/d+3/2*a^2*B*x+3/2/d*B*a^2*c+1/d*a^2*C*sin(d
*x+c)+2*a^2*B*sin(d*x+c)/d+2*a^2*C*x+2/d*C*a^2*c+1/d*a^2*C*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 0.941587, size = 136, normalized size = 1.55

$$\frac{(2dx+2c+\sin(2dx+2c))Ba^2+4(dx+c)Ba^2+8(dx+c)Ca^2+2Ca^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-\ldots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 4*(d*x + c)*B*a^2 + 8*(d*x +
c)*C*a^2 + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*B*a^
2*sin(d*x + c) + 4*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.516076, size = 194, normalized size = 2.2

$$\frac{(3B+4C)a^2dx + Ca^2\log(\sin(dx+c)+1) - Ca^2\log(-\sin(dx+c)+1) + \left(Ba^2\cos(dx+c) + 2(2B+C)a^2\right)\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*((3*B + 4*C)*a^2*d*x + C*a^2*log(sin(d*x + c) + 1) - C*a^2*log(-sin(d*x
 + c) + 1) + (B*a^2*cos(d*x + c) + 2*(2*B + C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17739, size = 196, normalized size = 2.23

$$\frac{2Ca^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 2Ca^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \left(3Ba^2+4Ca^2\right)(dx+c) + \frac{2\left(3Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*C*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*C*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (3*B*a^2 + 4*C*a^2)*(d*x + c) + 2*(3*B*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 2*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^2*tan(1/2*d*x + 1/2*c)
+ 2*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.320 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=102

$$\frac{2a^2(2B+3C)\sin(c+dx)}{3d}+\frac{a^2(2B+3C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(2B+3C)+\frac{B\sin(c+dx)\cos^2(c+dx)(a\sec(c-}{3d}$$

```
[Out] (a^2*(2*B + 3*C)*x)/2 + (2*a^2*(2*B + 3*C)*Sin[c + d*x])/(3*d) + (a^2*(2*B
+ 3*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*Cos[c + d*x]^2*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.2299, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4013, 3788, 2637, 4045, 8}

$$\frac{2a^2(2B+3C)\sin(c+dx)}{3d}+\frac{a^2(2B+3C)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(2B+3C)+\frac{B\sin(c+dx)\cos^2(c+dx)(a\sec(c-}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (a^2*(2*B + 3*C)*x)/2 + (2*a^2*(2*B + 3*C)*Sin[c + d*x])/(3*d) + (a^2*(2*B
+ 3*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*Cos[c + d*x]^2*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \cos^3(c+dx)(a+a\sec(c+dx))^2(B+C\,\mathrm{s}$$
$$= \frac{B\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+d}{3d}$$
$$= \frac{B\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+d}{3d}$$
$$= \frac{2a^2(2B+3C)\sin(c+dx)}{3d} + \frac{a^2(2B+3C)\,\mathrm{cc}}{}$$
$$= \frac{1}{2}a^2(2B+3C)x + \frac{2a^2(2B+3C)\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.168917, size = 61, normalized size = 0.6

$$\frac{a^2(3(7B+8C)\sin(c+dx)+3(2B+C)\sin(2(c+dx))+B\sin(3(c+dx))+12Bdx+18Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^2*(12*B*d*x + 18*C*d*x + 3*(7*B + 8*C)*Sin[c + d*x] + 3*(2*B + C)*Sin[2*
(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.084, size = 116, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Ba^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + 2\,Ba^2\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2)+a^2C\left(\frac{\cos(dx+c)\sin(}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*B*a^2*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^2*sin
(d*x+c)+2*a^2*C*sin(d*x+c)+a^2*C*(d*x+c))
```

Maxima [A] time = 0.93984, size = 149, normalized size = 1.46

$$-\frac{4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2-6\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^2-12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 6*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 - 12*(d*x + c)*
C*a^2 - 12*B*a^2*sin(d*x + c) - 24*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.483474, size = 165, normalized size = 1.62

$$\frac{3\,(2\,B+3\,C)a^2dx+\left(2\,Ba^2\cos{(dx+c)}^2+3\,(2\,B+C)a^2\cos{(dx+c)}+2\,(5\,B+6\,C)a^2\right)\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(2*B + 3*C)*a^2*d*x + (2*B*a^2*cos(d*x + c)^2 + 3*(2*B + C)*a^2*cos(
d*x + c) + 2*(5*B + 6*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1778, size = 192, normalized size = 1.88

$$\frac{3\left(2\,Ba^2+3\,Ca^2\right)(dx+c)+\frac{2\left(6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(3*(2*B*a^2 + 3*C*a^2)*(d*x + c) + 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 +
9*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 16*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*C*a^2
*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*tan(1/2*d*x + 1/2*c) + 15*C*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.321 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=135

$$\frac{a^2(4B+5C)\sin(c+dx)}{3d}+\frac{a^2(5B+4C)\sin(c+dx)\cos^2(c+dx)}{12d}+\frac{a^2(7B+8C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{B\sin(c+d}{}$$

```
[Out] (a^2*(7*B + 8*C)*x)/8 + (a^2*(4*B + 5*C)*Sin[c + d*x])/(3*d) + (a^2*(7*B +
8*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*B + 4*C)*Cos[c + d*x]^2*Sin
[c + d*x])/(12*d) + (B*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(4*d)
```

Rubi [A] time = 0.306812, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^2(4B+5C)\sin(c+dx)}{3d}+\frac{a^2(5B+4C)\sin(c+dx)\cos^2(c+dx)}{12d}+\frac{a^2(7B+8C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{B\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (a^2*(7*B + 8*C)*x)/8 + (a^2*(4*B + 5*C)*Sin[c + d*x])/(3*d) + (a^2*(7*B +
8*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*B + 4*C)*Cos[c + d*x]^2*Sin
[c + d*x])/(12*d) + (B*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(4*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+a\sec(c+dx))^2(B+C\sec($$
$$= \frac{B\cos^3(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx}{4d}$$
$$= \frac{a^2(5B+4C)\cos^2(c+dx)\sin(c+dx)}{12d} + \frac{B\cos^3}{}$$
$$= \frac{a^2(5B+4C)\cos^2(c+dx)\sin(c+dx)}{12d} + \frac{B\cos^3}{}$$
$$= \frac{a^2(4B+5C)\sin(c+dx)}{3d} + \frac{a^2(7B+8C)\cos(c}{8d}$$
$$= \frac{1}{8}a^2(7B+8C)x + \frac{a^2(4B+5C)\sin(c+dx)}{3d} + \frac{a}{}$$

Mathematica [A] time = 0.360068, size = 86, normalized size = 0.64

$$\frac{a^2(24(6B+7C)\sin(c+dx)+48(B+C)\sin(2(c+dx))+16B\sin(3(c+dx))+3B\sin(4(c+dx))+84Bc+84Bdx+8C\,\mathrm{si}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^2*(84*B*c + 84*B*d*x + 96*C*d*x + 24*(6*B + 7*C)*Sin[c + d*x] + 48*(B +
C)*Sin[2*(c + d*x)] + 16*B*Sin[3*(c + d*x)] + 8*C*Sin[3*(c + d*x)] + 3*B*Si
n[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.091, size = 154, normalized size = 1.1

$$\frac{1}{d}\left(Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2Ba^2(2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*
a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^2*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+a^2*C*sin(d*x+c))
```

Maxima [A] time = 0.946342, size = 194, normalized size = 1.44

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Ba^{2}-24\left(2\,dx+2\,c+\mathrm{s}\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*
c))*B*a^2 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 48*(2*d*x + 2*c +
sin(2*d*x + 2*c))*C*a^2 - 96*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.494474, size = 213, normalized size = 1.58

$$\frac{3\,(7\,B+8\,C)a^{2}dx+\left(6\,Ba^{2}\cos\left(dx+c\right)^{3}+8\,(2\,B+C)a^{2}\cos\left(dx+c\right)^{2}+3\,(7\,B+8\,C)a^{2}\cos\left(dx+c\right)+8\,(4\,B+5\,C)a^{2}\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/24*(3*(7*B + 8*C)*a^2*d*x + (6*B*a^2*cos(d*x + c)^3 + 8*(2*B + C)*a^2*cos
(d*x + c)^2 + 3*(7*B + 8*C)*a^2*cos(d*x + c) + 8*(4*B + 5*C)*a^2)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15471, size = 238, normalized size = 1.76

$$\frac{3\left(7\,Ba^{2}+8\,Ca^{2}\right)(dx+c)+\frac{2\left(21\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+24\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+77\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+88\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+83\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(7*B*a^2 + 8*C*a^2)*(d*x + c) + 2*(21*B*a^2*tan(1/2*d*x + 1/2*c)^7
+ 24*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 88*C*
a^2*tan(1/2*d*x + 1/2*c)^5 + 83*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 136*C*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 75*B*a^2*tan(1/2*d*x + 1/2*c) + 72*C*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.322 $\int \cos^6(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=160

$$-\frac{a^2(9B+10C)\sin^3(c+dx)}{15d}+\frac{a^2(9B+10C)\sin(c+dx)}{5d}+\frac{a^2(6B+5C)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(6B+7C)\sin(c}{8}$$

```
[Out] (a^2*(6*B + 7*C)*x)/8 + (a^2*(9*B + 10*C)*Sin[c + d*x])/(5*d) + (a^2*(6*B +
 7*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(6*B + 5*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (B*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(9*B + 10*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.329132, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^2(9B+10C)\sin^3(c+dx)}{15d}+\frac{a^2(9B+10C)\sin(c+dx)}{5d}+\frac{a^2(6B+5C)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(6B+7C)\sin(c}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (a^2*(6*B + 7*C)*x)/8 + (a^2*(9*B + 10*C)*Sin[c + d*x])/(5*d) + (a^2*(6*B +
 7*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(6*B + 5*C)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (B*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(9*B + 10*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^5(c+dx)(a+a\sec(c+dx))^2(B+C\sec($$
$$= \frac{B\cos^4(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx}{5d}$$
$$= \frac{a^2(6B+5C)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\cos^4}{}$$
$$= \frac{a^2(6B+5C)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\cos^4}{}$$
$$= \frac{a^2(6B+7C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(6B+}{}$$
$$= \frac{1}{8}a^2(6B+7C)x + \frac{a^2(9B+10C)\sin(c+dx)}{5d} +$$

Mathematica [A] time = 0.374035, size = 108, normalized size = 0.68

$$\frac{a^2(60(11B+12C)\sin(c+dx)+240(B+C)\sin(2(c+dx))+90B\sin(3(c+dx))+30B\sin(4(c+dx))+6B\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^2*(360*B*c + 360*B*d*x + 420*C*d*x + 60*(11*B + 12*C)*Sin[c + d*x] + 240
*(B + C)*Sin[2*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 80*C*Sin[3*(c + d*x)] +
 30*B*Sin[4*(c + d*x)] + 15*C*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(48
0*d)
```

Maple [A] time = 0.099, size = 186, normalized size = 1.2

$$\frac{1}{d}\left(\frac{Ba^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*B*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*C*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*B*a^2*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a^2*C*(2+cos(d*x+c)^2)*
sin(d*x+c)+1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*C*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.947882, size = 240, normalized size = 1.5

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^2-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^2+30(12dx+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2 -
160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 30*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))
*C*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2)/d
```

Fricas [A] time = 0.525635, size = 271, normalized size = 1.69

$$\frac{15(6B+7C)a^2dx+\left(24Ba^2\cos(dx+c)^4+30(2B+C)a^2\cos(dx+c)^3+8(9B+10C)a^2\cos(dx+c)^2+15(6B+}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(6*B + 7*C)*a^2*d*x + (24*B*a^2*cos(d*x + c)^4 + 30*(2*B + C)*a^2
*cos(d*x + c)^3 + 8*(9*B + 10*C)*a^2*cos(d*x + c)^2 + 15*(6*B + 7*C)*a^2*co
s(d*x + c) + 16*(9*B + 10*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20324, size = 284, normalized size = 1.78

$$\frac{15\left(6\,Ba^2+7\,Ca^2\right)(dx+c)+\frac{2\left(90\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+105\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+420\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+490\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+864\,Ba^2\tan\left(\frac{1}{2}\right.\right.}{\left(\mathrm{t}\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(6*B*a^2 + 7*C*a^2)*(d*x + c) + 2*(90*B*a^2*tan(1/2*d*x + 1/2*c)^
9 + 105*C*a^2*tan(1/2*d*x + 1/2*c)^9 + 420*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 4
90*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*C*
a^2*tan(1/2*d*x + 1/2*c)^5 + 540*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 790*C*a^2*t
an(1/2*d*x + 1/2*c)^3 + 390*B*a^2*tan(1/2*d*x + 1/2*c) + 375*C*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.323 $\int \sec(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=163

$$\frac{a^3(15B+13C)\tan^3(c+dx)}{60d}+\frac{a^3(15B+13C)\tan(c+dx)}{5d}+\frac{a^3(15B+13C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(15B+13C)\,\text{ta}}{}$$

```
[Out] (a^3*(15*B + 13*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(15*B + 13*C)*Tan[c
+ d*x])/(5*d) + (3*a^3*(15*B + 13*C)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + ((
5*B - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (C*(a + a*Sec[c + d*
x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(15*B + 13*C)*Tan[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.312872, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4072, 4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(15B+13C)\tan^3(c+dx)}{60d}+\frac{a^3(15B+13C)\tan(c+dx)}{5d}+\frac{a^3(15B+13C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(15B+13C)\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (a^3*(15*B + 13*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(15*B + 13*C)*Tan[c
+ d*x])/(5*d) + (3*a^3*(15*B + 13*C)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + ((
5*B - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (C*(a + a*Sec[c + d*
x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(15*B + 13*C)*Tan[c + d*x]^3)/(60*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \int \sec^2(c+dx)(a+a\sec(c+dx))^3(B+C\sec(c$$

$$= \frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{5ad} + \frac{\int \sec(c+a}{}$$

$$= \frac{(5B-C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a}{}$$

$$= \frac{(5B-C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a}{}$$

$$= \frac{(5B-C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a}{}$$

$$= \frac{a^3(15B+13C)\tanh^{-1}(\sin(c+dx))}{20d} + \frac{3a^3(15B}{}$$

$$= \frac{a^3(15B+13C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(15B+}{}$$

Mathematica [B] time = 0.80796, size = 391, normalized size = 2.4

$$-\frac{a^3\sec^5(c+dx)\left(150(15B+13C)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] -(a^3*Sec[c + d*x]^5*(225*B*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 195*C*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
```

```
] + 150*(15*B + 13*C)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 75*(15*B + 13*C)*Cos[3*(c +
 d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) - 225*B*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 195*C*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 1200*B*Sin[c + d*x] - 1600*C*Sin[c + d*x] - 1140*B*Sin[2*(c + d*x)] - 150
0*C*Sin[2*(c + d*x)] - 1560*B*Sin[3*(c + d*x)] - 1520*C*Sin[3*(c + d*x)] -
450*B*Sin[4*(c + d*x)] - 390*C*Sin[4*(c + d*x)] - 360*B*Sin[5*(c + d*x)] -
304*C*Sin[5*(c + d*x)]))/(1920*d)
```

Maple [A] time = 0.049, size = 234, normalized size = 1.4

$$3\frac{Ba^3\tan(dx+c)}{d}+\frac{13a^3C\sec(dx+c)\tan(dx+c)}{8d}+\frac{13a^3C\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{15Ba^3\sec(dx+c)\tan(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/d*B*a^3*tan(d*x+c)+13/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+13/8/d*a^3*C*ln(sec
(d*x+c)+tan(d*x+c))+15/8/d*B*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*B*a^3*ln(sec(
d*x+c)+tan(d*x+c))+38/15*a^3*C*tan(d*x+c)/d+19/15/d*a^3*C*tan(d*x+c)*sec(d*
x+c)^2+1/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^
3+1/4/d*B*a^3*tan(d*x+c)*sec(d*x+c)^3+1/5/d*a^3*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [B] time = 0.958117, size = 455, normalized size = 2.79

$$240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ca^3+240\left(\tan(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*C*a^3 - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 45*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 180*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 60*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.522507, size = 431, normalized size = 2.64

$$15(15B+13C)a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-15(15B+13C)a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/240*(15*(15*B + 13*C)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(15*B
 + 13*C)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(45*B + 38*C)*a^3
*cos(d*x + c)^4 + 15*(15*B + 13*C)*a^3*cos(d*x + c)^3 + 8*(15*B + 19*C)*a^3
*cos(d*x + c)^2 + 30*(B + 3*C)*a^3*cos(d*x + c) + 24*C*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int B\sec^2(c+dx)\,dx+\int 3B\sec^3(c+dx)\,dx+\int 3B\sec^4(c+dx)\,dx+\int B\sec^5(c+dx)\,dx+\int C\sec^3(c+dx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**3*(Integral(B*sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + I
ntegral(3*B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x) + Integral
(C*sec(c + d*x)**3, x) + Integral(3*C*sec(c + d*x)**4, x) + Integral(3*C*se
c(c + d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.19213, size = 332, normalized size = 2.04

$$\frac{15\left(15\,Ba^3+13\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(15\,Ba^3+13\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(225\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/120*(15*(15*B*a^3 + 13*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(15
*B*a^3 + 13*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(225*B*a^3*tan(1/
2*d*x + 1/2*c)^9 + 195*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 1050*B*a^3*tan(1/2*d*
x + 1/2*c)^7 - 910*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1920*B*a^3*tan(1/2*d*x +
1/2*c)^5 + 1664*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 1830*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1330*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 735*B*a^3*tan(1/2*d*x + 1/2*c)
+ 765*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.324 $\int (a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=125

$$\frac{a^3(4B+3C)\tan^3(c+dx)}{12d}+\frac{a^3(4B+3C)\tan(c+dx)}{d}+\frac{5a^3(4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(4B+3C)\tan(c+a}{8d}$$

```
[Out] (5*a^3*(4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(4*B + 3*C)*Tan[c +
d*x])/d + (3*a^3*(4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (C*(a + a*S
ec[c + d*x])^3*Tan[c + d*x])/(4*d) + (a^3*(4*B + 3*C)*Tan[c + d*x]^3)/(12*d
)
```

Rubi [A] time = 0.139477, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4054, 12, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(4B+3C)\tan^3(c+dx)}{12d}+\frac{a^3(4B+3C)\tan(c+dx)}{d}+\frac{5a^3(4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(4B+3C)\tan(c+a}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (5*a^3*(4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(4*B + 3*C)*Tan[c +
d*x])/d + (3*a^3*(4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (C*(a + a*S
ec[c + d*x])^3*Tan[c + d*x])/(4*d) + (a^3*(4*B + 3*C)*Tan[c + d*x]^3)/(12*d
)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{\int a(4B+3C)\sec(c+ \ldots}{}$$
$$= \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}(4B+3C)\int \sec(c+ \ldots$$
$$= \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}(4B+3C)\int \left(a^3\sec(\ldots\right.$$
$$= \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\left(a^3(4B+3C)\right)\int \sec \ldots$$
$$= \frac{a^3(4B+3C)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^3(4B+3C)\sec(c+d \ldots}{8d}$$
$$= \frac{5a^3(4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(4B+3C)\tan(c+ \ldots}{d}$$

Mathematica [A] time = 0.450373, size = 81, normalized size = 0.65

$$\frac{a^3\left(15(4B+3C)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8(B+3C)\tan^2(c+dx)+9(4B+5C)\sec(c+dx)+96(B+C)+6C \ldots\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(15*(4*B + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(96*(B + C) + 9*(
4*B + 5*C)*Sec[c + d*x] + 6*C*Sec[c + d*x]^3 + 8*(B + 3*C)*Tan[c + d*x]^2))
)/(24*d)
```

Maple [A] time = 0.051, size = 188, normalized size = 1.5

$$\frac{5\,Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+3\,\frac{a^3C\tan(dx+c)}{d}+\frac{11\,Ba^3\tan(dx+c)}{3\,d}+\frac{15\,a^3C\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{15 \ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 5/2/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*tan(d*x+c)/d+11/3/d*B*a^3*tan
(d*x+c)+15/8/d*a^3*C*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*C*ln(sec(d*x+c)+tan(d
```

```
*x+c))+3/2/d*B*a^3*sec(d*x+c)*tan(d*x+c)+1/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+
1/3/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [B] time = 0.954426, size = 354, normalized size = 2.83

$$\frac{16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^3+48\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ca^3-3\,Ca^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a^3 - 3*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^3*log(sec(d
*x + c) + tan(d*x + c)) + 144*B*a^3*tan(d*x + c) + 48*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.524279, size = 366, normalized size = 2.93

$$\frac{15\,(4\,B+3\,C)a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\,(4\,B+3\,C)a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(11\,B\right.}{48\,d\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(15*(4*B + 3*C)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(4*B + 3
*C)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(11*B + 9*C)*a^3*cos(d
*x + c)^3 + 9*(4*B + 5*C)*a^3*cos(d*x + c)^2 + 8*(B + 3*C)*a^3*cos(d*x + c)
 + 6*C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int B\sec\left(c+dx\right)dx+\int 3B\sec^2\left(c+dx\right)dx+\int 3B\sec^3\left(c+dx\right)dx+\int B\sec^4\left(c+dx\right)dx+\int C\sec^2\left(c+dx\right)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**3*(Integral(B*sec(c + d*x), x) + Integral(3*B*sec(c + d*x)**2, x) + Inte
gral(3*B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x) + Integral(C*
sec(c + d*x)**2, x) + Integral(3*C*sec(c + d*x)**3, x) + Integral(3*C*sec(c
 + d*x)**4, x) + Integral(C*sec(c + d*x)**5, x))
```

Giac [A] time = 1.19281, size = 286, normalized size = 2.29

$$15\left(4\,Ba^3+3\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(4\,Ba^3+3\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(15*(4*B*a^3 + 3*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*B*a
^3 + 3*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*B*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 45*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 220*B*a^3*tan(1/2*d*x + 1/2*
c)^5 - 165*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3
+ 219*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 132*B*a^3*tan(1/2*d*x + 1/2*c) - 147*C
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.325 $\int \cos(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=111

$$\frac{5a^3(B+C)\tan(c+dx)}{2d}+\frac{a^3(7B+5C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3B+5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+a^3Bx+$$

```
[Out] a^3*B*x + (a^3*(7*B + 5*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(B + C)*Ta
n[c + d*x])/(2*d) + (a*C*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((3*B
 + 5*C)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.204806, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^3(B+C)\tan(c+dx)}{2d}+\frac{a^3(7B+5C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3B+5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+a^3Bx+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^3*B*x + (a^3*(7*B + 5*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(B + C)*Ta
n[c + d*x])/(2*d) + (a*C*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((3*B
 + 5*C)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+a\sec(c+dx))^3(B+C\sec(c+dx))\,dx$$
$$= \frac{aC(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}\int (a+$$
$$= \frac{aC(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{(3B+5C)}{}$$
$$= a^3Bx + \frac{aC(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{(3}{}$$
$$= a^3Bx + \frac{a^3(7B+5C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aC(}{}$$
$$= a^3Bx + \frac{a^3(7B+5C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a^3}{}$$

Mathematica [B] time = 6.40118, size = 772, normalized size = 6.95

$$a^3\left(\frac{(\cos(c+dx)+1)^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(9B\sin\left(\frac{dx}{2}\right)+11C\sin\left(\frac{dx}{2}\right)\right)}{24d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{(\cos(c+dx)+1)^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(9B\sin\left(\frac{dx}{2}\right)+11C\,\mathrm{s}\right.}{24d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] a^3*((B*x*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/8 + ((-7*B - 5*C)*(1 +
 Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*
x)/2]^6)/(16*d) + ((7*B + 5*C)*(1 + Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2]
+ Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(16*d) + (C*(1 + Cos[c + d*x])^
3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/
2]^6*(3*B*Cos[c/2] + 10*C*Cos[c/2] - 3*B*Sin[c/2] - 8*C*Sin[c/2]))/(96*d*(C
os[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((1 + Co
s[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(9*B*Sin[(d*x)/2] + 11*C*Sin[(d*x)/2]))/
(24*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (C
*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] +
Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((1 + Cos[c + d*x]
)^3*Sec[c/2 + (d*x)/2]^6*(-3*B*Cos[c/2] - 10*C*Cos[c/2] - 3*B*Sin[c/2] - 8*
C*Sin[c/2]))/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d
*x)/2])^2) + ((1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(9*B*Sin[(d*x)/2] +
 11*C*Sin[(d*x)/2]))/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[
c/2 + (d*x)/2])))
```

Maple [A] time = 0.084, size = 158, normalized size = 1.4

$$a^3Bx + \frac{Ba^3c}{d} + \frac{5\,a^3C\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2\,d} + \frac{7\,Ba^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2\,d} + \frac{11\,a^3C\tan\left(dx+c\right)}{3\,d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*B*x+1/d*B*a^3*c+5/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*B*a^3*ln(se
c(d*x+c)+tan(d*x+c))+11/3*a^3*C*tan(d*x+c)/d+3/d*B*a^3*tan(d*x+c)+3/2/d*a^3
*C*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^3*sec(d*x+c)*tan(d*x+c)+1/3/d*a^3*C*tan(
d*x+c)*sec(d*x+c)^2
```

Maxima [B] time = 0.95599, size = 286, normalized size = 2.58

$$12\,(dx+c)Ba^3 + 4\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Ca^3 - 3\,Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 3*B*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 36*B*a^3*tan(d*x + c) + 36*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.525894, size = 356, normalized size = 3.21

$$\frac{12\,Ba^3dx\cos\left(dx+c\right)^3 + 3\,(7\,B+5\,C)a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right) - 3\,(7\,B+5\,C)a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(\right.\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(12*B*a^3*d*x*cos(d*x + c)^3 + 3*(7*B + 5*C)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(7*B + 5*C)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(9*B + 11*C)*a^3*cos(d*x + c)^2 + 3*(B + 3*C)*a^3*cos(d*x + c) + 2*C*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20201, size = 255, normalized size = 2.3

$$\frac{6\,(dx+c)Ba^3+3\left(7\,Ba^3+5\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Ba^3+5\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,Ba^3\tan\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*B*a^3 + 3*(7*B*a^3 + 5*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(7*B*a^3 + 5*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*B
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 40*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*B*a^3*tan(1/2*d
*x + 1/2*c) + 33*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

3.326 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=108

$$\frac{a^3(6B+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(B+2C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}+a^3x(3B+C)-\frac{5a^3C\sin(c+dx)}{2d}+\frac{a}{}$$

```
[Out] a^3*(3*B + C)*x + (a^3*(6*B + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*C*
Sin[c + d*x])/(2*d) + (a*C*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((B
 + 2*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/d
```

Rubi [A] time = 0.311784, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4072, 4018, 3996, 3770}

$$\frac{a^3(6B+7C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(B+2C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}+a^3x(3B+C)-\frac{5a^3C\sin(c+dx)}{2d}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] a^3*(3*B + C)*x + (a^3*(6*B + 7*C)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*C*
Sin[c + d*x])/(2*d) + (a*C*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((B
 + 2*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+a\sec(c+dx))^3(B+C\sec(c$$
$$= \frac{aC(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{1}{2}\int \cos($$
$$= \frac{aC(a+a\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{(B+2C)}{}$$
$$= -\frac{5a^3C\sin(c+dx)}{2d} + \frac{aC(a+a\sec(c+dx))^2\sin}{2d}$$
$$= a^3(3B+C)x - \frac{5a^3C\sin(c+dx)}{2d} + \frac{aC(a+a\sec}{}$$
$$= a^3(3B+C)x + \frac{a^3(6B+7C)\tanh^{-1}(\sin(c+dx}{2d}$$

Mathematica [A] time = 1.93798, size = 208, normalized size = 1.93

$$a^3\left(4(B+3C)\tan(c+dx)+4B\sin(c+dx)-12B\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+12B\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)+\cos\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^3*(12*B*c + 4*c*C + 12*B*d*x + 4*C*d*x - 12*B*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 14*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*B*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + 14*C*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + C/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - C/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 4*B*Sin[c + d*x] + 4*(B + 3*C)*Tan[c + d*x]))/(4*d)
```

Maple [A] time = 0.085, size = 144, normalized size = 1.3

$$\frac{Ba^3\sin(dx+c)}{d} + a^3Cx + \frac{Ca^3c}{d} + 3a^3Bx + 3\frac{Ba^3c}{d} + \frac{7a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 3\frac{Ba^3\ln(\sec(dx+c)+\text{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*B*sin(d*x+c)/d+a^3*C*x+1/d*C*a^3*c+3*a^3*B*x+3/d*B*a^3*c+7/2/d*a^3*C*ln
(sec(d*x+c)+tan(d*x+c))+3/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*tan(d*x
+c)/d+1/d*B*a^3*tan(d*x+c)+1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.945402, size = 223, normalized size = 2.06

$$12(dx+c)Ba^3 + 4(dx+c)Ca^3 - Ca^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 6Ba^3(\log(\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a^3 + 4*(d*x + c)*C*a^3 - C*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 4*B*a^3*tan(d*x + c)
+ 12*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.568991, size = 342, normalized size = 3.17

$$\frac{4\,(3\,B+C)a^3dx\cos\left(dx+c\right)^2+(6\,B+7\,C)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(6\,B+7\,C)a^3\cos\left(dx+c\right)^2\log\left(-\text{si}\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/4*(4*(3*B + C)*a^3*d*x*cos(d*x + c)^2 + (6*B + 7*C)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (6*B + 7*C)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*B*a^3*cos(d*x + c)^2 + 2*(B + 3*C)*a^3*cos(d*x + c) + C*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21921, size = 259, normalized size = 2.4

$$\frac{\frac{4\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(3\,Ba^3+Ca^3\right)(dx+c)+\left(6\,Ba^3+7\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,Ba^3+7\,Ca^3\right)\log\left(\left|\tan\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*B*a^3
 + C*a^3)*(d*x + c) + (6*B*a^3 + 7*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
```

```
) - (6*B*a^3 + 7*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a^3*tan
(1/2*d*x + 1/2*c)^3 + 5*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x
+ 1/2*c) - 7*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.327 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=117

$$\frac{a^3(B+3C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(B-2C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{5a^3B\sin(c+dx)}{2d}+\frac{1}{2}a^3x(7B+6C)+$$

```
[Out] (a^3*(7*B + 6*C)*x)/2 + (a^3*(B + 3*C)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*
Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((B - 2*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.334513, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4017, 4018, 3996, 3770}

$$\frac{a^3(B+3C)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(B-2C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{5a^3B\sin(c+dx)}{2d}+\frac{1}{2}a^3x(7B+6C)+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (a^3*(7*B + 6*C)*x)/2 + (a^3*(B + 3*C)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*
Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((B - 2*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+a\sec(c+dx))^3(B+C\sec($$
$$= \frac{aB\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{aB\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{5a^3B\sin(c+dx)}{2d} + \frac{aB\cos(c+dx)(a+a\sec(c}{2d}$$
$$= \frac{1}{2}a^3(7B+6C)x + \frac{5a^3B\sin(c+dx)}{2d} + \frac{aB\cos(c}{}$$
$$= \frac{1}{2}a^3(7B+6C)x + \frac{a^3(B+3C)\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [B] time = 1.68187, size = 272, normalized size = 2.32

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(3B+C)\sin(c)\cos(dx)}{d}+\frac{4(3B+C)\cos(c)\sin(dx)}{d}-\frac{4(B+3C)\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*(7*B + 6*C)*x - (4*(B + 3*C
)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(B + 3*C)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/d + (4*(3*B + C)*Cos[d*x]*Sin[c])/d + (B*Cos[2*
d*x]*Sin[2*c])/d + (4*(3*B + C)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin[2*d*x]
)/d + (4*C*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) + (4*C*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))))/32
```

Maple [A] time = 0.079, size = 145, normalized size = 1.2

$$\frac{Ba^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{7a^3Bx}{2}+\frac{7Ba^3c}{2d}+\frac{a^3C\sin(dx+c)}{d}+3\frac{Ba^3\sin(dx+c)}{d}+3a^3Cx+3\frac{Ca^3c}{d}+3\frac{a^3C\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*a^3*sin(d*x+c)*cos(d*x+c)+7/2*a^3*B*x+7/2/d*B*a^3*c+a^3*C*sin(d*x+c
)/d+3*a^3*B*sin(d*x+c)/d+3*a^3*C*x+3/d*C*a^3*c+3/d*a^3*C*ln(sec(d*x+c)+tan(
d*x+c))+1/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*C*tan(d*x+c)/d
```

Maxima [A] time = 0.94463, size = 189, normalized size = 1.62

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ba^3 + 12\,(dx + c)Ba^3 + 12\,(dx + c)Ca^3 + 2\,Ba^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 12*(d*x + c)*B*a^3 + 12*(d*x
+ c)*C*a^3 + 2*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*C*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a^3*sin(d*x + c)
 + 4*C*a^3*sin(d*x + c) + 4*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.559321, size = 323, normalized size = 2.76

$$\frac{(7\,B + 6\,C)a^3dx\cos(dx + c) + (B + 3\,C)a^3\cos(dx + c)\log(\sin(dx + c) + 1) - (B + 3\,C)a^3\cos(dx + c)\log(-\sin(dx \cdots}{2\,d\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*((7*B + 6*C)*a^3*d*x*cos(d*x + c) + (B + 3*C)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (B + 3*C)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (B*a^3*
cos(d*x + c)^2 + 2*(3*B + C)*a^3*cos(d*x + c) + 2*C*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21781, size = 259, normalized size = 2.21

$$-\frac{\frac{4\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \left(7\,Ba^3 + 6\,Ca^3\right)(dx + c) - 2\left(Ba^3 + 3\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 2\left(Ba^3 + 3\,Ca^3\right)\log\left(\left|\tan \cdots\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/2*(4*C*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (7*B*a^3
+ 6*C*a^3)*(d*x + c) - 2*(B*a^3 + 3*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(B*a^3 + 3*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*B*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 2*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*B*a^3*tan(1/2*d*x
 + 1/2*c) + 2*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.328 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=125

$$\frac{5a^3(B+C)\sin(c+dx)}{2d}+\frac{(5B+3C)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(5B+7C)+\frac{a^3C\tanh^{-1}(\sin}{d}$$

```
[Out] (a^3*(5*B + 7*C)*x)/2 + (a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(B + C)*Si
n[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(3*d) + ((5*B + 3*C)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rubi [A] time = 0.339268, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4072, 4017, 3996, 3770}

$$\frac{5a^3(B+C)\sin(c+dx)}{2d}+\frac{(5B+3C)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(5B+7C)+\frac{a^3C\tanh^{-1}(\sin}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (a^3*(5*B + 7*C)*x)/2 + (a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(B + C)*Si
n[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(3*d) + ((5*B + 3*C)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+a\sec(c+dx))^3(B+C\sec($$
$$= \frac{aB\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d}$$
$$= \frac{aB\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{3d}$$
$$= \frac{5a^3(B+C)\sin(c+dx)}{2d} + \frac{aB\cos^2(c+dx)(a+$$
$$= \frac{1}{2}a^3(5B+7C)x + \frac{5a^3(B+C)\sin(c+dx)}{2d} + \frac{aB$$
$$= \frac{1}{2}a^3(5B+7C)x + \frac{a^3C\tanh^{-1}(\sin(c+dx))}{d} + \frac{5$$

Mathematica [A] time = 0.249272, size = 113, normalized size = 0.9

$$\frac{a^3\left(9(5B+4C)\sin(c+dx)+3(3B+C)\sin(2(c+dx))+B\sin(3(c+dx))+30Bdx-12C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^3*(30*B*d*x + 42*C*d*x - 12*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(5*B + 4*C)*Sin[c + d*x]
 + 3*(3*B + C)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.088, size = 153, normalized size = 1.2

$$\frac{B(\cos(dx+c))^2\sin(dx+c)a^3}{3d} + \frac{11Ba^3\sin(dx+c)}{3d} + \frac{a^3C\sin(dx+c)\cos(dx+c)}{2d} + \frac{7a^3Cx}{2} + \frac{7a^3Cc}{2d} + \frac{3Ba^3\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*B*cos(d*x+c)^2*sin(d*x+c)*a^3+11/3*a^3*B*sin(d*x+c)/d+1/2/d*a^3*C*sin
(d*x+c)*cos(d*x+c)+7/2*a^3*C*x+7/2/d*C*a^3*c+3/2/d*B*a^3*sin(d*x+c)*cos(d*x
+c)+5/2*a^3*B*x+5/2/d*B*a^3*c+3*a^3*C*sin(d*x+c)/d+1/d*a^3*C*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 0.948515, size = 200, normalized size = 1.6

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3-9(2dx+2c+\sin(2dx+2c))Ba^3-12(dx+c)Ba^3-3(2dx+2c+\sin(2dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^3 - 12*(d*x + c)*B*a^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*
C*a^3 - 36*(d*x + c)*C*a^3 - 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) - 36*B*a^3*sin(d*x + c) - 36*C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.530216, size = 254, normalized size = 2.03

$$\frac{3\,(5\,B+7\,C)a^3dx+3\,Ca^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Ca^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ba^3\cos\left(dx+c\right)^2+3\,(3\,B+C)a^3\cdots\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(5*B + 7*C)*a^3*d*x + 3*C*a^3*log(sin(d*x + c) + 1) - 3*C*a^3*log(-s
in(d*x + c) + 1) + (2*B*a^3*cos(d*x + c)^2 + 3*(3*B + C)*a^3*cos(d*x + c) +
 2*(11*B + 9*C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2378, size = 243, normalized size = 1.94

$$\frac{6\,Ca^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Ca^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(5\,Ba^3+7\,Ca^3\right)(dx+c)+\frac{2\left(15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(6*C*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*C*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(5*B*a^3 + 7*C*a^3)*(d*x + c) + 2*(15*B*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^3*tan(1/2*d*x + 1/
2*c)^3 + 36*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*B*a^3*tan(1/2*d*x + 1/2*c) +
21*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.329 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=124

$$-\frac{a^3(3B+4C)\sin^3(c+dx)}{12d}+\frac{a^3(3B+4C)\sin(c+dx)}{d}+\frac{3a^3(3B+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(3B+4C)+\frac{B\sin}{}$$

```
[Out] (5*a^3*(3*B + 4*C)*x)/8 + (a^3*(3*B + 4*C)*Sin[c + d*x])/d + (3*a^3*(3*B +
4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^3*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d) - (a^3*(3*B + 4*C)*Sin[c + d*x]^3)/(12*d)
```

Rubi [A] time = 0.251688, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{a^3(3B+4C)\sin^3(c+dx)}{12d}+\frac{a^3(3B+4C)\sin(c+dx)}{d}+\frac{3a^3(3B+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(3B+4C)+\frac{B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```



```
[Out] (5*a^3*(3*B + 4*C)*x)/8 + (a^3*(3*B + 4*C)*Sin[c + d*x])/d + (3*a^3*(3*B +
4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^3*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d) - (a^3*(3*B + 4*C)*Sin[c + d*x]^3)/(12*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \cos^4(c+dx)(a+a\sec(c+dx))^3(B+C\,\mathrm{se}$$

$$= \frac{B\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+d}{4d}$$

$$= \frac{B\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+d}{4d}$$

$$= \frac{1}{4}a^3(3B+4C)x + \frac{B\cos^3(c+dx)(a+a\sec(}{4d}$$

$$= \frac{1}{4}a^3(3B+4C)x + \frac{3a^3(3B+4C)\sin(c+dx)}{4d}$$

$$= \frac{5}{8}a^3(3B+4C)x + \frac{a^3(3B+4C)\sin(c+dx)}{d}$$

Mathematica [A] time = 0.273803, size = 86, normalized size = 0.69

$$\frac{a^3(24(13B+15C)\sin(c+dx)+24(4B+3C)\sin(2(c+dx))+24B\sin(3(c+dx))+3B\sin(4(c+dx))+180Bdx+8C}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^3*(180*B*d*x + 240*C*d*x + 24*(13*B + 15*C)*Sin[c + d*x] + 24*(4*B + 3*C
)*Sin[2*(c + d*x)] + 24*B*Sin[3*(c + d*x)] + 8*C*Sin[3*(c + d*x)] + 3*B*Sin
[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.089, size = 176, normalized size = 1.4

$$\frac{1}{d}\left(Ba^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+Ba^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)+\frac{a^3C(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a
^3*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a^
3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+B*a^3*sin(d*x+c)+3*a^3*C*sin(d*x+c)+a^3*C*(d*x+c))
```

Maxima [A] time = 0.949358, size = 225, normalized size = 1.81

$$-\frac{96\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)Ba^3 - 3\left(12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\sin(2\,dx + 2\,c)\right)Ba^3 - 72\left(2\,dx + 2\,c + \sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/96*(96*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*
c))*B*a^3 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 72*(2*d*x + 2*c +
sin(2*d*x + 2*c))*C*a^3 - 96*(d*x + c)*C*a^3 - 96*B*a^3*sin(d*x + c) - 288*
C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.503874, size = 216, normalized size = 1.74

$$\frac{15\,(3\,B+4\,C)a^3dx + \left(6\,Ba^3\cos(dx+c)^3 + 8\,(3\,B+C)a^3\cos(dx+c)^2 + 9\,(5\,B+4\,C)a^3\cos(dx+c) + 8\,(9\,B+11\,C)a^3\right)\mathrm{s}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/24*(15*(3*B + 4*C)*a^3*d*x + (6*B*a^3*cos(d*x + c)^3 + 8*(3*B + C)*a^3*co
s(d*x + c)^2 + 9*(5*B + 4*C)*a^3*cos(d*x + c) + 8*(9*B + 11*C)*a^3)*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20704, size = 238, normalized size = 1.92

$$\frac{15\left(3\,Ba^3 + 4\,Ca^3\right)(dx+c) + \frac{2\left(45\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 60\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 165\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 220\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 219\,Ba^3\tan\left(\frac{1}{2}\,d\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(15*(3*B*a^3 + 4*C*a^3)*(d*x + c) + 2*(45*B*a^3*tan(1/2*d*x + 1/2*c)^7
 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 165*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 220
*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 292*C*a^
3*tan(1/2*d*x + 1/2*c)^3 + 147*B*a^3*tan(1/2*d*x + 1/2*c) + 132*C*a^3*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.330 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=176

$$\frac{a^3(38B+45C)\sin(c+dx)}{15d}+\frac{a^3(43B+45C)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^3(13B+15C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{(7B+5}{}$$

```
[Out] (a^3*(13*B + 15*C)*x)/8 + (a^3*(38*B + 45*C)*Sin[c + d*x])/(15*d) + (a^3*(1
3*B + 15*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*(43*B + 45*C)*Cos[c + d
*x]^2*Sin[c + d*x])/(60*d) + (a*B*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + ((7*B + 5*C)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin
[c + d*x])/(20*d)
```

Rubi [A] time = 0.446618, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^3(38B+45C)\sin(c+dx)}{15d}+\frac{a^3(43B+45C)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^3(13B+15C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{(7B+5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (a^3*(13*B + 15*C)*x)/8 + (a^3*(38*B + 45*C)*Sin[c + d*x])/(15*d) + (a^3*(1
3*B + 15*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*(43*B + 45*C)*Cos[c + d
*x]^2*Sin[c + d*x])/(60*d) + (a*B*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + ((7*B + 5*C)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin
[c + d*x])/(20*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^5(c+dx)(a+a\sec(c+dx))^3(B+C\text{ s}$$

$$= \frac{aB\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+}{5d}$$

$$= \frac{aB\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+}{5d}$$

$$= \frac{a^3(43B+45C)\cos^2(c+dx)\sin(c+dx)}{60d} + $$

$$= \frac{a^3(43B+45C)\cos^2(c+dx)\sin(c+dx)}{60d} + $$

$$= \frac{a^3(38B+45C)\sin(c+dx)}{15d} + \frac{a^3(13B+15C}{}$$

$$= \frac{1}{8}a^3(13B+15C)x + \frac{a^3(38B+45C)\sin(c+}{15d}$$

Mathematica [A] time = 0.429534, size = 108, normalized size = 0.61

$$\frac{a^3(60(23B+26C)\sin(c+dx)+480(B+C)\sin(2(c+dx))+170B\sin(3(c+dx))+45B\sin(4(c+dx))+6B\sin(5(c+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^3*(780*B*c + 780*B*d*x + 900*C*d*x + 60*(23*B + 26*C)*Sin[c + d*x] + 480
*(B + C)*Sin[2*(c + d*x)] + 170*B*Sin[3*(c + d*x)] + 120*C*Sin[3*(c + d*x)]
 + 45*B*Sin[4*(c + d*x)] + 15*C*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(
480*d)
```

Maple [A] time = 0.104, size = 223, normalized size = 1.3

$$\frac{1}{d}\left(\frac{Ba^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^3C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*B*a^3*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*C*(2+cos(d*x+c)^2)*sin(
d*x+c)+B*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+B*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*C*sin(d
*x+c))
```

Maxima [A] time = 0.94315, size = 288, normalized size = 1.64

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3+45\left(12\,dx+12\,c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 -
480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 45*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^3 - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 + 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*C*a^3 + 480*C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.500791, size = 278, normalized size = 1.58

$$\frac{15\,(13\,B+15\,C)a^3dx+\left(24\,Ba^3\cos(dx+c)^4+30\,(3\,B+C)a^3\cos(dx+c)^3+8\,(19\,B+15\,C)a^3\cos(dx+c)^2+15\,(13\,B+\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(13*B + 15*C)*a^3*d*x + (24*B*a^3*cos(d*x + c)^4 + 30*(3*B + C)*a
^3*cos(d*x + c)^3 + 8*(19*B + 15*C)*a^3*cos(d*x + c)^2 + 15*(13*B + 15*C)*a
^3*cos(d*x + c) + 8*(38*B + 45*C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20903, size = 284, normalized size = 1.61

$$\frac{15\left(13\,Ba^3+15\,Ca^3\right)(dx+c)+\frac{2\left(195\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+225\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,\cdots\right.}{\cdots}}{120\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(13*B*a^3 + 15*C*a^3)*(d*x + c) + 2*(195*B*a^3*tan(1/2*d*x + 1/2*
c)^9 + 225*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*B*a^3*tan(1/2*d*x + 1/2*c)^7
+ 1050*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1
920*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 1830
*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*B*a^3*tan(1/2*d*x + 1/2*c) + 735*C*a^3*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.331 $\int \cos^7(c+dx)(a+a\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=201

$$-\frac{a^3(17B+19C)\sin^3(c+dx)}{15d}+\frac{a^3(17B+19C)\sin(c+dx)}{5d}+\frac{a^3(21B+22C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^3(23B+26C)\mathrm{si}}{}$$

```
[Out] (a^3*(23*B + 26*C)*x)/16 + (a^3*(17*B + 19*C)*Sin[c + d*x])/(5*d) + (a^3*(2
3*B + 26*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(21*B + 22*C)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((4*B + 3*C)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(17*B + 19*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.479243, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^3(17B+19C)\sin^3(c+dx)}{15d}+\frac{a^3(17B+19C)\sin(c+dx)}{5d}+\frac{a^3(21B+22C)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^3(23B+26C)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (a^3*(23*B + 26*C)*x)/16 + (a^3*(17*B + 19*C)*Sin[c + d*x])/(5*d) + (a^3*(2
3*B + 26*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(21*B + 22*C)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((4*B + 3*C)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(17*B + 19*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \cos^6(c+dx)(a+a\sec(c+dx))^3(B+C\,\mathrm{s}$$

$$= \frac{aB\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+}{6d}$$

$$= \frac{aB\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+}{6d}$$

$$= \frac{a^3(21B+22C)\cos^3(c+dx)\sin(c+dx)}{40d}+$$

$$= \frac{a^3(21B+22C)\cos^3(c+dx)\sin(c+dx)}{40d}+$$

$$= \frac{a^3(23B+26C)\cos(c+dx)\sin(c+dx)}{16d}+$$

$$= \frac{1}{16}a^3(23B+26C)x+\frac{a^3(17B+19C)\sin(c+}{5d}$$

Mathematica [A] time = 0.481165, size = 134, normalized size = 0.67

$$\frac{a^3(120(21B+23C)\sin(c+dx)+15(63B+64C)\sin(2(c+dx))+380B\sin(3(c+dx))+135B\sin(4(c+dx))+36B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^3*(1380*B*c + 1380*B*d*x + 1560*C*d*x + 120*(21*B + 23*C)*Sin[c + d*x] +
 15*(63*B + 64*C)*Sin[2*(c + d*x)] + 380*B*Sin[3*(c + d*x)] + 340*C*Sin[3*(
c + d*x)] + 135*B*Sin[4*(c + d*x)] + 90*C*Sin[4*(c + d*x)] + 36*B*Sin[5*(c
+ d*x)] + 12*C*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.105, size = 266, normalized size = 1.3

$$\frac{1}{d}\left(Ba^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a^3C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3
/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*C*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*B*a^3*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*C*(2+cos(d*x+c)^2)*sin(d*x+
c)+1/3*B*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c))
```

Maxima [A] time = 0.953253, size = 354, normalized size = 1.76

$$\frac{192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 64*(3*sin(d*x + c)^
5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^3 - 960*(sin(d*x + c)^3 - 3*si
n(d*x + c))*C*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*C*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3)/d
```

Fricas [A] time = 0.512691, size = 332, normalized size = 1.65

$$\frac{15(23B+26C)a^3dx+\left(40Ba^3\cos(dx+c)^5+48(3B+C)a^3\cos(dx+c)^4+10(23B+18C)a^3\cos(dx+c)^3+16(17B\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(23*B + 26*C)*a^3*d*x + (40*B*a^3*cos(d*x + c)^5 + 48*(3*B + C)*a
^3*cos(d*x + c)^4 + 10*(23*B + 18*C)*a^3*cos(d*x + c)^3 + 16*(17*B + 19*C)*
a^3*cos(d*x + c)^2 + 15*(23*B + 26*C)*a^3*cos(d*x + c) + 32*(17*B + 19*C)*a
^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22676, size = 329, normalized size = 1.64

$$15\left(23\,Ba^3+26\,Ca^3\right)(dx+c)+\frac{2\left(345\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+390\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2210\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/240*(15*(23*B*a^3 + 26*C*a^3)*(d*x + c) + 2*(345*B*a^3*tan(1/2*d*x + 1/2*
c)^11 + 390*C*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*B*a^3*tan(1/2*d*x + 1/2*c)
^9 + 2210*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*B*a^3*tan(1/2*d*x + 1/2*c)^7
+ 5148*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 5
988*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 4190
*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*B*a^3*tan(1/2*d*x + 1/2*c) + 1530*C*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.332 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$-\frac{(3B-4C)\tan^3(c+dx)}{3ad}-\frac{(3B-4C)\tan(c+dx)}{ad}+\frac{3(B-C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(B-C)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

```
[Out] (3*(B - C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((3*B - 4*C)*Tan[c + d*x])/(a*d
) + (3*(B - C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((B - C)*Sec[c + d*x]^3
*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*B - 4*C)*Tan[c + d*x]^3)/(3*a
*d)
```

Rubi [A] time = 0.253334, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4019, 3787, 3768, 3770, 3767}

$$-\frac{(3B-4C)\tan^3(c+dx)}{3ad}-\frac{(3B-4C)\tan(c+dx)}{ad}+\frac{3(B-C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(B-C)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
]),x]
```

```
[Out] (3*(B - C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((3*B - 4*C)*Tan[c + d*x])/(a*d
) + (3*(B - C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((B - C)*Sec[c + d*x]^3
*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*B - 4*C)*Tan[c + d*x]^3)/(3*a
*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^3(c+dx)(3a(B-C)-$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3B-4C)\int \sec^4(c+dx)\,dx}{a}$$

$$= \frac{3(B-C)\sec(c+dx)\tan(c+dx)}{2ad} + \frac{(B-C)\sec^3(c+dx)\tan(c}{d(a+a\sec(c+dx))}$$

$$= \frac{3(B-C)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(3B-4C)\tan(c+dx)}{ad} + \frac{3(B-}{}$$

Mathematica [B] time = 1.11916, size = 550, normalized size = 4.2

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(27(B-C)\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(9*B*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] - 9*C*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 9*B*Cos[(7*(c + d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 9*C*Cos[(7*(c + d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 27*(B - C)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 27*(B - C)*Cos[(3*(c + d*x)
)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) - 9*B*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*C*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 9*B*Cos[(7*(c + d*x))/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*C
*Cos[(7*(c + d*x))/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 12*B*Sin[(
c + d*x)/2] + 18*B*Sin[(3*(c + d*x))/2] - 30*C*Sin[(3*(c + d*x))/2] - 6*B*S
in[(5*(c + d*x))/2] + 2*C*Sin[(5*(c + d*x))/2] + 12*B*Sin[(7*(c + d*x))/2]
- 16*C*Sin[(7*(c + d*x))/2]))/(24*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.062, size = 340, normalized size = 2.6

$$-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{3\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{B}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-1/3/a/d*C/(tan(1/2*d
*x+1/2*c)+1)^3-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*B+1/a/d/(tan(1/2*d*x+1/2*c)
+1)^2*C-3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)
*B-5/2/a/d/(tan(1/2*d*x+1/2*c)+1)*C+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B-1/3/a/
d*C/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d/(tan(1/2*d*x+1/2*c)-1)^2*C+1/2/a/d/(tan(
1/2*d*x+1/2*c)-1)^2*B+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-3/2/a/d*ln(tan(1/2
*d*x+1/2*c)-1)*B-5/2/a/d/(tan(1/2*d*x+1/2*c)-1)*C+3/2/a/d/(tan(1/2*d*x+1/2*
c)-1)*B
```

Maxima [B] time = 0.954362, size = 497, normalized size = 3.79

$$\frac{C\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(\ldots}{(\cos(dx\ldots}\right.}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin\ldots}{(\cos(d\ldots}}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/6*(C*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.519179, size = 417, normalized size = 3.18

$$\frac{9\left((B-C)\cos(dx+c)^4+(B-C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left((B-C)\cos(dx+c)^4+(B-C)\cos(dx+c)^3\right)\ldots}{12\left(ad\cos(dx+c\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/12*(9*((B - C)*cos(d*x + c)^4 + (B - C)*cos(d*x + c)^3)*log(sin(d*x + c)
+ 1) - 9*((B - C)*cos(d*x + c)^4 + (B - C)*cos(d*x + c)^3)*log(-sin(d*x + c
) + 1) - 2*(4*(3*B - 4*C)*cos(d*x + c)^3 + (3*B - 7*C)*cos(d*x + c)^2 - (3*
B - C)*cos(d*x + c) - 2*C)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
+ c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^4(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(B*sec(c + d*x)**4/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x
)**5/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.18516, size = 246, normalized size = 1.88

$$\frac{\frac{9(B-C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{9(B-C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{6\left(B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} + \frac{2\left(9B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15C\tan\left(\frac{1}{2}dx\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/6*(9*(B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(B - C)*log(abs(tan
(1/2*d*x + 1/2*c) - 1))/a - 6*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2
*c))/a + 2*(9*B*tan(1/2*d*x + 1/2*c)^5 - 15*C*tan(1/2*d*x + 1/2*c)^5 - 12*B
*tan(1/2*d*x + 1/2*c)^3 + 16*C*tan(1/2*d*x + 1/2*c)^3 + 3*B*tan(1/2*d*x + 1
/2*c) - 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.333 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{2(B-C)\tan(c+dx)}{ad}-\frac{(2B-3C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(2B-3C)\tan(c+dx)\sec(c}{2ad}$$

```
[Out] -((2*B - 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + (2*(B - C)*Tan[c + d*x])/(a*
d) - ((2*B - 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((B - C)*Sec[c + d*x
]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.239867, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4019, 3787, 3767, 8, 3768, 3770}

$$\frac{2(B-C)\tan(c+dx)}{ad}-\frac{(2B-3C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(2B-3C)\tan(c+dx)\sec(c}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
]),x]
```


```
[Out] -((2*B - 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + (2*(B - C)*Tan[c + d*x])/(a*
d) - ((2*B - 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((B - C)*Sec[c + d*x
]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx &= \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{a+a\sec(c+dx)}\,dx \\
&= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \sec^2(c+dx)(2a(B-C)-
\\
&= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{(2B-3C)\int \sec^3(c+dx)\,dx}{a} \\
&= -\frac{(2B-3C)\sec(c+dx)\tan(c+dx)}{2ad}+\frac{(B-C)\sec^2(c+dx)\tan
\\
&= -\frac{(2B-3C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{2(B-C)\tan(c+dx)}{ad}-(2B
\end{aligned}$$

Mathematica [B] time = 0.664843, size = 383, normalized size = 3.55

$$\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(2(2B-3C)\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^2*(2*B*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - 3*C*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + 2*(2*B - 3*C)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2*B - 3*C)
*Cos[(3*(c + d*x))/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]) - 2*B*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 3*C*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + 4*B*Sin[(c + d*x)/2] - 2*C*Sin[(c + d*x)/2] + 2*C*Sin[
(3*(c + d*x))/2] + 4*B*Sin[(5*(c + d*x))/2] - 4*C*Sin[(5*(c + d*x))/2]))/(4
*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.054, size = 252, normalized size = 2.3

$$\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3\,C}{2\,ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-1/2/a/d/(tan(1/2*d*x+
1/2*c)+1)^2*C+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C-1/a/d*ln(tan(1/2*d*x+1/2*c
)+1)*B+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*C-1/a/d/(tan(1/2*d*x+1/2*c)+1)*B+1/2/
a/d/(tan(1/2*d*x+1/2*c)-1)^2*C+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/a/d/(tan(
1/2*d*x+1/2*c)-1)*B-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C+1/a/d*ln(tan(1/2*d*x
+1/2*c)-1)*B
```

Maxima [B] time = 0.949285, size = 381, normalized size = 3.53

$$-\frac{C\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2\,B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+}{\cos(dx+c)}\right.}{a}\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -1/2*(C*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.513692, size = 386, normalized size = 3.57

$$-\frac{\left((2\,B-3\,C)\cos\left(dx+c\right)^3+(2\,B-3\,C)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((2\,B-3\,C)\cos\left(dx+c\right)^3+(2\,B-3\,C)\cos\right.}{4\left(ad\cos\left(dx+c\right)^3+ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/4*(((2*B - 3*C)*cos(d*x + c)^3 + (2*B - 3*C)*cos(d*x + c)^2)*log(sin(d*x
 + c) + 1) - ((2*B - 3*C)*cos(d*x + c)^3 + (2*B - 3*C)*cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(4*(B - C)*cos(d*x + c)^2 + (2*B - C)*cos(d*x + c) +
 C)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^3(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{C\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(B*sec(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x
)**4/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.15507, size = 211, normalized size = 1.95

$$-\frac{\frac{(2B-3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{(2B-3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{2\left(B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} + \frac{2\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3C\tan\left(\right.\right.}{\left(\mathrm{t}\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/2*((2*B - 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (2*B - 3*C)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x
+ 1/2*c))/a + 2*(2*B*tan(1/2*d*x + 1/2*c)^3 - 3*C*tan(1/2*d*x + 1/2*c)^3 -
2*B*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 - 1)^2*a))/d
```

3.334 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{C\tan(c+dx)}{ad}$$

```
[Out] ((B - C)*ArcTanh[Sin[c + d*x]])/(a*d) + (C*Tan[c + d*x])/(a*d) - ((B - C)*T
an[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.165813, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 4008, 3787, 3770, 3767, 8}

$$\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(B-C)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{C\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
,x]
```

```
[Out] ((B - C)*ArcTanh[Sin[c + d*x]])/(a*d) + (C*Tan[c + d*x])/(a*d) - ((B - C)*T
an[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{(B-C)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec(c+dx)(-a(B-C)-aC\sec(c+a$$
$$= -\frac{(B-C)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{(B-C)\int\sec(c+dx)\,dx}{a} + \frac{C\int\sec^2(c$$
$$= \frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(B-C)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{C\,\mathrm{Subst}($$
$$= \frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad} + \frac{C\tan(c+dx)}{ad} - \frac{(B-C)\tan(c+}{d(a+a\sec(c+}$$

Mathematica [B] time = 0.482103, size = 234, normalized size = 3.77

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-2\sin\left(\frac{1}{2}(c+dx)\right)(C-(B-2C)\cos(c+dx))+(B-C)\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] -((Cos[(c + d*x)/2]*((B - C)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (B - C)*Cos[(3*(
c + d*x))/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]) - 2*(C - (B - 2*C)*Cos[c + d*x])*Sin[(c + d*x)/2]))
/(a*d*(1 + Cos[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])))
```

Maple [B] time = 0.046, size = 163, normalized size = 2.6

$$-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-1/a/d/(tan(1/2*d*x+1
/2*c)+1)*C+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*
C-1/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B+1/a/d*ln(
```

```
tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.941696, size = 265, normalized size = 4.27

$$-\frac{C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] -(C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [B] time = 0.505733, size = 319, normalized size = 5.15

$$\frac{\left((B-C)\cos(dx+c)^2+(B-C)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((B-C)\cos(dx+c)^2+(B-C)\cos(dx+c)\right)\log(-}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 1/2*(((B - C)*cos(d*x + c)^2 + (B - C)*cos(d*x + c))*log(sin(d*x + c) + 1)
- ((B - C)*cos(d*x + c)^2 + (B - C)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
2*((B - 2*C)*cos(d*x + c) - C)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(
d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^2(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{C\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(B*sec(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x
)**3/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.14993, size = 147, normalized size = 2.37

$$\frac{\frac{(B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{(B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] ((B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (B - C)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a - (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a - 2
*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.335 $\int \frac{B \sec(c+dx)+C \sec^2(c+dx)}{a+a \sec(c+dx)} \, dx$

Optimal. Leaf size=44

$$\frac{(B-C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a*d) + ((B - C)*Tan[c + d*x])/(a*d*(1 + Sec[c +
d*x]))
```

Rubi [A] time = 0.0728113, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4050, 3770, 12, 3794}

$$\frac{(B-C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a*d) + ((B - C)*Tan[c + d*x])/(a*d*(1 + Sec[c +
d*x]))
```

Rule 4050

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e
 + f*x], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Csc[e + f*x])/(a + b*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(aB-aC)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{a} + \frac{C\int \sec(c+dx)\,dx}{a}$$
$$= \frac{C\tanh^{-1}(\sin(c+dx))}{ad} + (B-C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{C\tanh^{-1}(\sin(c+dx))}{ad} + \frac{(B-C)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.184871, size = 106, normalized size = 2.41

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-C)\sin\left(\frac{1}{2}(c+dx)\right)+C\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(C*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (B - C)*Sin[(c + d*x)
/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.044, size = 78, normalized size = 1.8

$$\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C-1/a/d*ln(tan(1/
2*d*x+1/2*c)-1)*C-1/a/d*C*tan(1/2*d*x+1/2*c)
```

Maxima [B] time = 0.932191, size = 134, normalized size = 3.05

$$\frac{C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] (C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.495314, size = 197, normalized size = 4.48

$$\frac{(C\cos(dx+c)+C)\log(\sin(dx+c)+1)-(C\cos(dx+c)+C)\log(-\sin(dx+c)+1)+2(B-C)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((C*cos(d*x + c) + C)*log(sin(d*x + c) + 1) - (C*cos(d*x + c) + C)*log(
-sin(d*x + c) + 1) + 2*(B - C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\sec(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{C\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(B*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**
2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.14828, size = 95, normalized size = 2.16

$$\frac{\frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}+\frac{B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] (C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - C*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a + (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a)/d
```

3.336 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{Bx}{a}-\frac{(B-C)\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] (B*x)/a - ((B - C)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.131612, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {4072, 3919, 3794}

$$\frac{Bx}{a}-\frac{(B-C)\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
,x]
```

```
[Out] (B*x)/a - ((B - C)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx &= \int \frac{B+C\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\
&= \frac{Bx}{a}-(B-C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\
&= \frac{Bx}{a}-\frac{(B-C)\tan(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.130997, size = 72, normalized size = 2.06

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(Bdx\cos\left(c+\frac{dx}{2}\right)+2(C-B)\sin\left(\frac{dx}{2}\right)+Bdx\cos\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(B*d*x*Cos[(d*x)/2] + B*d*x*Cos[c + (d*x)/2] + 2
*(-B + C)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.069, size = 56, normalized size = 1.6

$$-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{B\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad}+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+2/a/d*B*arctan(tan(1/2*d*x+1/2*c))+1/a/d*C*tan(
1/2*d*x+1/2*c)
```

Maxima [B] time = 1.43019, size = 99, normalized size = 2.83

$$\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{C\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] (B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + C*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.470496, size = 105, normalized size = 3.

$$\frac{Bdx\cos\left(dx+c\right)+Bdx-(B-C)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] (B*d*x*cos(d*x + c) + B*d*x - (B - C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\cos(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{C\cos(c+dx)\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*c
os(c + d*x)*sec(c + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.14913, size = 59, normalized size = 1.69

$$\frac{\frac{(dx+c)B}{a} - \frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] ((d*x + c)*B/a - (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a)/d
```

3.337 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{(2B-C)\sin(c+dx)}{ad}-\frac{(B-C)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(B-C)}{a}$$

```
[Out] -(((B - C)*x)/a) + ((2*B - C)*Sin[c + d*x])/(a*d) - ((B - C)*Sin[c + d*x])/
(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.195895, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4020, 3787, 2637, 8}

$$\frac{(2B-C)\sin(c+dx)}{ad}-\frac{(B-C)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(B-C)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
]),x]
```

```
[Out] -(((B - C)*x)/a) + ((2*B - C)*Sin[c + d*x])/(a*d) - ((B - C)*Sin[c + d*x])/
(d*(a + a*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos(c+dx)(B+C\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$
$$=-\frac{(B-C)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \cos(c+dx)(a(2B-C)-a(B-C)\mathrm{se}}{a^2}$$
$$=-\frac{(B-C)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(B-C)\int 1\,dx}{a}+\frac{(2B-C)\int \cos(c+d}{a}$$
$$=-\frac{(B-C)x}{a}+\frac{(2B-C)\sin(c+dx)}{ad}-\frac{(B-C)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.342519, size = 76, normalized size = 1.27

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)(B\sin(c+dx)+dx(C-B))+(B-C)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((B - C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-B
+ C)*d*x + B*Sin[c + d*x])))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.083, size = 108, normalized size = 1.8

$$\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{B\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\frac{B\arctan(\tan(1/2\,dx+c/2))}{ad}+2\frac{\arctan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)+2/a/d*B*tan(1/2*d*x+1
/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*B*arctan(tan(1/2*d*x+1/2*c))+2/a/d*arc
tan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.4173, size = 193, normalized size = 3.22

$$-\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -(B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.475969, size = 149, normalized size = 2.48

$$-\frac{(B-C)dx\cos(dx+c)+(B-C)dx-(B\cos(dx+c)+2B-C)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -((B - C)*d*x*cos(d*x + c) + (B - C)*d*x - (B*cos(d*x + c) + 2*B - C)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\cos^2(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{C\cos^2(c+dx)\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(
C*cos(c + d*x)**2*sec(c + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.12124, size = 107, normalized size = 1.78

$$-\frac{\frac{(dx+c)(B-C)}{a}-\frac{B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -((d*x + c)*(B - C)/a - (B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1/2*c))/a
 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.338 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$-\frac{2(B-C)\sin(c+dx)}{ad}+\frac{(3B-2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(B-C)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3B-2C)}{2a}$$

```
[Out] ((3*B - 2*C)*x)/(2*a) - (2*(B - C)*Sin[c + d*x])/(a*d) + ((3*B - 2*C)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((B - C)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rubi [A] time = 0.231741, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4020, 3787, 2635, 8, 2637}

$$-\frac{2(B-C)\sin(c+dx)}{ad}+\frac{(3B-2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(B-C)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3B-2C)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
]),x]
```

```
[Out] ((3*B - 2*C)*x)/(2*a) - (2*(B - C)*Sin[c + d*x])/(a*d) + ((3*B - 2*C)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((B - C)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \cos^2(c+dx)(a(3B-2C)-2}{a^2}$$
$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(3B-2C)\int \cos^2(c+dx)\,dx}{a} -$$
$$= -\frac{2(B-C)\sin(c+dx)}{ad} + \frac{(3B-2C)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(B-}{}$$
$$= \frac{(3B-2C)x}{2a} - \frac{2(B-C)\sin(c+dx)}{ad} + \frac{(3B-2C)\cos(c+dx)\sin(c}{2ad}$$

Mathematica [B] time = 0.409703, size = 197, normalized size = 2.01

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4dx(3B-2C)\cos\left(c+\frac{dx}{2}\right)-4B\sin\left(c+\frac{dx}{2}\right)-3B\sin\left(c+\frac{3dx}{2}\right)-3B\sin\left(2c+\frac{3dx}{2}\right)+B\sin\left(2c+\right.\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(3*B - 2*C)*d*x*Cos[(d*x)/2] + 4*(3*B - 2*C)*
d*x*Cos[c + (d*x)/2] - 20*B*Sin[(d*x)/2] + 20*C*Sin[(d*x)/2] - 4*B*Sin[c +
(d*x)/2] + 4*C*Sin[c + (d*x)/2] - 3*B*Sin[c + (3*d*x)/2] + 4*C*Sin[c + (3*d
*x)/2] - 3*B*Sin[2*c + (3*d*x)/2] + 4*C*Sin[2*c + (3*d*x)/2] + B*Sin[2*c +
(5*d*x)/2] + B*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.093, size = 211, normalized size = 2.2

$$-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\frac{(\tan(1/2\,dx+c/2))^3B}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+2\frac{C(\tan(1/2\,dx+c/2))^3}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{B}{ad}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/
```

```
2*d*x+1/2*c)^3*C-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+2/a/
d/(1+tan(1/2*d*x+1/2*c)^2)^2*C*tan(1/2*d*x+1/2*c)+3/a/d*B*arctan(tan(1/2*d*
x+1/2*c))-2/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.4336, size = 304, normalized size = 3.1

$$-\frac{B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+C\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -(B*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + C*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.484015, size = 203, normalized size = 2.07

$$\frac{(3B-2C)dx\cos(dx+c)+(3B-2C)dx+\left(B\cos(dx+c)^2-(B-2C)\cos(dx+c)-4B+4C\right)\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*((3*B - 2*C)*d*x*cos(d*x + c) + (3*B - 2*C)*d*x + (B*cos(d*x + c)^2 - (
B - 2*C)*cos(d*x + c) - 4*B + 4*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.131, size = 166, normalized size = 1.69

$$\frac{\frac{(dx+c)(3B-2C)}{a}-\frac{2\left(B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((d*x + c)*(3*B - 2*C)/a - 2*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x +
1/2*c))/a - 2*(3*B*tan(1/2*d*x + 1/2*c)^3 - 2*C*tan(1/2*d*x + 1/2*c)^3 + B*
tan(1/2*d*x + 1/2*c) - 2*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 +
 1)^2*a))/d
```

3.339 $\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(4B-3C)\sin^3(c+dx)}{3ad}+\frac{(4B-3C)\sin(c+dx)}{ad}-\frac{3(B-C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(B-C)\sin(c+dx)\cos^2(c+d}{d(a\sec(c+dx)+a)}$$

```
[Out] (-3*(B - C)*x)/(2*a) + ((4*B - 3*C)*Sin[c + d*x])/(a*d) - (3*(B - C)*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d) - ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x])) - ((4*B - 3*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.241275, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4020, 3787, 2633, 2635, 8}

$$-\frac{(4B-3C)\sin^3(c+dx)}{3ad}+\frac{(4B-3C)\sin(c+dx)}{ad}-\frac{3(B-C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(B-C)\sin(c+dx)\cos^2(c+d}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
]),x]
```


```
[Out] (-3*(B - C)*x)/(2*a) + ((4*B - 3*C)*Sin[c + d*x])/(a*d) - (3*(B - C)*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d) - ((B - C)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x])) - ((4*B - 3*C)*Sin[c + d*x]^3)/(3*a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^3(c+dx)(B+C\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \cos^3(c+dx)(a(4B-3C)-}{a^2}$$
$$= -\frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(4B-3C)\int \cos^3(c+dx)\,dx}{a}$$
$$= -\frac{3(B-C)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(B-C)\cos^2(c+dx)\sin(c+}{d(a+a\sec(c+dx))}$$
$$= -\frac{3(B-C)x}{2a} + \frac{(4B-3C)\sin(c+dx)}{ad} - \frac{3(B-C)\cos(c+dx)\sin(c}{2ad}$$

Mathematica [B] time = 0.603744, size = 249, normalized size = 2.04

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(B-C)\cos\left(c+\frac{dx}{2}\right)+21B\sin\left(c+\frac{dx}{2}\right)+18B\sin\left(c+\frac{3dx}{2}\right)+18B\sin\left(2c+\frac{3dx}{2}\right)-2B\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(B - C)*d*x*Cos[(d*x)/2] - 36*(B - C)*d*x*C
os[c + (d*x)/2] + 69*B*Sin[(d*x)/2] - 60*C*Sin[(d*x)/2] + 21*B*Sin[c + (d*x
)/2] - 12*C*Sin[c + (d*x)/2] + 18*B*Sin[c + (3*d*x)/2] - 9*C*Sin[c + (3*d*x
)/2] + 18*B*Sin[2*c + (3*d*x)/2] - 9*C*Sin[2*c + (3*d*x)/2] - 2*B*Sin[2*c +
 (5*d*x)/2] + 3*C*Sin[2*c + (5*d*x)/2] - 2*B*Sin[3*c + (5*d*x)/2] + 3*C*Sin
[3*c + (5*d*x)/2] + B*Sin[3*c + (7*d*x)/2] + B*Sin[4*c + (7*d*x)/2]))/(24*a
*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.096, size = 281, normalized size = 2.3

$$\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{C\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}}{ad\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{3}}+5\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}B}{ad\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{3}}-4\,\frac{C\,(\mathrm{ta}}{ad\left(1+(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*C+5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)^5*B-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*C+16/
3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B-1/a/d/(1+tan(1/2*d*
x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)+3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1
/2*d*x+1/2*c)-3/a/d*B*arctan(tan(1/2*d*x+1/2*c))+3/a/d*arctan(tan(1/2*d*x+1
/2*c))*C
```

Maxima [B] time = 1.43261, size = 419, normalized size = 3.43

$$\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3C\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\operatorname{arcta}}{}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/3*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*C*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.493189, size = 243, normalized size = 1.99

$$-\frac{9\,(B-C)dx\cos\left(dx+c\right)+9\,(B-C)dx-\left(2\,B\cos\left(dx+c\right)^3-(B-3\,C)\cos\left(dx+c\right)^2+(7\,B-3\,C)\cos\left(dx+c\right)+16\,B\right.}{6\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/6*(9*(B - C)*d*x*cos(d*x + c) + 9*(B - C)*d*x - (2*B*cos(d*x + c)^3 - (B
 - 3*C)*cos(d*x + c)^2 + (7*B - 3*C)*cos(d*x + c) + 16*B - 12*C)*sin(d*x +
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15318, size = 204, normalized size = 1.67

$$-\frac{\frac{9\,(dx+c)(B-C)}{a}-\frac{6\left(B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+16\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,B\,\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*(B - C)/a - 6*(B*tan(1/2*d*x + 1/2*c) - C*tan(1/2*d*x + 1
/2*c))/a - 2*(15*B*tan(1/2*d*x + 1/2*c)^5 - 9*C*tan(1/2*d*x + 1/2*c)^5 + 16
*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^3 + 9*B*tan(1/2*d*x +
 1/2*c) - 3*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.340 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=156

$$\frac{2(5B-8C)\tan(c+dx)}{3a^2d}-\frac{(4B-7C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(5B-8C)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(4B-7C)\tan(c+}{2a^2d}$$

```
[Out] -((4*B - 7*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (2*(5*B - 8*C)*Tan[c + d*x
])/(3*a^2*d) - ((4*B - 7*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((5*B -
8*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((B - C)*S
ec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.38068, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4019, 3787, 3767, 8, 3768, 3770}

$$\frac{2(5B-8C)\tan(c+dx)}{3a^2d}-\frac{(4B-7C)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(5B-8C)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(4B-7C)\tan(c+}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^2,x]
```

```
[Out] -((4*B - 7*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (2*(5*B - 8*C)*Tan[c + d*x
])/(3*a^2*d) - ((4*B - 7*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((5*B -
8*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((B - C)*S
ec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)(3a(B-C)-a(2B-5C)\sec(c}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{(5B-8C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(B-C)\sec^3(c+dx)\tan(c+}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(5B-8C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(B-C)\sec^3(c+dx)\tan(c+}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(4B-7C)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{(5B-8C)\sec^2(c+dx)\tan(}{3a^2d(1+\sec(c+dx)}$$

$$= -\frac{(4B-7C)\tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{2(5B-8C)\tan(c+dx)}{3a^2d} - \frac{(4B}{}$$

Mathematica [B] time = 1.48753, size = 379, normalized size = 2.43

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-8(5B-8C)\tan^3\left(\frac{1}{2}(c+dx)\right)+(26B-44C)\tan\left(\frac{1}{2}(c+dx)\right)-64(B-C)\sin^8\left(\frac{1}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(3*(4*B - 7*C)*(Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 8*(B + 5*C)*
Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 64*(B - C)*Csc[c + d*x]^5*Sin[(c + d*x)
/2]^8 - 128*C*Csc[c + d*x]^7*Sin[(c + d*x)/2]^12 + (26*B - 44*C)*Tan[(c + d
*x)/2] - 6*(4*B - 7*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])*Tan[(c + d*x)/2]^2 - 8*(5*B - 8*C)*Tan[(c
 + d*x)/2]^3 + 3*(4*B - 7*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Tan[(c + d*x)/2]^4 + (14*B - 20*C +
```

```
B*Sec[(c + d*x)/2]^2)*Tan[(c + d*x)/2]^5))/(6*a^2*d)
```

Maple [B] time = 0.066, size = 294, normalized size = 1.9

$$\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*B*tan(1/2*d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2/(tan(1/2*d*x+1
/2*c)+1)*B+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-2/d/a^2*ln(tan(1/2*d*x+1/2*c)
+1)*B+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^2*C/(tan(1/2*d*x+1/2*c)+
1)^2-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C+2/
d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d
/a^2*C/(tan(1/2*d*x+1/2*c)-1)^2
```

Maxima [B] time = 0.963206, size = 454, normalized size = 2.91

$$-\frac{C\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(d}{(\cos(dx}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.518364, size = 566, normalized size = 3.63

$$-\frac{3\left((4B-7C)\cos(dx+c)^4+2(4B-7C)\cos(dx+c)^3+(4B-7C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((4B-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/12*(3*((4*B - 7*C)*cos(d*x + c)^4 + 2*(4*B - 7*C)*cos(d*x + c)^3 + (4*B
- 7*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((4*B - 7*C)*cos(d*x + c)^
4 + 2*(4*B - 7*C)*cos(d*x + c)^3 + (4*B - 7*C)*cos(d*x + c)^2)*log(-sin(d*x
 + c) + 1) - 2*(4*(5*B - 8*C)*cos(d*x + c)^3 + (28*B - 43*C)*cos(d*x + c)^2
 + 6*(B - C)*cos(d*x + c) + 3*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^
2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,
x)
```

```
[Out] (Integral(B*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(C*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.22009, size = 267, normalized size = 1.71

$$-\frac{\frac{3(4B-7C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{3(4B-7C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{6\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3C\tan\left(\frac{1}{2}d\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/6*(3*(4*B - 7*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(4*B - 7*C)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(2*B*tan(1/2*d*x + 1/2*c)^3 - 5*
C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (B*a^4*tan(1/2*d*x + 1/2*c)^3 -
 C*a^4*tan(1/2*d*x + 1/2*c)^3 + 15*B*a^4*tan(1/2*d*x + 1/2*c) - 21*C*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

3.341 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{(B-4C)\tan(c+dx)}{3a^2d}+\frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-2C)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] ((B - 2*C)*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((B - 4*C)*Tan[c + d*x])/(3*a^2
*d) - ((B - 2*C)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((B - C)*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.335695, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(B-4C)\tan(c+dx)}{3a^2d}+\frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-2C)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^2,x]
```

```
[Out] ((B - 2*C)*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((B - 4*C)*Tan[c + d*x])/(3*a^2
*d) - ((B - 2*C)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((B - C)*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^2(c+dx)(2a(B-C)-a(B-4C)\sec(c+\cdots}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(B-2C)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec\cdots}{\cdots}$$

$$= -\frac{(B-2C)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(B-\cdots}{\cdots}$$

$$= \frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(B-2C)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(B-C)\,\mathrm{s}\cdots}{3d(a\cdots}$$

$$= \frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(B-4C)\tan(c+dx)}{3a^2d} - \frac{(B-2C)\cdots}{a^2d(1+\mathrm{s}\cdots}$$

Mathematica [B] time = 1.04893, size = 245, normalized size = 2.27

$$\cos^2\left(\tfrac{1}{2}(c+dx)\right)\sec(c+dx)\left((4B-7C)\tan^3\left(\tfrac{1}{2}(c+dx)\right)+(13C-4B)\tan\left(\tfrac{1}{2}(c+dx)\right)+16(B-C)\sin^8\left(\tfrac{1}{2}(c+dx)\right)\csc^5\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-3*(B - 2*C)*(Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4*(B - C)*Csc[c
 + d*x]^3*Sin[(c + d*x)/2]^4 + 16*(B - C)*Csc[c + d*x]^5*Sin[(c + d*x)/2]^8
 + (-4*B + 13*C)*Tan[(c + d*x)/2] + 3*(B - 2*C)*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Tan[(c + d*x)/2]
^2 + (4*B - 7*C)*Tan[(c + d*x)/2]^3))/(3*a^2*d)
```

Maple [A] time = 0.053, size = 205, normalized size = 1.9

$$-\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*B*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+1/d/a^2*ln(tan(1/2*d*
x+1/2*c)+1)*B-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^2/(tan(1/2*d*x+1/2*c
)+1)*C-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*
C-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.953393, size = 329, normalized size = 3.05

$$\frac{C\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx}{(\cos(dx+}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(C*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

Fricas [A] time = 0.504261, size = 501, normalized size = 4.64

$$\frac{3\left((B-2C)\cos(dx+c)^3+2(B-2C)\cos(dx+c)^2+(B-2C)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((B-2C)\cos\right.}{6\left(a^2d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*((B - 2*C)*cos(d*x + c)^3 + 2*(B - 2*C)*cos(d*x + c)^2 + (B - 2*C)*c
os(d*x + c))*log(sin(d*x + c) + 1) - 3*((B - 2*C)*cos(d*x + c)^3 + 2*(B - 2
*C)*cos(d*x + c)^2 + (B - 2*C)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*
(2*B - 5*C)*cos(d*x + c)^2 + (5*B - 14*C)*cos(d*x + c) - 3*C)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,
x)
```

```
[Out] (Integral(B*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(C*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.18023, size = 204, normalized size = 1.89

$$\frac{\frac{6(B-2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{6(B-2C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} - \frac{12C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2} - \frac{Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 9Ba^4\tan(}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*(B - 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(B - 2*C)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 12*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)*a^2) - (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x +
 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c) - 15*C*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.342 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{(2B-5C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((2*B - 5*C)*Tan[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((B - C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.233121, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4072, 4008, 3998, 3770, 3794}

$$\frac{(2B-5C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^2,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((2*B - 5*C)*Tan[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((B - C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a(B-C)-3aC\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= -\frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2B-5C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a} + \frac{C\int \sec(c$$
$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2B-5C)\tan(c}{3d\left(a^2+a^2\sec(c\right.}$$

Mathematica [A] time = 0.758428, size = 106, normalized size = 1.34

$$\frac{(B-4C)\tan\left(\frac{1}{2}(c+dx)\right)+4(B-C)\sin^4\left(\frac{1}{2}(c+dx)\right)\csc^3(c+dx)+3C\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(c\right.\right.}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```

```
[Out] (3*C*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + 4*(B - C)*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (B - 4*C)*
Tan[(c + d*x)/2])/(3*a^2*d)
```

Maple [A] time = 0.049, size = 119, normalized size = 1.5

$$\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{3C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{C}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2
*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan(1/2*d*x
+1/2*c)-1)*C+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [A] time = 0.957943, size = 196, normalized size = 2.48

$$-\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right) - \frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] -1/6*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.502423, size = 338, normalized size = 4.28

$$\frac{3\left(C\cos\left(dx+c\right)^{2}+2\,C\cos\left(dx+c\right)+C\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(C\cos\left(dx+c\right)^{2}+2\,C\cos\left(dx+c\right)+C\right)\log\left(-\sin\right.}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/6*(3*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*log(sin(d*x + c) + 1) - 3*
(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*log(-sin(d*x + c) + 1) + 2*((B -
4*C)*cos(d*x + c) + 2*B - 5*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\sec^{2}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{C\sec^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(B*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(C*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.16945, size = 151, normalized size = 1.91

$$\frac{\frac{6\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{6\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}+\frac{Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] 1/6*(6*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*C*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x + 1/2
*c)^3 + 3*B*a^4*tan(1/2*d*x + 1/2*c) - 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.343 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=62

$$\frac{(B+2C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(B-C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] ((B + 2*C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((B - C)*Tan[c + d*
x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.073853, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4052, 12, 3794}

$$\frac{(B+2C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(B-C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((B + 2*C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((B - C)*Tan[c + d*
x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{a(B+2C)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= \frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(B+2C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(B+2C)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.287011, size = 46, normalized size = 0.74

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left((C-B)\sec^2\left(\frac{1}{2}(c+dx)\right)+2(2B+C)\right)}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((2*(2*B + C) + (-B + C)*Sec[(c + d*x)/2]^2)*Tan[(c + d*x)/2])/(6*a^2*d)
```

Maple [A] time = 0.052, size = 60, normalized size = 1.

$$\frac{1}{2\,da^2}\left(-\frac{B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3*B+1/3*C*tan(1/2*d*x+1/2*c)^3+B*tan(1/2
*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.947883, size = 126, normalized size = 2.03

$$\frac{\frac{C\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(C*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + B*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.466989, size = 144, normalized size = 2.32

$$\frac{((2\,B+C)\cos(dx+c)+B+2\,C)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*((2*B + C)*cos(d*x + c) + B + 2*C)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(B*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(C*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.1259, size = 81, normalized size = 1.31

$$-\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(B*tan(1/2*d*x + 1/2*c)^3 - C*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x
 + 1/2*c) - 3*C*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.344 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{(4B-C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(B-C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (B*x)/a^2 - ((4*B - C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((B - C
)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.177613, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4072, 3922, 3919, 3794}

$$-\frac{(4B-C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(B-C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^2,x]
```


```
[Out] (B*x)/a^2 - ((4*B - C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((B - C
)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{B+C\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-3aB+a(B-C)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= \frac{Bx}{a^2} - \frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(4B-C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}$$
$$= \frac{Bx}{a^2} - \frac{(B-C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(4B-C)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

Mathematica [B] time = 0.394118, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(12B\sin\left(c+\frac{dx}{2}\right)-10B\sin\left(c+\frac{3dx}{2}\right)+9Bdx\cos\left(c+\frac{dx}{2}\right)+3Bdx\cos\left(c+\frac{3dx}{2}\right)+3Bdx\cos\left(2c+\right.\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*B*d*x*Cos[(d*x)/2] + 9*B*d*x*Cos[c + (d*x)/
2] + 3*B*d*x*Cos[c + (3*d*x)/2] + 3*B*d*x*Cos[2*c + (3*d*x)/2] - 18*B*Sin[(
d*x)/2] + 6*C*Sin[(d*x)/2] + 12*B*Sin[c + (d*x)/2] - 6*C*Sin[c + (d*x)/2] -
 10*B*Sin[c + (3*d*x)/2] + 4*C*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.079, size = 97, normalized size = 1.4

$$\frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{B\arctan\left(\tan\left(1/2\,dx\right.\right.}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*B*tan(1/2*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*B*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.43539, size = 162, normalized size = 2.31

$$-\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{C\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - C*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.480935, size = 228, normalized size = 3.26

$$\frac{3\,Bdx\cos\left(dx+c\right)^2+6\,Bdx\cos\left(dx+c\right)+3\,Bdx-\left(\left(5\,B-2\,C\right)\cos\left(dx+c\right)+4\,B-C\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/3*(3*B*d*x*cos(d*x + c)^2 + 6*B*d*x*cos(d*x + c) + 3*B*d*x - ((5*B - 2*C)
*cos(d*x + c) + 4*B - C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\cos\left(c+dx\right)\sec\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{C\cos\left(c+dx\right)\sec^2\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1
), x) + Integral(C*cos(c + d*x)*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c
+ d*x) + 1), x))/a**2
```

Giac [A] time = 1.12187, size = 115, normalized size = 1.64

$$\frac{\frac{6\,(dx+c)B}{a^2}+\frac{Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*B/a^2 + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 9*B*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.345 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=98

$$\frac{2(5B-2C)\sin(c+dx)}{3a^2d}-\frac{(2B-C)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2B-C)}{a^2}-\frac{(B-C)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -(((2*B - C)*x)/a^2) + (2*(5*B - 2*C)*Sin[c + d*x])/(3*a^2*d) - ((2*B - C)*
Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((B - C)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.315175, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4020, 3787, 2637, 8}

$$\frac{2(5B-2C)\sin(c+dx)}{3a^2d}-\frac{(2B-C)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2B-C)}{a^2}-\frac{(B-C)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^2,x]
```


```
[Out] -(((2*B - C)*x)/a^2) + (2*(5*B - 2*C)*Sin[c + d*x])/(3*a^2*d) - ((2*B - C)*
Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((B - C)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(B-C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(a(4B-C)-2a(B-C)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{(2B-C)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(B-C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \cos(c+dx}{}$$

$$= -\frac{(2B-C)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(B-C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2(5B-2C)}{}$$

$$= -\frac{(2B-C)x}{a^2} + \frac{2(5B-2C)\sin(c+dx)}{3a^2d} - \frac{(2B-C)\sin(c+dx)}{a^2d(1+\sec(c+dx))} -$$

Mathematica [B] time = 0.609427, size = 245, normalized size = 2.5

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-18dx(2B-C)\cos\left(c+\frac{dx}{2}\right)-30B\sin\left(c+\frac{dx}{2}\right)+41B\sin\left(c+\frac{3dx}{2}\right)+9B\sin\left(2c+\frac{3dx}{2}\right)+3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-18*(2*B - C)*d*x*Cos[(d*x)/2] - 18*(2*B - C)*d
*x*Cos[c + (d*x)/2] - 12*B*d*x*Cos[c + (3*d*x)/2] + 6*C*d*x*Cos[c + (3*d*x)
/2] - 12*B*d*x*Cos[2*c + (3*d*x)/2] + 6*C*d*x*Cos[2*c + (3*d*x)/2] + 66*B*S
in[(d*x)/2] - 36*C*Sin[(d*x)/2] - 30*B*Sin[c + (d*x)/2] + 24*C*Sin[c + (d*x
)/2] + 41*B*Sin[c + (3*d*x)/2] - 20*C*Sin[c + (3*d*x)/2] + 9*B*Sin[2*c + (3
*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] + 3*B*Sin[3*c + (5*d*x)/2]))/(12*a^2*d*
(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.086, size = 149, normalized size = 1.5

$$-\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{B\tan(1/2}{da^2\left(1+(\tan(1/\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*B*tan(1/2*d*x
+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*B*arctan(tan(1/2*d*x+1/2*c))+2/d/a
^2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.42961, size = 258, normalized size = 2.63

$$\frac{B\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-C\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 C*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 0.521093, size = 296, normalized size = 3.02

$$-\frac{3\,(2\,B-C)dx\cos\left(dx+c\right)^2+6\,(2\,B-C)dx\cos\left(dx+c\right)+3\,(2\,B-C)dx-\left(3\,B\cos\left(dx+c\right)^2+(14\,B-5\,C)\cos\left(dx+c\right)\right.}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/3*(3*(2*B - C)*d*x*cos(d*x + c)^2 + 6*(2*B - C)*d*x*cos(d*x + c) + 3*(2*
B - C)*d*x - (3*B*cos(d*x + c)^2 + (14*B - 5*C)*cos(d*x + c) + 10*B - 4*C)*
sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15623, size = 163, normalized size = 1.66

$$-\frac{\frac{6\,(dx+c)(2\,B-C)}{a^2}-\frac{12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*(2*B - C)/a^2 - 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
 1/2*c)^2 + 1)*a^2) + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d*x + 1
/2*c)^3 - 15*B*a^4*tan(1/2*d*x + 1/2*c) + 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

3.346 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=143

$$-\frac{2(8B-5C)\sin(c+dx)}{3a^2d}+\frac{(7B-4C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8B-5C)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7B-4C)}{2a^2}-\frac{(B-\cdots}{3\cdots}$$

```
[Out] ((7*B - 4*C)*x)/(2*a^2) - (2*(8*B - 5*C)*Sin[c + d*x])/(3*a^2*d) + ((7*B -
4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*B - 5*C)*Cos[c + d*x]*Sin[c
 + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((B - C)*Cos[c + d*x]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.377956, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4020, 3787, 2635, 8, 2637}

$$-\frac{2(8B-5C)\sin(c+dx)}{3a^2d}+\frac{(7B-4C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8B-5C)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7B-4C)}{2a^2}-\frac{(B-\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^2,x]
```


```
[Out] ((7*B - 4*C)*x)/(2*a^2) - (2*(8*B - 5*C)*Sin[c + d*x])/(3*a^2*d) + ((7*B -
4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*B - 5*C)*Cos[c + d*x]*Sin[c
 + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((B - C)*Cos[c + d*x]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(a(5B-2C)-3a(B-C)}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(8B-5C)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(B-C)\cos(c+dx)\sin(c}{3d(a+a\sec(c+dx)}$$

$$= -\frac{(8B-5C)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(B-C)\cos(c+dx)\sin(c}{3d(a+a\sec(c+dx)}$$

$$= -\frac{2(8B-5C)\sin(c+dx)}{3a^2d} + \frac{(7B-4C)\cos(c+dx)\sin(c+dx)}{2a^2d}$$

$$= \frac{(7B-4C)x}{2a^2} - \frac{2(8B-5C)\sin(c+dx)}{3a^2d} + \frac{(7B-4C)\cos(c+dx)}{2a^2d}$$

Mathematica [B] time = 0.754213, size = 315, normalized size = 2.2

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(7B-4C)\cos\left(c+\frac{dx}{2}\right)+147B\sin\left(c+\frac{dx}{2}\right)-239B\sin\left(c+\frac{3dx}{2}\right)-63B\sin\left(2c+\frac{3dx}{2}\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*B - 4*C)*d*x*Cos[(d*x)/2] + 36*(7*B - 4*C
)*d*x*Cos[c + (d*x)/2] + 84*B*d*x*Cos[c + (3*d*x)/2] - 48*C*d*x*Cos[c + (3*
d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] - 48*C*d*x*Cos[2*c + (3*d*x)/2] - 3
81*B*Sin[(d*x)/2] + 264*C*Sin[(d*x)/2] + 147*B*Sin[c + (d*x)/2] - 120*C*Sin
[c + (d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 164*C*Sin[c + (3*d*x)/2] - 63*B*
Sin[2*c + (3*d*x)/2] + 36*C*Sin[2*c + (3*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2
] + 12*C*Sin[2*c + (5*d*x)/2] - 15*B*Sin[3*c + (5*d*x)/2] + 12*C*Sin[3*c +
(5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2]))/(48*a^2*
d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.091, size = 252, normalized size = 1.8

$$\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 5\frac{(\tan(1/2dx+}{da^2\left(1+(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*B*tan(1/2*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*C*ta
n(1/2*d*x+1/2*c)^3-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+
2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*C*tan(1/2*d*x+1/2*c)+7/d/a^2*B*arctan(ta
n(1/2*d*x+1/2*c))-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.43435, size = 382, normalized size = 2.67

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-C\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - C*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

Fricas [A] time = 0.496026, size = 342, normalized size = 2.39

$$\frac{3(7B-4C)dx\cos(dx+c)^2+6(7B-4C)dx\cos(dx+c)+3(7B-4C)dx+\left(3B\cos(dx+c)^3-6(B-C)\cos(dx+c)^{\cdots}\right.}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(7*B - 4*C)*d*x*cos(d*x + c)^2 + 6*(7*B - 4*C)*d*x*cos(d*x + c) + 3*
(7*B - 4*C)*d*x + (3*B*cos(d*x + c)^3 - 6*(B - C)*cos(d*x + c)^2 - (43*B -
28*C)*cos(d*x + c) - 32*B + 20*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13709, size = 221, normalized size = 1.55

$$\frac{\frac{3\,(dx+c)(7\,B-4\,C)}{a^2}-\frac{6\left(5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*B - 4*C)/a^2 - 6*(5*B*tan(1/2*d*x + 1/2*c)^3 - 2*C*tan(
1/2*d*x + 1/2*c)^3 + 3*B*tan(1/2*d*x + 1/2*c) - 2*C*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4
*tan(1/2*d*x + 1/2*c)^3 - 21*B*a^4*tan(1/2*d*x + 1/2*c) + 15*C*a^4*tan(1/2*
d*x + 1/2*c))/a^6)/d
```

3.347 $\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{4(3B-2C)\sin^3(c+dx)}{3a^2d}+\frac{4(3B-2C)\sin(c+dx)}{a^2d}-\frac{(10B-7C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10B-7C)\sin(c+dx)\cos^2(}{3a^2d(\sec(c+dx)+1)}$$

```
[Out] -((10*B - 7*C)*x)/(2*a^2) + (4*(3*B - 2*C)*Sin[c + d*x])/(a^2*d) - ((10*B -
 7*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*B - 7*C)*Cos[c + d*x]^2*S
in[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((B - C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(3*B - 2*C)*Sin[c + d*x]^3)/(3*a^2
*d)
```

Rubi [A] time = 0.401685, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4020, 3787, 2633, 2635, 8}

$$-\frac{4(3B-2C)\sin^3(c+dx)}{3a^2d}+\frac{4(3B-2C)\sin(c+dx)}{a^2d}-\frac{(10B-7C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10B-7C)\sin(c+dx)\cos^2(}{3a^2d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^2,x]
```


```
[Out] -((10*B - 7*C)*x)/(2*a^2) + (4*(3*B - 2*C)*Sin[c + d*x])/(a^2*d) - ((10*B -
 7*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*B - 7*C)*Cos[c + d*x]^2*S
in[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((B - C)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(3*B - 2*C)*Sin[c + d*x]^3)/(3*a^2
*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^3(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(B-C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(3a(2B-C)-4a(B-C}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(10B-7C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(B-C)\cos^2(c+dx)\text{si}}{3d(a+a\sec(c+}$$

$$= -\frac{(10B-7C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(B-C)\cos^2(c+dx)\text{si}}{3d(a+a\sec(c+}$$

$$= -\frac{(10B-7C)\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{(10B-7C)\cos^2(c+dx}{3a^2d(1+\sec(c+}$$

$$= -\frac{(10B-7C)x}{2a^2} + \frac{4(3B-2C)\sin(c+dx)}{a^2d} - \frac{(10B-7C)\cos(c+}{2a^2d}$$

Mathematica [B] time = 0.708838, size = 369, normalized size = 2.17

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(10B-7C)\cos\left(c+\frac{dx}{2}\right)-156B\sin\left(c+\frac{dx}{2}\right)+342B\sin\left(c+\frac{3dx}{2}\right)+118B\sin\left(2c+\frac{3dx}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(10*B - 7*C)*d*x*Cos[(d*x)/2] - 36*(10*B -
7*C)*d*x*Cos[c + (d*x)/2] - 120*B*d*x*Cos[c + (3*d*x)/2] + 84*C*d*x*Cos[c +
 (3*d*x)/2] - 120*B*d*x*Cos[2*c + (3*d*x)/2] + 84*C*d*x*Cos[2*c + (3*d*x)/2
] + 516*B*Sin[(d*x)/2] - 381*C*Sin[(d*x)/2] - 156*B*Sin[c + (d*x)/2] + 147*
C*Sin[c + (d*x)/2] + 342*B*Sin[c + (3*d*x)/2] - 239*C*Sin[c + (3*d*x)/2] +
118*B*Sin[2*c + (3*d*x)/2] - 63*C*Sin[2*c + (3*d*x)/2] + 30*B*Sin[2*c + (5*
d*x)/2] - 15*C*Sin[2*c + (5*d*x)/2] + 30*B*Sin[3*c + (5*d*x)/2] - 15*C*Sin[
3*c + (5*d*x)/2] - 3*B*Sin[3*c + (7*d*x)/2] + 3*C*Sin[3*c + (7*d*x)/2] - 3*
B*Sin[4*c + (7*d*x)/2] + 3*C*Sin[4*c + (7*d*x)/2] + B*Sin[4*c + (9*d*x)/2]
+ B*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 0.089, size = 322, normalized size = 1.9

$$-\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+10\frac{(\tan(1/2dx+}{da^2\left(1+(\tan(1/2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3+9/2/d/a^
2*B*tan(1/2*d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)+10/d/a^2/(1+tan(1/2*d
*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B-5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^5*C+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*
c)^3*B-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)^3+6/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)-3/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*C*tan(1/2*d*x+1/2*c)-10/d/a^2*B*arctan(tan(1/2*d*x+1/2*c))+7/d/a^2*ar
ctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.45624, size = 502, normalized size = 2.95

$$\frac{B\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-C\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 0.502698, size = 389, normalized size = 2.29

$$-\frac{3(10B-7C)dx\cos(dx+c)^2+6(10B-7C)dx\cos(dx+c)+3(10B-7C)dx-\left(2B\cos(dx+c)^4-(2B-3C)\cos(d\right.}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(10*B - 7*C)*d*x*cos(d*x + c)^2 + 6*(10*B - 7*C)*d*x*cos(d*x + c) +
 3*(10*B - 7*C)*d*x - (2*B*cos(d*x + c)^4 - (2*B - 3*C)*cos(d*x + c)^3 + 6*
(2*B - C)*cos(d*x + c)^2 + (66*B - 43*C)*cos(d*x + c) + 48*B - 32*C)*sin(d*
x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14133, size = 259, normalized size = 1.52

$$-\frac{\frac{3\,(dx+c)(10\,B-7\,C)}{a^2}-\frac{2\left(30\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(10*B - 7*C)/a^2 - 2*(30*B*tan(1/2*d*x + 1/2*c)^5 - 15*C*
tan(1/2*d*x + 1/2*c)^5 + 40*B*tan(1/2*d*x + 1/2*c)^3 - 24*C*tan(1/2*d*x + 1
/2*c)^3 + 18*B*tan(1/2*d*x + 1/2*c) - 9*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^2) + (B*a^4*tan(1/2*d*x + 1/2*c)^3 - C*a^4*tan(1/2*d
*x + 1/2*c)^3 - 27*B*a^4*tan(1/2*d*x + 1/2*c) + 21*C*a^4*tan(1/2*d*x + 1/2*
c))/a^6)/d
```

3.348 $\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{8(9B-19C)\tan(c+dx)}{15a^3d}-\frac{(6B-13C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{4(9B-19C)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(6B-13C)\tan(c}{2a}$$

```
[Out] -((6*B - 13*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (8*(9*B - 19*C)*Tan[c + d
*x])/(15*a^3*d) - ((6*B - 13*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) + ((B
- C)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*B - 11
*C)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (4*(9*B
- 19*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.552848, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4019, 3787, 3767, 8, 3768, 3770}

$$\frac{8(9B-19C)\tan(c+dx)}{15a^3d}-\frac{(6B-13C)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{4(9B-19C)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(6B-13C)\tan(c}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^3,x]
```

```
[Out] -((6*B - 13*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) + (8*(9*B - 19*C)*Tan[c + d
*x])/(15*a^3*d) - ((6*B - 13*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) + ((B
- C)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*B - 11
*C)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (4*(9*B
- 19*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec^5(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)(4a(B-C)-a(2B-7C)\,\mathrm{s}}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6B-11C)\sec^3(c+dx)\tan}{15ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6B-11C)\sec^3(c+dx)\tan}{15ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6B-11C)\sec^3(c+dx)\tan}{15ad(a+a\sec(c+dx}$$

$$= -\frac{(6B-13C)\sec(c+dx)\tan(c+dx)}{2a^3d} + \frac{(B-C)\sec^4(c+dx)\,\mathrm{ta}}{5d(a+a\sec(c+d}$$

$$= -\frac{(6B-13C)\tanh^{-1}(\sin(c+dx))}{2a^3d} + \frac{8(9B-19C)\tan(c+dx)}{15a^3d} -$$

Mathematica [B] time = 1.97206, size = 428, normalized size = 2.12

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-64(9B-19C)\tan^3\left(\frac{1}{2}(c+dx)\right)+4(87B-197C)\tan\left(\frac{1}{2}(c+dx)\right)+16(12B+13C)\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(30*(6*B - 13*C)*(Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*(12*B +
 13*C)*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 4*(87*B - 197*C)*Tan[(c + d*x)/2
] + (-21*B + 31*C + (24*B - 34*C)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c +
 d*x)/2] - 60*(6*B - 13*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Tan[(c + d*x)/2]^2 - 64*(9*B - 19*C)*
Tan[(c + d*x)/2]^3 - (-6*B + 11*C + (12*B - 17*C)*Cos[c + d*x])*Sec[(c + d*
x)/2]^4*Tan[(c + d*x)/2]^3 + 30*(6*B - 13*C)*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Tan[(c + d*x)/2]^4
+ (228*B - 428*C + 3*(B - C)*Sec[(c + d*x)/2]^4)*Tan[(c + d*x)/2]^5))/(60*a
^3*d)
```

Maple [A] time = 0.069, size = 334, normalized size = 1.7

$$\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{2\,C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17\,B}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*B-2/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*B*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*C*tan(1/2*d*x+1/2*c)+7/2/d/a^3/(tan(1/2*d*x+1/2*
c)+1)*C-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B
+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^3*C/(tan(1/2*d*x+1/2*c)+1)^2
+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C+7
/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)*C-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^
3*C/(tan(1/2*d*x+1/2*c)-1)^2
```

Maxima [A] time = 0.976019, size = 509, normalized size = 2.52

$$-\frac{C\left(\frac{60\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-3\,B\left(\frac{}{\left(a^3\right.}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(C*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 0.525794, size = 757, normalized size = 3.75

$$-\frac{15\left((6B-13C)\cos(dx+c)^5+3(6B-13C)\cos(dx+c)^4+3(6B-13C)\cos(dx+c)^3+(6B-13C)\cos(dx+c)^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/60*(15*((6*B - 13*C)*cos(d*x + c)^5 + 3*(6*B - 13*C)*cos(d*x + c)^4 + 3*
(6*B - 13*C)*cos(d*x + c)^3 + (6*B - 13*C)*cos(d*x + c)^2)*log(sin(d*x + c)
 + 1) - 15*((6*B - 13*C)*cos(d*x + c)^5 + 3*(6*B - 13*C)*cos(d*x + c)^4 + 3
*(6*B - 13*C)*cos(d*x + c)^3 + (6*B - 13*C)*cos(d*x + c)^2)*log(-sin(d*x +
c) + 1) - 2*(16*(9*B - 19*C)*cos(d*x + c)^4 + 3*(114*B - 239*C)*cos(d*x + c
)^3 + (234*B - 479*C)*cos(d*x + c)^2 + 15*(2*B - 3*C)*cos(d*x + c) + 15*C)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(
d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,
x)
```

```
[Out] (Integral(B*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.19174, size = 315, normalized size = 1.56

$$-\frac{\frac{30(6B-13C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{30(6B-13C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/60*(30*(6*B - 13*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(6*B - 1
3*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(2*B*tan(1/2*d*x + 1/2*c)^
3 - 7*C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) + 5*C*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*B*a^12*tan(1/2*d*x + 1
/2*c)^5 - 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^12*tan(1/2*d*x + 1/2*c)^
3 - 40*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*B*a^12*tan(1/2*d*x + 1/2*c) - 46
5*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.349 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=156

$$-\frac{(7B-27C)\tan(c+dx)}{15a^3d}+\frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(B-3C)\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(B-C)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+$$

```
[Out] ((B - 3*C)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((7*B - 27*C)*Tan[c + d*x])/(15
*a^3*d) + ((B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3
) + ((4*B - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^
2) - ((B - 3*C)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.498398, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4019, 4008, 3787, 3770, 3767, 8}

$$-\frac{(7B-27C)\tan(c+dx)}{15a^3d}+\frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(B-3C)\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(B-C)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^3,x]
```


```
[Out] ((B - 3*C)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((7*B - 27*C)*Tan[c + d*x])/(15
*a^3*d) + ((B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3
) + ((4*B - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^
2) - ((B - 3*C)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
```

```
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)(3a(B-C)-a(B-6C)\mathrm{se}}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4B-9C)\sec^2(c+dx)\tan}{15ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4B-9C)\sec^2(c+dx)\tan}{15ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4B-9C)\sec^2(c+dx)\tan}{15ad(a+a\sec(c+dx}$$

$$= \frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(B-C)\sec^3(c+dx)\tan(c+d}{5d(a+a\sec(c+dx))^3}$$

$$= \frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(7B-27C)\tan(c+dx)}{15a^3d} + \frac{(B-}{}$$

Mathematica [A] time = 1.64531, size = 294, normalized size = 1.88

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left((22B-57C)\tan^3\left(\frac{1}{2}(c+dx)\right)+(87C-22B)\tan\left(\frac{1}{2}(c+dx)\right)+96(B-C)\sin^{10}\left(\frac{1}{2}(c+a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-15*(B - 3*C)*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4*(7*B - 12*C)
*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 96*(B - C)*Csc[c + d*x]^7*Sin[(c + d*x
)/2]^10 + (-22*B + 87*C)*Tan[(c + d*x)/2] - ((-4*B + 9*C + (7*B - 12*C)*Cos
[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/4 + 15*(B - 3*C)*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
*Tan[(c + d*x)/2]^2 + (22*B - 57*C)*Tan[(c + d*x)/2]^3))/(15*a^3*d)
```

Maple [A] time = 0.057, size = 245, normalized size = 1.6

$$-\frac{B}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{3\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{2\,d a^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,B}{4\,d a^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*B+1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*B*tan
(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+1/d/a^3*ln(tan(1/2*d*x+1/2*
c)+1)*B-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*C
-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*C-1/d/
a^3/(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [A] time = 0.96901, size = 386, normalized size = 2.47

$$\frac{3\,C\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-B\left(\frac{105\sin(}{\cos(dx+}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/60*(3*C*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 0.521296, size = 668, normalized size = 4.28

$$15\left((B-3\,C)\cos\left(dx+c\right)^4+3\,(B-3\,C)\cos\left(dx+c\right)^3+3\,(B-3\,C)\cos\left(dx+c\right)^2+(B-3\,C)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/30*(15*((B - 3*C)*cos(d*x + c)^4 + 3*(B - 3*C)*cos(d*x + c)^3 + 3*(B - 3*
C)*cos(d*x + c)^2 + (B - 3*C)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((B
- 3*C)*cos(d*x + c)^4 + 3*(B - 3*C)*cos(d*x + c)^3 + 3*(B - 3*C)*cos(d*x +
c)^2 + (B - 3*C)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(11*B - 36*C)*
cos(d*x + c)^3 + 3*(17*B - 57*C)*cos(d*x + c)^2 + (32*B - 117*C)*cos(d*x +
c) - 15*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,
x)
```

```
[Out] (Integral(B*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.21224, size = 251, normalized size = 1.61

$$\frac{\frac{60(B-3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{60(B-3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} - \frac{120C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3} - \frac{3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ca^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 -}{}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(60*(B - 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(B - 3*C)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 120*C*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 - 1)*a^3) - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan
(1/2*d*x + 1/2*c)^5 + 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*C*a^12*tan(1/2*
d*x + 1/2*c)^3 + 105*B*a^12*tan(1/2*d*x + 1/2*c) - 255*C*a^12*tan(1/2*d*x +
 1/2*c))/a^15)/d
```

3.350 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{(4B-29C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(2B-7C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((2*B - 7*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((4*B - 29*C)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.403038, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4019, 4008, 3998, 3770, 3794}

$$\frac{(4B-29C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(2B-7C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((2*B - 7*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((4*B - 29*C)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^2(c+dx)(2a(B-C)+5aC\sec(c+}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2B-7C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} -$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2B-7C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} +$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(}{15}$$

Mathematica [A] time = 0.659561, size = 136, normalized size = 1.09

$$\frac{2(B-11C)\tan\left(\frac{1}{2}(c+dx)\right)+24(B-C)\sin^6\left(\frac{1}{2}(c+dx)\right)\csc^5(c+dx)+4(2B-7C)\sin^4\left(\frac{1}{2}(c+dx)\right)\csc^3(c+dx)+15}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^3,x]
```

```
[Out] (15*C*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]) + 4*(2*B - 7*C)*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 24*(B
 - C)*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + 2*(B - 11*C)*Tan[(c + d*x)/2])/(1
5*a^3*d)
```

Maple [A] time = 0.054, size = 159, normalized size = 1.3

$$\frac{B}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{7\,C}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{4\,da^3}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/6/d/a^3*tan(1/2*d*x+1/2*c)^3*B+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d/a
^3*C*tan(1/2*d*x+1/2*c)^5-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)+1/4/d/a^3*B*tan(1/
2*d*x+1/2*c)-1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-1/d/a^3*ln(tan(1/2*d*x+1/2*c)
-1)*C+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C
```

Maxima [A] time = 0.952644, size = 252, normalized size = 2.02

$$-\frac{C\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(C*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.502417, size = 481, normalized size = 3.85

$$\frac{15\left(C\cos(dx+c)^3+3\,C\cos(dx+c)^2+3\,C\cos(dx+c)+C\right)\log(\sin(dx+c)+1)-15\left(C\cos(dx+c)^3+3\,C\cos(dx+\right.}{30\left(a^3d\cos(dx+c)^3+3\,a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/30*(15*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*log
(sin(d*x + c) + 1) - 15*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*
x + c) + C)*log(-sin(d*x + c) + 1) + 2*(2*(B - 11*C)*cos(d*x + c)^2 + 3*(2*
B - 17*C)*cos(d*x + c) + 7*B - 32*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,
x)
```

```
[Out] (Integral(B*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.1692, size = 198, normalized size = 1.58

$$\frac{\frac{60\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,Ca^{12}\tan(}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(60*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*C*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 + (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan(1/2*
d*x + 1/2*c)^5 + 10*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*C*a^12*tan(1/2*d*x +
 1/2*c)^3 + 15*B*a^12*tan(1/2*d*x + 1/2*c) - 105*C*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

3.351 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(3B+7C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3B-8C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(B-C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -((B - C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*B - 8*C)*Tan[c +
 d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*B + 7*C)*Tan[c + d*x])/(15*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.250912, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4072, 4008, 4000, 3794}

$$\frac{(3B+7C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3B-8C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(B-C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^3,x]
```

```
[Out] -((B - C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((3*B - 8*C)*Tan[c +
 d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*B + 7*C)*Tan[c + d*x])/(15*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a(B-C)-5aC\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3B-8C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3B+7C}{}$$

$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3B-8C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3B+7}{15d\left(a^3+\right.}$$

Mathematica [A] time = 0.171977, size = 70, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(6(3B+2C)\cos(c+dx)+(3B+2C)\cos(2(c+dx))+9B+16C)}{120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```

```
[Out] ((9*B + 16*C + 6*(3*B + 2*C)*Cos[c + d*x] + (3*B + 2*C)*Cos[2*(c + d*x)])*S
ec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(120*a^3*d)
```

Maple [A] time = 0.053, size = 64, normalized size = 0.6

$$\frac{1}{4\,da^3}\left(\frac{-B+C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(-B+C)*tan(1/2*d*x+1/2*c)^5+2/3*C*tan(1/2*d*x+1/2*c)^3+C*tan
(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.959214, size = 155, normalized size = 1.52

$$\frac{\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] 1/60*(C*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.460994, size = 227, normalized size = 2.23

$$\frac{\left((3B+2C)\cos(dx+c)^2+3(3B+2C)\cos(dx+c)+3B+7C\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/15*((3*B + 2*C)*cos(d*x + c)^2 + 3*(3*B + 2*C)*cos(d*x + c) + 3*B + 7*C)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{B\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(B*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(C*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.17823, size = 101, normalized size = 0.99

$$-\frac{3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-15C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] -1/60*(3*B*tan(1/2*d*x + 1/2*c)^5 - 3*C*tan(1/2*d*x + 1/2*c)^5 - 10*C*tan(1
/2*d*x + 1/2*c)^3 - 15*B*tan(1/2*d*x + 1/2*c) - 15*C*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

3.352 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2B+3C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2B+3C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{(B-C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] ((B - C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((2*B + 3*C)*Tan[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((2*B + 3*C)*Tan[c + d*x])/(15*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.109576, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4052, 12, 3796, 3794}

$$\frac{(2B+3C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2B+3C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{(B-C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((B - C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((2*B + 3*C)*Tan[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((2*B + 3*C)*Tan[c + d*x])/(15*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{a(2B+3C)\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2B+3C)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}$$
$$= \frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2B+3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2B+3C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}$$
$$= \frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2B+3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2B+3C)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

Mathematica [A] time = 0.359352, size = 70, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(6(2B+3C)\cos(c+dx)+(7B+3C)\cos(2(c+dx))+11B+9C)}{120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((11*B + 9*C + 6*(2*B + 3*C)*Cos[c + d*x] + (7*B + 3*C)*Cos[2*(c + d*x)])*S
ec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(120*a^3*d)
```

Maple [A] time = 0.056, size = 64, normalized size = 0.6

$$\frac{1}{4\,da^3}\left(\frac{B-C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,B}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(B-C)*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3*B+C*tan(
1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.957598, size = 155, normalized size = 1.52

$$\frac{\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,C\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*C*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.461763, size = 227, normalized size = 2.23

$$\frac{\left((7B+3C)\cos(dx+c)^2+3(2B+3C)\cos(dx+c)+2B+3C\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((7*B + 3*C)*cos(d*x + c)^2 + 3*(2*B + 3*C)*cos(d*x + c) + 2*B + 3*C)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int\frac{C\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(B*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.16376, size = 101, normalized size = 0.99

$$\frac{3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(3*B*tan(1/2*d*x + 1/2*c)^5 - 3*C*tan(1/2*d*x + 1/2*c)^5 - 10*B*tan(1/
2*d*x + 1/2*c)^3 + 15*B*tan(1/2*d*x + 1/2*c) + 15*C*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

3.353 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$-\frac{2(11B-C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Bx}{a^3}-\frac{(7B-2C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(B-C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] (B*x)/a^3 - ((B - C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*B - 2
*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (2*(11*B - C)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.252286, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4072, 3922, 3919, 3794}

$$-\frac{2(11B-C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Bx}{a^3}-\frac{(7B-2C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(B-C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^3,x]
```

```
[Out] (B*x)/a^3 - ((B - C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*B - 2
*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (2*(11*B - C)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{B+C\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5aB+2a(B-C)\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$
$$= -\frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7B-2C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2B-a^2}{a-}}{}$$
$$= \frac{Bx}{a^3} - \frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7B-2C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(2(11}{}$$
$$= \frac{Bx}{a^3} - \frac{(B-C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7B-2C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{2(11}{15d(}$$

Mathematica [B] time = 0.570751, size = 241, normalized size = 2.23

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(270B\sin\left(c+\frac{dx}{2}\right)-230B\sin\left(c+\frac{3dx}{2}\right)+90B\sin\left(2c+\frac{3dx}{2}\right)-64B\sin\left(2c+\frac{5dx}{2}\right)+150Bdx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*B*d*x*Cos[(d*x)/2] + 150*B*d*x*Cos[c + (d
*x)/2] + 75*B*d*x*Cos[c + (3*d*x)/2] + 75*B*d*x*Cos[2*c + (3*d*x)/2] + 15*B
*d*x*Cos[2*c + (5*d*x)/2] + 15*B*d*x*Cos[3*c + (5*d*x)/2] - 370*B*Sin[(d*x)
/2] + 80*C*Sin[(d*x)/2] + 270*B*Sin[c + (d*x)/2] - 60*C*Sin[c + (d*x)/2] -
230*B*Sin[c + (3*d*x)/2] + 40*C*Sin[c + (3*d*x)/2] + 90*B*Sin[2*c + (3*d*x)
/2] - 30*C*Sin[2*c + (3*d*x)/2] - 64*B*Sin[2*c + (5*d*x)/2] + 14*C*Sin[2*c
+ (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.088, size = 137, normalized size = 1.3

$$-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7\,B}{4\,da^3}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*B*tan
(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*B*arctan(tan(1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.45584, size = 216, normalized size = 2.

$$-\frac{B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - C*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3)/d
```

Fricas [A] time = 0.481348, size = 351, normalized size = 3.25

$$\frac{15\,Bdx\cos(dx+c)^3+45\,Bdx\cos(dx+c)^2+45\,Bdx\cos(dx+c)+15\,Bdx-\left((32\,B-7\,C)\cos(dx+c)^2+3\,(17\,B-2\,C)\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/15*(15*B*d*x*cos(d*x + c)^3 + 45*B*d*x*cos(d*x + c)^2 + 45*B*d*x*cos(d*x
+ c) + 15*B*d*x - ((32*B - 7*C)*cos(d*x + c)^2 + 3*(17*B - 2*C)*cos(d*x + c
) + 22*B - 2*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{B\cos(c+dx)\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int\frac{C\cos(c+dx)\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2
+ 3*sec(c + d*x) + 1), x) + Integral(C*cos(c + d*x)*sec(c + d*x)**2/(sec(c
+ d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.15711, size = 163, normalized size = 1.51

$$\frac{\frac{60\,(dx+c)B}{a^3}-\frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,Ca^{12}\tan\left(\frac{1}{2}\,dx\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*B/a^3 - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan(
1/2*d*x + 1/2*c)^5 - 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 10*C*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*B*a^12*tan(1/2*d*x + 1/2*c) - 15*C*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

3.354 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=136

$$\frac{2(36B-11C)\sin(c+dx)}{15a^3d}-\frac{(3B-C)\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(3B-C)}{a^3}-\frac{(9B-4C)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(B-C)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -(((3*B - C)*x)/a^3) + (2*(36*B - 11*C)*Sin[c + d*x])/(15*a^3*d) - ((B - C)
*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*B - 4*C)*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((3*B - C)*Sin[c + d*x])/(d*(a^3 + a^3*Sec[
c + d*x]))
```

Rubi [A] time = 0.44351, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4020, 3787, 2637, 8}

$$\frac{2(36B-11C)\sin(c+dx)}{15a^3d}-\frac{(3B-C)\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(3B-C)}{a^3}-\frac{(9B-4C)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(B-C)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^3,x]
```


```
[Out] -(((3*B - C)*x)/a^3) + (2*(36*B - 11*C)*Sin[c + d*x])/(15*a^3*d) - ((B - C)
*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*B - 4*C)*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((3*B - C)*Sin[c + d*x])/(d*(a^3 + a^3*Sec[
c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^3}dx$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(a(6B-C)-3a(B-C)\sec(c+dx))}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9B-4C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \int \frac{\cos(c+}{}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9B-4C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3B-C}{d\left(a^3+a\right.}$$

$$= -\frac{(B-C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9B-4C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3B-C}{d\left(a^3+a\right.}$$

$$= -\frac{(3B-C)x}{a^3} + \frac{2(36B-11C)\sin(c+dx)}{15a^3d} - \frac{(B-C)\sin(c+dx}{5d(a+a\sec(c+dx}$$

Mathematica [B] time = 1.00983, size = 365, normalized size = 2.68

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-300dx(3B-C)\cos\left(c+\frac{dx}{2}\right)-1125B\sin\left(c+\frac{dx}{2}\right)+1215B\sin\left(c+\frac{3dx}{2}\right)-225B\sin\left(2c+\frac{3d}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-300*(3*B - C)*d*x*Cos[(d*x)/2] - 300*(3*B - C)
*d*x*Cos[c + (d*x)/2] - 450*B*d*x*Cos[c + (3*d*x)/2] + 150*C*d*x*Cos[c + (3
*d*x)/2] - 450*B*d*x*Cos[2*c + (3*d*x)/2] + 150*C*d*x*Cos[2*c + (3*d*x)/2]
- 90*B*d*x*Cos[2*c + (5*d*x)/2] + 30*C*d*x*Cos[2*c + (5*d*x)/2] - 90*B*d*x*
Cos[3*c + (5*d*x)/2] + 30*C*d*x*Cos[3*c + (5*d*x)/2] + 1755*B*Sin[(d*x)/2]
- 740*C*Sin[(d*x)/2] - 1125*B*Sin[c + (d*x)/2] + 540*C*Sin[c + (d*x)/2] + 1
215*B*Sin[c + (3*d*x)/2] - 460*C*Sin[c + (3*d*x)/2] - 225*B*Sin[2*c + (3*d*
x)/2] + 180*C*Sin[2*c + (3*d*x)/2] + 363*B*Sin[2*c + (5*d*x)/2] - 128*C*Sin
[2*c + (5*d*x)/2] + 75*B*Sin[3*c + (5*d*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] +
 15*B*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.109, size = 189, normalized size = 1.4

$$\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{B}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{17\,B}{4\,da^3}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5-1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*B+1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*B*tan
(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*B*tan(1/2*d*x+1/2*c)
/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^3*B*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arct
an(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.44386, size = 312, normalized size = 2.29

$$\frac{3B\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-C\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+3\cdots}{a^3}\cdots\right.}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - C*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.493415, size = 431, normalized size = 3.17

$$-\frac{15(3B-C)dx\cos(dx+c)^3+45(3B-C)dx\cos(dx+c)^2+45(3B-C)dx\cos(dx+c)+15(3B-C)dx-\left(15B\cos(\cdots\right.}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/15*(15*(3*B - C)*d*x*cos(d*x + c)^3 + 45*(3*B - C)*d*x*cos(d*x + c)^2 +
45*(3*B - C)*d*x*cos(d*x + c) + 15*(3*B - C)*d*x - (15*B*cos(d*x + c)^3 + (
117*B - 32*C)*cos(d*x + c)^2 + 3*(57*B - 17*C)*cos(d*x + c) + 72*B - 22*C)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1791, size = 212, normalized size = 1.56

$$-\frac{\frac{60\,(dx+c)(3\,B-C)}{a^3}-\frac{120\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+255}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*(3*B - C)/a^3 - 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 + 1)*a^3) - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^12*tan(1/
2*d*x + 1/2*c)^5 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*C*a^12*tan(1/2*d*x
 + 1/2*c)^3 + 255*B*a^12*tan(1/2*d*x + 1/2*c) - 105*C*a^12*tan(1/2*d*x + 1/
2*c))/a^15)/d
```

3.355 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{8(19B-9C)\sin(c+dx)}{15a^3d}+\frac{(13B-6C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4(19B-9C)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13B-6C)}{2a^3}-$$

```
[Out] ((13*B - 6*C)*x)/(2*a^3) - (8*(19*B - 9*C)*Sin[c + d*x])/(15*a^3*d) + ((13*
B - 6*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((B - C)*Cos[c + d*x]*Sin[c
 + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*B - 6*C)*Cos[c + d*x]*Sin[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (4*(19*B - 9*C)*Cos[c + d*x]*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.545247, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4020, 3787, 2635, 8, 2637}

$$-\frac{8(19B-9C)\sin(c+dx)}{15a^3d}+\frac{(13B-6C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4(19B-9C)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13B-6C)}{2a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^3,x]
```


```
[Out] ((13*B - 6*C)*x)/(2*a^3) - (8*(19*B - 9*C)*Sin[c + d*x])/(15*a^3*d) + ((13*
B - 6*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((B - C)*Cos[c + d*x]*Sin[c
 + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*B - 6*C)*Cos[c + d*x]*Sin[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (4*(19*B - 9*C)*Cos[c + d*x]*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(a(7B-2C)-4a(B-C)}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11B-6C)\cos(c+dx)\sin(}{15ad(a+a\sec(c+dx}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11B-6C)\cos(c+dx)\sin(}{15ad(a+a\sec(c+dx}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11B-6C)\cos(c+dx)\sin(}{15ad(a+a\sec(c+dx}$$

$$= -\frac{8(19B-9C)\sin(c+dx)}{15a^3d} + \frac{(13B-6C)\cos(c+dx)\sin(c+dx}{2a^3d}$$

$$= \frac{(13B-6C)x}{2a^3} - \frac{8(19B-9C)\sin(c+dx)}{15a^3d} + \frac{(13B-6C)\cos(c+}{2a^3d}$$

Mathematica [B] time = 0.718959, size = 435, normalized size = 2.33

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(13B-6C)\cos\left(c+\frac{dx}{2}\right)+7560B\sin\left(c+\frac{dx}{2}\right)-9230B\sin\left(c+\frac{3dx}{2}\right)+930B\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*B - 6*C)*d*x*Cos[(d*x)/2] + 600*(13*B -
 6*C)*d*x*Cos[c + (d*x)/2] + 3900*B*d*x*Cos[c + (3*d*x)/2] - 1800*C*d*x*Cos
[c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (3*d*x)/2] - 1800*C*d*x*Cos[2*c + (3
*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)/2] - 360*C*d*x*Cos[2*c + (5*d*x)/2]
+ 780*B*d*x*Cos[3*c + (5*d*x)/2] - 360*C*d*x*Cos[3*c + (5*d*x)/2] - 12760*B
*Sin[(d*x)/2] + 7020*C*Sin[(d*x)/2] + 7560*B*Sin[c + (d*x)/2] - 4500*C*Sin[
c + (d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] + 4860*C*Sin[c + (3*d*x)/2] + 930*
B*Sin[2*c + (3*d*x)/2] - 900*C*Sin[2*c + (3*d*x)/2] - 2782*B*Sin[2*c + (5*d
```

```
*x)/2] + 1452*C*Sin[2*c + (5*d*x)/2] - 750*B*Sin[3*c + (5*d*x)/2] + 300*C*S
in[3*c + (5*d*x)/2] - 105*B*Sin[3*c + (7*d*x)/2] + 60*C*Sin[3*c + (7*d*x)/2
] - 105*B*Sin[4*c + (7*d*x)/2] + 60*C*Sin[4*c + (7*d*x)/2] + 15*B*Sin[4*c +
 (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.102, size = 292, normalized size = 1.6

$$-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,B}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31\,B}{4\,da^3}\tan\left(\frac{d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/20/d/a^3*C*tan(1/2*d*x+1/2*c)^5+2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*B-1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*B*ta
n(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2
*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*C*tan(1/
2*d*x+1/2*c)^3-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+2/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*C*tan(1/2*d*x+1/2*c)+13/d/a^3*B*arctan(tan(1
/2*d*x+1/2*c))-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [A] time = 1.45823, size = 435, normalized size = 2.33

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3\,C\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)\right.}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*C*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.520928, size = 495, normalized size = 2.65

$$\frac{15\,(13\,B-6\,C)dx\cos\left(dx+c\right)^3+45\,(13\,B-6\,C)dx\cos\left(dx+c\right)^2+45\,(13\,B-6\,C)dx\cos\left(dx+c\right)+15\,(13\,B-6\,C)dx+\left(1\right.}{30\left(a^3d\cos\left(dx+c\right)^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/30*(15*(13*B - 6*C)*d*x*cos(d*x + c)^3 + 45*(13*B - 6*C)*d*x*cos(d*x + c)
^2 + 45*(13*B - 6*C)*d*x*cos(d*x + c) + 15*(13*B - 6*C)*d*x + (15*B*cos(d*x
 + c)^4 - 15*(3*B - 2*C)*cos(d*x + c)^3 - (479*B - 234*C)*cos(d*x + c)^2 -
3*(239*B - 114*C)*cos(d*x + c) - 304*B + 144*C)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15859, size = 270, normalized size = 1.44

$$\frac{\frac{30\,(dx+c)(13\,B-6\,C)}{a^3} - \frac{60\left(7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} - \frac{3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ca^{12}\tan\left(\frac{1}{2}\,dx\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*B - 6*C)/a^3 - 60*(7*B*tan(1/2*d*x + 1/2*c)^3 - 2*C*
tan(1/2*d*x + 1/2*c)^3 + 5*B*tan(1/2*d*x + 1/2*c) - 2*C*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*B*a^12*tan(1/2*d*x + 1/2*c)^5
- 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*C
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*B*a^12*tan(1/2*d*x + 1/2*c) - 255*C*a^12
*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.356 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c-\right.$

Optimal. Leaf size=230

$$\frac{2a(11B+10C)\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{16a(11B+10C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{32(11B+10C)\tan(c+dx)(a\sec(}{1155ad}$$

```
[Out] (32*a*(11*B + 10*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a*
(11*B + 10*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*a*(11*B + 10*C)*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*C*Sec[c + d*x]^5*Tan[c + d*x])/(11*d*Sqrt[a + a*Sec[c + d*x]
]) - (64*(11*B + 10*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3465*d) + (3
2*(11*B + 10*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1155*a*d)
```

Rubi [A] time = 0.485266, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a(11B+10C)\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{16a(11B+10C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{32(11B+10C)\tan(c+dx)(a\sec(}{1155ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (32*a*(11*B + 10*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a*
(11*B + 10*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*a*(11*B + 10*C)*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*C*Sec[c + d*x]^5*Tan[c + d*x])/(11*d*Sqrt[a + a*Sec[c + d*x]
]) - (64*(11*B + 10*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3465*d) + (3
2*(11*B + 10*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1155*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
```

```
1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^5(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec$$

$$= \frac{2aC\sec^5(c+dx)\tan(c+dx)}{11d\sqrt{a+a\sec(c+dx)}} + \frac{1}{11}(11B+10$$

$$= \frac{2a(11B+10C)\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + 2$$

$$= \frac{16a(11B+10C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{16a(11B+10C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{16a(11B+10C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{32a(11B+10C)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}} + \frac{16a(11B+10}{693a}$$

Mathematica [A] time = 5.88604, size = 115, normalized size = 0.5

$$\frac{2a\tan(c+dx)\left(35(11B+10C)\sec^4(c+dx)+40(11B+10C)\sec^3(c+dx)+48(11B+10C)\sec^2(c+dx)+64(11B+1\right.}{3465d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a*(128*(11*B + 10*C) + 64*(11*B + 10*C)*Sec[c + d*x] + 48*(11*B + 10*C)*
Sec[c + d*x]^2 + 40*(11*B + 10*C)*Sec[c + d*x]^3 + 35*(11*B + 10*C)*Sec[c +
 d*x]^4 + 315*C*Sec[c + d*x]^5)*Tan[c + d*x])/(3465*d*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [A] time = 0.403, size = 160, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(1408B(\cos(dx+c))^5+1280C(\cos(dx+c))^5+704B(\cos(dx+c))^4+640C(\cos(dx+c))^4+ \cdots\right.}{3465 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(1408*B*cos(d*x+c)^5+1280*C*cos(d*x+c)^5+704*B*co
s(d*x+c)^4+640*C*cos(d*x+c)^4+528*B*cos(d*x+c)^3+480*C*cos(d*x+c)^3+440*B*c
os(d*x+c)^2+400*C*cos(d*x+c)^2+385*B*cos(d*x+c)+350*C*cos(d*x+c)+315*C)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.506013, size = 373, normalized size = 1.62

$$\frac{2\left(128(11B+10C)\cos(dx+c)^5+64(11B+10C)\cos(dx+c)^4+48(11B+10C)\cos(dx+c)^3+40(11B+10C)\cos\cdots\right.}{3465\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 2/3465*(128*(11*B + 10*C)*cos(d*x + c)^5 + 64*(11*B + 10*C)*cos(d*x + c)^4
+ 48*(11*B + 10*C)*cos(d*x + c)^3 + 40*(11*B + 10*C)*cos(d*x + c)^2 + 35*(1
1*B + 10*C)*cos(d*x + c) + 315*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.62683, size = 424, normalized size = 1.84

$$-\frac{2\left(3465\sqrt{2}Ba^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+3465\sqrt{2}Ca^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(8085\sqrt{2}Ba^{6}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+5775\sqrt{2}Ca^{6}\mathrm{s}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -2/3465*(3465*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 3465*sqrt(2)*C*a^6*sgn(cos(
d*x + c)) - (8085*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 5775*sqrt(2)*C*a^6*sgn(
cos(d*x + c)) - (14322*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 16170*sqrt(2)*C*a^
6*sgn(cos(d*x + c)) - (13266*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 8910*sqrt(2)
*C*a^6*sgn(cos(d*x + c)) - (4741*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 5885*sqr
t(2)*C*a^6*sgn(cos(d*x + c)) - (1177*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 755*
sqrt(2)*C*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^
2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*d)
```

3.357 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=187

$$\frac{2a(9B+8C)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4(9B+8C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{8(9B+8C)\tan(c+dx)\sqrt{a\sec(c}}{315d}$$

```
[Out] (4*a*(9*B + 8*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9*B
+ 8*C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
C*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(9*B + 8
*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (4*(9*B + 8*C)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.418851, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a(9B+8C)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4(9B+8C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{8(9B+8C)\tan(c+dx)\sqrt{a\sec(c}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (4*a*(9*B + 8*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9*B
+ 8*C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
C*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(9*B + 8
*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (4*(9*B + 8*C)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec$$

$$= \frac{2aC\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(9B+8C)$$

$$= \frac{2a(9B+8C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aC}{}$$

$$= \frac{2a(9B+8C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aC}{}$$

$$= \frac{2a(9B+8C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aC}{}$$

$$= \frac{4a(9B+8C)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{2a(9B+8C)\sec}{63d\sqrt{a+}}$$

Mathematica [A] time = 0.536767, size = 98, normalized size = 0.52

$$\frac{2a\tan(c+dx)\left(5(9B+8C)\sec^3(c+dx)+6(9B+8C)\sec^2(c+dx)+8(9B+8C)\sec(c+dx)+16(9B+8C)+35C\sec\right.}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a*(16*(9*B + 8*C) + 8*(9*B + 8*C)*Sec[c + d*x] + 6*(9*B + 8*C)*Sec[c + d
*x]^2 + 5*(9*B + 8*C)*Sec[c + d*x]^3 + 35*C*Sec[c + d*x]^4)*Tan[c + d*x])/(
315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.398, size = 138, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(144B(\cos(dx+c))^4+128C(\cos(dx+c))^4+72B(\cos(dx+c))^3+64C(\cos(dx+c))^3+54B(\right.}{315d(\cos(dx+c))^4\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(144*B*cos(d*x+c)^4+128*C*cos(d*x+c)^4+72*B*cos(d*
x+c)^3+64*C*cos(d*x+c)^3+54*B*cos(d*x+c)^2+48*C*cos(d*x+c)^2+45*B*cos(d*x+c
)+40*C*cos(d*x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/si
n(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.505987, size = 308, normalized size = 1.65

$$\frac{2\left(16(9B+8C)\cos(dx+c)^4+8(9B+8C)\cos(dx+c)^3+6(9B+8C)\cos(dx+c)^2+5(9B+8C)\cos(dx+c)+35\right.}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 2/315*(16*(9*B + 8*C)*cos(d*x + c)^4 + 8*(9*B + 8*C)*cos(d*x + c)^3 + 6*(9*
B + 8*C)*cos(d*x + c)^2 + 5*(9*B + 8*C)*cos(d*x + c) + 35*C)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\sec(c+dx)+1)}\,(B+C\sec(c+dx))\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(B + C*sec(c + d*x))*sec(c + d*x)**4, x
)
```

Giac [A] time = 4.56227, size = 362, normalized size = 1.94

$$\frac{2\left(315\sqrt{2}Ba^5\text{sgn}\left(\cos\left(dx+c\right)\right)+315\sqrt{2}Ca^5\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(630\sqrt{2}Ba^5\text{sgn}\left(\cos\left(dx+c\right)\right)+420\sqrt{2}Ca^5\text{sgn}\left(\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 315*sqrt(2)*C*a^5*sgn(cos(d*x
+ c)) - (630*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 420*sqrt(2)*C*a^5*sgn(cos(d*
x + c)) - (756*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 882*sqrt(2)*C*a^5*sgn(cos(
d*x + c)) - (522*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 324*sqrt(2)*C*a^5*sgn(co
s(d*x + c)) - (81*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 107*sqrt(2)*C*a^5*sgn(c
os(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1
/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.358 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=144

$$\frac{2(7B+6C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{4(7B+6C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7B+6C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (2*a*(7*B + 6*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(7*B + 6*C)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*B + 6*C)*(a + a*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.359372, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4016, 3800, 4001, 3792}

$$\frac{2(7B+6C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{4(7B+6C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7B+6C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (2*a*(7*B + 6*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(7*B + 6*C)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*B + 6*C)*(a + a*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec$$
$$= \frac{2aC\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(7B+6C)$$
$$= \frac{2aC\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2(7B+6C)}{}$$
$$= \frac{2aC\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{4(7B+6C)}{}$$
$$= \frac{2a(7B+6C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sec^3(c+dx}{7d\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.281068, size = 81, normalized size = 0.56

$$\frac{2a\tan(c+dx)\left(3(7B+6C)\sec^2(c+dx)+4(7B+6C)\sec(c+dx)+8(7B+6C)+15C\sec^3(c+dx)\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a*(8*(7*B + 6*C) + 4*(7*B + 6*C)*Sec[c + d*x] + 3*(7*B + 6*C)*Sec[c + d*
x]^2 + 15*C*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])]
)
```

Maple [A] time = 0.317, size = 116, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(56B(\cos(dx+c))^3+48C(\cos(dx+c))^3+28B(\cos(dx+c))^2+24C(\cos(dx+c))^2+21B\right.}{105d(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(56*B*cos(d*x+c)^3+48*C*cos(d*x+c)^3+28*B*cos(d*x+
c)^2+24*C*cos(d*x+c)^2+21*B*cos(d*x+c)+18*C*cos(d*x+c)+15*C)*(a*(cos(d*x+c)
```

```
+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.49372, size = 265, normalized size = 1.84

$$\frac{2\left(8\left(7B+6C\right)\cos\left(dx+c\right)^{3}+4\left(7B+6C\right)\cos\left(dx+c\right)^{2}+3\left(7B+6C\right)\cos\left(dx+c\right)+15C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 2/105*(8*(7*B + 6*C)*cos(d*x + c)^3 + 4*(7*B + 6*C)*cos(d*x + c)^2 + 3*(7*B
 + 6*C)*cos(d*x + c) + 15*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(B+C\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(B + C*sec(c + d*x))*sec(c + d*x)**3, x
)
```

Giac [A] time = 4.45322, size = 300, normalized size = 2.08

$$-\frac{2\left(105\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(175\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\left(\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -2/105*(105*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^4*sgn(cos(d*x
 + c)) - (175*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^4*sgn(cos(d
*x + c)) - (119*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 147*sqrt(2)*C*a^4*sgn(cos
(d*x + c)) - (49*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 27*sqrt(2)*C*a^4*sgn(cos
(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.359 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=101

$$\frac{2(5B-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a(5B+7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] (2*a*(5*B + 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(5*B -
2*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.276975, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4072, 4010, 4001, 3792}

$$\frac{2(5B-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a(5B+7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (2*a*(5*B + 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(5*B -
2*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
```

```
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec($$

$$= \frac{2C(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{2\int \text{se}}{}$$

$$= \frac{2(5B-2C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} +$$

$$= \frac{2a(5B+7C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2(5B-2C)\sqrt{a+}}{}$$

Mathematica [A] time = 0.305914, size = 80, normalized size = 0.79

$$\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}((5B+4C)\cos(c+dx)+(5B+4C)\cos(2(c+dx))+5B+7C)}{15d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + 
 d*x]^2),x]
```

```
[Out] (2*(5*B + 7*C + (5*B + 4*C)*Cos[c + d*x] + (5*B + 4*C)*Cos[2*(c + d*x)])*Se
c[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(15*d*(1 + Cos[c + d*x]
))
```

Maple [A] time = 0.285, size = 94, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))\left(10B(\cos(dx+c))^2+8C(\cos(dx+c))^2+5B\cos(dx+c)+4C\cos(dx+c)+3C\right)}{15d(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos}{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(10*B*cos(d*x+c)^2+8*C*cos(d*x+c)^2+5*B*cos(d*x+c)+
4*C*cos(d*x+c)+3*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*
x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.492669, size = 217, normalized size = 2.15

$$\frac{2\left(2\,(5\,B+4\,C)\cos\left(dx+c\right)^{2}+(5\,B+4\,C)\cos\left(dx+c\right)+3\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/15*(2*(5*B + 4*C)*cos(d*x + c)^2 + (5*B + 4*C)*cos(d*x + c) + 3*C)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*
x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(B+C\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(B + C*sec(c + d*x))*sec(c + d*x)**2, x
)
```

Giac [A] time = 4.41245, size = 238, normalized size = 2.36

$$\frac{2\left(15\sqrt{2}Ba^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\sqrt{2}Ca^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(20\sqrt{2}Ba^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+10\sqrt{2}Ca^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right.\right.\right.}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(2)*B*a^3*sgn(cos(d*x + c)) + 15*sqrt(2)*C*a^3*sgn(cos(d*x + c
)) - (20*sqrt(2)*B*a^3*sgn(cos(d*x + c)) + 10*sqrt(2)*C*a^3*sgn(cos(d*x + c
)) - (5*sqrt(2)*B*a^3*sgn(cos(d*x + c)) + 7*sqrt(2)*C*a^3*sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*
tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.360 $\int \sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=62

$$\frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*a*(3*B + C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0842717, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4054, 12, 3792}

$$\frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(3*B + C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2\int \frac{1}{2}a(3B+C)\sec\\ &= \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}(3B+C)\int \sec(c\\ &= \frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+\end{aligned}$$

Mathematica [A] time = 0.159898, size = 43, normalized size = 0.69

$$\frac{2a\tan(c+dx)(3B+C\sec(c+dx)+2C)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(3*B + 2*C + C*Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [A] time = 0.275, size = 70, normalized size = 1.1

$$-\frac{(-2+2\cos(dx+c))(3B\cos(dx+c)+2C\cos(dx+c)+C)}{3d\sin(dx+c)\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(3*B*cos(d*x+c)+2*C*cos(d*x+c)+C)*(a*(cos(d*x+c)+1)/
cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.496403, size = 169, normalized size = 2.73

$$\frac{2((3B+2C)\cos(dx+c)+C)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/3*((3*B + 2*C)*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(B+C\sec\left(c+dx\right)\right)\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(B + C*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 4.31345, size = 174, normalized size = 2.81

$$-\frac{2\left(3\sqrt{2}Ba^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+3\sqrt{2}Ca^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(3\sqrt{2}Ba^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\sqrt{2}Ca^{2}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)}{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -2/3*(3*sqrt(2)*B*a^2*sgn(cos(d*x + c)) + 3*sqrt(2)*C*a^2*sgn(cos(d*x + c))
 - (3*sqrt(2)*B*a^2*sgn(cos(d*x + c)) + sqrt(2)*C*a^2*sgn(cos(d*x + c)))*ta
n(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.361 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*Sqrt[a]*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.171062, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 3915, 3774, 203, 3792}

$$\frac{2\sqrt{a}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
```

```
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sqrt{a+a\sec(c+dx)}(B+C\sec(c+dx))\,dx$$
$$= B\int \sqrt{a+a\sec(c+dx)}\,dx + C\int \sec(c+dx$$
$$= \frac{2aC\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2aB)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,d\right.}{d}$$
$$= \frac{2\sqrt{a}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aC\tan(c+}{d\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.276155, size = 76, normalized size = 1.15

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2C\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*C*Sin[(c + d*x)/2]))/d
```

Maple [B] time = 0.255, size = 118, normalized size = 1.8

$$-\frac{1}{d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(B\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+2*C*cos(d*x+c)-2*C)/sin(d*x+c)
```

Maxima [B] time = 1.63858, size = 198, normalized size = 3.

$$\frac{B\sqrt{a}\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(
d*x + c))/d
```

Fricas [A] time = 0.554384, size = 620, normalized size = 9.39

$$\left[\frac{(B\cos(dx+c)+B)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,C\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] [((B*cos(d*x + c) + B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c) + d), -2*((B*cos(d*x + c) + B)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.362 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=68

$$\frac{\sqrt{a}(B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(B + 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*B*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.209768, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4072, 4015, 3774, 203}

$$\frac{\sqrt{a}(B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (Sqrt[a]*(B + 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*B*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec(c+$$
$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(B+2C)\int\sqrt{a+a\sec$$
$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(a(B+2C))\operatorname{Subst}\left(\int\frac{}{a+}\right.}{}$$
$$= \frac{\sqrt{a}(B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{aB\sin(}{d\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.241721, size = 93, normalized size = 1.37

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(B+2C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(B
 + 2*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[c + d*x]]*Sin[(c +
d*x)/2]))/(2*d)
```

Maple [B] time = 0.316, size = 198, normalized size = 2.9

$$-\frac{1}{2d\sin(dx+c)}\left(B\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+2C\sqrt{-2\frac{\mathrm{c}}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*C*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*B*cos(d*x+c)
^2-2*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.97053, size = 1268, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] 1/4*(4*C*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
```

```
1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

Fricas [A] time = 0.636405, size = 694, normalized size = 10.21

$$\left[\frac{2\,B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+((B+2\,C)\cos(dx+c)+B+2\,C)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \cdots}{\cos(dx \cdots}\right.}{2\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + ((B + 2*C)*cos(d*x + c) + B + 2*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
+ a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (B*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - ((B + 2*C)*cos
(d*x + c) + B + 2*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.32136, size = 450, normalized size = 6.62

$$-\frac{\left(B\sqrt{-a}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+2\,C\sqrt{-a}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -1/2*((B*sqrt(-a)*sgn(cos(d*x + c)) + 2*C*sqrt(-a)*sgn(cos(d*x + c)))*log(a
bs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
- a*(2*sqrt(2) + 3))) - (B*sqrt(-a)*sgn(cos(d*x + c)) + 2*C*sqrt(-a)*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a*sgn(cos(d*
x + c)) - B*sqrt(-a)*a^2*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.363 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=117

$$\frac{a(3B+4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(3B+4C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(3*B + 4*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*(3*B + 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*B*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.281341, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4015, 3805, 3774, 203}

$$\frac{a(3B+4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(3B+4C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (Sqrt[a]*(3*B + 4*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*(3*B + 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*B*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec(c$$
$$= \frac{aB\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(3B+4C)\int \cos$$
$$= \frac{a(3B+4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\cos(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+c}}$$
$$= \frac{a(3B+4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\cos(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+c}}$$
$$= \frac{\sqrt{a}(3B+4C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a(3B+4}{4d\sqrt{a+}}$$

Mathematica [C] time = 0.366543, size = 117, normalized size = 1.

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2B\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)+C\left(\cos(c+dx\right.\right.}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] ((C*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])
 + 2*B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]]
)
```

Maple [B] time = 0.379, size = 398, normalized size = 3.4

$$\frac{1}{16\,d\cos(dx+c)\sin(dx+c)}\left(3\,B\cos(dx+c)\sin(dx+c)\sqrt{2}\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(3*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))+4*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))+3*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*
```

```
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+
4*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*B*cos(d*
x+c)^4-4*B*cos(d*x+c)^3-16*C*cos(d*x+c)^3+12*B*cos(d*x+c)^2+16*C*cos(d*x+c)
^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 2.2573, size = 2499, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - 1)))*B + 4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
```

```
c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C)/d
```

Fricas [A] time = 0.646046, size = 801, normalized size = 6.85

$$\left[\frac{((3B+4C)\cos(dx+c)+3B+4C)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2B\cos(d\right.}{8\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/8*(((3*B + 4*C)*cos(d*x + c) + 3*B + 4*C)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*B*cos(d*x + c)^2 +
(3*B + 4*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c) + d), -1/4*(((3*B + 4*C)*cos(d*x + c) + 3*B + 4*C)*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - (2*B*cos(d*x + c)^2 + (3*B + 4*C)*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.61646, size = 851, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -1/8*((3*B*sqrt(-a)*sgn(cos(d*x + c)) + 4*C*sqrt(-a)*sgn(cos(d*x + c)))*log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 - a*(2*sqrt(2) + 3))) - (3*B*sqrt(-a)*sgn(cos(d*x + c)) + 4*C*sqrt(-a)*sg
n(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a*sgn(co
s(d*x + c)) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*C*sqrt(-a)*a*sgn(cos(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^2*sgn(cos(d*x +
 c)) + 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^4*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 17*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^3*sgn(cos(d*x + c))
- 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 4*C
*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.364 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=160

$$\frac{a(5B+6C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5B+6C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(5B+6C)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\cos}{3d\sqrt{a\sec(c+d}}$$

```
[Out] (Sqrt[a]*(5*B + 6*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*B + 6*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*(5*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (
a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.33617, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4015, 3805, 3774, 203}

$$\frac{a(5B+6C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5B+6C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(5B+6C)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\cos}{3d\sqrt{a\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (Sqrt[a]*(5*B + 6*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*B + 6*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*(5*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (
a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}(B+C\sec$$

$$= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(5B+6C)\int$$

$$= \frac{a(5B+6C)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aB\cos}{3d}$$

$$= \frac{a(5B+6C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(5B+6C)\cos(c}{12d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{a(5B+6C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(5B+6C)\cos(c}{12d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{\sqrt{a}(5B+6C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a(5B}{8d\sqrt{a}}$$

Mathematica [C] time = 0.187851, size = 70, normalized size = 0.44

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(B\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)+C\text{Hypergeometric2F1}\left(\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*(C*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]] + B*Hypergeometric2F
1[1/2, 4, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/
2])/d
```

Maple [B] time = 0.42, size = 580, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d*(15*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*sin(d*x+c)+18*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
```

```
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*sin(d*x+c)+30*B*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)+36*C*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*sin(d*x+c)+15*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)+18*C*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x
+c)+64*B*cos(d*x+c)^6+16*B*cos(d*x+c)^5+96*C*cos(d*x+c)^5+40*B*cos(d*x+c)^4
+48*C*cos(d*x+c)^4-120*B*cos(d*x+c)^3-144*C*cos(d*x+c)^3)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [B] time = 2.84536, size = 4024, normalized size = 25.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] 1/96*((4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
```

```
 + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arct
an2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B + 6*(2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) -
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2
*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*
c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) +
 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C)/d
```

Fricas [A] time = 0.652492, size = 898, normalized size = 5.61

$$\left[\frac{3\left((5B+6C)\cos(dx+c)+5B+6C\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8B\cos(\right.}{48\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((5*B + 6*C)*cos(d*x + c) + 5*B + 6*C)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*B*cos(d*x + c)^3
 + 2*(5*B + 6*C)*cos(d*x + c)^2 + 3*(5*B + 6*C)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(3*((5
*B + 6*C)*cos(d*x + c) + 5*B + 6*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*B*cos(d*x + c)^3
+ 2*(5*B + 6*C)*cos(d*x + c)^2 + 3*(5*B + 6*C)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.70739, size = 1156, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*B*sqrt(-a)*sgn(cos(d*x + c)) + 6*C*sqrt(-a)*sgn(cos(d*x + c)))*
log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a*(2*sqrt(2) + 3))) - 3*(5*B*sqrt(-a)*sgn(cos(d*x + c)) + 6*C*sqrt(-
a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(63*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a
*sgn(cos(d*x + c)) - 30*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
```

```
x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a*sgn(cos(d*x + c)) - 369*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^2*sgn(
cos(d*x + c)) + 66*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1638*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^3*sgn(cos
(d*x + c)) + 756*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 1074*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^4*sgn(cos(d
*x + c)) - 732*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^5*sgn(cos(d*x
+ c)) + 138*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 13*B*sqrt(-a)*a^6*sgn(cos(d*x +
 c)) - 6*C*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.365 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=234

$$\frac{2a^2(11B+12C)\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(187B+168C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(187B+168C)\tan(c+dx)}{495d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (4*a^2*(187*B + 168*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(187*B + 168*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a^2*(11*B + 12*C)*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a + a
*Sec[c + d*x]]) - (8*a*(187*B + 168*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x
])/(3465*d) + (2*a*C*Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/
(11*d) + (4*(187*B + 168*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1155*
d)
```

Rubi [A] time = 0.637845, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4018, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a^2(11B+12C)\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(187B+168C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(187B+168C)\tan(c+dx)}{495d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (4*a^2*(187*B + 168*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(187*B + 168*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a^2*(11*B + 12*C)*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a + a
*Sec[c + d*x]]) - (8*a*(187*B + 168*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x
])/(3465*d) + (2*a*C*Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/
(11*d) + (4*(187*B + 168*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1155*
d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}(B+C$$

$$= \frac{2aC\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c}{11d}$$

$$= \frac{2a^2(11B+12C)\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(187B+168C)\sec^3(c+dx)\tan(c+dx}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(187B+168C)\sec^3(c+dx)\tan(c+dx}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(187B+168C)\sec^3(c+dx)\tan(c+dx}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a^2(187B+168C)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(187}{}$$

Mathematica [A] time = 6.10408, size = 113, normalized size = 0.48

$$\frac{2a^2\tan(c+dx)\left(35(11B+21C)\sec^4(c+dx)+(935B+840C)\sec^3(c+dx)+6(187B+168C)\sec^2(c+dx)+8(187B+1\right.}{3465d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*a^2*(2992*B + 2688*C + 8*(187*B + 168*C)*Sec[c + d*x] + 6*(187*B + 168*C
)*Sec[c + d*x]^2 + (935*B + 840*C)*Sec[c + d*x]^3 + 35*(11*B + 21*C)*Sec[c
+ d*x]^4 + 315*C*Sec[c + d*x]^5)*Tan[c + d*x])/(3465*d*Sqrt[a*(1 + Sec[c +
d*x])])
```

Maple [A] time = 0.463, size = 161, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(2992B(\cos(dx+c))^5+2688C(\cos(dx+c))^5+1496B(\cos(dx+c))^4+1344C(\cos(dx+c))^4\right.}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(2992*B*cos(d*x+c)^5+2688*C*cos(d*x+c)^5+1496*B
*cos(d*x+c)^4+1344*C*cos(d*x+c)^4+1122*B*cos(d*x+c)^3+1008*C*cos(d*x+c)^3+9
35*B*cos(d*x+c)^2+840*C*cos(d*x+c)^2+385*B*cos(d*x+c)+735*C*cos(d*x+c)+315*
C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.51914, size = 394, normalized size = 1.68

$$\frac{2\left(16(187B+168C)a\cos(dx+c)^5+8(187B+168C)a\cos(dx+c)^4+6(187B+168C)a\cos(dx+c)^3+5(187B+16\right.}{3465\left(d\cos(dx+c)^6+d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/3465*(16*(187*B + 168*C)*a*cos(d*x + c)^5 + 8*(187*B + 168*C)*a*cos(d*x +
 c)^4 + 6*(187*B + 168*C)*a*cos(d*x + c)^3 + 5*(187*B + 168*C)*a*cos(d*x +
c)^2 + 35*(11*B + 21*C)*a*cos(d*x + c) + 315*C*a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.91038, size = 424, normalized size = 1.81

$$-\frac{4\left(3465\sqrt{2}Ba^7\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+3465\sqrt{2}Ca^7\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-\left(9240\sqrt{2}Ba^7\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+6930\sqrt{2}Ca^7\operatorname{s}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -4/3465*(3465*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 3465*sqrt(2)*C*a^7*sgn(cos(
d*x + c)) - (9240*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 6930*sqrt(2)*C*a^7*sgn(
cos(d*x + c)) - (14784*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 15246*sqrt(2)*C*a^
7*sgn(cos(d*x + c)) - (13662*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 11088*sqrt(2
)*C*a^7*sgn(cos(d*x + c)) - (5687*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 5313*sq
rt(2)*C*a^7*sgn(cos(d*x + c)) - 2*(517*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 48
3*sqrt(2)*C*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c
)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*d)
```

3.366 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c-$

Optimal. Leaf size=189

$$\frac{2a^2(9B+10C)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(39B+34C)\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{2(39B+34C)\tan(c+dx)(a\sec(c+dx)+a)^{3/}}{105d}$$

```
[Out] (2*a^2*(39*B + 34*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(9*B + 10*C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*a*(39*B + 34*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*
C*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(39*B +
34*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d)
```

Rubi [A] time = 0.564349, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4018, 4016, 3800, 4001, 3792}

$$\frac{2a^2(9B+10C)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(39B+34C)\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{2(39B+34C)\tan(c+dx)(a\sec(c+dx)+a)^{3/}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (2*a^2*(39*B + 34*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(9*B + 10*C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*a*(39*B + 34*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*
C*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(39*B +
34*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
```

```
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}(B+C$$

$$= \frac{2aC\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c}{9d}$$

$$= \frac{2a^2(9B+10C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a^2(9B+10C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a^2(9B+10C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{2a^2(39B+34C)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(9B+1}{63}$$

Mathematica [A] time = 0.687083, size = 100, normalized size = 0.53

$$\frac{2a^2\tan(c+dx)\left(5(9B+17C)\sec^3(c+dx)+3(39B+34C)\sec^2(c+dx)+4(39B+34C)\sec(c+dx)+8(39B+34C)\right.}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*a^2*(8*(39*B + 34*C) + 4*(39*B + 34*C)*Sec[c + d*x] + 3*(39*B + 34*C)*Se
c[c + d*x]^2 + 5*(9*B + 17*C)*Sec[c + d*x]^3 + 35*C*Sec[c + d*x]^4)*Tan[c +
 d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.287, size = 139, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(312B(\cos(dx+c))^4+272C(\cos(dx+c))^4+156B(\cos(dx+c))^3+136C(\cos(dx+c))^3+11\right.}{315d(\cos(dx+c))^4\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(312*B*cos(d*x+c)^4+272*C*cos(d*x+c)^4+156*B*cos
(d*x+c)^3+136*C*cos(d*x+c)^3+117*B*cos(d*x+c)^2+102*C*cos(d*x+c)^2+45*B*cos
(d*x+c)+85*C*cos(d*x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c
)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.537587, size = 329, normalized size = 1.74

$$\frac{2\left(8(39B+34C)a\cos(dx+c)^4+4(39B+34C)a\cos(dx+c)^3+3(39B+34C)a\cos(dx+c)^2+5(9B+17C)a\cos(d\right.}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/315*(8*(39*B + 34*C)*a*cos(d*x + c)^4 + 4*(39*B + 34*C)*a*cos(d*x + c)^3
+ 3*(39*B + 34*C)*a*cos(d*x + c)^2 + 5*(9*B + 17*C)*a*cos(d*x + c) + 35*C*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 +
d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.77483, size = 362, normalized size = 1.92

$$\frac{4\left(315\sqrt{2}Ba^6\operatorname{sgn}(\cos(dx+c)) + 315\sqrt{2}Ca^6\operatorname{sgn}(\cos(dx+c)) - \left(735\sqrt{2}Ba^6\operatorname{sgn}(\cos(dx+c)) + 525\sqrt{2}Ca^6\operatorname{sgn}(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] 4/315*(315*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 315*sqrt(2)*C*a^6*sgn(cos(d*x
+ c)) - (735*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 525*sqrt(2)*C*a^6*sgn(cos(d*
x + c)) - (819*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 819*sqrt(2)*C*a^6*sgn(cos(
d*x + c)) - (513*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 423*sqrt(2)*C*a^6*sgn(co
s(d*x + c)) - 2*(57*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 47*sqrt(2)*C*a^6*sgn(
cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.367 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=138

$$\frac{8a^2(21B+19C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7B-2C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2a(21B+19C)\tan(c+dx)\sqrt{a\sec(c+dx)}}{105d}$$

```
[Out] (8*a^2*(21*B + 19*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(21*B + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*B - 2*
C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + a*Sec[c + d*
x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.353351, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4010, 4001, 3793, 3792}

$$\frac{8a^2(21B+19C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7B-2C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2a(21B+19C)\tan(c+dx)\sqrt{a\sec(c+dx)}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (8*a^2*(21*B + 19*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(21*B + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*B - 2*
C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + a*Sec[c + d*
x])^(5/2)*Tan[c + d*x])/(7*a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}(B+C$$

$$= \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{2\int}{}$$

$$= \frac{2(7B-2C)(a+a\sec(c+dx))^{3/2}\tan(c+dx}{35d}$$

$$= \frac{2a(21B+19C)\sqrt{a+a\sec(c+dx)}\tan(c+}{105d}$$

$$= \frac{8a^2(21B+19C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(21B+1}{}$$

Mathematica [A] time = 0.372721, size = 82, normalized size = 0.59

$$\frac{2a^2\tan(c+dx)\left(3(7B+13C)\sec^2(c+dx)+(63B+52C)\sec(c+dx)+2(63B+52C)+15C\sec^3(c+dx)\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a^2*(2*(63*B + 52*C) + (63*B + 52*C)*Sec[c + d*x] + 3*(7*B + 13*C)*Sec[c
 + d*x]^2 + 15*C*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [A] time = 0.265, size = 117, normalized size = 0.9

$$-\frac{2a(-1+\cos(dx+c))\left(126B(\cos(dx+c))^3+104C(\cos(dx+c))^3+63B(\cos(dx+c))^2+52C(\cos(dx+c))^2+2}{105d(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(126*B*cos(d*x+c)^3+104*C*cos(d*x+c)^3+63*B*cos(
d*x+c)^2+52*C*cos(d*x+c)^2+21*B*cos(d*x+c)+39*C*cos(d*x+c)+15*C)*(a*(cos(d*
x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.510531, size = 279, normalized size = 2.02

$$\frac{2\left(2\,(63\,B+52\,C)a\cos\left(dx+c\right)^{3}+(63\,B+52\,C)a\cos\left(dx+c\right)^{2}+3\,(7\,B+13\,C)a\cos\left(dx+c\right)+15\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 2/105*(2*(63*B + 52*C)*a*cos(d*x + c)^3 + (63*B + 52*C)*a*cos(d*x + c)^2 +
3*(7*B + 13*C)*a*cos(d*x + c) + 15*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.67613, size = 300, normalized size = 2.17

$$-\frac{4\left(105\sqrt{2}Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ca^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(210\sqrt{2}Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+140\sqrt{2}Ca^{5}\mathrm{sgn}\left(\cos\left(\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -4/105*(105*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^5*sgn(cos(d*x
 + c)) - (210*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 140*sqrt(2)*C*a^5*sgn(cos(d
*x + c)) - (147*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 133*sqrt(2)*C*a^5*sgn(cos
(d*x + c)) - 2*(21*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 19*sqrt(2)*C*a^5*sgn(c
os(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.368 $\int (a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=101

$$\frac{8a^2(5B+3C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(5B+3C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (8*a^2*(5*B + 3*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(5*
B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.129032, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4054, 12, 3793, 3792}

$$\frac{8a^2(5B+3C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(5B+3C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (8*a^2*(5*B + 3*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(5*
B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
 !LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sec(c + dx))^{3/2} \left(B\sec(c + dx) + C\sec^2(c + dx)\right) dx = \frac{2C(a + a\sec(c + dx))^{3/2}\tan(c + dx)}{5d} + \frac{2\int \frac{1}{2}a(5B + 3C}{}$$
$$= \frac{2C(a + a\sec(c + dx))^{3/2}\tan(c + dx)}{5d} + \frac{1}{5}(5B + 3C)\int$$
$$= \frac{2a(5B + 3C)\sqrt{a + a\sec(c + dx)}\tan(c + dx)}{15d} + \frac{2C(a +}{}$$
$$= \frac{8a^2(5B + 3C)\tan(c + dx)}{15d\sqrt{a + a\sec(c + dx)}} + \frac{2a(5B + 3C)\sqrt{a + a\sec(c}}{15d}$$

Mathematica [A] time = 0.253683, size = 62, normalized size = 0.61

$$\frac{2a^2\tan(c + dx)\left((5B + 9C)\sec(c + dx) + 25B + 3C\sec^2(c + dx) + 18C\right)}{15d\sqrt{a(\sec(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a^2*(25*B + 18*C + (5*B + 9*C)*Sec[c + d*x] + 3*C*Sec[c + d*x]^2)*Tan[c
+ d*x])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.253, size = 95, normalized size = 0.9

$$-\frac{2a(-1 + \cos(dx + c))\left(25B(\cos(dx + c))^2 + 18C(\cos(dx + c))^2 + 5B\cos(dx + c) + 9C\cos(dx + c) + 3C\right)}{15d(\cos(dx + c))^2\sin(dx + c)}\sqrt{\frac{a(\mathrm{co}}{\mathrm{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(25*B*cos(d*x+c)^2+18*C*cos(d*x+c)^2+5*B*cos(d*x+
c)+9*C*cos(d*x+c)+3*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin
(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.498846, size = 225, normalized size = 2.23

$$\frac{2\left((25\,B+18\,C)a\cos\left(dx+c\right)^{2}+(5\,B+9\,C)a\cos\left(dx+c\right)+3\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/15*((25*B + 18*C)*a*cos(d*x + c)^2 + (5*B + 9*C)*a*cos(d*x + c) + 3*C*a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*
cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\left(\sec\left(c+dx\right)+1\right))^{\frac{3}{2}}\left(B+C\sec\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(B + C*sec(c + d*x))*sec(c + d*x), x
)
```

Giac [A] time = 4.52612, size = 238, normalized size = 2.36

$$\frac{4\left(15\,\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\,\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(25\,\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\,\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\left(dx+c\right.\right.\right.\right.}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{\;}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] 4/15*(15*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 15*sqrt(2)*C*a^4*sgn(cos(d*x + c
)) - (25*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 15*sqrt(2)*C*a^4*sgn(cos(d*x + c
)) - 2*(5*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 3*sqrt(2)*C*a^4*sgn(cos(d*x + c
)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((
a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.369 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=105

$$\frac{2a^2(3B+4C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*a^(3/2)*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(3*B + 4*C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.242768, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(3B+4C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (2*a^(3/2)*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(3*B + 4*C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+a\sec(c+dx))^{3/2}(B+C\sec(c+dx))\,dx$$
$$= \frac{2aC\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \sqrt{a}$$
$$= \frac{2aC\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+(aB)\int$$
$$= \frac{2a^2(3B+4C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2aC\sqrt{a+a\sec(c}{3}$$
$$= \frac{2a^{3/2}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2(3B+4C)\,\mathrm{t}}{3d\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 0.482679, size = 102, normalized size = 0.97

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((3B+5C)\cos(c+dx)+C)+3\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}\right.\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*B*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(C + (3*B + 5*C)*Cos[
c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] time = 0.267, size = 237, normalized size = 2.3

$$\frac{a}{6d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3B\cos(dx+c)\sin(dx+c)\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/6/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*B*cos(d*x+c)*sin(d*x+c)*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+3*B*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-12*B*cos(d*x+c)^2-20*C*cos(d*x+c)^2+12*
B*cos(d*x+c)+16*C*cos(d*x+c)+4*C)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 1.84859, size = 1347, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*d)
```

Fricas [A] time = 0.556835, size = 814, normalized size = 7.75

$$\left[\frac{3\left(Ba\cos\left(dx+c\right)^{2}+Ba\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(\left(3\,B+\right.\right.}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] [1/3*(3*(B*a*cos(d*x + c)^2 + B*a*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((3*B + 5*C)*a*cos(
d*x + c) + C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2 + d*cos(d*x + c)), -2/3*(3*(B*a*cos(d*x + c)^2 + B*a*cos(d*x +
 c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c))) - ((3*B + 5*C)*a*cos(d*x + c) + C*a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.370 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2} \left(B\sec(c+dx)+C\sec^2(c$

Optimal. Leaf size=103

$$\frac{a^2(B-2C)\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(3B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] (a^(3/2)*(3*B + 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (a^2*(B - 2*C)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.356627, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4018, 4015, 3774, 203}

$$\frac{a^2(B-2C)\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(3B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^(3/2)*(3*B + 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (a^2*(B - 2*C)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+a\sec(c+dx))^{3/2}(B+C\sec$$

$$= \frac{2aC\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + 2\int \cos$$

$$= \frac{a^2(B-2C)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sqrt{a+a\sec(c+}}{d}$$

$$= \frac{a^2(B-2C)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2aC\sqrt{a+a\sec(c+}}{d}$$

$$= \frac{a^{3/2}(3B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2(B-}{d\sqrt{a}}$$

Mathematica [A] time = 0.426962, size = 97, normalized size = 0.94

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(3B+2C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(B\cos(c\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(3*B + 2*C)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*C + B*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.289, size = 212, normalized size = 2.1

$$-\frac{a}{2d\sin(dx+c)}\left(3B\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+2C\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/2/d*a*(3*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
2*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*B*cos(d*
```

```
x+c)^2-2*B*cos(d*x+c)+4*C*cos(d*x+c)-4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)
```

Maxima [B] time = 2.18631, size = 2431, normalized size = 23.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
```

```
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A] time = 0.653416, size = 755, normalized size = 7.33

$$\left[\frac{((3B+2C)a\cos(dx+c)+(3B+2C)a)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2(Ba\cos}{2(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/2*(((3*B + 2*C)*a*cos(d*x + c) + (3*B + 2*C)*a)*sqrt(-a)*log((2*a*cos(d*
x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*
sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(B*a*cos(d*x + c
) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d), -(((3*B + 2*C)*a*cos(d*x + c) + (3*B + 2*C)*a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - (B*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.38393, size = 544, normalized size = 5.28

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}Ca^2\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+\left(3B\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))+2C\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a^2*sgn(cos(d*x + c))
*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (3*B*sqrt(-a)*a*sgn(
cos(d*x + c)) + 2*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))
) - (3*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*C*sqrt(-a)*a*sgn(cos(d*x + c)))*l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - B*
sqrt(-a)*a^3*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.371 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=119

$$\frac{a^2(5B+4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7B+12C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (a^(3/2)*(7*B + 12*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(5*B + 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a*B*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.377637, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4017, 4015, 3774, 203}

$$\frac{a^2(5B+4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7B+12C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^(3/2)*(7*B + 12*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(5*B + 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a*B*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}(B+$$

$$= \frac{aB\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{2d}$$

$$= \frac{a^2(5B+4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{aB\cos(c+dx)}{}$$

$$= \frac{a^2(5B+4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{aB\cos(c+dx)}{}$$

$$= \frac{a^{3/2}(7B+12C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^2}{4}$$

Mathematica [A] time = 0.710625, size = 101, normalized size = 0.85

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\cos(c+dx)\sqrt{\sec(c+dx)-1}(2B\cos(c+dx)+7B+4C)+(7B+12C)\tan^{-1}\left(\right.\right.}{4d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a*((7*B + 12*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]] + Cos[c + d*x]*(7*B + 4*C
+ 2*B*Cos[c + d*x])*Sqrt[-1 + Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan
[(c + d*x)/2])/(4*d*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.3, size = 399, normalized size = 3.4

$$\frac{a}{16\,d\cos(dx+c)\sin(dx+c)}\left(7\,B\cos(dx+c)\sin(dx+c)\sqrt{2}\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/16/d*a*(7*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))+12*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
```

```
/cos(d*x+c))+7*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+
c)+12*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*B*co
s(d*x+c)^4-20*B*cos(d*x+c)^3-16*C*cos(d*x+c)^3+28*B*cos(d*x+c)^2+16*C*cos(d
*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.692929, size = 833, normalized size = 7.

$$\left[\frac{((7B+12C)a\cos(dx+c)+(7B+12C)a)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2Ba\right.}{8(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/8*(((7*B + 12*C)*a*cos(d*x + c) + (7*B + 12*C)*a)*sqrt(-a)*log((2*a*cos(
d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*B*a*cos(d*x
 + c)^2 + (7*B + 4*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((7*B + 12*C)*a*cos(d*x + c) +
 (7*B + 12*C)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*B*a*cos(d*x + c)^2 + (7*B + 4*C)*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.68795, size = 863, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/8*((7*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*C*sqrt(-a)*a*sgn(cos(d*x + c))
)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3))) - (7*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*C*sqr
t(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a
)*a^2*sgn(cos(d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 95*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^3
*sgn(cos(d*x + c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 53*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^4*sgn(
cos(d*x + c)) + 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 5*B*sqrt(-a)*a^5*sgn(cos
(d*x + c)) - 4*C*sqrt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.372 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=164

$$\frac{a^2(11B+14C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11B+14C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(7B+6C)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+}{}$$

```
[Out] (a^(3/2)*(11*B + 14*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(11*B + 14*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(7*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*B*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.470371, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4017, 4015, 3805, 3774, 203}

$$\frac{a^2(11B+14C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11B+14C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(7B+6C)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^(3/2)*(11*B + 14*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(11*B + 14*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(7*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*B*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}(B+$$

$$= \frac{aB\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{3d}$$

$$= \frac{a^2(7B+6C)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aB}{}$$

$$= \frac{a^2(11B+14C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(7B+6C)}{12d\sqrt{}}$$

$$= \frac{a^2(11B+14C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(7B+6C)}{12d\sqrt{}}$$

$$= \frac{a^{3/2}(11B+14C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a}{}$$

Mathematica [C] time = 11.3871, size = 740, normalized size = 4.51

$$a\left(\frac{B(\cos(c+dx)+1)\tan(c+dx)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)}{d(\sec(c+dx)+1)}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] a*((C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x]
)*Sec[c/2 + (d*x)/2]^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])])/(2*Sqrt
[2]*d) - (C*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*Sec[(c + d*x)/2]*Sqrt[a
*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c +
```

```
 d*x]] + Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2]))
/(16*d) + (B*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c +
d*x]]*Sec[c/2 + (d*x)/2]^2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(d*(1 +
 Sec[c + d*x])) + (B*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(ArcTanh[Sqrt[
1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c
 + d*x])]*Tan[c + d*x])/(4*d*Sqrt[1 + Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])
+ (C*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(ArcTanh[Sqrt[1 - Sec[c + d*x]
]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c
+ d*x])/(2*d*Sqrt[1 + Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]) + (B*(1 + Cos[c
+ d*x])*Sec[c/2 + (d*x)/2]^2*Sqrt[a*(1 + Sec[c + d*x])]*(Sec[(c + d*x)/2]*S
qrt[1 + Sec[c + d*x]]*(-3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos
[c + d*x]] + 7*Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*
x))/2] + 2*Sin[(7*(c + d*x))/2]) + (12*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + C
os[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/Sqrt[-Tan[c + d*x]^2]))/(
96*d*Sqrt[1 + Sec[c + d*x]]))
```

Maple [B] time = 0.381, size = 581, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d*a*(33*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*sin(d*x+c)+42*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)+66*B*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*sin(d*x+c)+84*C*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*sin(d*x+c)+33*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*sin(d*x+c)+42*C*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d
*x+c)+64*B*cos(d*x+c)^6+112*B*cos(d*x+c)^5+96*C*cos(d*x+c)^5+88*B*cos(d*x+c
)^4+240*C*cos(d*x+c)^4-264*B*cos(d*x+c)^3-336*C*cos(d*x+c)^3)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.679484, size = 949, normalized size = 5.79

$$\left[\frac{3\left((11\,B+14\,C)a\cos\left(dx+c\right)+(11\,B+14\,C)a\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+}{48\left(d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((11*B + 14*C)*a*cos(d*x + c) + (11*B + 14*C)*a)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*B*a*co
s(d*x + c)^3 + 2*(11*B + 6*C)*a*cos(d*x + c)^2 + 3*(11*B + 14*C)*a*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
 + d), -1/24*(3*((11*B + 14*C)*a*cos(d*x + c) + (11*B + 14*C)*a)*sqrt(a)*ar
ctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - (8*B*a*cos(d*x + c)^3 + 2*(11*B + 6*C)*a*cos(d*x + c)^2 + 3*(11*B
+ 14*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.01428, size = 1166, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/48*(3*(11*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 14*C*sqrt(-a)*a*sgn(cos(d*x +
 c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(11*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 1
4*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(3
3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*
B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 42*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 303*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*s
```

qrt(-a)*a^3*sgn(cos(d*x + c)) - 822*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 3780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 2508*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 309*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 498*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 30*C*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d

3.373 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=209

$$\frac{a^2(75B+88C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75B+88C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(9B+8C)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(75B}{}$$

```
[Out] (a^(3/2)*(75*B + 88*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a^2*(75*B + 88*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (a^2*(75*B + 88*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(9*B + 8*C)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*S
ec[c + d*x]]) + (a*B*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(4*d)
```

Rubi [A] time = 0.557501, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4017, 4015, 3805, 3774, 203}

$$\frac{a^2(75B+88C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75B+88C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(9B+8C)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(75B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^(3/2)*(75*B + 88*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a^2*(75*B + 88*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (a^2*(75*B + 88*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(9*B + 8*C)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*S
ec[c + d*x]]) + (a*B*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(4*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}(B+C\,\mathrm{se}$$

$$= \frac{aB\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{a^2(9B+8C)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aB\,\mathrm{co}}{}$$

$$= \frac{a^2(75B+88C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(9}{}$$

$$= \frac{a^2(75B+88C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(75B+88C)}{96d\sqrt{a}}$$

$$= \frac{a^2(75B+88C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(75B+88C)}{96d\sqrt{a}}$$

$$= \frac{a^{3/2}(75B+88C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a^2(7}{64}$$

Mathematica [C] time = 11.9911, size = 1031, normalized size = 4.93

$$a\left(-\frac{C(\cos(c+dx)+1)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}\sqrt{\cos(c+dx)}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right.\right.}{16d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] a*((C*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x]
)*Sec[c/2 + (d*x)/2]^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]])/(2*Sqrt
[2]*d) - (C*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*Sec[(c + d*x)/2]*Sqrt[a
*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c +
 d*x]] + Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2]))
/(16*d) + (C*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*Sec[(c + d*x)/2]*Sqrt[
a*(1 + Sec[c + d*x])]*(-3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos
[c + d*x]] + 7*Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*
x))/2] + 2*Sin[(7*(c + d*x))/2]))/(96*d) - (B*(1 + Cos[c + d*x])*Sec[c/2 +
(d*x)/2]^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*ArcSin[Sq
rt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 5*Sin[(c + d*x)/2] - 16*Sin[(3
*(c + d*x))/2] - 9*Sin[(5*(c + d*x))/2] - 8*Sin[(7*(c + d*x))/2] - 6*Sin[(9
*(c + d*x))/2]))/(768*d) + (3*B*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3
, 3/2, 1 - Sec[c + d*x]]*Sec[c/2 + (d*x)/2]^2*Sqrt[a*(1 + Sec[c + d*x])]*Ta
n[c + d*x])/(4*d*(1 + Sec[c + d*x])) + (B*(1 + Cos[c + d*x])*Sec[c/2 + (d*x
)/2]^2*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x
]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(4*d*Sqrt[1 + Sec[c + d*x]]*Sq
rt[-Tan[c + d*x]^2]) + (3*C*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(ArcTan
h[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1
+ Sec[c + d*x])]*Tan[c + d*x])/(8*d*Sqrt[1 + Sec[c + d*x]]*Sqrt[-Tan[c + d*
x]^2]) + (B*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*Sqrt[a*(1 + Sec[c + d*x
])]*(Sec[(c + d*x)/2]*Sqrt[1 + Sec[c + d*x]]*(-3*Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 7*Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x)
)/2] + 3*Sin[(5*(c + d*x))/2] + 2*Sin[(7*(c + d*x))/2]) + (12*(ArcTanh[Sqrt
[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/Sq
rt[-Tan[c + d*x]^2]))/(96*d*Sqrt[1 + Sec[c + d*x]]))
```

Maple [B] time = 0.332, size = 763, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/3072/d*a*(-225*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(7/2)*2^(1/2)-264*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)*2^(1/2)-675*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(7/2)*2^(1/2)-792*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-675*B*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-792*C*sin(d*x+c)*cos(d*x+c)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-225*B*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(7/2)*2^(1/2)*sin(d*x+c)-264*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(7/2)*sin(d*x+c)+768*B*cos(d*x+c)^8+1152*B*cos(d*x+c)^7+1024*C*cos(d
*x+c)^7+480*B*cos(d*x+c)^6+1792*C*cos(d*x+c)^6+1200*B*cos(d*x+c)^5+1408*C*c
os(d*x+c)^5-3600*B*cos(d*x+c)^4-4224*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.766863, size = 1049, normalized size = 5.02

$$\left[\frac{3\left((75B+88C)a\cos(dx+c)+(75B+88C)a\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/384*(3*((75*B + 88*C)*a*cos(d*x + c) + (75*B + 88*C)*a)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*B*a*
cos(d*x + c)^4 + 8*(15*B + 8*C)*a*cos(d*x + c)^3 + 2*(75*B + 88*C)*a*cos(d*
x + c)^2 + 3*(75*B + 88*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((75*B + 88*C)*a*cos(
d*x + c) + (75*B + 88*C)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*B*a*cos(d*x + c)^4 + 8*(
15*B + 8*C)*a*cos(d*x + c)^3 + 2*(75*B + 88*C)*a*cos(d*x + c)^2 + 3*(75*B +
 88*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.26761, size = 1469, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/384*(3*(75*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*C*sqrt(-a)*a*sgn(cos(d*x
+ c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(75*B*sqrt(-a)*a*sgn(cos(d*x + c)) +
88*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(
225*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
4*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 264*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 6
261*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
2*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 4008*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) +
35925*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^10*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 33960*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^4*sgn(cos(d*x + c))
 - 127449*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 131784*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^5*sgn(cos(d*x +
c)) + 101667*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 108312*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^6*sgn(cos(d*x
 + c)) - 26079*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 29432*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^7*sgn(cos(d*
x + c)) + 3303*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 3384*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^8*sgn(cos(d*x
 + c)) - 147*B*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 152*C*sqrt(-a)*a^9*sgn(cos(
d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*a + a^2)^4)/d
```

3.374 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=282

$$\frac{2a^2(13B+16C)\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{143d}+\frac{2a^3(299B+280C)\tan(c+dx)\sec^4(c+dx)}{1287d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(4615B}{9}$$

```
[Out] (4*a^3*(4615*B + 4184*C)*Tan[c + d*x])/(6435*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a^3*(4615*B + 4184*C)*Sec[c + d*x]^3*Tan[c + d*x])/(9009*d*Sqrt[a + a*Se
c[c + d*x]]) + (2*a^3*(299*B + 280*C)*Sec[c + d*x]^4*Tan[c + d*x])/(1287*d*
Sqrt[a + a*Sec[c + d*x]]) - (8*a^2*(4615*B + 4184*C)*Sqrt[a + a*Sec[c + d*x
]]*Tan[c + d*x])/(45045*d) + (2*a^2*(13*B + 16*C)*Sec[c + d*x]^4*Sqrt[a + a
*Sec[c + d*x]]*Tan[c + d*x])/(143*d) + (4*a*(4615*B + 4184*C)*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(15015*d) + (2*a*C*Sec[c + d*x]^4*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(13*d)
```

Rubi [A] time = 0.839559, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4018, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a^2(13B+16C)\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{143d}+\frac{2a^3(299B+280C)\tan(c+dx)\sec^4(c+dx)}{1287d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(4615B}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (4*a^3*(4615*B + 4184*C)*Tan[c + d*x])/(6435*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a^3*(4615*B + 4184*C)*Sec[c + d*x]^3*Tan[c + d*x])/(9009*d*Sqrt[a + a*Se
c[c + d*x]]) + (2*a^3*(299*B + 280*C)*Sec[c + d*x]^4*Tan[c + d*x])/(1287*d*
Sqrt[a + a*Sec[c + d*x]]) - (8*a^2*(4615*B + 4184*C)*Sqrt[a + a*Sec[c + d*x
]]*Tan[c + d*x])/(45045*d) + (2*a^2*(13*B + 16*C)*Sec[c + d*x]^4*Sqrt[a + a
*Sec[c + d*x]]*Tan[c + d*x])/(143*d) + (4*a*(4615*B + 4184*C)*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(15015*d) + (2*a*C*Sec[c + d*x]^4*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(13*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}(B+C\sec$$

$$= \frac{2aC\sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c}{13d}$$

$$= \frac{2a^2(13B+16C)\sec^4(c+dx)\sqrt{a+a\sec(c+d}}{143d}$$

$$= \frac{2a^3(299B+280C)\sec^4(c+dx)\tan(c+dx)}{1287d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a^3(4615B+4184C)\sec^3(c+dx)\tan(c+dx}{9009d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(4615B+4184C)\sec^3(c+dx)\tan(c+dx}{9009d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(4615B+4184C)\sec^3(c+dx)\tan(c+dx}{9009d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a^3(4615B+4184C)\tan(c+dx)}{6435d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(4615}{}$$

Mathematica [A] time = 0.478743, size = 131, normalized size = 0.46

$$\frac{2a^3\tan(c+dx)\left(315(13B+38C)\sec^5(c+dx)+35(416B+523C)\sec^4(c+dx)+5(4615B+4184C)\sec^3(c+dx)+6(46\right.}{45045d\sqrt{a(\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*a^3*(73840*B + 66944*C + 8*(4615*B + 4184*C)*Sec[c + d*x] + 6*(4615*B +
4184*C)*Sec[c + d*x]^2 + 5*(4615*B + 4184*C)*Sec[c + d*x]^3 + 35*(416*B + 5
23*C)*Sec[c + d*x]^4 + 315*(13*B + 38*C)*Sec[c + d*x]^5 + 3465*C*Sec[c + d*
x]^6)*Tan[c + d*x])/(45045*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.365, size = 185, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(73840B(\cos(dx+c))^6+66944C(\cos(dx+c))^6+36920B(\cos(dx+c))^5+33472C(\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(73840*B*cos(d*x+c)^6+66944*C*cos(d*x+c)^6+3
6920*B*cos(d*x+c)^5+33472*C*cos(d*x+c)^5+27690*B*cos(d*x+c)^4+25104*C*cos(d
*x+c)^4+23075*B*cos(d*x+c)^3+20920*C*cos(d*x+c)^3+14560*B*cos(d*x+c)^2+1830
5*C*cos(d*x+c)^2+4095*B*cos(d*x+c)+11970*C*cos(d*x+c)+3465*C)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^6/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.538267, size = 479, normalized size = 1.7

$$\frac{2\left(16\,(4615\,B+4184\,C)a^2\cos\left(dx+c\right)^6+8\,(4615\,B+4184\,C)a^2\cos\left(dx+c\right)^5+6\,(4615\,B+4184\,C)a^2\cos\left(dx+c\right)^4+\right.}{45045}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/45045*(16*(4615*B + 4184*C)*a^2*cos(d*x + c)^6 + 8*(4615*B + 4184*C)*a^2*
cos(d*x + c)^5 + 6*(4615*B + 4184*C)*a^2*cos(d*x + c)^4 + 5*(4615*B + 4184*
C)*a^2*cos(d*x + c)^3 + 35*(416*B + 523*C)*a^2*cos(d*x + c)^2 + 315*(13*B +
 38*C)*a^2*cos(d*x + c) + 3465*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.25256, size = 486, normalized size = 1.72

$$\frac{8\left(45045\,\sqrt{2}Ba^9\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+45045\,\sqrt{2}Ca^9\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(150150\,\sqrt{2}Ba^9\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+120120\,\sqrt{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] 8/45045*(45045*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 45045*sqrt(2)*C*a^9*sgn(co
s(d*x + c)) - (150150*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 120120*sqrt(2)*C*a^
9*sgn(cos(d*x + c)) - (300300*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 294294*sqrt
(2)*C*a^9*sgn(cos(d*x + c)) - (356070*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 310
596*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - (232375*sqrt(2)*B*a^9*sgn(cos(d*x + c
)) + 212069*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - 4*(21125*sqrt(2)*B*a^9*sgn(co
s(d*x + c)) + 19279*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - 2*(1625*sqrt(2)*B*a^9
*sgn(cos(d*x + c)) + 1483*sqrt(2)*C*a^9*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c
)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(
(a*tan(1/2*d*x + 1/2*c)^2 - a)^6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.375 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=237

$$\frac{2a^3(209B+194C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(11B+14C)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2a^3(803}{495d}$$

```
[Out] (2*a^3*(803*B + 710*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^3*(209*B + 194*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) - (4*a^2*(803*B + 710*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(346
5*d) + (2*a^2*(11*B + 14*C)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(99*d) + (2*a*(803*B + 710*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(1155*d) + (2*a*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(11*d)
```

Rubi [A] time = 0.758672, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4018, 4016, 3800, 4001, 3792}

$$\frac{2a^3(209B+194C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(11B+14C)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2a^3(803}{495d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (2*a^3*(803*B + 710*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^3*(209*B + 194*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) - (4*a^2*(803*B + 710*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(346
5*d) + (2*a^2*(11*B + 14*C)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(99*d) + (2*a*(803*B + 710*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(1155*d) + (2*a*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(11*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}(B+C\,\mathrm{se}$$

$$= \frac{2aC\sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c\,-}{11d}$$

$$= \frac{2a^2(11B+14C)\sec^3(c+dx)\sqrt{a+a\sec(c+d}}{99d}$$

$$= \frac{2a^3(209B+194C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a^3(209B+194C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a^3(209B+194C)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{2a^3(803B+710C)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(209B+}{69}$$

Mathematica [B] time = 6.16619, size = 487, normalized size = 2.05

$$\frac{2B\tan(c+dx)\sec^3(c+dx)(a(\sec(c+dx)+1))^{5/2}}{9d(\sec(c+dx)+1)^2} + \frac{38B\tan(c+dx)\sec^3(c+dx)(a(\sec(c+dx)+1))^{5/2}}{63d(\sec(c+dx)+1)^3} + \frac{146B\tan(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (1168*B*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(315*d*(1 + Sec[c + d*x]
)^3) + (2272*C*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(693*d*(1 + Sec[c
 + d*x])^3) + (584*B*Sec[c + d*x]*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x]
)/(315*d*(1 + Sec[c + d*x])^3) + (1136*C*Sec[c + d*x]*(a*(1 + Sec[c + d*x])
)^(5/2)*Tan[c + d*x])/(693*d*(1 + Sec[c + d*x])^3) + (146*B*Sec[c + d*x]^2*
(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(105*d*(1 + Sec[c + d*x])^3) + (
284*C*Sec[c + d*x]^2*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(231*d*(1 +
 Sec[c + d*x])^3) + (38*B*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c
 + d*x])/(63*d*(1 + Sec[c + d*x])^3) + (710*C*Sec[c + d*x]^3*(a*(1 + Sec[c
+ d*x]))^(5/2)*Tan[c + d*x])/(693*d*(1 + Sec[c + d*x])^3) + (46*C*Sec[c + d
*x]^4*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(99*d*(1 + Sec[c + d*x])^3
) + (2*B*Sec[c + d*x]^3*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c + d*x])/(9*d*(1
+ Sec[c + d*x])^2) + (2*C*Sec[c + d*x]^4*(a*(1 + Sec[c + d*x]))^(5/2)*Tan[c
 + d*x])/(11*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.382, size = 163, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(6424B(\cos(dx+c))^5+5680C(\cos(dx+c))^5+3212B(\cos(dx+c))^4+2840C(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(6424*B*cos(d*x+c)^5+5680*C*cos(d*x+c)^5+3212
*B*cos(d*x+c)^4+2840*C*cos(d*x+c)^4+2409*B*cos(d*x+c)^3+2130*C*cos(d*x+c)^3
+1430*B*cos(d*x+c)^2+1775*C*cos(d*x+c)^2+385*B*cos(d*x+c)+1120*C*cos(d*x+c)
+315*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.561687, size = 409, normalized size = 1.73

$$\frac{2\left(8(803B+710C)a^2\cos(dx+c)^5+4(803B+710C)a^2\cos(dx+c)^4+3(803B+710C)a^2\cos(dx+c)^3+5(286B\right.}{3465\left(d\cos(dx+c)^6+d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/3465*(8*(803*B + 710*C)*a^2*cos(d*x + c)^5 + 4*(803*B + 710*C)*a^2*cos(d*
x + c)^4 + 3*(803*B + 710*C)*a^2*cos(d*x + c)^3 + 5*(286*B + 355*C)*a^2*cos
(d*x + c)^2 + 35*(11*B + 32*C)*a^2*cos(d*x + c) + 315*C*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.1038, size = 424, normalized size = 1.79

$$-\frac{8\left(3465\sqrt{2}Ba^8\text{sgn}\left(\cos\left(dx+c\right)\right)+3465\sqrt{2}Ca^8\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(10395\sqrt{2}Ba^8\text{sgn}\left(\cos\left(dx+c\right)\right)+8085\sqrt{2}Ca^8\text{sgn}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -8/3465*(3465*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 3465*sqrt(2)*C*a^8*sgn(cos(
d*x + c)) - (10395*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 8085*sqrt(2)*C*a^8*sgn
(cos(d*x + c)) - (15939*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 15015*sqrt(2)*C*a
^8*sgn(cos(d*x + c)) - (14157*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 12375*sqrt(
2)*C*a^8*sgn(cos(d*x + c)) - 4*(1573*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 1375
*sqrt(2)*C*a^8*sgn(cos(d*x + c)) - 2*(143*sqrt(2)*B*a^8*sgn(cos(d*x + c)) +
 125*sqrt(2)*C*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*d)
```

3.376 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c$

Optimal. Leaf size=175

$$\frac{16a^2(15B+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(15B+13C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(9B-2C)\tan(c+dx)(a\sec(c+}{63d}$$

```
[Out] (64*a^3*(15*B + 13*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(15*B + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(15
*B + 13*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) + (2*(9*B - 2*C
)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*C*(a + a*Sec[c + d*x
])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.409784, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4010, 4001, 3793, 3792}

$$\frac{16a^2(15B+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(15B+13C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(9B-2C)\tan(c+dx)(a\sec(c+}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (64*a^3*(15*B + 13*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(15*B + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(15
*B + 13*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) + (2*(9*B - 2*C
)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*C*(a + a*Sec[c + d*x
])^(7/2)*Tan[c + d*x])/(9*a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}(B+C\sec$$

$$= \frac{2C(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad} + \frac{2\int \sec}{}$$

$$= \frac{2(9B-2C)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d} +$$

$$= \frac{2a(15B+13C)(a+a\sec(c+dx))^{3/2}\tan(c+d}{105d}$$

$$= \frac{16a^2(15B+13C)\sqrt{a+a\sec(c+dx)}\tan(c+d}{315d}$$

$$= \frac{64a^3(15B+13C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(15B+13}{}$$

Mathematica [A] time = 0.647338, size = 96, normalized size = 0.55

$$\frac{2a^3\tan(c+dx)\left(5(9B+26C)\sec^3(c+dx)+3(60B+73C)\sec^2(c+dx)+(345B+292C)\sec(c+dx)+690B+35C\sec^4\right.}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a^3*(690*B + 584*C + (345*B + 292*C)*Sec[c + d*x] + 3*(60*B + 73*C)*Sec[
c + d*x]^2 + 5*(9*B + 26*C)*Sec[c + d*x]^3 + 35*C*Sec[c + d*x]^4)*Tan[c + d
*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.295, size = 141, normalized size = 0.8

$$-\frac{2a^2(-1+\cos(dx+c))\left(690B(\cos(dx+c))^4+584C(\cos(dx+c))^4+345B(\cos(dx+c))^3+292C(\cos(dx+c))^3+1\right.}{315d(\cos(dx+c))^4\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(690*B*cos(d*x+c)^4+584*C*cos(d*x+c)^4+345*B*c
os(d*x+c)^3+292*C*cos(d*x+c)^3+180*B*cos(d*x+c)^2+219*C*cos(d*x+c)^2+45*B*c
os(d*x+c)+130*C*cos(d*x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*
x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.511441, size = 346, normalized size = 1.98

$$\frac{2\left(2\,(345\,B+292\,C)a^2\cos\left(dx+c\right)^4+(345\,B+292\,C)a^2\cos\left(dx+c\right)^3+3\,(60\,B+73\,C)a^2\cos\left(dx+c\right)^2+5\,(9\,B+26}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 2/315*(2*(345*B + 292*C)*a^2*cos(d*x + c)^4 + (345*B + 292*C)*a^2*cos(d*x +
 c)^3 + 3*(60*B + 73*C)*a^2*cos(d*x + c)^2 + 5*(9*B + 26*C)*a^2*cos(d*x + c
) + 35*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [A] time = 5.02846, size = 362, normalized size = 2.07

$$8\left(315\sqrt{2}Ba^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+315\sqrt{2}Ca^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(840\sqrt{2}Ba^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+630\sqrt{2}Ca^7\mathrm{sgn}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 8/315*(315*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 315*sqrt(2)*C*a^7*sgn(cos(d*x
+ c)) - (840*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 630*sqrt(2)*C*a^7*sgn(cos(d*
x + c)) - (945*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 819*sqrt(2)*C*a^7*sgn(cos(
d*x + c)) - 4*(135*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 117*sqrt(2)*C*a^7*sgn(
cos(d*x + c)) - 2*(15*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 13*sqrt(2)*C*a^7*sg
n(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x
+ 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.377 $\int (a+a\sec(c+dx))^{5/2} \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=138

$$\frac{64a^3(7B+5C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(7B+5C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7B+5C)\tan(c+dx)(a\sec(c+dx)}{35d}$$

```
[Out] (64*a^3*(7*B + 5*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^
2*(7*B + 5*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*a*(7*B +
5*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + a*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.174504, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4054, 12, 3793, 3792}

$$\frac{64a^3(7B+5C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(7B+5C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7B+5C)\tan(c+dx)(a\sec(c+dx)}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (64*a^3*(7*B + 5*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^
2*(7*B + 5*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*a*(7*B +
5*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + a*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
 !LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{2\int \frac{1}{2}a(7B+5C)\,\mathrm{s}}{}$$
$$= \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{1}{7}(7B+5C)\int \sec$$
$$= \frac{2a(7B+5C)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2C(a+a}{}$$
$$= \frac{16a^2(7B+5C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} + \frac{2a(7B+}{}$$
$$= \frac{64a^3(7B+5C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(7B+5C)\sqrt{a+a\sec(c}}{105d}$$

Mathematica [A] time = 0.35133, size = 79, normalized size = 0.57

$$\frac{2a^3\tan(c+dx)\left(3(7B+20C)\sec^2(c+dx)+(98B+115C)\sec(c+dx)+301B+15C\sec^3(c+dx)+230C\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a^3*(301*B + 230*C + (98*B + 115*C)*Sec[c + d*x] + 3*(7*B + 20*C)*Sec[c
+ d*x]^2 + 15*C*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [A] time = 0.271, size = 119, normalized size = 0.9

$$-\frac{2a^2(-1+\cos(dx+c))\left(301B(\cos(dx+c))^3+230C(\cos(dx+c))^3+98B(\cos(dx+c))^2+115C(\cos(dx+c))^2+21}{105d(\cos(dx+c))^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(301*B*cos(d*x+c)^3+230*C*cos(d*x+c)^3+98*B*co
s(d*x+c)^2+115*C*cos(d*x+c)^2+21*B*cos(d*x+c)+60*C*cos(d*x+c)+15*C)*(a*(cos
(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.495352, size = 292, normalized size = 2.12

$$\frac{2\left((301\,B+230\,C)a^2\cos\left(dx+c\right)^3+(98\,B+115\,C)a^2\cos\left(dx+c\right)^2+3\,(7\,B+20\,C)a^2\cos\left(dx+c\right)+15\,Ca^2\right)\sqrt{\frac{a\cos(dx}{\cos(dx}}}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 2/105*((301*B + 230*C)*a^2*cos(d*x + c)^3 + (98*B + 115*C)*a^2*cos(d*x + c)
^2 + 3*(7*B + 20*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.79268, size = 300, normalized size = 2.17

$$-\frac{8\left(105\,\sqrt{2}Ba^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\,\sqrt{2}Ca^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(245\,\sqrt{2}Ba^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+175\,\sqrt{2}Ca^6\mathrm{sgn}\left(\mathrm{c}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] -8/105*(105*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^6*sgn(cos(d*x
 + c)) - (245*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 175*sqrt(2)*C*a^6*sgn(cos(d
*x + c)) - 4*(49*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 35*sqrt(2)*C*a^6*sgn(cos
(d*x + c)) - 2*(7*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 5*sqrt(2)*C*a^6*sgn(cos
(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.378 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2} \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=142

$$\frac{2a^3(35B+32C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(5B+8C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a^{5/2}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c}{}$$

```
[Out] (2*a^(5/2)*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(35*B + 32*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(5*B + 8*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*C*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.317288, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(35B+32C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(5B+8C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a^{5/2}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (2*a^(5/2)*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(35*B + 32*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(5*B + 8*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*C*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+a\sec(c+dx))^{5/2}(B+C\sec(c+dx))$$

$$= \frac{2aC(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}$$

$$= \frac{2a^2(5B+8C)\sqrt{a+a\sec(c+dx)}\tan(c+d}{15d}$$

$$= \frac{2a^2(5B+8C)\sqrt{a+a\sec(c+dx)}\tan(c+d}{15d}$$

$$= \frac{2a^3(35B+32C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(5B+8}{}$$

$$= \frac{2a^{5/2}B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3(35B+}{15d\sqrt{a+}}$$

Mathematica [A] time = 0.857512, size = 128, normalized size = 0.9

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(5B+14C)\cos(c+dx)+(40B+43C)\cos(2(c}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(30*Sqrt[2]
*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(40*B + 49*C + 2
*(5*B + 14*C)*Cos[c + d*x] + (40*B + 43*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/
2]))/(30*d)
```

Maple [B] time = 0.281, size = 341, normalized size = 2.4

$$-\frac{a^2}{60\,d\sin(dx+c)(\cos(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\,B(\cos(dx+c))^2\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{5/2}\mathrm{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/60/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*B*cos(d*x+c)^2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+30*B*cos(d*x+c)*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+15*B*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+320*B*cos(d*x+c)^3+344*C*cos(
d*x+c)^3-280*B*cos(d*x+c)^2-232*C*cos(d*x+c)^2-40*B*cos(d*x+c)-88*C*cos(d*x
+c)-24*C)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [B] time = 1.93491, size = 1885, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 0.572533, size = 954, normalized size = 6.72

$$\left[\frac{15\left(Ba^2\cos(dx+c)^3 + Ba^2\cos(dx+c)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx+c) - a}{\cos(dx+c)+1}\right) + 2\left((4\right.}{15\left(d\cos(dx+c)^3 + d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] [1/15*(15*(B*a^2*cos(d*x + c)^3 + B*a^2*cos(d*x + c)^2)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((40*B + 43
*C)*a^2*cos(d*x + c)^2 + (5*B + 14*C)*a^2*cos(d*x + c) + 3*C*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), -2/15*(15*(B*a^2*cos(d*x + c)^3 + B*a^2*cos(d*x + c)^2)*sqrt(a)*ar
ctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - ((40*B + 43*C)*a^2*cos(d*x + c)^2 + (5*B + 14*C)*a^2*cos(d*x + c)
+ 3*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.379 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=143

$$-\frac{a^3(3B+14C)\sin(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(B+2C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{a^{5/2}(5B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\,\mathrm{si}}{}$$

```
[Out] (a^(5/2)*(5*B + 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d - (a^3*(3*B + 14*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(B + 2*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*C*(a + a*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.515374, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4018, 4015, 3774, 203}

$$-\frac{a^3(3B+14C)\sin(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(B+2C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{a^{5/2}(5B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aC\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^(5/2)*(5*B + 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d - (a^3*(3*B + 14*C)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(B + 2*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*C*(a + a*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+a\sec(c+dx))^{5/2}(B+C$$

$$= \frac{2aC(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2}{3}$$

$$= \frac{2a^2(B+2C)\sqrt{a+a\sec(c+dx)}\sin(c+dx}{d}$$

$$= -\frac{a^3(3B+14C)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(B+2C)}{}$$

$$= -\frac{a^3(3B+14C)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(B+2C)}{}$$

$$= \frac{a^{5/2}(5B+2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^3(}{3}$$

Mathematica [A] time = 0.806314, size = 126, normalized size = 0.88

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(5B+2C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+\sin\left(\frac{1}{2}(c\right.\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(5
*B + 2*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (3*B + 4*C
+ 4*(3*B + 8*C)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*
d)
```

Maple [B] time = 0.364, size = 256, normalized size = 1.8

$$-\frac{a^2}{6d\cos(dx+c)\sin(dx+c)}\left(15B\cos(dx+c)\sqrt{2}\sin(dx+c)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/6/d*a^2*(15*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))+6*C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))+6*B*cos(d*x+c)^3+6*B*cos(d*x+c)^2+32*C*cos(d*x+c)^2-12*B*cos
(d*x+c)-28*C*cos(d*x+c)-4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)
/sin(d*x+c)
```

Maxima [B] time = 2.37032, size = 3753, normalized size = 26.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x +
2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x
 + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) +
 4*a^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c
) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c)
 - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*
a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)
^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
 + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*s
qrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
```

```
)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*c
os(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(
(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) +
 a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))*C/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.659229, size = 968, normalized size = 6.77

$$\left[\frac{3\left((5B+2C)a^2\cos(dx+c)^2+(5B+2C)a^2\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos\cdots}{\cos(dx+c)+1}\right.\cdots}{6\left(d\cos(dx+c)^2+d\cos\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/6*(3*((5*B + 2*C)*a^2*cos(d*x + c)^2 + (5*B + 2*C)*a^2*cos(d*x + c))*sqr
t(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
 + 2*(3*B*a^2*cos(d*x + c)^2 + 2*(3*B + 8*C)*a^2*cos(d*x + c) + 2*C*a^2)*sq
```

```
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*c
os(d*x + c)), -1/3*(3*((5*B + 2*C)*a^2*cos(d*x + c)^2 + (5*B + 2*C)*a^2*cos
(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c))) - (3*B*a^2*cos(d*x + c)^2 + 2*(3*B + 8*C)*a^2*c
os(d*x + c) + 2*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.68909, size = 644, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/6*(3*(5*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2*C*sqrt(-a)*a^2*sgn(cos(d*x
+ c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) +
 2*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 12*sqrt(2
)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - B*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2) + 4*(3*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 9*sqrt(2)*C*a^4*sgn(cos(d*x +
c)) - (3*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 7*sqrt(2)*C*a^4*sgn(cos(d*x + c)
))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.380 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=154

$$\frac{a^3(9B-4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(B-4C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19B+20C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin}{}$$

```
[Out] (a^(5/2)*(19*B + 20*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(9*B - 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
- (a^2*(B - 4*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*B*Cos[c
+ d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.528559, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4017, 4018, 4015, 3774, 203}

$$\frac{a^3(9B-4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(B-4C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19B+20C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^(5/2)*(19*B + 20*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(9*B - 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
- (a^2*(B - 4*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*B*Cos[c
+ d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}(B+C\,\mathrm{se}$$

$$= \frac{aB\cos(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+d}{2d}$$

$$= -\frac{a^2(B-4C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d} +$$

$$= \frac{a^3(9B-4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(B-4C)\sqrt{a+a}}{}$$

$$= \frac{a^3(9B-4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(B-4C)\sqrt{a+a}}{}$$

$$= \frac{a^{5/2}(19B+20C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^3(9}{4d}$$

Mathematica [A] time = 0.824339, size = 116, normalized size = 0.75

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(19B+20C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)((11B}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(19*B + 20*C)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(B + 8*C + (11*B + 4*C
)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.388, size = 410, normalized size = 2.7

$$\frac{a^2}{16\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(19\,B\cos(dx+c)\sin(dx+c)\sqrt{2}\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\text{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/16/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(19*B*cos(d*x+c)*sin(d*x+c)*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+20*C*cos(d*x+c)*sin(d*x+c
)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+19*B*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+20*C*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*B*cos(d*x+c)^4-36*B*cos(d*x+c)^3-16*C*c
os(d*x+c)^3+44*B*cos(d*x+c)^2-16*C*cos(d*x+c)^2+32*C*cos(d*x+c))/cos(d*x+c)
/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.655749, size = 890, normalized size = 5.78

$$\left[\frac{\left((19\,B+20\,C)a^2\cos(dx+c)+(19\,B+20\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)\cdot}{8\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/8*(((19*B + 20*C)*a^2*cos(d*x + c) + (19*B + 20*C)*a^2)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*B*a^2
*cos(d*x + c)^2 + (11*B + 4*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((19*B +
20*C)*a^2*cos(d*x + c) + (19*B + 20*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*B*a^2*cos
```

```
(d*x + c)^2 + (11*B + 4*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)

[Out] Timed out
```

Giac [B] time = 7.092, size = 957, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")

[Out] -1/8*(16*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a^3*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (19*B*sqrt(-a)*a^2*
sgn(cos(d*x + c)) + 20*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2
) + 3))) - (19*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 20*C*sqrt(-a)*a^2*sgn(cos
(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(19*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^3*sgn(cos(
d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^4*sgn(cos(d*x
+ c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 89*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c))
 + 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 9*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) -
4*C*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.381 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=164

$$\frac{a^3(49B+54C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(25B+38C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(3B+2C)\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+}}{4d}$$

```
[Out] (a^(5/2)*(25*B + 38*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*B + 54*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(3*B + 2*C)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
4*d) + (a*B*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.566515, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {4072, 4017, 4015, 3774, 203}

$$\frac{a^3(49B+54C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(25B+38C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(3B+2C)\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^(5/2)*(25*B + 38*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*B + 54*C)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(3*B + 2*C)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
4*d) + (a*B*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}(B+C\,\mathrm{se}$$

$$= \frac{aB\cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+}{3d}$$

$$= \frac{a^2(3B+2C)\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\mathrm{si}}{4d}$$

$$= \frac{a^3(49B+54C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(3B+2C)\,\mathrm{co}}{}$$

$$= \frac{a^3(49B+54C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(3B+2C)\,\mathrm{co}}{}$$

$$= \frac{a^{5/2}(25B+38C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{a^3(4}{24}$$

Mathematica [A] time = 1.29786, size = 121, normalized size = 0.74

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\cos(c+dx)\sqrt{\sec(c+dx)-1}(2(17B+6C)\cos(c+dx)+4B\cos(2(c+dx))+79B\right.}{24d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(3*(25*B + 38*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]] + Cos[c + d*x]*(79*B
+ 66*C + 2*(17*B + 6*C)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sqrt[-1 + Sec[
c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(24*d*Sqrt[-1 + Sec
[c + d*x]])
```

Maple [B] time = 0.567, size = 583, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d*a^2*(75*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*sin(d*x+c)+114*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*sin(d*x+c)+150*B*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*sin(d*x+c)+228*C*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)/cos(d*x+c))*sin(d*x+c)+75*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)+114*C*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*sin(d*x+c)+64*B*cos(d*x+c)^6+208*B*cos(d*x+c)^5+96*C*cos(d*x+c)^5+328*B*co
s(d*x+c)^4+432*C*cos(d*x+c)^4-600*B*cos(d*x+c)^3-528*C*cos(d*x+c)^3)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.655, size = 976, normalized size = 5.95

$$\left[\frac{3\left((25\,B+38\,C)a^2\cos\left(dx+c\right)+(25\,B+38\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{48\left(d\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((25*B + 38*C)*a^2*cos(d*x + c) + (25*B + 38*C)*a^2)*sqrt(-a)*log(
(2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*B*
a^2*cos(d*x + c)^3 + 2*(17*B + 6*C)*a^2*cos(d*x + c)^2 + 3*(25*B + 22*C)*a^
2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d), -1/24*(3*((25*B + 38*C)*a^2*cos(d*x + c) + (25*B + 38*C)*a
^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - (8*B*a^2*cos(d*x + c)^3 + 2*(17*B + 6*C)*a^2*cos(d*x
 + c)^2 + 3*(25*B + 22*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.6083, size = 1177, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/48*(3*(25*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 38*C*sqrt(-a)*a^2*sgn(cos(d
*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(25*B*sqrt(-a)*a^2*sgn(cos(d*x + c
)) + 38*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sq
rt(2)*(75*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^10*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 114*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^3*sgn(cos(d*x + c
)) - 1125*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1710*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^4*sgn(cos(d*x + c)
) + 6174*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 6804*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^5*sgn(cos(d*x + c))
 - 4314*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 4284*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^6*sgn(cos(d*x + c))
+ 807*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 858*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 4
9*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 54*C*sqrt(-a)*a^8*sgn(cos(d*x + c)))/(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
+ a^2)^3)/d
```

3.382 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=209

$$\frac{a^3(163B+200C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163B+200C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(11B+8C)\sin(c+dx)\cos^2(c+dx)\sqrt{a\,s}}{24d}$$

```
[Out] (a^(5/2)*(163*B + 200*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^3*(163*B + 200*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a^3*(95*B + 104*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*S
ec[c + d*x]]) + (a^2*(11*B + 8*C)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(24*d) + (a*B*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c
+ d*x])/(4*d)
```

Rubi [A] time = 0.666967, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4017, 4015, 3805, 3774, 203}

$$\frac{a^3(163B+200C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163B+200C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(11B+8C)\sin(c+dx)\cos^2(c+dx)\sqrt{a\,s}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^(5/2)*(163*B + 200*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(64*d) + (a^3*(163*B + 200*C)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a^3*(95*B + 104*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*S
ec[c + d*x]]) + (a^2*(11*B + 8*C)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(24*d) + (a*B*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c
+ d*x])/(4*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}(B+C\,\mathrm{se}$$

$$= \frac{aB\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+}{4d}$$

$$= \frac{a^2(11B+8C)\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}}{24d}$$

$$= \frac{a^3(95B+104C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{a^3(163B+200C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(95B+104}{96d}$$

$$= \frac{a^3(163B+200C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(95B+104}{96d}$$

$$= \frac{a^{5/2}(163B+200C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{a}{}$$

Mathematica [C] time = 1.29094, size = 366, normalized size = 1.75

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4608B\sqrt{1-\sec(c+dx)}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)+7680C\sqrt{1-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(6075*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 6600*C*ArcTanh[Sqrt[1 - Sec[
c + d*x]]] + 2079*B*Sqrt[1 - Sec[c + d*x]] + 1240*C*Sqrt[1 - Sec[c + d*x]]
+ 7641*B*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 6360*C*Cos[c + d*x]*Sqrt[1 -
 Sec[c + d*x]] + 2097*B*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1240*C*Co
s[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 522*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec
[c + d*x]] - 80*C*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 18*B*Cos[4*(c +
 d*x)]*Sqrt[1 - Sec[c + d*x]] + 7680*C*Hypergeometric2F1[1/2, 4, 3/2, 1 - S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 4608*B*Hypergeometric2F1[1/2, 5, 3/2,
 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c
 + d*x])/(2880*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.42, size = 765, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3072/d*a^2*(489*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(7/2)*2^(1/2)+600*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)*2^(1/2)+1467*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)+1800*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1467*B*sin(d*x+c)*cos(d*x+c)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1800*C*sin(d*x+c)*cos(d*x+c)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+489*B*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+600*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*sin(d*x+c)-768*B*cos(d*x+c)^8-2176*B*cos(d*x+c)^7-1024*C*c
os(d*x+c)^7-2272*B*cos(d*x+c)^6-3328*C*cos(d*x+c)^6-2608*B*cos(d*x+c)^5-524
8*C*cos(d*x+c)^5+7824*B*cos(d*x+c)^4+9600*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.749236, size = 1103, normalized size = 5.28

$$\left[\frac{3\left((163\,B+200\,C)a^2\cos\left(dx+c\right)+(163\,B+200\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/384*(3*((163*B + 200*C)*a^2*cos(d*x + c) + (163*B + 200*C)*a^2)*sqrt(-a)
*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*
(48*B*a^2*cos(d*x + c)^4 + 8*(23*B + 8*C)*a^2*cos(d*x + c)^3 + 2*(163*B + 1
36*C)*a^2*cos(d*x + c)^2 + 3*(163*B + 200*C)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*(
(163*B + 200*C)*a^2*cos(d*x + c) + (163*B + 200*C)*a^2)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
(48*B*a^2*cos(d*x + c)^4 + 8*(23*B + 8*C)*a^2*cos(d*x + c)^3 + 2*(163*B + 1
36*C)*a^2*cos(d*x + c)^2 + 3*(163*B + 200*C)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.96438, size = 1480, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/384*(3*(163*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*C*sqrt(-a)*a^2*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*B*sqrt(-a)*a^2*sgn(cos(d*x
 + c)) + 200*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) +
 4*sqrt(2)*(489*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^14*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 600*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^3*sgn(cos(d
```

```
*x + c)) - 10269*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^12*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 12600*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^4*sgn(co
s(d*x + c)) + 69885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^10*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 103992*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^5*sg
n(cos(d*x + c)) - 259233*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 339864*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^6
*sgn(cos(d*x + c)) + 209979*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 262920*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*
a^7*sgn(cos(d*x + c)) - 55511*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 73640*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)
*a^8*sgn(cos(d*x + c)) + 6687*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 8808*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*
a^9*sgn(cos(d*x + c)) - 299*B*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 392*C*sqrt(
-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.383 $\int \cos^6(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=254

$$\frac{a^3(283B+326C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283B+326C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(13B+10C)\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec}}{40d}$$

```
[Out] (a^(5/2)*(283*B + 326*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*B + 326*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(283*B + 326*C)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^3*(157*B + 170*C)*Cos[c + d*x]^2*Sin[c + d*x])/(240*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(13*B + 10*C)*Cos[c + d*x]^3*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^4*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.749418, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4017, 4015, 3805, 3774, 203}

$$\frac{a^3(283B+326C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283B+326C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(13B+10C)\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^(5/2)*(283*B + 326*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*B + 326*C)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(283*B + 326*C)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^3*(157*B + 170*C)*Cos[c + d*x]^2*Sin[c + d*x])/(240*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(13*B + 10*C)*Cos[c + d*x]^3*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^4*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}(B+$$

$$= \frac{aB\cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c}{5d}$$

$$= \frac{a^2(13B+10C)\cos^3(c+dx)\sqrt{a+a\sec(c+}}{40d}$$

$$= \frac{a^3(157B+170C)\cos^2(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(283B+326C)\cos(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(283B+326C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(283B}{1}$$

$$= \frac{a^3(283B+326C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(283B}{1}$$

$$= \frac{a^{5/2}(283B+326C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d}$$

Mathematica [C] time = 1.74728, size = 416, normalized size = 1.64

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(15360B\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},6,\frac{3}{2},1-\sec(c+dx)\right)+21504C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(25935*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 28350*C*ArcTanh[Sqrt[1 - Se
c[c + d*x]]] + 11651*B*Sqrt[1 - Sec[c + d*x]] + 9702*C*Sqrt[1 - Sec[c + d*x
]] + 37029*B*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 35658*C*Cos[c + d*x]*Sqr
t[1 - Sec[c + d*x]] + 12653*B*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 978
6*C*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 3818*B*Cos[3*(c + d*x)]*Sqrt[
1 - Sec[c + d*x]] + 2436*C*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1002*B
*Cos[4*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 84*C*Cos[4*(c + d*x)]*Sqrt[1 - S
ec[c + d*x]] + 72*B*Cos[5*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 21504*C*Hyper
geometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 15360*
B*Hypergeometric2F1[1/2, 6, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*
Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(13440*d*(1 + Cos[c + d*x])*Sqrt[1
 - Sec[c + d*x]])
```

Maple [B] time = 0.366, size = 947, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/61440/d*a^2*(4245*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)
*cos(d*x+c)^4*2^(1/2)+4890*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(
d*x+c)*cos(d*x+c)^4*2^(1/2)+16980*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)+19560*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)+25470*B*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)+29340*C*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)+16980*B*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+19560*C*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4245*B*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+4890*C*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+12288*B*cos(d*x+c)^10+3
2256*B*cos(d*x+c)^9+15360*C*cos(d*x+c)^9+27904*B*cos(d*x+c)^8+43520*C*cos(d
*x+c)^8+18112*B*cos(d*x+c)^7+45440*C*cos(d*x+c)^7+45280*B*cos(d*x+c)^6+5216
0*C*cos(d*x+c)^6-135840*B*cos(d*x+c)^5-156480*C*cos(d*x+c)^5)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.764589, size = 1227, normalized size = 4.83

$$\left[15\left((283\,B + 326\,C)a^2\cos\left(dx + c\right) + (283\,B + 326\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(\ldots}{\cos(dx+c)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/3840*(15*((283*B + 326*C)*a^2*cos(d*x + c) + (283*B + 326*C)*a^2)*sqrt(-
a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) +
2*(384*B*a^2*cos(d*x + c)^5 + 48*(29*B + 10*C)*a^2*cos(d*x + c)^4 + 8*(283*
B + 230*C)*a^2*cos(d*x + c)^3 + 10*(283*B + 326*C)*a^2*cos(d*x + c)^2 + 15*
(283*B + 326*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c) + d), -1/1920*(15*((283*B + 326*C)*a^2*cos(d*x
 + c) + (283*B + 326*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (384*B*a^2*cos(d*x + c)^5 +
 48*(29*B + 10*C)*a^2*cos(d*x + c)^4 + 8*(283*B + 230*C)*a^2*cos(d*x + c)^3
 + 10*(283*B + 326*C)*a^2*cos(d*x + c)^2 + 15*(283*B + 326*C)*a^2*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.22258, size = 1782, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(283*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*C*sqrt(-a)*a^2*sgn(
cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(283*B*sqrt(-a)*a^2*sgn(cos(
d*x + c)) + 326*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(4245*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^18*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 4890*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*sqrt(-a)*a^3*sgn(
cos(d*x + c)) - 114615*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^16*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 132030*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*sqrt(-a)*a^4
*sgn(cos(d*x + c)) + 1298820*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 1319880*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(
-a)*a^5*sgn(cos(d*x + c)) - 6176700*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 688812
0*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*
C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 16394598*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^7*sgn(cos(d*x + c))
+ 18352620*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^10*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 14042770*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^8*sgn(cos(d*
x + c)) - 15746180*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 4791060*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^9*sgn(
cos(d*x + c)) + 5497320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 860300*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^10
*sgn(cos(d*x + c)) - 959320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^10*sgn(cos(d*x + c)) + 75885*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*
a^11*sgn(cos(d*x + c)) + 84810*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^11*sgn(cos(d*x + c)) - 2671*B*sqrt
(-a)*a^12*sgn(cos(d*x + c)) - 2990*C*sqrt(-a)*a^12*sgn(cos(d*x + c)))/((sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^
2)^5)/d
```

3.384 $\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=243

$$\frac{2(9B-C)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}-\frac{2(3B-19C)\tan(c+dx)\sec^2(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2}{}$$

```
[Out] (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(111*B - 143*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*S
ec[c + d*x]]) - (2*(3*B - 19*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*(9*B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec
[c + d*x]]) + (2*(93*B - 29*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*
a*d)
```

Rubi [A] time = 0.876197, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4021, 4010, 4001, 3795, 203}

$$\frac{2(9B-C)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}-\frac{2(3B-19C)\tan(c+dx)\sec^2(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c +
 d*x]],x]
```

```
[Out] (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(111*B - 143*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*S
ec[c + d*x]]) - (2*(3*B - 19*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*(9*B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec
[c + d*x]]) + (2*(93*B - 29*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*
a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^5(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2C\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^4(c+dx)\left(4aC+\frac{1}{2}a(9B-C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{9a}$$

$$= \frac{2(9B-C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{2(3B-19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(9B-C)\sec^3(c+dx)\,\mathrm{t}}{63d\sqrt{a+a\sec(c+}}$$

$$= -\frac{2(3B-19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(9B-C)\sec^3(c+dx)\,\mathrm{t}}{63d\sqrt{a+a\sec(c+}}$$

$$= -\frac{4(111B-143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} - \frac{2(3B-19C)\sec^2(c+dx)\tan(c+}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{4(111B-143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} - \frac{2(3B-19C)\sec^2(c+dx)\tan(c+}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(111B-143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 1.17028, size = 183, normalized size = 0.75

$$\frac{\tan(c+dx)\left(\frac{1}{4}\sqrt{1-\sec(c+dx)}\sec^4(c+dx)((918B-214C)\cos(c+dx)-8(69B-157C)\cos(2(c+dx))+186B\cos(3\right.}{315d\sqrt{1-\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*S
ec[c + d*x]],x]
```

```
[Out] ((315*Sqrt[2]*(B - C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + ((-423*B +
1279*C + (918*B - 214*C)*Cos[c + d*x] - 8*(69*B - 157*C)*Cos[2*(c + d*x)] +
 186*B*Cos[3*(c + d*x)] - 58*C*Cos[3*(c + d*x)] - 129*B*Cos[4*(c + d*x)] +
257*C*Cos[4*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^4)/4)*Tan[c + d
*x])/(315*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.43, size = 975, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5040/d/a*(315*B*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c
)^4*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))-315*C*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^4*l
n(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)+1260*B*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^3*ln(((
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-12
60*C*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^3*ln(((-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+1890*B
*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^2*ln(((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-1890*C*sin
(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^2*ln(((-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+1260*B*sin(d*x
+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-1260*C*sin(d*x+c)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+315*B*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(9/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))*sin(d*x+c)-315*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*l
n(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)*sin(d*x+c)+4128*B*cos(d*x+c)^5-8224*C*cos(d*x+c)^5-7104*B*cos(d*x+c)^4+91
52*C*cos(d*x+c)^4+3264*B*cos(d*x+c)^3-2752*C*cos(d*x+c)^3-1728*B*cos(d*x+c)
^2+1984*C*cos(d*x+c)^2+1440*B*cos(d*x+c)-1280*C*cos(d*x+c)+1120*C)*(a*(cos(
d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.661966, size = 1211, normalized size = 4.98

$$\left[-\frac{315\sqrt{2}\left((B-C)a\cos(dx+c)^5+(B-C)a\cos(dx+c)^4\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [-1/630*(315*sqrt(2)*((B - C)*a*cos(d*x + c)^5 + (B - C)*a*cos(d*x + c)^4)*
sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a
)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((129*B - 257*C)*cos(d*x + c)^4 - (93*
B - 29*C)*cos(d*x + c)^3 + 3*(3*B - 19*C)*cos(d*x + c)^2 - 5*(9*B - C)*cos(
d*x + c) - 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d
*cos(d*x + c)^5 + a*d*cos(d*x + c)^4), -1/315*(2*((129*B - 257*C)*cos(d*x +
 c)^4 - (93*B - 29*C)*cos(d*x + c)^3 + 3*(3*B - 19*C)*cos(d*x + c)^2 - 5*(9
*B - C)*cos(d*x + c) - 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c) + 315*sqrt(2)*((B - C)*a*cos(d*x + c)^5 + (B - C)*a*cos(d*x + c)^4)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\sec^5(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 9.22317, size = 528, normalized size = 2.17

$$\frac{\frac{315(\sqrt{2}B-\sqrt{2}C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\left(\frac{315\sqrt{2}Ca^4}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\left(420\sqrt{2}Ba^4\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-840\sqrt{2}Ca^4\mathrm{sgn}\left(\tan\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/315*(315*(sqrt(2)*B - sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) +
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)) - 2*(315*sqrt(2)*C*a^4/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + (420*sqrt(2
)*B*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 840*sqrt(2)*C*a^4*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1) - (756*sqrt(2)*B*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 163
8*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (612*sqrt(2)*B*a^4*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1) - 936*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 -
1) - (276*sqrt(2)*B*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 383*sqrt(2)*C*a^4
*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2
*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)
/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.385 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=202

$$\frac{2(7B-C)\tan(c+dx)\sec^2(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(7B-31C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}-$$

```
[Out] -((Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(49*B - 37*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*
Sec[c + d*x]]) + (2*(7*B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(7*B - 31*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.693954, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4021, 4010, 4001, 3795, 203}

$$\frac{2(7B-C)\tan(c+dx)\sec^2(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(7B-31C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c +
 d*x]],x]
```

```
[Out] -((Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(49*B - 37*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*
Sec[c + d*x]]) + (2*(7*B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(7*B - 31*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
```

```
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$=\frac{2C\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}+\frac{2\int\frac{\sec^3(c+dx)\left(3aC+\frac{1}{2}a(7B-C)\sec(c+d$$

$$=\frac{2(7B-C)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sec^3(c+dx)\tan(c+$$

$$=\frac{2(7B-C)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sec^3(c+dx)\tan(c+$$

$$=\frac{4(49B-37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2(7B-C)\sec^2(c+dx)\tan(c+dx$$

$$=\frac{4(49B-37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2(7B-C)\sec^2(c+dx)\tan(c+dx$$

$$=-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{4(49B-37C)\tan(c+$$

Mathematica [A] time = 0.527371, size = 140, normalized size = 0.69

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(3(7B-C)\sec^2(c+dx)+(31C-7B)\sec(c+dx)+91B+15C\sec^3(c+dx)-43C\right)-}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*S
ec[c + d*x]],x]
```

```
[Out] ((-105*Sqrt[2]*(B - C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 -
 Sec[c + d*x]]*(91*B - 43*C + (-7*B + 31*C)*Sec[c + d*x] + 3*(7*B - C)*Sec[
c + d*x]^2 + 15*C*Sec[c + d*x]^3))*Tan[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*
x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.371, size = 785, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/840/d/a*(105*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x
+c)^3-105*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3
+315*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-315*
C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+315*B*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-315*C*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+105*B*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-105*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2
)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x
+c))*sin(d*x+c)-1456*B*cos(d*x+c)^4+688*C*cos(d*x+c)^4+1568*B*cos(d*x+c)^3-
1184*C*cos(d*x+c)^3-448*B*cos(d*x+c)^2+544*C*cos(d*x+c)^2+336*B*cos(d*x+c)-
288*C*cos(d*x+c)+240*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/si
n(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.623026, size = 1116, normalized size = 5.52

$$\left[-\frac{105\sqrt{2}\left((B-C)a\cos(dx+c)^4+(B-C)a\cos(dx+c)^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{210\left(ad\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [-1/210*(105*sqrt(2)*((B - C)*a*cos(d*x + c)^4 + (B - C)*a*cos(d*x + c)^3)*
sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/
a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((91*B - 43*C)*cos(d*x + c)^3 - (7*B
- 31*C)*cos(d*x + c)^2 + 3*(7*B - C)*cos(d*x + c) + 15*C)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)
^3), 1/105*(2*((91*B - 43*C)*cos(d*x + c)^3 - (7*B - 31*C)*cos(d*x + c)^2 +
 3*(7*B - C)*cos(d*x + c) + 15*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c) + 105*sqrt(2)*((B - C)*a*cos(d*x + c)^4 + (B - C)*a*cos(d*x + c
)^3)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\sec^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 9.10852, size = 387, normalized size = 1.92

$$-\frac{\frac{105\sqrt{2}(B-C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\left(\frac{105\sqrt{2}Ba^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\left(\left(\frac{\sqrt{2}(119Ba^3-92Ca^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{7\sqrt{2}(37Ba^3-16C}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -1/105*(105*sqrt(2)*(B - C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
- 2*(105*sqrt(2)*B*a^3/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - ((sqrt(2)*(119*B*a
^3 - 92*C*a^3)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 7*s
qrt(2)*(37*B*a^3 - 16*C*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x +
 1/2*c)^2 + 35*sqrt(2)*(7*B*a^3 - 4*C*a^3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 -
a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.386 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(5B-C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}-\frac{4(5B-7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c}{5d\sqrt{a}}$$

```
[Out] (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(5*B - 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*(5*B - C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.514366, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4021, 4010, 4001, 3795, 203}

$$\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(5B-C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}-\frac{4(5B-7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c}{5d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c +
 d*x]],x]
```

```
[Out] (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(5*B - 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*(5*B - C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
```

```
0] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^2(c+dx)\left(2aC+\frac{1}{2}a(5B-C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{5a}$$

$$= \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5B-C)\sqrt{a+a\sec(c+dx)}\tan(c}{15ad}$$

$$= -\frac{4(5B-7C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5B-}{}$$

$$= -\frac{4(5B-7C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5B-}{}$$

$$= \frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(5B-7C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2(}{}$$

Mathematica [A] time = 0.376557, size = 123, normalized size = 0.77

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((5B-C)\sec(c+dx)-5B+3C\sec^2(c+dx)+13C\right)+15\sqrt{2}(B-C)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx}}{\sqrt{2}}\right.\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*S
ec[c + d*x]],x]
```

```
[Out] ((15*Sqrt[2]*(B - C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - S
ec[c + d*x]]*(-5*B + 13*C + (5*B - C)*Sec[c + d*x] + 3*C*Sec[c + d*x]^2))*T
```

```
an[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.332, size = 595, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/60/d/a*(15*B*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x
+c)-15*C*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+30
*B*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-30*C*cos(d
*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+15*B*ln(((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-15*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*sin(d*x+c)+40*B*cos(d*x+c)^3-104*C*cos(d*x+c)^3-80*B*cos(d*x+c)^2+1
12*C*cos(d*x+c)^2+40*B*cos(d*x+c)-32*C*cos(d*x+c)+24*C)*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)^2}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^2/sqrt(a*sec(d*x
 + c) + a), x)
```

Fricas [A] time = 0.652144, size = 1019, normalized size = 6.41

$$\left[-\frac{15\sqrt{2}\left((B-C)a\cos\left(dx+c\right)^3 + (B-C)a\cos\left(dx+c\right)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\,c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{30\left(ad\cos\left(dx+c\right)^3+ad\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(2)*((B - C)*a*cos(d*x + c)^3 + (B - C)*a*cos(d*x + c)^2)*sq
rt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*
cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*B - 13*C)*cos(d*x + c)^2 - (5*B - C)
*cos(d*x + c) - 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/15*(2*((5*B - 13*C)*cos(d*x +
 c)^2 - (5*B - C)*cos(d*x + c) - 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c) + 15*sqrt(2)*((B - C)*a*cos(d*x + c)^3 + (B - C)*a*cos(d*x
+ c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\sec^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 8.94, size = 366, normalized size = 2.3

$$\frac{\frac{15\left(\sqrt{2}B-\sqrt{2}C\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\left(\left(10\sqrt{2}Ba^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-20\sqrt{2}Ca^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\left(10\sqrt{2}Ba\right.\right.\right.}{}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/15*(15*(sqrt(2)*B - sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - 2*((10*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 20*sqrt(2)*C*
a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (10*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1) - 17*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*C*a^2/sgn(tan(1/2*d*x + 1
/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.387 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

```
[Out] -((Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(3*B - 2*C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.303758, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4010, 4001, 3795, 203}

$$-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d
*x]],x]
```


```
[Out] -((Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(3*B - 2*C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$=\frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}+\frac{2\int \frac{\sec(c+dx)\left(\frac{aC}{2}+\frac{1}{2}a(3B-2C)\sec(c+$$

$$=\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}+(-B$$

$$=\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}+\frac{(2($$

$$=-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2$$

Mathematica [A] time = 0.296168, size = 106, normalized size = 0.9

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}(3B+C\sec(c+dx)-C)-3\sqrt{2}(B-C)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{3d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```

```
[Out] ((-3*Sqrt[2]*(B - C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - S
ec[c + d*x]]*(3*B - C + C*Sec[c + d*x]))*Tan[c + d*x])/(3*d*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.303, size = 405, normalized size = 3.4

$$-\frac{1}{6\,ad\sin(dx+c)\cos(dx+c)}\left(-3\,B\cos(dx+c)\sin(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+}{\cos(dx+c)}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d/a*(-3*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*l
n(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
```

```
)+3*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-3*B*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+3*C*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))*sin(d*x+c)+12*B*cos(d*x+c)^2-4*C*cos(d*x+c)^2-12*B*cos(d*x
+c)+8*C*cos(d*x+c)-4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(
d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)/sqrt(a*sec(d*x +
 c) + a), x)
```

Fricas [A] time = 0.642421, size = 917, normalized size = 7.77

$$\left[-\frac{3\sqrt{2}\left((B-C)a\cos\left(dx+c\right)^{2}+(B-C)a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^{2}-2\cos\cdots}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\cdots\right.}{6\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(2)*((B - C)*a*cos(d*x + c)^2 + (B - C)*a*cos(d*x + c))*sqrt(-
1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos
(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) - 4*((3*B - C)*cos(d*x + c) + C)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x +
 c)), 1/3*(2*((3*B - C)*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) + 3*sqrt(2)*((B - C)*a*cos(d*x + c)^2 + (B - C)*a*cos(d
*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A] time = 8.88951, size = 251, normalized size = 2.13

$$-\frac{\frac{3\sqrt{2}(B-C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(\frac{\sqrt{2}(3Ba-2Ca)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{3\sqrt{2}Ba}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(2)*(B - C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*
(sqrt(2)*(3*B*a - 2*C*a)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - 3*sqrt(2)*B*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.388 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (2*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0949332, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4054, 12, 3795, 203}

$$\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (2*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
 !LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}dx = \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2\int \frac{a(B-C)\sec(c+dx)}{2\sqrt{a+a\sec(c+dx)}}dx}{a}$$
$$= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(B-C)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}dx$$
$$= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2(B-C))\,\text{Subst}\left(\int \frac{1}{2a+x^2}dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.172361, size = 88, normalized size = 1.13

$$\frac{\tan(c+dx)\left(\sqrt{2}(B-C)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2C\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((Sqrt[2]*(B - C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*C*Sqrt[1 - Se
c[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [B] time = 0.252, size = 200, normalized size = 2.6

$$\frac{1}{ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(B\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\sqrt{-2\frac{c}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(B*ln(((-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2
*C*cos(d*x+c)+2*C)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{\sqrt{a\sec(dx+c)+a}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 0.586968, size = 751, normalized size = 9.63

$$\left[-\frac{\sqrt{2}((B-C)a\cos(dx+c)+(B-C)a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4C\sqrt{\cdots}}{2(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(2)*((B - C)*a*cos(d*x + c) + (B - C)*a)*sqrt(-1/a)*log((2*sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
+ c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - 4*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*
cos(d*x + c) + a*d), (2*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) - sqrt(2)*((B - C)*a*cos(d*x + c) + (B - C)*a)*arctan(sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a)
)/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 8.82177, size = 194, normalized size = 2.49

$$\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\left(\sqrt{2}B-\sqrt{2}C\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] (2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*tan(1/2*d*x + 1/2*c)/((a*t
an(1/2*d*x + 1/2*c)^2 - a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (sqrt(2)*B -
sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.389 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.186105, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 3920, 3774, 203, 3795}

$$\frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d
*x]],x]
```

```
[Out] (2*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{B+C\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{B\int\sqrt{a+a\sec(c+dx)}\,dx}{a} - (B-C)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= -\frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{(2(B-C))\,\mathrm{Subst}\,(}{}$$
$$= \frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.300487, size = 92, normalized size = 1.01

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((C-B)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (-B + C)*ArcTan[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [B] time = 0.249, size = 194, normalized size = 2.1

$$-\frac{1}{ad}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(B\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+B\ln\left(\frac{1}{\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))+B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))-C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-cos(d*x+c)+1)/sin(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.61816, size = 814, normalized size = 8.95

$$\left[-\frac{\sqrt{2}(B-C)a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+2B\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\ldots}}{\ldots}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(B - C)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 -
2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*B*sqrt(-a)*l
og((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d)
, (sqrt(2)*(B - C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*B*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\cos(c+dx)\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x)/sqrt(a*(sec(c + d*x
) + 1)), x)
```

Giac [B] time = 10.8623, size = 302, normalized size = 3.32

$$-\frac{\frac{\sqrt{2}(B-C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2B\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2B\log\left(\left|\left(\sqrt{-a}\ldots\right.\right.\right.}{\ldots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*(B - C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*B*lo
g(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*B*l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.390 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{(B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -(((B - 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x
]])
```

Rubi [A] time = 0.320073, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4022, 3920, 3774, 203, 3795}

$$-\frac{(B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c +
 d*x]],x]
```


```
[Out] -(((B - 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x
]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{B\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-\frac{1}{2}a(B-2C)+\frac{1}{2}aB\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a} \\
&= \frac{B\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(B-2C)\int\sqrt{a+a\sec(c+dx)}\,dx}{2a} + (B
\\
&= \frac{B\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(B-2C)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+}{\sqrt{a+a\sec(}}\right.}{d}
\\
&= -\frac{(B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{2}\sqrt{a+a\sec}}\right.}{\sqrt{a}d}
\end{aligned}$$

Mathematica [C] time = 26.4385, size = 10104, normalized size = 84.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*S
ec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.345, size = 353, normalized size = 3.

$$\frac{1}{2\,ad\sin(dx+c)}\left(B\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-2C\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*(B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*C
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*B*ln(((-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*C*ln(((-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-2*B*cos(d*x+c)^2+2*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/co
s(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^2}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/sqrt(a*sec(d*x
 + c) + a), x)
```

Fricas [A] time = 3.24055, size = 1214, normalized size = 10.2

$$\left[\frac{2B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right) - \sqrt{2}\left((B-C)a\cos\left(dx+c\right)+(B-C)a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\,s}{\cos(dx+c)^2 -}\right.}{2\,(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 - sqrt(2)*((B - C)*a*cos(d*x + c) + (B - C)*a)*sqrt(-1/a)*log((2*sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c)
 + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + ((B - 2*C)*cos(d*x + c) + B - 2*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2
 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d*cos(d*x + c) + a*d), (
B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + ((B -
 2*C)*cos(d*x + c) + B - 2*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((B - C)*a*cos(d*x
 + c) + (B - C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.3762, size = 531, normalized size = 4.46

$$\frac{\frac{\sqrt{2}(B-C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{(B-2C)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{(B-2C}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*(B - C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (B - 2*
C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
(B - 2*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2*B*sqrt(-a) - B*sqrt(-a)*a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sgn(tan(1/2*d*x + 1/2*
c)^2 - 1)))/d
```

3.391 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{(B-4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{(7B-4C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((7*B - 4*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((B - 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*
Sec[c + d*x]]) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]
])
```

Rubi [A] time = 0.470853, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4022, 3920, 3774, 203, 3795}

$$-\frac{(B-4C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{(7B-4C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{B\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c +
 d*x]],x]
```

```
[Out] ((7*B - 4*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sq
rt[a]*d) - (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((B - 4*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*
Sec[c + d*x]]) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]
])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{B\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{1}{2}a(B-4C)+\frac{3}{2}aB\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{2a}$$

$$= -\frac{(B-4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{1}{4}a^2(7B}{}$$

$$= -\frac{(B-4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(7B-4C}{}$$

$$= -\frac{(B-4C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(7B-4C}{}$$

$$= \frac{(7B-4C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{2}\sqrt{a+a\sec}}\right.}{\sqrt{a}d}$$

Mathematica [A] time = 0.411016, size = 135, normalized size = 0.82

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}(2B\cos(c+dx)-B+4C)+(7B-4C)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}(B\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*S
ec[c + d*x]],x]
```

```
[Out] (((7*B - 4*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*(B - C)*ArcTanh[S
qrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-B + 4*C + 2*B*Cos[c + d*x])
*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(
1 + Sec[c + d*x])])
```

Maple [B] time = 0.388, size = 717, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d/a*(-7*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))+4*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))-8*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2
)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x
+c))-7*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+8*C*c
os(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+4*C*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*B*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))*sin(d*x+c)+8*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(
((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*
sin(d*x+c)+8*B*cos(d*x+c)^4-12*B*cos(d*x+c)^3+16*C*cos(d*x+c)^3+4*B*cos(d*x
+c)^2-16*C*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin
(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^2 + B \sec \left(dx+c\right)\right) \cos \left(dx+c\right)^3}{\sqrt{a \sec \left(dx+c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^3/sqrt(a*sec(d*x
 + c) + a), x)
```

Fricas [A] time = 5.94974, size = 1323, normalized size = 8.02

$$\left[-\frac{4\sqrt{2}((B-C)a\cos\left(dx+c\right)+(B-C)a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-((7B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(2)*((B - C)*a*cos(d*x + c) + (B - C)*a)*sqrt(-1/a)*log(-(2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d
*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - ((7*B - 4*C)*cos(d*x + c) + 7*B - 4*C)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*B*cos(d*x
+ c)^2 - (B - 4*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(((7*B - 4*C)*cos(d*x + c) + 7*B
 - 4*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) - (2*B*cos(d*x + c)^2 - (B - 4*C)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*((B - C)*a*c
os(d*x + c) + (B - C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.5572, size = 876, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(2)*(B - C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (7*B
 - 4*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)) - (7*B - 4*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)) + 4*sqrt(2)*(17*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a) - 57*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a + 76*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a
)*a + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*B*sqrt(-a)*a^2 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^2 - 3*B*sqrt(-a)*a^3 + 4*C*sqrt(-a)*a^3
)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4
 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
```

```
2*a + a^2)^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.392 $\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=206

$$\frac{(7B-2C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9B-14C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(B-6C)\sin(c+dx}{12d\sqrt{a\sec(c+}$$

```
[Out] -((9*B - 14*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*B - 2*C)*Sin[c + d*x])/(8*d*Sqrt[a
+ a*Sec[c + d*x]]) - ((B - 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a
*Sec[c + d*x]]) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rubi [A] time = 0.645478, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4022, 3920, 3774, 203, 3795}

$$\frac{(7B-2C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9B-14C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(B-6C)\sin(c+dx}{12d\sqrt{a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c +
 d*x]],x]
```


```
[Out] -((9*B - 14*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*
Sqrt[a]*d) + (Sqrt[2]*(B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*B - 2*C)*Sin[c + d*x])/(8*d*Sqrt[a
+ a*Sec[c + d*x]]) - ((B - 6*C)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a
*Sec[c + d*x]]) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\cos^3(c+dx)(B+C\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$=\frac{B\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{\int \frac{\cos^2(c+dx)\left(-\frac{1}{2}a(B-6C)+\frac{5}{2}aB\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$=-\frac{(B-6C)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{B\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+$$

$$=\frac{(7B-2C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}-\frac{(B-6C)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{B\cos}{3d}$$

$$=\frac{(7B-2C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}-\frac{(B-6C)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{B\cos}{3d}$$

$$=\frac{(7B-2C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}-\frac{(B-6C)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{B\cos}{3d}$$

$$=-\frac{(9B-14C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+}{\sqrt{2}\sqrt{a+a\sec}}\right.}{\sqrt{a}d}$$

Mathematica [A] time = 0.762773, size = 150, normalized size = 0.73

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left(-2(B-6C)\cos(c+dx)+8B\cos^2(c+dx)+21B-6C\right)+(42C-27B)\tanh^{-1}\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*S
ec[c + d*x]],x]
```

```
[Out] (((-27*B + 42*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 24*Sqrt[2]*(B - C)*ArcTa
nh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(21*B - 6*C - 2*(B - 6*C)
*Cos[c + d*x] + 8*B*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(
24*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.349, size = 1067, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d/a*(-27*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*sin(d*x+c)+42*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*sin(d*x+c)-48*B*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*sin(d*x+c)-54*B*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*sin(d*x+c)+48*C*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*
sin(d*x+c)+84*C*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)-96*B*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*
x+c)-27*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+96*C
*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+
1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+42*C*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-48*B*ln(((-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+48*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*sin(d*x+c)+64*B*cos(d*x+c)^6-80*B*cos(d*x+c)^5+96*C*cos(d*x+c)^5+184*B
*cos(d*x+c)^4-144*C*cos(d*x+c)^4-168*B*cos(d*x+c)^3+48*C*cos(d*x+c)^3)*(a*(
cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^4}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^4/sqrt(a*sec(d*x
 + c) + a), x)
```

Fricas [A] time = 5.97143, size = 1426, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [-1/48*(24*sqrt(2)*((B - C)*a*cos(d*x + c) + (B - C)*a)*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 3*((9*B - 14*C)*cos(d*x + c) + 9*B - 14*C)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(8*B*cos
(d*x + c)^3 - 2*(B - 6*C)*cos(d*x + c)^2 + 3*(7*B - 2*C)*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d),
 1/24*(3*((9*B - 14*C)*cos(d*x + c) + 9*B - 14*C)*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (8*B*c
os(d*x + c)^3 - 2*(B - 6*C)*cos(d*x + c)^2 + 3*(7*B - 2*C)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 24*sqrt(2)*((B - C)*a*
cos(d*x + c) + (B - C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.7257, size = 1142, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(2)*(B - C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*(
9*B - 14*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)) - 3*(9*B - 14*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(165*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a) - 102*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a) - 1323*(sqrt(-a
```

```
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*
a + 954*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*C*sqrt(-a)*a + 3906*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^2 - 2268*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^2 - 2118*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^3
+ 1044*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^4*C*sqrt(-a)*a^3 + 393*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^4 - 222*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^4 - 31*B*sqrt(-a)*a^5
+ 18*C*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*a + a^2)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.393 $\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(15B-19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(273B-397C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{210a^2d}+\frac{(B-C)\tan(c+dx)\sec^4(c+d}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -((15*B - 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Sec[c + d*x]^4*Tan[c + d*x])/(2*d*
(a + a*Sec[c + d*x])^(3/2)) + ((651*B - 799*C)*Tan[c + d*x])/(105*a*d*Sqrt[
a + a*Sec[c + d*x]]) + ((63*B - 67*C)*Sec[c + d*x]^2*Tan[c + d*x])/(70*a*d*
Sqrt[a + a*Sec[c + d*x]]) - ((7*B - 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(14*
a*d*Sqrt[a + a*Sec[c + d*x]]) - ((273*B - 397*C)*Sqrt[a + a*Sec[c + d*x]]*T
an[c + d*x])/(210*a^2*d)
```

Rubi [A] time = 0.919155, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4019, 4021, 4010, 4001, 3795, 203}

$$-\frac{(15B-19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(273B-397C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{210a^2d}+\frac{(B-C)\tan(c+dx)\sec^4(c+d}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(3/2),x]
```


```
[Out] -((15*B - 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Sec[c + d*x]^4*Tan[c + d*x])/(2*d*
(a + a*Sec[c + d*x])^(3/2)) + ((651*B - 799*C)*Tan[c + d*x])/(105*a*d*Sqrt[
a + a*Sec[c + d*x]]) + ((63*B - 67*C)*Sec[c + d*x]^2*Tan[c + d*x])/(70*a*d*
Sqrt[a + a*Sec[c + d*x]]) - ((7*B - 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(14*
a*d*Sqrt[a + a*Sec[c + d*x]]) - ((273*B - 397*C)*Sqrt[a + a*Sec[c + d*x]]*T
an[c + d*x])/(210*a^2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^5(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^4(c+dx)\left(4a(B-C)-\frac{1}{2}a(7B-11C)\sec\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7B-11C)\sec^3(c+dx)\tan(c}{14ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(63B-67C)\sec^2(c+dx)\tan(}{70ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(63B-67C)\sec^2(c+dx)\tan(}{70ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(651B-799C)\tan(c+dx)}{105ad\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(651B-799C)\tan(c+dx)}{105ad\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(15B-19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(B-C)\sec^4(c+dx)\tan(}{2d(a+a\sec(c+dx))}$$

Mathematica [A] time = 1.23021, size = 204, normalized size = 0.78

$$\tan(c+dx)\left(\frac{1}{4}\sqrt{1-\sec(c+dx)}\sec^4(c+dx)(24(217B-213C)\cos(c+dx)+60(63B-67C)\cos(2(c+dx))+1512B\cos(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] ((-210*Sqrt[2]*(15*B - 19*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c
 + d*x)/2]^2*Sec[c + d*x] + ((2751*B - 2339*C + 24*(217*B - 213*C)*Cos[c +
d*x] + 60*(63*B - 67*C)*Cos[2*(c + d*x)] + 1512*B*Cos[3*(c + d*x)] - 1608*C
*Cos[3*(c + d*x)] + 1029*B*Cos[4*(c + d*x)] - 1201*C*Cos[4*(c + d*x)])*Sqrt
[1 - Sec[c + d*x]]*Sec[c + d*x]^4)/4)*Tan[c + d*x])/(420*d*Sqrt[1 - Sec[c +
 d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.37, size = 983, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3360/d/a^2*(-1+cos(d*x+c))*(-1575*B*cos(d*x+c)^4*sin(d*x+c)*ln(((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*
```

```
x+c)/(cos(d*x+c)+1))^(7/2)+1995*C*cos(d*x+c)^4*sin(d*x+c)*ln(((-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)-6300*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)^3+7980*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c
)*cos(d*x+c)^3-9450*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*co
s(d*x+c)^2+11970*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d
*x+c)^2-6300*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c
)+7980*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-1575
*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+1995*C*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+16464*B*cos(d*x+c)^5-19216*C*cos(d*x+c
)^5-4368*B*cos(d*x+c)^4+6352*C*cos(d*x+c)^4-13440*B*cos(d*x+c)^3+16000*C*co
s(d*x+c)^3+2688*B*cos(d*x+c)^2-3712*C*cos(d*x+c)^2-1344*B*cos(d*x+c)+1536*C
*cos(d*x+c)-960*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x
+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.672287, size = 1419, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [1/840*(105*sqrt(2)*((15*B - 19*C)*cos(d*x + c)^5 + 2*(15*B - 19*C)*cos(d*x
 + c)^4 + (15*B - 19*C)*cos(d*x + c)^3)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d
*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
4*((1029*B - 1201*C)*cos(d*x + c)^4 + 12*(63*B - 67*C)*cos(d*x + c)^3 - 28*
(3*B - 7*C)*cos(d*x + c)^2 + 12*(7*B - 3*C)*cos(d*x + c) + 60*C)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d
*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3), 1/420*(105*sqrt(2)*((15*B - 19*C)*
cos(d*x + c)^5 + 2*(15*B - 19*C)*cos(d*x + c)^4 + (15*B - 19*C)*cos(d*x + c
)^3)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) + 2*((1029*B - 1201*C)*cos(d*x + c)^4 + 12*(6
3*B - 67*C)*cos(d*x + c)^3 - 28*(3*B - 7*C)*cos(d*x + c)^2 + 12*(7*B - 3*C)
```

```
*cos(d*x + c) + 60*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))
/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^5(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.28776, size = 593, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] -1/420*(105*(15*sqrt(2)*B - 19*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1
/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)) - ((((105*(sqrt(2)*B*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) -
sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^3 -
 4*(693*sqrt(2)*B*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 877*sqrt(2)*C*a^5*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)*tan(1/2*d*x + 1/2*c)^2 + 14*(453*sqrt(
2)*B*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 517*sqrt(2)*C*a^5*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1))/a^3)*tan(1/2*d*x + 1/2*c)^2 - 140*(39*sqrt(2)*B*a^5*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1) - 47*sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1))/a^3)*tan(1/2*d*x + 1/2*c)^2 + 1785*(sqrt(2)*B*a^5*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1) - sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)*tan(1/2*d
*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)))/d
```

3.394 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(11B-15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(35B-39C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}+\frac{(B-C)\tan(c+dx)\sec^3(c+dx}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((11*B - 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((65*B - 93*C)*Tan[c + d*x])/(15*a*d*Sqrt[a +
a*Sec[c + d*x]]) - ((5*B - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
 + a*Sec[c + d*x]]) + ((35*B - 39*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])
/(30*a^2*d)
```

Rubi [A] time = 0.725143, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4019, 4021, 4010, 4001, 3795, 203}

$$\frac{(11B-15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(35B-39C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}+\frac{(B-C)\tan(c+dx)\sec^3(c+dx}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(3/2),x]
```


```
[Out] ((11*B - 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((65*B - 93*C)*Tan[c + d*x])/(15*a*d*Sqrt[a +
a*Sec[c + d*x]]) - ((5*B - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
 + a*Sec[c + d*x]]) + ((35*B - 39*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])
/(30*a^2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^3(c+dx)\left(3a(B-C)-\frac{1}{2}a(5B-9C\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5B-9C)\sec^2(c+dx)\tan}{10ad\sqrt{a+a\sec(c+d}}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5B-9C)\sec^2(c+dx)\tan}{10ad\sqrt{a+a\sec(c+d}}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65B-93C)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65B-93C)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{(11B-15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(B-C)\sec^3(c+dx)\tan}{2d(a+a\sec(c+dx}$$

Mathematica [A] time = 2.34895, size = 160, normalized size = 0.74

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(4(5B-3C)\sec^2(c+dx)-12(5B-9C)\sec(c+dx)-95B+12C\sec^3(c+dx)+147C\right)\right.}{30d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] ((15*Sqrt[2]*(11*B - 15*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
 d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-95*B + 147*C - 12*(5*B -
 9*C)*Sec[c + d*x] + 4*(5*B - 3*C)*Sec[c + d*x]^2 + 12*C*Sec[c + d*x]^3))*T
an[c + d*x])/(30*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.335, size = 793, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/240/d/a^2*(-1+cos(d*x+c))*(165*B*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(5/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))-225*C*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(5/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))+495*B*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*sin(d*x+c)-675*C*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
```

```
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*s
in(d*x+c)+495*B*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+
c)-675*C*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+165*
B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-225*C*ln(((-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+760*B*cos(d*x+c)^4-1176*C*cos(d*x+c)^4-2
80*B*cos(d*x+c)^3+312*C*cos(d*x+c)^3-640*B*cos(d*x+c)^2+960*C*cos(d*x+c)^2+
160*B*cos(d*x+c)-192*C*cos(d*x+c)+96*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)
/cos(d*x+c)^2/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.649612, size = 1315, normalized size = 6.09

$$\left[\frac{15\sqrt{2}\left((11B-15C)\cos(dx+c)^4+2(11B-15C)\cos(dx+c)^3+(11B-15C)\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx}{\cos(d}}}{}\right.}{120\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(2)*((11*B - 15*C)*cos(d*x + c)^4 + 2*(11*B - 15*C)*cos(d*x
+ c)^3 + (11*B - 15*C)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d
*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
4*((95*B - 147*C)*cos(d*x + c)^3 + 12*(5*B - 9*C)*cos(d*x + c)^2 - 4*(5*B -
 3*C)*cos(d*x + c) - 12*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2
), -1/60*(15*sqrt(2)*((11*B - 15*C)*cos(d*x + c)^4 + 2*(11*B - 15*C)*cos(d*
x + c)^3 + (11*B - 15*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((95*
B - 147*C)*cos(d*x + c)^3 + 12*(5*B - 9*C)*cos(d*x + c)^2 - 4*(5*B - 3*C)*c
os(d*x + c) - 12*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.14024, size = 421, normalized size = 1.95

$$\frac{\frac{15\sqrt{2}(11B-15C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\left(\left(\left(\frac{15\sqrt{2}(Ba^3-Ca^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}(245Ba^3-381Ca^3)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\cdots}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] 1/60*(15*sqrt(2)*(11*B - 15*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - (((15*sqrt(2)*(B*a^3 - C*a^3)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(245*B*a^3 - 381*C*a^3)/(a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*(73*B*a^3 - 105*C*a
^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 15*sqrt
(2)*(9*B*a^3 - 17*C*a^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x
 + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)))/d
```

3.395 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(7B-11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3B-7C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+$$

```
[Out] -((7*B - 11*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) + ((9*B - 13*C)*Tan[c + d*x])/(3*a*d*Sqrt[a + a*
Sec[c + d*x]]) - ((3*B - 7*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2
*d)
```

Rubi [A] time = 0.555913, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4019, 4010, 4001, 3795, 203}

$$-\frac{(7B-11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3B-7C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(3/2),x]
```

```
[Out] -((7*B - 11*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) + ((9*B - 13*C)*Tan[c + d*x])/(3*a*d*Sqrt[a + a*
Sec[c + d*x]]) - ((3*B - 7*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2
*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
```

```
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$=\frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sec^2(c+dx)\left(2a(B-C)-\frac{1}{2}a(3B-7C\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$=\frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(3B-7C)\sqrt{a+a\sec(c+d}}{6a^2d}$$

$$=\frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(9B-13C)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}}-$$

$$=\frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(9B-13C)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}}-$$

$$=-\frac{(7B-11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(B-C)\sec^2(c+dx)\,\mathrm{ta}}{2d(a+a\sec(c+d}$$

Mathematica [A] time = 1.3986, size = 141, normalized size = 0.82

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(12(B-C)\sec(c+dx)+15B+4C\sec^2(c+dx)-19C\right)-3\sqrt{2}(7B-11C)\cos^2\left(\frac{1}{2}(c+d\right.\right.}{6d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] ((-3*Sqrt[2]*(7*B - 11*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(15*B - 19*C + 12*(B - C)*S
ec[c + d*x] + 4*C*Sec[c + d*x]^2))*Tan[c + d*x])/(6*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.295, size = 603, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/24/d/a^2*(-1+cos(d*x+c))*(21*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln((
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*c
os(d*x+c)^2*sin(d*x+c)-33*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x
+c)^2*sin(d*x+c)+42*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))-66*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln
(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))
+21*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-33*C*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-60*B*cos(d*x+c)^3+76*C*cos(d*x+c)^3+1
2*B*cos(d*x+c)^2-28*C*cos(d*x+c)^2+48*B*cos(d*x+c)-64*C*cos(d*x+c)+16*C)*(a
*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.618492, size = 1189, normalized size = 6.95

$$\left[\frac{3\sqrt{2}\left((7B-11C)\cos(dx+c)^3+2(7B-11C)\cos(dx+c)^2+(7B-11C)\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\cdots}{\cos\cdots}\right.}{24\left(a^2d\cos(dx+c)^3+2a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((7*B - 11*C)*cos(d*x + c)^3 + 2*(7*B - 11*C)*cos(d*x + c)
^2 + (7*B - 11*C)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^
2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((15*B
 - 19*C)*cos(d*x + c)^2 + 12*(B - C)*cos(d*x + c) + 4*C)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
 + c)^2 + a^2*d*cos(d*x + c)), 1/12*(3*sqrt(2)*((7*B - 11*C)*cos(d*x + c)^3
 + 2*(7*B - 11*C)*cos(d*x + c)^2 + (7*B - 11*C)*cos(d*x + c))*sqrt(a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) + 2*((15*B - 19*C)*cos(d*x + c)^2 + 12*(B - C)*cos(d*x + c) + 4
*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 9.02966, size = 400, normalized size = 2.34

$$\frac{\frac{\left(\left(\frac{3\left(\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\sqrt{2}Ca\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{2\left(15\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-23\sqrt{2}Ca\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] 1/12*(((3*(sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*C*a*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a - 2*(15*sqrt(2)*B*a*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1) - 23*sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))/a)*tan(1/2*d*x + 1/2*c)^2 + 27*(sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a)*tan(1/2*d*x + 1/2*c)
/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*(
7*sqrt(2)*B - 11*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
))/d
```

3.396 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3B-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(B-C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{2C\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((3*B - 7*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + (2*C*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.319377, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4008, 4001, 3795, 203}

$$\frac{(3B-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(B-C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{2C\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^(3/2),x]
```

```
[Out] ((3*B - 7*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + (2*C*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$=-\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\int \frac{\sec(c+dx)\left(-\frac{3}{2}a(B-C)-2aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$=-\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2C\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{(3B-7C)\ \ldots}{\ldots}$$

$$=-\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2C\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}-\frac{(3B-7C)\ \ldots}{\ldots}$$

$$=\frac{(3B-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\ldots$$

Mathematica [A] time = 0.727204, size = 125, normalized size = 1.06

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}(-B+4C\sec(c+dx)+5C)+\sqrt{2}(3B-7C)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec\ldots}}{\ldots}\right)\right)}{2d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```

```
[Out] ((Sqrt[2]*(3*B - 7*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)
/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-B + 5*C + 4*C*Sec[c + d*x]))*
Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.26, size = 405, normalized size = 3.4

$$-\frac{-1+\cos(dx+c)}{4\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\,B\sin(dx+c)\cos(dx+c)\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*B*sin(d*x
+c)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+
```

```
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-7*C*sin(d*x+c)*cos(d
*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*B*ln(((-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-7*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+2*B*cos(d*x+c)^2-10*C*cos(d*x+c)^2-2*B*cos(d*x+c)+2*C*cos(d*x+c)+8*C)
/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)/(a*sec(d*x + c)
+ a)^(3/2), x)
```

Fricas [A] time = 0.596805, size = 1003, normalized size = 8.5

$$\left[\frac{\sqrt{2}\left((3B-7C)\cos\left(dx+c\right)^{2}+2\,(3B-7C)\cos\left(dx+c\right)+3B-7C\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos\ldots}{\cos(dx+c)^{2}+2\cos(dx+c)+\ldots}\right.\ldots}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+\ldots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*B - 7*C)*cos(d*x + c)^2 + 2*(3*B - 7*C)*cos(d*x + c) + 3*
B - 7*C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((B - 5*C)*cos(d*x + c) - 4*
C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*B - 7*C)*cos(d*x + c
)^2 + 2*(3*B - 7*C)*cos(d*x + c) + 3*B - 7*C)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*
((B - 5*C)*cos(d*x + c) - 4*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A] time = 8.79642, size = 257, normalized size = 2.18

$$-\frac{\frac{\left(\frac{\sqrt{2}\left(Ba^2-Ca^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{\sqrt{2}\left(Ba^2-9\,Ca^2\right)}{a^3\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}-\frac{\sqrt{2}(3\,B-7\,C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] -1/4*((sqrt(2)*(B*a^2 - C*a^2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)) - sqrt(2)*(B*a^2 - 9*C*a^2)/(a^3*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - sqrt(2)
*(3*B - 7*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.397 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{(B+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(B-C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((B + 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2))
```

Rubi [A] time = 0.105751, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4052, 12, 3795, 203}

$$\frac{(B+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(B-C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((B + 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) + ((B - C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2))
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int -\frac{a(B+3C)\sec(c+dx)}{2\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$
$$= \frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(B+3C)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$
$$= \frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(B+3C)\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}$$
$$= \frac{(B+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.784075, size = 127, normalized size = 1.46

$$\frac{2(B-C)\sin(c+dx)\sqrt{1-\sec(c+dx)}+2\sqrt{2}(B+3C)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)}{4ad(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(B - C)*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 2*Sqrt[2]*(B + 3*C)*ArcTan
h[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Tan[c + d*x])/(4*a*d*(
1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.24, size = 402, normalized size = 4.6

$$\frac{1}{4\,da^2\,(\cos(dx+c)+1)\sin(dx+c)}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}\left(B\sin(dx+c)\cos(dx+c)\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx}{\cos(dx+}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(B*sin(d*x+c)*cos(d*x+c)*ln((
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*C*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+3*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*B*cos(d*x+c)^
2+2*C*cos(d*x+c)^2+2*B*cos(d*x+c)-2*C*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+B\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [B] time = 0.604896, size = 957, normalized size = 11.

$$\left[\frac{4\,(B-C)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}\left((B+3\,C)\cos(dx+c)^2+2\,(B+3\,C)\cos(dx+c)+B+3\,C\right)\sqrt{-a}\,\text{l}}{8\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(4*(B - C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - sqrt(2)*((B + 3*C)*cos(d*x + c)^2 + 2*(B + 3*C)*cos(d*x + c) + B +
 3*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(2*(B - C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - sqrt(2)*((B + 3*C)*cos(d*x + c)^2 + 2*(B + 3*C)
*cos(d*x + c) + B + 3*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x
)
```

Giac [B] time = 8.6494, size = 208, normalized size = 2.39

$$\frac{\frac{\left(\sqrt{2}B+3\sqrt{2}C\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\left(\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\sqrt{2}Ca\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/4*((sqrt(2)*B + 3*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - (sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*C*a*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1
/2*c)/a^3)/d
```

3.398 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(5B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(B-C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2))
```

Rubi [A] time = 0.269656, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 3922, 3920, 3774, 203, 3795}

$$-\frac{(5B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(B-C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^(3/2),x]
```


```
[Out] (2*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*B - C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{B+C\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2aB+\frac{1}{2}a(B-C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{B\int \sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{(5B-$$

$$= -\frac{(B-C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+}{\sqrt{a+a\sec(}}\right.}{ad}$$

$$= \frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5B-C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} -$$

Mathematica [A] time = 1.57632, size = 147, normalized size = 1.16

$$\frac{\csc(c+dx)\left(2(C-B)\sin^2\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}(5B-C)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)+8B\cos^2}{2ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```

```
[Out] (Csc[c + d*x]*(8*B*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sqrt[
-1 + Sec[c + d*x]] - Sqrt[2]*(5*B - C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[
2]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] + 2*(-B + C)*Sin[(c + d*x)/2
]^2))/(2*a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.234, size = 552, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-4*B*cos(d*x+c)*2^(1/2)*sin(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-5*B*sin(d*x+c)*cos(d*x+c)*
ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-4*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+C*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-5*B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+C
*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*B*cos(d*x+c)^2-2*C*co
s(d*x+c)^2-2*B*cos(d*x+c)+2*C*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/(a*sec(d*x + c)
+ a)^(3/2), x)
```

Fricas [B] time = 7.92313, size = 1416, normalized size = 11.15

$$\left[-\frac{4\left(B-C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right) - \sqrt{2}\left(\left(5B-C\right)\cos\left(dx+c\right)^2 + 2\left(5B-C\right)\cos\left(dx+c\right) + 5B-C\right)\sqrt{-a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(4*(B - C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - sqrt(2)*((5*B - C)*cos(d*x + c)^2 + 2*(5*B - C)*cos(d*x + c) + 5*
B - C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x
 + c) + B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(
d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4
*(2*(B - C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) - sqrt(2)*((5*B - C)*cos(d*x + c)^2 + 2*(5*B - C)*cos(d*x + c) + 5*B - C
```

```
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c))) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\cos(c + dx)\sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a*(sec(c + d*x) +
1))**(3/2), x)
```

Giac [B] time = 11.0498, size = 417, normalized size = 3.28

$$-\frac{\frac{\sqrt{2}(5B-C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{8B\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{8B\log\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2)*(5*B - C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 8*
B*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
 8*B*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - 2*(sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*C*a*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/
2*c)/a^3)/d
```

3.399 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(3B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9B-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3B-C)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(B-C)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -(((3*B - 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*B - 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Sin[c + d*x])/(2*d*(a + a
*Sec[c + d*x])^(3/2)) + ((3*B - C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c +
d*x]])
```

Rubi [A] time = 0.493935, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(3B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9B-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3B-C)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(B-C)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(3/2),x]
```

```
[Out] -(((3*B - 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*B - 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Sin[c + d*x])/(2*d*(a + a
*Sec[c + d*x])^(3/2)) + ((3*B - C)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c +
d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
```

```
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(B-C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(a(3B-C)-\frac{3}{2}a(B-C)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3B-C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \int \frac{-a^2(3B-}{}$$

$$= -\frac{(B-C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3B-C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(9B-5C}{}$$

$$= -\frac{(B-C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3B-C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(9B-5C}{}$$

$$= -\frac{(3B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(9B-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{2}\sqrt{a+a\sec}}\right.}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 27.0136, size = 10898, normalized size = 64.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.291, size = 713, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(-1+cos(d*x+c))*(-6*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)/cos(d*x+c))+4*C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))-9*B*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)-6*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+5*C*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+4*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*B*cos(
d*x+c)^3-9*B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)
+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+5*C*ln(((-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*B*cos(d*x+c)^2-2*C*cos(d*x+c)
^2-6*B*cos(d*x+c)+2*C*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d
*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^{2} + B \sec\left(dx+c\right)\right) \cos\left(dx+c\right)^{2}}{\left(a \sec\left(dx+c\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/(a*sec(d*x + c
) + a)^(3/2), x)
```

Fricas [A] time = 10.519, size = 1575, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((9*B - 5*C)*cos(d*x + c)^2 + 2*(9*B - 5*C)*cos(d*x + c) + 9*
B - 5*C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*B - 2*C)*cos(d*x + c)^2
+ 2*(3*B - 2*C)*cos(d*x + c) + 3*B - 2*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2
+ 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(2*B*cos(d*x + c)^2 + (3*
B - C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((9*B
- 5*C)*cos(d*x + c)^2 + 2*(9*B - 5*C)*cos(d*x + c) + 9*B - 5*C)*sqrt(a)*arc
tan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - 4*((3*B - 2*C)*cos(d*x + c)^2 + 2*(3*B - 2*C)*cos(d*x + c)
+ 3*B - 2*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*B*cos(d*x + c)^2 + (3*B - C)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.400 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(19B-12C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13B-9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7B-6C)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2B-C)\sin(c+dx)}{2ad\sqrt{a\sec(c+}}$$

```
[Out] ((19*B - 12*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(3/2)*d) - ((13*B - 9*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Cos[c + d*x]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*B - 6*C)*Sin[c + d*x])/(4*a*d*Sq
rt[a + a*Sec[c + d*x]]) + ((2*B - C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt
[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.68525, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(19B-12C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13B-9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7B-6C)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2B-C)\sin(c+dx)}{2ad\sqrt{a\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(3/2),x]
```

```
[Out] ((19*B - 12*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(3/2)*d) - ((13*B - 9*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((B - C)*Cos[c + d*x]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*B - 6*C)*Sin[c + d*x])/(4*a*d*Sq
rt[a + a*Sec[c + d*x]]) + ((2*B - C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(2B-C)-\frac{5}{2}a(B-C)\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(2B-C)\cos(c+dx)\sin(c}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7B-6C)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2}{}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7B-6C)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2}{}$$

$$= -\frac{(B-C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7B-6C)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{(2}{}$$

$$= \frac{(19B-12C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13B-9C)\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a+a}}\right.}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 2.3293, size = 395, normalized size = 1.79

$$\frac{\sec(c+dx)\left(-40B\sqrt{1-\sec(c+dx)}(\sin(c+dx)+\tan(c+dx))\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)+(91B-4\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*(-52*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Sin[c
+ d*x] + 36*Sqrt[2]*C*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Sin[c + d*x]
- 13*B*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 24*C*Sqrt[1 - Sec[c + d*x]]*Si
n[c + d*x] + 18*B*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + (13*
B*Sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)])/2 + 8*C*Sqrt[1 - Sec[c + d*x]]*S
in[2*(c + d*x)] - 52*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[
c + d*x] + 36*Sqrt[2]*C*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x
] + (91*B - 48*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + Tan[c + d
*x]) - 40*B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]]*(Sin[c + d*x] + Tan[c + d*x])))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1
 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.351, size = 1075, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/16/d/a^2*(-1+cos(d*x+c))*(19*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))-12*C*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+26*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))*cos(d*x+c)^2*sin(d*x+c)+38*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))-18*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*
ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c
))*cos(d*x+c)^2*sin(d*x+c)-24*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)/cos(d*x+c))+52*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d
*x+c)+1)/sin(d*x+c))+19*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)
*sin(d*x+c)-36*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))-12*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+26*B*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-8*B*cos(d*x+c)^5-18*C*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+20*B*cos(d*x+c)^4-16*C*cos
(d*x+c)^4+16*B*cos(d*x+c)^3-8*C*cos(d*x+c)^3-28*B*cos(d*x+c)^2+24*C*cos(d*x
```

```
+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^3}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^3/(a*sec(d*x + c
) + a)^(3/2), x)
```

Fricas [A] time = 15.2902, size = 1673, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((13*B - 9*C)*cos(d*x + c)^2 + 2*(13*B - 9*C)*cos(d*x + c) +
13*B - 9*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c)
 - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*B - 12*C)*cos(d*x + c)^
2 + 2*(19*B - 12*C)*cos(d*x + c) + 19*B - 12*C)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*B*cos(d*x + c)^3
 - (3*B - 4*C)*cos(d*x + c)^2 - (7*B - 6*C)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d), 1/4*(sqrt(2)*((13*B - 9*C)*cos(d*x + c)^2 + 2*(13*B - 9*C)
*cos(d*x + c) + 13*B - 9*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((19*B - 12*C)*cos(d
*x + c)^2 + 2*(19*B - 12*C)*cos(d*x + c) + 19*B - 12*C)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(2*B*cos(d*x + c)^3 - (3*B - 4*C)*cos(d*x + c)^2 - (7*B - 6*C)*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.401 $\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(85B-157C)\tan(c+dx)\sec^2(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(163B-283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(475B-787C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{240a^3d}$$

```
[Out] ((163*B - 283*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Sec[c + d*x]^4*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) + ((13*B - 21*C)*Sec[c + d*x]^3*Tan[c + d*x])
/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((985*B - 1729*C)*Tan[c + d*x])/(120
*a^2*d*Sqrt[a + a*Sec[c + d*x]]) - ((85*B - 157*C)*Sec[c + d*x]^2*Tan[c + d
*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((475*B - 787*C)*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(240*a^3*d)
```

Rubi [A] time = 0.928138, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4019, 4021, 4010, 4001, 3795, 203}

$$-\frac{(85B-157C)\tan(c+dx)\sec^2(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(163B-283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(475B-787C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(5/2),x]
```


```
[Out] ((163*B - 283*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Sec[c + d*x]^4*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) + ((13*B - 21*C)*Sec[c + d*x]^3*Tan[c + d*x])
/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((985*B - 1729*C)*Tan[c + d*x])/(120
*a^2*d*Sqrt[a + a*Sec[c + d*x]]) - ((85*B - 157*C)*Sec[c + d*x]^2*Tan[c + d
*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((475*B - 787*C)*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(240*a^3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^5(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^4(c+dx)\left(4a(B-C)-\frac{1}{2}a(5B-13\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(13B-21C)\sec^3(c+dx)\operatorname{ta}}{16ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(13B-21C)\sec^3(c+dx)\operatorname{ta}}{16ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(13B-21C)\sec^3(c+dx)\operatorname{ta}}{16ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(13B-21C)\sec^3(c+dx)\operatorname{ta}}{16ad(a+a\sec(c+dx}$$

$$= \frac{(B-C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(13B-21C)\sec^3(c+dx)\operatorname{ta}}{16ad(a+a\sec(c+dx}$$

$$= \frac{(163B-283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(B-C)\sec^4(c+dx)}{4d(a+a\sec(c+}$$

Mathematica [A] time = 2.60335, size = 177, normalized size = 0.68

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(160(B-C)\sec^3(c+dx)-32(25B-49C)\sec^2(c+dx)-5(503B-911C)\sec(c+dx)-\right.\right.}{240d\sqrt{1-\sec(c+dx)}(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(5/2),x]
```

```
[Out] ((30*Sqrt[2]*(163*B - 283*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c
 + d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-1495*B + 2671*C - 5*
(503*B - 911*C)*Sec[c + d*x] - 32*(25*B - 49*C)*Sec[c + d*x]^2 + 160*(B - C
)*Sec[c + d*x]^3 + 96*C*Sec[c + d*x]^4))*Tan[c + d*x])/(240*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.325, size = 985, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/1920/d/a^3*(-1+cos(d*x+c))^2*(2445*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)
*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))*sin(d*x+c)*cos(d*x+c)^4-4245*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*ln(
```

```
((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*
sin(d*x+c)*cos(d*x+c)^4+9780*B*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(5/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))-16980*C*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(5/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))+14670*B*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*sin(d*x+c)-25470*C*cos(d*x+c)^2*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*
sin(d*x+c)+9780*B*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*
x+c)-16980*C*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+
2445*B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/si
n(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-4245*C*ln(((-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+11960*B*cos(d*x+c)^5-21368*C*cos(d
*x+c)^5+8160*B*cos(d*x+c)^4-15072*C*cos(d*x+c)^4-13720*B*cos(d*x+c)^3+23896
*C*cos(d*x+c)^3-7680*B*cos(d*x+c)^2+13824*C*cos(d*x+c)^2+1280*B*cos(d*x+c)-
2048*C*cos(d*x+c)+768*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/c
os(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 0.686998, size = 1597, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="fricas")

[Out] [1/960*(15*sqrt(2)*((163*B - 283*C)*cos(d*x + c)^5 + 3*(163*B - 283*C)*cos(
d*x + c)^4 + 3*(163*B - 283*C)*cos(d*x + c)^3 + (163*B - 283*C)*cos(d*x + c
)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((1495*B - 2671*C)*cos(d*x + c)^
4 + 5*(503*B - 911*C)*cos(d*x + c)^3 + 32*(25*B - 49*C)*cos(d*x + c)^2 - 16
0*(B - C)*cos(d*x + c) - 96*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x
+ c)^3 + a^3*d*cos(d*x + c)^2), -1/480*(15*sqrt(2)*((163*B - 283*C)*cos(d*x
 + c)^5 + 3*(163*B - 283*C)*cos(d*x + c)^4 + 3*(163*B - 283*C)*cos(d*x + c)
^3 + (163*B - 283*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((1495*B
- 2671*C)*cos(d*x + c)^4 + 5*(503*B - 911*C)*cos(d*x + c)^3 + 32*(25*B - 49
```

```
*C)*cos(d*x + c)^2 - 160*(B - C)*cos(d*x + c) - 96*C)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x +
c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5
/2),x)

[Out] Timed out
```

Giac [A] time = 10.0303, size = 593, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="giac")

[Out] -1/480*((((15*(2*(sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*C
*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^2 + (21*sqrt
(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 29*sqrt(2)*C*a^2*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1))/a^2)*tan(1/2*d*x + 1/2*c)^2 - (3685*sqrt(2)*B*a^2*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1) - 6733*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1))/a^2)*tan(1/2*d*x + 1/2*c)^2 + 5*(1133*sqrt(2)*B*a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1) - 1973*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^2)*ta
n(1/2*d*x + 1/2*c)^2 - 15*(155*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
) - 291*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^2)*tan(1/2*d*x + 1
/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
) - 15*(163*sqrt(2)*B - 283*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*
c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))/d
```

3.402 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{(75B-163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(39B-95C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}+\frac{(93B-197C)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(B$$

```
[Out] -((75*B - 163*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) + ((9*B - 17*C)*Sec[c + d*x]^2*Tan[c + d*x])/
(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((93*B - 197*C)*Tan[c + d*x])/(24*a^2
*d*Sqrt[a + a*Sec[c + d*x]]) - ((39*B - 95*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[
c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.747935, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4019, 4010, 4001, 3795, 203}

$$-\frac{(75B-163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(39B-95C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}+\frac{(93B-197C)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(B$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(5/2),x]
```

```
[Out] -((75*B - 163*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*
d*(a + a*Sec[c + d*x])^(5/2)) + ((9*B - 17*C)*Sec[c + d*x]^2*Tan[c + d*x])/
(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((93*B - 197*C)*Tan[c + d*x])/(24*a^2
*d*Sqrt[a + a*Sec[c + d*x]]) - ((39*B - 95*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[
c + d*x])/(48*a^3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^3(c+dx)\left(3a(B-C)-\frac{1}{2}a(3B-11\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9B-17C)\sec^2(c+dx)\tan}{16ad(a+a\sec(c+dx)}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9B-17C)\sec^2(c+dx)\tan}{16ad(a+a\sec(c+dx)}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9B-17C)\sec^2(c+dx)\tan}{16ad(a+a\sec(c+dx)}$$

$$= \frac{(B-C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9B-17C)\sec^2(c+dx)\tan}{16ad(a+a\sec(c+dx)}$$

$$= -\frac{(75B-163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(B-C)\sec^3(c+dx)}{4d(a+a\sec(c+}$$

Mathematica [A] time = 2.59787, size = 161, normalized size = 0.75

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32(3B-5C)\sec^2(c+dx)+(255B-503C)\sec(c+dx)+147B+32C\sec^3(c+dx)-2\right.\right.}{48d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(5/2),x]
```

```
[Out] ((-6*Sqrt[2]*(75*B - 163*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c
+ d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(147*B - 299*C + (255*B
 - 503*C)*Sec[c + d*x] + 32*(3*B - 5*C)*Sec[c + d*x]^2 + 32*C*Sec[c + d*x]^
3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)
)
```

Maple [B] time = 0.306, size = 795, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d/a^3*(-1+cos(d*x+c))^2*(-225*B*cos(d*x+c)^3*sin(d*x+c)*ln(((-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)+489*C*cos(d*x+c)^3*sin(d*x+c)*ln(((-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(3/2)-675*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*
x+c)^2*sin(d*x+c)+1467*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)
^2*sin(d*x+c)-675*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))+1467*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln
(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))
-225*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+489*C*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(3/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+588*B*cos(d*x+c)^4-1196*C*cos(d*x+c
)^4+432*B*cos(d*x+c)^3-816*C*cos(d*x+c)^3-636*B*cos(d*x+c)^2+1372*C*cos(d*x
+c)^2-384*B*cos(d*x+c)+768*C*cos(d*x+c)-128*C)*(a*(cos(d*x+c)+1)/cos(d*x+c)
)^(1/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.652811, size = 1474, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(2)*((75*B - 163*C)*cos(d*x + c)^4 + 3*(75*B - 163*C)*cos(d*x
 + c)^3 + 3*(75*B - 163*C)*cos(d*x + c)^2 + (75*B - 163*C)*cos(d*x + c))*sq
rt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*((147*B - 299*C)*cos(d*x + c)^3 + (255*B
 - 503*C)*cos(d*x + c)^2 + 32*(3*B - 5*C)*cos(d*x + c) + 32*C)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/96*(3*sqrt(
2)*((75*B - 163*C)*cos(d*x + c)^4 + 3*(75*B - 163*C)*cos(d*x + c)^3 + 3*(75
*B - 163*C)*cos(d*x + c)^2 + (75*B - 163*C)*cos(d*x + c))*sqrt(a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) + 2*((147*B - 299*C)*cos(d*x + c)^3 + (255*B - 503*C)*cos(d*x + c)^
2 + 32*(3*B - 5*C)*cos(d*x + c) + 32*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 9.78534, size = 420, normalized size = 1.94

$$\frac{\frac{\left(\left(\left(3\left(\frac{2\sqrt{2}(Ba^5-Ca^5)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}(15Ba^5-23Ca^5)}{a^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}(75Ba^5-167Ca^5)}{a^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3\sqrt{2}(83Ba^5-155Ca^5)}{a^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] 1/96*(((3*(2*sqrt(2)*(B*a^5 - C*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^6*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(15*B*a^5 - 23*C*a^5)/(a^6*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(75*B*a^5 - 167*C*a^5
)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)
*(83*B*a^5 - 155*C*a^5)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x
+ 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
```

```
)) - 3*sqrt(2)*(75*B - 163*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)))/d
```

3.403 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(19B-75C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(B-9C)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(5B-13C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((19*B - 75*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*
(a + a*Sec[c + d*x])^(5/2)) - ((5*B - 13*C)*Tan[c + d*x])/(16*a*d*(a + a*Se
c[c + d*x])^(3/2)) - ((B - 9*C)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rubi [A] time = 0.564363, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4019, 4008, 4001, 3795, 203}

$$\frac{(19B-75C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(B-9C)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(B-C)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(5B-13C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(5/2),x]
```

```
[Out] ((19*B - 75*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*
(a + a*Sec[c + d*x])^(5/2)) - ((5*B - 13*C)*Tan[c + d*x])/(16*a*d*(a + a*Se
c[c + d*x])^(3/2)) - ((B - 9*C)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^2(c+dx)\left(2a(B-C)-\frac{1}{2}a(B-9C)\sec(\ldots\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5B-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \ldots$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5B-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \ldots$$

$$= \frac{(B-C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5B-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \ldots$$

$$= \frac{(19B-75C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(B-C)\sec^2(c+dx)\tan(c\ldots}{4d(a+a\sec(c+dx))^{5\ldots}}$$

Mathematica [A] time = 1.47091, size = 144, normalized size = 0.85

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left((85C-13B)\sec(c+dx)-9B+32C\sec^2(c+dx)+49C\right)+2\sqrt{2}(19B-75C)\cos^4\left(\frac{1}{2}(c+\ldots\right.\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(5/2),x]
```

```
[Out] ((2*Sqrt[2]*(19*B - 75*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-9*B + 49*C + (-13*B + 8
5*C)*Sec[c + d*x] + 32*C*Sec[c + d*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.281, size = 597, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(19*B*cos(
d*x+c)^2*sin(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-75*C*cos(d*x+c)
^2*sin(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+38*B*sin(d*x+c)*cos(d
*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-150*C*sin(d*x+c)*cos(d*x+c)*l
n(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+18*B*cos(d*x+c)^3+19*B*ln(((-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-98*C*cos(d*x+c)^3-75*C*ln(((-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*B*cos(d*x+c)^2-72*C*cos(d*x+c)^2-26*
B*cos(d*x+c)+106*C*cos(d*x+c)+64*C)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.627088, size = 1273, normalized size = 7.53

$$\left[\frac{\sqrt{2}\left((19\,B - 75\,C)\cos\left(dx + c\right)^{3} + 3\,(19\,B - 75\,C)\cos\left(dx + c\right)^{2} + 3\,(19\,B - 75\,C)\cos\left(dx + c\right) + 19\,B - 75\,C\right)\sqrt{-a}\log\left(\right.}{64\left(a^{3}d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*B - 75*C)*cos(d*x + c)^3 + 3*(19*B - 75*C)*cos(d*x + c)
^2 + 3*(19*B - 75*C)*cos(d*x + c) + 19*B - 75*C)*sqrt(-a)*log(-(2*sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) -
 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((9*B - 49*C)*cos(d*x + c)^2 + (13*B - 85*C)*cos(d*x + c) - 32*
C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2
)*((19*B - 75*C)*cos(d*x + c)^3 + 3*(19*B - 75*C)*cos(d*x + c)^2 + 3*(19*B
- 75*C)*cos(d*x + c) + 19*B - 75*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((9*B - 49
*C)*cos(d*x + c)^2 + (13*B - 85*C)*cos(d*x + c) - 32*C)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 9.55333, size = 390, normalized size = 2.31

$$-\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Ba^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\sqrt{2}Ca^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{9\sqrt{2}Ba^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-17\sqrt{2}Ca^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{32 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] -1/32*(((2*(sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*C*a^6*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^8 + (9*sqrt(2)*B*a
^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 17*sqrt(2)*C*a^6*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1))/a^8)*tan(1/2*d*x + 1/2*c)^2 - (11*sqrt(2)*B*a^6*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1) - 83*sqrt(2)*C*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*t
an(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) - (19*sqrt(2)*B - 7
5*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.404 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5B-13C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(B-C)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((5*B + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d
*x])^(5/2)) + ((5*B - 13*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2
))
```

Rubi [A] time = 0.33442, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4008, 4000, 3795, 203}

$$\frac{(5B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5B-13C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(B-C)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^(5/2),x]
```


```
[Out] ((5*B + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d
*x])^(5/2)) + ((5*B - 13*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2
))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{5}{2}a(B-C)-4aC\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5B-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(5B+19C}{}$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5B-13C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(5B+19C}{}$$

$$= \frac{(5B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(}{16}$$

Mathematica [A] time = 1.43266, size = 131, normalized size = 1.04

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}((5B-13C)\sec(c+dx)+B-9C)+2\sqrt{2}(5B+19C)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(5/2),x]
```

```
[Out] ((2*Sqrt[2]*(5*B + 19*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d
*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(B - 9*C + (5*B - 13*C)*Se
c[c + d*x]))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(5/2))
```

Maple [B] time = 0.261, size = 602, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-5*B*cos(d*
x+c)^2*sin(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d
```

```
*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-19*C*cos(d*x+c)^2
*sin(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+
1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-10*B*sin(d*x+c)*cos(d*x
+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-38*C*sin(d*x+c)*cos(d*x+c)*ln((
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*B*cos(d*x+c)^3-5*B*ln(((-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-18*C*cos(d*x+c)^3-19*C*ln(((-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*B*cos(d*x+c)^2-8*C*cos(d*x+c)^2-10*B*cos(
d*x+c)+26*C*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.617093, size = 1233, normalized size = 9.79

$$\left[-\frac{\sqrt{2}\left((5B+19C)\cos(dx+c)^3+3(5B+19C)\cos(dx+c)^2+3(5B+19C)\cos(dx+c)+5B+19C\right)\sqrt{-a}\log\left(\frac{2\sqrt{}}{}\right.}{64\left(a^3d\cos(dx+c)^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*B + 19*C)*cos(d*x + c)^3 + 3*(5*B + 19*C)*cos(d*x + c)^
2 + 3*(5*B + 19*C)*cos(d*x + c) + 5*B + 19*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) - 4*((B - 9*C)*cos(d*x + c)^2 + (5*B - 13*C)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*B + 19*C)
*cos(d*x + c)^3 + 3*(5*B + 19*C)*cos(d*x + c)^2 + 3*(5*B + 19*C)*cos(d*x +
c) + 5*B + 19*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((B - 9*C)*cos(d*x + c)^2 + (
5*B - 13*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A] time = 9.29425, size = 258, normalized size = 2.05

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\left(\frac{2\sqrt{2}\left(Ba^5 - Ca^5\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)} + \frac{\sqrt{2}\left(3\,Ba^5 - 11\,Ca^5\right)}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{\sqrt{2}(5\,B + 19\,C)\log\left(\left|-\sqrt{-a}\tan\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(B*a^5 - C*a^5)*tan(1
/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(3*B*a^5
- 11*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - s
qrt(2)*(5*B + 19*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.405 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(3B+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3B+5C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{(B-C)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((3*B + 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*
x])^(5/2)) + ((3*B + 5*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.151674, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4052, 12, 3796, 3795, 203}

$$\frac{(3B+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(3B+5C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{(B-C)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*B + 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(16*Sqrt[2]*a^(5/2)*d) + ((B - C)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*
x])^(5/2)) + ((3*B + 5*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int -\frac{a(3B+5C)\sec(c+dx)}{2(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3B+5C)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3B+5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(3B+5C)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3B+5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(3B+5C)\,\text{Subst}\left(\int \frac{1}{2a+x^2}}{16a^2}$$

$$= \frac{(3B+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3B+5C)\tan(}{16ad(a+a\sec(c}$$

Mathematica [C] time = 1.48807, size = 206, normalized size = 1.63

$$\frac{64B\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{1-\sec(c+dx)}\sec(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+C}{32a^2d(\cos(c+dx)+1)^2\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (40*Sqrt[2]*C*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Se
c[c + d*x]*Sin[(c + d*x)/2] + 64*B*Cos[(c + d*x)/2]^5*Hypergeometric2F1[1/2
, 3, 3/2, (1 - Sec[c + d*x])/2]*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Sin[(c
+ d*x)/2] + C*Sqrt[1 - Sec[c + d*x]]*(10*Sin[c + d*x] + Sin[2*(c + d*x)]))/
(32*a^2*d*(1 + Cos[c + d*x])^2*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [B] time = 0.207, size = 594, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*B*cos(d*x+c)^2*sin(d*x+c)
*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*C*cos(d*x+c)^2*sin(d*x+c)*ln(((-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*B*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+
```

```
c)/(cos(d*x+c)+1))^(1/2)+10*C*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)+3*B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-co
s(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-14*
B*cos(d*x+c)^3+5*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(
d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*C*c
os(d*x+c)^3+8*B*cos(d*x+c)^2-8*C*cos(d*x+c)^2+6*B*cos(d*x+c)+10*C*cos(d*x+c
))/(cos(d*x+c)+1)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.616432, size = 1219, normalized size = 9.67

$$\left[-\frac{\sqrt{2}\left((3B+5C)\cos(dx+c)^3+3(3B+5C)\cos(dx+c)^2+3(3B+5C)\cos(dx+c)+3B+5C\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}}{\cdots}\right.}{64\left(a^3d\cos(dx+c)^3+3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*B + 5*C)*cos(d*x + c)^3 + 3*(3*B + 5*C)*cos(d*x + c)^2
+ 3*(3*B + 5*C)*cos(d*x + c) + 3*B + 5*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos
(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
- 4*((7*B + C)*cos(d*x + c)^2 + (3*B + 5*C)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*B + 5*C)*cos(d
*x + c)^3 + 3*(3*B + 5*C)*cos(d*x + c)^2 + 3*(3*B + 5*C)*cos(d*x + c) + 3*B
 + 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((7*B + C)*cos(d*x + c)^2 + (3*B + 5*C
)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x
)
```

Giac [A] time = 8.97639, size = 258, normalized size = 2.05

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\left(Ba^{5}-Ca^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(5\,Ba^{5}+3\,Ca^{5}\right)}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(3\,B+5\,C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\right.\right.\right.}{\sqrt{-a}a^{2}\mathrm{sgn}\left(\mathrm{t}\right.}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(B*a^5 - C*a^5)*tan(1/
2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(5*B*a^5 +
 3*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + sqr
t(2)*(3*B + 5*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.406 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{(43B-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11B-3C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(B-C)\tan(c+dx)}{4d(a\sec(c+dx)+a)}$$

```
[Out] (2*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*B - 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*B - 3*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rubi [A] time = 0.346006, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 3922, 3920, 3774, 203, 3795}

$$-\frac{(43B-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11B-3C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(B-C)\tan(c+dx)}{4d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])
^(5/2),x]
```

```
[Out] (2*B*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*B - 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*B - 3*C)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{B+C\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4aB+\frac{3}{2}a(B-C)\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11B-3C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2B-\frac{1}{4}a^2 \cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11B-3C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{B\int\sqrt{a+\cdots}}{\cdots}$$

$$= -\frac{(B-C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11B-3C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2B)\,\mathrm{Subs}\cdots}{\cdots}$$

$$= \frac{2B\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43B-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \cdots$$

Mathematica [C] time = 26.6628, size = 10133, normalized size = 61.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.24, size = 824, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-32*B*cos(d*x+c)^2*sin(d*x+
c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-43*B*cos(d*x+c)^2*sin(
d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/si
n(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-64*B*cos(d*x+c)*2^(1/2)*sin(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+3*C*cos(d*x+c)^2*sin(d*x+c
)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-86*B*sin(d*x+c)*cos(d*x+c)*ln(((-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-32*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+6*C*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+30*B*cos(d*x+c)^3-43*B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)-14*C*cos(d*x+c)^3+3*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)-8*B*cos(d*x+c)^2+8*C*cos(d*x+c)^2-22*B*cos(d*x+c)+6*C*cos(d*x+
c))/(cos(d*x+c)+1)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/(a*sec(d*x + c)
+ a)^(5/2), x)
```

Fricas [B] time = 16.8055, size = 1754, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*B - 3*C)*cos(d*x + c)^3 + 3*(43*B - 3*C)*cos(d*x + c)^2
 + 3*(43*B - 3*C)*cos(d*x + c) + 43*B - 3*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*
cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 64*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt
(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
- 4*((15*B - 7*C)*cos(d*x + c)^2 + (11*B - 3*C)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
```

```
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*B - 3*C)*c
os(d*x + c)^3 + 3*(43*B - 3*C)*cos(d*x + c)^2 + 3*(43*B - 3*C)*cos(d*x + c)
 + 43*B - 3*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 64*(B*cos(d*x + c)^3 + 3*B*cos(d*
x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((15*B - 7*C)*cos(d*x
 + c)^2 + (11*B - 3*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [B] time = 11.7508, size = 471, normalized size = 2.87

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\sqrt{2}\left(Ba^{5}-Ca^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(13\,Ba^{5}-5\,Ca^{5}\right)}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(43\,B-3\,C)\log\left(\left(\left(\sqrt{-a}\,\mathrm{ta}\cdots\right.\right.\right.}{\sqrt{-a}a^{2}\mathrm{sg}\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(B*a^5 - C*a^5)*tan
(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(13*B*a
^5 - 5*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) +
 sqrt(2)*(43*B - 3*C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 64
*B*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - 64*B*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)))/d
```

3.407 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{(35B-11C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(5B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115B-43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15B-7C)\,\mathrm{s}}{16ad(a\sec(c+}$$

```
[Out] -(((5*B - 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*B - 43*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Sin[c + d*x])/(4*d*(a
 + a*Sec[c + d*x])^(5/2)) - ((15*B - 7*C)*Sin[c + d*x])/(16*a*d*(a + a*Sec[
c + d*x])^(3/2)) + ((35*B - 11*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```

Rubi [A] time = 0.67209, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(35B-11C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(5B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115B-43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15B-7C)\,\mathrm{s}}{16ad(a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x
])^(5/2),x]
```


```
[Out] -(((5*B - 2*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*B - 43*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((B - C)*Sin[c + d*x])/(4*d*(a
 + a*Sec[c + d*x])^(5/2)) - ((15*B - 7*C)*Sin[c + d*x])/(16*a*d*(a + a*Sec[
c + d*x])^(3/2)) + ((35*B - 11*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(a(5B-C)-\frac{5}{2}a(B-C)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15B-7C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \int \frac{\cos(c+d}{}$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15B-7C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35B-1}{16a^2d\sqrt{a}}$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15B-7C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35B-1}{16a^2d\sqrt{a}}$$

$$= -\frac{(B-C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15B-7C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35B-1}{16a^2d\sqrt{a}}$$

$$= -\frac{(5B-2C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(115B-43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{2}\sqrt{a+a\,\mathrm{se}}}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 26.9155, size = 10956, normalized size = 52.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c
 + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.348, size = 1065, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(80*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-32*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)+115*B*cos(d*x+c)^2*sin(d*x+c)*ln(((-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+160*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-43*C*cos(d*x+c)^2*sin(d*x+c)*ln(((-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)-64*C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))+230*B*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+80*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-32*B*cos(d*x+c)^4-86*C*sin(d*x+c)*cos(d*x+c)*ln(((-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-32*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^
(1/2)*sin(d*x+c)+115*B*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-7
8*B*cos(d*x+c)^3-43*C*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+30
*C*cos(d*x+c)^3+40*B*cos(d*x+c)^2-8*C*cos(d*x+c)^2+70*B*cos(d*x+c)-22*C*cos
(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx + c\right)^{2} + B \sec\left(dx + c\right)\right) \cos\left(dx + c\right)^{2}}{\left(a \sec\left(dx + c\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/(a*sec(d*x + c
) + a)^(5/2), x)
```

Fricas [A] time = 22.2534, size = 1948, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((115*B - 43*C)*cos(d*x + c)^3 + 3*(115*B - 43*C)*cos(d*x +
c)^2 + 3*(115*B - 43*C)*cos(d*x + c) + 115*B - 43*C)*sqrt(-a)*log(-(2*sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1)) + 32*((5*B - 2*C)*cos(d*x + c)^3 + 3*(5*B - 2*C)*cos(d*x + c)^2
 + 3*(5*B - 2*C)*cos(d*x + c) + 5*B - 2*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2
 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(16*B*cos(d*x + c)^3 + 5
*(11*B - 3*C)*cos(d*x + c)^2 + (35*B - 11*C)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((115*B - 43*C)*c
os(d*x + c)^3 + 3*(115*B - 43*C)*cos(d*x + c)^2 + 3*(115*B - 43*C)*cos(d*x
+ c) + 115*B - 43*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 32*((5*B - 2*C)*cos(d*x + c
)^3 + 3*(5*B - 2*C)*cos(d*x + c)^2 + 3*(5*B - 2*C)*cos(d*x + c) + 5*B - 2*C
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 2*(16*B*cos(d*x + c)^3 + 5*(11*B - 3*C)*cos(d*x + c)^2
 + (35*B - 11*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.408 $\int \sec^3(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=152

$$\frac{a(5A+5B+4C)\tan^3(c+dx)}{15d}+\frac{a(5A+5B+4C)\tan(c+dx)}{5d}+\frac{a(4A+3(B+C))\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3(B\cdots}{}$$

```
[Out] (a*(4*A + 3*(B + C))*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 5*B + 4*C)*Ta
n[c + d*x])/(5*d) + (a*(4*A + 3*(B + C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (a*(B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*C*Sec[c + d*x]^4*Tan[c
+ d*x])/(5*d) + (a*(5*A + 5*B + 4*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.213046, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4076, 4047, 3767, 4046, 3768, 3770}

$$\frac{a(5A+5B+4C)\tan^3(c+dx)}{15d}+\frac{a(5A+5B+4C)\tan(c+dx)}{5d}+\frac{a(4A+3(B+C))\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3(B\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (a*(4*A + 3*(B + C))*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 5*B + 4*C)*Ta
n[c + d*x])/(5*d) + (a*(4*A + 3*(B + C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (a*(B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*C*Sec[c + d*x]^4*Tan[c
+ d*x])/(5*d) + (a*(5*A + 5*B + 4*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
```

```
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3$$
$$= \frac{aC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3$$
$$= \frac{a(B+C)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{aC}{}$$
$$= \frac{a(5A+5B+4C)\tan(c+dx)}{5d}+\frac{a(4A+}{}$$
$$= \frac{a(4A+3(B+C))\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [A] time = 1.00636, size = 101, normalized size = 0.66

$$\frac{a\left(15(4A+3(B+C))\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5(A+B+2C)\tan^2(c+dx)+15(A+B+C)+3C\tan^4(\right.\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*(15*(4*A + 3*(B + C))*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(4*A + 3*
(B + C))*Sec[c + d*x] + 30*(B + C)*Sec[c + d*x]^3 + 8*(15*(A + B + C) + 5*(
A + B + 2*C)*Tan[c + d*x]^2 + 3*C*Tan[c + d*x]^4))))/(120*d)
```

Maple [B] time = 0.056, size = 287, normalized size = 1.9

$$\frac{Aa\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2Ba\tan(dx+c)}{3d}+\frac{Ba\tan(dx+c)(\sec(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B
*a*tan(d*x+c)+1/3/d*B*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*C*sec(d*x+c)^3*tan(d*
x+c)/d+3/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+
```

```
2/3/d*A*a*tan(d*x+c)+1/3/d*A*a*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*a*tan(d*x+c)
*sec(d*x+c)^3+3/8/d*B*a*sec(d*x+c)*tan(d*x+c)+3/8/d*B*a*ln(sec(d*x+c)+tan(d
*x+c))+8/15*a*C*tan(d*x+c)/d+1/5*a*C*sec(d*x+c)^4*tan(d*x+c)/d+4/15*a*C*sec
(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.953166, size = 359, normalized size = 2.36

$$80\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa+80\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba+16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 80*(tan(d*x + c)^3 + 3*ta
n(d*x + c))*B*a + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c
))*C*a - 15*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
15*C*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
 c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*A*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)))/d
```

Fricas [A] time = 0.531814, size = 437, normalized size = 2.88

$$15\left(4\,A+3\,B+3\,C\right)a\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,A+3\,B+3\,C\right)a\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(4*A + 3*B + 3*C)*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*
A + 3*B + 3*C)*a*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*A + 5*B +
 4*C)*a*cos(d*x + c)^4 + 15*(4*A + 3*B + 3*C)*a*cos(d*x + c)^3 + 8*(5*A + 5
*B + 4*C)*a*cos(d*x + c)^2 + 30*(B + C)*a*cos(d*x + c) + 24*C*a)*sin(d*x +
c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^{3}\left(c+dx\right)dx+\int A\sec^{4}\left(c+dx\right)dx+\int B\sec^{4}\left(c+dx\right)dx+\int B\sec^{5}\left(c+dx\right)dx+\int C\sec^{5}\left(c+dx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integr
al(B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x) + Integral(C*sec(
```

```
c + d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [B] time = 1.30164, size = 404, normalized size = 2.66

$$15\,(4\,Aa+3\,Ba+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,Aa+3\,Ba+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,Aa\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(4*A*a + 3*B*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(
4*A*a + 3*B*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a*tan(1
/2*d*x + 1/2*c)^9 + 45*B*a*tan(1/2*d*x + 1/2*c)^9 + 45*C*a*tan(1/2*d*x + 1/
2*c)^9 - 200*A*a*tan(1/2*d*x + 1/2*c)^7 - 290*B*a*tan(1/2*d*x + 1/2*c)^7 -
130*C*a*tan(1/2*d*x + 1/2*c)^7 + 400*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*B*a*t
an(1/2*d*x + 1/2*c)^5 + 464*C*a*tan(1/2*d*x + 1/2*c)^5 - 440*A*a*tan(1/2*d*
x + 1/2*c)^3 - 350*B*a*tan(1/2*d*x + 1/2*c)^3 - 190*C*a*tan(1/2*d*x + 1/2*c
)^3 + 180*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c) + 195*C*a
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.409 $\int \sec^2(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=127

$$\frac{a(3A+2(B+C))\tan(c+dx)}{3d}+\frac{a(4A+4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+4B+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a(}{}$$

```
[Out] (a*(4*A + 4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(3*A + 2*(B + C))*Ta
n[c + d*x])/(3*d) + (a*(4*A + 4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (a*(B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*C*Sec[c + d*x]^3*Tan[c
+ d*x])/(4*d)
```

Rubi [A] time = 0.190692, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4076, 4047, 3768, 3770, 4046, 3767, 8}

$$\frac{a(3A+2(B+C))\tan(c+dx)}{3d}+\frac{a(4A+4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+4B+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (a*(4*A + 4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(3*A + 2*(B + C))*Ta
n[c + d*x])/(3*d) + (a*(4*A + 4*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (a*(B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (a*C*Sec[c + d*x]^3*Tan[c
+ d*x])/(4*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec$$
$$= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec$$
$$= \frac{a(4A+4B+3C)\sec(c+dx)\tan(c+dx)}{8d}$$
$$= \frac{a(4A+4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+$$
$$= \frac{a(4A+4B+3C)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Mathematica [A] time = 0.622798, size = 84, normalized size = 0.66

$$\frac{a\left(3(4A+4B+3C)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(4A+4B+3C)\sec(c+dx)+24(A+B+C)+8(B+C)\tan\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*(3*(4*A + 4*B + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(A + B + C
) + 3*(4*A + 4*B + 3*C)*Sec[c + d*x] + 6*C*Sec[c + d*x]^3 + 8*(B + C)*Tan[c
 + d*x]^2)))/(24*d)
```

Maple [A] time = 0.049, size = 223, normalized size = 1.8

$$\frac{Aa\tan(dx+c)}{d}+\frac{Ba\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2aC\tan(dx+c)}{3d}+\frac{aC(\sec(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*tan(d*x+c)+1/2/d*B*a*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a*ln(sec(d*x+c)+
tan(d*x+c))+2/3*a*C*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*A*
a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*a*tan(d
*x+c)+1/3/d*B*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*C*sec(d*x+c)^3*tan(d*x+c)/d+3
/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.961057, size = 294, normalized size = 2.31

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca-3Ca\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*C*a - 3*C*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a*tan(d*x + c))/d
```

Fricas [A] time = 0.514318, size = 375, normalized size = 2.95

$$\frac{3(4A+4B+3C)a\cos(dx+c)^4\log(\sin(dx+c)+1)-3(4A+4B+3C)a\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8\right.}{48d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(4*A + 4*B + 3*C)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A +
 4*B + 3*C)*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(3*A + 2*B + 2*C
)*a*cos(d*x + c)^3 + 3*(4*A + 4*B + 3*C)*a*cos(d*x + c)^2 + 8*(B + C)*a*cos
(d*x + c) + 6*C*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec^2(c+dx)\,dx+\int A\sec^3(c+dx)\,dx+\int B\sec^3(c+dx)\,dx+\int B\sec^4(c+dx)\,dx+\int C\sec^4(c+dx)\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integr
al(B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x) + Integral(C*sec(
c + d*x)**4, x) + Integral(C*sec(c + d*x)**5, x))
```

Giac [B] time = 1.29903, size = 343, normalized size = 2.7

$$3\,(4\,Aa+4\,Ba+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,Aa+4\,Ba+3\,Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,Aa\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(4*A*a + 4*B*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A
*a + 4*B*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*A*a*tan(1/2*
d*x + 1/2*c)^7 + 12*B*a*tan(1/2*d*x + 1/2*c)^7 + 9*C*a*tan(1/2*d*x + 1/2*c)
^7 - 60*A*a*tan(1/2*d*x + 1/2*c)^5 - 28*B*a*tan(1/2*d*x + 1/2*c)^5 - 49*C*a
*tan(1/2*d*x + 1/2*c)^5 + 84*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*
x + 1/2*c)^3 + 31*C*a*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*tan(1/2*d*x + 1/2*c)
- 36*B*a*tan(1/2*d*x + 1/2*c) - 39*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^4)/d
```

3.410 $\int \sec(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=92

$$\frac{a(3A+3B+2C)\tan(c+dx)}{3d}+\frac{a(2A+B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(B+C)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tan(c+d}{3}$$

```
[Out] (a*(2*A + B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 3*B + 2*C)*Tan[c
+ d*x])/(3*d) + (a*(B + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*C*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.120209, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4076, 4047, 3767, 8, 4046, 3770}

$$\frac{a(3A+3B+2C)\tan(c+dx)}{3d}+\frac{a(2A+B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(B+C)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aC\tan(c+d}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (a*(2*A + B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 3*B + 2*C)*Tan[c
+ d*x])/(3*d) + (a*(B + C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*C*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c$$
$$= \frac{aC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c$$
$$= \frac{a(B+C)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aC\,\mathrm{se}}{}$$
$$= \frac{a(2A+B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(}{}$$

Mathematica [B] time = 4.35116, size = 485, normalized size = 5.27

$$\frac{a\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4(3A+3B+2C)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4(3A+3B+2C)\sin\left(\right.}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a*Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-6*(2*A + B + C)
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(2*A + B + C)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + (2*C*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((3*B + 4*C)*Cos[c/2] - (3*B + 2*C)*S
in[c/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) +
(4*(3*A + 3*B + 2*C)*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])) + (2*C*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^3) - ((3*B + 4*C)*Cos[c/2] + (3*B + 2*C)*Sin[c
/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(
3*A + 3*B + 2*C)*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]))))/(6*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.048, size = 160, normalized size = 1.7

$$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba\tan(dx+c)}{d}+\frac{aC\sec(dx+c)\tan(dx+c)}{2d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a*tan(d*x+c)+1/2*a*C*sec(d*x+c)*tan
(d*x+c)/d+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*tan(d*x+c)+1/2/d*B*a*
sec(d*x+c)*tan(d*x+c)+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*C*tan(d*x+c
)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.947713, size = 209, normalized size = 2.27

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca-3\,Ba\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ca\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*C*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*A*a*log(sec(d*x + c) + tan(d*x + c)) + 12*A*a*tan(d*x + c) +
12*B*a*tan(d*x + c))/d
```

Fricas [A] time = 0.516922, size = 312, normalized size = 3.39

$$\frac{3\,(2\,A+B+C)a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+B+C)a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(3\,A+\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(3*(2*A + B + C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + B +
 C)*a*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A + 3*B + 2*C)*a*cos(
d*x + c)^2 + 3*(B + C)*a*cos(d*x + c) + 2*C*a)*sin(d*x + c))/(d*cos(d*x + c
)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\sec^{2}\left(c+dx\right)dx+\int B\sec^{2}\left(c+dx\right)dx+\int B\sec^{3}\left(c+dx\right)dx+\int C\sec^{3}\left(c+dx\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sec(c + d*x)**2, x) + Integral(
B*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**3, x) + Integral(C*sec(c +
 d*x)**3, x) + Integral(C*sec(c + d*x)**4, x))
```

Giac [B] time = 1.24508, size = 277, normalized size = 3.01

$$3\,(2\,Aa+Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,Aa+Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(2*A*a + B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a +
B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*
c)^5 + 3*B*a*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*tan(1/2*d*x + 1/2*c)^5 - 12*A*a
*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 6*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c) + 9*C
*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.411 $\int (a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=63

$$\frac{a(2A+2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+aAx+\frac{a(B+C)\tan(c+dx)}{d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] a*A*x + (a*(2*A + 2*B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(B + C)*Tan[c
+ d*x])/d + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0642943, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4048, 3770, 3767, 8}

$$\frac{a(2A+2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+aAx+\frac{a(B+C)\tan(c+dx)}{d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (a*(2*A + 2*B + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(B + C)*Tan[c
+ d*x])/d + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{aC\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \Big(2aA+a(2A+2$$
$$=aAx+\frac{aC\sec(c+dx)\tan(c+dx)}{2d}+(a(B+C))\int \sec$$
$$=aAx+\frac{a(2A+2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\sec$$
$$=aAx+\frac{a(2A+2B+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(B+$$

Mathematica [B] time = 1.76347, size = 305, normalized size = 4.84

$$\frac{a\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-\frac{2(2A+2B+C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(2A+2B+C)\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}{d}\right.}{2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(4*A*x - (2*(2*A
+ 2*B + C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(2*A + 2*B + C)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + C/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2) + (4*(B + C)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])) - C/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) + (4*(B + C)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))))/(2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)
]))
```

Maple [A] time = 0.048, size = 117, normalized size = 1.9

$$aAx+\frac{Aac}{d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a*A*x+1/d*A*a*c+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+a*C*tan(d*x+c)/d+1/d*A*a*
ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a*tan(d*x+c)+1/2*a*C*sec(d*x+c)*tan(d*x+c)/
d+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.938048, size = 157, normalized size = 2.49

$$\frac{4(dx+c)Aa-Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Aa\log(\sec(dx+c)+\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a - C*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*A*a*log(sec(d*x + c) + tan(d*x +
c)) + 4*B*a*log(sec(d*x + c) + tan(d*x + c)) + 4*B*a*tan(d*x + c) + 4*C*a*t
an(d*x + c))/d
```

Fricas [A] time = 0.534177, size = 292, normalized size = 4.63

$$\frac{4\,Aadx\cos\left(dx+c\right)^{2}+\left(2\,A+2\,B+C\right)a\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,A+2\,B+C\right)a\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx\right.\right.}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*A*a*d*x*cos(d*x + c)^2 + (2*A + 2*B + C)*a*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (2*A + 2*B + C)*a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(
2*(B + C)*a*cos(d*x + c) + C*a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\,dx+\int A\sec\left(c+dx\right)dx+\int B\sec\left(c+dx\right)dx+\int B\sec^{2}\left(c+dx\right)dx+\int C\sec^{2}\left(c+dx\right)dx+\int C\sec^{3}\left(c+dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A, x) + Integral(A*sec(c + d*x), x) + Integral(B*sec(c + d*x),
x) + Integral(B*sec(c + d*x)**2, x) + Integral(C*sec(c + d*x)**2, x) + Inte
gral(C*sec(c + d*x)**3, x))
```

Giac [B] time = 1.30011, size = 190, normalized size = 3.02

$$\frac{2\left(dx+c\right)Aa+\left(2\,Aa+2\,Ba+Ca\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Aa+2\,Ba+Ca\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Ba\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + (2*A*a + 2*B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (2*A*a + 2*B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a*
tan(1/2*d*x + 1/2*c)^3 + C*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1
/2*c) - 3*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.412 $\int \cos(c+dx)(a+a\sec(c+dx)) \left(A + B\sec(c + dx) + C\sec^2\right.$

Optimal. Leaf size=46

$$ax(A+B)+\frac{aA\sin(c+dx)}{d}+\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\tan(c+dx)}{d}$$

```
[Out] a*(A + B)*x + (a*(B + C)*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d +
(a*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.116204, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4076, 4047, 8, 4045, 3770}

$$ax(A+B)+\frac{aA\sin(c+dx)}{d}+\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] a*(A + B)*x + (a*(B + C)*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d +
(a*C*Tan[c + d*x])/d
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aC\tan(c+dx)}{d}+\int \cos(c+dx)\left(aA+a(A\right.$$
$$= \frac{aC\tan(c+dx)}{d}+(a(A+B))\int 1\,dx+\int \text{co}$$
$$= a(A+B)x+\frac{aA\sin(c+dx)}{d}+\frac{aC\tan(c+d}{d}$$
$$= a(A+B)x+\frac{a(B+C)\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.0421884, size = 71, normalized size = 1.54

$$\frac{aA\sin(c)\cos(dx)}{d}+\frac{aA\cos(c)\sin(dx)}{d}+aAx+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+aBx+\frac{aC\tan(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] a*A*x + a*B*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])
/d + (a*A*Cos[d*x]*Sin[c])/d + (a*A*Cos[c]*Sin[d*x])/d + (a*C*Tan[c + d*x])
/d
```

Maple [A] time = 0.076, size = 88, normalized size = 1.9

$$aAx+aBx+\frac{Aa\sin(dx+c)}{d}+\frac{Aac}{d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bac}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a*A*x+a*B*x+a*A*sin(d*x+c)/d+1/d*A*a*c+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+1/
d*B*a*c+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+a*C*tan(d*x+c)/d
```

Maxima [A] time = 0.947619, size = 124, normalized size = 2.7

$$\frac{2(dx+c)Aa+2(dx+c)Ba+Ba(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + 2*(d*x + c)*B*a + B*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2
*A*a*sin(d*x + c) + 2*C*a*tan(d*x + c))/d
```

Fricas [A] time = 0.523306, size = 257, normalized size = 5.59

$$\frac{2(A+B)adx\cos(dx+c)+(B+C)a\cos(dx+c)\log(\sin(dx+c)+1)-(B+C)a\cos(dx+c)\log(-\sin(dx+c)+1)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(A + B)*a*d*x*cos(d*x + c) + (B + C)*a*cos(d*x + c)*log(sin(d*x + c)
 + 1) - (B + C)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a*cos(d*x + c)
 + C*a)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\cos(c+dx)\,dx+\int A\cos(c+dx)\sec(c+dx)\,dx+\int B\cos(c+dx)\sec(c+dx)\,dx+\int B\cos(c+dx)\sec^2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a*(Integral(A*cos(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c +
 d*x)**2, x) + Integral(C*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(C*cos
(c + d*x)*sec(c + d*x)**3, x))
```

Giac [B] time = 1.18872, size = 181, normalized size = 3.93

$$\frac{(Aa+Ba)(dx+c)+(Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(Ba+Ca)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] ((A*a + B*a)*(d*x + c) + (B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
B*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)
^3 - C*a*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - C*a*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.413 $\int \cos^2(c+dx)(a+a\sec(c+dx)) \left(A + B\sec(c + dx) + C\sec^2(\right.$

Optimal. Leaf size=62

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(A+2(B+C))+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*(A + 2*(B + C))*x)/2 + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*(A + B)*Sin[c
+ d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.149771, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4074, 4047, 8, 4045, 3770}

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(A+2(B+C))+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a*(A + 2*(B + C))*x)/2 + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*(A + B)*Sin[c
+ d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}\int \cos(c$$
$$= \frac{aA\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}\int \cos(c$$
$$= \frac{1}{2}a(A+2(B+C))x+\frac{a(A+B)\sin(c+}{d}$$
$$= \frac{1}{2}a(A+2(B+C))x+\frac{aC\tanh^{-1}(\sin(c}{d}$$

Mathematica [A] time = 0.141427, size = 59, normalized size = 0.95

$$\frac{a\left(4(A+B)\sin(c+dx)+A\sin(2(c+dx))+2Ac+2Adx+4Bdx+4C\tanh^{-1}(\sin(c+dx))+4Cdx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*(2*A*c + 2*A*d*x + 4*B*d*x + 4*C*d*x + 4*C*ArcTanh[Sin[c + d*x]] + 4*(A
+ B)*Sin[c + d*x] + A*Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.09, size = 100, normalized size = 1.6

$$\frac{Aa\cos(dx+c)\sin(dx+c)}{2d}+\frac{aAx}{2}+\frac{Aac}{2d}+\frac{Ba\sin(dx+c)}{d}+aCx+\frac{Cac}{d}+\frac{Aa\sin(dx+c)}{d}+aBx+\frac{Bac}{d}+\frac{aC\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2*a*A*cos(d*x+c)*sin(d*x+c)/d+1/2*a*A*x+1/2/d*A*a*c+a*B*sin(d*x+c)/d+a*C*
x+1/d*C*a*c+a*A*sin(d*x+c)/d+a*B*x+1/d*B*a*c+1/d*a*C*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 0.939819, size = 120, normalized size = 1.94

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa+4\,(dx+c)Ba+4\,(dx+c)Ca+2\,Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*B*a + 4*(d*x + c)*C
*a + 2*C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a*sin(d*x
+ c) + 4*B*a*sin(d*x + c))/d
```

Fricas [A] time = 0.519659, size = 184, normalized size = 2.97

$$\frac{(A+2B+2C)adx+Ca\log(\sin(dx+c)+1)-Ca\log(-\sin(dx+c)+1)+(Aa\cos(dx+c)+2(A+B)a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*((A + 2*B + 2*C)*a*d*x + C*a*log(sin(d*x + c) + 1) - C*a*log(-sin(d*x +
 c) + 1) + (A*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.24857, size = 177, normalized size = 2.85

$$\frac{2\,Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Aa+2\,Ba+2\,Ca)(dx+c)+\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,B}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*C*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (A*a + 2*B*a + 2*C*a)*(d*x + c) + 2*(A*a*tan(1/2*d*x + 1/2*
c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.414 $\int \cos^3(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=82

$$\frac{a(2A+3(B+C))\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B+2C)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] (a*(A + B + 2*C)*x)/2 + (a*(2*A + 3*(B + C))*Sin[c + d*x])/(3*d) + (a*(A +
B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*
d)
```

Rubi [A] time = 0.17611, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4074, 4047, 2637, 4045, 8}

$$\frac{a(2A+3(B+C))\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B+2C)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a*(A + B + 2*C)*x)/2 + (a*(2*A + 3*(B + C))*Sin[c + d*x])/(3*d) + (a*(A +
B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*
d)
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c
$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c
$$
$$= \frac{a(2A+3(B+C))\sin(c+dx)}{3d} + \frac{a(A+B)c
$$
$$= \frac{1}{2}a(A+B+2C)x + \frac{a(2A+3(B+C))\sin(}{3d}
$$

Mathematica [A] time = 0.228515, size = 64, normalized size = 0.78

$$\frac{a(3(3A+4(B+C))\sin(c+dx)+3(A+B)\sin(2(c+dx))+A\sin(3(c+dx))+6Adx+6Bdx+12Cdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*(6*A*d*x + 6*B*d*x + 12*C*d*x + 3*(3*A + 4*(B + C))*Sin[c + d*x] + 3*(A
+ B)*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.087, size = 102, normalized size = 1.2

$$\frac{1}{d}\left(\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c)+a*
C*sin(d*x+c)+a*C*(d*x+c))
```

Maxima [A] time = 0.93669, size = 132, normalized size = 1.61

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-3(2dx+2c+\sin(2dx+2c))Aa-3(2dx+2c+\sin(2dx+2c))Ba-12(dx+}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 12*(d*x + c)*C*a -
```

```
12*B*a*sin(d*x + c) - 12*C*a*sin(d*x + c))/d
```

Fricas [A] time = 0.497724, size = 162, normalized size = 1.98

$$\frac{3\,(A+B+2\,C)adx+\left(2\,Aa\cos\left(dx+c\right)^{2}+3\,(A+B)a\cos\left(dx+c\right)+2\,(2\,A+3\,B+3\,C)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(A + B + 2*C)*a*d*x + (2*A*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x +
c) + 2*(2*A + 3*B + 3*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.2487, size = 231, normalized size = 2.82

$$\frac{3\,(Aa+Ba+2\,Ca)(dx+c)+\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(3*(A*a + B*a + 2*C*a)*(d*x + c) + 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*
B*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*tan(1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d
*x + 1/2*c)^3 + 12*B*a*tan(1/2*d*x + 1/2*c)^3 + 12*C*a*tan(1/2*d*x + 1/2*c)
^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c) + 6*C*a*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.415 $\int \cos^4(c+dx)(a+a\sec(c+dx)) \left(A + B\sec(c + dx) + C\sec^2(\right.$

Optimal. Leaf size=102

$$\frac{a(A+B+C)\sin(c+dx)}{d} + \frac{a(3A+4(B+C))\sin(c+dx)\cos(c+dx)}{8d} - \frac{a(A+B)\sin^3(c+dx)}{3d} + \frac{1}{8}ax(3A+4(B+C)) +$$

```
[Out] (a*(3*A + 4*(B + C))*x)/8 + (a*(A + B + C)*Sin[c + d*x])/d + (a*(3*A + 4*(B
 + C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])
/(4*d) - (a*(A + B)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.210859, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4074, 4047, 2635, 8, 4044, 3013}

$$\frac{a(A+B+C)\sin(c+dx)}{d} + \frac{a(3A+4(B+C))\sin(c+dx)\cos(c+dx)}{8d} - \frac{a(A+B)\sin^3(c+dx)}{3d} + \frac{1}{8}ax(3A+4(B+C)) +$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a*(3*A + 4*(B + C))*x)/8 + (a*(A + B + C)*Sin[c + d*x])/d + (a*(3*A + 4*(B
 + C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])
/(4*d) - (a*(A + B)*Sin[c + d*x]^3)/(3*d)
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{1}{4}\int \cos^3\\ &= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{1}{4}\int \cos^3\\ &= \frac{a(3A+4(B+C))\cos(c+dx)\sin(c+d}{8d}\\ &= \frac{1}{8}a(3A+4(B+C))x+\frac{a(3A+4(B+C}{}\\ &= \frac{1}{8}a(3A+4(B+C))x+\frac{a(A+B+C)\sin}{d}\end{aligned}$$

Mathematica [A] time = 0.403058, size = 97, normalized size = 0.95

$$\frac{a(24(3A+3B+4C)\sin(c+dx)+24(A+B+C)\sin(2(c+dx))+8A\sin(3(c+dx))+3A\sin(4(c+dx))+24Ac+3}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*(24*A*c + 48*B*c + 36*A*d*x + 48*B*d*x + 48*C*d*x + 24*(3*A + 3*B + 4*C)
*Sin[c + d*x] + 24*(A + B + C)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 8*
B*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.109, size = 141, normalized size = 1.4

$$\frac{1}{d}\left(Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{Ba\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*
x+1/2*c)+a*C*sin(d*x+c))
```

Maxima [A] time = 0.937492, size = 178, normalized size = 1.75

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa-3\left(12dx+12c+\sin\left(4dx+4c\right)+8\sin\left(2dx+2c\right)\right)Aa+32\left(\sin\left(dx+c\right)^{3}-3\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))
*B*a - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 24*(2*d*x + 2*c + sin(2*d*
x + 2*c))*C*a - 96*C*a*sin(d*x + c))/d
```

Fricas [A] time = 0.504472, size = 221, normalized size = 2.17

$$\frac{3\left(3A+4B+4C\right)adx+\left(6Aa\cos\left(dx+c\right)^{3}+8\left(A+B\right)a\cos\left(dx+c\right)^{2}+3\left(3A+4B+4C\right)a\cos\left(dx+c\right)+8\left(2A+2B\right.\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/24*(3*(3*A + 4*B + 4*C)*a*d*x + (6*A*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d
*x + c)^2 + 3*(3*A + 4*B + 4*C)*a*cos(d*x + c) + 8*(2*A + 2*B + 3*C)*a)*sin
(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.24325, size = 294, normalized size = 2.88

$$\frac{3\left(3Aa+4Ba+4Ca\right)\left(dx+c\right)+\frac{2\left(9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+12Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+12Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+49Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+28Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a + 4*C*a)*(d*x + c) + 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7
 + 12*B*a*tan(1/2*d*x + 1/2*c)^7 + 12*C*a*tan(1/2*d*x + 1/2*c)^7 + 49*A*a*t
an(1/2*d*x + 1/2*c)^5 + 28*B*a*tan(1/2*d*x + 1/2*c)^5 + 60*C*a*tan(1/2*d*x
+ 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x + 1/2*c)^3
+ 84*C*a*tan(1/2*d*x + 1/2*c)^3 + 39*A*a*tan(1/2*d*x + 1/2*c) + 36*B*a*tan(
1/2*d*x + 1/2*c) + 36*C*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1
)^4)/d
```

3.416 $\int \cos^5(c+dx)(a+a\sec(c+dx)) \left(A + B\sec(c+dx) + C\sec^2(\right.$

Optimal. Leaf size=141

$$-\frac{a(4A+5(B+C))\sin^3(c+dx)}{15d}+\frac{a(4A+5(B+C))\sin(c+dx)}{5d}+\frac{a(3(A+B)+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{a(A+B)}{}$$

```
[Out] (a*(3*(A + B) + 4*C)*x)/8 + (a*(4*A + 5*(B + C))*Sin[c + d*x])/(5*d) + (a*(
3*(A + B) + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]
^3*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A
+ 5*(B + C))*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.225113, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4074, 4047, 2633, 4045, 2635, 8}

$$-\frac{a(4A+5(B+C))\sin^3(c+dx)}{15d}+\frac{a(4A+5(B+C))\sin(c+dx)}{5d}+\frac{a(3(A+B)+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{a(A+B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (a*(3*(A + B) + 4*C)*x)/8 + (a*(4*A + 5*(B + C))*Sin[c + d*x])/(5*d) + (a*(
3*(A + B) + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]
^3*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A
+ 5*(B + C))*Sin[c + d*x]^3)/(15*d)
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4$$
$$= \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4$$
$$= \frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aA}{}$$
$$= \frac{a(4A+5(B+C))\sin(c+dx)}{5d} + \frac{a(3(A}{}$$
$$= \frac{1}{8}a(3(A+B)+4C)x + \frac{a(4A+5(B+C}{5d}$$

Mathematica [A] time = 0.442144, size = 94, normalized size = 0.67

$$\frac{a\left(-160(2A+B+C)\sin^3(c+dx)+480(A+B+C)\sin(c+dx)+15(4(3A+3B+4C)(c+dx)+8(A+B+C)\sin(2\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a*(480*(A + B + C)*Sin[c + d*x] - 160*(2*A + B + C)*Sin[c + d*x]^3 + 96*A*
Sin[c + d*x]^5 + 15*(4*(3*A + 3*B + 4*C)*(c + d*x) + 8*(A + B + C)*Sin[2*(c
 + d*x)] + (A + B)*Sin[4*(c + d*x)])))/(480*d)
```

Maple [A] time = 0.105, size = 173, normalized size = 1.2

$$\frac{1}{d}\left(\frac{Aa\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)
```

```
+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 0.948338, size = 224, normalized size = 1.59

$$\frac{32 \left(3 \sin \left(dx+c\right)^5-10 \sin \left(dx+c\right)^3+15 \sin \left(dx+c\right)\right)Aa+15 \left(12 dx+12 c+\sin \left(4 dx+4 c\right)+8 \sin \left(2 dx+2 c\right)\right)Aa-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 160*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*B*a + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*si
n(2*d*x + 2*c))*B*a - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a + 120*(2*d*
x + 2*c + sin(2*d*x + 2*c))*C*a)/d
```

Fricas [A] time = 0.512884, size = 282, normalized size = 2.

$$\frac{15 \left(3 A+3 B+4 C\right)adx+\left(24 Aa \cos \left(dx+c\right)^4+30 \left(A+B\right)a \cos \left(dx+c\right)^3+8 \left(4 A+5 B+5 C\right)a \cos \left(dx+c\right)^2+15 \left(3 A\right.\right.}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(3*A + 3*B + 4*C)*a*d*x + (24*A*a*cos(d*x + c)^4 + 30*(A + B)*a*c
os(d*x + c)^3 + 8*(4*A + 5*B + 5*C)*a*cos(d*x + c)^2 + 15*(3*A + 3*B + 4*C)
*a*cos(d*x + c) + 16*(4*A + 5*B + 5*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.27956, size = 355, normalized size = 2.52

$$\frac{15 \left(3 Aa+3 Ba+4 Ca\right)\left(dx+c\right)+\frac{2 \left(45 Aa \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^9+45 Ba \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^9+60 Ca \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^9+130 Aa \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^7+290 Ba \tan \left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(3*A*a + 3*B*a + 4*C*a)*(d*x + c) + 2*(45*A*a*tan(1/2*d*x + 1/2*c
)^9 + 45*B*a*tan(1/2*d*x + 1/2*c)^9 + 60*C*a*tan(1/2*d*x + 1/2*c)^9 + 130*A
*a*tan(1/2*d*x + 1/2*c)^7 + 290*B*a*tan(1/2*d*x + 1/2*c)^7 + 200*C*a*tan(1/
2*d*x + 1/2*c)^7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*B*a*tan(1/2*d*x + 1
/2*c)^5 + 400*C*a*tan(1/2*d*x + 1/2*c)^5 + 190*A*a*tan(1/2*d*x + 1/2*c)^3 +
 350*B*a*tan(1/2*d*x + 1/2*c)^3 + 440*C*a*tan(1/2*d*x + 1/2*c)^3 + 195*A*a*
tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c) + 180*C*a*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.417 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c + dx) + C\sec^2\right.$

Optimal. Leaf size=222

$$\frac{a^2(10A + 9B + 8C)\tan^3(c + dx)}{15d} + \frac{a^2(10A + 9B + 8C)\tan(c + dx)}{5d} + \frac{a^2(14A + 12B + 11C)\tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^2(1\ldots}{}$$

```
[Out] (a^2*(14*A + 12*B + 11*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^2*(10*A + 9*B
+ 8*C)*Tan[c + d*x])/(5*d) + (a^2*(14*A + 12*B + 11*C)*Sec[c + d*x]*Tan[c +
 d*x])/(16*d) + (a^2*(10*A + 12*B + 9*C)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d
) + (C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((3*B +
C)*Sec[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(15*d) + (a^2*(10*
A + 9*B + 8*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.429762, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4088, 4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^2(10A + 9B + 8C)\tan^3(c + dx)}{15d} + \frac{a^2(10A + 9B + 8C)\tan(c + dx)}{5d} + \frac{a^2(14A + 12B + 11C)\tanh^{-1}(\sin(c + dx))}{16d} + \frac{a^2(1\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(14*A + 12*B + 11*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^2*(10*A + 9*B
+ 8*C)*Tan[c + d*x])/(5*d) + (a^2*(14*A + 12*B + 11*C)*Sec[c + d*x]*Tan[c +
 d*x])/(16*d) + (a^2*(10*A + 12*B + 9*C)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d
) + (C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((3*B +
C)*Sec[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(15*d) + (a^2*(10*
A + 9*B + 8*C)*Tan[c + d*x]^3)/(15*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^3(c+dx)(a+a\sec(c+dx))^2\tan(}{6d}$$

$$= \frac{C\sec^3(c+dx)(a+a\sec(c+dx))^2\tan(}{6d}$$

$$= \frac{a^2(10A+12B+9C)\sec^3(c+dx)\tan(}{40d}$$

$$= \frac{a^2(10A+12B+9C)\sec^3(c+dx)\tan(}{40d}$$

$$= \frac{a^2(14A+12B+11C)\sec(c+dx)\tan(}{16d}$$

$$= \frac{a^2(14A+12B+11C)\tanh^{-1}(\sin(c+}{16d}$$

Mathematica [A] time = 3.29793, size = 359, normalized size = 1.62

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(15(14A+12B+11C)\cos^6(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^4*Sec[c + d*x]^6*(15*(14*A + 12*B + 11*C)*Cos[c + d*x]^6*(Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) - 40*C*Sec[c]*Sin[d*x] - 8*Cos[c + d*x]*Sec[c]*(5*C*Sin[c] + 6*(B + 2*C)*
Sin[d*x]) - 2*Cos[c + d*x]^3*Sec[c]*(5*(6*A + 12*B + 11*C)*Sin[c] + 8*(10*A
 + 9*B + 8*C)*Sin[d*x]) - Cos[c + d*x]^5*Sec[c]*(15*(14*A + 12*B + 11*C)*Si
n[c] + 32*(10*A + 9*B + 8*C)*Sin[d*x]) - 2*Cos[c + d*x]^2*Sec[c]*(24*(B + 2
*C)*Sin[c] + 5*(6*A + 12*B + 11*C)*Sin[d*x]) - Cos[c + d*x]^4*Sec[c]*(16*(1
0*A + 9*B + 8*C)*Sin[c] + 15*(14*A + 12*B + 11*C)*Sin[d*x])))/(480*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.068, size = 386, normalized size = 1.7

$$\frac{7a^2A\sec(dx+c)\tan(dx+c)}{8d}+\frac{7a^2A\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{6Ba^2\tan(dx+c)}{5d}+\frac{3Ba^2\tan(dx+c)(\sec(d}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 7/8/d*a^2*A*sec(d*x+c)*tan(d*x+c)+7/8/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+6/5
/d*B*a^2*tan(d*x+c)+3/5/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+11/24/d*a^2*C*tan(d
*x+c)*sec(d*x+c)^3+11/16/d*a^2*C*sec(d*x+c)*tan(d*x+c)+11/16/d*a^2*C*ln(sec
(d*x+c)+tan(d*x+c))+4/3/d*a^2*A*tan(d*x+c)+2/3/d*a^2*A*tan(d*x+c)*sec(d*x+c
)^2+1/2/d*B*a^2*tan(d*x+c)*sec(d*x+c)^3+3/4/d*B*a^2*sec(d*x+c)*tan(d*x+c)+3
/4/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+16/15/d*a^2*C*tan(d*x+c)+2/5/d*a^2*C*t
an(d*x+c)*sec(d*x+c)^4+8/15/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^2*A*tan
(d*x+c)*sec(d*x+c)^3+1/5/d*B*a^2*tan(d*x+c)*sec(d*x+c)^4+1/6/d*a^2*C*tan(d*
x+c)*sec(d*x+c)^5
```

Maxima [B] time = 0.977191, size = 644, normalized size = 2.9

$$\frac{320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^2+160\left(\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(320*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 32*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^2 + 160*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^2 + 64*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c)
)*C*a^2 - 5*C*a^2*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 30*A*a^2*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 30*C*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x +
c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*l
og(sin(d*x + c) - 1)) - 120*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.555197, size = 535, normalized size = 2.41

$$\frac{15\,(14\,A+12\,B+11\,C)a^2\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\,(14\,A+12\,B+11\,C)a^2\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/480*(15*(14*A + 12*B + 11*C)*a^2*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 1
5*(14*A + 12*B + 11*C)*a^2*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(32*(1
0*A + 9*B + 8*C)*a^2*cos(d*x + c)^5 + 15*(14*A + 12*B + 11*C)*a^2*cos(d*x +
 c)^4 + 16*(10*A + 9*B + 8*C)*a^2*cos(d*x + c)^3 + 10*(6*A + 12*B + 11*C)*a
^2*cos(d*x + c)^2 + 48*(B + 2*C)*a^2*cos(d*x + c) + 40*C*a^2)*sin(d*x + c))
/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\sec^3\left(c+dx\right)dx+\int 2A\sec^4\left(c+dx\right)dx+\int A\sec^5\left(c+dx\right)dx+\int B\sec^4\left(c+dx\right)dx+\int 2B\sec^5\left(c+dx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*sec(c + d*x)**4, x) + I
ntegral(A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**4, x) + Integral(2
*B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x) + Integral(C*sec(c
+ d*x)**5, x) + Integral(2*C*sec(c + d*x)**6, x) + Integral(C*sec(c + d*x)*
*7, x))
```

Giac [A] time = 1.36714, size = 529, normalized size = 2.38

$$\frac{15\left(14\,Aa^2+12\,Ba^2+11\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(14\,Aa^2+12\,Ba^2+11\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(14*A*a^2 + 12*B*a^2 + 11*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 15*(14*A*a^2 + 12*B*a^2 + 11*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(210*A*a^2*tan(1/2*d*x + 1/2*c)^11 + 180*B*a^2*tan(1/2*d*x + 1/2*c)^11
 + 165*C*a^2*tan(1/2*d*x + 1/2*c)^11 - 1190*A*a^2*tan(1/2*d*x + 1/2*c)^9 -
1020*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 935*C*a^2*tan(1/2*d*x + 1/2*c)^9 + 2580
*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 2568*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 1986*C*
a^2*tan(1/2*d*x + 1/2*c)^7 - 3180*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 2808*B*a^2
```

```
*tan(1/2*d*x + 1/2*c)^5 - 3006*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 2330*A*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 1860*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 1305*C*a^2*tan(1
/2*d*x + 1/2*c)^3 - 750*A*a^2*tan(1/2*d*x + 1/2*c) - 780*B*a^2*tan(1/2*d*x
+ 1/2*c) - 795*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/
d
```

3.418 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c + dx) + C\sec\right.$

Optimal. Leaf size=190

$$\frac{a^2(8A + 7B + 6C)\tan(c + dx)}{6d} + \frac{a^2(8A + 7B + 6C)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(8A + 7B + 6C)\tan(c + dx)\sec(c + dx)}{24d}$$

```
[Out] (a^2*(8*A + 7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*A + 7*B + 6*C
)*Tan[c + d*x])/(6*d) + (a^2*(8*A + 7*B + 6*C)*Sec[c + d*x]*Tan[c + d*x])/(
24*d) + ((20*A - 5*B + 6*C)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + (
C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(5*d) + ((5*B + 2*C)*
(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*a*d)
```

Rubi [A] time = 0.41012, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4088, 4010, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(8A + 7B + 6C)\tan(c + dx)}{6d} + \frac{a^2(8A + 7B + 6C)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(8A + 7B + 6C)\tan(c + dx)\sec(c + dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```



```
[Out] (a^2*(8*A + 7*B + 6*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*A + 7*B + 6*C
)*Tan[c + d*x])/(6*d) + (a^2*(8*A + 7*B + 6*C)*Sec[c + d*x]*Tan[c + d*x])/(
24*d) + ((20*A - 5*B + 6*C)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + (
C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(5*d) + ((5*B + 2*C)*
(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*a*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
```

```
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^2\tan(c+}{5d}$$

$$= \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^2\tan(c+}{5d}$$

$$= \frac{(20A-5B+6C)(a+a\sec(c+dx))^2\tan(}{60d}$$

$$= \frac{(20A-5B+6C)(a+a\sec(c+dx))^2\tan(}{60d}$$

$$= \frac{a^2(8A+7B+6C)\sec(c+dx)\tan(c+dx)}{24d}$$

$$= \frac{a^2(8A+7B+6C)\tanh^{-1}(\sin(c+dx))}{8d} +$$

Mathematica [B] time = 3.06567, size = 417, normalized size = 2.19

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(240(8A+7B+6C)\cos^5(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^4*Sec[c + d*x]^5*(240*(8*A + 7*B + 6*C)*Cos[c + d*x]^5*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
- Sec[c]*(80*(16*A + 14*B + 15*C)*Sin[d*x] - 240*(3*A + 2*B + C)*Sin[2*c +
d*x] + 240*A*Sin[c + 2*d*x] + 330*B*Sin[c + 2*d*x] + 420*C*Sin[c + 2*d*x] +
 240*A*Sin[3*c + 2*d*x] + 330*B*Sin[3*c + 2*d*x] + 420*C*Sin[3*c + 2*d*x] +
 880*A*Sin[2*c + 3*d*x] + 800*B*Sin[2*c + 3*d*x] + 720*C*Sin[2*c + 3*d*x] -
 120*A*Sin[4*c + 3*d*x] + 120*A*Sin[3*c + 4*d*x] + 105*B*Sin[3*c + 4*d*x] +
 90*C*Sin[3*c + 4*d*x] + 120*A*Sin[5*c + 4*d*x] + 105*B*Sin[5*c + 4*d*x] +
90*C*Sin[5*c + 4*d*x] + 200*A*Sin[4*c + 5*d*x] + 160*B*Sin[4*c + 5*d*x] + 1
44*C*Sin[4*c + 5*d*x])))/(3840*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c +
 d*x)]))
```

Maple [A] time = 0.061, size = 315, normalized size = 1.7

$$\frac{5a^2A\tan(dx+c)}{3d}+\frac{7Ba^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{7Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{6a^2C\tan(dx+c)}{5d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 5/3/d*a^2*A*tan(d*x+c)+7/8/d*B*a^2*sec(d*x+c)*tan(d*x+c)+7/8/d*B*a^2*ln(sec
(d*x+c)+tan(d*x+c))+6/5/d*a^2*C*tan(d*x+c)+3/5/d*a^2*C*tan(d*x+c)*sec(d*x+c
)^2+1/d*a^2*A*sec(d*x+c)*tan(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+4/3
/d*B*a^2*tan(d*x+c)+2/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^2*C*tan(d*x
+c)*sec(d*x+c)^3+3/4/d*a^2*C*sec(d*x+c)*tan(d*x+c)+3/4/d*a^2*C*ln(sec(d*x+c
)+tan(d*x+c))+1/3/d*a^2*A*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*a^2*tan(d*x+c)*se
c(d*x+c)^3+1/5/d*a^2*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [B] time = 0.967308, size = 486, normalized size = 2.56

$$\frac{80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+160\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2+16\left(3\tan(dx+c)^5+10\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 160*(tan(d*x + c)^3 + 3
*tan(d*x + c))*B*a^2 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*C*a^2 + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 - 15*B*a^2*(2*(3
*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*C*a^2*(2*(3*sin(d*
x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(
sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^2*(2*sin(d*x + c)/(s
in(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 240*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.536121, size = 463, normalized size = 2.44

$$15(8A+7B+6C)a^2\cos(dx+c)^5\log(\sin(dx+c)+1)-15(8A+7B+6C)a^2\cos(dx+c)^5\log(-\sin(dx+c)+1)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*A + 7*B + 6*C)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(
8*A + 7*B + 6*C)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(25*A + 2
0*B + 18*C)*a^2*cos(d*x + c)^4 + 15*(8*A + 7*B + 6*C)*a^2*cos(d*x + c)^3 +
8*(5*A + 10*B + 9*C)*a^2*cos(d*x + c)^2 + 30*(B + 2*C)*a^2*cos(d*x + c) + 2
4*C*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\sec^2(c+dx)\,dx+\int 2A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int B\sec^3(c+dx)\,dx+\int 2B\sec^4(c+dx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*sec(c + d*x)**3, x) + I
ntegral(A*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**3, x) + Integral(2
*B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x) + Integral(C*sec(c
+ d*x)**4, x) + Integral(2*C*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)*
*6, x))
```

Giac [A] time = 1.31246, size = 460, normalized size = 2.42

$$15\left(8Aa^2+7Ba^2+6Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(8Aa^2+7Ba^2+6Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2(120}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(15*(8*A*a^2 + 7*B*a^2 + 6*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(8*A*a^2 + 7*B*a^2 + 6*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(
120*A*a^2*tan(1/2*d*x + 1/2*c)^9 + 105*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 90*C*
a^2*tan(1/2*d*x + 1/2*c)^9 - 560*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 490*B*a^2*t
an(1/2*d*x + 1/2*c)^7 - 420*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 1120*A*a^2*tan(1
/2*d*x + 1/2*c)^5 + 800*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 864*C*a^2*tan(1/2*d*
x + 1/2*c)^5 - 1040*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 790*B*a^2*tan(1/2*d*x +
1/2*c)^3 - 540*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 360*A*a^2*tan(1/2*d*x + 1/2*c
) + 375*B*a^2*tan(1/2*d*x + 1/2*c) + 390*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.419 $\int \sec(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c + dx) + C\sec$

Optimal. Leaf size=147

$$\frac{a^2(12A + 8B + 7C)\tan(c + dx)}{6d} + \frac{a^2(12A + 8B + 7C)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(12A + 8B + 7C)\tan(c + dx)\sec(c +}{24d}$$

```
[Out] (a^2*(12*A + 8*B + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(12*A + 8*B + 7
*C)*Tan[c + d*x])/(6*d) + (a^2*(12*A + 8*B + 7*C)*Sec[c + d*x]*Tan[c + d*x]
)/(24*d) + ((4*B - C)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a +
 a*Sec[c + d*x])^3*Tan[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.234871, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4082, 4001, 3788, 3767, 8, 4046, 3770}

$$\frac{a^2(12A + 8B + 7C)\tan(c + dx)}{6d} + \frac{a^2(12A + 8B + 7C)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(12A + 8B + 7C)\tan(c + dx)\sec(c +}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a^2*(12*A + 8*B + 7*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(12*A + 8*B + 7
*C)*Tan[c + d*x])/(6*d) + (a^2*(12*A + 8*B + 7*C)*Sec[c + d*x]*Tan[c + d*x]
)/(24*d) + ((4*B - C)*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a +
 a*Sec[c + d*x])^3*Tan[c + d*x])/(4*a*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4ad}+\frac{\int \sec(}{}$$
$$=\frac{(4B-C)(a+a\sec(c+dx))^2\tan(c+dx)}{12d}+$$
$$=\frac{(4B-C)(a+a\sec(c+dx))^2\tan(c+dx)}{12d}+$$
$$=\frac{a^2(12A+8B+7C)\sec(c+dx)\tan(c+dx)}{24d}$$
$$=\frac{a^2(12A+8B+7C)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Mathematica [B] time = 2.41386, size = 386, normalized size = 2.63

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(24(12A+8B+7C)\cos^4(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^4*Sec[c + d*x]^4*(24*(12*A + 8*B + 7*C)*Cos[c + d*x]^4*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
- Sec[c]*(-24*(6*A + 5*B + 4*C)*Sin[c] + 3*(4*A + 8*B + 15*C)*Sin[d*x] + 12
*A*Sin[2*c + d*x] + 24*B*Sin[2*c + d*x] + 45*C*Sin[2*c + d*x] + 144*A*Sin[c
 + 2*d*x] + 136*B*Sin[c + 2*d*x] + 128*C*Sin[c + 2*d*x] - 48*A*Sin[3*c + 2*
d*x] - 24*B*Sin[3*c + 2*d*x] + 12*A*Sin[2*c + 3*d*x] + 24*B*Sin[2*c + 3*d*x
] + 21*C*Sin[2*c + 3*d*x] + 12*A*Sin[4*c + 3*d*x] + 24*B*Sin[4*c + 3*d*x] +
 21*C*Sin[4*c + 3*d*x] + 48*A*Sin[3*c + 4*d*x] + 40*B*Sin[3*c + 4*d*x] + 32
*C*Sin[3*c + 4*d*x])))/(384*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*
x)]))
```

Maple [A] time = 0.057, size = 246, normalized size = 1.7

$$\frac{3\,a^{2}A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2\,d}+\frac{5\,Ba^{2}\tan\left(dx+c\right)}{3\,d}+\frac{7\,a^{2}C\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}+\frac{7\,a^{2}C\ln\left(\sec\left(dx+c\right)\cdot\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+5/3/d*B*a^2*tan(d*x+c)+7/8/d*a^2*C*se
c(d*x+c)*tan(d*x+c)+7/8/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*A*tan(d*x
+c)+1/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3
/d*a^2*C*tan(d*x+c)+2/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^2*A*sec(d*x
+c)*tan(d*x+c)+1/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^2*C*tan(d*x+c)*s
ec(d*x+c)^3
```

Maxima [B] time = 0.959035, size = 417, normalized size = 2.84

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{2}+32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca^{2}-3\,Ca^{2}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a^2 - 3*C*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*C*a^2*(2*sin(d*
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)
) + 48*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 96*A*a^2*tan(d*x + c) + 48*
B*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.518893, size = 397, normalized size = 2.7

$$\frac{3\,(12\,A+8\,B+7\,C)a^{2}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(12\,A+8\,B+7\,C)a^{2}\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right.}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(12*A + 8*B + 7*C)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(12
*A + 8*B + 7*C)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(6*A + 5*B
 + 4*C)*a^2*cos(d*x + c)^3 + 3*(4*A + 8*B + 7*C)*a^2*cos(d*x + c)^2 + 8*(B
+ 2*C)*a^2*cos(d*x + c) + 6*C*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int A \sec(c+dx)\,dx + \int 2A \sec^2(c+dx)\,dx + \int A \sec^3(c+dx)\,dx + \int B \sec^2(c+dx)\,dx + \int 2B \sec^3(c+dx)\,dx \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sec(c + d*x)**2, x) + Inte
gral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**2, x) + Integral(2*B*
sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x) + Integral(C*sec(c + d
*x)**3, x) + Integral(2*C*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**5,
 x))
```

Giac [B] time = 1.26903, size = 392, normalized size = 2.67

$$3\left(12\,Aa^2 + 8\,Ba^2 + 7\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(12\,Aa^2 + 8\,Ba^2 + 7\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(36\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(12*A*a^2 + 8*B*a^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 3*(12*A*a^2 + 8*B*a^2 + 7*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3
6*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 21*C*a^2
*tan(1/2*d*x + 1/2*c)^7 - 132*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 88*B*a^2*tan(1
/2*d*x + 1/2*c)^5 - 77*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 156*A*a^2*tan(1/2*d*x
 + 1/2*c)^3 + 136*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 83*C*a^2*tan(1/2*d*x + 1/2
*c)^3 - 60*A*a^2*tan(1/2*d*x + 1/2*c) - 72*B*a^2*tan(1/2*d*x + 1/2*c) - 75*
C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.420 $\int(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=120

$$\frac{a^2(2A+3B+2C)\tan(c+dx)}{2d}+\frac{a^2(4A+3B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{(3B+2C)\tan(c+dx)\left(a^2\sec(c+\right.}{6d}$$

```
[Out] a^2*A*x + (a^2*(4*A + 3*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A +
 3*B + 2*C)*Tan[c + d*x])/(2*d) + (C*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(
3*d) + ((3*B + 2*C)*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.157222, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4054, 3917, 3914, 3767, 8, 3770}

$$\frac{a^2(2A+3B+2C)\tan(c+dx)}{2d}+\frac{a^2(4A+3B+2C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{(3B+2C)\tan(c+dx)\left(a^2\sec(c+\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a^2*A*x + (a^2*(4*A + 3*B + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A +
 3*B + 2*C)*Tan[c + d*x])/(2*d) + (C*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(
3*d) + ((3*B + 2*C)*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx = \frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{\int (a + a\sec(c+d}{}$$

$$= \frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{(3B+2C)\left(a^2 + a\right.}{}$$

$$= a^2Ax + \frac{C(a+a\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{(3B+2C)}{}$$

$$= a^2Ax + \frac{a^2(4A+3B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{C(a}{}$$

$$= a^2Ax + \frac{a^2(4A+3B+2C)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(2}{}$$

Mathematica [B] time = 5.68545, size = 542, normalized size = 4.52

$$\frac{a^2\cos^4(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4(3A+6B+5C)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(12*A*x - (6*(4*A + 3*B + 2*C)*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (6*(4*A + 3*B + 2*C)*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]])/d + (2*C*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^3) + ((3*B + 7*C)*Cos[c/2] - (3*B + 5*C)*Sin[c/2]
)/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(3
*A + 6*B + 5*C)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])) + (2*C*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^3) - ((3*B + 7*C)*Cos[c/2] + (3*B + 5*C)*Sin[c/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*
(3*A + 6*B + 5*C)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]))))/(24*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]
))
```

Maple [A] time = 0.056, size = 193, normalized size = 1.6

$$a^2Ax + \frac{Aa^2c}{d} + \frac{3\,Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + \frac{5\,a^2C\tan(dx+c)}{3\,d} + 2\,\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\,\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^2*A*x+1/d*A*a^2*c+3/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+5/3/d*a^2*C*tan(d
*x+c)+2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a^2*tan(d*x+c)+1/d*a^2*C*se
c(d*x+c)*tan(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*A*tan(d*x+c
)+1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.951897, size = 284, normalized size = 2.37

$$\frac{12\,(dx+c)Aa^2+4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ca^2-3\,Ba^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^2 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 - 3*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 6*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^2*log(sec(d*x + c) + tan(d*x +
 c)) + 12*B*a^2*log(sec(d*x + c) + tan(d*x + c)) + 12*A*a^2*tan(d*x + c) +
24*B*a^2*tan(d*x + c) + 12*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.526347, size = 379, normalized size = 3.16

$$\frac{12\,Aa^2dx\cos\left(dx+c\right)^3+3\,(4\,A+3\,B+2\,C)a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,A+3\,B+2\,C)a^2\cos\left(dx+c\right)}{12\,d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/12*(12*A*a^2*d*x*cos(d*x + c)^3 + 3*(4*A + 3*B + 2*C)*a^2*cos(d*x + c)^3*
log(sin(d*x + c) + 1) - 3*(4*A + 3*B + 2*C)*a^2*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(2*(3*A + 6*B + 5*C)*a^2*cos(d*x + c)^2 + 3*(B + 2*C)*a^2*co
s(d*x + c) + 2*C*a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int A\,dx+\int 2A\sec\left(c+dx\right)dx+\int A\sec^2\left(c+dx\right)dx+\int B\sec\left(c+dx\right)dx+\int 2B\sec^2\left(c+dx\right)dx+\int B\sec^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**2*(Integral(A, x) + Integral(2*A*sec(c + d*x), x) + Integral(A*sec(c + d
*x)**2, x) + Integral(B*sec(c + d*x), x) + Integral(2*B*sec(c + d*x)**2, x)
 + Integral(B*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**2, x) + Integr
```

```
al(2*C*sec(c + d*x)**3, x) + Integral(C*sec(c + d*x)**4, x))
```

Giac [B] time = 1.29302, size = 338, normalized size = 2.82

$$\frac{6\,(dx+c)Aa^2+3\left(4\,Aa^2+3\,Ba^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,Aa^2+3\,Ba^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^2 + 3*(4*A*a^2 + 3*B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) - 3*(4*A*a^2 + 3*B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*
c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2
4*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 16*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*
tan(1/2*d*x + 1/2*c) + 15*B*a^2*tan(1/2*d*x + 1/2*c) + 18*C*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.421 $\int \cos(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=121

$$-\frac{a^2(2A-2B-3C)\tan(c+dx)}{2d}+\frac{a^2(2A+4B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(2A+B)-\frac{(2A-C)\tan(c+dx)\left(a^2\right.}{2d}$$

```
[Out] a^2*(2*A + B)*x + (a^2*(2*A + 4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*
(a + a*Sec[c + d*x])^2*Sin[c + d*x])/d - (a^2*(2*A - 2*B - 3*C)*Tan[c + d*x
])/(2*d) - ((2*A - C)*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.216887, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4086, 3917, 3914, 3767, 8, 3770}

$$-\frac{a^2(2A-2B-3C)\tan(c+dx)}{2d}+\frac{a^2(2A+4B+3C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(2A+B)-\frac{(2A-C)\tan(c+dx)\left(a^2\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a^2*(2*A + B)*x + (a^2*(2*A + 4*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*
(a + a*Sec[c + d*x])^2*Sin[c + d*x])/d - (a^2*(2*A - 2*B - 3*C)*Tan[c + d*x
])/(2*d) - ((2*A - C)*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^2\sin(c+dx)}{d} + \frac{\int(a+}{}$$

$$= \frac{A(a+a\sec(c+dx))^2\sin(c+dx)}{d} - \frac{(2A-}{}$$

$$= a^2(2A+B)x + \frac{A(a+a\sec(c+dx))^2\sin(c}{d}$$

$$= a^2(2A+B)x + \frac{a^2(2A+4B+3C)\tanh^{-1}(s}{2d}$$

$$= a^2(2A+B)x + \frac{a^2(2A+4B+3C)\tanh^{-1}(s}{2d}$$

Mathematica [B] time = 3.5695, size = 365, normalized size = 3.02

$$a^2\cos^4(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-\frac{2(2A+4B+3C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin(}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a^2*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(4*(2*A + B)*x - (2*(2*A + 4*B + 3*C)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(2*A + 4*B + 3*C)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + (4*A*Cos[d*x]*Sin[c])/d + (4*A*Cos[c]*Sin[d*x])/d
 + C/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(B + 2*C)*Sin[(d*x)/2
])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - C/(d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(B + 2*C)*Sin[(d*x)/2])/(d*(Co
s[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(8*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.103, size = 166, normalized size = 1.4

$$\frac{a^2A\sin(dx+c)}{d} + a^2Bx + \frac{Ba^2c}{d} + \frac{3a^2C\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 2a^2Ax + 2\frac{Aa^2c}{d} + 2\frac{Ba^2\ln(\sec(dx+c)+\mathrm{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*sin(d*x+c)+a^2*B*x+1/d*B*a^2*c+3/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+
c))+2*a^2*A*x+2/d*A*a^2*c+2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*C*tan
(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^2*tan(d*x+c)+1/2/d*a^2*
C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.954943, size = 259, normalized size = 2.14

$$\frac{8\,(dx+c)Aa^2 + 4\,(dx+c)Ba^2 - Ca^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Aa^2(\log\left(\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*(8*(d*x + c)*A*a^2 + 4*(d*x + c)*B*a^2 - C*a^2*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 4*A*a^2*sin(d*x + c) + 4*B*a^2*tan(d*x + c) + 8*C*a^2*tan(d*x
+ c))/d
```

Fricas [A] time = 0.530149, size = 358, normalized size = 2.96

$$\frac{4\,(2\,A+B)a^2dx\cos\left(dx+c\right)^2 + (2\,A+4\,B+3\,C)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - (2\,A+4\,B+3\,C)a^2\cos\left(dx+\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/4*(4*(2*A + B)*a^2*d*x*cos(d*x + c)^2 + (2*A + 4*B + 3*C)*a^2*cos(d*x + c
)^2*log(sin(d*x + c) + 1) - (2*A + 4*B + 3*C)*a^2*cos(d*x + c)^2*log(-sin(d
*x + c) + 1) + 2*(2*A*a^2*cos(d*x + c)^2 + 2*(B + 2*C)*a^2*cos(d*x + c) + C
*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.23479, size = 275, normalized size = 2.27

$$\frac{\frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(2\,Aa^2+Ba^2\right)(dx+c)+\left(2\,Aa^2+4\,Ba^2+3\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Aa^2+4\,Ba^2+3\,C\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(2*A*a^2
 + B*a^2)*(d*x + c) + (2*A*a^2 + 4*B*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1
/2*c) + 1)) - (2*A*a^2 + 4*B*a^2 + 3*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(2*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^3 -
2*B*a^2*tan(1/2*d*x + 1/2*c) - 5*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^2)/d
```

3.422 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c + dx) + C\text{se}\right.$

Optimal. Leaf size=128

$$\frac{a^2(3A+2B-2C)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(3A+4B+2C)-\frac{(A-2C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{a^2(B+2C)\tan}{}$$

```
[Out] (a^2*(3*A + 4*B + 2*C)*x)/2 + (a^2*(B + 2*C)*ArcTanh[Sin[c + d*x]])/d + (a^
2*(3*A + 2*B - 2*C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*
x])^2*Sin[c + d*x])/(2*d) - ((A - 2*C)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(2*d)
```

Rubi [A] time = 0.287309, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4086, 4018, 3996, 3770}

$$\frac{a^2(3A+2B-2C)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(3A+4B+2C)-\frac{(A-2C)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{a^2(B+2C)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(3*A + 4*B + 2*C)*x)/2 + (a^2*(B + 2*C)*ArcTanh[Sin[c + d*x]])/d + (a^
2*(3*A + 2*B - 2*C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*
x])^2*Sin[c + d*x])/(2*d) - ((A - 2*C)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(2*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+}{2d}$$
$$= \frac{A\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+}{2d}$$
$$= \frac{a^2(3A+2B-2C)\sin(c+dx)}{2d} + \frac{A\cos(c}{}$$
$$= \frac{1}{2}a^2(3A+4B+2C)x + \frac{a^2(3A+2B-2C}{2d}$$
$$= \frac{1}{2}a^2(3A+4B+2C)x + \frac{a^2(B+2C)\tanh^{-}}{d}$$

Mathematica [B] time = 3.65387, size = 329, normalized size = 2.57

$$\frac{a^2\cos^2(c+dx)(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4(2A+B)\sin(c)\cos(dx)}{d}+\frac{4(2A+B)\cos(c}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*Cos[c + d*x]^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(2*(3*A + 4*B + 2*C)*x - (4*(B + 2*C)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(B + 2*C)*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]])/d + (4*(2*A + B)*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])
/d + (4*(2*A + B)*Cos[c]*Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/d + (4*C*Sin
[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
+ (4*C*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]))))/(8*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.091, size = 160, normalized size = 1.3

$$\frac{a^2A\cos(dx+c)\sin(dx+c)}{2d} + \frac{3a^2Ax}{2} + \frac{3a^2Ac}{2d} + \frac{Ba^2\sin(dx+c)}{d} + a^2Cx + \frac{Ca^2c}{d} + 2\frac{a^2A\sin(dx+c)}{d} + 2a^2Bx + 2\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*a^2*A*cos(d*x+c)*sin(d*x+c)+3/2*a^2*A*x+3/2/d*A*a^2*c+a^2*B*sin(d*x+c
)/d+a^2*C*x+1/d*C*a^2*c+2/d*a^2*A*sin(d*x+c)+2*a^2*B*x+2/d*B*a^2*c+2/d*a^2*
C*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*t
an(d*x+c)
```

Maxima [A] time = 0.958634, size = 204, normalized size = 1.59

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+4\,(dx+c)Aa^2+8\,(dx+c)Ba^2+4\,(dx+c)Ca^2+2\,Ba^2(\log(\sin(dx+c)+1)-\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*A*a^2 + 8*(d*x +
c)*B*a^2 + 4*(d*x + c)*C*a^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 4*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*A
*a^2*sin(d*x + c) + 4*B*a^2*sin(d*x + c) + 4*C*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.529299, size = 331, normalized size = 2.59

$$\frac{(3\,A+4\,B+2\,C)a^2dx\cos(dx+c)+(B+2\,C)a^2\cos(dx+c)\log(\sin(dx+c)+1)-(B+2\,C)a^2\cos(dx+c)\log(-\sin}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/2*((3*A + 4*B + 2*C)*a^2*d*x*cos(d*x + c) + (B + 2*C)*a^2*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - (B + 2*C)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (
A*a^2*cos(d*x + c)^2 + 2*(2*A + B)*a^2*cos(d*x + c) + 2*C*a^2)*sin(d*x + c)
)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29523, size = 267, normalized size = 2.09

$$-\frac{\frac{4\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\left(3\,Aa^2+4\,Ba^2+2\,Ca^2\right)(dx+c)-2\left(Ba^2+2\,Ca^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(Ba^2+2\,Ca^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -1/2*(4*C*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (3*A*a^2
+ 4*B*a^2 + 2*C*a^2)*(d*x + c) - 2*(B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) + 2*(B*a^2 + 2*C*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(
3*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*t
an(1/2*d*x + 1/2*c) + 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 + 1)^2)/d
```

3.423 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\text{se}\right.$

Optimal. Leaf size=134

$$\frac{a^2(2A+3B+2C)\sin(c+dx)}{2d} + \frac{(2A+3B)\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{6d} + \frac{1}{2}a^2x(2A+3B+4C) + \frac{a^2}{}$$

```
[Out] (a^2*(2*A + 3*B + 4*C)*x)/2 + (a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A +
 3*B + 2*C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*
Sin[c + d*x])/(3*d) + ((2*A + 3*B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Si
n[c + d*x])/(6*d)
```

Rubi [A] time = 0.291759, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4086, 4017, 3996, 3770}

$$\frac{a^2(2A+3B+2C)\sin(c+dx)}{2d} + \frac{(2A+3B)\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{6d} + \frac{1}{2}a^2x(2A+3B+4C) + \frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```



```
[Out] (a^2*(2*A + 3*B + 4*C)*x)/2 + (a^2*C*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A +
 3*B + 2*C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*
Sin[c + d*x])/(3*d) + ((2*A + 3*B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Si
n[c + d*x])/(6*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+}{3d}$$
$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+}{3d}$$
$$= \frac{a^2(2A+3B+2C)\sin(c+dx)}{2d} + \frac{A\cos^2(c}{}$$
$$= \frac{1}{2}a^2(2A+3B+4C)x + \frac{a^2(2A+3B+2C}{2d}$$
$$= \frac{1}{2}a^2(2A+3B+4C)x + \frac{a^2C\tanh^{-1}(\sin(c}{d}$$

Mathematica [A] time = 0.290327, size = 121, normalized size = 0.9

$$\frac{a^2\left(3(7A+8B+4C)\sin(c+dx)+3(2A+B)\sin(2(c+dx))+A\sin(3(c+dx))+12Adx+18Bdx-12C\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(12*A*d*x + 18*B*d*x + 24*C*d*x - 12*C*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 12*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(7*A + 8*B +
4*C)*Sin[c + d*x] + 3*(2*A + B)*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12
*d)
```

Maple [A] time = 0.103, size = 181, normalized size = 1.4

$$\frac{A(\cos(dx+c))^2\sin(dx+c)a^2}{3d} + \frac{5a^2A\sin(dx+c)}{3d} + \frac{Ba^2\cos(dx+c)\sin(dx+c)}{2d} + \frac{3a^2Bx}{2} + \frac{3Ba^2c}{2d} + \frac{a^2C\sin(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^2+5/3/d*a^2*A*sin(d*x+c)+1/2*a^2*B*cos(d*
x+c)*sin(d*x+c)/d+3/2*a^2*B*x+3/2/d*B*a^2*c+1/d*a^2*C*sin(d*x+c)+1/d*a^2*A*
cos(d*x+c)*sin(d*x+c)+a^2*A*x+1/d*A*a^2*c+2*a^2*B*sin(d*x+c)/d+2*a^2*C*x+2/
d*C*a^2*c+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.946821, size = 216, normalized size = 1.61

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-6(2dx+2c+\sin(2dx+2c))Aa^2-3(2dx+2c+\sin(2dx+2c))Ba^2-12(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 6*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 12*(d*x + c)*
B*a^2 - 24*(d*x + c)*C*a^2 - 6*C*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) - 12*A*a^2*sin(d*x + c) - 24*B*a^2*sin(d*x + c) - 12*C*a^2*sin(d*
x + c))/d
```

Fricas [A] time = 0.528278, size = 269, normalized size = 2.01

$$\frac{3\,(2\,A+3\,B+4\,C)a^2dx+3\,Ca^2\log\left(\sin\left(dx+c\right)+1\right)-3\,Ca^2\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Aa^2\cos\left(dx+c\right)^2+3\,(2\,A\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A + 3*B + 4*C)*a^2*d*x + 3*C*a^2*log(sin(d*x + c) + 1) - 3*C*a^2*
log(-sin(d*x + c) + 1) + (2*A*a^2*cos(d*x + c)^2 + 3*(2*A + B)*a^2*cos(d*x
+ c) + 2*(5*A + 6*B + 3*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27996, size = 317, normalized size = 2.37

$$6\,Ca^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Ca^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(2\,Aa^2+3\,Ba^2+4\,Ca^2\right)(dx+c)+\frac{2\left(6\,Aa^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/6*(6*C*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*C*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*A*a^2 + 3*B*a^2 + 4*C*a^2)*(d*x + c) + 2*(6*A*a^2*
tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d
```

```
*x + 1/2*c)^5 + 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a^2*tan(1/2*d*x + 1/
2*c)^3 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) +
15*B*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

3.424 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\sec\right.$

Optimal. Leaf size=149

$$\frac{a^2(7A+8B+12C)\sin(c+dx)}{6d} + \frac{a^2(7A+8B+12C)\sin(c+dx)\cos(c+dx)}{24d} + \frac{1}{8}a^2x(7A+8B+12C) + \frac{(A+2B)\sin}{}$$

```
[Out] (a^2*(7*A + 8*B + 12*C)*x)/8 + (a^2*(7*A + 8*B + 12*C)*Sin[c + d*x])/(6*d)
+ (a^2*(7*A + 8*B + 12*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((A + 2*B)*Co
s[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^3
*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.327804, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4086, 4013, 3788, 2637, 4045, 8}

$$\frac{a^2(7A+8B+12C)\sin(c+dx)}{6d} + \frac{a^2(7A+8B+12C)\sin(c+dx)\cos(c+dx)}{24d} + \frac{1}{8}a^2x(7A+8B+12C) + \frac{(A+2B)\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(7*A + 8*B + 12*C)*x)/8 + (a^2*(7*A + 8*B + 12*C)*Sin[c + d*x])/(6*d)
+ (a^2*(7*A + 8*B + 12*C)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((A + 2*B)*Co
s[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^3
*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(4*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^2\sin(c+$$
$$= \frac{(A+2B)\cos^2(c+dx)(a+a\sec(c+dx))^2}{6d}$$
$$= \frac{(A+2B)\cos^2(c+dx)(a+a\sec(c+dx))^2}{6d}$$
$$= \frac{a^2(7A+8B+12C)\sin(c+dx)}{6d} + \frac{a^2(7A+$$
$$= \frac{1}{8}a^2(7A+8B+12C)x + \frac{a^2(7A+8B+12}{6d}$$

Mathematica [A] time = 0.33116, size = 95, normalized size = 0.64

$$\frac{a^2(24(6A+7B+8C)\sin(c+dx)+24(2A+2B+C)\sin(2(c+dx))+16A\sin(3(c+dx))+3A\sin(4(c+dx))+84Adx+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(84*A*d*x + 96*B*d*x + 144*C*d*x + 24*(6*A + 7*B + 8*C)*Sin[c + d*x] +
 24*(2*A + 2*B + C)*Sin[2*(c + d*x)] + 16*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c
 + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.102, size = 203, normalized size = 1.4

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2A\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{Ba^2(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3
*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^
2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*B*a^2*(1/2*cos(d*x+c)*sin(d
```

```
*x+c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^2*
sin(d*x+c)+2*a^2*C*sin(d*x+c)+a^2*C*(d*x+c))
```

Maxima [A] time = 0.950691, size = 257, normalized size = 1.72

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{2}-24\left(2\,dx+2\,c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*
c))*A*a^2 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 48*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 - 96*(d
*x + c)*C*a^2 - 96*B*a^2*sin(d*x + c) - 192*C*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.503528, size = 239, normalized size = 1.6

$$\frac{3\left(7\,A+8\,B+12\,C\right)a^{2}dx+\left(6\,Aa^{2}\cos\left(dx+c\right)^{3}+8\left(2\,A+B\right)a^{2}\cos\left(dx+c\right)^{2}+3\left(7\,A+8\,B+4\,C\right)a^{2}\cos\left(dx+c\right)+8\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/24*(3*(7*A + 8*B + 12*C)*a^2*d*x + (6*A*a^2*cos(d*x + c)^3 + 8*(2*A + B)*
a^2*cos(d*x + c)^2 + 3*(7*A + 8*B + 4*C)*a^2*cos(d*x + c) + 8*(4*A + 5*B +
6*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26549, size = 335, normalized size = 2.25

$$\frac{3\left(7\,Aa^{2}+8\,Ba^{2}+12\,Ca^{2}\right)\left(dx+c\right)+\frac{2\left(21\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+24\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+36\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+77\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+88\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2 + 12*C*a^2)*(d*x + c) + 2*(21*A*a^2*tan(1/2*d*x
+ 1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 36*C*a^2*tan(1/2*d*x + 1/2*c
)^7 + 77*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 88*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 1
32*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136*B*a
^2*tan(1/2*d*x + 1/2*c)^3 + 156*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*A*a^2*tan
(1/2*d*x + 1/2*c) + 72*B*a^2*tan(1/2*d*x + 1/2*c) + 60*C*a^2*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.425 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\text{se}\right.$

Optimal. Leaf size=187

$$\frac{a^2(18A+20B+25C)\sin(c+dx)}{15d}+\frac{a^2(18A+25B+20C)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^2(6A+7B+8C)\sin(c+dx)\,c}{8d}$$

```
[Out] (a^2*(6*A + 7*B + 8*C)*x)/8 + (a^2*(18*A + 20*B + 25*C)*Sin[c + d*x])/(15*d
) + (a^2*(6*A + 7*B + 8*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(18*A +
25*B + 20*C)*Cos[c + d*x]^2*Sin[c + d*x])/(60*d) + (A*Cos[c + d*x]^4*(a + a
*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((2*A + 5*B)*Cos[c + d*x]^3*(a^2 + a
^2*Sec[c + d*x])*Sin[c + d*x])/(20*d)
```

Rubi [A] time = 0.414596, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^2(18A+20B+25C)\sin(c+dx)}{15d}+\frac{a^2(18A+25B+20C)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^2(6A+7B+8C)\sin(c+dx)\,c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(6*A + 7*B + 8*C)*x)/8 + (a^2*(18*A + 20*B + 25*C)*Sin[c + d*x])/(15*d
) + (a^2*(6*A + 7*B + 8*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(18*A +
25*B + 20*C)*Cos[c + d*x]^2*Sin[c + d*x])/(60*d) + (A*Cos[c + d*x]^4*(a + a
*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((2*A + 5*B)*Cos[c + d*x]^3*(a^2 + a
^2*Sec[c + d*x])*Sin[c + d*x])/(20*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
```

```
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+}{5d}$$
$$= \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+}{5d}$$
$$= \frac{a^2(18A+25B+20C)\cos^2(c+dx)\sin(c+}{60d}$$
$$= \frac{a^2(18A+25B+20C)\cos^2(c+dx)\sin(c+}{60d}$$
$$= \frac{a^2(18A+20B+25C)\sin(c+dx)}{15d} + \frac{a^2(6A}{}$$
$$= \frac{1}{8}a^2(6A+7B+8C)x + \frac{a^2(18A+20B+2}{15d}$$

Mathematica [A] time = 0.603488, size = 132, normalized size = 0.71

$$\frac{a^2(60(11A+12B+14C)\sin(c+dx)+240(A+B+C)\sin(2(c+dx))+90A\sin(3(c+dx))+30A\sin(4(c+dx))+6A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(240*A*c + 420*B*c + 360*A*d*x + 420*B*d*x + 480*C*d*x + 60*(11*A + 12
*B + 14*C)*Sin[c + d*x] + 240*(A + B + C)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c
+ d*x)] + 80*B*Sin[3*(c + d*x)] + 40*C*Sin[3*(c + d*x)] + 30*A*Sin[4*(c + d
*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.11, size = 247, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*C*(2+cos(d*x+
c)^2)*sin(d*x+c)+2*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*
d*x+3/8*c)+2/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*C*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*C*sin(d*x+c))
```

Maxima [A] time = 0.957742, size = 319, normalized size = 1.71

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2+30(12dx+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 30*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))
*B*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 160*(sin(d*x + c)^3 - 3*sin(d
*x + c))*C*a^2 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 + 480*C*a^2*sin
(d*x + c))/d
```

Fricas [A] time = 0.509778, size = 305, normalized size = 1.63

$$\frac{15(6A+7B+8C)a^2dx+\left(24Aa^2\cos(dx+c)^4+30(2A+B)a^2\cos(dx+c)^3+8(9A+10B+5C)a^2\cos(dx+c)^2\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/120*(15*(6*A + 7*B + 8*C)*a^2*d*x + (24*A*a^2*cos(d*x + c)^4 + 30*(2*A +
B)*a^2*cos(d*x + c)^3 + 8*(9*A + 10*B + 5*C)*a^2*cos(d*x + c)^2 + 15*(6*A +
 7*B + 8*C)*a^2*cos(d*x + c) + 8*(18*A + 20*B + 25*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24251, size = 404, normalized size = 2.16

$$15\left(6\,Aa^2 + 7\,Ba^2 + 8\,Ca^2\right)(dx+c) + \frac{2\left(90\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 105\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 120\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 420\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 490\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(15*(6*A*a^2 + 7*B*a^2 + 8*C*a^2)*(d*x + c) + 2*(90*A*a^2*tan(1/2*d*x
 + 1/2*c)^9 + 105*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 120*C*a^2*tan(1/2*d*x + 1/
2*c)^9 + 420*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 490*B*a^2*tan(1/2*d*x + 1/2*c)^
7 + 560*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 8
00*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 1120*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 540*A
*a^2*tan(1/2*d*x + 1/2*c)^3 + 790*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 1040*C*a^2
*tan(1/2*d*x + 1/2*c)^3 + 390*A*a^2*tan(1/2*d*x + 1/2*c) + 375*B*a^2*tan(1/
2*d*x + 1/2*c) + 360*C*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
1)^5)/d
```

3.426 $\int \cos^6(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c + dx) + C \mathrm{se}\right.$

Optimal. Leaf size=213

$$-\frac{a^2(8A+9B+10C)\sin^3(c+dx)}{15d}+\frac{a^2(8A+9B+10C)\sin(c+dx)}{5d}+\frac{a^2(9A+12B+10C)\sin(c+dx)\cos^3(c+dx)}{40d}$$

```
[Out] (a^2*(11*A + 12*B + 14*C)*x)/16 + (a^2*(8*A + 9*B + 10*C)*Sin[c + d*x])/(5*
d) + (a^2*(11*A + 12*B + 14*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(9*
A + 12*B + 10*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + (A*Cos[c + d*x]^5*(a
 + a*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + ((A + 3*B)*Cos[c + d*x]^4*(a^2 +
 a^2*Sec[c + d*x])*Sin[c + d*x])/(15*d) - (a^2*(8*A + 9*B + 10*C)*Sin[c + d
*x]^3)/(15*d)
```

Rubi [A] time = 0.441033, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^2(8A+9B+10C)\sin^3(c+dx)}{15d}+\frac{a^2(8A+9B+10C)\sin(c+dx)}{5d}+\frac{a^2(9A+12B+10C)\sin(c+dx)\cos^3(c+dx)}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(11*A + 12*B + 14*C)*x)/16 + (a^2*(8*A + 9*B + 10*C)*Sin[c + d*x])/(5*
d) + (a^2*(11*A + 12*B + 14*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(9*
A + 12*B + 10*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + (A*Cos[c + d*x]^5*(a
 + a*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + ((A + 3*B)*Cos[c + d*x]^4*(a^2 +
 a^2*Sec[c + d*x])*Sin[c + d*x])/(15*d) - (a^2*(8*A + 9*B + 10*C)*Sin[c + d
*x]^3)/(15*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+}{6d}$$

$$= \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+}{6d}$$

$$= \frac{a^2(9A+12B+10C)\cos^3(c+dx)\sin(c+}{40d}$$

$$= \frac{a^2(9A+12B+10C)\cos^3(c+dx)\sin(c+}{40d}$$

$$= \frac{a^2(11A+12B+14C)\cos(c+dx)\sin(c+}{16d}$$

$$= \frac{1}{16}a^2(11A+12B+14C)x + \frac{a^2(8A+9B+}{}$$

Mathematica [A] time = 0.966495, size = 170, normalized size = 0.8

$$\frac{a^2(120(10A+11B+12C)\sin(c+dx)+15(31A+32(B+C))\sin(2(c+dx))+200A\sin(3(c+dx))+75A\sin(4(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^2*(240*A*c + 720*B*c + 660*A*d*x + 720*B*d*x + 840*C*d*x + 120*(10*A + 1
1*B + 12*C)*Sin[c + d*x] + 15*(31*A + 32*(B + C))*Sin[2*(c + d*x)] + 200*A*
Sin[3*(c + d*x)] + 180*B*Sin[3*(c + d*x)] + 160*C*Sin[3*(c + d*x)] + 75*A*S
```

```
in[4*(c + d*x)] + 60*B*Sin[4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 24*A*Sin[
5*(c + d*x)] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.122, size = 304, normalized size = 1.4

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{Ba^2\sin(dx+c)}{5}\left(\frac{8}{3}+(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*B*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a
^2*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/5*a^2*A
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*B*a^2*(1/4*(cos(d*x+c)^3+
3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a^2*C*(2+cos(d*x+c)^2)*sin(d*
x+c)+a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 0.960133, size = 400, normalized size = 1.88

$$\frac{128\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/960*(128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*A*a^2 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2
 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 60*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x +
c))*C*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^
2 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2)/d
```

Fricas [A] time = 0.523407, size = 373, normalized size = 1.75

$$\frac{15(11A+12B+14C)a^2dx+\left(40Aa^2\cos(dx+c)^5+48(2A+B)a^2\cos(dx+c)^4+10(11A+12B+6C)a^2\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(11*A + 12*B + 14*C)*a^2*d*x + (40*A*a^2*cos(d*x + c)^5 + 48*(2*A
 + B)*a^2*cos(d*x + c)^4 + 10*(11*A + 12*B + 6*C)*a^2*cos(d*x + c)^3 + 16*(
```

```
8*A + 9*B + 10*C)*a^2*cos(d*x + c)^2 + 15*(11*A + 12*B + 14*C)*a^2*cos(d*x
+ c) + 32*(8*A + 9*B + 10*C)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30301, size = 473, normalized size = 2.22

$$15\left(11\,Aa^2+12\,Ba^2+14\,Ca^2\right)(dx+c)+\frac{2\left(165\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+180\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+210\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+935\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(11*A*a^2 + 12*B*a^2 + 14*C*a^2)*(d*x + c) + 2*(165*A*a^2*tan(1/2
*d*x + 1/2*c)^11 + 180*B*a^2*tan(1/2*d*x + 1/2*c)^11 + 210*C*a^2*tan(1/2*d*
x + 1/2*c)^11 + 935*A*a^2*tan(1/2*d*x + 1/2*c)^9 + 1020*B*a^2*tan(1/2*d*x +
 1/2*c)^9 + 1190*C*a^2*tan(1/2*d*x + 1/2*c)^9 + 1986*A*a^2*tan(1/2*d*x + 1/
2*c)^7 + 2568*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 2580*C*a^2*tan(1/2*d*x + 1/2*c
)^7 + 3006*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 2808*B*a^2*tan(1/2*d*x + 1/2*c)^5
 + 3180*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 1305*A*a^2*tan(1/2*d*x + 1/2*c)^3 +
1860*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 2330*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 795
*A*a^2*tan(1/2*d*x + 1/2*c) + 780*B*a^2*tan(1/2*d*x + 1/2*c) + 750*C*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.427 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\text{se}\right.$

Optimal. Leaf size=274

$$\frac{a^3(133A+119B+108C)\tan^3(c+dx)}{105d}+\frac{a^3(133A+119B+108C)\tan(c+dx)}{35d}+\frac{a^3(26A+23B+21C)\tanh^{-1}(\sin(c}{16d}$$

```
[Out] (a^3*(26*A + 23*B + 21*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(133*A + 119
*B + 108*C)*Tan[c + d*x])/(35*d) + (a^3*(26*A + 23*B + 21*C)*Sec[c + d*x]*T
an[c + d*x])/(16*d) + (a^3*(154*A + 147*B + 129*C)*Sec[c + d*x]^3*Tan[c + d
*x])/(280*d) + (C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(7*d)
 + ((7*B + 3*C)*Sec[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(42
*a*d) + ((3*A + 4*B + 3*C)*Sec[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Tan[c +
d*x])/(15*d) + (a^3*(133*A + 119*B + 108*C)*Tan[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.598019, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4088, 4018, 3997, 3787, 3768, 3770, 3767}

$$\frac{a^3(133A+119B+108C)\tan^3(c+dx)}{105d}+\frac{a^3(133A+119B+108C)\tan(c+dx)}{35d}+\frac{a^3(26A+23B+21C)\tanh^{-1}(\sin(c}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^3*(26*A + 23*B + 21*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(133*A + 119
*B + 108*C)*Tan[c + d*x])/(35*d) + (a^3*(26*A + 23*B + 21*C)*Sec[c + d*x]*T
an[c + d*x])/(16*d) + (a^3*(154*A + 147*B + 129*C)*Sec[c + d*x]^3*Tan[c + d
*x])/(280*d) + (C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(7*d)
 + ((7*B + 3*C)*Sec[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(42
*a*d) + ((3*A + 4*B + 3*C)*Sec[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Tan[c +
d*x])/(15*d) + (a^3*(133*A + 119*B + 108*C)*Tan[c + d*x]^3)/(105*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{C\sec^3(c+dx)(a+a\sec(c+dx))^3\tan(c+}{7d}$$

$$=\frac{C\sec^3(c+dx)(a+a\sec(c+dx))^3\tan(c+}{7d}$$

$$=\frac{C\sec^3(c+dx)(a+a\sec(c+dx))^3\tan(c+}{7d}$$

$$=\frac{a^3(154A+147B+129C)\sec^3(c+dx)\tan}{280d}$$

$$=\frac{a^3(154A+147B+129C)\sec^3(c+dx)\tan}{280d}$$

$$=\frac{a^3(26A+23B+21C)\sec(c+dx)\tan(c+}{16d}$$

$$=\frac{a^3(26A+23B+21C)\tanh^{-1}(\sin(c+dx)}{16d}$$

Mathematica [A] time = 6.16201, size = 402, normalized size = 1.47

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(105(26A+23B+21C)\cos^7(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^6*Sec[c + d*x]^7*(105*(26*A + 23*B + 21*C)*Cos[c + d*x]^7*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) - 240*C*Sec[c]*Sin[d*x] - 40*Cos[c + d*x]*Sec[c]*(6*C*Sin[c] + 7*(B + 3*
C)*Sin[d*x]) - 2*Cos[c + d*x]^3*Sec[c]*(24*(7*A + 21*B + 27*C)*Sin[c] + 35*
(18*A + 23*B + 21*C)*Sin[d*x]) - Cos[c + d*x]^5*Sec[c]*(16*(133*A + 119*B +
 108*C)*Sin[c] + 105*(26*A + 23*B + 21*C)*Sin[d*x]) - 8*Cos[c + d*x]^2*Sec[
c]*(35*(B + 3*C)*Sin[c] + 6*(7*A + 21*B + 27*C)*Sin[d*x]) - 2*Cos[c + d*x]^
4*Sec[c]*(35*(18*A + 23*B + 21*C)*Sin[c] + 8*(133*A + 119*B + 108*C)*Sin[d*
x]) - Cos[c + d*x]^6*Sec[c]*(105*(26*A + 23*B + 21*C)*Sin[c] + 32*(133*A +
119*B + 108*C)*Sin[d*x])))/(6720*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c
 + d*x)]))
```

Maple [A] time = 0.074, size = 455, normalized size = 1.7

$$\frac{23\,Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{16\,d}+\frac{23\,Ba^3\tan(dx+c)(\sec(dx+c))^3}{24\,d}+\frac{23\,Ba^3\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{3\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 23/16/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+23/24/d*B*a^3*tan(d*x+c)*sec(d*x+c)
^3+23/16/d*B*a^3*sec(d*x+c)*tan(d*x+c)+3/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+
1/2/d*a^3*C*tan(d*x+c)*sec(d*x+c)^5+1/6/d*B*a^3*tan(d*x+c)*sec(d*x+c)^5+34/
15/d*B*a^3*tan(d*x+c)+17/15/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+38/15/d*A*a^3*t
an(d*x+c)+19/15/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+72/35*a^3*C*tan(d*x+c)/d+27
/35/d*a^3*C*tan(d*x+c)*sec(d*x+c)^4+36/35/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+3
/5/d*B*a^3*tan(d*x+c)*sec(d*x+c)^4+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+1/7/
d*a^3*C*tan(d*x+c)*sec(d*x+c)^6+13/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+7/8/d*a^
3*C*tan(d*x+c)*sec(d*x+c)^3+21/16/d*a^3*C*sec(d*x+c)*tan(d*x+c)+13/8/d*A*a^
3*ln(sec(d*x+c)+tan(d*x+c))+21/16/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [B] time = 0.989039, size = 876, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/3360*(224*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3
+ 3360*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 672*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 1120*(tan(d*x + c)^3 + 3*tan(d*x
 + c))*B*a^3 + 96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3
 + 35*tan(d*x + c))*C*a^3 + 672*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*
tan(d*x + c))*C*a^3 - 35*B*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 +
33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1)
```

```
- 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 105*C*a^3*(2*(15*
sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*s
in(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(s
in(d*x + c) - 1)) - 630*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 630*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1)) - 210*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 840*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.556788, size = 617, normalized size = 2.25

$$\frac{105 (26 A + 23 B + 21 C)a^3 \cos (dx + c)^7 \log (\sin (dx + c) + 1) - 105 (26 A + 23 B + 21 C)a^3 \cos (dx + c)^7 \log (- \sin (dx +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/3360*(105*(26*A + 23*B + 21*C)*a^3*cos(d*x + c)^7*log(sin(d*x + c) + 1) -
 105*(26*A + 23*B + 21*C)*a^3*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 2*(32
*(133*A + 119*B + 108*C)*a^3*cos(d*x + c)^6 + 105*(26*A + 23*B + 21*C)*a^3*
cos(d*x + c)^5 + 16*(133*A + 119*B + 108*C)*a^3*cos(d*x + c)^4 + 70*(18*A +
 23*B + 21*C)*a^3*cos(d*x + c)^3 + 48*(7*A + 21*B + 27*C)*a^3*cos(d*x + c)^
2 + 280*(B + 3*C)*a^3*cos(d*x + c) + 240*C*a^3)*sin(d*x + c))/(d*cos(d*x +
c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int A \sec^3 (c + dx) \, dx + \int 3A \sec^4 (c + dx) \, dx + \int 3A \sec^5 (c + dx) \, dx + \int A \sec^6 (c + dx) \, dx + \int B \sec^4 (c + dx) \, dx \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**3, x) + Integral(3*A*sec(c + d*x)**4, x) + I
ntegral(3*A*sec(c + d*x)**5, x) + Integral(A*sec(c + d*x)**6, x) + Integral
(B*sec(c + d*x)**4, x) + Integral(3*B*sec(c + d*x)**5, x) + Integral(3*B*se
c(c + d*x)**6, x) + Integral(B*sec(c + d*x)**7, x) + Integral(C*sec(c + d*x
)**5, x) + Integral(3*C*sec(c + d*x)**6, x) + Integral(3*C*sec(c + d*x)**7,
 x) + Integral(C*sec(c + d*x)**8, x))
```

Giac [A] time = 1.3511, size = 598, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(26*A*a^3 + 23*B*a^3 + 21*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 105*(26*A*a^3 + 23*B*a^3 + 21*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(2730*A*a^3*tan(1/2*d*x + 1/2*c)^13 + 2415*B*a^3*tan(1/2*d*x + 1/2*
c)^13 + 2205*C*a^3*tan(1/2*d*x + 1/2*c)^13 - 18200*A*a^3*tan(1/2*d*x + 1/2*
c)^11 - 16100*B*a^3*tan(1/2*d*x + 1/2*c)^11 - 14700*C*a^3*tan(1/2*d*x + 1/2
*c)^11 + 51506*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 45563*B*a^3*tan(1/2*d*x + 1/2
*c)^9 + 41601*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 77952*A*a^3*tan(1/2*d*x + 1/2*
c)^7 - 72576*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 62592*C*a^3*tan(1/2*d*x + 1/2*c
)^7 + 71246*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 62853*B*a^3*tan(1/2*d*x + 1/2*c)
^5 + 63231*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 40040*A*a^3*tan(1/2*d*x + 1/2*c)^
3 - 33180*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 25620*C*a^3*tan(1/2*d*x + 1/2*c)^3
 + 10710*A*a^3*tan(1/2*d*x + 1/2*c) + 11025*B*a^3*tan(1/2*d*x + 1/2*c) + 11
235*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.428 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=216

$$\frac{a^3(30A+26B+23C)\tan^3(c+dx)}{120d}+\frac{a^3(30A+26B+23C)\tan(c+dx)}{10d}+\frac{a^3(30A+26B+23C)\tanh^{-1}(\sin(c+dx))}{16d}+$$

```
[Out] (a^3*(30*A + 26*B + 23*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(30*A + 26*B
 + 23*C)*Tan[c + d*x])/(10*d) + (3*a^3*(30*A + 26*B + 23*C)*Sec[c + d*x]*Ta
n[c + d*x])/(80*d) + ((30*A - 6*B + 7*C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x
])/(120*d) + (C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(6*d) +
 ((2*B + C)*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(10*a*d) + (a^3*(30*A + 26
*B + 23*C)*Tan[c + d*x]^3)/(120*d)
```

Rubi [A] time = 0.455123, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4088, 4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(30A+26B+23C)\tan^3(c+dx)}{120d}+\frac{a^3(30A+26B+23C)\tan(c+dx)}{10d}+\frac{a^3(30A+26B+23C)\tanh^{-1}(\sin(c+dx))}{16d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^3*(30*A + 26*B + 23*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(30*A + 26*B
 + 23*C)*Tan[c + d*x])/(10*d) + (3*a^3*(30*A + 26*B + 23*C)*Sec[c + d*x]*Ta
n[c + d*x])/(80*d) + ((30*A - 6*B + 7*C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x
])/(120*d) + (C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(6*d) +
 ((2*B + C)*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(10*a*d) + (a^3*(30*A + 26
*B + 23*C)*Tan[c + d*x]^3)/(120*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^3\tan(}{6d}$$

$$= \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^3\tan(}{6d}$$

$$= \frac{(30A-6B+7C)(a+a\sec(c+dx))^3\text{ta}}{120d}$$

$$= \frac{(30A-6B+7C)(a+a\sec(c+dx))^3\text{ta}}{120d}$$

$$= \frac{(30A-6B+7C)(a+a\sec(c+dx))^3\text{ta}}{120d}$$

$$= \frac{a^3(30A+26B+23C)\tanh^{-1}(\sin(c+}{40d}$$

$$= \frac{a^3(30A+26B+23C)\tanh^{-1}(\sin(c+}{16d}$$

Mathematica [A] time = 4.24088, size = 359, normalized size = 1.66

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(15(30A+26B+23C)\cos^6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^6*Sec[c + d*x]^6*(15*(30*A + 26*B + 23*C)*Cos[c + d*x]^6*(Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) - 40*C*Sec[c]*Sin[d*x] - 8*Cos[c + d*x]*Sec[c]*(5*C*Sin[c] + 6*(B + 3*C)*
Sin[d*x]) - 2*Cos[c + d*x]^3*Sec[c]*(5*(6*A + 18*B + 23*C)*Sin[c] + 8*(15*A
 + 19*B + 17*C)*Sin[d*x]) - 2*Cos[c + d*x]^2*Sec[c]*(24*(B + 3*C)*Sin[c] +
5*(6*A + 18*B + 23*C)*Sin[d*x]) - Cos[c + d*x]^4*Sec[c]*(16*(15*A + 19*B +
17*C)*Sin[c] + 15*(30*A + 26*B + 23*C)*Sin[d*x]) - Cos[c + d*x]^5*Sec[c]*(1
5*(30*A + 26*B + 23*C)*Sin[c] + 16*(45*A + 38*B + 34*C)*Sin[d*x])))/(960*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.067, size = 385, normalized size = 1.8

$$3\,\frac{Aa^3\tan(dx+c)}{d}+\frac{13\,Ba^3\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{13\,Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{34\,a^3C\tan(dx+c)}{15\,d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/d*A*a^3*tan(d*x+c)+13/8/d*B*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*B*a^3*ln(sec
(d*x+c)+tan(d*x+c))+34/15*a^3*C*tan(d*x+c)/d+17/15/d*a^3*C*tan(d*x+c)*sec(d
*x+c)^2+15/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*A*a^3*ln(sec(d*x+c)+tan(d
*x+c))+38/15/d*B*a^3*tan(d*x+c)+19/15/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+23/24
/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3+23/16/d*a^3*C*sec(d*x+c)*tan(d*x+c)+23/16/
d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4/d*B
*a^3*tan(d*x+c)*sec(d*x+c)^3+3/5/d*a^3*C*tan(d*x+c)*sec(d*x+c)^4+1/4/d*A*a^
3*tan(d*x+c)*sec(d*x+c)^3+1/5/d*B*a^3*tan(d*x+c)*sec(d*x+c)^4+1/6/d*a^3*C*t
an(d*x+c)*sec(d*x+c)^5
```

Maxima [B] time = 0.982393, size = 755, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 32*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 480*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 + 96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c)
)*C*a^3 + 160*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 5*C*a^3*(2*(15*sin(
d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d
*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d
*x + c) - 1)) - 30*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 90*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
```

```
- 90*C*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 360*A
*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)) - 120*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*A*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.543938, size = 540, normalized size = 2.5

$$\frac{15\,(30\,A + 26\,B + 23\,C)a^3\cos\left(dx + c\right)^6\log\left(\sin\left(dx + c\right) + 1\right) - 15\,(30\,A + 26\,B + 23\,C)a^3\cos\left(dx + c\right)^6\log\left(-\sin\left(dx + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/480*(15*(30*A + 26*B + 23*C)*a^3*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 1
5*(30*A + 26*B + 23*C)*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(4
5*A + 38*B + 34*C)*a^3*cos(d*x + c)^5 + 15*(30*A + 26*B + 23*C)*a^3*cos(d*x
 + c)^4 + 16*(15*A + 19*B + 17*C)*a^3*cos(d*x + c)^3 + 10*(6*A + 18*B + 23*
C)*a^3*cos(d*x + c)^2 + 48*(B + 3*C)*a^3*cos(d*x + c) + 40*C*a^3)*sin(d*x +
 c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\sec^2\left(c + dx\right)dx + \int 3A\sec^3\left(c + dx\right)dx + \int 3A\sec^4\left(c + dx\right)dx + \int A\sec^5\left(c + dx\right)dx + \int B\sec^3\left(c + dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sec(c + d*x)**3, x) + I
ntegral(3*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral
(B*sec(c + d*x)**3, x) + Integral(3*B*sec(c + d*x)**4, x) + Integral(3*B*se
c(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x) + Integral(C*sec(c + d*x
)**4, x) + Integral(3*C*sec(c + d*x)**5, x) + Integral(3*C*sec(c + d*x)**6,
 x) + Integral(C*sec(c + d*x)**7, x))
```

Giac [A] time = 1.36912, size = 529, normalized size = 2.45

$$\frac{15\left(30\,Aa^3 + 26\,Ba^3 + 23\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(30\,Aa^3 + 26\,Ba^3 + 23\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(30*A*a^3 + 26*B*a^3 + 23*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 15*(30*A*a^3 + 26*B*a^3 + 23*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(450*A*a^3*tan(1/2*d*x + 1/2*c)^11 + 390*B*a^3*tan(1/2*d*x + 1/2*c)^11
 + 345*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 2550*A*a^3*tan(1/2*d*x + 1/2*c)^9 -
2210*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 1955*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 594
0*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 5148*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 4554*C
*a^3*tan(1/2*d*x + 1/2*c)^7 - 7500*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 5988*B*a^
3*tan(1/2*d*x + 1/2*c)^5 - 5814*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 5130*A*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4190*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 3165*C*a^3*tan(
1/2*d*x + 1/2*c)^3 - 1470*A*a^3*tan(1/2*d*x + 1/2*c) - 1530*B*a^3*tan(1/2*d
*x + 1/2*c) - 1575*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^6)/d
```

3.429 $\int \sec(c+dx)(a+a\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=175

$$\frac{a^3(20A+15B+13C)\tan^3(c+dx)}{60d}+\frac{a^3(20A+15B+13C)\tan(c+dx)}{5d}+\frac{a^3(20A+15B+13C)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (a^3*(20*A + 15*B + 13*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(20*A + 15*B
+ 13*C)*Tan[c + d*x])/(5*d) + (3*a^3*(20*A + 15*B + 13*C)*Sec[c + d*x]*Tan[
c + d*x])/(40*d) + ((5*B - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) +
 (C*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(20*A + 15*B + 13*C
)*Tan[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.276564, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4082, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(20A+15B+13C)\tan^3(c+dx)}{60d}+\frac{a^3(20A+15B+13C)\tan(c+dx)}{5d}+\frac{a^3(20A+15B+13C)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (a^3*(20*A + 15*B + 13*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(20*A + 15*B
+ 13*C)*Tan[c + d*x])/(5*d) + (3*a^3*(20*A + 15*B + 13*C)*Sec[c + d*x]*Tan[
c + d*x])/(40*d) + ((5*B - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) +
 (C*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(20*A + 15*B + 13*C
)*Tan[c + d*x]^3)/(60*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{5ad}+\frac{\int \sec(}{}$$
$$=\frac{(5B-C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d}+$$
$$=\frac{(5B-C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d}+$$
$$=\frac{(5B-C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d}+$$
$$=\frac{a^3(20A+15B+13C)\tanh^{-1}(\sin(c+dx))}{20d}$$
$$=\frac{a^3(20A+15B+13C)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [B] time = 3.60817, size = 431, normalized size = 2.46

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(240(20A+15B+13C)\cos^5(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^6*Sec[c + d*x]^5*(240*(20*A + 15*B + 13*C)*Cos[c + d*x]^5*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) - Sec[c]*(80*(34*A + 30*B + 29*C)*Sin[d*x] - 240*(7*A + 5*B + 3*C)*Sin[2
*c + d*x] + 360*A*Sin[c + 2*d*x] + 570*B*Sin[c + 2*d*x] + 750*C*Sin[c + 2*d
*x] + 360*A*Sin[3*c + 2*d*x] + 570*B*Sin[3*c + 2*d*x] + 750*C*Sin[3*c + 2*d
*x] + 1840*A*Sin[2*c + 3*d*x] + 1680*B*Sin[2*c + 3*d*x] + 1520*C*Sin[2*c +
3*d*x] - 360*A*Sin[4*c + 3*d*x] - 120*B*Sin[4*c + 3*d*x] + 180*A*Sin[3*c +
```

```
4*d*x] + 225*B*Sin[3*c + 4*d*x] + 195*C*Sin[3*c + 4*d*x] + 180*A*Sin[5*c +
4*d*x] + 225*B*Sin[5*c + 4*d*x] + 195*C*Sin[5*c + 4*d*x] + 440*A*Sin[4*c +
5*d*x] + 360*B*Sin[4*c + 5*d*x] + 304*C*Sin[4*c + 5*d*x])))/(7680*d*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.069, size = 316, normalized size = 1.8

$$\frac{5\,Aa^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2\,d}+3\,\frac{Ba^3\tan\left(dx+c\right)}{d}+\frac{13\,a^3C\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}+\frac{13\,a^3C\ln\left(\sec\left(dx+c\right)\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^3*tan(d*x+c)+13/8/d*a^3*C*sec
(d*x+c)*tan(d*x+c)+13/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*A*a^3*tan(
d*x+c)+15/8/d*B*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*B*a^3*ln(sec(d*x+c)+tan(d*
x+c))+38/15*a^3*C*tan(d*x+c)/d+19/15/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+3/2/d*
A*a^3*sec(d*x+c)*tan(d*x+c)+1/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^3*C*t
an(d*x+c)*sec(d*x+c)^3+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*a^3*tan(
d*x+c)*sec(d*x+c)^3+1/5/d*a^3*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [B] time = 0.974619, size = 593, normalized size = 3.39

$$\frac{80\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+240\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^3+16\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 240*(tan(d*x + c)^3 + 3
*tan(d*x + c))*B*a^3 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*C*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 15*B*a^3*(2*(
3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1)
- 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 45*C*a^3*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^3*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 180*
B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(si
n(d*x + c) - 1)) - 60*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*log(sec(d*x + c) + tan(d
*x + c)) + 720*A*a^3*tan(d*x + c) + 240*B*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.527647, size = 477, normalized size = 2.73

$$\frac{15\left(20\,A+15\,B+13\,C\right)a^3\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(20\,A+15\,B+13\,C\right)a^3\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(20*A + 15*B + 13*C)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 1
5*(20*A + 15*B + 13*C)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(55
*A + 45*B + 38*C)*a^3*cos(d*x + c)^4 + 15*(12*A + 15*B + 13*C)*a^3*cos(d*x
+ c)^3 + 8*(5*A + 15*B + 19*C)*a^3*cos(d*x + c)^2 + 30*(B + 3*C)*a^3*cos(d*
x + c) + 24*C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\sec(c+dx)\,dx+\int 3A\sec^2(c+dx)\,dx+\int 3A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int B\sec^2(c+dx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sec(c + d*x)**2, x) + Inte
gral(3*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integral(B*
sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + Integral(3*B*sec(c
 + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**
3, x) + Integral(3*C*sec(c + d*x)**4, x) + Integral(3*C*sec(c + d*x)**5, x)
 + Integral(C*sec(c + d*x)**6, x))
```

Giac [B] time = 1.34055, size = 460, normalized size = 2.63

$$\frac{15\left(20\,Aa^3+15\,Ba^3+13\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(20\,Aa^3+15\,Ba^3+13\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(20*A*a^3 + 15*B*a^3 + 13*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 15*(20*A*a^3 + 15*B*a^3 + 13*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(300*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 +
 195*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 1400*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 105
0*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 910*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 2560*A*
a^3*tan(1/2*d*x + 1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1664*C*a^3
*tan(1/2*d*x + 1/2*c)^5 - 2120*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 1830*B*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 1330*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 660*A*a^3*tan(1/
2*d*x + 1/2*c) + 735*B*a^3*tan(1/2*d*x + 1/2*c) + 765*C*a^3*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.430 $\int (a+a\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)$

Optimal. Leaf size=162

$$\frac{5a^3(4A+4B+3C)\tan(c+dx)}{8d} + \frac{a^3(28A+20B+15C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(12A+20B+15C)\tan(c+dx)\left(a^3 s\right.}{24d}$$

```
[Out] a^3*A*x + (a^3*(28*A + 20*B + 15*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(
4*A + 4*B + 3*C)*Tan[c + d*x])/(8*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c + d*
x])/(4*d) + ((4*B + 3*C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*a*d)
+ ((12*A + 20*B + 15*C)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.237257, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4054, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^3(4A+4B+3C)\tan(c+dx)}{8d} + \frac{a^3(28A+20B+15C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(12A+20B+15C)\tan(c+dx)\left(a^3 s\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a^3*A*x + (a^3*(28*A + 20*B + 15*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^3*(
4*A + 4*B + 3*C)*Tan[c + d*x])/(8*d) + (C*(a + a*Sec[c + d*x])^3*Tan[c + d*
x])/(4*d) + ((4*B + 3*C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*a*d)
+ ((12*A + 20*B + 15*C)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx = \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{\int (a+a\sec(c+dx)}{}$$

$$= \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{(4B+3C)\left(a^2+a\right.}{}$$

$$= \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{(4B+3C)\left(a^2+a\right.}{}$$

$$= a^3Ax + \frac{C(a+a\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{(4B+3C)}{}$$

$$= a^3Ax + \frac{a^3(28A+20B+15C)\tanh^{-1}(\sin(c+dx))}{8d} + $$

$$= a^3Ax + \frac{a^3(28A+20B+15C)\tanh^{-1}(\sin(c+dx))}{8d} + $$

Mathematica [B] time = 3.09256, size = 464, normalized size = 2.86

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(\sec(c)(12A\sin(2c+dx)+216A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
d*x)/2]^6*Sec[c + d*x]^4*(-24*(28*A + 20*B + 15*C)*Cos[c + d*x]^4*(Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + Sec[c]*(72*A*d*x*Cos[c] + 48*A*d*x*Cos[c + 2*d*x] + 48*A*d*x*Cos[3*c +
2*d*x] + 12*A*d*x*Cos[3*c + 4*d*x] + 12*A*d*x*Cos[5*c + 4*d*x] - 216*A*Sin[
c] - 264*B*Sin[c] - 216*C*Sin[c] + 12*A*Sin[d*x] + 36*B*Sin[d*x] + 69*C*Sin
[d*x] + 12*A*Sin[2*c + d*x] + 36*B*Sin[2*c + d*x] + 69*C*Sin[2*c + d*x] + 2
16*A*Sin[c + 2*d*x] + 280*B*Sin[c + 2*d*x] + 264*C*Sin[c + 2*d*x] - 72*A*Si
n[3*c + 2*d*x] - 72*B*Sin[3*c + 2*d*x] - 24*C*Sin[3*c + 2*d*x] + 12*A*Sin[2
*c + 3*d*x] + 36*B*Sin[2*c + 3*d*x] + 45*C*Sin[2*c + 3*d*x] + 12*A*Sin[4*c
+ 3*d*x] + 36*B*Sin[4*c + 3*d*x] + 45*C*Sin[4*c + 3*d*x] + 72*A*Sin[3*c + 4
*d*x] + 88*B*Sin[3*c + 4*d*x] + 72*C*Sin[3*c + 4*d*x])))/(768*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.065, size = 262, normalized size = 1.6

$$a^3Ax + \frac{Aa^3c}{d} + \frac{5\,Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + 3\,\frac{a^3C\tan(dx+c)}{d} + \frac{7\,Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + 11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*x+1/d*A*a^3*c+5/2/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*tan(d*x+c
)/d+7/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*B*a^3*tan(d*x+c)+15/8/d*a^
3*C*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^3*
tan(d*x+c)+3/2/d*B*a^3*sec(d*x+c)*tan(d*x+c)+1/d*a^3*C*tan(d*x+c)*sec(d*x+c
)^2+1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/3/d*B*a^3*tan(d*x+c)*sec(d*x+c)^2+1
/4/d*a^3*C*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [B] time = 0.960431, size = 475, normalized size = 2.93

$$48\,(dx+c)Aa^3+16\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ba^3+48\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ca^3-3\,Ca^3\left(\frac{2\left(3\,\sin(dx+\right.}{\sin(dx+c)^4-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/48*(48*(d*x + c)*A*a^3 + 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*
(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 - 3*C*a^3*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 - 36*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) + 144*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 48*B*
a^3*log(sec(d*x + c) + tan(d*x + c)) + 144*A*a^3*tan(d*x + c) + 144*B*a^3*t
an(d*x + c) + 48*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.558428, size = 447, normalized size = 2.76

$$48\,Aa^3dx\cos\left(dx+c\right)^4+3\,(28\,A+20\,B+15\,C)a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(28\,A+20\,B+15\,C)a^3\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/48*(48*A*a^3*d*x*cos(d*x + c)^4 + 3*(28*A + 20*B + 15*C)*a^3*cos(d*x + c)
^4*log(sin(d*x + c) + 1) - 3*(28*A + 20*B + 15*C)*a^3*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(8*(9*A + 11*B + 9*C)*a^3*cos(d*x + c)^3 + 3*(4*A + 12
*B + 15*C)*a^3*cos(d*x + c)^2 + 8*(B + 3*C)*a^3*cos(d*x + c) + 6*C*a^3)*sin
(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int A\,dx+\int 3A\sec\left(c+dx\right)dx+\int 3A\sec^2\left(c+dx\right)dx+\int A\sec^3\left(c+dx\right)dx+\int B\sec\left(c+dx\right)dx+\int 3B\sec\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**3*(Integral(A, x) + Integral(3*A*sec(c + d*x), x) + Integral(3*A*sec(c +
 d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x), x)
 + Integral(3*B*sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + In
tegral(B*sec(c + d*x)**4, x) + Integral(C*sec(c + d*x)**2, x) + Integral(3*
C*sec(c + d*x)**3, x) + Integral(3*C*sec(c + d*x)**4, x) + Integral(C*sec(c
 + d*x)**5, x))
```

Giac [A] time = 1.32838, size = 406, normalized size = 2.51

$$\frac{24\,(dx+c)Aa^3+3\left(28\,Aa^3+20\,Ba^3+15\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(28\,Aa^3+20\,Ba^3+15\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/24*(24*(d*x + c)*A*a^3 + 3*(28*A*a^3 + 20*B*a^3 + 15*C*a^3)*log(abs(tan(1
/2*d*x + 1/2*c) + 1)) - 3*(28*A*a^3 + 20*B*a^3 + 15*C*a^3)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(60*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*B*a^3*tan(1/2*
d*x + 1/2*c)^7 + 45*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 204*A*a^3*tan(1/2*d*x +
1/2*c)^5 - 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 165*C*a^3*tan(1/2*d*x + 1/2*c
)^5 + 228*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 +
 219*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 84*A*a^3*tan(1/2*d*x + 1/2*c) - 132*B*a
^3*tan(1/2*d*x + 1/2*c) - 147*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

3.431 $\int \cos(c+dx)(a+a\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=156

$$\frac{a^3(6A+7B+5C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A-3B-5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+a^3x(3A+B)-\frac{(3A-C}{}$$

```
[Out] a^3*(3*A + B)*x + (a^3*(6*A + 7*B + 5*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*
(a + a*Sec[c + d*x])^3*Sin[c + d*x])/d + (5*a^3*(B + C)*Tan[c + d*x])/(2*d)
 - ((3*A - C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*a*d) - ((6*A - 3*
B - 5*C)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.284327, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4086, 3917, 3914, 3767, 8, 3770}

$$\frac{a^3(6A+7B+5C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A-3B-5C)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+a^3x(3A+B)-\frac{(3A-C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] a^3*(3*A + B)*x + (a^3*(6*A + 7*B + 5*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*
(a + a*Sec[c + d*x])^3*Sin[c + d*x])/d + (5*a^3*(B + C)*Tan[c + d*x])/(2*d)
 - ((3*A - C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*a*d) - ((6*A - 3*
B - 5*C)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} + \frac{\int(a+}{}$$

$$= \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} - \frac{(3A-}{}$$

$$= \frac{A(a+a\sec(c+dx))^3\sin(c+dx)}{d} - \frac{(3A-}{}$$

$$= a^3(3A+B)x + \frac{A(a+a\sec(c+dx))^3\sin(c}{d}$$

$$= a^3(3A+B)x + \frac{a^3(6A+7B+5C)\tanh^{-1}(s}{2d}$$

$$= a^3(3A+B)x + \frac{a^3(6A+7B+5C)\tanh^{-1}(s}{2d}$$

Mathematica [B] time = 6.45498, size = 1503, normalized size = 9.63

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] ((3*A + B)*x*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(4*(A + 2*C + 2*B*Cos[c + d*x] + A*Co
s[2*c + 2*d*x])) + ((-6*A - 7*B - 5*C)*Cos[c + d*x]^5*Log[Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) + ((6*A + 7*B + 5*C)*Cos[c + d*x]^5*Log[Cos[c/2 + (d*x)/2] +
 Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])) + (A*Cos[d*x]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c
+ d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(4*d*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (A*Cos[c]*Cos[c + d*x]^5*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sin[d*x])/(4*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (C*Cos[
c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(24*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])^3) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B
```

```
*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3*B*Cos[c/2] + 10*C*Cos[c/2] - 3*B*Sin[c
/2] - 8*C*Sin[c/2]))/(48*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos
[c + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*(3*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2] + 11*C*Sin[(d*x)/
2]))/(12*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2] - Si
n[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (C*Cos[c + d*x]^5*Sec[
c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*Sin[(d*x)/2])/(24*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c
 + d*x]^5*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*(-3*B*Cos[c/2] - 10*C*Cos[c/2] - 3*B*Sin[c/2] - 8*C*Sin[
c/2]))/(48*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2] +
Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^5*Se
c[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*(3*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2] + 11*C*Sin[(d*x)/2]))/(12*d*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c
/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.107, size = 226, normalized size = 1.5

$$\frac{Aa^3\sin(dx+c)}{d} + a^3Bx + \frac{Ba^3c}{d} + \frac{5\,a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + 3\,a^3Ax + 3\,\frac{Aa^3c}{d} + \frac{7\,Ba^3\ln(\sec(dx+c) -}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*sin(d*x+c)/d+a^3*B*x+1/d*B*a^3*c+5/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c)
)+3*a^3*A*x+3/d*A*a^3*c+7/2/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3*a^3*C*ta
n(d*x+c)/d+3/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^3*tan(d*x+c)+3/2/d*a
^3*C*sec(d*x+c)*tan(d*x+c)+1/d*A*a^3*tan(d*x+c)+1/2/d*B*a^3*sec(d*x+c)*tan(
d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.965363, size = 370, normalized size = 2.37

$$\frac{36\,(dx+c)Aa^3 + 12\,(dx+c)Ba^3 + 4\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Ca^3 - 3\,Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/12*(36*(d*x + c)*A*a^3 + 12*(d*x + c)*B*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d
*x + c))*C*a^3 - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 9*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*A*a^3*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 12*A*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c) + 36*B*a^3*tan(d*x
+ c) + 36*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.557569, size = 425, normalized size = 2.72

$$\frac{12\,(3\,A+B)a^3dx\cos\left(dx+c\right)^3+3\,(6\,A+7\,B+5\,C)a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(6\,A+7\,B+5\,C)a^3\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/12*(12*(3*A + B)*a^3*d*x*cos(d*x + c)^3 + 3*(6*A + 7*B + 5*C)*a^3*cos(d*x
 + c)^3*log(sin(d*x + c) + 1) - 3*(6*A + 7*B + 5*C)*a^3*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(6*A*a^3*cos(d*x + c)^3 + 2*(3*A + 9*B + 11*C)*a^3*c
os(d*x + c)^2 + 3*(B + 3*C)*a^3*cos(d*x + c) + 2*C*a^3)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.3687, size = 389, normalized size = 2.49

$$\frac{\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(3\,Aa^3+Ba^3\right)(dx+c)+3\left(6\,Aa^3+7\,Ba^3+5\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(6\,Aa^3+7\,Ba^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(3*A*a^
3 + B*a^3)*(d*x + c) + 3*(6*A*a^3 + 7*B*a^3 + 5*C*a^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 3*(6*A*a^3 + 7*B*a^3 + 5*C*a^3)*log(abs(tan(1/2*d*x + 1/2*
c) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c
)^5 + 15*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 3
6*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*
tan(1/2*d*x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c) + 33*C*a^3*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.432 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\,\mathrm{se}\right.$

Optimal. Leaf size=171

$$\frac{a^3(2A+6B+7C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{(A-2B-4C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d} + \frac{1}{2}a^3x(7A+6B+2C) + \frac{5}{}$$

```
[Out] (a^3*(7*A + 6*B + 2*C)*x)/2 + (a^3*(2*A + 6*B + 7*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (5*a^3*(A - C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c
+ d*x])^3*Sin[c + d*x])/(2*d) - ((A - C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c +
 d*x])/(2*a*d) - ((A - 2*B - 4*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2
*d)
```

Rubi [A] time = 0.425446, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4086, 4018, 3996, 3770}

$$\frac{a^3(2A+6B+7C)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{(A-2B-4C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d} + \frac{1}{2}a^3x(7A+6B+2C) + \frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^3*(7*A + 6*B + 2*C)*x)/2 + (a^3*(2*A + 6*B + 7*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (5*a^3*(A - C)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + a*Sec[c
+ d*x])^3*Sin[c + d*x])/(2*d) - ((A - C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c +
 d*x])/(2*a*d) - ((A - 2*B - 4*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2
*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
```

```
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{A\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+}{2d}$$

$$=\frac{A\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+}{2d}$$

$$=\frac{A\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+}{2d}$$

$$=\frac{5a^3(A-C)\sin(c+dx)}{2d}+\frac{A\cos(c+dx)(a}{}$$

$$=\frac{1}{2}a^3(7A+6B+2C)x+\frac{5a^3(A-C)\sin(c}{2d}$$

$$=\frac{1}{2}a^3(7A+6B+2C)x+\frac{a^3(2A+6B+7C)}{}$$

Mathematica [B] time = 5.89112, size = 406, normalized size = 2.37

$$a^3\cos^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-\frac{2(2A+6B+7C)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^3*Cos[c + d*x]^5*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(2*(7*A + 6*B + 2*C)*x - (2*(2*A + 6*B + 7*C)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(2*A + 6*B + 7*C)*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(3*A + B)*Cos[d*x]*Sin[c])/d + (A*Cos[
2*d*x]*Sin[2*c])/d + (4*(3*A + B)*Cos[c]*Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*
x])/d + C/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(B + 3*C)*Sin[(d
*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - C
/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(B + 3*C)*Sin[(d*x)/2])/(
d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(16*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.108, size = 219, normalized size = 1.3

$$\frac{Aa^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{7a^3Ax}{2}+\frac{7Aa^3c}{2d}+\frac{Ba^3\sin(dx+c)}{d}+a^3Cx+\frac{Ca^3c}{d}+3\frac{Aa^3\sin(dx+c)}{d}+3a^3Bx+3\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^3*sin(d*x+c)*cos(d*x+c)+7/2*a^3*A*x+7/2/d*A*a^3*c+a^3*B*sin(d*x+c
)/d+a^3*C*x+1/d*C*a^3*c+3*a^3*A*sin(d*x+c)/d+3*a^3*B*x+3/d*B*a^3*c+7/2/d*a^
3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*C*t
an(d*x+c)/d+1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^3*tan(d*x+c)+1/2/d*
a^3*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.964186, size = 320, normalized size = 1.87

$$(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^3 + 12\,(dx + c)Aa^3 + 12\,(dx + c)Ba^3 + 4\,(dx + c)Ca^3 - Ca^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*A*a^3 + 12*(d*x
+ c)*B*a^3 + 4*(d*x + c)*C*a^3 - C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^3*(log(sin(d*x +
c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 12*A*a^3*sin(d*x + c) + 4*B*a^3*sin(d*x + c) + 4*B*a^3*tan(d*x + c) + 12*C
*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.553861, size = 410, normalized size = 2.4

$$2\,(7\,A + 6\,B + 2\,C)a^3dx\cos(dx + c)^2 + (2\,A + 6\,B + 7\,C)a^3\cos(dx + c)^2\log(\sin(dx + c) + 1) - (2\,A + 6\,B + 7\,C)a^3\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/4*(2*(7*A + 6*B + 2*C)*a^3*d*x*cos(d*x + c)^2 + (2*A + 6*B + 7*C)*a^3*cos
(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A + 6*B + 7*C)*a^3*cos(d*x + c)^2*lo
g(-sin(d*x + c) + 1) + 2*(A*a^3*cos(d*x + c)^3 + 2*(3*A + B)*a^3*cos(d*x +
c)^2 + 2*(B + 3*C)*a^3*cos(d*x + c) + C*a^3)*sin(d*x + c))/(d*cos(d*x + c)^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24962, size = 378, normalized size = 2.21

$$\left(7\,Aa^3+6\,Ba^3+2\,Ca^3\right)(dx+c)+\left(2\,Aa^3+6\,Ba^3+7\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Aa^3+6\,Ba^3+7\,Ca^3\right)\log\left(\left|\mathrm{t}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/2*((7*A*a^3 + 6*B*a^3 + 2*C*a^3)*(d*x + c) + (2*A*a^3 + 6*B*a^3 + 7*C*a^3
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A*a^3 + 6*B*a^3 + 7*C*a^3)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a^3*
tan(1/2*d*x + 1/2*c)^7 - 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^3*tan(1/2*d
*x + 1/2*c)^5 - 3*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^3*tan(1/2*d*x + 1/2*
c)^3 + 9*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^3*tan(1/2*d*x + 1/2*c) + 4*B*
a^3*tan(1/2*d*x + 1/2*c) + 7*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^4 - 1)^2)/d
```

3.433 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec$

Optimal. Leaf size=169

$$-\frac{(5A+3B-6C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(A+B)\sin(c+dx)\cos(c+dx)\left(a^2\sec\right.}{2ad}$$

```
[Out] (a^3*(5*A + 7*B + 6*C)*x)/2 + (a^3*(B + 3*C)*ArcTanh[Sin[c + d*x]])/d + (5*
a^3*(A + B)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*
Sin[c + d*x])/(3*d) + ((A + B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(2*a*d) - ((5*A + 3*B - 6*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x
])/(6*d)
```

Rubi [A] time = 0.441559, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4086, 4017, 4018, 3996, 3770}

$$-\frac{(5A+3B-6C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(A+B)\sin(c+dx)\cos(c+dx)\left(a^2\sec\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^3*(5*A + 7*B + 6*C)*x)/2 + (a^3*(B + 3*C)*ArcTanh[Sin[c + d*x]])/d + (5*
a^3*(A + B)*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*
Sin[c + d*x])/(3*d) + ((A + B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(2*a*d) - ((5*A + 3*B - 6*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x
])/(6*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
```

```
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+}{3d}$$

$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+}{3d}$$

$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+}{3d}$$

$$= \frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{A\cos^2(c+dx)(}{}$$

$$= \frac{1}{2}a^3(5A+7B+6C)x + \frac{5a^3(A+B)\sin(c\cdot}{2d}$$

$$= \frac{1}{2}a^3(5A+7B+6C)x + \frac{a^3(B+3C)\tanh^{-}}{d}$$

Mathematica [B] time = 2.18545, size = 379, normalized size = 2.24

$$\frac{a^3\cos^2(c+dx)(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{3\sin(c)(15A+4(3B+C))\cos(dx)}{d}+\frac{3\cos(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^3*Cos[c + d*x]^2*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(6*(5*A + 7*B + 6*C)*x - (12*(B + 3*C)*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]])/d + (12*(B + 3*C)*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]])/d + (3*(15*A + 4*(3*B + C))*Cos[d*x]*Sin[c])/d + (3*(3*A +
B)*Cos[2*d*x]*Sin[2*c])/d + (A*Cos[3*d*x]*Sin[3*c])/d + (3*(15*A + 4*(3*B +
 C))*Cos[c]*Sin[d*x])/d + (3*(3*A + B)*Cos[2*c]*Sin[2*d*x])/d + (A*Cos[3*c]
*Sin[3*d*x])/d + (12*C*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) + (12*C*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(48*(A + 2*C + 2*B*Cos[c + d*x] + A*
```

```
Cos[2*(c + d*x)]))
```

Maple [A] time = 0.099, size = 221, normalized size = 1.3

$$\frac{A(\cos(dx+c))^2\sin(dx+c)\,a^3}{3d}+\frac{11\,Aa^3\sin(dx+c)}{3d}+\frac{Ba^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{7\,a^3Bx}{2}+\frac{7\,Ba^3c}{2d}+\frac{a^3C\sin(\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3+11/3*a^3*A*sin(d*x+c)/d+1/2/d*B*a^3*sin
(d*x+c)*cos(d*x+c)+7/2*a^3*B*x+7/2/d*B*a^3*c+a^3*C*sin(d*x+c)/d+3/2/d*A*a^3
*sin(d*x+c)*cos(d*x+c)+5/2*a^3*A*x+5/2/d*A*a^3*c+3*a^3*B*sin(d*x+c)/d+3*a^3
*C*x+3/d*C*a^3*c+3/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^3*ln(sec(d*x+c
)+tan(d*x+c))+a^3*C*tan(d*x+c)/d
```

Maxima [A] time = 0.963204, size = 284, normalized size = 1.68

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3-12\,(dx+c)Aa^3-3\,(2\,dx+2\,c+\sin(2\,d\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^3 - 36*(d*x + c)*B*a^3 - 36*(d*x + c)*C*a^3 - 6*B*a^3*(log(sin(d*x + c)
 + 1) - log(sin(d*x + c) - 1)) - 18*C*a^3*(log(sin(d*x + c) + 1) - log(sin(
d*x + c) - 1)) - 36*A*a^3*sin(d*x + c) - 36*B*a^3*sin(d*x + c) - 12*C*a^3*s
in(d*x + c) - 12*C*a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.537418, size = 398, normalized size = 2.36

$$\frac{3\,(5\,A+7\,B+6\,C)a^3dx\cos(dx+c)+3\,(B+3\,C)a^3\cos(dx+c)\log(\sin(dx+c)+1)-3\,(B+3\,C)a^3\cos(dx+c)\log\ldots}{6\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(5*A + 7*B + 6*C)*a^3*d*x*cos(d*x + c) + 3*(B + 3*C)*a^3*cos(d*x + c
)*log(sin(d*x + c) + 1) - 3*(B + 3*C)*a^3*cos(d*x + c)*log(-sin(d*x + c) +
1) + (2*A*a^3*cos(d*x + c)^3 + 3*(3*A + B)*a^3*cos(d*x + c)^2 + 2*(11*A + 9
*B + 3*C)*a^3*cos(d*x + c) + 6*C*a^3)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30036, size = 379, normalized size = 2.24

$$-\frac{\frac{12\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\left(5\,Aa^3+7\,Ba^3+6\,Ca^3\right)(dx+c)-6\left(Ba^3+3\,Ca^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6\left(Ba^3+3\,Ca^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -1/6*(12*C*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(5*A*a
^3 + 7*B*a^3 + 6*C*a^3)*(d*x + c) - 6*(B*a^3 + 3*C*a^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) + 6*(B*a^3 + 3*C*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*
a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^3*tan
(1/2*d*x + 1/2*c)^3 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d*
x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c) + 6*C*a^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.434 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\,\text{se}\right.$

Optimal. Leaf size=183

$$\frac{5a^3(3A+4(B+C))\sin(c+dx)}{8d} + \frac{(15A+20B+12C)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{24d} + \frac{(3A+4B)\sin(\ldots}{}$$

```
[Out] (a^3*(15*A + 20*B + 28*C)*x)/8 + (a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(
3*A + 4*(B + C))*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x
])^3*Sin[c + d*x])/(4*d) + ((3*A + 4*B)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(12*a*d) + ((15*A + 20*B + 12*C)*Cos[c + d*x]*(a^3 + a
^3*Sec[c + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.441221, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4086, 4017, 3996, 3770}

$$\frac{5a^3(3A+4(B+C))\sin(c+dx)}{8d} + \frac{(15A+20B+12C)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{24d} + \frac{(3A+4B)\sin(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^3*(15*A + 20*B + 28*C)*x)/8 + (a^3*C*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(
3*A + 4*(B + C))*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x
])^3*Sin[c + d*x])/(4*d) + ((3*A + 4*B)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(12*a*d) + ((15*A + 20*B + 12*C)*Cos[c + d*x]*(a^3 + a
^3*Sec[c + d*x])*Sin[c + d*x])/(24*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
```

```
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+}{4d}$$

$$= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+}{4d}$$

$$= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+}{4d}$$

$$= \frac{5a^3(3A+4(B+C))\sin(c+dx)}{8d} + \frac{A\cos^3}{}$$

$$= \frac{1}{8}a^3(15A+20B+28C)x + \frac{5a^3(3A+4(B}{}$$

$$= \frac{1}{8}a^3(15A+20B+28C)x + \frac{a^3C\tanh^{-1}(\mathrm{si}}{d}$$

Mathematica [A] time = 0.420321, size = 147, normalized size = 0.8

$$\frac{a^3\left(24(13A+15B+12C)\sin(c+dx)+24(4A+3B+C)\sin(2(c+dx))+24A\sin(3(c+dx))+3A\sin(4(c+dx))+180A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^3*(180*A*d*x + 240*B*d*x + 336*C*d*x - 96*C*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 96*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(13*A + 1
5*B + 12*C)*Sin[c + d*x] + 24*(4*A + 3*B + C)*Sin[2*(c + d*x)] + 24*A*Sin[3
*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.116, size = 251, normalized size = 1.4

$$\frac{Aa^3\sin(dx+c)(\cos(dx+c))^3}{4d} + \frac{15\,Aa^3\sin(dx+c)\cos(dx+c)}{8d} + \frac{15\,a^3Ax}{8} + \frac{15\,Aa^3c}{8d} + \frac{B(\cos(dx+c))^2\sin(dx+c)\,a}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/4/d*A*a^3*sin(d*x+c)*cos(d*x+c)^3+15/8/d*A*a^3*sin(d*x+c)*cos(d*x+c)+15/8
*a^3*A*x+15/8/d*A*a^3*c+1/3/d*B*cos(d*x+c)^2*sin(d*x+c)*a^3+11/3*a^3*B*sin(
d*x+c)/d+1/2/d*a^3*C*sin(d*x+c)*cos(d*x+c)+7/2*a^3*C*x+7/2/d*C*a^3*c+1/d*A*
```

```
cos(d*x+c)^2*sin(d*x+c)*a^3+3*a^3*A*sin(d*x+c)/d+3/2/d*B*a^3*sin(d*x+c)*cos
(d*x+c)+5/2*a^3*B*x+5/2/d*B*a^3*c+3*a^3*C*sin(d*x+c)/d+1/d*a^3*C*ln(sec(d*x
+c)+tan(d*x+c))
```

Maxima [A] time = 0.954555, size = 324, normalized size = 1.77

$$-\frac{96\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{3}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-72\left(2\,dx+2\,c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/96*(96*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*
c))*A*a^3 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 72*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^3 - 96*(d*x + c)*B*a^3 - 24*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*C*a^3 - 288*(d*x + c)*C*a^3 - 48*C*a^3*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) - 96*A*a^3*sin(d*x + c) - 288*B*a^3*sin(d*x + c) - 288*
C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.538279, size = 336, normalized size = 1.84

$$\frac{3\left(15\,A+20\,B+28\,C\right)a^{3}dx+12\,Ca^{3}\log\left(\sin\left(dx+c\right)+1\right)-12\,Ca^{3}\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Aa^{3}\cos\left(dx+c\right)^{3}+8\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/24*(3*(15*A + 20*B + 28*C)*a^3*d*x + 12*C*a^3*log(sin(d*x + c) + 1) - 12*
C*a^3*log(-sin(d*x + c) + 1) + (6*A*a^3*cos(d*x + c)^3 + 8*(3*A + B)*a^3*co
s(d*x + c)^2 + 3*(15*A + 12*B + 4*C)*a^3*cos(d*x + c) + 8*(9*A + 11*B + 9*C
)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33623, size = 386, normalized size = 2.11

$$24\,Ca^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Ca^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(15\,Aa^3+20\,Ba^3+28\,Ca^3\right)(dx+c)+\frac{2\left(45\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/24*(24*C*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*C*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(15*A*a^3 + 20*B*a^3 + 28*C*a^3)*(d*x + c) + 2*(45
*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^3*
tan(1/2*d*x + 1/2*c)^7 + 165*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 220*B*a^3*tan(1
/2*d*x + 1/2*c)^5 + 204*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*
x + 1/2*c)^3 + 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 228*C*a^3*tan(1/2*d*x + 1
/2*c)^3 + 147*A*a^3*tan(1/2*d*x + 1/2*c) + 132*B*a^3*tan(1/2*d*x + 1/2*c) +
 84*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.435 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\text{se}\right.$

Optimal. Leaf size=179

$$-\frac{a^3(13A+15B+20C)\sin^3(c+dx)}{60d}+\frac{a^3(13A+15B+20C)\sin(c+dx)}{5d}+\frac{3a^3(13A+15B+20C)\sin(c+dx)\cos(c+}{40d}$$

```
[Out] (a^3*(13*A + 15*B + 20*C)*x)/8 + (a^3*(13*A + 15*B + 20*C)*Sin[c + d*x])/(5
*d) + (3*a^3*(13*A + 15*B + 20*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((3*A
 + 5*B)*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos
[c + d*x]^4*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(5*d) - (a^3*(13*A + 15*B
+ 20*C)*Sin[c + d*x]^3)/(60*d)
```

Rubi [A] time = 0.365264, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{a^3(13A+15B+20C)\sin^3(c+dx)}{60d}+\frac{a^3(13A+15B+20C)\sin(c+dx)}{5d}+\frac{3a^3(13A+15B+20C)\sin(c+dx)\cos(c+}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^3*(13*A + 15*B + 20*C)*x)/8 + (a^3*(13*A + 15*B + 20*C)*Sin[c + d*x])/(5
*d) + (3*a^3*(13*A + 15*B + 20*C)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((3*A
 + 5*B)*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos
[c + d*x]^4*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(5*d) - (a^3*(13*A + 15*B
+ 20*C)*Sin[c + d*x]^3)/(60*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^3\sin(c+}{5d}$$

$$= \frac{(3A+5B)\cos^3(c+dx)(a+a\sec(c+dx))}{20d}$$

$$= \frac{(3A+5B)\cos^3(c+dx)(a+a\sec(c+dx))}{20d}$$

$$= \frac{1}{20}a^3(13A+15B+20C)x + \frac{(3A+5B)\,\mathrm{co}}{}$$

$$= \frac{1}{20}a^3(13A+15B+20C)x + \frac{3a^3(13A+15}{}$$

$$= \frac{1}{8}a^3(13A+15B+20C)x + \frac{a^3(13A+15B}{}$$

Mathematica [A] time = 0.430481, size = 130, normalized size = 0.73

$$\frac{a^3(60(23A+26B+30C)\sin(c+dx)+120(4A+4B+3C)\sin(2(c+dx))+170A\sin(3(c+dx))+45A\sin(4(c+dx))+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^3*(780*A*d*x + 900*B*d*x + 1200*C*d*x + 60*(23*A + 26*B + 30*C)*Sin[c +
d*x] + 120*(4*A + 4*B + 3*C)*Sin[2*(c + d*x)] + 170*A*Sin[3*(c + d*x)] + 12
0*B*Sin[3*(c + d*x)] + 40*C*Sin[3*(c + d*x)] + 45*A*Sin[4*(c + d*x)] + 15*B
*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.109, size = 295, normalized size = 1.7

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3Aa^3\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*A*a^3*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^3*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a^3*(2+cos(d*x+c)^2)*sin(
d*x+c)+B*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x
+c)+A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*B*a^3*(1/2*cos(d*x+c)
*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+B*a^3*sin(d*x+c)+3*a^3*C*sin(d*x+c)+a^3*C*(d*x+c))
```

Maxima [A] time = 0.961152, size = 381, normalized size = 2.13

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3+45\,(12\,dx+12\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 45*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^3 - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 360*(2*d*x +
2*c + sin(2*d*x + 2*c))*C*a^3 + 480*(d*x + c)*C*a^3 + 480*B*a^3*sin(d*x + c
) + 1440*C*a^3*sin(d*x + c))/d
```

Fricas [A] time = 0.529727, size = 315, normalized size = 1.76

$$\frac{15\,(13\,A+15\,B+20\,C)a^3dx+\left(24\,Aa^3\cos(dx+c)^4+30\,(3\,A+B)a^3\cos(dx+c)^3+8\,(19\,A+15\,B+5\,C)a^3\cos(dx\cdots}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/120*(15*(13*A + 15*B + 20*C)*a^3*d*x + (24*A*a^3*cos(d*x + c)^4 + 30*(3*A
 + B)*a^3*cos(d*x + c)^3 + 8*(19*A + 15*B + 5*C)*a^3*cos(d*x + c)^2 + 15*(1
3*A + 15*B + 12*C)*a^3*cos(d*x + c) + 8*(38*A + 45*B + 55*C)*a^3)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29428, size = 404, normalized size = 2.26

$$\frac{15\left(13\,Aa^3+15\,Ba^3+20\,Ca^3\right)(dx+c)+\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+225\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+300\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3 + 20*C*a^3)*(d*x + c) + 2*(195*A*a^3*tan(1/2
*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 300*C*a^3*tan(1/2*d*x
+ 1/2*c)^9 + 910*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 1050*B*a^3*tan(1/2*d*x + 1/
2*c)^7 + 1400*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x + 1/2*c
)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 2560*C*a^3*tan(1/2*d*x + 1/2*c)^5
 + 1330*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 1830*B*a^3*tan(1/2*d*x + 1/2*c)^3 +
2120*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c) + 735*B*
a^3*tan(1/2*d*x + 1/2*c) + 660*C*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^5)/d
```

3.436 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=235

$$\frac{a^3(34A+38B+45C)\sin(c+dx)}{15d}+\frac{a^3(73A+86B+90C)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{a^3(23A+26B+30C)\sin(c+dx)}{16d}$$

```
[Out] (a^3*(23*A + 26*B + 30*C)*x)/16 + (a^3*(34*A + 38*B + 45*C)*Sin[c + d*x])/(
15*d) + (a^3*(23*A + 26*B + 30*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*
(73*A + 86*B + 90*C)*Cos[c + d*x]^2*Sin[c + d*x])/(120*d) + (A*Cos[c + d*x]
^5*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + ((A + 2*B)*Cos[c + d*x]^4*(
a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(10*a*d) + ((31*A + 42*B + 30*C)*Co
s[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(120*d)
```

Rubi [A] time = 0.591015, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^3(34A+38B+45C)\sin(c+dx)}{15d}+\frac{a^3(73A+86B+90C)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{a^3(23A+26B+30C)\sin(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^3*(23*A + 26*B + 30*C)*x)/16 + (a^3*(34*A + 38*B + 45*C)*Sin[c + d*x])/(
15*d) + (a^3*(23*A + 26*B + 30*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*
(73*A + 86*B + 90*C)*Cos[c + d*x]^2*Sin[c + d*x])/(120*d) + (A*Cos[c + d*x]
^5*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + ((A + 2*B)*Cos[c + d*x]^4*(
a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(10*a*d) + ((31*A + 42*B + 30*C)*Co
s[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(120*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+$$

$$= \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+}{6d}$$

$$= \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+}{6d}$$

$$= \frac{a^3(73A+86B+90C)\cos^2(c+dx)\sin(c+}{120d}$$

$$= \frac{a^3(73A+86B+90C)\cos^2(c+dx)\sin(c+}{120d}$$

$$= \frac{a^3(34A+38B+45C)\sin(c+dx)}{15d}+\frac{a^3(23}{}$$

$$= \frac{1}{16}a^3(23A+26B+30C)x+\frac{a^3(34A+38}{}$$

Mathematica [A] time = 0.752729, size = 170, normalized size = 0.72

$$\frac{a^3(120(21A+23B+26C)\sin(c+dx)+15(63A+64(B+C))\sin(2(c+dx))+380A\sin(3(c+dx))+135A\sin(4(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^3*(900*A*c + 1560*B*c + 1380*A*d*x + 1560*B*d*x + 1800*C*d*x + 120*(21*A
 + 23*B + 26*C)*Sin[c + d*x] + 15*(63*A + 64*(B + C))*Sin[2*(c + d*x)] + 38
0*A*Sin[3*(c + d*x)] + 340*B*Sin[3*(c + d*x)] + 240*C*Sin[3*(c + d*x)] + 13
5*A*Sin[4*(c + d*x)] + 90*B*Sin[4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 36*A
*Sin[5*(c + d*x)] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.125, size = 364, normalized size = 1.6

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{Ba^3\sin(dx+c)}{5}\left(\frac{8}{3}+(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a
^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*A*a^3
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*B*a^3*(1/4*(cos(d*x+c)^3+
3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)
+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^3
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+1/3*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+
1/2*d*x+1/2*c)+a^3*C*sin(d*x+c))
```

Maxima [A] time = 0.971601, size = 478, normalized size = 2.03

$$\frac{192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 64*(3*sin(d*x + c)^
5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*si
n(d*x + c))*B*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*B*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 960*(sin(d*x + c)^
3 - 3*sin(d*x + c))*C*a^3 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*
d*x + 2*c))*C*a^3 + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 + 960*C*a^3*
sin(d*x + c))/d
```

Fricas [A] time = 0.584421, size = 378, normalized size = 1.61

$$\frac{15(23A+26B+30C)a^3dx+\left(40Aa^3\cos(dx+c)^5+48(3A+B)a^3\cos(dx+c)^4+10(23A+18B+6C)a^3\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(23*A + 26*B + 30*C)*a^3*d*x + (40*A*a^3*cos(d*x + c)^5 + 48*(3*A
 + B)*a^3*cos(d*x + c)^4 + 10*(23*A + 18*B + 6*C)*a^3*cos(d*x + c)^3 + 16*(
17*A + 19*B + 15*C)*a^3*cos(d*x + c)^2 + 15*(23*A + 26*B + 30*C)*a^3*cos(d*
x + c) + 16*(34*A + 38*B + 45*C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32686, size = 473, normalized size = 2.01

$$15\left(23\,Aa^3 + 26\,Ba^3 + 30\,Ca^3\right)(dx+c) + \frac{2\left(345\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 390\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 450\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 1955\,Aa^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(23*A*a^3 + 26*B*a^3 + 30*C*a^3)*(d*x + c) + 2*(345*A*a^3*tan(1/2
*d*x + 1/2*c)^11 + 390*B*a^3*tan(1/2*d*x + 1/2*c)^11 + 450*C*a^3*tan(1/2*d*
x + 1/2*c)^11 + 1955*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 2210*B*a^3*tan(1/2*d*x
+ 1/2*c)^9 + 2550*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*A*a^3*tan(1/2*d*x + 1
/2*c)^7 + 5148*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 5940*C*a^3*tan(1/2*d*x + 1/2*
c)^7 + 5814*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 5988*B*a^3*tan(1/2*d*x + 1/2*c)^
5 + 7500*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
 4190*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 5130*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 15
75*A*a^3*tan(1/2*d*x + 1/2*c) + 1530*B*a^3*tan(1/2*d*x + 1/2*c) + 1470*C*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.437 $\int \cos^7(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\operatorname{se}\right.$

Optimal. Leaf size=265

$$-\frac{a^3(108A+119B+133C)\sin^3(c+dx)}{105d}+\frac{a^3(108A+119B+133C)\sin(c+dx)}{35d}+\frac{a^3(129A+147B+154C)\sin(c+d}{280d}$$

```
[Out] (a^3*(21*A + 23*B + 26*C)*x)/16 + (a^3*(108*A + 119*B + 133*C)*Sin[c + d*x]
)/(35*d) + (a^3*(21*A + 23*B + 26*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a
^3*(129*A + 147*B + 154*C)*Cos[c + d*x]^3*Sin[c + d*x])/(280*d) + (A*Cos[c
+ d*x]^6*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(7*d) + ((3*A + 7*B)*Cos[c +
d*x]^5*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(42*a*d) + ((3*A + 4*B + 3*
C)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d) - (a^3*(108
*A + 119*B + 133*C)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.610614, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^3(108A+119B+133C)\sin^3(c+dx)}{105d}+\frac{a^3(108A+119B+133C)\sin(c+dx)}{35d}+\frac{a^3(129A+147B+154C)\sin(c+d}{280d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^3*(21*A + 23*B + 26*C)*x)/16 + (a^3*(108*A + 119*B + 133*C)*Sin[c + d*x]
)/(35*d) + (a^3*(21*A + 23*B + 26*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a
^3*(129*A + 147*B + 154*C)*Cos[c + d*x]^3*Sin[c + d*x])/(280*d) + (A*Cos[c
+ d*x]^6*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(7*d) + ((3*A + 7*B)*Cos[c +
d*x]^5*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(42*a*d) + ((3*A + 4*B + 3*
C)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d) - (a^3*(108
*A + 119*B + 133*C)*Sin[c + d*x]^3)/(105*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+}{7d}$$

$$= \frac{A\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+}{7d}$$

$$= \frac{A\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+}{7d}$$

$$= \frac{a^3(129A+147B+154C)\cos^3(c+dx)\sin}{280d}$$

$$= \frac{a^3(129A+147B+154C)\cos^3(c+dx)\sin}{280d}$$

$$= \frac{a^3(21A+23B+26C)\cos(c+dx)\sin(c+}{16d}$$

$$= \frac{1}{16}a^3(21A+23B+26C)x + \frac{a^3(108A+11}{}$$

Mathematica [A] time = 1.39099, size = 204, normalized size = 0.77

$$\frac{a^3(105(155A+168B+184C)\sin(c+dx)+105(61A+63B+64C)\sin(2(c+dx))+2835A\sin(3(c+dx))+1155A\sin(4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^3*(3360*A*c + 9660*B*c + 8820*A*d*x + 9660*B*d*x + 10920*C*d*x + 105*(15
5*A + 168*B + 184*C)*Sin[c + d*x] + 105*(61*A + 63*B + 64*C)*Sin[2*(c + d*x
)] + 2835*A*Sin[3*(c + d*x)] + 2660*B*Sin[3*(c + d*x)] + 2380*C*Sin[3*(c +
d*x)] + 1155*A*Sin[4*(c + d*x)] + 945*B*Sin[4*(c + d*x)] + 630*C*Sin[4*(c +
 d*x)] + 399*A*Sin[5*(c + d*x)] + 252*B*Sin[5*(c + d*x)] + 84*C*Sin[5*(c +
d*x)] + 105*A*Sin[6*(c + d*x)] + 35*B*Sin[6*(c + d*x)] + 15*A*Sin[7*(c + d*
x)]))/(6720*d)
```

Maple [A] time = 0.132, size = 427, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/7*A*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)+B*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*a^3*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3
*A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+3/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*A*a^3*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*B*a^3*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A
*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a^3
*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 0.98335, size = 574, normalized size = 2.17

$$-\frac{192\left(5\sin\left(dx+c\right)^7-21\sin\left(dx+c\right)^5+35\sin\left(dx+c\right)^3-35\sin\left(dx+c\right)\right)Aa^3-1344\left(3\sin\left(dx+c\right)^5-10\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/6720*(192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*A*a^3 - 1344*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*A*a^3 + 105*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x +
 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c)
+ 8*sin(2*d*x + 2*c))*A*a^3 - 1344*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 +
15*sin(d*x + c))*B*a^3 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4
*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^3 + 2240*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*B*a^3 - 630*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*
B*a^3 - 448*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^3
+ 6720*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 630*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 - 1680*(2*d*x + 2*c + sin(2*d*x + 2
*c))*C*a^3)/d
```

Fricas [A] time = 0.579458, size = 454, normalized size = 1.71

$$\frac{105\,(21\,A+23\,B+26\,C)a^3dx+\left(240\,Aa^3\cos\left(dx+c\right)^6+280\,(3\,A+B)a^3\cos\left(dx+c\right)^5+48\,(27\,A+21\,B+7\,C)a^3\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/1680*(105*(21*A + 23*B + 26*C)*a^3*d*x + (240*A*a^3*cos(d*x + c)^6 + 280*
(3*A + B)*a^3*cos(d*x + c)^5 + 48*(27*A + 21*B + 7*C)*a^3*cos(d*x + c)^4 +
70*(21*A + 23*B + 18*C)*a^3*cos(d*x + c)^3 + 16*(108*A + 119*B + 133*C)*a^3
*cos(d*x + c)^2 + 105*(21*A + 23*B + 26*C)*a^3*cos(d*x + c) + 32*(108*A + 1
19*B + 133*C)*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33252, size = 541, normalized size = 2.04

$$\frac{105\left(21\,Aa^3+23\,Ba^3+26\,Ca^3\right)(dx+c)+\frac{2\left(2205\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2415\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2730\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+14700\,Aa^3\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(21*A*a^3 + 23*B*a^3 + 26*C*a^3)*(d*x + c) + 2*(2205*A*a^3*tan(
1/2*d*x + 1/2*c)^13 + 2415*B*a^3*tan(1/2*d*x + 1/2*c)^13 + 2730*C*a^3*tan(1
/2*d*x + 1/2*c)^13 + 14700*A*a^3*tan(1/2*d*x + 1/2*c)^11 + 16100*B*a^3*tan(
1/2*d*x + 1/2*c)^11 + 18200*C*a^3*tan(1/2*d*x + 1/2*c)^11 + 41601*A*a^3*tan
(1/2*d*x + 1/2*c)^9 + 45563*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 51506*C*a^3*tan(
1/2*d*x + 1/2*c)^9 + 62592*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 72576*B*a^3*tan(1
/2*d*x + 1/2*c)^7 + 77952*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 63231*A*a^3*tan(1/
2*d*x + 1/2*c)^5 + 62853*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 71246*C*a^3*tan(1/2
*d*x + 1/2*c)^5 + 25620*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 33180*B*a^3*tan(1/2*
d*x + 1/2*c)^3 + 40040*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 11235*A*a^3*tan(1/2*d
*x + 1/2*c) + 11025*B*a^3*tan(1/2*d*x + 1/2*c) + 10710*C*a^3*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.438 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\mathrm{se}\right.$

Optimal. Leaf size=252

$$\frac{2a^4(56A + 49B + 44C)\tan^3(c + dx)}{105d} + \frac{4a^4(56A + 49B + 44C)\tan(c + dx)}{35d} + \frac{a^4(56A + 49B + 44C)\tanh^{-1}(\sin(c + }{16d}$$

```
[Out] (a^4*(56*A + 49*B + 44*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(56*A + 49
*B + 44*C)*Tan[c + d*x])/(35*d) + (27*a^4*(56*A + 49*B + 44*C)*Sec[c + d*x]
*Tan[c + d*x])/(560*d) + (a^4*(56*A + 49*B + 44*C)*Sec[c + d*x]^3*Tan[c + d
*x])/(280*d) + ((42*A - 7*B + 8*C)*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(21
0*d) + (C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(7*d) + ((7*B
 + 4*C)*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/(42*a*d) + (2*a^4*(56*A + 49*B
 + 44*C)*Tan[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.520057, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4088, 4010, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{2a^4(56A + 49B + 44C)\tan^3(c + dx)}{105d} + \frac{4a^4(56A + 49B + 44C)\tan(c + dx)}{35d} + \frac{a^4(56A + 49B + 44C)\tanh^{-1}(\sin(c + }{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^4*(56*A + 49*B + 44*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(56*A + 49
*B + 44*C)*Tan[c + d*x])/(35*d) + (27*a^4*(56*A + 49*B + 44*C)*Sec[c + d*x]
*Tan[c + d*x])/(560*d) + (a^4*(56*A + 49*B + 44*C)*Sec[c + d*x]^3*Tan[c + d
*x])/(280*d) + ((42*A - 7*B + 8*C)*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(21
0*d) + (C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(7*d) + ((7*B
 + 4*C)*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/(42*a*d) + (2*a^4*(56*A + 49*B
 + 44*C)*Tan[c + d*x]^3)/(105*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^4\tan(c+}{7d}$$

$$= \frac{C\sec^2(c+dx)(a+a\sec(c+dx))^4\tan(c+}{7d}$$

$$= \frac{(42A-7B+8C)(a+a\sec(c+dx))^4\tan(}{210d}$$

$$= \frac{(42A-7B+8C)(a+a\sec(c+dx))^4\tan(}{210d}$$

$$= \frac{(42A-7B+8C)(a+a\sec(c+dx))^4\tan(}{210d}$$

$$= \frac{a^4(56A+49B+44C)\tanh^{-1}(\sin(c+dx)}{70d}$$

$$= \frac{2a^4(56A+49B+44C)\tanh^{-1}(\sin(c+dx}{35d}$$

$$= \frac{a^4(56A+49B+44C)\tanh^{-1}(\sin(c+dx)}{16d}$$

Mathematica [B] time = 6.45561, size = 1087, normalized size = 4.31

$$\frac{(-56A-49B-44C)\cos^6(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^4\left(C\sec^2(c+dx)+B\sec(c+d\right.}{128d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((-56*A - 49*B - 44*C)*Cos[c + d*x]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2))/(128*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
 + ((56*A + 49*B + 44*C)*Cos[c + d*x]^6*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2))/(128*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)) + (C*Sec[c]*Sec[c/2 + (d*x)/2]^8*Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[d*x])/(56*d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + (Sec[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c +
d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*C*Sin[c] + 7*B*Sin[d*x]
+ 28*C*Sin[d*x]))/(336*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
 + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*(35*B*Sin[c] + 140*C*Sin[c] + 42*A*Sin[d*x
] + 168*B*Sin[d*x] + 288*C*Sin[d*x]))/(1680*d*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*Sec[c]*Sec[c/2 + (d*x)/2]^8*(a + a*
Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(168*A*Sin[c] + 672
*B*Sin[c] + 1152*C*Sin[c] + 840*A*Sin[d*x] + 1435*B*Sin[d*x] + 1540*C*Sin[d
*x]))/(6720*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c +
 d*x]^3*Sec[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*(840*A*Sin[c] + 1435*B*Sin[c] + 1540*C*Sin[c] + 190
4*A*Sin[d*x] + 2016*B*Sin[d*x] + 1816*C*Sin[d*x]))/(6720*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^4*Sec[c]*Sec[c/2 + (d*x)
/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3808*
A*Sin[c] + 4032*B*Sin[c] + 3632*C*Sin[c] + 5880*A*Sin[d*x] + 5145*B*Sin[d*x
] + 4620*C*Sin[d*x]))/(13440*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])) + (Cos[c + d*x]^5*Sec[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(5880*A*Sin[c] + 5145*B*Sin[c] + 4
620*C*Sin[c] + 9296*A*Sin[d*x] + 8064*B*Sin[d*x] + 7264*C*Sin[d*x]))/(13440
*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 0.077, size = 454, normalized size = 1.8

$$\frac{Ba^4\tan(dx+c)(\sec(dx+c))^5}{6d}+\frac{454\,a^4C\tan(dx+c)}{105\,d}+\frac{227\,a^4C\tan(dx+c)(\sec(dx+c))^2}{105\,d}+\frac{24\,Ba^4\tan(dx+c)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/6/d*B*a^4*tan(d*x+c)*sec(d*x+c)^5+454/105/d*a^4*C*tan(d*x+c)+227/105/d*a^
4*C*tan(d*x+c)*sec(d*x+c)^2+24/5/d*B*a^4*tan(d*x+c)+12/5/d*B*a^4*tan(d*x+c)
*sec(d*x+c)^2+7/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+11/6/d*a^4*C*tan(d*x+c)*sec
(d*x+c)^3+11/4/d*a^4*C*sec(d*x+c)*tan(d*x+c)+41/24/d*B*a^4*tan(d*x+c)*sec(d
*x+c)^3+1/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+2/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)
^5+49/16/d*B*a^4*sec(d*x+c)*tan(d*x+c)+34/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^
2+48/35/d*a^4*C*tan(d*x+c)*sec(d*x+c)^4+4/5/d*B*a^4*tan(d*x+c)*sec(d*x+c)^4
```

```
+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/7/d*a^4*C*tan(d*x+c)*sec(d*x+c)^6+11
/4/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+83/15/d*A*a^4*tan(d*x+c)+49/16/d*B*a^4
*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [B] time = 1.00252, size = 987, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/3360*(224*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4
+ 6720*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 896*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 4480*(tan(d*x + c)^3 + 3*tan(d*x
 + c))*B*a^4 + 96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3
 + 35*tan(d*x + c))*C*a^4 + 1344*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15
*tan(d*x + c))*C*a^4 + 1120*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 - 35*B*
a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x +
 c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1)) - 140*C*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x
 + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x +
 c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*A*a
^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^
2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 1260*B*a^4*(2
*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1
) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*C*a^4*(2*(3*si
n(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*
log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 3360*A*a^4*(2*sin(d*x +
c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) -
840*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) + 3360*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.55581, size = 617, normalized size = 2.45

$$\frac{105\,(56\,A+49\,B+44\,C)a^4\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\,(56\,A+49\,B+44\,C)a^4\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/3360*(105*(56*A + 49*B + 44*C)*a^4*cos(d*x + c)^7*log(sin(d*x + c) + 1) -
 105*(56*A + 49*B + 44*C)*a^4*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 2*(16
*(581*A + 504*B + 454*C)*a^4*cos(d*x + c)^6 + 105*(56*A + 49*B + 44*C)*a^4*
cos(d*x + c)^5 + 16*(238*A + 252*B + 227*C)*a^4*cos(d*x + c)^4 + 70*(24*A +
 41*B + 44*C)*a^4*cos(d*x + c)^3 + 48*(7*A + 28*B + 48*C)*a^4*cos(d*x + c)^
2 + 280*(B + 4*C)*a^4*cos(d*x + c) + 240*C*a^4)*sin(d*x + c))/(d*cos(d*x +
c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4 \left(\int A \sec^2{(c+dx)}\,dx + \int 4A\sec^3{(c+dx)}\,dx + \int 6A\sec^4{(c+dx)}\,dx + \int 4A\sec^5{(c+dx)}\,dx + \int A\sec^6{(c+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + I
ntegral(6*A*sec(c + d*x)**4, x) + Integral(4*A*sec(c + d*x)**5, x) + Integr
al(A*sec(c + d*x)**6, x) + Integral(B*sec(c + d*x)**3, x) + Integral(4*B*se
c(c + d*x)**4, x) + Integral(6*B*sec(c + d*x)**5, x) + Integral(4*B*sec(c +
 d*x)**6, x) + Integral(B*sec(c + d*x)**7, x) + Integral(C*sec(c + d*x)**4,
 x) + Integral(4*C*sec(c + d*x)**5, x) + Integral(6*C*sec(c + d*x)**6, x) +
 Integral(4*C*sec(c + d*x)**7, x) + Integral(C*sec(c + d*x)**8, x))
```

Giac [A] time = 1.30492, size = 598, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(56*A*a^4 + 49*B*a^4 + 44*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 105*(56*A*a^4 + 49*B*a^4 + 44*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - 2*(5880*A*a^4*tan(1/2*d*x + 1/2*c)^13 + 5145*B*a^4*tan(1/2*d*x + 1/2*
c)^13 + 4620*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 39200*A*a^4*tan(1/2*d*x + 1/2*
c)^11 - 34300*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 30800*C*a^4*tan(1/2*d*x + 1/2
*c)^11 + 110936*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 97069*B*a^4*tan(1/2*d*x + 1/
2*c)^9 + 87164*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 172032*A*a^4*tan(1/2*d*x + 1/
2*c)^7 - 150528*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 135168*C*a^4*tan(1/2*d*x + 1
/2*c)^7 + 159656*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 134099*B*a^4*tan(1/2*d*x +
1/2*c)^5 + 126084*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 86240*A*a^4*tan(1/2*d*x +
1/2*c)^3 - 73220*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 58800*C*a^4*tan(1/2*d*x + 1
/2*c)^3 + 21000*A*a^4*tan(1/2*d*x + 1/2*c) + 21735*B*a^4*tan(1/2*d*x + 1/2*
c) + 22260*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.439 $\int \sec(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec^2(\right.$

Optimal. Leaf size=209

$$\frac{2a^4(10A+8B+7C)\tan^3(c+dx)}{15d}+\frac{4a^4(10A+8B+7C)\tan(c+dx)}{5d}+\frac{7a^4(10A+8B+7C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(}{}$$

```
[Out] (7*a^4*(10*A + 8*B + 7*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(10*A + 8*
B + 7*C)*Tan[c + d*x])/(5*d) + (27*a^4*(10*A + 8*B + 7*C)*Sec[c + d*x]*Tan[
c + d*x])/(80*d) + (a^4*(10*A + 8*B + 7*C)*Sec[c + d*x]^3*Tan[c + d*x])/(40
*d) + ((6*B - C)*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(30*d) + (C*(a + a*Se
c[c + d*x])^5*Tan[c + d*x])/(6*a*d) + (2*a^4*(10*A + 8*B + 7*C)*Tan[c + d*x
]^3)/(15*d)
```

Rubi [A] time = 0.328311, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4082, 4001, 3791, 3770, 3767, 8, 3768}

$$\frac{2a^4(10A+8B+7C)\tan^3(c+dx)}{15d}+\frac{4a^4(10A+8B+7C)\tan(c+dx)}{5d}+\frac{7a^4(10A+8B+7C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (7*a^4*(10*A + 8*B + 7*C)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(10*A + 8*
B + 7*C)*Tan[c + d*x])/(5*d) + (27*a^4*(10*A + 8*B + 7*C)*Sec[c + d*x]*Tan[
c + d*x])/(80*d) + (a^4*(10*A + 8*B + 7*C)*Sec[c + d*x]^3*Tan[c + d*x])/(40
*d) + ((6*B - C)*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(30*d) + (C*(a + a*Se
c[c + d*x])^5*Tan[c + d*x])/(6*a*d) + (2*a^4*(10*A + 8*B + 7*C)*Tan[c + d*x
]^3)/(15*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

```
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{C(a+a\sec(c+dx))^5\tan(c+dx)}{6ad}+\frac{\int s}{}$$
$$=\frac{(6B-C)(a+a\sec(c+dx))^4\tan(c+dx}{30d}$$
$$=\frac{(6B-C)(a+a\sec(c+dx))^4\tan(c+dx}{30d}$$
$$=\frac{(6B-C)(a+a\sec(c+dx))^4\tan(c+dx}{30d}$$
$$=\frac{a^4(10A+8B+7C)\tanh^{-1}(\sin(c+dx))}{10d}$$
$$=\frac{2a^4(10A+8B+7C)\tanh^{-1}(\sin(c+dx}{5d}$$
$$=\frac{7a^4(10A+8B+7C)\tanh^{-1}(\sin(c+dx}{16d}$$

Mathematica [A] time = 5.92255, size = 359, normalized size = 1.72

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(105(10A+8B+7C)\cos^6(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
 d*x)/2]^8*Sec[c + d*x]^6*(105*(10*A + 8*B + 7*C)*Cos[c + d*x]^6*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 - 40*C*Sec[c]*Sin[d*x] - 8*Cos[c + d*x]*Sec[c]*(5*C*Sin[c] + 6*(B + 4*C)*S
```

```
in[d*x]) - 2*Cos[c + d*x]^3*Sec[c]*(5*(6*A + 24*B + 41*C)*Sin[c] + 16*(10*A
 + 17*B + 18*C)*Sin[d*x]) - 2*Cos[c + d*x]^2*Sec[c]*(24*(B + 4*C)*Sin[c] +
5*(6*A + 24*B + 41*C)*Sin[d*x]) - Cos[c + d*x]^4*Sec[c]*(32*(10*A + 17*B +
18*C)*Sin[c] + 15*(54*A + 56*B + 49*C)*Sin[d*x]) - Cos[c + d*x]^5*Sec[c]*(1
5*(54*A + 56*B + 49*C)*Sin[c] + 16*(100*A + 83*B + 72*C)*Sin[d*x])))/(1920*
d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.08, size = 385, normalized size = 1.8

$$\frac{49\,a^4C\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{Ba^4\tan(dx+c)\left(\sec(dx+c)\right)^3}{d}+\frac{Aa^4\tan(dx+c)\left(\sec(dx+c)\right)^3}{4\,d}+\frac{a^4C\tan(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 49/16/d*a^4*C*sec(d*x+c)*tan(d*x+c)+1/d*B*a^4*tan(d*x+c)*sec(d*x+c)^3+1/4/d
*A*a^4*tan(d*x+c)*sec(d*x+c)^3+1/6/d*a^4*C*tan(d*x+c)*sec(d*x+c)^5+7/2/d*B*
a^4*sec(d*x+c)*tan(d*x+c)+27/8/d*A*a^4*sec(d*x+c)*tan(d*x+c)+41/24/d*a^4*C*
tan(d*x+c)*sec(d*x+c)^3+24/5/d*a^4*C*tan(d*x+c)+12/5/d*a^4*C*tan(d*x+c)*sec
(d*x+c)^2+83/15/d*B*a^4*tan(d*x+c)+34/15/d*B*a^4*tan(d*x+c)*sec(d*x+c)^2+20
/3/d*A*a^4*tan(d*x+c)+4/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+4/5/d*a^4*C*tan(d
*x+c)*sec(d*x+c)^4+1/5/d*B*a^4*tan(d*x+c)*sec(d*x+c)^4+49/16/d*a^4*C*ln(sec
(d*x+c)+tan(d*x+c))+7/2/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+35/8/d*A*a^4*ln(s
ec(d*x+c)+tan(d*x+c))
```

Maxima [B] time = 0.98382, size = 861, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 32*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c
))*C*a^4 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 - 5*C*a^4*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 30*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 120*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)
) - 180*C*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72
0*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 480*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*C*a^4*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*A*a^
4*log(sec(d*x + c) + tan(d*x + c)) + 1920*A*a^4*tan(d*x + c) + 480*B*a^4*ta
n(d*x + c))/d
```

Fricas [A] time = 0.543912, size = 539, normalized size = 2.58

$$105\,(10\,A+8\,B+7\,C)a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-105\,(10\,A+8\,B+7\,C)a^4\cos(dx+c)^6\log(-\sin(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/480*(105*(10*A + 8*B + 7*C)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 10
5*(10*A + 8*B + 7*C)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(100
*A + 83*B + 72*C)*a^4*cos(d*x + c)^5 + 15*(54*A + 56*B + 49*C)*a^4*cos(d*x
+ c)^4 + 32*(10*A + 17*B + 18*C)*a^4*cos(d*x + c)^3 + 10*(6*A + 24*B + 41*C
)*a^4*cos(d*x + c)^2 + 48*(B + 4*C)*a^4*cos(d*x + c) + 40*C*a^4)*sin(d*x +
c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int A\sec(c+dx)\,dx+\int 4A\sec^2(c+dx)\,dx+\int 6A\sec^3(c+dx)\,dx+\int 4A\sec^4(c+dx)\,dx+\int A\sec^5(c+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*sec(c + d*x)**2, x) + Inte
gral(6*A*sec(c + d*x)**3, x) + Integral(4*A*sec(c + d*x)**4, x) + Integral(
A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**2, x) + Integral(4*B*sec(c
 + d*x)**3, x) + Integral(6*B*sec(c + d*x)**4, x) + Integral(4*B*sec(c + d*
x)**5, x) + Integral(B*sec(c + d*x)**6, x) + Integral(C*sec(c + d*x)**3, x)
 + Integral(4*C*sec(c + d*x)**4, x) + Integral(6*C*sec(c + d*x)**5, x) + In
tegral(4*C*sec(c + d*x)**6, x) + Integral(C*sec(c + d*x)**7, x))
```

Giac [B] time = 1.33016, size = 529, normalized size = 2.53

$$105\left(10\,Aa^4+8\,Ba^4+7\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(10\,Aa^4+8\,Ba^4+7\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/240*(105*(10*A*a^4 + 8*B*a^4 + 7*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 105*(10*A*a^4 + 8*B*a^4 + 7*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
 2*(1050*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11
+ 735*C*a^4*tan(1/2*d*x + 1/2*c)^11 - 5950*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 4
760*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 4165*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 1386
```

```
0*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 9702*
C*a^4*tan(1/2*d*x + 1/2*c)^7 - 16860*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 13488*B
*a^4*tan(1/2*d*x + 1/2*c)^5 - 11802*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 10690*A*
a^4*tan(1/2*d*x + 1/2*c)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 7355*C*a^4
*tan(1/2*d*x + 1/2*c)^3 - 2790*A*a^4*tan(1/2*d*x + 1/2*c) - 3000*B*a^4*tan(
1/2*d*x + 1/2*c) - 3105*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^6)/d
```

3.440 $\int (a+a\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)$

Optimal. Leaf size=195

$$\frac{a^4(40A+35B+28C)\tan(c+dx)}{8d}+\frac{a^4(48A+35B+28C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(20A+35B+28C)\tan(c+dx)\left(a\right.}{60d}$$

```
[Out] a^4*A*x + (a^4*(48*A + 35*B + 28*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(40
*A + 35*B + 28*C)*Tan[c + d*x])/(8*d) + (a*(5*B + 4*C)*(a + a*Sec[c + d*x])
^3*Tan[c + d*x])/(20*d) + (C*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(5*d) + (
(20*A + 35*B + 28*C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((32
*A + 35*B + 28*C)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.304017, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4054, 3917, 3914, 3767, 8, 3770}

$$\frac{a^4(40A+35B+28C)\tan(c+dx)}{8d}+\frac{a^4(48A+35B+28C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(20A+35B+28C)\tan(c+dx)\left(a\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] a^4*A*x + (a^4*(48*A + 35*B + 28*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(40
*A + 35*B + 28*C)*Tan[c + d*x])/(8*d) + (a*(5*B + 4*C)*(a + a*Sec[c + d*x])
^3*Tan[c + d*x])/(20*d) + (C*(a + a*Sec[c + d*x])^4*Tan[c + d*x])/(5*d) + (
(20*A + 35*B + 28*C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((32
*A + 35*B + 28*C)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```


Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
 !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{C(a+a\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{\int (a+a\sec(c+d}{}$$

$$= \frac{a(5B+4C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a+a}{}$$

$$= \frac{a(5B+4C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a+a}{}$$

$$= \frac{a(5B+4C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{C(a+a}{}$$

$$= a^4Ax + \frac{a(5B+4C)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} +$$

$$= a^4Ax + \frac{a^4(48A+35B+28C)\tanh^{-1}(\sin(c+dx))}{8d} +$$

$$= a^4Ax + \frac{a^4(48A+35B+28C)\tanh^{-1}(\sin(c+dx))}{8d} +$$

Mathematica [B] time = 5.09068, size = 538, normalized size = 2.76

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(\sec(c)(-3120A\sin(2c+dx)+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
d*x)/2]^8*Sec[c + d*x]^5*(-240*(48*A + 35*B + 28*C)*Cos[c + d*x]^5*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) + Sec[c]*(600*A*d*x*Cos[d*x] + 600*A*d*x*Cos[2*c + d*x] + 300*A*d*x*Cos[
2*c + 3*d*x] + 300*A*d*x*Cos[4*c + 3*d*x] + 60*A*d*x*Cos[4*c + 5*d*x] + 60*
A*d*x*Cos[6*c + 5*d*x] + 4880*A*Sin[d*x] + 5120*B*Sin[d*x] + 4720*C*Sin[d*x
] - 3120*A*Sin[2*c + d*x] - 2880*B*Sin[2*c + d*x] - 1920*C*Sin[2*c + d*x] +
 480*A*Sin[c + 2*d*x] + 930*B*Sin[c + 2*d*x] + 1320*C*Sin[c + 2*d*x] + 480*
A*Sin[3*c + 2*d*x] + 930*B*Sin[3*c + 2*d*x] + 1320*C*Sin[3*c + 2*d*x] + 328
0*A*Sin[2*c + 3*d*x] + 3520*B*Sin[2*c + 3*d*x] + 3200*C*Sin[2*c + 3*d*x] -
720*A*Sin[4*c + 3*d*x] - 480*B*Sin[4*c + 3*d*x] - 120*C*Sin[4*c + 3*d*x] +
240*A*Sin[3*c + 4*d*x] + 405*B*Sin[3*c + 4*d*x] + 420*C*Sin[3*c + 4*d*x] +
240*A*Sin[5*c + 4*d*x] + 405*B*Sin[5*c + 4*d*x] + 420*C*Sin[5*c + 4*d*x] +
800*A*Sin[4*c + 5*d*x] + 800*B*Sin[4*c + 5*d*x] + 664*C*Sin[4*c + 5*d*x]))
```

```
/(15360*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.074, size = 331, normalized size = 1.7

$$a^4Ax + \frac{Aa^4c}{d} + \frac{35\,Ba^4\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{83\,a^4C\tan(dx+c)}{15d} + 6\,\frac{Aa^4\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^4*A*x+1/d*A*a^4*c+35/8/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+83/15/d*a^4*C*ta
n(d*x+c)+6/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*B*a^4*tan(d*x+c)+7/2/d*
a^4*C*sec(d*x+c)*tan(d*x+c)+7/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*A*
a^4*tan(d*x+c)+27/8/d*B*a^4*sec(d*x+c)*tan(d*x+c)+34/15/d*a^4*C*tan(d*x+c)*
sec(d*x+c)^2+2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+4/3/d*B*a^4*tan(d*x+c)*sec(d*x
+c)^2+1/d*a^4*C*tan(d*x+c)*sec(d*x+c)^3+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2
+1/4/d*B*a^4*tan(d*x+c)*sec(d*x+c)^3+1/5/d*a^4*C*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [B] time = 0.966995, size = 651, normalized size = 3.34

$$\frac{80\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Aa^4 + 240\,(dx+c)Aa^4 + 320\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Ba^4 + 16\left(3\,\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 240*(d*x + c)*A*a^4 + 3
20*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 16*(3*tan(d*x + c)^5 + 10*tan(
d*x + c)^3 + 15*tan(d*x + c))*C*a^4 + 480*(tan(d*x + c)^3 + 3*tan(d*x + c))
*C*a^4 - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*C*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 240*A
*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)) - 360*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 240*C*a^4*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 960*A*a^4*l
og(sec(d*x + c) + tan(d*x + c)) + 240*B*a^4*log(sec(d*x + c) + tan(d*x + c)
) + 1440*A*a^4*tan(d*x + c) + 960*B*a^4*tan(d*x + c) + 240*C*a^4*tan(d*x +
c))/d
```

Fricas [A] time = 0.561003, size = 521, normalized size = 2.67

$$\frac{240\,Aa^4dx\cos(dx+c)^5 + 15\,(48\,A + 35\,B + 28\,C)a^4\cos(dx+c)^5\log(\sin(dx+c)+1) - 15\,(48\,A + 35\,B + 28\,C)a^4\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(240*A*a^4*d*x*cos(d*x + c)^5 + 15*(48*A + 35*B + 28*C)*a^4*cos(d*x +
 c)^5*log(sin(d*x + c) + 1) - 15*(48*A + 35*B + 28*C)*a^4*cos(d*x + c)^5*lo
g(-sin(d*x + c) + 1) + 2*(8*(100*A + 100*B + 83*C)*a^4*cos(d*x + c)^4 + 15*
(16*A + 27*B + 28*C)*a^4*cos(d*x + c)^3 + 8*(5*A + 20*B + 34*C)*a^4*cos(d*x
 + c)^2 + 30*(B + 4*C)*a^4*cos(d*x + c) + 24*C*a^4)*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int A\,dx+\int 4A\sec(c+dx)\,dx+\int 6A\sec^2(c+dx)\,dx+\int 4A\sec^3(c+dx)\,dx+\int A\sec^4(c+dx)\,dx+\int B\sec(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] a**4*(Integral(A, x) + Integral(4*A*sec(c + d*x), x) + Integral(6*A*sec(c +
 d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**
4, x) + Integral(B*sec(c + d*x), x) + Integral(4*B*sec(c + d*x)**2, x) + In
tegral(6*B*sec(c + d*x)**3, x) + Integral(4*B*sec(c + d*x)**4, x) + Integra
l(B*sec(c + d*x)**5, x) + Integral(C*sec(c + d*x)**2, x) + Integral(4*C*sec
(c + d*x)**3, x) + Integral(6*C*sec(c + d*x)**4, x) + Integral(4*C*sec(c +
d*x)**5, x) + Integral(C*sec(c + d*x)**6, x))
```

Giac [A] time = 1.29353, size = 475, normalized size = 2.44

$$\frac{120\,(dx+c)Aa^4+15\left(48\,Aa^4+35\,Ba^4+28\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(48\,Aa^4+35\,Ba^4+28\,Ca^4\right)\log\left(\left|\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/120*(120*(d*x + c)*A*a^4 + 15*(48*A*a^4 + 35*B*a^4 + 28*C*a^4)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 15*(48*A*a^4 + 35*B*a^4 + 28*C*a^4)*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) - 2*(600*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 525*B*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 420*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 2720*A*a^4*tan(1/
2*d*x + 1/2*c)^7 - 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 1960*C*a^4*tan(1/2*d
*x + 1/2*c)^7 + 4720*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 4480*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 + 3584*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 3680*A*a^4*tan(1/2*d*x + 1
/2*c)^3 - 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3160*C*a^4*tan(1/2*d*x + 1/2*
c)^3 + 1080*A*a^4*tan(1/2*d*x + 1/2*c) + 1395*B*a^4*tan(1/2*d*x + 1/2*c) +
1500*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.441 $\int \cos(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\sec\right.$

Optimal. Leaf size=196

$$\frac{5a^4(4A+8B+7C)\tan(c+dx)}{8d} + \frac{a^4(52A+48B+35C)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(12A-4B-7C)\tan(c+dx)\left(a^2\sec\right.}{12d}$$

```
[Out] a^4*(4*A + B)*x + (a^4*(52*A + 48*B + 35*C)*ArcTanh[Sin[c + d*x]])/(8*d) +
(A*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/d + (5*a^4*(4*A + 8*B + 7*C)*Tan[c
+ d*x])/(8*d) - (a*(4*A - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) - (
(12*A - 4*B - 7*C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - ((12*A
 - 32*B - 35*C)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.382038, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4086, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^4(4A+8B+7C)\tan(c+dx)}{8d} + \frac{a^4(52A+48B+35C)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{(12A-4B-7C)\tan(c+dx)\left(a^2\sec\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a^4*(4*A + B)*x + (a^4*(52*A + 48*B + 35*C)*ArcTanh[Sin[c + d*x]])/(8*d) +
(A*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/d + (5*a^4*(4*A + 8*B + 7*C)*Tan[c
+ d*x])/(8*d) - (a*(4*A - C)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) - (
(12*A - 4*B - 7*C)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - ((12*A
 - 32*B - 35*C)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d} + \frac{\int(a+}{}$$

$$= \frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d} - \frac{a(4A-}{}$$

$$= \frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d} - \frac{a(4A-}{}$$

$$= \frac{A(a+a\sec(c+dx))^4\sin(c+dx)}{d} - \frac{a(4A-}{}$$

$$= a^4(4A+B)x + \frac{A(a+a\sec(c+dx))^4\sin(c}{d}$$

$$= a^4(4A+B)x + \frac{a^4(52A+48B+35C)\tanh}{8d}$$

$$= a^4(4A+B)x + \frac{a^4(52A+48B+35C)\tanh}{8d}$$

Mathematica [B] time = 4.65642, size = 530, normalized size = 2.7

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(\sec(c)(72dx(4A+B)\cos(c)+48dx(4A+B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a^4*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]^8*(1 + Sec[c
+ d*x])^4*(-24*(52*A + 48*B + 35*C)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*(72*
(4*A + B)*d*x*Cos[c] + 48*(4*A + B)*d*x*Cos[c + 2*d*x] + 192*A*d*x*Cos[3*c
+ 2*d*x] + 48*B*d*x*Cos[3*c + 2*d*x] + 48*A*d*x*Cos[3*c + 4*d*x] + 12*B*d*x
*Cos[3*c + 4*d*x] + 48*A*d*x*Cos[5*c + 4*d*x] + 12*B*d*x*Cos[5*c + 4*d*x] -
 288*A*Sin[c] - 480*B*Sin[c] - 480*C*Sin[c] + 24*A*Sin[d*x] + 48*B*Sin[d*x]
 + 105*C*Sin[d*x] + 24*A*Sin[2*c + d*x] + 48*B*Sin[2*c + d*x] + 105*C*Sin[2
*c + d*x] + 288*A*Sin[c + 2*d*x] + 496*B*Sin[c + 2*d*x] + 544*C*Sin[c + 2*d
*x] - 96*A*Sin[3*c + 2*d*x] - 144*B*Sin[3*c + 2*d*x] - 96*C*Sin[3*c + 2*d*x
] + 30*A*Sin[2*c + 3*d*x] + 48*B*Sin[2*c + 3*d*x] + 81*C*Sin[2*c + 3*d*x] +
```

```
30*A*Sin[4*c + 3*d*x] + 48*B*Sin[4*c + 3*d*x] + 81*C*Sin[4*c + 3*d*x] + 96
*A*Sin[3*c + 4*d*x] + 160*B*Sin[3*c + 4*d*x] + 160*C*Sin[3*c + 4*d*x] + 6*A
*Sin[4*c + 5*d*x] + 6*A*Sin[6*c + 5*d*x])))/(1536*d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.117, size = 294, normalized size = 1.5

$$\frac{Aa^4\sin(dx+c)}{d}+Ba^4x+\frac{Ba^4c}{d}+\frac{35\,a^4C\ln(\sec(dx+c)+\tan(dx+c))}{8d}+4\,a^4Ax+4\,\frac{Aa^4c}{d}+6\,\frac{Ba^4\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+B*a^4*x+1/d*B*a^4*c+35/8/d*a^4*C*ln(sec(d*x+c)+tan(d*x
+c))+4*a^4*A*x+4/d*A*a^4*c+6/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*a^4*C
*tan(d*x+c)+13/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*B*a^4*tan(d*x+c)+
27/8/d*a^4*C*sec(d*x+c)*tan(d*x+c)+4/d*A*a^4*tan(d*x+c)+2/d*B*a^4*sec(d*x+c
)*tan(d*x+c)+4/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*A*a^4*sec(d*x+c)*tan
(d*x+c)+1/3/d*B*a^4*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^4*C*tan(d*x+c)*sec(d*x+
c)^3
```

Maxima [B] time = 0.978593, size = 562, normalized size = 2.87

$$192\,(dx+c)Aa^4+16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^4+48\,(dx+c)Ba^4+64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/48*(192*(d*x + c)*A*a^4 + 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 48
*(d*x + c)*B*a^4 + 64*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 - 3*C*a^4*(2*
(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1)
 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 - 48*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 72*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 144*A*a^4*(log(sin(d*x + c)
+ 1) - log(sin(d*x + c) - 1)) + 96*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 24*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
48*A*a^4*sin(d*x + c) + 192*A*a^4*tan(d*x + c) + 288*B*a^4*tan(d*x + c) + 1
92*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.565522, size = 493, normalized size = 2.52

$$48\,(4\,A+B)a^4dx\cos(dx+c)^4+3\,(52\,A+48\,B+35\,C)a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(52\,A+48\,B+35\,C)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/48*(48*(4*A + B)*a^4*d*x*cos(d*x + c)^4 + 3*(52*A + 48*B + 35*C)*a^4*cos(
d*x + c)^4*log(sin(d*x + c) + 1) - 3*(52*A + 48*B + 35*C)*a^4*cos(d*x + c)^
4*log(-sin(d*x + c) + 1) + 2*(24*A*a^4*cos(d*x + c)^4 + 32*(3*A + 5*B + 5*C
)*a^4*cos(d*x + c)^3 + 3*(4*A + 16*B + 27*C)*a^4*cos(d*x + c)^2 + 8*(B + 4*
C)*a^4*cos(d*x + c) + 6*C*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.32555, size = 458, normalized size = 2.34

$$\frac{\frac{48\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 24\left(4\,Aa^4 + Ba^4\right)(dx+c) + 3\left(52\,Aa^4 + 48\,Ba^4 + 35\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(52\,Aa^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 24*(4*A*
a^4 + B*a^4)*(d*x + c) + 3*(52*A*a^4 + 48*B*a^4 + 35*C*a^4)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - 3*(52*A*a^4 + 48*B*a^4 + 35*C*a^4)*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(84*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d
*x + 1/2*c)^7 + 105*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 276*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 385*C*a^4*tan(1/2*d*x + 1/2*c
)^5 + 300*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 +
 511*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 108*A*a^4*tan(1/2*d*x + 1/2*c) - 216*B*
a^4*tan(1/2*d*x + 1/2*c) - 279*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 - 1)^4)/d
```

3.442 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\,\text{se}\right.$

Optimal. Leaf size=209

$$\frac{5a^4(A-B-2C)\sin(c+dx)}{2d}+\frac{a^4(8A+13B+12C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(A-B-2C)\sin(c+dx)\left(a^2\sec(c+dx)+\right.}{2d}$$

```
[Out] (a^4*(13*A + 8*B + 2*C)*x)/2 + (a^4*(8*A + 13*B + 12*C)*ArcTanh[Sin[c + d*x
]])/(2*d) + (5*a^4*(A - B - 2*C)*Sin[c + d*x])/(2*d) - (a*(3*A - 2*C)*(a +
a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x]
)^4*Sin[c + d*x])/(2*d) - ((A - B - 2*C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c +
 d*x])/(2*d) + ((3*A + 18*B + 22*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rubi [A] time = 0.565657, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4086, 4018, 3996, 3770}

$$\frac{5a^4(A-B-2C)\sin(c+dx)}{2d}+\frac{a^4(8A+13B+12C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(A-B-2C)\sin(c+dx)\left(a^2\sec(c+dx)+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^4*(13*A + 8*B + 2*C)*x)/2 + (a^4*(8*A + 13*B + 12*C)*ArcTanh[Sin[c + d*x
]])/(2*d) + (5*a^4*(A - B - 2*C)*Sin[c + d*x])/(2*d) - (a*(3*A - 2*C)*(a +
a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]*(a + a*Sec[c + d*x]
)^4*Sin[c + d*x])/(2*d) - ((A - B - 2*C)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c +
 d*x])/(2*d) + ((3*A + 18*B + 22*C)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^4\sin(c+}{2d}$$
$$= -\frac{a(3A-2C)(a+a\sec(c+dx))^3\sin(c+}{6d}$$
$$= -\frac{a(3A-2C)(a+a\sec(c+dx))^3\sin(c+}{6d}$$
$$= -\frac{a(3A-2C)(a+a\sec(c+dx))^3\sin(c+}{6d}$$
$$= \frac{5a^4(A-B-2C)\sin(c+dx)}{2d} - \frac{a(3A-2C}{}$$
$$= \frac{1}{2}a^4(13A+8B+2C)x + \frac{5a^4(A-B-2C)}{2d}$$
$$= \frac{1}{2}a^4(13A+8B+2C)x + \frac{a^4(8A+13B+1}{}$$

Mathematica [B] time = 4.62698, size = 524, normalized size = 2.51

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(\sec(c)(36dx(13A+8B+2C)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[(c +
d*x)/2]^8*Sec[c + d*x]^3*(-96*(8*A + 13*B + 12*C)*Cos[c + d*x]^3*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 + Sec[c]*(36*(13*A + 8*B + 2*C)*d*x*Cos[d*x] + 36*(13*A + 8*B + 2*C)*d*x*C
os[2*c + d*x] + 156*A*d*x*Cos[2*c + 3*d*x] + 96*B*d*x*Cos[2*c + 3*d*x] + 24
*C*d*x*Cos[2*c + 3*d*x] + 156*A*d*x*Cos[4*c + 3*d*x] + 96*B*d*x*Cos[4*c + 3
*d*x] + 24*C*d*x*Cos[4*c + 3*d*x] + 102*A*Sin[d*x] + 384*B*Sin[d*x] + 672*C
*Sin[d*x] - 42*A*Sin[2*c + d*x] - 192*B*Sin[2*c + d*x] - 288*C*Sin[2*c + d*
x] + 96*A*Sin[c + 2*d*x] + 48*B*Sin[c + 2*d*x] + 96*C*Sin[c + 2*d*x] + 96*A
*Sin[3*c + 2*d*x] + 48*B*Sin[3*c + 2*d*x] + 96*C*Sin[3*c + 2*d*x] + 57*A*Si
n[2*c + 3*d*x] + 192*B*Sin[2*c + 3*d*x] + 320*C*Sin[2*c + 3*d*x] + 9*A*Sin[
4*c + 3*d*x] + 48*A*Sin[3*c + 4*d*x] + 12*B*Sin[3*c + 4*d*x] + 48*A*Sin[5*c
 + 4*d*x] + 12*B*Sin[5*c + 4*d*x] + 3*A*Sin[4*c + 5*d*x] + 3*A*Sin[6*c + 5*
d*x])))/(1536*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.125, size = 279, normalized size = 1.3

$$\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13a^4Ax}{2}+\frac{13Aa^4c}{2d}+\frac{Ba^4\sin(dx+c)}{d}+a^4Cx+\frac{Ca^4c}{d}+4\frac{Aa^4\sin(dx+c)}{d}+4Ba^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*a^4*A*x+13/2/d*A*a^4*c+1/d*B*a^4*sin
(d*x+c)+a^4*C*x+1/d*C*a^4*c+4/d*A*a^4*sin(d*x+c)+4*B*a^4*x+4/d*B*a^4*c+6/d*
a^4*C*ln(sec(d*x+c)+tan(d*x+c))+13/2/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3
/d*a^4*C*tan(d*x+c)+4/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a^4*tan(d*x+c
)+2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+1/d*A*a^4*tan(d*x+c)+1/2/d*B*a^4*sec(d*x+
c)*tan(d*x+c)+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.966271, size = 432, normalized size = 2.07

$$\frac{3(2dx+2c+\sin(2dx+2c))Aa^4+72(dx+c)Aa^4+48(dx+c)Ba^4+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^4+12(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 72*(d*x + c)*A*a^4 + 48*(d
*x + c)*B*a^4 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 12*(d*x + c)*C*
a^4 - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)) - 12*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(sin(d*x + c)
 + 1) - log(sin(d*x + c) - 1)) + 36*B*a^4*(log(sin(d*x + c) + 1) - log(sin(
d*x + c) - 1)) + 24*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 48*A*a^4*sin(d*x + c) + 12*B*a^4*sin(d*x + c) + 12*A*a^4*tan(d*x + c) + 48
*B*a^4*tan(d*x + c) + 72*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.560103, size = 487, normalized size = 2.33

$$\frac{6(13A+8B+2C)a^4dx\cos(dx+c)^3+3(8A+13B+12C)a^4\cos(dx+c)^3\log(\sin(dx+c)+1)-3(8A+13B+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(13*A + 8*B + 2*C)*a^4*d*x*cos(d*x + c)^3 + 3*(8*A + 13*B + 12*C)*a
^4*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(8*A + 13*B + 12*C)*a^4*cos(d*x
 + c)^3*log(-sin(d*x + c) + 1) + 2*(3*A*a^4*cos(d*x + c)^4 + 6*(4*A + B)*a^
4*cos(d*x + c)^3 + 2*(3*A + 12*B + 20*C)*a^4*cos(d*x + c)^2 + 3*(B + 4*C)*a
^4*cos(d*x + c) + 2*C*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31841, size = 468, normalized size = 2.24

$$3\left(13\,Aa^4+8\,Ba^4+2\,Ca^4\right)(dx+c)+3\left(8\,Aa^4+13\,Ba^4+12\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(8\,Aa^4+13\,Ba^4+12\,C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/6*(3*(13*A*a^4 + 8*B*a^4 + 2*C*a^4)*(d*x + c) + 3*(8*A*a^4 + 13*B*a^4 + 1
2*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*A*a^4 + 13*B*a^4 + 12*C*
a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 6*(7*A*a^4*tan(1/2*d*x + 1/2*c)^3
 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) + 2*B*a^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 2*(6*A*a^4*tan(1/2*d
*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*C*a^4*tan(1/2*d*x + 1/
2*c)^5 - 12*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^4*tan(1/2*d*x + 1/2*c)^3
- 76*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4
*tan(1/2*d*x + 1/2*c) + 54*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^3)/d
```

3.443 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\text{se}\right.$

Optimal. Leaf size=217

$$\frac{5a^4(2A+B-C)\sin(c+dx)}{2d}+\frac{a^4(2A+8B+13C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(2A+B-C)\sin(c+dx)\left(a^2\sec(c+dx)+\right.}{2d}$$

```
[Out] (a^4*(12*A + 13*B + 8*C)*x)/2 + (a^4*(2*A + 8*B + 13*C)*ArcTanh[Sin[c + d*x
]])/(2*d) + (5*a^4*(2*A + B - C)*Sin[c + d*x])/(2*d) + (a*(4*A + 3*B)*Cos[c
 + d*x]*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a +
 a*Sec[c + d*x])^4*Sin[c + d*x])/(3*d) - ((2*A + B - C)*(a^2 + a^2*Sec[c +
d*x])^2*Sin[c + d*x])/(2*d) - ((8*A - 3*B - 18*C)*(a^4 + a^4*Sec[c + d*x])*
Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.601841, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4086, 4017, 4018, 3996, 3770}

$$\frac{5a^4(2A+B-C)\sin(c+dx)}{2d}+\frac{a^4(2A+8B+13C)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(2A+B-C)\sin(c+dx)\left(a^2\sec(c+dx)+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```



```
[Out] (a^4*(12*A + 13*B + 8*C)*x)/2 + (a^4*(2*A + 8*B + 13*C)*ArcTanh[Sin[c + d*x
]])/(2*d) + (5*a^4*(2*A + B - C)*Sin[c + d*x])/(2*d) + (a*(4*A + 3*B)*Cos[c
 + d*x]*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a +
 a*Sec[c + d*x])^4*Sin[c + d*x])/(3*d) - ((2*A + B - C)*(a^2 + a^2*Sec[c +
d*x])^2*Sin[c + d*x])/(2*d) - ((8*A - 3*B - 18*C)*(a^4 + a^4*Sec[c + d*x])*
Sin[c + d*x])/(6*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^4\sin(c+}{3d}$$
$$= \frac{a(4A+3B)\cos(c+dx)(a+a\sec(c+dx))}{6d}$$
$$= \frac{a(4A+3B)\cos(c+dx)(a+a\sec(c+dx))}{6d}$$
$$= \frac{a(4A+3B)\cos(c+dx)(a+a\sec(c+dx))}{6d}$$
$$= \frac{5a^4(2A+B-C)\sin(c+dx)}{2d} + \frac{a(4A+3B}{}$$
$$= \frac{1}{2}a^4(12A+13B+8C)x + \frac{5a^4(2A+B-C}{2d}$$
$$= \frac{1}{2}a^4(12A+13B+8C)x + \frac{a^4(2A+8B+1}{}$$

Mathematica [B] time = 6.31162, size = 1518, normalized size = 7.

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((12*A + 13*B + 8*C)*x*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d
*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(16*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])) + ((-2*A - 8*B - 13*C)*Cos[c + d*x]^6*Log[Cos[c
/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x
])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(16*d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])) + ((2*A + 8*B + 13*C)*Cos[c + d*x]^6*Log[Cos[c/
2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(16*d*(A + 2*C + 2*B*Cos[c + d
```

```
*x] + A*Cos[2*c + 2*d*x])) + ((27*A + 16*B + 4*C)*Cos[d*x]*Cos[c + d*x]^6*S
ec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sin[c])/(32*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
((4*A + B)*Cos[2*d*x]*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*
x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[2*c])/(32*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (A*Cos[3*d*x]*Cos[c + d*x]^6*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*Sin[3*c])/(96*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((2
7*A + 16*B + 4*C)*Cos[c]*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c +
 d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[d*x])/(32*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((4*A + B)*Cos[2*c]*Cos[c + d*x]
^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*Sin[2*d*x])/(32*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])) + (A*Cos[3*c]*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[3*d*x])/(96*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^6*Sec[c/2 + (d*x)/2
]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(32*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2 + (d*x)/2] - Sin[
c/2 + (d*x)/2])^2) + (Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*
x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(B*Sin[(d*x)/2] + 4*C*Sin[(d*
x)/2]))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2] -
Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (C*Cos[c + d*x]^6*Se
c[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2))/(32*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^6*Sec[c/2 + (d*x)/2]^8*(a
 + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(B*Sin[(d*x)/2
] + 4*C*Sin[(d*x)/2]))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.122, size = 280, normalized size = 1.3

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^4}{3d}+\frac{20\,Aa^4\sin(dx+c)}{3d}+\frac{Ba^4\sin(dx+c)\cos(dx+c)}{2d}+\frac{13\,Ba^4x}{2}+\frac{13\,Ba^4c}{2d}+\frac{a^4C\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+1/2/d*B*a^4*sin
(d*x+c)*cos(d*x+c)+13/2*B*a^4*x+13/2/d*B*a^4*c+1/d*a^4*C*sin(d*x+c)+2/d*A*a
^4*cos(d*x+c)*sin(d*x+c)+6*a^4*A*x+6/d*A*a^4*c+4/d*B*a^4*sin(d*x+c)+4*a^4*C
*x+4/d*C*a^4*c+13/2/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a^4*ln(sec(d*x+
c)+tan(d*x+c))+4/d*a^4*C*tan(d*x+c)+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d
*B*a^4*tan(d*x+c)+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.965466, size = 400, normalized size = 1.84

$$-\frac{4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^4-12\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4-48\,(dx+c)Aa^4-3\,(2\,dx+2\,c+\sin(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 12*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a^4 - 48*(d*x + c)*A*a^4 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))
*B*a^4 - 72*(d*x + c)*B*a^4 - 48*(d*x + c)*C*a^4 + 3*C*a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*A
*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 24*B*a^4*(log(sin(d*
x + c) + 1) - log(sin(d*x + c) - 1)) - 36*C*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 72*A*a^4*sin(d*x + c) - 48*B*a^4*sin(d*x + c) - 12*C
*a^4*sin(d*x + c) - 12*B*a^4*tan(d*x + c) - 48*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.565319, size = 486, normalized size = 2.24

$$\frac{6\,(12\,A+13\,B+8\,C)a^4dx\cos\left(dx+c\right)^2+3\,(2\,A+8\,B+13\,C)a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+8\,B+13\,C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(12*A + 13*B + 8*C)*a^4*d*x*cos(d*x + c)^2 + 3*(2*A + 8*B + 13*C)*a
^4*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*A + 8*B + 13*C)*a^4*cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*a^4*cos(d*x + c)^4 + 3*(4*A + B)*a^4
*cos(d*x + c)^3 + 2*(20*A + 12*B + 3*C)*a^4*cos(d*x + c)^2 + 6*(B + 4*C)*a^
4*cos(d*x + c) + 3*C*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3056, size = 468, normalized size = 2.16

$$\frac{3\left(12\,Aa^4+13\,Ba^4+8\,Ca^4\right)(dx+c)+3\left(2\,Aa^4+8\,Ba^4+13\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^4+8\,Ba^4+13\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/6*(3*(12*A*a^4 + 13*B*a^4 + 8*C*a^4)*(d*x + c) + 3*(2*A*a^4 + 8*B*a^4 + 1
3*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^4 + 8*B*a^4 + 13*C*a
^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*(2*B*a^4*tan(1/2*d*x + 1/2*c)^3
```

```
+ 7*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^4*tan(1/2*d*x + 1/2*c) - 9*C*a^4*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(30*A*a^4*tan(1/2*d
*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^4*tan(1/2*d*x + 1/2
*c)^5 + 76*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 +
 12*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4
*tan(1/2*d*x + 1/2*c) + 6*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

3.444 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2$

Optimal. Leaf size=217

$$\frac{5a^4(7A+8B+4C)\sin(c+dx)}{8d}-\frac{(35A+32B-12C)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{24d}+\frac{(7A+8B+4C)\sin(c+dx)\cos}{8}$$

```
[Out] (a^4*(35*A + 48*B + 52*C)*x)/8 + (a^4*(B + 4*C)*ArcTanh[Sin[c + d*x]])/d +
(5*a^4*(7*A + 8*B + 4*C)*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^2*(a
 + a*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c +
 d*x])^4*Sin[c + d*x])/(4*d) + ((7*A + 8*B + 4*C)*Cos[c + d*x]*(a^2 + a^2*S
ec[c + d*x])^2*Sin[c + d*x])/(8*d) - ((35*A + 32*B - 12*C)*(a^4 + a^4*Sec[c
 + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.621899, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4086, 4017, 4018, 3996, 3770}

$$\frac{5a^4(7A+8B+4C)\sin(c+dx)}{8d}-\frac{(35A+32B-12C)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{24d}+\frac{(7A+8B+4C)\sin(c+dx)\cos}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^4*(35*A + 48*B + 52*C)*x)/8 + (a^4*(B + 4*C)*ArcTanh[Sin[c + d*x]])/d +
(5*a^4*(7*A + 8*B + 4*C)*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^2*(a
 + a*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c +
 d*x])^4*Sin[c + d*x])/(4*d) + ((7*A + 8*B + 4*C)*Cos[c + d*x]*(a^2 + a^2*S
ec[c + d*x])^2*Sin[c + d*x])/(8*d) - ((35*A + 32*B - 12*C)*(a^4 + a^4*Sec[c
 + d*x])*Sin[c + d*x])/(24*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^4\sin(}{4d}$$

$$= \frac{a(A+B)\cos^2(c+dx)(a+a\sec(c+dx}{3d}$$

$$= \frac{a(A+B)\cos^2(c+dx)(a+a\sec(c+dx}{3d}$$

$$= \frac{a(A+B)\cos^2(c+dx)(a+a\sec(c+dx}{3d}$$

$$= \frac{5a^4(7A+8B+4C)\sin(c+dx)}{8d} + \frac{a(A}{}$$

$$= \frac{1}{8}a^4(35A+48B+52C)x + \frac{5a^4(7A+8}{}$$

$$= \frac{1}{8}a^4(35A+48B+52C)x + \frac{a^4(B+4C)}{}$$

Mathematica [B] time = 6.2291, size = 1436, normalized size = 6.62

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] a^4*(((35*A + 48*B + 52*C)*x*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[c/2 +
(d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(64*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*c + 2*d*x])) + ((-B - 4*C)*Cos[c + d*x]^2*(1 + Cos[c + d
*x])^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])) + ((B + 4*C)*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Log[Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
```

```
d*x])) + ((28*A + 27*B + 16*C)*Cos[d*x]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4
*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(32*d
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((7*A + 4*B + C)*Cos[
2*d*x]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*Sin[2*c])/(32*d*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])) + ((4*A + B)*Cos[3*d*x]*Cos[c + d*x]^2*(1 + Cos[c + d*
x])^4*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[3*c]
)/(96*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (A*Cos[4*d*x]*
Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*Sin[4*c])/(256*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) + ((28*A + 27*B + 16*C)*Cos[c]*Cos[c + d*x]^2*(1 + Cos[c + d
*x])^4*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[d*x
])/(32*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((7*A + 4*B +
 C)*Cos[2*c]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[2*d*x])/(32*d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + ((4*A + B)*Cos[3*c]*Cos[c + d*x]^2*(1 + Cos[
c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Si
n[3*d*x])/(96*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (A*Cos
[4*c]*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*Sin[4*d*x])/(256*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*Sec[c/2 + (
d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[(d*x)/2])/(8*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2
+ (d*x)/2] - Sin[c/2 + (d*x)/2])) + (C*Cos[c + d*x]^2*(1 + Cos[c + d*x])^4*
Sec[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[(d*x)/2])/
(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c/2] + Sin[c/2]
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])))
```

Maple [A] time = 0.11, size = 289, normalized size = 1.3

$$\frac{B\sin(dx+c)(\cos(dx+c))^2a^4}{3d}+\frac{20Ba^4\sin(dx+c)}{3d}+\frac{35a^4Ax}{8}+\frac{Aa^4\sin(dx+c)(\cos(dx+c))^3}{4d}+\frac{27Aa^4\cos(dx+c}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*B*a^4*sin(d*x+c)+35/8*a^4*A*x+1/
4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+27/8/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*a
^4*C*x+1/2/d*a^4*C*sin(d*x+c)*cos(d*x+c)+6*B*a^4*x+2/d*B*a^4*sin(d*x+c)*cos
(d*x+c)+1/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+35/8/d*A*a^4*c+13/2/d*C*a^4*c+6
/d*B*a^4*c+4/d*a^4*C*sin(d*x+c)+4/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*
A*a^4*sin(d*x+c)+1/d*a^4*C*tan(d*x+c)+4/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4
```

Maxima [A] time = 0.968961, size = 392, normalized size = 1.81

$$-\frac{128\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa^4-144(2dx+2c+s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 144*(2*d*x + 2*c + sin(2*d*x +
2*c))*A*a^4 - 96*(d*x + c)*A*a^4 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a
^4 - 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 384*(d*x + c)*B*a^4 - 24*(
2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 - 576*(d*x + c)*C*a^4 - 48*B*a^4*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 192*C*a^4*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) - 384*A*a^4*sin(d*x + c) - 576*B*a^4*sin(d*x +
 c) - 384*C*a^4*sin(d*x + c) - 96*C*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.553484, size = 466, normalized size = 2.15

$$\frac{3\,(35\,A+48\,B+52\,C)a^4dx\cos\left(dx+c\right)+12\,(B+4\,C)a^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-12\,(B+4\,C)a^4\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/24*(3*(35*A + 48*B + 52*C)*a^4*d*x*cos(d*x + c) + 12*(B + 4*C)*a^4*cos(d*
x + c)*log(sin(d*x + c) + 1) - 12*(B + 4*C)*a^4*cos(d*x + c)*log(-sin(d*x +
 c) + 1) + (6*A*a^4*cos(d*x + c)^4 + 8*(4*A + B)*a^4*cos(d*x + c)^3 + 3*(27
*A + 16*B + 4*C)*a^4*cos(d*x + c)^2 + 32*(5*A + 5*B + 3*C)*a^4*cos(d*x + c)
 + 24*C*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29502, size = 448, normalized size = 2.06

$$-\frac{\frac{48\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\left(35\,Aa^4+48\,Ba^4+52\,Ca^4\right)(dx+c)-24\left(Ba^4+4\,Ca^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+24\left(Ba^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -1/24*(48*C*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(35*A
*a^4 + 48*B*a^4 + 52*C*a^4)*(d*x + c) - 24*(B*a^4 + 4*C*a^4)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) + 24*(B*a^4 + 4*C*a^4)*log(abs(tan(1/2*d*x + 1/2*c) -
```

```
1)) - 2*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^
7 + 84*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 42
4*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 276*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a
^4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 300*C*a^4*ta
n(1/2*d*x + 1/2*c)^3 + 279*A*a^4*tan(1/2*d*x + 1/2*c) + 216*B*a^4*tan(1/2*d
*x + 1/2*c) + 108*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^
4)/d
```

3.445 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\,\text{se}\right.$

Optimal. Leaf size=225

$$\frac{a^4(28A + 35B + 40C)\sin(c + dx)}{8d} + \frac{(28A + 35B + 20C)\sin(c + dx)\cos^2(c + dx)\left(a^2\sec(c + dx) + a^2\right)^2}{60d} + \frac{(28A + 35}{}$$

```
[Out] (a^4*(28*A + 35*B + 48*C)*x)/8 + (a^4*C*ArcTanh[Sin[c + d*x]])/d + (a^4*(28
*A + 35*B + 40*C)*Sin[c + d*x])/(8*d) + (a*(4*A + 5*B)*Cos[c + d*x]^3*(a +
a*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d
*x])^4*Sin[c + d*x])/(5*d) + ((28*A + 35*B + 20*C)*Cos[c + d*x]^2*(a^2 + a^
2*Sec[c + d*x])^2*Sin[c + d*x])/(60*d) + ((28*A + 35*B + 32*C)*Cos[c + d*x]
*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(24*d)
```

Rubi [A] time = 0.582938, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4086, 4017, 3996, 3770}

$$\frac{a^4(28A + 35B + 40C)\sin(c + dx)}{8d} + \frac{(28A + 35B + 20C)\sin(c + dx)\cos^2(c + dx)\left(a^2\sec(c + dx) + a^2\right)^2}{60d} + \frac{(28A + 35}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^4*(28*A + 35*B + 48*C)*x)/8 + (a^4*C*ArcTanh[Sin[c + d*x]])/d + (a^4*(28
*A + 35*B + 40*C)*Sin[c + d*x])/(8*d) + (a*(4*A + 5*B)*Cos[c + d*x]^3*(a +
a*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d
*x])^4*Sin[c + d*x])/(5*d) + ((28*A + 35*B + 20*C)*Cos[c + d*x]^2*(a^2 + a^
2*Sec[c + d*x])^2*Sin[c + d*x])/(60*d) + ((28*A + 35*B + 32*C)*Cos[c + d*x]
*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(24*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
```

```
f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+\ldots}{5d}$$
$$= \frac{a(4A+5B)\cos^3(c+dx)(a+a\sec(c+dx)\ldots}{20d}$$
$$= \frac{a(4A+5B)\cos^3(c+dx)(a+a\sec(c+dx)\ldots}{20d}$$
$$= \frac{a(4A+5B)\cos^3(c+dx)(a+a\sec(c+dx)\ldots}{20d}$$
$$= \frac{a^4(28A+35B+40C)\sin(c+dx)}{8d}+\frac{a(4A\ldots}{}$$
$$= \frac{1}{8}a^4(28A+35B+48C)x+\frac{a^4(28A+35B\ldots}{}$$
$$= \frac{1}{8}a^4(28A+35B+48C)x+\frac{a^4C\tanh^{-1}(\mathrm{si}\ldots}{d}$$

Mathematica [A] time = 0.617114, size = 182, normalized size = 0.81

$$\frac{a^4\left(60(49A+56B+54C)\sin(c+dx)+120(8A+7B+4C)\sin(2(c+dx))+290A\sin(3(c+dx))+60A\sin(4(c+dx))+\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^4*(1680*A*d*x + 2100*B*d*x + 2880*C*d*x - 480*C*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 480*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*(49*
A + 56*B + 54*C)*Sin[c + d*x] + 120*(8*A + 7*B + 4*C)*Sin[2*(c + d*x)] + 29
0*A*Sin[3*(c + d*x)] + 160*B*Sin[3*(c + d*x)] + 40*C*Sin[3*(c + d*x)] + 60*
A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.125, size = 320, normalized size = 1.4

$$\frac{4B\sin(dx+c)(\cos(dx+c))^2a^4}{3d}+\frac{20Ba^4\sin(dx+c)}{3d}+\frac{35Ba^4c}{8d}+\frac{7Aa^4c}{2d}+\frac{35Ba^4x}{8}+\frac{Ba^4\sin(dx+c)(\cos(dx+c))^3\ldots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 4/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*B*a^4*sin(d*x+c)+35/8/d*B*a^4*c+
7/2/d*A*a^4*c+35/8*B*a^4*x+1/4/d*B*a^4*sin(d*x+c)*cos(d*x+c)^3+27/8/d*B*a^4
*sin(d*x+c)*cos(d*x+c)+7/2*a^4*A*x+1/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+7/2/d*
A*a^4*cos(d*x+c)*sin(d*x+c)+6*a^4*C*x+2/d*a^4*C*sin(d*x+c)*cos(d*x+c)+20/3/
d*a^4*C*sin(d*x+c)+1/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+83/15/d*A*a^4*sin(d*
x+c)+34/15/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+6/d*C*a^4*c+1/5/d*A*a^4*sin(d*x+
c)*cos(d*x+c)^4+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^4
```

Maxima [A] time = 0.967566, size = 448, normalized size = 1.99

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa^{4}-960\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}+60\,(12\,dx+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 60*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^4 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^4 + 480*(d*x + c)*B*a^4 - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))
*C*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 1920*(d*x + c)*C*a^4
+ 240*C*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 480*A*a^4*sin
(d*x + c) + 1920*B*a^4*sin(d*x + c) + 2880*C*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.561305, size = 408, normalized size = 1.81

$$\frac{15\,(28\,A+35\,B+48\,C)a^{4}dx+60\,Ca^{4}\log\left(\sin\left(dx+c\right)+1\right)-60\,Ca^{4}\log\left(-\sin\left(dx+c\right)+1\right)+\left(24\,Aa^{4}\cos\left(dx+c\right)^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/120*(15*(28*A + 35*B + 48*C)*a^4*d*x + 60*C*a^4*log(sin(d*x + c) + 1) - 6
0*C*a^4*log(-sin(d*x + c) + 1) + (24*A*a^4*cos(d*x + c)^4 + 30*(4*A + B)*a^
4*cos(d*x + c)^3 + 8*(34*A + 20*B + 5*C)*a^4*cos(d*x + c)^2 + 15*(28*A + 27
*B + 16*C)*a^4*cos(d*x + c) + 8*(83*A + 100*B + 100*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34333, size = 455, normalized size = 2.02

$$120\,Ca^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-120\,Ca^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\left(28\,Aa^4+35\,Ba^4+48\,Ca^4\right)(dx+c)+\frac{2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(120*C*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 120*C*a^4*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 15*(28*A*a^4 + 35*B*a^4 + 48*C*a^4)*(d*x + c) + 2
*(420*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 600
*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 1960*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 2450*B*
a^4*tan(1/2*d*x + 1/2*c)^7 + 2720*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4
*tan(1/2*d*x + 1/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 4720*C*a^4*ta
n(1/2*d*x + 1/2*c)^5 + 3160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 3950*B*a^4*tan(1
/2*d*x + 1/2*c)^3 + 3680*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*
d*x + 1/2*c) + 1395*B*a^4*tan(1/2*d*x + 1/2*c) + 1080*C*a^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.446 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\,\mathrm{se}\right.$

Optimal. Leaf size=213

$$-\frac{2a^4(7A+8B+10C)\sin^3(c+dx)}{15d}+\frac{4a^4(7A+8B+10C)\sin(c+dx)}{5d}+\frac{a^4(7A+8B+10C)\sin(c+dx)\cos^3(c+dx)}{40d}$$

```
[Out] (7*a^4*(7*A + 8*B + 10*C)*x)/16 + (4*a^4*(7*A + 8*B + 10*C)*Sin[c + d*x])/(
5*d) + (27*a^4*(7*A + 8*B + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*
(7*A + 8*B + 10*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + ((2*A + 3*B)*Cos[c
 + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(15*d) + (A*Cos[c + d*x]^5*(
a + a*Sec[c + d*x])^4*Sin[c + d*x])/(6*d) - (2*a^4*(7*A + 8*B + 10*C)*Sin[c
 + d*x]^3)/(15*d)
```

Rubi [A] time = 0.40923, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4013, 3791, 2637, 2635, 8, 2633}

$$-\frac{2a^4(7A+8B+10C)\sin^3(c+dx)}{15d}+\frac{4a^4(7A+8B+10C)\sin(c+dx)}{5d}+\frac{a^4(7A+8B+10C)\sin(c+dx)\cos^3(c+dx)}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (7*a^4*(7*A + 8*B + 10*C)*x)/16 + (4*a^4*(7*A + 8*B + 10*C)*Sin[c + d*x])/(
5*d) + (27*a^4*(7*A + 8*B + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*
(7*A + 8*B + 10*C)*Cos[c + d*x]^3*Sin[c + d*x])/(40*d) + ((2*A + 3*B)*Cos[c
 + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(15*d) + (A*Cos[c + d*x]^5*(
a + a*Sec[c + d*x])^4*Sin[c + d*x])/(6*d) - (2*a^4*(7*A + 8*B + 10*C)*Sin[c
 + d*x]^3)/(15*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

```
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^4\sin(c+}{6d}$$
$$= \frac{(2A+3B)\cos^4(c+dx)(a+a\sec(c+dx))}{15d}$$
$$= \frac{(2A+3B)\cos^4(c+dx)(a+a\sec(c+dx))}{15d}$$
$$= \frac{1}{10}a^4(7A+8B+10C)x+\frac{(2A+3B)\cos^4}{}$$
$$= \frac{1}{10}a^4(7A+8B+10C)x+\frac{2a^4(7A+8B+}{5}$$
$$= \frac{2}{5}a^4(7A+8B+10C)x+\frac{4a^4(7A+8B+1}{5}$$
$$= \frac{7}{16}a^4(7A+8B+10C)x+\frac{4a^4(7A+8B+}{5}$$

Mathematica [A] time = 0.496599, size = 163, normalized size = 0.77

$$\frac{a^4(120(44A+49B+56C)\sin(c+dx)+15(127A+128B+112C)\sin(2(c+dx))+720A\sin(3(c+dx))+225A\sin(4(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^4*(2940*A*d*x + 3360*B*d*x + 4200*C*d*x + 120*(44*A + 49*B + 56*C)*Sin[c
 + d*x] + 15*(127*A + 128*B + 112*C)*Sin[2*(c + d*x)] + 720*A*Sin[3*(c + d*
x)] + 580*B*Sin[3*(c + d*x)] + 320*C*Sin[3*(c + d*x)] + 225*A*Sin[4*(c + d*
x)] + 120*B*Sin[4*(c + d*x)] + 30*C*Sin[4*(c + d*x)] + 48*A*Sin[5*(c + d*x)
```

```
] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [B] time = 0.13, size = 416, normalized size = 2.

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4Aa^4\sin(dx+c)}{5}\left(\frac{8}{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1
/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*B*a^4*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^4*C*(1/4*(cos(d*x+c)^3+3/2*co
s(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+2
*B*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*
a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*B*a^4*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+6*a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a^
4*sin(d*x+c)+4*a^4*C*sin(d*x+c)+a^4*C*(d*x+c))
```

Maxima [B] time = 0.972799, size = 540, normalized size = 2.54

$$\frac{256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 180*(12*d*
x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 240*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*A*a^4 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*B*a^4 - 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 120*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2
*c + sin(2*d*x + 2*c))*B*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4
 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4 + 1440*
(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 + 960*(d*x + c)*C*a^4 + 960*B*a^4*si
n(d*x + c) + 3840*C*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.528742, size = 378, normalized size = 1.77

$$\frac{105(7A+8B+10C)a^4dx+\left(40Aa^4\cos(dx+c)^5+48(4A+B)a^4\cos(dx+c)^4+10(41A+24B+6C)a^4\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(105*(7*A + 8*B + 10*C)*a^4*d*x + (40*A*a^4*cos(d*x + c)^5 + 48*(4*A
+ B)*a^4*cos(d*x + c)^4 + 10*(41*A + 24*B + 6*C)*a^4*cos(d*x + c)^3 + 32*(1
8*A + 17*B + 10*C)*a^4*cos(d*x + c)^2 + 15*(49*A + 56*B + 54*C)*a^4*cos(d*x
 + c) + 16*(72*A + 83*B + 100*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3357, size = 473, normalized size = 2.22

$$105\left(7\,Aa^4+8\,Ba^4+10\,Ca^4\right)(dx+c)+\frac{2\left(735\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+840\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1050\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,Aa^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4 + 10*C*a^4)*(d*x + c) + 2*(735*A*a^4*tan(1/2*
d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 1050*C*a^4*tan(1/2*d*
x + 1/2*c)^11 + 4165*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 4760*B*a^4*tan(1/2*d*x
+ 1/2*c)^9 + 5950*C*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x + 1
/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 13860*C*a^4*tan(1/2*d*x + 1/
2*c)^7 + 11802*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 13488*B*a^4*tan(1/2*d*x + 1/2
*c)^5 + 16860*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/2*c
)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 10690*C*a^4*tan(1/2*d*x + 1/2*c)^
3 + 3105*A*a^4*tan(1/2*d*x + 1/2*c) + 3000*B*a^4*tan(1/2*d*x + 1/2*c) + 279
0*C*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.447 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\sec\right.$

Optimal. Leaf size=278

$$\frac{a^4(454A + 504B + 581C)\sin(c + dx)}{105d} + \frac{a^4(988A + 1113B + 1232C)\sin(c + dx)\cos^2(c + dx)}{840d} + \frac{a^4(44A + 49B + 56C)}{1}$$

```
[Out] (a^4*(44*A + 49*B + 56*C)*x)/16 + (a^4*(454*A + 504*B + 581*C)*Sin[c + d*x]
)/(105*d) + (a^4*(44*A + 49*B + 56*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (
a^4*(988*A + 1113*B + 1232*C)*Cos[c + d*x]^2*Sin[c + d*x])/(840*d) + (a*(4*
A + 7*B)*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(42*d) + (A*Co
s[c + d*x]^6*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(7*d) + ((16*A + 21*B + 1
4*C)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(70*d) + ((436
*A + 511*B + 504*C)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(
840*d)
```

Rubi [A] time = 0.763355, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4017, 3996, 3787, 2635, 8, 2637}

$$\frac{a^4(454A + 504B + 581C)\sin(c + dx)}{105d} + \frac{a^4(988A + 1113B + 1232C)\sin(c + dx)\cos^2(c + dx)}{840d} + \frac{a^4(44A + 49B + 56C)}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^4*(44*A + 49*B + 56*C)*x)/16 + (a^4*(454*A + 504*B + 581*C)*Sin[c + d*x]
)/(105*d) + (a^4*(44*A + 49*B + 56*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (
a^4*(988*A + 1113*B + 1232*C)*Cos[c + d*x]^2*Sin[c + d*x])/(840*d) + (a*(4*
A + 7*B)*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(42*d) + (A*Co
s[c + d*x]^6*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(7*d) + ((16*A + 21*B + 1
4*C)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(70*d) + ((436
*A + 511*B + 504*C)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(
840*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^6(c+dx)(a+a\sec(c+dx))^4\sin(c+}{7d}$$

$$= \frac{a(4A+7B)\cos^5(c+dx)(a+a\sec(c+dx)}{42d}$$

$$= \frac{a(4A+7B)\cos^5(c+dx)(a+a\sec(c+dx)}{42d}$$

$$= \frac{a(4A+7B)\cos^5(c+dx)(a+a\sec(c+dx)}{42d}$$

$$= \frac{a^4(988A+1113B+1232C)\cos^2(c+dx)\,s}{840d}$$

$$= \frac{a^4(988A+1113B+1232C)\cos^2(c+dx)\,s}{840d}$$

$$= \frac{a^4(454A+504B+581C)\sin(c+dx)}{105d}+\frac{a}{}$$

$$= \frac{1}{16}a^4(44A+49B+56C)x+\frac{a^4(454A+5}{}$$

Mathematica [A] time = 1.00692, size = 204, normalized size = 0.73

$$\frac{a^4(105(323A+352B+392C)\sin(c+dx)+105(124A+127B+128C)\sin(2(c+dx))+5495A\sin(3(c+dx))+2100A\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^4*(11760*A*c + 20580*B*c + 18480*A*d*x + 20580*B*d*x + 23520*C*d*x + 105
*(323*A + 352*B + 392*C)*Sin[c + d*x] + 105*(124*A + 127*B + 128*C)*Sin[2*(
c + d*x)] + 5495*A*Sin[3*(c + d*x)] + 5040*B*Sin[3*(c + d*x)] + 4060*C*Sin[
3*(c + d*x)] + 2100*A*Sin[4*(c + d*x)] + 1575*B*Sin[4*(c + d*x)] + 840*C*Si
n[4*(c + d*x)] + 651*A*Sin[5*(c + d*x)] + 336*B*Sin[5*(c + d*x)] + 84*C*Sin
[5*(c + d*x)] + 140*A*Sin[6*(c + d*x)] + 35*B*Sin[6*(c + d*x)] + 15*A*Sin[7
*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.141, size = 490, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/7*A*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)+B*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
*A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+4/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4
*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*A*a^4*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*B*a^4*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)
+4*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*B
*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+1/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^4*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+a^4*C*sin(d*x+c))
```

Maxima [A] time = 0.985393, size = 652, normalized size = 2.35

$$-\frac{192\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)Aa^4-2688\left(3\sin(dx+c)^5-10\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/6720*(192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*A*a^4 - 2688*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*A*a^4 + 140*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x +
 4*c) - 48*sin(2*d*x + 2*c))*A*a^4 + 2240*(sin(d*x + c)^3 - 3*sin(d*x + c))
*A*a^4 - 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4
- 1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 + 35*
(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*B*a^4 + 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 - 1260*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 1680*(2*d*x + 2*c +
 sin(2*d*x + 2*c))*B*a^4 - 448*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
```

```
in(d*x + c))*C*a^4 + 13440*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 - 840*(1
2*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4 - 6720*(2*d*x +
 2*c + sin(2*d*x + 2*c))*C*a^4 - 6720*C*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.529753, size = 454, normalized size = 1.63

$$\frac{105\,(44\,A+49\,B+56\,C)a^4dx+\left(240\,Aa^4\cos\left(dx+c\right)^6+280\,(4\,A+B)a^4\cos\left(dx+c\right)^5+48\,(48\,A+28\,B+7\,C)a^4\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/1680*(105*(44*A + 49*B + 56*C)*a^4*d*x + (240*A*a^4*cos(d*x + c)^6 + 280*
(4*A + B)*a^4*cos(d*x + c)^5 + 48*(48*A + 28*B + 7*C)*a^4*cos(d*x + c)^4 +
70*(44*A + 41*B + 24*C)*a^4*cos(d*x + c)^3 + 16*(227*A + 252*B + 238*C)*a^4
*cos(d*x + c)^2 + 105*(44*A + 49*B + 56*C)*a^4*cos(d*x + c) + 16*(454*A + 5
04*B + 581*C)*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32531, size = 541, normalized size = 1.95

$$\frac{105\left(44\,Aa^4+49\,Ba^4+56\,Ca^4\right)(dx+c)+\frac{2\left(4620\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5145\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5880\,Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+30800\,Aa^4\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(44*A*a^4 + 49*B*a^4 + 56*C*a^4)*(d*x + c) + 2*(4620*A*a^4*tan(
1/2*d*x + 1/2*c)^13 + 5145*B*a^4*tan(1/2*d*x + 1/2*c)^13 + 5880*C*a^4*tan(1
/2*d*x + 1/2*c)^13 + 30800*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 34300*B*a^4*tan(
1/2*d*x + 1/2*c)^11 + 39200*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 87164*A*a^4*tan
(1/2*d*x + 1/2*c)^9 + 97069*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 110936*C*a^4*tan
(1/2*d*x + 1/2*c)^9 + 135168*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 150528*B*a^4*ta
n(1/2*d*x + 1/2*c)^7 + 172032*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 126084*A*a^4*t
an(1/2*d*x + 1/2*c)^5 + 134099*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 159656*C*a^4*
tan(1/2*d*x + 1/2*c)^5 + 58800*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 73220*B*a^4*t
```

```
an(1/2*d*x + 1/2*c)^3 + 86240*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 22260*A*a^4*ta
n(1/2*d*x + 1/2*c) + 21735*B*a^4*tan(1/2*d*x + 1/2*c) + 21000*C*a^4*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.448 $\int \cos^8(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=303

$$-\frac{a^4(208A+227B+252C)\sin^3(c+dx)}{105d}+\frac{a^4(208A+227B+252C)\sin(c+dx)}{35d}+\frac{a^4(2007A+2208B+2408C)\sin(c+}{2240d}$$

```
[Out] (a^4*(323*A + 352*B + 392*C)*x)/128 + (a^4*(208*A + 227*B + 252*C)*Sin[c +
d*x])/(35*d) + (a^4*(323*A + 352*B + 392*C)*Cos[c + d*x]*Sin[c + d*x])/(128
*d) + (a^4*(2007*A + 2208*B + 2408*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2240*d)
 + (a*(A + 2*B)*Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(14*d)
+ (A*Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(8*d) + ((61*A + 8
0*B + 56*C)*Cos[c + d*x]^5*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(336*d)
 + (7*(7*A + 8*(B + C))*Cos[c + d*x]^4*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x
])/(120*d) - (a^4*(208*A + 227*B + 252*C)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.794197, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4086, 4017, 3996, 3787, 2633, 2635, 8}

$$-\frac{a^4(208A+227B+252C)\sin^3(c+dx)}{105d}+\frac{a^4(208A+227B+252C)\sin(c+dx)}{35d}+\frac{a^4(2007A+2208B+2408C)\sin(c+}{2240d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a^4*(323*A + 352*B + 392*C)*x)/128 + (a^4*(208*A + 227*B + 252*C)*Sin[c +
d*x])/(35*d) + (a^4*(323*A + 352*B + 392*C)*Cos[c + d*x]*Sin[c + d*x])/(128
*d) + (a^4*(2007*A + 2208*B + 2408*C)*Cos[c + d*x]^3*Sin[c + d*x])/(2240*d)
 + (a*(A + 2*B)*Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(14*d)
+ (A*Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*Sin[c + d*x])/(8*d) + ((61*A + 8
0*B + 56*C)*Cos[c + d*x]^5*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(336*d)
 + (7*(7*A + 8*(B + C))*Cos[c + d*x]^4*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x
])/(120*d) - (a^4*(208*A + 227*B + 252*C)*Sin[c + d*x]^3)/(105*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^8(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^7(c+dx)(a+a\sec(c+dx))^4\sin(}{8d}$$

$$= \frac{a(A+2B)\cos^6(c+dx)(a+a\sec(c+d}{14d}$$

$$= \frac{a(A+2B)\cos^6(c+dx)(a+a\sec(c+d}{14d}$$

$$= \frac{a(A+2B)\cos^6(c+dx)(a+a\sec(c+d}{14d}$$

$$= \frac{a^4(2007A+2208B+2408C)\cos^3(c+}{2240d}$$

$$= \frac{a^4(2007A+2208B+2408C)\cos^3(c+}{2240d}$$

$$= \frac{a^4(323A+352B+392C)\cos(c+dx)\,s}{128d}$$

$$= \frac{1}{128}a^4(323A+352B+392C)x+\frac{a^4(2\cdots}{}$$

Mathematica [A] time = 1.9343, size = 237, normalized size = 0.78

$$\frac{a^4(1680(300A+323B+352C)\sin(c+dx)+1680(120A+124B+127C)\sin(2(c+dx))+91840A\sin(3(c+dx))+3948}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a^4*(106680*A*c + 295680*B*c + 271320*A*d*x + 295680*B*d*x + 329280*C*d*x
+ 1680*(300*A + 323*B + 352*C)*Sin[c + d*x] + 1680*(120*A + 124*B + 127*C)*
Sin[2*(c + d*x)] + 91840*A*Sin[3*(c + d*x)] + 87920*B*Sin[3*(c + d*x)] + 80
640*C*Sin[3*(c + d*x)] + 39480*A*Sin[4*(c + d*x)] + 33600*B*Sin[4*(c + d*x)
] + 25200*C*Sin[4*(c + d*x)] + 14784*A*Sin[5*(c + d*x)] + 10416*B*Sin[5*(c
+ d*x)] + 5376*C*Sin[5*(c + d*x)] + 4480*A*Sin[6*(c + d*x)] + 2240*B*Sin[6*
(c + d*x)] + 560*C*Sin[6*(c + d*x)] + 960*A*Sin[7*(c + d*x)] + 240*B*Sin[7*
(c + d*x)] + 105*A*Sin[8*(c + d*x)]))/(107520*d)
```

Maple [B] time = 0.221, size = 577, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos
(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c)+1/7*B*a^4*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+a^4*C*(1/6*(cos(d*x+c)^5+5/4*cos(d
*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4/7*A*a^4*(16/5+cos(d*
x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+4*B*a^4*(1/6*(cos(d*x+
c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4/5*a^4*
C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/6*(cos(d*x+c)^5
+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+6/5*B*a^4*(8
/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4*C*(1/4*(cos(d*x+c)^3+3/2
*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(
d*x+c)^2)*sin(d*x+c)+4*B*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/8*d*x+3/8*c)+4/3*a^4*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/4*(cos(d*x+c)
^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a^4*(2+cos(d*x+c)^2)*sin
(d*x+c)+a^4*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [B] time = 0.99332, size = 782, normalized size = 2.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/107520*(12288*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3
- 35*sin(d*x + c))*A*a^4 - 28672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15
*sin(d*x + c))*A*a^4 + 35*(128*sin(2*d*x + 2*c)^3 - 840*d*x - 840*c - 3*sin
```

```
(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*A*a^4 + 3360*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
2*c))*A*a^4 - 3360*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*
A*a^4 + 3072*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*B*a^4 - 43008*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*B*a^4 + 2240*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x
 + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 + 35840*(sin(d*x + c)^3 - 3*sin(d*x +
c))*B*a^4 - 13440*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B
*a^4 - 28672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*C*a^4
 + 560*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(
2*d*x + 2*c))*C*a^4 + 143360*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 - 2016
0*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^4 - 26880*(2*
d*x + 2*c + sin(2*d*x + 2*c))*C*a^4)/d
```

Fricas [A] time = 0.547145, size = 536, normalized size = 1.77

$$105\,(323\,A+352\,B+392\,C)a^4dx+\left(1680\,Aa^4\cos\left(dx+c\right)^7+1920\,(4\,A+B)a^4\cos\left(dx+c\right)^6+280\,(55\,A+32\,B+8\,\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/13440*(105*(323*A + 352*B + 392*C)*a^4*d*x + (1680*A*a^4*cos(d*x + c)^7 +
 1920*(4*A + B)*a^4*cos(d*x + c)^6 + 280*(55*A + 32*B + 8*C)*a^4*cos(d*x +
c)^5 + 1536*(13*A + 12*B + 7*C)*a^4*cos(d*x + c)^4 + 70*(323*A + 352*B + 32
8*C)*a^4*cos(d*x + c)^3 + 128*(208*A + 227*B + 252*C)*a^4*cos(d*x + c)^2 +
105*(323*A + 352*B + 392*C)*a^4*cos(d*x + c) + 256*(208*A + 227*B + 252*C)*
a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31838, size = 610, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/13440*(105*(323*A*a^4 + 352*B*a^4 + 392*C*a^4)*(d*x + c) + 2*(33915*A*a^4
*tan(1/2*d*x + 1/2*c)^15 + 36960*B*a^4*tan(1/2*d*x + 1/2*c)^15 + 41160*C*a^
```

```
4*tan(1/2*d*x + 1/2*c)^15 + 260015*A*a^4*tan(1/2*d*x + 1/2*c)^13 + 283360*B
*a^4*tan(1/2*d*x + 1/2*c)^13 + 315560*C*a^4*tan(1/2*d*x + 1/2*c)^13 + 86596
3*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 943712*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 10
50952*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 1632119*A*a^4*tan(1/2*d*x + 1/2*c)^9
+ 1778656*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 1980776*C*a^4*tan(1/2*d*x + 1/2*c)
^9 + 1872009*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 2090016*B*a^4*tan(1/2*d*x + 1/2
*c)^7 + 2277016*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 1442133*A*a^4*tan(1/2*d*x +
1/2*c)^5 + 1479072*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 1658552*C*a^4*tan(1/2*d*x
 + 1/2*c)^5 + 528465*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 648480*B*a^4*tan(1/2*d*
x + 1/2*c)^3 + 759640*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 181125*A*a^4*tan(1/2*d
*x + 1/2*c) + 178080*B*a^4*tan(1/2*d*x + 1/2*c) + 173880*C*a^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

3.449 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=183

$$-\frac{(3A-4B+4C)\tan^3(c+dx)}{3ad}-\frac{(3A-4B+4C)\tan(c+dx)}{ad}+\frac{3(4A-4B+5C)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{(A-B+C}{d(a}$$

```
[Out] (3*(4*A - 4*B + 5*C)*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((3*A - 4*B + 4*C)*Ta
n[c + d*x])/(a*d) + (3*(4*A - 4*B + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d)
 + ((4*A - 4*B + 5*C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d) - ((A - B + C)*S
ec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*A - 4*B + 4*C)*T
an[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.21822, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4084, 3787, 3767, 3768, 3770}

$$-\frac{(3A-4B+4C)\tan^3(c+dx)}{3ad}-\frac{(3A-4B+4C)\tan(c+dx)}{ad}+\frac{3(4A-4B+5C)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{(A-B+C}{d(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```


```
[Out] (3*(4*A - 4*B + 5*C)*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((3*A - 4*B + 4*C)*Ta
n[c + d*x])/(a*d) + (3*(4*A - 4*B + 5*C)*Sec[c + d*x]*Tan[c + d*x])/(8*a*d)
 + ((4*A - 4*B + 5*C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d) - ((A - B + C)*S
ec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*A - 4*B + 4*C)*T
an[c + d*x]^3)/(3*a*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^4(c+dx)(-a(3$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-4B+4C)\int \text{s}}{a}$$

$$= \frac{(4A-4B+5C)\sec^3(c+dx)\tan(c+dx)}{4ad} - \frac{(A-B+C)\sec^4(}{d(a+a\,\text{s}}$$

$$= -\frac{(3A-4B+4C)\tan(c+dx)}{ad} + \frac{3(4A-4B+5C)\sec(c+dx)}{8ad}$$

$$= \frac{3(4A-4B+5C)\tanh^{-1}(\sin(c+dx))}{8ad} - \frac{(3A-4B+4C)\tan(}{ad}$$

Mathematica [B] time = 6.39341, size = 1099, normalized size = 6.01

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x]),x]
```

```
[Out] (-3*(4*A - 4*B + 5*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Log[Cos[c/2 + (d*x)
/2] - Sin[c/2 + (d*x)/2]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(2*d*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (3*(
4*A - 4*B + 5*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Log[Cos[c/2 + (d*x)/2] +
 Sin[c/2 + (d*x)/2]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(2*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2
+ (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*(-60*A*Sin[(d*x)/2] + 108*B*Sin[(d*x)/2] - 75*C*Sin[(d*x)/2] - 60*A*
Sin[(3*d*x)/2] + 124*B*Sin[(3*d*x)/2] - 91*C*Sin[(3*d*x)/2] + 204*A*Sin[c -
 (d*x)/2] - 252*B*Sin[c - (d*x)/2] + 219*C*Sin[c - (d*x)/2] - 60*A*Sin[c +
(d*x)/2] + 12*B*Sin[c + (d*x)/2] + 21*C*Sin[c + (d*x)/2] + 84*A*Sin[2*c + (
d*x)/2] - 132*B*Sin[2*c + (d*x)/2] + 165*C*Sin[2*c + (d*x)/2] + 36*A*Sin[c
+ (3*d*x)/2] + 28*B*Sin[c + (3*d*x)/2] + 5*C*Sin[c + (3*d*x)/2] + 36*A*Sin[
2*c + (3*d*x)/2] - 36*B*Sin[2*c + (3*d*x)/2] + 69*C*Sin[2*c + (3*d*x)/2] +
132*A*Sin[3*c + (3*d*x)/2] - 132*B*Sin[3*c + (3*d*x)/2] + 165*C*Sin[3*c + (
3*d*x)/2] - 156*A*Sin[c + (5*d*x)/2] + 220*B*Sin[c + (5*d*x)/2] - 211*C*Sin
[c + (5*d*x)/2] - 60*A*Sin[2*c + (5*d*x)/2] + 124*B*Sin[2*c + (5*d*x)/2] -
115*C*Sin[2*c + (5*d*x)/2] - 60*A*Sin[3*c + (5*d*x)/2] + 60*B*Sin[3*c + (5*
d*x)/2] - 51*C*Sin[3*c + (5*d*x)/2] + 36*A*Sin[4*c + (5*d*x)/2] - 36*B*Sin[
4*c + (5*d*x)/2] + 45*C*Sin[4*c + (5*d*x)/2] - 12*A*Sin[2*c + (7*d*x)/2] +
28*B*Sin[2*c + (7*d*x)/2] - 19*C*Sin[2*c + (7*d*x)/2] + 12*A*Sin[3*c + (7*d
*x)/2] + 4*B*Sin[3*c + (7*d*x)/2] + 5*C*Sin[3*c + (7*d*x)/2] + 12*A*Sin[4*c
```

```
 + (7*d*x)/2] - 12*B*Sin[4*c + (7*d*x)/2] + 21*C*Sin[4*c + (7*d*x)/2] + 36*
A*Sin[5*c + (7*d*x)/2] - 36*B*Sin[5*c + (7*d*x)/2] + 45*C*Sin[5*c + (7*d*x)
/2] - 48*A*Sin[3*c + (9*d*x)/2] + 64*B*Sin[3*c + (9*d*x)/2] - 64*C*Sin[3*c
+ (9*d*x)/2] - 24*A*Sin[4*c + (9*d*x)/2] + 40*B*Sin[4*c + (9*d*x)/2] - 40*C
*Sin[4*c + (9*d*x)/2] - 24*A*Sin[5*c + (9*d*x)/2] + 24*B*Sin[5*c + (9*d*x)/
2] - 24*C*Sin[5*c + (9*d*x)/2]))/(192*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*(a + a*Sec[c + d*x]))
```

Maple [B] time = 0.074, size = 576, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d/(tan(1/2*d*x+1/2*c)-1)^2*B+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*A+3/2/a/
d/(tan(1/2*d*x+1/2*c)+1)*A+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*A-1/4/a/d*C/(tan(
1/2*d*x+1/2*c)+1)^4-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*A+1/4/a/d*C/(tan(1/2*d
*x+1/2*c)-1)^4+5/6/a/d/(tan(1/2*d*x+1/2*c)-1)^3*C-1/3/a/d/(tan(1/2*d*x+1/2*
c)-1)^3*B+5/6/a/d/(tan(1/2*d*x+1/2*c)+1)^3*C-1/3/a/d/(tan(1/2*d*x+1/2*c)+1)
^3*B+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*A-
1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)+15/8/a/d*ln(tan(1/2*d
*x+1/2*c)+1)*C+25/8/a/d/(tan(1/2*d*x+1/2*c)+1)*C-15/8/a/d*ln(tan(1/2*d*x+1/
2*c)-1)*C+25/8/a/d/(tan(1/2*d*x+1/2*c)-1)*C+1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/
d/(tan(1/2*d*x+1/2*c)+1)^2*B-15/8/a/d/(tan(1/2*d*x+1/2*c)+1)^2*C-3/2/a/d*ln
(tan(1/2*d*x+1/2*c)+1)*B-5/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B+15/8/a/d/(tan(1/2
*d*x+1/2*c)-1)^2*C+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B-5/2/a/d/(tan(1/2*d*x+
1/2*c)-1)*B
```

Maxima [B] time = 0.979095, size = 825, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -1/24*(C*(2*(21*sin(d*x + c)/(cos(d*x + c) + 1) - 109*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 75*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a + 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 24*sin(d
*x + c)/(a*(cos(d*x + c) + 1))) - 4*B*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1)
 - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5)/(a - 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a + 6*sin(d*x + c)/(a*(cos(d*x + c) + 1))) + 12*A*(2*(sin(d*x + c)/(cos
(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) +
```

```
1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.542855, size = 545, normalized size = 2.98

$$\frac{9\left((4A-4B+5C)\cos(dx+c)^5+(4A-4B+5C)\cos(dx+c)^4\right)\log(\sin(dx+c)+1)-9\left((4A-4B+5C)\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] 1/48*(9*((4*A - 4*B + 5*C)*cos(d*x + c)^5 + (4*A - 4*B + 5*C)*cos(d*x + c)^
4)*log(sin(d*x + c) + 1) - 9*((4*A - 4*B + 5*C)*cos(d*x + c)^5 + (4*A - 4*B
 + 5*C)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(16*(3*A - 4*B + 4*C)*co
s(d*x + c)^4 + (12*A - 28*B + 19*C)*cos(d*x + c)^3 - (12*A - 4*B + 13*C)*co
s(d*x + c)^2 - 2*(4*B - C)*cos(d*x + c) - 6*C)*sin(d*x + c))/(a*d*cos(d*x +
 c)^5 + a*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{C\sec^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x
)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**5/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**6/(sec(c + d*x) + 1)
, x))/a
```

Giac [A] time = 1.3149, size = 385, normalized size = 2.1

$$\frac{\frac{9(4A-4B+5C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9(4A-4B+5C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{24\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(36\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/24*(9*(4*A - 4*B + 5*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(4*A - 4
*B + 5*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 24*(A*tan(1/2*d*x + 1/2*c)
 - B*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*(36*A*tan(1/2*d*x
 + 1/2*c)^7 - 60*B*tan(1/2*d*x + 1/2*c)^7 + 75*C*tan(1/2*d*x + 1/2*c)^7 - 8
4*A*tan(1/2*d*x + 1/2*c)^5 + 124*B*tan(1/2*d*x + 1/2*c)^5 - 115*C*tan(1/2*d
*x + 1/2*c)^5 + 60*A*tan(1/2*d*x + 1/2*c)^3 - 100*B*tan(1/2*d*x + 1/2*c)^3
```

```
+ 109*C*tan(1/2*d*x + 1/2*c)^3 - 12*A*tan(1/2*d*x + 1/2*c) + 36*B*tan(1/2*d
*x + 1/2*c) - 21*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a)
)/d
```

3.450 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{(3A-3B+4C)\tan^3(c+dx)}{3ad}+\frac{(3A-3B+4C)\tan(c+dx)}{ad}-\frac{(2A-3B+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A-B+C)\tan(}{d(a\sec(c}$$

```
[Out] -((2*A - 3*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((3*A - 3*B + 4*C)*Tan
[c + d*x])/(a*d) - ((2*A - 3*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) -
((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((3*A
- 3*B + 4*C)*Tan[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.198302, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4084, 3787, 3768, 3770, 3767}

$$\frac{(3A-3B+4C)\tan^3(c+dx)}{3ad}+\frac{(3A-3B+4C)\tan(c+dx)}{ad}-\frac{(2A-3B+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A-B+C)\tan(}{d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] -((2*A - 3*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((3*A - 3*B + 4*C)*Tan
[c + d*x])/(a*d) - ((2*A - 3*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) -
((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((3*A
- 3*B + 4*C)*Tan[c + d*x]^3)/(3*a*d)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \sec^3(c+dx)(-$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{(2A-3B+3C)}{}$$

$$= -\frac{(2A-3B+3C)\sec(c+dx)\tan(c+dx)}{2ad}-\frac{(A-B+C)\,\mathrm{s}}{d(a+}$$

$$= -\frac{(2A-3B+3C)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-3B+4C)}{ad}$$

Mathematica [B] time = 6.31357, size = 898, normalized size = 6.07

$$\frac{2(2A-3B+3C)\cos(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(\sec(c+dx)a+a)}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x]),x]
```

```
[Out] (2*(2*A - 3*B + 3*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Log[Cos[c/2 + (d*x)/
2] - Sin[c/2 + (d*x)/2]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*(2*A
 - 3*B + 3*C)*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Log[Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d*
x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*(-6*A*Sin[(d*x)/2] + 6*B*Sin[(d*x)/2] + 6*C*Sin[(d*x)/2] + 30*A*Sin[(3*d*
x)/2] - 27*B*Sin[(3*d*x)/2] + 39*C*Sin[(3*d*x)/2] - 12*A*Sin[c - (d*x)/2] +
 12*B*Sin[c - (d*x)/2] - 24*C*Sin[c - (d*x)/2] - 6*A*Sin[c + (d*x)/2] + 6*B
*Sin[c + (d*x)/2] - 6*C*Sin[c + (d*x)/2] - 24*A*Sin[2*c + (d*x)/2] + 24*B*S
in[2*c + (d*x)/2] - 24*C*Sin[2*c + (d*x)/2] + 12*A*Sin[c + (3*d*x)/2] - 9*B
*Sin[c + (3*d*x)/2] + 21*C*Sin[c + (3*d*x)/2] + 12*A*Sin[2*c + (3*d*x)/2] -
 9*B*Sin[2*c + (3*d*x)/2] + 9*C*Sin[2*c + (3*d*x)/2] - 6*A*Sin[3*c + (3*d*x
)/2] + 9*B*Sin[3*c + (3*d*x)/2] - 9*C*Sin[3*c + (3*d*x)/2] + 6*A*Sin[c + (5
*d*x)/2] - 3*B*Sin[c + (5*d*x)/2] + 7*C*Sin[c + (5*d*x)/2] + 3*B*Sin[2*c +
(5*d*x)/2] + C*Sin[2*c + (5*d*x)/2] + 3*B*Sin[3*c + (5*d*x)/2] - 3*C*Sin[3*
c + (5*d*x)/2] - 6*A*Sin[4*c + (5*d*x)/2] + 9*B*Sin[4*c + (5*d*x)/2] - 9*C*
Sin[4*c + (5*d*x)/2] + 12*A*Sin[2*c + (7*d*x)/2] - 12*B*Sin[2*c + (7*d*x)/2
] + 16*C*Sin[2*c + (7*d*x)/2] + 6*A*Sin[3*c + (7*d*x)/2] - 6*B*Sin[3*c + (7
*d*x)/2] + 10*C*Sin[3*c + (7*d*x)/2] + 6*A*Sin[4*c + (7*d*x)/2] - 6*B*Sin[4
*c + (7*d*x)/2] + 6*C*Sin[4*c + (7*d*x)/2]))/(24*d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x]))
```

Maple [B] time = 0.073, size = 442, normalized size = 3.

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{3ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{B}{2ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)-1/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3*C-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*B
+1/a/d/(tan(1/2*d*x+1/2*c)+1)^2*C-3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C+3/2/a/
d*ln(tan(1/2*d*x+1/2*c)+1)*B-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A-5/2/a/d/(tan(
1/2*d*x+1/2*c)+1)*C+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B-1/a/d/(tan(1/2*d*x+1/2
*c)+1)*A-1/3/a/d/(tan(1/2*d*x+1/2*c)-1)^3*C+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^
2*B-1/a/d/(tan(1/2*d*x+1/2*c)-1)^2*C+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C-3/2
/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*A-5/2/a/d/(t
an(1/2*d*x+1/2*c)-1)*C+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*B-1/a/d/(tan(1/2*d*x+
1/2*c)-1)*A
```

Maxima [B] time = 0.964716, size = 655, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] 1/6*(C*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - 6*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x +
 c) + 1))))/d
```

Fricas [A] time = 0.530972, size = 489, normalized size = 3.3

$$-\frac{3\left((2A-3B+3C)\cos(dx+c)^4+(2A-3B+3C)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((2A-3B+3C)\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -1/12*(3*((2*A - 3*B + 3*C)*cos(d*x + c)^4 + (2*A - 3*B + 3*C)*cos(d*x + c)
^3)*log(sin(d*x + c) + 1) - 3*((2*A - 3*B + 3*C)*cos(d*x + c)^4 + (2*A - 3*
B + 3*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(4*(3*A - 3*B + 4*C)*co
s(d*x + c)^3 + (6*A - 3*B + 7*C)*cos(d*x + c)^2 + (3*B - C)*cos(d*x + c) +
2*C)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{B\sec^4(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{C\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x
)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**4/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**5/(sec(c + d*x) + 1)
, x))/a
```

Giac [A] time = 1.25946, size = 328, normalized size = 2.22

$$-\frac{\frac{3(2A-3B+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3(2A-3B+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] -1/6*(3*(2*A - 3*B + 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*(2*A - 3
*B + 3*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c)
- B*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a + 2*(6*A*tan(1/2*d*x +
 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 + 15*C*tan(1/2*d*x + 1/2*c)^5 - 12*A
*tan(1/2*d*x + 1/2*c)^3 + 12*B*tan(1/2*d*x + 1/2*c)^3 - 16*C*tan(1/2*d*x +
1/2*c)^3 + 6*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c) + 9*C*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.451 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-2B+2C)\tan(c+dx)}{ad}+\frac{(2A-2B+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(2A-2B$$

```
[Out] ((2*A - 2*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((A - 2*B + 2*C)*Tan[c
+ d*x])/(a*d) + ((2*A - 2*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A
 - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.191991, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 3787, 3767, 8, 3768, 3770}

$$-\frac{(A-2B+2C)\tan(c+dx)}{ad}+\frac{(2A-2B+3C)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(2A-2B$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] ((2*A - 2*B + 3*C)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((A - 2*B + 2*C)*Tan[c
+ d*x])/(a*d) + ((2*A - 2*B + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A
 - B + C)*Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^2(c+dx)(-$$
$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-2B+2C)\int}{a}$$
$$= \frac{(2A-2B+3C)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B+C)\sec}{d(a+a}$$
$$= \frac{(2A-2B+3C)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(A-2B+2C)\tan}{ad}$$

Mathematica [B] time = 4.28866, size = 392, normalized size = 3.29

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-4\sec\left(\frac{c}{2}\right)(A-B+C)\sin\left(\frac{dx}{2}\right)-2(2A-2B+3C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-2*
(2*A - 2*B + 3*C)*Cos[(c + d*x)/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2*(2*A - 2*B + 3*C)*Cos[(c + d*x)/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 4*(A - B + C)*Sec[c/2]*Sin[(d*x)/2] + (C*Cos[(c + d*x)/2])/(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^2 + (4*(B - C)*Cos[(c + d*x)/2]*Sin[(d*x)/2])
/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (C*Cos[(c
+ d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(B - C)*Cos[(c + d*
x)/2]*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]))))/(a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(1 + Sec[c
+ d*x]))
```

Maple [B] time = 0.064, size = 311, normalized size = 2.6

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3\,C}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+1/2*c)-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*C+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)*C-1/a/d/(tan(1/2*d*x+1/2*c)+1)*B+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*A+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*C+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/a/d/(tan(1/2*d*x+1/2*c)-1)*B-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*A

Maxima [B] time = 0.956857, size = 481, normalized size = 4.04

$$-\frac{C\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.}{a}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -1/2*(C*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 2
*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.5103, size = 428, normalized size = 3.6

$$\frac{\left((2A-2B+3C)\cos(dx+c)^3+(2A-2B+3C)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((2A-2B+3C)\cos(dx+c)^3\right.}{4\left(ad\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] 1/4*(((2*A - 2*B + 3*C)*cos(d*x + c)^3 + (2*A - 2*B + 3*C)*cos(d*x + c)^2)*
log(sin(d*x + c) + 1) - ((2*A - 2*B + 3*C)*cos(d*x + c)^3 + (2*A - 2*B + 3*
C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(2*(A - 2*B + 2*C)*cos(d*x +
c)^2 - (2*B - C)*cos(d*x + c) - C)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{B\sec^3(c+dx)}{\sec(c+dx)+1}\,dx+\int \frac{C\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x
)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**3/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**4/(sec(c + d*x) + 1)
, x))/a
```

Giac [A] time = 1.24403, size = 234, normalized size = 1.97

$$\frac{\frac{(2A-2B+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(2A-2B+3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(2B\cdots\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/2*((2*A - 2*B + 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (2*A - 2*B +
3*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*t
an(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*(2*B*tan(1/2*d*x + 1/2*
c)^3 - 3*C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*
x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.452 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{(A-B+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{C\tan(c+dx)}{ad}$$

```
[Out] ((B - C)*ArcTanh[Sin[c + d*x]])/(a*d) + (C*Tan[c + d*x])/(a*d) + ((A - B +
C)*Tan[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Rubi [A] time = 0.168264, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4082, 3998, 3770, 3794}

$$\frac{(A-B+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{C\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x]),x]
```

```
[Out] ((B - C)*ArcTanh[Sin[c + d*x]])/(a*d) + (C*Tan[c + d*x])/(a*d) + ((A - B +
C)*Tan[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \frac{C\tan(c+dx)}{ad} + \frac{\int \frac{\sec(c+dx)(aA+a(B-C)\sec(c+dx))}{a+a\sec(c+dx)}dx}{a}$$
$$= \frac{C\tan(c+dx)}{ad} + \frac{(B-C)\int\sec(c+dx)\,dx}{a} + (A-B+C)\int$$
$$= \frac{(B-C)\tanh^{-1}(\sin(c+dx))}{ad} + \frac{C\tan(c+dx)}{ad} + \frac{(A-B+C}{d(a+a\,s}$$

Mathematica [B] time = 1.39599, size = 255, normalized size = 4.05

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)(A-B+C)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\cdots\right.\right.}{ad(\sec(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((
A - B + C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-((B - C)*(Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]))
+ (C*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.059, size = 180, normalized size = 2.9

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{B}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)-1/a/d/(tan(1/2*d*x+1/2*c)+1)*C+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B-1/a/d
*ln(tan(1/2*d*x+1/2*c)+1)*C-1/a/d/(tan(1/2*d*x+1/2*c)-1)*C-1/a/d*ln(tan(1/2
*d*x+1/2*c)-1)*B+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.942921, size = 294, normalized size = 4.67

$$-\frac{C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+\cdots}\right.}{a}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -(C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - A*sin(d*x + c)/(a*(cos(d*x
+ c) + 1)))/d
```

Fricas [B] time = 0.512863, size = 324, normalized size = 5.14

$$\frac{\left((B-C)\cos\left(dx+c\right)^2+(B-C)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((B-C)\cos\left(dx+c\right)^2+(B-C)\cos\left(dx+c\right)\right)\log\left(-\right.}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*(((B - C)*cos(d*x + c)^2 + (B - C)*cos(d*x + c))*log(sin(d*x + c) + 1)
- ((B - C)*cos(d*x + c)^2 + (B - C)*cos(d*x + c))*log(-sin(d*x + c) + 1) +
2*((A - B + 2*C)*cos(d*x + c) + C)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*
cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^2\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int\frac{C\sec^3\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**
2/(sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**3/(sec(c + d*x) + 1), x
))/a
```

Giac [A] time = 1.28263, size = 161, normalized size = 2.56

$$\frac{\frac{(B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{(B-C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] ((B - C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (B - C)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c) + C*ta
n(1/2*d*x + 1/2*c))/a - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*a))/d
```

3.453 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{(A-B+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B + C)*Tan[c + d*x])/(a*d
*(1 + Sec[c + d*x]))
```

Rubi [A] time = 0.115726, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4050, 3770, 3919, 3794}

$$-\frac{(A-B+C)\tan(c+dx)}{ad(\sec(c+dx)+1)}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B + C)*Tan[c + d*x])/(a*d
*(1 + Sec[c + d*x]))
```

Rule 4050

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e
 + f*x], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Csc[e + f*x])/(a + b*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{aA+(aB-aC)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{a} + \frac{C\int \sec(c+dx)\,dx}{a}$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{ad} + (-A+B-C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.493384, size = 163, normalized size = 3.13

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(Adx-C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{ad(\cos(c+dx)+1)(A\cos(2(c+dx))+A+2B\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*(Cos[(c + d*x)/
2]*(A*d*x - C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + C*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]]) - (A - B + C)*Sec[c/2]*Sin[(d*x)/2]))/(a*d*(1 + C
os[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.063, size = 115, normalized size = 2.2

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{A\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad}+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+2/a/d*A*arctan(tan(1/2*d*x+1/2*c))+1/a/d*B*tan(
1/2*d*x+1/2*c)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*C-1/a/d*C*tan(1/2*d*x+1/2*c)-
1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 1.41614, size = 197, normalized size = 3.79

$$\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+C\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] (A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + C*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*si
```

```
n(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.507555, size = 247, normalized size = 4.75

$$\frac{2\,Adx\cos\left(dx+c\right)+2\,Adx+\left(C\cos\left(dx+c\right)+C\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(C\cos\left(dx+c\right)+C\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*A*d*x*cos(d*x + c) + 2*A*d*x + (C*cos(d*x + c) + C)*log(sin(d*x + c)
 + 1) - (C*cos(d*x + c) + C)*log(-sin(d*x + c) + 1) - 2*(A - B + C)*sin(d*x
 + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^{2}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)
+ 1), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.20756, size = 124, normalized size = 2.38

$$\frac{\frac{(dx+c)A}{a}+\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] ((d*x + c)*A/a + C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c) +
C*tan(1/2*d*x + 1/2*c))/a)/d
```

3.454 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{(2A-B+C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(A-B)}{a}$$

```
[Out] -(((A - B)*x)/a) + ((2*A - B + C)*Sin[c + d*x])/(a*d) - ((A - B + C)*Sin[c
+ d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.130308, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4084, 3787, 2637, 8}

$$\frac{(2A-B+C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(A-B)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x]),x]
```

```
[Out] -(((A - B)*x)/a) + ((2*A - B + C)*Sin[c + d*x])/(a*d) - ((A - B + C)*Sin[c
+ d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}dx = -\frac{(A-B+C)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int\cos(c+dx)(a(2A-B+C)-}{a^2}$$
$$= -\frac{(A-B+C)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(A-B)\int 1\,dx}{a}+\frac{(2A-B+C)}{}$$
$$= -\frac{(A-B)x}{a}+\frac{(2A-B+C)\sin(c+dx)}{ad}-\frac{(A-B+C)\sin(}{d(a+a\sec(c+}$$

Mathematica [A] time = 0.401857, size = 77, normalized size = 1.24

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)(A-B+C)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)(dx(B-A)+A\sin(c+dx))\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B + C)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(
(-A + B)*d*x + A*Sin[c + d*x])))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.093, size = 125, normalized size = 2.

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{A\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\frac{A\arctan(\tan(1/2\,dx}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)+2/a/d*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/a/d*A*arctan(ta
n(1/2*d*x+1/2*c))+2/a/d*arctan(tan(1/2*d*x+1/2*c))*B
```

Maxima [B] time = 1.42891, size = 223, normalized size = 3.6

$$-\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-\frac{C\sin(dx}{a(\cos(dx+}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -(A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))) - C*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 0.479795, size = 154, normalized size = 2.48

$$-\frac{(A-B)dx\cos(dx+c)+(A-B)dx-(A\cos(dx+c)+2A-B+C)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -((A - B)*d*x*cos(d*x + c) + (A - B)*d*x - (A*cos(d*x + c) + 2*A - B + C)*s
in(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{C\cos(c+dx)\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*cos(c + d*x)*sec(c + d*x)**
2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.18937, size = 122, normalized size = 1.97

$$-\frac{\frac{(dx+c)(A-B)}{a}-\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c) +
C*tan(1/2*d*x + 1/2*c))/a - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 + 1)*a))/d
```

3.455 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(2A-2B+C)\sin(c+dx)}{ad}+\frac{(3A-2B+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B+C)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A$$

```
[Out] ((3*A - 2*B + 2*C)*x)/(2*a) - ((2*A - 2*B + C)*Sin[c + d*x])/(a*d) + ((3*A
- 2*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d*x]
*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.181688, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4084, 3787, 2635, 8, 2637}

$$-\frac{(2A-2B+C)\sin(c+dx)}{ad}+\frac{(3A-2B+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B+C)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] ((3*A - 2*B + 2*C)*x)/(2*a) - ((2*A - 2*B + C)*Sin[c + d*x])/(a*d) + ((3*A
- 2*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d*x]
*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \cos^2(c+dx)(a(3A$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(2A-2B+C)\int \cos}{a}$$

$$= -\frac{(2A-2B+C)\sin(c+dx)}{ad}+\frac{(3A-2B+2C)\cos(c+dx)\sin}{2ad}$$

$$= \frac{(3A-2B+2C)x}{2a}-\frac{(2A-2B+C)\sin(c+dx)}{ad}+\frac{(3A-2B+$$

Mathematica [A] time = 0.504956, size = 213, normalized size = 1.97

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(4dx(3A-2B+2C)\cos\left(c+\frac{dx}{2}\right)+4dx(3A-2B+2C)\cos\left(\frac{dx}{2}\right)-4A\sin\left(c+\frac{dx}{2}\right)-3A\sin\left(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(3*A - 2*B + 2*C)*d*x*Cos[(d*x)/2] + 4*(3*A -
 2*B + 2*C)*d*x*Cos[c + (d*x)/2] - 20*A*Sin[(d*x)/2] + 20*B*Sin[(d*x)/2] -
16*C*Sin[(d*x)/2] - 4*A*Sin[c + (d*x)/2] + 4*B*Sin[c + (d*x)/2] - 3*A*Sin[c
 + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2] - 3*A*Sin[2*c + (3*d*x)/2] + 4*B*Sin
[2*c + (3*d*x)/2] + A*Sin[2*c + (5*d*x)/2] + A*Sin[3*c + (5*d*x)/2]))/(8*a*
d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.099, size = 248, normalized size = 2.3

$$-\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\frac{(\tan(1/2\,dx+c/2))^3 A}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+2\frac{(\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+
1/2*c)-3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+2/a/d/(1+tan
(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
2*A*tan(1/2*d*x+1/2*c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c
)+3/a/d*A*arctan(tan(1/2*d*x+1/2*c))-2/a/d*arctan(tan(1/2*d*x+1/2*c))*B+2/a
/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.43265, size = 369, normalized size = 3.42

$$-\frac{A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -(A*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - C*(2*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.486207, size = 227, normalized size = 2.1

$$\frac{(3A-2B+2C)dx\cos(dx+c)+(3A-2B+2C)dx+\left(A\cos(dx+c)^2-(A-2B)\cos(dx+c)-4A+4B-2C\right)s}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] 1/2*((3*A - 2*B + 2*C)*d*x*cos(d*x + c) + (3*A - 2*B + 2*C)*d*x + (A*cos(d*
x + c)^2 - (A - 2*B)*cos(d*x + c) - 4*A + 4*B - 2*C)*sin(d*x + c))/(a*d*cos
(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos^2(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\cos^2(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{C\cos^2(c+dx)\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x
)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*cos(c + d*x)**2*sec(c
 + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [A] time = 1.18685, size = 185, normalized size = 1.71

$$\frac{\frac{(dx+c)(3A-2B+2C)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2B\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*(3*A - 2*B + 2*C)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*
d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*(3*A*tan(1/2*d*x + 1/2*c)^3 -
2*B*tan(1/2*d*x + 1/2*c)^3 + A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.456 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{(4A-3B+3C)\sin^3(c+dx)}{3ad}+\frac{(4A-3B+3C)\sin(c+dx)}{ad}-\frac{(3A-3B+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B+C}{d(}$$

```
[Out] -((3*A - 3*B + 2*C)*x)/(2*a) + ((4*A - 3*B + 3*C)*Sin[c + d*x])/(a*d) - ((3
*A - 3*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d
*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((4*A - 3*B + 3*C)*Sin[c + d
*x]^3)/(3*a*d)
```

Rubi [A] time = 0.196548, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4084, 3787, 2633, 2635, 8}

$$-\frac{(4A-3B+3C)\sin^3(c+dx)}{3ad}+\frac{(4A-3B+3C)\sin(c+dx)}{ad}-\frac{(3A-3B+2C)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B+C}{d(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] -((3*A - 3*B + 2*C)*x)/(2*a) + ((4*A - 3*B + 3*C)*Sin[c + d*x])/(a*d) - ((3
*A - 3*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B + C)*Cos[c + d
*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((4*A - 3*B + 3*C)*Sin[c + d
*x]^3)/(3*a*d)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \cos^3(c+dx)(a(4A$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(3A-3B+2C)\int c}{a}$$

$$= -\frac{(3A-3B+2C)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A-B+C)\cos^2}{d(a+a\,s}$$

$$= -\frac{(3A-3B+2C)x}{2a}+\frac{(4A-3B+3C)\sin(c+dx)}{ad}-\frac{(3A-3B}{}$$

Mathematica [B] time = 1.0246, size = 307, normalized size = 2.21

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-12dx(3A-3B+2C)\cos\left(c+\frac{dx}{2}\right)-12dx(3A-3B+2C)\cos\left(\frac{dx}{2}\right)+21A\sin\left(c+\frac{dx}{2}\right)+18A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-12*(3*A - 3*B + 2*C)*d*x*Cos[(d*x)/2] - 12*(3*
A - 3*B + 2*C)*d*x*Cos[c + (d*x)/2] + 69*A*Sin[(d*x)/2] - 60*B*Sin[(d*x)/2]
 + 60*C*Sin[(d*x)/2] + 21*A*Sin[c + (d*x)/2] - 12*B*Sin[c + (d*x)/2] + 12*C
*Sin[c + (d*x)/2] + 18*A*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 12*C
*Sin[c + (3*d*x)/2] + 18*A*Sin[2*c + (3*d*x)/2] - 9*B*Sin[2*c + (3*d*x)/2]
+ 12*C*Sin[2*c + (3*d*x)/2] - 2*A*Sin[2*c + (5*d*x)/2] + 3*B*Sin[2*c + (5*d
*x)/2] - 2*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin[3*c + (5*d*x)/2] + A*Sin[3*c +
(7*d*x)/2] + A*Sin[4*c + (7*d*x)/2]))/(24*a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.102, size = 420, normalized size = 3.

$$\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{C}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{(\tan(1/2\,dx+c/2))^5 B}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+5\,\frac{(\tan(1/2\,dx+c/2))^5}{ad\left(1+(\tan(1/2\,dx+c/\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)+1/a/d*C*tan(1/2*d*x+1
/2*c)-3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B+5/a/d/(1+tan(
1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^5*C-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)
^3*B+16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+4/a/d/(1+ta
```

```
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*C-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)
^3*B*tan(1/2*d*x+1/2*c)+3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*
c)+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)-3/a/d*A*arctan(tan
(1/2*d*x+1/2*c))+3/a/d*arctan(tan(1/2*d*x+1/2*c))*B-2/a/d*arctan(tan(1/2*d*
x+1/2*c))*C
```

Maxima [B] time = 1.43742, size = 540, normalized size = 3.88

$$\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\cdots}{}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] 1/3*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*B*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*C*(
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d
*x + c) + 1))))/d
```

Fricas [A] time = 0.496041, size = 288, normalized size = 2.07

$$-\frac{3(3A-3B+2C)dx\cos(dx+c)+3(3A-3B+2C)dx-\left(2A\cos(dx+c)^3-(A-3B)\cos(dx+c)^2+(7A-3B\cdots\right.}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(3*A - 3*B + 2*C)*d*x*cos(d*x + c) + 3*(3*A - 3*B + 2*C)*d*x - (2*A
*cos(d*x + c)^3 - (A - 3*B)*cos(d*x + c)^2 + (7*A - 3*B + 6*C)*cos(d*x + c)
 + 16*A - 12*B + 12*C)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x
)
```

[Out] Timed out

Giac [A] time = 1.24446, size = 279, normalized size = 2.01

$$-\frac{\frac{3\,(dx+c)(3\,A-3\,B+2\,C)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+\cdots\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(3*A - 3*B + 2*C)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1
/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*(15*A*tan(1/2*d*x + 1/2*c)^
5 - 9*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 + 16*A*tan(1/2*
d*x + 1/2*c)^3 - 12*B*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1/2*d*x + 1/2*c)^3
+ 9*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.457 $\int \frac{\cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{(4A-4B+3C)\sin^3(c+dx)}{3ad}-\frac{(4A-4B+3C)\sin(c+dx)}{ad}+\frac{(5A-4B+4C)\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3(5A-4B}{}$$

```
[Out] (3*(5*A - 4*B + 4*C)*x)/(8*a) - ((4*A - 4*B + 3*C)*Sin[c + d*x])/(a*d) + (3
*(5*A - 4*B + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((5*A - 4*B + 4*C)*
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - ((A - B + C)*Cos[c + d*x]^3*Sin[c +
d*x])/(d*(a + a*Sec[c + d*x])) + ((4*A - 4*B + 3*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.212429, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4084, 3787, 2635, 8, 2633}

$$\frac{(4A-4B+3C)\sin^3(c+dx)}{3ad}-\frac{(4A-4B+3C)\sin(c+dx)}{ad}+\frac{(5A-4B+4C)\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3(5A-4B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x]),x]
```

```
[Out] (3*(5*A - 4*B + 4*C)*x)/(8*a) - ((4*A - 4*B + 3*C)*Sin[c + d*x])/(a*d) + (3
*(5*A - 4*B + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + ((5*A - 4*B + 4*C)*
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - ((A - B + C)*Cos[c + d*x]^3*Sin[c +
d*x])/(d*(a + a*Sec[c + d*x])) + ((4*A - 4*B + 3*C)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \cos^4(c+dx)(a(5A$$

$$= -\frac{(A-B+C)\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(4A-4B+3C)\int c}{a}$$

$$= \frac{(5A-4B+4C)\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{(A-B+C)\cos^3(}{d(a+a\,\mathrm{se}}$$

$$= -\frac{(4A-4B+3C)\sin(c+dx)}{ad} + \frac{3(5A-4B+4C)\cos(c+dx)}{8ad}$$

$$= \frac{3(5A-4B+4C)x}{8a} - \frac{(4A-4B+3C)\sin(c+dx)}{ad} + \frac{3(5A-4}{}$$

Mathematica [B] time = 1.00996, size = 393, normalized size = 2.26

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(72dx(5A-4B+4C)\cos\left(c+\frac{dx}{2}\right)+72dx(5A-4B+4C)\cos\left(\frac{dx}{2}\right)-168A\sin\left(c+\frac{dx}{2}\right)-120A\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(72*(5*A - 4*B + 4*C)*d*x*Cos[(d*x)/2] + 72*(5*A
 - 4*B + 4*C)*d*x*Cos[c + (d*x)/2] - 552*A*Sin[(d*x)/2] + 552*B*Sin[(d*x)/2
] - 480*C*Sin[(d*x)/2] - 168*A*Sin[c + (d*x)/2] + 168*B*Sin[c + (d*x)/2] -
96*C*Sin[c + (d*x)/2] - 120*A*Sin[c + (3*d*x)/2] + 144*B*Sin[c + (3*d*x)/2]
 - 72*C*Sin[c + (3*d*x)/2] - 120*A*Sin[2*c + (3*d*x)/2] + 144*B*Sin[2*c + (
3*d*x)/2] - 72*C*Sin[2*c + (3*d*x)/2] + 40*A*Sin[2*c + (5*d*x)/2] - 16*B*Si
n[2*c + (5*d*x)/2] + 24*C*Sin[2*c + (5*d*x)/2] + 40*A*Sin[3*c + (5*d*x)/2]
- 16*B*Sin[3*c + (5*d*x)/2] + 24*C*Sin[3*c + (5*d*x)/2] - 5*A*Sin[3*c + (7*
d*x)/2] + 8*B*Sin[3*c + (7*d*x)/2] - 5*A*Sin[4*c + (7*d*x)/2] + 8*B*Sin[4*c
 + (7*d*x)/2] + 3*A*Sin[4*c + (9*d*x)/2] + 3*A*Sin[5*c + (9*d*x)/2]))/(192*
a*d*(1 + Cos[c + d*x]))
```

Maple [B] time = 0.111, size = 526, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*C*tan(1/2*d*x+
1/2*c)-25/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A-3/a/d/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*C+5/a/d/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^7*B-115/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^5*A-7/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*C+31/3/
a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*B-109/12/a/d/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A-5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*ta
n(1/2*d*x+1/2*c)^3*C+25/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
^3*B-7/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*A*tan(1/2*d*x+1/2*c)-1/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^4*C*tan(1/2*d*x+1/2*c)+3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*B*
tan(1/2*d*x+1/2*c)+15/4/a/d*A*arctan(tan(1/2*d*x+1/2*c))-3/a/d*arctan(tan(1
/2*d*x+1/2*c))*B+3/a/d*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.45223, size = 709, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] -1/12*(A*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 4*B*((9*sin(d*x +
 c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*x + c)/(a*
(cos(d*x + c) + 1))) + 12*C*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.50898, size = 344, normalized size = 1.98

$$\frac{9\,(5\,A-4\,B+4\,C)dx\cos\left(dx+c\right)+9\,(5\,A-4\,B+4\,C)dx+\left(6\,A\cos\left(dx+c\right)^{4}-2\,(A-4\,B)\cos\left(dx+c\right)^{3}+(13\,A-4}{24\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] 1/24*(9*(5*A - 4*B + 4*C)*d*x*cos(d*x + c) + 9*(5*A - 4*B + 4*C)*d*x + (6*A
*cos(d*x + c)^4 - 2*(A - 4*B)*cos(d*x + c)^3 + (13*A - 4*B + 12*C)*cos(d*x
+ c)^2 - (19*A - 28*B + 12*C)*cos(d*x + c) - 64*A + 64*B - 48*C)*sin(d*x +
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.27924, size = 336, normalized size = 1.93

$$\frac{\frac{9\,(dx+c)(5\,A-4\,B+4\,C)}{a}-\frac{24\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(75\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-60\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+36\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*(5*A - 4*B + 4*C)/a - 24*(A*tan(1/2*d*x + 1/2*c) - B*tan(
1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/a - 2*(75*A*tan(1/2*d*x + 1/2*c)
^7 - 60*B*tan(1/2*d*x + 1/2*c)^7 + 36*C*tan(1/2*d*x + 1/2*c)^7 + 115*A*tan(
1/2*d*x + 1/2*c)^5 - 124*B*tan(1/2*d*x + 1/2*c)^5 + 84*C*tan(1/2*d*x + 1/2*
c)^5 + 109*A*tan(1/2*d*x + 1/2*c)^3 - 100*B*tan(1/2*d*x + 1/2*c)^3 + 60*C*t
an(1/2*d*x + 1/2*c)^3 + 21*A*tan(1/2*d*x + 1/2*c) - 36*B*tan(1/2*d*x + 1/2*
c) + 12*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.458 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$\frac{(5A-8B+12C)\tan^3(c+dx)}{3a^2d}+\frac{(5A-8B+12C)\tan(c+dx)}{a^2d}-\frac{(4A-7B+10C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(4A-7B+}{3}$$

```
[Out] -((4*A - 7*B + 10*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + ((5*A - 8*B + 12*C)
*Tan[c + d*x])/(a^2*d) - ((4*A - 7*B + 10*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((4*A - 7*B + 10*C)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec
[c + d*x])) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c +
 d*x])^2) + ((5*A - 8*B + 12*C)*Tan[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.363064, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4019, 3787, 3768, 3770, 3767}

$$\frac{(5A-8B+12C)\tan^3(c+dx)}{3a^2d}+\frac{(5A-8B+12C)\tan(c+dx)}{a^2d}-\frac{(4A-7B+10C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(4A-7B+}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```



```
[Out] -((4*A - 7*B + 10*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + ((5*A - 8*B + 12*C)
*Tan[c + d*x])/(a^2*d) - ((4*A - 7*B + 10*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((4*A - 7*B + 10*C)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec
[c + d*x])) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c +
 d*x])^2) + ((5*A - 8*B + 12*C)*Tan[c + d*x]^3)/(3*a^2*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^4(c+dx)(-a(A-4B+}{a+a}}{}$$
$$= -\frac{(4A-7B+10C)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\text{se}}{3d(a+}$$
$$= -\frac{(4A-7B+10C)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\text{se}}{3d(a+}$$
$$= -\frac{(4A-7B+10C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(4A-7B+10C}{3a^2d(}$$
$$= -\frac{(4A-7B+10C)\tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{(5A-8B+12C)\text{ta}}{a^2d}$$

Mathematica [B] time = 6.42222, size = 1069, normalized size = 5.51

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^2,x]
```

```
[Out] (4*(4*A - 7*B + 10*C)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2
 + (d*x)/2]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (4*(4*A - 7*B + 1
0*C)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c
]*Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-48*A*Sin[(d*x)/2
] + 45*B*Sin[(d*x)/2] - 6*C*Sin[(d*x)/2] + 132*A*Sin[(3*d*x)/2] - 201*B*Sin
[(3*d*x)/2] + 310*C*Sin[(3*d*x)/2] - 120*A*Sin[c - (d*x)/2] + 195*B*Sin[c -
 (d*x)/2] - 306*C*Sin[c - (d*x)/2] + 48*A*Sin[c + (d*x)/2] - 51*B*Sin[c + (
```

```
d*x)/2] + 42*C*Sin[c + (d*x)/2] - 120*A*Sin[2*c + (d*x)/2] + 189*B*Sin[2*c
+ (d*x)/2] - 270*C*Sin[2*c + (d*x)/2] - 8*A*Sin[c + (3*d*x)/2] - B*Sin[c +
(3*d*x)/2] + 50*C*Sin[c + (3*d*x)/2] + 72*A*Sin[2*c + (3*d*x)/2] - 81*B*Sin
[2*c + (3*d*x)/2] + 90*C*Sin[2*c + (3*d*x)/2] - 68*A*Sin[3*c + (3*d*x)/2] +
 119*B*Sin[3*c + (3*d*x)/2] - 170*C*Sin[3*c + (3*d*x)/2] + 84*A*Sin[c + (5*
d*x)/2] - 129*B*Sin[c + (5*d*x)/2] + 198*C*Sin[c + (5*d*x)/2] - 9*B*Sin[2*c
 + (5*d*x)/2] + 42*C*Sin[2*c + (5*d*x)/2] + 48*A*Sin[3*c + (5*d*x)/2] - 57*
B*Sin[3*c + (5*d*x)/2] + 66*C*Sin[3*c + (5*d*x)/2] - 36*A*Sin[4*c + (5*d*x)
/2] + 63*B*Sin[4*c + (5*d*x)/2] - 90*C*Sin[4*c + (5*d*x)/2] + 48*A*Sin[2*c
+ (7*d*x)/2] - 75*B*Sin[2*c + (7*d*x)/2] + 114*C*Sin[2*c + (7*d*x)/2] + 6*A
*Sin[3*c + (7*d*x)/2] - 15*B*Sin[3*c + (7*d*x)/2] + 36*C*Sin[3*c + (7*d*x)/
2] + 30*A*Sin[4*c + (7*d*x)/2] - 39*B*Sin[4*c + (7*d*x)/2] + 48*C*Sin[4*c +
 (7*d*x)/2] - 12*A*Sin[5*c + (7*d*x)/2] + 21*B*Sin[5*c + (7*d*x)/2] - 30*C*
Sin[5*c + (7*d*x)/2] + 20*A*Sin[3*c + (9*d*x)/2] - 32*B*Sin[3*c + (9*d*x)/2
] + 48*C*Sin[3*c + (9*d*x)/2] + 6*A*Sin[4*c + (9*d*x)/2] - 12*B*Sin[4*c + (
9*d*x)/2] + 22*C*Sin[4*c + (9*d*x)/2] + 14*A*Sin[5*c + (9*d*x)/2] - 20*B*Si
n[5*c + (9*d*x)/2] + 26*C*Sin[5*c + (9*d*x)/2]))/(48*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 0.079, size = 506, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*
d*x+1/2*c)+9/2/d/a^2*C*tan(1/2*d*x+1/2*c)+3/2/d/a^2*C/(tan(1/2*d*x+1/2*c)+1
)^2-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2*B-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A+5
/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B-5/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-2/d/a^2*
ln(tan(1/2*d*x+1/2*c)+1)*A+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B-5/d/a^2*ln(
tan(1/2*d*x+1/2*c)+1)*C-1/3/d/a^2*C/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/a^2/(tan
(1/2*d*x+1/2*c)-1)^2*B-3/2/d/a^2*C/(tan(1/2*d*x+1/2*c)-1)^2-1/d/a^2/(tan(1/
2*d*x+1/2*c)-1)*A+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B-5/d/a^2/(tan(1/2*d*x+1
/2*c)-1)*C+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A-7/2/d/a^2*ln(tan(1/2*d*x+1/2*
c)-1)*B+5/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-1/3/d/a^2*C/(tan(1/2*d*x+1/2*c)-
1)^3
```

Maxima [B] time = 0.980584, size = 765, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] 1/6*(C*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
```

```
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) + A*((15*sin(d*x + c)/
(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.5293, size = 689, normalized size = 3.55

$$-\frac{3\left((4A-7B+10C)\cos(dx+c)^5+2(4A-7B+10C)\cos(dx+c)^4+(4A-7B+10C)\cos(dx+c)^3\right)\log(\sin(dx+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] -1/12*(3*((4*A - 7*B + 10*C)*cos(d*x + c)^5 + 2*(4*A - 7*B + 10*C)*cos(d*x
+ c)^4 + (4*A - 7*B + 10*C)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((4*A
 - 7*B + 10*C)*cos(d*x + c)^5 + 2*(4*A - 7*B + 10*C)*cos(d*x + c)^4 + (4*A
- 7*B + 10*C)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(4*(5*A - 8*B + 12
*C)*cos(d*x + c)^4 + (28*A - 43*B + 66*C)*cos(d*x + c)^3 + 6*(A - B + 2*C)*
cos(d*x + c)^2 + (3*B - 2*C)*cos(d*x + c) + 2*C)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{C\sec^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(C*sec(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.27143, size = 409, normalized size = 2.11

$$-\frac{\frac{3(4A-7B+10C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(4A-7B+10C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{2\left(6A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+30C\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(3*(4*A - 7*B + 10*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(4*A
- 7*B + 10*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(6*A*tan(1/2*d*x +
 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 30*C*tan(1/2*d*x + 1/2*c)^5 - 12*
A*tan(1/2*d*x + 1/2*c)^3 + 24*B*tan(1/2*d*x + 1/2*c)^3 - 40*C*tan(1/2*d*x +
 1/2*c)^3 + 6*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c) + 18*C*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x
 + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 +
 15*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*tan(1/2*d*x + 1/2*c) + 27*C*a^4*t
an(1/2*d*x + 1/2*c))/a^6)/d
```

3.459 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=169

$$-\frac{2(2A-5B+8C)\tan(c+dx)}{3a^2d}+\frac{(2A-4B+7C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(2A-5B+8C)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(2A$$

```
[Out] ((2*A - 4*B + 7*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(2*A - 5*B + 8*C)*
Tan[c + d*x])/(3*a^2*d) + ((2*A - 4*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((2*A - 5*B + 8*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[
c + d*x])) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rubi [A] time = 0.337234, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4084, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{2(2A-5B+8C)\tan(c+dx)}{3a^2d}+\frac{(2A-4B+7C)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(2A-5B+8C)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(2A$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```


```
[Out] ((2*A - 4*B + 7*C)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(2*A - 5*B + 8*C)*
Tan[c + d*x])/(3*a^2*d) + ((2*A - 4*B + 7*C)*Sec[c + d*x]*Tan[c + d*x])/(2*
a^2*d) - ((2*A - 5*B + 8*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[
c + d*x])) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)(3a(B-\cdots}{a\cdots}}{\cdots}$$

$$= -\frac{(2A-5B+8C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\cdots}{3d(a\cdots}$$

$$= -\frac{(2A-5B+8C)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\cdots}{3d(a\cdots}$$

$$= \frac{(2A-4B+7C)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(2A-5B+8C\cdots}{3a^2d(\cdots}$$

$$= \frac{(2A-4B+7C)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{2(2A-5B+8C)\,\mathrm{t}\cdots}{3a^2d}$$

Mathematica [B] time = 6.32553, size = 901, normalized size = 5.33

$$-\frac{4(2A-4B+7C)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(\sec(c+dx)a+a)^2} + \frac{4(2A-4B\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^2,x]
```

```
[Out] (-4*(2*A - 4*B + 7*C)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2
 + (d*x)/2]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (4*(2*A - 4*B + 7
*C)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(A +
```

```
B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]
*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(20*A*Sin[(d*x)/2]
- 14*B*Sin[(d*x)/2] + 14*C*Sin[(d*x)/2] - 22*A*Sin[(3*d*x)/2] + 64*B*Sin[(3
*d*x)/2] - 97*C*Sin[(3*d*x)/2] + 36*A*Sin[c - (d*x)/2] - 84*B*Sin[c - (d*x)
/2] + 126*C*Sin[c - (d*x)/2] - 36*A*Sin[c + (d*x)/2] + 42*B*Sin[c + (d*x)/2
] - 42*C*Sin[c + (d*x)/2] + 20*A*Sin[2*c + (d*x)/2] - 56*B*Sin[2*c + (d*x)/
2] + 98*C*Sin[2*c + (d*x)/2] + 18*A*Sin[c + (3*d*x)/2] - 6*B*Sin[c + (3*d*x
)/2] + 3*C*Sin[c + (3*d*x)/2] - 22*A*Sin[2*c + (3*d*x)/2] + 34*B*Sin[2*c +
(3*d*x)/2] - 37*C*Sin[2*c + (3*d*x)/2] + 18*A*Sin[3*c + (3*d*x)/2] - 36*B*S
in[3*c + (3*d*x)/2] + 63*C*Sin[3*c + (3*d*x)/2] - 18*A*Sin[c + (5*d*x)/2] +
 48*B*Sin[c + (5*d*x)/2] - 75*C*Sin[c + (5*d*x)/2] + 6*A*Sin[2*c + (5*d*x)/
2] + 6*B*Sin[2*c + (5*d*x)/2] - 15*C*Sin[2*c + (5*d*x)/2] - 18*A*Sin[3*c +
(5*d*x)/2] + 30*B*Sin[3*c + (5*d*x)/2] - 39*C*Sin[3*c + (5*d*x)/2] + 6*A*Si
n[4*c + (5*d*x)/2] - 12*B*Sin[4*c + (5*d*x)/2] + 21*C*Sin[4*c + (5*d*x)/2]
- 8*A*Sin[2*c + (7*d*x)/2] + 20*B*Sin[2*c + (7*d*x)/2] - 32*C*Sin[2*c + (7*
d*x)/2] + 6*B*Sin[3*c + (7*d*x)/2] - 12*C*Sin[3*c + (7*d*x)/2] - 8*A*Sin[4*
c + (7*d*x)/2] + 14*B*Sin[4*c + (7*d*x)/2] - 20*C*Sin[4*c + (7*d*x)/2]))/(2
4*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
2)
```

Maple [B] time = 0.074, size = 373, normalized size = 2.2

$$-\frac{A}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6\,d a^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3\,A}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\,B}{2\,d a^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^
2*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2
*d*x+1/2*c)-7/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B
+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*A-2/d/
a^2*ln(tan(1/2*d*x+1/2*c)+1)*B+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a
^2*C/(tan(1/2*d*x+1/2*c)+1)^2-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B+5/2/d/a^2/(t
an(1/2*d*x+1/2*c)-1)*C-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A+2/d/a^2*ln(tan(1/
2*d*x+1/2*c)-1)*B-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^2*C/(tan(1/2
*d*x+1/2*c)-1)^2
```

Maxima [B] time = 0.972351, size = 582, normalized size = 3.44

$$-\frac{C\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+\cdots}}{a^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] -1/6*(C*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
```

```
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))) + A*((9*sin(d*x + c)/(cos(d*x + c) + 1) +
sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Fricas [A] time = 0.525568, size = 630, normalized size = 3.73

$$\frac{3\left((2A-4B+7C)\cos(dx+c)^4+2(2A-4B+7C)\cos(dx+c)^3+(2A-4B+7C)\cos(dx+c)^2\right)\log(\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] 1/12*(3*((2*A - 4*B + 7*C)*cos(d*x + c)^4 + 2*(2*A - 4*B + 7*C)*cos(d*x + c
)^3 + (2*A - 4*B + 7*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((2*A - 4
*B + 7*C)*cos(d*x + c)^4 + 2*(2*A - 4*B + 7*C)*cos(d*x + c)^3 + (2*A - 4*B
+ 7*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*(2*A - 5*B + 8*C)*cos(
d*x + c)^3 + (10*A - 28*B + 43*C)*cos(d*x + c)^2 - 6*(B - C)*cos(d*x + c) -
 3*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*
cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{C\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(C*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.26907, size = 317, normalized size = 1.88

$$\frac{\frac{3(2A-4B+7C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(2A-4B+7C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{6\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(2*A - 4*B + 7*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(2*A -
4*B + 7*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 6*(2*B*tan(1/2*d*x + 1/
2*c)^3 - 5*C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) + 3*C*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x +
 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 9
*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan(1/2*d*x + 1/2*c) + 21*C*a^4*tan(
1/2*d*x + 1/2*c))/a^6)/d
```

3.460 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$\frac{(A-B+4C)\tan(c+dx)}{3a^2d}+\frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-2C)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] ((B - 2*C)*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((A - B + 4*C)*Tan[c + d*x])/(3
*a^2*d) - ((B - 2*C)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B + C
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.282613, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4008, 3787, 3770, 3767, 8}

$$\frac{(A-B+4C)\tan(c+dx)}{3a^2d}+\frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(B-2C)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```



```
[Out] ((B - 2*C)*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((A - B + 4*C)*Tan[c + d*x])/(3
*a^2*d) - ((B - 2*C)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B + C
)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^2(c+dx)(a(A+2B-2}{a+a\,s}$$

$$= -\frac{(B-2C)\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^2(c+dx)\tan(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(B-2C)\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^2(c+dx)\tan(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(B-2C)\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-}{}$$

$$= \frac{(B-2C)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{(A-B+4C)\tan(c+dx)}{3a^2d} -$$

Mathematica [B] time = 2.07581, size = 312, normalized size = 2.79

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\tan\left(\frac{c}{2}\right)(A-B+C)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)(A-B+C)\sin\left(\frac{c}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((A - B + C)*Se
c[c/2]*Sin[(d*x)/2] + 2*(A - 4*B + 7*C)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*
x)/2] + Cos[(c + d*x)/2]^3*(-6*(B - 2*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (6*C*Sin[d*x])/((Cos[
c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (A - B + C)*Cos[(c + d*x)/2]*Ta
n[c/2]))/(3*a^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(1 + Se
c[c + d*x])^2)
```

Maple [B] time = 0.063, size = 243, normalized size = 2.2

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{3B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3+1/2/d/a^2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*
d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*
B-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*C-1/d/a
^2*ln(tan(1/2*d*x+1/2*c)-1)*B+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*C-1/d/a^2/(t
an(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.96502, size = 387, normalized size = 3.46

$$\frac{C\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx}{(\cos(dx+}}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] 1/6*(C*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2) + A*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.522094, size = 513, normalized size = 4.58

$$\frac{3\left((B-2\,C)\cos(dx+c)^3+2\,(B-2\,C)\cos(dx+c)^2+(B-2\,C)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((B-2\,C)\cos\right.}{6\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] 1/6*(3*((B - 2*C)*cos(d*x + c)^3 + 2*(B - 2*C)*cos(d*x + c)^2 + (B - 2*C)*c
os(d*x + c))*log(sin(d*x + c) + 1) - 3*((B - 2*C)*cos(d*x + c)^3 + 2*(B - 2
*C)*cos(d*x + c)^2 + (B - 2*C)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*((A
 - 4*B + 10*C)*cos(d*x + c)^2 + (2*A - 5*B + 14*C)*cos(d*x + c) + 3*C)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x +
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{B\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{C\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(C*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.2245, size = 244, normalized size = 2.18

$$\frac{\frac{6\,(B-2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,(B-2\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{12\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/6*(6*(B - 2*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(B - 2*C)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 12*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x +
 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) - 9
*B*a^4*tan(1/2*d*x + 1/2*c) + 15*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.461 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{(A+2B-5C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{(A-B+C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((A + 2*B - 5*C)*Tan[c + d*x])/(3*a^2*d
*(1 + Sec[c + d*x])) + ((A - B + C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])
^2)
```

Rubi [A] time = 0.171757, antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4078, 3998, 3770, 3794}

$$\frac{(2A+B-4C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B+C)\tan(c+dx)\sec(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^2,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((2*A + B - 4*C)*Tan[c + d*x])/(3*a^2*d
*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*
Sec[c + d*x])^2)
```

Rule 4078

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*B - b*C - 2*A*b*(m + 1) - (b*B*(m + 2) -
a*(A*(m + 2) - C*(m - 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(a(2A+B-C)+3}{a+a\sec(c+d}}{3a^2}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A+B-4C)\int \frac{\sec}{a+a\,\mathrm{s}}}{3a}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [B] time = 0.827799, size = 219, normalized size = 2.7

$$-\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(\tan\left(\frac{c}{2}\right)(A-B+C)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)(A-B+C)\sin\right.}{3a^2d(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*(6*C*Cos[(c +
d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + (A - B + C)*Sec[c/2]*Sin[(d*x)/2] - 2*(2*A + B - 4*C
)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A - B + C)*Cos[(c + d*x)/2]*T
an[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*(c + d*x)]))
```

Maple [B] time = 0.066, size = 157, normalized size = 1.9

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{3C}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{C}{da^2}\ln\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/d/a^2*
ln(tan(1/2*d*x+1/2*c)+1)*C-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan(1/
2*d*x+1/2*c)-1)*C+1/2/d/a^2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*B*tan(1/2*d*x+1/
2*c)-1/6/d/a^2*C*tan(1/2*d*x+1/2*c)^3
```

Maxima [B] time = 0.959797, size = 257, normalized size = 3.17

$$-\frac{C\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/6*(C*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - A*(3*sin(d*x + c)/(cos(d*x + c
) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.526687, size = 351, normalized size = 4.33

$$\frac{3\left(C\cos\left(dx+c\right)^{2}+2\,C\cos\left(dx+c\right)+C\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(C\cos\left(dx+c\right)^{2}+2\,C\cos\left(dx+c\right)+C\right)\log\left(-\sin\right.}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*log(sin(d*x + c) + 1) - 3*
(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*log(-sin(d*x + c) + 1) + 2*((2*A
+ B - 4*C)*cos(d*x + c) + A + 2*B - 5*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{2}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{C\sec^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x
)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(B*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral
(C*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.29305, size = 194, normalized size = 2.4

$$\frac{\frac{6\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{6\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}-\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+Ca^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/6*(6*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*C*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
```

```
*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) - 3*B*a
^4*tan(1/2*d*x + 1/2*c) + 9*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.462 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{(4A-B-2C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A-B+C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (A*x)/a^2 - ((4*A - B - 2*C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (
(A - B + C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.12873, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4052, 3919, 3794}

$$-\frac{(4A-B-2C)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A-B+C)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (A*x)/a^2 - ((4*A - B - 2*C)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (
(A - B + C)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{(A-B+C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{-3aA+a(A-B-2C)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{Ax}{a^2}-\frac{(A-B+C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(4A-B-2C)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\
&= \frac{Ax}{a^2}-\frac{(A-B+C)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(4A-B-2C)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}
\end{aligned}$$

Mathematica [B] time = 0.504773, size = 175, normalized size = 2.36

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(12A\sin\left(c+\frac{dx}{2}\right)-10A\sin\left(c+\frac{3dx}{2}\right)+9Adx\cos\left(c+\frac{dx}{2}\right)+3Adx\cos\left(c+\frac{3dx}{2}\right)+3Adx\cos\left(2c\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*A*d*x*Cos[(d*x)/2] + 9*A*d*x*Cos[c + (d*x)/
2] + 3*A*d*x*Cos[c + (3*d*x)/2] + 3*A*d*x*Cos[2*c + (3*d*x)/2] - 18*A*Sin[(
d*x)/2] + 6*B*Sin[(d*x)/2] + 6*C*Sin[(d*x)/2] + 12*A*Sin[c + (d*x)/2] - 6*B
*Sin[c + (d*x)/2] - 10*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2] + 2*C*
Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.069, size = 135, normalized size = 1.8

$$\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*B*tan(1/2*
d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*A*arctan(tan(1/2*d*x+1/2*
c))
```

Maxima [B] time = 1.4278, size = 221, normalized size = 2.99

$$-\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{C\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - C*(3*sin(d*
x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - B*(3
*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2
)/d
```

Fricas [A] time = 0.473313, size = 242, normalized size = 3.27

$$\frac{3Adx\cos(dx+c)^2+6Adx\cos(dx+c)+3Adx-((5A-2B-C)\cos(dx+c)+4A-B-2C)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] 1/3*(3*A*d*x*cos(d*x + c)^2 + 6*A*d*x*cos(d*x + c) + 3*A*d*x - ((5*A - 2*B
- C)*cos(d*x + c) + 4*A - B - 2*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{B\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx + \int \frac{C\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c +
 d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)*
*2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A] time = 1.28628, size = 157, normalized size = 2.12

$$\frac{\frac{6(dx+c)A}{a^2} + \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 9Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/6*(6*(d*x + c)*A/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) +
3*B*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.463 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{(10A-4B+C)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2A-B)}{a^2}-\frac{(A-B+C)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -(((2*A - B)*x)/a^2) + ((10*A - 4*B + C)*Sin[c + d*x])/(3*a^2*d) - ((2*A -
B)*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.25863, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4084, 4020, 3787, 2637, 8}

$$\frac{(10A-4B+C)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2A-B)}{a^2}-\frac{(A-B+C)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^2,x]
```

```
[Out] -(((2*A - B)*x)/a^2) + ((10*A - 4*B + C)*Sin[c + d*x])/(3*a^2*d) - ((2*A -
B)*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}dx = -\frac{(A-B+C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(a(4A-B+C)-a(2A-2B-C)}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \cos(}{}$$

$$= -\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{(2A-}{}$$

$$= -\frac{(2A-B)x}{a^2} + \frac{(10A-4B+C)\sin(c+dx)}{3a^2d} - \frac{(2A-B)\sin(}{a^2d(1+\sec(c}$$

Mathematica [B] time = 0.831113, size = 279, normalized size = 2.79

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-18dx(2A-B)\cos\left(c+\frac{dx}{2}\right)-18dx(2A-B)\cos\left(\frac{dx}{2}\right)-30A\sin\left(c+\frac{dx}{2}\right)+41A\sin\left(c+\frac{3dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-18*(2*A - B)*d*x*Cos[(d*x)/2] - 18*(2*A - B)*d
*x*Cos[c + (d*x)/2] - 12*A*d*x*Cos[c + (3*d*x)/2] + 6*B*d*x*Cos[c + (3*d*x)
/2] - 12*A*d*x*Cos[2*c + (3*d*x)/2] + 6*B*d*x*Cos[2*c + (3*d*x)/2] + 66*A*S
in[(d*x)/2] - 36*B*Sin[(d*x)/2] + 12*C*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2]
 + 24*B*Sin[c + (d*x)/2] - 12*C*Sin[c + (d*x)/2] + 41*A*Sin[c + (3*d*x)/2]
- 20*B*Sin[c + (3*d*x)/2] + 8*C*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3*d*x)/
2] + 3*A*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2]))/(12*a^2*d*(1 + C
os[c + d*x])^2)
```

Maple [A] time = 0.098, size = 187, normalized size = 1.9

$$-\frac{A}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{C}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5A}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3B}{2da^2}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^
2*C*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2
*d*x+1/2*c)+1/2/d/a^2*C*tan(1/2*d*x+1/2*c)+2/d/a^2*A*tan(1/2*d*x+1/2*c)/(1+
tan(1/2*d*x+1/2*c)^2)-4/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^2*B*arctan
```

```
(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.44759, size = 317, normalized size = 3.17

$$\frac{A\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) + C*(3*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A] time = 0.487391, size = 309, normalized size = 3.09

$$-\frac{3\,(2\,A-B)dx\cos(dx+c)^2+6\,(2\,A-B)dx\cos(dx+c)+3\,(2\,A-B)dx-\left(3\,A\cos(dx+c)^2+(14\,A-5\,B+2\,C)\cos(d\right.}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/3*(3*(2*A - B)*d*x*cos(d*x + c)^2 + 6*(2*A - B)*d*x*cos(d*x + c) + 3*(2*
A - B)*d*x - (3*A*cos(d*x + c)^2 + (14*A - 5*B + 2*C)*cos(d*x + c) + 10*A -
 4*B + C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\cos(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{C\cos(c+dx)\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x
)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
+ Integral(C*cos(c + d*x)*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x)
 + 1), x))/a**2
```

Giac [A] time = 1.26524, size = 205, normalized size = 2.05

$$-\frac{\frac{6\,(dx+c)(2\,A-B)}{a^2}-\frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Ca^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba^4\tan(\frac{1}{2}\,dx}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*(2*A - B)/a^2 - 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
 1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1
/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 9*
B*a^4*tan(1/2*d*x + 1/2*c) - 3*C*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.464 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=156

$$-\frac{2(8A-5B+2C)\sin(c+dx)}{3a^2d}+\frac{(7A-4B+2C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8A-5B+2C)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(}{}$$

```
[Out] ((7*A - 4*B + 2*C)*x)/(2*a^2) - (2*(8*A - 5*B + 2*C)*Sin[c + d*x])/(3*a^2*d
) + ((7*A - 4*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*A - 5*B +
 2*C)*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B + C
)*Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.334872, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4020, 3787, 2635, 8, 2637}

$$-\frac{2(8A-5B+2C)\sin(c+dx)}{3a^2d}+\frac{(7A-4B+2C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8A-5B+2C)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```

```
[Out] ((7*A - 4*B + 2*C)*x)/(2*a^2) - (2*(8*A - 5*B + 2*C)*Sin[c + d*x])/(3*a^2*d
) + ((7*A - 4*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*A - 5*B +
 2*C)*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B + C
)*Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(a(5A-2\ldots}{a+\ldots}}{\ldots}$$
$$= -\frac{(8A-5B+2C)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\,\mathrm{c}\ldots}{3d(a+\ldots}$$
$$= -\frac{(8A-5B+2C)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\,\mathrm{c}\ldots}{3d(a+\ldots}$$
$$= -\frac{2(8A-5B+2C)\sin(c+dx)}{3a^2d} + \frac{(7A-4B+2C)\cos(c+\ldots}{2a^2d}$$
$$= \frac{(7A-4B+2C)x}{2a^2} - \frac{2(8A-5B+2C)\sin(c+dx)}{3a^2d} + \frac{(7A-\ldots}{\ldots}$$

Mathematica [B] time = 1.51753, size = 377, normalized size = 2.42

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(36dx(7A-4B+2C)\cos\left(c+\frac{dx}{2}\right)+36dx(7A-4B+2C)\cos\left(\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-23\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(36*(7*A - 4*B + 2*C)*d*x*Cos[(d*x)/2] + 36*(7
*A - 4*B + 2*C)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 48*B*d
*x*Cos[c + (3*d*x)/2] + 24*C*d*x*Cos[c + (3*d*x)/2] + 84*A*d*x*Cos[2*c + (3
*d*x)/2] - 48*B*d*x*Cos[2*c + (3*d*x)/2] + 24*C*d*x*Cos[2*c + (3*d*x)/2] -
381*A*Sin[(d*x)/2] + 264*B*Sin[(d*x)/2] - 144*C*Sin[(d*x)/2] + 147*A*Sin[c
+ (d*x)/2] - 120*B*Sin[c + (d*x)/2] + 96*C*Sin[c + (d*x)/2] - 239*A*Sin[c +
 (3*d*x)/2] + 164*B*Sin[c + (3*d*x)/2] - 80*C*Sin[c + (3*d*x)/2] - 63*A*Sin
[2*c + (3*d*x)/2] + 36*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d*x)/2] +
 12*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 12*B*Sin[3*c + (5*
d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] + 3*A*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

Maple [B] time = 0.106, size = 309, normalized size = 2.

$$\frac{A}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{B}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7\,A}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{5\,B}{2\,da^2}\tan\left(\frac{dx}{2}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B+1/6/d/a^2
*C*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*
d*x+1/2*c)-3/2/d/a^2*C*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
2*tan(1/2*d*x+1/2*c)^3*A+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2
*c)^3*B-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+2/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+7/d/a^2*A*arctan(tan(1/2*d*x+1
/2*c))-4/d/a^2*B*arctan(tan(1/2*d*x+1/2*c))+2/d/a^2*arctan(tan(1/2*d*x+1/2*
c))*C
```

Maxima [B] time = 1.44225, size = 475, normalized size = 3.04

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a^2}\cdots\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))) + C*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A] time = 0.49705, size = 383, normalized size = 2.46

$$\frac{3(7A-4B+2C)dx\cos(dx+c)^2+6(7A-4B+2C)dx\cos(dx+c)+3(7A-4B+2C)dx+\left(3A\cos(dx+c)^3-6(\cdots\right.}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(7*A - 4*B + 2*C)*d*x*cos(d*x + c)^2 + 6*(7*A - 4*B + 2*C)*d*x*cos(d
*x + c) + 3*(7*A - 4*B + 2*C)*d*x + (3*A*cos(d*x + c)^3 - 6*(A - B)*cos(d*x
 + c)^2 - (43*A - 28*B + 10*C)*cos(d*x + c) - 32*A + 20*B - 8*C)*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24307, size = 267, normalized size = 1.71

$$\frac{\frac{3(dx+c)(7A-4B+2C)}{a^2} - \frac{6\left(5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a^2} + \frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ba^4\tan\left(\frac{1}{2}dx+\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A - 4*B + 2*C)/a^2 - 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*
B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
 B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 21*A*a^4*tan
(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c) - 9*C*a^4*tan(1/2*d*x + 1
/2*c))/a^6)/d
```

3.465 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$-\frac{(12A-8B+5C)\sin^3(c+dx)}{3a^2d}+\frac{(12A-8B+5C)\sin(c+dx)}{a^2d}-\frac{(10A-7B+4C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10A-7B}{3}$$

```
[Out] -((10*A - 7*B + 4*C)*x)/(2*a^2) + ((12*A - 8*B + 5*C)*Sin[c + d*x])/(a^2*d)
 - ((10*A - 7*B + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*A - 7*B
+ 4*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B
+ C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - ((12*A - 8
*B + 5*C)*Sin[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.362868, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4020, 3787, 2633, 2635, 8}

$$-\frac{(12A-8B+5C)\sin^3(c+dx)}{3a^2d}+\frac{(12A-8B+5C)\sin(c+dx)}{a^2d}-\frac{(10A-7B+4C)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10A-7B}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^2,x]
```


```
[Out] -((10*A - 7*B + 4*C)*x)/(2*a^2) + ((12*A - 8*B + 5*C)*Sin[c + d*x])/(a^2*d)
 - ((10*A - 7*B + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*A - 7*B
+ 4*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B
+ C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - ((12*A - 8
*B + 5*C)*Sin[c + d*x]^3)/(3*a^2*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\cos^3(c+dx)(3a(2A\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(10A-7B+4C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\cdots}{3d(a\cdots}$$

$$= -\frac{(10A-7B+4C)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\cdots}{3d(a\cdots}$$

$$= -\frac{(10A-7B+4C)\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{(10A-7B+\cdots}{3a^2\cdots}$$

$$= -\frac{(10A-7B+4C)x}{2a^2}+\frac{(12A-8B+5C)\sin(c+dx)}{a^2d}-\frac{(10\cdots}{\cdots}$$

Mathematica [B] time = 1.81922, size = 473, normalized size = 2.56

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-36dx(10A-7B+4C)\cos\left(c+\frac{dx}{2}\right)-36dx(10A-7B+4C)\cos\left(\frac{dx}{2}\right)-156A\sin\left(c+\frac{dx}{2}\right)+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-36*(10*A - 7*B + 4*C)*d*x*Cos[(d*x)/2] - 36*
(10*A - 7*B + 4*C)*d*x*Cos[c + (d*x)/2] - 120*A*d*x*Cos[c + (3*d*x)/2] + 84
*B*d*x*Cos[c + (3*d*x)/2] - 48*C*d*x*Cos[c + (3*d*x)/2] - 120*A*d*x*Cos[2*c
 + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] - 48*C*d*x*Cos[2*c + (3*d*x)/
2] + 516*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] + 264*C*Sin[(d*x)/2] - 156*A*S
in[c + (d*x)/2] + 147*B*Sin[c + (d*x)/2] - 120*C*Sin[c + (d*x)/2] + 342*A*S
in[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 164*C*Sin[c + (3*d*x)/2] + 1
18*A*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 36*C*Sin[2*c + (3*d
*x)/2] + 30*A*Sin[2*c + (5*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 12*C*Sin[2
*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[3*c + (5*d*x)/2] + 1
2*C*Sin[3*c + (5*d*x)/2] - 3*A*Sin[3*c + (7*d*x)/2] + 3*B*Sin[3*c + (7*d*x)
/2] - 3*A*Sin[4*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2] + A*Sin[4*c + (9*
```

```
d*x)/2] + A*Sin[5*c + (9*d*x)/2]))/(192*a^2*d)
```

Maple [B] time = 0.112, size = 482, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*B-1/6/d/a^
2*C*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2
*d*x+1/2*c)+5/2/d/a^2*C*tan(1/2*d*x+1/2*c)+10/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^3*tan(1/2*d*x+1/2*c)^5*A-5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)^5*B+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*C+40/3/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A-8/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*C*t
an(1/2*d*x+1/2*c)^3+6/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)
-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)+2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)-10/d/a^2*A*arctan(tan(1/2*d*x+1/2*c))
+7/d/a^2*B*arctan(tan(1/2*d*x+1/2*c))-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.45625, size = 657, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2) + C*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2 + 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1))))/d
```

Fricas [A] time = 0.512127, size = 437, normalized size = 2.36

$$-\frac{3\,(10\,A-7\,B+4\,C)dx\cos\left(dx+c\right)^{2}+6\,(10\,A-7\,B+4\,C)dx\cos\left(dx+c\right)+3\,(10\,A-7\,B+4\,C)dx-\left(2\,A\cos\left(dx+c\right)^{4}\right.}{6\left(a^{2}d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] -1/6*(3*(10*A - 7*B + 4*C)*d*x*cos(d*x + c)^2 + 6*(10*A - 7*B + 4*C)*d*x*co
s(d*x + c) + 3*(10*A - 7*B + 4*C)*d*x - (2*A*cos(d*x + c)^4 - (2*A - 3*B)*c
os(d*x + c)^3 + 6*(2*A - B + C)*cos(d*x + c)^2 + (66*A - 43*B + 28*C)*cos(d
*x + c) + 48*A - 32*B + 20*C)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.251, size = 359, normalized size = 1.94

$$-\frac{\frac{3\,(dx+c)(10\,A-7\,B+4\,C)}{a^2}-\frac{2\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,C\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/6*(3*(d*x + c)*(10*A - 7*B + 4*C)/a^2 - 2*(30*A*tan(1/2*d*x + 1/2*c)^5 -
 15*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 + 40*A*tan(1/2*d*
x + 1/2*c)^3 - 24*B*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1/2*d*x + 1/2*c)^3 +
18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3
- B*a^4*tan(1/2*d*x + 1/2*c)^3 + C*a^4*tan(1/2*d*x + 1/2*c)^3 - 27*A*a^4*ta
n(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2*d*x + 1/2*c) - 15*C*a^4*tan(1/2*d*x +
 1/2*c))/a^6)/d
```

3.466 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$-\frac{2(11A-36B+76C)\tan(c+dx)}{15a^3d}+\frac{(2A-6B+13C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(11A-36B+76C)\tan(c+dx)\sec^2(c+d}{15d\left(a^3\sec(c+dx)+a^3\right)}$$

```
[Out] ((2*A - 6*B + 13*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(11*A - 36*B + 76
*C)*Tan[c + d*x])/(15*a^3*d) + ((2*A - 6*B + 13*C)*Sec[c + d*x]*Tan[c + d*x
])/(2*a^3*d) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c
+ d*x])^3) - ((A - 6*B + 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*
Sec[c + d*x])^2) - ((11*A - 36*B + 76*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d
*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.516282, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4084, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{2(11A-36B+76C)\tan(c+dx)}{15a^3d}+\frac{(2A-6B+13C)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(11A-36B+76C)\tan(c+dx)\sec^2(c+d}{15d\left(a^3\sec(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```


```
[Out] ((2*A - 6*B + 13*C)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (2*(11*A - 36*B + 76
*C)*Tan[c + d*x])/(15*a^3*d) + ((2*A - 6*B + 13*C)*Sec[c + d*x]*Tan[c + d*x
])/(2*a^3*d) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c
+ d*x])^3) - ((A - 6*B + 11*C)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*
Sec[c + d*x])^2) - ((11*A - 36*B + 76*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d
*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)(a(A+4}{(}}{}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A-6B+11C)\,s}{15ad(a+}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A-6B+11C)\,s}{15ad(a+}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A-6B+11C)\,s}{15ad(a+}$$

$$= \frac{(2A-6B+13C)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A-B+C)\,s}{5d(a+}$$

$$= \frac{(2A-6B+13C)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{2(11A-36B+7}{15a^3}$$

Mathematica [B] time = 6.45349, size = 1081, normalized size = 5.

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^3,x]
```

```
[Out] (-8*(2*A - 6*B + 13*C)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/
2 + (d*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (8*
(2*A - 6*B + 13*C)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/
2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*(490*A*Sin[(d*x)/2] - 870*B*Sin[(d*x)/2] + 1235*C*Sin[(d*x)/2] - 5
30*A*Sin[(3*d*x)/2] + 1830*B*Sin[(3*d*x)/2] - 3805*C*Sin[(3*d*x)/2] + 654*A
*Sin[c - (d*x)/2] - 2094*B*Sin[c - (d*x)/2] + 4329*C*Sin[c - (d*x)/2] - 654
*A*Sin[c + (d*x)/2] + 1314*B*Sin[c + (d*x)/2] - 1989*C*Sin[c + (d*x)/2] + 4
90*A*Sin[2*c + (d*x)/2] - 1650*B*Sin[2*c + (d*x)/2] + 3575*C*Sin[2*c + (d*x
)/2] + 350*A*Sin[c + (3*d*x)/2] - 450*B*Sin[c + (3*d*x)/2] + 475*C*Sin[c +
(3*d*x)/2] - 530*A*Sin[2*c + (3*d*x)/2] + 1230*B*Sin[2*c + (3*d*x)/2] - 200
5*C*Sin[2*c + (3*d*x)/2] + 350*A*Sin[3*c + (3*d*x)/2] - 1050*B*Sin[3*c + (3
*d*x)/2] + 2275*C*Sin[3*c + (3*d*x)/2] - 378*A*Sin[c + (5*d*x)/2] + 1278*B*
Sin[c + (5*d*x)/2] - 2673*C*Sin[c + (5*d*x)/2] + 150*A*Sin[2*c + (5*d*x)/2]
 - 90*B*Sin[2*c + (5*d*x)/2] - 105*C*Sin[2*c + (5*d*x)/2] - 378*A*Sin[3*c +
 (5*d*x)/2] + 918*B*Sin[3*c + (5*d*x)/2] - 1593*C*Sin[3*c + (5*d*x)/2] + 15
0*A*Sin[4*c + (5*d*x)/2] - 450*B*Sin[4*c + (5*d*x)/2] + 975*C*Sin[4*c + (5*
d*x)/2] - 190*A*Sin[2*c + (7*d*x)/2] + 630*B*Sin[2*c + (7*d*x)/2] - 1325*C*
Sin[2*c + (7*d*x)/2] + 30*A*Sin[3*c + (7*d*x)/2] + 60*B*Sin[3*c + (7*d*x)/2
] - 255*C*Sin[3*c + (7*d*x)/2] - 190*A*Sin[4*c + (7*d*x)/2] + 480*B*Sin[4*c
 + (7*d*x)/2] - 875*C*Sin[4*c + (7*d*x)/2] + 30*A*Sin[5*c + (7*d*x)/2] - 90
*B*Sin[5*c + (7*d*x)/2] + 195*C*Sin[5*c + (7*d*x)/2] - 44*A*Sin[3*c + (9*d*
x)/2] + 144*B*Sin[3*c + (9*d*x)/2] - 304*C*Sin[3*c + (9*d*x)/2] + 30*B*Sin[
4*c + (9*d*x)/2] - 90*C*Sin[4*c + (9*d*x)/2] - 44*A*Sin[5*c + (9*d*x)/2] +
114*B*Sin[5*c + (9*d*x)/2] - 214*C*Sin[5*c + (9*d*x)/2]))/(240*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 0.082, size = 433, normalized size = 2.

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{2\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d
/a^3*C*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*tan(
1/2*d*x+1/2*c)^3*B-2/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan(1/2*d*x
+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-31/4/d/a^3*C*tan(1/2*d*x+1/2*c)+7/2
/d/a^3/(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B+1/d/a^3*ln
(tan(1/2*d*x+1/2*c)+1)*A-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B+13/2/d/a^3*ln(t
an(1/2*d*x+1/2*c)+1)*C-1/2/d/a^3*C/(tan(1/2*d*x+1/2*c)+1)^2+7/2/d/a^3/(tan(
1/2*d*x+1/2*c)-1)*C-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3*ln(tan(1/2*d*x
+1/2*c)-1)*A+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-13/2/d/a^3*ln(tan(1/2*d*x+1
/2*c)-1)*C+1/2/d/a^3*C/(tan(1/2*d*x+1/2*c)-1)^2
```

Maxima [B] time = 0.988907, size = 666, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] -1/60*(C*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3) + A*((105*sin(d*x + c)/(co
s(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A] time = 0.532569, size = 849, normalized size = 3.93

$$\frac{15\left((2A-6B+13C)\cos(dx+c)^5+3(2A-6B+13C)\cos(dx+c)^4+3(2A-6B+13C)\cos(dx+c)^3+(2A-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/60*(15*((2*A - 6*B + 13*C)*cos(d*x + c)^5 + 3*(2*A - 6*B + 13*C)*cos(d*x
+ c)^4 + 3*(2*A - 6*B + 13*C)*cos(d*x + c)^3 + (2*A - 6*B + 13*C)*cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 15*((2*A - 6*B + 13*C)*cos(d*x + c)^5 + 3*(2
*A - 6*B + 13*C)*cos(d*x + c)^4 + 3*(2*A - 6*B + 13*C)*cos(d*x + c)^3 + (2*
A - 6*B + 13*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*(11*A - 36*B
+ 76*C)*cos(d*x + c)^4 + 3*(34*A - 114*B + 239*C)*cos(d*x + c)^3 + (64*A -
234*B + 479*C)*cos(d*x + c)^2 - 15*(2*B - 3*C)*cos(d*x + c) - 15*C)*sin(d*x
 + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c
)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,d\cdots}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**6/(sec(c + d*x)
**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.25943, size = 389, normalized size = 1.8

$$\frac{\frac{30\,(2\,A-6\,B+13\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(2\,A-6\,B+13\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{60\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(30*(2*A - 6*B + 13*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(2*
A - 6*B + 13*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 60*(2*B*tan(1/2*d*
x + 1/2*c)^3 - 7*C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) + 5*C*
tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(
1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x
 + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^12*tan(1/2*d*x + 1/
2*c)^3 + 40*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c)
 - 255*B*a^12*tan(1/2*d*x + 1/2*c) + 465*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.467 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$\frac{(2A-7B+27C)\tan(c+dx)}{15a^3d}+\frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(B-3C)\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-B+C)\tan(c+dx)\,\mathrm{se}}{5d(a\sec(c+dx)+}$$

```
[Out] ((B - 3*C)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((2*A - 7*B + 27*C)*Tan[c + d*x
])/(15*a^3*d) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c
 + d*x])^3) + ((A + 4*B - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*
Sec[c + d*x])^2) - ((B - 3*C)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.45006, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4084, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{(2A-7B+27C)\tan(c+dx)}{15a^3d}+\frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(B-3C)\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-B+C)\tan(c+dx)\,\mathrm{se}}{5d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```

```
[Out] ((B - 3*C)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((2*A - 7*B + 27*C)*Tan[c + d*x
])/(15*a^3*d) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c
 + d*x])^3) + ((A + 4*B - 9*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*
Sec[c + d*x])^2) - ((B - 3*C)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)(a(2A+3B-}{(a+a\,s}}{}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A+4B-9C)\sec^2}{15ad(a+a\,s}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A+4B-9C)\sec^2}{15ad(a+a\,s}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A+4B-9C)\sec^2}{15ad(a+a\,s}$$

$$= \frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B+C)\sec^3(c+dx)\tan}{5d(a+a\sec(c+dx))^3}$$

$$= \frac{(B-3C)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(2A-7B+27C)\tan(c+dx}{15a^3d}$$

Mathematica [B] time = 6.3667, size = 839, normalized size = 5.21

$$\frac{16(3C-B)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(\sec(c+dx)a+a)^3} - \frac{16(3C-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^3,x]
```

```
[Out] (16*(-B + 3*C)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x
)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (16*(-B + 3
*C)*Cos[c/2 + (d*x)/2]^6*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c
 + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]*Se
c[c/2]*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-20*A
*Sin[(d*x)/2] + 160*B*Sin[(d*x)/2] - 255*C*Sin[(d*x)/2] + 22*A*Sin[(3*d*x)/
2] - 167*B*Sin[(3*d*x)/2] + 567*C*Sin[(3*d*x)/2] - 10*A*Sin[c - (d*x)/2] +
170*B*Sin[c - (d*x)/2] - 600*C*Sin[c - (d*x)/2] + 10*A*Sin[c + (d*x)/2] - 1
70*B*Sin[c + (d*x)/2] + 375*C*Sin[c + (d*x)/2] - 20*A*Sin[2*c + (d*x)/2] +
160*B*Sin[2*c + (d*x)/2] - 480*C*Sin[2*c + (d*x)/2] + 75*B*Sin[c + (3*d*x)/
2] - 60*C*Sin[c + (3*d*x)/2] + 22*A*Sin[2*c + (3*d*x)/2] - 167*B*Sin[2*c +
(3*d*x)/2] + 402*C*Sin[2*c + (3*d*x)/2] + 75*B*Sin[3*c + (3*d*x)/2] - 225*C
*Sin[3*c + (3*d*x)/2] + 10*A*Sin[c + (5*d*x)/2] - 95*B*Sin[c + (5*d*x)/2] +
 315*C*Sin[c + (5*d*x)/2] + 15*B*Sin[2*c + (5*d*x)/2] + 30*C*Sin[2*c + (5*d
*x)/2] + 10*A*Sin[3*c + (5*d*x)/2] - 95*B*Sin[3*c + (5*d*x)/2] + 240*C*Sin[
3*c + (5*d*x)/2] + 15*B*Sin[4*c + (5*d*x)/2] - 45*C*Sin[4*c + (5*d*x)/2] +
2*A*Sin[2*c + (7*d*x)/2] - 22*B*Sin[2*c + (7*d*x)/2] + 72*C*Sin[2*c + (7*d*
x)/2] + 15*C*Sin[3*c + (7*d*x)/2] + 2*A*Sin[4*c + (7*d*x)/2] - 22*B*Sin[4*c
 + (7*d*x)/2] + 57*C*Sin[4*c + (7*d*x)/2]))/(60*d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 0.072, size = 303, normalized size = 1.9

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{3\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/20/d/
a^3*C*tan(1/2*d*x+1/2*c)^5+1/6/d/a^3*tan(1/2*d*x+1/2*c)^3*A-1/3/d/a^3*tan(1
/2*d*x+1/2*c)^3*B+1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(1/2*d*x+
1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+1/d/a
^3*ln(tan(1/2*d*x+1/2*c)+1)*B-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/a^3/(t
an(1/2*d*x+1/2*c)+1)*C-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B+3/d/a^3*ln(tan(1/
2*d*x+1/2*c)-1)*C-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [B] time = 0.981516, size = 473, normalized size = 2.94

$$\frac{3\,C\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-B\left(\frac{105}{\cos}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] 1/60*(3*C*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
```

```
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3) + A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/
d
```

Fricas [A] time = 0.521837, size = 690, normalized size = 4.29

$$\frac{15\left((B-3C)\cos(dx+c)^4+3(B-3C)\cos(dx+c)^3+3(B-3C)\cos(dx+c)^2+(B-3C)\cos(dx+c)\right)\log(\sin(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/30*(15*((B - 3*C)*cos(d*x + c)^4 + 3*(B - 3*C)*cos(d*x + c)^3 + 3*(B - 3*
C)*cos(d*x + c)^2 + (B - 3*C)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((B
- 3*C)*cos(d*x + c)^4 + 3*(B - 3*C)*cos(d*x + c)^3 + 3*(B - 3*C)*cos(d*x +
c)^2 + (B - 3*C)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*(A - 11*B + 36
*C)*cos(d*x + c)^3 + 3*(2*A - 17*B + 57*C)*cos(d*x + c)^2 + (7*A - 32*B + 1
17*C)*cos(d*x + c) + 15*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**5/(sec(c + d*x)
**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.2796, size = 316, normalized size = 1.96

$$\frac{\frac{60(B-3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60(B-3C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{120C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}+\frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3C}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(60*(B - 3*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(B - 3*C)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 120*C*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 - 1)*a^3) + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan
(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 + 10*A*a^12*tan(1/2*d
*x + 1/2*c)^3 - 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*C*a^12*tan(1/2*d*x +
1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/2*c)
 + 255*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.468 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$\frac{(6A+4B-29C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(3A+2B-7C)\tan(c}{15ad(a\sec(c+dx)+}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x
])/(5*d*(a + a*Sec[c + d*x])^3) - ((3*A + 2*B - 7*C)*Tan[c + d*x])/(15*a*d*
(a + a*Sec[c + d*x])^2) + ((6*A + 4*B - 29*C)*Tan[c + d*x])/(15*d*(a^3 + a^
3*Sec[c + d*x]))
```

Rubi [A] time = 0.350824, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4084, 4008, 3998, 3770, 3794}

$$\frac{(6A+4B-29C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(3A+2B-7C)\tan(c}{15ad(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x
])/(5*d*(a + a*Sec[c + d*x])^3) - ((3*A + 2*B - 7*C)*Tan[c + d*x])/(15*a*d*
(a + a*Sec[c + d*x])^2) + ((6*A + 4*B - 29*C)*Tan[c + d*x])/(15*d*(a^3 + a^
3*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^2(c+dx)(a(3A+}{(a+a}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(3A+2B-7C)}{15ad(a+a\sec}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(3A+2B-7C)}{15ad(a+a\sec}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B+C)\sec^2(c+dx)\tan(c+}{5d(a+a\sec(c+dx))^3}$$

Mathematica [B] time = 1.61372, size = 277, normalized size = 2.1

$$-\frac{\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(240C\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^3,x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*(240*C*Cos[(c + d*x)/2]^6*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) - Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 4*B - 29*C)*Sin[(d*x)/2] - 15*(A
- 5*C)*Sin[c + (d*x)/2] + 15*A*Sin[c + (3*d*x)/2] + 10*B*Sin[c + (3*d*x)/2]
 - 95*C*Sin[c + (3*d*x)/2] + 15*C*Sin[2*c + (3*d*x)/2] + 3*A*Sin[2*c + (5*d
*x)/2] + 2*B*Sin[2*c + (5*d*x)/2] - 22*C*Sin[2*c + (5*d*x)/2])))/(15*a^3*d*
(1 + Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.069, size = 197, normalized size = 1.5

$$\frac{B}{6\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{C}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + \frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{C}{20\,da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/6/d/a^3*tan(1/2*d*x+1/2*c)^3*B+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*C+1/20/d/
a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*C*t
an(1/2*d*x+1/2*c)^5-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)-1/d/a^3*ln(tan(1/2*d*x+1
/2*c)-1)*C+1/4/d/a^3*A*tan(1/2*d*x+1/2*c)+1/4/d/a^3*B*tan(1/2*d*x+1/2*c)-1/
3/d/a^3*C*tan(1/2*d*x+1/2*c)^3
```

Maxima [A] time = 0.967131, size = 313, normalized size = 2.37

$$-\frac{C\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}-}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] -1/60*(C*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 3*A*(5*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.511033, size = 505, normalized size = 3.83

$$\frac{15\left(C\cos(dx+c)^3+3\,C\cos(dx+c)^2+3\,C\cos(dx+c)+C\right)\log(\sin(dx+c)+1)-15\left(C\cos(dx+c)^3+3\,C\cos(dx+\right.}{30\left(a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/30*(15*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*log
(sin(d*x + c) + 1) - 15*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*
x + c) + C)*log(-sin(d*x + c) + 1) + 2*((3*A + 2*B - 22*C)*cos(d*x + c)^2 +
 3*(3*A + 2*B - 17*C)*cos(d*x + c) + 3*A + 7*B - 32*C)*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{C\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**4/(sec(c + d*x)
**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.31097, size = 243, normalized size = 1.84

$$\frac{\frac{60\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,Ba^{12}\tan\left(\frac{1}{2}\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(60*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*C*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 10*B*a^12*tan(1/2*d*x +
1/2*c)^3 + 20*C*a^12*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^12*tan(1/2*d*x + 1/2*c
) - 15*B*a^12*tan(1/2*d*x + 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.469 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=110

$$\frac{(2A+3B+7C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-B+C)\tan(c+dx)\sec(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(A-C)\tan(c+dx)}{3ad(a\sec(c+dx)+a)^2}$$

```
[Out] -((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((A
 - C)*Tan[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 3*B + 7*C)*Tan
[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.207278, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4078, 4000, 3794}

$$\frac{(2A+3B+7C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-B+C)\tan(c+dx)\sec(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(A-C)\tan(c+dx)}{3ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^3,x]
```

```
[Out] -((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((A
 - C)*Tan[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 3*B + 7*C)*Tan
[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4078

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*B - b*C - 2*A*b*(m + 1) - (b*B*(m + 2) -
a*(A*(m + 2) - C*(m - 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(a(4A+B-C}{(a+a\,s}}{}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-C)\tan(c+}{3ad(a+a\sec(c+}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-C)\tan(c+}{3ad(a+a\sec(c+}$$

Mathematica [A] time = 0.566438, size = 156, normalized size = 1.42

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-15(2A+B)\sin\left(c+\frac{dx}{2}\right)+5(8A+3B+4C)\sin\left(\frac{dx}{2}\right)+20A\sin\left(c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{3}{}\right.\right.}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(5*(8*A + 3*B + 4*C)*Sin[(d*x)/2] - 15*(2*A +
B)*Sin[c + (d*x)/2] + 20*A*Sin[c + (3*d*x)/2] + 15*B*Sin[c + (3*d*x)/2] + 1
0*C*Sin[c + (3*d*x)/2] - 15*A*Sin[2*c + (3*d*x)/2] + 7*A*Sin[2*c + (5*d*x)/
2] + 3*B*Sin[2*c + (5*d*x)/2] + 2*C*Sin[2*c + (5*d*x)/2]))/(240*a^3*d)
```

Maple [A] time = 0.068, size = 113, normalized size = 1.

$$\frac{1}{4\,da^3}\left(\frac{A}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,A}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{2\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5*A-1/5*tan(1/2*d*x+1/2*c)^5*B+1/5*C*tan(
1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3*A+2/3*C*tan(1/2*d*x+1/2*c)^3+A*ta
n(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.976994, size = 242, normalized size = 2.2

$$\frac{\frac{C\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/60*(C*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + A*(15*sin(d*x + c)
```

```
/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c)/(cos(d*x + c) + 1) - s
in(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.459965, size = 251, normalized size = 2.28

$$\frac{\left((7A+3B+2C)\cos(dx+c)^2+3(2A+3B+2C)\cos(dx+c)+2A+3B+7C\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/15*((7*A + 3*B + 2*C)*cos(d*x + c)^2 + 3*(2*A + 3*B + 2*C)*cos(d*x + c) +
 2*A + 3*B + 7*C)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int\frac{B\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int\frac{C\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x
)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1), x) + Integral(C*sec(c + d*x)**3/(sec(c + d*x)**3
 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.27846, size = 155, normalized size = 1.41

$$\frac{3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+10C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 3*C*tan(1/2
*d*x + 1/2*c)^5 - 10*A*tan(1/2*d*x + 1/2*c)^3 + 10*C*tan(1/2*d*x + 1/2*c)^3
 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c) + 15*C*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.470 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=115

$$-\frac{(22A-2B-3C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-2B-3C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B+C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] (A*x)/a^3 - ((A - B + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A
 - 2*B - 3*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((22*A - 2*B
- 3*C)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.196376, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4052, 3922, 3919, 3794}

$$-\frac{(22A-2B-3C)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-2B-3C)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B+C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (A*x)/a^3 - ((A - B + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A
 - 2*B - 3*C)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((22*A - 2*B
- 3*C)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5aA+a(2A-2B-3C)\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B-3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2A-a^2(7A-2B}{a+a\sec(}}{15}$$

$$= \frac{Ax}{a^3} - \frac{(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B-3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(22A-2B}{}$$

$$= \frac{Ax}{a^3} - \frac{(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B-3C)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(22A-2B}{15d\left(a^3+\right.}$$

Mathematica [B] time = 0.908352, size = 289, normalized size = 2.51

$$\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(270A\sin\left(c+\frac{dx}{2}\right)-230A\sin\left(c+\frac{3dx}{2}\right)+90A\sin\left(2c+\frac{3dx}{2}\right)-64A\sin\left(2c+\frac{5dx}{2}\right)+150Adx\,\mathrm{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*A*d*x*Cos[(d*x)/2] + 150*A*d*x*Cos[c + (d
*x)/2] + 75*A*d*x*Cos[c + (3*d*x)/2] + 75*A*d*x*Cos[2*c + (3*d*x)/2] + 15*A
*d*x*Cos[2*c + (5*d*x)/2] + 15*A*d*x*Cos[3*c + (5*d*x)/2] - 370*A*Sin[(d*x)
/2] + 80*B*Sin[(d*x)/2] + 30*C*Sin[(d*x)/2] + 270*A*Sin[c + (d*x)/2] - 60*B
*Sin[c + (d*x)/2] - 30*C*Sin[c + (d*x)/2] - 230*A*Sin[c + (3*d*x)/2] + 40*B
*Sin[c + (3*d*x)/2] + 30*C*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c + (3*d*x)/2] -
 30*B*Sin[2*c + (3*d*x)/2] - 64*A*Sin[2*c + (5*d*x)/2] + 14*B*Sin[2*c + (5*
d*x)/2] + 6*C*Sin[2*c + (5*d*x)/2]))/(480*a^3*d)
```

Maple [A] time = 0.079, size = 175, normalized size = 1.5

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{6\,da^3}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d
/a^3*C*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^3*tan(
1/2*d*x+1/2*c)^3*B-7/4/d/a^3*A*tan(1/2*d*x+1/2*c)+1/4/d/a^3*B*tan(1/2*d*x+1
/2*c)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.43891, size = 277, normalized size = 2.41

$$-\frac{A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}-\frac{3C\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(}{(\cos(d}\right.}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3 - 3*C*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*
x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.48512, size = 375, normalized size = 3.26

$$\frac{15\,Adx\cos\left(dx+c\right)^{3}+45\,Adx\cos\left(dx+c\right)^{2}+45\,Adx\cos\left(dx+c\right)+15\,Adx-\left(\left(32\,A-7\,B-3\,C\right)\cos\left(dx+c\right)^{2}+3\,(1\cdots}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] 1/15*(15*A*d*x*cos(d*x + c)^3 + 45*A*d*x*cos(d*x + c)^2 + 45*A*d*x*cos(d*x
+ c) + 15*A*d*x - ((32*A - 7*B - 3*C)*cos(d*x + c)^2 + 3*(17*A - 2*B - 3*C)
*cos(d*x + c) + 22*A - 2*B - 3*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int\frac{C\sec^{2}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,d\cdots}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)
+ Integral(B*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x
)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.30671, size = 207, normalized size = 1.8

$$\frac{\frac{60\,(dx+c)A}{a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+10\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,Aa^{12}\tan\left(\cdots\right.}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] 1/60*(60*(d*x + c)*A/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*
x + 1/2*c)^3 + 10*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x +
1/2*c) - 15*B*a^12*tan(1/2*d*x + 1/2*c) - 15*C*a^12*tan(1/2*d*x + 1/2*c))/a
^15)/d
```

3.471 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$\frac{2(36A-11B+C)\sin(c+dx)}{15a^3d}-\frac{(3A-B)\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(3A-B)}{a^3}-\frac{(9A-4B-C)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B+C)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -(((3*A - B)*x)/a^3) + (2*(36*A - 11*B + C)*Sin[c + d*x])/(15*a^3*d) - ((A
- B + C)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - 4*B - C)*Sin[
c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((3*A - B)*Sin[c + d*x])/(d*(a^
3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.403664, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4084, 4020, 3787, 2637, 8}

$$\frac{2(36A-11B+C)\sin(c+dx)}{15a^3d}-\frac{(3A-B)\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(3A-B)}{a^3}-\frac{(9A-4B-C)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B+C)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^3,x]
```

```
[Out] -(((3*A - B)*x)/a^3) + (2*(36*A - 11*B + C)*Sin[c + d*x])/(15*a^3*d) - ((A
- B + C)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - 4*B - C)*Sin[
c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((3*A - B)*Sin[c + d*x])/(d*(a^
3 + a^3*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(a(6A-B+C)-a(3A-3B-2C)\sec}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B-C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \int$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B-C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3A}{d\left(a^3\right.}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B-C)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3A}{d\left(a^3\right.}$$

$$= -\frac{(3A-B)x}{a^3} + \frac{2(36A-11B+C)\sin(c+dx)}{15a^3d} - \frac{(A-B+C)\sin}{5d(a+a\sec(}$$

Mathematica [B] time = 1.63649, size = 419, normalized size = 2.97

$$\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-300dx(3A-B)\cos\left(c+\frac{dx}{2}\right)-300dx(3A-B)\cos\left(\frac{dx}{2}\right)-1125A\sin\left(c+\frac{dx}{2}\right)+1215A\sin\left(c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(-300*(3*A - B)*d*x*Cos[(d*x)/2] - 300*(3*A -
B)*d*x*Cos[c + (d*x)/2] - 450*A*d*x*Cos[c + (3*d*x)/2] + 150*B*d*x*Cos[c +
(3*d*x)/2] - 450*A*d*x*Cos[2*c + (3*d*x)/2] + 150*B*d*x*Cos[2*c + (3*d*x)/2
] - 90*A*d*x*Cos[2*c + (5*d*x)/2] + 30*B*d*x*Cos[2*c + (5*d*x)/2] - 90*A*d*
x*Cos[3*c + (5*d*x)/2] + 30*B*d*x*Cos[3*c + (5*d*x)/2] + 1755*A*Sin[(d*x)/2
] - 740*B*Sin[(d*x)/2] + 160*C*Sin[(d*x)/2] - 1125*A*Sin[c + (d*x)/2] + 540
*B*Sin[c + (d*x)/2] - 120*C*Sin[c + (d*x)/2] + 1215*A*Sin[c + (3*d*x)/2] -
460*B*Sin[c + (3*d*x)/2] + 80*C*Sin[c + (3*d*x)/2] - 225*A*Sin[2*c + (3*d*x
)/2] + 180*B*Sin[2*c + (3*d*x)/2] - 60*C*Sin[2*c + (3*d*x)/2] + 363*A*Sin[2
*c + (5*d*x)/2] - 128*B*Sin[2*c + (5*d*x)/2] + 28*C*Sin[2*c + (5*d*x)/2] +
75*A*Sin[3*c + (5*d*x)/2] + 15*A*Sin[3*c + (7*d*x)/2] + 15*A*Sin[4*c + (7*d
*x)/2]))/(960*a^3*d)
```

Maple [A] time = 0.11, size = 247, normalized size = 1.8

$$\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{3\,da^3}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/20/d/
a^3*C*tan(1/2*d*x+1/2*c)^5-1/2/d/a^3*tan(1/2*d*x+1/2*c)^3*A+1/3/d/a^3*tan(1
/2*d*x+1/2*c)^3*B-1/6/d/a^3*C*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan(1/2*d*x
+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+1/4/d/a^3*C*tan(1/2*d*x+1/2*c)+2/d/a
^3*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^3*A*arctan(tan(1/2*d
*x+1/2*c))+2/d/a^3*B*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.45872, size = 398, normalized size = 2.82

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-B\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
C*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A] time = 0.498218, size = 455, normalized size = 3.23

$$-\frac{15\,(3\,A-B)dx\cos\left(dx+c\right)^3+45\,(3\,A-B)dx\cos\left(dx+c\right)^2+45\,(3\,A-B)dx\cos\left(dx+c\right)+15\,(3\,A-B)dx-\left(15\,A\,c\right.}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/15*(15*(3*A - B)*d*x*cos(d*x + c)^3 + 45*(3*A - B)*d*x*cos(d*x + c)^2 +
45*(3*A - B)*d*x*cos(d*x + c) + 15*(3*A - B)*d*x - (15*A*cos(d*x + c)^3 + (
117*A - 32*B + 7*C)*cos(d*x + c)^2 + 3*(57*A - 17*B + 2*C)*cos(d*x + c) + 7
2*A - 22*B + 2*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.22984, size = 278, normalized size = 1.97

$$-\frac{\frac{60\,(dx+c)(3\,A-B)}{a^3}-\frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ca^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,Ba^{12}\,\mathrm{t}\cdots}{\cdots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*(3*A - B)/a^3 - 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/
2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x
+ 1/2*c)^3 + 20*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*C*a^12*tan(1/2*d*x + 1/2
*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/2*c) +
 15*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.472 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{2(76A-36B+11C)\sin(c+dx)}{15a^3d}+\frac{(13A-6B+2C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(76A-36B+11C)\sin(c+dx)\cos(c}{15d\left(a^3\sec(c+dx)+a^3\right)}$$

```
[Out] ((13*A - 6*B + 2*C)*x)/(2*a^3) - (2*(76*A - 36*B + 11*C)*Sin[c + d*x])/(15*
a^3*d) + ((13*A - 6*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B
 + C)*Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A - 6*
B + C)*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((76*A
- 36*B + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.517399, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4020, 3787, 2635, 8, 2637}

$$-\frac{2(76A-36B+11C)\sin(c+dx)}{15a^3d}+\frac{(13A-6B+2C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(76A-36B+11C)\sin(c+dx)\cos(c}{15d\left(a^3\sec(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```



```
[Out] ((13*A - 6*B + 2*C)*x)/(2*a^3) - (2*(76*A - 36*B + 11*C)*Sin[c + d*x])/(15*
a^3*d) + ((13*A - 6*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B
 + C)*Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A - 6*
B + C)*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((76*A
- 36*B + 11*C)*Cos[c + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(a(7A-2B+2C}{(a+a\sec}}{}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B+C)\cos(c}{15ad(a+a\sec}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B+C)\cos(c}{15ad(a+a\sec}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B+C)\cos(c}{15ad(a+a\sec}$$

$$= -\frac{2(76A-36B+11C)\sin(c+dx)}{15a^3d} + \frac{(13A-6B+2C)\cos(c+}{2a^3d}$$

$$= \frac{(13A-6B+2C)x}{2a^3} - \frac{2(76A-36B+11C)\sin(c+dx)}{15a^3d} + \frac{(13A}{}$$

Mathematica [B] time = 1.58546, size = 557, normalized size = 2.77

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(600dx(13A-6B+2C)\cos\left(c+\frac{dx}{2}\right)+600dx(13A-6B+2C)\cos\left(\frac{dx}{2}\right)+7560A\sin\left(c+\frac{dx}{2}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(600*(13*A - 6*B + 2*C)*d*x*Cos[(d*x)/2] + 600
*(13*A - 6*B + 2*C)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] -
1800*B*d*x*Cos[c + (3*d*x)/2] + 600*C*d*x*Cos[c + (3*d*x)/2] + 3900*A*d*x*C
os[2*c + (3*d*x)/2] - 1800*B*d*x*Cos[2*c + (3*d*x)/2] + 600*C*d*x*Cos[2*c +
 (3*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 360*B*d*x*Cos[2*c + (5*d*x)/
2] + 120*C*d*x*Cos[2*c + (5*d*x)/2] + 780*A*d*x*Cos[3*c + (5*d*x)/2] - 360*
B*d*x*Cos[3*c + (5*d*x)/2] + 120*C*d*x*Cos[3*c + (5*d*x)/2] - 12760*A*Sin[(
d*x)/2] + 7020*B*Sin[(d*x)/2] - 2960*C*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/
2] - 4500*B*Sin[c + (d*x)/2] + 2160*C*Sin[c + (d*x)/2] - 9230*A*Sin[c + (3*
```

```
d*x)/2] + 4860*B*Sin[c + (3*d*x)/2] - 1840*C*Sin[c + (3*d*x)/2] + 930*A*Sin
[2*c + (3*d*x)/2] - 900*B*Sin[2*c + (3*d*x)/2] + 720*C*Sin[2*c + (3*d*x)/2]
 - 2782*A*Sin[2*c + (5*d*x)/2] + 1452*B*Sin[2*c + (5*d*x)/2] - 512*C*Sin[2*
c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] + 300*B*Sin[3*c + (5*d*x)/2] -
105*A*Sin[3*c + (7*d*x)/2] + 60*B*Sin[3*c + (7*d*x)/2] - 105*A*Sin[4*c + (7
*d*x)/2] + 60*B*Sin[4*c + (7*d*x)/2] + 15*A*Sin[4*c + (9*d*x)/2] + 15*A*Sin
[5*c + (9*d*x)/2]))/(3840*a^3*d)
```

Maple [A] time = 0.128, size = 369, normalized size = 1.8

$$-\frac{A}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{B}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{C}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2\,A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{B}{2\,da^3}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A+1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B-1/20/d
/a^3*C*tan(1/2*d*x+1/2*c)^5+2/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A-1/2/d/a^3*tan(
1/2*d*x+1/2*c)^3*B+1/3/d/a^3*C*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*tan(1/2*d*
x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-7/4/d/a^3*C*tan(1/2*d*x+1/2*c)-7/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+2/d/a^3/(1+tan(1/2*d
*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*A*
tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+
13/d/a^3*A*arctan(tan(1/2*d*x+1/2*c))-6/d/a^3*B*arctan(tan(1/2*d*x+1/2*c))+
2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.46466, size = 555, normalized size = 2.76

$$-\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3B\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(d\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*B*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3) + C*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3
- 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A] time = 0.511017, size = 556, normalized size = 2.77

$$\frac{15\,(13\,A-6\,B+2\,C)dx\cos\left(dx+c\right)^{3}+45\,(13\,A-6\,B+2\,C)dx\cos\left(dx+c\right)^{2}+45\,(13\,A-6\,B+2\,C)dx\cos\left(dx+c\right)+15}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] 1/30*(15*(13*A - 6*B + 2*C)*d*x*cos(d*x + c)^3 + 45*(13*A - 6*B + 2*C)*d*x*
cos(d*x + c)^2 + 45*(13*A - 6*B + 2*C)*d*x*cos(d*x + c) + 15*(13*A - 6*B +
2*C)*d*x + (15*A*cos(d*x + c)^4 - 15*(3*A - 2*B)*cos(d*x + c)^3 - (479*A -
234*B + 64*C)*cos(d*x + c)^2 - 3*(239*A - 114*B + 34*C)*cos(d*x + c) - 304*
A + 144*B - 44*C)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22925, size = 340, normalized size = 1.69

$$\frac{\frac{30\,(dx+c)(13\,A-6\,B+2\,C)}{a^3}-\frac{60\left(7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{12}\tan\left(\frac{1}{2}\,d\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*A - 6*B + 2*C)/a^3 - 60*(7*A*tan(1/2*d*x + 1/2*c)^3
- 2*B*tan(1/2*d*x + 1/2*c)^3 + 5*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5 -
 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*C
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12
*tan(1/2*d*x + 1/2*c) + 105*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.473 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{4(34A-19B+9C)\sin^3(c+dx)}{15a^3d}+\frac{4(34A-19B+9C)\sin(c+dx)}{5a^3d}-\frac{(23A-13B+6C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(2$$

```
[Out] -((23*A - 13*B + 6*C)*x)/(2*a^3) + (4*(34*A - 19*B + 9*C)*Sin[c + d*x])/(5*
a^3*d) - ((23*A - 13*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A -
B + C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A -
 8*B + 3*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) -
((23*A - 13*B + 6*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d
*x])) - (4*(34*A - 19*B + 9*C)*Sin[c + d*x]^3)/(15*a^3*d)
```

Rubi [A] time = 0.544866, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4020, 3787, 2633, 2635, 8}

$$-\frac{4(34A-19B+9C)\sin^3(c+dx)}{15a^3d}+\frac{4(34A-19B+9C)\sin(c+dx)}{5a^3d}-\frac{(23A-13B+6C)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^3,x]
```



```
[Out] -((23*A - 13*B + 6*C)*x)/(2*a^3) + (4*(34*A - 19*B + 9*C)*Sin[c + d*x])/(5*
a^3*d) - ((23*A - 13*B + 6*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A -
B + C)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A -
 8*B + 3*C)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) -
((23*A - 13*B + 6*C)*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d
*x])) - (4*(34*A - 19*B + 9*C)*Sin[c + d*x]^3)/(15*a^3*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(a(8A-3B+}{(a+a\sec}}{5}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B+3C)\text{co}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B+3C)\text{co}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B+3C)\text{co}}{15ad(a+a}$$

$$= -\frac{(23A-13B+6C)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A-B+C)\text{co}}{5d(a+}$$

$$= -\frac{(23A-13B+6C)x}{2a^3} + \frac{4(34A-19B+9C)\sin(c+dx)}{5a^3d} - \frac{(23}{}$$

Mathematica [B] time = 2.64782, size = 655, normalized size = 2.76

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-600dx(23A-13B+6C)\cos\left(c+\frac{dx}{2}\right)-600dx(23A-13B+6C)\cos\left(\frac{dx}{2}\right)-11110A\sin\left(c+\frac{dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(-600*(23*A - 13*B + 6*C)*d*x*Cos[(d*x)/2] - 6
00*(23*A - 13*B + 6*C)*d*x*Cos[c + (d*x)/2] - 6900*A*d*x*Cos[c + (3*d*x)/2]
 + 3900*B*d*x*Cos[c + (3*d*x)/2] - 1800*C*d*x*Cos[c + (3*d*x)/2] - 6900*A*d
*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (3*d*x)/2] - 1800*C*d*x*Cos[
2*c + (3*d*x)/2] - 1380*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5
```

```
*d*x)/2] - 360*C*d*x*Cos[2*c + (5*d*x)/2] - 1380*A*d*x*Cos[3*c + (5*d*x)/2]
 + 780*B*d*x*Cos[3*c + (5*d*x)/2] - 360*C*d*x*Cos[3*c + (5*d*x)/2] + 20410*
A*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] + 7020*C*Sin[(d*x)/2] - 11110*A*Sin[c
 + (d*x)/2] + 7560*B*Sin[c + (d*x)/2] - 4500*C*Sin[c + (d*x)/2] + 15380*A*S
in[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] + 4860*C*Sin[c + (3*d*x)/2] -
 380*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] - 900*C*Sin[2*c +
(3*d*x)/2] + 4777*A*Sin[2*c + (5*d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 14
52*C*Sin[2*c + (5*d*x)/2] + 1625*A*Sin[3*c + (5*d*x)/2] - 750*B*Sin[3*c + (
5*d*x)/2] + 300*C*Sin[3*c + (5*d*x)/2] + 230*A*Sin[3*c + (7*d*x)/2] - 105*B
*Sin[3*c + (7*d*x)/2] + 60*C*Sin[3*c + (7*d*x)/2] + 230*A*Sin[4*c + (7*d*x)
/2] - 105*B*Sin[4*c + (7*d*x)/2] + 60*C*Sin[4*c + (7*d*x)/2] - 20*A*Sin[4*c
 + (9*d*x)/2] + 15*B*Sin[4*c + (9*d*x)/2] - 20*A*Sin[5*c + (9*d*x)/2] + 15*
B*Sin[5*c + (9*d*x)/2] + 5*A*Sin[5*c + (11*d*x)/2] + 5*A*Sin[6*c + (11*d*x)
/2]))/(3840*a^3*d)
```

Maple [B] time = 0.122, size = 542, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*A-1/20/d/a^3*tan(1/2*d*x+1/2*c)^5*B+1/20/d/
a^3*C*tan(1/2*d*x+1/2*c)^5-5/6/d/a^3*tan(1/2*d*x+1/2*c)^3*A+2/3/d/a^3*tan(1
/2*d*x+1/2*c)^3*B-1/2/d/a^3*C*tan(1/2*d*x+1/2*c)^3+49/4/d/a^3*A*tan(1/2*d*x
+1/2*c)-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)+17/4/d/a^3*C*tan(1/2*d*x+1/2*c)+17/
d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A-7/d/a^3/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^5*C+76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^3*A-12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B+4/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)^3+11/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/
2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)-23/d/a
^3*A*arctan(tan(1/2*d*x+1/2*c))+13/d/a^3*B*arctan(tan(1/2*d*x+1/2*c))-6/d/a
^3*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.46487, size = 738, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] 1/60*(A*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
```

```
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 3*C*(40*sin(d*x + c)/((a^3
+ a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*
x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^3))/d
```

Fricas [A] time = 0.519655, size = 613, normalized size = 2.59

$$-\frac{15\,(23\,A-13\,B+6\,C)dx\cos(dx+c)^3+45\,(23\,A-13\,B+6\,C)dx\cos(dx+c)^2+45\,(23\,A-13\,B+6\,C)dx\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] -1/30*(15*(23*A - 13*B + 6*C)*d*x*cos(d*x + c)^3 + 45*(23*A - 13*B + 6*C)*d
*x*cos(d*x + c)^2 + 45*(23*A - 13*B + 6*C)*d*x*cos(d*x + c) + 15*(23*A - 13
*B + 6*C)*d*x - (10*A*cos(d*x + c)^5 - 15*(A - B)*cos(d*x + c)^4 + 5*(19*A
- 9*B + 6*C)*cos(d*x + c)^3 + (869*A - 479*B + 234*C)*cos(d*x + c)^2 + 3*(4
29*A - 239*B + 114*C)*cos(d*x + c) + 544*A - 304*B + 144*C)*sin(d*x + c))/(
a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19437, size = 432, normalized size = 1.82

$$-\frac{\frac{30\,(dx+c)(23\,A-13\,B+6\,C)}{a^3}-\frac{20\left(51\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+76\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,C\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -1/60*(30*(d*x + c)*(23*A - 13*B + 6*C)/a^3 - 20*(51*A*tan(1/2*d*x + 1/2*c)
^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/2*d*x + 1/2*c)^5 + 76*A*tan(1/
```

```
2*d*x + 1/2*c)^3 - 36*B*tan(1/2*d*x + 1/2*c)^3 + 12*C*tan(1/2*d*x + 1/2*c)^
3 + 33*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1
/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^12*tan(1/2*d*x + 1/2*c)^5
 - 50*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*B*a^12*tan(1/2*d*x + 1/2*c)^3 - 30
*C*a^12*tan(1/2*d*x + 1/2*c)^3 + 735*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^
12*tan(1/2*d*x + 1/2*c) + 255*C*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.474 $\int \frac{\sec^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=254

$$-\frac{8(20A-83B+216C)\tan(c+dx)}{105a^4d}+\frac{(2A-8B+21C)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{(10A-52B+129C)\tan(c+dx)\sec^3(c+}{105a^4d(\sec(c+dx)+1)^2}$$

```
[Out] ((2*A - 8*B + 21*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(20*A - 83*B + 21
6*C)*Tan[c + d*x])/(105*a^4*d) + ((2*A - 8*B + 21*C)*Sec[c + d*x]*Tan[c + d
*x])/(2*a^4*d) - ((10*A - 52*B + 129*C)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a
^4*d*(1 + Sec[c + d*x])^2) - (4*(20*A - 83*B + 216*C)*Sec[c + d*x]^2*Tan[c
+ d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^5*Tan[c
+ d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((B - 2*C)*Sec[c + d*x]^4*Tan[c + d*
x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.689982, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4084, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{8(20A-83B+216C)\tan(c+dx)}{105a^4d}+\frac{(2A-8B+21C)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{(10A-52B+129C)\tan(c+dx)\sec^3(c+}{105a^4d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^4,x]
```

```
[Out] ((2*A - 8*B + 21*C)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(20*A - 83*B + 21
6*C)*Tan[c + d*x])/(105*a^4*d) + ((2*A - 8*B + 21*C)*Sec[c + d*x]*Tan[c + d
*x])/(2*a^4*d) - ((10*A - 52*B + 129*C)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a
^4*d*(1 + Sec[c + d*x])^2) - (4*(20*A - 83*B + 216*C)*Sec[c + d*x]^2*Tan[c
+ d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^5*Tan[c
+ d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((B - 2*C)*Sec[c + d*x]^4*Tan[c + d*
x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \sec^5(c+dx)(a(2A+\cdots}{\cdots}$$

$$= -\frac{(A-B+C)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(B-2C)\sec^4(c\cdots}{5ad(a+a\,s\cdots}$$

$$= -\frac{(10A-52B+129C)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B\cdots}{7\cdots}$$

$$= -\frac{(10A-52B+129C)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B\cdots}{7\cdots}$$

$$= -\frac{(10A-52B+129C)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B\cdots}{7\cdots}$$

$$= \frac{(2A-8B+21C)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(10A-52B+\cdots}{105\cdots}$$

$$= \frac{(2A-8B+21C)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{8(20A-83B+2\cdots}{105\cdots}$$

Mathematica [B] time = 6.47611, size = 1322, normalized size = 5.2

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^4,x]
```

```
[Out] (-16*(2*A - 8*B + 21*C)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c
/2 + (d*x)/2]]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) +
(16*(2*A - 8*B + 21*C)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/
2 + (d*x)/2]]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (
4*Cos[c/2 + (d*x)/2]^2*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*(A*Sin[c/2] - B*Sin[c/2] + C*Sin[c/2]))/(7*d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (8*Cos[c/2 + (d*x
)/2]^4*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(10*
A*Sin[c/2] - 17*B*Sin[c/2] + 24*C*Sin[c/2]))/(35*d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (16*Cos[c/2 + (d*x)/2]^
6*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(55*A*Sin
[c/2] - 139*B*Sin[c/2] + 258*C*Sin[c/2]))/(105*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (4*Cos[c/2 + (d*x)/2]*Sec
[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*Sin[(d*x)/2
] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (8*Cos[c/2 + (d*x)/2]^3*Sec[c/2
]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(10*A*Sin[(d*x)/2]
 - 17*B*Sin[(d*x)/2] + 24*C*Sin[(d*x)/2]))/(35*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (16*Cos[c/2 + (d*x)/2]^5*
Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(55*A*Sin[(
d*x)/2] - 139*B*Sin[(d*x)/2] + 258*C*Sin[(d*x)/2]))/(105*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (32*Cos[c/2 + (
d*x)/2]^7*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(
160*A*Sin[(d*x)/2] - 559*B*Sin[(d*x)/2] + 1308*C*Sin[(d*x)/2]))/(105*d*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) + (16
*C*Cos[c/2 + (d*x)/2]^8*Sec[c]*Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*Sin[d*x])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
a + a*Sec[c + d*x])^4) + (16*Cos[c/2 + (d*x)/2]^8*Sec[c]*Sec[c + d*x]^3*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(C*Sin[c] + 2*B*Sin[d*x] - 8*C*Sin[d*x
]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x
])^4)
```

Maple [B] time = 0.086, size = 493, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/56/d
/a^4*C*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5*A+7/40/d/a^4*tan
(1/2*d*x+1/2*c)^5*B-9/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-11/24/d/a^4*A*tan(1/2
*d*x+1/2*c)^3+23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-13/8/d/a^4*C*tan(1/2*d*x+1
/2*c)^3-15/8/d/a^4*A*tan(1/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-111
/8/d/a^4*C*tan(1/2*d*x+1/2*c)+9/2/d/a^4*C/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4/(t
an(1/2*d*x+1/2*c)+1)*B+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A-4/d/a^4*ln(tan(1/
2*d*x+1/2*c)+1)*B+21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d/a^4*C/(tan(1/
2*d*x+1/2*c)+1)^2+9/2/d/a^4*C/(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/(tan(1/2*d*x+1
/2*c)-1)*B-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A+4/d/a^4*ln(tan(1/2*d*x+1/2*c)
-1)*B-21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/a^4*C/(tan(1/2*d*x+1/2*c)
```

```
-1)^2
```

Maxima [B] time = 1.00343, size = 751, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] -1/840*(3*C*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) - B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)
+ 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 1
68*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [A] time = 0.547221, size = 1062, normalized size = 4.18

$$\frac{105\left((2A-8B+21C)\cos(dx+c)^6+4(2A-8B+21C)\cos(dx+c)^5+6(2A-8B+21C)\cos(dx+c)^4+4(2A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/420*(105*((2*A - 8*B + 21*C)*cos(d*x + c)^6 + 4*(2*A - 8*B + 21*C)*cos(d*
x + c)^5 + 6*(2*A - 8*B + 21*C)*cos(d*x + c)^4 + 4*(2*A - 8*B + 21*C)*cos(d
*x + c)^3 + (2*A - 8*B + 21*C)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*
((2*A - 8*B + 21*C)*cos(d*x + c)^6 + 4*(2*A - 8*B + 21*C)*cos(d*x + c)^5 +
6*(2*A - 8*B + 21*C)*cos(d*x + c)^4 + 4*(2*A - 8*B + 21*C)*cos(d*x + c)^3 +
 (2*A - 8*B + 21*C)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(16*(20*A -
83*B + 216*C)*cos(d*x + c)^5 + (1070*A - 4472*B + 11619*C)*cos(d*x + c)^4 +
 4*(310*A - 1318*B + 3411*C)*cos(d*x + c)^3 + 4*(130*A - 592*B + 1509*C)*co
s(d*x + c)^2 - 210*(B - 2*C)*cos(d*x + c) - 105*C)*sin(d*x + c))/(a^4*d*cos
(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{}{\sec^4(c+dx)+4\,\mathrm{s}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**6/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) +
Integral(C*sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
 d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.32497, size = 458, normalized size = 1.8

$$\frac{\frac{420\,(2A-8B+21C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{420\,(2A-8B+21C)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{840\left(2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(420*(2*A - 8*B + 21*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*
(2*A - 8*B + 21*C)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 840*(2*B*tan(1/
2*d*x + 1/2*c)^3 - 9*C*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c) +
7*C*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24
*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(
1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 189*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*
x + 1/2*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 1365*C*a^24*tan(1/2*d*x
+ 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 5145*B*a^24*tan(1/2*d*x + 1
/2*c) + 11655*C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.475 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=204

$$\frac{(6A-55B+244C)\tan(c+dx)}{105a^4d}+\frac{(3A+25B-88C)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{(B-4C)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(B\cdots}{a^4c\cdots}$$

```
[Out] ((B - 4*C)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((6*A - 55*B + 244*C)*Tan[c + d
*x])/(105*a^4*d) + ((3*A + 25*B - 88*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a
^4*d*(1 + Sec[c + d*x])^2) - ((B - 4*C)*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d
*x])) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + ((2*A + 5*B - 12*C)*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c
 + d*x])^3)
```

Rubi [A] time = 0.629072, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4084, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{(6A-55B+244C)\tan(c+dx)}{105a^4d}+\frac{(3A+25B-88C)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{(B-4C)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(B\cdots}{a^4c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^4,x]
```

```
[Out] ((B - 4*C)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((6*A - 55*B + 244*C)*Tan[c + d
*x])/(105*a^4*d) + ((3*A + 25*B - 88*C)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a
^4*d*(1 + Sec[c + d*x])^2) - ((B - 4*C)*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d
*x])) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + ((2*A + 5*B - 12*C)*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c
 + d*x])^3)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^4(c+dx)(a(3A+4B-}{(a+a\,s}}{}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(2A+5B-12C)\,\mathrm{se}}{35ad(a+a}$$

$$= \frac{(3A+25B-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\,\mathrm{se}}{7d(a+}$$

$$= \frac{(3A+25B-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\,\mathrm{se}}{7d(a+}$$

$$= \frac{(3A+25B-88C)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\,\mathrm{se}}{7d(a+}$$

$$= \frac{(B-4C)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(3A+25B-88C)\sec^2(c+}{105a^4d(1+\sec(c+}$$

$$= \frac{(B-4C)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(6A-55B+244C)\tan(c+}{105a^4d}$$

Mathematica [B] time = 6.39385, size = 1208, normalized size = 5.92

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^4,x]
```

```
[Out] (32*(-B + 4*C)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x
)/2]]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (32*(-B +
 4*C)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec
[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) + (4*Cos[c/2 + (d*x)
/2]^2*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*Si
n[c/2] - B*Sin[c/2] + C*Sin[c/2]))/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c/2]*S
ec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3*A*Sin[c/2] - 10*B*
Sin[c/2] + 17*C*Sin[c/2]))/(35*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(a + a*Sec[c + d*x])^4) + (16*Cos[c/2 + (d*x)/2]^6*Sec[c/2]*Sec[c +
 d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*A*Sin[c/2] - 55*B*Sin[c/
2] + 139*C*Sin[c/2]))/(105*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*(a + a*Sec[c + d*x])^4) + (4*Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c + d*x]^2
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] +
 C*Sin[(d*x)/2]))/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^4) + (8*Cos[c/2 + (d*x)/2]^3*Sec[c/2]*Sec[c + d*x]^2*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2]
+ 17*C*Sin[(d*x)/2]))/(35*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^4) + (16*Cos[c/2 + (d*x)/2]^5*Sec[c/2]*Sec[c + d*x]
^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*A*Sin[(d*x)/2] - 55*B*Sin[(d*
x)/2] + 139*C*Sin[(d*x)/2]))/(105*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])*(a + a*Sec[c + d*x])^4) + (32*Cos[c/2 + (d*x)/2]^7*Sec[c/2]*Sec[
c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*A*Sin[(d*x)/2] - 160*
B*Sin[(d*x)/2] + 559*C*Sin[(d*x)/2]))/(105*d*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) + (32*C*Cos[c/2 + (d*x)/2]^8*Se
c[c]*Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[d*x])/(d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [A] time = 0.076, size = 363, normalized size = 1.8

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+1/56/d/
a^4*C*tan(1/2*d*x+1/2*c)^7+3/40/d/a^4*tan(1/2*d*x+1/2*c)^5*A-1/8/d/a^4*tan(
1/2*d*x+1/2*c)^5*B+7/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+1/8/d/a^4*A*tan(1/2*d*
x+1/2*c)^3-11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+23/24/d/a^4*C*tan(1/2*d*x+1/2
*c)^3+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+49/8/d
/a^4*C*tan(1/2*d*x+1/2*c)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-4/d/a^4*ln(tan
(1/2*d*x+1/2*c)+1)*C-1/d/a^4*C/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4*ln(tan(1/2*d*
x+1/2*c)-1)*B+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C-1/d/a^4*C/(tan(1/2*d*x+1/2
*c)-1)
```

Maxima [B] time = 0.994374, size = 555, normalized size = 2.72

$$C\left(\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] 1/840*(C*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) + 3*A*(35*sin(d*x + c)/(cos(d
*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.530904, size = 872, normalized size = 4.27

$$105\left((B-4\,C)\cos\left(dx+c\right)^5+4\,(B-4\,C)\cos\left(dx+c\right)^4+6\,(B-4\,C)\cos\left(dx+c\right)^3+4\,(B-4\,C)\cos\left(dx+c\right)^2+(B-4\,C)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/210*(105*((B - 4*C)*cos(d*x + c)^5 + 4*(B - 4*C)*cos(d*x + c)^4 + 6*(B -
4*C)*cos(d*x + c)^3 + 4*(B - 4*C)*cos(d*x + c)^2 + (B - 4*C)*cos(d*x + c))*
log(sin(d*x + c) + 1) - 105*((B - 4*C)*cos(d*x + c)^5 + 4*(B - 4*C)*cos(d*x
 + c)^4 + 6*(B - 4*C)*cos(d*x + c)^3 + 4*(B - 4*C)*cos(d*x + c)^2 + (B - 4*
C)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*(3*A - 80*B + 332*C)*cos(d*x
 + c)^4 + (24*A - 535*B + 2236*C)*cos(d*x + c)^3 + (39*A - 620*B + 2636*C)*
cos(d*x + c)^2 + 4*(9*A - 65*B + 296*C)*cos(d*x + c) + 105*C)*sin(d*x + c))
/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 +
4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{B\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{}{\sec^4(c+dx)+4\sec^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) +
Integral(C*sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
 d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.36198, size = 385, normalized size = 1.89

$$\frac{\frac{840\,(B-4\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,(B-4\,C)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{1680\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(840*(B - 4*C)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(B - 4*C)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 1680*C*tan(1/2*d*x + 1/2*c)/((tan
(1/2*d*x + 1/2*c)^2 - 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^
24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 63*A*a^24*ta
n(1/2*d*x + 1/2*c)^5 - 105*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*C*a^24*tan(1
/2*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2*
d*x + 1/2*c)^3 + 805*C*a^24*tan(1/2*d*x + 1/2*c)^3 + 105*A*a^24*tan(1/2*d*x
 + 1/2*c) - 1575*B*a^24*tan(1/2*d*x + 1/2*c) + 5145*C*a^24*tan(1/2*d*x + 1/
2*c))/a^28)/d
```

3.476 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=173

$$\frac{(16A+12B-215C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(8A+6B-55C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(A-B+C)\tan(c+dx)\,\mathrm{se}}{7d(a\sec(c+dx)+}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((8*A + 6*B - 55*C)*Tan[c + d*x])/(105*
a^4*d*(1 + Sec[c + d*x])^2) + ((16*A + 12*B - 215*C)*Tan[c + d*x])/(105*a^4
*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a
+ a*Sec[c + d*x])^4) + ((4*A + 3*B - 10*C)*Sec[c + d*x]^2*Tan[c + d*x])/(35
*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.496624, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4019, 4008, 3998, 3770, 3794}

$$\frac{(16A+12B-215C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(8A+6B-55C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{C\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(A-B+C)\tan(c+dx)\,\mathrm{se}}{7d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^4,x]
```


```
[Out] (C*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((8*A + 6*B - 55*C)*Tan[c + d*x])/(105*
a^4*d*(1 + Sec[c + d*x])^2) + ((16*A + 12*B - 215*C)*Tan[c + d*x])/(105*a^4
*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a
+ a*Sec[c + d*x])^4) + ((4*A + 3*B - 10*C)*Sec[c + d*x]^2*Tan[c + d*x])/(35
*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{\int \frac{\sec^3(c+dx)(a(4A+}{(a+a}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A+3B-10C}{35ad(a}$$

$$= -\frac{(8A+6B-55C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(A-B+C)\sec^3(c+dx}{7d(a+a\sec(c+}$$

$$= -\frac{(8A+6B-55C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(A-B+C)\sec^3(c+dx}{7d(a+a\sec(c+}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(8A+6B-55C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-$$

Mathematica [A] time = 2.81439, size = 335, normalized size = 1.94

$$-\frac{\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(6720C\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^4,x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*(6720*C*Cos[(c + d*x)/2]^8*(Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]) - Cos[(c + d*x)/2]*Sec[c/2]*(70*(2*A + 3*B - 49*C)*Sin[(d*x)/2] - 70*(
2*A - 31*C)*Sin[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 126*B*Sin[c + (3*
```

```
d*x)/2] - 2625*C*Sin[c + (3*d*x)/2] + 735*C*Sin[2*c + (3*d*x)/2] + 56*A*Sin
[2*c + (5*d*x)/2] + 42*B*Sin[2*c + (5*d*x)/2] - 1015*C*Sin[2*c + (5*d*x)/2]
 + 105*C*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] + 6*B*Sin[3*c + (7
*d*x)/2] - 160*C*Sin[3*c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.077, size = 277, normalized size = 1.6

$$\frac{A}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{B}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{C}{da^4}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{3\,B}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{A}{40\,da^4}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/24/d/a^4*A*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3+1/d/a^4*
ln(tan(1/2*d*x+1/2*c)+1)*C+3/40/d/a^4*tan(1/2*d*x+1/2*c)^5*B-1/40/d/a^4*tan
(1/2*d*x+1/2*c)^5*A-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5-15/8/d/a^4*C*tan(1/2*d
*x+1/2*c)-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7
*B-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/8/d/a^4*A*tan(1/2*d*x+1/2*c)+1/8/d/
a^4*B*tan(1/2*d*x+1/2*c)-11/24/d/a^4*C*tan(1/2*d*x+1/2*c)^3-1/56/d/a^4*C*ta
n(1/2*d*x+1/2*c)^7
```

Maxima [A] time = 0.991262, size = 423, normalized size = 2.45

$$-\frac{5\,C\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-\frac{A\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c}{(\cos(dx+c)+1}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] -1/840*(5*C*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - A*(105*sin(d*x + c
)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 -
 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.537018, size = 660, normalized size = 3.82

$$\frac{105\left(C\cos(dx+c)^4+4\,C\cos(dx+c)^3+6\,C\cos(dx+c)^2+4\,C\cos(dx+c)+C\right)\log(\sin(dx+c)+1)-105\left(C\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/210*(105*(C*cos(d*x + c)^4 + 4*C*cos(d*x + c)^3 + 6*C*cos(d*x + c)^2 + 4*
C*cos(d*x + c) + C)*log(sin(d*x + c) + 1) - 105*(C*cos(d*x + c)^4 + 4*C*cos
(d*x + c)^3 + 6*C*cos(d*x + c)^2 + 4*C*cos(d*x + c) + C)*log(-sin(d*x + c)
+ 1) + 2*(2*(4*A + 3*B - 80*C)*cos(d*x + c)^3 + (32*A + 24*B - 535*C)*cos(d
*x + c)^2 + (52*A + 39*B - 620*C)*cos(d*x + c) + 13*A + 36*B - 260*C)*sin(d
*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
 c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx + \int \frac{B\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx + \int \frac{}{\sec^4(c+dx)+4\mathrm{s}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) +
Integral(C*sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
 d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.23529, size = 335, normalized size = 1.94

$$\frac{\frac{840\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4} - \frac{840\,C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4} - \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 21\,Aa^{24}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(840*C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*A*a^24*tan(1/2*
d*x + 1/2*c)^5 - 63*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*C*a^24*tan(1/2*d*x
+ 1/2*c)^5 - 35*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*x + 1/
2*c)^3 + 385*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c
) - 105*B*a^24*tan(1/2*d*x + 1/2*c) + 1575*C*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

3.477 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=148

$$\frac{(8A+13B+36C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}+\frac{(23A-2B-54C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{(6A+B-8C}{35ad(a\sec(}$$

```
[Out] ((23*A - 2*B - 54*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + ((8*A
 + 13*B + 36*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B + C)
*Sec[c + d*x]^2*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((6*A + B - 8*
C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.406298, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4084, 4008, 4000, 3794}

$$\frac{(8A+13B+36C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}+\frac{(23A-2B-54C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{(6A+B-8C}{35ad(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^4,x]
```



```
[Out] ((23*A - 2*B - 54*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + ((8*A
 + 13*B + 36*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B + C)
*Sec[c + d*x]^2*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((6*A + B - 8*
C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^2(c+dx)(a(5A+}{(a}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(6A+B-8C)\,\mathrm{t}}{35ad(a+a\sec(}$$

$$= \frac{(23A-2B-54C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\sec^2(c+dx)}{7d(a+a\sec(c+}$$

$$= \frac{(23A-2B-54C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\sec^2(c+dx)}{7d(a+a\sec(c+}$$

Mathematica [A] time = 0.683712, size = 200, normalized size = 1.35

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-35(5A+4B)\sin\left(c+\frac{dx}{2}\right)+70(4A+2B+3C)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(70*(4*A + 2*B + 3*C)*Sin[(d*x)/2] - 35*(5*A +
 4*B)*Sin[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2
] + 126*C*Sin[c + (3*d*x)/2] - 105*A*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c +
(5*d*x)/2] + 56*B*Sin[2*c + (5*d*x)/2] + 42*C*Sin[2*c + (5*d*x)/2] + 13*A*S
in[3*c + (7*d*x)/2] + 8*B*Sin[3*c + (7*d*x)/2] + 6*C*Sin[3*c + (7*d*x)/2]))
/(6720*a^4*d)
```

Maple [A] time = 0.075, size = 106, normalized size = 0.7

$$\frac{1}{8\,da^4}\left(\frac{A-B+C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{-A+3\,C-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-A+B+3\,C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B+C)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*C-B)*tan(1/2*d*x+1/2*
c)^5+1/3*(-A+B+3*C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x
+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.986233, size = 350, normalized size = 2.36

$$\frac{\frac{B\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1}+\frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{A\left(\frac{105 \sin(dx+c)}{\cos(dx+c)+1}-\frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3\,C\left(\frac{35 \sin(dx+c)}{\cos(dx+c)+1}+\frac{35 \sin}{(\cos(d}\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 + 3*C*(35*sin(d*x + c)/(cos(d*x
+ c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.472272, size = 343, normalized size = 2.32

$$\frac{\left((13\,A+8\,B+6\,C)\cos(dx+c)^3+4\,(13\,A+8\,B+6\,C)\cos(dx+c)^2+(32\,A+52\,B+39\,C)\cos(dx+c)+8\,A+13\,B+\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/105*((13*A + 8*B + 6*C)*cos(d*x + c)^3 + 4*(13*A + 8*B + 6*C)*cos(d*x + c
)^2 + (32*A + 52*B + 39*C)*cos(d*x + c) + 8*A + 13*B + 36*C)*sin(d*x + c)/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{}{\sec^4(c+dx)+4\sec^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) +
Integral(C*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
 d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.21144, size = 231, normalized size = 1.56

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-21\,B\tan\left(\frac{1}{2}\,dx+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 15*C*tan
(1/2*d*x + 1/2*c)^7 - 21*A*tan(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*
c)^5 + 63*C*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*c)^3 + 35*B*tan
(1/2*d*x + 1/2*c)^3 + 105*C*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/
2*c) + 105*B*tan(1/2*d*x + 1/2*c) + 105*C*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.478 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{(6A+8B+13C)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{(6A+8B+13C)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(8A-B-6C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B+C)\tan(c+dx)\,\mathrm{s}}{7d(a\sec(c+dx)+}$$

```
[Out] -((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((8
*A - B - 6*C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((6*A + 8*B +
 13*C)*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((6*A + 8*B + 13*
C)*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.260498, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4078, 4000, 3796, 3794}

$$\frac{(6A+8B+13C)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{(6A+8B+13C)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(8A-B-6C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B+C)\tan(c+dx)\,\mathrm{s}}{7d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^4,x]
```

```
[Out] -((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((8
*A - B - 6*C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((6*A + 8*B +
 13*C)*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((6*A + 8*B + 13*
C)*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 4078

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*B - b*C - 2*A*b*(m + 1) - (b*B*(m + 2) -
a*(A*(m + 2) - C*(m - 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)(a(6A+B-C}{(a+a}}{}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(8A-B-6C)\tan}{35ad(a+a\sec(c}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(8A-B-6C)\tan}{35ad(a+a\sec(c}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(8A-B-6C)\tan}{35ad(a+a\sec(c}$$

Mathematica [A] time = 0.762154, size = 231, normalized size = 1.5

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-35(18A+5B+4C)\sin\left(c+\frac{dx}{2}\right)+70(9A+4B+2C)\sin\left(\frac{dx}{2}\right)+441A\sin\left(c+\frac{3dx}{2}\right)-315A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(70*(9*A + 4*B + 2*C)*Sin[(d*x)/2] - 35*(18*A
+ 5*B + 4*C)*Sin[c + (d*x)/2] + 441*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3
*d*x)/2] + 168*C*Sin[c + (3*d*x)/2] - 315*A*Sin[2*c + (3*d*x)/2] - 105*B*Si
n[2*c + (3*d*x)/2] + 147*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*x)/2]
 + 56*C*Sin[2*c + (5*d*x)/2] - 105*A*Sin[3*c + (5*d*x)/2] + 36*A*Sin[3*c +
(7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2] + 8*C*Sin[3*c + (7*d*x)/2]))/(6720*a
^4*d)
```

Maple [A] time = 0.078, size = 108, normalized size = 0.7

$$\frac{1}{8da^4}\left(\frac{-A+B-C}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3A-C-B}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{-3A-B+C}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+A\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B-C)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-C-B)*tan(1/2*d*x+1/2*
c)^5+1/3*(-3*A-B+C)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x
+1/2*c)+C*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.989747, size = 350, normalized size = 2.27

$$\frac{\frac{C\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+3A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin}{(\cos(d}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x,
algorithm="maxima")
```

```
[Out] 1/840*(C*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x
+ c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.473608, size = 343, normalized size = 2.23

$$\frac{\left((36A+13B+8C)\cos(dx+c)^3+(39A+52B+32C)\cos(dx+c)^2+4(6A+8B+13C)\cos(dx+c)+6A+8B+\right.}{105\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] 1/105*((36*A + 13*B + 8*C)*cos(d*x + c)^3 + (39*A + 52*B + 32*C)*cos(d*x +
c)^2 + 4*(6*A + 8*B + 13*C)*cos(d*x + c) + 6*A + 8*B + 13*C)*sin(d*x + c)/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{B\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{}{\sec^4(c+dx)+4\sec^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x
)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)
**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Int
egral(C*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*
x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A] time = 1.2063, size = 231, normalized size = 1.5

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+15\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+21\,B\tan\left(\frac{1}{2}\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 15*C*ta
n(1/2*d*x + 1/2*c)^7 - 63*A*tan(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2
*c)^5 + 21*C*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2*c)^3 + 35*B*t
an(1/2*d*x + 1/2*c)^3 - 35*C*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1
/2*c) - 105*B*tan(1/2*d*x + 1/2*c) - 105*C*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.479 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=148

$$-\frac{2(80A-3B-4C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-6B-8C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{(10A-3B-4C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B+C)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] (A*x)/a^4 - ((55*A - 6*B - 8*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])
^2) - (2*(80*A - 3*B - 4*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) -
((A - B + C)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((10*A - 3*B - 4*
C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.283938, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4052, 3922, 3919, 3794}

$$-\frac{2(80A-3B-4C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-6B-8C)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{(10A-3B-4C)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B+C)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (A*x)/a^4 - ((55*A - 6*B - 8*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])
^2) - (2*(80*A - 3*B - 4*C)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) -
((A - B + C)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((10*A - 3*B - 4*
C)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{-7aA+a(3A-3B-4C)\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-3B-4C)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{\int \frac{35a^2A-2a^2(}{(a-}}{}$$

$$= -\frac{(55A-6B-8C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-3B}{35ad(a+}$$

$$= \frac{Ax}{a^4} - \frac{(55A-6B-8C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-}{35ad}$$

$$= \frac{Ax}{a^4} - \frac{(55A-6B-8C)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(10A-}{35ad}$$

Mathematica [B] time = 1.36193, size = 405, normalized size = 2.74

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(8260A\sin\left(c+\frac{dx}{2}\right)-7140A\sin\left(c+\frac{3dx}{2}\right)+3780A\sin\left(2c+\frac{3dx}{2}\right)-2800A\sin\left(2c+\frac{5dx}{2}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*A*d*x*Cos[(d*x)/2] + 3675*A*d*x*Cos[c +
(d*x)/2] + 2205*A*d*x*Cos[c + (3*d*x)/2] + 2205*A*d*x*Cos[2*c + (3*d*x)/2]
+ 735*A*d*x*Cos[2*c + (5*d*x)/2] + 735*A*d*x*Cos[3*c + (5*d*x)/2] + 105*A*d
*x*Cos[3*c + (7*d*x)/2] + 105*A*d*x*Cos[4*c + (7*d*x)/2] - 9940*A*Sin[(d*x)
/2] + 1260*B*Sin[(d*x)/2] + 560*C*Sin[(d*x)/2] + 8260*A*Sin[c + (d*x)/2] -
1260*B*Sin[c + (d*x)/2] - 350*C*Sin[c + (d*x)/2] - 7140*A*Sin[c + (3*d*x)/2
] + 882*B*Sin[c + (3*d*x)/2] + 336*C*Sin[c + (3*d*x)/2] + 3780*A*Sin[2*c +
(3*d*x)/2] - 630*B*Sin[2*c + (3*d*x)/2] - 210*C*Sin[2*c + (3*d*x)/2] - 2800
*A*Sin[2*c + (5*d*x)/2] + 294*B*Sin[2*c + (5*d*x)/2] + 182*C*Sin[2*c + (5*d
*x)/2] + 840*A*Sin[3*c + (5*d*x)/2] - 210*B*Sin[3*c + (5*d*x)/2] - 520*A*Si
n[3*c + (7*d*x)/2] + 72*B*Sin[3*c + (7*d*x)/2] + 26*C*Sin[3*c + (7*d*x)/2])
)/(13440*a^4*d)
```

Maple [A] time = 0.085, size = 255, normalized size = 1.7

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{A}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3\,B}{40\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+1/56/d/
a^4*C*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5*A+3/40/d/a^4*tan(
1/2*d*x+1/2*c)^5*B-1/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*A*tan(1/2*
```

```
d*x+1/2*c)^3-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3-1/24/d/a^4*C*tan(1/2*d*x+1/2*
c)^3-15/8/d/a^4*A*tan(1/2*d*x+1/2*c)+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)+1/8/d/a
^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.45776, size = 386, normalized size = 2.61

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{C\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(d}{(\cos(dx+}}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - C*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.49751, size = 512, normalized size = 3.46

$$\frac{105\,Adx\cos(dx+c)^4+420\,Adx\cos(dx+c)^3+630\,Adx\cos(dx+c)^2+420\,Adx\cos(dx+c)+105\,Adx-\left((260\,A-36\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] 1/105*(105*A*d*x*cos(d*x + c)^4 + 420*A*d*x*cos(d*x + c)^3 + 630*A*d*x*cos(
d*x + c)^2 + 420*A*d*x*cos(d*x + c) + 105*A*d*x - ((260*A - 36*B - 13*C)*co
s(d*x + c)^3 + (620*A - 39*B - 52*C)*cos(d*x + c)^2 + (535*A - 24*B - 32*C)
*cos(d*x + c) + 160*A - 6*B - 8*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{B\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{}{\sec^4(c+dx)+4\sec^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*se
c(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c +
d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(C*sec(c +
d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c
+ d*x) + 1), x))/a**4
```

Giac [A] time = 1.17882, size = 297, normalized size = 2.01

$$\frac{840\,(dx+c)A}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+63\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,Ca^{24}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/840*(840*(d*x + c)*A/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(
1/2*d*x + 1/2*c)^5 + 63*B*a^24*tan(1/2*d*x + 1/2*c)^5 - 21*C*a^24*tan(1/2*d
*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*x
+ 1/2*c)^3 - 35*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1
/2*c) + 105*B*a^24*tan(1/2*d*x + 1/2*c) + 105*C*a^24*tan(1/2*d*x + 1/2*c))/
a^28)/d
```

3.480 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=176

$$\frac{2(332A-80B+3C)\sin(c+dx)}{105a^4d}-\frac{(88A-25B-3C)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(4A-B)\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{x(4A-B)}{a^4}-\frac{(12A-5B-2C}{35ad(a\sec(c}$$

```
[Out] -(((4*A - B)*x)/a^4) + (2*(332*A - 80*B + 3*C)*Sin[c + d*x])/(105*a^4*d) -
((88*A - 25*B - 3*C)*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((4*A
 - B)*Sin[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])
/(7*d*(a + a*Sec[c + d*x])^4) - ((12*A - 5*B - 2*C)*Sin[c + d*x])/(35*a*d*(
a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.559175, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4084, 4020, 3787, 2637, 8}

$$\frac{2(332A-80B+3C)\sin(c+dx)}{105a^4d}-\frac{(88A-25B-3C)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(4A-B)\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{x(4A-B)}{a^4}-\frac{(12A-5B-2C}{35ad(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^4,x]
```



```
[Out] -(((4*A - B)*x)/a^4) + (2*(332*A - 80*B + 3*C)*Sin[c + d*x])/(105*a^4*d) -
((88*A - 25*B - 3*C)*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((4*A
 - B)*Sin[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])
/(7*d*(a + a*Sec[c + d*x])^4) - ((12*A - 5*B - 2*C)*Sin[c + d*x])/(35*a*d*(
a + a*Sec[c + d*x])^3)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(a(8A-B+C)-a(4A-4B-3C}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5B-2C)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} +$$

$$= -\frac{(88A-25B-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4}$$

$$= -\frac{(88A-25B-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4}$$

$$= -\frac{(88A-25B-3C)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d(a+a\sec(c+dx))^4}$$

$$= -\frac{(4A-B)x}{a^4} + \frac{2(332A-80B+3C)\sin(c+dx)}{105a^4d} - \frac{(88A-2}{105a^4}$$

Mathematica [B] time = 1.92801, size = 567, normalized size = 3.22

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-7350dx(4A-B)\cos\left(c+\frac{dx}{2}\right)-7350dx(4A-B)\cos\left(\frac{dx}{2}\right)-46130A\sin\left(c+\frac{dx}{2}\right)+46116A$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(-7350*(4*A - B)*d*x*Cos[(d*x)/2] - 7350*(4*A
- B)*d*x*Cos[c + (d*x)/2] - 17640*A*d*x*Cos[c + (3*d*x)/2] + 4410*B*d*x*Cos
[c + (3*d*x)/2] - 17640*A*d*x*Cos[2*c + (3*d*x)/2] + 4410*B*d*x*Cos[2*c + (
3*d*x)/2] - 5880*A*d*x*Cos[2*c + (5*d*x)/2] + 1470*B*d*x*Cos[2*c + (5*d*x)/
2] - 5880*A*d*x*Cos[3*c + (5*d*x)/2] + 1470*B*d*x*Cos[3*c + (5*d*x)/2] - 84
0*A*d*x*Cos[3*c + (7*d*x)/2] + 210*B*d*x*Cos[3*c + (7*d*x)/2] - 840*A*d*x*C
os[4*c + (7*d*x)/2] + 210*B*d*x*Cos[4*c + (7*d*x)/2] + 60830*A*Sin[(d*x)/2]
 - 19880*B*Sin[(d*x)/2] + 2520*C*Sin[(d*x)/2] - 46130*A*Sin[c + (d*x)/2] +
16520*B*Sin[c + (d*x)/2] - 2520*C*Sin[c + (d*x)/2] + 46116*A*Sin[c + (3*d*x
)/2] - 14280*B*Sin[c + (3*d*x)/2] + 1764*C*Sin[c + (3*d*x)/2] - 18060*A*Sin
[2*c + (3*d*x)/2] + 7560*B*Sin[2*c + (3*d*x)/2] - 1260*C*Sin[2*c + (3*d*x)/
2] + 19292*A*Sin[2*c + (5*d*x)/2] - 5600*B*Sin[2*c + (5*d*x)/2] + 588*C*Sin
[2*c + (5*d*x)/2] - 2100*A*Sin[3*c + (5*d*x)/2] + 1680*B*Sin[3*c + (5*d*x)/
2] - 420*C*Sin[3*c + (5*d*x)/2] + 3791*A*Sin[3*c + (7*d*x)/2] - 1040*B*Sin[
3*c + (7*d*x)/2] + 144*C*Sin[3*c + (7*d*x)/2] + 735*A*Sin[4*c + (7*d*x)/2]
```

```
+ 105*A*Sin[4*c + (9*d*x)/2] + 105*A*Sin[5*c + (9*d*x)/2]))/(26880*a^4*d)
```

Maple [A] time = 0.122, size = 307, normalized size = 1.7

$$-\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{B}{8\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B-1/56/d
/a^4*C*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5*A-1/8/d/a^4*tan
(1/2*d*x+1/2*c)^5*B+3/40/d/a^4*C*tan(1/2*d*x+1/2*c)^5-23/24/d/a^4*A*tan(1/2
*d*x+1/2*c)^3+11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-1/8/d/a^4*C*tan(1/2*d*x+1/
2*c)^3+49/8/d/a^4*A*tan(1/2*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+1/8/
d/a^4*C*tan(1/2*d*x+1/2*c)+2/d/a^4*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*
c)^2)-8/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^4*arctan(tan(1/2*d*x+1/2*c
))*B
```

Maxima [B] time = 1.46788, size = 481, normalized size = 2.73

$$\frac{A\left(\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5\,B\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\ldots}{\ldots}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x,
algorithm="maxima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4) + 3*C*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A] time = 0.512233, size = 608, normalized size = 3.45

$$-\frac{105\,(4\,A-B)dx\cos(dx+c)^4+420\,(4\,A-B)dx\cos(dx+c)^3+630\,(4\,A-B)dx\cos(dx+c)^2+420\,(4\,A-B)dx\cos(d\ldots}{105\,(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] -1/105*(105*(4*A - B)*d*x*cos(d*x + c)^4 + 420*(4*A - B)*d*x*cos(d*x + c)^3
 + 630*(4*A - B)*d*x*cos(d*x + c)^2 + 420*(4*A - B)*d*x*cos(d*x + c) + 105*
(4*A - B)*d*x - (105*A*cos(d*x + c)^4 + 4*(296*A - 65*B + 9*C)*cos(d*x + c)
^3 + (2636*A - 620*B + 39*C)*cos(d*x + c)^2 + (2236*A - 535*B + 24*C)*cos(d
*x + c) + 664*A - 160*B + 6*C)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*
d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**4,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.16047, size = 346, normalized size = 1.97

$$-\frac{\frac{840\,(dx+c)(4\,A-B)}{a^4}-\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+15\,Ca^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*(4*A - B)/a^4 - 1680*A*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*t
an(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1
/2*d*x + 1/2*c)^5 + 105*B*a^24*tan(1/2*d*x + 1/2*c)^5 - 63*C*a^24*tan(1/2*d
*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2*d*x
+ 1/2*c)^3 + 105*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x +
1/2*c) + 1575*B*a^24*tan(1/2*d*x + 1/2*c) - 105*C*a^24*tan(1/2*d*x + 1/2*c)
)/a^28)/d
```

3.481 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=239

$$-\frac{8(216A-83B+20C)\sin(c+dx)}{105a^4d}+\frac{(21A-8B+2C)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{4(216A-83B+20C)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}$$

```
[Out] ((21*A - 8*B + 2*C)*x)/(2*a^4) - (8*(216*A - 83*B + 20*C)*Sin[c + d*x])/(10
5*a^4*d) + ((21*A - 8*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129
*A - 52*B + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^
2) - (4*(216*A - 83*B + 20*C)*Cos[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Se
c[c + d*x])) - ((A - B + C)*Cos[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c +
d*x])^4) - ((2*A - B)*Cos[c + d*x]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x]
)^3)
```

Rubi [A] time = 0.698801, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4020, 3787, 2635, 8, 2637}

$$-\frac{8(216A-83B+20C)\sin(c+dx)}{105a^4d}+\frac{(21A-8B+2C)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{4(216A-83B+20C)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^4,x]
```


```
[Out] ((21*A - 8*B + 2*C)*x)/(2*a^4) - (8*(216*A - 83*B + 20*C)*Sin[c + d*x])/(10
5*a^4*d) + ((21*A - 8*B + 2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129
*A - 52*B + 10*C)*Cos[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^
2) - (4*(216*A - 83*B + 20*C)*Cos[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Se
c[c + d*x])) - ((A - B + C)*Cos[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c +
d*x])^4) - ((2*A - B)*Cos[c + d*x]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x]
)^3)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(a(9A-2\ldots}{(a\ldots}}{}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(2A-B)\cos(c+\ldots}{5ad(a+a\sec\ldots}$$

$$= -\frac{(129A-52B+10C)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+\ldots}{7d\ldots}$$

$$= -\frac{(129A-52B+10C)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+\ldots}{7d\ldots}$$

$$= -\frac{(129A-52B+10C)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B+\ldots}{7d\ldots}$$

$$= -\frac{8(216A-83B+20C)\sin(c+dx)}{105a^4d} + \frac{(21A-8B+2C)\,\mathrm{co}\ldots}{2a\ldots}$$

$$= \frac{(21A-8B+2C)x}{2a^4} - \frac{8(216A-83B+20C)\sin(c+dx)}{105a^4d} + \ldots$$

Mathematica [A] time = 5.25318, size = 345, normalized size = 1.44

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(210\cos^7\left(\frac{1}{2}(c+dx)\right)(4(B-4A)\sin(c+dx)+2dx(21A-8B\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^4,x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*(15*(A - B + C)
*Sec[c/2]*Sin[(d*x)/2] - 6*(39*A - 32*B + 25*C)*Cos[(c + d*x)/2]^2*Sec[c/2]
```

```
*Sin[(d*x)/2] + 4*(447*A - 286*B + 160*C)*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(
d*x)/2] - 8*(1653*A - 764*B + 260*C)*Cos[(c + d*x)/2]^6*Sec[c/2]*Sin[(d*x)/
2] + 210*Cos[(c + d*x)/2]^7*(2*(21*A - 8*B + 2*C)*d*x + 4*(-4*A + B)*Sin[c
+ d*x] + A*Sin[2*(c + d*x)]) + 15*(A - B + C)*Cos[(c + d*x)/2]*Tan[c/2] - 6
*(39*A - 32*B + 25*C)*Cos[(c + d*x)/2]^3*Tan[c/2] + 4*(447*A - 286*B + 160*
C)*Cos[(c + d*x)/2]^5*Tan[c/2]))/(105*a^4*d*(1 + Cos[c + d*x])^4*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.128, size = 429, normalized size = 1.8

$$\frac{A}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{B}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{C}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9\,A}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{7\,B}{40\,da^4}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*B+1/56/d/
a^4*C*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5*A+7/40/d/a^4*tan
(1/2*d*x+1/2*c)^5*B-1/8/d/a^4*C*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*A*tan(1/2*d
*x+1/2*c)^3-23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+11/24/d/a^4*C*tan(1/2*d*x+1/
2*c)^3-111/8/d/a^4*A*tan(1/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-15/
8/d/a^4*C*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)^3*A+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-7/d/a
^4/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)+2/d/a^4/(1+tan(1/2*d*x+1
/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+21/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))-8/d/a
^4*arctan(tan(1/2*d*x+1/2*c))*B+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [B] time = 1.47432, size = 640, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4) + 5*C*((315*sin(d*x + c)/(cos(d
*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arct
an(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A] time = 0.524789, size = 730, normalized size = 3.05

$$\frac{105\,(21\,A-8\,B+2\,C)dx\cos{(dx+c)}^4+420\,(21\,A-8\,B+2\,C)dx\cos{(dx+c)}^3+630\,(21\,A-8\,B+2\,C)dx\cos{(dx+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] 1/210*(105*(21*A - 8*B + 2*C)*d*x*cos(d*x + c)^4 + 420*(21*A - 8*B + 2*C)*d
*x*cos(d*x + c)^3 + 630*(21*A - 8*B + 2*C)*d*x*cos(d*x + c)^2 + 420*(21*A -
 8*B + 2*C)*d*x*cos(d*x + c) + 105*(21*A - 8*B + 2*C)*d*x + (105*A*cos(d*x
+ c)^5 - 210*(2*A - B)*cos(d*x + c)^4 - 4*(1509*A - 592*B + 130*C)*cos(d*x
+ c)^3 - 4*(3411*A - 1318*B + 310*C)*cos(d*x + c)^2 - (11619*A - 4472*B + 1
070*C)*cos(d*x + c) - 3456*A + 1328*B - 320*C)*sin(d*x + c))/(a^4*d*cos(d*x
 + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x
 + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16637, size = 408, normalized size = 1.71

$$\frac{\frac{420\,(dx+c)(21\,A-8\,B+2\,C)}{a^4}-\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^2\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/840*(420*(d*x + c)*(21*A - 8*B + 2*C)/a^4 - 840*(9*A*tan(1/2*d*x + 1/2*c)
^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + 7*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*
x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x +
 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 15*C*a^24*tan(1/2*d*x + 1/2*
c)^7 - 189*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*tan(1/2*d*x + 1/2*c)^
5 - 105*C*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2*c)^3
- 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 385*C*a^24*tan(1/2*d*x + 1/2*c)^3 - 1
1655*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/2*d*x + 1/2*c) - 1575*
C*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.482 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=239

$$\frac{2a(99A+88B+80C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4(99A+88B+80C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+88B}{}$$

```
[Out] (4*a*(99*A + 88*B + 80*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a*(99*A + 88*B + 80*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Se
c[c + d*x]]) + (2*a*(11*B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a +
a*Sec[c + d*x]]) - (8*(99*A + 88*B + 80*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(3465*d) + (2*C*Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]
)/(11*d) + (4*(99*A + 88*B + 80*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])
/(1155*a*d)
```

Rubi [A] time = 0.55846, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4088, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a(99A+88B+80C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4(99A+88B+80C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{1155ad}-\frac{8(99A+88B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (4*a*(99*A + 88*B + 80*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a*(99*A + 88*B + 80*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Se
c[c + d*x]]) + (2*a*(11*B + C)*Sec[c + d*x]^4*Tan[c + d*x])/(99*d*Sqrt[a +
a*Sec[c + d*x]]) - (8*(99*A + 88*B + 80*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(3465*d) + (2*C*Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]
)/(11*d) + (4*(99*A + 88*B + 80*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])
/(1155*a*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(}{11d}$$

$$= \frac{2a(11B+C)\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a(99A+88B+80C)\sec^3(c+dx)\tan(}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a(99A+88B+80C)\sec^3(c+dx)\tan(}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a(99A+88B+80C)\sec^3(c+dx)\tan(}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a(99A+88B+80C)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A] time = 1.72971, size = 185, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}((2871A+3322B+3020C)\cos(c+dx)+13(99A+88B+80C)\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] ((1089*A + 968*B + 1510*C + (2871*A + 3322*B + 3020*C)*Cos[c + d*x] + 13*(9
9*A + 88*B + 80*C)*Cos[2*(c + d*x)] + 1287*A*Cos[3*(c + d*x)] + 1144*B*Cos[
3*(c + d*x)] + 1040*C*Cos[3*(c + d*x)] + 198*A*Cos[4*(c + d*x)] + 176*B*Cos
[4*(c + d*x)] + 160*C*Cos[4*(c + d*x)] + 198*A*Cos[5*(c + d*x)] + 176*B*Cos
[5*(c + d*x)] + 160*C*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c +
d*x])]*Tan[(c + d*x)/2])/(3465*d)
```

Maple [A] time = 0.433, size = 204, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))\left(1584\,A(\cos(dx+c))^5+1408\,B(\cos(dx+c))^5+1280\,C(\cos(dx+c))^5+792\,A(\cos(dx+c))^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(1584*A*cos(d*x+c)^5+1408*B*cos(d*x+c)^5+1280*C*c
os(d*x+c)^5+792*A*cos(d*x+c)^4+704*B*cos(d*x+c)^4+640*C*cos(d*x+c)^4+594*A*
cos(d*x+c)^3+528*B*cos(d*x+c)^3+480*C*cos(d*x+c)^3+495*A*cos(d*x+c)^2+440*B
*cos(d*x+c)^2+400*C*cos(d*x+c)^2+385*B*cos(d*x+c)+350*C*cos(d*x+c)+315*C)*(
a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.517519, size = 405, normalized size = 1.69

$$\frac{2\left(16\,(99\,A+88\,B+80\,C)\cos(dx+c)^5+8\,(99\,A+88\,B+80\,C)\cos(dx+c)^4+6\,(99\,A+88\,B+80\,C)\cos(dx+c)^3+5\right.}{3465\left(d\cos(dx+c)^6+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3465*(16*(99*A + 88*B + 80*C)*cos(d*x + c)^5 + 8*(99*A + 88*B + 80*C)*cos
(d*x + c)^4 + 6*(99*A + 88*B + 80*C)*cos(d*x + c)^3 + 5*(99*A + 88*B + 80*C
)*cos(d*x + c)^2 + 35*(11*B + 10*C)*cos(d*x + c) + 315*C)*sqrt((a*cos(d*x +
```

```
c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.98062, size = 554, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -2/3465*(3465*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 3465*sqrt(2)*B*a^6*sgn(cos(
d*x + c)) + 3465*sqrt(2)*C*a^6*sgn(cos(d*x + c)) - (10395*sqrt(2)*A*a^6*sgn
(cos(d*x + c)) + 8085*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 5775*sqrt(2)*C*a^6*
sgn(cos(d*x + c)) - (15246*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 14322*sqrt(2)*
B*a^6*sgn(cos(d*x + c)) + 16170*sqrt(2)*C*a^6*sgn(cos(d*x + c)) - (14058*sq
rt(2)*A*a^6*sgn(cos(d*x + c)) + 13266*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 891
0*sqrt(2)*C*a^6*sgn(cos(d*x + c)) - (6633*sqrt(2)*A*a^6*sgn(cos(d*x + c)) +
 4741*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 5885*sqrt(2)*C*a^6*sgn(cos(d*x + c)
) - (891*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 1177*sqrt(2)*B*a^6*sgn(cos(d*x +
 c)) + 755*sqrt(2)*C*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2
*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.483 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx)+C\sec$

Optimal. Leaf size=193

$$\frac{2(21A+18B+16C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+18B+16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2a(21A+}{45a}$$

```
[Out] (2*a*(21*A + 18*B + 16*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*(9*B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*(21*A + 18*B + 16*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) +
(2*C*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(21*A
 + 18*B + 16*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.473151, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4088, 4016, 3800, 4001, 3792}

$$\frac{2(21A+18B+16C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{4(21A+18B+16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2a(21A+}{45a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*a*(21*A + 18*B + 16*C)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*(9*B + C)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*(21*A + 18*B + 16*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) +
(2*C*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(21*A
 + 18*B + 16*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
```

```
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\tan(}{9d}$$
$$= \frac{2a(9B+C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} +$$
$$= \frac{2a(9B+C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} +$$
$$= \frac{2a(9B+C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} -$$
$$= \frac{2a(21A+18B+16C)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A] time = 1.84672, size = 153, normalized size = 0.79

$$\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}(2(63A+99B+88C)\cos(c+dx)+11(21A+18B+16C)\cos(2(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] ((189*A + 162*B + 214*C + 2*(63*A + 99*B + 88*C)*Cos[c + d*x] + 11*(21*A +
18*B + 16*C)*Cos[2*(c + d*x)] + 42*A*Cos[3*(c + d*x)] + 36*B*Cos[3*(c + d*x
)] + 32*C*Cos[3*(c + d*x)] + 42*A*Cos[4*(c + d*x)] + 36*B*Cos[4*(c + d*x)]
+ 32*C*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c +
 d*x)/2])/(315*d)
```

Maple [A] time = 0.37, size = 171, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))\left(168A(\cos(dx+c))^4+144B(\cos(dx+c))^4+128C(\cos(dx+c))^4+84A(\cos(dx+c))^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(168*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+128*C*cos(d
*x+c)^4+84*A*cos(d*x+c)^3+72*B*cos(d*x+c)^3+64*C*cos(d*x+c)^3+63*A*cos(d*x+
c)^2+54*B*cos(d*x+c)^2+48*C*cos(d*x+c)^2+45*B*cos(d*x+c)+40*C*cos(d*x+c)+35
*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.505792, size = 343, normalized size = 1.78

$$\frac{2\left(8\left(21\,A+18\,B+16\,C\right)\cos\left(dx+c\right)^{4}+4\left(21\,A+18\,B+16\,C\right)\cos\left(dx+c\right)^{3}+3\left(21\,A+18\,B+16\,C\right)\cos\left(dx+c\right)^{2}+5\left(9\right.\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*(21*A + 18*B + 16*C)*cos(d*x + c)^4 + 4*(21*A + 18*B + 16*C)*cos(d
*x + c)^3 + 3*(21*A + 18*B + 16*C)*cos(d*x + c)^2 + 5*(9*B + 8*C)*cos(d*x +
 c) + 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
 + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2
)*sec(c + d*x)**3, x)
```

Giac [B] time = 4.8382, size = 470, normalized size = 2.44

$$2\left(315\,\sqrt{2}Aa^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+315\,\sqrt{2}Ba^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+315\,\sqrt{2}Ca^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(840\,\sqrt{2}Aa^5\mathrm{sgn}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^5*sgn(cos(d*x
+ c)) + 315*sqrt(2)*C*a^5*sgn(cos(d*x + c)) - (840*sqrt(2)*A*a^5*sgn(cos(d*
x + c)) + 630*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 420*sqrt(2)*C*a^5*sgn(cos(d
*x + c)) - (882*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 756*sqrt(2)*B*a^5*sgn(cos
(d*x + c)) + 882*sqrt(2)*C*a^5*sgn(cos(d*x + c)) - (504*sqrt(2)*A*a^5*sgn(c
os(d*x + c)) + 522*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 324*sqrt(2)*C*a^5*sgn(
cos(d*x + c)) - (147*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 81*sqrt(2)*B*a^5*sgn
(cos(d*x + c)) + 107*sqrt(2)*C*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^
2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)*d)
```

3.484 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=147

$$\frac{2(35A-14B+18C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(35A+49B+27C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7B+C)\tan(c+dx)(a\sec}{35ad}$$

```
[Out] (2*a*(35*A + 49*B + 27*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*(35*A - 14*B + 18*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2
*C*Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(7*d) + (2*(7*B +
C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.427648, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {4088, 4010, 4001, 3792}

$$\frac{2(35A-14B+18C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(35A+49B+27C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7B+C)\tan(c+dx)(a\sec}{35ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*a*(35*A + 49*B + 27*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*(35*A - 14*B + 18*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2
*C*Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(7*d) + (2*(7*B +
C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\tan(}{7d}$$

$$= \frac{2C\sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\tan(}{7d}$$

$$= \frac{2(35A-14B+18C)\sqrt{a+a\sec(c+dx)}}{105d}$$

$$= \frac{2a(35A+49B+27C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(}{}$$

Mathematica [A] time = 1.31726, size = 119, normalized size = 0.81

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}(3(35A+42B+36C)\cos(c+dx)+(35A+28B+24C)\cos(2(c+dx}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] ((35*A + 28*B + 54*C + 3*(35*A + 42*B + 36*C)*Cos[c + d*x] + (35*A + 28*B +
 24*C)*Cos[2*(c + d*x)] + 35*A*Cos[3*(c + d*x)] + 28*B*Cos[3*(c + d*x)] + 2
4*C*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*
x)/2])/(105*d)
```

Maple [A] time = 0.334, size = 138, normalized size = 0.9

$$-\frac{(-2+2\cos(dx+c))\left(70A(\cos(dx+c))^3+56B(\cos(dx+c))^3+48C(\cos(dx+c))^3+35A(\cos(dx+c))^2+28B}{105d(\cos(dx+c))^3\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(70*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+48*C*cos(d*x+
c)^3+35*A*cos(d*x+c)^2+28*B*cos(d*x+c)^2+24*C*cos(d*x+c)^2+21*B*cos(d*x+c)+
18*C*cos(d*x+c)+15*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(
d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.494984, size = 286, normalized size = 1.95

$$\frac{2\left(2\left(35A+28B+24C\right)\cos\left(dx+c\right)^{3}+\left(35A+28B+24C\right)\cos\left(dx+c\right)^{2}+3\left(7B+6C\right)\cos\left(dx+c\right)+15C\right)\sqrt{\frac{a\cos(dx}{\cos(d}}}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/105*(2*(35*A + 28*B + 24*C)*cos(d*x + c)^3 + (35*A + 28*B + 24*C)*cos(d*x
 + c)^2 + 3*(7*B + 6*C)*cos(d*x + c) + 15*C)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2
)*sec(c + d*x)**2, x)
```

Giac [B] time = 4.65004, size = 386, normalized size = 2.63

$$-\frac{2\left(105\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ca^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(245\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -2/105*(105*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d*x
 + c)) + 105*sqrt(2)*C*a^4*sgn(cos(d*x + c)) - (245*sqrt(2)*A*a^4*sgn(cos(d
*x + c)) + 175*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*C*a^4*sgn(cos(
d*x + c)) - (175*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 119*sqrt(2)*B*a^4*sgn(co
s(d*x + c)) + 147*sqrt(2)*C*a^4*sgn(cos(d*x + c)) - (35*sqrt(2)*A*a^4*sgn(c
os(d*x + c)) + 49*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 27*sqrt(2)*C*a^4*sgn(co
s(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.485 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=104

$$\frac{2a(15A+5B+7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(5B-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] (2*a*(15*A + 5*B + 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
(5*B - 2*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec
[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.209287, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {4082, 4001, 3792}

$$\frac{2a(15A+5B+7C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(5B-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2C\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (2*a*(15*A + 5*B + 7*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
(5*B - 2*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a + a*Sec
[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{2}{}$$
$$= \frac{2(5B-2C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}$$
$$= \frac{2a(15A+5B+7C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2(5B}{}$$

Mathematica [A] time = 0.786311, size = 83, normalized size = 0.8

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}((15A+10B+8C)\cos(2(c+dx))+15A+2(5B+4C)\cos(c+dx)+}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((15*A + 10*B + 14*C + 2*(5*B + 4*C)*Cos[c + d*x] + (15*A + 10*B + 8*C)*Cos
[2*(c + d*x)])*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/
(15*d)
```

Maple [A] time = 0.319, size = 105, normalized size = 1.

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^2+10B(\cos(dx+c))^2+8C(\cos(dx+c))^2+5B\cos(dx+c)+4C\cos(d\right.}{15d(\cos(dx+c))^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+8*C*cos(d*x+c)
^2+5*B*cos(d*x+c)+4*C*cos(d*x+c)+3*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/c
os(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.494679, size = 225, normalized size = 2.16

$$\frac{2\left((15\,A+10\,B+8\,C)\cos\left(dx+c\right)^{2}+(5\,B+4\,C)\cos\left(dx+c\right)+3\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 2/15*((15*A + 10*B + 8*C)*cos(d*x + c)^2 + (5*B + 4*C)*cos(d*x + c) + 3*C)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*
cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/
2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2
)*sec(c + d*x), x)
```

Giac [B] time = 4.54192, size = 302, normalized size = 2.9

$$2\left(15\sqrt{2}Aa^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\sqrt{2}Ba^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+15\sqrt{2}Ca^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(30\sqrt{2}Aa^{3}\mathrm{sgn}\left(\cos\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 15*sqrt(2)*B*a^3*sgn(cos(d*x + c
)) + 15*sqrt(2)*C*a^3*sgn(cos(d*x + c)) - (30*sqrt(2)*A*a^3*sgn(cos(d*x + c
)) + 20*sqrt(2)*B*a^3*sgn(cos(d*x + c)) + 10*sqrt(2)*C*a^3*sgn(cos(d*x + c)
) - (15*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 5*sqrt(2)*B*a^3*sgn(cos(d*x + c))
 + 7*sqrt(2)*C*a^3*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.486 $\int \sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx\right.$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*(3*B + C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sqrt[a +
a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.152305, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4054, 3915, 3774, 203, 3792}

$$\frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*(3*B + C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sqrt[a +
a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2\int \sqrt{a+a\sec(c-}{}$$
$$= \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + A\int \sqrt{a+a\sec(c}$$
$$= \frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c}{3d}$$
$$= \frac{2\sqrt{a}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a(3B+C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.687608, size = 101, normalized size = 1.01

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}A\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\sin\left(\frac{1}{2}(c+dx)\right)((3B-}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*A*ArcS
in[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(C + (3*B + 2*C)*Cos[c
+ d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] time = 0.321, size = 236, normalized size = 2.4

$$\frac{1}{6d\sin(dx+c)\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\mathrm{cc}}{\mathrm{cos}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+3*A*2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-12*B*cos(d*x+c)^2-8*C*cos(d*x+c)^2+12*B*c
os(d*x+c)+4*C*cos(d*x+c)+4*C)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.551226, size = 792, normalized size = 7.92

$$\left[\frac{3\left(A\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(\left(3\,B+2\right.\right.}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/3*(3*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^
2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((3*B + 2*C)*cos(d*x +
c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^2 + d*cos(d*x + c)), -2/3*(3*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(a)
*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c))) - ((3*B + 2*C)*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2
), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.487 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=98

$$\frac{\sqrt{a}(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(A-2C)\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d - (a*(A - 2*C)*Tan[c + d*x
])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.210891, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4086, 3915, 3774, 203, 3792}

$$\frac{\sqrt{a}(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(A-2C)\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d - (a*(A - 2*C)*Tan[c + d*x
])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{\int \sqrt{a}}{}$$
$$= \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(A$$
$$= \frac{A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} - \frac{a(A}{d\sqrt{}}$$
$$= \frac{\sqrt{a}(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{A\sqrt{}}{}$$

Mathematica [A] time = 0.395689, size = 94, normalized size = 0.96

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(A+2B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx)\right)(A\cos(\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(A + 2*B)*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*C + A*Cos[c + d*x])*Sin[(c
+ d*x)/2]))/(2*d)
```

Maple [B] time = 0.351, size = 210, normalized size = 2.1

$$-\frac{1}{2\,d\sin(dx+c)}\left(A\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\,\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,B\sqrt{2}\mathrm{Ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*B*
2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)
^2-2*A*cos(d*x+c)+4*C*cos(d*x+c)-4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/s
in(d*x+c)
```

Maxima [B] time = 1.97047, size = 1268, normalized size = 12.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] 1/4*(4*B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A)/d
```

Fricas [A] time = 0.667057, size = 717, normalized size = 7.32

$$\left[\frac{((A+2B)\cos(dx+c)+A+2B)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2(A\cos(dx+c}{2(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/2*(((A + 2*B)*cos(d*x + c) + A + 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 -
 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(A*cos(d*x + c) + 2*C)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(
```

```
((A + 2*B)*cos(d*x + c) + A + 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (A*cos(d*x + c) + 2*C
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/
2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.38662, size = 531, normalized size = 5.42

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}Ca\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+\left(A\sqrt{-a}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+2\,B\sqrt{-a}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\log\left(\left|\left(\sqrt{-}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a*sgn(cos(d*x + c))*t
an(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (A*sqrt(-a)*sgn(cos(d*
x + c)) + 2*B*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (A*s
qrt(-a)*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqr
t(2) - 3))) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a*sgn(cos(d*x + c)) - A*sqrt(-a)*a^2*sgn
(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*a + a^2))/d
```

3.488 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=117

$$\frac{\sqrt{a}(3A+4B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (Sqrt[a]*(3*A + 4*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
 d*x]]])/(4*d) + (a*(A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
+ (A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.277488, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {4086, 4015, 3774, 203}

$$\frac{\sqrt{a}(3A+4B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (Sqrt[a]*(3*A + 4*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
 d*x]]])/(4*d) + (a*(A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
+ (A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{2d}$$

$$= \frac{a(A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)}{}$$

$$= \frac{a(A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)}{}$$

$$= \frac{\sqrt{a}(3A+4B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{4d}$$

Mathematica [A] time = 0.489035, size = 113, normalized size = 0.97

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(3A+4B+8C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(3
*A + 4*B + 8*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(3*
A + 4*B + 2*A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.383, size = 548, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(3*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(3/2)+4*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))+8*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))+3*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)
+4*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+8*C*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)^4-4*
A*cos(d*x+c)^3-16*B*cos(d*x+c)^3+12*A*cos(d*x+c)^2+16*B*cos(d*x+c)^2)*(a*(c
os(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] time = 2.40706, size = 2695, normalized size = 23.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*C*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 1)))*A + 4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*a
```

```
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
1)))*B)/d
```

Fricas [A] time = 0.913163, size = 833, normalized size = 7.12

$$\left[\frac{((3A+4B+8C)\cos(dx+c)+3A+4B+8C)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{8(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/8*(((3*A + 4*B + 8*C)*cos(d*x + c) + 3*A + 4*B + 8*C)*sqrt(-a)*log((2*a*
cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d
*x + c)^2 + (3*A + 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((3*A + 4*B + 8*C)*cos(d*x + c
) + 3*A + 4*B + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c)^2 + (3*A + 4*B)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.67674, size = 891, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/8*((3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sgn(cos(d*x + c)) + 8*
C*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(-a)*sg
n(cos(d*x + c)) + 4*B*sqrt(-a)*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*sgn(cos(d*x
 + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a*sgn(cos(d*x + c
)) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^6*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 76
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*
sqrt(-a)*a^2*sgn(cos(d*x + c)) - 17*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 36*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(
-a)*a^3*sgn(cos(d*x + c)) + A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 4*B*sqrt(-a)
*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.489 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=163

$$\frac{a(5A+6B+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5A+6B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{A\,\mathrm{si}}{}$$

```
[Out] (Sqrt[a]*(5*A + 6*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
 d*x]]])/(8*d) + (a*(5*A + 6*B + 8*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a*(A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c +
d*x]]) + (A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.370184, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4086, 4015, 3805, 3774, 203}

$$\frac{a(5A+6B+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(5A+6B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{A\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (Sqrt[a]*(5*A + 6*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
 d*x]]])/(8*d) + (a*(5*A + 6*B + 8*C)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a*(A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c +
d*x]]) + (A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+\ldots}{3d}$$
$$= \frac{a(A+6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos\ldots}{\ldots}$$
$$= \frac{a(5A+6B+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(A+6B)\ldots}{12d\ldots}$$
$$= \frac{a(5A+6B+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(A+6B)\ldots}{12d\ldots}$$
$$= \frac{\sqrt{a}(5A+6B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \ldots$$

Mathematica [C] time = 0.482351, size = 152, normalized size = 0.93

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2A\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)+2B\sqrt{1-\sec(\ldots}\right.}{d\sqrt{1-\sec\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] ((C*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])
 + 2*B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]] + 2*A*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*
x]])
```

Maple [B] time = 0.362, size = 832, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d*(15*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))+18*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*sin(d*x+c)+24*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*sin(d*x+c)+30*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))+36*B*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*sin(d*x+c)+48*C*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*sin(d*x+c)+15*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*sin(d*x+c)+18*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*s
in(d*x+c)+24*C*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)
+64*A*cos(d*x+c)^6+16*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+40*A*cos(d*x+c)^4+48
*B*cos(d*x+c)^4+192*C*cos(d*x+c)^4-120*A*cos(d*x+c)^3-144*B*cos(d*x+c)^3-19
2*C*cos(d*x+c)^3)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+
c)
```

Maxima [B] time = 3.143, size = 5090, normalized size = 31.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] 1/96*((4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
```

```
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arct
an2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + 6*(2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) -
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2
*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*x + 2*
c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) +
 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B + 24*(2*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x +
 c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a)
+ sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*C)/d
```

Fricas [A] time = 0.912951, size = 946, normalized size = 5.8

$$\left[\frac{3\left((5A+6B+8C)\cos(dx+c)+5A+6B+8C\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-}{\cos(dx+c)+1}\right.}{48\,(d\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((5*A + 6*B + 8*C)*cos(d*x + c) + 5*A + 6*B + 8*C)*sqrt(-a)*log((2
*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*co
s(d*x + c)^3 + 2*(5*A + 6*B)*cos(d*x + c)^2 + 3*(5*A + 6*B + 8*C)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d), -1/24*(3*((5*A + 6*B + 8*C)*cos(d*x + c) + 5*A + 6*B + 8*C)*sqrt(a)*a
rctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) - (8*A*cos(d*x + c)^3 + 2*(5*A + 6*B)*cos(d*x + c)^2 + 3*(5*A + 6*B
 + 8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))
```

```
/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.98312, size = 1596, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 6*B*sqrt(-a)*sgn(cos(d*x + c)) +
 8*C*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-
a)*sgn(cos(d*x + c)) + 6*B*sqrt(-a)*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(63*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a*sgn(cos(
d*x + c)) - 30*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^10*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 72*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a*sgn(cos(d*x + c
)) - 369*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^8*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 66*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) -
 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1638*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 7
56*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*
B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 3024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 107
4*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A
*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 732*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1776*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*s
qrt(-a)*a^4*sgn(cos(d*x + c)) + 171*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 138*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt
(-a)*a^5*sgn(cos(d*x + c)) + 360*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 13*A*sqrt(
-a)*a^6*sgn(cos(d*x + c)) - 6*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 24*C*sqrt(
-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.490 $\int \cos^4(c+dx)\sqrt{a + a\sec(c+dx)}\left(A + B\sec(c+dx) + C\,\mathrm{s}\right.$

Optimal. Leaf size=209

$$\frac{a(35A+40B+48C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}} + \frac{\sqrt{a}(35A+40B+48C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d} + \frac{a(35A+40B+48C)\sin(c+dx)\,\mathrm{c}}{96d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(35*A + 40*B + 48*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(64*d) + (a*(35*A + 40*B + 48*C)*Sin[c + d*x])/(64*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*(35*A + 40*B + 48*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*
Sqrt[a + a*Sec[c + d*x]]) + (a*(A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d
*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(4*d)
```

Rubi [A] time = 0.45412, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4086, 4015, 3805, 3774, 203}

$$\frac{a(35A+40B+48C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}} + \frac{\sqrt{a}(35A+40B+48C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d} + \frac{a(35A+40B+48C)\sin(c+dx)\,\mathrm{c}}{96d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (Sqrt[a]*(35*A + 40*B + 48*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(64*d) + (a*(35*A + 40*B + 48*C)*Sin[c + d*x])/(64*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*(35*A + 40*B + 48*C)*Cos[c + d*x]*Sin[c + d*x])/(96*d*
Sqrt[a + a*Sec[c + d*x]]) + (a*(A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d
*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(4*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+a}{4d}$$

$$= \frac{a(A+8B)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{A\,\mathrm{co}}{}$$

$$= \frac{a(35A+40B+48C)\cos(c+dx)\sin(c+d}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a(35A+40B+48C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a(35A}{}$$

$$= \frac{a(35A+40B+48C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a(35A}{}$$

$$= \frac{\sqrt{a}(35A+40B+48C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx}}\right.}{64d}$$

Mathematica [C] time = 0.238655, size = 90, normalized size = 0.43

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(A\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)+B\text{Hypergeometric2F1}\left(\frac{1}{2},4\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*(C*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]] + B*Hypergeometric2F
1[1/2, 4, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec
[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [B] time = 0.398, size = 1105, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3072/d*(105*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(7/2)*2^(1/2)+120*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(7/2)*2^(1/2)+144*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*2^(1/2)+315*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)+360*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+432*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+315*A*sin(d*x+c)*cos(d*x+c)*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+360*B*sin(d*x+c)*cos(d*x+c)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+432*C*sin(d*x+c)*cos(d*x+c)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+105*A*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+120*B*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+144*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-128*A*cos(d*x+c)^7-1024*B*cos(d
*x+c)^7-224*A*cos(d*x+c)^6-256*B*cos(d*x+c)^6-1536*C*cos(d*x+c)^6-560*A*cos
(d*x+c)^5-640*B*cos(d*x+c)^5-768*C*cos(d*x+c)^5+1680*A*cos(d*x+c)^4+1920*B*
cos(d*x+c)^4+2304*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.29231, size = 1084, normalized size = 5.19

$$\left[\frac{3\left((35\,A+40\,B+48\,C)\cos\left(dx+c\right)+35\,A+40\,B+48\,C\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos\cdots}{\cos(dx+c)+1}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*((35*A + 40*B + 48*C)*cos(d*x + c) + 35*A + 40*B + 48*C)*sqrt(-a)
*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*
(48*A*cos(d*x + c)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 2*(35*A + 40*B + 48*C
)*cos(d*x + c)^2 + 3*(35*A + 40*B + 48*C)*cos(d*x + c))*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((35*A +
 40*B + 48*C)*cos(d*x + c) + 35*A + 40*B + 48*C)*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*c
os(d*x + c)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 2*(35*A + 40*B + 48*C)*cos(d
*x + c)^2 + 3*(35*A + 40*B + 48*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.16694, size = 2049, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/384*(3*(35*A*sqrt(-a)*sgn(cos(d*x + c)) + 40*B*sqrt(-a)*sgn(cos(d*x + c)
) + 48*C*sqrt(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(35*A*s
qrt(-a)*sgn(cos(d*x + c)) + 40*B*sqrt(-a)*sgn(cos(d*x + c)) + 48*C*sqrt(-a)
*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(279*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a*
sgn(cos(d*x + c)) - 504*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 240*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a*sgn(c
os(d*x + c)) + 285*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^12*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 5976*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^2*sgn(c
os(d*x + c)) - 1968*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^12*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 4605*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^3*sgn(
cos(d*x + c)) - 31320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^10*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 2640*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^3*sg
```

n(cos(d*x + c)) + 37281*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 90168*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 41616*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 35643*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 66024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 42288*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 9175*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 16904*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 12528*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 1311*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 1992*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 1392*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 43*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 104*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 48*C*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d

3.491 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=243

$$\frac{2a^2(99A+110B+84C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(429A+374B+336C)\tan(c+dx)}{495d\sqrt{a\sec(c+dx)+a}}+\frac{2(429A+374B+336C)\,t}{}$$

```
[Out] (2*a^2*(429*A + 374*B + 336*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]
]) + (2*a^2*(99*A + 110*B + 84*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[
a + a*Sec[c + d*x]]) - (4*a*(429*A + 374*B + 336*C)*Sqrt[a + a*Sec[c + d*x]
]*Tan[c + d*x])/(3465*d) + (2*a*(11*B + 3*C)*Sec[c + d*x]^3*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(99*d) + (2*(429*A + 374*B + 336*C)*(a + a*Sec[c +
d*x])^(3/2)*Tan[c + d*x])/(1155*d) + (2*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x
])^(3/2)*Tan[c + d*x])/(11*d)
```

Rubi [A] time = 0.694628, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4088, 4018, 4016, 3800, 4001, 3792}

$$\frac{2a^2(99A+110B+84C)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(429A+374B+336C)\tan(c+dx)}{495d\sqrt{a\sec(c+dx)+a}}+\frac{2(429A+374B+336C)\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*a^2*(429*A + 374*B + 336*C)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]
]) + (2*a^2*(99*A + 110*B + 84*C)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[
a + a*Sec[c + d*x]]) - (4*a*(429*A + 374*B + 336*C)*Sqrt[a + a*Sec[c + d*x]
]*Tan[c + d*x])/(3465*d) + (2*a*(11*B + 3*C)*Sec[c + d*x]^3*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(99*d) + (2*(429*A + 374*B + 336*C)*(a + a*Sec[c +
d*x])^(3/2)*Tan[c + d*x])/(1155*d) + (2*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x
])^(3/2)*Tan[c + d*x])/(11*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)(a+a\sec(c+dx))^{3/2}}{11d}$$

$$= \frac{2a(11B+3C)\sec^3(c+dx)\sqrt{a+a\sec}}{99d}$$

$$= \frac{2a^2(99A+110B+84C)\sec^3(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(99A+110B+84C)\sec^3(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(99A+110B+84C)\sec^3(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(429A+374B+336C)\tan(c+d}{495d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 2.03983, size = 185, normalized size = 0.76

$$a\tan\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}((12441A+12386B+12684C)\cos(c+dx)+(4422A+4862B+4$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a*(3564*A + 4114*B + 4956*C + (12441*A + 12386*B + 12684*C)*Cos[c + d*x] +
 (4422*A + 4862*B + 4368*C)*Cos[2*(c + d*x)] + 5577*A*Cos[3*(c + d*x)] + 48
62*B*Cos[3*(c + d*x)] + 4368*C*Cos[3*(c + d*x)] + 858*A*Cos[4*(c + d*x)] +
748*B*Cos[4*(c + d*x)] + 672*C*Cos[4*(c + d*x)] + 858*A*Cos[5*(c + d*x)] +
748*B*Cos[5*(c + d*x)] + 672*C*Cos[5*(c + d*x)])*Sec[c + d*x]^5*Sqrt[a*(1 +
 Sec[c + d*x])]*Tan[(c + d*x)/2])/(6930*d)
```

Maple [A] time = 0.36, size = 205, normalized size = 0.8

$$-\frac{2a(-1+\cos(dx+c))\left(3432A(\cos(dx+c))^5+2992B(\cos(dx+c))^5+2688C(\cos(dx+c))^5+1716A(\cos(dx+c))\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(3432*A*cos(d*x+c)^5+2992*B*cos(d*x+c)^5+2688*C
*cos(d*x+c)^5+1716*A*cos(d*x+c)^4+1496*B*cos(d*x+c)^4+1344*C*cos(d*x+c)^4+1
287*A*cos(d*x+c)^3+1122*B*cos(d*x+c)^3+1008*C*cos(d*x+c)^3+495*A*cos(d*x+c)
^2+935*B*cos(d*x+c)^2+840*C*cos(d*x+c)^2+385*B*cos(d*x+c)+735*C*cos(d*x+c)+
315*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.523222, size = 435, normalized size = 1.79

$$\frac{2\left(8(429A+374B+336C)a\cos(dx+c)^5+4(429A+374B+336C)a\cos(dx+c)^4+3(429A+374B+336C)a\cos\right.}{3465\left(d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/3465*(8*(429*A + 374*B + 336*C)*a*cos(d*x + c)^5 + 4*(429*A + 374*B + 336
*C)*a*cos(d*x + c)^4 + 3*(429*A + 374*B + 336*C)*a*cos(d*x + c)^3 + 5*(99*A
 + 187*B + 168*C)*a*cos(d*x + c)^2 + 35*(11*B + 21*C)*a*cos(d*x + c) + 315*
C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6
```

```
+ d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.19118, size = 554, normalized size = 2.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -4/3465*(3465*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 3465*sqrt(2)*B*a^7*sgn(cos(
d*x + c)) + 3465*sqrt(2)*C*a^7*sgn(cos(d*x + c)) - (11550*sqrt(2)*A*a^7*sgn
(cos(d*x + c)) + 9240*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 6930*sqrt(2)*C*a^7*
sgn(cos(d*x + c)) - (17094*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 14784*sqrt(2)*
B*a^7*sgn(cos(d*x + c)) + 15246*sqrt(2)*C*a^7*sgn(cos(d*x + c)) - (14652*sq
rt(2)*A*a^7*sgn(cos(d*x + c)) + 13662*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 110
88*sqrt(2)*C*a^7*sgn(cos(d*x + c)) - (6897*sqrt(2)*A*a^7*sgn(cos(d*x + c))
+ 5687*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 5313*sqrt(2)*C*a^7*sgn(cos(d*x + c
)) - 2*(627*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 517*sqrt(2)*B*a^7*sgn(cos(d*x
 + c)) + 483*sqrt(2)*C*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1
/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*
d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.492 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=187

$$\frac{8a^2(63A+57B+47C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(63A-18B+22C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{315d}+\frac{2a(63A+57B+47C)\mathrm{ta}}{3}$$

```
[Out] (8*a^2*(63*A + 57*B + 47*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
+ (2*a*(63*A + 57*B + 47*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d)
+ (2*(63*A - 18*B + 22*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*d)
+ (2*C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(9*d) + (2*(
3*B + C)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(21*a*d)
```

Rubi [A] time = 0.515844, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4088, 4010, 4001, 3793, 3792}

$$\frac{8a^2(63A+57B+47C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2(63A-18B+22C)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{315d}+\frac{2a(63A+57B+47C)\mathrm{ta}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (8*a^2*(63*A + 57*B + 47*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
+ (2*a*(63*A + 57*B + 47*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d)
+ (2*(63*A - 18*B + 22*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*d)
+ (2*C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(9*d) + (2*(
3*B + C)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(21*a*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{3/2}}{9d}$$
$$= \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{3/2}}{9d}$$
$$= \frac{2(63A-18B+22C)(a+a\sec(c+dx}{315d}$$
$$= \frac{2a(63A+57B+47C)\sqrt{a+a\sec(c+}}{315d}$$
$$= \frac{8a^2(63A+57B+47C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+$$

Mathematica [A] time = 2.14844, size = 152, normalized size = 0.81

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}((567A+648B+748C)\cos(c+dx)+(882A+858B+748C)\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a*(693*A + 702*B + 752*C + (567*A + 648*B + 748*C)*Cos[c + d*x] + (882*A +
 858*B + 748*C)*Cos[2*(c + d*x)] + 189*A*Cos[3*(c + d*x)] + 156*B*Cos[3*(c
+ d*x)] + 136*C*Cos[3*(c + d*x)] + 189*A*Cos[4*(c + d*x)] + 156*B*Cos[4*(c
+ d*x)] + 136*C*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]
*Tan[(c + d*x)/2])/(630*d)
```

Maple [A] time = 0.315, size = 172, normalized size = 0.9

$$-\frac{2a(-1+\cos(dx+c))\left(378A(\cos(dx+c))^4+312B(\cos(dx+c))^4+272C(\cos(dx+c))^4+189A(\cos(dx+c))^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(378*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+272*C*cos
(d*x+c)^4+189*A*cos(d*x+c)^3+156*B*cos(d*x+c)^3+136*C*cos(d*x+c)^3+63*A*cos
(d*x+c)^2+117*B*cos(d*x+c)^2+102*C*cos(d*x+c)^2+45*B*cos(d*x+c)+85*C*cos(d*
x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.50904, size = 363, normalized size = 1.94

$$\frac{2\left(2\,(189\,A+156\,B+136\,C)a\cos\left(dx+c\right)^{4}+(189\,A+156\,B+136\,C)a\cos\left(dx+c\right)^{3}+3\,(21\,A+39\,B+34\,C)a\cos\left(dx+\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/315*(2*(189*A + 156*B + 136*C)*a*cos(d*x + c)^4 + (189*A + 156*B + 136*C)
*a*cos(d*x + c)^3 + 3*(21*A + 39*B + 34*C)*a*cos(d*x + c)^2 + 5*(9*B + 17*C
)*a*cos(d*x + c) + 35*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.03251, size = 470, normalized size = 2.51

$$4\left(315\sqrt{2}Aa^6\text{sgn}\left(\cos\left(dx+c\right)\right)+315\sqrt{2}Ba^6\text{sgn}\left(\cos\left(dx+c\right)\right)+315\sqrt{2}Ca^6\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(945\sqrt{2}Aa^6\text{sgn}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] 4/315*(315*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^6*sgn(cos(d*x
+ c)) + 315*sqrt(2)*C*a^6*sgn(cos(d*x + c)) - (945*sqrt(2)*A*a^6*sgn(cos(d*
x + c)) + 735*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 525*sqrt(2)*C*a^6*sgn(cos(d
*x + c)) - (1071*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 819*sqrt(2)*B*a^6*sgn(co
s(d*x + c)) + 819*sqrt(2)*C*a^6*sgn(cos(d*x + c)) - (567*sqrt(2)*A*a^6*sgn(
cos(d*x + c)) + 513*sqrt(2)*B*a^6*sgn(cos(d*x + c)) + 423*sqrt(2)*C*a^6*sgn
(cos(d*x + c)) - 2*(63*sqrt(2)*A*a^6*sgn(cos(d*x + c)) + 57*sqrt(2)*B*a^6*s
gn(cos(d*x + c)) + 47*sqrt(2)*C*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)
*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)*d)
```

3.493 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=144

$$\frac{8a^2(35A+21B+19C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(35A+21B+19C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2(7B-2C)\tan(c+dx)(a}{35d}$$

```
[Out] (8*a^2*(35*A + 21*B + 19*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
+ (2*a*(35*A + 21*B + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d)
+ (2*(7*B - 2*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a
+ a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.283473, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4082, 4001, 3793, 3792}

$$\frac{8a^2(35A+21B+19C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(35A+21B+19C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2(7B-2C)\tan(c+dx)(a}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (8*a^2*(35*A + 21*B + 19*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
+ (2*a*(35*A + 21*B + 19*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d)
+ (2*(7*B - 2*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a
+ a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} +$$
$$= \frac{2(7B-2C)(a+a\sec(c+dx))^{3/2}\tan(c}{35d}$$
$$= \frac{2a(35A+21B+19C)\sqrt{a+a\sec(c+a}}{105d}$$
$$= \frac{8a^2(35A+21B+19C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A] time = 1.63287, size = 120, normalized size = 0.83

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}((525A+462B+468C)\cos(c+dx)+2(35A+63B+52C)\cos(2(c}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (a*(70*A + 126*B + 164*C + (525*A + 462*B + 468*C)*Cos[c + d*x] + 2*(35*A +
 63*B + 52*C)*Cos[2*(c + d*x)] + 175*A*Cos[3*(c + d*x)] + 126*B*Cos[3*(c +
d*x)] + 104*C*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*T
an[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.296, size = 139, normalized size = 1.

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(175\,A\,(\cos(dx+c))^3+126\,B\,(\cos(dx+c))^3+104\,C\,(\cos(dx+c))^3+35\,A\,(\cos(dx+c))^2+\right.}{105\,d\,(\cos(dx+c))^3\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(175*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+104*C*cos
(d*x+c)^3+35*A*cos(d*x+c)^2+63*B*cos(d*x+c)^2+52*C*cos(d*x+c)^2+21*B*cos(d*
x+c)+39*C*cos(d*x+c)+15*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3
/sin(d*x+c)
```

Maple [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.501427, size = 300, normalized size = 2.08

$$\frac{2\left((175A+126B+104C)a\cos(dx+c)^3+(35A+63B+52C)a\cos(dx+c)^2+3(7B+13C)a\cos(dx+c)+15Ca\right)\sqrt{\cdots}}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/105*((175*A + 126*B + 104*C)*a*cos(d*x + c)^3 + (35*A + 63*B + 52*C)*a*co
s(d*x + c)^2 + 3*(7*B + 13*C)*a*cos(d*x + c) + 15*C*a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.89244, size = 386, normalized size = 2.68

$$-\frac{4\left(105\sqrt{2}Aa^5\mathrm{sgn}(\cos(dx+c))+105\sqrt{2}Ba^5\mathrm{sgn}(\cos(dx+c))+105\sqrt{2}Ca^5\mathrm{sgn}(\cos(dx+c))-\left(280\sqrt{2}Aa^5\mathrm{sgn}(\cos\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -4/105*(105*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^5*sgn(cos(d*x
 + c)) + 105*sqrt(2)*C*a^5*sgn(cos(d*x + c)) - (280*sqrt(2)*A*a^5*sgn(cos(d
*x + c)) + 210*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 140*sqrt(2)*C*a^5*sgn(cos(
d*x + c)) - (245*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 147*sqrt(2)*B*a^5*sgn(co
s(d*x + c)) + 133*sqrt(2)*C*a^5*sgn(cos(d*x + c)) - 2*(35*sqrt(2)*A*a^5*sgn
(cos(d*x + c)) + 21*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 19*sqrt(2)*C*a^5*sgn(
cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.494 $\int (a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=142

$$\frac{2a^2(15A+20B+12C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(5B+3C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+$$

```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(15*A + 20*B + 12*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a*(5*B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a +
a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.23624, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4054, 3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(15A+20B+12C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(5B+3C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(15*A + 20*B + 12*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a*(5*B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*C*(a +
a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2\int(a+a\sec(}{}$$

$$=\frac{2a(5B+3C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2C(a+}{}$$

$$=\frac{2a(5B+3C)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2C(a+}{}$$

$$=\frac{2a^2(15A+20B+12C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a(5B+3C)\sqrt{}}{}$$

$$=\frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2(15A+20B+12C}{15d\sqrt{a+a\sec(}}$$

Mathematica [A] time = 1.35286, size = 132, normalized size = 0.93

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((15A+25B+18C)\cos(2(c+dx))+15A+2(5B+}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(30*Sqrt[2]*A
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(15*A + 25*B + 24*
C + 2*(5*B + 9*C)*Cos[c + d*x] + (15*A + 25*B + 18*C)*Cos[2*(c + d*x)])*Sin
[(c + d*x)/2]))/(30*d)
```

Maple [B] time = 0.322, size = 361, normalized size = 2.5

$$-\frac{a}{60\,d\,(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\,A\sqrt{2}\,(\cos(dx+c))^2\sin(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{5/2}\mathrm{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/60/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*A*2^(1/2)*cos(d*x+c)^2*si
n(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+30*A*2^(1/2)*cos(d*x+c)*
sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+15*A*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+120*A*cos(d*x+c)^3+200*B*cos(d*
x+c)^3+144*C*cos(d*x+c)^3-120*A*cos(d*x+c)^2-160*B*cos(d*x+c)^2-72*C*cos(d*
x+c)^2-40*B*cos(d*x+c)-48*C*cos(d*x+c)-24*C)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.573072, size = 944, normalized size = 6.65

$$\left[\frac{15\left(Aa\cos\left(dx+c\right)^{3}+Aa\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left((15\right.}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/15*(15*(A*a*cos(d*x + c)^3 + A*a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((15*A + 25*B +
 18*C)*a*cos(d*x + c)^2 + (5*B + 9*C)*a*cos(d*x + c) + 3*C*a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)
^2), -2/15*(15*(A*a*cos(d*x + c)^3 + A*a*cos(d*x + c)^2)*sqrt(a)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
 ((15*A + 25*B + 18*C)*a*cos(d*x + c)^2 + (5*B + 9*C)*a*cos(d*x + c) + 3*C*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3
+ d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x) + C*sec(c + d*x)
**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.495 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=144

$$-\frac{a^2(3A-6B-8C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (a^2*(3*A - 6*B - 8*
C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (a*(3*A - 2*C)*Sqrt[a + a
*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.310406, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4086, 3917, 3915, 3774, 203, 3792}

$$-\frac{a^2(3A-6B-8C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(3A-2C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (a^2*(3*A - 6*B - 8*
C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (a*(3*A - 2*C)*Sqrt[a + a
*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} + \frac{\int(a\ldots}{\ldots}$$

$$= \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{a(3\ldots}{\ldots}$$

$$= \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{a(3\ldots}{\ldots}$$

$$= \frac{A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{a^2(3\ldots}{\ldots}$$

$$= \frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{A(\ldots}{\ldots}$$

Mathematica [A] time = 1.94935, size = 115, normalized size = 0.8

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}(\sec(c+dx)(3A\cos(2(c+dx))+3A+4C)+4(3B+5C))+6(3\ldots\right.}{6d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[a*(1 + Sec[c + d*x])]*(6*(3*A + 2*B)*ArcTan[Sqrt[-1 + Sec[c + d*x]]
] + Sqrt[-1 + Sec[c + d*x]]*(4*(3*B + 5*C) + (3*A + 4*C + 3*A*Cos[2*(c + d*
x)])*Sec[c + d*x]))*Tan[(c + d*x)/2])/(6*d*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.371, size = 409, normalized size = 2.8

$$-\frac{a}{12\,d\cos(dx+c)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-9A\sqrt{2}\sin(dx+c)\cos(dx+c)\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/12/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-9*A*2^(1/2)*sin(d*x+c)*cos(
d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-6*B*cos(d*x+c)*sin(d*x+c)*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-9*A*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-6*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*2^(1/2)*sin(d*x+c)+12*A*cos(d*x+c)^3-12*A*cos(d*x+c)^2+24*B*cos(
d*x+c)^2+40*C*cos(d*x+c)^2-24*B*cos(d*x+c)-32*C*cos(d*x+c)-8*C)/cos(d*x+c)/
sin(d*x+c)
```

Maxima [B] time = 2.19349, size = 2431, normalized size = 16.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
```

```
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A] time = 0.654096, size = 941, normalized size = 6.53

$$\left[\frac{3\left((3A+2B)a\cos(dx+c)^2+(3A+2B)a\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)}{\cos(dx+c)+1}\right.}{6\left(d\cos(dx+c)^2+d\cos(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/6*(3*((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqrt(-a
)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2
*(3*A*a*cos(d*x + c)^2 + 2*(3*B + 5*C)*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c
)), -1/3*(3*((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - (3*A*a*cos(d*x + c)^2 + 2*(3*B + 5*C)*a*cos(d*x + c) + 2*C*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
+ d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.54185, size = 633, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/6*(3*(3*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)
))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - a*(2*sqrt(2) + 3))) - 3*(3*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*B*s
qrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 12*sqrt(2)*(3*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt
(-a)*a^2*sgn(cos(d*x + c)) - A*sqrt(-a)*a^3*sgn(cos(d*x + c)))/((sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2) + 4*
(3*sqrt(2)*B*a^3*sgn(cos(d*x + c)) + 6*sqrt(2)*C*a^3*sgn(cos(d*x + c)) - (3
*sqrt(2)*B*a^3*sgn(cos(d*x + c)) + 4*sqrt(2)*C*a^3*sgn(cos(d*x + c)))*tan(1
/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.496 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=157

$$\frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7A+12B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}-\frac{a(A-4C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (a^(3/2)*(7*A + 12*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(4*d) + (a^2*(5*A + 4*B - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[
c + d*x]]) - (a*(A - 4*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (A
*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.450759, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4086, 4018, 4015, 3774, 203}

$$\frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(7A+12B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}-\frac{a(A-4C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (a^(3/2)*(7*A + 12*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(4*d) + (a^2*(5*A + 4*B - 8*C)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[
c + d*x]]) - (a*(A - 4*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (A
*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^{3/2}\sin}{2d}$$

$$= -\frac{a(A-4C)\sqrt{a+a\sec(c+dx)}\sin(c}{2d}$$

$$= \frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a(A}{}$$

$$= \frac{a^2(5A+4B-8C)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a(A}{}$$

$$= \frac{a^{3/2}(7A+12B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{a+a\sec(}}\right.}{4d}$$

Mathematica [A] time = 0.760699, size = 117, normalized size = 0.75

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(7A+12B+8C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+dx\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(7*A + 12*B + 8*C)*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(A + 8*C + (7*A + 4
*B)*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.395, size = 569, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/16/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(7*A*2^(1/2)*sin(d*x+c)*cos(d*
x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+12*B*cos(d*x+c)*sin(d*x+c)*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+8*C*cos(d*x+c)*sin(d*x+c)*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+7*A*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+12*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*2^(1/2)*sin(d*x+c)+8*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*2^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)^4-20*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3+2
8*A*cos(d*x+c)^2+16*B*cos(d*x+c)^2-32*C*cos(d*x+c)^2+32*C*cos(d*x+c))/cos(d
*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.924091, size = 887, normalized size = 5.65

$$\left[\frac{((7A+12B+8C)a\cos(dx+c)+(7A+12B+8C)a)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)}{\cos(dx+c)+1}\right.}{8(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/8*(((7*A + 12*B + 8*C)*a*cos(d*x + c) + (7*A + 12*B + 8*C)*a)*sqrt(-a)*l
og((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2
*A*a*cos(d*x + c)^2 + (7*A + 4*B)*a*cos(d*x + c) + 8*C*a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((7*A + 12
*B + 8*C)*a*cos(d*x + c) + (7*A + 12*B + 8*C)*a)*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a*
cos(d*x + c)^2 + (7*A + 4*B)*a*cos(d*x + c) + 8*C*a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.76439, size = 990, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/8*(16*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a^2*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (7*A*sqrt(-a)*a*sgn
(cos(d*x + c)) + 12*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*a*sgn(cos
(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - (7*A*sqrt(-a)*a*sgn(cos(d*x + c))
+ 12*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 8*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 12*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sq
rt(-a)*a^2*sgn(cos(d*x + c)) - 95*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 76*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a
)*a^3*sgn(cos(d*x + c)) + 53*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 36*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^4
*sgn(cos(d*x + c)) - 5*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 4*B*sqrt(-a)*a^5*
sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.497 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=165

$$\frac{a^2(19A+30B+24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11A+14B+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(A+2B)\sin(c+dx)\cos(c+dx)\sqrt{}}{4d}$$

```
[Out] (a^(3/2)*(11*A + 14*B + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(8*d) + (a^2*(19*A + 30*B + 24*C)*Sin[c + d*x])/(24*d*Sqrt[a +
a*Sec[c + d*x]]) + (a*(A + 2*B)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(4*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(3*d)
```

Rubi [A] time = 0.481126, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4086, 4017, 4015, 3774, 203}

$$\frac{a^2(19A+30B+24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(11A+14B+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(A+2B)\sin(c+dx)\cos(c+dx)\sqrt{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a^(3/2)*(11*A + 14*B + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(8*d) + (a^2*(19*A + 30*B + 24*C)*Sin[c + d*x])/(24*d*Sqrt[a +
a*Sec[c + d*x]]) + (a*(A + 2*B)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(4*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(3*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\text{s}}{3d}$$

$$= \frac{a(A+2B)\cos(c+dx)\sqrt{a+a\sec(c+}}{4d}$$

$$= \frac{a^2(19A+30B+24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+$$

$$= \frac{a^2(19A+30B+24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+$$

$$= \frac{a^{3/2}(11A+14B+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a+a\sec}}\right.}{8d}$$

Mathematica [A] time = 1.67123, size = 124, normalized size = 0.75

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\cos(c+dx)\sqrt{\sec(c+dx)-1}(2(11A+6B)\cos(c+dx)+4A\cos(2(c+dx))+3\right.}{24d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a*((33*A + 42*B + 72*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]] + Cos[c + d*x]*(37
*A + 42*B + 24*C + 2*(11*A + 6*B)*Cos[c + d*x] + 4*A*Cos[2*(c + d*x)])*Sqrt
[-1 + Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(24*d*Sqr
t[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.297, size = 833, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d*a*(33*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))+42*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)+72*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*sin(d*x+c)+66*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))+84*B*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*sin(d*x+c)+144*C*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*sin(d*x+c)+33*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*sin(d*x+c)+42*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)+72*C*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x
+c)+64*A*cos(d*x+c)^6+112*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+88*A*cos(d*x+c)^
4+240*B*cos(d*x+c)^4+192*C*cos(d*x+c)^4-264*A*cos(d*x+c)^3-336*B*cos(d*x+c)
^3-192*C*cos(d*x+c)^3)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin
(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.916013, size = 1003, normalized size = 6.08

$$\left[\frac{3\left((11A+14B+24C)a\cos(dx+c)+(11A+14B+24C)a\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\,\mathrm{co}}{\cos(dx+c)+1}\right.}{48\,(d\,\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((11*A + 14*B + 24*C)*a*cos(d*x + c) + (11*A + 14*B + 24*C)*a)*sqr
t(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
 + 2*(8*A*a*cos(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A + 14
```

```
*B + 8*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c) + d), -1/24*(3*((11*A + 14*B + 24*C)*a*cos(d*x + c) +
(11*A + 14*B + 24*C)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*a*cos(d*x + c)^3 + 2*(11*A
+ 6*B)*a*cos(d*x + c)^2 + 3*(11*A + 14*B + 8*C)*a*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.28482, size = 1612, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 14*B*sqrt(-a)*a*sgn(cos(d*x +
 c)) + 24*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(1
1*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 14*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 24*C
*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(33*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*s
qrt(-a)*a^2*sgn(cos(d*x + c)) + 42*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 72*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt
(-a)*a^2*sgn(cos(d*x + c)) - 303*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 822*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a
)*a^3*sgn(cos(d*x + c)) - 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*
a^4*sgn(cos(d*x + c)) + 3780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 3024*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a
^4*sgn(cos(d*x + c)) - 1806*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 2508*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^
5*sgn(cos(d*x + c)) - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 309*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^6*
sgn(cos(d*x + c)) + 498*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 360*(sqrt(-a)*tan(1
```

```
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^6*sgn
(cos(d*x + c)) - 19*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 30*B*sqrt(-a)*a^7*sg
n(cos(d*x + c)) - 24*C*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.498 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\mathrm{s}\right.$

Optimal. Leaf size=215

$$\frac{a^2(75A+88B+112C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75A+88B+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(39A+56B+48C)\sin(c+}{96d\sqrt{a\sec(c+dx)}}$$

```
[Out] (a^(3/2)*(75*A + 88*B + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(64*d) + (a^2*(75*A + 88*B + 112*C)*Sin[c + d*x])/(64*d*Sqrt[a
 + a*Sec[c + d*x]]) + (a^2*(39*A + 56*B + 48*C)*Cos[c + d*x]*Sin[c + d*x])/
(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(3*A + 8*B)*Cos[c + d*x]^2*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.58606, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4017, 4015, 3805, 3774, 203}

$$\frac{a^2(75A+88B+112C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(75A+88B+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(39A+56B+48C)\sin(c+}{96d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (a^(3/2)*(75*A + 88*B + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(64*d) + (a^2*(75*A + 88*B + 112*C)*Sin[c + d*x])/(64*d*Sqrt[a
 + a*Sec[c + d*x]]) + (a^2*(39*A + 56*B + 48*C)*Cos[c + d*x]*Sin[c + d*x])/
(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(3*A + 8*B)*Cos[c + d*x]^2*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\sin(\cdots}{4d}$$

$$= \frac{a(3A+8B)\cos^2(c+dx)\sqrt{a+a\sec(c+\cdots}}{24d}$$

$$= \frac{a^2(39A+56B+48C)\cos(c+dx)\sin(c\cdots}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(75A+88B+112C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cdots}{\cdots}$$

$$= \frac{a^2(75A+88B+112C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cdots}{\cdots}$$

$$= \frac{a^{3/2}(75A+88B+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c\cdots}{\sqrt{a+a\sec(\cdots}}\right.}{64d}$$

Mathematica [A] time = 1.74926, size = 157, normalized size = 0.73

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(75A+88B+112C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(c+d\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(75*A + 88*B + 11
2*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (285*A + 296*B +
 336*C + 2*(93*A + 88*B + 48*C)*Cos[c + d*x] + 4*(15*A + 8*B)*Cos[2*(c + d*
x)] + 12*A*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(
384*d)
```

Maple [B] time = 0.342, size = 1106, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3072/d*a*(225*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)+264*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*2^(1/2)+336*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(7/2)*2^(1/2)+675*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+792*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1008*C*sin(d*x+c)*cos(d*x+c)^2*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+675*A*sin(d*x+c)*cos(d*x+c)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+792*B*sin(d*x+c)*cos(d*x+c)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+1008*C*sin(d*x+c)*cos(d*x+
c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+225*A*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+264*B*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+336*C*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(7/2)*sin(d*x+c)-768*A*cos(d*x+c)^8-1152*A*cos(d*x+c)^7-1024*B*
cos(d*x+c)^7-480*A*cos(d*x+c)^6-1792*B*cos(d*x+c)^6-1536*C*cos(d*x+c)^6-120
0*A*cos(d*x+c)^5-1408*B*cos(d*x+c)^5-3840*C*cos(d*x+c)^5+3600*A*cos(d*x+c)^
4+4224*B*cos(d*x+c)^4+5376*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.3044, size = 1133, normalized size = 5.27

$$\left[\frac{3\left((75\,A + 88\,B + 112\,C)a\cos\left(dx + c\right) + (75\,A + 88\,B + 112\,C)a\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*((75*A + 88*B + 112*C)*a*cos(d*x + c) + (75*A + 88*B + 112*C)*a)*
sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
1)) + 2*(48*A*a*cos(d*x + c)^4 + 8*(15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A
+ 88*B + 48*C)*a*cos(d*x + c)^2 + 3*(75*A + 88*B + 112*C)*a*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d),
-1/192*(3*((75*A + 88*B + 112*C)*a*cos(d*x + c) + (75*A + 88*B + 112*C)*a)*
sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) - (48*A*a*cos(d*x + c)^4 + 8*(15*A + 8*B)*a*cos(d*x + c)^3
 + 2*(75*A + 88*B + 48*C)*a*cos(d*x + c)^2 + 3*(75*A + 88*B + 112*C)*a*cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.69475, size = 2066, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/384*(3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*B*sqrt(-a)*a*sgn(cos(d*x
+ c)) + 112*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*
(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 11
2*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
```

```
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(2
25*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14
*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 264*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 33
6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*
C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 6261*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 40
08*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12
*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 8592*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 3
5925*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
10*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 33960*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^4*sgn(cos(d*x + c))
+ 70032*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^10*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 127449*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^5*sgn(cos(d*x + c
)) - 131784*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^8*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 208080*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^5*sgn(cos(d*x
+ c)) + 101667*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 108312*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^6*sgn(cos(d
*x + c)) + 154608*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^6*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 26079*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^7*sgn(cos
(d*x + c)) - 29432*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 44208*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^7*sgn(co
s(d*x + c)) + 3303*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 3384*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^8*sgn(cos
(d*x + c)) + 5424*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 147*A*sqrt(-a)*a^9*sgn(co
s(d*x + c)) - 152*B*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 240*C*sqrt(-a)*a^9*sgn
(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*a + a^2)^4)/d
```

3.499 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{3/2}\left(A + B\sec(c + dx) + C\sec$

Optimal. Leaf size=263

$$\frac{a^2(133A + 150B + 176C)\sin(c + dx)}{128d\sqrt{a\sec(c + dx) + a}} + \frac{a^{3/2}(133A + 150B + 176C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d} + \frac{a^2(67A + 90B + 80C)\sin(c}{240d\sqrt{a\sec(c + a}}$$

```
[Out] (a^(3/2)*(133*A + 150*B + 176*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*S
ec[c + d*x]]])/(128*d) + (a^2*(133*A + 150*B + 176*C)*Sin[c + d*x])/(128*d*
Sqrt[a + a*Sec[c + d*x]]) + (a^2*(133*A + 150*B + 176*C)*Cos[c + d*x]*Sin[c
 + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(67*A + 90*B + 80*C)*Cos[c
 + d*x]^2*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(3*A + 10*B)*
Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (A*Cos[c + d
*x]^4*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.672605, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4017, 4015, 3805, 3774, 203}

$$\frac{a^2(133A + 150B + 176C)\sin(c + dx)}{128d\sqrt{a\sec(c + dx) + a}} + \frac{a^{3/2}(133A + 150B + 176C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d} + \frac{a^2(67A + 90B + 80C)\sin(c}{240d\sqrt{a\sec(c + a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a^(3/2)*(133*A + 150*B + 176*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*S
ec[c + d*x]]])/(128*d) + (a^2*(133*A + 150*B + 176*C)*Sin[c + d*x])/(128*d*
Sqrt[a + a*Sec[c + d*x]]) + (a^2*(133*A + 150*B + 176*C)*Cos[c + d*x]*Sin[c
 + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(67*A + 90*B + 80*C)*Cos[c
 + d*x]^2*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(3*A + 10*B)*
Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (A*Cos[c + d
*x]^4*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\text{s}}{5d}$$

$$= \frac{a(3A+10B)\cos^3(c+dx)\sqrt{a+a\sec(}}{40d}$$

$$= \frac{a^2(67A+90B+80C)\cos^2(c+dx)\text{si}}{240d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(133A+150B+176C)\cos(c+dx)}{192d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(133A+150B+176C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(133A+150B+176C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{3/2}(133A+150B+176C)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{128d}$$

Mathematica [A] time = 2.67023, size = 182, normalized size = 0.69

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(15\sqrt{2}(133A+150B+176C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[2]*(133*A + 150*B +
 176*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (2671*A + 285
0*B + 2960*C + 2*(1007*A + 930*B + 880*C)*Cos[c + d*x] + 4*(181*A + 150*B +
 80*C)*Cos[2*(c + d*x)] + 228*A*Cos[3*(c + d*x)] + 120*B*Cos[3*(c + d*x)] +
 48*A*Cos[4*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(3840*
d)
```

Maple [B] time = 0.365, size = 1379, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/61440/d*a*(13500*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*
cos(d*x+c)^2*2^(1/2)+15840*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(
d*x+c)*cos(d*x+c)^2*2^(1/2)+12288*A*cos(d*x+c)^10+16896*A*cos(d*x+c)^9+2048
0*C*cos(d*x+c)^8+15360*B*cos(d*x+c)^9+9000*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+10560*C*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+1995*A*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^4*sin(d*x+c)+7980*A*2^(1/2)*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^3*sin(d*x+c)+11970*A*2^(1/2)*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^2*sin(d*x+c)+7980*A*2^
(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)*sin(d*x+c)+2250*
B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)
+2640*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^4*2
^(1/2)+9000*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+
c)^3*2^(1/2)+10560*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*c
os(d*x+c)^3*2^(1/2)+21280*A*cos(d*x+c)^6-63840*A*cos(d*x+c)^5-72000*B*cos(d
*x+c)^5-84480*C*cos(d*x+c)^5+4864*A*cos(d*x+c)^8+8512*A*cos(d*x+c)^7+9600*B
*cos(d*x+c)^7+24000*B*cos(d*x+c)^6+28160*C*cos(d*x+c)^6+2250*B*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2640*C*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+1995*A*2^(1/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+23040*B*cos(d*x+c)^8+35840*C*cos(d*x+c)^
7)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.31934, size = 1295, normalized size = 4.92

$$\left[\frac{15\left((133\,A+150\,B+176\,C)a\cos\left(dx+c\right)+(133\,A+150\,B+176\,C)a\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/3840*(15*((133*A + 150*B + 176*C)*a*cos(d*x + c) + (133*A + 150*B + 176*
C)*a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x +
 c) + 1)) + 2*(384*A*a*cos(d*x + c)^5 + 48*(19*A + 10*B)*a*cos(d*x + c)^4 +
 8*(133*A + 150*B + 80*C)*a*cos(d*x + c)^3 + 10*(133*A + 150*B + 176*C)*a*c
os(d*x + c)^2 + 15*(133*A + 150*B + 176*C)*a*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/1920*(15*((1
33*A + 150*B + 176*C)*a*cos(d*x + c) + (133*A + 150*B + 176*C)*a)*sqrt(a)*a
rctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) - (384*A*a*cos(d*x + c)^5 + 48*(19*A + 10*B)*a*cos(d*x + c)^4 + 8*(
133*A + 150*B + 80*C)*a*cos(d*x + c)^3 + 10*(133*A + 150*B + 176*C)*a*cos(d
*x + c)^2 + 15*(133*A + 150*B + 176*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.00739, size = 2519, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/3840*(15*(133*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 150*B*sqrt(-a)*a*sgn(cos(
d*x + c)) + 176*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))
- 15*(133*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 150*B*sqrt(-a)*a*sgn(cos(d*x + c
)) + 176*C*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqr
t(2)*(1995*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^18*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2250*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*sqrt(-a)*a^2*sgn(cos(d*x +
 c)) + 2640*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^18*C*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 38505*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*A*sqrt(-a)*a^3*sgn(cos(d*x
 + c)) - 76110*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^16*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 55920*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*sqrt(-a)*a^3*sgn(cos(
d*x + c)) + 561660*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^14*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 737160*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^4*sgn
(cos(d*x + c)) + 582720*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 2684100*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a
^5*sgn(cos(d*x + c)) - 3492600*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 3395520*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqr
t(-a)*a^5*sgn(cos(d*x + c)) + 7371738*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 9022
860*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
0*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 9329760*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^6*sgn(cos(d*x + c))
 - 6407470*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^8*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 7635300*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^7*sgn(cos(d*x
+ c)) - 8110880*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^8*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 2176620*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^8*sgn(cos
(d*x + c)) + 2614440*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^6*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 2882880*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^8*sg
n(cos(d*x + c)) - 399860*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 460440*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^9
*sgn(cos(d*x + c)) - 498880*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 34035*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a
^10*sgn(cos(d*x + c)) + 41850*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^10*sgn(cos(d*x + c)) + 42960*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a
)*a^10*sgn(cos(d*x + c)) - 1201*A*sqrt(-a)*a^11*sgn(cos(d*x + c)) - 1470*B*
sqrt(-a)*a^11*sgn(cos(d*x + c)) - 1520*C*sqrt(-a)*a^11*sgn(cos(d*x + c)))/(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
+ a^2)^5)/d
```

3.500 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2} \left(A + B\sec(c + dx) + C \mathrm{s}\right.$

Optimal. Leaf size=294

$$\frac{2a^3(2717A + 2522B + 2224C)\tan(c + dx)\sec^3(c + dx)}{9009d\sqrt{a\sec(c + dx) + a}} + \frac{2a^2(143A + 182B + 136C)\tan(c + dx)\sec^3(c + dx)\sqrt{a\sec(}}{1287d}$$

```
[Out] (2*a^3*(10439*A + 9230*B + 8368*C)*Tan[c + d*x])/(6435*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a^3*(2717*A + 2522*B + 2224*C)*Sec[c + d*x]^3*Tan[c + d*x])/(9
009*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a^2*(10439*A + 9230*B + 8368*C)*Sqrt[a
 + a*Sec[c + d*x]]*Tan[c + d*x])/(45045*d) + (2*a^2*(143*A + 182*B + 136*C)
*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(1287*d) + (2*a*(104
39*A + 9230*B + 8368*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(15015*d)
+ (2*a*(13*B + 5*C)*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])
/(143*d) + (2*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(13
*d)
```

Rubi [A] time = 0.90489, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4088, 4018, 4016, 3800, 4001, 3792}

$$\frac{2a^3(2717A + 2522B + 2224C)\tan(c + dx)\sec^3(c + dx)}{9009d\sqrt{a\sec(c + dx) + a}} + \frac{2a^2(143A + 182B + 136C)\tan(c + dx)\sec^3(c + dx)\sqrt{a\sec(}}{1287d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*a^3*(10439*A + 9230*B + 8368*C)*Tan[c + d*x])/(6435*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a^3*(2717*A + 2522*B + 2224*C)*Sec[c + d*x]^3*Tan[c + d*x])/(9
009*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a^2*(10439*A + 9230*B + 8368*C)*Sqrt[a
 + a*Sec[c + d*x]]*Tan[c + d*x])/(45045*d) + (2*a^2*(143*A + 182*B + 136*C)
*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(1287*d) + (2*a*(104
39*A + 9230*B + 8368*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(15015*d)
+ (2*a*(13*B + 5*C)*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])
/(143*d) + (2*C*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(13
*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
```

```
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\tan}{13d}$$

$$= \frac{2a(13B+5C)\sec^3(c+dx)(a+a\sec(c+}{143d}$$

$$= \frac{2a^2(143A+182B+136C)\sec^3(c+dx)\sqrt{}}{1287d}$$

$$= \frac{2a^3(2717A+2522B+2224C)\sec^3(c+}{9009d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(2717A+2522B+2224C)\sec^3(c+}{9009d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(2717A+2522B+2224C)\sec^3(c+}{9009d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(10439A+9230B+8368C)\tan(c+}{6435d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 2.14288, size = 222, normalized size = 0.76

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(\sec(c+dx)+1)}(70(4576A+5083B+5552C)\cos(c+dx)+14(32747A+31850B\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(322751*A + 325910*B + 343612*C + 70*(4576*A + 5083*B + 5552*C)*Cos[c
+ d*x] + 14*(32747*A + 31850*B + 30334*C)*Cos[2*(c + d*x)] + 141570*A*Cos[3
*(c + d*x)] + 138450*B*Cos[3*(c + d*x)] + 125520*C*Cos[3*(c + d*x)] + 15658
5*A*Cos[4*(c + d*x)] + 138450*B*Cos[4*(c + d*x)] + 125520*C*Cos[4*(c + d*x)
] + 20878*A*Cos[5*(c + d*x)] + 18460*B*Cos[5*(c + d*x)] + 16736*C*Cos[5*(c
+ d*x)] + 20878*A*Cos[6*(c + d*x)] + 18460*B*Cos[6*(c + d*x)] + 16736*C*Cos
[6*(c + d*x)])*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/
(180180*d)
```

Maple [A] time = 0.37, size = 240, normalized size = 0.8

$$-\frac{2a^2(-1+\cos(dx+c))\left(83512A(\cos(dx+c))^6+73840B(\cos(dx+c))^6+66944C(\cos(dx+c))^6+41756A(\cos\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(83512*A*cos(d*x+c)^6+73840*B*cos(d*x+c)^6+6
6944*C*cos(d*x+c)^6+41756*A*cos(d*x+c)^5+36920*B*cos(d*x+c)^5+33472*C*cos(d
*x+c)^5+31317*A*cos(d*x+c)^4+27690*B*cos(d*x+c)^4+25104*C*cos(d*x+c)^4+1859
0*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+20920*C*cos(d*x+c)^3+5005*A*cos(d*x+c
)^2+14560*B*cos(d*x+c)^2+18305*C*cos(d*x+c)^2+4095*B*cos(d*x+c)+11970*C*cos
(d*x+c)+3465*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^6/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.570552, size = 541, normalized size = 1.84

$$\frac{2\left(8(10439A+9230B+8368C)a^2\cos(dx+c)^6+4(10439A+9230B+8368C)a^2\cos(dx+c)^5+3(10439A+92\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/45045*(8*(10439*A + 9230*B + 8368*C)*a^2*cos(d*x + c)^6 + 4*(10439*A + 92
30*B + 8368*C)*a^2*cos(d*x + c)^5 + 3*(10439*A + 9230*B + 8368*C)*a^2*cos(d
*x + c)^4 + 5*(3718*A + 4615*B + 4184*C)*a^2*cos(d*x + c)^3 + 35*(143*A + 4
16*B + 523*C)*a^2*cos(d*x + c)^2 + 315*(13*B + 38*C)*a^2*cos(d*x + c) + 346
5*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c)^7 + d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.7069, size = 637, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] 8/45045*(45045*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 45045*sqrt(2)*B*a^9*sgn(co
s(d*x + c)) + 45045*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - (180180*sqrt(2)*A*a^9
*sgn(cos(d*x + c)) + 150150*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 120120*sqrt(2
)*C*a^9*sgn(cos(d*x + c)) - (342342*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 30030
0*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 294294*sqrt(2)*C*a^9*sgn(cos(d*x + c))
- (391248*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 356070*sqrt(2)*B*a^9*sgn(cos(d*
x + c)) + 310596*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - (265837*sqrt(2)*A*a^9*sg
n(cos(d*x + c)) + 232375*sqrt(2)*B*a^9*sgn(cos(d*x + c)) + 212069*sqrt(2)*C
*a^9*sgn(cos(d*x + c)) - 4*(24167*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 21125*s
qrt(2)*B*a^9*sgn(cos(d*x + c)) + 19279*sqrt(2)*C*a^9*sgn(cos(d*x + c)) - 2*
(1859*sqrt(2)*A*a^9*sgn(cos(d*x + c)) + 1625*sqrt(2)*B*a^9*sgn(cos(d*x + c)
) + 1483*sqrt(2)*C*a^9*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d
*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.501 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=229

$$\frac{16a^2(165A+143B+125C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3465d}+\frac{64a^3(165A+143B+125C)\tan(c+dx)}{3465d\sqrt{a\sec(c+dx)+a}}+\frac{2(99A-22B}{}$$

```
[Out] (64*a^3*(165*A + 143*B + 125*C)*Tan[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*
x]]) + (16*a^2*(165*A + 143*B + 125*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x
])/(3465*d) + (2*a*(165*A + 143*B + 125*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c
 + d*x])/(1155*d) + (2*(99*A - 22*B + 26*C)*(a + a*Sec[c + d*x])^(5/2)*Tan[
c + d*x])/(693*d) + (2*C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*Tan[c +
d*x])/(11*d) + (2*(11*B + 5*C)*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(99
*a*d)
```

Rubi [A] time = 0.586802, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4088, 4010, 4001, 3793, 3792}

$$\frac{16a^2(165A+143B+125C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3465d}+\frac{64a^3(165A+143B+125C)\tan(c+dx)}{3465d\sqrt{a\sec(c+dx)+a}}+\frac{2(99A-22B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (64*a^3*(165*A + 143*B + 125*C)*Tan[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*
x]]) + (16*a^2*(165*A + 143*B + 125*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x
])/(3465*d) + (2*a*(165*A + 143*B + 125*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c
 + d*x])/(1155*d) + (2*(99*A - 22*B + 26*C)*(a + a*Sec[c + d*x])^(5/2)*Tan[
c + d*x])/(693*d) + (2*C*Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*Tan[c +
d*x])/(11*d) + (2*(11*B + 5*C)*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(99
*a*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\tan}{11d}$$

$$= \frac{2C\sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\tan}{11d}$$

$$= \frac{2(99A-22B+26C)(a+a\sec(c+dx))^{5/}}{693d}$$

$$= \frac{2a(165A+143B+125C)(a+a\sec(c+a}{1155d}$$

$$= \frac{16a^2(165A+143B+125C)\sqrt{a+a\sec(c}}{3465d}$$

$$= \frac{64a^3(165A+143B+125C)\tan(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 1.66857, size = 188, normalized size = 0.82

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\sec(c+dx)+1)}((49830A+49654B+50140C)\cos(c+dx)+4(4290A+4642B+46}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(13365*A + 15356*B + 18140*C + (49830*A + 49654*B + 50140*C)*Cos[c + d
*x] + 4*(4290*A + 4642*B + 4615*C)*Cos[2*(c + d*x)] + 22935*A*Cos[3*(c + d*
x)] + 20878*B*Cos[3*(c + d*x)] + 18460*C*Cos[3*(c + d*x)] + 3795*A*Cos[4*(c
 + d*x)] + 3212*B*Cos[4*(c + d*x)] + 2840*C*Cos[4*(c + d*x)] + 3795*A*Cos[5
*(c + d*x)] + 3212*B*Cos[5*(c + d*x)] + 2840*C*Cos[5*(c + d*x)])*Sec[c + d*
x]^5*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(13860*d)
```

Maple [A] time = 0.33, size = 207, normalized size = 0.9

$$-\frac{2a^2(-1+\cos(dx+c))\left(7590A(\cos(dx+c))^5+6424B(\cos(dx+c))^5+5680C(\cos(dx+c))^5+3795A(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(7590*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+5680
*C*cos(d*x+c)^5+3795*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+2840*C*cos(d*x+c)^4
+1980*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3+2130*C*cos(d*x+c)^3+495*A*cos(d*x+
c)^2+1430*B*cos(d*x+c)^2+1775*C*cos(d*x+c)^2+385*B*cos(d*x+c)+1120*C*cos(d*
x+c)+315*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.543928, size = 456, normalized size = 1.99

$$\frac{2\left(2(3795A+3212B+2840C)a^2\cos(dx+c)^5+(3795A+3212B+2840C)a^2\cos(dx+c)^4+3(660A+803B+\right.}{3465\left(d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] 2/3465*(2*(3795*A + 3212*B + 2840*C)*a^2*cos(d*x + c)^5 + (3795*A + 3212*B
+ 2840*C)*a^2*cos(d*x + c)^4 + 3*(660*A + 803*B + 710*C)*a^2*cos(d*x + c)^3
 + 5*(99*A + 286*B + 355*C)*a^2*cos(d*x + c)^2 + 35*(11*B + 32*C)*a^2*cos(d
*x + c) + 315*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.5264, size = 554, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -8/3465*(3465*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 3465*sqrt(2)*B*a^8*sgn(cos(
d*x + c)) + 3465*sqrt(2)*C*a^8*sgn(cos(d*x + c)) - (12705*sqrt(2)*A*a^8*sgn
(cos(d*x + c)) + 10395*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 8085*sqrt(2)*C*a^8
*sgn(cos(d*x + c)) - (19635*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 15939*sqrt(2)
*B*a^8*sgn(cos(d*x + c)) + 15015*sqrt(2)*C*a^8*sgn(cos(d*x + c)) - (16335*s
qrt(2)*A*a^8*sgn(cos(d*x + c)) + 14157*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 12
375*sqrt(2)*C*a^8*sgn(cos(d*x + c)) - 4*(1815*sqrt(2)*A*a^8*sgn(cos(d*x + c
)) + 1573*sqrt(2)*B*a^8*sgn(cos(d*x + c)) + 1375*sqrt(2)*C*a^8*sgn(cos(d*x
+ c)) - 2*(165*sqrt(2)*A*a^8*sgn(cos(d*x + c)) + 143*sqrt(2)*B*a^8*sgn(cos(
d*x + c)) + 125*sqrt(2)*C*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*ta
n(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1
/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^5*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.502 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2} \left(A + B\sec(c+dx) + C\,\mathrm{se}\right.$

Optimal. Leaf size=184

$$\frac{16a^2(21A+15B+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d} + \frac{64a^3(21A+15B+13C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}} + \frac{2a(21A+15B+13C}{}$$

```
[Out] (64*a^3*(21*A + 15*B + 13*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
 + (16*a^2*(21*A + 15*B + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315
*d) + (2*a*(21*A + 15*B + 13*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1
05*d) + (2*(9*B - 2*C)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2
*C*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.3447, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4082, 4001, 3793, 3792}

$$\frac{16a^2(21A+15B+13C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d} + \frac{64a^3(21A+15B+13C)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}} + \frac{2a(21A+15B+13C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```



```
[Out] (64*a^3*(21*A + 15*B + 13*C)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
 + (16*a^2*(21*A + 15*B + 13*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315
*d) + (2*a*(21*A + 15*B + 13*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(1
05*d) + (2*(9*B - 2*C)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2
*C*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad} + \frac{2}{}$$

$$= \frac{2(9B-2C)(a+a\sec(c+dx))^{5/2}\tan(c+}{63d}$$

$$= \frac{2a(21A+15B+13C)(a+a\sec(c+dx))^{3/}}{105d}$$

$$= \frac{16a^2(21A+15B+13C)\sqrt{a+a\sec(c+dx}}{315d}$$

$$= \frac{64a^3(21A+15B+13C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{16}{}$$

Mathematica [A] time = 2.44974, size = 156, normalized size = 0.85

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\sec(c+dx)+1)}(2(882A+1215B+1396C)\cos(c+dx)+4(966A+870B+803C)\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (a^2*(2961*A + 2790*B + 2908*C + 2*(882*A + 1215*B + 1396*C)*Cos[c + d*x] +
 4*(966*A + 870*B + 803*C)*Cos[2*(c + d*x)] + 588*A*Cos[3*(c + d*x)] + 690*
B*Cos[3*(c + d*x)] + 584*C*Cos[3*(c + d*x)] + 903*A*Cos[4*(c + d*x)] + 690*
B*Cos[4*(c + d*x)] + 584*C*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Sqrt[a*(1 + Sec
[c + d*x])]*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.29, size = 174, normalized size = 1.

$$-\frac{2a^2(-1+\cos(dx+c))\left(903A(\cos(dx+c))^4+690B(\cos(dx+c))^4+584C(\cos(dx+c))^4+294A(\cos(dx+c))^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(903*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+584*C*c
os(d*x+c)^4+294*A*cos(d*x+c)^3+345*B*cos(d*x+c)^3+292*C*cos(d*x+c)^3+63*A*c
os(d*x+c)^2+180*B*cos(d*x+c)^2+219*C*cos(d*x+c)^2+45*B*cos(d*x+c)+130*C*cos
(d*x+c)+35*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.533747, size = 374, normalized size = 2.03

$$\frac{2\left((903\,A + 690\,B + 584\,C)a^2\cos(dx+c)^4 + (294\,A + 345\,B + 292\,C)a^2\cos(dx+c)^3 + 3\,(21\,A + 60\,B + 73\,C)a^2\cos\right.}{315\left(d\cos(dx+c)^5 + d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] 2/315*((903*A + 690*B + 584*C)*a^2*cos(d*x + c)^4 + (294*A + 345*B + 292*C)
*a^2*cos(d*x + c)^3 + 3*(21*A + 60*B + 73*C)*a^2*cos(d*x + c)^2 + 5*(9*B +
26*C)*a^2*cos(d*x + c) + 35*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.36341, size = 470, normalized size = 2.55

$$\frac{8\left(315\sqrt{2}Aa^7\mathrm{sgn}(\cos(dx+c)) + 315\sqrt{2}Ba^7\mathrm{sgn}(\cos(dx+c)) + 315\sqrt{2}Ca^7\mathrm{sgn}(\cos(dx+c)) - \left(1050\sqrt{2}Aa^7\mathrm{sgn}(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] 8/315*(315*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^7*sgn(cos(d*x
+ c)) + 315*sqrt(2)*C*a^7*sgn(cos(d*x + c)) - (1050*sqrt(2)*A*a^7*sgn(cos(d
*x + c)) + 840*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 630*sqrt(2)*C*a^7*sgn(cos(
d*x + c)) - (1323*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 945*sqrt(2)*B*a^7*sgn(c
os(d*x + c)) + 819*sqrt(2)*C*a^7*sgn(cos(d*x + c)) - 4*(189*sqrt(2)*A*a^7*s
gn(cos(d*x + c)) + 135*sqrt(2)*B*a^7*sgn(cos(d*x + c)) + 117*sqrt(2)*C*a^7*
sgn(cos(d*x + c)) - 2*(21*sqrt(2)*A*a^7*sgn(cos(d*x + c)) + 15*sqrt(2)*B*a^
7*sgn(cos(d*x + c)) + 13*sqrt(2)*C*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2
*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)*d)
```

3.503 $\int (a+a\sec(c+dx))^{5/2} \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)$

Optimal. Leaf size=182

$$\frac{2a^3(245A+224B+160C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}} + \frac{2a^2(35A+56B+40C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d} + \frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{a\ldots}}\right)}{d}$$

```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(245*A + 224*B + 160*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a^2*(35*A + 56*B + 40*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105
*d) + (2*a*(7*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2
*C*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.324399, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4054, 3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(245A+224B+160C)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}} + \frac{2a^2(35A+56B+40C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d} + \frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\ldots}{\sqrt{a\ldots}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(245*A + 224*B + 160*C)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a^2*(35*A + 56*B + 40*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105
*d) + (2*a*(7*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2
*C*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```


Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
 !LtQ[m, -2^(-1)]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{2\int (a+a\sec(}{}$$

$$= \frac{2a(7B+5C)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2C(}{}$$

$$= \frac{2a^2(35A+56B+40C)\sqrt{a+a\sec(c+dx)}\tan(c+dx}{105d}$$

$$= \frac{2a^2(35A+56B+40C)\sqrt{a+a\sec(c+dx)}\tan(c+dx}{105d}$$

$$= \frac{2a^3(245A+224B+160C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(35A+}{}$$

$$= \frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3(245A+224B+1}{105d\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 2.43599, size = 170, normalized size = 0.93

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((840A+987B+930C)\cos(c+dx)+2(35A+98B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(420*Sqrt[2
]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 2*(70*A + 196*B +
 290*C + (840*A + 987*B + 930*C)*Cos[c + d*x] + 2*(35*A + 98*B + 115*C)*Cos
[2*(c + d*x)] + 280*A*Cos[3*(c + d*x)] + 301*B*Cos[3*(c + d*x)] + 230*C*Cos
[3*(c + d*x)])*Sin[(c + d*x)/2]))/(420*d)
```

Maple [B] time = 0.355, size = 476, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/840/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-105*A*sin(d*x+c)*cos(d*x
+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-315*A*sin(d*x+c)*co
s(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-315*A*sin(d*x+
c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-105*A*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)+4480*A*cos(d*x+c)^4
+4816*B*cos(d*x+c)^4+3680*C*cos(d*x+c)^4-3920*A*cos(d*x+c)^3-3248*B*cos(d*x
+c)^3-1840*C*cos(d*x+c)^3-560*A*cos(d*x+c)^2-1232*B*cos(d*x+c)^2-880*C*cos(
d*x+c)^2-336*B*cos(d*x+c)-720*C*cos(d*x+c)-240*C)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.595144, size = 1111, normalized size = 6.1

$$\left[\frac{105\left(Aa^{2}\cos\left(dx+c\right)^{4}+Aa^{2}\cos\left(dx+c\right)^{3}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\Big(}{105}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] [1/105*(105*(A*a^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((280*A +
 301*B + 230*C)*a^2*cos(d*x + c)^3 + (35*A + 98*B + 115*C)*a^2*cos(d*x + c)
^2 + 3*(7*B + 20*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/105*(
105*(A*a^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((280*
A + 301*B + 230*C)*a^2*cos(d*x + c)^3 + (35*A + 98*B + 115*C)*a^2*cos(d*x +
 c)^2 + 3*(7*B + 20*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.504 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=184

$$\frac{a^3(15A+70B+64C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(15A-10B-16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right.}{d}$$

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d + (a^3*(15*A + 70*B +
64*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(15*A - 10*B - 1
6*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) - (a*(5*A - 2*C)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.40408, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4086, 3917, 3915, 3774, 203, 3792}

$$\frac{a^3(15A+70B+64C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(15A-10B-16C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d + (a^3*(15*A + 70*B +
64*C)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(15*A - 10*B - 1
6*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) - (a*(5*A - 2*C)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} + \frac{\int(a}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{a(5A}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{a^2(1}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{a^2(1}{}$$

$$= \frac{A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{d} + \frac{a^3(1}{}$$

$$= \frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{A(}{}$$

Mathematica [A] time = 1.75414, size = 209, normalized size = 1.14

$$\frac{\cos(c+dx)(a(\sec(c+dx)+1))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)((45A+40B+112C)\cos(c+dx)+4(15A+}{(\cos(c+}\right.}{30d(A\cos(2(c+dx))+A+2B\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]*(a*(1 + Sec[c + d*x]))^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((60*(5*A + 2*B)*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Sin[c + d*x])/(Sqr
t[-1 + Sec[c + d*x]]*(1 + Sec[c + d*x])^3) + (Cos[c + d*x]*(60*A + 160*B +
196*C + (45*A + 40*B + 112*C)*Cos[c + d*x] + 4*(15*A + 40*B + 43*C)*Cos[2*(
c + d*x)] + 15*A*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(1 + Cos[c + d*x])^2))
```

```
/(30*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 0.368, size = 604, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/120/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(75*A*2^(1/2)*cos(d*x+c)^2
*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+30*B*cos(d*x+c)^2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+150*A*2^(1/2)*cos
(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+60*B*cos(d*x+c
)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+75*A*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+30*B*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+120*A*cos(d*x+c)^4+120*A*cos(d
*x+c)^3+640*B*cos(d*x+c)^3+688*C*cos(d*x+c)^3-240*A*cos(d*x+c)^2-560*B*cos(
d*x+c)^2-464*C*cos(d*x+c)^2-80*B*cos(d*x+c)-176*C*cos(d*x+c)-48*C)/sin(d*x+
c)/cos(d*x+c)^2
```

Maxima [B] time = 2.40302, size = 3753, normalized size = 20.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x +
2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x
 + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) +
 4*a^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c
) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c)
 - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*
a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
```

cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)
^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
 + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*c
os(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(
(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) +
 a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d

---

Fricas [A] time = 0.681727, size = 1108, normalized size = 6.02

$$\left[\frac{15\left((5A+2B)a^2\cos(dx+c)^3+(5A+2B)a^2\cos(dx+c)^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a}{\cos(dx+c)+1}\right.}{30(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] [1/30*(15*((5*A + 2*B)*a^2*cos(d*x + c)^3 + (5*A + 2*B)*a^2*cos(d*x + c)^2)
*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
 1)) + 2*(15*A*a^2*cos(d*x + c)^3 + 2*(15*A + 40*B + 43*C)*a^2*cos(d*x + c)
^2 + 2*(5*B + 14*C)*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), -1/15*(15
*((5*A + 2*B)*a^2*cos(d*x + c)^3 + (5*A + 2*B)*a^2*cos(d*x + c)^2)*sqrt(a)*
arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) - (15*A*a^2*cos(d*x + c)^3 + 2*(15*A + 40*B + 43*C)*a^2*cos(d*x +
c)^2 + 2*(5*B + 14*C)*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.86492, size = 771, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] -1/30*(15*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a^2*sgn(cos(d*
x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)
) + 2*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 60*sqr
t(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
```

```
))^2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - A*sqrt(-a)*a^4*sgn(cos(d*x + c)))/(
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
+ a^2) - 4*(15*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 45*sqrt(2)*B*a^5*sgn(cos(d
*x + c)) + 60*sqrt(2)*C*a^5*sgn(cos(d*x + c)) - (30*sqrt(2)*A*a^5*sgn(cos(d
*x + c)) + 80*sqrt(2)*B*a^5*sgn(cos(d*x + c)) + 80*sqrt(2)*C*a^5*sgn(cos(d*
x + c)) - (15*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 35*sqrt(2)*B*a^5*sgn(cos(d*
x + c)) + 32*sqrt(2)*C*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1
/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.505 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=197

$$\frac{a^3(27A-12B-56C)\sin(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-4B-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19A+20B+8C)\tan^{-1}\left(\cdot\right.}{4d}$$

```
[Out] (a^(5/2)*(19*A + 20*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]])/(4*d) + (a^3*(27*A - 12*B - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a + a
*Sec[c + d*x]]) - (a^2*(A - 4*B - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x
])/(2*d) - (a*(3*A - 4*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) +
(A*Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.631031, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4086, 4018, 4015, 3774, 203}

$$\frac{a^3(27A-12B-56C)\sin(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-4B-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{a^{5/2}(19A+20B+8C)\tan^{-1}\left(\cdot\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (a^(5/2)*(19*A + 20*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]])/(4*d) + (a^3*(27*A - 12*B - 56*C)*Sin[c + d*x])/(12*d*Sqrt[a + a
*Sec[c + d*x]]) - (a^2*(A - 4*B - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x
])/(2*d) - (a*(3*A - 4*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) +
(A*Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c}{2d}$$

$$= -\frac{a(3A-4C)(a+a\sec(c+dx))^{3/2}\sin(c}{6d}$$

$$= -\frac{a^2(A-4B-8C)\sqrt{a+a\sec(c+dx)}\sin}{2d}$$

$$= \frac{a^3(27A-12B-56C)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(}{}$$

$$= \frac{a^3(27A-12B-56C)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(}{}$$

$$= \frac{a^{5/2}(19A+20B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a+a\sec(c+}}\right.}{4d}$$

Mathematica [A] time = 1.19124, size = 152, normalized size = 0.77

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(6\sqrt{2}(19A+20B+8C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2\sin\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(1
9*A + 20*B + 8*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(
33*A + 12*B + 16*C + (9*A + 48*B + 128*C)*Cos[c + d*x] + 3*(11*A + 4*B)*Cos
[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.388, size = 583, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(57*A*2^(1/2)*sin(d*x+c)*cos
(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)+60*B*cos(d*x+c)*sin(d*x+c
)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+24*C*cos(d*x+c)*sin(d*x
+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+57*A*2^(1/2)*arctanh(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+60*B*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+24*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*2^(1/2)*sin(d*x+c)-24*A*cos(d*x+c)^4-108*A*cos(d*x+c)^3-48*B*cos(d*
x+c)^3+132*A*cos(d*x+c)^2-48*B*cos(d*x+c)^2-256*C*cos(d*x+c)^2+96*B*cos(d*x
+c)+224*C*cos(d*x+c)+32*C)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.955547, size = 1111, normalized size = 5.64

$$\left[\frac{3\left((19\,A+20\,B+8\,C)a^2\cos\left(dx+c\right)^2+(19\,A+20\,B+8\,C)a^2\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos}{\cos(dx+c)}\right.}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((19*A + 20*B + 8*C)*a^2*cos(d*x + c)^2 + (19*A + 20*B + 8*C)*a^2*
cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(c
os(d*x + c) + 1)) + 2*(6*A*a^2*cos(d*x + c)^3 + 3*(11*A + 4*B)*a^2*cos(d*x
+ c)^2 + 8*(3*B + 8*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), -1/12*(3
*((19*A + 20*B + 8*C)*a^2*cos(d*x + c)^2 + (19*A + 20*B + 8*C)*a^2*cos(d*x
+ c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - (6*A*a^2*cos(d*x + c)^3 + 3*(11*A + 4*B)*a^2*cos(d
```

```
*x + c)^2 + 8*(3*B + 8*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.08242, size = 1095, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/24*(3*(19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos(d
*x + c)) + 8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) -
 3*(19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos(d*x + c
)) + 8*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 16*(3
*sqrt(2)*B*a^4*sgn(cos(d*x + c)) + 9*sqrt(2)*C*a^4*sgn(cos(d*x + c)) - (3*s
qrt(2)*B*a^4*sgn(cos(d*x + c)) + 7*sqrt(2)*C*a^4*sgn(cos(d*x + c)))*tan(1/2
*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)) + 12*sqrt(2)*(19*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(d*x +
c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 171*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x + c))
- 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
4*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 89*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 36*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*s
qrt(-a)*a^5*sgn(cos(d*x + c)) - 9*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 4*B*sq
rt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.506 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\text{s}\right.$

Optimal. Leaf size=207

$$\frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(3A+2B-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{a^{5/2}(25A+38B+40C)\tan^{-}}{8d}$$

```
[Out] (a^(5/2)*(25*A + 38*B + 40*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(8*d) + (a^3*(49*A + 54*B - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a +
a*Sec[c + d*x]]) - (a^2*(3*A + 2*B - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(4*d) + (a*(5*A + 6*B)*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c
+ d*x])/(12*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d)
```

Rubi [A] time = 0.658396, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4017, 4018, 4015, 3774, 203}

$$\frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(3A+2B-8C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{a^{5/2}(25A+38B+40C)\tan^{-}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (a^(5/2)*(25*A + 38*B + 40*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(8*d) + (a^3*(49*A + 54*B - 24*C)*Sin[c + d*x])/(24*d*Sqrt[a +
a*Sec[c + d*x]]) - (a^2*(3*A + 2*B - 8*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(4*d) + (a*(5*A + 6*B)*Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c
+ d*x])/(12*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d)
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\sin(}{3d}$$

$$= \frac{a(5A+6B)\cos(c+dx)(a+a\sec(c+dx}{12d}$$

$$= -\frac{a^2(3A+2B-8C)\sqrt{a+a\sec(c+dx)}\,\mathrm{si}}{4d}$$

$$= \frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(3}{}$$

$$= \frac{a^3(49A+54B-24C)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(3}{}$$

$$= \frac{a^{5/2}(25A+38B+40C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+}{\sqrt{a+a\sec(c}}\right.}{8d}$$

Mathematica [A] time = 1.70356, size = 140, normalized size = 0.68

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{\sec(c+dx)-1}(3(27A+22B+8C)\cos(c+dx)+(17A+6B)\cos(2(c+dx))+\right.}{24d\sqrt{\sec(c+dx)-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*(3*(25*A + 38*B + 40*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]] + (17*A + 6*B
+ 48*C + 3*(27*A + 22*B + 8*C)*Cos[c + d*x] + (17*A + 6*B)*Cos[2*(c + d*x)]
 + 2*A*Cos[3*(c + d*x)])*Sqrt[-1 + Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])
]*Tan[(c + d*x)/2])/(24*d*Sqrt[-1 + Sec[c + d*x]])
```

Maple [B] time = 0.323, size = 846, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(75*A*2^(1/2)*cos(d*x+c)^2
*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+114*B*cos(d*x+c)^2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+120*C*cos(d*x+c)
^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+150*A*2^(1/
2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+228*B*co
s(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+240*C
*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+75
*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+114*B*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+120*C*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+64*A*cos(d*x+c)^6+208*A*cos
(d*x+c)^5+96*B*cos(d*x+c)^5+328*A*cos(d*x+c)^4+432*B*cos(d*x+c)^4+192*C*cos
(d*x+c)^4-600*A*cos(d*x+c)^3-528*B*cos(d*x+c)^3+192*C*cos(d*x+c)^3-384*C*co
s(d*x+c)^2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.926529, size = 1060, normalized size = 5.12

$$\left[\frac{3\left((25\,A+38\,B+40\,C)a^2\cos\left(dx+c\right)+(25\,A+38\,B+40\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((25*A + 38*B + 40*C)*a^2*cos(d*x + c) + (25*A + 38*B + 40*C)*a^2)
*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
 1)) + 2*(8*A*a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x + c)^2 + 3*(2
5*A + 22*B + 8*C)*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(3*((25*A + 38*B + 40
*C)*a^2*cos(d*x + c) + (25*A + 38*B + 40*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*a^
2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x + c)^2 + 3*(25*A + 22*B + 8*C
)*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.79896, size = 1712, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/48*(96*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*a^3*sgn(cos(d*x + c
))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 3*(25*A*sqrt(-a)*a
^2*sgn(cos(d*x + c)) + 38*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 40*C*sqrt(-a)*
a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(25*A*sqrt(-a)*a^2*sg
n(cos(d*x + c)) + 38*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 40*C*sqrt(-a)*a^2*s
gn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(75*(sqrt(-a)*tan(
```

```
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^3*s
gn(cos(d*x + c)) + 114*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 72*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^3*sgn
(cos(d*x + c)) - 1125*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^8*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1710*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^4*sgn(
cos(d*x + c)) - 888*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^8*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 6174*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^5*sgn(co
s(d*x + c)) + 6804*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 3024*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^5*sgn(cos
(d*x + c)) - 4314*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^4*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 4284*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^6*sgn(cos(
d*x + c)) - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^4*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 807*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^7*sgn(cos(d*
x + c)) + 858*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 360*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^7*sgn(cos(d*x +
 c)) - 49*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 54*B*sqrt(-a)*a^8*sgn(cos(d*x
+ c)) - 24*C*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.507 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=215

$$\frac{a^3(299A+392B+432C)\sin(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163A+200B+304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+24B+16C)\sin(c}{}$$

```
[Out] (a^(5/2)*(163*A + 200*B + 304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*S
ec[c + d*x]]])/(64*d) + (a^3*(299*A + 392*B + 432*C)*Sin[c + d*x])/(192*d*S
qrt[a + a*Sec[c + d*x]]) + (a^2*(17*A + 24*B + 16*C)*Cos[c + d*x]*Sqrt[a +
a*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (a*(5*A + 8*B)*Cos[c + d*x]^2*(a + a
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c
+ d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.699186, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4086, 4017, 4015, 3774, 203}

$$\frac{a^3(299A+392B+432C)\sin(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(163A+200B+304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(17A+24B+16C)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (a^(5/2)*(163*A + 200*B + 304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*S
ec[c + d*x]]])/(64*d) + (a^3*(299*A + 392*B + 432*C)*Sin[c + d*x])/(192*d*S
qrt[a + a*Sec[c + d*x]]) + (a^2*(17*A + 24*B + 16*C)*Cos[c + d*x]*Sqrt[a +
a*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (a*(5*A + 8*B)*Cos[c + d*x]^2*(a + a
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c
+ d*x])^(5/2)*Sin[c + d*x])/(4*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\text{s}}{4d}$$

$$= \frac{a(5A+8B)\cos^2(c+dx)(a+a\sec(c}{24d}$$

$$= \frac{a^2(17A+24B+16C)\cos(c+dx)\sqrt{a}}{32d}$$

$$= \frac{a^3(299A+392B+432C)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(299A+392B+432C)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(163A+200B+304C)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{64d}$$

Mathematica [A] time = 2.33545, size = 156, normalized size = 0.73

$$a^2\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}(163A+200B+304C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt
[2]*(163*A + 200*B + 304*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c
 + d*x]]*(581*A + 632*B + 528*C + (362*A + 272*B + 96*C)*Cos[c + d*x] + 4*(
23*A + 8*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(3
84*d)
```

Maple [B] time = 0.309, size = 1108, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/3072/d*a^2*(-489*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(7/2)*2^(1/2)-600*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(7/2)*2^(1/2)-912*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)*2^(1/2)-1467*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-1800*B*sin(d*x+c)*cos(d*x+c)^2*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-2736*C*sin(d*x+c)*cos(d*x+c)^2*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-1467*A*sin(d*x+c)*cos(d*x+
c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-1800*B*sin(d*x+c)*cos(
d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-2736*C*sin(d*x+c)*
cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-489*A*arctanh(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)-600*B*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)*sin(d*x+c)-912*C*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+768*A*cos(d*x+c)^8+2176*A*cos(d*x+c)^7
+1024*B*cos(d*x+c)^7+2272*A*cos(d*x+c)^6+3328*B*cos(d*x+c)^6+1536*C*cos(d*x
+c)^6+2608*A*cos(d*x+c)^5+5248*B*cos(d*x+c)^5+6912*C*cos(d*x+c)^5-7824*A*co
s(d*x+c)^4-9600*B*cos(d*x+c)^4-8448*C*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d
*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.29515, size = 1187, normalized size = 5.52

$$\left[\frac{3\left((163\,A+200\,B+304\,C)a^2\cos\left(dx+c\right)+(163\,A+200\,B+304\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*((163*A + 200*B + 304*C)*a^2*cos(d*x + c) + (163*A + 200*B + 304*
C)*a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x
 + c) + 1)) + 2*(48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^
3 + 2*(163*A + 136*B + 48*C)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B + 176*C)
*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d), -1/192*(3*((163*A + 200*B + 304*C)*a^2*cos(d*x + c) + (
163*A + 200*B + 304*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*a^2*cos(d*x + c)^4 + 8
*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 136*B + 48*C)*a^2*cos(d*x + c
)^2 + 3*(163*A + 200*B + 176*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.35373, size = 2082, normalized size = 9.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/384*(3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*B*sqrt(-a)*a^2*sgn(co
s(d*x + c)) + 304*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3
))) - 3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*B*sqrt(-a)*a^2*sgn(cos(
d*x + c)) + 304*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(489*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
```

```
/2*c)^2 + a))^14*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 600*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^3*sgn(co
s(d*x + c)) + 912*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^14*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 10269*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^4*sgn(c
os(d*x + c)) - 12600*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^12*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 19152*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^4*sg
n(cos(d*x + c)) + 69885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 103992*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^
5*sgn(cos(d*x + c)) + 137424*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 259233*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a
)*a^6*sgn(cos(d*x + c)) - 339864*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 374544*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt
(-a)*a^6*sgn(cos(d*x + c)) + 209979*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 262920*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*s
qrt(-a)*a^7*sgn(cos(d*x + c)) + 266928*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 5551
1*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A
*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 73640*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 758
88*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*
C*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 6687*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 880
8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B
*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 9456*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 299*
A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 392*B*sqrt(-a)*a^10*sgn(cos(d*x + c)) -
 432*C*sqrt(-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.508 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,s\right.$

Optimal. Leaf size=261

$$\frac{a^3(283A+326B+400C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283A+326B+400C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(79A+110B+80C)\,s}{}$$

```
[Out] (a^(5/2)*(283*A + 326*B + 400*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*S
ec[c + d*x]]])/(128*d) + (a^3*(283*A + 326*B + 400*C)*Sin[c + d*x])/(128*d*
Sqrt[a + a*Sec[c + d*x]]) + (a^3*(787*A + 950*B + 1040*C)*Cos[c + d*x]*Sin[
c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(79*A + 110*B + 80*C)*Cos
[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(240*d) + (a*(A + 2*B)*C
os[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d) + (A*Cos[c + d
*x]^4*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.810299, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4017, 4015, 3805, 3774, 203}

$$\frac{a^3(283A+326B+400C)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(283A+326B+400C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^2(79A+110B+80C)\,s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (a^(5/2)*(283*A + 326*B + 400*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*S
ec[c + d*x]]])/(128*d) + (a^3*(283*A + 326*B + 400*C)*Sin[c + d*x])/(128*d*
Sqrt[a + a*Sec[c + d*x]]) + (a^3*(787*A + 950*B + 1040*C)*Cos[c + d*x]*Sin[
c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(79*A + 110*B + 80*C)*Cos
[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(240*d) + (a*(A + 2*B)*C
os[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d) + (A*Cos[c + d
*x]^4*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\sin(}{5d}$$

$$= \frac{a(A+2B)\cos^3(c+dx)(a+a\sec(c+dx)}{8d}$$

$$= \frac{a^2(79A+110B+80C)\cos^2(c+dx)\sqrt{a+}}{240d}$$

$$= \frac{a^3(787A+950B+1040C)\cos(c+dx)\,\mathrm{s}}{960d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(283A+326B+400C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(283A+326B+400C)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^{5/2}(283A+326B+400C)\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{ta}}{\sqrt{a+a\,\mathrm{s}}}\right.}{128d}$$

Mathematica [A] time = 2.39056, size = 183, normalized size = 0.7

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(15\sqrt{2}(283A+326B+400C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\sin\left(\frac{3}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[2]*(283*A + 326*B
 + 400*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (5521*A + 5
810*B + 6320*C + (3874*A + 3620*B + 2720*C)*Cos[c + d*x] + 4*(331*A + 230*B
 + 80*C)*Cos[2*(c + d*x)] + 348*A*Cos[3*(c + d*x)] + 120*B*Cos[3*(c + d*x)]
 + 48*A*Cos[4*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384
0*d)

Maple [B] time = 0.368, size = 1381, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)

[Out] -1/61440/d*a^2*(29340*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c
)*cos(d*x+c)^2*2^(1/2)+36000*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh
(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*si
n(d*x+c)*cos(d*x+c)^2*2^(1/2)+12288*A*cos(d*x+c)^10+32256*A*cos(d*x+c)^9+20
480*C*cos(d*x+c)^8+15360*B*cos(d*x+c)^9+19560*B*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+24000*C*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4245*A*2^(1/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^4*sin(d*x+c)+16980*A*2^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^3*sin(d*x+c)+25470*A*2^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)^2*sin(d*x+c)+16980
*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*cos(d*x+c)*sin(d*x+c)+
4890*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^4*2^
(1/2)+6000*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c
)^4*2^(1/2)+19560*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*co
s(d*x+c)^3*2^(1/2)+24000*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2
2^(1/2)(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*
x+c)*cos(d*x+c)^3*2^(1/2)+45280*A*cos(d*x+c)^6-135840*A*cos(d*x+c)^5-156480
*B*cos(d*x+c)^5-192000*C*cos(d*x+c)^5+27904*A*cos(d*x+c)^8+18112*A*cos(d*x+
c)^7+45440*B*cos(d*x+c)^7+52160*B*cos(d*x+c)^6+104960*C*cos(d*x+c)^6+4890*B
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+6000*C*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+4245*A*2^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+43520*B*cos(d*x+c)^8+66560*
C*cos(d*x+c)^7)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.32708, size = 1332, normalized size = 5.1

$$\left[\frac{15\left((283\,A + 326\,B + 400\,C)a^2\cos\left(dx+c\right) + (283\,A + 326\,B + 400\,C)a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/3840*(15*((283*A + 326*B + 400*C)*a^2*cos(d*x + c) + (283*A + 326*B + 40
0*C)*a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d
*x + c) + 1)) + 2*(384*A*a^2*cos(d*x + c)^5 + 48*(29*A + 10*B)*a^2*cos(d*x
+ c)^4 + 8*(283*A + 230*B + 80*C)*a^2*cos(d*x + c)^3 + 10*(283*A + 326*B +
272*C)*a^2*cos(d*x + c)^2 + 15*(283*A + 326*B + 400*C)*a^2*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -
1/1920*(15*((283*A + 326*B + 400*C)*a^2*cos(d*x + c) + (283*A + 326*B + 400
*C)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c))) - (384*A*a^2*cos(d*x + c)^5 + 48*(29*A + 10*B)*a^
2*cos(d*x + c)^4 + 8*(283*A + 230*B + 80*C)*a^2*cos(d*x + c)^3 + 10*(283*A
+ 326*B + 272*C)*a^2*cos(d*x + c)^2 + 15*(283*A + 326*B + 400*C)*a^2*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.92671, size = 2535, normalized size = 9.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/3840*(15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(
cos(d*x + c)) + 400*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) +
 3))) - 15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(c
os(d*x + c)) + 400*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) -
3))) + 4*sqrt(2)*(4245*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^18*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 4890*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*sqrt(-a)*a^3*s
gn(cos(d*x + c)) + 6000*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^18*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 114615*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*A*sqrt(-a)*a^
4*sgn(cos(d*x + c)) - 132030*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^16*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 162000*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*C*sqrt(-
a)*a^4*sgn(cos(d*x + c)) + 1298820*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 1319880
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B
*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 1801920*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*C*sqrt(-a)*a^5*sgn(cos(d*x + c)) -
6176700*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^12*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 6888120*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^6*sgn(cos(d*x +
 c)) - 9764160*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^12*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 16394598*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^7*sgn(c
os(d*x + c)) + 18352620*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 24060960*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*
a^7*sgn(cos(d*x + c)) - 14042770*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 15746180*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sq
rt(-a)*a^8*sgn(cos(d*x + c)) - 19910240*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 479
1060*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
6*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 5497320*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^9*sgn(cos(d*x + c))
+ 7135680*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^6*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 860300*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^10*sgn(cos(d*x +
 c)) - 959320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*B*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 1268800*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^10*sgn(cos
(d*x + c)) + 75885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*A*sqrt(-a)*a^11*sgn(cos(d*x + c)) + 84810*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^11*sgn(
cos(d*x + c)) + 111600*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*C*sqrt(-a)*a^11*sgn(cos(d*x + c)) - 2671*A*sqrt(-a)*a^1
2*sgn(cos(d*x + c)) - 2990*B*sqrt(-a)*a^12*sgn(cos(d*x + c)) - 3920*C*sqrt(
-a)*a^12*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
```

```
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^5)/d
```

3.509 $\int \cos^6(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=311

$$\frac{a^3(1015A+1132B+1304C)\sin(c+dx)}{512d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(1015A+1132B+1304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{512d}+\frac{a^2(115A+156B+}{}$$

```
[Out] (a^(5/2)*(1015*A + 1132*B + 1304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a +
a*Sec[c + d*x]]])/(512*d) + (a^3*(1015*A + 1132*B + 1304*C)*Sin[c + d*x])/(
512*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1015*A + 1132*B + 1304*C)*Cos[c + d
*x]*Sin[c + d*x])/(768*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(545*A + 628*B +
680*C)*Cos[c + d*x]^2*Sin[c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2
*(115*A + 156*B + 120*C)*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*
x])/(480*d) + (a*(5*A + 12*B)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(60*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*
x])/(6*d)
```

Rubi [A] time = 0.894864, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4017, 4015, 3805, 3774, 203}

$$\frac{a^3(1015A+1132B+1304C)\sin(c+dx)}{512d\sqrt{a\sec(c+dx)+a}}+\frac{a^{5/2}(1015A+1132B+1304C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{512d}+\frac{a^2(115A+156B+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (a^(5/2)*(1015*A + 1132*B + 1304*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a +
a*Sec[c + d*x]]])/(512*d) + (a^3*(1015*A + 1132*B + 1304*C)*Sin[c + d*x])/(
512*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1015*A + 1132*B + 1304*C)*Cos[c + d
*x]*Sin[c + d*x])/(768*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(545*A + 628*B +
680*C)*Cos[c + d*x]^2*Sin[c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2
*(115*A + 156*B + 120*C)*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*
x])/(480*d) + (a*(5*A + 12*B)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(60*d) + (A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*
x])/(6*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
```

```
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+a\sec(c+dx))^{5/2}\sin(}{6d}$$

$$= \frac{a(5A+12B)\cos^4(c+dx)(a+a\sec(c+}{60d}$$

$$= \frac{a^2(115A+156B+120C)\cos^3(c+dx)\sqrt{}}{480d}$$

$$= \frac{a^3(545A+628B+680C)\cos^2(c+dx)\,\mathrm{si}}{960d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(1015A+1132B+1304C)\cos(c+dx)}{768d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(1015A+1132B+1304C)\sin(c+dx)}{512d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(1015A+1132B+1304C)\sin(c+dx)}{512d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(1015A+1132B+1304C)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{512d}$$

Mathematica [A] time = 3.42134, size = 217, normalized size = 0.7

$$a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(15\sqrt{2}(1015A+1132B+1304C)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\left(\mathrm{s}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[2]*(1015*A + 1132
*B + 1304*C)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (20965*A
 + 22084*B + 23240*C + 2*(8085*A + 7748*B + 7240*C)*Cos[c + d*x] + 4*(1575*
A + 1324*B + 920*C)*Cos[2*(c + d*x)] + 2140*A*Cos[3*(c + d*x)] + 1392*B*Cos
[3*(c + d*x)] + 480*C*Cos[3*(c + d*x)] + 560*A*Cos[4*(c + d*x)] + 192*B*Cos
[4*(c + d*x)] + 80*A*Cos[5*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x
))/2])))/(15360*d)
```

Maple [B] time = 0.404, size = 1654, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/491520/d*a^2*(195600*C*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))+76125*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)/cos(d*x+c))-158720*A*cos(d*x+c)^10-74240*A*cos(d*x+c)^9-
363520*C*cos(d*x+c)^8-223232*B*cos(d*x+c)^9+974400*A*cos(d*x+c)^6+16980*B*2
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*sin(d*x+c)+15225*A*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-98304*B*cos(d*x+c)^
11-258048*B*cos(d*x+c)^10-129920*A*cos(d*x+c)^8-324800*A*cos(d*x+c)^7-36224
0*B*cos(d*x+c)^7+1086720*B*cos(d*x+c)^6+97800*C*cos(d*x+c)*sin(d*x+c)*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+15225*A*cos(d*x+c)^5*sin(d*x+
c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+19560*C*cos(d*x+c)^5*
sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+76125*A*cos(d
*x+c)^4*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+97800
*C*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))+152250*A*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))+195600*C*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))+152250*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))+16980*B*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
```

```
11/2)*2^(1/2)*cos(d*x+c)^5*sin(d*x+c)+84900*B*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(11/2)*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+169800*B*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(11/2)*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+169800*B*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+84900*B*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)+19560
*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-81920*A*co
s(d*x+c)^12-204800*A*cos(d*x+c)^11-122880*C*cos(d*x+c)^10-348160*C*cos(d*x+
c)^9+1251840*C*cos(d*x+c)^6-144896*B*cos(d*x+c)^8-417280*C*cos(d*x+c)^7)*(a
*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.336, size = 1507, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] [1/15360*(15*((1015*A + 1132*B + 1304*C)*a^2*cos(d*x + c) + (1015*A + 1132*
B + 1304*C)*a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)
/(cos(d*x + c) + 1)) + 2*(1280*A*a^2*cos(d*x + c)^6 + 128*(35*A + 12*B)*a^2
*cos(d*x + c)^5 + 48*(145*A + 116*B + 40*C)*a^2*cos(d*x + c)^4 + 8*(1015*A
+ 1132*B + 920*C)*a^2*cos(d*x + c)^3 + 10*(1015*A + 1132*B + 1304*C)*a^2*co
s(d*x + c)^2 + 15*(1015*A + 1132*B + 1304*C)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/7680*(15
*((1015*A + 1132*B + 1304*C)*a^2*cos(d*x + c) + (1015*A + 1132*B + 1304*C)*
a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c))) - (1280*A*a^2*cos(d*x + c)^6 + 128*(35*A + 12*B)*a^2*
cos(d*x + c)^5 + 48*(145*A + 116*B + 40*C)*a^2*cos(d*x + c)^4 + 8*(1015*A +
 1132*B + 920*C)*a^2*cos(d*x + c)^3 + 10*(1015*A + 1132*B + 1304*C)*a^2*cos
(d*x + c)^2 + 15*(1015*A + 1132*B + 1304*C)*a^2*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 9.33477, size = 2989, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] -1/15360*(15*(1015*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1132*B*sqrt(-a)*a^2*s
gn(cos(d*x + c)) + 1304*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(
2) + 3))) - 15*(1015*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 1132*B*sqrt(-a)*a^2
*sgn(cos(d*x + c)) + 1304*C*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqr
t(2) - 3))) + 4*sqrt(2)*(15225*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^22*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 16980*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^22*B*sqrt(
-a)*a^3*sgn(cos(d*x + c)) + 19560*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^22*C*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 502425*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^20*A*s
qrt(-a)*a^4*sgn(cos(d*x + c)) - 560340*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^20*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 645
480*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
0*C*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 6518495*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*A*sqrt(-a)*a^5*sgn(cos(d*x + c))
 + 7963020*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^18*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 8467800*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*C*sqrt(-a)*a^5*sgn(cos(d*
x + c)) - 49683495*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^16*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 56336940*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*B*sqrt(-a)*a^6*s
gn(cos(d*x + c)) - 59757720*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^16*C*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 191286330*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt
(-a)*a^7*sgn(cos(d*x + c)) + 219014472*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 244
004880*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^14*C*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 418895130*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^8*sgn(cos(d*x
+ c)) - 474348232*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^12*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 531000080*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*sqrt(-a)*a^8*s
gn(cos(d*x + c)) + 374587230*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 421769112*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqr
t(-a)*a^9*sgn(cos(d*x + c)) + 473308080*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 15
4254030*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 174597720*(sqrt(-a)*tan(1/2*d*x +
```

```
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^10*sgn(cos(d*x
 + c)) - 198757680*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^8*C*sqrt(-a)*a^10*sgn(cos(d*x + c)) + 35939005*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^11*s
gn(cos(d*x + c)) + 40114980*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^11*sgn(cos(d*x + c)) + 45352200*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-
a)*a^11*sgn(cos(d*x + c)) - 4649085*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^12*sgn(cos(d*x + c)) - 527312
4*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B
*sqrt(-a)*a^12*sgn(cos(d*x + c)) - 5884680*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*C*sqrt(-a)*a^12*sgn(cos(d*x + c)) +
 324435*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*A*sqrt(-a)*a^13*sgn(cos(d*x + c)) + 367644*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^13*sgn(cos(d*x +
c)) + 411000*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*C*sqrt(-a)*a^13*sgn(cos(d*x + c)) - 9435*A*sqrt(-a)*a^14*sgn(cos(
d*x + c)) - 10684*B*sqrt(-a)*a^14*sgn(cos(d*x + c)) - 11960*C*sqrt(-a)*a^14
*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2*a + a^2)^6)/d
```

3.510 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=254

$$\frac{2(21A-3B+19C)\tan(c+dx)\sec^2(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(21A-93B+29C)\tan(c}{315a}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(Sqrt[a]*d)) + (4*(147*A - 111*B + 143*C)*Tan[c + d*x])/(315
*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(21*A - 3*B + 19*C)*Sec[c + d*x]^2*Tan[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(9*B - C)*Sec[c + d*x]^3*Tan[
c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^4*Tan[c + d*x
])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (2*(21*A - 93*B + 29*C)*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(315*a*d)
```

Rubi [A] time = 0.856623, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4088, 4021, 4010, 4001, 3795, 203}

$$\frac{2(21A-3B+19C)\tan(c+dx)\sec^2(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(21A-93B+29C)\tan(c}{315a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(Sqrt[a]*d)) + (4*(147*A - 111*B + 143*C)*Tan[c + d*x])/(315
*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(21*A - 3*B + 19*C)*Sec[c + d*x]^2*Tan[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(9*B - C)*Sec[c + d*x]^3*Tan[
c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^4*Tan[c + d*x
])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (2*(21*A - 93*B + 29*C)*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(315*a*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
```

```
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2C\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}+\frac{2\int\frac{\sec^4(c+dx)\left(\frac{1}{2}a(9A+8C)+\frac{1}{2}a(9B-C\right.}{\sqrt{a+a\sec(c+dx)}}}{9a}$$

$$=\frac{2(9B-C)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2C\sec^4(c+dx)\tan(c+}{9d\sqrt{a+a\sec(c+dx}}$$

$$=\frac{2(21A-3B+19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2(9B-C)\sec}{63d\sqrt{a}}$$

$$=\frac{2(21A-3B+19C)\sec^2(c+dx)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2(9B-C)\sec}{63d\sqrt{a}}$$

$$=\frac{4(147A-111B+143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{2(21A-3B+19C)\mathrm{se}}{105d\sqrt{a+a}}$$

$$=\frac{4(147A-111B+143C)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{2(21A-3B+19C)\mathrm{se}}{105d\sqrt{a+a}}$$

$$=-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{4(147A-111B+}{315d\sqrt{a+a}}$$

Mathematica [C] time = 29.6254, size = 7186, normalized size = 28.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.45, size = 1429, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/5040/d/a*(-1120*C-3264*B*cos(d*x+c)^3-9152*C*cos(d*x+c)^4-9408*A*cos(d*x
+c)^4+2752*C*cos(d*x+c)^3-1984*C*cos(d*x+c)^2+1280*C*cos(d*x+c)+315*A*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)-315*B*ln(-(-(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(9/2)*sin(d*x+c)+315*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
9/2)*sin(d*x+c)+8736*A*cos(d*x+c)^5-4128*B*cos(d*x+c)^5+1260*A*cos(d*x+c)*s
in(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-1260*B*cos(d*x+c)*sin(d
*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+8224*C*cos(d*x+c)^5+7104*B*
cos(d*x+c)^4+2688*A*cos(d*x+c)^3+1728*B*cos(d*x+c)^2-2016*A*cos(d*x+c)^2+12
60*C*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-315*B*
cos(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+315*C*cos
(d*x+c)^4*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+1890*A*cos(d
*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+co
s(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-1890*B*cos(d*x
+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+1890*C*cos(d*x+c
)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+1260*A*cos(d*x+c)^
3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-1260*B*cos(d*x+c)^3*
sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+1260*C*cos(d*x+c)^3*si
n(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)+315*A*cos(d*x+c)^4*sin(d
*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)-1440*B*cos(d*x+c))*(a*(cos(
d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.648656, size = 1289, normalized size = 5.07

$$\left[\frac{315\,\sqrt{2}\left((A-B+C)a\cos\left(dx+c\right)^5+(A-B+C)a\cos\left(dx+c\right)^4\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\,\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\,\cos(dx+}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(2)*((A - B + C)*a*cos(d*x + c)^5 + (A - B + C)*a*cos(d*x +
 c)^4)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((273*A - 129*B + 257*C)*cos(d*
x + c)^4 - (21*A - 93*B + 29*C)*cos(d*x + c)^3 + 3*(21*A - 3*B + 19*C)*cos(
d*x + c)^2 + 5*(9*B - C)*cos(d*x + c) + 35*C)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4), 1/315*(
2*((273*A - 129*B + 257*C)*cos(d*x + c)^4 - (21*A - 93*B + 29*C)*cos(d*x +
c)^3 + 3*(21*A - 3*B + 19*C)*cos(d*x + c)^2 + 5*(9*B - C)*cos(d*x + c) + 35
*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 315*sqrt(2)*((A
- B + C)*a*cos(d*x + c)^5 + (A - B + C)*a*cos(d*x + c)^4)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/
sqrt(a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^4\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**4/sqrt(a*(s
ec(c + d*x) + 1)), x)
```

Giac [B] time = 9.52989, size = 689, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/315*(315*(sqrt(2)*A - sqrt(2)*B + sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d
*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)) + 2*(315*sqrt(2)*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
+ 315*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (1050*sqrt(2)*A*a^4*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1) - 420*sqrt(2)*B*a^4*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1) + 840*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (1512*sqrt(2)
*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 756*sqrt(2)*B*a^4*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 1638*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (113
4*sqrt(2)*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 612*sqrt(2)*B*a^4*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1) + 936*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
) - (357*sqrt(2)*A*a^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 276*sqrt(2)*B*a^4*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 383*sqrt(2)*C*a^4*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2
*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c
)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.511 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(35A-7B+31C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}-\frac{4(35A-49B+37C)\tan}{105d\sqrt{a\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(Sqrt[a]*d) - (4*(35*A - 49*B + 37*C)*Tan[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) + (2*(7*B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sq
rt[a + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*(35*A - 7*B + 31*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d
*x])/(105*a*d)
```

Rubi [A] time = 0.637046, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4088, 4021, 4010, 4001, 3795, 203}

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(35A-7B+31C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}-\frac{4(35A-49B+37C)\tan}{105d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(Sqrt[a]*d) - (4*(35*A - 49*B + 37*C)*Tan[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) + (2*(7*B - C)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sq
rt[a + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*(35*A - 7*B + 31*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d
*x])/(105*a*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^3(c+dx)\left(\frac{1}{2}a(7A+6C)+\frac{1}{2}a(7\cdots\right.}{\sqrt{a+a\sec(c+dx\cdots}}}{7a}$$

$$= \frac{2(7B-C)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx)\tan\cdots}{7d\sqrt{a+a\sec(c+\cdots}}$$

$$= \frac{2(7B-C)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^3(c+dx)\tan\cdots}{7d\sqrt{a+a\sec(c+\cdots}}$$

$$= -\frac{4(35A-49B+37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7B-C)\sec^2(c+\cdots}{35d\sqrt{a+a\sec\cdots}}$$

$$= -\frac{4(35A-49B+37C)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7B-C)\sec^2(c+\cdots}{35d\sqrt{a+a\sec\cdots}}$$

$$= \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4(35A-49B+\cdots}{105d\sqrt{a+\cdots}}$$

Mathematica [C] time = 29.5461, size = 7134, normalized size = 34.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.39, size = 1144, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/840/d/a*(105*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(
d*x+c)^3-105*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^3*sin(d
*x+c)+105*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)
^3+315*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-
315*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*sin(d*x+c)+315
*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+315*A*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-315*B*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c)+315*C*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+105*A*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-105*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(7/2)*sin(d*x+c)+105*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d
*x+c)-560*A*cos(d*x+c)^4+1456*B*cos(d*x+c)^4-688*C*cos(d*x+c)^4+1120*A*cos(
d*x+c)^3-1568*B*cos(d*x+c)^3+1184*C*cos(d*x+c)^3-560*A*cos(d*x+c)^2+448*B*c
os(d*x+c)^2-544*C*cos(d*x+c)^2-336*B*cos(d*x+c)+288*C*cos(d*x+c)-240*C)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.633937, size = 1176, normalized size = 5.65

$$\left[\frac{105\sqrt{2}\left((A-B+C)a\cos(dx+c)^4+(A-B+C)a\cos(dx+c)^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos\cdots}{\cos(dx+c)^2+2\cos(dx+c)+\cdots}\right.}{2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(2)*((A - B + C)*a*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x +
 c)^3)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((35*A - 91*B + 43*C)*cos(d*x
+ c)^3 - (35*A - 7*B + 31*C)*cos(d*x + c)^2 - 3*(7*B - C)*cos(d*x + c) - 15
*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)
^4 + a*d*cos(d*x + c)^3), -1/105*(2*((35*A - 91*B + 43*C)*cos(d*x + c)^3 -
(35*A - 7*B + 31*C)*cos(d*x + c)^2 - 3*(7*B - C)*cos(d*x + c) - 15*C)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 105*sqrt(2)*((A - B + C)*
a*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x + c)^3)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/
(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/sqrt(a*(s
ec(c + d*x) + 1)), x)
```

Giac [A] time = 9.26495, size = 412, normalized size = 1.98

$$\frac{\frac{105\sqrt{2}(A-B+C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(\frac{105\sqrt{2}Ba^3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\left(\left(\frac{\sqrt{2}\left(70Aa^3-119Ba^3+92Ca^3\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{7\sqrt{2}(20A\cdots}{\mathrm{sgn}\left(\tan\cdots\right.}\cdots\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\cdots\right.\right.}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] 1/105*(105*sqrt(2)*(A - B + C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)) + 2*(105*sqrt(2)*B*a^3/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + ((sqrt(2)*(70*A
*a^3 - 119*B*a^3 + 92*C*a^3)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c
)^2 - 1) - 7*sqrt(2)*(20*A*a^3 - 37*B*a^3 + 16*C*a^3)/sgn(tan(1/2*d*x + 1/2
*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(2*A*a^3 - 7*B*a^3 + 4*C*a^
3)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
))/d
```

3.512 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(15A-10B+14C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(5B-C)\tan(c+dx)\sqrt{a\sec(c+dx)}}{15ad}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(Sqrt[a]*d)) + (2*(15*A - 10*B + 14*C)*Tan[c + d*x])/(15*d*S
qrt[a + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a +
a*Sec[c + d*x]]) + (2*(5*B - C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*
a*d)
```

Rubi [A] time = 0.451997, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4088, 4010, 4001, 3795, 203}

$$-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(15A-10B+14C)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(5B-C)\tan(c+dx)\sqrt{a\sec(c+dx)}}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(Sqrt[a]*d)) + (2*(15*A - 10*B + 14*C)*Tan[c + d*x])/(15*d*S
qrt[a + a*Sec[c + d*x]]) + (2*C*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a +
a*Sec[c + d*x]]) + (2*(5*B - C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*
a*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^2(c+dx)\left(\frac{1}{2}a(5A+4C)+\frac{1}{2}a(5B-C\right.}{\sqrt{a+a\sec(c+dx)}}}{5a}$$

$$= \frac{2C\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5B-C)\sqrt{a+a\sec(c+dx)}\,\mathrm{t}}{15ad}$$

$$= \frac{2(15A-10B+14C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(c+dx)\tan(c+d}{5d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2(15A-10B+14C)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sec^2(c+dx)\tan(c+d}{5d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(15A-10B+14}{15d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [C] time = 10.6567, size = 1666, normalized size = 10.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 a*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[(1 - 2*Sin
[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*(-(A*Sin[(c + d*x)/2]
)/(2*(1 - 2*Sin[(c + d*x)/2]^2)^(5/2)) + (2*B*Sin[(c + d*x)/2])/(5*(1 - 2*S
in[(c + d*x)/2]^2)^(5/2)) + (8*B*(Sin[(c + d*x)/2]/(1 - 2*Sin[(c + d*x)/2]^
2)^(3/2) + (2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2]))/15 - ((A -
 B + C)*Csc[(c + d*x)/2]^7*(4725*Sin[(c + d*x)/2]^2 - 48825*Sin[(c + d*x)/2
]^4 + 210105*Sin[(c + d*x)/2]^6 - 486630*Sin[(c + d*x)/2]^8 + 655812*Sin[(c
 + d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[(c + d*x)/2]^2/(-1
+ 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyperg
eometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c
 + d*x)/2]^2)]*Sin[(c + d*x)/2]^10 - 518760*Sin[(c + d*x)/2]^12 + 1770*Hype
rgeometric2F1[2, 9/2, 11/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]
```

```
*Sin[(c + d*x)/2]^12 + 226656*Sin[(c + d*x)/2]^14 - 1500*Hypergeometric2F1[
2, 9/2, 11/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)
/2]^14 - 42048*Sin[(c + d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Si
n[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^16 + 4725*Ar
cTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sqrt[Sin[(c + d
*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[(c + d*x)/2]
^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^2*Sqrt[Sin[(c + d*x)/2]^2
/(-1 + 2*Sin[(c + d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1
+ 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^4*Sqrt[Sin[(c + d*x)/2]^2/(-1 +
2*Sin[(c + d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin
[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^6*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(
c + d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c +
d*x)/2]^2)]]*Sin[(c + d*x)/2]^8*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*
x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2
]^2)]]*Sin[(c + d*x)/2]^10*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]
^2)] + 1080000*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]
]*Sin[(c + d*x)/2]^12*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]
- 414720*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[
(c + d*x)/2]^14*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 6912
0*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*
x)/2]^16*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 60*Cos[(c +
 d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[(c + d*x)/2]^2/(-1
 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^10*(-5 + 4*Sin[(c + d*x)/2]^2)))
/(675*(1 - 2*Sin[(c + d*x)/2]^2)^(7/2)*(-1 + 2*Sin[(c + d*x)/2]^2)) + (A*((
3*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^(5/2) + 4*(Sin[(c + d*x)/2]/
(1 - 2*Sin[(c + d*x)/2]^2)^(3/2) + (2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c +
 d*x)/2]^2])))/30))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Se
c[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.353, size = 859, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/60/d/a*(15*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(
d*x+c)-15*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+
c)+15*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+3
0*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-30*B*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)+30*C*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)+15*A*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-15*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(5/2)*sin(d*x+c)+15*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*
x+c)+120*A*cos(d*x+c)^3-40*B*cos(d*x+c)^3+104*C*cos(d*x+c)^3-120*A*cos(d*x+
c)^2+80*B*cos(d*x+c)^2-112*C*cos(d*x+c)^2-40*B*cos(d*x+c)+32*C*cos(d*x+c)-2
4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.618778, size = 1057, normalized size = 6.45

$$\left[\frac{15\sqrt{2}\left((A-B+C)a\cos(dx+c)^3+(A-B+C)a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{30\left(ad\cos(dx+c)^3+ad\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*x + c
)^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((15*A - 5*B + 13*C)*cos(d*x + c)
^2 + (5*B - C)*cos(d*x + c) + 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), 1/15*(2*((15*A - 5
*B + 13*C)*cos(d*x + c)^2 + (5*B - C)*cos(d*x + c) + 3*C)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*((A - B + C)*a*cos(d*x + c
)^3 + (A - B + C)*a*cos(d*x + c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x
+ c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/sqrt(a*(s
ec(c + d*x) + 1)), x)
```

Giac [B] time = 9.15661, size = 464, normalized size = 2.83

$$-\frac{\frac{15\left(\sqrt{2}A-\sqrt{2}B+\sqrt{2}C\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}+\frac{2\left(15\sqrt{2}Aa^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)+15\sqrt{2}Ca^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(sqrt(2)*A - sqrt(2)*B + sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x
 + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + 2*(15*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 1
5*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (30*sqrt(2)*A*a^2*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1) - 10*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
 + 20*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - (15*sqrt(2)*A*a^2*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1) - 10*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) + 17*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*
c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)
^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.513 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(Sqrt[a]*d) + (2*(3*B - 2*C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec
[c + d*x]]) + (2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.225303, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4082, 4001, 3795, 203}

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2C\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c
 + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(Sqrt[a]*d) + (2*(3*B - 2*C)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec
[c + d*x]]) + (2*C*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} + \frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}a(3A+C)+\right.}{\sqrt{a+a\sec}}}{3a}$$

$$= \frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx}{3ad}$$

$$= \frac{2(3B-2C)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2C\sqrt{a+a\sec(c+dx)}\tan(c+dx}{3ad}$$

$$= \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2(3B-2C)\tan(}{3d\sqrt{a+a\sec(c}}$$

Mathematica [C] time = 7.13237, size = 628, normalized size = 5.32

$$4\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{(A-B+C)\csc^5\left(\frac{1}{2}(c+dx)\right)\Big(}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[(1 - 2*Sin
[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*((2*B*Sin[(c + d*x)/2
])/(3*(1 - 2*Sin[(c + d*x)/2]^2)^(3/2)) - (4*A*Sin[(c + d*x)/2]^3)/(3*(1 -
2*Sin[(c + d*x)/2]^2)^(3/2)) + (4*B*Sin[(c + d*x)/2])/(3*Sqrt[1 - 2*Sin[(c
+ d*x)/2]^2]) + ((A - B + C)*Csc[(c + d*x)/2]^5*(-12*Cos[(c + d*x)/2]^4*Hyp
ergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[(c + d*x)/2]^2/(1 - 2*Sin[(c +
d*x)/2]^2))]*Sin[(c + d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[(
c + d*x)/2]^2/(1 - 2*Sin[(c + d*x)/2]^2))]*Sin[(c + d*x)/2]^8*(4 - 7*Sin[(c
 + d*x)/2]^2 + 3*Sin[(c + d*x)/2]^4) + 7*Sqrt[-(Sin[(c + d*x)/2]^2/(1 - 2*S
in[(c + d*x)/2]^2))]*(1 - 2*Sin[(c + d*x)/2]^2)^3*(15 - 20*Sin[(c + d*x)/2]
^2 + 8*Sin[(c + d*x)/2]^4)*((3 - 7*Sin[(c + d*x)/2]^2)*Sqrt[-(Sin[(c + d*x)
/2]^2/(1 - 2*Sin[(c + d*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[(c + d*x)/2]^2/(1
 - 2*Sin[(c + d*x)/2]^2))]]*(1 - 2*Sin[(c + d*x)/2]^2))))/(63*(1 - 2*Sin[(c
 + d*x)/2]^2)^(7/2))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.339, size = 563, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d/a*(3*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))-3*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+3*C*
cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+3*A*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-3*B*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*sin(d*x+c)+3*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*s
in(d*x+c)+12*B*cos(d*x+c)^2-4*C*cos(d*x+c)^2-12*B*cos(d*x+c)+8*C*cos(d*x+c)
-4*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(a*sec(d
*x + c) + a), x)
```

Fricas [A] time = 0.615628, size = 937, normalized size = 7.94

$$\left[\frac{3\sqrt{2}\left((A-B+C)a\cos\left(dx+c\right)^{2}+(A-B+C)a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^{2}}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{6\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*((A - B + C)*a*cos(d*x + c)^2 + (A - B + C)*a*cos(d*x + c))
*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1
/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*B - C)*cos(d*x + c) + C)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*co
s(d*x + c)), 1/3*(2*((3*B - C)*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c) - 3*sqrt(2)*((A - B + C)*a*cos(d*x + c)^2 + (A -
 B + C)*a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d
*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/sqrt(a*(sec(
c + d*x) + 1)), x)
```

Giac [A] time = 8.95486, size = 252, normalized size = 2.14

$$\frac{\frac{3\sqrt{2}(A-B+C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\left(\frac{\sqrt{2}(3Ba-2Ca)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{3\sqrt{2}Ba}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(2)*(A - B + C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
 2*(sqrt(2)*(3*B*a - 2*C*a)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)
^2 - 1) - 3*sqrt(2)*B*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*
c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.514 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(Sqrt[a]*d) + (2*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.172013, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4054, 3920, 3774, 203, 3795}

$$-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2C\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(Sqrt[a]*d) + (2*C*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\frac{aA}{2}+\frac{1}{2}a(B-C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{a} + (-A+B-C)\int \frac{}{\sqrt{a}}$$

$$= \frac{2C\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2A)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{(2(A}{}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{}{d\sqrt{}}$$

Mathematica [A] time = 1.18443, size = 123, normalized size = 1.04

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-(A-B+C)\sqrt{\cos(c+dx)}\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}A\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos}\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + a*Sec[c + d*x]],
x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]
]*Sqrt[Cos[c + d*x]] - (A - B + C)*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]] + 2*C*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sec[c + d*x]
)])
```

Maple [B] time = 0.299, size = 347, normalized size = 2.9

$$-\frac{1}{ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(A*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)+A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+C*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(
```

```
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*C*cos(d*x+c)-2*C)/sin(d*x+
c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.6248, size = 1172, normalized size = 9.93

$$\left[\frac{\sqrt{2}((A-B+C)a\cos(dx+c)+(A-B+C)a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-}{2(ad\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*log((
2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*s
in(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 2*(A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2
+ 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*C*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -(2*(A*cos(d*x + c
) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 2*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c) - sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sqrt(a*(sec(c + d*x) + 1)
), x)
```

Giac [B] time = 10.9916, size = 396, normalized size = 3.36

$$-\frac{\frac{\sqrt{2}(A-B+C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.\right.\right.}{\sqrt{-a}\,\mathrm{sgn}\left(\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/2*(sqrt(2)*(A - B + C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 4*
sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*C*tan(1/2*d*x + 1/2*c)/((a*tan(
1/2*d*x + 1/2*c)^2 - a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*A*log(abs((sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*
sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*A*log(abs((sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2
*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.515 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=120

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -(((A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c +
 d*x]])
```

Rubi [A] time = 0.239328, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4086, 3920, 3774, 203, 3795}

$$\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c
 + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c +
 d*x]])
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{\int \frac{-\frac{1}{2}a(A-2B)+\frac{1}{2}a(A+2C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}\\ &= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(A-2B)\int \sqrt{a+a\sec(c+dx)}\,dx}{2a}+\\ &= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{(A-2B)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\,\text{ta}}{\sqrt{a+a}}\right.}{d}\\ &= -\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\right.}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 1.14681, size = 120, normalized size = 1.

$$\frac{\sin(c+dx)\left(-\sqrt{2}(A-B+C)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(c+dx)-1}}{\sqrt{2}}\right)+(A-2B)\sqrt{\sec(c+dx)-1}\tan^{-1}\left(\sqrt{\sec(c+dx)-}\right.\right.}{d(\cos(c+dx)-1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```

```
[Out] ((A*(-1 + Cos[c + d*x]) + (A - 2*B)*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Sqrt[-1
 + Sec[c + d*x]] - Sqrt[2]*(A - B + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqrt[
2]]*Sqrt[-1 + Sec[c + d*x]])*Sin[c + d*x])/(d*(-1 + Cos[c + d*x])*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [B] time = 0.352, size = 430, normalized size = 3.6

$$-\frac{1}{2\,ad\sin(dx+c)}\left(-A\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\,\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,B\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d/a*(-A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2
*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
```

```
+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*ln(-(-(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*B*ln(-(-(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-2*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+2*A*cos(d*x+c)^2-2*A*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(a*sec(d
*x + c) + a), x)
```

Fricas [A] time = 16.585, size = 1237, normalized size = 10.31

$$\left[\frac{2A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+\sqrt{2}\left((A-B+C)a\cos\left(dx+c\right)+(A-B+C)a\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{}\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*log(-(2*
sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin
(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + ((A - 2*B)*cos(d*x + c) + A - 2*B)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d*cos(d*x + c)
+ a*d), (A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + ((A - 2*B)*cos(d*x + c) + A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B +
 C)*a*cos(d*x + c) + (A - B + C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x
+ c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx) + C\sec^2(c + dx))\cos(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)/sqrt(a*(sec(
c + d*x) + 1)), x)
```

Giac [B] time = 11.1194, size = 532, normalized size = 4.43

$$\frac{\frac{\sqrt{2}(A-B+C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{(A-2B)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{(A\cdots}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*(A - B + C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + (A
- 2*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) - (A - 2*B)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*A*sqrt(-a) - A*sqrt(-a)*a)/((((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))/d
```

3.516 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{(7A-4B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)}{2d\sqrt{a\sec(c}$$

```
[Out] ((7*A - 4*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Sin[c + d*x])/(4*d*S
qrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Se
c[c + d*x]])
```

Rubi [A] time = 0.397217, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4022, 3920, 3774, 203, 3795}

$$\frac{(7A-4B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)}{2d\sqrt{a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```

```
[Out] ((7*A - 4*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Sin[c + d*x])/(4*d*S
qrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Se
c[c + d*x]])
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{\int \frac{\cos(c+dx)\left(-\frac{1}{2}a(A-4B)+\frac{1}{2}a(3A+4C\right.}{\sqrt{a+a\sec(c+dx)}}}{2a}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{\int \frac{1}{4}}{}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+(-A$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{(2(}{}$$

$$= \frac{(7A-4B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\,\mathrm{ta}}{}$$

Mathematica [C] time = 27.6679, size = 16865, normalized size = 99.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.33, size = 1025, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/16/d/a*(-7*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(3/2)+4*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))-8*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))-8*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/
2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(
d*x+c))-7*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+8*
B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+4*B*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*C*cos(d*x+c)*sin(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-8*C*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-8*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))*sin(d*x+c)+8*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+
c)-8*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+8*A*cos(d*x+c)^
4-12*A*cos(d*x+c)^3+16*B*cos(d*x+c)^3+4*A*cos(d*x+c)^2-16*B*cos(d*x+c)^2)*(
a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(a*sec
(d*x + c) + a), x)
```

Fricas [A] time = 43.0983, size = 1374, normalized size = 8.13

$$\left[\frac{4\sqrt{2}\left((A-B+C)a\cos\left(dx+c\right)+(A-B+C)a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^{2}+2\cos(dx+c)-1}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*log
((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)
*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) - ((7*A - 4*B + 8*C)*cos(d*x + c) + 7*A - 4*B + 8*C)*sqr
t(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
 + 2*(2*A*cos(d*x + c)^2 - (A - 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(((7*A - 4*B +
 8*C)*cos(d*x + c) + 7*A - 4*B + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c)^
2 - (A - 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c) - 4*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.8042, size = 886, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(2)*(A - B + C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) +
(7*A - 4*B + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1)) - (7*A - 4*B + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(17*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a) - 12*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a) - 57*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-
a)*a + 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^4*B*sqrt(-a)*a + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^2 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^2 - 3*A*sqrt(-a)*a^3 +
4*B*sqrt(-a)*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
```

```
1/2*c)^2 + a))^2*a + a^2)^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.517 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{(7A-2B+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9A-14B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A$$

```
[Out] -((9*A - 14*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B + 8*C)*Sin[c + d*
x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) - ((A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/
(12*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt
[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.588598, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4022, 3920, 3774, 203, 3795}

$$\frac{(7A-2B+8C)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(9A-14B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```


```
[Out] -((9*A - 14*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B + 8*C)*Sin[c + d*
x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) - ((A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/
(12*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt
[a + a*Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(-\frac{1}{2}a(A-6B)+\frac{1}{2}a(5A+6C)s\right.}{\sqrt{a+a\sec(c+dx)}}}{3a}$$

$$= -\frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+d}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(7A-2B+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+d}{12d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(7A-2B+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+d}{12d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(7A-2B+8C)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+d}{12d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(9A-14B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B+C)\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.809781, size = 161, normalized size = 0.76

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left(-2(A-6B)\cos(c+dx)+3(7A-2B+8C)+8A\cos^2(c+dx)\right)-3(9A-14B\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 a*Sec[c + d*x]],x]
```

```
[Out] ((-3*(9*A - 14*B + 8*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 24*Sqrt[2]*(A - B
 + C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(3*(7*A - 2*B
```

```
+ 8*C) - 2*(A - 6*B)*Cos[c + d*x] + 8*A*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*
x]])*Tan[c + d*x])/(24*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.361, size = 1561, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/192/d/a*(-42*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*sin(d*x+c)+27*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))+24*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*sin(d*x+c)-48*B*cos(d*x+c)^3-84*B*cos(d*x+c)*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+48*C*cos(d*x+c)*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-192*C*cos(d*x+c)^4+48*A*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-48*B*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)*sin(d*x+c)+48*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/
2)*sin(d*x+c)-184*A*cos(d*x+c)^4+54*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+192*C*cos(d*x+c)^3+48*A*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)-48*B*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+48*C*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+96*A*ln(-(-(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-96*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*cos(d*x+c)*sin(d*x+c)+96*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2
)*cos(d*x+c)*sin(d*x+c)-64*A*cos(d*x+c)^6+80*A*cos(d*x+c)^5-96*B*cos(d*x+c)
^5-42*B*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+24*C*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+144*B*cos(d*x+c
)^4+168*A*cos(d*x+c)^3+27*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^2 + B \sec\left(dx+c\right) + A\right) \cos\left(dx+c\right)^3}{\sqrt{a \sec\left(dx+c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(a*sec
(d*x + c) + a), x)
```

Fricas [A] time = 42.7949, size = 1496, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*l
og(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x +
 c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) - 3*((9*A - 14*B + 8*C)*cos(d*x + c) + 9*A - 14*B + 8
*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c
) + 1)) + 2*(8*A*cos(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B
 + 8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))
/(a*d*cos(d*x + c) + a*d), 1/24*(3*((9*A - 14*B + 8*C)*cos(d*x + c) + 9*A -
 14*B + 8*C)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) + (8*A*cos(d*x + c)^3 - 2*(A - 6*B)*cos(d*x +
 c)^2 + 3*(7*A - 2*B + 8*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) - 24*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*
a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 11.8569, size = 1490, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(2)*(A - B + C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) +
 3*(9*A - 14*B + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)) - 3*(9*A - 14*B + 8*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(165*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a) - 102*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)
 + 72*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^10*C*sqrt(-a) - 1323*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^8*A*sqrt(-a)*a + 954*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a - 888*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a + 3906*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(
-a)*a^2 - 2268*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*B*sqrt(-a)*a^2 + 3024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^2 - 2118*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3 + 1044*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt
(-a)*a^3 - 1776*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^4*C*sqrt(-a)*a^3 + 393*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4 - 222*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^4 + 360*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*C*sqrt(-
a)*a^4 - 31*A*sqrt(-a)*a^5 + 18*B*sqrt(-a)*a^5 - 24*C*sqrt(-a)*a^5)/(((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2
)^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.518 $\int \frac{\cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=259

$$-\frac{(21A-56B+16C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{(107A-72B+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] ((107*A - 72*B + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((21*A - 56*B + 16*C)*Sin
[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + ((43*A - 8*B + 48*C)*Cos[c + d
*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) - ((A - 8*B)*Cos[c + d*x]
^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]^3*Sin[c
+ d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.776892, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4086, 4022, 3920, 3774, 203, 3795}

$$-\frac{(21A-56B+16C)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{(107A-72B+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64\sqrt{a}d}-\frac{\sqrt{2}(A-B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec
[c + d*x]],x]
```


```
[Out] ((107*A - 72*B + 112*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((21*A - 56*B + 16*C)*Sin
[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + ((43*A - 8*B + 48*C)*Cos[c + d
*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) - ((A - 8*B)*Cos[c + d*x]
^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]^3*Sin[c
+ d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^3(c+dx)\left(-\frac{1}{2}a(A-8B)+\frac{1}{2}a(7A+}{\sqrt{a+a\sec(c+dx)}}}{4a}$$

$$= -\frac{(A-8B)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^3(c+dx)\sin(c}{4d\sqrt{a+a\sec(c+}}$$

$$= \frac{(43A-8B+48C)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} - \frac{(A-8B)\cos}{24d\sqrt{a}}$$

$$= -\frac{(21A-56B+16C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{(43A-8B+48C)\cos}{96d\sqrt{a+a\sec}}$$

$$= -\frac{(21A-56B+16C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{(43A-8B+48C)\cos}{96d\sqrt{a+a\sec}}$$

$$= -\frac{(21A-56B+16C)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{(43A-8B+48C)\cos}{96d\sqrt{a+a\sec}}$$

$$= \frac{(107A-72B+112C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{a}d} - \frac{\sqrt{2}(A-B+}{}$$

Mathematica [A] time = 0.969472, size = 178, normalized size = 0.69

$$\frac{\tan(c+dx)\left(\cos(c+dx)\sqrt{1-\sec(c+dx)}\left(2(43A-8B+48C)\cos(c+dx)-8(A-8B)\cos^2(c+dx)+48A\cos^3(c+\right.\right.}{192d\sqrt{1-\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 a*Sec[c + d*x]],x]
```

```
[Out] ((3*(107*A - 72*B + 112*C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 192*Sqrt[2]*(A
 - B + C)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-63*A + 1
68*B - 48*C + 2*(43*A - 8*B + 48*C)*Cos[c + d*x] - 8*(A - 8*B)*Cos[c + d*x]
^2 + 48*A*Cos[c + d*x]^3)*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(192*d*Sqrt
[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.413, size = 2086, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3072/d/a*(321*A*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(7/2)*2^(1/2)-216*B*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*2^(1/2)+336*C*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(7/2)*2^(1/2)+321*A*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*
2^(1/2)*sin(d*x+c)-216*B*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)*
sin(d*x+c)+384*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+384*C
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-768*C*cos(d*x+c)^4+10
08*C*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1
/2)+1008*C*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*
2^(1/2)-1152*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*
x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*sin(d
*x+c)-1152*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c
)-384*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^3*sin(d*x+c)-1
008*A*cos(d*x+c)^4-1504*A*cos(d*x+c)^6+2384*A*cos(d*x+c)^5-2944*B*cos(d*x+c
)^5+2304*C*cos(d*x+c)^5+2688*B*cos(d*x+c)^4-768*A*cos(d*x+c)^8+896*A*cos(d*
x+c)^7-1024*B*cos(d*x+c)^7+1280*B*cos(d*x+c)^6-384*B*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(7/2)*sin(d*x+c)+963*A*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-648*B*sin(d*x+c)*cos(d*x+c)^2*arctanh(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+963*A*sin(d*x+c)*cos(d*x+c)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)-648*B*sin(d*x+c)*cos(d*x+c)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*2^(1/2)+336*C*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)-1536*C*cos(d*x+c)^6+384*A*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
```

```
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+384*C*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3+1152*A*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+1152*C*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+1152*A*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+1152*C*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)/cos(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{4}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^4/sqrt(a*sec
(d*x + c) + a), x)
```

Fricas [A] time = 64.5368, size = 1639, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/384*(192*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)
*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x
+ c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2
+ 2*cos(d*x + c) + 1)) - 3*((107*A - 72*B + 112*C)*cos(d*x + c) + 107*A - 7
2*B + 112*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(co
s(d*x + c) + 1)) + 2*(48*A*cos(d*x + c)^4 - 8*(A - 8*B)*cos(d*x + c)^3 + 2*
(43*A - 8*B + 48*C)*cos(d*x + c)^2 - 3*(21*A - 56*B + 16*C)*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*
d), -1/192*(3*((107*A - 72*B + 112*C)*cos(d*x + c) + 107*A - 72*B + 112*C)*
sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) - (48*A*cos(d*x + c)^4 - 8*(A - 8*B)*cos(d*x + c)^3 + 2*(4
3*A - 8*B + 48*C)*cos(d*x + c)^2 - 3*(21*A - 56*B + 16*C)*cos(d*x + c))*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 192*sqrt(2)*((A - B + C
)*a*cos(d*x + c) + (A - B + C)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x +
c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 12.3017, size = 1887, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] -1/384*(192*sqrt(2)*(A - B + C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) + 3*(107*A - 72*B + 112*C)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)) - 3*(107*A - 72*B + 112*C)*log(abs((sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)
))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 4*sqrt(2)*(1599*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a) -
1320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
14*B*sqrt(-a) + 816*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^14*C*sqrt(-a) - 18219*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a + 18504*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a - 12528*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*C*
sqrt(-a)*a + 91467*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^10*A*sqrt(-a)*a^2 - 96072*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^2 + 64752*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*C*sqrt(-a)*a^2 -
 177735*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^8*A*sqrt(-a)*a^3 + 215016*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^3 - 124848*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*C*sqrt(-a)*a^3 + 100413*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(
-a)*a^4 - 136056*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*B*sqrt(-a)*a^4 + 70032*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*C*sqrt(-a)*a^4 - 26881*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^5 + 3605
6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B
*sqrt(-a)*a^5 - 19152*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*C*sqrt(-a)*a^5 + 3321*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^6 - 4632*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^6 + 2
640*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
*C*sqrt(-a)*a^6 - 205*A*sqrt(-a)*a^7 + 248*B*sqrt(-a)*a^7 - 144*C*sqrt(-a)*
a^7)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2*a + a^2)^4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.519 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{(11A-15B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(245A-273B+397C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{210a^2d}-\frac{(A-B+C)\text{t}}{2d(a\text{se}}$$

```
[Out] ((11*A - 15*B + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d
*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((455*A - 651*B + 799*C)*Tan[c + d*
x])/(105*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((35*A - 63*B + 67*C)*Sec[c + d*x]
^2*Tan[c + d*x])/(70*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((7*A - 7*B + 11*C)*Se
c[c + d*x]^3*Tan[c + d*x])/(14*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((245*A - 27
3*B + 397*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(210*a^2*d)
```

Rubi [A] time = 0.877086, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4021, 4010, 4001, 3795, 203}

$$\frac{(11A-15B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(245A-273B+397C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{210a^2d}-\frac{(A-B+C)\text{t}}{2d(a\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```


```
[Out] ((11*A - 15*B + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]^4*Tan[c + d
*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((455*A - 651*B + 799*C)*Tan[c + d*
x])/(105*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((35*A - 63*B + 67*C)*Sec[c + d*x]
^2*Tan[c + d*x])/(70*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((7*A - 7*B + 11*C)*Se
c[c + d*x]^3*Tan[c + d*x])/(14*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((245*A - 27
3*B + 397*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(210*a^2*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
```

```
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\sec^4(c+dx)\left(-2a(A-2B\right.}{\sqrt{}}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(7A-7B+11C)\,\mathrm{se}}{14ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(35A-63B+67C)}{70ad\sqrt{a-}}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(35A-63B+67C)}{70ad\sqrt{a-}}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(455A-651B+79}{105ad\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(455A-651B+79}{105ad\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{(11A-15B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\,\mathrm{sec}}{2d(a+a}$$

Mathematica [C] time = 11.7739, size = 2746, normalized size = 9.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(3/2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[(1 - 2*S
in[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*((4*A*Sin[(c + d*x)
/2])/(7*(1 - 2*Sin[(c + d*x)/2]^2)^(7/2)) - ((A - B + C)*(1 - 2*Sin[(c + d*
x)/2]))/(28*(1 + Sin[(c + d*x)/2])*(1 - 2*Sin[(c + d*x)/2]^2)^(7/2)) + ((A
- B + C)*(1 + 2*Sin[(c + d*x)/2]))/(28*(1 - Sin[(c + d*x)/2])*(1 - 2*Sin[(c
 + d*x)/2]^2)^(7/2)) - ((A - B + C)*(315*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sq
rt[1 - 2*Sin[(c + d*x)/2]^2]] + (5 + 3*Sin[(c + d*x)/2])/((1 - Sin[(c + d*x
)/2])*(1 - 2*Sin[(c + d*x)/2]^2)^(5/2)) - (11 + 17*Sin[(c + d*x)/2])/((1 -
Sin[(c + d*x)/2])*(1 - 2*Sin[(c + d*x)/2]^2)^(3/2)) + (61 + 71*Sin[(c + d*x
)/2])/((1 - Sin[(c + d*x)/2])*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]) + (193*Sqrt[1
 - 2*Sin[(c + d*x)/2]^2])/(1 - Sin[(c + d*x)/2])))/70 + ((A - B + C)*(315*A
rcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2]] + (5 - 3*Sin
[(c + d*x)/2])/((1 + Sin[(c + d*x)/2])*(1 - 2*Sin[(c + d*x)/2]^2)^(5/2)) -
(11 - 17*Sin[(c + d*x)/2])/((1 + Sin[(c + d*x)/2])*(1 - 2*Sin[(c + d*x)/2]^
2)^(3/2)) + (61 - 71*Sin[(c + d*x)/2])/((1 + Sin[(c + d*x)/2])*Sqrt[1 - 2*S
in[(c + d*x)/2]^2]) + (193*Sqrt[1 - 2*Sin[(c + d*x)/2]^2])/(1 + Sin[(c + d*
x)/2])))/70 - ((7*A - 3*B - C)*Csc[(c + d*x)/2]^9*(363825*Sin[(c + d*x)/2]^
2 - 4729725*Sin[(c + d*x)/2]^4 + 26785605*Sin[(c + d*x)/2]^6 - 86790165*Sin
[(c + d*x)/2]^8 + 177677808*Sin[(c + d*x)/2]^10 - 239283044*Sin[(c + d*x)/2
]^12 + 52080*Hypergeometric2F1[2, 11/2, 13/2, Sin[(c + d*x)/2]^2/(-1 + 2*Si
n[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^12 + 560*HypergeometricPFQ[{2, 2, 2, 2,
 11/2}, {1, 1, 1, 13/2}, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Si
n[(c + d*x)/2]^12 + 213120160*Sin[(c + d*x)/2]^14 - 168280*Hypergeometric2F
1[2, 11/2, 13/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d
*x)/2]^14 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin
[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^14 - 12149702
4*Sin[(c + d*x)/2]^16 + 212520*Hypergeometric2F1[2, 11/2, 13/2, Sin[(c + d*
x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^16 + 3360*Hypergeomet
ricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[
(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^16 + 40125184*Sin[(c + d*x)/2]^18 - 12432
0*Hypergeometric2F1[2, 11/2, 13/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)
/2]^2)]*Sin[(c + d*x)/2]^18 - 2240*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1
, 1, 1, 13/2}, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x
)/2]^18 - 5840384*Sin[(c + d*x)/2]^20 + 28000*Hypergeometric2F1[2, 11/2, 13
/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^20 + 5
60*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[(c + d*x)/2]^
2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^20 + 363825*ArcTanh[Sqrt[Si
n[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sqrt[Sin[(c + d*x)/2]^2/(-1
+ 2*Sin[(c + d*x)/2]^2)] - 5336100*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*
Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^2*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Si
n[(c + d*x)/2]^2)] + 34636140*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(
c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^4*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c
+ d*x)/2]^2)] - 131060160*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c +
d*x)/2]^2)]]*Sin[(c + d*x)/2]^6*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*
x)/2]^2)] + 320535600*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)
/2]^2)]]*Sin[(c + d*x)/2]^8*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2
]^2)] - 530671680*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^
2)]]*Sin[(c + d*x)/2]^10*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2
)] + 604296000*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]
]*Sin[(c + d*x)/2]^12*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]
```

```
- 468948480*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*S
in[(c + d*x)/2]^14*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 2
37726720*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[
(c + d*x)/2]^16*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] - 7096
3200*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c +
 d*x)/2]^18*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 9461760*
ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)
/2]^20*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] - 1120*Cos[(c +
 d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[(c + d*x)/2
]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^12*(-6 + 5*Sin[(c + d*x)/
2]^2) + 280*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2}, {1, 13/2}, S
in[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^12*(103 - 1
64*Sin[(c + d*x)/2]^2 + 70*Sin[(c + d*x)/2]^4)))/(80850*(1 - 2*Sin[(c + d*x
)/2]^2)^(9/2)*(-1 + 2*Sin[(c + d*x)/2]^2)) + (8*A*((3*Sin[(c + d*x)/2])/(1
- 2*Sin[(c + d*x)/2]^2)^(5/2) + 4*(Sin[(c + d*x)/2]/(1 - 2*Sin[(c + d*x)/2]
^2)^(3/2) + (2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2])))/35))/(d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a*(1
+ Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.407, size = 1437, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3360/d/a^2*(-1+cos(d*x+c))*(-960*C-13440*B*cos(d*x+c)^3+1155*A*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+1995*C*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*sin(d*x+c)+6352*C*cos(d*x+c)^4-9450*B*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*sin(d*x+c)-6300*B*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c)-6300*B*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)*cos(d*x+c)^3*sin(d*x+c)+3920*A*cos(d*x+c)^4+16000*C*cos(d*x
+c)^3-3712*C*cos(d*x+c)^2+1536*C*cos(d*x+c)-10640*A*cos(d*x+c)^5+16464*B*co
s(d*x+c)^5-19216*C*cos(d*x+c)^5-4368*B*cos(d*x+c)^4+8960*A*cos(d*x+c)^3+268
8*B*cos(d*x+c)^2+1155*A*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2
)*cos(d*x+c)^4+1995*C*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*
cos(d*x+c)^4-1575*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)-15
75*B*sin(d*x+c)*cos(d*x+c)^4*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)-2240
*A*cos(d*x+c)^2+4620*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)
*cos(d*x+c)^3+7980*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*c
os(d*x+c)^3+6930*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos
(d*x+c)^2+11970*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(
d*x+c)^2+4620*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(
```

```
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*
x+c)+7980*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)
-1344*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*
x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.694546, size = 1535, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] [-1/840*(105*sqrt(2)*((11*A - 15*B + 19*C)*cos(d*x + c)^5 + 2*(11*A - 15*B
+ 19*C)*cos(d*x + c)^4 + (11*A - 15*B + 19*C)*cos(d*x + c)^3)*sqrt(-a)*log(
(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*si
n(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*((665*A - 1029*B + 1201*C)*cos(d*x + c)^4 + 12*(35*
A - 63*B + 67*C)*cos(d*x + c)^3 - 28*(5*A - 3*B + 7*C)*cos(d*x + c)^2 - 12*
(7*B - 3*C)*cos(d*x + c) - 60*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x
+ c)^3), -1/420*(105*sqrt(2)*((11*A - 15*B + 19*C)*cos(d*x + c)^5 + 2*(11*A
 - 15*B + 19*C)*cos(d*x + c)^4 + (11*A - 15*B + 19*C)*cos(d*x + c)^3)*sqrt(
a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))) + 2*((665*A - 1029*B + 1201*C)*cos(d*x + c)^4 + 12*(35*
A - 63*B + 67*C)*cos(d*x + c)^3 - 28*(5*A - 3*B + 7*C)*cos(d*x + c)^2 - 12*
(7*B - 3*C)*cos(d*x + c) - 60*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x
+ c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec{\left(c + dx \right)} + C\sec^{2}{\left(c + dx \right)}\right)\sec^{4}{\left(c + dx \right)}}{\left(a\left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a*(sec(c
 + d*x) + 1))**(3/2), x)
```

Giac [B] time = 9.75266, size = 756, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] 1/420*(105*(11*sqrt(2)*A - 15*sqrt(2)*B + 19*sqrt(2)*C)*log(abs(-sqrt(-a)*t
an(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)) - ((((105*(sqrt(2)*A*a^5*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1) - sqrt(2)*B*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a^5
*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^3 - 4*(455*sqrt(
2)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 693*sqrt(2)*B*a^5*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1) + 877*sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)
*tan(1/2*d*x + 1/2*c)^2 + 14*(305*sqrt(2)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - 453*sqrt(2)*B*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 517*sqrt(2)*C*a^
5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)*tan(1/2*d*x + 1/2*c)^2 - 140*(25*sq
rt(2)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 39*sqrt(2)*B*a^5*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1) + 47*sqrt(2)*C*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3
)*tan(1/2*d*x + 1/2*c)^2 + 105*(9*sqrt(2)*A*a^5*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - 17*sqrt(2)*B*a^5*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 17*sqrt(2)*C*a^5*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^3)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x
+ 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.520 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{(7A-11B+15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(15A-35B+39C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}-\frac{(A-B+C)\tan(}{2d(a\sec(c}$$

```
[Out] -((7*A - 11*B + 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d
*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((45*A - 65*B + 93*C)*Tan[c + d*x])
/(15*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 5*B + 9*C)*Sec[c + d*x]^2*Tan[
c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((15*A - 35*B + 39*C)*Sqrt[a
+ a*Sec[c + d*x]]*Tan[c + d*x])/(30*a^2*d)
```

Rubi [A] time = 0.682523, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4021, 4010, 4001, 3795, 203}

$$-\frac{(7A-11B+15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(15A-35B+39C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}-\frac{(A-B+C)\tan(}{2d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```


```
[Out] -((7*A - 11*B + 15*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]^3*Tan[c + d
*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((45*A - 65*B + 93*C)*Tan[c + d*x])
/(15*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 5*B + 9*C)*Sec[c + d*x]^2*Tan[
c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((15*A - 35*B + 39*C)*Sqrt[a
+ a*Sec[c + d*x]]*Tan[c + d*x])/(30*a^2*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sec^3(c+dx)\left(-a(A-3B+\right.}{\sqrt{a}}}{}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(5A-5B+9C)\sec}{10ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(5A-5B+9C)\sec}{10ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(45A-65B+93C)}{15ad\sqrt{a+a\sec}}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(45A-65B+93C)}{15ad\sqrt{a+a\sec}}$$

$$= -\frac{(7A-11B+15C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sec}{2d(a+a}$$

Mathematica [C] time = 8.84682, size = 2025, normalized size = 8.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(3/2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[(1 - 2*S
in[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*((4*A*Sin[(c + d*x)
/2])/(5*(1 - 2*Sin[(c + d*x)/2]^2)^(5/2)) - ((A - B + C)*(1 - 2*Sin[(c + d*
x)/2]))/(20*(1 + Sin[(c + d*x)/2])*(1 - 2*Sin[(c + d*x)/2]^2)^(5/2)) + ((A
- B + C)*(1 + 2*Sin[(c + d*x)/2]))/(20*(1 - Sin[(c + d*x)/2])*(1 - 2*Sin[(c
 + d*x)/2]^2)^(5/2)) + (16*A*(Sin[(c + d*x)/2]/(1 - 2*Sin[(c + d*x)/2]^2)^(
3/2) + (2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2]))/15 - ((A - B +
 C)*(-105*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2]] +
 (4 + 3*Sin[(c + d*x)/2])/((1 - Sin[(c + d*x)/2])*(1 - 2*Sin[(c + d*x)/2]^2
)^(3/2)) - (19 + 29*Sin[(c + d*x)/2])/((1 - Sin[(c + d*x)/2])*Sqrt[1 - 2*Si
n[(c + d*x)/2]^2]) - (67*Sqrt[1 - 2*Sin[(c + d*x)/2]^2])/(1 - Sin[(c + d*x)
/2])))/30 + ((A - B + C)*(-105*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[1 - 2*S
in[(c + d*x)/2]^2]] + (4 - 3*Sin[(c + d*x)/2])/((1 + Sin[(c + d*x)/2])*(1 -
 2*Sin[(c + d*x)/2]^2)^(3/2)) - (19 - 29*Sin[(c + d*x)/2])/((1 + Sin[(c + d
*x)/2])*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]) - (67*Sqrt[1 - 2*Sin[(c + d*x)/2]^2
])/(1 + Sin[(c + d*x)/2])))/30 + ((7*A - 3*B - C)*Csc[(c + d*x)/2]^7*(4725*
Sin[(c + d*x)/2]^2 - 48825*Sin[(c + d*x)/2]^4 + 210105*Sin[(c + d*x)/2]^6 -
 486630*Sin[(c + d*x)/2]^8 + 655812*Sin[(c + d*x)/2]^10 - 710*Hypergeometri
c2F1[2, 9/2, 11/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c +
 d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1
, 11/2}, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^1
0 - 518760*Sin[(c + d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[(
c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^12 + 226656*Sin
[(c + d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[(c + d*x)/2]^2/
(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^14 - 42048*Sin[(c + d*x)/2]^1
6 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c +
 d*x)/2]^2)]*Sin[(c + d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1
 + 2*Sin[(c + d*x)/2]^2)]]*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]
^2)] - 56700*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*
Sin[(c + d*x)/2]^2*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 2
91060*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c
+ d*x)/2]^4*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] - 833760*A
rcTanh[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/
2]^6*Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 1458000*ArcTanh
[Sqrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^8*S
qrt[Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[
Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^10*Sqrt[S
in[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[(
c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^12*Sqrt[Sin[(c
 + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[(c + d*
x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^14*Sqrt[Sin[(c + d*x
)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2
/(-1 + 2*Sin[(c + d*x)/2]^2)]]*Sin[(c + d*x)/2]^16*Sqrt[Sin[(c + d*x)/2]^2/
(-1 + 2*Sin[(c + d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
2, 9/2}, {1, 11/2}, Sin[(c + d*x)/2]^2/(-1 + 2*Sin[(c + d*x)/2]^2)]*Sin[(c
+ d*x)/2]^10*(-5 + 4*Sin[(c + d*x)/2]^2)))/(1350*(1 - 2*Sin[(c + d*x)/2]^2)
^(7/2)*(-1 + 2*Sin[(c + d*x)/2]^2))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Co
s[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.376, size = 1152, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/240/d/a^2*(-1+cos(d*x+c))*(105*A*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)-165*B*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(5/2)+225*C*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(5/2)+315*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x
+c)^2*sin(d*x+c)-495*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)
^2*sin(d*x+c)+675*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*
sin(d*x+c)+315*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d
*x+c)-495*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)
+675*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)+105*
A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-165*B*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+225*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(5/2)*sin(d*x+c)+600*A*cos(d*x+c)^4-760*B*cos(d*x+c)^4+1176*C*cos(d*x
+c)^4-120*A*cos(d*x+c)^3+280*B*cos(d*x+c)^3-312*C*cos(d*x+c)^3-480*A*cos(d*
x+c)^2+640*B*cos(d*x+c)^2-960*C*cos(d*x+c)^2-160*B*cos(d*x+c)+192*C*cos(d*x
+c)-96*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.652874, size = 1399, normalized size = 6.11

$$\left[-\frac{15\,\sqrt{2}\left((7\,A-11\,B+15\,C)\cos\left(dx+c\right)^4+2\,(7\,A-11\,B+15\,C)\cos\left(dx+c\right)^3+(7\,A-11\,B+15\,C)\cos\left(dx+c\right)^2\right)\sqrt{-a}\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*sqrt(2)*((7*A - 11*B + 15*C)*cos(d*x + c)^4 + 2*(7*A - 11*B + 1
5*C)*cos(d*x + c)^3 + (7*A - 11*B + 15*C)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*((75*A - 95*B + 147*C)*cos(d*x + c)^3 + 12*(5*A - 5*B
+ 9*C)*cos(d*x + c)^2 + 4*(5*B - 3*C)*cos(d*x + c) + 12*C)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d
*x + c)^3 + a^2*d*cos(d*x + c)^2), 1/60*(15*sqrt(2)*((7*A - 11*B + 15*C)*co
s(d*x + c)^4 + 2*(7*A - 11*B + 15*C)*cos(d*x + c)^3 + (7*A - 11*B + 15*C)*c
os(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((75*A - 95*B + 147*C)*cos(d*x
+ c)^3 + 12*(5*A - 5*B + 9*C)*cos(d*x + c)^2 + 4*(5*B - 3*C)*cos(d*x + c) +
 12*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec^{3}\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a*(sec(c
 + d*x) + 1))**(3/2), x)
```

Giac [A] time = 9.61757, size = 456, normalized size = 1.99

$$-\frac{\frac{15\sqrt{2}(7A-11B+15C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{\left(\left(\left(\frac{15\sqrt{2}(Aa^{3}-Ba^{3}+Ca^{3})\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{\sqrt{2}(165Aa^{3}-245Ba^{3}+381Ca^{3})}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}\right)\tan}{\left(a\tan\left(\frac{1}{2}\right.\right.}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] -1/60*(15*sqrt(2)*(7*A - 11*B + 15*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c
) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)) - (((15*sqrt(2)*(A*a^3 - B*a^3 + C*a^3)*tan(1/2*d*x + 1/2*c)^2/
(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(165*A*a^3 - 245*B*a^3 + 38
1*C*a^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 + 5*
sqrt(2)*(57*A*a^3 - 73*B*a^3 + 105*C*a^3)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)))*tan(1/2*d*x + 1/2*c)^2 - 15*sqrt(2)*(9*A*a^3 - 9*B*a^3 + 17*C*a^3)/(a
^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.521 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{(3A-7B+11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A-3B+7C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{(A-B+C)\tan(c+dx)\mathrm{se}}{2d(a\sec(c+dx)+a}$$

```
[Out] ((3*A - 7*B + 11*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x
])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((3*A - 9*B + 13*C)*Tan[c + d*x])/(3*
a*d*Sqrt[a + a*Sec[c + d*x]]) + ((3*A - 3*B + 7*C)*Sqrt[a + a*Sec[c + d*x]]
*Tan[c + d*x])/(6*a^2*d)
```

Rubi [A] time = 0.476456, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4084, 4010, 4001, 3795, 203}

$$\frac{(3A-7B+11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(3A-3B+7C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{(A-B+C)\tan(c+dx)\mathrm{se}}{2d(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B + 11*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d*x
])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((3*A - 9*B + 13*C)*Tan[c + d*x])/(3*
a*d*Sqrt[a + a*Sec[c + d*x]]) + ((3*A - 3*B + 7*C)*Sqrt[a + a*Sec[c + d*x]]
*Tan[c + d*x])/(6*a^2*d)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sec^2(c+dx)\left(2a(B-\right.}{\sqrt{}}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(3A-3B+7C)}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(3A-9B+13C)}{3ad\sqrt{a+a\,\mathrm{se}}}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(3A-9B+13C)}{3ad\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{(3A-7B+11C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\,\mathrm{se}}{2d(a+}$$

Mathematica [C] time = 25.2684, size = 7119, normalized size = 39.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.321, size = 867, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d/a^2*(-1+cos(d*x+c))*(9*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*c
os(d*x+c)^2*sin(d*x+c)-21*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d
*x+c)^2*sin(d*x+c)+33*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c
)^2*sin(d*x+c)+18*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))-42*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln
(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c
))+66*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+9*A
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-21*B*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+33*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))*sin(d*x+c)-12*A*cos(d*x+c)^3+60*B*cos(d*x+c)^3-76*C*cos(d*x+c)^3+1
2*A*cos(d*x+c)^2-12*B*cos(d*x+c)^2+28*C*cos(d*x+c)^2-48*B*cos(d*x+c)+64*C*c
os(d*x+c)-16*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.637374, size = 1257, normalized size = 6.94

$$\left[-\frac{3\sqrt{2}\left((3A-7B+11C)\cos(dx+c)^3+2(3A-7B+11C)\cos(dx+c)^2+(3A-7B+11C)\cos(dx+c)\right)\sqrt{-a}\log\left(2 \ldots\right.}{24\left(a^2d\cos \ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*((3*A - 7*B + 11*C)*cos(d*x + c)^3 + 2*(3*A - 7*B + 11*C)
*cos(d*x + c)^2 + (3*A - 7*B + 11*C)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
+ 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) + 4*((3*A - 15*B + 19*C)*cos(d*x + c)^2 - 12*(B - C)*cos(d*x + c)
- 4*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/12*(3*sqrt(2)*((
3*A - 7*B + 11*C)*cos(d*x + c)^3 + 2*(3*A - 7*B + 11*C)*cos(d*x + c)^2 + (3
```

```
*A - 7*B + 11*C)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((3*A - 15*B +
19*C)*cos(d*x + c)^2 - 12*(B - C)*cos(d*x + c) - 4*C)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x +
c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a*(sec(c
 + d*x) + 1))**(3/2), x)
```

Giac [B] time = 9.28646, size = 491, normalized size = 2.71

$$-\frac{\left(\left(\left(\frac{3\left(\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)+\sqrt{2}Ca\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{2\left(3\sqrt{2}Aa\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)-15\sqrt{2}Ba\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.\right.}{a}\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] -1/12*(((3*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*t
an(1/2*d*x + 1/2*c)^2/a - 2*(3*sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
- 15*sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 23*sqrt(2)*C*a*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1))/a)*tan(1/2*d*x + 1/2*c)^2 + 3*(sqrt(2)*A*a*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1) - 9*sqrt(2)*B*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 9
*sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a)*tan(1/2*d*x + 1/2*c)/((a*t
an(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*(3*sqrt
(2)*A - 7*sqrt(2)*B + 11*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c)
+ sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1)))/d
```

3.522 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{(A+3B-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B+C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{2C\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((A + 3*B - 7*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B + C)*Tan[c + d*x])/(2*d*(a + a*Sec
[c + d*x])^(3/2)) + (2*C*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.246218, antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4078, 4001, 3795, 203}

$$\frac{(A+3B-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B+5C)\tan(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\tan(c+dx)\sec(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^(3/2),x]
```

```
[Out] ((A + 3*B - 7*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2)) + ((A - B + 5*C)*Tan[c + d*x])/(2*a*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 4078

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*B - b*C - 2*A*b*(m + 1) - (b*B*(m + 2) -
a*(A*(m + 2) - C*(m - 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\sec(c+dx)\left(a(A+B-C\right.}{\sqrt{a+}}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+5C)\tan(}{2ad\sqrt{a+a\sec(c}}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+5C)\tan(}{2ad\sqrt{a+a\sec(c}}$$

$$= \frac{(A+3B-7C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\sec(c}{2d(a+a\,\mathrm{se}}$$

Mathematica [C] time = 6.48058, size = 748, normalized size = 6.23

$$4\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}\cos^3\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{(7A-3B-C)\sin\left(\frac{1}{2}(c+dx)\right)}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[(1 - 2*S
in[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*((3*(A - B + C)*Arc
Tan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2]])/2 - (3*(A - B
 + C)*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2]])/2 +
(4*A*Sin[(c + d*x)/2])/Sqrt[1 - 2*Sin[(c + d*x)/2]^2] - ((A - B + C)*(1 - 2
*Sin[(c + d*x)/2]))/(4*(1 + Sin[(c + d*x)/2])*Sqrt[1 - 2*Sin[(c + d*x)/2]^2
]) + ((A - B + C)*(1 + 2*Sin[(c + d*x)/2]))/(4*(1 - Sin[(c + d*x)/2])*Sqrt[
1 - 2*Sin[(c + d*x)/2]^2]) + ((A - B + C)*Sqrt[1 - 2*Sin[(c + d*x)/2]^2])/(
1 - Sin[(c + d*x)/2]) - ((A - B + C)*Sqrt[1 - 2*Sin[(c + d*x)/2]^2])/(1 + S
in[(c + d*x)/2]) + ((7*A - 3*B - C)*Sin[(c + d*x)/2]*((2*Cos[(c + d*x)/2]^2
*Hypergeometric2F1[2, 5/2, 7/2, -(Sin[(c + d*x)/2]^2/(1 - 2*Sin[(c + d*x)/2
]^2))]*Sin[(c + d*x)/2]^2)/(1 - 2*Sin[(c + d*x)/2]^2) + 5*Csc[(c + d*x)/2]^
4*Sqrt[-(Sin[(c + d*x)/2]^2/(1 - 2*Sin[(c + d*x)/2]^2))]*(1 - 2*Sin[(c + d*
x)/2]^2)^2*(3 - 2*Sin[(c + d*x)/2]^2)*(-ArcTanh[Sqrt[-(Sin[(c + d*x)/2]^2/(
1 - 2*Sin[(c + d*x)/2]^2))]] + Sqrt[-(Sin[(c + d*x)/2]^2/(1 - 2*Sin[(c + d*
x)/2]^2))])))/(10*(1 - 2*Sin[(c + d*x)/2]^2)^(3/2))))/(d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(
3/2))
```

Maple [B] time = 0.306, size = 583, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-A*cos(d*x+c
)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)*sin(d
*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/s
in(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+7*C*cos(d*x+c)*sin(d*x+c)*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-3*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+7*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)^
2-2*B*cos(d*x+c)^2+10*C*cos(d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(d*x+c)-2*C*cos(
d*x+c)-8*C)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.611697, size = 1046, normalized size = 8.72

$$\left[\frac{\sqrt{2}\left((A+3B-7C)\cos(dx+c)^2+2(A+3B-7C)\cos(dx+c)+A+3B-7C\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+ \cdots}{\cos(dx+ \cdots}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((A + 3*B - 7*C)*cos(d*x + c)^2 + 2*(A + 3*B - 7*C)*cos(d*x +
 c) + A + 3*B - 7*C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*co
s(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A - B + 5*C)*c
os(d*x + c) + 4*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((A + 3*
```

```
B - 7*C)*cos(d*x + c)^2 + 2*(A + 3*B - 7*C)*cos(d*x + c) + A + 3*B - 7*C)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 2*((A - B + 5*C)*cos(d*x + c) + 4*C)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a*(sec(c +
d*x) + 1))**(3/2), x)
```

Giac [A] time = 8.98315, size = 271, normalized size = 2.26

$$\frac{\frac{\left(\frac{\sqrt{2}\left(Aa^{2}-Ba^{2}+Ca^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(Aa^{2}-Ba^{2}+9\,Ca^{2}\right)}{a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}+\frac{\sqrt{2}(A+3\,B-7\,C)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*(A*a^2 - B*a^2 + C*a^2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(A*a^2 - B*a^2 + 9*C*a^2)/(a^3*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a) + sqrt(2)*(A + 3*B - 7*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)))/d
```

3.523 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{(5A-B-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - B - 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Tan[c + d*x])/(2*d*(a + a*Se
c[c + d*x])^(3/2))
```

Rubi [A] time = 0.194571, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4052, 3920, 3774, 203, 3795}

$$-\frac{(5A-B-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - B - 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Tan[c + d*x])/(2*d*(a + a*Se
c[c + d*x])^(3/2))
```


Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2aA+\frac{1}{2}a(A-B-3C)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{(5A-B-3C)}{}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} +$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B-3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A}{2d(}$$

Mathematica [C] time = 28.1837, size = 16094, normalized size = 122.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(3/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.249, size = 732, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(4*A*sin(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)+5*A*cos(d*x+c)*sin(d*x+c)*
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*A*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)-B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)-3*C*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+5*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-B*
```

```
ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*C*ln(-(-(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*B*cos(d*x+c)^2-2*C*co
s(d*x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c)+2*C*cos(d*x+c))/(cos(d*x+c)+1)/sin
(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(3/2
), x)
```

Fricas [B] time = 24.7142, size = 1625, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/16*(8*(A - B + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*
sin(d*x + c) - sqrt(2)*((5*A - B - 3*C)*cos(d*x + c)^2 + 2*(5*A - B - 3*C)*
cos(d*x + c) + 5*A - B - 3*C)*sqrt(-a)*log((17*a*cos(d*x + c)^3 + 4*sqrt(2)
*(3*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c) + 3*a*cos(d*x + c)^2 - 13*a*cos(d*x + c) + a)/(cos(d*
x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)) + 8*(A*cos(d*x + c)^2 +
2*A*cos(d*x + c) + A)*sqrt(-a)*log((8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^
2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d), -1/8*(4*(A - B + C)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sqrt(2)*((5*A - B - 3*C)*cos(d*x
 + c)^2 + 2*(5*A - B - 3*C)*cos(d*x + c) + 5*A - B - 3*C)*sqrt(a)*arctan(1/
4*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) - 1)/(sqr
t(a)*sin(d*x + c))) + 8*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt(a)*a
rctan(1/2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) - 1)/(sqr
t(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(
3/2), x)
```

Giac [B] time = 10.9264, size = 448, normalized size = 3.42

$$-\frac{\frac{\sqrt{2}(5A-B-3C)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{8A\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{8A\,\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] -1/8*(sqrt(2)*(5*A - B - 3*C)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
) + 8*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)) - 8*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)) - 2*(sqrt(2)*A*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.524 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{(9A-5B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A-B+C)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\sin}{2d(a\sec(c+dx}$$

```
[Out] -(((3*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*A - 5*B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d*x])/(2*
d*(a + a*Sec[c + d*x])^(3/2)) + ((3*A - B + C)*Sin[c + d*x])/(2*a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rubi [A] time = 0.416723, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4084, 4022, 3920, 3774, 203, 3795}

$$\frac{(9A-5B+C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(3A-B+C)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\sin}{2d(a\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*A - 5*B + C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d*x])/(2*
d*(a + a*Sec[c + d*x])^(3/2)) + ((3*A - B + C)*Sin[c + d*x])/(2*a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(a(3A-B+C)-\frac{1}{2}a(3A-3B-\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B+C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\int}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B+C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(3}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B+C)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(3}{}$$

$$= -\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B+C)\tan^{-1}\left(\right.}{2\sqrt{2}a^{3/2}}$$

Mathematica [A] time = 2.37418, size = 179, normalized size = 1.03

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(2\sin^2\left(\frac{1}{2}(c+dx)\right)(2A\cos(c+dx)+3A-B+C)+\sqrt{2}(9A-5B+C)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+d}\right.}{2ad(\cos(c+dx)-1)\sqrt{a(\sec}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```

```
[Out] -((-4*(3*A - 2*B)*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sqrt[-
1 + Sec[c + d*x]] + Sqrt[2]*(9*A - 5*B + C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/
Sqrt[2]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] + 2*(3*A - B + C + 2*A*
Cos[c + d*x])*Sin[(c + d*x)/2]^2)*Tan[(c + d*x)/2])/(2*a*d*(-1 + Cos[c + d*
x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.352, size = 889, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(-1+cos(d*x+c))*(6*A*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
s(d*x+c))*2^(1/2)*cos(d*x+c)-4*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))+6*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)+9*A*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-4*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-5*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+C*co
s(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+9*A*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*A*cos(d*x+c)^3-5*B*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*B*cos(d*x+c)^2-2*C*cos(d*x+c)^2+6
*A*cos(d*x+c)-2*B*cos(d*x+c)+2*C*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x +
 c) + a)^(3/2), x)
```

Fricas [A] time = 60.3448, size = 1619, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((9*A - 5*B + C)*cos(d*x + c)^2 + 2*(9*A - 5*B + C)*cos(d*x
+ c) + 9*A - 5*B + C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*co
s(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((3*A - 2*B)*cos
(d*x + c)^2 + 2*(3*A - 2*B)*cos(d*x + c) + 3*A - 2*B)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(2*A*cos(d*x
+ c)^2 + (3*A - B + C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4
*(sqrt(2)*((9*A - 5*B + C)*cos(d*x + c)^2 + 2*(9*A - 5*B + C)*cos(d*x + c)
+ 9*A - 5*B + C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 4*((3*A - 2*B)*cos(d*x + c)^2 +
 2*(3*A - 2*B)*cos(d*x + c) + 3*A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*A*cos(d*x
+ c)^2 + (3*A - B + C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\cos\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)/(a*(sec(c +
d*x) + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.525 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=232

$$\frac{(19A-12B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B+2C)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A}{}$$

```
[Out] ((19*A - 12*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*a^(3/2)*d) - ((13*A - 9*B + 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c +
 d*x]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A - 6*B + 2*C)*S
in[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((2*A - B + C)*Cos[c + d*x]
*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.622673, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4022, 3920, 3774, 203, 3795}

$$\frac{(19A-12B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B+5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B+2C)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```


```
[Out] ((19*A - 12*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*a^(3/2)*d) - ((13*A - 9*B + 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c +
 d*x]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A - 6*B + 2*C)*S
in[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((2*A - B + C)*Cos[c + d*x]
*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(2A-\ldots\right.}{\sqrt{\ldots}}}{\ldots}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(2A-B+C)\cos\ldots}{2ad\sqrt{a+a\ldots}}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B+2C)\sin\ldots}{4ad\sqrt{a+a\sec(\ldots}}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B+2C)\sin\ldots}{4ad\sqrt{a+a\sec(\ldots}}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B+2C)\sin\ldots}{4ad\sqrt{a+a\sec(\ldots}}$$

$$= \frac{(19A-12B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B+5C\ldots}{\ldots}$$

Mathematica [C] time = 28.1997, size = 17669, normalized size = 76.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.319, size = 1569, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/16/d/a^2*(-1+cos(d*x+c))*(-12*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*2^(1/2)*sin(d*x+c)+8*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*
sin(d*x+c)+38*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(3/2)+19*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)/cos(d*x+c))-24*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))-8*B*cos(d*x+c)^3+19*A*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*sin(d*x+c)+20*A*cos(d*x+c)^4+8*C*sin(d*x+c)*cos(d*x+c)^2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+8*C*cos(d*x+c)^3-8*C*cos(d*
x+c)^2+10*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+
c)-8*A*cos(d*x+c)^5-16*B*cos(d*x+c)^4+16*A*cos(d*x+c)^3+24*B*cos(d*x+c)^2+1
6*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
+26*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-18*B*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+10*C*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*sin(d*x+c)+26*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos
(d*x+c)^2*sin(d*x+c)-18*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x
+c)^2*sin(d*x+c)-12*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))-28*A*cos(d*x+c)^2+52*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-36*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)-1)/sin(d*x+c))+20*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c)))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(
d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^2/(a*sec(d*x
 + c) + a)^(3/2), x)
```

Fricas [A] time = 118.304, size = 1789, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((13*A - 9*B + 5*C)*cos(d*x + c)^2 + 2*(13*A - 9*B + 5*C)*co
s(d*x + c) + 13*A - 9*B + 5*C)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^
2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A -
 12*B + 8*C)*cos(d*x + c)^2 + 2*(19*A - 12*B + 8*C)*cos(d*x + c) + 19*A - 1
2*B + 8*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(
d*x + c) + 1)) - 2*(2*A*cos(d*x + c)^3 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A
- 6*B + 2*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*(
(13*A - 9*B + 5*C)*cos(d*x + c)^2 + 2*(13*A - 9*B + 5*C)*cos(d*x + c) + 13*
A - 9*B + 5*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((19*A - 12*B + 8*C)*cos(d*x + c)
^2 + 2*(19*A - 12*B + 8*C)*cos(d*x + c) + 19*A - 12*B + 8*C)*sqrt(a)*arctan
(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)
)) + (2*A*cos(d*x + c)^3 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B + 2*C)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.526 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{(47A-38B+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B+9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(21A-14B+12C)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -((47*A - 38*B + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*a^(3/2)*d) + ((17*A - 13*B + 9*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[
c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((21*A - 14*B +
 12*C)*Sin[c + d*x])/(8*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A - 12*B + 6*C
)*Cos[c + d*x]*Sin[c + d*x])/(12*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 3*
B + 3*C)*Cos[c + d*x]^2*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.834081, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4022, 3920, 3774, 203, 3795}

$$-\frac{(47A-38B+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B+9C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(21A-14B+12C)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(3/2),x]
```

```
[Out] -((47*A - 38*B + 24*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*a^(3/2)*d) + ((17*A - 13*B + 9*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[
c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((21*A - 14*B +
 12*C)*Sin[c + d*x])/(8*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A - 12*B + 6*C
)*Cos[c + d*x]*Sin[c + d*x])/(12*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 3*
B + 3*C)*Cos[c + d*x]^2*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\cos^3(c+dx)\left(a(5A-3B+\right.}{\sqrt{a}}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-3B+3C)\cos}{6ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(13A-12B+6C)\,c}{12ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(21A-14B+12C)}{8ad\sqrt{a+a\sec(}}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(21A-14B+12C)}{8ad\sqrt{a+a\sec(}}$$

$$= -\frac{(A-B+C)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(21A-14B+12C)}{8ad\sqrt{a+a\sec(}}$$

$$= -\frac{(47A-38B+24C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A-13B+9}{}$$

Mathematica [A] time = 2.97019, size = 221, normalized size = 0.78

$$\tan\left(\frac{1}{2}(c+dx)\right)\left(-4\sin^2\left(\frac{1}{2}(c+dx)\right)((43A-18B+24C)\cos(c+dx)-3(A-2B)\cos(2(c+dx))+2A\cos(3(c+dx))+6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(3/2),x]
```

```
[Out] ((12*(47*A - 38*B + 24*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Cos[(c + d*x)/2]^
2*Sqrt[-1 + Sec[c + d*x]] - 24*Sqrt[2]*(17*A - 13*B + 9*C)*ArcTan[Sqrt[-1 +
 Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Sqrt[-1 + Sec[c + d*x]] - 4*(60*
A - 36*B + 36*C + (43*A - 18*B + 24*C)*Cos[c + d*x] - 3*(A - 2*B)*Cos[2*(c
+ d*x)] + 2*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]^2)*Tan[(c + d*x)/2])/(48*a
*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.357, size = 2094, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/192/d/a^2*(-1+cos(d*x+c))*(342*B*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-423*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-216*C*cos(d*x+c)^2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+336*B*cos(d*x+c)^3+342*B*cos(d*
x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-216*C*cos
(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+96*C*c
os(d*x+c)^4-204*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+156*
B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-108*C*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+208*A*cos(d*x+c)^4-423*A*2^(1/2)*cos
(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))-288*C*cos(d*x+
c)^3-612*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)
-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c
)+468*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)-3
24*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)-612*
A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)+468*B*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-324*C*ln(-(-(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-112*A*cos(d*x+c)^6+
344*A*cos(d*x+c)^5-240*B*cos(d*x+c)^5-141*A*cos(d*x+c)^3*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+114*B*cos(d*x+c)^3*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-72*C*cos(d*x+c)^3*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+114*B*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
```

```
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-72*C*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+192*C*cos(d*x+c)^5-192*B*cos(d*x+c)^4-50
4*A*cos(d*x+c)^3+64*A*cos(d*x+c)^7+96*B*cos(d*x+c)^6-204*A*cos(d*x+c)^3*sin
(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+156*B*cos(d*x+c)^3*sin(d*
x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/si
n(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-108*C*cos(d*x+c)^3*sin(d*x+c
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-141*A*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/co
s(d*x+c)^2/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^3}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^3/(a*sec(d*x
 + c) + a)^(3/2), x)
```

Fricas [A] time = 118.417, size = 1935, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*sqrt(2)*((17*A - 13*B + 9*C)*cos(d*x + c)^2 + 2*(17*A - 13*B + 9*
C)*cos(d*x + c) + 17*A - 13*B + 9*C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x
+ c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*(
(47*A - 38*B + 24*C)*cos(d*x + c)^2 + 2*(47*A - 38*B + 24*C)*cos(d*x + c) +
 47*A - 38*B + 24*C)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
- a)/(cos(d*x + c) + 1)) - 2*(8*A*cos(d*x + c)^4 - 6*(A - 2*B)*cos(d*x + c)
^3 + (37*A - 18*B + 24*C)*cos(d*x + c)^2 + 3*(21*A - 14*B + 12*C)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/24*(6*sqrt(2)*((17*A - 13*B + 9*C
)*cos(d*x + c)^2 + 2*(17*A - 13*B + 9*C)*cos(d*x + c) + 17*A - 13*B + 9*C)*
sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) - 3*((47*A - 38*B + 24*C)*cos(d*x + c)^2 + 2*(47*A
 - 38*B + 24*C)*cos(d*x + c) + 47*A - 38*B + 24*C)*sqrt(a)*arctan(sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*
cos(d*x + c)^4 - 6*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B + 24*C)*cos(d*x
+ c)^2 + 3*(21*A - 14*B + 12*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
```

```
(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2
*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(3/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/
2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.527 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{(45A-85B+157C)\tan(c+dx)\sec^2(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A-163B+283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(195A-475B+787C)}{}$$

```
[Out] -((75*A - 163*B + 283*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]^4*Tan[c
 + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((5*A - 13*B + 21*C)*Sec[c + d*
x]^3*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((465*A - 985*B +
1729*C)*Tan[c + d*x])/(120*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((45*A - 85*B
+ 157*C)*Sec[c + d*x]^2*Tan[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]]) -
 ((195*A - 475*B + 787*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(240*a^3*d
)
```

Rubi [A] time = 0.901622, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4019, 4021, 4010, 4001, 3795, 203}

$$\frac{(45A-85B+157C)\tan(c+dx)\sec^2(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A-163B+283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(195A-475B+787C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(5/2),x]
```



```
[Out] -((75*A - 163*B + 283*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]^4*Tan[c
 + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((5*A - 13*B + 21*C)*Sec[c + d*
x]^3*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((465*A - 985*B +
1729*C)*Tan[c + d*x])/(120*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((45*A - 85*B
+ 157*C)*Sec[c + d*x]^2*Tan[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]]) -
 ((195*A - 475*B + 787*C)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(240*a^3*d
)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
```

```
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^4(c+dx)\left(4a(B-C)+\right.}{(a+a\,\mathrm{se}}}{}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B+21C)\,\mathrm{s}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B+21C)\,\mathrm{s}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B+21C)\,\mathrm{s}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B+21C)\,\mathrm{s}}{16ad(a+a}$$

$$= -\frac{(A-B+C)\sec^4(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B+21C)\,\mathrm{s}}{16ad(a+a}$$

$$= -\frac{(75A-163B+283C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C}{4d(a}$$

Mathematica [C] time = 25.1746, size = 7237, normalized size = 26.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.421, size = 1439, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/1920/d/a^3*(-1+cos(d*x+c))^2*(-768*C+13720*B*cos(d*x+c)^3+15072*C*cos(d*
x+c)^4+1125*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x
+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-2445*B*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+4245*C*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+4320*A*cos(d*x+c)^4-23896*C*cos(d*x+c)
^3-13824*C*cos(d*x+c)^2+2048*C*cos(d*x+c)+6750*A*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)-14670*B*ln(-(-(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
```

```
x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+25470*C*ln(-(-(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+4500*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-9780*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(5/2)*cos(d*x+c)*sin(d*x+c)+16980*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
5/2)*cos(d*x+c)*sin(d*x+c)+5880*A*cos(d*x+c)^5-11960*B*cos(d*x+c)^5+21368*C
*cos(d*x+c)^5-8160*B*cos(d*x+c)^4-6360*A*cos(d*x+c)^3+7680*B*cos(d*x+c)^2+4
500*A*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-978
0*B*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+16980
*C*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)+4245*C
*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c
)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^4+1125*A*s
in(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^4-2445*B*sin
(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^4-3840*A*cos(d
*x+c)^2-1280*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5
/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.698488, size = 1735, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/960*(15*sqrt(2)*((75*A - 163*B + 283*C)*cos(d*x + c)^5 + 3*(75*A - 163*
B + 283*C)*cos(d*x + c)^4 + 3*(75*A - 163*B + 283*C)*cos(d*x + c)^3 + (75*A
 - 163*B + 283*C)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x +
c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((7
35*A - 1495*B + 2671*C)*cos(d*x + c)^4 + 5*(255*A - 503*B + 911*C)*cos(d*x
+ c)^3 + 32*(15*A - 25*B + 49*C)*cos(d*x + c)^2 + 160*(B - C)*cos(d*x + c)
+ 96*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x
+ c)^2), 1/480*(15*sqrt(2)*((75*A - 163*B + 283*C)*cos(d*x + c)^5 + 3*(75*A
```

```
 - 163*B + 283*C)*cos(d*x + c)^4 + 3*(75*A - 163*B + 283*C)*cos(d*x + c)^3
+ (75*A - 163*B + 283*C)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((735
*A - 1495*B + 2671*C)*cos(d*x + c)^4 + 5*(255*A - 503*B + 911*C)*cos(d*x +
c)^3 + 32*(15*A - 25*B + 49*C)*cos(d*x + c)^2 + 160*(B - C)*cos(d*x + c) +
96*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x +
c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 10.4614, size = 755, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] 1/480*((((15*(2*(sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*
a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c
)^2 - 1))*tan(1/2*d*x + 1/2*c)^2/a^2 + (13*sqrt(2)*A*a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1) - 21*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 29*sqrt(
2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^2)*tan(1/2*d*x + 1/2*c)^2 - (17
25*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 3685*sqrt(2)*B*a^2*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1) + 6733*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2
- 1))/a^2)*tan(1/2*d*x + 1/2*c)^2 + 5*(549*sqrt(2)*A*a^2*sgn(tan(1/2*d*x +
1/2*c)^2 - 1) - 1133*sqrt(2)*B*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 1973*s
qrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^2)*tan(1/2*d*x + 1/2*c)^2 -
 15*(83*sqrt(2)*A*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 155*sqrt(2)*B*a^2*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1) + 291*sqrt(2)*C*a^2*sgn(tan(1/2*d*x + 1/2*c)
^2 - 1))/a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)) - 15*(75*sqrt(2)*A - 163*sqrt(2)*B + 283*sqr
t(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.528 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=227

$$\frac{(19A-75B+163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(15A-39B+95C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{(21A-93B+1\cdots}{24a^2d\sqrt{a\sec\cdots}}$$

```
[Out] ((19*A - 75*B + 163*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]^3*Tan[c +
 d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((A - 9*B + 17*C)*Sec[c + d*x]^2*
Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((21*A - 93*B + 197*C)*
Tan[c + d*x])/(24*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((15*A - 39*B + 95*C)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.695241, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4019, 4010, 4001, 3795, 203}

$$\frac{(19A-75B+163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(15A-39B+95C)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{(21A-93B+1\cdots}{24a^2d\sqrt{a\sec\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(5/2),x]
```


```
[Out] ((19*A - 75*B + 163*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]^3*Tan[c +
 d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((A - 9*B + 17*C)*Sec[c + d*x]^2*
Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((21*A - 93*B + 197*C)*
Tan[c + d*x])/(24*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((15*A - 39*B + 95*C)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(48*a^3*d)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^3(c+dx)\left(a(A+3B-3\right.}{(a+a}}{}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A-9B+17C)\sec}{16ad(a+a\,s}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A-9B+17C)\sec}{16ad(a+a\,s}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A-9B+17C)\sec}{16ad(a+a\,s}$$

$$= -\frac{(A-B+C)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(A-9B+17C)\sec}{16ad(a+a\,s}$$

$$= \frac{(19A-75B+163C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\,s}{4d(a+}$$

Mathematica [C] time = 25.6931, size = 7197, normalized size = 31.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.356, size = 1154, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d/a^3*(-1+cos(d*x+c))^2*(57*A*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)-225*B*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(3/2)+489*C*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(3/2)+171*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d
*x+c)^2*sin(d*x+c)-675*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+
c)^2*sin(d*x+c)+1467*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)
^2*sin(d*x+c)+171*A*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))-675*B*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))+1467*C*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(
-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+
57*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-225*B*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+489*C*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-
1)/sin(d*x+c))*sin(d*x+c)-108*A*cos(d*x+c)^4+588*B*cos(d*x+c)^4-1196*C*cos(
d*x+c)^4-48*A*cos(d*x+c)^3+432*B*cos(d*x+c)^3-816*C*cos(d*x+c)^3+156*A*cos(
d*x+c)^2-636*B*cos(d*x+c)^2+1372*C*cos(d*x+c)^2-384*B*cos(d*x+c)+768*C*cos(
d*x+c)-128*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.650645, size = 1590, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*((19*A - 75*B + 163*C)*cos(d*x + c)^4 + 3*(19*A - 75*B +
 163*C)*cos(d*x + c)^3 + 3*(19*A - 75*B + 163*C)*cos(d*x + c)^2 + (19*A - 7
5*B + 163*C)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2
*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((27*A - 14
7*B + 299*C)*cos(d*x + c)^3 + (39*A - 255*B + 503*C)*cos(d*x + c)^2 - 32*(3
*B - 5*C)*cos(d*x + c) - 32*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + a^3*d*cos(d*x + c)), -1/96*(3*sqrt(2)*((19*A - 75*B + 163*C)*cos(d
*x + c)^4 + 3*(19*A - 75*B + 163*C)*cos(d*x + c)^3 + 3*(19*A - 75*B + 163*C
)*cos(d*x + c)^2 + (19*A - 75*B + 163*C)*cos(d*x + c))*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c))) + 2*((27*A - 147*B + 299*C)*cos(d*x + c)^3 + (39*A - 255*B + 503*C)*co
s(d*x + c)^2 - 32*(3*B - 5*C)*cos(d*x + c) - 32*C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a*(sec(c
 + d*x) + 1))**(5/2), x)
```

Giac [A] time = 10.032, size = 455, normalized size = 2.

$$-\frac{\frac{\left(\left(\left(3\left(\frac{2\sqrt{2}\left(Aa^5 - Ba^5 + Ca^5\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)} + \frac{\sqrt{2}\left(7\,Aa^5 - 15\,Ba^5 + 23\,Ca^5\right)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - \frac{4\sqrt{2}\left(15\,Aa^5 - 75\,Ba^5 + 167\,Ca^5\right)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + \frac{3\sqrt{2}\left(11\,Aa^5 - 83\,Ba^5 + 155\,Ca^5\right)}{a^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)}\right)}{\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^6*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*(7*A*a^5 - 15*B*a^5 + 23*C*a^5)/(
```

```
a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(1
5*A*a^5 - 75*B*a^5 + 167*C*a^5)/(a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(
1/2*d*x + 1/2*c)^2 + 3*sqrt(2)*(11*A*a^5 - 83*B*a^5 + 155*C*a^5)/(a^6*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^
2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*sqrt(2)*(19*A - 75*B + 163*
C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.529 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$\frac{(5A+19B-75C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B+9C)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(3A+\ldots}{16ad\ldots}$$

```
[Out] ((5*A + 19*B - 75*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d
*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((3*A + 5*B - 13*C)*Tan[c + d*x])/(
16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((A - B + 9*C)*Tan[c + d*x])/(4*a^2*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.49072, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4084, 4008, 4001, 3795, 203}

$$\frac{(5A+19B-75C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B+9C)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(3A+\ldots}{16ad\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(5/2),x]
```


```
[Out] ((5*A + 19*B - 75*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]^2*Tan[c + d
*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((3*A + 5*B - 13*C)*Tan[c + d*x])/(
16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((A - B + 9*C)*Tan[c + d*x])/(4*a^2*d*
Sqrt[a + a*Sec[c + d*x]])
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^2(c+dx)\left(2a(A+\right.}{(a}}{}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(3A+5B-13C)}{16ad(a+a\sec}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(3A+5B-13C)}{16ad(a+a\sec}$$

$$= -\frac{(A-B+C)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(3A+5B-13C)}{16ad(a+a\sec}$$

$$= \frac{(5A+19B-75C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)}{4d(a+}$$

Mathematica [C] time = 25.0304, size = 7172, normalized size = 40.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.319, size = 870, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(-5*A*cos
(d*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-19*B*cos(d*x
+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+75*C*cos(d*x+c)^
2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-10*A*cos(d*x+c)*sin(
d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-38*B*cos(d*x+c)*sin(d*x+c)
*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*
x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+150*C*cos(d*x+c)*sin(d*x+c)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*A*cos(d*x+c)^3-5*A*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-18*B*cos(d*x+c)^3-19*B*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+98*C*cos(d*x+c)^3+75*C*ln(-(-(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+72
*C*cos(d*x+c)^2-10*A*cos(d*x+c)+26*B*cos(d*x+c)-106*C*cos(d*x+c)-64*C)/sin(
d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.626159, size = 1365, normalized size = 7.63

$$\left[\frac{\sqrt{2}\left((5\,A+19\,B-75\,C)\cos\left(dx+c\right)^{3}+3\,(5\,A+19\,B-75\,C)\cos\left(dx+c\right)^{2}+3\,(5\,A+19\,B-75\,C)\cos\left(dx+c\right)+5\,A+\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((5*A + 19*B - 75*C)*cos(d*x + c)^3 + 3*(5*A + 19*B - 75*C)*
cos(d*x + c)^2 + 3*(5*A + 19*B - 75*C)*cos(d*x + c) + 5*A + 19*B - 75*C)*sq
rt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A - 9*B + 49*C)*cos(d*x + c)^2 + (5*A
 - 13*B + 85*C)*cos(d*x + c) + 32*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
```

```
s(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A + 19*B - 75*C)*cos(d*x + c)^3 + 3
*(5*A + 19*B - 75*C)*cos(d*x + c)^2 + 3*(5*A + 19*B - 75*C)*cos(d*x + c) +
5*A + 19*B - 75*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((A - 9*B + 49*C)*cos(d*x +
 c)^2 + (5*A - 13*B + 85*C)*cos(d*x + c) + 32*C)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a*(sec(c
 + d*x) + 1))**(5/2), x)
```

Giac [B] time = 9.76439, size = 487, normalized size = 2.72

$$\frac{\left(\left(\left(\frac{2\left(\sqrt{2}Aa^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)-\sqrt{2}Ba^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)+\sqrt{2}Ca^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^8}+\frac{\sqrt{2}Aa^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)-9\sqrt{2}Ba^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}\cdots\right.\right.}{a^8}\cdots}{\sqrt{-a\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] 1/32*(((2*(sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - sqrt(2)*B*a^6*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1) + sqrt(2)*C*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 -
1))*tan(1/2*d*x + 1/2*c)^2/a^8 + (sqrt(2)*A*a^6*sgn(tan(1/2*d*x + 1/2*c)^2
- 1) - 9*sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) + 17*sqrt(2)*C*a^6*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*tan(1/2*d*x + 1/2*c)^2 - (3*sqrt(2)*A*
a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 11*sqrt(2)*B*a^6*sgn(tan(1/2*d*x + 1/
2*c)^2 - 1) + 83*sqrt(2)*C*a^6*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))/a^8)*tan(1/
2*d*x + 1/2*c)/sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a) + (5*sqrt(2)*A + 19*sqrt
(2)*B - 75*sqrt(2)*C)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.530 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{(3A+5B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(7A+B-9C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B+C)\tan(c+dx)\sec(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] ((3*A + 5*B + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x]
)/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((7*A + B - 9*C)*Tan[c + d*x])/(16*a*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.272791, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4078, 4000, 3795, 203}

$$\frac{(3A+5B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(7A+B-9C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B+C)\tan(c+dx)\sec(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B + 19*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c + d*x]*Tan[c + d*x]
)/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((7*A + B - 9*C)*Tan[c + d*x])/(16*a*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4078

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*B - b*C - 2*A*b*(m + 1) - (b*B*(m + 2) -
a*(A*(m + 2) - C*(m - 1)))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \int \frac{\sec(c+dx)\left(a(3A+B-\right.}{(a+a\,\text{s}}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A+B-9C)\tan}{16ad(a+a\sec(c+}$$

$$= -\frac{(A-B+C)\sec(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A+B-9C)\tan}{16ad(a+a\sec(c+}$$

$$= \frac{(3A+5B+19C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\,\text{se}}{4d(a+a}$$

Mathematica [C] time = 24.9314, size = 7163, normalized size = 52.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.292, size = 875, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*A*cos(d*
x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*B*cos(d*x+c)^
2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+19*C*cos(d*x+c)^2*si
n(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*A*cos(d*x+c)*sin(d*x+c
)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+10*B*cos(d*x+c)*sin(d*x+c)*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+38*C*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
```

```
2)*sin(d*x+c)-14*A*cos(d*x+c)^3+5*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-2*B*cos(d*x+c)^3+19*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+18*C*cos(d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+8*C*cos(d
*x+c)^2+6*A*cos(d*x+c)+10*B*cos(d*x+c)-26*C*cos(d*x+c))/(cos(d*x+c)+1)/sin(
d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.62427, size = 1330, normalized size = 9.71

$$\left[-\frac{\sqrt{2}\left((3\,A+5\,B+19\,C)\cos\left(dx+c\right)^3+3\,(3\,A+5\,B+19\,C)\cos\left(dx+c\right)^2+3\,(3\,A+5\,B+19\,C)\cos\left(dx+c\right)+3\,A+5\,\right.}{64\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A + 5*B + 19*C)*cos(d*x + c)^3 + 3*(3*A + 5*B + 19*C)*c
os(d*x + c)^2 + 3*(3*A + 5*B + 19*C)*cos(d*x + c) + 3*A + 5*B + 19*C)*sqrt(
-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) - 4*((7*A + B - 9*C)*cos(d*x + c)^2 + (3*A + 5*
B - 13*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
a^3*d), -1/32*(sqrt(2)*((3*A + 5*B + 19*C)*cos(d*x + c)^3 + 3*(3*A + 5*B +
19*C)*cos(d*x + c)^2 + 3*(3*A + 5*B + 19*C)*cos(d*x + c) + 3*A + 5*B + 19*C
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c))) - 2*((7*A + B - 9*C)*cos(d*x + c)^2 + (3*A + 5*B
 - 13*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a*(sec(c +
d*x) + 1))**(5/2), x)
```

Giac [A] time = 9.41366, size = 277, normalized size = 2.02

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\left(Aa^{5}-Ba^{5}+Ca^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\left(5\,Aa^{5}+3\,Ba^{5}-11\,Ca^{5}\right)}{a^{8}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}(3\,A+5\,B+19\,C)\,\mathrm{lo}}{}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5 + C*a^5
)*tan(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*(5
*A*a^5 + 3*B*a^5 - 11*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2
*d*x + 1/2*c) + sqrt(2)*(3*A + 5*B + 19*C)*log(abs(-sqrt(-a)*tan(1/2*d*x +
1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)))/d
```

3.531 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(43A-3B-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B-5C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B+C)\,\mathrm{ta}}{4d(a\sec(c+d}$$

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 3*B - 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Tan[c + d*x])/(4*d*(a +
a*Sec[c + d*x])^(5/2)) - ((11*A - 3*B - 5*C)*Tan[c + d*x])/(16*a*d*(a + a*S
ec[c + d*x])^(3/2))
```

Rubi [A] time = 0.277868, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4052, 3922, 3920, 3774, 203, 3795}

$$-\frac{(43A-3B-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B-5C)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B+C)\,\mathrm{ta}}{4d(a\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 3*B - 5*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Tan[c + d*x])/(4*d*(a +
a*Sec[c + d*x])^(5/2)) - ((11*A - 3*B - 5*C)*Tan[c + d*x])/(16*a*d*(a + a*S
ec[c + d*x])^(3/2))
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4aA+\frac{1}{2}a(3A-3B-5C)\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B-5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2A-\frac{1}{4}a^2(1 \ldots}{\sqrt{a \ldots}}}{\ldots}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B-5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a \ldots}}{\ldots}$$

$$= -\frac{(A-B+C)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B-5C)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst} \ldots}{\ldots}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B-5C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \ldots$$

Mathematica [C] time = 28.2715, size = 16181, normalized size = 94.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.244, size = 1097, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(32*A*sin(d*x+c)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^2*2^(1/2)+64*A*sin(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)+43*A*cos(d*x+c)^2*si
n(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1
)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*B*cos(d*x+c)^2*sin(d*x
+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5*C*cos(d*x+c)^2*sin(d*x+c)*l
n(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+32*A*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)+86*A*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-6*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-10*C*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)+43*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-30*A
*cos(d*x+c)^3-3*B*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+14*B
*cos(d*x+c)^3-5*C*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos
(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*C*
cos(d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+8*C*cos(d*x+c)^2+22*A*cos(d*
x+c)-6*B*cos(d*x+c)-10*C*cos(d*x+c))/(cos(d*x+c)+1)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 96.8348, size = 1845, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*A - 3*B - 5*C)*cos(d*x + c)^3 + 3*(43*A - 3*B - 5*C)*co
s(d*x + c)^2 + 3*(43*A - 3*B - 5*C)*cos(d*x + c) + 43*A - 3*B - 5*C)*sqrt(-
a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*
A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) -
 a)/(cos(d*x + c) + 1)) - 4*((15*A - 7*B - C)*cos(d*x + c)^2 + (11*A - 3*B
```

```
- 5*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3
*d), 1/32*(sqrt(2)*((43*A - 3*B - 5*C)*cos(d*x + c)^3 + 3*(43*A - 3*B - 5*C
)*cos(d*x + c)^2 + 3*(43*A - 3*B - 5*C)*cos(d*x + c) + 43*A - 3*B - 5*C)*sq
rt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*co
s(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((15*A - 7*B - C)*cos(d*x + c)^2 + (11
*A - 3*B - 5*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{\left(a\left(\sec(c + dx) + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(
5/2), x)
```

Giac [B] time = 11.2594, size = 490, normalized size = 2.87

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\left(\left(\frac{2\sqrt{2}\left(Aa^5 - Ba^5 + Ca^5\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)} - \frac{\sqrt{2}\left(13\,Aa^5 - 5\,Ba^5 - 3\,Ca^5\right)}{a^8\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{\sqrt{2}(43\,A - 3\,B - 5\,C \cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5 + C*
a^5)*tan(1/2*d*x + 1/2*c)^2/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)
*(13*A*a^5 - 5*B*a^5 - 3*C*a^5)/(a^8*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(
1/2*d*x + 1/2*c) + sqrt(2)*(43*A - 3*B - 5*C)*log((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)) + 64*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a^2*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)) - 64*A*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a^2*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.532 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(35A-11B+3C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(115A-43B+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(15A}{16ad(}$$

```
[Out] -(((5*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*A - 43*B + 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x
])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((15*A - 7*B - C)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2)) + ((35*A - 11*B + 3*C)*Sin[c + d*x])/(16*a^2
*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.610402, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4084, 4020, 4022, 3920, 3774, 203, 3795}

$$\frac{(35A-11B+3C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(115A-43B+3C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(15A}{16ad(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c + d
*x])^(5/2),x]
```


```
[Out] -(((5*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*A - 43*B + 3*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x
])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((15*A - 7*B - C)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2)) + ((35*A - 11*B + 3*C)*Sin[c + d*x])/(16*a^2
*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(a(5A-B+C)-\frac{1}{2}a(5A-5B-\cdots\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B-C)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B+3C)\tan\cdots}{16\sqrt{2}\cdots}$$

Mathematica [A] time = 5.28304, size = 181, normalized size = 0.83

$$\frac{-2\tan^3\left(\frac{1}{2}(c+dx)\right)((55A-15B+7C)\cos(c+dx)+8A\cos(2(c+dx))+43A-11B+3C)-\sqrt{2}(115A-43B+3C)\sin}{32a^2d(\cos(c+dx)-1)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```

```
[Out] (32*(5*A - 2*B)*ArcTan[Sqrt[-1 + Sec[c + d*x]]]*Sqrt[-1 + Sec[c + d*x]]*Sin
[c + d*x] - Sqrt[2]*(115*A - 43*B + 3*C)*ArcTan[Sqrt[-1 + Sec[c + d*x]]/Sqr
t[2]]*Sqrt[-1 + Sec[c + d*x]]*Sin[c + d*x] - 2*(43*A - 11*B + 3*C + (55*A -
 15*B + 7*C)*Cos[c + d*x] + 8*A*Cos[2*(c + d*x)])*Tan[(c + d*x)/2]^3)/(32*a
^2*d*(-1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.39, size = 1338, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(115*A*cos(d*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)-43*B*cos(d*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+3*C*cos(d*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+230*A*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-86*B*cos(d*x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+115*A*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-43*B*ln(-
(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*C*ln(-(-(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+30*B*cos(d*x+c)^3+70*A*cos(d*x+c)+160*A*sin(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)+80*A*si
n(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^2*2^(1/2)-32*
A*cos(d*x+c)^4-14*C*cos(d*x+c)^3+8*C*cos(d*x+c)^2+6*C*cos(d*x+c)+6*C*cos(d*
x+c)*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-78*A*cos(d*x+c)^3
-8*B*cos(d*x+c)^2-64*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))+80*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
sin(d*x+c)-32*B*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*2^(1/2)+40*A*cos(d*x+c)^2-32*B*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-22*B*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2
```

```
)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 129.546, size = 2043, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((115*A - 43*B + 3*C)*cos(d*x + c)^3 + 3*(115*A - 43*B + 3*
C)*cos(d*x + c)^2 + 3*(115*A - 43*B + 3*C)*cos(d*x + c) + 115*A - 43*B + 3*
C)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*((5*A - 2*B)*cos(d*x + c)^3 + 3*(5
*A - 2*B)*cos(d*x + c)^2 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B)*sqrt(-a)
*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*
(16*A*cos(d*x + c)^3 + (55*A - 15*B + 7*C)*cos(d*x + c)^2 + (35*A - 11*B +
3*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
), -1/32*(sqrt(2)*((115*A - 43*B + 3*C)*cos(d*x + c)^3 + 3*(115*A - 43*B +
3*C)*cos(d*x + c)^2 + 3*(115*A - 43*B + 3*C)*cos(d*x + c) + 115*A - 43*B +
3*C)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - 32*((5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2
*B)*cos(d*x + c)^2 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B)*sqrt(a)*arctan
(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)
)) - 2*(16*A*cos(d*x + c)^3 + (55*A - 15*B + 7*C)*cos(d*x + c)^2 + (35*A -
11*B + 3*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.533 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$-\frac{(63A-35B+11C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(39A-20B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B+43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{2}\sqrt{a\sec(c}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] ((39*A - 20*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*a^(5/2)*d) - ((219*A - 115*B + 43*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Co
s[c + d*x]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A - 11*B +
 3*C)*Cos[c + d*x]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((63
*A - 35*B + 11*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*
A - 15*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x
]])
```

Rubi [A] time = 0.868814, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4020, 4022, 3920, 3774, 203, 3795}

$$-\frac{(63A-35B+11C)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(39A-20B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B+43C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{2}\sqrt{a\sec(c}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec[c +
 d*x])^(5/2),x]
```



```
[Out] ((39*A - 20*B + 8*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*a^(5/2)*d) - ((219*A - 115*B + 43*C)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Co
s[c + d*x]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A - 11*B +
 3*C)*Cos[c + d*x]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((63
*A - 35*B + 11*C)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*
A - 15*B + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x
]])
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
```

```
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(3A-\right.}{(a+}}{}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B+3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B+3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B+3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B+3C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B+3C)}{16ad(a+}$$

$$= \frac{(39A-20B+8C)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B}{}$$

Mathematica [C] time = 28.4276, size = 17747, normalized size = 63.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*S
ec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.367, size = 2096, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/64/d/a^3*(-1+cos(d*x+c))^2*(80*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*2^(1/2)*sin(d*x+c)-32*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2
)*sin(d*x+c)-468*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)-468*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))+80*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x
+c)^3*sin(d*x+c)*2^(1/2)+240*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)
```

```
/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))-80*B*cos(d*x+c)^3-60*C*cos(d*x+c)^4-156*A*2^(1/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-300*A*cos(d*x+c)^4-
96*C*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))+16*C*cos(d*x+c)^3+44*C*cos(d*x+c)^2-129*C*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)
/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+32*A*cos(d*x+c)^6-112*A*cos(d*x+c)^5+6
4*B*cos(d*x+c)^5+156*B*cos(d*x+c)^4+128*A*cos(d*x+c)^3-140*B*cos(d*x+c)^2-1
56*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)*2^(1
/2)-96*C*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))-219*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+115*B*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-43*C*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d
*x+c)-1)/sin(d*x+c))*sin(d*x+c)-219*A*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2)-657*A*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(
-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c)
)*cos(d*x+c)^2*sin(d*x+c)+345*B*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*c
os(d*x+c)^2*sin(d*x+c)+240*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))-32*C*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+252*A*cos(d*x+c)^2-657*A*cos(d*x+c)*sin(d*x
+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+345*B*cos(d*x+c)*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))-129*C*cos(d*x+c)*sin(d*x+c)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(3/2)*ln(-(-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))+115*B*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(3/2)-43*C*sin(d*x+c)*cos(d*x+c)^3*ln(-(-(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(3/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c
)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.534 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=217

$$\frac{2a(7A+7B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(7A+7B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a}{}$$

```
[Out] (-2*a*(5*A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(7*A + 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*A + 3*(B + C))*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(7*A + 7*B + 5*C)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(21*d) + (2*a*(B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
+ (2*a*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.256471, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a(7A+7B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5A+3(B+C))\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(7A+7B+5C)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (-2*a*(5*A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(7*A + 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*A + 3*(B + C))*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(7*A + 7*B + 5*C)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(21*d) + (2*a*(B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
+ (2*a*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2aC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \mathrm{se}$$

$$= \frac{2aC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \mathrm{se}$$

$$= \frac{2a(7A+7B+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{21d}$$

$$= \frac{2a(5A+3(B+C))\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= \frac{2a(7A+7B+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{21d}$$

$$= -\frac{2a(5A+3(B+C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.}{5d}$$

Mathematica [C] time = 6.69553, size = 527, normalized size = 2.43

$$\frac{2a\csc(c)e^{-idx}\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)e^{2idx}(5A+3(B+C))\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*a*Cos[c + d*x]^2*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(7*Sqrt[
2]*(5*A + 3*(B + C))*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(35*A*(1 + E^((2
*I)*(c + d*x)))^2*(-1 + 3*E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + 3*E^((3*I
)*(c + d*x))) + 7*B*(-5 + 3*E^(I*(c + d*x)) - 5*E^((2*I)*(c + d*x)) + 27*E^
((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 33*E^((5*I)*(c + d*x)) + 5*E^((
6*I)*(c + d*x)) + 9*E^((7*I)*(c + d*x))) + C*(-25 + 21*E^(I*(c + d*x)) - 85
*E^((2*I)*(c + d*x)) + 189*E^((3*I)*(c + d*x)) + 85*E^((4*I)*(c + d*x)) + 2
31*E^((5*I)*(c + d*x)) + 25*E^((6*I)*(c + d*x)) + 63*E^((7*I)*(c + d*x))))*
Sqrt[Sec[c + d*x]])/(E^(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3) + 10*(7*A
 + 7*B + 5*C)*E^(I*d*x)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]]*Sin[c]))/(105*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*(c + d*x)]))
```

Maple [B] time = 8.038, size = 850, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/5*(1/2*C+1
/2*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4
-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin
(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*(1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+2*C*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca\sec\left(dx+c\right)^4+(B+C)a\sec\left(dx+c\right)^3+(A+B)a\sec\left(dx+c\right)^2+Aa\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^4 + (B + C)*a*sec(d*x + c)^3 + (A + B)*a*sec(d*x
 + c)^2 + A*a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

3.535 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=181

$$\frac{2a(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(5A+5B+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2a(5}{}$$

```
[Out] (-2*a*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 5*B + 3*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3*d) + (2*a*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.22118, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a(5A+5B+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+5B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```



```
[Out] (-2*a*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*a*(3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 5*B + 3*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3*d) + (2*a*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2aC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{}$$

$$= \frac{2aC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{}$$

$$= \frac{2a(5A+5B+3C)\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= \frac{2a(5A+5B+3C)\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= -\frac{2a(5A+5B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.}{5d}$$

Mathematica [C] time = 6.26046, size = 366, normalized size = 2.02

$$\frac{2ae^{-ic}\left(-1+e^{2ic}\right)\csc(c)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left((5A+5B+3C)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeomet}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*a*(-1 + E^((2*I)*c))*Csc[c]*(5*B + 5*C - 15*A*E^(I*(c + d*x)) - 15*B*E^(
I*(c + d*x)) - 3*C*E^(I*(c + d*x)) - 30*A*E^((3*I)*(c + d*x)) - 30*B*E^((3*
I)*(c + d*x)) - 24*C*E^((3*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 5*C*E^
((4*I)*(c + d*x)) - 15*A*E^((5*I)*(c + d*x)) - 15*B*E^((5*I)*(c + d*x)) - 9
*C*E^((5*I)*(c + d*x)) - (5*I)*(3*A + B + C)*(1 + E^((2*I)*(c + d*x)))^2*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 5*B + 3*C)*E^(I*(c + d*
```

```
x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*c)*(1
+ E^((2*I)*(c + d*x)))^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*
Sec[c + d*x]^(3/2))
```

Maple [B] time = 6.857, size = 741, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*C
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*(1/2*C+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+4*(1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(a\sec\left(dx+c\right) + a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Ca\sec\left(dx+c\right)^3 + (B+C)a\sec\left(dx+c\right)^2 + (A+B)a\sec\left(dx+c\right) + Aa\right)\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^2 + B \sec \left(dx+c\right) + A\right)\left(a \sec \left(dx+c\right) + a\right)\sqrt{\sec \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

3.536 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=143

$$\frac{2a(3A+3B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}$$

```
[Out] (2*a*(A - B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a*(3*A + 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/d + (2*a*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.219038, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a(3A+3B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[Sec
[c + d*x]],x]
```

```
[Out] (2*a*(A - B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a*(3*A + 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/d + (2*a*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{1}{2}a(3A+$$

$$= \frac{2aC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{3}{2}a(B+}{\sqrt{\sec(}}$$

$$= \frac{2a(B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2aC\sec^{\frac{3}{2}}(c+}{3}$$

$$= \frac{2a(3A+3B+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3d}$$

$$= \frac{2a(A-B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

Mathematica [C] time = 1.84693, size = 208, normalized size = 1.45

$$\frac{ae^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-i(A-B-C)\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((3*I)*A - (3*I)*B - (3*I)*C
+ (3*I)*A*Cos[2*(c + d*x)] - (3*I)*B*Cos[2*(c + d*x)] - (3*I)*C*Cos[2*(c +
d*x)] + 2*(3*A + 3*B + C)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - I*
(A - B - C)*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))] + 2*C*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + 3*C*Sin
[2*(c + d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 5.522, size = 516, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+4*(1/2*C+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right)\left(a \sec \left(dx+c\right)+a\right)}{\sqrt{\sec \left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca \sec \left(dx+c\right)^{3}+\left(B+C\right)a \sec \left(dx+c\right)^{2}+\left(A+B\right)a \sec \left(dx+c\right)+Aa}{\sqrt{\sec \left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

3.537 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{2a(A+3(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}$$

```
[Out] (2*a*(A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a*(A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
 + (2*a*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.209211, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a(A+3(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(3/2),x]
```

```
[Out] (2*a*(A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a*(A + 3*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
 + (2*a*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{1}{2}a(A+3(B+$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{3}{2}aC\sec^2(c+}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2aC\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+($$

$$= \frac{2a(A+3(B+C))\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3d}$$

$$= \frac{2a(A+B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

Mathematica [C] time = 1.8731, size = 183, normalized size = 1.33

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2i(A+B-C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((6*I)*A*Cos[c + d*x] + (6*I)
*B*Cos[c + d*x] - (6*I)*C*Cos[c + d*x] + 2*(A + 3*(B + C))*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - (2*I)*(A + B - C)*E^(I*(c + d*x))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
+ 6*C*Sin[c + d*x] + A*Sin[2*(c + d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 2.306, size = 380, normalized size = 2.8

$$-\frac{2a}{3d}\left(4A\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\cos\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*a*(4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*
A+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)*C)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+(B+C)a\sec\left(dx+c\right)^{2}+(A+B)a\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{A}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{A}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{B}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int B\sqrt{\sec\left(c+dx\right)}\,dx+\int C\sqrt{\sec\left(c+dx\right)}\,dx+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/
2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) +
 Integral(B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x) + In
tegral(C*sqrt(sec(c + d*x)), x) + Integral(C*sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

3.538 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=146

$$\frac{2a(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(3A+5(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{5d}$$

```
[Out] (2*a*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)) + (2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.216795, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(5/2),x]
```

```
[Out] (2*a*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)) + (2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{1}{2}a(3A+5(B$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{5}{2}aC\sec^2(c+}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}(a(A+$$

$$= \frac{2a(3A+5(B+C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{5d}$$

$$= \frac{2a(3A+5(B+C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{5d}$$

Mathematica [C] time = 1.80058, size = 177, normalized size = 1.21

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-4i(3A+5(B+C))e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(A + B + 3*C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (4*I)*(3*A + 5*(B + C))*E^(I*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))] + 2*Cos[c + d*x]*((6*I)*(3*A + 5*(B + C)) + 10*(A + B)*Sin[c + d
*x] + 3*A*Sin[2*(c + d*x)])))/(30*d*E^(I*d*x))
```

Maple [B] time = 2.182, size = 447, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-16*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+(B+C)a\sec\left(dx+c\right)^{2}+(A+B)a\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{B}{\sqrt{\sec(c+dx)}}\,dx+\int\frac{C}{\sqrt{\sec(c+dx)}}\,dx+\int C\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/
2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(A/sec(c + d*x)**(3/2), x)
+ Integral(B/sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)), x) +
Integral(C/sqrt(sec(c + d*x)), x) + Integral(C*sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(
d*x + c)^(5/2), x)
```

3.539 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{2a(5A+7(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(5A+7(B+C))\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(3(A+B)+$$

```
[Out] (2*a*(3*(A + B) + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(5*A + 7*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c
+ d*x]^(5/2)) + (2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*
(5*A + 7*(B + C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.24431, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a(5A+7(B+C))\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(3(A+B)+5C)\sqrt{c$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(7/2),x]
```

```
[Out] (2*a*(3*(A + B) + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*a*(5*A + 7*(B + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c
+ d*x]^(5/2)) + (2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*
(5*A + 7*(B + C))*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B) - \frac{1}{2}a(5A+7(B\cdots}{\sec\cdots}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B) - \frac{7}{2}aC\sec^2(c+\cdots}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5A\cdots}{2\cdots}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5A\cdots}{2\cdots}$$

$$= \frac{2a(3(A+B)+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}\cdots}}{5d}$$

Mathematica [C] time = 2.20212, size = 201, normalized size = 1.1

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56i(3A+3B+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(\cdots}\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(5*A + 7*(B + C))*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(3*A + 3*B + 5*C)*E^(I*(c +
d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))] + 2*Cos[c + d*x]*((84*I)*(3*A + 3*B + 5*C) + 5*(23*A + 28*(
```

```
B + C))*Sin[c + d*x] + 42*(A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])
))/(420*d*E^(I*d*x))
```

Maple [B] time = 2.415, size = 481, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(448*A+308*B+140*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-122*A-112*B-70*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+(B+C)a\sec\left(dx+c\right)^{2}+(A+B)a\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(7/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(
d*x + c)^(7/2), x)
```

3.540 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=215

$$\frac{2a(5(A+B)+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(7A+9(B+C))\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5(A+B)+\ldots}{21d\sqrt{\text{se}\ldots}}$$

```
[Out] (2*a*(7*A + 9*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*a*(5*(A + B) + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(9*d*Sec[c
 + d*x]^(7/2)) + (2*a*(A + B)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a
*(7*A + 9*(B + C))*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(5*(A + B
) + 7*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.282104, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4074, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a(7A+9(B+C))\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5(A+B)+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5(A+B)+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\ldots\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(9/2),x]
```


```
[Out] (2*a*(7*A + 9*(B + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*a*(5*(A + B) + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(9*d*Sec[c
 + d*x]^(7/2)) + (2*a*(A + B)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a
*(7*A + 9*(B + C))*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(5*(A + B
) + 7*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}-\frac{2}{9}\int \frac{-\frac{9}{2}a(A+B)-\frac{1}{2}a(7A+9(B \\
&= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}-\frac{2}{9}\int \frac{-\frac{9}{2}a(A+B)-\frac{9}{2}aC\sec^2(c+}{\sec^{\frac{7}{2}}(c+dx)} \\
&= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(7A}{4} \\
&= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(7A}{4} \\
&= \frac{2a(7A+9(B+C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{15d} \\
&= \frac{2a(7A+9(B+C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{15d}
\end{aligned}$$

Mathematica [C] time = 2.51103, size = 231, normalized size = 1.07

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(7A+9(B+C))e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(9/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(5*A + 5*B + 7*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(7*A + 9*(B + C))*E^(I*(c
+ d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))] + 2*Cos[c + d*x]*((1176*I)*A + (1512*I)*B + (1512*I)*C +
30*(23*A + 23*B + 28*C)*Sin[c + d*x] + 14*(19*A + 18*(B + C))*Sin[2*(c + d*
x)] + 90*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 35*A*Sin[4*(c + d*x)]
)))/(2520*d*E^(I*d*x))
```

Maple [B] time = 2.228, size = 512, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2960*A+720*B)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-3152*A-1584*B-504*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(1792*A+1344*B+924*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-408
*A-366*B-336*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+(B+C)a\sec\left(dx+c\right)^{2}+(A+B)a\sec\left(dx+c\right)+Aa}{\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(9/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(
d*x + c)^(9/2), x)
```

3.541 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=291

$$\frac{4a^2(7A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(21A+27B+19C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}$$

```
[Out] (-4*a^2*(12*A + 9*B + 8*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (4*a^2*(7*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(12*A + 9*B + 8*C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^2*(7*A + 6*B + 5*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(21*A + 27*B + 19*C)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(105*d) + (2*C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])
^2*Sin[c + d*x])/(9*d) + (2*(9*B + 4*C)*Sec[c + d*x]^(5/2)*(a^2 + a^2*Sec[c
 + d*x])*Sin[c + d*x])/(63*d)
```

Rubi [A] time = 0.495369, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4088, 4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a^2(21A+27B+19C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^2(7A+6B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{4a^2(12A+9B+8C)\text{s}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-4*a^2*(12*A + 9*B + 8*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (4*a^2*(7*A + 6*B + 5*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(12*A + 9*B + 8*C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^2*(7*A + 6*B + 5*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(21*A + 27*B + 19*C)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(105*d) + (2*C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])
^2*Sin[c + d*x])/(9*d) + (2*(9*B + 4*C)*Sec[c + d*x]^(5/2)*(a^2 + a^2*Sec[c
 + d*x])*Sin[c + d*x])/(63*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c}{9d}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c}{9d}$$

$$= \frac{2a^2(21A+27B+19C)\sec^{\frac{5}{2}}(c+dx)\sin(c}{105d}$$

$$= \frac{2a^2(21A+27B+19C)\sec^{\frac{5}{2}}(c+dx)\sin(c}{105d}$$

$$= \frac{4a^2(12A+9B+8C)\sqrt{\sec(c+dx)}\sin(c+}{15d}$$

$$= \frac{4a^2(12A+9B+8C)\sqrt{\sec(c+dx)}\sin(c+}{15d}$$

$$= -\frac{4a^2(12A+9B+8C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{15d}$$

Mathematica [C] time = 7.14858, size = 1270, normalized size = 4.36

$$\frac{4\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4(c+dx)\csc(c)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1}\right.}{15d(\cos(2c+2dx)A+A+2C+2B\cos(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]
*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*
(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4,
7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + (8*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sq
rt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec
[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(45*d*E^(I*d*x)*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (2*A*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2))/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*Sec[c + d*x]^(7/2)) + (4*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2))/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
Sec[c + d*x]^(7/2)) + (10*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2))/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x
]^(7/2)) + (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]
```

```
 + C*Sec[c + d*x]^2)*((2*(12*A + 9*B + 8*C)*Cos[d*x]*Csc[c])/(15*d) + (C*Se
c[c]*Sec[c + d*x]^4*Sin[d*x])/(9*d) + (Sec[c]*Sec[c + d*x]^3*(7*C*Sin[c] +
9*B*Sin[d*x] + 18*C*Sin[d*x]))/(63*d) + (Sec[c]*Sec[c + d*x]^2*(45*B*Sin[c]
 + 90*C*Sin[c] + 63*A*Sin[d*x] + 126*B*Sin[d*x] + 112*C*Sin[d*x]))/(315*d)
+ (Sec[c]*Sec[c + d*x]*(63*A*Sin[c] + 126*B*Sin[c] + 112*C*Sin[c] + 210*A*S
in[d*x] + 180*B*Sin[d*x] + 150*C*Sin[d*x]))/(315*d) + (2*(7*A + 6*B + 5*C)*
Tan[c])/(21*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c +
 d*x]^(7/2))
```

Maple [B] time = 10.199, size = 1183, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*(1/4*B+1/
2*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*C*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-1
4/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*
sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))-8/5*(1/4*A+1/2*B+1/4*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*(1/2*A+1/
4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\sec(dx+c)^5 + (B+2C)a^2\sec(dx+c)^4 + (A+2B+C)a^2\sec(dx+c)^3 + (2A+B)a^2\sec(dx+c)^2 + Aa\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*a^2*sec(d*x + c)^5 + (B + 2*C)*a^2*sec(d*x + c)^4 + (A + 2*B + C)*a^2*sec(d*x + c)^3 + (2*A + B)*a^2*sec(d*x + c)^2 + A*a^2*sec(d*x + c))*sqrt(sec(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)

3.542 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=255

$$\frac{4a^2(14A+7B+6C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(35A+49B+33C)\sin(c+dx)\sec^{\frac{3}{2}}(c-}{105d}$$

```
[Out] (-4*a^2*(5*A + 4*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^2*(14*A + 7*B + 6*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(5*A + 4*B + 3*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^2*(35*A + 49*B + 33*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(7*d) + (2*(7*B + 4*C)*Sec[c + d*x]^(3/2)*(a^2 + a^2*
Sec[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.505565, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4088, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a^2(35A+49B+33C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(5A+4B+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(14A+7B+6C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-4*a^2*(5*A + 4*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^2*(14*A + 7*B + 6*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(5*A + 4*B + 3*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^2*(35*A + 49*B + 33*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(7*d) + (2*(7*B + 4*C)*Sec[c + d*x]^(3/2)*(a^2 + a^2*
Sec[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin}{7d}$$

$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin}{7d}$$

$$= \frac{2a^2(35A+49B+33C)\sec^{\frac{3}{2}}(c+dx)\sin}{105d}$$

$$= \frac{2a^2(35A+49B+33C)\sec^{\frac{3}{2}}(c+dx)\sin}{105d}$$

$$= \frac{4a^2(5A+4B+3C)\sqrt{\sec(c+dx)}\sin(}{5d}$$

$$= \frac{4a^2(14A+7B+6C)\sqrt{\cos(c+dx)}F\left(\right.}{21d}$$

$$= -\frac{4a^2(5A+4B+3C)\sqrt{\cos(c+dx)}E\left(\right.}{5d}$$

Mathematica [C] time = 7.00799, size = 1216, normalized size = 4.77

$$\frac{\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^4(c+dx)\csc(c)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{}\right.}{3d(\cos(2c+2dx)A+A+2C+2B\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(
(2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])) + (4*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*
Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(
c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7
/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + (Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt
[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
 + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*A*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2))/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*Sec[c + d*x]^(7/2)) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2))/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec
[c + d*x]^(7/2)) + (4*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/
2)) + (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*
```

```
Sec[c + d*x]^2)*((2*(5*A + 4*B + 3*C)*Cos[d*x]*Csc[c])/(5*d) + (C*Sec[c]*Se
c[c + d*x]^3*Sin[d*x])/(7*d) + (Sec[c]*Sec[c + d*x]^2*(5*C*Sin[c] + 7*B*Sin
[d*x] + 14*C*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]*(21*B*Sin[c] + 42*C*S
in[c] + 35*A*Sin[d*x] + 70*B*Sin[d*x] + 60*C*Sin[d*x]))/(105*d) + ((7*A + 1
4*B + 12*C)*Tan[c])/(21*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 9., size = 934, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-
8/5*(1/4*B+1/2*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*(1/4*A+1/2*B+1/4*C)*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8*(1/2*A+1/4*B)*(-(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\sec(dx+c)^4+(B+2C)a^2\sec(dx+c)^3+(A+2B+C)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)*sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sq
rt(sec(d*x + c)), x)
```

3.543 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{4a^2(3A+2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(15A+25B+17C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}$$

```
[Out] (-4*a^2*(5*B + 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(3*A + 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(15*A + 25*B + 17*C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*
x])^2*Sin[c + d*x])/(5*d) + (2*(5*B + 4*C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Se
c[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.453359, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4088, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(15A+25B+17C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(3A+2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5B$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```

```
[Out] (-4*a^2*(5*B + 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(3*A + 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(15*A + 25*B + 17*C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*
x])^2*Sin[c + d*x])/(5*d) + (2*(5*B + 4*C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Se
c[c + d*x])*Sin[c + d*x])/(15*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d}+$$

$$= \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\sin(c+dx)}{5d}+$$

$$= \frac{2a^2(15A+25B+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+$$

$$= \frac{2a^2(15A+25B+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+$$

$$= \frac{2a^2(15A+25B+17C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+$$

$$= -\frac{4a^2(5B+4C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

Mathematica [C] time = 3.50829, size = 265, normalized size = 1.24

$$a^2e^{-idx}\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(40(3A+2B+C)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2i(5B+4C)e^{-i(c+d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-90*I)*B*Cos[c + d*x] - (
72*I)*C*Cos[c + d*x] - (30*I)*B*Cos[3*(c + d*x)] - (24*I)*C*Cos[3*(c + d*x)
] + 40*(3*A + 2*B + C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + ((2*I
)*(5*B + 4*C)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7
/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 15*A*Sin[c + d*x] + 30*B*Sin[c
 + d*x] + 36*C*Sin[c + d*x] + 10*B*Sin[2*(c + d*x)] + 20*C*Sin[2*(c + d*x)]
 + 15*A*Sin[3*(c + d*x)] + 30*B*Sin[3*(c + d*x)] + 24*C*Sin[3*(c + d*x)]))/
(30*d*E^(I*d*x))
```

Maple [B] time = 6.886, size = 908, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*C/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+8*(1/4*B+1/2*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8*(1/4
*A+1/2*B+1/4*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2 \sec(dx+c)^4 + (B+2C)a^2 \sec(dx+c)^3 + (A+2B+C)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + A}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sq
rt(sec(d*x + c)), x)
```

3.544 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=208

$$\frac{4a^2(2A+3B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a^2(A-3B-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}-2$$

```
[Out] (4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (4*a^2*(2*A + 3*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(A - 3*B - 5*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(3*d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Se
c[c + d*x]]) - (2*(A - C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.445799, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4086, 4018, 3997, 3787, 3771, 2639, 2641}

$$-\frac{2a^2(A-3B-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(A-C}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(3/2),x]
```


```
[Out] (4*a^2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (4*a^2*(2*A + 3*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(A - 3*B - 5*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(3*d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Se
c[c + d*x]]) - (2*(A - C)*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c
 + d*x])/(3*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(A-C)\sqrt{\mathrm{se}}}{}$$

$$= -\frac{2a^2(A-3B-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= -\frac{2a^2(A-3B-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= -\frac{2a^2(A-3B-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= \frac{4a^2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{d}$$

Mathematica [C] time = 2.45074, size = 209, normalized size = 1.

$$a^2e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(8(2A+3B+2C)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-4i(A-C)\left(1+e^{2i(}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((12*I)*A - (12*I)*C + (12*
I)*A*Cos[2*(c + d*x)] - (12*I)*C*Cos[2*(c + d*x)] + 8*(2*A + 3*B + 2*C)*Cos
[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - (4*I)*(A - C)*(1 + E^((2*I)*(c
+ d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + A*S
in[c + d*x] + 4*C*Sin[c + d*x] + 6*B*Sin[2*(c + d*x)] + 12*C*Sin[2*(c + d*x
)] + A*Sin[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

Maple [B] time = 6.223, size = 801, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-4*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+4*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*sin(1/2*d*x+1/2*c)^2-4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+12*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A+3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-3*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4+(B+2C)a^2\sec(dx+c)^3+(A+2B+C)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/se
c(d*x + c)^(3/2), x)
```

3.545 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{4a^2(A+2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a^2(7A+5B-15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+$$

```
[Out] (4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(A + 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(7*A + 5*B - 15*C)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (2*(4*A + 5*B)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*
x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.493026, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4086, 4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{2a^2(7A+5B-15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(A+2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(4A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(5/2),x]
```


```
[Out] (4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(A + 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*a^2*(7*A + 5*B - 15*C)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (2*(4*A + 5*B)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*
x])/(15*d*Sqrt[Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+\ldots}{\ldots}}{\ldots}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(4A+5B)\left(\ldots\right.}{\ldots}$$

$$= -\frac{2a^2(7A+5B-15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \ldots$$

$$= -\frac{2a^2(7A+5B-15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \ldots$$

$$= -\frac{2a^2(7A+5B-15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \ldots$$

$$= \frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c\ldots}}{5d}$$

Mathematica [C] time = 2.06259, size = 187, normalized size = 0.87

$$a^2\sqrt{\sec(c+dx)}\left(40(A+2B+3C)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-8i(4A+5B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeo}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*((96*I)*A*Cos[c + d*x] + (120*I)*B*Cos[c + d*x] + 4
0*(A + 2*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (8*I)*(4*A
 + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x] + 60*C*Sin[c + d*x] +
20*A*Sin[2*(c + d*x)] + 10*B*Sin[2*(c + d*x)] + 3*A*Sin[3*(c + d*x)]))/(30*
d)
```

Maple [B] time = 2.448, size = 595, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*a^2*(-12*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(16*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+5*B+15*C)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+5*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4 + (B+2C)a^2\sec(dx+c)^3 + (A+2B+C)a^2\sec(dx+c)^2 + (2A+B)a^2\sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(5
/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{2A}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{2B}{\sqrt{\sec(c+dx)}}\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(5/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(2*A/sec(c + d*x)**(3/2)
, x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(B/sec(c + d*x)**(3/2),
x) + Integral(2*B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x
) + Integral(C/sqrt(sec(c + d*x)), x) + Integral(2*C*sqrt(sec(c + d*x)), x)
 + Integral(C*sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/se
c(d*x + c)^(5/2), x)
```

3.546 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=219

$$\frac{4a^2(6A+7B+14C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^2(3A}{}$$

```
[Out] (4*a^2*(3*A + 4*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^2*(6*A + 7*B + 14*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(33*A + 49*B + 35*C)
*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(4*A + 7*B)*(a^2 + a^2*Sec[c + d*x
])*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.496946, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4086, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^2(6A+7B+14C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+4B+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(7/2),x]
```


```
[Out] (4*a^2*(3*A + 4*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^2*(6*A + 7*B + 14*C)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(33*A + 49*B + 35*C)
*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(4*A + 7*B)*(a^2 + a^2*Sec[c + d*x
])*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(4A+7B)\left(\right.}{}$$

$$= \frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec}{7d}$$

$$= \frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec}{7d}$$

$$= \frac{2a^2(33A+49B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec}{7d}$$

$$= \frac{4a^2(3A+4B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}$$

Mathematica [C] time = 2.25534, size = 189, normalized size = 0.86

$$a^2\sqrt{\sec(c+dx)}\left(-112i(3A+4B+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+80(6A+7(B+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(80*(6*A + 7*(B + 2*C))*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2] - (112*I)*(3*A + 4*B + 5*C)*E^(I*(c + d*x))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2
*Cos[c + d*x]*((504*I)*A + (672*I)*B + (840*I)*C + 5*(51*A + 28*(2*B + C))*
Sin[c + d*x] + 42*(2*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(42
0*d)
```

Maple [A] time = 2.378, size = 483, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(378*A+224*B+70*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-117*A-91*B-35*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-84*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+70*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2 \sec(dx+c)^4 + (B+2C)a^2 \sec(dx+c)^3 + (A+2B+C)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(7
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/se
c(d*x + c)^(7/2), x)
```

3.547 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=255

$$\frac{4a^2(5A+6B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+}{21}$$

```
[Out] (4*a^2*(8*A + 9*B + 12*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(15*d) + (4*a^2*(5*A + 6*B + 7*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(19*A + 27*B + 21*C
)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (4*a^2*(5*A + 6*B + 7*C)*Sin[c
 + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*
x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(4*A + 9*B)*(a^2 + a^2*Sec[c + d*x])*Sin[
c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.501663, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4086, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+6B+7C)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(5A+6B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F(}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(9/2),x]
```


```
[Out] (4*a^2*(8*A + 9*B + 12*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(15*d) + (4*a^2*(5*A + 6*B + 7*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(19*A + 27*B + 21*C
)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (4*a^2*(5*A + 6*B + 7*C)*Sin[c
 + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*
x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(4*A + 9*B)*(a^2 + a^2*Sec[c + d*x])*Sin[
c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^2}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(4A+9B)\left(a^2+\right.}{63}$$

$$= \frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+}{9d\sec^{\frac{7}{2}}}$$

$$= \frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a\sec(c+}{9d\sec^{\frac{7}{2}}}$$

$$= \frac{2a^2(19A+27B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(5A+6B+7}{21d\sqrt{\sec}}$$

$$= \frac{4a^2(8A+9B+12C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$

$$= \frac{4a^2(8A+9B+12C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$

Mathematica [C] time = 3.56378, size = 234, normalized size = 0.92

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(8A+9B+12C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(5*A + 6*B + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(8*A + 9*B + 12*C)*E^(I*
(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))] + Cos[c + d*x]*((2688*I)*A + (3024*I)*B + (4032*I)*C +
 30*(46*A + 51*B + 56*C)*Sin[c + d*x] + 14*(37*A + 36*B + 18*C)*Sin[2*(c +
d*x)] + 180*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 35*A*Sin[4*(c + d*
x)])))/(1260*d*E^(I*d*x))
```

Maple [A] time = 2.476, size = 514, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^
```

```
8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(1568*A+1134*B+672*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-38
7*A-351*B-273*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-168*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18
9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4 + (B+2\,C)a^2\sec(dx+c)^3 + (A+2\,B+C)a^2\sec(dx+c)^2 + (2\,A+B)a^2\sec(dx+c) + A}{\sec(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(9
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/se
c(d*x + c)^(9/2), x)
```

3.548 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=291

$$\frac{4a^2(50A+55B+66C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^2(7A+8B+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(89\ldots$$

```
[Out] (4*a^2*(7*A + 8*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(15*d) + (4*a^2*(50*A + 55*B + 66*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(89*A + 121*B +
99*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)) + (4*a^2*(7*A + 8*B + 9*C)*S
in[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (4*a^2*(50*A + 55*B + 66*C)*Sin[c
+ d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*
x])/(11*d*Sec[c + d*x]^(9/2)) + (2*(4*A + 11*B)*(a^2 + a^2*Sec[c + d*x])*Si
n[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.523717, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4086, 4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^2(7A+8B+9C)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(89A+121B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^2(50A+55B+66C)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{4a^2(5\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(11/2),x]
```


```
[Out] (4*a^2*(7*A + 8*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(15*d) + (4*a^2*(50*A + 55*B + 66*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a^2*(89*A + 121*B +
99*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)) + (4*a^2*(7*A + 8*B + 9*C)*S
in[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (4*a^2*(50*A + 55*B + 66*C)*Sin[c
+ d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^2*Sin[c + d*
x])/(11*d*Sec[c + d*x]^(9/2)) + (2*(4*A + 11*B)*(a^2 + a^2*Sec[c + d*x])*Si
n[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
```

```
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(4A+11B)(}{}$$

$$= \frac{2a^2(89A+121B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\,\mathrm{se}}{11a}$$

$$= \frac{2a^2(89A+121B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\,\mathrm{se}}{11a}$$

$$= \frac{2a^2(89A+121B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+8}{45d\,\mathrm{s}}$$

$$= \frac{2a^2(89A+121B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+8}{45d\,\mathrm{s}}$$

$$= \frac{4a^2(7A+8B+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

Mathematica [C] time = 4.54627, size = 270, normalized size = 0.93

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2464i(7A+8B+9C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(480*(50*A + 55*B + 66*C)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(7*A + 8*B + 9*C)*E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((51744*I)*A + (59136*I)*B + (66528*
I)*C + 30*(941*A + 22*(46*B + 51*C))*Sin[c + d*x] + 308*(38*A + 37*B + 36*C
)*Sin[2*(c + d*x)] + 4545*A*Sin[3*(c + d*x)] + 3960*B*Sin[3*(c + d*x)] + 19
80*C*Sin[3*(c + d*x)] + 1540*A*Sin[4*(c + d*x)] + 770*B*Sin[4*(c + d*x)] +
315*A*Sin[5*(c + d*x)])))/(27720*d*E^(I*d*x))
```

Maple [A] time = 2.298, size = 545, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-37520*A-6160*B)*sin(1/2*d*x+1/
```

```
2*c)^10*cos(1/2*d*x+1/2*c)+(57040*A+20240*B+3960*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-46192*A-26048*B-11484*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(22022*A+17248*B+12474*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-4563*A-4257*B-3861*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+750*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+825*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1848*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+990*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2 \sec(dx+c)^4 + (B+2C)a^2 \sec(dx+c)^3 + (A+2B+C)a^2 \sec(dx+c)^2 + (2A+B)a^2 \sec(dx+c) + Aa^2}{\sec(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sec(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/se
c(d*x + c)^(11/2), x)
```

3.549 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=343

$$\frac{4a^3(143A+121B+105C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(264A+253B+210C)\sin(c+dx)\text{se}}{1155d}$$

```
[Out] (-4*a^3*(21*A + 17*B + 15*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(143*A + 121*B + 105*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(21*A + 1
7*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(143*A + 121*B
 + 105*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(231*d) + (4*a^3*(264*A + 253*B
+ 210*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(1155*d) + (2*C*Sec[c + d*x]^(5/2
)*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(11*d) + (2*(11*B + 6*C)*Sec[c + d*x
]^(5/2)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(99*a*d) + (2*(99*A + 143*
B + 105*C)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(693*d
)
```

Rubi [A] time = 0.699487, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4088, 4018, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{4a^3(264A+253B+210C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d}+\frac{4a^3(143A+121B+105C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{231d}+\frac{2(99A+14}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-4*a^3*(21*A + 17*B + 15*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(143*A + 121*B + 105*C)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(21*A + 1
7*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(143*A + 121*B
 + 105*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(231*d) + (4*a^3*(264*A + 253*B
+ 210*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(1155*d) + (2*C*Sec[c + d*x]^(5/2
)*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(11*d) + (2*(11*B + 6*C)*Sec[c + d*x
]^(5/2)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(99*a*d) + (2*(99*A + 143*
B + 105*C)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(693*d
)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c}{11d}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c}{11d}$$

$$= \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\sin(c}{11d}$$

$$= \frac{4a^3(264A+253B+210C)\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{1155d}$$

$$= \frac{4a^3(264A+253B+210C)\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{1155d}$$

$$= \frac{4a^3(21A+17B+15C)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$= \frac{4a^3(21A+17B+15C)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$= -\frac{4a^3(21A+17B+15C)\sqrt{\cos(c+dx)}E\left(\right.}{15d}$$

Mathematica [C] time = 7.35393, size = 1324, normalized size = 3.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (7*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2))/(15*Sqrt[2]*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) + (17*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt
[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2))/(45*Sqrt[2]*d*E^(I*d*x)*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])) + (C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 +
 E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d
*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(3*Sqrt[2]*d*E^(I*d*x)*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (13*A*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2))/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Co
s[2*c + 2*d*x])*Sec[c + d*x]^(9/2)) + (11*B*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2))/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*Sec[c + d*x]^(9/2)) + (5*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2))/(11*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c +
```

```
d*x]^(9/2)) + (Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(((21*A + 17*B + 15*C)*Cos[d*x]*Csc[c])/(15*d) + (
C*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(22*d) + (Sec[c]*Sec[c + d*x]^4*(9*C*Sin[
c] + 11*B*Sin[d*x] + 33*C*Sin[d*x]))/(198*d) + (Sec[c]*Sec[c + d*x]^3*(77*B
*Sin[c] + 231*C*Sin[c] + 99*A*Sin[d*x] + 297*B*Sin[d*x] + 378*C*Sin[d*x]))/
(1386*d) + (Sec[c]*Sec[c + d*x]^2*(495*A*Sin[c] + 1485*B*Sin[c] + 1890*C*Si
n[c] + 2079*A*Sin[d*x] + 2618*B*Sin[d*x] + 2310*C*Sin[d*x]))/(6930*d) + (Se
c[c]*Sec[c + d*x]*(2079*A*Sin[c] + 2618*B*Sin[c] + 2310*C*Sin[c] + 4290*A*S
in[d*x] + 3630*B*Sin[d*x] + 3150*C*Sin[d*x]))/(6930*d) + ((143*A + 121*B +
105*C)*Tan[c])/(231*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sec[c + d*x]^(9/2))
```

Maple [B] time = 12.427, size = 1427, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(1/8*A+3
/8*B+3/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2
+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))-16/5*(3/8*A+3/8*B+1/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+16*(3/8*A+1
/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/352*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)
^2-1/2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-15/154*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+
15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+16*(1/8*B+3/8*C)*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d
*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))))+2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3 \sec(dx+c)^6 + (B+3C)a^3 \sec(dx+c)^5 + (A+3B+3C)a^3 \sec(dx+c)^4 + (3A+3B+C)a^3 \sec(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^6 + (B + 3*C)*a^3*sec(d*x + c)^5 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^4 + (3*A + 3*B + C)*a^3*sec(d*x + c)^3 + (3*A + B)*a^
3*sec(d*x + c)^2 + A*a^3*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right)(a \sec(dx+c) + a)^3 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*se
c(d*x + c)^(3/2), x)
```

3.550 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=307

$$\frac{4a^3(21A+13B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(42A+41B+32C)\sin(c+dx)\sec^{\frac{3}{2}}(c}{105d}$$

```
[Out] (-4*a^3*(27*A + 21*B + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(21*A + 13*B + 11*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(27*A + 21*B
+ 17*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(42*A + 41*B + 32*
C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*(3*B + 2*C)*Sec[c + d*x]^(3/2)*(
a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(21*a*d) + (2*(63*A + 99*B + 73*C)*
Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(315*d)
```

Rubi [A] time = 0.645568, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4088, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{4a^3(42A+41B+32C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(63A+99B+73C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-4*a^3*(27*A + 21*B + 17*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(15*d) + (4*a^3*(21*A + 13*B + 11*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(27*A + 21*B
+ 17*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(42*A + 41*B + 32*
C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*C*Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*(3*B + 2*C)*Sec[c + d*x]^(3/2)*(
a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(21*a*d) + (2*(63*A + 99*B + 73*C)*
Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(315*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3 \text{si}}{9d}$$
$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3 \text{si}}{9d}$$
$$= \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3 \text{si}}{9d}$$
$$= \frac{4a^3(42A+41B+32C)\sec^{\frac{3}{2}}(c+dx)\,\text{s}}{105d}$$
$$= \frac{4a^3(42A+41B+32C)\sec^{\frac{3}{2}}(c+dx)\,\text{s}}{105d}$$
$$= \frac{4a^3(27A+21B+17C)\sqrt{\sec(c+dx)}\,\text{s}}{15d}$$
$$= \frac{4a^3(21A+13B+11C)\sqrt{\cos(c+dx)}F}{21d}$$
$$= -\frac{4a^3(27A+21B+17C)\sqrt{\cos(c+dx)}}{15d}$$

Mathematica [C] time = 7.19545, size = 1267, normalized size = 4.13

$$\frac{3Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5(c+dx)\csc(c)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1}\right.}{5\sqrt{2}d(\cos(2c+2dx)A+A+2C+2B\cos(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (3*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2))/(5*Sqrt[2]*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])) + (7*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2))/(15*Sqrt[2]*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])) + (17*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^5*Csc[c]*(-3*Sqrt[1
+ E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1
[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(45*Sqrt[2]*d*E^(I*d*x)*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (A*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sec[c + d*x]^(9/2)) + (13*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2))/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sec[c + d*x]^(9/2)) + (11*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
```

```
ec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*
x]^(9/2)) + (Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*(((27*A + 21*B + 17*C)*Cos[d*x]*Csc[c])/(15*d) + (C*S
ec[c]*Sec[c + d*x]^4*Sin[d*x])/(18*d) + (Sec[c]*Sec[c + d*x]^3*(7*C*Sin[c]
+ 9*B*Sin[d*x] + 27*C*Sin[d*x]))/(126*d) + (Sec[c]*Sec[c + d*x]^2*(45*B*Sin
[c] + 135*C*Sin[c] + 63*A*Sin[d*x] + 189*B*Sin[d*x] + 238*C*Sin[d*x]))/(630
*d) + (Sec[c]*Sec[c + d*x]*(63*A*Sin[c] + 189*B*Sin[c] + 238*C*Sin[c] + 315
*A*Sin[d*x] + 390*B*Sin[d*x] + 330*C*Sin[d*x]))/(630*d) + ((21*A + 26*B + 2
2*C)*Tan[c])/(42*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Se
c[c + d*x]^(9/2))
```

Maple [B] time = 11.043, size = 1265, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*
(1/8*B+3/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))-16/5*(1/8*A+3/8*B+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*
x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)
-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+16*(3/8*A
+3/8*B+1/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/144*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*
x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1
/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+16*(3/8*A+1/
8*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2
-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(Ca^{3}\sec\left(dx+c\right)^{5}+\left(B+3\,C\right)a^{3}\sec\left(dx+c\right)^{4}+\left(A+3\,B+3\,C\right)a^{3}\sec\left(dx+c\right)^{3}+\left(3\,A+3\,B+C\right)a^{3}\sec\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sq
rt(sec(d*x + c)), x)
```

3.551 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=271

$$\frac{4a^3(35A+21B+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(140A+147B+106C)\sin(c+dx)\sqrt{\sec(}}{105d}$$

```
[Out] (-4*a^3*(5*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(35*A + 21*B + 13*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(140*A + 147*B +
106*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Sec[c + d*x]]*(
a + a*Sec[c + d*x])^3*Sin[c + d*x])/(7*d) + (2*(7*B + 6*C)*Sqrt[Sec[c + d*x
]]*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(35*a*d) + (2*(5*A + 9*B + 7*C)
*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.630653, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4088, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(140A+147B+106C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(5A+9B+7C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (-4*a^3*(5*A + 9*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (4*a^3*(35*A + 21*B + 13*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(140*A + 147*B +
106*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*Sqrt[Sec[c + d*x]]*(
a + a*Sec[c + d*x])^3*Sin[c + d*x])/(7*d) + (2*(7*B + 6*C)*Sqrt[Sec[c + d*x
]]*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(35*a*d) + (2*(5*A + 9*B + 7*C)
*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \cdots$$

$$= \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \cdots$$

$$= \frac{2C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\sin(c+dx)}{7d} + \cdots$$

$$= \frac{4a^3(140A+147B+106C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} \cdots$$

$$= \frac{4a^3(140A+147B+106C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} \cdots$$

$$= \frac{4a^3(140A+147B+106C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} \cdots$$

$$= -\frac{4a^3(5A+9B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) \cdots}{5d}$$

Mathematica [C] time = 5.46975, size = 359, normalized size = 1.32

$$\frac{a^3e^{-idx}\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(14i(5A+9B+7C)e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{7/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-630*I)*A - (1134*I)*B -
(882*I)*C - (840*I)*A*Cos[2*(c + d*x)] - (1512*I)*B*Cos[2*(c + d*x)] - (117
6*I)*C*Cos[2*(c + d*x)] - (210*I)*A*Cos[4*(c + d*x)] - (378*I)*B*Cos[4*(c +
 d*x)] - (294*I)*C*Cos[4*(c + d*x)] + 80*(35*A + 21*B + 13*C)*Cos[c + d*x]^
(7/2)*EllipticF[(c + d*x)/2, 2] + ((14*I)*(5*A + 9*B + 7*C)*(1 + E^((2*I)*(
c + d*x)))^(7/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^
((2*I)*(c + d*x)) + 70*A*Sin[c + d*x] + 210*B*Sin[c + d*x] + 380*C*Sin[c +
d*x] + 630*A*Sin[2*(c + d*x)] + 840*B*Sin[2*(c + d*x)] + 840*C*Sin[2*(c + d
*x)] + 70*A*Sin[3*(c + d*x)] + 210*B*Sin[3*(c + d*x)] + 260*C*Sin[3*(c + d*
x)] + 315*A*Sin[4*(c + d*x)] + 378*B*Sin[4*(c + d*x)] + 294*C*Sin[4*(c + d*
x)]))/(420*d*E^(I*d*x))
```

Maple [B] time = 9.23, size = 1099, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*(-1/56*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1
/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-16/5*(1/8*B+3/8*C)/
(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)+16*(1/8*A+3/8*B+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+16*(3/8*A+3/8*B+1/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
```

```
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3 \sec(dx+c)^5 + (B+3\,C)a^3 \sec(dx+c)^4 + (A+3\,B+3\,C)a^3 \sec(dx+c)^3 + (3\,A+3\,B+C)a^3 \sec(dx}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sq
rt(sec(d*x + c)), x)
```

3.552 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=271

$$\frac{4a^3(5A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(5A+20B+21C)\sin(c+dx)\sqrt{\sec(c+a}}{15d}$$

```
[Out] (4*a^3*(5*A - 5*B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A + 20*B + 21*C)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]) - (2*(5*A - 3*C)*Sqrt[Sec[c + d*x]]*(a^2 +
a^2*Sec[c + d*x])^2*Sin[c + d*x])/(15*a*d) - (2*(5*A - 5*B - 9*C)*Sqrt[Sec[
c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.630659, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4086, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(5A+20B+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(5A-5B-9C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (4*a^3*(5*A - 5*B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(5*A + 5*B + 3*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A + 20*B + 21*C)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]) - (2*(5*A - 3*C)*Sqrt[Sec[c + d*x]]*(a^2 +
a^2*Sec[c + d*x])^2*Sin[c + d*x])/(15*a*d) - (2*(5*A - 5*B - 9*C)*Sqrt[Sec[
c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\int\frac{(a+a\sec(c+dx))^3}{\cdots}}{\cdots}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2(5A-3C)\sqrt{\sec\cdots}}{\cdots}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2(5A-3C)\sqrt{\sec\cdots}}{\cdots}$$

$$= \frac{4a^3(5A+20B+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{2A(\cdots}{\cdots}$$

$$= \frac{4a^3(5A+20B+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{2A(\cdots}{\cdots}$$

$$= \frac{4a^3(5A+20B+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d}+\frac{2A(\cdots}{\cdots}$$

$$= \frac{4a^3(5A-5B-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec\cdots}}{5d}$$

Mathematica [C] time = 4.72114, size = 316, normalized size = 1.17

$$a^3e^{-idx}\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-4i(5A-5B-9C)e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((180*I)*A*Cos[c + d*x] - (
180*I)*B*Cos[c + d*x] - (324*I)*C*Cos[c + d*x] + (60*I)*A*Cos[3*(c + d*x)]
- (60*I)*B*Cos[3*(c + d*x)] - (108*I)*C*Cos[3*(c + d*x)] + 80*(5*A + 5*B +
3*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - ((4*I)*(5*A - 5*B - 9*C
)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])/E^(I*(c + d*x)) + 30*A*Sin[c + d*x] + 90*B*Sin[c + d*x] + 13
2*C*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 20*B*Sin[2*(c + d*x)] + 60*C*Sin
[2*(c + d*x)] + 30*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 108*C*Sin[3
*(c + d*x)] + 5*A*Sin[4*(c + d*x)]))/(60*d*E^(I*d*x))
```

Maple [B] time = 8.219, size = 1328, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 4/15*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(40*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+90*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4-120*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2
5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+27*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^4+100*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+60*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+60*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^4+108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+100*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2
*d*x+1/2*c)^2+190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+246*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-100*A*(sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*sin(1/2*d*x+1/2*c)^2-100*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^2-108*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)*A-72*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C-180*B*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^6-216*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + (B+3C)a^3\sec(dx+c)^4 + (A+3B+3C)a^3\sec(dx+c)^3 + (3A+3B+C)a^3\sec(dx+c}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/se
c(d*x + c)^(3/2), x)
```

3.553 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=270

$$\frac{4a^3(3A+5(B+C))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3(6A-5B-20C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-$$

```
[Out] (4*a^3*(9*A + 5*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(6*A - 5*B - 20*C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(6*A + 5*B)*(a^2 + a^2*Sec[c + d*x])^2*
Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]]) - (2*(9*A + 5*B - 5*C)*Sqrt[Sec[c
 + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.657914, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4086, 4017, 4018, 3997, 3787, 3771, 2639, 2641}

$$-\frac{4a^3(6A-5B-20C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}-\frac{2(9A+5B-5C)\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(5/2),x]
```


```
[Out] (4*a^3*(9*A + 5*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(3*A + 5*(B + C))*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(6*A - 5*B - 20*C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*(a + a*Sec[c + d*x])^3*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(6*A + 5*B)*(a^2 + a^2*Sec[c + d*x])^2*
Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]]) - (2*(9*A + 5*B - 5*C)*Sqrt[Sec[c
 + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6A+5B)\left(a^2+\right.}{15a}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6A+5B)\left(a^2+\right.}{15a}$$

$$= -\frac{4a^3(6A-5B-20C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + 2A($$

$$= -\frac{4a^3(6A-5B-20C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + 2A($$

$$= -\frac{4a^3(6A-5B-20C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + 2A($$

$$= \frac{4a^3(9A+5B-5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5d}$$

Mathematica [C] time = 3.06768, size = 275, normalized size = 1.02

$$\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-8i(9A+5B-5C)\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((216*I)*A + (120*I)*B - (1
20*I)*C + (216*I)*A*Cos[2*(c + d*x)] + (120*I)*B*Cos[2*(c + d*x)] - (120*I)
*C*Cos[2*(c + d*x)] + 80*(3*A + 5*(B + C))*Cos[c + d*x]^(3/2)*EllipticF[(c
+ d*x)/2, 2] - (8*I)*(9*A + 5*B - 5*C)*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 30*A*Sin[c + d*x] + 10
*B*Sin[c + d*x] + 40*C*Sin[c + d*x] + 6*A*Sin[2*(c + d*x)] + 60*B*Sin[2*(c
+ d*x)] + 180*C*Sin[2*(c + d*x)] + 30*A*Sin[3*(c + d*x)] + 10*B*Sin[3*(c +
d*x)] + 3*A*Sin[4*(c + d*x)]))/(60*d*E^(I*d*x))
```

Maple [B] time = 7.704, size = 950, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+54*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^
2+78*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+30*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*sin(1/2*d*x+1/2*c)^2-50*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+5
0*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-30*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^2-50*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+90*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-18*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-15*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-20*B*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-50*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5+(B+3\,C)a^3\sec(dx+c)^4+(A+3\,B+3\,C)a^3\sec(dx+c)^3+(3\,A+3\,B+C)a^3\sec(dx}{\sec(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/se
c(d*x + c)^(5/2), x)
```

3.554 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=271

$$\frac{4a^3(13A+21B+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^3(41A+42B-35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}$$

```
[Out] (4*a^3*(7*A + 9*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(13*A + 21*B + 35*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(41*A + 42*B - 35*
C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*(a + a*Sec[c + d*x])^3*S
in[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(6*A + 7*B)*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(35*a*d*Sec[c + d*x]^(3/2)) + (2*(7*A + 9*B + 5*C)*(a^
3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.632989, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4086, 4017, 3997, 3787, 3771, 2639, 2641}

$$-\frac{4a^3(41A+42B-35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d}+\frac{2(7A+9B+5C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(7/2),x]
```


```
[Out] (4*a^3*(7*A + 9*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(5*d) + (4*a^3*(13*A + 21*B + 35*C)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) - (4*a^3*(41*A + 42*B - 35*
C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*(a + a*Sec[c + d*x])^3*S
in[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(6*A + 7*B)*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(35*a*d*Sec[c + d*x]^(3/2)) + (2*(7*A + 9*B + 5*C)*(a^
3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(6A+7B)\left(\right.}{}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(6A+7B)\left(\right.}{}$$

$$= -\frac{4a^3(41A+42B-35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} +$$

$$= -\frac{4a^3(41A+42B-35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} +$$

$$= -\frac{4a^3(41A+42B-35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d} +$$

$$= \frac{4a^3(7A+9B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}$$

Mathematica [C] time = 3.08927, size = 266, normalized size = 0.98

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112i(7A+9B+5C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((2352*I)*A*Cos[c + d*x] +
(3024*I)*B*Cos[c + d*x] + (1680*I)*C*Cos[c + d*x] + 80*(13*A + 21*B + 35*C)
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(7*A + 9*B + 5*C)*E
^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, -E^((2*I)*(c + d*x))] + 126*A*Sin[c + d*x] + 42*B*Sin[c + d*x] + 840*C*S
in[c + d*x] + 550*A*Sin[2*(c + d*x)] + 420*B*Sin[2*(c + d*x)] + 140*C*Sin[2
*(c + d*x)] + 126*A*Sin[3*(c + d*x)] + 42*B*Sin[3*(c + d*x)] + 15*A*Sin[4*(
c + d*x)]))/(420*d*E^(I*d*x))
```

Maple [B] time = 2.711, size = 727, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*a^3*(120*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(36*A+7*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A+21*B+5*C)*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(104*A+63*B+70*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*A*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-147*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+105*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-189*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+175*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3 \sec(dx+c)^5 + (B+3C)a^3 \sec(dx+c)^4 + (A+3B+3C)a^3 \sec(dx+c)^3 + (3A+3B+C)a^3 \sec(dx+c)}{\sec(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^2+B \sec \left(dx+c\right)+A\right)\left(a \sec \left(dx+c\right)+a\right)^3}{\sec \left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/se
c(d*x + c)^(7/2), x)
```

3.555 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=271

$$\frac{4a^3(11A+13B+21C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(73A+99B+63C)\sin(c+dx)\left(a^3\sec(c+\right.}{315d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (4*a^3*(17*A + 21*B + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B + 21*C)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(32*A + 41*B +
 42*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])
^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(2*A + 3*B)*(a^2 + a^2*Sec[c
 + d*x])^2*Sin[c + d*x])/(21*a*d*Sec[c + d*x]^(5/2)) + (2*(73*A + 99*B + 63
*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.646838, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4086, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(73A+99B+63C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(11A+13B+21C)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(9/2),x]
```


```
[Out] (4*a^3*(17*A + 21*B + 27*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B + 21*C)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(32*A + 41*B +
 42*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])
^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(2*A + 3*B)*(a^2 + a^2*Sec[c
 + d*x])^2*Sin[c + d*x])/(21*a*d*Sec[c + d*x]^(5/2)) + (2*(73*A + 99*B + 63
*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(2A+3B)\left(a^2+\right.}{21}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(2A+3B)\left(a^2+\right.}{21}$$

$$= \frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+}{9d\sec}$$

$$= \frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+}{9d\sec}$$

$$= \frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2A(a+a\sec(c+}{9d\sec}$$

$$= \frac{4a^3(17A+21B+27C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

Mathematica [C] time = 3.04776, size = 214, normalized size = 0.79

$$\frac{a^3\sqrt{\sec(c+dx)}\left(-224i(17A+21B+27C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+480(11A+13\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(480*(11*A + 13*B + 21*C)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] - (224*I)*(17*A + 21*B + 27*C)*E^(I*(c + d*x))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))
] + 2*Cos[c + d*x]*((5712*I)*A + (7056*I)*B + (9072*I)*C + 30*(97*A + 107*B
 + 84*C)*Sin[c + d*x] + 14*(73*A + 54*B + 18*C)*Sin[2*(c + d*x)] + 270*A*Si
n[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 35*A*Sin[4*(c + d*x)])))/(2520*d)
```

Maple [A] time = 2.337, size = 514, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2200*A+360*B)*sin(1/2*d*x+1/2*c)^
```

```
8*cos(1/2*d*x+1/2*c)+(-3412*A-1296*B-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(2702*A+1806*B+882*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-73
8*A-624*B-378*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-357*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+195*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
441*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3 \sec(dx+c)^5 + (B+3C)a^3 \sec(dx+c)^4 + (A+3B+3C)a^3 \sec(dx+c)^3 + (3A+3B+C)a^3 \sec(dx}{\sec(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/se
c(d*x + c)^(9/2), x)
```

3.556 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=307

$$\frac{4a^3(105A+121B+143C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(210A+253B+264C)\sin(c+dx}{1155d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (4*a^3*(15*A + 17*B + 21*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B + 143*C)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(210*A + 2
53*B + 264*C)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(3/2)) + (4*a^3*(105*A + 1
21*B + 143*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c
+ d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (2*(6*A + 11*B)*(a^2 +
a^2*Sec[c + d*x])^2*Sin[c + d*x])/(99*a*d*Sec[c + d*x]^(7/2)) + (2*(105*A +
 143*B + 99*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(693*d*Sec[c + d*x]^(
5/2))
```

Rubi [A] time = 0.675096, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4086, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(105A+143B+99C)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(105A+121}{231d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(11/2),x]
```


```
[Out] (4*a^3*(15*A + 17*B + 21*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B + 143*C)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (4*a^3*(210*A + 2
53*B + 264*C)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(3/2)) + (4*a^3*(105*A + 1
21*B + 143*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c
+ d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (2*(6*A + 11*B)*(a^2 +
a^2*Sec[c + d*x])^2*Sin[c + d*x])/(99*a*d*Sec[c + d*x]^(7/2)) + (2*(105*A +
 143*B + 99*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(693*d*Sec[c + d*x]^(
5/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(6A+11B)(}{}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(6A+11B)(}{}$$

$$= \frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a}{1}$$

$$= \frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+a}{1}$$

$$= \frac{4a^3(210A+253B+264C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(105A}{}$$

$$= \frac{4a^3(15A+17B+21C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

$$= \frac{4a^3(15A+17B+21C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

Mathematica [C] time = 5.1811, size = 246, normalized size = 0.8

$$\frac{a^3\sqrt{\sec(c+dx)}\left(-2464i(15A+17B+21C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+480(105A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(480*(105*A + 121*B + 143*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2] - (2464*I)*(15*A + 17*B + 21*C)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((110880*I)*A + (125664*I)*B + (155232*I)*C + 30*(1953
*A + 2134*B + 2354*C)*Sin[c + d*x] + 308*(75*A + 73*B + 54*C)*Sin[2*(c + d*
x)] + 8505*A*Sin[3*(c + d*x)] + 5940*B*Sin[3*(c + d*x)] + 1980*C*Sin[3*(c +
 d*x)] + 2310*A*Sin[4*(c + d*x)] + 770*B*Sin[4*(c + d*x)] + 315*A*Sin[5*(c
+ d*x)])))/(27720*d)
```

Maple [A] time = 2.188, size = 545, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-43680*A-6160*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(77280*A+24200*B+3960*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-72240*A-37532*B-14256*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(39270*A+29722*B+19866*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-8820*A-8118*B-6864*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1575*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-3465*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1815*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3927*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2145*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4
851*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + (B+3C)a^3\sec(dx+c)^4 + (A+3B+3C)a^3\sec(dx+c)^3 + (3A+3B+C)a^3\sec(dx+c)}{\sec(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/se
c(d*x + c)^(11/2), x)
```

3.557 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=343

$$\frac{4a^3(95A+105B+121C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(175A+195B+221C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (4*a^3*(175*A + 195*B + 221*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(195*d) + (4*a^3*(95*A + 105*B + 121*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(236*A
 + 273*B + 286*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (4*a^3*(175*A
 + 195*B + 221*C)*Sin[c + d*x])/(585*d*Sec[c + d*x]^(3/2)) + (4*a^3*(95*A +
 105*B + 121*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[
c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c + d*x]^(11/2)) + (2*(6*A + 13*B)*(a^2
 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(143*a*d*Sec[c + d*x]^(9/2)) + (2*(145
*A + 195*B + 143*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(1287*d*Sec[c +
d*x]^(7/2))
```

Rubi [A] time = 0.717735, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4086, 4017, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{4a^3(175A+195B+221C)\sin(c+dx)}{585d\sec^{\frac{3}{2}}(c+dx)}+\frac{20a^3(236A+273B+286C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(145A+195B+143C)\sin(c+dx)}{1287d\sec^{\frac{7}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(13/2),x]
```


```
[Out] (4*a^3*(175*A + 195*B + 221*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(195*d) + (4*a^3*(95*A + 105*B + 121*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(236*A
 + 273*B + 286*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)) + (4*a^3*(175*A
 + 195*B + 221*C)*Sin[c + d*x])/(585*d*Sec[c + d*x]^(3/2)) + (4*a^3*(95*A +
 105*B + 121*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[
c + d*x])^3*Sin[c + d*x])/(13*d*Sec[c + d*x]^(11/2)) + (2*(6*A + 13*B)*(a^2
 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(143*a*d*Sec[c + d*x]^(9/2)) + (2*(145
*A + 195*B + 143*C)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(1287*d*Sec[c +
d*x]^(7/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+dx))^3 \cdots}{\cdots}}{\cdots}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2(6A+13B)\left(a^2 \cdots\right.}{143 \cdots}$$

$$= \frac{2A(a+a\sec(c+dx))^3\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2(6A+13B)\left(a^2 \cdots\right.}{143 \cdots}$$

$$= \frac{20a^3(236A+273B+286C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\,\mathrm{se}\cdots}{13\cdots}$$

$$= \frac{20a^3(236A+273B+286C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+a\,\mathrm{se}\cdots}{13\cdots}$$

$$= \frac{20a^3(236A+273B+286C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(175A\cdots}{5\cdots}$$

$$= \frac{20a^3(236A+273B+286C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(175A\cdots}{5\cdots}$$

$$= \frac{4a^3(175A+195B+221C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{195d}$$

Mathematica [C] time = 6.50075, size = 300, normalized size = 0.87

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-4928i(175A+195B+221C)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(13/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(12480*(95*A + 105*B + 121*
C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (4928*I)*(175*A + 195*B +
 221*C)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((2587200*I)*A + (2882880*
I)*B + (3267264*I)*C + 780*(1811*A + 1953*B + 2134*C)*Sin[c + d*x] + 77*(78
25*A + 7800*B + 7592*C)*Sin[2*(c + d*x)] + 251550*A*Sin[3*(c + d*x)] + 2211
30*B*Sin[3*(c + d*x)] + 154440*C*Sin[3*(c + d*x)] + 90860*A*Sin[4*(c + d*x)
] + 60060*B*Sin[4*(c + d*x)] + 20020*C*Sin[4*(c + d*x)] + 24570*A*Sin[5*(c
+ d*x)] + 8190*B*Sin[5*(c + d*x)] + 3465*A*Sin[6*(c + d*x)])))/(720720*d*E^
(I*d*x))
```

Maple [A] time = 2.474, size = 576, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(1058400*A+131040*B)*sin(1/2*
d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-2122400*A-567840*B-80080*C)*sin(1/2*d*x+
1/2*c)^10*cos(1/2*d*x+1/2*c)+(2331040*A+1004640*B+314600*C)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-1535860*A-939120*B-487916*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(633710*A+510510*B+386386*C)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-121230*A-114660*B-105534*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)+18525*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-40425*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+20475*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-45045*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+23595*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-51051*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13
/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + (B+3C)a^3\sec(dx+c)^4 + (A+3B+3C)a^3\sec(dx+c)^3 + (3A+3B+C)a^3\sec(dx}{\sec(dx+c)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13
/2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/se
c(d*x + c)^(13/2), x)
```

3.558 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{(3A-5B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-$$

```
[Out] (-3*(5*A - 5*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) - ((3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (3*(5*A - 5*B + 7*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*a*d) - ((3*A - 5*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d) + ((5*A - 5*B + 7*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d
) - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.27642, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 3787, 3768, 3771, 2641, 2639}

$$-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-5B+7C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}-\frac{(3A-5B+5C)\sin(c+dx)\sec^{\frac{3}{2}}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```

```
[Out] (-3*(5*A - 5*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) - ((3*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (3*(5*A - 5*B + 7*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*a*d) - ((3*A - 5*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d) + ((5*A - 5*B + 7*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d
) - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \sec^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(3A-5B+5C)\int s}{2a}$$

$$= -\frac{(3A-5B+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}+\frac{(5A-5B+7C)}{}$$

$$= \frac{3(5A-5B+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad}-\frac{(3A-5B+5C)}{}$$

$$= -\frac{(3A-5B+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}$$

$$= -\frac{3(5A-5B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5ad}$$

Mathematica [C] time = 7.87158, size = 1307, normalized size = 5.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a +
a*Sec[c + d*x])) - (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]
*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c +
```

```
d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (7*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1
+ E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*
Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*A*Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (10*B*
Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]
)) - (10*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c +
d*x)/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*S
ec[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*((6*(5*A - 5*B + 7*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (4*Sec[c/2]*Se
c[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8
*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(5*d) + (8*Sec[c]*Sec[c + d*x]*(3*C*Sin[
c] + 5*B*Sin[d*x] - 5*C*Sin[d*x]))/(15*d) - (4*(-2*B + 2*C + 3*A*Cos[c] - 5
*B*Cos[c] + 5*C*Cos[c])*Sec[c]*Tan[c/2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 8.685, size = 812, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*C/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1
/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+(2*B-2*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+
(-A+B-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*A-2*B+2*C)*
(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(
d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec
(d*x + c) + a), x)
```

3.559 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{(3A-3B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-3$$

```
[Out] ((A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d) + ((3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((A - 3*B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(a*d) + ((3*A - 3*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d
) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.24829, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-3B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{(A-3B+3C)\sin(c+dx)\sqrt{\sec(}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```


```
[Out] ((A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d) + ((3*A - 3*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - ((A - 3*B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(a*d) + ((3*A - 3*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d
) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \sec^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(A-3B+3C)\int \mathrm{se}}{2a}$$

$$= -\frac{(A-3B+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}+\frac{(3A-3B+5C)\,\mathrm{s}}{}$$

$$= -\frac{(A-3B+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}+\frac{(3A-3B+5C)\,\mathrm{s}}{}$$

$$= \frac{(A-3B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+$$

Mathematica [C] time = 7.43532, size = 1261, normalized size = 6.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] -(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((
2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c
/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2
```

```
*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Cos[c/2 +
(d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) - (2*B*Co
s[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) +
 (10*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
 + d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(
(-2*(A - 3*B + 3*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*Sec[c/2]*Sec[c/2 + (
d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*C*Sec[c]
*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(2*C + 3*A*Cos[c] - 3*B*Cos[c] + 5*C*Cos
[c])*Sec[c]*Tan[c/2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 6.934, size = 494, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A-B+C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)+(2*B-2*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*
x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec
(d*x + c) + a), x)
```

3.560 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=162

$$\frac{(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(A-B+3C}{}$$

```
[Out] -(((A - B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d)) + ((A + B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*d) + ((A - B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*d) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x
]))
```

Rubi [A] time = 0.213708, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(A-B+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(A+B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```


```
[Out] -(((A - B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a*d)) + ((A + B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*d) + ((A - B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*d) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x
]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a\right.}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A+B-C)\int \sqrt{\mathrm{se}}}{2a}$$

$$= \frac{(A-B+3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(A-B+C)\sec^{\frac{3}{2}}(c}{d(a+a\sec}$$

$$= \frac{(A+B-C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A}{}$$

$$= -\frac{(A-B+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} +$$

Mathematica [C] time = 6.80785, size = 1224, normalized size = 7.56

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
+ a*Sec[c + d*x])) - (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/
2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^
2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(
-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*S
```

```
ec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (2*B*C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
- (2*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
 d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2
*(A - B + 3*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/
2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d - (4*(A - B + C)*T
an[c/2])/d))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c
+ d*x]]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 4.813, size = 353, normalized size = 2.2

$$-\frac{1}{ad}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-B+3*C)*sin(1/2*d*x+1/2*c
)^4+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-B+5*C)*sin(1/2*
d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(
d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec
(d*x + c) + a), x)
```

3.561 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=133

$$-\frac{(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(3A-B+\cdots}{\cdots}$$

```
[Out] ((3*A - B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*d) - ((A - B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a
 + a*Sec[c + d*x]))
```

Rubi [A] time = 0.209064, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4084, 3787, 3771, 2639, 2641}

$$-\frac{(A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{(A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B+C)\sqrt{\cos\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[
c + d*x])),x]
```

```
[Out] ((3*A - B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*d) - ((A - B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a
 + a*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A-B+C)-\frac{1}{2}a(A-B-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B-C)\int\sqrt{\sec(c+dx)}\,dx}{2a} + \cdots$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c\cdots\right.}{2a}$$

$$= \frac{(3A-B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-B-C)\sqrt{\cos\cdots}}{\cdots}$$

Mathematica [C] time = 6.543, size = 1243, normalized size = 9.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])),x]
```

```
[Out] -((Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
+ a*Sec[c + d*x]))) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c
/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]
^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*
Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*A*Cos[c/
2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (2*
B*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2
]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]
)) + (2*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[
```

```
c + d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((-2*(2*A - B + C + A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*Sec[c/2]
*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d +
 (8*A*Cos[c]*Sin[d*x])/d + (4*(A - B + C)*Tan[c/2])/d))/((A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 1.951, size = 281, normalized size = 2.1

$$\frac{1}{ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+(2*A-2*B+2*C)*sin(1/2*d*x+1/2*c)^4+(-A+B-C)*sin(1/2*d*x+1/2*c)^2)/a/
cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{2}+a\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(
d*x + c)^2 + a*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{C\sec^{2}(c+dx)}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))/sec(d*x+c)**(1/
2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec
(c + d*x)/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(C*sec(c
 + d*x)**2/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^{2}+B\sec(dx+c)+A}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

3.562 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$\frac{(5A-3B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(5A-3B+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)}{d\sqrt{\sec(c+dx)}(a}$$

```
[Out] -(((3*A - 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d)) + ((5*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + ((5*A - 3*B + 3*C)*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a
 + a*Sec[c + d*x]))
```

Rubi [A] time = 0.233566, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 3787, 3769, 3771, 2641, 2639}

$$\frac{(5A-3B+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}+\frac{(5A-3B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])),x]
```


```
[Out] -(((3*A - 3*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d)) + ((5*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + ((5*A - 3*B + 3*C)*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a
 + a*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B+3C)-\frac{1}{2}a(3A-3B+C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{(3A-3B+C)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{(5A-}{}$$

$$= \frac{(5A-3B+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B+C)\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{(5A-3B}{}$$

$$= -\frac{(3A-3B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-3B+3}{3ad\sqrt{\mathrm{sec}}}$$

$$= -\frac{(3A-3B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-3B+3}{}$$

Mathematica [C] time = 6.68384, size = 1287, normalized size = 7.4

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a +
a*Sec[c + d*x])) - (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]
*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 +
```

```
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Co
s[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 +
 E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c
/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (10*A*Cos[c/2 +
(d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) - (2*B*
Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
 + (2*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x
)/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
+ d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((
2*(2*A - 2*B + C + A*Cos[2*c] - B*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/d +
 (4*A*Cos[2*d*x]*Sin[2*c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d
*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d - (8*(A - B)*Cos[c]*Sin[d*x])/
d + (4*A*Cos[2*c]*Sin[2*d*x])/(3*d) - (4*(A - B + C)*Tan[c/2])/d))/((A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
+ d*x]))
```

Maple [A] time = 2.449, size = 300, normalized size = 1.7

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\,(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+3*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B+6*C)*sin(1/2*d*x+1
/2*c)^4+(-7*A+3*B-3*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(
d*x + c)^3 + a*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^{\frac{5}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{B\sec\left(c+dx\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{C\sec^{2}\left(c+dx\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(B*se
c(c + d*x)/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(C*sec
(c + d*x)**2/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

3.563 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=214

$$-\frac{(5A-5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A-5B}{5ad\,\text{s}}$$

```
[Out] (3*(7*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*a*d) - ((5*A - 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + ((7*A - 5*B + 5*C)*Sin[c + d*x])/(5
*a*d*Sec[c + d*x]^(3/2)) - ((5*A - 5*B + 3*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec
[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c
 + d*x]))
```

Rubi [A] time = 0.253706, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A-5B+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{(5A-5B+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(5A-5B+3C)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])),x]
```


```
[Out] (3*(7*A - 5*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*a*d) - ((5*A - 5*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + ((7*A - 5*B + 5*C)*Sin[c + d*x])/(5
*a*d*Sec[c + d*x]^(3/2)) - ((5*A - 5*B + 3*C)*Sin[c + d*x])/(3*a*d*Sqrt[Sec
[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c
 + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(7A-5B+5C)-\frac{1}{2}a(5A-5B+3C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(5A-5B+3C)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(7A}{}$$

$$= \frac{(7A-5B+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(5A-5B+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B+C}{d\sec^{\frac{3}{2}}(c+dx)(}$$

$$= \frac{(7A-5B+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(5A-5B+3C)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B+C}{d\sec^{\frac{3}{2}}(c+dx)(}$$

$$= \frac{3(7A-5B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{(5A-5B+3}{}$$

Mathematica [C] time = 6.77207, size = 1350, normalized size = 6.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])),x]
```

```
[Out] (-7*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^
((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(a + a*Sec[c + d*x])) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc
[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*
```

```
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]
^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*
Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (10*A*Cos[c/2
 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2
]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (1
0*B*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
d*x])) - (2*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c
 + d*x)/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*(-((51*A - 40*B + 40*C + 33*A*Cos[2*c] - 20*B*Cos[2*c] + 20*C*Cos[2*c])
*Cos[d*x]*Csc[c/2]*Sec[c/2])/(10*d) - (4*(A - B)*Cos[2*d*x]*Sin[2*c])/(3*d)
 + (2*A*Cos[3*d*x]*Sin[3*c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[
(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (2*(33*A - 20*B + 20*C)*Co
s[c]*Sin[d*x])/(5*d) - (4*(A - B)*Cos[2*c]*Sin[2*d*x])/(3*d) + (2*A*Cos[3*c
]*Sin[3*d*x])/(5*d) + (4*(A - B + C)*Tan[c/2])/d))/((A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.247, size = 320, normalized size = 1.5

$$-\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(63*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-45*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-25*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+45*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/
2*d*x+1/2*c)^6+(-30*A+90*B-30*C)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B+15*C)*sin(
1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{4}+a\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(
d*x + c)^4 + a*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec
(d*x + c)^(5/2)), x)
```

3.564 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=250

$$\frac{5(9A-7B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21ad}-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(7A-7B-}{5ad\,\text{se}}$$

```
[Out] (-3*(7*A - 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) + (5*(9*A - 7*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*a*d) + ((9*A - 7*B + 7*C)*Sin[c + d*x]
)/(7*a*d*Sec[c + d*x]^(5/2)) - ((7*A - 7*B + 5*C)*Sin[c + d*x])/(5*a*d*Sec[
c + d*x]^(3/2)) + (5*(9*A - 7*B + 7*C)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d
*x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x
]))
```

Rubi [A] time = 0.281834, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 3787, 3769, 3771, 2641, 2639}

$$-\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(7A-7B+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(9A-7B+7C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(9A-7B+7C)\,\text{s}}{21ad\sqrt{\sec(c-}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*(a + a*Sec[
c + d*x])),x]
```


```
[Out] (-3*(7*A - 7*B + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a*d) + (5*(9*A - 7*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*a*d) + ((9*A - 7*B + 7*C)*Sin[c + d*x]
)/(7*a*d*Sec[c + d*x]^(5/2)) - ((7*A - 7*B + 5*C)*Sin[c + d*x])/(5*a*d*Sec[
c + d*x]^(3/2)) + (5*(9*A - 7*B + 7*C)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d
*x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x
]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(9A-7B+7C)-\frac{1}{2}a(7A-7B+5C)\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(7A-7B+5C)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a} + \frac{(9A}{}$$

$$= \frac{(9A-7B+7C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{(7A-7B+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-B+}{d\sec^{\frac{5}{2}}(c+dx)(}$$

$$= \frac{(9A-7B+7C)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{(7A-7B+5C)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(9A-7B+7}{21ad\sqrt{\sec}}$$

$$= -\frac{3(7A-7B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{(9A-7B+}{7ad\,\mathrm{se}}$$

$$= -\frac{3(7A-7B+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(9A-7B}{}$$

Mathematica [C] time = 6.90396, size = 1406, normalized size = 5.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*(a +
a*Sec[c + d*x])),x]
```

```
[Out] (7*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
```

```
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
a + a*Sec[c + d*x])) - (7*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Cs
c[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))
*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Sqrt[2]*C*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)
/2]^2*Cos[c + d*x]*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*
x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))
])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*E^(I*d*x)*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (30*A*Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[
c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(7*d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) -
 (10*B*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
 + d*x])) + (10*C*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*Elliptic
F[(c + d*x)/2, 2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/
(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(
a + a*Sec[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*(((51*A - 51*B + 40*C + 33*A*Cos[2*c] - 33*B*Cos[2*c] + 20*C*Cos[
2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(10*d) + (2*(27*A - 14*B + 14*C)*Cos[2*d*
x]*Sin[2*c])/(21*d) - (2*(A - B)*Cos[3*d*x]*Sin[3*c])/(5*d) + (A*Cos[4*d*x]
*Sin[4*c])/(7*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(
d*x)/2] + C*Sin[(d*x)/2]))/d - (2*(33*A - 33*B + 20*C)*Cos[c]*Sin[d*x])/(5*
d) + (2*(27*A - 14*B + 14*C)*Cos[2*c]*Sin[2*d*x])/(21*d) - (2*(A - B)*Cos[3
*c]*Sin[3*d*x])/(5*d) + (A*Cos[4*c]*Sin[4*d*x])/(7*d) - (4*(A - B + C)*Tan[
c/2])/d))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.282, size = 341, normalized size = 1.4

$$-\frac{1}{105\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(225*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-175*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+175*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*C*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-480*A*sin(1/2*d*x+1/2*c)^10+(864*A+336*
B)*sin(1/2*d*x+1/2*c)^8+(-888*A-392*B-280*C)*sin(1/2*d*x+1/2*c)^6+(930*A-21
0*B+630*C)*sin(1/2*d*x+1/2*c)^4+(-321*A+161*B-245*C)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{5}+a\sec\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(
d*x + c)^5 + a*sec(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec
(d*x + c)^(7/2)), x)
```

3.565 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=251

$$\frac{(2A-5B+10C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(A-4B+7C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(2A$$

```
[Out] ((A - 4*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*d) + ((2*A - 5*B + 10*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - 4*B + 7*C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(a^2*d) + ((2*A - 5*B + 10*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(3*a^2*d) - ((A - 4*B + 7*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1
 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.41211, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(A-4B+7C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(2A-5B+10C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{(A-4B+7C)\sin(c+dx)\sqrt{s}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```


```
[Out] ((A - 4*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*d) + ((2*A - 5*B + 10*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - ((A - 4*B + 7*C)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(a^2*d) + ((2*A - 5*B + 10*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(3*a^2*d) - ((A - 4*B + 7*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1
 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(A+5B}{a+}}{}$$

$$= -\frac{(A-4B+7C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^{\frac{7}{2}}}{3d(a+a\,\mathrm{s}}$$

$$= -\frac{(A-4B+7C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^{\frac{7}{2}}}{3d(a+a\,\mathrm{s}}$$

$$= -\frac{(A-4B+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{(2A-5B+10C)}{}$$

$$= -\frac{(A-4B+7C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{(2A-5B+10C)}{}$$

$$= \frac{(A-4B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \cdot$$

Mathematica [C] time = 7.84023, size = 1347, normalized size = 5.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(3*
d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
+ d*x])^2) + (8*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*S
qrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
a + a*Sec[c + d*x])^2) - (14*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3
*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (8*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c
+ d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (20*B*Cos[c/2 + (d*x)/2]^
4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c
 + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (40*C*Cos[c/
2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) +
 (Cos[c/2 + (d*x)/2]^4*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*((-4*(A - 4*B + 7*C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*Sec[c/2]*Sec
[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d)
 + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - 5*B*Sin[(d*x)/2] + 8*
C*Sin[(d*x)/2]))/(3*d) + (16*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (8*(2*
C + 2*A*Cos[c] - 5*B*Cos[c] + 10*C*Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (4*(A -
 B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 8.239, size = 751, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A-
B+C)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2
*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)
+4*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(4*C-2*B)*(cos(1/2*d*x+1/2*c)
```

```
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-8*C+4*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(
d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^2, x)
```

3.566 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=207

$$\frac{(A+2B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A+2B-5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(B-4C)\text{s}}{}$$

```
[Out] ((B - 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + ((A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a^2*d) - ((B - 4*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
a^2*d) + ((A + 2*B - 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Se
c[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rubi [A] time = 0.38067, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(A+2B-5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(A+2B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(B-4C)\sin(c+}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```

```
[Out] ((B - 4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + ((A + 2*B - 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a^2*d) - ((B - 4*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
a^2*d) + ((A + 2*B - 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Se
c[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A\right.}{}$$

$$=\frac{(A+2B-5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\sec}{3d(a+a}$$

$$=\frac{(A+2B-5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\sec}{3d(a+a}$$

$$=-\frac{(B-4C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{(A+2B-5C)\sec^{\frac{3}{2}}}{3a^2d(1+s}$$

$$=\frac{(A+2B-5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

$$=\frac{(B-4C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(A}{}$$

Mathematica [C] time = 4.44942, size = 567, normalized size = 2.74

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(4A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-2\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*Sqrt[2]*
B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*
x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I
)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) +
(8*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E
^(I*d*x) + 4*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]] + 8*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] -
 20*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 2*S
qrt[Sec[c + d*x]]*(6*(B - 4*C)*Cos[d*x]*Csc[c] - 2*(A + 2*B - 5*C)*Sec[c/2]
*Sec[(c + d*x)/2]*Sin[(d*x)/2] + (A - B + C)*Sec[c/2]*Sec[(c + d*x)/2]^3*Si
n[(d*x)/2] - 2*(A + 2*B - 5*C)*Tan[c/2] + (A - B + C)*Sec[(c + d*x)/2]^2*Ta
n[c/2])))/(3*a^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(1 + S
ec[c + d*x])^2)
```

Maple [B] time = 5.62, size = 559, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(-2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(B-4*C)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(A-10*B+43*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-7*B+37*C)*sin(1/2*d*x+1/2*c)^2)/sin
(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*
x+1/2*c)^2-1)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*
x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^2, x)
```

3.567 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=173

$$\frac{(2A+B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}-\frac{(A-C)\sqrt{\cos(c}{}$$

```
[Out] -(((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d)) + ((2*A + B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + ((A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.362718, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4019, 3787, 3771, 2639, 2641}

$$\frac{(2A+B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}-\frac{(A-C)\sqrt{\cos(c+dx)}\sqrt{s}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```

```
[Out] -(((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d)) + ((2*A + B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + ((A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(5\right.}{}$$

$$= \frac{(A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^{\frac{3}{2}}(c+}{3d(a+a\sec(c}$$

$$= \frac{(A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^{\frac{3}{2}}(c+}{3d(a+a\sec(c}$$

$$= \frac{(A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B+C)\sec^{\frac{3}{2}}(c+}{3d(a+a\sec(c}$$

$$= -\frac{(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2}{}$$

Mathematica [C] time = 6.82155, size = 1097, normalized size = 6.34

$$\frac{2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+d}}\right.}{3d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(\sec(c+dx)a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(3*d
*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
 d*x])^2) - (2*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sq
rt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((
```

```
2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^2) + (8*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/
2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*(a + a*Sec[c + d*x])^2) + (4*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (8*C*Cos[c/2 + (d*x)/2]^4*S
qrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c +
d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x
)/2]^4*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(A -
C)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(4*A*Sin[
(d*x)/2] - B*Sin[(d*x)/2] - 2*C*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2
+ (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (8
*(4*A - B - 2*C)*Tan[c/2])/(3*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[
c/2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])^2)
```

Maple [B] time = 2.318, size = 509, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3-12*C*cos(1/2*d*x+1/2*c)^6+4*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*c
os(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+16*C*cos(1/2*d*x+1/2*c)^4+9*A*
cos(1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-3*C*cos(1/2*d*x+1/2*c)^2-A+B-
C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*se
c(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec
(d*x + c) + a)^2, x)
```

3.568 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{(5A-2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(5A-2B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)}{}$$

```
[Out] ((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) - ((5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a^2*d) - ((5*A - 2*B - C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.364301, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4020, 3787, 3771, 2639, 2641}

$$-\frac{(5A-2B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{(5A-2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[
c + d*x])^2),x]
```

```
[Out] ((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) - ((5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*a^2*d) - ((5*A - 2*B - C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\frac{1}{2}a(7A-B+C)-\frac{3}{2}a(A-B-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}}{3a^2}$$

$$= -\frac{(5A-2B-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(5A-2B-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(5A-2B-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B-C)\sqrt{c}}{}$$

Mathematica [C] time = 6.89314, size = 1114, normalized size = 6.05

$$-\frac{8\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+}}\right.}{3d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(\sec(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] (-8*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(3*
d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
+ d*x])^2) + (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*S
qrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^(
```

```
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
a + a*Sec[c + d*x])^2) - (20*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[
c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (8*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c
+ d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (4*C*Cos[c/2 + (d*x)/2]^4
*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c
+ d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d
*x)/2]^4*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(3
*A - B + A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (8*Sec[c/2]*Sec[c/2 +
(d*x)/2]*(7*A*Sin[(d*x)/2] - 4*B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (4
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*
x)/2]))/(3*d) + (16*A*Cos[c]*Sin[d*x])/d + (8*(7*A - 4*B + C)*Tan[c/2])/(3*
d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.51, size = 509, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d
*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*cos(1/
2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*
cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4-2*C*cos(1/2*d*x+1/2*c)^4+15*
A*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2+3*C*cos(1/2*d*x+1/2*c)^2-A+
B-C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{3}+2\,a^{2}\sec\left(dx+c\right)^{2}+a^{2}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*se
c(d*x + c)^3 + 2*a^2*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sec^{\frac{5}{2}}\left(c+dx\right)+2\sec^{\frac{3}{2}}\left(c+dx\right)+\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{B\sec\left(c+dx\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)+2\sec^{\frac{3}{2}}\left(c+dx\right)+\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{C\sec^{2}\left(c+dx\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)+2\sec^{\frac{3}{2}}\left(c+dx\right)+\sqrt{\sec\left(c+dx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2/sec(d*x+c)**
(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x
))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3
/2) + sqrt(sec(c + d*x))), x) + Integral(C*sec(c + d*x)**2/(sec(c + d*x)**(
5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*s
qrt(sec(d*x + c))), x)
```

3.569 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=220

$$\frac{(10A-5B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(10A-5B+2C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A-4B+C}{3a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] -(((7*A - 4*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*d)) + ((10*A - 5*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((10*A - 5*B + 2*C)*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]) - ((7*A - 4*B + C)*Sin[c + d*x])/(3*a^2*d*Sqr
t[Sec[c + d*x]]*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sqrt[
Sec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.402946, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(10A-5B+2C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}+\frac{(10A-5B+2C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^2),x]
```


```
[Out] -(((7*A - 4*B + C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*d)) + ((10*A - 5*B + 2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((10*A - 5*B + 2*C)*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]) - ((7*A - 4*B + C)*Sin[c + d*x])/(3*a^2*d*Sqr
t[Sec[c + d*x]]*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sqrt[
Sec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A-B+C)-\frac{1}{2}a(5A-5B-C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}}{3a^2}$$

$$= -\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx)}$$

$$= -\frac{(7A-4B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx)}$$

$$= \frac{(10A-5B+2C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(7A-4B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{}{3d}$$

$$= -\frac{(7A-4B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(10A-5B}{3a^2d}$$

$$= -\frac{(7A-4B+C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(10A-5B}{}$$

Mathematica [C] time = 6.74433, size = 762, normalized size = 3.46

$$-\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-14\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-14*Sqrt[2
]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2
*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x)
+ (8*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))
/E^(I*d*x) - (2*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*S
qrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(
(2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))]))/E^(I*d*x) - 40*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]] + 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]] - 8*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]] - 2*Sqrt[Sec[c + d*x]]*(3*(5*A - 3*B + C + (2*A - B)*Cos[2*c])*Cos
[d*x]*Csc[c/2]*Sec[c/2] + 2*A*Cos[2*d*x]*Sin[2*c] - 2*(10*A - 7*B + 4*C)*Se
c[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + (A - B + C)*Sec[c/2]*Sec[(c + d*x)/2
]^3*Sin[(d*x)/2] - 12*(2*A - B)*Cos[c]*Sin[d*x] + 2*A*Cos[2*c]*Sin[2*d*x] -
 2*(10*A - 7*B + 4*C)*Tan[c/2] + (A - B + C)*Sec[(c + d*x)/2]^2*Tan[c/2])))
/(3*a^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(1 + Sec[c + d*
x])^2)
```

Maple [A] time = 3.004, size = 472, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+21*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+21*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)+16*A*sin(1/2*d*x+1/2*c)^8+(-76*A+24*B-12*C)*sin(1/2*d*x+1/2*c
)^6+(84*A-34*B+16*C)*sin(1/2*d*x+1/2*c)^4+(-25*A+11*B-5*C)*sin(1/2*d*x+1/2*
c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{4}+2\,a^{2}\sec\left(dx+c\right)^{3}+a^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*se
c(d*x + c)^4 + 2*a^2*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*s
ec(d*x + c)^(3/2)), x)
```

3.570 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=254

$$-\frac{5(3A-2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(3A-2B+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{(56A-35B\cdots}{15a^2d\,\mathrm{s}\cdots}$$

```
[Out] ((56*A - 35*B + 20*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a^2*d) - (5*(3*A - 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((56*A - 35*B + 20*C)*Sin[c +
d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*(3*A - 2*B + C)*Sin[c + d*x])/(3*a
^2*d*Sqrt[Sec[c + d*x]]) - ((3*A - 2*B + C)*Sin[c + d*x])/(a^2*d*Sec[c + d*
x]^(3/2)*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sec[c + d*x]
^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.425377, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(3A-2B+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{(56A-35B+20C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(3A-2B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{5(3A-2B+C)\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^2),x]
```


```
[Out] ((56*A - 35*B + 20*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*a^2*d) - (5*(3*A - 2*B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + ((56*A - 35*B + 20*C)*Sin[c +
d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*(3*A - 2*B + C)*Sin[c + d*x])/(3*a
^2*d*Sqrt[Sec[c + d*x]]) - ((3*A - 2*B + C)*Sin[c + d*x])/(a^2*d*Sec[c + d*
x]^(3/2)*(1 + Sec[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sec[c + d*x]
^(3/2)*(a + a*Sec[c + d*x])^2)
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
```

```
f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Sec[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B+C)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(11A-5B+5C)-\frac{1}{2}a(7A-7B+C)\sec(c+}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}}{3a^2}$$

$$= -\frac{(3A-2B+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= -\frac{(3A-2B+C)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= \frac{(56A-35B+20C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A}{a^2d\sec^{\frac{3}{2}}}$$

$$= \frac{(56A-35B+20C)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B+C)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A}{a^2d\sec^{\frac{3}{2}}}$$

$$= \frac{(56A-35B+20C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5(3A-}{}$$

Mathematica [C] time = 7.22942, size = 1442, normalized size = 5.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] (-112*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(
15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])^2) + (14*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 +
 E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^2) - (8*Sqrt[2]*C*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (20*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Co
s[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (40*B*Cos[c/2 + (d*x)/2
]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec
[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (20*C*Cos[
c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[
c/2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*
d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
 + (Cos[c/2 + (d*x)/2]^4*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*(-((151*A - 100*B + 60*C + 73*A*Cos[2*c] - 40*B*Cos[2*c] + 20*C*Co
s[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (8*(2*A - B)*Cos[2*d*x]*Sin[2*c
])/(3*d) + (4*A*Cos[3*d*x]*Sin[3*c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]
^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (8*Sec[c/2]*
Sec[c/2 + (d*x)/2]*(13*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2] + 7*C*Sin[(d*x)/2
]))/(3*d) + (4*(73*A - 40*B + 20*C)*Cos[c]*Sin[d*x])/(5*d) - (8*(2*A - B)*C
os[2*c]*Sin[2*d*x])/(3*d) + (4*A*Cos[3*c]*Sin[3*d*x])/(5*d) + (8*(13*A - 10
*B + 7*C)*Tan[c/2])/(3*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(
3*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*
x])^2)
```

Maple [A] time = 2.749, size = 491, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(75*A*EllipticF(cos(1/2
```

```
*d*x+1/2*c),2^(1/2))+168*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+60*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(75*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+168*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*cos(1/2*d*x+1/2*c)-96*A*sin(1/2*d*x+1/2*c)^10+(128*A+80*B)*sin(1/2*d*x
+1/2*c)^8+(328*A-380*B+120*C)*sin(1/2*d*x+1/2*c)^6+(-526*A+420*B-170*C)*sin
(1/2*d*x+1/2*c)^4+(171*A-125*B+55*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+
1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{5}+2\,a^{2}\sec\left(dx+c\right)^{4}+a^{2}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*se
c(d*x + c)^5 + 2*a^2*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*s
ec(d*x + c)^(5/2)), x)
```

3.571 $\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=308

$$\frac{(3A-13B+33C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(9A-49B+119C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}-$$

```
[Out] ((9*A - 49*B + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(10*a^3*d) + ((3*A - 13*B + 33*C)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((9*A - 49*B + 119*C)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + ((3*A - 13*B + 33*C)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(9/2)*Sin[c + d*x
])/(5*d*(a + a*Sec[c + d*x])^3) + ((B - 2*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(3*a*d*(a + a*Sec[c + d*x])^2) - ((9*A - 49*B + 119*C)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.62118, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(9A-49B+119C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-13B+33C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6a^3d}-\frac{(9A-49B+119C)\sin}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```


```
[Out] ((9*A - 49*B + 119*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(10*a^3*d) + ((3*A - 13*B + 33*C)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((9*A - 49*B + 119*C)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + ((3*A - 13*B + 33*C)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(9/2)*Sin[c + d*x
])/(5*d*(a + a*Sec[c + d*x])^3) + ((B - 2*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(3*a*d*(a + a*Sec[c + d*x])^2) - ((9*A - 49*B + 119*C)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
```

```
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = -\frac{(A-B+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{1}{2}a(3.$$

$$= -\frac{(A-B+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(B-2C)\sec^{\frac{7}{2}}(c}{3ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(B-2C)\sec^{\frac{7}{2}}(c}{3ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(B-2C)\sec^{\frac{7}{2}}(c}{3ad(a+a\,\mathrm{s}}$$

$$= -\frac{(9A-49B+119C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(3A-13}{}$$

$$= -\frac{(9A-49B+119C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(3A-13}{}$$

$$= \frac{(9A-49B+119C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{10a^3d}$$

Mathematica [C] time = 8.60798, size = 1462, normalized size = 4.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(a + a*Sec[c + d*x])^3) + (98*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-
3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (238*Sqrt[2]*C*Sqrt[E^
(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[
c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)
*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*
d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])
^3) + (4*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c +
d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])^3) - (52*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*Elli
pticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^3) + (44*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*
Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
```

```
 + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^
(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(9*A - 49*B + 119*C)*Cos
[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d
*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d
*x)/2]^3*(3*A*Sin[(d*x)/2] - 8*B*Sin[(d*x)/2] + 13*C*Sin[(d*x)/2]))/(15*d)
+ (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 13*B*Sin[(d*x)/2] + 29
*C*Sin[(d*x)/2]))/(3*d) + (32*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (8*(4
*C + 3*A*Cos[c] - 13*B*Cos[c] + 33*C*Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (8*(3
*A - 8*B + 13*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A - B + C)*Sec
[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 10.019, size = 1040, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/3*(4*
C-2*B)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1
/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-
1)+8*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-4*B+12*C)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(A-B+C)*(1/5*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+4/5*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+18/5*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-8/5*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18/5*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+(8*B-24*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(sec(
d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
 a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec
(d*x + c) + a)^3, x)
```

3.572 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=269

$$\frac{(A+3B-13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A+3B-13C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A+9B}{}$$

```
[Out] ((A + 9*B - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(10*a^3*d) + ((A + 3*B - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A + 9*B - 49*C)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(10*a^3*d) - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])
/(5*d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B - 8*C)*Sec[c + d*x]^(5/2)*Sin[c
 + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((A + 3*B - 13*C)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.584145, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4019, 3787, 3771, 2641, 3768, 2639}

$$\frac{(A+3B-13C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A+9B-49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A+3B-13C)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```

```
[Out] ((A + 9*B - 49*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(10*a^3*d) + ((A + 3*B - 13*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A + 9*B - 49*C)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(10*a^3*d) - ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])
/(5*d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B - 8*C)*Sec[c + d*x]^(5/2)*Sin[c
 + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((A + 3*B - 13*C)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
```

```
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A\right.}{}}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B-8C)}{15ad(a}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B-8C)}{15ad(a}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B-8C)}{15ad(a}$$

$$= -\frac{(A+9B-49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{(A-B+C)}{5d(a}$$

$$= \frac{(A+3B-13C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{6a^3d}$$

$$= \frac{(A+9B-49C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{10a^3d}$$

Mathematica [C] time = 7.40387, size = 1430, normalized size = 5.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])^3) - (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-
3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (98*Sqrt[2]*C*Sqrt[E^(I
*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/
2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(
-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*S
ec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*
x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3
) + (4*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*
x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])^3) + (4*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*Ellip
ticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(a + a*Sec[c + d*x])^3) - (52*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc
[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(
3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(A + 9*B - 49*C)*Cos[d*x]
*Csc[c/2]*Sec[c/2])/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2]
+ 3*B*Sin[(d*x)/2] - 13*C*Sin[(d*x)/2]))/(3*d) + (8*Sec[c/2]*Sec[c/2 + (d*x
)/2]^3*(2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2] - 8*C*Sin[(d*x)/2]))/(15*d) - (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d
*x)/2]))/(5*d) - (8*(-A - 3*B + 13*C)*Tan[c/2])/(3*d) + (8*(2*A + 3*B - 8*C
)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]
^4*Tan[c/2])/(5*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
+ a*Sec[c + d*x])^3)
```

Maple [B] time = 3.349, size = 789, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*EllipticF(c
```

```
os(1/2*d*x+1/2*c),2^(1/2))-27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-65*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(A+9*B-49*C)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+147*B-817*C)*sin(1/2*d*x+1/2*c)^6+6*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+43*B-248*C)*sin(1/2*d
*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+69*B-43
9*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(sec(
d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
 a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^3, x)
```

3.573 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=231

$$\frac{(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A-B-9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(A-B-$$

```
[Out] -((A - B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((4*A + B - 6*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((A - B - 9*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.548874, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4019, 3787, 3771, 2639, 2641}

$$\frac{(A-B-9C)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B-9C)\sqrt{\text{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```


```
[Out] -((A - B - 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((A + B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((4*A + B - 6*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((A - B - 9*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(7A+3\right.}{(a+}}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(4A+B-6C)\sec^{\frac{3}{2}}}{15ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(4A+B-6C)\sec^{\frac{3}{2}}}{15ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(4A+B-6C)\sec^{\frac{3}{2}}}{15ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(4A+B-6C)\sec^{\frac{3}{2}}}{15ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B-9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+$$

Mathematica [C] time = 7.10411, size = 1425, normalized size = 6.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(a + a*Sec[c + d*x])^3) - (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3
*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (6*Sqrt[2]*C*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
+ (4*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
 + d*x])^3) + (4*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*Ellipti
cF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[
c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2
)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(A - B - 9*C)*Cos[d*x]*Csc[c/
2]*Sec[c/2])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2] - 2
*B*Sin[(d*x)/2] - 3*C*Sin[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2
]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]
*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/(
3*d) + (8*(A + B + 3*C)*Tan[c/2])/(3*d) - (8*(7*A - 2*B - 3*C)*Sec[c/2 + (d
*x)/2]^2*Tan[c/2])/(15*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(
5*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*
x])^3)
```

Maple [B] time = 2.55, size = 624, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8+10*B*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8+30*C*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-54*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-2*A*cos(1/2*d*x+1/2*c)^6+22*B*cos(1/2*d*x+1/2*c)^6+138*C*cos(1/2*d*x+1/2*
```

```
c)^6-24*A*cos(1/2*d*x+1/2*c)^4-6*B*cos(1/2*d*x+1/2*c)^4-24*C*cos(1/2*d*x+1/
2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-7*B*cos(1/2*d*x+1/2*c)^2-3*C*cos(1/2*d*x+1
/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(sec(d*
x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^3, x)
```

3.574 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=231

$$\frac{(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(3A+B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(9A+B-C}{}$$

```
[Out] -((9*A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*A - B - 4*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + B + C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.555442, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(3A+B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A+B-C)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```

```
[Out] -((9*A + B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) + ((3*A + B + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*A - B - 4*C)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + B + C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(9\right.}{}}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-B-4C)\sqrt{}}{15ad(a}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-B-4C)\sqrt{}}{15ad(a}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-B-4C)\sqrt{}}{15ad(a}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-B-4C)\sqrt{}}{15ad(a}$$

$$= -\frac{(9A+B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{10a^3d}$$

Mathematica [C] time = 7.20596, size = 1431, normalized size = 6.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
a + a*Sec[c + d*x])^3) + (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*
Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeome
tric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (2*Sqrt[2]*C*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
+ (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
+ (4*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
 d*x])^3) + (4*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF
[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/
2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(3/2
)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(9*A + B - C)*Cos[d*x]*Csc[c/
2]*Sec[c/2])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] - B*S
in[(d*x)/2] - C*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/
2 + (d*x)/2]^3*(12*A*Sin[(d*x)/2] - 7*B*Sin[(d*x)/2] + 2*C*Sin[(d*x)/2]))/(
15*d) - (8*(9*A - B - C)*Tan[c/2])/(3*d) + (8*(12*A - 7*B + 2*C)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])
/(5*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
d*x])^3)
```

Maple [B] time = 2.505, size = 624, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*A*cos
```

```
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8+10*B*
cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8+10*C*cos(1/2*d*x+1/2*c)^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-198*A*cos(1/2*d*x+1/2*c)^6-2*B*cos(1/2*d*x+1/2*c)^6+22*C*cos(1/2*d*x+1/2*
c)^6+114*A*cos(1/2*d*x+1/2*c)^4-24*B*cos(1/2*d*x+1/2*c)^4-6*C*cos(1/2*d*x+1
/2*c)^4-27*A*cos(1/2*d*x+1/2*c)^2+17*B*cos(1/2*d*x+1/2*c)^2-7*C*cos(1/2*d*x
+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")

[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*se
c(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec
(d*x + c) + a)^3, x)
```

3.575 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=241

$$-\frac{(13A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(13A-3B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49\cdots}{\cdots}$$

```
[Out] ((49*A - 9*B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(10*a^3*d) - ((13*A - 3*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((8*A - 3*B - 2*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - 3*B - C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.562311, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4084, 4020, 3787, 3771, 2639, 2641}

$$-\frac{(13A-3B-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] ((49*A - 9*B - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(10*a^3*d) - ((13*A - 3*B - C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B + C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((8*A - 3*B - 2*C)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - 3*B - C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A-B+C)-\frac{5}{2}a(A-B-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{\sec(c+dx)}\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{\sec(c+dx)}\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{\sec(c+dx)}\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(8A-3B-2C)\sqrt{\sec(c+dx)}\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(49A-9B-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-3B-C}{}$$

Mathematica [C] time = 7.53288, size = 1449, normalized size = 6.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^3),x]
```

```
[Out] (-98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c
```

```
+ d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^3) + (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(
-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (2*Sqrt[2]*C*Sqrt[E^(I
*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/
2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(
-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*S
ec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*
x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3
) - (52*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d
*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Se
c[c + d*x])^3) + (4*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*Elli
pticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc
[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(
3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(39*A - 9*B - C + 10*A*Co
s[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*
(23*A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2
]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/
(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(17*A*Sin[(d*x)/2] - 12*B*Sin[(d*x
)/2] + 7*C*Sin[(d*x)/2]))/(15*d) + (32*A*Cos[c]*Sin[d*x])/d + (8*(23*A - 9*
B + C)*Tan[c/2])/(3*d) - (8*(17*A - 12*B + 7*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/
2])/(15*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.616, size = 624, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos
(1/2*d*x+1/2*c)^8+130*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+294*
A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8
-30*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-54*B*cos(1/2*d*x+1/2*c
)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C*cos(1/2*d*x+1/
2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*
d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4+24*C*cos
(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2-17*C*
cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
```

```
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{4}+3\,a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+a^{3}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*se
c(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + a^3*sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*s
qrt(sec(d*x + c))), x)
```

3.576 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=274

$$\frac{(33A-13B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(33A-13B+3C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(119A-}{30d\sqrt{\sec(c+}}$$

```
[Out] -((119*A - 49*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) + ((33*A - 13*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((33*A - 13*B + 3*C)*Sin[c
 + d*x])/(6*a^3*d*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sqr
t[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*S
qrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - ((119*A - 49*B + 9*C)*Sin[c + d
*x])/(30*d*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.599819, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{(33A-13B+3C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(119A-49B+9C)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{(33A-13B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] -((119*A - 49*B + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) + ((33*A - 13*B + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((33*A - 13*B + 3*C)*Sin[c
 + d*x])/(6*a^3*d*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sqr
t[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*S
qrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - ((119*A - 49*B + 9*C)*Sin[c + d
*x])/(30*d*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
```

```
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B+3C)-\frac{1}{2}a(7A-7B-3C)\sec(c}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+d}$$

$$= \frac{(33A-13B+3C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{}{3}$$

$$= -\frac{(119A-49B+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(33A}{}$$

$$= -\frac{(119A-49B+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(33A}{}$$

Mathematica [C] time = 7.55158, size = 1497, normalized size = 5.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^3),x]
```

```
[Out] (238*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^3) - (98*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*
(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (6*Sqrt[2]*C*Sqrt[E^
(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[
c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)
*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d
*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^
3) + (44*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c +
d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])^3) - (52*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*Elli
pticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
```

```
])*(a + a*Sec[c + d*x])^3) + (4*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*C
sc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(
3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(89*A - 39*B + 9*C + 30*A*
Cos[2*c] - 10*B*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (16*A*Cos[2*d
*x]*Sin[2*c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*
Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(43*
A*Sin[(d*x)/2] - 23*B*Sin[(d*x)/2] + 9*C*Sin[(d*x)/2]))/(3*d) + (8*Sec[c/2]
*Sec[c/2 + (d*x)/2]^3*(22*A*Sin[(d*x)/2] - 17*B*Sin[(d*x)/2] + 12*C*Sin[(d*
x)/2]))/(15*d) - (32*(3*A - B)*Cos[c]*Sin[d*x])/d + (16*A*Cos[2*c]*Sin[2*d*
x])/(3*d) - (8*(43*A - 23*B + 9*C)*Tan[c/2])/(3*d) + (8*(22*A - 17*B + 12*C
)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]
^4*Tan[c/2])/(5*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
+ a*Sec[c + d*x])^3)
```

Maple [B] time = 2.704, size = 638, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*co
s(1/2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-348*B*cos(1/2*d*x+1/2*c)^8-130*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*C*cos(1/2*
d*x+1/2*c)^8+30*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*C*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^6+578*B
*cos(1/2*d*x+1/2*c)^6-198*C*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4
-264*B*cos(1/2*d*x+1/2*c)^4+114*C*cos(1/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2
*c)^2+37*B*cos(1/2*d*x+1/2*c)^2-27*C*cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{5}+3\,a^{3}\sec\left(dx+c\right)^{4}+3\,a^{3}\sec\left(dx+c\right)^{3}+a^{3}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*se
c(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + a^3*sec(d*x +
c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*s
ec(d*x + c)^(3/2)), x)
```

3.577 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$-\frac{(63A-33B+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(63A-33B+13C)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33\ldots}{}$$

```
[Out] (7*(33*A - 17*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) - ((63*A - 33*B + 13*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(33*A - 17*B + 7*C)*Si
n[c + d*x])/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((63*A - 33*B + 13*C)*Sin[c + d
*x])/(6*a^3*d*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sec[c +
 d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - ((12*A - 7*B + 2*C)*Sin[c + d*x])/(15
*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - ((63*A - 33*B + 13*C)*Sin
[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.638633, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4084, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{(63A-33B+13C)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33A-17B+7C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(63A-33B+13C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{(63A-3\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] (7*(33*A - 17*B + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(10*a^3*d) - ((63*A - 33*B + 13*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(33*A - 17*B + 7*C)*Si
n[c + d*x])/(30*a^3*d*Sec[c + d*x]^(3/2)) - ((63*A - 33*B + 13*C)*Sin[c + d
*x])/(6*a^3*d*Sqrt[Sec[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sec[c +
 d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - ((12*A - 7*B + 2*C)*Sin[c + d*x])/(15
*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - ((63*A - 33*B + 13*C)*Sin
[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
```

```
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{5}{2}a(3A-B+C)-\frac{1}{2}a(9A-9B-C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B+2C)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B+2C)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B+2C)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= \frac{7(33A-17B+7C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(63A-33B+13C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A}{5d\sec^{\frac{3}{2}}}$$

$$= \frac{7(33A-17B+7C)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(63A-33B+13C)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A}{5d\sec^{\frac{3}{2}}}$$

$$= \frac{7(33A-17B+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(63A-33}{}$$

Mathematica [C] time = 7.74251, size = 1555, normalized size = 4.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + 
a*Sec[c + d*x])^3),x]
```

```
[Out] (-154*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
 d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2))/(5*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^3) + (238*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]
*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(I*d*x)*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (98*Sqrt[2]*C*Sqrt[
E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Co
s[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*
x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))
])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(15*d*E^(
I*d*x)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x
])^3) - (84*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c
 + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*
Sec[c + d*x])^3) + (44*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*E
```

```
llipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sin[c])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*(a + a*Sec[c + d*x])^3) - (52*C*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]
*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*Sin[c])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*
x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(329*A - 178*B + 78*C
 + 133*A*Cos[2*c] - 60*B*Cos[2*c] + 20*C*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/
2])/(5*d) - (16*(3*A - B)*Cos[2*d*x]*Sin[2*c])/(3*d) + (8*A*Cos[3*d*x]*Sin[
3*c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x
)/2] + C*Sin[(d*x)/2]))/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(27*A*Sin[
(d*x)/2] - 22*B*Sin[(d*x)/2] + 17*C*Sin[(d*x)/2]))/(15*d) + (8*Sec[c/2]*Sec
[c/2 + (d*x)/2]*(69*A*Sin[(d*x)/2] - 43*B*Sin[(d*x)/2] + 23*C*Sin[(d*x)/2])
)/(3*d) + (8*(133*A - 60*B + 20*C)*Cos[c]*Sin[d*x])/(5*d) - (16*(3*A - B)*C
os[2*c]*Sin[2*d*x])/(3*d) + (8*A*Cos[3*c]*Sin[3*d*x])/(5*d) + (8*(69*A - 43
*B + 23*C)*Tan[c/2])/(3*d) - (8*(27*A - 22*B + 17*C)*Sec[c/2 + (d*x)/2]^2*T
an[c/2])/(15*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.47, size = 666, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*co
s(1/2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10+160*B*cos(1/2*d*x+1/2*c)^10
-228*A*cos(1/2*d*x+1/2*c)^8-630*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-1386*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*B*cos(1/2*d
*x+1/2*c)^8+330*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+714*B*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C
*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-294*C*cos(1/2*d*x+1/2*c)^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6-1058*B*cos(1/2*d*x+1/
2*c)^6+578*C*cos(1/2*d*x+1/2*c)^6-744*A*cos(1/2*d*x+1/2*c)^4+474*B*cos(1/2*
d*x+1/2*c)^4-264*C*cos(1/2*d*x+1/2*c)^4+57*A*cos(1/2*d*x+1/2*c)^2-47*B*cos(
1/2*d*x+1/2*c)^2+37*C*cos(1/2*d*x+1/2*c)^2-3*A+3*B-3*C)/cos(1/2*d*x+1/2*c)^
5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^6+3\,a^3\sec\left(dx+c\right)^5+3\,a^3\sec\left(dx+c\right)^4+a^3\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*se
c(d*x + c)^6 + 3*a^3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + a^3*sec(d*x +
c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*s
ec(d*x + c)^(5/2)), x)
```

3.578 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\,s\right.$

Optimal. Leaf size=227

$$\frac{a(48A+40B+35C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+40B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+3}{}$$

```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(64*d) + (a*(48*A + 40*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A + 40*B + 35*C)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*B + C)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^
(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.518342, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4016, 3803, 3801, 215}

$$\frac{a(48A+40B+35C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+40B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(64*d) + (a*(48*A + 40*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A + 40*B + 35*C)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*B + C)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^
(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{4d}$$

$$= \frac{a(8B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{C\,\mathrm{se}}{}$$

$$= \frac{a(48A+40B+35C)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a(48A+40B+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{64d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a(48A+40B+35C)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{64d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{a}(48A+40B+35C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a+a\sec(c+d}}\right.}{64d}$$

Mathematica [A] time = 2.10504, size = 179, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((432A+77(8B+7C))\cos(c+dx)+4(48A+40B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(24*Sqrt[2]
*(48*A + 40*B + 35*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + 4*
(192*A + 160*B + 332*C + (432*A + 77*(8*B + 7*C))*Cos[c + d*x] + 4*(48*A +
40*B + 35*C)*Cos[2*(c + d*x)] + 144*A*Cos[3*(c + d*x)] + 120*B*Cos[3*(c + d
```

```
*x)] + 105*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(3072*d)
```

Maple [B] time = 0.443, size = 638, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/768/d*(144*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*2^(1/2)-144*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+120*B*cos(d*x+c)^4
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^
(1/2)-120*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))+105*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^4-105*C*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)
^4+288*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+240*B*sin(d*x+c)
*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+210*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(
cos(d*x+c)+1))^(1/2)+192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+160*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+140*C*sin(d*x+c)*
cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+128*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2)+112*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+96*
C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(5/2)*(a*(cos(d*x+c)
+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(co
s(d*x+c)^2-1)
```

Maxima [B] time = 4.19588, size = 8764, normalized size = 38.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/768*(48*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(
7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*
sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*
(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
```

), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt
(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + s
in(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c
)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1) + 8*(60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*
d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3
*sqrt(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1
68*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2
*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*si
n(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*c
os(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c)
+ 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin
(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d
*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin
(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2
 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*
x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x
 + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 +
 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x
+ 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) + 2) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*
cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x

+ 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))
) - 20*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 1
68*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(
sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(
2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*co
s(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(3*cos(
4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^
2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*
cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + (420*(sqrt(2)
*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c)
+ 4*sqrt(2)*sin(2*d*x + 2*c))*cos(15/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 140*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*
sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(13/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 1596*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x +
6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 500*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt
(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2
*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 500*(sqrt(2)*sin(8*d*x
+ 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2
)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 1596*(sq
rt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x +
4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x +
c))) - 140*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt
(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 420*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x
+ 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*
d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2
+ 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos
(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d
*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x +
4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c
)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin
(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*
sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 2) + 105*(2*(4*cos(6*d*x + 6*c) + 6*co
s(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c
)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16
*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos
(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*
(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x

```
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
+ 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 105*(2*(4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x +
 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36
*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin
(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 +
 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d
*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 105*(2*(4*cos(6*d*x + 6*c
) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)
^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin
(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 420*(sqrt(2)*cos(8*d*
x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(15/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 140*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)
*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(13/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 1596*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*c
os(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 500*(sqrt(2)*cos(
8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*
sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x +
 c))) + 500*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt
(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1596*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)
*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 140*(sqrt(2)*cos
(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4
*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 420*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqr
t(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4
*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2
 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*co
s(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*
d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*
sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*
c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*si
n(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 2.39991, size = 1338, normalized size = 5.89

$$\left[\frac{3\left((48A+40B+35C)\cos(dx+c)^4+(48A+40B+35C)\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-\right.}{}}{\cos(dx+c)^3+}\right.}{768\left(d\cos(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*((48*A + 40*B + 35*C)*cos(d*x + c)^4 + (48*A + 40*B + 35*C)*cos(d
*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
+ c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*(3*(48*A + 40*B + 35*C)*cos(d*x + c)^3 + 2*(48*A + 40*B + 35*C)*cos(d*x +
c)^2 + 8*(8*B + 7*C)*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^
3), 1/384*(3*((48*A + 40*B + 35*C)*cos(d*x + c)^4 + (48*A + 40*B + 35*C)*co
s(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
2*a)) + 2*(3*(48*A + 40*B + 35*C)*cos(d*x + c)^3 + 2*(48*A + 40*B + 35*C)*c
os(d*x + c)^2 + 8*(8*B + 7*C)*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(
d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
sec(d*x + c)^(5/2), x)
```

3.579 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx)+C\sec$

Optimal. Leaf size=179

$$\frac{a(8A+6B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+6B+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(6B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+}{12d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(8*d) + (a*(8*A + 6*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*
d*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
 d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.422774, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4016, 3803, 3801, 215}

$$\frac{a(8A+6B+5C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+6B+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(6B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(8*d) + (a*(8*A + 6*B + 5*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*
d*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*B + C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
 d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c\ldots}{3d}$$

$$= \frac{a(6B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \ldots$$

$$= \frac{a(8A+6B+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+\ldots}{8d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a(8A+6B+5C)\sec^{\frac{3}{2}}(c+dx)\sin(c+\ldots}{8d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{a}(8A+6B+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx\ldots}{\sqrt{a+a\sec(c+d\ldots}}\right.}{8d}$$

Mathematica [A] time = 1.18962, size = 141, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+6B+5C)\cos(2(c+dx))+24A+4(6B+\ldots\right.}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])]*(12*Sqrt[2]
*(8*A + 6*B + 5*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 4*(24
*A + 18*B + 31*C + 4*(6*B + 5*C)*Cos[c + d*x] + 3*(8*A + 6*B + 5*C)*Cos[2*(
c + d*x)])*Sin[(c + d*x)/2]))/(192*d)
```

Maple [B] time = 0.433, size = 543, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(-24*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+24*A*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)-18*
B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*c
os(d*x+c)^3*2^(1/2)+18*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)-15*C*arctan(1/4*2^(1/2)*(-2/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+15*C*arct
an(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x
+c)^3*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+36*B*c
os(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+30*C*sin(d*x+c)*cos(d*x+c)
^2*(-2/(cos(d*x+c)+1))^(1/2)+24*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2)+20*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*C*(-2/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(3/2)*(a*(cos(d*x+c)+1)/cos(d*x+
c))^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)
```

Maxima [B] time = 3.02556, size = 5403, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)
*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2
*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
```

```
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
+ 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos
(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*
cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arct
an2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c
)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*
d*x + 2*c) + 1) + (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c
) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*
sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(
2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*
c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*d*x +
 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(
2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x +
4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
 + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos
(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) +
 cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4
*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*
c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
 - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + c
os(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d
```

```
*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos
(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x
 + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 6
0*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(s
qrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(
2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*c
os(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(6*
d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*d*x +
 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.49327, size = 1200, normalized size = 6.7

$$\left[\frac{3\left((8A+6B+5C)\cos(dx+c)^3+(8A+6B+5C)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\cdots}{\sqrt{\cos(d\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.\cdots}{96\left(d\cos(dx+c)^3+d\cos(dx+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((8*A + 6*B + 5*C)*cos(d*x + c)^3 + (8*A + 6*B + 5*C)*cos(d*x + c)
^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
- 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(8*
A + 6*B + 5*C)*cos(d*x + c)^2 + 2*(6*B + 5*C)*cos(d*x + c) + 8*C)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
+ c)^3 + d*cos(d*x + c)^2), 1/48*(3*((8*A + 6*B + 5*C)*cos(d*x + c)^3 + (8*
A + 6*B + 5*C)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
```

```
a*cos(d*x + c) - 2*a)) + 2*(3*(8*A + 6*B + 5*C)*cos(d*x + c)^2 + 2*(6*B + 5
*C)*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^2 + B \sec \left(dx+c\right) + A\right) \sqrt{a \sec \left(dx+c\right) + a} \sec \left(dx+c\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
sec(d*x + c)^(3/2), x)
```

3.580 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\text{ s}\right.$

Optimal. Leaf size=131

$$\frac{\sqrt{a}(8A+4B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+}}{2d}$$

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(4*d) + (a*(4*B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a
 + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rubi [A] time = 0.33842, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4088, 4016, 3801, 215}

$$\frac{\sqrt{a}(8A+4B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]])/(4*d) + (a*(4*B + C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a
 + a*Sec[c + d*x]]) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c}{2d}$$

$$= \frac{a(4B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \cdots$$

$$= \frac{a(4B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \cdots$$

$$= \frac{\sqrt{a}(8A+4B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a+a\sec(c+a}}\right.}{4d}$$

Mathematica [A] time = 0.838574, size = 109, normalized size = 0.83

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(8A+4B+3C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(\right.}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(8*A + 4*B + 3*C)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*B + 3*C + 2*C*Sec[c + d*
x])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.47, size = 452, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/16/d*(8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))*2^(1/2)*cos(d*x+c)^2-8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2+4*B*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2-4
*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*
2^(1/2)*cos(d*x+c)^2+3*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2-3*C*arctan(1/4*2^(1/2)*(-2/(cos(
```

```
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2+8*B*cos(d*
x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+6*C*sin(d*x+c)*cos(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)+4*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))
^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*
x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.60528, size = 2925, normalized size = 22.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2)) - 4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*
d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d
*x + 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))
*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1) - (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
```

```
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)
*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(s
in(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(s
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.48598, size = 1077, normalized size = 8.22

$$\left[\frac{\left((8A+4B+3C)\cos(dx+c)^2+(8A+4B+3C)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\cdots}{\sqrt{\cos(\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right.}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*A + 4*B + 3*C)*cos(d*x + c)^2 + (8*A + 4*B + 3*C)*cos(d*x + c))*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((4*B + 3*
C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((8*A + 4*B
+ 3*C)*cos(d*x + c)^2 + (8*A + 4*B + 3*C)*cos(d*x + c))*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*((4*B + 3*C)*cos(d*x +
c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
sqrt(sec(d*x + c)), x)
```

3.581 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=119

$$\frac{a(2A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(2B+C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{C\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)}}{d}$$

```
[Out] (Sqrt[a]*(2*B + C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (a*(2*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d
*x]]) + (C*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.330895, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4088, 4015, 3801, 215}

$$\frac{a(2A-C)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(2B+C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{C\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[Sec[c + d*x]],x]
```


```
[Out] (Sqrt[a]*(2*B + C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (a*(2*A - C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d
*x]]) + (C*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx = \frac{C\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}+\int\frac{\sqrt{a+a}}{}$$

$$=\frac{a(2A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{C\sqrt{\sec(c+dx)}\sqrt{}}{}$$

$$=\frac{a(2A-C)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{C\sqrt{\sec(c+dx)}\sqrt{}}{}$$

$$=\frac{\sqrt{a}(2B+C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a(2A-C)\sqrt{\sec(c}}{d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 0.582514, size = 94, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2A+C\sec(c+dx))+\sqrt{2}(2B+C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2*B + C)*ArcTanh[Sqr
t[2]*Sin[(c + d*x)/2]] + 2*(2*A + C*Sec[c + d*x])*Sin[(c + d*x)/2]))/(2*d*S
qrt[Sec[c + d*x]])
```

Maple [B] time = 0.445, size = 344, normalized size = 2.9

$$-\frac{1}{4d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2B\cos(dx+c)\sqrt{2}\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-2(\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*B*cos(d*x+c)*2^(1/2)*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1+sin(d*x+c)))-2*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d
*x+c)))+C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-C*cos(d*x+c)
*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
```

```
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+8*A*cos(d*x+c)^2-8*A*cos(d*x+c)+
4*C*cos(d*x+c)-4*C)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.35329, size = 1246, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + 2*B*sqrt(a)*(log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)) - (4*sqrt(2)*cos(3/2*arc
tan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.783009, size = 919, normalized size = 7.72

$$\left[\frac{((2B+C)\cos(dx+c)+2B+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{4(d\cos(dx+c)+d)}+\frac{4(2A\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*B + C)*cos(d*x + c) + 2*B + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1
/2*(((2*B + C)*cos(d*x + c) + 2*B + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
 c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
sqrt(sec(d*x + c)), x)
```

3.582 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.320724, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4086, 4015, 3801, 215}

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(3/2),x]
```


```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{1}{2}\right.}{\sqrt{}}}{}$$

$$= \frac{2a(A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(}}{3d\sqrt{\mathrm{se}}}$$

$$= \frac{2a(A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(}}{3d\sqrt{\mathrm{se}}}$$

$$= \frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a(A+3B)\sqrt{\sec(c+d}}{3d\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.723258, size = 94, normalized size = 0.78

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(A\cos(c+dx)+2A+3B)+3\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*C*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]] + 2*(2*A + 3*B + A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d*S
qrt[Sec[c + d*x]])
```

Maple [B] time = 0.431, size = 210, normalized size = 1.8

$$-\frac{(\cos(dx+c))^2}{6d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-3C\sqrt{-2(\cos(dx+c)+1)^{-1}}\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-3*C*(-2/(cos(d*x+c)+1))^(1/2)*
2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))*sin(d*x+c)+3*C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+4*A*cos(d*x+c
)^2+4*A*cos(d*x+c)+12*B*cos(d*x+c)-8*A-12*B)*cos(d*x+c)^2*(1/cos(d*x+c))^(3
/2)/sin(d*x+c)
```

Maxima [B] time = 2.23262, size = 504, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*
sqrt(a)*sin(1/2*d*x + 1/2*c) + 3*C*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [A] time = 0.597208, size = 949, normalized size = 7.91

$$\left[\frac{3\,(C\cos(dx+c)+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(A\cos(dx+c)^2+(2\,A\cdots\right.}{\cdots}}{6\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(A*cos(d*x + c)^2 + (2*A + 3*B)*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c) + d), 1/3*(3*(C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)) + 2*(A*cos(d*x + c)^2 + (2*A + 3*B)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c) + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\,(\sec(c+dx)+1)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2
)/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
sec(d*x + c)^(3/2), x)
```

3.583 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=129

$$\frac{2a(7A+5B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*a*(7*A + 5*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*
x]^(3/2)) + (2*(A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[
Sec[c + d*x]])
```

Rubi [A] time = 0.353531, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4086, 4013, 3804}

$$\frac{2a(7A+5B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(5/2),x]
```



```
[Out] (2*a*(7*A + 5*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*
x]^(3/2)) + (2*(A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[
Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{1}{2}\right.}{}}{}$$

$$= \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(A+5B)\sqrt{a+a}}{15d\sqrt{}}$$

$$= \frac{2a(7A+5B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a}}{}$$

Mathematica [A] time = 0.499872, size = 77, normalized size = 0.6

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(4A+5B)\cos(c+dx)+3A\cos(2(c+dx))+19A+20B+30C)}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((19*A + 20*B + 30*C + 2*(4*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.426, size = 99, normalized size = 0.8

$$-\frac{(-2+2\cos(dx+c))\left(3A(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+c)+8A+10B+15C\right)(\cos(dx+c))^3}{15d\sin(dx+c)}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+8*A
+10*B+15*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))
^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.22649, size = 452, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
```

```
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 + 10*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a) + 120*sqrt(2)*
C*sqrt(a)*sin(1/2*d*x + 1/2*c))/d
```

Fricas [A] time = 0.482774, size = 251, normalized size = 1.95

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{3}+\left(4\,A+5\,B\right)\cos\left(dx+c\right)^{2}+\left(8\,A+10\,B+15\,C\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^3 + (4*A + 5*B)*cos(d*x + c)^2 + (8*A + 10*B + 15*C)
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos
(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
sec(d*x + c)^(5/2), x)
```

3.584 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{4a(24A+28B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*(A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sq
rt[a + a*Sec[c + d*x]]) + (4*a*(24*A + 28*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.42577, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4086, 4015, 3805, 3804}

$$\frac{4a(24A+28B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(7/2),x]
```

```
[Out] (2*a*(A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sq
rt[a + a*Sec[c + d*x]]) + (4*a*(24*A + 28*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\sec(c+d}}{}}{}$$

$$= \frac{2a(A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\operatorname{se}}}{7d\,\mathrm{s}}$$

$$= \frac{2a(A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(24A+2}{105d\sqrt{\sec(c}}$$

$$= \frac{2a(A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(24A+2}{105d\sqrt{\sec(c}}$$

Mathematica [A] time = 0.866304, size = 99, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((141A+28(4B+5C))\cos(c+dx)+6(6A+7B)\cos(2(c+dx))+15A\cos(3(c+}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((228*A + 266*B + 280*C + (141*A + 28*(4*B + 5*C))*Cos[c + d*x] + 6*(6*A +
7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*T
an[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.422, size = 130, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^3+18A(\cos(dx+c))^2+21B(\cos(dx+c))^2+24A\cos(dx+c)+28B\cos}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2)
,x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+35*C*cos(d*x+c)+48*A+56*B+70*C)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.34806, size = 822, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) + 14*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x
+ 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*c
os(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))) + 6*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))))*B*sqrt(a) + 140*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x +
 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C
*sqrt(a))/d
```

Fricas [A] time = 0.488662, size = 312, normalized size = 1.75

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{4}+3\left(6\,A+7\,B\right)\cos\left(dx+c\right)^{3}+\left(24\,A+28\,B+35\,C\right)\cos\left(dx+c\right)^{2}+2\left(24\,A+28\,B+35\,C\right)\cos\left(dx+\right.\right.}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^4 + 3*(6*A + 7*B)*cos(d*x + c)^3 + (24*A + 28*B +
35*C)*cos(d*x + c)^2 + 2*(24*A + 28*B + 35*C)*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x +
c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
sec(d*x + c)^(7/2), x)
```

3.585 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=226

$$\frac{2a(16A+18B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+18B+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+18B+21C)\sin(c}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+}$$

```
[Out] (2*a*(A + 9*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(315*d*Sq
rt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a*(16*A + 18*B + 21*C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.51009, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4086, 4015, 3805, 3804}

$$\frac{2a(16A+18B+21C)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+18B+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+18B+21C)\sin(c}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(9/2),x]
```


```
[Out] (2*a*(A + 9*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(315*d*Sq
rt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a*(16*A + 18*B + 21*C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{\sqrt{a+a\sec(c+d}}{}}{}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\,\mathrm{se}}}{9d\,\mathrm{s}}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+1}{105d\sec^{\frac{3}{2}}(c}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+1}{105d\sec^{\frac{3}{2}}(c}$$

$$= \frac{2a(A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+1}{105d\sec^{\frac{3}{2}}(c}$$

Mathematica [A] time = 1.35198, size = 121, normalized size = 0.54

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((752A+846B+672C)\cos(c+dx)+4(83A+54B+63C)\cos(2(c+dx))+80A}{1260d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((1321*A + 1368*B + 1596*C + (752*A + 846*B + 672*C)*Cos[c + d*x] + 4*(83*A
 + 54*B + 63*C)*Cos[2*(c + d*x)] + 80*A*Cos[3*(c + d*x)] + 90*B*Cos[3*(c +
d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])
/(1260*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.441, size = 163, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(35A(\cos(dx+c))^4+40A(\cos(dx+c))^3+45B(\cos(dx+c))^3+48A(\cos(dx+c))^2+54B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2)
,x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+40*A*cos(d*x+c)^3+45*B*cos(d*x+
c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+64*A*cos(d*x+c)+
72*B*cos(d*x+c)+84*C*cos(d*x+c)+128*A+144*B+168*C)*(a*(cos(d*x+c)+1)/cos(d*
x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.36974, size = 1185, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(1890*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c)))*sin(9/2*d*x + 9/2*c) + 420*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 252*cos(4/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 45*cos(2/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 1890*c
os(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2
*c))) - 420*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) - 252*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) - 45*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*sin(9/2*d*x + 9/2*c) + 45*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) + 18*sqrt(2)*(105*cos(6/7*
arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) +
 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*
x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))
*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 7*cos(7/2*d*x + 7/2*c)*sin
(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 10*sin(7/2*d*x
+ 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) +
 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 105*sin(
1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*B*sqrt(a) + 84*sq
rt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(
5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin(2/5*ar
ctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x + 5/2*c)
 + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(
1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.494847, size = 369, normalized size = 1.63

$$\frac{2\left(35\,A\cos\left(dx+c\right)^{5}+5\,(8\,A+9\,B)\cos\left(dx+c\right)^{4}+3\,(16\,A+18\,B+21\,C)\cos\left(dx+c\right)^{3}+4\,(16\,A+18\,B+21\,C)\cos\left(dx\right.}{315\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*A*cos(d*x + c)^5 + 5*(8*A + 9*B)*cos(d*x + c)^4 + 3*(16*A + 18*B
+ 21*C)*cos(d*x + c)^3 + 4*(16*A + 18*B + 21*C)*cos(d*x + c)^2 + 8*(16*A +
18*B + 21*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
sec(d*x + c)^(9/2), x)
```

3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=283

$$\frac{a^2(80A+90B+67C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+150B+133C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+150B}{128}$$

```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]])/(128*d) + (a^2*(176*A + 150*B + 133*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(128*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(176*A + 150*B + 133*C
)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*
(80*A + 90*B + 67*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec
[c + d*x]]) + (a*(10*B + 3*C)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(40*d) + (C*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d)
```

Rubi [A] time = 0.748112, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(80A+90B+67C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+150B+133C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+150B}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```



```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]])/(128*d) + (a^2*(176*A + 150*B + 133*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(128*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(176*A + 150*B + 133*C
)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*
(80*A + 90*B + 67*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec
[c + d*x]]) + (a*(10*B + 3*C)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(40*d) + (C*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,\mathrm{s}\ldots}{5d}$$

$$= \frac{a(10B+3C)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(\ldots}}{40d}$$

$$= \frac{a^2(80A+90B+67C)\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}\ldots}{240d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(176A+150B+133C)\sec^{\frac{5}{2}}(c+d\ldots}{192d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(176A+150B+133C)\sec^{\frac{3}{2}}(c+d\ldots}{128d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(176A+150B+133C)\sec^{\frac{3}{2}}(c+d\ldots}{128d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{3/2}(176A+150B+133C)\sinh^{-1}\left(\ldots\right.}{128d}$$

Mathematica [A] time = 3.89925, size = 211, normalized size = 0.75

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(12(880A+1070B+1273C)\cos(c+dx)+4(3280A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(9/2)*Sqrt[a*(1 + Sec[c + d*x])]*(240*Sqrt
[2]*(176*A + 150*B + 133*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^
5 + 4*(10480*A + 11550*B + 13313*C + 12*(880*A + 1070*B + 1273*C)*Cos[c + d
*x] + 4*(3280*A + 3450*B + 3059*C)*Cos[2*(c + d*x)] + 3520*A*Cos[3*(c + d*x
)] + 3000*B*Cos[3*(c + d*x)] + 2660*C*Cos[3*(c + d*x)] + 2640*A*Cos[4*(c +
d*x)] + 2250*B*Cos[4*(c + d*x)] + 1995*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]
))/(61440*d)
```

Maple [B] time = 0.417, size = 732, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] 1/7680/d*a*(2640*A*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-2640*A*cos(d*x+c)^5*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+2250*B*cos(d*
x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c
)))*2^(1/2)-2250*B*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+1995*C*cos(d*x+c)^5*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-1995*C*cos(d*
x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))*2^(1/2)+5280*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+4500*B
*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3990*C*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)^4+3520*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*
x+c)+1))^(1/2)+3000*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+266
0*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+1280*A*cos(d*x+c)^2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2400*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2)+2128*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+
960*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+1824*C*sin(d*x+c)*cos
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+768*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(cos(d*x+c)
+1))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.41543, size = 1543, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*((176*A + 150*B + 133*C)*a*cos(d*x + c)^5 + (176*A + 150*B + 13
3*C)*a*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*(15*(176*A + 150*B + 133*C)*a*cos(d*x + c)^4 + 10*(176*A + 150
*B + 133*C)*a*cos(d*x + c)^3 + 8*(80*A + 150*B + 133*C)*a*cos(d*x + c)^2 +
48*(10*B + 19*C)*a*cos(d*x + c) + 384*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)
^4), 1/3840*(15*((176*A + 150*B + 133*C)*a*cos(d*x + c)^5 + (176*A + 150*B
+ 133*C)*a*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)) + 2*(15*(176*A + 150*B + 133*C)*a*cos(d*x + c)^4 + 10*(1
76*A + 150*B + 133*C)*a*cos(d*x + c)^3 + 8*(80*A + 150*B + 133*C)*a*cos(d*x
 + c)^2 + 48*(10*B + 19*C)*a*cos(d*x + c) + 384*C*a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*co
s(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*sec(d*x + c)^(5/2), x)
```

3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\,s\right.$

Optimal. Leaf size=233

$$\frac{a^2(48A+56B+39C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(112A+88B+75C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+88B}{}$$

```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Se
c[c + d*x]]])/(64*d) + (a^2*(112*A + 88*B + 75*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(48*A + 56*B + 39*C)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*B + 3*C)*
Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (C*Sec[c
 + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.646121, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(48A+56B+39C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(112A+88B+75C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+88B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Se
c[c + d*x]]])/(64*d) + (a^2*(112*A + 88*B + 75*C)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(48*A + 56*B + 39*C)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*B + 3*C)*
Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (C*Sec[c
 + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(}{4d}$$

$$= \frac{a(8B+3C)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}{24d}$$

$$= \frac{a^2(48A+56B+39C)\sec^{\frac{5}{2}}(c+dx)\sin(}{96d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(112A+88B+75C)\sec^{\frac{3}{2}}(c+dx)\sin(}{64d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(112A+88B+75C)\sec^{\frac{3}{2}}(c+dx)\sin(}{64d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{3/2}(112A+88B+75C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{a+a\sec}}\right.}{64d}$$

Mathematica [A] time = 2.44121, size = 177, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((1008A+1048B+1155C)\cos(c+dx)+4(48A+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(24*Sqrt[
2]*(112*A + 88*B + 75*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 +
 4*(192*A + 352*B + 492*C + (1008*A + 1048*B + 1155*C)*Cos[c + d*x] + 4*(48
*A + 88*B + 75*C)*Cos[2*(c + d*x)] + 336*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(
c + d*x)] + 225*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(3072*d)
```

Maple [B] time = 0.394, size = 637, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/384/d*a*(-1+cos(d*x+c))*(336*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-336*A*cos(d*x+c)^4*2^(1
/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
+264*B*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))*2^(1/2)-264*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+225*C*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^4-22
5*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
*2^(1/2)*cos(d*x+c)^4+672*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/
2)+528*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+450*C*sin(d*x+c)
*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2)+352*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+300*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+128*B*cos(d*x+c)*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+240*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2)+96*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1
)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+
c)^2/cos(d*x+c)^2
```

Maxima [B] time = 4.52806, size = 10963, normalized size = 47.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] -1/768*(48*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
4*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*s
in(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*si
n(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*si
n(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2
```

(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(

3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(2*(2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 8*(132*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*
x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos
(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*
x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^
2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2
+ 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin
(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6
*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x
 + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x +
 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4
*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2
*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*
a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(
4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2
*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4
*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c)
+ a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a
*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*s
in(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x
 + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*
c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x
 + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqr

t(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)
*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*
a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c)
+ 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*
d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt
(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(300*(sqrt(2)*a*sin(
8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c)
+ 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 100*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(13/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*sin(8*d*x + 8*
c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2
)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(
2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(
2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*
x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt
(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d
*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 100*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d
*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(sqrt(2)*a*sin(8*d
*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4
*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*
x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*
x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*
c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) +
 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*si
n(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x
 + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*
d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*
d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x +
4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c)
 + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) +
a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*
sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d
*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos

```
(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(
4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(
6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x
+ 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*
c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c)
+ a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*
a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4
*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*co
s(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*si
n(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*
x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x +
4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c
) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) +
3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin
(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x +
6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2
)*a)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*a
*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x +
4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(13/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*
a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x +
 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*
a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 228*(sqrt(2)*a*cos(8*d*x + 8*c)
+ 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a
*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
+ 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*a*cos(8
*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) +
 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*
d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/
(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8
*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2
*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*
c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))
/d
```

Fricas [A] time = 2.38893, size = 1381, normalized size = 5.93

$$\left[\frac{3\left((112\,A+88\,B+75\,C)a\cos\left(dx+c\right)^{4}+(112\,A+88\,B+75\,C)a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}\right.}{}}{\cos(dx+c)^{3}}\right.}{768\left(d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*((112*A + 88*B + 75*C)*a*cos(d*x + c)^4 + (112*A + 88*B + 75*C)*a
*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(co
s(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*(3*(112*A + 88*B + 75*C)*a*cos(d*x + c)^3 + 2*(48*A + 88*B + 75*C)*a
*cos(d*x + c)^2 + 8*(8*B + 15*C)*a*cos(d*x + c) + 48*C*a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 +
 d*cos(d*x + c)^3), 1/384*(3*((112*A + 88*B + 75*C)*a*cos(d*x + c)^4 + (112
*A + 88*B + 75*C)*a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(112*A + 88*B + 75*C)*a*cos(d*x + c)^3 +
 2*(48*A + 88*B + 75*C)*a*cos(d*x + c)^2 + 8*(8*B + 15*C)*a*cos(d*x + c) +
48*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*sec(d*x + c)^(3/2), x)
```

3.588 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=181

$$\frac{a^2(24A+30B+19C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+14B+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(2B+C)\sin(c+}{}$$

```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(8*d) + (a^2*(24*A + 30*B + 19*C)*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(2*B + C)*Sec[c + d*x]^(3/2)*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sec[c + d*x]^(3/2)*(a + a*Sec
[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.548173, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4018, 4016, 3801, 215}

$$\frac{a^2(24A+30B+19C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+14B+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(2B+C)\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(8*d) + (a^2*(24*A + 30*B + 19*C)*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(2*B + C)*Sec[c + d*x]^(3/2)*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sec[c + d*x]^(3/2)*(a + a*Sec
[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin}{3d}$$

$$= \frac{a(2B+C)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+a}}{4d}$$

$$= \frac{a^2(24A+30B+19C)\sec^{\frac{3}{2}}(c+dx)\sin(}{24d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2(24A+30B+19C)\sec^{\frac{3}{2}}(c+dx)\sin(}{24d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{3/2}(24A+14B+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{a+a\sec(}}\right.}{8d}$$

Mathematica [A] time = 1.47986, size = 142, normalized size = 0.78

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+14B+11C)\cos(2(c+dx))+24A+4(6B+\right.}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])]*(12*Sqrt[
2]*(24*A + 14*B + 11*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 +
4*(24*A + 42*B + 49*C + 4*(6*B + 11*C)*Cos[c + d*x] + 3*(8*A + 14*B + 11*C)
*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(192*d)
```

Maple [B] time = 0.379, size = 546, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] 1/96/d*a*(72*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+s
in(d*x+c)))*cos(d*x+c)^3*2^(1/2)-72*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+42*B*arctan(1/4*2^(
1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1
/2)-42*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x
+c)))*cos(d*x+c)^3*2^(1/2)+33*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)-33*C*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+48
*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+84*B*cos(d*x+c)^2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+66*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+
c)+1))^(1/2)+24*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+44*C*sin(
d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*C*(-2/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.27295, size = 7760, normalized size = 42.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d
*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
 + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
```

```
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin
(2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - 6*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*s
qrt(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sq
rt(2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sq
rt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sq
rt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*s
in(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4
*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
```

cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt
(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a)/(2*(2*cos(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (132*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*
sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2
*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*
sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2
*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 33*(a*cos(6*d*x
+ 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x +
6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) +
a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*c
os(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos
(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4
*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*
c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c)
+ 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6
*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9
*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*
a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4
*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x
+ 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c

```
) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)
 + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)
^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*c
os(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*
d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2
*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*c
os(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*
(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*
sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x
 + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*
a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c)
+ 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(3*cos(4
*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*c
os(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c
) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.5024, size = 1251, normalized size = 6.91

$$\left[\frac{3\left((24\,A+14\,B+11\,C)a\cos\left(dx+c\right)^{3}+(24\,A+14\,B+11\,C)a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cdots\right.}{\cdots}}{\cos(dx+c)^{3}+\cdots}\right.}{96\left(d\cos\left(dx+c\right)^{3}+d\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((24*A + 14*B + 11*C)*a*cos(d*x + c)^3 + (24*A + 14*B + 11*C)*a*co
s(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d
*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
+ 4*(3*(8*A + 14*B + 11*C)*a*cos(d*x + c)^2 + 2*(6*B + 11*C)*a*cos(d*x + c)
 + 8*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(3*((24*A + 14*B + 11*C
)*a*cos(d*x + c)^3 + (24*A + 14*B + 11*C)*a*cos(d*x + c)^2)*sqrt(-a)*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
```

```
d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(8*A + 14*B + 11
*C)*a*cos(d*x + c)^2 + 2*(6*B + 11*C)*a*cos(d*x + c) + 8*C*a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right)(a \sec(dx+c) + a)^{\frac{3}{2}} \sqrt{\sec(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*sqrt(sec(d*x + c)), x)
```

3.589 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=183

$$\frac{a^2(8A-4B-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+12B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+3C)\sin(c+dx)\sqrt{\sec}}{4}$$

```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]])/(4*d) + (a^2*(8*A - 4*B - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(4*B + 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
 d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.524863, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4018, 4015, 3801, 215}

$$\frac{a^2(8A-4B-5C)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+12B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+3C)\sin(c+dx)\sqrt{\sec}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]])/(4*d) + (a^2*(8*A - 4*B - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*(4*B + 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
 d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d} +$$

$$= \frac{a(4B+3C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c}{4d}$$

$$= \frac{a^2(8A-4B-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a(4}{}$$

$$= \frac{a^2(8A-4B-5C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a(4}{}$$

$$= \frac{a^{3/2}(8A+12B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^2(}{}$$

Mathematica [A] time = 1.23574, size = 129, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(8A+12B+7C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sec^2(c+}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(8*A + 12*B + 7*C)*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*((4*B + 7*C)*Cos[c + d*x] + 2*(2*A +
C + 2*A*Cos[2*(c + d*x)]))*Sec[c + d*x]^2*Sin[(c + d*x)/2]))/(8*d*Sqrt[Sec[
c + d*x]])
```

Maple [B] time = 0.388, size = 533, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] 1/16/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*A*2^(1/2)*sin(d*x+c)*cos(d*
x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))-8*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x
+c)+1))^(1/2)+12*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-1
2*B*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+7*C*2^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-7*C*2^(1/2)*sin(d*x+c)*cos(d*x+c
)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
*(-2/(cos(d*x+c)+1))^(1/2)-32*A*cos(d*x+c)^3+32*A*cos(d*x+c)^2-16*B*cos(d*x
+c)^2-28*C*cos(d*x+c)^2+16*B*cos(d*x+c)+20*C*cos(d*x+c)+8*C)*(1/cos(d*x+c))
^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [B] time = 2.80073, size = 4942, normalized size = 27.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A
*sqrt(a) + 4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
 a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3
```

/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) +
 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*
sqrt(2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*
sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a
*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*s
in(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3

```
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4
*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sq
rt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sqrt(a)/(2*(2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d
```

<u>Fricas</u> [A] time = 1.4962, size = 1166, normalized size = 6.37

$$\left[\frac{\left((8\,A+12\,B+7\,C)a\cos\left(dx+c\right)^2+(8\,A+12\,B+7\,C)a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)}{\sqrt{\cos\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2\cdots}\right.}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*A + 12*B + 7*C)*a*cos(d*x + c)^2 + (8*A + 12*B + 7*C)*a*cos(d*x
+ c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^
```

```
2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(8*A
*a*cos(d*x + c)^2 + (4*B + 7*C)*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d
*cos(d*x + c)), 1/8*(((8*A + 12*B + 7*C)*a*cos(d*x + c)^2 + (8*A + 12*B + 7
*C)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)) + 2*(8*A*a*cos(d*x + c)^2 + (4*B + 7*C)*a*cos(d*x + c) + 2*C*a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sqrt(sec(d*x + c)), x)
```

3.590 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{a^2(8A+6B-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^2*(8*A + 6*B - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a
 + a*Sec[c + d*x]]) - (a*(2*A - 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.522638, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4018, 4015, 3801, 215}

$$\frac{a^2(8A+6B-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```



```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^2*(8*A + 6*B - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a
 + a*Sec[c + d*x]]) - (a*(2*A - 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
3*d*Sqrt[Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec}{}}{}$$

$$= -\frac{a(2A-3C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(}{3d}$$

$$= \frac{a^2(8A+6B-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a(2}{}$$

$$= \frac{a^2(8A+6B-3C)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a(2}{}$$

$$= \frac{a^{3/2}(2B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2(8A+6}{}$$

Mathematica [A] time = 1.07353, size = 122, normalized size = 0.69

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(5A+3B)\cos(c+dx)+A\cos(2(c+dx))+A\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2
]*(2*B + 3*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 3*C +
 2*(5*A + 3*B)*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

Maple [B] time = 0.405, size = 383, normalized size = 2.2

$$-\frac{a\cos(dx+c)}{12d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(6B\cos(dx+c)\sqrt{2}\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-2(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/12/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(6*B*cos(d*x+c)*2^(1/2)*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))-6*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))+9*C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arct
an(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-9*C*cos
(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+8*A*cos(d*x+c)^3+32*A*cos
(d*x+c)^2+24*B*cos(d*x+c)^2-40*A*cos(d*x+c)-24*B*cos(d*x+c)+12*C*cos(d*x+c)
-12*C)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.43957, size = 1964, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*B
*sqrt(a) + 4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/
2*c))*A*sqrt(a) + 3*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*
a*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2
```

```
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x +
 2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x +
1/2*c))*sin(2*d*x + 2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.798965, size = 1058, normalized size = 5.98

$$\left[\frac{3\left((2B+3C)a\cos(dx+c)+(2B+3C)a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{12\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((2*B + 3*C)*a*cos(d*x + c) + (2*B + 3*C)*a)*sqrt(a)*log((a*cos(d*
x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*A*a*cos(d*x + c)^2 + 2*(5*A
+ 3*B)*a*cos(d*x + c) + 3*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/6*(3*((2*B + 3*C)*a*co
s(d*x + c) + (2*B + 3*C)*a)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)) + 2*(2*A*a*cos(d*x + c)^2 + 2*(5*A + 3*B)*a*cos(d*x + c
) + 3*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(3/2), x)
```

3.591 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\sec(c}}{15d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(12*A + 20*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a +
 a*Sec[c + d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.51202, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4017, 4015, 3801, 215}

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\sec(c}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(5/2),x]
```


```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(12*A + 20*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a +
 a*Sec[c + d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+d}{}}{}$$

$$= \frac{2a(3A+5B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2A(a}{}$$

$$= \frac{2a^2(12A+20B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^2(12A+20B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2(12A+20B+15}{15d\sqrt{a}}$$

Mathematica [A] time = 1.84905, size = 162, normalized size = 0.94

$$\frac{\sec^3\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)(2(9A+5B)\cos(c+dx)+\right.}{15d\sec^{\frac{7}{2}}(c+dx)(A\cos(2(c+dx))+A+2B\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^(3/2)*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*(15*Sqrt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (39*A + 50*
B + 30*C + 2*(9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sin[(c + d*x)
/2]))/(15*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^
```

```
(7/2))
```

Maple [A] time = 0.411, size = 245, normalized size = 1.4

$$-\frac{a\left(\cos\left(dx+c\right)\right)^{3}}{30\,d\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(-15\,C\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] -1/30/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-15*C*(-2/(cos(d*x+c)+1))^(1
/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(
d*x+c)))*sin(d*x+c)+15*C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/
2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+12*A*cos
(d*x+c)^3+24*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+36*A*cos(d*x+c)+80*B*cos(d*x+
c)+60*C*cos(d*x+c)-72*A-100*B-60*C)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d
*x+c)
```

Maxima [B] time = 2.31892, size = 705, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x +
 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*
sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))))*A*sqrt(a) + 15*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sq
rt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d
*x + 1/2*c))*C*sqrt(a) + 20*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*s
in(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 0.627288, size = 1089, normalized size = 6.33

$$\left[\frac{15\,(Ca\cos(dx+c)+Ca)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3\,Aa\cos(dx+c)^3+(9\right.}{}}{30\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(C*a*cos(d*x + c) + C*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(
d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^
3 + cos(d*x + c)^2)) + 4*(3*A*a*cos(d*x + c)^3 + (9*A + 5*B)*a*cos(d*x + c)
^2 + (18*A + 25*B + 15*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/15*(15*(C*a
*cos(d*x + c) + C*a)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x
+ c) - 2*a)) + 2*(3*A*a*cos(d*x + c)^3 + (9*A + 5*B)*a*cos(d*x + c)^2 + (18
*A + 25*B + 15*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(5/2), x)
```

3.592 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{8a^2(19A+21B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+21B+35C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2(3A+7B}{}$$

```
[Out] (8*a^2*(19*A + 21*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(19*A + 21*B + 35*C)*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(3*A + 7*B)*(a + a*Sec[c + d*x])^(
3/2)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.465627, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4086, 4013, 3809, 3804}

$$\frac{8a^2(19A+21B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+21B+35C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2(3A+7B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(7/2),x]
```

```
[Out] (8*a^2*(19*A + 21*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(19*A + 21*B + 35*C)*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(3*A + 7*B)*(a + a*Sec[c + d*x])^(
3/2)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
```

```
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+a}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(3A+7B)(a}{}$$

$$= \frac{2a(19A+21B+35C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}$$

$$= \frac{8a^2(19A+21B+35C)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A] time = 0.965081, size = 100, normalized size = 0.55

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((253A+28(9B+5C))\cos(c+dx)+6(13A+7B)\cos(2(c+dx))+15A\cos(3(c+}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(494*A + 546*B + 700*C + (253*A + 28*(9*B + 5*C))*Cos[c + d*x] + 6*(13*A
 + 7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])
]*Tan[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.361, size = 131, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+39A(\cos(dx+c))^2+21B(\cos(dx+c))^2+52A\cos(dx+c)+63B\cos}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*
x+c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+35*C*cos(d*x+c)+104*A+126*B+175*C)*(
```

```
a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*
x+c)
```

Maxima [B] time = 2.31557, size = 743, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 42*sqrt(2)*(20*a*cos(4/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) +
5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*
x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin(5/2*d*x + 5/2*c) + 5*a*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*a*sin(1/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*B*sqrt(a) + 280*(sqrt(2)
*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 0.489389, size = 325, normalized size = 1.8

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{4}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B+35\,C)a\cos\left(dx+c\right)^{2}+(104\,A+126\,B+175\,C}{105\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^4 + 3*(13*A + 7*B)*a*cos(d*x + c)^3 + (52*A + 63
*B + 35*C)*a*cos(d*x + c)^2 + (104*A + 126*B + 175*C)*a*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(
cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(7/2), x)
```

3.593 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+156B+189C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+156B+18}{315d\sqrt{\sec(c+dx)}\sqrt{a\,s}}$$

```
[Out] (2*a^2*(52*A + 72*B + 63*C)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sin[c + d*x])/(315*d*Sq
rt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(136*A + 156*B + 189*C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(
A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) +
 (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.653402, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4017, 4015, 3805, 3804}

$$\frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+156B+189C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+156B+18}{315d\sqrt{\sec(c+dx)}\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(52*A + 72*B + 63*C)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sin[c + d*x])/(315*d*Sq
rt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(136*A + 156*B + 189*C)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(
A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) +
 (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2a(A+3B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+}{}$$

$$= \frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(A+3B)}{}$$

$$= \frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(136A+}{315d\sqrt{\sec(c}}$$

$$= \frac{2a^2(52A+72B+63C)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(136A+}{315d\sqrt{\sec(c}}$$

Mathematica [A] time = 1.55762, size = 123, normalized size = 0.53

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(799A+759B+756C)\cos(c+dx)+4(137A+117B+63C)\cos(2(c+dx))+17}{1260d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a*(2689*A + 2964*B + 3276*C + 2*(799*A + 759*B + 756*C)*Cos[c + d*x] + 4*(
137*A + 117*B + 63*C)*Cos[2*(c + d*x)] + 170*A*Cos[3*(c + d*x)] + 90*B*Cos[
3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d
*x)/2])/(1260*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.402, size = 164, normalized size = 0.7

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(35\,A\,(\cos(dx+c))^4+85\,A\,(\cos(dx+c))^3+45\,B\,(\cos(dx+c))^3+102\,A\,(\cos(dx+c))^2+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*
x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+136*A*cos(d*
x+c)+156*B*cos(d*x+c)+189*C*cos(d*x+c)+272*A+312*B+378*C)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.45184, size = 1226, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) + 6*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7
/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))
 - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*B*sqrt(a) + 252*sqrt(2)*(20*a*cos(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c
) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/
2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin(5/2*d*x + 5/2*c) + 5*a*
sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*a*sin(1/5
```

```
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.498864, size = 389, normalized size = 1.68

$$\frac{2\left(35\,Aa\cos\left(dx+c\right)^{5}+5\left(17\,A+9\,B\right)a\cos\left(dx+c\right)^{4}+3\left(34\,A+39\,B+21\,C\right)a\cos\left(dx+c\right)^{3}+\left(136\,A+156\,B+189\,C\right)a}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^5 + 5*(17*A + 9*B)*a*cos(d*x + c)^4 + 3*(34*A +
39*B + 21*C)*a*cos(d*x + c)^3 + (136*A + 156*B + 189*C)*a*cos(d*x + c)^2 +
2*(136*A + 156*B + 189*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(9/2), x)
```

3.594 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2a^2(336A+374B+429C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(336A+374B+429C)\sin(c+}{3465d\sqrt{a\sec(c+dx)}}$$

```
[Out] (2*a^2*(84*A + 110*B + 99*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a
 + a*Sec[c + d*x]]) + (2*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(1155*d*
Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*
C)*Sin[c + d*x])/(3465*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16
*a^2*(336*A + 374*B + 429*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*a*(3*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d
*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(11*d*Sec[c + d*x]^(9/2))
```

Rubi [A] time = 0.739475, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4017, 4015, 3805, 3804}

$$\frac{2a^2(336A+374B+429C)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(336A+374B+429C)\sin(c+}{3465d\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(11/2),x]
```

```
[Out] (2*a^2*(84*A + 110*B + 99*C)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a
 + a*Sec[c + d*x]]) + (2*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(1155*d*
Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*
C)*Sin[c + d*x])/(3465*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16
*a^2*(336*A + 374*B + 429*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*a*(3*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d
*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(11*d*Sec[c + d*x]^(9/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+d}{}}{}$$

$$= \frac{2a(3A+11B)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(3A+11B}{}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(336A+}{1155d\sec^{\frac{3}{2}}(}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(336A+}{1155d\sec^{\frac{3}{2}}(}$$

$$= \frac{2a^2(84A+110B+99C)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(336A+}{1155d\sec^{\frac{3}{2}}(}$$

Mathematica [A] time = 2.33363, size = 158, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((34734A+44(799B+759C))\cos(c+dx)+8(1743A+1507B+1287C)\cos(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(11/2),x]
```

```
[Out] (a*(55482*A + 59158*B + 65208*C + (34734*A + 44*(799*B + 759*C))*Cos[c + d*
x] + 8*(1743*A + 1507*B + 1287*C)*Cos[2*(c + d*x)] + 4935*A*Cos[3*(c + d*x)
] + 3740*B*Cos[3*(c + d*x)] + 1980*C*Cos[3*(c + d*x)] + 1470*A*Cos[4*(c + d
*x)] + 770*B*Cos[4*(c + d*x)] + 315*A*Cos[5*(c + d*x)])*Sqrt[a*(1 + Sec[c +
 d*x])]*Tan[(c + d*x)/2])/(27720*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.403, size = 197, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(315A(\cos(dx+c))^5+735A(\cos(dx+c))^4+385B(\cos(dx+c))^4+840A(\cos(dx+c))^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2
),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+735*A*cos(d*x+c)^4+385*B*co
s(d*x+c)^4+840*A*cos(d*x+c)^3+935*B*cos(d*x+c)^3+495*C*cos(d*x+c)^3+1008*A*
cos(d*x+c)^2+1122*B*cos(d*x+c)^2+1287*C*cos(d*x+c)^2+1344*A*cos(d*x+c)+1496
*B*cos(d*x+c)+1716*C*cos(d*x+c)+2688*A+2992*B+3432*C)*(a*(cos(d*x+c)+1)/cos
(d*x+c))^(1/2)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [B] time = 2.51371, size = 1604, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 1/110880*(21*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(
11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*c
os(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d
*x + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x
+ 11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d
*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c)
, cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/
2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55
*a*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2
*d*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(
11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) + 22*sqrt(2)*(3780*a*c
```

```
os(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9
/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2
*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 3780*a*cos(9/2*d*x +
9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 1050*
a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/
2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(9/2*d*x + 9/2*c) + 135*a*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 378*a*sin(5/
9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1050*a*sin(1/3*arc
tan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 3780*a*sin(1/9*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*B*sqrt(a) + 132*sqrt(2)*(735*
a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x
+ 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)
))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*x + 7/2*c)*sin(6/7*
arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 175*a*cos(7/2*d*x +
7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 63*a*
cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.506846, size = 460, normalized size = 1.62

$$\frac{2\left(315\,Aa\cos\left(dx+c\right)^{6}+35\,(21\,A+11\,B)a\cos\left(dx+c\right)^{5}+5\,(168\,A+187\,B+99\,C)a\cos\left(dx+c\right)^{4}+3\,(336\,A+374\,B+\right.}{3465\,(d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*A*a*cos(d*x + c)^6 + 35*(21*A + 11*B)*a*cos(d*x + c)^5 + 5*(168
*A + 187*B + 99*C)*a*cos(d*x + c)^4 + 3*(336*A + 374*B + 429*C)*a*cos(d*x +
 c)^3 + 4*(336*A + 374*B + 429*C)*a*cos(d*x + c)^2 + 8*(336*A + 374*B + 429
*C)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((
d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(11/2), x)
```

3.595 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=333

$$\frac{a^2(120A+156B+115C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{480d}+\frac{a^3(680A+628B+545C)\sin(c+dx)\sec^{\frac{7}{2}}(c+d}{960d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a +
 a*Sec[c + d*x]]])/(512*d) + (a^3*(1304*A + 1132*B + 1015*C)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(512*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1304*A + 1132*B
 + 1015*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(768*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^3*(680*A + 628*B + 545*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(960*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^2*(120*A + 156*B + 115*C)*Sec[c + d*x]^(7/2)*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(480*d) + (a*(12*B + 5*C)*Sec[c + d*x
]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(60*d) + (C*Sec[c + d*x]^(
7/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.949499, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(120A+156B+115C)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{480d}+\frac{a^3(680A+628B+545C)\sin(c+dx)\sec^{\frac{7}{2}}(c+d}{960d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a +
 a*Sec[c + d*x]]])/(512*d) + (a^3*(1304*A + 1132*B + 1015*C)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(512*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1304*A + 1132*B
 + 1015*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(768*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^3*(680*A + 628*B + 545*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(960*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^2*(120*A + 156*B + 115*C)*Sec[c + d*x]^(7/2)*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(480*d) + (a*(12*B + 5*C)*Sec[c + d*x
]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(60*d) + (C*Sec[c + d*x]^(
7/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
```

```
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(}{6d}$$

$$= \frac{a(12B+5C)\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+}{60d}$$

$$= \frac{a^2(120A+156B+115C)\sec^{\frac{7}{2}}(c+dx)\sqrt{}}{480d}$$

$$= \frac{a^3(680A+628B+545C)\sec^{\frac{7}{2}}(c+dx)\,s}{960d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(1304A+1132B+1015C)\sec^{\frac{5}{2}}(c+d}{768d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(1304A+1132B+1015C)\sec^{\frac{3}{2}}(c+d}{512d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(1304A+1132B+1015C)\sec^{\frac{3}{2}}(c+d}{512d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(1304A+1132B+1015C)\sinh^{-1}\left(-\right.}{512d}$$

Mathematica [A] time = 4.68739, size = 245, normalized size = 0.74

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((283920A+303048B+321370C)\cos(c+dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^(11/2)*Sqrt[a*(1 + Sec[c + d*x])]*(480*S
qrt[2]*(1304*A + 1132*B + 1015*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c +
 d*x]^6 + 4*(93600*A + 112464*B + 137060*C + (283920*A + 303048*B + 321370*
C)*Cos[c + d*x] + 16*(7480*A + 8444*B + 8555*C)*Cos[2*(c + d*x)] + 127240*A
*Cos[3*(c + d*x)] + 121124*B*Cos[3*(c + d*x)] + 108605*C*Cos[3*(c + d*x)] +
 26080*A*Cos[4*(c + d*x)] + 22640*B*Cos[4*(c + d*x)] + 20300*C*Cos[4*(c + d
*x)] + 19560*A*Cos[5*(c + d*x)] + 16980*B*Cos[5*(c + d*x)] + 15225*C*Cos[5*
(c + d*x)])*Sin[(c + d*x)/2]))/(491520*d)
```

Maple [B] time = 0.451, size = 827, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] 1/30720/d*a^2*(19560*A*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-19560*A*cos(d*x+c)^6*2^(1/2)*arcta
n(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+16980*B*
cos(d*x+c)^6*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin
(d*x+c)))*2^(1/2)-16980*B*cos(d*x+c)^6*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+15225*C*cos(d*x+c)^6*2^(1/2)*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-15225
*C*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))+39120*A*cos(d*x+c)^5*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+33960*B*cos(d*x+c)^5*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+30450*C*cos(d
*x+c)^5*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+26080*A*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)^4+22640*B*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+20300*C*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+14720*
A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+18112*B*sin(d*x+c)*cos(
d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+16240*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos
(d*x+c)+1))^(1/2)+3840*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+
11136*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+13920*C*sin(d*x+c
)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+3072*B*cos(d*x+c)*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2)+8960*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
+2560*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))
^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^3/sin(d*x+
c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.40478, size = 1756, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*((1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^6 + (1304*A + 113
2*B + 1015*C)*a^2*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(15*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^5 +
 10*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^4 + 8*(920*A + 1132*B + 101
5*C)*a^2*cos(d*x + c)^3 + 48*(40*A + 116*B + 145*C)*a^2*cos(d*x + c)^2 + 12
8*(12*B + 35*C)*a^2*cos(d*x + c) + 1280*C*a^2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^6 + d*cos(d*x
+ c)^5), 1/15360*(15*((1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^6 + (1304
*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^5)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
```

```
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(15*(1304*A + 1132*B + 1015*C)*a^2*co
s(d*x + c)^5 + 10*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^4 + 8*(920*A
+ 1132*B + 1015*C)*a^2*cos(d*x + c)^3 + 48*(40*A + 116*B + 145*C)*a^2*cos(d
*x + c)^2 + 128*(12*B + 35*C)*a^2*cos(d*x + c) + 1280*C*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*sec(d*x + c)^(5/2), x)
```

3.596 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=281

$$\frac{a^3(1040A+950B+787C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{960d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(400A+326B+283C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+}{}$$

```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]])/(128*d) + (a^3*(400*A + 326*B + 283*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(128*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1040*A + 950*B + 787*
C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2
*(80*A + 110*B + 79*C)*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(240*d) + (a*(2*B + C)*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*
Sin[c + d*x])/(8*d) + (C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[
c + d*x])/(5*d)
```

Rubi [A] time = 0.858446, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(1040A+950B+787C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{960d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(400A+326B+283C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]])/(128*d) + (a^3*(400*A + 326*B + 283*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(128*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1040*A + 950*B + 787*
C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(960*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2
*(80*A + 110*B + 79*C)*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(240*d) + (a*(2*B + C)*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*
Sin[c + d*x])/(8*d) + (C*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[
c + d*x])/(5*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(}{5d}$$

$$= \frac{a(2B+C)\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx}{8d}$$

$$= \frac{a^2(80A+110B+79C)\sec^{\frac{5}{2}}(c+dx)\sqrt{a}}{240d}$$

$$= \frac{a^3(1040A+950B+787C)\sec^{\frac{5}{2}}(c+dx)}{960d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(400A+326B+283C)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{128d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(400A+326B+283C)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{128d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(400A+326B+283C)\sinh^{-1}\left(\frac{\sqrt{a}\,\mathrm{t}}{\sqrt{a+a}}\right.}{128d}$$

Mathematica [A] time = 3.21727, size = 213, normalized size = 0.76

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(12(1360A+1950B+2343C)\cos(c+dx)+4(6$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^(9/2)*Sqrt[a*(1 + Sec[c + d*x])]*(240*Sq
rt[2]*(400*A + 326*B + 283*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x
]^5 + 4*(20560*A + 22030*B + 24863*C + 12*(1360*A + 1950*B + 2343*C)*Cos[c
+ d*x] + 4*(6640*A + 6730*B + 6509*C)*Cos[2*(c + d*x)] + 5440*A*Cos[3*(c +
d*x)] + 6520*B*Cos[3*(c + d*x)] + 5660*C*Cos[3*(c + d*x)] + 6000*A*Cos[4*(c
 + d*x)] + 4890*B*Cos[4*(c + d*x)] + 4245*C*Cos[4*(c + d*x)])*Sin[(c + d*x)
/2]))/(61440*d)
```

Maple [B] time = 0.402, size = 732, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/3840/d*a^2*(-1+cos(d*x+c))*(6000*A*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-6000*A*cos(d*x+c)^5
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^
(1/2)+4890*B*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))*2^(1/2)-4890*B*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+4245*C*cos(d*x+c)^5
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^
(1/2)-4245*C*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1-sin(d*x+c)))*2^(1/2)+12000*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)^4+9780*B*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8490
*C*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+5440*A*sin(d*x+c)*cos(
d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+6520*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(
d*x+c)+1))^(1/2)+5660*C*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+1
280*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3680*B*cos(d*x+c)^2
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4528*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(c
os(d*x+c)+1))^(1/2)+960*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2
784*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+768*C*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(
3/2)/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^3/sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.43503, size = 1581, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*((400*A + 326*B + 283*C)*a^2*cos(d*x + c)^5 + (400*A + 326*B +
283*C)*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 4*(15*(400*A + 326*B + 283*C)*a^2*cos(d*x + c)^4 + 10*(272*A
 + 326*B + 283*C)*a^2*cos(d*x + c)^3 + 8*(80*A + 230*B + 283*C)*a^2*cos(d*x
 + c)^2 + 48*(10*B + 29*C)*a^2*cos(d*x + c) + 384*C*a^2)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^5 +
d*cos(d*x + c)^4), 1/3840*(15*((400*A + 326*B + 283*C)*a^2*cos(d*x + c)^5 +
 (400*A + 326*B + 283*C)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos
(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(15*(400*A + 326*B + 283*C)*a^2*co
s(d*x + c)^4 + 10*(272*A + 326*B + 283*C)*a^2*cos(d*x + c)^3 + 8*(80*A + 23
0*B + 283*C)*a^2*cos(d*x + c)^2 + 48*(10*B + 29*C)*a^2*cos(d*x + c) + 384*C
*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*sec(d*x + c)^(3/2), x)
```

3.597 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,s\right.$

Optimal. Leaf size=233

$$\frac{a^3(432A+392B+299C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+24B+17C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{32d}+$$

```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]])/(64*d) + (a^3*(432*A + 392*B + 299*C)*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(16*A + 24*B + 17*C)*Se
c[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (a*(8*B +
5*C)*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (
C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.752273, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4018, 4016, 3801, 215}

$$\frac{a^3(432A+392B+299C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+24B+17C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{32d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]])/(64*d) + (a^3*(432*A + 392*B + 299*C)*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(16*A + 24*B + 17*C)*Se
c[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (a*(8*B +
5*C)*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (
C*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}{4d}$$

$$= \frac{a(8B+5C)\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}{24d}$$

$$= \frac{a^2(16A+24B+17C)\sec^{\frac{3}{2}}(c+dx)\sqrt{}}{32d}$$

$$= \frac{a^3(432A+392B+299C)\sec^{\frac{3}{2}}(c+}{192d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{a^3(432A+392B+299C)\sec^{\frac{3}{2}}(c+}{192d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{a^{5/2}(304A+200B+163C)\sinh^{-1}\left(\right.}{64d}$$

Mathematica [A] time = 2.15909, size = 179, normalized size = 0.77

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((1584A+2056B+2203C)\cos(c+dx)+4(48A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(24*Sqr
t[2]*(304*A + 200*B + 163*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]
^4 + 4*(192*A + 544*B + 844*C + (1584*A + 2056*B + 2203*C)*Cos[c + d*x] + 4
*(48*A + 136*B + 163*C)*Cos[2*(c + d*x)] + 528*A*Cos[3*(c + d*x)] + 600*B*C
```

```
os[3*(c + d*x)] + 489*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(3072*d)
```

Maple [B] time = 0.375, size = 641, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] 1/768/d*a^2*(912*A*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-912*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+600*B*cos(d*x+
c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)*2^(1/2)-600*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+489*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^4-489*C*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*
x+c)^4+1056*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+1200*B*sin(
d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+978*C*sin(d*x+c)*cos(d*x+c)^3
*(-2/(cos(d*x+c)+1))^(1/2)+192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2)+544*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+652*C*sin(d
*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+128*B*cos(d*x+c)*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)+368*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)+96*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))
^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+
c)^3*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.37755, size = 1435, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*((304*A + 200*B + 163*C)*a^2*cos(d*x + c)^4 + (304*A + 200*B + 16
3*C)*a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
```

```
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*(3*(176*A + 200*B + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 13
6*B + 163*C)*a^2*cos(d*x + c)^2 + 8*(8*B + 23*C)*a^2*cos(d*x + c) + 48*C*a^
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/384*(3*((304*A + 200*B + 163*C)*a
^2*cos(d*x + c)^4 + (304*A + 200*B + 163*C)*a^2*cos(d*x + c)^3)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(176*A + 200
*B + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 136*B + 163*C)*a^2*cos(d*x + c)^
2 + 8*(8*B + 23*C)*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x
 + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*sqrt(sec(d*x + c)), x)
```

3.598 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=233

$$\frac{a^3(24A-54B-49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+42B+31C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{24d}+\frac{a^5}{}$$

```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(8*d) + (a^3*(24*A - 54*B - 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*A + 42*B + 31*C)*Sqrt[Sec[c
 + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(6*B + 5*C)*Sqr
t[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (C*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.725627, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4018, 4015, 3801, 215}

$$\frac{a^3(24A-54B-49C)\sin(c+dx)\sqrt{\sec(c+dx)}}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+42B+31C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{24d}+\frac{a^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```


```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/(8*d) + (a^3*(24*A - 54*B - 49*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*A + 42*B + 31*C)*Sqrt[Sec[c
 + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(6*B + 5*C)*Sqr
t[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (C*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d} +$$

$$= \frac{a(6B+5C)\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\sin}{12d}$$

$$= \frac{a^2(24A+42B+31C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}{24d}$$

$$= \frac{a^3(24A-54B-49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(24A-54B-49C)\sqrt{\sec(c+dx)}\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^{5/2}(40A+38B+25C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} +$$

Mathematica [A] time = 1.75494, size = 158, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(4(18A+6B+17C)\cos(c+dx)+3(8A+22B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])]*(12*Sqr
t[2]*(40*A + 38*B + 25*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3
```

```
+ 4*(24*A + 66*B + 91*C + 4*(18*A + 6*B + 17*C)*Cos[c + d*x] + 3*(8*A + 22*
B + 25*C)*Cos[2*(c + d*x)] + 24*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(192
*d)
```

Maple [B] time = 0.399, size = 568, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] 1/96/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(120*A*(-2/(cos(d*x+c)+1))^(
1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3-120*A*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)*2
^(1/2)*cos(d*x+c)^3+114*B*2^(1/2)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d
*x+c)-114*B*2^(1/2)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/
2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+75*C*(-2
/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3-75*C*(-2/(cos(d*x+c)+1)
)^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c)))*sin(d*x+c)*2^(1/2)*cos(d*x+c)^3-192*A*cos(d*x+c)^4+96*A*cos(d*x+c)^3-2
64*B*cos(d*x+c)^3-300*C*cos(d*x+c)^3+96*A*cos(d*x+c)^2+216*B*cos(d*x+c)^2+1
64*C*cos(d*x+c)^2+48*B*cos(d*x+c)+104*C*cos(d*x+c)+32*C)*(1/cos(d*x+c))^(1/
2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.53903, size = 1349, normalized size = 5.79

$$\left[\frac{3\left(\left(40A+38B+25C\right)a^{2}\cos\left(dx+c\right)^{3}+\left(40A+38B+25C\right)a^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)\cdots\right.}{\cdots}}{\cos(dx+c)^{3}\cdots}\right.\cdots}{96\left(d\cos\left(dx+\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((40*A + 38*B + 25*C)*a^2*cos(d*x + c)^3 + (40*A + 38*B + 25*C)*a^
2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(c
os(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^
2)) + 4*(48*A*a^2*cos(d*x + c)^3 + 3*(8*A + 22*B + 25*C)*a^2*cos(d*x + c)^2
 + 2*(6*B + 17*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x +
 c)^2), 1/48*(3*((40*A + 38*B + 25*C)*a^2*cos(d*x + c)^3 + (40*A + 38*B + 2
5*C)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(
d*x + c) - 2*a)) + 2*(48*A*a^2*cos(d*x + c)^3 + 3*(8*A + 22*B + 25*C)*a^2*c
os(d*x + c)^2 + 2*(6*B + 17*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3
+ d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(a\sec\left(dx+c\right) + a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sqrt(sec(d*x + c)), x)
```

3.599 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=233

$$\frac{a^3(56A+12B-27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-12B-21C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{12d}+\frac{a^{5/2}(\cdots}{\cdots}$$

```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(4*d) + (a^3*(56*A + 12*B - 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(8*A - 12*B - 21*C)*Sqrt[Sec[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(12*d) - (a*(4*A - 3*C)*Sqrt[
Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (2*A*(a + a*
Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.741483, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4018, 4015, 3801, 215}

$$\frac{a^3(56A+12B-27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{12d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-12B-21C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{12d}+\frac{a^{5/2}(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]])/(4*d) + (a^3*(56*A + 12*B - 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(8*A - 12*B - 21*C)*Sqrt[Sec[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(12*d) - (a*(4*A - 3*C)*Sqrt[
Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (2*A*(a + a*
Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeqQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{(a+a\sec}{}}{}$$

$$= -\frac{a(4A-3C)\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\text{si}}{6d}$$

$$= -\frac{a^2(8A-12B-21C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}{12d}$$

$$= \frac{a^3(56A+12B-27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{a^3(56A+12B-27C)\sqrt{\sec(c+dx)}\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{a^{5/2}(8A+20B+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{a^{}}{}$$

Mathematica [A] time = 1.27296, size = 155, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(3(2A+4B+11C)\cos(c+dx)+4(8A+3B)\text{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt
[2]*(8*A + 20*B + 19*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 +
```

```
4*(32*A + 12*B + 6*C + 3*(2*A + 4*B + 11*C)*Cos[c + d*x] + 4*(8*A + 3*B)*Co
s[2*(c + d*x)] + 2*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

Maple [B] time = 0.351, size = 549, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-24*A*2^(1/2)*sin(d*x+c)*c
os(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+24*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(c
os(d*x+c)+1))^(1/2)-60*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*2^(1/2)*sin(d*
x+c)+60*B*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-57*C*2^(1/
2)*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+57*C*2^(1/2)*sin(d*x+c)*c
os(d*x+c)^2*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)+32*A*cos(d*x+c)^4+224*A*cos(d*x+c)^3+96*
B*cos(d*x+c)^3-256*A*cos(d*x+c)^2-48*B*cos(d*x+c)^2+132*C*cos(d*x+c)^2-48*B
*cos(d*x+c)-108*C*cos(d*x+c)-24*C)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.50063, size = 1308, normalized size = 5.61

$$\left[\frac{3\left((8\,A + 20\,B + 19\,C)a^2\cos\left(dx + c\right)^2 + (8\,A + 20\,B + 19\,C)a^2\cos\left(dx + c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - \frac{4\left(\cos(dx+c)^2 - 2\,\mathrm{c}\cdots\right.}{\cdots}}{\cos(dx+c)^3 + \mathrm{cos}\cdots}\right.}{48\left(d\cos\left(dx + c\right)^2 + \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*A + 20*B + 19*C)*a^2*cos(d*x + c)^2 + (8*A + 20*B + 19*C)*a^2*
cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d
*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
+ 4*(8*A*a^2*cos(d*x + c)^3 + 8*(8*A + 3*B)*a^2*cos(d*x + c)^2 + 3*(4*B + 1
1*C)*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/24*(3
*((8*A + 20*B + 19*C)*a^2*cos(d*x + c)^2 + (8*A + 20*B + 19*C)*a^2*cos(d*x
+ c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) +
2*(8*A*a^2*cos(d*x + c)^3 + 8*(8*A + 3*B)*a^2*cos(d*x + c)^2 + 3*(4*B + 11*
C)*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^{2} + B \sec\left(dx+c\right) + A\right)\left(a \sec\left(dx+c\right) + a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(3/2), x)
```

3.600 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$\frac{a^3(64A+70B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A+10B-15C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{a^{5/}}{}$$

```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^3*(64*A + 70*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[a + a*Sec[c + d*x]]) - (a^2*(16*A + 10*B - 15*C)*Sqrt[Sec[c + d*x]]*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*(A + B)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.71922, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4086, 4017, 4018, 4015, 3801, 215}

$$\frac{a^3(64A+70B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A+10B-15C)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{a^{5/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(5/2),x]
```

```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^3*(64*A + 70*B + 15*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[a + a*Sec[c + d*x]]) - (a^2*(16*A + 10*B - 15*C)*Sqrt[Sec[c + d*x]]*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*(A + B)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec}{}}{}$$

$$= \frac{2a(A+B)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2A}{}$$

$$= -\frac{a^2(16A+10B-15C)\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c}}{15d}$$

$$= \frac{a^3(64A+70B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{a^3(64A+70B+15C)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{a^{5/2}(2B+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3(64A+}{}$$

Mathematica [A] time = 1.1362, size = 149, normalized size = 0.67

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)\left((181A+160B+60C)\cos(c+dx)+2(14A+5B)\right.\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(30*Sqr
t[2]*(2*B + 5*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(28*A +
 10*B + 30*C + (181*A + 160*B + 60*C)*Cos[c + d*x] + 2*(14*A + 5*B)*Cos[2*(
c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(60*d)
```

Maple [B] time = 0.36, size = 420, normalized size = 1.9

$$-\frac{a^2(\cos(dx+c))^2}{60\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-30\,B\cos(dx+c)\sqrt{2}\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] -1/60/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-30*B*cos(d*x+c)*2^(1/2)*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+30*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1+sin(d*x+c)))-75*C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+7
5*C*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+24*A*cos(d*x+c)^4+
88*A*cos(d*x+c)^3+40*B*cos(d*x+c)^3+232*A*cos(d*x+c)^2+280*B*cos(d*x+c)^2+1
20*C*cos(d*x+c)^2-344*A*cos(d*x+c)-320*B*cos(d*x+c)-60*C*cos(d*x+c)-60*C)*c
os(d*x+c)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.819469, size = 1214, normalized size = 5.44

$$\left[\frac{15\left((2B+5C)a^2\cos(dx+c)+(2B+5C)a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{60\left(d\cos(dx+c)+d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((2*B + 5*C)*a^2*cos(d*x + c) + (2*B + 5*C)*a^2)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(6*A*a^2*cos(d*x + c)^3 +
2*(14*A + 5*B)*a^2*cos(d*x + c)^2 + 2*(43*A + 40*B + 15*C)*a^2*cos(d*x + c)
 + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(d*cos(d*x + c) + d), 1/30*(15*((2*B + 5*C)*a^2*cos(d*x + c) + (
2*B + 5*C)*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
- 2*a)) + 2*(6*A*a^2*cos(d*x + c)^3 + 2*(14*A + 5*B)*a^2*cos(d*x + c)^2 + 2
*(43*A + 40*B + 15*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\left(a\sec(dx+c)+a\right)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(5/2), x)
```

3.601 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=222

$$\frac{2a^3(160A+224B+245C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(40A+56B+35C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}C\,\mathrm{si}}{}$$

```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(160*A + 224*B + 245*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt
[a + a*Sec[c + d*x]]) + (2*a^2*(40*A + 56*B + 35*C)*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*a*(5*A + 7*B)*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.696677, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4017, 4015, 3801, 215}

$$\frac{2a^3(160A+224B+245C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(40A+56B+35C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}C\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(7/2),x]
```


```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(160*A + 224*B + 245*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt
[a + a*Sec[c + d*x]]) + (2*a^2*(40*A + 56*B + 35*C)*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*a*(5*A + 7*B)*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2\int\frac{(a+a\sec}{}}{}$$

$$=\frac{2a(5A+7B)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{}$$

$$=\frac{2a^2(40A+56B+35C)\sqrt{a+a\sec(c+dx)}\sin(c+}{105d\sqrt{\sec(c+dx)}}$$

$$=\frac{2a^3(160A+224B+245C)\sqrt{\sec(c+dx)}\sin(c+d}{105d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{2a^3(160A+224B+245C)\sqrt{\sec(c+dx)}\sin(c+d}{105d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{2a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3(160A+224B}{105}$$

Mathematica [A] time = 6.35336, size = 194, normalized size = 0.87

$$\frac{\sec^5\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-140(8A+4B+C)\sin^3\left(\frac{1}{2}(c+dx)\right)+\right.}{210d\sec^{\frac{9}{2}}(c+dx)(A\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*(105*Sqrt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 210*(4*A +
 4*B + 3*C)*Sin[(c + d*x)/2] - 140*(8*A + 4*B + C)*Sin[(c + d*x)/2]^3 + 168
*(5*A + B)*Sin[(c + d*x)/2]^5 - 240*A*Sin[(c + d*x)/2]^7))/(210*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [A] time = 0.436, size = 280, normalized size = 1.3

$$-\frac{a^2(\cos(dx+c))^4}{210\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(60\,A(\cos(dx+c))^4-105\,C\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x)
```

```
[Out] -1/210/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(60*A*cos(d*x+c)^4-105*C*(
-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)+105*C*(-2/(cos(d*x+c)+1))^(1/2)*2^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*sin(d*x+c)+180*A*cos(d*x+c)^3+84*B*cos(d*x+c)^3+220*A*cos(d*x+c)^2+308*B
*cos(d*x+c)^2+140*C*cos(d*x+c)^2+460*A*cos(d*x+c)+812*B*cos(d*x+c)+980*C*co
s(d*x+c)-920*A-1204*B-1120*C)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.52191, size = 1318, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*
cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^
2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin
(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 70*sqrt(
2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin
(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
```

```
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x + 3/2*c)
+ 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sq
rt(a) + 28*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 0.642632, size = 1257, normalized size = 5.66

$$\left[\frac{105\left(Ca^2\cos\left(dx+c\right)+Ca^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(15\,Aa^2\cos\right.}{}}{210\left(d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*(15*A*a^2*cos(d*x + c)^4 + 3*(20*A + 7*B)*a^
2*cos(d*x + c)^3 + (115*A + 98*B + 35*C)*a^2*cos(d*x + c)^2 + (230*A + 301*
B + 280*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/105*(105*(C*a^2*cos(d*x
+ c) + C*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
2*a)) + 2*(15*A*a^2*cos(d*x + c)^4 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^3 + (1
15*A + 98*B + 35*C)*a^2*cos(d*x + c)^2 + (230*A + 301*B + 280*C)*a^2*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(7/2), x)
```

3.602 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=231

$$\frac{64a^3(13A+15B+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(13A+15B+21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{315d\sqrt{\sec(c+dx)}}+\frac{2a(13A$$

```
[Out] (64*a^3*(13*A + 15*B + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Sec[c + d*x]]) + (16*a^2*(13*A + 15*B + 21*C)*Sqrt[a + a*Sec[c + d*x]]
*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]) + (2*a*(13*A + 15*B + 21*C)*(a +
a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (2*A*(a +
a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(5*A + 9*
B)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.555245, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4086, 4013, 3809, 3804}

$$\frac{64a^3(13A+15B+21C)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(13A+15B+21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{315d\sqrt{\sec(c+dx)}}+\frac{2a(13A$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(9/2),x]
```

```
[Out] (64*a^3*(13*A + 15*B + 21*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Sec[c + d*x]]) + (16*a^2*(13*A + 15*B + 21*C)*Sqrt[a + a*Sec[c + d*x]]
*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]) + (2*a*(13*A + 15*B + 21*C)*(a +
a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)) + (2*A*(a +
a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(5*A + 9*
B)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+}{}}{}$$

$$= \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(5A+9B)(a}{}$$

$$= \frac{2a(13A+15B+21C)(a+a\sec(c+dx))^{3/2}\sin(c+d}{105d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{16a^2(13A+15B+21C)\sqrt{a+a\sec(c+dx)}\sin(c+d}{315d\sqrt{\sec(c+dx)}}$$

$$= \frac{64a^3(13A+15B+21C)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{1}{}$$

Mathematica [A] time = 1.63969, size = 124, normalized size = 0.54

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((3116A+3030B+2352C)\cos(c+dx)+4(254A+180B+63C)\cos(2(c+dx))\cdots}{1260d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^2*(5653*A + 6240*B + 7476*C + (3116*A + 3030*B + 2352*C)*Cos[c + d*x] +
4*(254*A + 180*B + 63*C)*Cos[2*(c + d*x)] + 260*A*Cos[3*(c + d*x)] + 90*B*C
os[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
+ d*x)/2])/(1260*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.371, size = 166, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(35A(\cos(dx+c))^4+130A(\cos(dx+c))^3+45B(\cos(dx+c))^3+219A(\cos(dx+c))^2+18\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+45*B*cos
(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+292*A*cos
(d*x+c)+345*B*cos(d*x+c)+294*C*cos(d*x+c)+584*A+690*B+903*C)*(a*(cos(d*x+c)
+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.41028, size = 1085, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) + 30*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7
/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4
/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c
) + 21*a^2*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin
(7/2*d*x + 7/2*c) - 315*a^2*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7
/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*
sin(7/2*d*x + 7/2*c) + 21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) + 77*a^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 315*a^2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))))*B*sqrt(a) + 168*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^
2*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*C*sqrt(a))/d
```

Fricas [A] time = 0.497328, size = 400, normalized size = 1.73

$$\frac{2\left(35\,Aa^2\cos\left(dx+c\right)^5+5\,(26\,A+9\,B)a^2\cos\left(dx+c\right)^4+3\,(73\,A+60\,B+21\,C)a^2\cos\left(dx+c\right)^3+(292\,A+345\,B+\cdots}{315\,(d\cos\left(dx+c\right)+d)\sqrt{\cos\left(dx\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^5 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^4 + 3*(73*
A + 60*B + 21*C)*a^2*cos(d*x + c)^3 + (292*A + 345*B + 294*C)*a^2*cos(d*x +
 c)^2 + (584*A + 690*B + 903*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(9/2), x)
```

3.603 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(32A+44B+33C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(2840A+3\cdots}{3\cdots}$$

```
[Out] (2*a^3*(1160*A + 1364*B + 1485*C)*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sin[c + d*x])
/(3465*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(2840*A + 32
12*B + 3795*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(231*d*Sec[c + d*x]^(5/2)) + (2*a*(5*A + 11*B)*(a + a*Sec[c + d*x])^(3/2)*
Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```

Rubi [A] time = 0.863486, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4017, 4015, 3805, 3804}

$$\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(32A+44B+33C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(2840A+3\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(11/2),x]
```


```
[Out] (2*a^3*(1160*A + 1364*B + 1485*C)*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sin[c + d*x])
/(3465*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(2840*A + 32
12*B + 3795*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])
/(231*d*Sec[c + d*x]^(5/2)) + (2*a*(5*A + 11*B)*(a + a*Sec[c + d*x])^(3/2)*
Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec(c+\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a(5A+11B)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2A\cdots}{\cdots}$$

$$= \frac{2a^2(32A+44B+33C)\sqrt{a+a\sec(c+dx)}\sin(c+dx\cdots}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(32\cdots}{\cdots}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(284\cdots}{3465\cdots}$$

$$= \frac{2a^3(1160A+1364B+1485C)\sin(c+dx)}{3465d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(284\cdots}{3465\cdots}$$

Mathematica [A] time = 1.52795, size = 157, normalized size = 0.55

$$a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((69890A+68552B+66660C)\cos(c+dx)+16(1625A+1397B+990C)\cos(2(\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^2*(114640*A + 124366*B + 137280*C + (69890*A + 68552*B + 66660*C)*Cos[c
+ d*x] + 16*(1625*A + 1397*B + 990*C)*Cos[2*(c + d*x)] + 8675*A*Cos[3*(c +
d*x)] + 5720*B*Cos[3*(c + d*x)] + 1980*C*Cos[3*(c + d*x)] + 2240*A*Cos[4*(c
 + d*x)] + 770*B*Cos[4*(c + d*x)] + 315*A*Cos[5*(c + d*x)])*Sqrt[a*(1 + Sec
[c + d*x])]*Tan[(c + d*x)/2])/(27720*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.387, size = 199, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(315A(\cos(dx+c))^5+1120A(\cos(dx+c))^4+385B(\cos(dx+c))^4+1775A(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2
),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B
*cos(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+495*C*cos(d*x+c)^3+21
30*A*cos(d*x+c)^2+2409*B*cos(d*x+c)^2+1980*C*cos(d*x+c)^2+2840*A*cos(d*x+c)
+3212*B*cos(d*x+c)+3795*C*cos(d*x+c)+5680*A+6424*B+7590*C)*(a*(cos(d*x+c)+1
)/cos(d*x+c))^(1/2)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [B] time = 2.54516, size = 1709, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 34
65*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 318
78*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11
/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1
287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2
*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 1
1/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
```

```
 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*
x + 11/2*c))))*A*sqrt(a) + 22*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) +
756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/
2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arc
tan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x +
9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a
^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(si
n(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*B*sqrt(a) + 660*sqrt(2)*(315*a^
2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x
+ 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos(7/2*d*x + 7/2*c)*sin
(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 77*a^2*cos(7/2*
d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*
d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*sin(5/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/7*arctan2(sin(7/2*d*x
 + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.508561, size = 482, normalized size = 1.7

$$\frac{2\left(315\,Aa^2\cos\left(dx+c\right)^6+35\left(32\,A+11\,B\right)a^2\cos\left(dx+c\right)^5+5\left(355\,A+286\,B+99\,C\right)a^2\cos\left(dx+c\right)^4+3\left(710\,A+803\,\ldots\right.}{3465\,(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*A*a^2*cos(d*x + c)^6 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^5 + 5*
(355*A + 286*B + 99*C)*a^2*cos(d*x + c)^4 + 3*(710*A + 803*B + 660*C)*a^2*c
os(d*x + c)^3 + (2840*A + 3212*B + 3795*C)*a^2*cos(d*x + c)^2 + 2*(2840*A +
 3212*B + 3795*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(11/2), x)
```

3.604 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=334

$$\frac{2a^3(8368A+9230B+10439C)\sin(c+dx)}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+182B+143C)\sin}{1287d\sec}$$

```
[Out] (2*a^3*(2224*A + 2522*B + 2717*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x]
)/(15015*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(8368*A +
9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^3*(8368*A + 9230*B + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(45045*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1287*d*Sec[c + d*x]^(7/2)) + (2*a*(
5*A + 13*B)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(143*d*Sec[c + d*x]^(9
/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(13*d*Sec[c + d*x]^(11
/2))
```

Rubi [A] time = 0.940532, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4017, 4015, 3805, 3804}

$$\frac{2a^3(8368A+9230B+10439C)\sin(c+dx)}{15015d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+182B+143C)\sin}{1287d\sec}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(13/2),x]
```

```
[Out] (2*a^3*(2224*A + 2522*B + 2717*C)*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2)*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x]
)/(15015*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(8368*A +
9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^3*(8368*A + 9230*B + 10439*C)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(45045*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1287*d*Sec[c + d*x]^(7/2)) + (2*a*(
5*A + 13*B)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(143*d*Sec[c + d*x]^(9
/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(13*d*Sec[c + d*x]^(11
/2))
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\sec}{}}{}$$

$$= \frac{2a(5A+13B)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{143d\sec^{\frac{9}{2}}(c+dx)} +$$

$$= \frac{2a^2(136A+182B+143C)\sqrt{a+a\sec(c+dx)}\sin(}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(}{15}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(}{15}$$

$$= \frac{2a^3(2224A+2522B+2717C)\sin(c+dx)}{9009d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(}{15}$$

Mathematica [A] time = 1.61267, size = 190, normalized size = 0.57

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(4(453146A+454285B+445588C)\cos(c+dx)+(746519A+676000B+58115}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(13/2),x]
```

```
[Out] (a^2*(2798182*A + 2980640*B + 3233516*C + 4*(453146*A + 454285*B + 445588*C
)*Cos[c + d*x] + (746519*A + 676000*B + 581152*C)*Cos[2*(c + d*x)] + 287060
*A*Cos[3*(c + d*x)] + 225550*B*Cos[3*(c + d*x)] + 148720*C*Cos[3*(c + d*x)]
 + 94010*A*Cos[4*(c + d*x)] + 58240*B*Cos[4*(c + d*x)] + 20020*C*Cos[4*(c +
 d*x)] + 23940*A*Cos[5*(c + d*x)] + 8190*B*Cos[5*(c + d*x)] + 3465*A*Cos[6*
(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(720720*d*Sqrt[Sec
[c + d*x]])
```

Maple [A] time = 0.418, size = 232, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(3465A(\cos(dx+c))^6+11970A(\cos(dx+c))^5+4095B(\cos(dx+c))^5+18305A(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2
),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(3465*A*cos(d*x+c)^6+11970*A*cos(d*x+c)^5+40
95*B*cos(d*x+c)^5+18305*A*cos(d*x+c)^4+14560*B*cos(d*x+c)^4+5005*C*cos(d*x+
c)^4+20920*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18590*C*cos(d*x+c)^3+25104*A
*cos(d*x+c)^2+27690*B*cos(d*x+c)^2+31317*C*cos(d*x+c)^2+33472*A*cos(d*x+c)+
36920*B*cos(d*x+c)+41756*C*cos(d*x+c)+66944*A+73840*B+83512*C)*(a*(cos(d*x+
c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^7*(1/cos(d*x+c))^(13/2)/sin(d*x+c)
```

Maxima [B] time = 2.6364, size = 2109, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(13/2),x, algorithm="maxima")
```

```
[Out] 1/2882880*(sqrt(2)*(3783780*a^2*cos(12/13*arctan2(sin(13/2*d*x + 13/2*c), c
os(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 1066065*a^2*cos(10/13*arct
an2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c)
 + 459459*a^2*cos(8/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*
c)))*sin(13/2*d*x + 13/2*c) + 193050*a^2*cos(6/13*arctan2(sin(13/2*d*x + 13
/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 70070*a^2*cos(4/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13
/2*c) + 20475*a^2*cos(2/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c)))*sin(13/2*d*x + 13/2*c) - 3783780*a^2*cos(13/2*d*x + 13/2*c)*sin(12
```

```
/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 1066065*a^2*
cos(13/2*d*x + 13/2*c)*sin(10/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d
*x + 13/2*c))) - 459459*a^2*cos(13/2*d*x + 13/2*c)*sin(8/13*arctan2(sin(13/
2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 193050*a^2*cos(13/2*d*x + 13/2*
c)*sin(6/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 7007
0*a^2*cos(13/2*d*x + 13/2*c)*sin(4/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) - 20475*a^2*cos(13/2*d*x + 13/2*c)*sin(2/13*arctan2(sin
(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 6930*a^2*sin(13/2*d*x + 13/
2*c) + 20475*a^2*sin(11/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c))) + 70070*a^2*sin(9/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x
+ 13/2*c))) + 193050*a^2*sin(7/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*
d*x + 13/2*c))) + 459459*a^2*sin(5/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) + 1066065*a^2*sin(3/13*arctan2(sin(13/2*d*x + 13/2*c),
cos(13/2*d*x + 13/2*c))) + 3783780*a^2*sin(1/13*arctan2(sin(13/2*d*x + 13/2
*c), cos(13/2*d*x + 13/2*c))))*A*sqrt(a) + 130*sqrt(2)*(31878*a^2*cos(10/11
*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11
/2*c) + 8778*a^2*cos(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11
/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*a^2*cos(6/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11
*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11
/2*c) + 385*a^2*cos(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c)))*sin(11/2*d*x + 11/2*c) - 31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/
2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2
*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2
*c))) + 126*a^2*sin(11/2*d*x + 11/2*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*
d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1287*a^2*sin(7/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3465*a^2*sin(5/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 8778*a^2*sin(3/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 31878*a^2*sin(1/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*B*sqrt(a) + 572*sqrt(2)*(
8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9
/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*
x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 8190*
a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c
)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin
(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/
2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))))*C*sqrt(a))/d
```

Fricas [A] time = 0.51812, size = 567, normalized size = 1.7

$$\frac{2\left(3465\,Aa^{2}\cos\left(dx+c\right)^{7}+315\,(38\,A+13\,B)a^{2}\cos\left(dx+c\right)^{6}+35\,(523\,A+416\,B+143\,C)a^{2}\cos\left(dx+c\right)^{5}+5\,(4184\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(13/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*A*a^2*cos(d*x + c)^7 + 315*(38*A + 13*B)*a^2*cos(d*x + c)^6 +
 35*(523*A + 416*B + 143*C)*a^2*cos(d*x + c)^5 + 5*(4184*A + 4615*B + 3718*
C)*a^2*cos(d*x + c)^4 + 3*(8368*A + 9230*B + 10439*C)*a^2*cos(d*x + c)^3 +
4*(8368*A + 9230*B + 10439*C)*a^2*cos(d*x + c)^2 + 8*(8368*A + 9230*B + 104
39*C)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(13/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(13/2), x)
```

3.605 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=241

$$\frac{(8A-2B+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A-14B+9C)\sinh^{-1}\left(\right.}{8\sqrt{a}d}$$

```
[Out] -((8*A - 14*B + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((8*A -
2*B + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ ((6*B - C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]
]) + (C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.803534, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4088, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(8A-2B+7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A-14B+9C)\sinh^{-1}\left(\right.}{8\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```



```
[Out] -((8*A - 14*B + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((8*A -
2*B + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ ((6*B - C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]
]) + (C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{C\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(6A+5C)+\frac{1}{2}a(6B-C)\,\mathrm{s}\right.}{\sqrt{a+a\sec(c+dx)}}}{3a}$$

$$= \frac{(6B-C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{C\sec^{\frac{7}{2}}(c+dx)\sin(c+d}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(8A-2B+7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sec^{\frac{5}{2}}(c}{12d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{(8A-2B+7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sec^{\frac{5}{2}}(c}{12d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{(8A-2B+7C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sec^{\frac{5}{2}}(c}{12d\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{(8A-14B+9C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}(A-B+C)\,\mathrm{t}}{}$$

Mathematica [A] time = 1.42648, size = 198, normalized size = 0.82

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(48(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\sqrt{2}(8A-14B+9C\right.}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(48*(A - B + C)*A
rcTanh[Sin[(c + d*x)/2]] - 3*Sqrt[2]*(8*A - 14*B + 9*C)*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*Sec[c + d*x]*(3*(8*A - 2*B + 7*C) + 2*(6*B - C)*Sec[c +
d*x] + 8*C*Sec[c + d*x]^2)*Sin[(c + d*x)/2]))/(12*d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.417, size = 640, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/48/d/a*(-1+cos(d*x+c))*(-24*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+24*A*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+4
2*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
*cos(d*x+c)^3*2^(1/2)-42*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)-27*C*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+27*C*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d
*x+c)^3*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+96*A
*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-12*B*cos(d*x
+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-96*B*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+42*C*sin(d*x+c)*cos(d*x+c)^2*(-2/(cos(d*
x+c)+1))^(1/2)+96*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)^3+24*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-4*C*sin(d*x+c)*
cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/(-2/(cos(d*x+
c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 3.36399, size = 7028, normalized size = 29.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
```

+ 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x +
2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - 6*(4*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*sin(4*d*x
+ 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x
+ c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
+ 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4
*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 7*(2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
+ 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sq
rt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4
*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^

```
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(sqrt(2)*cos(4*
d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 +
4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2
+ 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*sqrt(a)) + (84*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4
*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 100*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) +
 3*sqrt(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 312*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin
(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 312*(sqrt(2)*
sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))
*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 100*(sqrt(2)*sin(6*d*x + 6*
c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arcta
n2(sin(d*x + c), cos(d*x + c))) - 84*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*
sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(
6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) +
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c
)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2
*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*
d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + si
n(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^
2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*
x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(
2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2
+ 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x
 + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x
+ 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 +
18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2) - 48*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 +
```

```
9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4
*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*si
n(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*c
os(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*s
in(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 48*(sqrt(2)*c
os(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*
c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*
sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x
 + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqr
t(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - 84*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt
(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(11/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 100*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)
*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 312*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos
(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 312*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d
*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 100*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x +
 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 84*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c)
 + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))))*C/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 1.80223, size = 1809, normalized size = 7.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((8*A - 14*B + 9*C)*cos(d*x + c)^3 + (8*A - 14*B + 9*C)*cos(d*x +
c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 48*sqr
t(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*x + c)^2)*log(-(co
s(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1))/sqrt(a) + 4*(3*(8*A - 2*B + 7*C)*cos(d*x + c)^2 + 2*(6*B -
C)*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/48*(48*s
qrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*x + c)^2)*sqrt(-
1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt
(cos(d*x + c))/sin(d*x + c)) + 3*((8*A - 14*B + 9*C)*cos(d*x + c)^3 + (8*A
```

```
- 14*B + 9*C)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a
*cos(d*x + c) - 2*a)) - 2*(3*(8*A - 2*B + 7*C)*cos(d*x + c)^2 + 2*(6*B - C)
*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a
*sec(d*x + c) + a), x)
```

3.606 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(8A-4B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4B-C)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{4d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((8*A - 4*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((4*B - C)
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c
 + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.598531, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4088, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(8A-4B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}+\frac{(4B-C)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{4d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```

```
[Out] ((8*A - 4*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((4*B - C)
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (C*Sec[c
 + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(4A+3C)+\frac{1}{2}a(4B-\right.}{\sqrt{a+a\sec(c+dx)}}}{2a}$$

$$=\frac{(4B-C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+}}$$

$$=\frac{(4B-C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+}}$$

$$=\frac{(4B-C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{C\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+}}$$

$$=\frac{(8A-4B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{ad}}-\frac{\sqrt{2}(A-B+C)}{}$$

Mathematica [A] time = 0.836023, size = 174, normalized size = 0.89

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-8(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(8A-4B+7C)\right.}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}(A\cos(2(c+dx))+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-8*(A - B + C)*A
rcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(8*A - 4*B + 7*C)*ArcTanh[Sqrt[2]*Sin[(c
 + d*x)/2]] + 2*Sec[c + d*x]*(4*B - C + 2*C*Sec[c + d*x])*Sin[(c + d*x)/2])
)/(2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)
*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.39, size = 549, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/16/d/a*(8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*2^(1/2)*cos(d*x+c)^2-8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2-4*B*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2
+4*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)*2^(1/2)*cos(d*x+c)^2+7*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2-7*C*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^2-16*A*cos
(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+16*B*cos(d*x+c)^
2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+8*B*cos(d*x+c)*sin(d*x+c
)*(-2/(cos(d*x+c)+1))^(1/2)-16*C*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))-2*C*sin(d*x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*C
*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*
(1/cos(d*x+c))^(3/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1
)
```

Maxima [B] time = 2.87679, size = 4047, normalized size = 20.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
```

(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) + 4*(4*sqrt(2)*cos(3/2*arcta
n2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*
sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^
2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log
(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
 - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sqrt(a)) - (4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(
2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*si
n(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))
*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c)
 + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*
cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos

```
(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c
)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x +
 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arct
an2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
c))))*C/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 1.76519, size = 1671, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*A - 4*B + 7*C)*cos(d*x + c)^2 + (8*A - 4*B + 7*C)*cos(d*x + c))*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 8*sqrt(2)*((
A - B + C)*a*cos(d*x + c)^2 + (A - B + C)*a*cos(d*x + c))*log(-(cos(d*x + c
)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1))/sqrt(a) + 4*((4*B - C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*c
os(d*x + c)), 1/8*(8*sqrt(2)*((A - B + C)*a*cos(d*x + c)^2 + (A - B + C)*a*
cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + ((8*A - 4*B + 7*C)*cos(d
*x + c)^2 + (8*A - 4*B + 7*C)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(
d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*((4*B - C)*cos(d*x + c) + 2*C)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*c
os(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right) \sec \left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a \sec \left(dx+c\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a
*sec(d*x + c) + a), x)
```

3.607 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((2*B - C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[
a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (C*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.417567, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4088, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```


```
[Out] ((2*B - C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[
a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (C*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(2A+C)+\frac{1}{2}a(2B-\cdots\right.}{\sqrt{a+a\sec(c+dx)}}}{a}$$

$$= \frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2B-C)\int\sqrt{\sec(c+dx)}\sqrt{a\cdots}}{2a}$$

$$= \frac{C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2B-C)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,d\cdots\right.}{ad}$$

$$= \frac{(2B-C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{2}(A-B+C)\tanh^{-1}\cdots}{\sqrt{\cdots}}$$

Mathematica [A] time = 0.623653, size = 107, normalized size = 0.76

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(2B-C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\cdots\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(2*(A - B + C)*ArcTanh[Sin[(c + d*x)/2
]] + Sqrt[2]*(2*B - C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*C*Sec[c + d*x]
*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.409, size = 378, normalized size = 2.7

$$\frac{(\cos(dx+c))^2-1}{4\,ad\,(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2\,B\sqrt{2}\cos(dx+c)\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/4/d/a*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*B*2^(1/
2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))-2*B*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))-C*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+C*2^(1/2)*cos(d*x+c)*arcta
n(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+4*A*cos(
d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*B*cos(d*x+c)*arct
an(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+4*C*cos(d*x+c)*arctan(1/2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+2*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))
*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.57597, size = 1947, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - 2*(sqrt(2)*log(cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(
2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2))*B/sqrt(a) - (4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*
sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
```

```
 c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*co
s(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2
 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C/((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.890274, size = 1424, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B - C)*cos(d*x + c) + 2*B - C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)) - 2*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A -
B + C)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*C*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/
2*(2*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
c))/sin(d*x + c)) - ((2*B - C)*cos(d*x + c) + 2*B - C)*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*C*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec (dx+c)^2 + B \sec (dx+c) + A\right)\sqrt{\sec (dx+c)}}{\sqrt{a \sec (dx+c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a
*sec(d*x + c) + a), x)
```

3.608 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.388291, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4086, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]),x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}aC\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (-A+B-C)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx + \frac{C}{}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2C)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} +$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} +$$

Mathematica [A] time = 0.502301, size = 96, normalized size = 0.7

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(\right.\right.\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(-((A - B + C)*ArcTanh[Sin[(c + d*x)
/2]]) + Sqrt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sin[(c + d*x)/2])
)/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.351, size = 319, normalized size = 2.3

$$-\frac{1}{2ad\sin(dx+c)}\left(-C\sqrt{-2(\cos(dx+c)+1)^{-1}}\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/2/d/a*(-C*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)+C*(-2/(cos(d*x+c)+1)
)^(1/2)*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))*sin(d*x+c)-2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*
(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-2*C*(-2/(cos(d*x+c)+1
))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*A*co
s(d*x+c)-4*A)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/(1/cos(d*x+c))
^(1/2)
```

Maxima [B] time = 2.40783, size = 902, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a) + (sqrt(2)*log(cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt
(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2))*C/sqrt(a))/d
```

Fricas [A] time = 0.678405, size = 1373, normalized size = 9.95

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(C\cos\left(dx+c\right)+C\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\ldots}}{\sqrt{\cos(dx+\ldots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2\left(ad\cos\left(dx+c\right)\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)) + sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(
-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B + C
)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (
C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*sqrt(sec(d*x + c))), x)
```

3.609 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.361446, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4086, 4013, 3808, 206}

$$\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*Sqrt[a + a*
Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2\int\frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(2A+3C)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$
$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}-\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}-\frac{2(A-3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 0.586395, size = 88, normalized size = 0.62

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(3(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)(A\cos(c+dx)-A+3B)\right)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*(A - B + C)*ArcTanh[Sin[(c + d*x)
/2]] + 2*(-A + 3*B + A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d*Sqrt[a*(1 + Se
c[c + d*x])])
```

Maple [A] time = 0.401, size = 229, normalized size = 1.6

$$-\frac{(\cos(dx+c))^2}{3\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/3/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-3*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)
+3*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*sin(d*x+c)+2*A*cos(d*x+c)^2-4*A*cos(d*x+c)+6*B*cos(d*x+c)+2*A-6*B)*cos
```

```
(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.31069, size = 641, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*B/sqrt(a) - 3*(s
qrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*C/sqrt(a))/d
```

Fricas [A] time = 0.540836, size = 960, normalized size = 6.71

$$\left[\frac{\frac{3\sqrt{2}\left((A-B+C)a\cos(dx+c)+(A-B+C)a\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(A\cos(dx+c)^2-(A-3B)\cos(dx+c)\right)\sqrt{\frac{a\cos\cdots}{\cos\cdots}}\cdots}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos(dx+c)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1))/sqrt(a) + 4*(A*cos(d*x + c)^2 - (A - 3*B)*cos(d*x + c))*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*
x + c) + a*d), -1/3*(3*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)
*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/
a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(A*cos(d*x + c)^2 - (A - 3*B)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
```

```
+ c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(sqrt(a*(sec(c + d*x) + 1
))*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + B\sec(dx + c) + A}{\sqrt{a\sec(dx + c) + a}\sec(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*sec(d*x + c)^(3/2)), x)
```

3.610 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=191

$$\frac{2(13A-5B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-5B)\sin(c+}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*S
ec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(1
5*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A - 5*B + 15*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.570631, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4022, 4013, 3808, 206}

$$\frac{2(13A-5B+15C)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-5B)\sin(c+}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*Sqrt[a + a*
Sec[c + d*x]]),x]
```


```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*S
ec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(1
5*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A - 5*B + 15*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+\frac{1}{2}a(4A+5C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a}$$

Mathematica [A] time = 0.885045, size = 155, normalized size = 0.81

$$-\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(15(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+20(A+B)\sin^3\left(\right.\right.}{15d\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}(A\cos(2c+2dx)+A+2B\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(15*(A - B + C
)*ArcTanh[Sin[(c + d*x)/2]] - 30*(A + C)*Sin[(c + d*x)/2] + 20*(A + B)*Sin[
(c + d*x)/2]^3 - 24*A*Sin[(c + d*x)/2]^5))/(15*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.427, size = 263, normalized size = 1.4

$$\frac{(\cos(dx+c))^3}{15\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/15/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*arctan(1/2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-6*A*cos(d*x+c
)^3-15*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))
^(1/2)*B*sin(d*x+c)+15*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+8*A*cos(d*x+c)^2-10*B*cos(d*x+c)^2-28*A
*cos(d*x+c)+20*B*cos(d*x+c)-30*C*cos(d*x+c)+26*A-10*B+30*C)*cos(d*x+c)^3*(1
/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.45713, size = 1002, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) - 10*(3*sqrt(2)*cos(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*
cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*
sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin
(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*B/sqrt(a) - 30*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2
)*sin(1/2*d*x + 1/2*c))*C/sqrt(a))/d
```

Fricas [A] time = 0.544631, size = 1069, normalized size = 5.6

$$\left[\frac{\frac{15\sqrt{2}((A-B+C)a\cos(dx+c)+(A-B+C)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(3A\cos(dx+c)^3-(A-5B)\cos(dx+c)\right.}{}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*
x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a) + 4*(3*A*cos(d*x + c)^3 - (A - 5*B)*cos(d*x + c)^2 + (1
3*A - 5*B + 15*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((
A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c
)) + 2*(3*A*cos(d*x + c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5*B + 15*C)
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*sec(d*x + c)^(5/2)), x)
```

3.611 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=237

$$-\frac{2(43A-91B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(}{\sqrt{2}\sqrt{}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[
c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d
*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sin[
c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A -
91*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x
]])
```

Rubi [A] time = 0.758292, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4086, 4022, 4013, 3808, 206}

$$-\frac{2(43A-91B+35C)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B+35C)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(}{\sqrt{2}\sqrt{}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*Sqrt[a + a*
Sec[c + d*x]]),x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[
c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d
*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sin[
c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A -
91*B + 35*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x
]])
```


Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-7B)+\frac{1}{2}a(6A+7C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 1.56284, size = 175, normalized size = 0.74

$$\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-140(2A+B+C)\sin^3\left(\frac{1}{2}(c+dx)\right)+105(A-B+C)\tanh^{-1}\right.}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}(A\cos(2c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(105*(A - B + C
)*ArcTanh[Sin[(c + d*x)/2]] + 210*B*Sin[(c + d*x)/2] - 140*(2*A + B + C)*Si
n[(c + d*x)/2]^3 + 168*(2*A + B)*Sin[(c + d*x)/2]^5 - 240*A*Sin[(c + d*x)/2
]^7))/(105*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]
^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.377, size = 296, normalized size = 1.3

$$-\frac{(\cos(dx+c))^4}{105\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(30\,A(\cos(dx+c))^4+105\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/105/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(30*A*cos(d*x+c)^4+105*arcta
n(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin
(d*x+c)-36*A*cos(d*x+c)^3-105*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+42*B*cos(d*x+c)^3+105*C*(-2/(cos
(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+
c)+68*A*cos(d*x+c)^2-56*B*cos(d*x+c)^2+70*C*cos(d*x+c)^2-148*A*cos(d*x+c)+1
96*B*cos(d*x+c)-140*C*cos(d*x+c)+86*A-182*B+70*C)*cos(d*x+c)^4*(1/cos(d*x+c
))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.58547, size = 1465, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) - 14*s
```

```
qrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin
(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin(2/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))))*B/sqrt(a) + 140*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/
2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2
)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d
*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))))*C/sqrt(a))/d
```

Fricas [A] time = 0.566275, size = 1187, normalized size = 5.01

$$\left[\frac{\frac{105\sqrt{2}((A-B+C)a\cos(dx+c)+(A-B+C)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\left(15A\cos(dx+c)^4-3(A-7B)\cos(d\right.}{210(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(
d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1))/sqrt(a) + 4*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3
 + (31*A - 7*B + 35*C)*cos(d*x + c)^2 - (43*A - 91*B + 35*C)*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
*d*cos(d*x + c) + a*d), -1/105*(105*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (
A - B + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(15*A*cos(d*x + c)^4 -
 3*(A - 7*B)*cos(d*x + c)^3 + (31*A - 7*B + 35*C)*cos(d*x + c)^2 - (43*A -
91*B + 35*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{\sqrt{a \sec (dx + c) + a} \sec (dx + c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*sec(d*x + c)^(7/2)), x)
```

3.612 $\int \frac{\sqrt{\sec(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{2}(a-b)(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2aB+2Ab-bB)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{bB\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (b*B
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.472165, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4088, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(a-b)(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2aB+2Ab-bB)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{bB\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c + d*x]^
2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (b*B
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(2aA+\right.}{\sqrt{}}}{}$$

$$= \frac{bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + ((a-b)(A-B))\int$$

$$= \frac{bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2(a-b)(A-B))\,\mathrm{S}}{}$$

$$= \frac{(2Ab+2aB-bB)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}(a}{}$$

Mathematica [A] time = 0.515238, size = 118, normalized size = 0.78

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2(a-b)(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(2aB+2Ab-bB)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c +
 d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(2*(a - b)*(A - B)*ArcTanh[Sin[(c + d*
x)/2]] + Sqrt[2]*(2*A*b + 2*a*B - b*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] +
2*b*B*Sec[c + d*x]*Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.419, size = 515, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*a+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*sec(d
*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2*A*2^(1/
2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*b-2*A*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*b+2*B*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*a-B*2^(1/2)*cos(d*x+
c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*
b-2*B*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1-sin(d*x+c)))*a+B*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*b+4*A*cos(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*a-4*A*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*b-4*B*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))*a+4*B*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(
1/2))*b+2*B*(-2/(cos(d*x+c)+1))^(1/2)*b*sin(d*x+c))*(-2/(cos(d*x+c)+1))^(1/
2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.99421, size = 2589, normalized size = 17.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A*sqrt(a) - 2*(sqrt(2)*log(cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(
2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2))*B*sqrt(a) - 2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
```

```
))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))*A*b/sqrt(a) - (4*sqrt(2)*
cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(
2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos
(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))))*B*b/((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [B] time = 1.83382, size = 1563, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*b*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)) - (2*B*a + (2*A - B)*b + (2*B*a + (2*A - B)*b)*cos(d*x + c))*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d
*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*((A - B
)*a^2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*cos(d*x + c))*log(-(cos(d
*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*((A - B)*a^
2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*cos(d*x + c))*sqrt(-1/a)*arct
an(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
```

```
+ c))/sin(d*x + c)) - 2*B*b*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) - (2*B*a + (2*A - B)*b + (2*B*a + (2*A - B)*b)*cos
(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(sec(d*
x + c))/sqrt(a*sec(d*x + c) + a), x)
```

3.613 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{(5A-9B+13C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(8A-12B+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\,\mathrm{s}}{2d(a\sec(c+dx)+}$$

```
[Out] ((8*A - 12*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) -
 ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3
/2)) - ((2*A - 6*B + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a*d*Sqrt[a +
a*Sec[c + d*x]]) + ((A - B + 2*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*d*S
qrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.90626, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4084, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(5A-9B+13C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(8A-12B+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\,\mathrm{s}}{2d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```

```
[Out] ((8*A - 12*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) -
 ((A - B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3
/2)) - ((2*A - 6*B + 7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a*d*Sqrt[a +
a*Sec[c + d*x]]) + ((A - B + 2*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*d*S
qrt[a + a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
```

```
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \int \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(A-5\right.}{\sqrt{a}}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+2C)\sec^{\frac{5}{2}}(}{2ad\sqrt{a+a\,s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A-6B+7C)\sec}{4ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A-6B+7C)\sec}{4ad\sqrt{a+a}}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A-6B+7C)\sec}{4ad\sqrt{a+a}}$$

$$= \frac{(8A-12B+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - (5A-9B+13C)$$

Mathematica [A] time = 2.2754, size = 239, normalized size = 0.92

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-2(5A-9B+13C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{\sqrt{2}(8A-12B+19C)\cos}{}\right.}{d\sqrt{\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-2*(5*A - 9*B
+ 13*C)*ArcTanh[Sin[(c + d*x)/2]] - (Sqrt[2]*(8*A - 12*B + 19*C)*ArcTanh[Sq
rt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + ((-2*A + 6*B - 3*C + (8*B - 6*
C)*Cos[c + d*x] + (-2*A + 6*B - 7*C)*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Sin[(
c + d*x)/2])/2)/(-1 + Sin[(c + d*x)/2]^2)))/(d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.363, size = 741, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 1/16/d/a^2*(8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-8*A*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+
c)-12*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+
```

```
c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+12*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+19*C
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*co
s(d*x+c)^2*2^(1/2)*sin(d*x+c)-19*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+4*A*(-2/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)^3-20*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-12*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)^3+36*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)+14*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-52*C*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-4*A*cos(d*x+c)^2*(-2/
(cos(d*x+c)+1))^(1/2)+4*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-8*C*cos(d*
x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+8*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-1
0*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*C*(-2/(cos(d*x+c)+1))^(1/2))*cos
(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.23739, size = 2103, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(2)*((5*A - 9*B + 13*C)*cos(d*x + c)^3 + 2*(5*A - 9*B + 13*C)*
cos(d*x + c)^2 + (5*A - 9*B + 13*C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x +
 c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + ((8*A - 12*B + 19*C)*cos(d*x + c)^3 + 2*(8*A - 12*B + 19*C)*
cos(d*x + c)^2 + (8*A - 12*B + 19*C)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a
)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((2*A - 6*B + 7*C)*cos(d*x + c)^2
- (4*B - 3*C)*cos(d*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c
)^2 + a^2*d*cos(d*x + c)), 1/8*(2*sqrt(2)*((5*A - 9*B + 13*C)*cos(d*x + c)^
3 + 2*(5*A - 9*B + 13*C)*cos(d*x + c)^2 + (5*A - 9*B + 13*C)*cos(d*x + c))*
sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))/(a*sin(d*x + c))) + ((8*A - 12*B + 19*C)*cos(d*x + c)^3 +
2*(8*A - 12*B + 19*C)*cos(d*x + c)^2 + (8*A - 12*B + 19*C)*cos(d*x + c))*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((2*A
- 6*B + 7*C)*cos(d*x + c)^2 - (4*B - 3*C)*cos(d*x + c) - 2*C)*sqrt((a*cos(d
```

```
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x
+ c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^(3/2), x)
```

3.614 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(A-5B+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ((2*B - 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
3/2)*d) + ((A - 5*B + 9*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((A -
B + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.603515, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4084, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(A-5B+9C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```

```
[Out] ((2*B - 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
3/2)*d) + ((A - 5*B + 9*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((A -
B + 3*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(A+3B\right.}{\sqrt{a+}}}{}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+3C)\sec^{\frac{3}{2}}(}{2ad\sqrt{a+a\,s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+3C)\sec^{\frac{3}{2}}(}{2ad\sqrt{a+a\,s}}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+3C)\sec^{\frac{3}{2}}(}{2ad\sqrt{a+a\,s}}$$

$$= \frac{(2B-3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B+9C)\tanh^{-1}\left(\right.}{2\sqrt{2}a^{3/2}}$$

Mathematica [A] time = 2.1853, size = 177, normalized size = 0.88

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)(A-B+2C\sec(c+dx)+3C)+(A-5B+9C)\cos\left(\frac{1}{2}(c+dx)\right)\right.}{ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}(A\cos(2(c+dx))+A}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((A - 5*B + 9*C)*ArcTanh[Sin[(c +
d*x)/2]]*Cos[(c + d*x)/2] + 2*Sqrt[2]*(2*B - 3*C)*ArcTanh[Sqrt[2]*Sin[(c +
d*x)/2]]*Cos[(c + d*x)/2] + (A - B + 3*C + 2*C*Sec[c + d*x])*Tan[(c + d*x)/
2]))/(a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(3
/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.389, size = 561, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 1/4/d/a^2*(-2*B*2^(1/2)*sin(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+2*B*2^(1/2)*sin(d*x+c)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+3
*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))-3*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+A*sin(d*x+c)*c
os(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*cos(d*x+c)^2*(
-2/(cos(d*x+c)+1))^(1/2)-5*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)+B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+9*C*sin(d
*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*C*cos(d
*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-B*
(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
+2*C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(cos(d*x
+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)
^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.03804, size = 1793, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((A - 5*B + 9*C)*cos(d*x + c)^2 + 2*(A - 5*B + 9*C)*cos(d*x +
 c) + A - 5*B + 9*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*((2*B - 3*C)
*cos(d*x + c)^2 + 2*(2*B - 3*C)*cos(d*x + c) + 2*B - 3*C)*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((A - B + 3*C)*cos(d*x + c)
 + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*
((A - 5*B + 9*C)*cos(d*x + c)^2 + 2*(A - 5*B + 9*C)*cos(d*x + c) + A - 5*B
+ 9*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((2*B - 3*C)*cos(d*x + c)^2 +
 2*(2*B - 3*C)*cos(d*x + c) + 2*B - 3*C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
 + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((A - B + 3*C)*cos(d*x + c) + 2*C)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*
d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^2 + B \sec\left(dx+c\right) + A\right) \sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a \sec\left(dx+c\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^(3/2), x)
```

3.615 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{(3A+B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((3*A + B - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.406162, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4084, 4023, 3808, 206, 3801, 215}

$$\frac{(3A+B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((3*A + B - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(3A+\ldots}{\sqrt{a+a\sec\ldots}}}{2\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+B-5C)\int \ldots}{4a\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A+B-5C)\,\mathrm{Sub}\ldots}{\ldots}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(3A+B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c\ldots}}{\sqrt{2}\sqrt{a+\ldots}}\right)}{2\sqrt{2}a^{3/2}d}\ldots$$

Mathematica [A] time = 1.30354, size = 175, normalized size = 1.17

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{A-B+C}{\sin\left(\frac{1}{2}(c+dx)\right)-1}+\frac{A-B+C}{\sin\left(\frac{1}{2}(c+dx)\right)+1}+2(3A+B-5C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c\ldots\right.\right.\right.}{d\sqrt{\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+dx))+A+2B\cos(c+dx)+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(2*(3*A + B - 5
*C)*ArcTanh[Sin[(c + d*x)/2]] + 8*Sqrt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2
]] + (A - B + C)/(-1 + Sin[(c + d*x)/2]) + (A - B + C)/(1 + Sin[(c + d*x)/2
])))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.371, size = 384, normalized size = 2.6

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-2C\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 1/4/d/a^2*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+
c)*(-2*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))*sin(d*x+c)+2*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2)+3*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-B*
(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c
)+1))^(1/2))*sin(d*x+c)+C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*C*arctan(1
/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-A*(-2/(cos(d*x+c)+1))^(
1/2)+B*(-2/(cos(d*x+c)+1))^(1/2)-C*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+
c)+1))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.715002, size = 1656, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((3*A + B - 5*C)*cos(d*x + c)^2 + 2*(3*A + B - 5*C)*cos(d*x
+ c) + 3*A + B - 5*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(A - B + C)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) -
4*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 -
 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d), -1/4*(sqrt(2)*((3*A + B - 5*C)*cos(d*x + c)^2 + 2*(3*A + B - 5*C
)*cos(d*x + c) + 3*A + B - 5*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A -
 B + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c) - 4*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec
(d*x + c) + a)^(3/2), x)
```

3.616 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{(7A-3B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\sin(c+dx)\sqrt{\mathrm{se}}}{2d(a\sec(c+dx)+a}$$

```
[Out] -((7*A - 3*B - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A - B + C)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.382554, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4084, 4013, 3808, 206}

$$-\frac{(7A-3B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\sin(c+dx)\sqrt{\mathrm{se}}}{2d(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[
c + d*x])^(3/2)),x]
```

```
[Out] -((7*A - 3*B - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A - B + C)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A-B+C)-a(A-B-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B+C)\sqrt{\sec(c+dx)}\sin(c+}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B+C)\sqrt{\sec(c+dx)}\sin(c+}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(7A-3B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\sqrt{\sec(c+dx)}\,\mathrm{si}}{2d(a+a\sec(c+dx)}$$

Mathematica [A] time = 1.11864, size = 147, normalized size = 0.91

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2(-7A+3B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\tan\left(\frac{1}{2}(c+dx)\right)\mathrm{se}\right.}{d\sqrt{\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+dx))+A+2B\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(2*(-7*A + 3*B
+ C)*ArcTanh[Sin[(c + d*x)/2]] + 2*(5*A - B + C + 4*A*Cos[c + d*x])*Sec[(c
+ d*x)/2]*Tan[(c + d*x)/2]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c +
d*x)])*Sqrt[Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.366, size = 397, normalized size = 2.5

$$-\frac{-1+\cos(dx+c)}{4da^2(\sin(dx+c))^3}\left(7A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/4/d/a^2*(-1+cos(d*x+c))*(7*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-3*B*sin(d*x+c)*cos(d*
x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
```

```
1/2)-C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)+7*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-C*(-2/(cos(d*x+c
)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-8*A
*cos(d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(d*x+c)-2*C*cos(d*x+c)+10*A-2*B+2*C)*(a
*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1/cos(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.545915, size = 1157, normalized size = 7.19

$$\left[-\frac{\sqrt{2}\left((7\,A-3\,B-C)\cos\left(dx+c\right)^2+2\,(7\,A-3\,B-C)\cos\left(dx+c\right)+7\,A-3\,B-C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(}{\cos}}}{\cos(}\right.}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - 3*B - C)*cos(d*x + c)^2 + 2*(7*A - 3*B - C)*cos(d*x
+ c) + 7*A - 3*B - C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*
x + c)^2 + (5*A - B + C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d), 1/4*(sqrt(2)*((7*A - 3*B - C)*cos(d*x + c)^2 + 2*(7*A - 3*
B - C)*cos(d*x + c) + 7*A - 3*B - C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2
*(4*A*cos(d*x + c)^2 + (5*A - B + C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec{(dx+c)}^2+B\sec{(dx+c)}+A}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\sec{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/
2)*sqrt(sec(d*x + c))), x)
```

3.617 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{(11A-7B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(19A-15B+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A-3B+3C)\,\mathrm{s}}{6ad\sqrt{\sec(c+dx)}\sqrt{a\,\mathrm{s}}}$$

```
[Out] ((11*A - 7*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c
+ d*x])/(2*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A - 3*B +
 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (
(19*A - 15*B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[
c + d*x]])
```

Rubi [A] time = 0.57464, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4084, 4022, 4013, 3808, 206}

$$\frac{(11A-7B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(19A-15B+3C)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A-3B+3C)\,\mathrm{s}}{6ad\sqrt{\sec(c+dx)}\sqrt{a\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^(3/2)),x]
```

```
[Out] ((11*A - 7*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c
+ d*x])/(2*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A - 3*B +
 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (
(19*A - 15*B + 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[
c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\frac{1}{2}a(7A-3B+3C)-2a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}+\frac{(7A-3B+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}+\frac{(7A-3B+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}+\frac{(7A-3B+3C)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(11A-7B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B+C)\sin(c+}{2d\sqrt{\sec(c+dx)}(a+a\sec(}$$

Mathematica [A] time = 1.52552, size = 126, normalized size = 0.59

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(-12(A-B)\cos(c+dx)+2A\cos(2(c+dx))-17A+15B-3C)+6(11A\right.}{12ad\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(6*(11*A - 7*B + 3*C)*ArcTanh[Sin[(c +
 d*x)/2]]*Cos[(c + d*x)/2]^2 + 2*(-17*A + 15*B - 3*C - 12*(A - B)*Cos[c + d
*x] + 2*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(12*a*d*Sqrt[a*(1 + Sec[c +
d*x])])
```

Maple [B] time = 0.379, size = 427, normalized size = 2.

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^2}{12\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(33\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 1/12/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(33*A*sin(d*
x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d
*x+c)+1))^(1/2)-21*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+9*C*sin(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+33*arc
tan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*s
in(d*x+c)+8*A*cos(d*x+c)^3-21*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+9*C*(-2/(cos(d*x+c)+1))^(1/2)*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-32*A*cos(d*x+c)^2
+24*B*cos(d*x+c)^2-14*A*cos(d*x+c)+6*B*cos(d*x+c)-6*C*cos(d*x+c)+38*A-30*B+
6*C)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.558188, size = 1281, normalized size = 6.01

$$\left[\frac{3\sqrt{2}\left((11\,A-7\,B+3\,C)\cos(dx+c)^2+2\,(11\,A-7\,B+3\,C)\cos(dx+c)+11\,A-7\,B+3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}}{}\right.}{24\left(a^2d\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((11*A - 7*B + 3*C)*cos(d*x + c)^2 + 2*(11*A - 7*B + 3*C)*
cos(d*x + c) + 11*A - 7*B + 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*
(4*A*cos(d*x + c)^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A - 15*B + 3*C)*cos(d
```

```
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt
(2)*((11*A - 7*B + 3*C)*cos(d*x + c)^2 + 2*(11*A - 7*B + 3*C)*cos(d*x + c)
+ 11*A - 7*B + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*A*cos(d*x + c
)^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A - 15*B + 3*C)*cos(d*x + c))*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*co
s(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/
2)*sec(d*x + c)^(3/2)), x)
```

3.618 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{(15A-11B+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\sin(c+}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+}$$

```
[Out] -((15*A - 11*B + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[
c + d*x])/(2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A - 5*B
 + 5*C)*Sin[c + d*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
- ((39*A - 35*B + 15*C)*Sin[c + d*x])/(30*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + ((147*A - 95*B + 75*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
30*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.752957, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4084, 4022, 4013, 3808, 206}

$$-\frac{(15A-11B+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A-B+C)\sin(c+}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^(3/2)),x]
```

```
[Out] -((15*A - 11*B + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[
c + d*x])/(2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A - 5*B
 + 5*C)*Sin[c + d*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
- ((39*A - 35*B + 15*C)*Sin[c + d*x])/(30*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + ((147*A - 95*B + 75*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
30*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(9A-5B+5C)-a(3A-3B+C)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,d}{2a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B+5C)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(15A-11B+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B+C)\sin(c}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec}$$

Mathematica [A] time = 2.0798, size = 148, normalized size = 0.56

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(3(39A+20(C-B))\cos(c+dx)+(10B-6A)\cos(2(c+dx))+3A\cos(3\right.}{60ad\sqrt{a(\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(-30*(15*A - 11*B + 7*C)*ArcTanh[Sin[(
c + d*x)/2]]*Cos[(c + d*x)/2]^2 + 2*(141*A - 85*B + 75*C + 3*(39*A + 20*(-B
 + C))*Cos[c + d*x] + (-6*A + 10*B)*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)]
)*Sin[(c + d*x)/2]))/(60*a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.402, size = 460, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/60/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(225*A*sin(
d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos
(d*x+c)+1))^(1/2)-24*A*cos(d*x+c)^4-165*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+105*C*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)+225*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(
cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+48*A*cos(d*x+c)^3-165*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)-40*B*c
os(d*x+c)^3+105*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))*sin(d*x+c)-240*A*cos(d*x+c)^2+160*B*cos(d*x+c)^2-120*C*co
s(d*x+c)^2-78*A*cos(d*x+c)+70*B*cos(d*x+c)-30*C*cos(d*x+c)+294*A-190*B+150*
C)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.568519, size = 1411, normalized size = 5.37

$$\left[\frac{15\sqrt{2}\left((15A - 11B + 7C)\cos(dx+c)^2 + 2(15A - 11B + 7C)\cos(dx+c) + 15A - 11B + 7C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 \cdots}{\cdots}\right.}{120\left(a \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(2)*((15*A - 11*B + 7*C)*cos(d*x + c)^2 + 2*(15*A - 11*B + 7
*C)*cos(d*x + c) + 15*A - 11*B + 7*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sq
rt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
 + 4*(12*A*cos(d*x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B +
5*C)*cos(d*x + c)^2 + (147*A - 95*B + 75*C)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/60*(15*sqrt(2)*((15*A - 11*B + 7*C)*c
os(d*x + c)^2 + 2*(15*A - 11*B + 7*C)*cos(d*x + c) + 15*A - 11*B + 7*C)*sqr
t(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*x + c)^4 - 4*(3*A - 5*B)*co
s(d*x + c)^3 + 12*(9*A - 5*B + 5*C)*cos(d*x + c)^2 + (147*A - 95*B + 75*C)*
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/
2)*sec(d*x + c)^(5/2)), x)
```

3.619 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{(3A-11B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(3A-43B+115C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2B-5C)\sinh^{-1}\left(\right.}{a^{5/2}d}$$

```
[Out] ((2*B - 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
5/2)*d) + ((3*A - 43*B + 115*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A -
B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) +
((A + 7*B - 15*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d
*x])^(3/2)) + ((3*A - 11*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2
*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.824856, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4084, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(3A-11B+35C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(3A-43B+115C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2B-5C)\sinh^{-1}\left(\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```



```
[Out] ((2*B - 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
5/2)*d) + ((3*A - 43*B + 115*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A -
B + C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) +
((A + 7*B - 15*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d
*x])^(3/2)) + ((3*A - 11*B + 35*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2
*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
```

```
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}dx = -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}a(3\right.}{(}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)}{16ad(a+}$$

$$= -\frac{(A-B+C)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)}{16ad(a+}$$

$$= \frac{(2B-5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B+115C)\,\mathrm{ta}}{16}$$

Mathematica [A] time = 3.54744, size = 222, normalized size = 0.87

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left((6A-86B+230C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\right.}{4d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*((6*A - 86*B + 230*C)*ArcTanh[Sin[(c + d*x)/2]] + 32*Sqrt[2]*(2*B - 5*C
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + ((3*A - 11*B + 67*C + 2*(7*A - 15*B +
 55*C)*Cos[c + d*x] + (3*A - 11*B + 35*C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2
]^3*Sec[c + d*x]*Tan[(c + d*x)/2])/2))/(4*d*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.415, size = 982, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/16/d/a^3*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)*(-1+cos(d*x+c))^2*(16*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-16*B*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2
^(1/2)*sin(d*x+c)-40*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
```

```
x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+40*C*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*
sin(d*x+c)+3*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)-3*A*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-43*B*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+16*B*2^(1/2)*sin
(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c)))*cos(d*x+c)-16*B*2^(1/2)*sin(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+11*B*(-2/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)^3+115*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos
(d*x+c)^2*sin(d*x+c)-40*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+40*C*sin(d*x+c)*2^(1/2
)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin
(d*x+c)))-35*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+3*A*sin(d*x+c)*cos(d*
x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)^2*(-2/
(cos(d*x+c)+1))^(1/2)-43*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2))*cos(d*x+c)+4*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+115*C*sin
(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-20*C*co
s(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+7*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)-15*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+39*C*cos(d*x+c)*(-2/(cos(d*x+c
)+1))^(1/2)+16*C*(-2/(cos(d*x+c)+1))^(1/2))/(-2/(cos(d*x+c)+1))^(1/2)/sin(d
*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.18421, size = 2217, normalized size = 8.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((3*A - 43*B + 115*C)*cos(d*x + c)^3 + 3*(3*A - 43*B + 115*C
)*cos(d*x + c)^2 + 3*(3*A - 43*B + 115*C)*cos(d*x + c) + 3*A - 43*B + 115*C
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 16*((2*B - 5*C)*cos(d*x + c)^3 +
3*(2*B - 5*C)*cos(d*x + c)^2 + 3*(2*B - 5*C)*cos(d*x + c) + 2*B - 5*C)*sqrt
(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(
d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((3*A - 11*B +
 35*C)*cos(d*x + c)^2 + (7*A - 15*B + 55*C)*cos(d*x + c) + 16*C)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(
sqrt(2)*((3*A - 43*B + 115*C)*cos(d*x + c)^3 + 3*(3*A - 43*B + 115*C)*cos(d
```

```
*x + c)^2 + 3*(3*A - 43*B + 115*C)*cos(d*x + c) + 3*A - 43*B + 115*C)*sqrt(
-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))/(a*sin(d*x + c))) - 16*((2*B - 5*C)*cos(d*x + c)^3 + 3*(2*B - 5
*C)*cos(d*x + c)^2 + 3*(2*B - 5*C)*cos(d*x + c) + 2*B - 5*C)*sqrt(-a)*arcta
n(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((3*A - 11*B + 35*
C)*cos(d*x + c)^2 + (7*A - 15*B + 55*C)*cos(d*x + c) + 16*C)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^(5/2), x)
```

3.620 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(5A+3B-43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((5*A + 3*B - 43*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec
[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*
B - 11*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rubi [A] time = 0.596285, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4084, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(5A+3B-43C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(A-B+C)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```


```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((5*A + 3*B - 43*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec
[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*
B - 11*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a(5\ldots\right.}{(a+a\ldots}}{\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B-11C\ldots}{16ad(a+\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B-11C\ldots}{16ad(a+\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B-11C\ldots}{16ad(a+\ldots}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(5A+3B-43C)\tanh^{-1}\left(\frac{\sqrt{a}\ldots}{\ldots}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 1.60902, size = 204, normalized size = 1.01

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2(5A+3B-43C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin\left(\frac{1}{2}\right.}{}\right.}{4d(a(\sec(c+dx)+1))^{5/2}(A\cos(2(c+dx))+A+2B\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*(2*(5*A + 3*B - 43*C)*ArcTanh[Sin[(c + d*x)/2]] + (64*Sqrt[2]*C*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^4 + (A + 7*B - 15*C + (5*A + 3*
B - 11*C)*Cos[c + d*x])*Sin[(c + d*x)/2])/(-1 + Sin[(c + d*x)/2]^2)^2))/(4*
d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^
(5/2))
```

Maple [B] time = 0.381, size = 684, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/16/d/a^3*(-1+cos(d*x+c))^2*(-16*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+16*C*sin(d*x
+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))+5*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))-5*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+3*B*sin(d*x+
c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*B*(-2/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)^2-43*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))-16*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)+16*C*2^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+1
1*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+5*A*arctan(1/2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*sin(d*x+c)+4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*
B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-4*B*(-2/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)-43*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2))*sin(d*x+c)+4*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+A*(-2/(cos(d*x+c
)+1))^(1/2)+7*B*(-2/(cos(d*x+c)+1))^(1/2)-15*C*(-2/(cos(d*x+c)+1))^(1/2))*(
a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*
x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.767072, size = 2064, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*A + 3*B - 43*C)*cos(d*x + c)^3 + 3*(5*A + 3*B - 43*C)*c
os(d*x + c)^2 + 3*(5*A + 3*B - 43*C)*cos(d*x + c) + 5*A + 3*B - 43*C)*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^
2 + 3*C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) - 4*((5*A + 3*B - 11*C)*cos(d*x + c)^2 + (A + 7*B - 15*C)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c
) + a^3*d), -1/32*(sqrt(2)*((5*A + 3*B - 43*C)*cos(d*x + c)^3 + 3*(5*A + 3*
B - 43*C)*cos(d*x + c)^2 + 3*(5*A + 3*B - 43*C)*cos(d*x + c) + 5*A + 3*B -
43*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 32*(C*cos(d*x + c)^3 + 3*C*cos(d
*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)) - 2*((5*A + 3*B - 11*C)*cos(d*x + c)^2 + (A + 7*B
 - 15*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^(5/2), x)
```

3.621 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=163

$$\frac{(19A+5B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B+C)\sin(c+dx)\,s}{4d(a\sec(c+dx)+}$$

```
[Out] ((19*A + 5*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((9*A - B -
7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.408229, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4084, 4012, 3808, 206}

$$\frac{(19A+5B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B+C)\sin(c+dx)\,s}{4d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```

```
[Out] ((19*A + 5*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((9*A - B -
7*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
```

```
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(7A+\ldots\right.}{(a+a\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(9A-B-7C)\sec^{\frac{3}{2}}\ldots}{16ad(a+a\,\mathrm{s}\ldots}$$

$$= -\frac{(A-B+C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(9A-B-7C)\sec^{\frac{3}{2}}\ldots}{16ad(a+a\,\mathrm{s}\ldots}$$

$$= \frac{(19A+5B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+\ldots}{4d(\ldots}$$

Mathematica [A] time = 1.9, size = 119, normalized size = 0.73

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(8(19A+5B+3C)\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sin\left(\frac{1}{2}(c+dx)\right)((13A-5\ldots\right.}{64ad(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(8*(19*A + 5*B + 3*C)*ArcTanh[Sin[(c +
 d*x)/2]]*Cos[(c + d*x)/2]^4 - 4*(9*A - B - 7*C + (13*A - 5*B - 3*C)*Cos[c
+ d*x])*Sin[(c + d*x)/2]))/(64*a*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.374, size = 482, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/16/d/a^3*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x
+c)*(-1+cos(d*x+c))^2*(13*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+19*A*sin
(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*B*(-2
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+5*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1
/2)+3*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
```

```
2))-4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+19*A*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
+5*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-4*C*cos(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2))*sin(d*x+c)-9*A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c)+1))^(1/
2)+7*C*(-2/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.54422, size = 1400, normalized size = 8.59

$$\left[\frac{\sqrt{2}\left((19\,A+5\,B+3\,C)\cos\left(dx+c\right)^3+3\,(19\,A+5\,B+3\,C)\cos\left(dx+c\right)^2+3\,(19\,A+5\,B+3\,C)\cos\left(dx+c\right)+19\,A+\right.}{64\left(a^3d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 5*B + 3*C)*cos(d*x + c)^3 + 3*(19*A + 5*B + 3*C)*co
s(d*x + c)^2 + 3*(19*A + 5*B + 3*C)*cos(d*x + c) + 19*A + 5*B + 3*C)*sqrt(a
)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((13*A - 5*B - 3*C)*cos(d*x + c)^2 + (9
*A - B - 7*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*A + 5*B + 3*C)*cos(d*x +
 c)^3 + 3*(19*A + 5*B + 3*C)*cos(d*x + c)^2 + 3*(19*A + 5*B + 3*C)*cos(d*x
+ c) + 19*A + 5*B + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 5*
B - 3*C)*cos(d*x + c)^2 + (9*A - B - 7*C)*cos(d*x + c))*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec
(d*x + c) + a)^(5/2), x)
```

3.622 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=211

$$\frac{(49A-9B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B-3C)\sin(c}{16ad(a\sec(c+}$$

```
[Out] -((75*A - 19*B - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 5
*B - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2)) + ((49*A - 9*B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a +
 a*Sec[c + d*x]])
```

Rubi [A] time = 0.5896, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4084, 4020, 4013, 3808, 206}

$$\frac{(49A-9B+C)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(13A-5B-3C)\sin(c}{16ad(a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + a*Sec[
c + d*x])^(5/2)),x]
```

```
[Out] -((75*A - 19*B - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 5
*B - 3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2)) + ((49*A - 9*B + C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a +
 a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B+C)-2a(A-B-C)\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sqrt{\sec(c+dx)}\sin(}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sqrt{\sec(c+dx)}\sin(}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B-3C)\sqrt{\sec(c+dx)}\sin(}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(75A-19B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+C)\sqrt{\sec(c+d}}{4d(a+a\sec(c+}$$

Mathematica [A] time = 1.53791, size = 128, normalized size = 0.61

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((85A-13B+5C)\cos(c+dx)+16A\cos(2(c+dx))+65A-9B+C)-\right.}{64ad(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(-8*(75*A - 19*B - 5*C)*ArcTanh[Sin[(c
 + d*x)/2]]*Cos[(c + d*x)/2]^4 + 4*(65*A - 9*B + C + (85*A - 13*B + 5*C)*Co
s[c + d*x] + 16*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(64*a*d*(a*(1 + Sec[
c + d*x]))^(3/2))
```

Maple [B] time = 0.378, size = 594, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(75*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-19*B*sin(d*x+c)*
cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+
c)+1))^(1/2)-5*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+150*A*sin(d*x+c)*cos(d*x+c)*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-38*B*s
in(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(
cos(d*x+c)+1))^(1/2)-10*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+75*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-64*A*cos(d*
x+c)^3-19*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+
1))^(1/2)*B*sin(d*x+c)-5*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-106*A*cos(d*x+c)^2+26*B*cos(d*x+c)^2-
10*C*cos(d*x+c)^2+72*A*cos(d*x+c)-8*B*cos(d*x+c)+8*C*cos(d*x+c)+98*A-18*B+2
*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.557281, size = 1476, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B - 5*C)*cos(d*x + c)^3 + 3*(75*A - 19*B - 5*C)
*cos(d*x + c)^2 + 3*(75*A - 19*B - 5*C)*cos(d*x + c) + 75*A - 19*B - 5*C)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 + (85*A - 13*
B + 5*C)*cos(d*x + c)^2 + (49*A - 9*B + C)*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^
```

```
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*(
(75*A - 19*B - 5*C)*cos(d*x + c)^3 + 3*(75*A - 19*B - 5*C)*cos(d*x + c)^2 +
 3*(75*A - 19*B - 5*C)*cos(d*x + c) + 75*A - 19*B - 5*C)*sqrt(-a)*arctan(sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(
a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^3 + (85*A - 13*B + 5*C)*cos(d*x + c
)^2 + (49*A - 9*B + C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(a \sec(dx+c) + a)^{\frac{5}{2}} \sqrt{\sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/
2)*sqrt(sec(d*x + c))), x)
```

3.623 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(299A-147B+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(95A-39B+15C)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B+19C)\tan}{16\sqrt{}}$$

```
[Out] ((163*A - 75*B + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Si
n[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A -
9*B + C)*Sin[c + d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2
)) + ((95*A - 39*B + 15*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) - ((299*A - 147*B + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.802, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4084, 4020, 4022, 4013, 3808, 206}

$$-\frac{(299A-147B+27C)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(95A-39B+15C)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B+19C)\tan}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] ((163*A - 75*B + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Si
n[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A -
9*B + C)*Sin[c + d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2
)) + ((95*A - 39*B + 15*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) - ((299*A - 147*B + 27*C)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}+\frac{\int \frac{\frac{1}{2}a(11A-3B+3C)-a(3A-3B-C)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}-\frac{(17A-9B+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}-\frac{(17A-9B+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}-\frac{(17A-9B+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}-\frac{(17A-9B+C)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx}$$

$$= \frac{(163A-75B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B+C)\sin}{4d\sqrt{\sec(c+dx)}(a+a\,s}$$

Mathematica [A] time = 2.0878, size = 146, normalized size = 0.56

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((-479A+255B-39C)\cos(c+dx)+(48B-80A)\cos(2(c+dx))+8\right.}{192ad(a(\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(24*(163*A - 75*B + 19*C)*ArcTanh[Sin[
(c + d*x)/2]]*Cos[(c + d*x)/2]^4 + 4*(-379*A + 195*B - 27*C + (-479*A + 255
*B - 39*C)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)] + 8*A*Cos[3*(c +
d*x)])*Sin[(c + d*x)/2]))/(192*a*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.391, size = 624, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] -1/96/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*A*si
n(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)-225*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(
-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+57*C*arctan(1/2*sin(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x
+c)^2+978*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+64*A*cos(d*x+c)^4-450*B*sin(d*x+c)*cos(d*
x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
1/2)+114*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+489*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-384*A*cos(d*x+c)^3-225*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*
sin(d*x+c)+192*B*cos(d*x+c)^3+57*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-686*A*cos(d*x+c)^2+318*B*cos(
d*x+c)^2-78*C*cos(d*x+c)^2+408*A*cos(d*x+c)-216*B*cos(d*x+c)+24*C*cos(d*x+c
)+598*A-294*B+54*C)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.572841, size = 1615, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(2)*((163*A - 75*B + 19*C)*cos(d*x + c)^3 + 3*(163*A - 75*B +
 19*C)*cos(d*x + c)^2 + 3*(163*A - 75*B + 19*C)*cos(d*x + c) + 163*A - 75*B
 + 19*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c)
 - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*A*cos(d*x + c)^4 - 3
2*(5*A - 3*B)*cos(d*x + c)^3 - (503*A - 255*B + 39*C)*cos(d*x + c)^2 - (299
*A - 147*B + 27*C)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163*A - 75*B + 19*C)
*cos(d*x + c)^3 + 3*(163*A - 75*B + 19*C)*cos(d*x + c)^2 + 3*(163*A - 75*B
+ 19*C)*cos(d*x + c) + 163*A - 75*B + 19*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c
))) - 2*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^3 - (503*A - 255
*B + 39*C)*cos(d*x + c)^2 - (299*A - 147*B + 27*C)*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/
2)*sec(d*x + c)^(3/2)), x)
```

3.624 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=313

$$\frac{(157A-85B+45C)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B+735C)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B+195}{240a^2d\sqrt{\sec(c+dx)}\sqrt{a}}$$

```
[Out] -((283*A - 163*B + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*
Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A
- 13*B + 5*C)*Sin[c + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])
^(3/2)) + ((157*A - 85*B + 45*C)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]]) - ((787*A - 475*B + 195*C)*Sin[c + d*x])/(240*a^
2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((2671*A - 1495*B + 735*
C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.990029, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4084, 4020, 4022, 4013, 3808, 206}

$$\frac{(157A-85B+45C)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B+735C)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B+195}{240a^2d\sqrt{\sec(c+dx)}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^(5/2)),x]
```

```
[Out] -((283*A - 163*B + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*
Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A
- 13*B + 5*C)*Sin[c + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])
^(3/2)) + ((157*A - 85*B + 45*C)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]]) - ((787*A - 475*B + 195*C)*Sin[c + d*x])/(240*a^
2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((2671*A - 1495*B + 735*
C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
```

```
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(13A-5B+5C)-4a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5C)\sin(c+\ldots}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+\ldots}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5C)\sin(c+\ldots}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+\ldots}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5C)\sin(c+\ldots}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+\ldots}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5C)\sin(c+\ldots}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+\ldots}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5C)\sin(c+\ldots}{16ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+\ldots}$$

$$= -\frac{(283A-163B+75C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B+\ldots}{4d\sec^{\frac{3}{2}}(c+dx)(\ldots}$$

Mathematica [A] time = 3.29578, size = 221, normalized size = 0.71

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(15(283A-163B+75C)\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+\ldots\right.\right.\right.}{60a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -(Sec[(c + d*x)/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(15*(283*A - 163
*B + 75*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^4 - ((3491*A - 1895*B
 + 975*C + 5*(887*A - 479*B + 255*C)*Cos[c + d*x] + 16*(52*A - 25*B + 15*C)
*Cos[2*(c + d*x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)] + 12*A*Co
s[4*(c + d*x)])*Sin[(c + d*x)/2])/2))/(60*a*d*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.423, size = 657, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/480/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(4245*A*s
in(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2
/(cos(d*x+c)+1))^(1/2)-192*A*cos(d*x+c)^5-2445*B*sin(d*x+c)*cos(d*x+c)^2*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+11
25*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*cos(d*x+c)^2+8490*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+512*A*cos(d*x+c)^
4-4890*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*(-2/(cos(d*x+c)+1))^(1/2)-320*B*cos(d*x+c)^4+2250*C*sin(d*x+c)*arctan(
1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)+4245*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1
))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*arctan(1/2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+1920*B*cos(d*x+
c)^3+1125*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2))*sin(d*x+c)-960*C*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*
x+c)^2-1590*C*cos(d*x+c)^2+3768*A*cos(d*x+c)-2040*B*cos(d*x+c)+1080*C*cos(d
*x+c)+5342*A-2990*B+1470*C)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.586198, size = 1747, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/960*(15*sqrt(2)*((283*A - 163*B + 75*C)*cos(d*x + c)^3 + 3*(283*A - 163*
B + 75*C)*cos(d*x + c)^2 + 3*(283*A - 163*B + 75*C)*cos(d*x + c) + 283*A -
163*B + 75*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x
 + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(96*A*cos(d*x + c)^
5 - 160*(A - B)*cos(d*x + c)^4 + 32*(49*A - 25*B + 15*C)*cos(d*x + c)^3 + 5
*(911*A - 503*B + 255*C)*cos(d*x + c)^2 + (2671*A - 1495*B + 735*C)*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B + 75*C)*cos(d*x + c)^3 + 3*(283
*A - 163*B + 75*C)*cos(d*x + c)^2 + 3*(283*A - 163*B + 75*C)*cos(d*x + c) +
 283*A - 163*B + 75*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x
 + c)^5 - 160*(A - B)*cos(d*x + c)^4 + 32*(49*A - 25*B + 15*C)*cos(d*x + c)
^3 + 5*(911*A - 503*B + 255*C)*cos(d*x + c)^2 + (2671*A - 1495*B + 735*C)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
```

```
+ c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/
2)*sec(d*x + c)^(5/2)), x)
```

3.625 $\int (a+a\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=446

$$-\frac{3^{3/4}(5B+2C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\right.}{10\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*C*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*Sqrt[2]*A*AppellF1
[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]) + (3*(5*B + 2*C)*(a
+ a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(10*d*(1 + Sec[c + d*x])) - (3^(3/4)*
(5*B + 2*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*
(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)*d*(1
 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/
3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))^2)])
```

Rubi [A] time = 0.722644, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4054, 3924, 3779, 3778, 136, 3828, 3827, 50, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}+\frac{3(5B+2C)\tan(c+dx)(a\,\mathrm{se}}{10d(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*C*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*Sqrt[2]*A*AppellF1
[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]) + (3*(5*B + 2*C)*(a
+ a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(10*d*(1 + Sec[c + d*x])) - (3^(3/4)*
(5*B + 2*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*
(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)*d*(1
 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/
3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))^2)])
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
```

```
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\int(a+a\sec(}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + A\int(a+a\sec$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{\left(A(a+a\sec(\right.}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(A(a+a\sec(\right.}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2}\right.}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2}\right.}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3\sqrt{2}AF_1\left(\frac{7}{6};\frac{1}{2}\right.}{}$$

Mathematica [B] time = 21.0617, size = 5449, normalized size = 12.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.182, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^{\frac{2}{3}} \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(2/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}} \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*(A + B*sec(c + d*x) + C*sec(c + d*x)
**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(2/3
), x)
```

3.626 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=390

$$-\frac{3^{3/4}(2B-C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{s}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{s}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (3*C*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sqrt[2]*A*AppellF1
[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d
*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) - (3^(3/4)*(2*B - C)*El
lipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3
)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.451648, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4054, 3924, 3779, 3778, 136, 3828, 3827, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}-\frac{3^{3/4}(2B-C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+\right.}{2\sqrt[3]{2}d(1-\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(1/3),x]
```


```
[Out] (3*C*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sqrt[2]*A*AppellF1
[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d
*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) - (3^(3/4)*(2*B - C)*El
lipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3
)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```

```
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\int \frac{\frac{2aA}{3}+\frac{1}{3}a(2B-C)\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + A\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx + \frac{1}{2}(2B-C)\int \frac{\text{s}}{\sqrt[3]{a\,\cdot}}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}} + \frac{((2B-}{}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{(A\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{5/6}}\,dx, x, \sec(\right.}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(}}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c\right.}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+}}$$

$$= \frac{3C\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c\right.}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+}}$$

Mathematica [B] time = 19.5951, size = 2931, normalized size = 7.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(1/3
),x]
```

```
[Out] (3*C*Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*
x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Tan[(c + d*x)/2])/(d*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(1/3))
 + (2^(2/3)*Cos[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec
[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*Cos[c + d*x]*Se
c[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3) + Sec[(c + d*x)/2]^2*(B*(1 + Sec[
c + d*x])^(2/3) - (C*(1 + Sec[c + d*x])^(2/3))/2))*Tan[(c + d*x)/2]*(-((2*A
 - 2*B + C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2) + (27*(2*A
 + 2*B - C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(1/3)*((Sec[(c + d*x)/2]^2*(Co
s[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(-((2*A - 2*B + C)*AppellF1[3/2, 2/3,
1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2) + (27*(2*A + 2*B - C)*AppellF1[1/2, 2/3,
1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*App
ellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*Ap
pellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*Appel
```

```
lF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*
x)/2]^2)))/(3*2^(1/3)) + (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*T
an[(c + d*x)/2]*(-((2*A - 2*B + C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/
2]^2)^(2/3)*Tan[(c + d*x)/2]) - (2*A - 2*B + C)*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*A
ppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/5) - (2*(2*A - 2*B + C)*AppellF1[3/2, 2/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c
+ d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (27*(2*A + 2*B - C)*Appel
lF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x
)/2]*Sin[(c + d*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (27*(2*A + 2*B - C)*Cos[(c + d*x)/2
]^2*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9
))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Ta
n[(c + d*x)/2]^2) - (27*(2*A + 2*B - C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/
3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3,
1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2] + 9*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/
5 + (2*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/5 + AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/3 + (2*2^(2/3)*
Tan[(c + d*x)/2]*(-((2*A - 2*B + C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c
 + d*x)/2]^2) + (27*(2*A + 2*B - C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c
+ d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*
Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3))))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(A + B\sec\left(dx + c\right) + C\left(\sec\left(dx + c\right)\right)^{2}\right)\frac{1}{\sqrt[3]{a + a\sec\left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(
1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3
), x)
```

3.627 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=402

$$\frac{3^{3/4}(A-B-4C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3}}{\sqrt[3]{2}-(1+\sqrt{3}}\right.\right.}{5\sqrt[3]{2}ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^(4/3)) + (3*Sqrt[2]
*A*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c
 + d*x])/(a*d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)
*(A - B - 4*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/
4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 - Sec[c + d*x])*(a + a
*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
 + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.469029, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4052, 3924, 3779, 3778, 136, 3828, 3827, 63, 225}

$$\frac{3\sqrt{2}A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{ad\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}-\frac{3(A-B+C)\tan(c+dx)}{5d(a\sec(c+dx)+a)^{4/3}}+\frac{3^{3/4}(A-B-4C)\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(4/3),x]
```


```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^(4/3)) + (3*Sqrt[2]
*A*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c
 + d*x])/(a*d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)
*(A - B - 4*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/
4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 - Sec[c + d*x])*(a + a
*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
 + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```

```
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx = -\frac{3(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} - \frac{3\int \frac{-\frac{5aA}{3}+\frac{1}{3}a(A-B-4C)\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{5a^2}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{A\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{a} - \frac{(A-B-4C)\int \frac{s}{\sqrt[3]{a}}}{5a}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{a\sqrt[3]{a+a\sec(c+dx)}} - \frac{(}{}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} - \frac{(A\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{5/6}}\,dx, x\right.}{ad\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\right.}{ad\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\,\mathrm{s}}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{5d(a+a\sec(c+dx))^{4/3}} + \frac{3\sqrt{2}AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\right.}{ad\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\,\mathrm{s}}}$$

Mathematica [B] time = 19.5334, size = 3029, normalized size = 7.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(4/3
),x]
```

```
[Out] (Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^
(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-6*Sec[(c + d*x)/2]*(A*Sin[
(c + d*x)/2] - B*Sin[(c + d*x)/2] + C*Sin[(c + d*x)/2]))/5 + (3*Sec[(c + d*
x)/2]^3*(A*Sin[(c + d*x)/2] - B*Sin[(c + d*x)/2] + C*Sin[(c + d*x)/2]))/5))
/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x])
)^(4/3)) + (2*2^(2/3)*Cos[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3
)*(1 + Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*Cos[c
 + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3) + Sec[(c + d*x)/2]^2*(-
(A*(1 + Sec[c + d*x])^(2/3))/5 + (B*(1 + Sec[c + d*x])^(2/3))/5 + (4*C*(1 +
 Sec[c + d*x])^(2/3))/5))*Tan[(c + d*x)/2]*((-6*A + B + 4*C)*AppellF1[3/2,
2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c
+ d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B + 4*C)*AppellF1[1/2, 2
/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9
*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-
3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*A
ppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
+ d*x)/2]^2)))/(15*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a*(
1 + Sec[c + d*x]))^(4/3)*((2^(2/3)*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*S
ec[c + d*x])^(2/3)*((-6*A + B + 4*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(
c + d*x)/2]^2 + (27*(4*A + B + 4*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x
```

```
)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/15 + (2*2
^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*((-6*A + B
+ 4*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]
 + (-6*A + B + 4*C)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2
]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
/5) + (2*(-6*A + B + 4*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) +
 Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(1/3)) - (27*(4*A + B + 4*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(9*A
ppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*
AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*App
ellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
d*x)/2]^2) + (27*(4*A + B + 4*C)*Cos[(c + d*x)/2]^2*(-(AppellF1[3/2, 2/3, 2
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, 2/3, 1,
 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (27*(4*
A + B + 4*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-(AppellF1[
3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d
*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3,
1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2])*Tan[(c + d*x)/2]^2)^2))/15 + (4*2^(2/3)*Tan[(c + d*x)/2]*((-6*A +
B + 4*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B
 + 4*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(
c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(45*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(1/3))))
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int \left(A+B\sec\left(dx+c\right)+C\left(\sec\left(dx+c\right)\right)^{2}\right)\left(a+a\sec\left(dx+c\right)\right)^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(
4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A}{\left(a\sec\left(dx + c\right) + a\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(4/3
), x)
```

3.628 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{7/3}}\,dx$

Optimal. Leaf size=466

$$\frac{3^{3/4}(4A-4B-7C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-}{\sqrt[3]{2}-(1+}\right.\right.}{55\sqrt[3]{2}a^2d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(11*d*(a + a*Sec[c + d*x])^(7/3)) - (3*(4*A -
 4*B - 7*C)*Tan[c + d*x])/(55*a^2*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])
^(1/3)) - (3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1
+ Sec[c + d*x]]*Tan[c + d*x])/(5*a^2*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c +
d*x])*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*(4*A - 4*B - 7*C)*EllipticF[Ar
cCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c +
d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c
+ d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c
+ d*x])/(55*2^(1/3)*a^2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqr
t[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.518451, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4052, 3924, 3779, 3778, 136, 3828, 3827, 51, 63, 225}

$$-\frac{3\sqrt{2}A\tan(c+dx)F_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5a^2d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}-\frac{3(4A-4B-7C)\tan(c+dx)}{55a^2d(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(7/3),x]
```


```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(11*d*(a + a*Sec[c + d*x])^(7/3)) - (3*(4*A -
 4*B - 7*C)*Tan[c + d*x])/(55*a^2*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])
^(1/3)) - (3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1
+ Sec[c + d*x]]*Tan[c + d*x])/(5*a^2*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c +
d*x])*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*(4*A - 4*B - 7*C)*EllipticF[Ar
cCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c +
d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c
+ d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c
+ d*x])/(55*2^(1/3)*a^2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqr
t[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```


Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
```

```
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{7/3}}\,dx = -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3\int \frac{-\frac{11aA}{3}+\frac{1}{3}a(4A-4B-7C)\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx}{11a^2}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} + \frac{A\int \frac{1}{(a+a\sec(c+dx))^{4/3}}\,dx}{a} - \frac{(4A-4B-7C)\int$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} + \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{(1+\sec(c+dx))^{4/3}}\,dx}{a^2\sqrt[3]{a+a\sec(c+dx)}} -$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{(A\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{11/6}}\,dx,\right.}{a^2d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3(4A-4B-7C)\tan(c+dx)}{55a^2d(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3(4A-4B-7C)\tan(c+dx)}{55a^2d(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{11d(a+a\sec(c+dx))^{7/3}} - \frac{3(4A-4B-7C)\tan(c+dx)}{55a^2d(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [B] time = 19.6769, size = 3111, normalized size = 6.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(7/3
),x]
```

```
[Out] (Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^
(7/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-6*Sec[(c + d*x)/2]*(20*A*S
in[(c + d*x)/2] - 9*B*Sin[(c + d*x)/2] - 2*C*Sin[(c + d*x)/2]))/55 - (3*Sec
[(c + d*x)/2]^5*(A*Sin[(c + d*x)/2] - B*Sin[(c + d*x)/2] + C*Sin[(c + d*x)/
```

```
2]))/22 + (3*Sec[(c + d*x)/2]^3*(25*A*Sin[(c + d*x)/2] - 14*B*Sin[(c + d*x)
/2] + 3*C*Sin[(c + d*x)/2]))/55))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(7/3)) + (2*2^(2/3)*Cos[c + d*x]^2*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(7/3)*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*(A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*
x])^(2/3) + Sec[(c + d*x)/2]^2*((-3*A*(1 + Sec[c + d*x])^(2/3))/11 + (4*B*(
1 + Sec[c + d*x])^(2/3))/55 + (7*C*(1 + Sec[c + d*x])^(2/3))/55))*Tan[(c +
d*x)/2]*((-70*A + 4*B + 7*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)
/2]^2 + (27*(40*A + 4*B + 7*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(165*d*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(7/3)*((2
^(2/3)*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*((-70*A +
 4*B + 7*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(40*A
 + 4*B + 7*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/165 + (2*2^(2/3)*(Cos[(c + d*x)
/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*((-70*A + 4*B + 7*C)*AppellF1[3/
2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (-70*A + 4*B +
7*C)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*Appell
F1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*(-70*A
 + 4*B + 7*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^(1/3)) - (27*(40*A + 4*B + 7*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(9*AppellF1[1
/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[
3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2
, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
) + (27*(40*A + 4*B + 7*C)*Cos[(c + d*x)/2]^2*(-(AppellF1[3/2, 2/3, 2, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, 2/3, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (27*(40*A + 4
*B + 7*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-(AppellF1[3/2
, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)
/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1,
7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
```

```
/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2])*Tan[(c + d*x)/2]^2)^2))/165 + (4*2^(2/3)*Tan[(c + d*x)/2]*((-70*A + 4
*B + 7*C)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(40*A +
 4*B + 7*C)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*S
in[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(495*(Cos
[(c + d*x)/2]^2*Sec[c + d*x])^(1/3))))
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \left(A+B\sec\left(dx+c\right)+C\left(\sec\left(dx+c\right)\right)^{2}\right)\left(a+a\sec\left(dx+c\right)\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + B \sec (dx+c) + A}{(a \sec (dx+c) + a)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(7/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(7/3
), x)
```

3.629 $\int (a+a\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=839

$$\frac{3C\tan(c+dx)(\sec(c+dx)a+a)^{4/3}}{7d}+\frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)(\sec(c+dx)+1)\,\mathrm{ta}}{11d\sqrt{1-\sec(c+dx)}}$$

```
[Out] (3*a*(7*B + 4*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(28*d) + (3*Sqrt[
2]*a*A*AppellF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]
*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 -
 Sec[c + d*x]]) + (3*C*(a + a*Sec[c + d*x])^(4/3)*Tan[c + d*x])/(7*d) - (15
*(1 + Sqrt[3])*a*(7*B + 4*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(28*d
*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)
)) + (15*3^(1/4)*a*(7*B + 4*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1
 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]
, (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x
])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x
])/(14*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*(7*B + 4
*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(
1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(28*2^(2/3)*d*(1 - Sec[c
 + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3)
 - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))^2)])
```

Rubi [A] time = 1.06779, antiderivative size = 839, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {4054, 3924, 3779, 3778, 136, 3828, 3827, 50, 63, 308, 225, 1881}

$$\frac{3C\tan(c+dx)(\sec(c+dx)a+a)^{4/3}}{7d}+\frac{3\sqrt{2}aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)(\sec(c+dx)+1)\,\mathrm{ta}}{11d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*a*(7*B + 4*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(28*d) + (3*Sqrt[
2]*a*A*AppellF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]
*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 -
 Sec[c + d*x]]) + (3*C*(a + a*Sec[c + d*x])^(4/3)*Tan[c + d*x])/(7*d) - (15
*(1 + Sqrt[3])*a*(7*B + 4*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(28*d
*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)
)) + (15*3^(1/4)*a*(7*B + 4*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1
 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))]
```

```
, (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x
])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x
])/(14*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*(7*B + 4
*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(
1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(28*2^(2/3)*d*(1 - Sec[c
 + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3)
 - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))^2)])
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{4/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} + \frac{3\int (a+a\sec(}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} + A\int (a+a\sec$$

$$= \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} + \frac{\left(aA\sqrt[3]{a+a\sec}\right.}{}$$

$$= \frac{3C(a+a\sec(c+dx))^{4/3}\tan(c+dx)}{7d} - \frac{\left(aA\sqrt[3]{a+a\sec}\right.}{}$$

$$= \frac{3a(7B+4C)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{28d} + \frac{3\sqrt{2}aA}{}$$

$$= \frac{3a(7B+4C)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{28d} + \frac{3\sqrt{2}aA}{}$$

$$= \frac{3a(7B+4C)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{28d} + \frac{3\sqrt{2}aA}{}$$

$$= \frac{3a(7B+4C)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{28d} + \frac{3\sqrt{2}aA}{}$$

Mathematica [B] time = 20.3094, size = 4995, normalized size = 5.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c + d*x
]))^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((3*(28*A + 35*B + 20*C)*
Sin[c + d*x])/14 + (3*Sec[c + d*x]*(7*B*Sin[c + d*x] + 8*C*Sin[c + d*x]))/1
4 + (6*C*Sec[c + d*x]*Tan[c + d*x])/7))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*(1 + Sec[c + d*x])^(4/3)) + (Cos[c + d*x]^2*(a*(1 + Sec[c
 + d*x]))^(4/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(4*A*(1 + Sec[c + d
*x])^(1/3) + (5*B*(1 + Sec[c + d*x])^(1/3))/2 + (10*C*(1 + Sec[c + d*x])^(1
/3))/7 + Cos[c + d*x]*(-6*A*(1 + Sec[c + d*x])^(1/3) - (15*B*(1 + Sec[c + d
*x])^(1/3))/2 - (30*C*(1 + Sec[c + d*x])^(1/3))/7))*Tan[(c + d*x)/2]*(-(((2
8*A + 35*B + 20*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) +
(Cos[(c + d*x)/2]^2*(18*(28*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]^2 + 27*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(-4*(14*A + 35*B + 20*C) + 3*(28*A + 35*B + 20*C)*Tan[(c + d*x)/
2]^2)))/((-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(21*2^(2/3)*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(
```

2/3)*(1 + Sec[c + d*x])^(4/3)*((Sec[(c + d*x)/2]^2*(-(((28*A + 35*B + 20*C)
*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c
 + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) + (Cos[(c + d*x)/2]^
2*(18*(28*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 27*
AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*(14
*A + 35*B + 20*C) + 3*(28*A + 35*B + 20*C)*Tan[(c + d*x)/2]^2)))/((-1 + Tan
[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])*Tan[(c + d*x)/2]^2))))/(42*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(2/3)) + (Tan[(c + d*x)/2]*(-(((28*A + 35*B + 20*C)*AppellF1[3/2, 1/3,
1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) - ((28*A + 35*B + 20*C)
*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4
/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + (2*(28*A + 35*
B + 20*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5
/3)) - (Tan[(c + d*x)/2]*(18*(28*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]^2 + 27*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(-4*(14*A + 35*B + 20*C) + 3*(28*A + 35*B + 20*C)*Tan[(c +
d*x)/2]^2)))/((-1 + Tan[(c + d*x)/2]^2)^2*(-9*AppellF1[1/2, 1/3, 1, 3/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) - (Cos[(c + d*x)/2]
*Sin[(c + d*x)/2]*(18*(28*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2]^2 + 27*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(-4*(14*A + 35*B + 20*C) + 3*(28*A + 35*B + 20*C)*Tan[(c + d*x)/2]
^2)))/((-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) - (Cos[(c + d*x)/2]^2*(18*(2
8*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 27*AppellF1
[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*(14*A + 35*
B + 20*C) + 3*(28*A + 35*B + 20*C)*Tan[(c + d*x)/2]^2))*(2*(3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2] - 9*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4
*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 +
 (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/((-1 + Tan[(c + d*x)/2]^2)*(-9*Appell
F1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*Appell
F1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/

```
2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^
2)^2) + (Cos[(c + d*x)/2]^2*(81*(28*A + 35*B + 20*C)*AppellF1[1/2, 1/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2] + 18*(28*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] -
 18*(28*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2]^2 + 18*(2
8*A + 35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3 + 27*(-(Appel
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9)*(-4*(14*A +
35*B + 20*C) + 3*(28*A + 35*B + 20*C)*Tan[(c + d*x)/2]^2) + 18*(28*A + 35*B
 + 20*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2*((3*AppellF1[5/
2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1
[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/((-1 + Tan[(
c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2])*Tan[(c + d*x)/2]^2))))/(21*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(2/3)) - (2^(1/3)*Tan[(c + d*x)/2]*(-(((28*A + 35*B + 20*C)*AppellF1[3/2,
 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/
(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) + (Cos[(c + d*x)/2]^2*(18*(28*A +
35*B + 20*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 27*AppellF1[1/2,
1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*(14*A + 35*B + 20
*C) + 3*(28*A + 35*B + 20*C)*Tan[(c + d*x)/2]^2)))/((-1 + Tan[(c + d*x)/2]^
2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
(c + d*x)/2]^2)))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[
(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(63*(Cos[(c + d*x)/2]^2*Sec[c +
d*x])^(5/3))))
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \left(a + a\sec\left(dx + c\right)\right)^{\frac{4}{3}} \left(A + B\sec\left(dx + c\right) + C\left(\sec\left(dx + c\right)\right)^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(a\sec\left(dx+c\right) + a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3
), x)
```

3.630 $\int \sqrt[3]{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=786

$$\frac{3^{3/4}\left(1-\sqrt{3}\right)(4B+C)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\text{Ellipt}}{4\,2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*A*AppellF1
[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqrt[3])*(
4*B + C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*(1 + Sec[c + d*x])^(
2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*3^(1/4)*(4*B
+ C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*
Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/3)*d*(1 - Sec[c
 + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3)
 - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(4*B + C)*EllipticF[ArcCos[(2^(1/3) - (
1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(4*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*
Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.844691, antiderivative size = 786, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {4054, 3924, 3779, 3778, 136, 3828, 3827, 63, 308, 225, 1881}

$$\frac{3\sqrt{2}A\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5d\sqrt{1-\sec(c+dx)}}-\frac{3\left(1+\sqrt{3}\right)(4B+C)\tan(c+}{4d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*C*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*A*AppellF1
[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c +
d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 + Sqrt[3])*(
4*B + C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*(1 + Sec[c + d*x])^(
2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*3^(1/4)*(4*B
+ C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*
Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/3)*d*(1 - Sec[c
```

```
 + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3)
 - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(4*B + C)*EllipticF[ArcCos[(2^(1/3) - (
1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(4*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*
Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 4054

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(b*(m + 1)), Int[
(a + b*Csc[e + f*x])^m*Simp[A*b*(m + 1) + (a*C*m + b*B*(m + 1))*Csc[e + f*x
], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &&
  !LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt[3]{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\int\sqrt[3]{a+a\sec}}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + A\int\sqrt[3]{a+a\,\mathrm{se}}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{\left(A\sqrt[3]{a+a\sec(\right.}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} - \frac{\left(A\sqrt[3]{a+a\sec(\right.}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},\right.}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},\right.}{}$$

$$= \frac{3C\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},\right.}{}$$

Mathematica [B] time = 19.7487, size = 4191, normalized size = 5.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c + d*x
]))^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((3*(4*B + C)*Sin[c + d*x
])/2 + (3*C*Tan[c + d*x])/2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(1 + Sec[c + d*x])^(1/3)) + (Cos[c + d*x]^2*(a*(1 + Sec[c + d*x]))^
(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(2*A*(1 + Sec[c + d*x])^(1/3)
 + 2*B*(1 + Sec[c + d*x])^(1/3) + (C*(1 + Sec[c + d*x])^(1/3))/2 + Cos[c +
d*x]*(-6*B*(1 + Sec[c + d*x])^(1/3) - (3*C*(1 + Sec[c + d*x])^(1/3))/2))*Ta
n[(c + d*x)/2]*(-(((4*B + C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)) + (9*((3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*(8*A - 4*B - C + (8*A - 7*(4*B + C))*Cos[c + d*x]))/2 + 2*(4*B +
C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
 AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c
 + d*x]*Tan[(c + d*x)/2]^2))/((-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1
/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1
/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*
2^(2/3)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/
2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(1/3)*((Sec[(c + d*x)/2]^2*(-((
(4*B + C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) + (9*((3*Ap
pellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(8*A - 4*
B - C + (8*A - 7*(4*B + C))*Cos[c + d*x]))/2 + 2*(4*B + C)*(3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3,
```

```
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*
x)/2]^2))/((-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(6*2^(2/3)*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^(2/3)) + (Tan[(c + d*x)/2]*(-(((4*B + C)*AppellF1[3/
2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) - ((4*B + C)*Ta
n[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3,
 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + (2*(4*B + C)*Appe
llF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*
x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3)) - (9*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]*((3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*(8*A - 4*B - C + (8*A - 7*(4*B + C))*Cos[c + d
*x]))/2 + 2*(4*B + C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/((-1 + Tan[(c + d*x)/2]^2)^
2*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
 AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2)) - (9*((3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*(8*A - 4*B - C + (8*A - 7*(4*B + C))*Cos[c + d*x]))/2 + 2
*(4*B + C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-(A
ppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[
(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3,
1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3,
 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/5))))/((-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) + (9*((-3*(8*A
 - 7*(4*B + C))*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sin[c + d*x])/2 + 2*(4*B + C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2] - 2*(4*B + C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2])*Sin[c + d*x]*Tan[(c + d*x)/2]^2 + (3*(8*A - 4*B - C + (8*A - 7*(
4*B + C))*Cos[c + d*x])*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2,
 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/9))/2 + 2*(4*B + C)*Cos[c + d*x]*Tan[(c + d*x)/2]^2*((3*Ap
pellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6
*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x
```

```
)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/((-
1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(2/3)) - (2^(1/3)*Tan[(c + d*x)/2]*(-(((4*B + C)*AppellF1[3/2, 1/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) + (9*((3*AppellF1[1/2, 1/3, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(8*A - 4*B - C + (8*A - 7*(4*B + C)
)*Cos[c + d*x]))/2 + 2*(4*B + C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/((-1 + Tan[(c +
d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])*Tan[(c + d*x)/2]^2)))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]
) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*S
ec[c + d*x])^(5/3))))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + a \sec (dx + c)} \left(A + B \sec (dx + c) + C (\sec (dx + c))^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^{2} + B \sec (dx + c) + A\right)(a \sec (dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(1/3)*(A + B*sec(c + d*x) + C*sec(c + d*x)
**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3
), x)
```

3.631 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=803

$$\frac{3\sqrt[3]{2}\sqrt[4]{3}(A-B+2C)E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{\sec(c+dx)a+a}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec\cdots}{\cdots}}}{ad(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) + (3*Sqrt[2]*A
*AppellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a
*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 +
 Sqrt[3])*(A - B + 2*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(a*d*(1 +
Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (
3*2^(1/3)*3^(1/4)*(A - B + 2*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x]
)^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*
x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*
x])/(a*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d
*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(A - B + 2*C)*Ellipt
icF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x
])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 +
Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1
 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*a*d*(1 - Sec[c + d*x])*(1
 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec
[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.887594, antiderivative size = 803, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {4052, 3924, 3779, 3778, 136, 3828, 3827, 63, 308, 225, 1881}

$$\frac{3\sqrt[3]{2}\sqrt[4]{3}(A-B+2C)E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{\sec(c+dx)a+a}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec\cdots}{\cdots}}}{ad(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) + (3*Sqrt[2]*A
*AppellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a
*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]) - (3*(1 +
 Sqrt[3])*(A - B + 2*C)*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(a*d*(1 +
Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (
3*2^(1/3)*3^(1/4)*(A - B + 2*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x]
)^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*
x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*
```

```
x])/(a*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d
*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(A - B + 2*C)*Ellipt
icF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x
])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 +
Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1
 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*a*d*(1 - Sec[c + d*x])*(1
 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec
[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx = -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\int \sqrt[3]{a+a\sec(c+dx)}\left(-\frac{aA}{3}-\frac{1}{3}a(A-B+2C\right.}{a^2}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{A\int \sqrt[3]{a+a\sec(c+dx)}\,dx}{a} + \frac{(A-B+2C)\int}{}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\right)\int \sqrt[3]{1+\sec(c+dx)}\,dx}{a\sqrt[3]{1+\sec(c+dx)}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int\right.}{ad\sqrt{1-\sec(c+dx)}(1+\sec(}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\text{se}\right.}{5ad\sqrt{1-\text{sec}}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\text{se}\right.}{5ad\sqrt{1-\text{sec}}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} + \frac{3\sqrt{2}AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\text{se}\right.}{5ad\sqrt{1-\text{sec}}}$$

Mathematica [B] time = 19.6808, size = 4253, normalized size = 5.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(2/3
),x]
```

```
[Out] (Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^
(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-6*Sec[(c + d*x)/2]*(A*Sin[(
c + d*x)/2] - B*Sin[(c + d*x)/2] + C*Sin[(c + d*x)/2]) + 6*(A - B + 2*C)*Si
n[c + d*x]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a*(1 + S
ec[c + d*x]))^(2/3)) - (2*2^(1/3)*Cos[c + d*x]^2*(1 + Sec[c + d*x])^(2/3)*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(4*A*(1 + Sec[c + d*x])^(1/3) - 2*B*
(1 + Sec[c + d*x])^(1/3) + 4*C*(1 + Sec[c + d*x])^(1/3) + Cos[c + d*x]*(-6*
A*(1 + Sec[c + d*x])^(1/3) + 6*B*(1 + Sec[c + d*x])^(1/3) - 12*C*(1 + Sec[c
 + d*x])^(1/3)))*Tan[(c + d*x)/2]*(((A - B + 2*C)*AppellF1[3/2, 1/3, 1, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x
]*Sec[(c + d*x)/2]^2)^(2/3) + (9*(-3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*(A + B - 2*C + (-5*A + 7*(B - 2*C))*Cos[c + d
*x]) - 4*(A - B + 2*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^
2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
(c + d*x)/2]^2))))/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(-(2^(1
/3)*Sec[(c + d*x)/2]^2*(((A - B + 2*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(2/3) + (9*(-3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
```

an[(c + d*x)/2]^2]*(A + B - 2*C + (-5*A + 7*(B - 2*C))*Cos[c + d*x]) - 4*(A - B + 2*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (2*2^(1/3)*Tan[(c + d*x)/2]*(((A - B + 2*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + ((A - B + 2*C)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) - (2*(A - B + 2*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3)) - (9*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(-3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(A + B - 2*C + (-5*A + 7*(B - 2*C))*Cos[c + d*x]) - 4*(A - B + 2*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)^2*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) - (9*(-3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(A + B - 2*C + (-5*A + 7*(B - 2*C))*Cos[c + d*x]) - 4*(A - B + 2*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) + (9*(3*(-5*A + 7*(B - 2*C))*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[c + d*x] - 4*(A - B + 2*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 4*(A - B + 2*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[c + d*x]*Tan[(c + d*x)/2]^2 - 3*(A + B - 2*C + (-5*A + 7*(B - 2*C))*Cos[c + d*x])*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) - 4*(A - B + 2*C)*Cos[c + d*x]*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]

```
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)/2]^2)*(
-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2))))/(3*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (4*2^(1/3)*Tan
[(c + d*x)/2]*(((A - B + 2*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^(2/3) + (9*(-3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*(A + B - 2*C + (-5*A + 7*(B - 2*C))*Cos[c + d*x]) - 4*(A - B + 2*
C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
 AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c
 + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))*(-
(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c
 + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/3))))
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int \left(A + B\sec\left(dx + c\right) + C\left(\sec\left(dx + c\right)\right)^{2}\right)\left(a + a\sec\left(dx + c\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(
2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + B\sec(dx + c) + A}{(a\sec(dx + c) + a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(2/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3
), x)
```

3.632 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=856

$$\frac{3\sqrt[3]{2}\sqrt[4]{3}(2A-2B-5C)E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+}{(}}}{7ad(1-\sec(c+dx))(\sec(c+dx)a+a)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (3*(2*A -
2*B - 5*C)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*
AppellF1[-1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) - (3*(1 + Sq
rt[3])*(2*A - 2*B - 5*C)*(1 + Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a +
 a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)))
+ (3*2^(1/3)*3^(1/4)*(2*A - 2*B - 5*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c
 + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c
 + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 +
Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(2*A - 2*B
- 5*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 +
Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec
[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(
1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))^2)])
```

Rubi [A] time = 0.973871, antiderivative size = 856, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {4052, 3924, 3779, 3778, 136, 3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{3\sqrt[3]{2}\sqrt[4]{3}(2A-2B-5C)E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+}{(}}}{7ad(1-\sec(c+dx))(\sec(c+dx)a+a)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*(A - B + C)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (3*(2*A -
2*B - 5*C)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*
AppellF1[-1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) - (3*(1 + Sq
rt[3])*(2*A - 2*B - 5*C)*(1 + Sec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a +
 a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)))
+ (3*2^(1/3)*3^(1/4)*(2*A - 2*B - 5*C)*EllipticE[ArcCos[(2^(1/3) - (1 - Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
```

```
^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c
 + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c
 + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 +
Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*(2*A - 2*B
- 5*C)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 +
Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec
[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(
1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))^2)])
```

Rule 4052

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((a*A -
b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[
1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*(2*m + 1) + (b
*B*(m + 1) - a*(A*(m + 1) - C*m))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
e, f, A, B, C}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3\int \frac{-\frac{7aA}{3}+\frac{1}{3}a(2A-2B-5C)\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a^2}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{A\int \frac{1}{(a+a\sec(c+dx))^{2/3}}\,dx}{a} - \frac{(2A-2B-5C)\int}{7}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{\left(A(1+\sec(c+dx))^{2/3}\right)\int \frac{1}{(1+\sec(c+dx))^{2/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{\left(A\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\mathrm{Subst}\left(\int\right.}{ad\sqrt{1-\sec(c+dx)}(a+a}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-2B-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(\right.}{}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-2B-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(\right.}{}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-2B-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(\right.}{}$$

$$= -\frac{3(A-B+C)\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{3(2A-2B-5C)\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}AF_1\left(\right.}{}$$

Mathematica [B] time = 19.6848, size = 4383, normalized size = 5.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^(5/3
),x]
```

```
[Out] (Cos[c + d*x]^2*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^
(5/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-6*Sec[(c + d*x)/2]*(10*A*S
in[(c + d*x)/2] - 3*B*Sin[(c + d*x)/2] - 4*C*Sin[(c + d*x)/2]))/7 + (3*Sec[
(c + d*x)/2]^3*(A*Sin[(c + d*x)/2] - B*Sin[(c + d*x)/2] + C*Sin[(c + d*x)/2
]))/7 + (6*(9*A - 2*B - 5*C)*Sin[c + d*x])/7))/(d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*(a*(1 + Sec[c + d*x]))^(5/3)) + (2*2^(1/3)*Cos[c +
 d*x]^2*(1 + Sec[c + d*x])^(5/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((
32*A*(1 + Sec[c + d*x])^(1/3))/7 - (4*B*(1 + Sec[c + d*x])^(1/3))/7 - (10*C
*(1 + Sec[c + d*x])^(1/3))/7 + Cos[c + d*x]*((-54*A*(1 + Sec[c + d*x])^(1/3
))/7 + (12*B*(1 + Sec[c + d*x])^(1/3))/7 + (30*C*(1 + Sec[c + d*x])^(1/3))/
7))*Tan[(c + d*x)/2]*(-(((9*A - 2*B - 5*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^(2/3)) + (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*(5*A + 2*B + 5*C - 7*(7*A - 2*B - 5*C)*Cos[c + d*x])
 + 4*(9*A - 2*B - 5*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^
2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
```

(c + d*x)/2]^2))))/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((2^(1/3)*Sec[(c + d*x)/2]^2*(-(((9*A - 2*B - 5*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) + (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(5*A + 2*B + 5*C - 7*(7*A - 2*B - 5*C)*Cos[c + d*x]) + 4*(9*A - 2*B - 5*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(21*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (2*2^(1/3)*Tan[(c + d*x)/2]*(-(((9*A - 2*B - 5*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) - ((9*A - 2*B - 5*C)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + (2*(9*A - 2*B - 5*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3)) - (9*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(5*A + 2*B + 5*C - 7*(7*A - 2*B - 5*C)*Cos[c + d*x]) + 4*(9*A - 2*B - 5*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)^2*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(5*A + 2*B + 5*C - 7*(7*A - 2*B - 5*C)*Cos[c + d*x]) + 4*(9*A - 2*B - 5*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) + (9*(21*(7*A - 2*B - 5*C)*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[c + d*x] + 4*(9*A - 2*B - 5*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 4*(9*A - 2*B - 5*C)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[c + d*x]*Tan[(c + d*x)/2]^2 + 3*(5*A + 2*B + 5*C - 7*(7*A - 2*B - 5*C)*Cos[c + d*x])*(-(AppellF1[3/2, 1/3, 2, 5/2,

```
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 4*(9*A - 2*B - 5*C)*Cos[c + d*x
]*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2,
 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2
, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3,
 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3,
 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(21*(C
os[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (4*2^(1/3)*Tan[(c + d*x)/2]*(-(((9
*A - 2*B - 5*C)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) + (9*
(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(5*A
 + 2*B + 5*C - 7*(7*A - 2*B - 5*C)*Cos[c + d*x]) + 4*(9*A - 2*B - 5*C)*(3*A
ppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - Appell
F1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]
*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))*(-(Cos[(c
 + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]
*Tan[c + d*x]))/(63*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/3))))
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int (A + B\sec(dx+c) + C(\sec(dx+c))^2)(a + a\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a*(sec(c + d*x) + 1))**(
5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + B\sec(dx + c) + A}{(a\sec(dx + c) + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(5/3
), x)
```

3.633 $\int \sec^m(c+dx)(a+a\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=259

$$\frac{2^{n+\frac{1}{2}}\tan(c+dx)(A(m+n+1)-B(m+n+1)+C(m-n))(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{1}{2};1-m,\frac{1}{2}-\right.}{d(m+n+1)}$$

```
[Out] (C*Sec[c + d*x]^(1 + m)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)
) + (2^(3/2 + n)*(C*n + B*(1 + m + n))*AppellF1[1/2, 1 - m, -1/2 - n, 3/2,
1 - Sec[c + d*x], (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a +
a*Sec[c + d*x])^n*Tan[c + d*x])/(d*(1 + m + n)) + (2^(1/2 + n)*(C*(m - n) +
 A*(1 + m + n) - B*(1 + m + n))*AppellF1[1/2, 1 - m, 1/2 - n, 3/2, 1 - Sec[
c + d*x], (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c
+ d*x])^n*Tan[c + d*x])/(d*(1 + m + n))
```

Rubi [A] time = 0.612415, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4088, 4023, 3828, 3825, 133}

$$\frac{2^{n+\frac{1}{2}}\tan(c+dx)(A(m+n+1)-B(m+n+1)+C(m-n))(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{1}{2};1-m,\frac{1}{2}-\right.}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (C*Sec[c + d*x]^(1 + m)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + m + n)
) + (2^(3/2 + n)*(C*n + B*(1 + m + n))*AppellF1[1/2, 1 - m, -1/2 - n, 3/2,
1 - Sec[c + d*x], (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a +
a*Sec[c + d*x])^n*Tan[c + d*x])/(d*(1 + m + n)) + (2^(1/2 + n)*(C*(m - n) +
 A*(1 + m + n) - B*(1 + m + n))*AppellF1[1/2, 1 - m, 1/2 - n, 3/2, 1 - Sec[
c + d*x], (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec[c
+ d*x])^n*Tan[c + d*x])/(d*(1 + m + n))
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^m(c+dx)(a+a\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(}{d(1+m+n)}$$
$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(}{d(1+m+n)}$$
$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(}{d(1+m+n)}$$
$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(}{d(1+m+n)}$$
$$= \frac{C\sec^{1+m}(c+dx)(a+a\sec(c+dx))^n\sin(}{d(1+m+n)}$$

Mathematica [F] time = 4.19021, size = 0, normalized size = 0.

$$\int \sec^m(c+dx)(a+a\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^m*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] Integrate[Sec[c + d*x]^m*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2), x]
```

Maple [F] time = 1.272, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (a+a\sec(dx+c))^n \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*se
c(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec
(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+a*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*se
c(d*x + c)^m, x)
```

3.634 $\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=258

$$\frac{(An+B(n+1)-C(n+1))\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(\frac{1}{2}-\right.}{dn(n+1)(\sec(c+dx)+1)}$$

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) + ((A*n
+ B*(1 + n) - C*(1 + n))*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c +
d*x])/(1 - Sec[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec
[c + d*x]))^(1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1
+ Sec[c + d*x])) + (2^(3/2 + n)*C*AppellF1[1/2, 1 + n, -1/2 - n, 3/2, 1 - S
ec[c + d*x], (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec
[c + d*x])^n*Tan[c + d*x])/d
```

Rubi [A] time = 0.555473, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4086, 4023, 3828, 3825, 132, 133}

$$\frac{(An+B(n+1)-C(n+1))\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2}-n,-n;1-n;-\frac{2\sec}{1-\sec}\right.}{dn(n+1)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2),x]
```

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) + ((A*n
+ B*(1 + n) - C*(1 + n))*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c +
d*x])/(1 - Sec[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec
[c + d*x]))^(1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1
+ Sec[c + d*x])) + (2^(3/2 + n)*C*AppellF1[1/2, 1 + n, -1/2 - n, 3/2, 1 - S
ec[c + d*x], (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*(a + a*Sec
[c + d*x])^n*Tan[c + d*x])/d
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n \sin}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n \sin}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n \sin}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n \sin}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n \sin}{d(1+n)}$$

Mathematica [F] time = 2.49207, size = 0, normalized size = 0.

$$\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2), x]
```

Maple [F] time = 0.4, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{-1-n} (a + a \sec (dx + c))^n \left(A + B \sec (dx + c) + C (\sec (dx + c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^2 + B \sec (dx + c) + A\right)(a \sec (dx + c) + a)^n \sec (dx + c)^{-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*se
c(d*x + c)^(-n - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \sec (dx + c)^2 + B \sec (dx + c) + A\right)(a \sec (dx + c) + a)^n \sec (dx + c)^{-n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec
(d*x + c)^(-n - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^n\sec\left(dx+c\right)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*se
c(d*x + c)^(-n - 1), x)
```

3.635 $\int \left(\frac{\sec^{-n}(c+dx)(a+a\sec(c+dx))^n(-a(B+An+Bn)-aC(1+n)\sec(c+dx))}{a(1+n)} + \sec\right.$

Optimal. Leaf size=38

$$\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a\sec(c+dx)+a)^n}{d(n+1)}$$

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Rubi [A] time = 0.981481, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 102, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4023, 3828, 3825, 132, 133, 4086}

$$\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a\sec(c+dx)+a)^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^n*(-(a*(B + A*n + B*n)) - a*C*(1 + n)*Sec[c + d*x
]))/(a*(1 + n)*Sec[c + d*x]^n) + Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])
^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] && !IntegerQ[n]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
& !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rubi steps

$$\int \left(\frac{\sec^{-n}(c+dx)(a+a\sec(c+dx))^n(-a(B+An+Bn)-aC(1+n)\sec(c+dx))}{a(1+n)} + \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n \left(\right.\right.$$

Mathematica [A] time = 0.154944, size = 38, normalized size = 1.

$$\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a(\sec(c+dx)+1))^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^n*(-(a*(B + A*n + B*n)) - a*C*(1 + n)*Sec[c
 + d*x]))/(a*(1 + n)*Sec[c + d*x]^n) + Sec[c + d*x]^(-1 - n)*(a + a*Sec[c +
 d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (A*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Maple [F] time = 1.303, size = 0, normalized size = 0.

$$\int \frac{(a+a\sec(dx+c))^n(-a(An+Bn+B)-aC(1+n)\sec(dx+c))}{(1+n)a(\sec(dx+c))^n}+(\sec(dx+c))^{-1-n}(a+a\sec(dx+c))^n\left(A+B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(-a*(A*n+B*n+B)-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d
*x+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*(-a*(A*n+B*n+B)-a*C*(1+n)*sec(d*x+c))/a/(1+n)/(sec(d
*x+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x)
```

Maxima [B] time = 19.4958, size = 419, normalized size = 11.03

$$\frac{\left(\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)+1\right)^n Aa^n\cos\left(-(dn+d)x+2n\arctan\left(\sin(dx+c),\cos(dx+c)+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(-a*(A*n+B*n+B)-a*C*(1+n)*sec(d*x+c))/a/(1+n)/
(sec(d*x+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d
*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(-(d
*n + d)*x + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1) - c)*sin(c*n) - (co
s(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(-(d*n - d)*
x + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1) + c)*sin(c*n) - (cos(d*x +
c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(c*n)*sin(-(d*n + d)
*x + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1) - c) + (cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*A*a^n*cos(c*n)*sin(-(d*n - d)*x + 2*n
*arctan2(sin(d*x + c), cos(d*x + c) + 1) + c))/((d*n + d)*2^n*cos(c*n)^2 +
(d*n + d)*2^n*sin(c*n)^2)
```

Fricas [A] time = 0.595195, size = 142, normalized size = 3.74

$$\frac{A\left(\frac{a\cos(dx+c)+a}{\cos(dx+c)}\right)^n\left(\frac{1}{\cos(dx+c)}\right)^{-n-1}\sin(dx+c)}{(dn+d)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(-a*(A*n+B*n+B)-a*C*(1+n)*sec(d*x+c))/a/(1+n)/
(sec(d*x+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d
*x+c)^2),x, algorithm="fricas")
```

```
[Out] A*((a*cos(d*x + c) + a)/cos(d*x + c))^n*(1/cos(d*x + c))^(-n - 1)*sin(d*x +
 c)/((d*n + d)*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(-a*(A*n+B*n+B)-a*C*(1+n)*sec(d*x+c))/a/(1+n)
/(sec(d*x+c)**n)+sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+B*sec(d*x+c)+C*s
ec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)^{-n-1}-\frac{\left(Ca(n+1)\sec\left(dx+c\right)+(An+Bn+B)\right.}{a(n+1)\sec\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(-a*(A*n+B*n+B)-a*C*(1+n)*sec(d*x+c))/a/(1+n)/
(sec(d*x+c)^n)+sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)+C*sec(d
*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*se
c(d*x + c)^(-n - 1) - (C*a*(n + 1)*sec(d*x + c) + (A*n + B*n + B)*a)*(a*sec
(d*x + c) + a)^n/(a*(n + 1)*sec(d*x + c)^n), x)
```

3.636 $\int (a+a\sec(c+dx))^m \left(B - C + B\sec(c + dx) + C\sec^2(c + \right.$

Optimal. Leaf size=171

$$\frac{C2^{m+\frac{3}{2}}\tan(c+dx)(\sec(c+dx)+1)^{-m-\frac{1}{2}}(a\sec(c+dx)+a)^m\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{d}$$

```
[Out] (Sqrt[2]*(B - C)*AppellF1[3/2 + m, 1/2, 1, 5/2 + m, (1 + Sec[c + d*x])/2, 1
 + Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^m*Tan[c + d*x])/(d
*(3 + 2*m)*Sqrt[1 - Sec[c + d*x]]) + (2^(3/2 + m)*C*Hypergeometric2F1[1/2,
-1/2 - m, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - m)*(a + a*S
ec[c + d*x])^m*Tan[c + d*x])/d
```

Rubi [A] time = 0.263587, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4041, 3924, 3779, 3778, 136, 3828, 3827, 69}

$$\frac{\sqrt{2}(B-C)\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^m F_1\left(m+\frac{3}{2};\frac{1}{2},1;m+\frac{5}{2};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d(2m+3)\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^m*(B - C + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*(B - C)*AppellF1[3/2 + m, 1/2, 1, 5/2 + m, (1 + Sec[c + d*x])/2, 1
 + Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^m*Tan[c + d*x])/(d
*(3 + 2*m)*Sqrt[1 - Sec[c + d*x]]) + (2^(3/2 + m)*C*Hypergeometric2F1[1/2,
-1/2 - m, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - m)*(a + a*S
ec[c + d*x])^m*Tan[c + d*x])/d
```

Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3924

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
```

```
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^m \left(B-C+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{\int (a+a\sec(c+dx))^{1+m}(a(B-C)+aC\sec(c+dx}{a^2}$$

$$= \frac{(B-C)\int (a+a\sec(c+dx))^{1+m}\,dx}{a} + \frac{C\int \sec(c+}{}$$

$$= ((B-C)(1+\sec(c+dx))^{-m}(a+a\sec(c+dx))^m)\int$$

$$= -\frac{\left((B-C)(1+\sec(c+dx))^{-\frac{1}{2}-m}(a+a\sec(c+dx))\right.}{d\sqrt{1-}}$$

$$= \frac{\sqrt{2}(B-C)F_1\left(\frac{3}{2}+m;\frac{1}{2},1;\frac{5}{2}+m;\frac{1}{2}(1+\sec(c+dx\right.}{d(3}$$

Mathematica [B] time = 16.9767, size = 2582, normalized size = 15.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^m*(B - C + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] (2^(1 + m)*Cos[(c + d*x)/2]*Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
m*(1 + Sec[c + d*x])^(-1 - m)*(a*(1 + Sec[c + d*x]))^(1 + m)*(B - C + C*Sec
[c + d*x])*(2*C*Sec[c + d*x]^2*(1 + Sec[c + d*x])^m + Sec[c + d*x]*(2*B*(1
+ Sec[c + d*x])^m - 2*C*(1 + Sec[c + d*x])^m))*Sin[(c + d*x)/2]*(((B - C)*H
ypergeometric2F1[1/2, 1 + m, 3/2, Tan[(c + d*x)/2]^2] + 2*C*Hypergeometric2
F1[1/2, 2 + m, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^
m + (3*(B - C)*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[(c + d*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(a*d*(C + B*Cos[c + d*x] - C*Cos[c +
d*x])*(2^m*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^m*(((B - C)
*Hypergeometric2F1[1/2, 1 + m, 3/2, Tan[(c + d*x)/2]^2] + 2*C*Hypergeometri
c2F1[1/2, 2 + m, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^m + (3*(B - C)*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Cos[(c + d*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) + 2^(1 + m)*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^m*Tan[(c + d*x)/2]*(m*((B - C)*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(c + d*x)/2]^2] + 2*C*Hypergeometric2F1[1/2, 2 + m, 3/2, Tan[(c + d*x)
/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + m)*(-(Sec[(c + d*x)/2]^2*Si
n[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) - (3*(B - C
)*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c
+ d*x)/2]*Sin[(c + d*x)/2])/(3*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (3*(B - C)*Cos[(c + d*x)/2]^2*(-(Ap
pellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/3 + (m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(3*App
ellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF
1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - m*AppellF1[3/2
, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]
^2) - (3*(B - C)*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Cos[(c + d*x)/2]^2*(-2*(AppellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 3*(-(AppellF1[
3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/3 + (m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3) - 2*Tan[(c + d
*x)/2]^2*((-6*AppellF1[5/2, m, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*m*AppellF1[5/2, 1 + m, 2,
7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])/5 - m*((-3*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + m)*AppellF1[
5/2, 2 + m, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])/5))))/(3*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^m*(C*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-Hypergeometric2F1[1/2, 2 + m, 3/
2, Tan[(c + d*x)/2]^2] + (1 - Tan[(c + d*x)/2]^2)^(-2 - m)) + ((B - C)*Csc[
(c + d*x)/2]*Sec[(c + d*x)/2]*(-Hypergeometric2F1[1/2, 1 + m, 3/2, Tan[(c +
 d*x)/2]^2] + (1 - Tan[(c + d*x)/2]^2)^(-1 - m)))/2)) + 2^(1 + m)*m*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(-1 + m)*Tan[(c + d*x)/2]*(((B - C)*Hypergeometr
```

```
ic2F1[1/2, 1 + m, 3/2, Tan[(c + d*x)/2]^2] + 2*C*Hypergeometric2F1[1/2, 2 +
 m, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m + (3*(B -
 C)*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(
c + d*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]
) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^m \left(B-C+B\sec(dx+c)+C(\sec(dx+c))^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^m*(B-C+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+a*sec(d*x+c))^m*(B-C+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+B-C\right)(a\sec(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^m*(B-C+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + B - C)*(a*sec(d*x + c) + a)^
m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec(dx+c)^2+B\sec(dx+c)+B-C\right)(a\sec(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^m*(B-C+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + B - C)*(a*sec(d*x + c) + a)^m
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^m (\sec(c+dx)+1)(B+C\sec(c+dx)-C)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**m*(B-C+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**m*(sec(c + d*x) + 1)*(B + C*sec(c + d*x) -
 C), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+B-C\right)\left(a\sec\left(dx+c\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^m*(B-C+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + B - C)*(a*sec(d*x + c) + a)^
m, x)
```

3.637 $\int \sec^3(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=140

$$\frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aC\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{b(5A+4C)\tan}{15d}$$

```
[Out] (a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(5*A + 4*C)*Tan[c + d*x])/
(5*d) + (a*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^3*Tan[c + d*x])/(4*d) + (b*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (b*(5*A
+ 4*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.174958, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4077, 4047, 3767, 4046, 3768, 3770}

$$\frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aC\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{b(5A+4C)\tan}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(5*A + 4*C)*Tan[c + d*x])/
(5*d) + (a*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^3*Tan[c + d*x])/(4*d) + (b*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (b*(5*A
+ 4*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] && !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{bC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3(c+dx)\left(5aA+\right.$$

$$= \frac{bC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3(c+dx)\left(5aA+\right.$$

$$= \frac{aC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{bC\sec^4(c+dx)\tan(c+}{5d}$$

$$= \frac{b(5A+4C)\tan(c+dx)}{5d}+\frac{a(4A+3C)\sec(c+dx)\tan(}{8d}$$

$$= \frac{a(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(5A+4C)\tan(c+}{5d}$$

Mathematica [A] time = 0.854788, size = 96, normalized size = 0.69

$$\frac{\tan(c+dx)\left(15a(4A+3C)\sec(c+dx)+30aC\sec^3(c+dx)+8b\left(5(A+2C)\tan^2(c+dx)+15(A+C)+3C\tan^4(c+\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (15*a*(4*A + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*a*(4*A + 3*C)*Se
c[c + d*x] + 30*a*C*Sec[c + d*x]^3 + 8*b*(15*(A + C) + 5*(A + 2*C)*Tan[c +
d*x]^2 + 3*C*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.039, size = 192, normalized size = 1.4

$$\frac{Aa\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aC(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3aC\sec(dx+}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/4*a*C
*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec
(d*x+c)+tan(d*x+c))+2/3/d*A*b*tan(d*x+c)+1/3/d*A*b*tan(d*x+c)*sec(d*x+c)^2+
8/15*b*C*tan(d*x+c)/d+1/5*b*C*sec(d*x+c)^4*tan(d*x+c)/d+4/15*b*C*sec(d*x+c)
^2*tan(d*x+c)/d
```

Maxima [A] time = 0.990882, size = 236, normalized size = 1.69

$$\frac{80 \left(\tan (dx+c)^3 + 3 \tan (dx+c)\right)Ab + 16 \left(3 \tan (dx+c)^5 + 10 \tan (dx+c)^3 + 15 \tan (dx+c)\right)Cb - 15\,Ca\left(\frac{2\left(3 \sin(dx+c}{\sin(dx+c)^4-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b + 16*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*C*b - 15*C*a*(2*(3*sin(d*x + c)^3 - 5*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) - 60*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.577245, size = 389, normalized size = 2.78

$$\frac{15\,(4\,A + 3\,C)a \cos (dx+c)^5 \log (\sin (dx+c) + 1) - 15\,(4\,A + 3\,C)a \cos (dx+c)^5 \log (-\sin (dx+c) + 1) + 2\left(16\,(5\,A + 4\right.}{240\,d \cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*A + 3*C)*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*A + 3*
C)*a*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*A + 4*C)*b*cos(d*x +
c)^4 + 15*(4*A + 3*C)*a*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 3
0*C*a*cos(d*x + c) + 24*C*b)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C \sec^2 (c+dx)\right)(a + b \sec (c+dx)) \sec^3 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.23531, size = 451, normalized size = 3.22

$$\frac{15\,(4\,Aa + 3\,Ca) \log \left(\left|\tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\,(4\,Aa + 3\,Ca) \log \left(\left|\tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(60\,Aa \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 + 75\,Ca \tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(4*A*a + 3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a +
3*C*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*A*a*tan(1/2*d*x + 1/2*c)^
9 + 75*C*a*tan(1/2*d*x + 1/2*c)^9 - 120*A*b*tan(1/2*d*x + 1/2*c)^9 - 120*C*
b*tan(1/2*d*x + 1/2*c)^9 - 120*A*a*tan(1/2*d*x + 1/2*c)^7 - 30*C*a*tan(1/2*
d*x + 1/2*c)^7 + 320*A*b*tan(1/2*d*x + 1/2*c)^7 + 160*C*b*tan(1/2*d*x + 1/2
*c)^7 - 400*A*b*tan(1/2*d*x + 1/2*c)^5 - 464*C*b*tan(1/2*d*x + 1/2*c)^5 + 1
20*A*a*tan(1/2*d*x + 1/2*c)^3 + 30*C*a*tan(1/2*d*x + 1/2*c)^3 + 320*A*b*tan
(1/2*d*x + 1/2*c)^3 + 160*C*b*tan(1/2*d*x + 1/2*c)^3 - 60*A*a*tan(1/2*d*x +
 1/2*c) - 75*C*a*tan(1/2*d*x + 1/2*c) - 120*A*b*tan(1/2*d*x + 1/2*c) - 120*
C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.638 $\int \sec^2(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=117

$$\frac{a(3A+2C)\tan(c+dx)}{3d}+\frac{aC\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{b(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(4A+3C)\tan(c+dx)\sec}{8d}$$

```
[Out] (b*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(3*A + 2*C)*Tan[c + d*x])/
(3*d) + (b*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (b*C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.163016, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4077, 4047, 3768, 3770, 4046, 3767, 8}

$$\frac{a(3A+2C)\tan(c+dx)}{3d}+\frac{aC\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{b(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(4A+3C)\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (b*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(3*A + 2*C)*Tan[c + d*x])/
(3*d) + (b*(4*A + 3*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*Sec[c + d*x]
^2*Tan[c + d*x])/(3*d) + (b*C*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{bC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^2(c+dx)\left(4aA+\right.$$
$$= \frac{bC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^2(c+dx)\left(4aA+\right.$$
$$= \frac{b(4A+3C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aC\sec^2(c+dx)\,\mathrm{t}}{3d}$$
$$= \frac{b(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(4A+3C)\sec(c+}{8d}$$
$$= \frac{b(4A+3C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+2C)\tan(c+}{3d}$$

Mathematica [A] time = 0.490301, size = 80, normalized size = 0.68

$$\frac{\tan(c+dx)\left(8a\left(3(A+C)+C\tan^2(c+dx)\right)+3b(4A+3C)\sec(c+dx)+6bC\sec^3(c+dx)\right)+3b(4A+3C)\tanh^{-1}(}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*b*(4*A + 3*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(4*A + 3*C)*Sec[
c + d*x] + 6*b*C*Sec[c + d*x]^3 + 8*a*(3*(A + C) + C*Tan[c + d*x]^2)))/(24*
d)
```

Maple [A] time = 0.035, size = 149, normalized size = 1.3

$$\frac{Aa\tan(dx+c)}{d}+\frac{2\,aC\tan(dx+c)}{3\,d}+\frac{C\left(\sec(dx+c)\right)^2a\tan(dx+c)}{3\,d}+\frac{A\sec(dx+c)\,b\tan(dx+c)}{2\,d}+\frac{Ab\ln(\sec(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*tan(d*x+c)+2/3*a*C*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d+1
/2/d*A*b*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/4*b*C*
sec(d*x+c)^3*tan(d*x+c)/d+3/8*b*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*C*b*ln(sec(
```

```
d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.961949, size = 205, normalized size = 1.75

$$\frac{16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ca-3\,Cb\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a - 3*C*b*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*A*b*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a*tan(d*x +
 c))/d
```

Fricas [A] time = 0.562531, size = 335, normalized size = 2.86

$$\frac{3\left(4\,A+3\,C\right)b\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,A+3\,C\right)b\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(3\,A+2\,C\right)\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(4*A + 3*C)*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A + 3*C)*
b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(3*A + 2*C)*a*cos(d*x + c)^3
 + 3*(4*A + 3*C)*b*cos(d*x + c)^2 + 8*C*a*cos(d*x + c) + 6*C*b)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+C\sec^2\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.22292, size = 410, normalized size = 3.5

$$3\left(4\,Ab+3\,Cb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,Ab+3\,Cb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Ca\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(4*A*b + 3*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*b + 3*C
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*A*a*tan(1/2*d*x + 1/2*c)^7 +
 24*C*a*tan(1/2*d*x + 1/2*c)^7 - 12*A*b*tan(1/2*d*x + 1/2*c)^7 - 15*C*b*tan
(1/2*d*x + 1/2*c)^7 - 72*A*a*tan(1/2*d*x + 1/2*c)^5 - 40*C*a*tan(1/2*d*x +
1/2*c)^5 + 12*A*b*tan(1/2*d*x + 1/2*c)^5 - 9*C*b*tan(1/2*d*x + 1/2*c)^5 + 7
2*A*a*tan(1/2*d*x + 1/2*c)^3 + 40*C*a*tan(1/2*d*x + 1/2*c)^3 + 12*A*b*tan(1
/2*d*x + 1/2*c)^3 - 9*C*b*tan(1/2*d*x + 1/2*c)^3 - 24*A*a*tan(1/2*d*x + 1/2
*c) - 24*C*a*tan(1/2*d*x + 1/2*c) - 12*A*b*tan(1/2*d*x + 1/2*c) - 15*C*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.639 $\int \sec(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=86

$$\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}+\frac{b(3A+2C)\tan(c+dx)}{3d}+\frac{bC\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] (a*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(3*A + 2*C)*Tan[c + d*x])/(3
*d) + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*C*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rubi [A] time = 0.102999, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4077, 4047, 3767, 8, 4046, 3770}

$$\frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)\sec(c+dx)}{2d}+\frac{b(3A+2C)\tan(c+dx)}{3d}+\frac{bC\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(3*A + 2*C)*Tan[c + d*x])/(3
*d) + (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*C*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
```

```
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{bC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)\left(3aA+b\right.$$
$$= \frac{bC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)\left(3aA+3\right.$$
$$= \frac{aC\sec(c+dx)\tan(c+dx)}{2d}+\frac{bC\sec^2(c+dx)\tan(c+dx}{3d}$$
$$= \frac{a(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(3A+2C)\tan(c+dx}{3d}$$

Mathematica [A] time = 0.305646, size = 59, normalized size = 0.69

$$\frac{\tan(c+dx)\left(3aC\sec(c+dx)+6b(A+C)+2bC\tan^2(c+dx)\right)+3a(2A+C)\tanh^{-1}(\sin(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*a*(2*A + C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*b*(A + C) + 3*a*C*Se
c[c + d*x] + 2*b*C*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.035, size = 108, normalized size = 1.3

$$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\sec(dx+c)\tan(dx+c)}{2d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Ab\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*C
*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b*tan(d*x+c)+2/3*b*C*tan(d*x+c)/d+1/3*b*C*
sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.973692, size = 135, normalized size = 1.57

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cb-3\,Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,Aa\log(}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b - 3*C*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*A*a*l
og(sec(d*x + c) + tan(d*x + c)) + 12*A*b*tan(d*x + c))/d
```

Fricas [A] time = 0.571301, size = 285, normalized size = 3.31

$$\frac{3\,(2\,A+C)a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+C)a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(3\,A+2\,C)b\cos\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*A + C)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + C)*a*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A + 2*C)*b*cos(d*x + c)^2 + 3
*C*a*cos(d*x + c) + 2*C*b)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+C\sec^{2}\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 1.23619, size = 248, normalized size = 2.88

$$\frac{3\,(2\,Aa+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,Aa+Ca)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*A*a + C*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a + C*a)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*C*a*tan(1/2*d*x + 1/2*c)^5 - 6*A*b
*tan(1/2*d*x + 1/2*c)^5 - 6*C*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*b*tan(1/2*d*x
 + 1/2*c)^3 + 4*C*b*tan(1/2*d*x + 1/2*c)^3 - 3*C*a*tan(1/2*d*x + 1/2*c) - 6
*A*b*tan(1/2*d*x + 1/2*c) - 6*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^3)/d
```

3.640 $\int (a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=58

$$aAx+\frac{aC\tan(c+dx)}{d}+\frac{b(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] a*A*x + (b*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*Tan[c + d*x])/d +
(b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0536912, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4049, 3770, 3767, 8}

$$aAx+\frac{aC\tan(c+dx)}{d}+\frac{b(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (b*(2*A + C)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*C*Tan[c + d*x])/d +
(b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4049

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x])/(2*f), x] + D
ist[1/2, Int[Simp[2*A*a + b*(2*A + C)*Csc[e + f*x] + 2*a*C*Csc[e + f*x]^2,
x], x], x] /; FreeQ[{a, b, e, f, A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx &= \frac{bC\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2aA+b(2A+C)\sec(c+dx)+2\right.\\ &= aAx+\frac{bC\sec(c+dx)\tan(c+dx)}{2d}+(aC)\int \sec^2(c+dx)\,dx+\frac{1}{2}(b(2\\ &= aAx+\frac{b(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\sec(c+dx)\tan(c+dx)}{2d}\\ &= aAx+\frac{b(2A+C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aC\tan(c+dx)}{d}+\frac{bC\sec(c\end{aligned}$$

Mathematica [A] time = 0.019705, size = 67, normalized size = 1.16

$$aAx+\frac{aC\tan(c+dx)}{d}+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (b*C*ArcTanh[Sin[c + d*x]])/(2*d) +
 (a*C*Tan[c + d*x])/d + (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.035, size = 85, normalized size = 1.5

$$aAx+\frac{Aac}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cb\sec(dx+c)\tan(dx+c)}{2d}+\frac{Cb\ln(\sec(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a*A*x+1/d*A*a*c+a*C*tan(d*x+c)/d+1/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/2*b*C*
sec(d*x+c)*tan(d*x+c)/d+1/2/d*C*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.964559, size = 119, normalized size = 2.05

$$\frac{4(dx+c)Aa-Cb\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Ab\log(\sec(dx+c)+\tan(dx+c))+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a - C*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*A*b*log(sec(d*x + c) + tan(d*x +
c)) + 4*C*a*tan(d*x + c))/d
```

Fricas [A] time = 0.572936, size = 267, normalized size = 4.6

$$\frac{4Aadx\cos(dx+c)^2+(2A+C)b\cos(dx+c)^2\log(\sin(dx+c)+1)-(2A+C)b\cos(dx+c)^2\log(-\sin(dx+c)+1)+}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*A*a*d*x*cos(d*x + c)^2 + (2*A + C)*b*cos(d*x + c)^2*log(sin(d*x + c)
 + 1) - (2*A + C)*b*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*C*a*cos(d*
x + c) + C*b)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2{(c + dx)}\right)(a + b\sec{(c + dx)})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.15396, size = 181, normalized size = 3.12

$$\frac{2(dx + c)Aa + (2Ab + Cb)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (2Ab + Cb)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(2Ca\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right.}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + (2*A*b + C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
2*A*b + C*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*C*a*tan(1/2*d*x + 1/
2*c)^3 - C*b*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1/2*d*x + 1/2*c) - C*b*tan(
1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.641 $\int \cos(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$\frac{aA\sin(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+Abx+\frac{bC\tan(c+dx)}{d}$$

```
[Out] A*b*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d + (b*C*Tan[c +
 d*x])/d
```

Rubi [A] time = 0.0960225, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4077, 4047, 8, 4045, 3770}

$$\frac{aA\sin(c+dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+Abx+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] A*b*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d + (b*C*Tan[c +
 d*x])/d
```

Rule 4077

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Cs
c[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 2)), x] + Dist[1/(n + 2
), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2) + b*(C*(n + 1) + A*(n + 2))*Csc[
e + f*x] + a*C*(n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
A, C, n}, x] &&  !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{bC\tan(c+dx)}{d}+\int \cos(c+dx)\left(aA+Ab\sec(c+dx)+\right.$$
$$= \frac{bC\tan(c+dx)}{d}+(Ab)\int 1\,dx+\int \cos(c+dx)\left(aA+aC\right.$$
$$= Abx+\frac{aA\sin(c+dx)}{d}+\frac{bC\tan(c+dx)}{d}+(aC)\int \sec(c$$
$$= Abx+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)}{d}+\frac{bC\tan}{}$$

Mathematica [A] time = 0.0195565, size = 54, normalized size = 1.29

$$\frac{aA\sin(c)\cos(dx)}{d}+\frac{aA\cos(c)\sin(dx)}{d}+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+Abx+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] A*b*x + (a*C*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (a*A*Cos[
c]*Sin[d*x])/d + (b*C*Tan[c + d*x])/d
```

Maple [A] time = 0.047, size = 57, normalized size = 1.4

$$Abx+\frac{A\sin(dx+c)a}{d}+\frac{Abc}{d}+\frac{Cb\tan(dx+c)}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] A*b*x+a*A*sin(d*x+c)/d+1/d*A*b*c+b*C*tan(d*x+c)/d+1/d*a*C*ln(sec(d*x+c)+tan
(d*x+c))
```

Maxima [A] time = 0.950508, size = 80, normalized size = 1.9

$$\frac{2(dx+c)Ab+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2Aa\sin(dx+c)+2Cb\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*A*b + C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*A*a*sin(d*x + c) + 2*C*b*tan(d*x + c))/d
```

Fricas [B] time = 0.540632, size = 232, normalized size = 5.52

$$\frac{2Abdx\cos(dx+c)+Ca\cos(dx+c)\log(\sin(dx+c)+1)-Ca\cos(dx+c)\log(-\sin(dx+c)+1)+2(Aa\cos(dx+}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*A*b*d*x*cos(d*x + c) + C*a*cos(d*x + c)*log(sin(d*x + c) + 1) - C*a*
cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a*cos(d*x + c) + C*b)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)\cos\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [B] time = 1.1475, size = 161, normalized size = 3.83

$$\frac{(dx+c)Ab + Ca\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - Ca\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3} - Cb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3} - Aa\tan\left(\right.\right.}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] ((d*x + c)*A*b + C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*a*log(abs(tan(1
/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - C*b*tan(1/2*d*x + 1
/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^4 - 1))/d
```

3.642 $\int \cos^2(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=58

$$\frac{aA\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+2C)+\frac{Ab\sin(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*(A + 2*C)*x)/2 + (b*C*ArcTanh[Sin[c + d*x]])/d + (A*b*Sin[c + d*x])/d +
(a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.122537, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4075, 4047, 8, 4045, 3770}

$$\frac{aA\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+2C)+\frac{Ab\sin(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(A + 2*C)*x)/2 + (b*C*ArcTanh[Sin[c + d*x]])/d + (A*b*Sin[c + d*x])/d +
(a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)\left(-2Ab-a(A\right.$$
$$= \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)\left(-2Ab-2bC\right.$$
$$= \frac{1}{2}a(A+2C)x + \frac{Ab\sin(c+dx)}{d} + \frac{aA\cos(c+dx)\sin(c+d}{2d}$$
$$= \frac{1}{2}a(A+2C)x + \frac{bC\tanh^{-1}(\sin(c+dx))}{d} + \frac{Ab\sin(c+dx)}{d}$$

Mathematica [A] time = 0.131695, size = 73, normalized size = 1.26

$$\frac{aA(c+dx)}{2d} + \frac{aA\sin(2(c+dx))}{4d} + aCx + \frac{Ab\sin(c)\cos(dx)}{d} + \frac{Ab\cos(c)\sin(dx)}{d} + \frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*C*x + (a*A*(c + d*x))/(2*d) + (b*C*ArcTanh[Sin[c + d*x]])/d + (A*b*Cos[d*
x]*Sin[c])/d + (A*b*Cos[c]*Sin[d*x])/d + (a*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.061, size = 77, normalized size = 1.3

$$\frac{Aa\cos(dx+c)\sin(dx+c)}{2d} + \frac{aAx}{2} + \frac{Aac}{2d} + aCx + \frac{Cac}{d} + \frac{Ab\sin(dx+c)}{d} + \frac{Cb\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2*a*A*cos(d*x+c)*sin(d*x+c)/d+1/2*a*A*x+1/2/d*A*a*c+a*C*x+1/d*C*a*c+A*b*s
in(d*x+c)/d+1/d*C*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.982577, size = 95, normalized size = 1.64

$$\frac{(2dx+2c+\sin(2dx+2c))Aa + 4(dx+c)Ca + 2Cb(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 4Ab\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*C*a + 2*C*b*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*b*sin(d*x + c))/d
```

Fricas [A] time = 0.517982, size = 167, normalized size = 2.88

$$\frac{(A+2C)adx + Cb\log(\sin(dx+c)+1) - Cb\log(-\sin(dx+c)+1) + (Aa\cos(dx+c)+2Ab)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/2*((A + 2*C)*a*d*x + C*b*log(sin(d*x + c) + 1) - C*b*log(-sin(d*x + c) +
1) + (A*a*cos(d*x + c) + 2*A*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24247, size = 171, normalized size = 2.95

$$\frac{2\,Cb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Cb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Aa+2\,Ca)(dx+c)-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,Ab\tan\cdots\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*C*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*C*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (A*a + 2*C*a)*(d*x + c) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2
*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.643 $\int \cos^3(c+dx)(a+b\sec(c+dx)) \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=77

$$\frac{a(2A+3C)\sin(c+dx)}{3d} + \frac{aA\sin(c+dx)\cos^2(c+dx)}{3d} + \frac{Ab\sin(c+dx)\cos(c+dx)}{2d} + \frac{1}{2}bx(A+2C)$$

```
[Out] (b*(A + 2*C)*x)/2 + (a*(2*A + 3*C)*Sin[c + d*x])/(3*d) + (A*b*Cos[c + d*x]*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.140967, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4075, 4047, 2637, 4045, 8}

$$\frac{a(2A+3C)\sin(c+dx)}{3d} + \frac{aA\sin(c+dx)\cos^2(c+dx)}{3d} + \frac{Ab\sin(c+dx)\cos(c+dx)}{2d} + \frac{1}{2}bx(A+2C)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (b*(A + 2*C)*x)/2 + (a*(2*A + 3*C)*Sin[c + d*x])/(3*d) + (A*b*Cos[c + d*x]*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)\left(-3Ab\right.$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)\left(-3Ab\right.$$
$$= \frac{a(2A+3C)\sin(c+dx)}{3d} + \frac{Ab\cos(c+dx)\sin(c+dx)}{2d} +$$
$$= \frac{1}{2}b(A+2C)x + \frac{a(2A+3C)\sin(c+dx)}{3d} + \frac{Ab\cos(c+d}{2}$$

Mathematica [A] time = 0.11832, size = 64, normalized size = 0.83

$$\frac{3a(3A+4C)\sin(c+dx)+aA\sin(3(c+dx))+3Ab\sin(2(c+dx))+6Abc+6Abdx+12bCdx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (6*A*b*c + 6*A*b*d*x + 12*b*C*d*x + 3*a*(3*A + 4*C)*Sin[c + d*x] + 3*A*b*Si
n[2*(c + d*x)] + a*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.057, size = 68, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\sin(dx+c)+Cb(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*sin(d*x+c)+C*b*(d*x+c))
```

Maxima [A] time = 0.955525, size = 90, normalized size = 1.17

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-3(2dx+2c+\sin(2dx+2c))Ab-12(dx+c)Cb-12Ca\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*A*b - 12*(d*x + c)*C*b - 12*C*a*sin(d*x + c))/d
```

Fricas [A] time = 0.486873, size = 140, normalized size = 1.82

$$\frac{3(A+2C)bdx+\left(2Aa\cos(dx+c)^2+3Ab\cos(dx+c)+2(2A+3C)a\right)\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(A + 2*C)*b*d*x + (2*A*a*cos(d*x + c)^2 + 3*A*b*cos(d*x + c) + 2*(2*
A + 3*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14409, size = 207, normalized size = 2.69

$$\frac{3\,(Ab+2\,Cb)(dx+c)+\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,Aa\tan(\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(A*b + 2*C*b)*(d*x + c) + 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*ta
n(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1
/2*c)^3 + 12*C*a*tan(1/2*d*x + 1/2*c)^3 + 6*A*a*tan(1/2*d*x + 1/2*c) + 6*C*
a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 + 1)^3)/d
```

3.644 $\int \cos^4(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{a(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{8}ax(3A+4C)+\frac{b(A+C)\sin(c+dx)}{d}-\frac{Ab\sin$$

```
[Out] (a*(3*A + 4*C)*x)/8 + (b*(A + C)*Sin[c + d*x])/d + (a*(3*A + 4*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (A*b*Si
n[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.172779, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4075, 4047, 2635, 8, 4044, 3013}

$$\frac{a(3A+4C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{1}{8}ax(3A+4C)+\frac{b(A+C)\sin(c+dx)}{d}-\frac{Ab\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(3*A + 4*C)*x)/8 + (b*(A + C)*Sin[c + d*x])/d + (a*(3*A + 4*C)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (A*b*Si
n[c + d*x]^3)/(3*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
```

```
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)\left(-4Ab-a\right.$$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)\left(-4Ab-4\right.$$
$$= \frac{a(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA\cos^3(c+dx)\sin(}{4d}$$
$$= \frac{1}{8}a(3A+4C)x + \frac{a(3A+4C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA}{}$$
$$= \frac{1}{8}a(3A+4C)x + \frac{b(A+C)\sin(c+dx)}{d} + \frac{a(3A+4C)\cos(c}{8}$$

Mathematica [A] time = 0.209407, size = 84, normalized size = 0.88

$$\frac{24a(A+C)\sin(2(c+dx))+3aA\sin(4(c+dx))+36aAc+36aAdx+48acC+48aCdx+24b(3A+4C)\sin(c+dx)+8}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (36*a*A*c + 48*a*c*C + 36*a*A*d*x + 48*a*C*d*x + 24*b*(3*A + 4*C)*Sin[c + d
*x] + 24*a*(A + C)*Sin[2*(c + d*x)] + 8*A*b*Sin[3*(c + d*x)] + 3*a*A*Sin[4*
(c + d*x)])/(96*d)
```

Maple [A] time = 0.066, size = 96, normalized size = 1.

$$\frac{1}{d}\left(Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+aC\left(\frac{\cos(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+C*sin(d*x+c)*b)
```

Maxima [A] time = 0.95724, size = 122, normalized size = 1.28

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa+24(2dx+2c+\sin(2dx+2c))Ca-32\left(\sin(dx+c)^3-3\sin\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a + 24*(2
*d*x + 2*c + sin(2*d*x + 2*c))*C*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A
*b + 96*C*b*sin(d*x + c))/d
```

Fricas [A] time = 0.49443, size = 189, normalized size = 1.99

$$\frac{3(3A+4C)adx + \left(6Aa\cos(dx+c)^3 + 8Ab\cos(dx+c)^2 + 3(3A+4C)a\cos(dx+c) + 8(2A+3C)b\right)\sin(dx+c}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(3*A + 4*C)*a*d*x + (6*A*a*cos(d*x + c)^3 + 8*A*b*cos(d*x + c)^2 +
3*(3*A + 4*C)*a*cos(d*x + c) + 8*(2*A + 3*C)*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20293, size = 367, normalized size = 3.86

$$3(3Aa+4Ca)(dx+c) - \frac{2\left(15Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 12Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 24Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 24Cb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a + 4*C*a)*(d*x + c) - 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 + 12*C
*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 - 24*C*b*tan(1/2*
d*x + 1/2*c)^7 - 9*A*a*tan(1/2*d*x + 1/2*c)^5 + 12*C*a*tan(1/2*d*x + 1/2*c)
^5 - 40*A*b*tan(1/2*d*x + 1/2*c)^5 - 72*C*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a*
tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/2*d*x
 + 1/2*c)^3 - 72*C*b*tan(1/2*d*x + 1/2*c)^3 - 15*A*a*tan(1/2*d*x + 1/2*c) -
 12*C*a*tan(1/2*d*x + 1/2*c) - 24*A*b*tan(1/2*d*x + 1/2*c) - 24*C*b*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.645 $\int \cos^5(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=131

$$-\frac{a(4A+5C)\sin^3(c+dx)}{15d}+\frac{a(4A+5C)\sin(c+dx)}{5d}+\frac{aA\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{b(3A+4C)\sin(c+dx)\cos(c+dx}{8d}$$

```
[Out] (b*(3*A + 4*C)*x)/8 + (a*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (b*(3*A + 4*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (A*b*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*C)*Sin[c + d*x]^3)/(
15*d)
```

Rubi [A] time = 0.172793, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4075, 4047, 2633, 4045, 2635, 8}

$$-\frac{a(4A+5C)\sin^3(c+dx)}{15d}+\frac{a(4A+5C)\sin(c+dx)}{5d}+\frac{aA\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{b(3A+4C)\sin(c+dx)\cos(c+dx}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (b*(3*A + 4*C)*x)/8 + (a*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (b*(3*A + 4*C)*C
os[c + d*x]*Sin[c + d*x])/(8*d) + (A*b*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*C)*Sin[c + d*x]^3)/(
15*d)
```

Rule 4075

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(A*a*Cot[
e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])
^(n + 1)*Simp[A*b*n + a*(C*n + A*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)\left(-5Ab\right.$$
$$= \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx)\left(-5Ab\right.$$
$$= \frac{Ab\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aA\cos^4(c+dx)\sin(c+}{5d}$$
$$= \frac{a(4A+5C)\sin(c+dx)}{5d} + \frac{b(3A+4C)\cos(c+dx)\sin(c}{8d}$$
$$= \frac{1}{8}b(3A+4C)x + \frac{a(4A+5C)\sin(c+dx)}{5d} + \frac{b(3A+4C)}{}$$

Mathematica [A] time = 0.310102, size = 89, normalized size = 0.68

$$\frac{-160a(2A+C)\sin^3(c+dx)+480a(A+C)\sin(c+dx)+96aA\sin^5(c+dx)+15b(4(3A+4C)(c+dx)+8(A+C)\mathrm{si}}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (480*a*(A + C)*Sin[c + d*x] - 160*a*(2*A + C)*Sin[c + d*x]^3 + 96*a*A*Sin[c
 + d*x]^5 + 15*b*(4*(3*A + 4*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + A*
Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.067, size = 117, normalized size = 0.9

$$\frac{1}{d}\left(\frac{A\sin(dx+c)\,a}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+Ab\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\,\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*C*(2+cos(d*x+c)^2)*
sin(d*x+c)+C*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.979189, size = 153, normalized size = 1.17

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Aa-160\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ca+15\left(12\,dx+12\,c+\mathrm{s}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a - 16
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a + 15*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*A*b + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*b)/d
```

Fricas [A] time = 0.507375, size = 242, normalized size = 1.85

$$\frac{15\left(3\,A+4\,C\right)bdx+\left(24\,Aa\cos\left(dx+c\right)^{4}+30\,Ab\cos\left(dx+c\right)^{3}+8\left(4\,A+5\,C\right)a\cos\left(dx+c\right)^{2}+15\left(3\,A+4\,C\right)b\cos\left(dx+\right.\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*A + 4*C)*b*d*x + (24*A*a*cos(d*x + c)^4 + 30*A*b*cos(d*x + c)^
3 + 8*(4*A + 5*C)*a*cos(d*x + c)^2 + 15*(3*A + 4*C)*b*cos(d*x + c) + 16*(4*
A + 5*C)*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16463, size = 408, normalized size = 3.11

$$15\left(3\,Ab+4\,Cb\right)\left(dx+c\right)+\frac{2\left(120\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+120\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-60\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+160\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*A*b + 4*C*b)*(d*x + c) + 2*(120*A*a*tan(1/2*d*x + 1/2*c)^9 + 1
20*C*a*tan(1/2*d*x + 1/2*c)^9 - 75*A*b*tan(1/2*d*x + 1/2*c)^9 - 60*C*b*tan(
```

```
1/2*d*x + 1/2*c)^9 + 160*A*a*tan(1/2*d*x + 1/2*c)^7 + 320*C*a*tan(1/2*d*x +
 1/2*c)^7 - 30*A*b*tan(1/2*d*x + 1/2*c)^7 - 120*C*b*tan(1/2*d*x + 1/2*c)^7
+ 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*C*a*tan(1/2*d*x + 1/2*c)^5 + 160*A*a
*tan(1/2*d*x + 1/2*c)^3 + 320*C*a*tan(1/2*d*x + 1/2*c)^3 + 30*A*b*tan(1/2*d
*x + 1/2*c)^3 + 120*C*b*tan(1/2*d*x + 1/2*c)^3 + 120*A*a*tan(1/2*d*x + 1/2*
c) + 120*C*a*tan(1/2*d*x + 1/2*c) + 75*A*b*tan(1/2*d*x + 1/2*c) + 60*C*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.646 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=226

$$\frac{\left(2a^2b^2(5A+3C)+a^4C+2b^4(5A+4C)\right)\tan(c+dx)}{15b^2d}+\frac{\left(a^2C+2b^2(5A+4C)\right)\tan(c+dx)(a+b\sec(c+dx))^2}{30b^2d}+\frac{a\left(2a^2\right.}{}$$

```
[Out] (a*b*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(4*d) + ((a^4*C + 2*a^2*b^2*(5*A +
3*C) + 2*b^4*(5*A + 4*C))*Tan[c + d*x])/(15*b^2*d) + (a*(20*A*b^2 + 2*a^2*C
 + 13*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(60*b*d) + ((a^2*C + 2*b^2*(5*A + 4
*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(30*b^2*d) - (a*C*(a + b*Sec[c +
d*x])^3*Tan[c + d*x])/(10*b^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^3*T
an[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.502938, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4093, 4082, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(2a^2b^2(5A+3C)+a^4C+2b^4(5A+4C)\right)\tan(c+dx)}{15b^2d}+\frac{\left(a^2C+2b^2(5A+4C)\right)\tan(c+dx)(a+b\sec(c+dx))^2}{30b^2d}+\frac{a\left(2a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*b*(4*A + 3*C)*ArcTanh[Sin[c + d*x]])/(4*d) + ((a^4*C + 2*a^2*b^2*(5*A +
3*C) + 2*b^4*(5*A + 4*C))*Tan[c + d*x])/(15*b^2*d) + (a*(20*A*b^2 + 2*a^2*C
 + 13*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(60*b*d) + ((a^2*C + 2*b^2*(5*A + 4
*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(30*b^2*d) - (a*C*(a + b*Sec[c +
d*x])^3*Tan[c + d*x])/(10*b^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^3*T
an[c + d*x])/(5*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
```

```
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec(c+dx)(a+b\sec(c+dx))^3\tan(c+dx)}{5bd} + \frac{\int \sec(\ldots}{\ldots}$$

$$= -\frac{aC(a+b\sec(c+dx))^3\tan(c+dx)}{10b^2d} + \frac{C\sec(c+dx)(a\ldots}{\ldots}$$

$$= \frac{\left(a^2C+2b^2(5A+4C)\right)(a+b\sec(c+dx))^2\tan(c+dx)}{30b^2d}\ldots$$

$$= \frac{a\left(20Ab^2+2a^2C+13b^2C\right)\sec(c+dx)\tan(c+dx)}{60bd} + \ldots$$

$$= \frac{a\left(20Ab^2+2a^2C+13b^2C\right)\sec(c+dx)\tan(c+dx)}{60bd} + \ldots$$

$$= \frac{ab(4A+3C)\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a\left(20Ab^2+2a^2C\ldots\right.}{\ldots}$$

$$= \frac{ab(4A+3C)\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\left(a^4C+2a^2b^2(5A\ldots\right.}{\ldots}$$

Mathematica [A] time = 2.29767, size = 275, normalized size = 1.22

$$-\frac{\sec^5(c+dx)\left(A\cos^2(c+dx)+C\right)\left(60ab(4A+3C)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\ldots\right.\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -((C + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(60*a*b*(4*A + 3*C)*Cos[c + d*x]^5*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) - (90*a^2*A + 100*A*b^2 + 100*a^2*C + 128*b^2*C + 15*a*b*(12*A +
 17*C)*Cos[c + d*x] + 24*(5*a^2*(A + C) + b^2*(5*A + 4*C))*Cos[2*(c + d*x)]
 + 60*a*A*b*Cos[3*(c + d*x)] + 45*a*b*C*Cos[3*(c + d*x)] + 30*a^2*A*Cos[4*(
c + d*x)] + 20*A*b^2*Cos[4*(c + d*x)] + 20*a^2*C*Cos[4*(c + d*x)] + 16*b^2*
C*Cos[4*(c + d*x)])*Sin[c + d*x]))/(120*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.044, size = 257, normalized size = 1.1

$$\frac{a^2 A \tan(dx+c)}{d} + \frac{2a^2 C \tan(dx+c)}{3d} + \frac{a^2 C \tan(dx+c)(\sec(dx+c))^2}{3d} + \frac{Aab\sec(dx+c)\tan(dx+c)}{d} + \frac{Aab\ln(\sec($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*tan(d*x+c)+2/3/d*a^2*C*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+
c)^2+1/d*A*a*b*sec(d*x+c)*tan(d*x+c)+1/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/
2/d*a*b*C*tan(d*x+c)*sec(d*x+c)^3+3/4*a*b*C*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a
*b*C*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*A*b^2*tan(d*x+c)+1/3/d*A*b^2*tan(d*x+c
)*sec(d*x+c)^2+8/15*b^2*C*tan(d*x+c)/d+1/5/d*b^2*C*tan(d*x+c)*sec(d*x+c)^4+
4/15/d*b^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.0013, size = 292, normalized size = 1.29

$$\frac{40\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ca^2 + 40\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Ab^2 + 8\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 + 40*(tan(d*x + c)^3 + 3*
tan(d*x + c))*A*b^2 + 8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
+ c))*C*b^2 - 15*C*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 60*A*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)) + 120*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.539279, size = 455, normalized size = 2.01

$$\frac{15(4A+3C)ab\cos(dx+c)^5\log(\sin(dx+c)+1) - 15(4A+3C)ab\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\left(15(4A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(4*A + 3*C)*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*A +
3*C)*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(15*(4*A + 3*C)*a*b*cos(
d*x + c)^3 + 4*(5*(3*A + 2*C)*a^2 + 2*(5*A + 4*C)*b^2)*cos(d*x + c)^4 + 30*
C*a*b*cos(d*x + c) + 12*C*b^2 + 4*(5*C*a^2 + (5*A + 4*C)*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^2\sec^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**2*sec(c + d*x)**2, x
)
```

Giac [B] time = 1.20978, size = 718, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/60*(15*(4*A*a*b + 3*C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a
*b + 3*C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a^2*tan(1/2*d*x
+ 1/2*c)^9 + 60*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*a*b*tan(1/2*d*x + 1/2*c
)^9 - 75*C*a*b*tan(1/2*d*x + 1/2*c)^9 + 60*A*b^2*tan(1/2*d*x + 1/2*c)^9 + 6
0*C*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 160*C*a
^2*tan(1/2*d*x + 1/2*c)^7 + 120*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 30*C*a*b*tan
(1/2*d*x + 1/2*c)^7 - 160*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 80*C*b^2*tan(1/2*d
*x + 1/2*c)^7 + 360*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 200*C*a^2*tan(1/2*d*x +
1/2*c)^5 + 200*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 232*C*b^2*tan(1/2*d*x + 1/2*c
)^5 - 240*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 160*C*a^2*tan(1/2*d*x + 1/2*c)^3 -
 120*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 30*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 160*A
*b^2*tan(1/2*d*x + 1/2*c)^3 - 80*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 60*A*a^2*ta
n(1/2*d*x + 1/2*c) + 60*C*a^2*tan(1/2*d*x + 1/2*c) + 60*A*a*b*tan(1/2*d*x +
 1/2*c) + 75*C*a*b*tan(1/2*d*x + 1/2*c) + 60*A*b^2*tan(1/2*d*x + 1/2*c) + 6
0*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.647 $\int \sec(c+dx)(a+b\sec(c+dx))^2 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=170

$$\frac{a\left(a^2(-C)+12Ab^2+8b^2C\right)\tan(c+dx)}{6bd}+\frac{\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\left(2a^2C-3b^2(4A+3C)\right)}{2}$$

```
[Out] ((4*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(12*
A*b^2 - a^2*C + 8*b^2*C)*Tan[c + d*x])/(6*b*d) - ((2*a^2*C - 3*b^2*(4*A + 3
*C))*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (a*C*(a + b*Sec[c + d*x])^2*Tan[c
+ d*x])/(12*b*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.308335, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4083, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(a^2(-C)+12Ab^2+8b^2C\right)\tan(c+dx)}{6bd}+\frac{\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\left(2a^2C-3b^2(4A+3C)\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] ((4*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(12*
A*b^2 - a^2*C + 8*b^2*C)*Tan[c + d*x])/(6*b*d) - ((2*a^2*C - 3*b^2*(4*A + 3
*C))*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (a*C*(a + b*Sec[c + d*x])^2*Tan[c
+ d*x])/(12*b*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*b*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4bd}+\frac{\int \sec(c+dx)(a+b\ldots}{}$$
$$= -\frac{aC(a+b\sec(c+dx))^2\tan(c+dx)}{12bd}+\frac{C(a+b\sec(c+\ldots}{4\ldots}$$
$$= -\frac{\left(2a^2C-3b^2(4A+3C)\right)\sec(c+dx)\tan(c+dx)}{24d}-\frac{aC(\ldots}{}$$
$$= -\frac{\left(2a^2C-3b^2(4A+3C)\right)\sec(c+dx)\tan(c+dx)}{24d}-\frac{aC(\ldots}{}$$
$$= \frac{\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(2\ldots}{}$$
$$= \frac{\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\ldots}{}$$

Mathematica [B] time = 6.31943, size = 1123, normalized size = 6.61

$$\frac{\left(-8Aa^2-4Ca^2-4Ab^2-3b^2C\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^2\left(C\sec^2(c+dx)+A\right)\cos\ldots}{4d(b+a\cos(c+dx))^2(\cos(2c+2dx)A+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((-8*a^2*A - 4*A*b^2 - 4*a^2*C - 3*b^2*C)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/(4*d*
(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((8*a^2*A + 4*A*b^
2 + 4*a^2*C + 3*b^2*C)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/(4*d*(b + a*Cos[c + d*x]
)^2*(A + 2*C + A*Cos[2*c + 2*d*x])) + (b^2*C*Cos[c + d*x]^4*(a + b*Sec[c +
d*x])^2*(A + C*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Co
s[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + ((12*A*b^2 + 12*
a^2*C + 8*a*b*C + 9*b^2*C)*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + C*Sec
[c + d*x]^2))/(24*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*(
```

```
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*a*b*C*Cos[c + d*x]^4*(a + b*Se
c[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(3*d*(b + a*Cos[c +
d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^3) - (b^2*C*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))
/(8*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^4) + (2*a*b*C*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*
(A + C*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(3*d*(b + a*Cos[c + d*x])^2*(A + 2
*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-12*A
*b^2 - 12*a^2*C - 8*a*b*C - 9*b^2*C)*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*
(A + C*Sec[c + d*x]^2))/(24*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c +
 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*Cos[c + d*x]^4*(a +
b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*(3*a*A*b*Sin[(c + d*x)/2] + 2*a*b*
C*Sin[(c + d*x)/2]))/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d
*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Cos[c + d*x]^4*(a + b*Sec[
c + d*x])^2*(A + C*Sec[c + d*x]^2)*(3*a*A*b*Sin[(c + d*x)/2] + 2*a*b*C*Sin[
(c + d*x)/2]))/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.044, size = 229, normalized size = 1.4

$$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2C\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^2C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+2\frac{Aab\tan(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/2/d
*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a*b*tan(d*x+c)+4/3/d*a*b*C*tan(d*x+c
)+2/3/d*a*b*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*A*b^2*sec(d*x+c)*tan(d*x+c)+1/2
/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^2*C*tan(d*x+c)*sec(d*x+c)^3+3/8/
d*b^2*C*sec(d*x+c)*tan(d*x+c)+3/8/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.988969, size = 304, normalized size = 1.79

$$32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cab-3Cb^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b - 3*C*b^2*(2*(3*sin(d*x +
c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 48*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 96*A*a*b*tan(d*x +
c))/d
```

Fricas [A] time = 0.537459, size = 424, normalized size = 2.49

$$\frac{3\left(4\left(2A+C\right)a^{2}+\left(4A+3C\right)b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\left(2A+C\right)a^{2}+\left(4A+3C\right)b^{2}\right)\cos\left(dx+c\right)^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(4*(2*A + C)*a^2 + (4*A + 3*C)*b^2)*cos(d*x + c)^4*log(sin(d*x + c)
 + 1) - 3*(4*(2*A + C)*a^2 + (4*A + 3*C)*b^2)*cos(d*x + c)^4*log(-sin(d*x +
 c) + 1) + 2*(16*(3*A + 2*C)*a*b*cos(d*x + c)^3 + 16*C*a*b*cos(d*x + c) + 6
*C*b^2 + 3*(4*C*a^2 + (4*A + 3*C)*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos
(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+C\sec^{2}\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{2}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.21912, size = 575, normalized size = 3.38

$$\frac{3\left(8Aa^{2}+4Ca^{2}+4Ab^{2}+3Cb^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8Aa^{2}+4Ca^{2}+4Ab^{2}+3Cb^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(8*A*a^2 + 4*C*a^2 + 4*A*b^2 + 3*C*b^2)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) - 3*(8*A*a^2 + 4*C*a^2 + 4*A*b^2 + 3*C*b^2)*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) + 2*(12*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*tan(1/2*d*x + 1
/2*c)^7 - 48*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^7
 + 15*C*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 144*
A*a*b*tan(1/2*d*x + 1/2*c)^5 + 80*C*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^2*t
an(1/2*d*x + 1/2*c)^5 + 9*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*C*a^2*tan(1/2*d
*x + 1/2*c)^3 - 144*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 80*C*a*b*tan(1/2*d*x + 1
/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*C*b^2*tan(1/2*d*x + 1/2*c)^3
+ 12*C*a^2*tan(1/2*d*x + 1/2*c) + 48*A*a*b*tan(1/2*d*x + 1/2*c) + 48*C*a*b*
tan(1/2*d*x + 1/2*c) + 12*A*b^2*tan(1/2*d*x + 1/2*c) + 15*C*b^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.648 $\int (a+b\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=103

$$\frac{\left(2C\left(a^2+b^2\right)+3Ab^2\right)\tan(c+dx)}{3d}+a^2Ax+\frac{ab(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{abC\tan(c+dx)\sec(c+dx)}{3d}+\frac{C\tan(c$$

```
[Out] a^2*A*x + (a*b*(2*A + C)*ArcTanh[Sin[c + d*x]])/d + ((3*A*b^2 + 2*(a^2 + b^
2)*C)*Tan[c + d*x])/(3*d) + (a*b*C*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (C*(a
 + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.139301, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4057, 4048, 3770, 3767, 8}

$$\frac{\left(2C\left(a^2+b^2\right)+3Ab^2\right)\tan(c+dx)}{3d}+a^2Ax+\frac{ab(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{abC\tan(c+dx)\sec(c+dx)}{3d}+\frac{C\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a^2*A*x + (a*b*(2*A + C)*ArcTanh[Sin[c + d*x]])/d + ((3*A*b^2 + 2*(a^2 + b^
2)*C)*Tan[c + d*x])/(3*d) + (a*b*C*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (C*(a
 + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\sec(c+dx))\left(3aA+\right.$$
$$=\frac{abC\sec(c+dx)\tan(c+dx)}{3d}+\frac{C(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+$$
$$=a^2Ax+\frac{abC\sec(c+dx)\tan(c+dx)}{3d}+\frac{C(a+b\sec(c+dx))^2\tan(c\cdot}{3d}$$
$$=a^2Ax+\frac{ab(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{abC\sec(c+dx)\tan(c+}{3d}$$
$$=a^2Ax+\frac{ab(2A+C)\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(3Ab^2+2\left(a^2+b^2\right)C\right)t}{3d}$$

Mathematica [B] time = 1.24115, size = 242, normalized size = 2.35

$$\sec^3(c+dx)\left(2\sin(c+dx)\left(\left(3a^2C+3Ab^2+2b^2C\right)\cos(2(c+dx))+3a^2C+6abC\cos(c+dx)+3Ab^2+4b^2C\right)+9a\,c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sec[c + d*x]^3*(9*a*Cos[c + d*x]*(a*A*(c + d*x) - b*(2*A + C)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + b*(2*A + C)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + 3*a*Cos[3*(c + d*x)]*(a*A*(c + d*x) - b*(2*A + C)*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + b*(2*A + C)*Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]]) + 2*(3*A*b^2 + 3*a^2*C + 4*b^2*C + 6*a*b*C*Cos[c + d*x] + (3*A*b^2
 + 3*a^2*C + 2*b^2*C)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

Maple [A] time = 0.045, size = 145, normalized size = 1.4

$$a^2Ax+\frac{Aa^2c}{d}+\frac{a^2C\tan(dx+c)}{d}+2\frac{Aab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{abC\sec(dx+c)\tan(dx+c)}{d}+\frac{abC\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^2*A*x+1/d*A*a^2*c+1/d*a^2*C*tan(d*x+c)+2/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c)
)+a*b*C*sec(d*x+c)*tan(d*x+c)/d+1/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b
^2*tan(d*x+c)+2/3*b^2*C*tan(d*x+c)/d+1/3/d*b^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.988857, size = 174, normalized size = 1.69

$$\frac{6(dx+c)Aa^2+2\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cb^2-3Cab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^2 + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b^2 - 3*C*a*
b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 12*A*a*b*log(sec(d*x + c) + tan(d*x + c)) + 6*C*a^2*tan(d*x
+ c) + 6*A*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.531963, size = 347, normalized size = 3.37

$$\frac{6\,Aa^2dx\cos\left(dx+c\right)^3+3\,(2\,A+C)ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+C)ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*A*a^2*d*x*cos(d*x + c)^3 + 3*(2*A + C)*a*b*cos(d*x + c)^3*log(sin(d*
x + c) + 1) - 3*(2*A + C)*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(3*
C*a*b*cos(d*x + c) + C*b^2 + (3*C*a^2 + (3*A + 2*C)*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+C\sec^2\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.24665, size = 354, normalized size = 3.44

$$\frac{3\,(dx+c)Aa^2+3\,(2\,Aab+Cab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,Aab+Cab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Ca^2\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*A*a^2 + 3*(2*A*a*b + C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 3*(2*A*a*b + C*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*C*a^2*
tan(1/2*d*x + 1/2*c)^5 - 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^2*tan(1/2*d
*x + 1/2*c)^5 + 3*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^2*tan(1/2*d*x + 1/2*
c)^3 - 6*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*
C*a^2*tan(1/2*d*x + 1/2*c) + 3*C*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2
*d*x + 1/2*c) + 3*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
3)/d
```

3.649 $\int \cos(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=109

$$\frac{\left(C\left(2a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2ab(A-C)\tan(c+dx)}{d}+\frac{A\sin(c+dx)(a+b\sec(c+dx))^2}{d}+2aAbx-$$

```
[Out] 2*a*A*b*x + ((2*A*b^2 + (2*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (2*a*b*(A - C)*Tan[c + d*x])/d - (
b^2*(2*A - C)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.164549, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4095, 4048, 3770, 3767, 8}

$$\frac{\left(C\left(2a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2ab(A-C)\tan(c+dx)}{d}+\frac{A\sin(c+dx)(a+b\sec(c+dx))^2}{d}+2aAbx-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] 2*a*A*b*x + ((2*A*b^2 + (2*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (2*a*b*(A - C)*Tan[c + d*x])/d - (
b^2*(2*A - C)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^2\sin(c+dx)}{d} + \int (a+b\sec(c+dx))\,(2$$
$$= \frac{A(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \frac{b^2(2A-C)\sec(c+dx)}{2d}$$
$$= 2aAbx + \frac{A(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \frac{b^2(2A-C)\,\mathrm{se}}{}$$
$$= 2aAbx + \frac{\left(2Ab^2+\left(2a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{A(}{}$$
$$= 2aAbx + \frac{\left(2Ab^2+\left(2a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{A(}{}$$

Mathematica [B] time = 0.87785, size = 352, normalized size = 3.23

$$\sec^2(c+dx)\left(\left(a^2A+2b^2C\right)\sin(c+dx)+\cos(2(c+dx))\left(-\left(C\left(2a^2+b^2\right)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]

[Out] (Sec[c + d*x]^2*(4*a*A*b*c + 4*a*A*b*d*x - 2*A*b^2*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] - 2*a^2*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b^2*C
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*b^2*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 2*a^2*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2*
C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(4*a*A*b*(c +
 d*x) - (2*A*b^2 + (2*a^2 + b^2)*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + (2*A*b^2 + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) +
 (a^2*A + 2*b^2*C)*Sin[c + d*x] + 4*a*b*C*Sin[2*(c + d*x)] + a^2*A*Sin[3*(c
 + d*x)]))/(4*d)
```

Maple [A] time = 0.063, size = 133, normalized size = 1.2

$$\frac{a^2A\sin(dx+c)}{d} + \frac{a^2C\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2aAbx + 2\frac{Aabc}{d} + 2\frac{abC\tan(dx+c)}{d} + \frac{Ab^2\ln(\sec(dx+c)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)

[Out] 1/d*a^2*A*sin(d*x+c)+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2*a*A*b*x+2/d*A*a*
b*c+2/d*a*b*C*tan(d*x+c)+1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*b^2*C*se
c(d*x+c)*tan(d*x+c)+1/2/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.989743, size = 189, normalized size = 1.73

$$\frac{8(dx+c)Aab - Cb^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 2Ca^2(\log(\sin(dx+c)+1) - \log(\sin}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/4*(8*(d*x + c)*A*a*b - C*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*A*a^2*sin(d*x + c) + 8*C*a*b*tan(d*x + c))/d
```

Fricas [A] time = 0.537493, size = 347, normalized size = 3.18

$$\frac{8\,Aabdx\cos\left(dx+c\right)^{2}+\left(2\,Ca^{2}+\left(2\,A+C\right)b^{2}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ca^{2}+\left(2\,A+C\right)b^{2}\right)\cos\left(dx+c\right.}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(8*A*a*b*d*x*cos(d*x + c)^2 + (2*C*a^2 + (2*A + C)*b^2)*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (2*C*a^2 + (2*A + C)*b^2)*cos(d*x + c)^2*log(-sin(d
*x + c) + 1) + 2*(2*A*a^2*cos(d*x + c)^2 + 4*C*a*b*cos(d*x + c) + C*b^2)*si
n(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17603, size = 258, normalized size = 2.37

$$\frac{4\left(dx+c\right)Aab+\frac{4\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\left(2\,Ca^{2}+2\,Ab^{2}+Cb^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ca^{2}+2\,Ab^{2}+Cb^{2}\right)\log\left(\right|}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(d*x + c)*A*a*b + 4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)
^2 + 1) + (2*C*a^2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
(2*C*a^2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(4*C*a*b
*tan(1/2*d*x + 1/2*c)^3 - C*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b*tan(1/2*d*
x + 1/2*c) - C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.650 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=103

$$\frac{1}{2}x\left(a^2(A+2C)+2Ab^2\right)+\frac{aAb\sin(c+dx)}{d}+\frac{A\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{2abC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] ((2*A*b^2 + a^2*(A + 2*C))*x)/2 + (2*a*b*C*ArcTanh[Sin[c + d*x]])/d + (a*A*
b*Sin[c + d*x])/d + (A*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2
*d) - (b^2*(A - 2*C)*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.289607, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4095, 4076, 4047, 8, 4045, 3770}

$$\frac{1}{2}x\left(a^2(A+2C)+2Ab^2\right)+\frac{aAb\sin(c+dx)}{d}+\frac{A\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{2abC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((2*A*b^2 + a^2*(A + 2*C))*x)/2 + (2*a*b*C*ArcTanh[Sin[c + d*x]])/d + (a*A*
b*Sin[c + d*x])/d + (A*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2
*d) - (b^2*(A - 2*C)*Tan[c + d*x])/(2*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{1}{2}\int \mathrm{co}$$
$$= \frac{A\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}-\frac{b^2(A-}{}$$
$$= \frac{A\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}-\frac{b^2(A-}{}$$
$$= \frac{1}{2}\left(2Ab^2+a^2(A+2C)\right)x+\frac{aAb\sin(c+dx)}{d}+\frac{A\cos(c}{}$$
$$= \frac{1}{2}\left(2Ab^2+a^2(A+2C)\right)x+\frac{2abC\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.748399, size = 130, normalized size = 1.26

$$\frac{2(c+dx)\left(a^2(A+2C)+2Ab^2\right)+\tan(c+dx)\left(a^2A\cos(2(c+dx))+a^2A+4b^2C\right)+8aAb\sin(c+dx)-8abC\log\left(\cos\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(2*A*b^2 + a^2*(A + 2*C))*(c + d*x) - 8*a*b*C*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 8*a*b*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*A*b
*Sin[c + d*x] + (a^2*A + 4*b^2*C + a^2*A*Cos[2*(c + d*x)])*Tan[c + d*x])/(4
*d)
```

Maple [A] time = 0.059, size = 120, normalized size = 1.2

$$\frac{a^2A\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2Ax}{2}+\frac{a^2Ac}{2d}+a^2Cx+\frac{Ca^2c}{d}+2\frac{Aab\sin(dx+c)}{d}+2\frac{abC\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*a^2*A*cos(d*x+c)*sin(d*x+c)+1/2*a^2*A*x+1/2/d*A*a^2*c+a^2*C*x+1/d*C*a
^2*c+2*a*A*b*sin(d*x+c)/d+2/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+A*b^2*x+1/d*A
*b^2*c+b^2*C*tan(d*x+c)/d
```

Maxima [A] time = 1.02662, size = 134, normalized size = 1.3

$$\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Aa^2+4\,(dx+c)Ca^2+4\,(dx+c)Ab^2+4\,Cab(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)})}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*C*a^2 + 4*(d*x +
c)*A*b^2 + 4*C*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*A*a*
b*sin(d*x + c) + 4*C*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.5304, size = 309, normalized size = 3.

$$\frac{2\,Cab\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-2\,Cab\cos{(dx+c)}\log{(-\sin{(dx+c)}+1)}+\left((A+2\,C)a^2+2\,Ab^2\right)dx\cos{(dx+}}{2\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*C*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 2*C*a*b*cos(d*x + c)*log(
-sin(d*x + c) + 1) + ((A + 2*C)*a^2 + 2*A*b^2)*d*x*cos(d*x + c) + (A*a^2*co
s(d*x + c)^2 + 4*A*a*b*cos(d*x + c) + 2*C*b^2)*sin(d*x + c))/(d*cos(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19432, size = 236, normalized size = 2.29

$$\frac{4\,Cab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,Cab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\left(Aa^2+2\,Ca^2+2\,Ab^2\right)(dx+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/2*(4*C*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*C*a*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*C*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (A*a^2 + 2*C*a^2 + 2*A*b^2)*(d*x + c) - 2*(A*a^2*tan(1/2*d*x + 1/2*c)^
3 - 4*A*a*b*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x + 1/2*c) - 4*A*a*b*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.651 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=112

$$\frac{\left(a^2(2A+3C)+2Ab^2\right)\sin(c+dx)}{3d}+\frac{aAb\sin(c+dx)\cos(c+dx)}{3d}+\frac{A\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}+abx$$

```
[Out] a*b*(A + 2*C)*x + (b^2*C*ArcTanh[Sin[c + d*x]])/d + ((2*A*b^2 + a^2*(2*A +
3*C))*Sin[c + d*x])/(3*d) + (a*A*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (A*Co
s[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.291247, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4095, 4074, 4047, 8, 4045, 3770}

$$\frac{\left(a^2(2A+3C)+2Ab^2\right)\sin(c+dx)}{3d}+\frac{aAb\sin(c+dx)\cos(c+dx)}{3d}+\frac{A\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*b*(A + 2*C)*x + (b^2*C*ArcTanh[Sin[c + d*x]])/d + ((2*A*b^2 + a^2*(2*A +
3*C))*Sin[c + d*x])/(3*d) + (a*A*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (A*Co
s[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int$$
$$= \frac{aAb\cos(c+dx)\sin(c+dx)}{3d}+\frac{A\cos^2(c+dx)(a+b\sec}{3d}$$
$$= \frac{aAb\cos(c+dx)\sin(c+dx)}{3d}+\frac{A\cos^2(c+dx)(a+b\sec}{3d}$$
$$= ab(A+2C)x+\frac{\left(2Ab^2+a^2(2A+3C)\right)\sin(c+dx)}{3d}+$$
$$= ab(A+2C)x+\frac{b^2C\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(2Ab^2+a^2\right.}{}$$

Mathematica [A] time = 0.252356, size = 144, normalized size = 1.29

$$\frac{3\left(a^2(3A+4C)+4Ab^2\right)\sin(c+dx)+a^2A\sin(3(c+dx))+6aAb\sin(2(c+dx))+12aAbc+12aAbdx+24abcC+24}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (12*a*A*b*c + 24*a*b*c*C + 12*a*A*b*d*x + 24*a*b*C*d*x - 12*b^2*C*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^2*C*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 3*(4*A*b^2 + a^2*(3*A + 4*C))*Sin[c + d*x] + 6*a*A*b*Sin[2*(c +
d*x)] + a^2*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.067, size = 137, normalized size = 1.2

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^2}{3d}+\frac{2a^2A\sin(dx+c)}{3d}+\frac{a^2C\sin(dx+c)}{d}+\frac{Aab\cos(dx+c)\sin(dx+c)}{d}+aAbx+\frac{Aa}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^2+2/3/d*a^2*A*sin(d*x+c)+1/d*a^2*C*sin(d*
x+c)+a*A*b*cos(d*x+c)*sin(d*x+c)/d+a*A*b*x+1/d*A*a*b*c+2*a*b*C*x+2/d*C*a*b*
c+1/d*A*b^2*sin(d*x+c)+1/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.964166, size = 151, normalized size = 1.35

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aab-12\left(dx+c\right)Cab-3\,Cb^{2}\left(\log\left(\sin\left(dx+c\right)+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d*
x + 2*c))*A*a*b - 12*(d*x + c)*C*a*b - 3*C*b^2*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) - 6*C*a^2*sin(d*x + c) - 6*A*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.53749, size = 250, normalized size = 2.23

$$\frac{6\left(A+2\,C\right)abdx+3\,Cb^{2}\log\left(\sin\left(dx+c\right)+1\right)-3\,Cb^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(Aa^{2}\cos\left(dx+c\right)^{2}+3\,Aab\cos\left(dx+c\right)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(6*(A + 2*C)*a*b*d*x + 3*C*b^2*log(sin(d*x + c) + 1) - 3*C*b^2*log(-sin
(d*x + c) + 1) + 2*(A*a^2*cos(d*x + c)^2 + 3*A*a*b*cos(d*x + c) + (2*A + 3*
C)*a^2 + 3*A*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21707, size = 346, normalized size = 3.09

$$\frac{3\,Cb^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,Cb^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(Aab+2\,Cab\right)\left(dx+c\right)+\frac{2\left(3\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/3*(3*C*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*C*b^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(A*a*b + 2*C*a*b)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x
```

```
+ 1/2*c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*a*b*tan(1/2*d*x + 1/2*c)^
5 + 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a
^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^2*tan(1/
2*d*x + 1/2*c) + 3*C*a^2*tan(1/2*d*x + 1/2*c) + 3*A*a*b*tan(1/2*d*x + 1/2*c
) + 3*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.652 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=145

$$\frac{\left(a^2(3A+4C)+2Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2(3A+4C)+4b^2(A+2C)\right)+\frac{2ab(2A+3C)\sin(c+dx)}{3d}+\frac{aAb\,\mathrm{s}}{}$$

```
[Out] ((4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/8 + (2*a*b*(2*A + 3*C)*Sin[c + d*x]
)/(3*d) + ((2*A*b^2 + a^2*(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a*A*b*Cos[c + d*x]^2*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c +
 d*x])^2*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.381924, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4095, 4074, 4047, 2637, 4045, 8}

$$\frac{\left(a^2(3A+4C)+2Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2(3A+4C)+4b^2(A+2C)\right)+\frac{2ab(2A+3C)\sin(c+dx)}{3d}+\frac{aAb\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/8 + (2*a*b*(2*A + 3*C)*Sin[c + d*x]
)/(3*d) + ((2*A*b^2 + a^2*(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a*A*b*Cos[c + d*x]^2*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c +
 d*x])^2*Sin[c + d*x])/(4*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}+\frac{1}{4}\int c$$
$$= \frac{aAb\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{A\cos^3(c+dx)(a+b\,\mathrm{se}}{4}$$
$$= \frac{aAb\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{A\cos^3(c+dx)(a+b\,\mathrm{se}}{4}$$
$$= \frac{2ab(2A+3C)\sin(c+dx)}{3d}+\frac{\left(2Ab^2+a^2(3A+4C)\right)\mathrm{co}}{8d}$$
$$= \frac{1}{8}\left(4b^2(A+2C)+a^2(3A+4C)\right)x+\frac{2ab(2A+3C)\sin}{3d}$$

Mathematica [A] time = 0.391724, size = 104, normalized size = 0.72

$$\frac{12(c+dx)\left(a^2(3A+4C)+4b^2(A+2C)\right)+24\left(a^2(A+C)+Ab^2\right)\sin(2(c+dx))+3a^2A\sin(4(c+dx))+48ab(3A+4}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (12*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*(c + d*x) + 48*a*b*(3*A + 4*C)*Sin[
c + d*x] + 24*(A*b^2 + a^2*(A + C))*Sin[2*(c + d*x)] + 16*a*A*b*Sin[3*(c +
d*x)] + 3*a^2*A*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.071, size = 140, normalized size = 1.

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2Aab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3
*A*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*C*sin(d*x+c)+
```

```
b^2*C*(d*x+c))
```

Maxima [A] time = 0.996142, size = 176, normalized size = 1.21

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Aa^2+24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ca^2-64\left(\sin\left(dx+c\right)^3-3\,\mathrm{si}\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 + 24*
(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x + c
))*A*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 + 96*(d*x + c)*C*b^2 +
 192*C*a*b*sin(d*x + c))/d
```

Fricas [A] time = 0.510309, size = 248, normalized size = 1.71

$$\frac{3\left((3\,A+4\,C)a^2+4\,(A+2\,C)b^2\right)dx+\left(6\,Aa^2\cos\left(dx+c\right)^3+16\,Aab\cos\left(dx+c\right)^2+16\,(2\,A+3\,C)ab+3\left((3\,A+4\,C)a^2\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/24*(3*((3*A + 4*C)*a^2 + 4*(A + 2*C)*b^2)*d*x + (6*A*a^2*cos(d*x + c)^3 +
 16*A*a*b*cos(d*x + c)^2 + 16*(2*A + 3*C)*a*b + 3*((3*A + 4*C)*a^2 + 4*A*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16697, size = 510, normalized size = 3.52

$$\frac{3\left(3\,Aa^2+4\,Ca^2+4\,Ab^2+8\,Cb^2\right)(dx+c)-\frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 4*C*a^2 + 4*A*b^2 + 8*C*b^2)*(d*x + c) - 2*(15*A*a^2*tan
(1/2*d*x + 1/2*c)^7 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*tan(1/2*d*
x + 1/2*c)^7 - 48*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x + 1/2
*c)^7 - 9*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^5 -
80*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 144*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*b
^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^2*tan(1
/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 144*C*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^2*tan(1/2*d*x + 1/2*
c) - 12*C*a^2*tan(1/2*d*x + 1/2*c) - 48*A*a*b*tan(1/2*d*x + 1/2*c) - 48*C*a
*b*tan(1/2*d*x + 1/2*c) - 12*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 + 1)^4)/d
```

3.653 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=161

$$-\frac{\left(a^2(4A+5C)+2Ab^2\right)\sin^3(c+dx)}{15d}+\frac{\left(a^2+b^2\right)(4A+5C)\sin(c+dx)}{5d}+\frac{ab(3A+4C)\sin(c+dx)\cos(c+dx)}{4d}+\frac{aAb\,\mathrm{s}}{}$$

```
[Out] (a*b*(3*A + 4*C)*x)/4 + ((a^2 + b^2)*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (a*b
*(3*A + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*A*b*Cos[c + d*x]^3*Sin[c
 + d*x])/(10*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5
*d) - ((2*A*b^2 + a^2*(4*A + 5*C))*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.399549, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{\left(a^2(4A+5C)+2Ab^2\right)\sin^3(c+dx)}{15d}+\frac{\left(a^2+b^2\right)(4A+5C)\sin(c+dx)}{5d}+\frac{ab(3A+4C)\sin(c+dx)\cos(c+dx)}{4d}+\frac{aAb\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*b*(3*A + 4*C)*x)/4 + ((a^2 + b^2)*(4*A + 5*C)*Sin[c + d*x])/(5*d) + (a*b
*(3*A + 4*C)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*A*b*Cos[c + d*x]^3*Sin[c
 + d*x])/(10*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5
*d) - ((2*A*b^2 + a^2*(4*A + 5*C))*Sin[c + d*x]^3)/(15*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{1}{5}\int c$$
$$= \frac{aAb\cos^3(c+dx)\sin(c+dx)}{10d}+\frac{A\cos^4(c+dx)(a+b\,\mathrm{se}}{5}$$
$$= \frac{aAb\cos^3(c+dx)\sin(c+dx)}{10d}+\frac{A\cos^4(c+dx)(a+b\,\mathrm{se}}{5}$$
$$= \frac{ab(3A+4C)\cos(c+dx)\sin(c+dx)}{4d}+\frac{aAb\cos^3(c+d}{10}$$
$$= \frac{1}{4}ab(3A+4C)x+\frac{ab(3A+4C)\cos(c+dx)\sin(c+dx)}{4d}$$
$$= \frac{1}{4}ab(3A+4C)x+\frac{\left(a^2+b^2\right)(4A+5C)\sin(c+dx)}{5d}+\frac{a}{}$$

Mathematica [A] time = 0.45228, size = 126, normalized size = 0.78

$$\frac{30\left(a^2(5A+6C)+2b^2(3A+4C)\right)\sin(c+dx)+5\left(a^2(5A+4C)+4Ab^2\right)\sin(3(c+dx))+3a^2A\sin(5(c+dx))+60ab}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (60*a*b*(3*A + 4*C)*(c + d*x) + 30*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*Si
n[c + d*x] + 120*a*b*(A + C)*Sin[2*(c + d*x)] + 5*(4*A*b^2 + a^2*(5*A + 4*C
))*Sin[3*(c + d*x)] + 15*a*A*b*Sin[4*(c + d*x)] + 3*a^2*A*Sin[5*(c + d*x)])
/(240*d)
```

Maple [A] time = 0.077, size = 158, normalized size = 1.

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+\frac{a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+2Aab\left(1/4\left((\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/3*a^2*C*(2+
cos(d*x+c)^2)*sin(d*x+c)+2*A*a*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)+3/8*d*x+3/8*c)+2*a*b*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*A*
b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*C*sin(d*x+c))
```

Maxima [A] time = 0.989982, size = 208, normalized size = 1.29

$$\frac{16\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)Aa^{2}-80\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Ca^{2}+15\left(12\,dx+12\,c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 15*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*A*a*b + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C
*a*b - 80*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^2 + 240*C*b^2*sin(d*x + c))
/d
```

Fricas [A] time = 0.513828, size = 300, normalized size = 1.86

$$\frac{15\,(3\,A+4\,C)abdx+\left(12\,Aa^{2}\cos\left(dx+c\right)^{4}+30\,Aab\cos\left(dx+c\right)^{3}+15\,(3\,A+4\,C)ab\cos\left(dx+c\right)+8\,(4\,A+5\,C)a^{2}+20\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/60*(15*(3*A + 4*C)*a*b*d*x + (12*A*a^2*cos(d*x + c)^4 + 30*A*a*b*cos(d*x
+ c)^3 + 15*(3*A + 4*C)*a*b*cos(d*x + c) + 8*(4*A + 5*C)*a^2 + 20*(2*A + 3*
C)*b^2 + 4*((4*A + 5*C)*a^2 + 5*A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19396, size = 672, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/60*(15*(3*A*a*b + 4*C*a*b)*(d*x + c) + 2*(60*A*a^2*tan(1/2*d*x + 1/2*c)^9
 + 60*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*A*a*b*tan(1/2*d*x + 1/2*c)^9 - 60*C
*a*b*tan(1/2*d*x + 1/2*c)^9 + 60*A*b^2*tan(1/2*d*x + 1/2*c)^9 + 60*C*b^2*ta
n(1/2*d*x + 1/2*c)^9 + 80*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 160*C*a^2*tan(1/2*
d*x + 1/2*c)^7 - 30*A*a*b*tan(1/2*d*x + 1/2*c)^7 - 120*C*a*b*tan(1/2*d*x +
1/2*c)^7 + 160*A*b^2*tan(1/2*d*x + 1/2*c)^7 + 240*C*b^2*tan(1/2*d*x + 1/2*c
)^7 + 232*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 200*C*a^2*tan(1/2*d*x + 1/2*c)^5 +
 200*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 360*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 80*A
*a^2*tan(1/2*d*x + 1/2*c)^3 + 160*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 30*A*a*b*t
an(1/2*d*x + 1/2*c)^3 + 120*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 160*A*b^2*tan(1/
2*d*x + 1/2*c)^3 + 240*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 60*A*a^2*tan(1/2*d*x
+ 1/2*c) + 60*C*a^2*tan(1/2*d*x + 1/2*c) + 75*A*a*b*tan(1/2*d*x + 1/2*c) +
60*C*a*b*tan(1/2*d*x + 1/2*c) + 60*A*b^2*tan(1/2*d*x + 1/2*c) + 60*C*b^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.654 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=306

$$\frac{a\left(a^2b^2(30A+17C)+2a^4C+24b^4(5A+4C)\right)\tan(c+dx)}{60b^2d}+\frac{b\left(6a^2(4A+3C)+b^2(6A+5C)\right)\tanh^{-1}(\sin(c+dx))}{16d}+\left(2\right.$$

```
[Out] (b*(6*a^2*(4*A + 3*C) + b^2*(6*A + 5*C))*ArcTanh[Sin[c + d*x]])/(16*d) + (a
*(2*a^4*C + 24*b^4*(5*A + 4*C) + a^2*b^2*(30*A + 17*C))*Tan[c + d*x])/(60*b
^2*d) + ((4*a^4*C + 12*a^2*b^2*(5*A + 3*C) + 15*b^4*(6*A + 5*C))*Sec[c + d*
x]*Tan[c + d*x])/(240*b*d) + (a*(30*A*b^2 + 2*a^2*C + 21*b^2*C)*(a + b*Sec[
c + d*x])^2*Tan[c + d*x])/(120*b^2*d) + ((2*a^2*C + 5*b^2*(6*A + 5*C))*(a +
 b*Sec[c + d*x])^3*Tan[c + d*x])/(120*b^2*d) - (a*C*(a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(15*b^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^4*Tan[c + d
*x])/(6*b*d)
```

Rubi [A] time = 0.719236, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4093, 4082, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(a^2b^2(30A+17C)+2a^4C+24b^4(5A+4C)\right)\tan(c+dx)}{60b^2d}+\frac{b\left(6a^2(4A+3C)+b^2(6A+5C)\right)\tanh^{-1}(\sin(c+dx))}{16d}+\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (b*(6*a^2*(4*A + 3*C) + b^2*(6*A + 5*C))*ArcTanh[Sin[c + d*x]])/(16*d) + (a
*(2*a^4*C + 24*b^4*(5*A + 4*C) + a^2*b^2*(30*A + 17*C))*Tan[c + d*x])/(60*b
^2*d) + ((4*a^4*C + 12*a^2*b^2*(5*A + 3*C) + 15*b^4*(6*A + 5*C))*Sec[c + d*
x]*Tan[c + d*x])/(240*b*d) + (a*(30*A*b^2 + 2*a^2*C + 21*b^2*C)*(a + b*Sec[
c + d*x])^2*Tan[c + d*x])/(120*b^2*d) + ((2*a^2*C + 5*b^2*(6*A + 5*C))*(a +
 b*Sec[c + d*x])^3*Tan[c + d*x])/(120*b^2*d) - (a*C*(a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(15*b^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^4*Tan[c + d
*x])/(6*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec(c+dx)(a+b\sec(c+dx))^4\tan(c+dx)}{6bd} + \frac{\int \sec(\ldots}{\ldots}$$

$$= -\frac{aC(a+b\sec(c+dx))^4\tan(c+dx)}{15b^2d} + \frac{C\sec(c+dx)(a\ldots}{\ldots}$$

$$= \frac{\left(2a^2C+5b^2(6A+5C)\right)(a+b\sec(c+dx))^3\tan(c+dx\ldots}{120b^2d}$$

$$= \frac{a\left(30Ab^2+2a^2C+21b^2C\right)(a+b\sec(c+dx))^2\tan(c+\ldots}{120b^2d}$$

$$= \frac{\left(4a^4C+12a^2b^2(5A+3C)+15b^4(6A+5C)\right)\sec(c+d\ldots}{240bd}$$

$$= \frac{\left(4a^4C+12a^2b^2(5A+3C)+15b^4(6A+5C)\right)\sec(c+d\ldots}{240bd}$$

$$= \frac{b\left(6a^2(4A+3C)+b^2(6A+5C)\right)\tanh^{-1}(\sin(c+dx))}{16d} \ldots$$

$$= \frac{b\left(6a^2(4A+3C)+b^2(6A+5C)\right)\tanh^{-1}(\sin(c+dx))}{16d} \ldots$$

Mathematica [A] time = 3.61659, size = 407, normalized size = 1.33

$$\frac{\sec^6(c+dx)\left(A\cos^2(c+dx)+C\right)\left(2\sin(c+dx)\left(16a\left(a^2(75A+80C)+24b^2(10A+11C)\right)\cos(c+dx)+20b\left(18a^2(4A+\dots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((C + A*Cos[c + d*x]^2)*Sec[c + d*x]^6*(-240*b*(6*a^2*(4*A + 3*C) + b^2*(6*
A + 5*C))*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(1080*a^2*A*b + 510*A*b^3 + 1530*a^
2*b*C + 745*b^3*C + 16*a*(24*b^2*(10*A + 11*C) + a^2*(75*A + 80*C))*Cos[c +
 d*x] + 20*b*(18*a^2*(4*A + 5*C) + 5*b^2*(6*A + 5*C))*Cos[2*(c + d*x)] + 60
0*a^3*A*Cos[3*(c + d*x)] + 1680*a*A*b^2*Cos[3*(c + d*x)] + 560*a^3*C*Cos[3*
(c + d*x)] + 1344*a*b^2*C*Cos[3*(c + d*x)] + 360*a^2*A*b*Cos[4*(c + d*x)] +
 90*A*b^3*Cos[4*(c + d*x)] + 270*a^2*b*C*Cos[4*(c + d*x)] + 75*b^3*C*Cos[4*
(c + d*x)] + 120*a^3*A*Cos[5*(c + d*x)] + 240*a*A*b^2*Cos[5*(c + d*x)] + 80
*a^3*C*Cos[5*(c + d*x)] + 192*a*b^2*C*Cos[5*(c + d*x)])*Sin[c + d*x]))/(192
0*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.051, size = 430, normalized size = 1.4

$$\frac{Aa^3\tan(dx+c)}{d}+\frac{2a^3C\tan(dx+c)}{3d}+\frac{a^3C\tan(dx+c)(\sec(dx+c))^2}{3d}+\frac{3Aa^2b\sec(dx+c)\tan(dx+c)}{2d}+\frac{3Aa^2b\ln\dots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^3*tan(d*x+c)+2/3*a^3*C*tan(d*x+c)/d+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+
c)^2+3/2/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(d*x+c)+tan(d*
x+c))+3/4/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)^3+9/8/d*a^2*b*C*sec(d*x+c)*tan(d*
x+c)+9/8/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a*b^2*tan(d*x+c)+1/d*A*a
*b^2*tan(d*x+c)*sec(d*x+c)^2+8/5/d*C*a*b^2*tan(d*x+c)+3/5/d*C*a*b^2*tan(d*x
+c)*sec(d*x+c)^4+4/5/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*A*b^3*tan(d*x+
c)*sec(d*x+c)^3+3/8/d*A*b^3*sec(d*x+c)*tan(d*x+c)+3/8/d*A*b^3*ln(sec(d*x+c)
+tan(d*x+c))+1/6/d*C*b^3*tan(d*x+c)*sec(d*x+c)^5+5/24/d*C*b^3*tan(d*x+c)*se
c(d*x+c)^3+5/16/d*C*b^3*sec(d*x+c)*tan(d*x+c)+5/16/d*C*b^3*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 1.02067, size = 521, normalized size = 1.7

$$\frac{160\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^3+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab^2+96\left(3\tan(dx+c)^5+10\tan(dx\dots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/480*(160*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 + 480*(tan(d*x + c)^3 +
3*tan(d*x + c))*A*a*b^2 + 96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan
```

```
(d*x + c))*C*a*b^2 - 5*C*b^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33
*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) -
 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 90*C*a^2*b*(2*(3*si
n(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*
log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*A*b^3*(2*(3*sin(d*x +
 c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin
(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 360*A*a^2*b*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*A*
a^3*tan(d*x + c))/d
```

Fricas [A] time = 0.576212, size = 636, normalized size = 2.08

$$\frac{15\left(6(4A+3C)a^2b+(6A+5C)b^3\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left(6(4A+3C)a^2b+(6A+5C)b^3\right)\cos(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/480*(15*(6*(4*A + 3*C)*a^2*b + (6*A + 5*C)*b^3)*cos(d*x + c)^6*log(sin(d*
x + c) + 1) - 15*(6*(4*A + 3*C)*a^2*b + (6*A + 5*C)*b^3)*cos(d*x + c)^6*log
(-sin(d*x + c) + 1) + 2*(16*(5*(3*A + 2*C)*a^3 + 6*(5*A + 4*C)*a*b^2)*cos(d
*x + c)^5 + 144*C*a*b^2*cos(d*x + c) + 15*(6*(4*A + 3*C)*a^2*b + (6*A + 5*C
)*b^3)*cos(d*x + c)^4 + 40*C*b^3 + 16*(5*C*a^3 + 3*(5*A + 4*C)*a*b^2)*cos(d
*x + c)^3 + 10*(18*C*a^2*b + (6*A + 5*C)*b^3)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2(c + dx)\right)(a + b\sec(c + dx))^3\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**3*sec(c + d*x)**2, x
)
```

Giac [B] time = 1.27429, size = 1258, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(15*(24*A*a^2*b + 18*C*a^2*b + 6*A*b^3 + 5*C*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 15*(24*A*a^2*b + 18*C*a^2*b + 6*A*b^3 + 5*C*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*(240*A*a^3*tan(1/2*d*x + 1/2*c)^11 + 240*C*a
```

^3*tan(1/2*d*x + 1/2*c)^11 - 360*A*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 450*C*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 720*A*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 150*A*b^3*tan(1/2*d*x + 1/2*c)^11 - 165*C*b^3*tan(1/2*d*x + 1/2*c)^11 - 1200*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 880*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 1080*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 630*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 2640*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 1680*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 210*A*b^3*tan(1/2*d*x + 1/2*c)^9 - 25*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 2400*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 720*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 180*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4320*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 3744*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 450*C*b^3*tan(1/2*d*x + 1/2*c)^7 - 2400*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 1440*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 720*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 180*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 4320*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3744*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 880*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 1080*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 630*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 2640*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 1680*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 210*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 25*C*b^3*tan(1/2*d*x + 1/2*c)^3 - 240*A*a^3*tan(1/2*d*x + 1/2*c) - 240*C*a^3*tan(1/2*d*x + 1/2*c) - 360*A*a^2*b*tan(1/2*d*x + 1/2*c) - 450*C*a^2*b*tan(1/2*d*x + 1/2*c) - 720*A*a*b^2*tan(1/2*d*x + 1/2*c) - 720*C*a*b^2*tan(1/2*d*x + 1/2*c) - 150*A*b^3*tan(1/2*d*x + 1/2*c) - 165*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

3.655 $\int \sec(c+dx)(a+b\sec(c+dx))^3 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=234

$$-\frac{\left(-4a^2b^2(20A+13C)+3a^4C-4b^4(5A+4C)\right)\tan(c+dx)}{30bd}+\frac{a\left(4a^2(2A+C)+3b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (a*(4*a^2*(2*A + C) + 3*b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) - ((3
*a^4*C - 4*b^4*(5*A + 4*C) - 4*a^2*b^2*(20*A + 13*C))*Tan[c + d*x])/(30*b*d
) + (a*(100*A*b^2 - 6*a^2*C + 71*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(120*d)
- ((3*a^2*C - 4*b^2*(5*A + 4*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b
*d) - (a*C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + (C*(a + b*Sec[c
+ d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.488304, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4083, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{\left(-4a^2b^2(20A+13C)+3a^4C-4b^4(5A+4C)\right)\tan(c+dx)}{30bd}+\frac{a\left(4a^2(2A+C)+3b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(4*a^2*(2*A + C) + 3*b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) - ((3
*a^4*C - 4*b^4*(5*A + 4*C) - 4*a^2*b^2*(20*A + 13*C))*Tan[c + d*x])/(30*b*d
) + (a*(100*A*b^2 - 6*a^2*C + 71*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(120*d)
- ((3*a^2*C - 4*b^2*(5*A + 4*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b
*d) - (a*C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + (C*(a + b*Sec[c
+ d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
```

```
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^4\tan(c+dx)}{5bd}+\frac{\int \sec(c+dx)(a+b\sec}{}$$

$$= -\frac{aC(a+b\sec(c+dx))^3\tan(c+dx)}{20bd}+\frac{C(a+b\sec(c+dx)}{5bd}$$

$$= -\frac{\left(3a^2C-4b^2(5A+4C)\right)(a+b\sec(c+dx))^2\tan(c+dx)}{60bd}$$

$$= \frac{a\left(100Ab^2-6a^2C+71b^2C\right)\sec(c+dx)\tan(c+dx)}{120d}-\frac{(3a^2}{}$$

$$= \frac{a\left(100Ab^2-6a^2C+71b^2C\right)\sec(c+dx)\tan(c+dx)}{120d}-\frac{(3a^2}{}$$

$$= \frac{a\left(4a^2(2A+C)+3b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(}{}$$

$$= \frac{a\left(4a^2(2A+C)+3b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(3}{}$$

Mathematica [A] time = 1.88146, size = 324, normalized size = 1.38

$$-\frac{\sec^5(c+dx)\left(A\cos^2(c+dx)+C\right)\left(120a\left(4a^2(2A+C)+3b^2(4A+3C)\right)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -((C + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(120*a*(4*a^2*(2*A + C) + 3*b^2*(4*
A + 3*C))*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2*(540*a^2*A*b + 200*A*b^3 + 600*a^2*
b*C + 256*b^3*C + 45*a*(12*A*b^2 + 4*a^2*C + 17*b^2*C)*Cos[c + d*x] + 48*b*
(15*a^2*(A + C) + b^2*(5*A + 4*C))*Cos[2*(c + d*x)] + 180*a*A*b^2*Cos[3*(c
```

```
+ d*x)] + 60*a^3*C*Cos[3*(c + d*x)] + 135*a*b^2*C*Cos[3*(c + d*x)] + 180*a^
2*A*b*Cos[4*(c + d*x)] + 40*A*b^3*Cos[4*(c + d*x)] + 120*a^2*b*C*Cos[4*(c +
 d*x)] + 32*b^3*C*Cos[4*(c + d*x)])*Sin[c + d*x]))/(480*d*(A + 2*C + A*Cos[
2*(c + d*x)]))
```

Maple [A] time = 0.056, size = 338, normalized size = 1.4

$$\frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3C\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3\frac{Aa^2b\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+1/2/d
*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^2*b*tan(d*x+c)+2/d*a^2*b*C*tan(d*x
+c)+1/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)^2+3/2/d*A*a*b^2*sec(d*x+c)*tan(d*x+c)
+3/2/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*C*a*b^2*tan(d*x+c)*sec(d*x+c
)^3+9/8/d*C*a*b^2*sec(d*x+c)*tan(d*x+c)+9/8/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x
+c))+2/3/d*A*b^3*tan(d*x+c)+1/3/d*A*b^3*tan(d*x+c)*sec(d*x+c)^2+8/15/d*C*b^
3*tan(d*x+c)+1/5/d*C*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*C*b^3*tan(d*x+c)*se
c(d*x+c)^2
```

Maxima [A] time = 1.00404, size = 390, normalized size = 1.67

$$\frac{240\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^2b+80\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ab^3+16\left(3\,\tan(dx+c)^5+10\,\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b + 80*(tan(d*x + c)^3 +
 3*tan(d*x + c))*A*b^3 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(
d*x + c))*C*b^3 - 45*C*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 60*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 180*A*a*b^2*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*log(s
ec(d*x + c) + tan(d*x + c)) + 720*A*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 0.553672, size = 552, normalized size = 2.36

$$\frac{15\left(4(2A+C)a^3+3(4A+3C)ab^2\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(4(2A+C)a^3+3(4A+3C)ab^2\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*(2*A + C)*a^3 + 3*(4*A + 3*C)*a*b^2)*cos(d*x + c)^5*log(sin(d*
x + c) + 1) - 15*(4*(2*A + C)*a^3 + 3*(4*A + 3*C)*a*b^2)*cos(d*x + c)^5*log
(-sin(d*x + c) + 1) + 2*(90*C*a*b^2*cos(d*x + c) + 8*(15*(3*A + 2*C)*a^2*b
+ 2*(5*A + 4*C)*b^3)*cos(d*x + c)^4 + 24*C*b^3 + 15*(4*C*a^3 + 3*(4*A + 3*C
)*a*b^2)*cos(d*x + c)^3 + 8*(15*C*a^2*b + (5*A + 4*C)*b^3)*cos(d*x + c)^2)*
sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^3\sec\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] time = 1.24651, size = 886, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(8*A*a^3 + 4*C*a^3 + 12*A*a*b^2 + 9*C*a*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(8*A*a^3 + 4*C*a^3 + 12*A*a*b^2 + 9*C*a*b^2)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*(60*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 360*A*a^2*b
*tan(1/2*d*x + 1/2*c)^9 - 360*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 180*A*a*b^2*
tan(1/2*d*x + 1/2*c)^9 + 225*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 120*A*b^3*tan
(1/2*d*x + 1/2*c)^9 - 120*C*b^3*tan(1/2*d*x + 1/2*c)^9 - 120*C*a^3*tan(1/2*
d*x + 1/2*c)^7 + 1440*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 960*C*a^2*b*tan(1/2*
d*x + 1/2*c)^7 - 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 90*C*a*b^2*tan(1/2*d*
x + 1/2*c)^7 + 320*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 160*C*b^3*tan(1/2*d*x + 1
/2*c)^7 - 2160*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1200*C*a^2*b*tan(1/2*d*x +
1/2*c)^5 - 400*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 464*C*b^3*tan(1/2*d*x + 1/2*c
)^5 + 120*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 1440*A*a^2*b*tan(1/2*d*x + 1/2*c)^
3 + 960*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^3
 + 90*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 320*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
60*C*b^3*tan(1/2*d*x + 1/2*c)^3 - 60*C*a^3*tan(1/2*d*x + 1/2*c) - 360*A*a^2
*b*tan(1/2*d*x + 1/2*c) - 360*C*a^2*b*tan(1/2*d*x + 1/2*c) - 180*A*a*b^2*ta
n(1/2*d*x + 1/2*c) - 225*C*a*b^2*tan(1/2*d*x + 1/2*c) - 120*A*b^3*tan(1/2*d
*x + 1/2*c) - 120*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
5)/d
```

3.656 $\int (a+b\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=167

$$\frac{a\left(C\left(a^2+4b^2\right)+6Ab^2\right)\tan(c+dx)}{2d}+\frac{b\left(12a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2C+b^2(4A+3C\right.}{}$$

```
[Out] a^3*A*x + (b*(12*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8
*d) + (a*(6*A*b^2 + (a^2 + 4*b^2)*C)*Tan[c + d*x])/(2*d) + (b*(2*a^2*C + b^
2*(4*A + 3*C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*(a + b*Sec[c + d*x])
^2*Tan[c + d*x])/(4*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.313253, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4057, 4056, 4048, 3770, 3767, 8}

$$\frac{a\left(C\left(a^2+4b^2\right)+6Ab^2\right)\tan(c+dx)}{2d}+\frac{b\left(12a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2C+b^2(4A+3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a^3*A*x + (b*(12*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8
*d) + (a*(6*A*b^2 + (a^2 + 4*b^2)*C)*Tan[c + d*x])/(2*d) + (b*(2*a^2*C + b^
2*(4*A + 3*C))*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*C*(a + b*Sec[c + d*x])
^2*Tan[c + d*x])/(4*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx &= \frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\sec(c+dx))^2\left(4aA+b(\right.\\ &= \frac{aC(a+b\sec(c+dx))^2\tan(c+dx)}{4d}+\frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4d}\\ &= \frac{b\left(2a^2C+b^2(4A+3C)\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aC(a+b\sec(c+dx)}{4d}\\ &= a^3Ax+\frac{b\left(2a^2C+b^2(4A+3C)\right)\sec(c+dx)\tan(c+dx)}{8d}+aC(a+b\,\mathrm{se}\\ &= a^3Ax+\frac{b\left(12a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2\right.}{}\\ &= a^3Ax+\frac{b\left(12a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(6A\right.}{}\end{aligned}$$

Mathematica [B] time = 6.41372, size = 1241, normalized size = 7.43

$$\frac{\left(-4Ab^3-3Cb^3-24a^2Ab-12a^2Cb\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3\left(C\sec^2(c+dx)+A\right)\mathrm{co}}{4d(b+a\cos(c+dx))^3(\cos(2c+2dx)A+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a^3*A*(c + d*x)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x
]^2))/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((-24*a^2
*A*b - 4*A*b^3 - 12*a^2*b*C - 3*b^3*C)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(4*d*(b
+ a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((24*a^2*A*b + 4*A*b^
3 + 12*a^2*b*C + 3*b^3*C)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/(4*d*(b + a*Cos[c + d
*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])) + (b^3*C*Cos[c + d*x]^5*(a + b*Sec[c
 + d*x])^3*(A + C*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A
*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + ((4*A*b^3 + 1
2*a^2*b*C + 4*a*b^2*C + 3*b^3*C)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A +
 C*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*
x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (a*b^2*C*Cos[c + d*x]^5*(a +
 b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(d*(b + a*Cos[c
 + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3) - (b^3*C*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^
```

```
2))/(8*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^4) + (a*b^2*C*Cos[c + d*x]^5*(a + b*Sec[c + d*x])
^3*(A + C*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(A +
2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-4*A
*b^3 - 12*a^2*b*C - 4*a*b^2*C - 3*b^3*C)*Cos[c + d*x]^5*(a + b*Sec[c + d*x]
)^3*(A + C*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*
c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*Cos[c + d*x]^5*(a
 + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*(3*a*A*b^2*Sin[(c + d*x)/2] + a
^3*C*Sin[(c + d*x)/2] + 2*a*b^2*C*Sin[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x]
)^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) +
 (2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*(3*a*A*b^2
*Sin[(c + d*x)/2] + a^3*C*Sin[(c + d*x)/2] + 2*a*b^2*C*Sin[(c + d*x)/2]))/(
d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))
```

Maple [A] time = 0.049, size = 267, normalized size = 1.6

$$a^3Ax+\frac{Aa^3c}{d}+\frac{a^3C\tan(dx+c)}{d}+3\frac{Aa^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^2bC\sec(dx+c)\tan(dx+c)}{2d}+\frac{3a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*x+1/d*A*a^3*c+a^3*C*tan(d*x+c)/d+3/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c)
)+3/2/d*a^2*b*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c
))+3/d*A*a*b^2*tan(d*x+c)+2/d*C*a*b^2*tan(d*x+c)+1/d*C*a*b^2*tan(d*x+c)*sec
(d*x+c)^2+1/2/d*A*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d
*x+c))+1/4/d*C*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*C*b^3*sec(d*x+c)*tan(d*x+c
)+3/8/d*C*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.963603, size = 343, normalized size = 2.05

$$16(dx+c)Aa^3+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cab^2-Cb^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*(d*x + c)*A*a^3 + 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b^2 - C
*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c
)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*C*a^2*b*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)) - 4*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^2*b*log(sec(d*x + c) + tan(d*x + c
)) + 16*C*a^3*tan(d*x + c) + 48*A*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.557219, size = 485, normalized size = 2.9

$$16Aa^3dx\cos(dx+c)^4+\left(12(2A+C)a^2b+(4A+3C)b^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-\left(12(2A+C)a^2b+(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(16*A*a^3*d*x*cos(d*x + c)^4 + (12*(2*A + C)*a^2*b + (4*A + 3*C)*b^3)*
cos(d*x + c)^4*log(sin(d*x + c) + 1) - (12*(2*A + C)*a^2*b + (4*A + 3*C)*b^
3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*C*a*b^2*cos(d*x + c) + 2*C*
b^3 + 8*(C*a^3 + (3*A + 2*C)*a*b^2)*cos(d*x + c)^3 + (12*C*a^2*b + (4*A + 3
*C)*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.25045, size = 710, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)*A*a^3 + (24*A*a^2*b + 12*C*a^2*b + 4*A*b^3 + 3*C*b^3)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - (24*A*a^2*b + 12*C*a^2*b + 4*A*b^3 + 3*C*b
^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*C*a^3*tan(1/2*d*x + 1/2*c)^7
- 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 2
4*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 4*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*b^3
*tan(1/2*d*x + 1/2*c)^7 - 24*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*b*tan(
1/2*d*x + 1/2*c)^5 - 72*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 40*C*a*b^2*tan(1/2
*d*x + 1/2*c)^5 + 4*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*C*b^3*tan(1/2*d*x + 1/
2*c)^5 + 24*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^2*b*tan(1/2*d*x + 1/2*c)^
3 + 72*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 40*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 +
 4*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*C*a^3*
tan(1/2*d*x + 1/2*c) - 12*C*a^2*b*tan(1/2*d*x + 1/2*c) - 24*A*a*b^2*tan(1/2
*d*x + 1/2*c) - 24*C*a*b^2*tan(1/2*d*x + 1/2*c) - 4*A*b^3*tan(1/2*d*x + 1/2
*c) - 5*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.657 $\int \cos(c+dx)(a+b\sec(c+dx))^3 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=167

$$-\frac{b\left(a^2(6A-8C)-b^2(3A+2C)\right)\tan(c+dx)}{3d}+\frac{a\left(2a^2C+6Ab^2+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+3a^2Abx-\frac{ab^2(6A-$$

```
[Out] 3*a^2*A*b*x + (a*(6*A*b^2 + 2*a^2*C + 3*b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d)
 + (A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d - (b*(a^2*(6*A - 8*C) - b^2*(3
*A + 2*C))*Tan[c + d*x])/(3*d) - (a*b^2*(6*A - 5*C)*Sec[c + d*x]*Tan[c + d*
x])/(6*d) - (b*(3*A - C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.312066, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4095, 4056, 4048, 3770, 3767, 8}

$$-\frac{b\left(a^2(6A-8C)-b^2(3A+2C)\right)\tan(c+dx)}{3d}+\frac{a\left(2a^2C+6Ab^2+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+3a^2Abx-\frac{ab^2(6A-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] 3*a^2*A*b*x + (a*(6*A*b^2 + 2*a^2*C + 3*b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d)
 + (A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d - (b*(a^2*(6*A - 8*C) - b^2*(3
*A + 2*C))*Tan[c + d*x])/(3*d) - (a*b^2*(6*A - 5*C)*Sec[c + d*x]*Tan[c + d*
x])/(6*d) - (b*(3*A - C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} + \int (a+b\sec(c+dx))^2 ($$

$$= \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b(3A-C)(a+b\sec(c}{3d}$$

$$= \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{ab^2(6A-5C)\sec(c+}{6d}$$

$$= 3a^2Abx + \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{ab^2(6A-5C}{}$$

$$= 3a^2Abx + \frac{a\left(6Ab^2+2a^2C+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d} +$$

$$= 3a^2Abx + \frac{a\left(6Ab^2+2a^2C+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d} +$$

Mathematica [A] time = 1.61292, size = 325, normalized size = 1.95

$$\frac{\sec^3(c+dx)\left(2\sin(c+dx)\left(9a\left(a^2A+2b^2C\right)\cos(c+dx)+2\left(9a^2bC+3Ab^3+2b^3C\right)\cos(2(c+dx))+3a^3A\cos(3(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sec[c + d*x]^3*(9*a*Cos[c + d*x]*(6*a*A*b*(c + d*x) - (6*A*b^2 + 2*a^2*C +
 3*b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + (6*A*b^2 + 2*a^2*C + 3
*b^2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*a*Cos[3*(c + d*x)]*(6
*a*A*b*(c + d*x) - (6*A*b^2 + 2*a^2*C + 3*b^2*C)*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + (6*A*b^2 + 2*a^2*C + 3*b^2*C)*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]) + 2*(6*A*b^3 + 18*a^2*b*C + 8*b^3*C + 9*a*(a^2*A + 2*b^2*C)*C
os[c + d*x] + 2*(3*A*b^3 + 9*a^2*b*C + 2*b^3*C)*Cos[2*(c + d*x)] + 3*a^3*A*
Cos[3*(c + d*x)])*Sin[c + d*x]))/(24*d)
```

Maple [A] time = 0.069, size = 195, normalized size = 1.2

$$\frac{Aa^3\sin(dx+c)}{d} + \frac{a^3C\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3a^2Abx + 3\frac{Aa^2bc}{d} + 3\frac{a^2bC\tan(dx+c)}{d} + 3\frac{Aab^2\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*sin(d*x+c)/d+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*A*b*x+3/d*A*a^
2*b*c+3/d*a^2*b*C*tan(d*x+c)+3/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*C*
a*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b
^3*tan(d*x+c)+2/3/d*C*b^3*tan(d*x+c)+1/3/d*C*b^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.970288, size = 244, normalized size = 1.46

$$\frac{36 (dx + c)Aa^2b + 4\left(\tan (dx + c)^3 + 3 \tan (dx + c)\right)Cb^3 - 9 Cab^2\left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log (\sin (dx + c) + 1) + \log (\sin (dx +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(36*(d*x + c)*A*a^2*b + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b^3 - 9*
C*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 18*A*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^3*s
in(d*x + c) + 36*C*a^2*b*tan(d*x + c) + 12*A*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.555508, size = 440, normalized size = 2.63

$$\frac{36 Aa^2bdx \cos (dx + c)^3 + 3\left(2 Ca^3 + 3 (2 A + C)ab^2\right) \cos (dx + c)^3 \log (\sin (dx + c) + 1) - 3\left(2 Ca^3 + 3 (2 A + C)ab^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(36*A*a^2*b*d*x*cos(d*x + c)^3 + 3*(2*C*a^3 + 3*(2*A + C)*a*b^2)*cos(d
*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*C*a^3 + 3*(2*A + C)*a*b^2)*cos(d*x +
 c)^3*log(-sin(d*x + c) + 1) + 2*(6*A*a^3*cos(d*x + c)^3 + 9*C*a*b^2*cos(d*
x + c) + 2*C*b^3 + 2*(9*C*a^2*b + (3*A + 2*C)*b^3)*cos(d*x + c)^2)*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24784, size = 435, normalized size = 2.6

$$18\,(dx+c)Aa^2b+\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+3\left(2\,Ca^3+6\,Aab^2+3\,Cab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Ca^3+6\,Aab^2+3\,C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/6*(18*(d*x + c)*A*a^2*b + 12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/
2*c)^2 + 1) + 3*(2*C*a^3 + 6*A*a*b^2 + 3*C*a*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 3*(2*C*a^3 + 6*A*a*b^2 + 3*C*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - 2*(18*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 9*C*a*b^2*tan(1/2*d*x + 1/
2*c)^5 + 6*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^3*tan(1/2*d*x + 1/2*c)^5 -
36*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*C*b
^3*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 9*C*a*b^2*tan
(1/2*d*x + 1/2*c) + 6*A*b^3*tan(1/2*d*x + 1/2*c) + 6*C*b^3*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.658 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=168

$$\frac{b\left(C\left(6a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2(A+2C)+6Ab^2\right)-\frac{3ab^2(3A-2C)\tan(c+dx)}{2d}+\frac{3Ab\sin(c}{}$$

```
[Out] (a*(6*A*b^2 + a^2*(A + 2*C))*x)/2 + (b*(2*A*b^2 + (6*a^2 + b^2)*C)*ArcTanh[
Sin[c + d*x]])/(2*d) + (3*A*b*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) +
(A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(2*d) - (3*a*b^2*(3*A
- 2*C)*Tan[c + d*x])/(2*d) - (b^3*(4*A - C)*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Rubi [A] time = 0.393104, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4095, 4094, 4048, 3770, 3767, 8}

$$\frac{b\left(C\left(6a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2(A+2C)+6Ab^2\right)-\frac{3ab^2(3A-2C)\tan(c+dx)}{2d}+\frac{3Ab\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*(6*A*b^2 + a^2*(A + 2*C))*x)/2 + (b*(2*A*b^2 + (6*a^2 + b^2)*C)*ArcTanh[
Sin[c + d*x]])/(2*d) + (3*A*b*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) +
(A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(2*d) - (3*a*b^2*(3*A
- 2*C)*Tan[c + d*x])/(2*d) - (b^3*(4*A - C)*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{2d}+\frac{1}{2}\int \cos($$
$$= \frac{3Ab(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{A\cos(c+dx)(a+b}{}$$
$$= \frac{3Ab(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{A\cos(c+dx)(a+b}{}$$
$$= \frac{1}{2}a\left(6Ab^2+a^2(A+2C)\right)x+\frac{3Ab(a+b\sec(c+dx))^2\sin(}{2d}$$
$$= \frac{1}{2}a\left(6Ab^2+a^2(A+2C)\right)x+\frac{b\left(2Ab^2+\left(6a^2+b^2\right)C\right)\tan}{2d}$$
$$= \frac{1}{2}a\left(6Ab^2+a^2(A+2C)\right)x+\frac{b\left(2Ab^2+\left(6a^2+b^2\right)C\right)\tan}{2d}$$

Mathematica [A] time = 1.91745, size = 287, normalized size = 1.71

$$\frac{2a(c+dx)\left(a^2(A+2C)+6Ab^2\right)-2b\left(C\left(6a^2+b^2\right)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\left(C\left(6a^2+b^2\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(6*A*b^2 + a^2*(A + 2*C))*(c + d*x) - 2*b*(2*A*b^2 + (6*a^2 + b^2)*C)*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*b*(2*A*b^2 + (6*a^2 + b^2)*C)*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*C)/(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2 + (12*a*b^2*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) - (b^3*C)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (12*a*b^2*C*
Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 12*a^2*A*b*Sin[c
+ d*x] + a^3*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.07, size = 196, normalized size = 1.2

$$\frac{Aa^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{a^3Ax}{2}+\frac{Aa^3c}{2d}+a^3Cx+\frac{Ca^3c}{d}+3\frac{Aa^2b\sin(dx+c)}{d}+3\frac{a^2bC\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^3*sin(d*x+c)*cos(d*x+c)+1/2*a^3*A*x+1/2/d*A*a^3*c+a^3*C*x+1/d*C*a
^3*c+3/d*A*a^2*b*sin(d*x+c)+3/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+3*A*a*b^2
*x+3/d*A*a*b^2*c+3/d*C*a*b^2*tan(d*x+c)+1/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))
+1/2/d*C*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*C*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01732, size = 242, normalized size = 1.44

$$(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^3 + 4\,(dx + c)Ca^3 + 12\,(dx + c)Aab^2 - Cb^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 4*(d*x + c)*C*a^3 + 12*(d*x +
 c)*A*a*b^2 - C*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
 + 1) + log(sin(d*x + c) - 1)) + 6*C*a^2*b*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) + 2*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
 12*A*a^2*b*sin(d*x + c) + 12*C*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.558954, size = 419, normalized size = 2.49

$$2\left((A + 2\,C)a^3 + 6\,Aab^2\right)dx\cos(dx + c)^2 + \left(6\,Ca^2b + (2\,A + C)b^3\right)\cos(dx + c)^2\log(\sin(dx + c) + 1) - \left(6\,Ca^2b + (2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*((A + 2*C)*a^3 + 6*A*a*b^2)*d*x*cos(d*x + c)^2 + (6*C*a^2*b + (2*A +
 C)*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (6*C*a^2*b + (2*A + C)*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^3*cos(d*x + c)^3 + 6*A*a^2*
b*cos(d*x + c)^2 + 6*C*a*b^2*cos(d*x + c) + C*b^3)*sin(d*x + c))/(d*cos(d*x
 + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24466, size = 522, normalized size = 3.11

$$\left(Aa^3 + 2\,Ca^3 + 6\,Aab^2\right)(dx + c) + \left(6\,Ca^2b + 2\,Ab^3 + Cb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(6\,Ca^2b + 2\,Ab^3 + Cb^3\right)\log\left(\left|\mathrm{ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/2*((A*a^3 + 2*C*a^3 + 6*A*a*b^2)*(d*x + c) + (6*C*a^2*b + 2*A*b^3 + C*b^3
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (6*C*a^2*b + 2*A*b^3 + C*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^3*tan(1/2*d*x + 1/2*c)^7 - 6*A*a^2*b*
tan(1/2*d*x + 1/2*c)^7 + 6*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - C*b^3*tan(1/2*d
*x + 1/2*c)^7 - 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b*tan(1/2*d*x + 1/
2*c)^5 + 6*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*C*b^3*tan(1/2*d*x + 1/2*c)^5
+ 3*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 6*C*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*b^3*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/
2*d*x + 1/2*c) - 6*A*a^2*b*tan(1/2*d*x + 1/2*c) - 6*C*a*b^2*tan(1/2*d*x + 1
/2*c) - C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.659 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=163

$$\frac{a\left(a^2(2A+3C)+3Ab^2\right)\sin(c+dx)}{3d}+\frac{1}{2}bx\left(3a^2(A+2C)+2Ab^2\right)+\frac{A\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^3}{3d}+$$

```
[Out] (b*(2*A*b^2 + 3*a^2*(A + 2*C))*x)/2 + (3*a*b^2*C*ArcTanh[Sin[c + d*x]])/d +
 (a*(3*A*b^2 + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*d) + (A*b*Cos[c + d*x]*(a
+ b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c +
d*x])^3*Sin[c + d*x])/(3*d) - (b^3*(5*A - 6*C)*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.523604, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4076, 4047, 8, 4045, 3770}

$$\frac{a\left(a^2(2A+3C)+3Ab^2\right)\sin(c+dx)}{3d}+\frac{1}{2}bx\left(3a^2(A+2C)+2Ab^2\right)+\frac{A\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^3}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (b*(2*A*b^2 + 3*a^2*(A + 2*C))*x)/2 + (3*a*b^2*C*ArcTanh[Sin[c + d*x]])/d +
 (a*(3*A*b^2 + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*d) + (A*b*Cos[c + d*x]*(a
+ b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c +
d*x])^3*Sin[c + d*x])/(3*d) - (b^3*(5*A - 6*C)*Tan[c + d*x])/(6*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
```

```
!LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{3d}+\frac{1}{3}\int \cos$$

$$= \frac{Ab\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{A\cos^2(c}{}$$

$$= \frac{Ab\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{A\cos^2(c}{}$$

$$= \frac{Ab\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{A\cos^2(c}{}$$

$$= \frac{1}{2}b\left(2Ab^2+3a^2(A+2C)\right)x+\frac{a\left(3Ab^2+a^2(2A+3C)\right)\sin}{3d}$$

$$= \frac{1}{2}b\left(2Ab^2+3a^2(A+2C)\right)x+\frac{3ab^2C\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.911944, size = 184, normalized size = 1.13

$$\frac{3a\left(a^2(3A+4C)+12Ab^2\right)\sin(c+dx)+9a^2Ab\sin(2(c+dx))+18a^2Abc+18a^2Abdx+a^3A\sin(3(c+dx))+36a^2bcC+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (18*a^2*A*b*c + 12*A*b^3*c + 36*a^2*b*c*C + 18*a^2*A*b*d*x + 12*A*b^3*d*x +
 36*a^2*b*C*d*x - 36*a*b^2*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 36*
a*b^2*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*a*(12*A*b^2 + a^2*(3*A
 + 4*C))*Sin[c + d*x] + 9*a^2*A*b*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)]
 + 12*b^3*C*Tan[c + d*x])/(12*d)
```

Maple [A] time = 0.069, size = 183, normalized size = 1.1

$$\frac{A(\cos(dx+c))^2\sin(dx+c)a^3}{3d}+\frac{2Aa^3\sin(dx+c)}{3d}+\frac{a^3C\sin(dx+c)}{d}+\frac{3Aa^2b\sin(dx+c)\cos(dx+c)}{2d}+\frac{3a^2Abx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3+2/3*a^3*A*sin(d*x+c)/d+a^3*C*sin(d*x+c)
/d+3/2/d*A*a^2*b*sin(d*x+c)*cos(d*x+c)+3/2*a^2*A*b*x+3/2/d*A*a^2*b*c+3*a^2*
b*C*x+3/d*C*a^2*b*c+3/d*A*a*b^2*sin(d*x+c)+3/d*C*a*b^2*ln(sec(d*x+c)+tan(d*
x+c))+A*b^3*x+1/d*A*b^3*c+1/d*C*b^3*tan(d*x+c)
```

Maxima [A] time = 0.977301, size = 190, normalized size = 1.17

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-9(2dx+2c+\sin(2dx+2c))Aa^2b-36(dx+c)Ca^2b-12(dx+c)Ab^3-18}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^2*b - 36*(d*x + c)*C*a^2*b - 12*(d*x + c)*A*b^3 - 18*C*a*b^2
*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 12*C*a^3*sin(d*x + c) -
36*A*a*b^2*sin(d*x + c) - 12*C*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.550193, size = 394, normalized size = 2.42

$$\frac{9Cab^2\cos(dx+c)\log(\sin(dx+c)+1)-9Cab^2\cos(dx+c)\log(-\sin(dx+c)+1)+3\left(3(A+2C)a^2b+2Ab^3\right)dx}{6d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(9*C*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 9*C*a*b^2*cos(d*x + c)*
log(-sin(d*x + c) + 1) + 3*(3*(A + 2*C)*a^2*b + 2*A*b^3)*d*x*cos(d*x + c) +
 (2*A*a^3*cos(d*x + c)^3 + 9*A*a^2*b*cos(d*x + c)^2 + 6*C*b^3 + 2*((2*A + 3
*C)*a^3 + 9*A*a*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23739, size = 413, normalized size = 2.53

$$18\,Cab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,Cab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,Cb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+3\left(3\,Aa^2b+6\,Ca^2b+2\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(18*C*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*C*a*b^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 12*C*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*
c)^2 - 1) + 3*(3*A*a^2*b + 6*C*a^2*b + 2*A*b^3)*(d*x + c) + 2*(6*A*a^3*tan(
1/2*d*x + 1/2*c)^5 + 6*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^3*tan(1/2*d*x + 1/2
*c)^3 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3
 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 6*C*a^3*tan(1/2*d*x + 1/2*c) + 9*A*a^2*b*
tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

3.660 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=182

$$\frac{b\left(a^2(4A+6C)+Ab^2\right)\sin(c+dx)}{2d}+\frac{a\left(a^2(3A+4C)+2Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(a^2(3A+4C)+12b^2(A\right.$$

```
[Out] (a*(12*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/8 + (b^3*C*ArcTanh[Sin[c + d*x]]
)/d + (b*(A*b^2 + a^2*(4*A + 6*C))*Sin[c + d*x])/(2*d) + (a*(2*A*b^2 + a^2*
(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (A*b*Cos[c + d*x]^2*(a + b*
Sec[c + d*x])^2*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x]
)^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.558659, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4074, 4047, 8, 4045, 3770}

$$\frac{b\left(a^2(4A+6C)+Ab^2\right)\sin(c+dx)}{2d}+\frac{a\left(a^2(3A+4C)+2Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(a^2(3A+4C)+12b^2(A\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(12*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/8 + (b^3*C*ArcTanh[Sin[c + d*x]]
)/d + (b*(A*b^2 + a^2*(4*A + 6*C))*Sin[c + d*x])/(2*d) + (a*(2*A*b^2 + a^2*
(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (A*b*Cos[c + d*x]^2*(a + b*
Sec[c + d*x])^2*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x]
)^3*Sin[c + d*x])/(4*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^{\cdots}$$
$$= \frac{Ab\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}+\frac{A\cos^3(\cdots}{\cdots}$$
$$= \frac{a\left(2Ab^2+a^2(3A+4C)\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{Ab\cos\cdots}{\cdots}$$
$$= \frac{a\left(2Ab^2+a^2(3A+4C)\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{Ab\cos\cdots}{\cdots}$$
$$= \frac{1}{8}a\left(12b^2(A+2C)+a^2(3A+4C)\right)x+\frac{b\left(Ab^2+a^2(4A+\cdots\right.}{2d}$$
$$= \frac{1}{8}a\left(12b^2(A+2C)+a^2(3A+4C)\right)x+\frac{b^3C\tanh^{-1}(\sin(c\cdots}{d}$$

Mathematica [A] time = 0.55403, size = 177, normalized size = 0.97

$$\frac{4a(c+dx)\left(a^2(3A+4C)+12b^2(A+2C)\right)+8a\left(a^2(A+C)+3Ab^2\right)\sin(2(c+dx))+8b\left(3a^2(3A+4C)+4Ab^2\right)\sin(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*a*(12*b^2*(A + 2*C) + a^2*(3*A + 4*C))*(c + d*x) - 32*b^3*C*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 32*b^3*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 8*b*(4*A*b^2 + 3*a^2*(3*A + 4*C))*Sin[c + d*x] + 8*a*(3*A*b^2 + a^2
*(A + C))*Sin[2*(c + d*x)] + 8*a^2*A*b*Sin[3*(c + d*x)] + a^3*A*Sin[4*(c +
```

```
d*x)])/(32*d)
```

Maple [A] time = 0.073, size = 252, normalized size = 1.4

$$\frac{Aa^3\sin(dx+c)(\cos(dx+c))^3}{4d}+\frac{3Aa^3\sin(dx+c)\cos(dx+c)}{8d}+\frac{3a^3Ax}{8}+\frac{3Aa^3c}{8d}+\frac{a^3C\sin(dx+c)\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/4/d*A*a^3*sin(d*x+c)*cos(d*x+c)^3+3/8/d*A*a^3*sin(d*x+c)*cos(d*x+c)+3/8*a
^3*A*x+3/8/d*A*a^3*c+1/2/d*a^3*C*sin(d*x+c)*cos(d*x+c)+1/2*a^3*C*x+1/2/d*C*
a^3*c+1/d*A*cos(d*x+c)^2*sin(d*x+c)*a^2*b+2/d*A*a^2*b*sin(d*x+c)+3/d*a^2*b*
C*sin(d*x+c)+3/2/d*A*a*b^2*sin(d*x+c)*cos(d*x+c)+3/2*A*a*b^2*x+3/2/d*A*a*b^
2*c+3*C*a*b^2*x+3/d*C*a*b^2*c+1/d*A*b^3*sin(d*x+c)+1/d*C*b^3*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 0.993957, size = 235, normalized size = 1.29

$$\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3+8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^3-32\left(\sin(dx+c)^3-3\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 8*(2*
d*x + 2*c + sin(2*d*x + 2*c))*C*a^3 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*
A*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 + 96*(d*x + c)*C*a*b^
2 + 16*C*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*C*a^2*b*s
in(d*x + c) + 32*A*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.554237, size = 354, normalized size = 1.95

$$\frac{4\,Cb^3\log(\sin(dx+c)+1)-4\,Cb^3\log(-\sin(dx+c)+1)+\left((3\,A+4\,C)a^3+12\,(A+2\,C)ab^2\right)dx+\left(2\,Aa^3\cos(dx+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/8*(4*C*b^3*log(sin(d*x + c) + 1) - 4*C*b^3*log(-sin(d*x + c) + 1) + ((3*A
 + 4*C)*a^3 + 12*(A + 2*C)*a*b^2)*d*x + (2*A*a^3*cos(d*x + c)^3 + 8*A*a^2*b
*cos(d*x + c)^2 + 8*(2*A + 3*C)*a^2*b + 8*A*b^3 + ((3*A + 4*C)*a^3 + 12*A*a
*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26949, size = 679, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/8*(8*C*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*C*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (3*A*a^3 + 4*C*a^3 + 12*A*a*b^2 + 24*C*a*b^2)*(d*x + c)
 - 2*(5*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 4*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*
A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*a
*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^3*tan(
1/2*d*x + 1/2*c)^5 + 4*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^2*b*tan(1/2*d*
x + 1/2*c)^5 - 72*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b^2*tan(1/2*d*x +
 1/2*c)^5 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^
3 - 4*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 72
*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 24*A*
b^3*tan(1/2*d*x + 1/2*c)^3 - 5*A*a^3*tan(1/2*d*x + 1/2*c) - 4*C*a^3*tan(1/2
*d*x + 1/2*c) - 24*A*a^2*b*tan(1/2*d*x + 1/2*c) - 24*C*a^2*b*tan(1/2*d*x +
1/2*c) - 12*A*a*b^2*tan(1/2*d*x + 1/2*c) - 8*A*b^3*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.661 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=218

$$\frac{a\left(2a^2(4A+5C)+15b^2(2A+3C)\right)\sin(c+dx)}{15d}+\frac{a\left(2a^2(4A+5C)+3Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{3b\left(5a^2(3A+\right.}{}$$

```
[Out] (b*(4*b^2*(A + 2*C) + 3*a^2*(3*A + 4*C))*x)/8 + (a*(15*b^2*(2*A + 3*C) + 2*
a^2*(4*A + 5*C))*Sin[c + d*x])/(15*d) + (3*b*(2*A*b^2 + 5*a^2*(3*A + 4*C))*
Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a*(3*A*b^2 + 2*a^2*(4*A + 5*C))*Cos[c
+ d*x]^2*Sin[c + d*x])/(30*d) + (3*A*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^
2*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(5*d)
```

Rubi [A] time = 0.650911, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4074, 4047, 2637, 4045, 8}

$$\frac{a\left(2a^2(4A+5C)+15b^2(2A+3C)\right)\sin(c+dx)}{15d}+\frac{a\left(2a^2(4A+5C)+3Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{3b\left(5a^2(3A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (b*(4*b^2*(A + 2*C) + 3*a^2*(3*A + 4*C))*x)/8 + (a*(15*b^2*(2*A + 3*C) + 2*
a^2*(4*A + 5*C))*Sin[c + d*x])/(15*d) + (3*b*(2*A*b^2 + 5*a^2*(3*A + 4*C))*
Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a*(3*A*b^2 + 2*a^2*(4*A + 5*C))*Cos[c
+ d*x]^2*Sin[c + d*x])/(30*d) + (3*A*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^
2*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(5*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx &= \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4\\ &= \frac{3Ab\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{20d}+\frac{A\cos^4}{}\\ &= \frac{a\left(3Ab^2+2a^2(4A+5C)\right)\cos^2(c+dx)\sin(c+dx)}{30d}+\frac{3Ab}{}\\ &= \frac{a\left(3Ab^2+2a^2(4A+5C)\right)\cos^2(c+dx)\sin(c+dx)}{30d}+\frac{3Ab}{}\\ &= \frac{a\left(15b^2(2A+3C)+2a^2(4A+5C)\right)\sin(c+dx)}{15d}+\frac{3b\left(2Ab\right.}{}\\ &= \frac{1}{8}b\left(4b^2(A+2C)+3a^2(3A+4C)\right)x+\frac{a\left(15b^2(2A+3C)\right.}{}\end{aligned}$$

Mathematica [A] time = 0.644841, size = 155, normalized size = 0.71

$$\frac{60b(c+dx)\left(3a^2(3A+4C)+4b^2(A+2C)\right)+60a\left(a^2(5A+6C)+6b^2(3A+4C)\right)\sin(c+dx)+10a\left(a^2(5A+4C)+12A\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (60*b*(4*b^2*(A + 2*C) + 3*a^2*(3*A + 4*C))*(c + d*x) + 60*a*(6*b^2*(3*A +
4*C) + a^2*(5*A + 6*C))*Sin[c + d*x] + 120*b*(A*b^2 + 3*a^2*(A + C))*Sin[2*
(c + d*x)] + 10*a*(12*A*b^2 + a^2*(5*A + 4*C))*Sin[3*(c + d*x)] + 45*a^2*A*
```

```
b*Sin[4*(c + d*x)] + 6*a^3*A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.073, size = 201, normalized size = 0.9

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3Aa^2b\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*A*a^2*b*(1/
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a*b^2*(2+cos(d*
x+c)^2)*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b^3*(1/2*cos(d*x
+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+
1/2*c)+3*C*a*b^2*sin(d*x+c)+C*b^3*(d*x+c))
```

Maxima [A] time = 1.00878, size = 262, normalized size = 1.2

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca^3+45(12dx+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
160*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 45*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b + 360*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*C*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 + 120*(2*d*x + 2*
c + sin(2*d*x + 2*c))*A*b^3 + 480*(d*x + c)*C*b^3 + 1440*C*a*b^2*sin(d*x +
c))/d
```

Fricas [A] time = 0.527668, size = 367, normalized size = 1.68

$$\frac{15\left(3(3A+4C)a^2b+4(A+2C)b^3\right)dx+\left(24Aa^3\cos(dx+c)^4+90Aa^2b\cos(dx+c)^3+16(4A+5C)a^3+120(2A\right.}{120a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/120*(15*(3*(3*A + 4*C)*a^2*b + 4*(A + 2*C)*b^3)*d*x + (24*A*a^3*cos(d*x +
 c)^4 + 90*A*a^2*b*cos(d*x + c)^3 + 16*(4*A + 5*C)*a^3 + 120*(2*A + 3*C)*a*
b^2 + 8*((4*A + 5*C)*a^3 + 15*A*a*b^2)*cos(d*x + c)^2 + 15*(3*(3*A + 4*C)*a
^2*b + 4*A*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23341, size = 818, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/120*(15*(9*A*a^2*b + 12*C*a^2*b + 4*A*b^3 + 8*C*b^3)*(d*x + c) + 2*(120*A
*a^3*tan(1/2*d*x + 1/2*c)^9 + 120*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*
b*tan(1/2*d*x + 1/2*c)^9 - 180*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2
*tan(1/2*d*x + 1/2*c)^9 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan
(1/2*d*x + 1/2*c)^9 + 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 320*C*a^3*tan(1/2*
d*x + 1/2*c)^7 - 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 360*C*a^2*b*tan(1/2*d*
x + 1/2*c)^7 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a*b^2*tan(1/2*d*
x + 1/2*c)^7 - 120*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1
/2*c)^5 + 400*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2
*c)^5 + 2160*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*tan(1/2*d*x + 1/2*c
)^3 + 320*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3
+ 360*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 +
 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
20*A*a^3*tan(1/2*d*x + 1/2*c) + 120*C*a^3*tan(1/2*d*x + 1/2*c) + 225*A*a^2*
b*tan(1/2*d*x + 1/2*c) + 180*C*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan
(1/2*d*x + 1/2*c) + 360*C*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.662 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=257

$$-\frac{b\left(3a^2(4A+5C)+Ab^2\right)\sin^3(c+dx)}{15d}+\frac{b\left(9a^2(4A+5C)+b^2(11A+15C)\right)\sin(c+dx)}{15d}+\frac{a\left(5a^2(5A+6C)+6Ab^2\right)}{120}$$

```
[Out] (a*(6*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*x)/16 + (b*(9*a^2*(4*A + 5*C) + b^
2*(11*A + 15*C))*Sin[c + d*x])/(15*d) + (a*(6*b^2*(3*A + 4*C) + a^2*(5*A +
6*C))*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A*b^2 + 5*a^2*(5*A + 6*C))*
Cos[c + d*x]^3*Sin[c + d*x])/(120*d) + (A*b*Cos[c + d*x]^4*(a + b*Sec[c + d
*x])^2*Sin[c + d*x])/(10*d) + (A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*Sin[
c + d*x])/(6*d) - (b*(A*b^2 + 3*a^2*(4*A + 5*C))*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.752351, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4095, 4094, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{b\left(3a^2(4A+5C)+Ab^2\right)\sin^3(c+dx)}{15d}+\frac{b\left(9a^2(4A+5C)+b^2(11A+15C)\right)\sin(c+dx)}{15d}+\frac{a\left(5a^2(5A+6C)+6Ab^2\right)}{120}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(6*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*x)/16 + (b*(9*a^2*(4*A + 5*C) + b^
2*(11*A + 15*C))*Sin[c + d*x])/(15*d) + (a*(6*b^2*(3*A + 4*C) + a^2*(5*A +
6*C))*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A*b^2 + 5*a^2*(5*A + 6*C))*
Cos[c + d*x]^3*Sin[c + d*x])/(120*d) + (A*b*Cos[c + d*x]^4*(a + b*Sec[c + d
*x])^2*Sin[c + d*x])/(10*d) + (A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*Sin[
c + d*x])/(6*d) - (b*(A*b^2 + 3*a^2*(4*A + 5*C))*Sin[c + d*x]^3)/(15*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^5$$

$$= \frac{Ab\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{10d}+\frac{A\cos^5(}{}$$

$$= \frac{a\left(6Ab^2+5a^2(5A+6C)\right)\cos^3(c+dx)\sin(c+dx)}{120d}+\frac{Ab\,c}{}$$

$$= \frac{a\left(6Ab^2+5a^2(5A+6C)\right)\cos^3(c+dx)\sin(c+dx)}{120d}+\frac{Ab\,c}{}$$

$$= \frac{a\left(6b^2(3A+4C)+a^2(5A+6C)\right)\cos(c+dx)\sin(c+dx)}{16d}$$

$$= \frac{1}{16}a\left(6b^2(3A+4C)+a^2(5A+6C)\right)x+\frac{a\left(6b^2(3A+4C)\right.}{}$$

$$= \frac{1}{16}a\left(6b^2(3A+4C)+a^2(5A+6C)\right)x+\frac{b\left(9a^2(4A+5C)\right.}{}$$

Mathematica [A] time = 1.12658, size = 253, normalized size = 0.98

$$\frac{15a\left(a^2(15A+16C)+48b^2(A+C)\right)\sin(2(c+dx))+120b\left(3a^2(5A+6C)+2b^2(3A+4C)\right)\sin(c+dx)+300a^2Ab\sin(3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (300*a^3*A*c + 1080*a*A*b^2*c + 360*a^3*c*C + 1440*a*b^2*c*C + 300*a^3*A*d*
x + 1080*a*A*b^2*d*x + 360*a^3*C*d*x + 1440*a*b^2*C*d*x + 120*b*(2*b^2*(3*A
 + 4*C) + 3*a^2*(5*A + 6*C))*Sin[c + d*x] + 15*a*(48*b^2*(A + C) + a^2*(15*
A + 16*C))*Sin[2*(c + d*x)] + 300*a^2*A*b*Sin[3*(c + d*x)] + 80*A*b^3*Sin[3
*(c + d*x)] + 240*a^2*b*C*Sin[3*(c + d*x)] + 45*a^3*A*Sin[4*(c + d*x)] + 90
*a*A*b^2*Sin[4*(c + d*x)] + 30*a^3*C*Sin[4*(c + d*x)] + 36*a^2*A*b*Sin[5*(c
 + d*x)] + 5*a^3*A*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.084, size = 249, normalized size = 1.

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+a^3C\left(\frac{\sin(dx+c)}{4}\left((\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+a^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*
x+3/8*c)+3/5*A*a^2*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*b*C
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+3/8*d*x+3/8*c)+3*C*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
)+1/3*A*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+C*b^3*sin(d*x+c))
```

Maxima [A] time = 1.02243, size = 328, normalized size = 1.28

$$-\frac{5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Aa^3-30(12dx+12c+\sin(4dx+4c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^3 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*C*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c
))*A*a^2*b + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b - 90*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^2 - 720*(2*d*x + 2*c + s
in(2*d*x + 2*c))*C*a*b^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^3 - 96
0*C*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.553126, size = 450, normalized size = 1.75

$$\frac{15\left((5A+6C)a^3+6(3A+4C)ab^2\right)dx+\left(40Aa^3\cos(dx+c)^5+144Aa^2b\cos(dx+c)^4+96(4A+5C)a^2b+80(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*((5*A + 6*C)*a^3 + 6*(3*A + 4*C)*a*b^2)*d*x + (40*A*a^3*cos(d*x +
 c)^5 + 144*A*a^2*b*cos(d*x + c)^4 + 96*(4*A + 5*C)*a^2*b + 80*(2*A + 3*C)*
b^3 + 10*((5*A + 6*C)*a^3 + 18*A*a*b^2)*cos(d*x + c)^3 + 16*(3*(4*A + 5*C)*
a^2*b + 5*A*b^3)*cos(d*x + c)^2 + 15*((5*A + 6*C)*a^3 + 6*(3*A + 4*C)*a*b^2
)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26488, size = 1191, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(15*(5*A*a^3 + 6*C*a^3 + 18*A*a*b^2 + 24*C*a*b^2)*(d*x + c) - 2*(165*
A*a^3*tan(1/2*d*x + 1/2*c)^11 + 150*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*A*a
^2*b*tan(1/2*d*x + 1/2*c)^11 - 720*C*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*A*
a*b^2*tan(1/2*d*x + 1/2*c)^11 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 240*A
*b^3*tan(1/2*d*x + 1/2*c)^11 - 240*C*b^3*tan(1/2*d*x + 1/2*c)^11 - 25*A*a^3
*tan(1/2*d*x + 1/2*c)^9 + 210*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*A*a^2*b*t
an(1/2*d*x + 1/2*c)^9 - 2640*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 630*A*a*b^2*t
an(1/2*d*x + 1/2*c)^9 + 1080*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*A*b^3*tan
(1/2*d*x + 1/2*c)^9 - 1200*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^3*tan(1/2
*d*x + 1/2*c)^7 + 60*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*A*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*A*a*b^2*tan(1/2*d*
x + 1/2*c)^7 + 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*A*b^3*tan(1/2*d*x
+ 1/2*c)^7 - 2400*C*b^3*tan(1/2*d*x + 1/2*c)^7 - 450*A*a^3*tan(1/2*d*x + 1/
2*c)^5 - 60*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*A*a^2*b*tan(1/2*d*x + 1/2*c
)^5 - 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 180*A*a*b^2*tan(1/2*d*x + 1/2*c
)^5 - 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*A*b^3*tan(1/2*d*x + 1/2*c)^
5 - 2400*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2
10*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 264
0*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 108
0*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 880*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 1200*
C*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*A*a^3*tan(1/2*d*x + 1/2*c) - 150*C*a^3*t
an(1/2*d*x + 1/2*c) - 720*A*a^2*b*tan(1/2*d*x + 1/2*c) - 720*C*a^2*b*tan(1/
2*d*x + 1/2*c) - 450*A*a*b^2*tan(1/2*d*x + 1/2*c) - 360*C*a*b^2*tan(1/2*d*x
 + 1/2*c) - 240*A*b^3*tan(1/2*d*x + 1/2*c) - 240*C*b^3*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.663 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=381

$$\frac{\left(a^4b^2(42A+23C)+8a^2b^4(49A+39C)+2a^6C+8b^6(7A+6C)\right)\tan(c+dx)}{105b^2d}+\frac{ab\left(a^2(8A+6C)+b^2(6A+5C)\right)\tan}{4d}$$

```
[Out] (a*b*(b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*ArcTanh[Sin[c + d*x]])/(4*d) + ((2
*a^6*C + 8*b^6*(7*A + 6*C) + a^4*b^2*(42*A + 23*C) + 8*a^2*b^4*(49*A + 39*C
))*Tan[c + d*x])/(105*b^2*d) + (a*(4*a^4*C + 12*a^2*b^2*(7*A + 4*C) + b^4*(
406*A + 333*C))*Sec[c + d*x]*Tan[c + d*x])/(420*b*d) + ((2*a^4*C + 8*b^4*(7
*A + 6*C) + 3*a^2*b^2*(14*A + 9*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(2
10*b^2*d) + (a*(42*A*b^2 + 2*a^2*C + 31*b^2*C)*(a + b*Sec[c + d*x])^3*Tan[c
 + d*x])/(210*b^2*d) + ((a^2*C + 3*b^2*(7*A + 6*C))*(a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(105*b^2*d) - (a*C*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(21*b
^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.982717, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4093, 4082, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(a^4b^2(42A+23C)+8a^2b^4(49A+39C)+2a^6C+8b^6(7A+6C)\right)\tan(c+dx)}{105b^2d}+\frac{ab\left(a^2(8A+6C)+b^2(6A+5C)\right)\tan}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*b*(b^2*(6*A + 5*C) + a^2*(8*A + 6*C))*ArcTanh[Sin[c + d*x]])/(4*d) + ((2
*a^6*C + 8*b^6*(7*A + 6*C) + a^4*b^2*(42*A + 23*C) + 8*a^2*b^4*(49*A + 39*C
))*Tan[c + d*x])/(105*b^2*d) + (a*(4*a^4*C + 12*a^2*b^2*(7*A + 4*C) + b^4*(
406*A + 333*C))*Sec[c + d*x]*Tan[c + d*x])/(420*b*d) + ((2*a^4*C + 8*b^4*(7
*A + 6*C) + 3*a^2*b^2*(14*A + 9*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(2
10*b^2*d) + (a*(42*A*b^2 + 2*a^2*C + 31*b^2*C)*(a + b*Sec[c + d*x])^3*Tan[c
 + d*x])/(210*b^2*d) + ((a^2*C + 3*b^2*(7*A + 6*C))*(a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(105*b^2*d) - (a*C*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(21*b
^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(7*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{C\sec(c+dx)(a+b\sec(c+dx))^5\tan(c+dx)}{7bd} + \frac{\int \sec(}{}$$
$$= -\frac{aC(a+b\sec(c+dx))^5\tan(c+dx)}{21b^2d} + \frac{C\sec(c+dx)(a}{}$$
$$= \frac{\left(a^2C+3b^2(7A+6C)\right)(a+b\sec(c+dx))^4\tan(c+dx)}{105b^2d}$$
$$= \frac{a\left(42Ab^2+2a^2C+31b^2C\right)(a+b\sec(c+dx))^3\tan(c+}{210b^2d}$$
$$= \frac{\left(2a^4C+8b^4(7A+6C)+3a^2b^2(14A+9C)\right)(a+b\sec(}{210b^2d}$$
$$= \frac{a\left(4a^4C+12a^2b^2(7A+4C)+b^4(406A+333C)\right)\sec(c}{420bd}$$
$$= \frac{a\left(4a^4C+12a^2b^2(7A+4C)+b^4(406A+333C)\right)\sec(c}{420bd}$$
$$= \frac{ab\left(b^2(6A+5C)+a^2(8A+6C)\right)\tanh^{-1}(\sin(c+dx))}{4d}$$
$$= \frac{ab\left(b^2(6A+5C)+a^2(8A+6C)\right)\tanh^{-1}(\sin(c+dx))}{4d}$$

Mathematica [A] time = 2.72815, size = 371, normalized size = 0.97

$$-\frac{\sec^6(c+dx)\left(A\cos^2(c+dx)+C\right)\left(-2b^2\left(3\left(6C\left(7a^2+b^2\right)+7Ab^2\right)\sin(2(c+dx))+140abC\sin(c+dx)+30b^2C\tan\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -((C + A*Cos[c + d*x]^2)*Sec[c + d*x]^6*(105*a*b*(b^2*(6*A + 5*C) + a^2*(8*
A + 6*C))*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 70*a*b*(6*A*b^2 + 6*a^2*C + 5*b^2*C)*
Cos[c + d*x]^2*Sin[c + d*x] - 4*(35*a^4*C + 42*a^2*b^2*(5*A + 4*C) + 4*b^4*
(7*A + 6*C))*Cos[c + d*x]^3*Sin[c + d*x] - 105*a*b*(b^2*(6*A + 5*C) + a^2*(
8*A + 6*C))*Cos[c + d*x]^4*Sin[c + d*x] - 4*(35*a^4*(3*A + 2*C) + 84*a^2*b^
2*(5*A + 4*C) + 8*b^4*(7*A + 6*C))*Cos[c + d*x]^5*Sin[c + d*x] - 2*b^2*(140
*a*b*C*Sin[c + d*x] + 3*(7*A*b^2 + 6*(7*a^2 + b^2)*C)*Sin[2*(c + d*x)] + 30
*b^2*C*Tan[c + d*x])))/(210*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.061, size = 591, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 8/15/d*A*b^4*tan(d*x+c)+16/35/d*C*b^4*tan(d*x+c)+2/3/d*a^4*C*tan(d*x+c)+1/d
*A*a^4*tan(d*x+c)+1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+2/3/d*C*a*b^3*tan(d*x
+c)*sec(d*x+c)^5+1/5/d*A*b^4*tan(d*x+c)*sec(d*x+c)^4+5/4/d*C*a*b^3*ln(sec(d
*x+c)+tan(d*x+c))+4/d*A*a^2*b^2*tan(d*x+c)+4/15/d*A*b^4*tan(d*x+c)*sec(d*x+
```

```
c)^2+1/7/d*C*b^4*tan(d*x+c)*sec(d*x+c)^6+6/35/d*C*b^4*tan(d*x+c)*sec(d*x+c)
^4+8/35/d*C*b^4*tan(d*x+c)*sec(d*x+c)^2+16/5/d*C*a^2*b^2*tan(d*x+c)+2/d*A*a
^3*b*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+3/2/
d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+6/5/d*C*a^2*b^2*tan(d*x+c)*sec(d*x+c)^4
+8/5/d*C*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+2/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+
1/d*a^3*b*C*tan(d*x+c)*sec(d*x+c)^3+3/2/d*a^3*b*C*sec(d*x+c)*tan(d*x+c)+5/6
/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)^3+5/4/d*C*a*b^3*sec(d*x+c)*tan(d*x+c)+2/d*
A*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+1/d*A*a*b^3*tan(d*x+c)*sec(d*x+c)^3+3/2/d
*A*a*b^3*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.00559, size = 637, normalized size = 1.67

$$\frac{280\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^4+1680\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2b^2+336\left(3\tan(dx+c)^5+10\tan(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/840*(280*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 1680*(tan(d*x + c)^3 +
 3*tan(d*x + c))*A*a^2*b^2 + 336*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15
*tan(d*x + c))*C*a^2*b^2 + 56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*ta
n(d*x + c))*A*b^4 + 24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x +
 c)^3 + 35*tan(d*x + c))*C*b^4 - 35*C*a*b^3*(2*(15*sin(d*x + c)^5 - 40*sin(
d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*
x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 210*
C*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 210*A*a*
b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)
^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 840*A*a^3*b*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)) + 840*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.609102, size = 783, normalized size = 2.06

$$\frac{105\left(2(4A+3C)a^3b+(6A+5C)ab^3\right)\cos(dx+c)^7\log(\sin(dx+c)+1)-105\left(2(4A+3C)a^3b+(6A+5C)ab^3\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/840*(105*(2*(4*A + 3*C)*a^3*b + (6*A + 5*C)*a*b^3)*cos(d*x + c)^7*log(sin
(d*x + c) + 1) - 105*(2*(4*A + 3*C)*a^3*b + (6*A + 5*C)*a*b^3)*cos(d*x + c)
^7*log(-sin(d*x + c) + 1) + 2*(4*(35*(3*A + 2*C)*a^4 + 84*(5*A + 4*C)*a^2*b
^2 + 8*(7*A + 6*C)*b^4)*cos(d*x + c)^6 + 280*C*a*b^3*cos(d*x + c) + 105*(2*
(4*A + 3*C)*a^3*b + (6*A + 5*C)*a*b^3)*cos(d*x + c)^5 + 60*C*b^4 + 4*(35*C*
a^4 + 42*(5*A + 4*C)*a^2*b^2 + 4*(7*A + 6*C)*b^4)*cos(d*x + c)^4 + 70*(6*C*
a^3*b + (6*A + 5*C)*a*b^3)*cos(d*x + c)^3 + 12*(42*C*a^2*b^2 + (7*A + 6*C)*
b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^4 \sec^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**4*sec(c + d*x)**2, x
)
```

Giac [B] time = 1.28149, size = 1728, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/420*(105*(8*A*a^3*b + 6*C*a^3*b + 6*A*a*b^3 + 5*C*a*b^3)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - 105*(8*A*a^3*b + 6*C*a^3*b + 6*A*a*b^3 + 5*C*a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/2*d*x + 1/2*c)^13 + 4
20*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 840*A*a^3*b*tan(1/2*d*x + 1/2*c)^13 - 10
50*C*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 2520*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^13
 + 2520*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 - 1050*A*a*b^3*tan(1/2*d*x + 1/2*
c)^13 - 1155*C*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 420*A*b^4*tan(1/2*d*x + 1/2*
c)^13 + 420*C*b^4*tan(1/2*d*x + 1/2*c)^13 - 2520*A*a^4*tan(1/2*d*x + 1/2*c)
^11 - 1960*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 3360*A*a^3*b*tan(1/2*d*x + 1/2*c
)^11 + 2520*C*a^3*b*tan(1/2*d*x + 1/2*c)^11 - 11760*A*a^2*b^2*tan(1/2*d*x +
 1/2*c)^11 - 8400*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 2520*A*a*b^3*tan(1/2*
d*x + 1/2*c)^11 + 980*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 1400*A*b^4*tan(1/2*
d*x + 1/2*c)^11 - 840*C*b^4*tan(1/2*d*x + 1/2*c)^11 + 6300*A*a^4*tan(1/2*d*
x + 1/2*c)^9 + 4060*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 4200*A*a^3*b*tan(1/2*d*x
 + 1/2*c)^9 - 1890*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 24360*A*a^2*b^2*tan(1/2
*d*x + 1/2*c)^9 + 18984*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 1890*A*a*b^3*tan
(1/2*d*x + 1/2*c)^9 - 2975*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 3164*A*b^4*tan(
1/2*d*x + 1/2*c)^9 + 3612*C*b^4*tan(1/2*d*x + 1/2*c)^9 - 8400*A*a^4*tan(1/2
*d*x + 1/2*c)^7 - 5040*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 30240*A*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 26208*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 4368*A*b^4*tan
(1/2*d*x + 1/2*c)^7 - 2544*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 6300*A*a^4*tan(1/
2*d*x + 1/2*c)^5 + 4060*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 4200*A*a^3*b*tan(1/2
*d*x + 1/2*c)^5 + 1890*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 24360*A*a^2*b^2*tan
(1/2*d*x + 1/2*c)^5 + 18984*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1890*A*a*b^3
*tan(1/2*d*x + 1/2*c)^5 + 2975*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3164*A*b^4*
tan(1/2*d*x + 1/2*c)^5 + 3612*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 2520*A*a^4*tan
(1/2*d*x + 1/2*c)^3 - 1960*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 3360*A*a^3*b*tan(
1/2*d*x + 1/2*c)^3 - 2520*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 11760*A*a^2*b^2*
tan(1/2*d*x + 1/2*c)^3 - 8400*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2520*A*a*b
^3*tan(1/2*d*x + 1/2*c)^3 - 980*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1400*A*b^4
*tan(1/2*d*x + 1/2*c)^3 - 840*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 420*A*a^4*tan(
1/2*d*x + 1/2*c) + 420*C*a^4*tan(1/2*d*x + 1/2*c) + 840*A*a^3*b*tan(1/2*d*x
 + 1/2*c) + 1050*C*a^3*b*tan(1/2*d*x + 1/2*c) + 2520*A*a^2*b^2*tan(1/2*d*x
+ 1/2*c) + 2520*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 1050*A*a*b^3*tan(1/2*d*x +
```

```
 1/2*c) + 1155*C*a*b^3*tan(1/2*d*x + 1/2*c) + 420*A*b^4*tan(1/2*d*x + 1/2*c
) + 420*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.664 $\int \sec(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=310

$$-\frac{a\left(-a^2b^2(190A+121C)+4a^4C-32b^4(5A+4C)\right)\tan(c+dx)}{60bd}+\frac{\left(12a^2b^2(4A+3C)+8a^4(2A+C)+b^4(6A+5C)\right)}{16d}$$

```
[Out] ((8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*ArcTanh[Sin[c
 + d*x]])/(16*d) - (a*(4*a^4*C - 32*b^4*(5*A + 4*C) - a^2*b^2*(190*A + 121*
C))*Tan[c + d*x])/(60*b*d) - ((8*a^4*C - 15*b^4*(6*A + 5*C) - 2*a^2*b^2*(13
0*A + 89*C))*Sec[c + d*x]*Tan[c + d*x])/(240*d) + (a*(70*A*b^2 - 4*a^2*C +
53*b^2*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(120*b*d) - ((4*a^2*C - 5*b^
2*(6*A + 5*C))*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(120*b*d) - (a*C*(a + b
*Sec[c + d*x])^4*Tan[c + d*x])/(30*b*d) + (C*(a + b*Sec[c + d*x])^5*Tan[c +
 d*x])/(6*b*d)
```

Rubi [A] time = 0.704779, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4083, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{a\left(-a^2b^2(190A+121C)+4a^4C-32b^4(5A+4C)\right)\tan(c+dx)}{60bd}+\frac{\left(12a^2b^2(4A+3C)+8a^4(2A+C)+b^4(6A+5C)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*ArcTanh[Sin[c
 + d*x]])/(16*d) - (a*(4*a^4*C - 32*b^4*(5*A + 4*C) - a^2*b^2*(190*A + 121*
C))*Tan[c + d*x])/(60*b*d) - ((8*a^4*C - 15*b^4*(6*A + 5*C) - 2*a^2*b^2*(13
0*A + 89*C))*Sec[c + d*x]*Tan[c + d*x])/(240*d) + (a*(70*A*b^2 - 4*a^2*C +
53*b^2*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(120*b*d) - ((4*a^2*C - 5*b^
2*(6*A + 5*C))*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(120*b*d) - (a*C*(a + b
*Sec[c + d*x])^4*Tan[c + d*x])/(30*b*d) + (C*(a + b*Sec[c + d*x])^5*Tan[c +
 d*x])/(6*b*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c+dx)(a+b\sec}{}$$

$$= -\frac{aC(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} + \frac{C(a+b\sec(c+dx)}{6bd}$$

$$= -\frac{\left(4a^2C-5b^2(6A+5C)\right)(a+b\sec(c+dx))^3\tan(c+dx)}{120bd}$$

$$= \frac{a\left(70Ab^2-4a^2C+53b^2C\right)(a+b\sec(c+dx))^2\tan(c+dx)}{120bd}$$

$$= -\frac{\left(8a^4C-15b^4(6A+5C)-2a^2b^2(130A+89C)\right)\sec(c+dx}{240d}$$

$$= -\frac{\left(8a^4C-15b^4(6A+5C)-2a^2b^2(130A+89C)\right)\sec(c+dx}{240d}$$

$$= \frac{\left(8a^4(2A+C)+12a^2b^2(4A+3C)+b^4(6A+5C)\right)\tanh^{-1}(}{16d}$$

$$= \frac{\left(8a^4(2A+C)+12a^2b^2(4A+3C)+b^4(6A+5C)\right)\tanh^{-1}(}{16d}$$

Mathematica [A] time = 2.90493, size = 460, normalized size = 1.48

$$-\frac{\sec^6(c+dx)\left(A\cos^2(c+dx)+C\right)\left(240\left(12a^2b^2(4A+3C)+8a^4(2A+C)+b^4(6A+5C)\right)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] -((C + A*Cos[c + d*x]^2)*Sec[c + d*x]^6*(240*(8*a^4*(2*A + C) + 12*a^2*b^2*
(4*A + 3*C) + b^4*(6*A + 5*C))*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2*(2160*a^2*A*b^
2 + 510*A*b^4 + 360*a^4*C + 3060*a^2*b^2*C + 745*b^4*C + 64*a*b*(8*b^2*(10*
A + 11*C) + a^2*(75*A + 80*C))*Cos[c + d*x] + 20*(24*a^4*C + 36*a^2*b^2*(4*
A + 5*C) + 5*b^4*(6*A + 5*C))*Cos[2*(c + d*x)] + 2400*a^3*A*b*Cos[3*(c + d*
x)] + 2240*a*A*b^3*Cos[3*(c + d*x)] + 2240*a^3*b*C*Cos[3*(c + d*x)] + 1792*
a*b^3*C*Cos[3*(c + d*x)] + 720*a^2*A*b^2*Cos[4*(c + d*x)] + 90*A*b^4*Cos[4*
(c + d*x)] + 120*a^4*C*Cos[4*(c + d*x)] + 540*a^2*b^2*C*Cos[4*(c + d*x)] +
75*b^4*C*Cos[4*(c + d*x)] + 480*a^3*A*b*Cos[5*(c + d*x)] + 320*a*A*b^3*Cos[
5*(c + d*x)] + 320*a^3*b*C*Cos[5*(c + d*x)] + 256*a*b^3*C*Cos[5*(c + d*x)])
*Sin[c + d*x]))/(1920*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.059, size = 511, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)+1/2/d
*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^3*b*tan(d*x+c)+8/3/d*a^3*b*C*tan(d
*x+c)+4/3/d*a^3*b*C*tan(d*x+c)*sec(d*x+c)^2+3/d*A*a^2*b^2*sec(d*x+c)*tan(d*
x+c)+3/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*C*a^2*b^2*tan(d*x+c)*sec
(d*x+c)^3+9/4/d*C*a^2*b^2*sec(d*x+c)*tan(d*x+c)+9/4/d*C*a^2*b^2*ln(sec(d*x+
c)+tan(d*x+c))+8/3/d*A*a*b^3*tan(d*x+c)+4/3/d*A*a*b^3*tan(d*x+c)*sec(d*x+c)
^2+32/15/d*C*a*b^3*tan(d*x+c)+4/5/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)^4+16/15/d
*C*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*A*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*
A*b^4*sec(d*x+c)*tan(d*x+c)+3/8/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*C*b
^4*tan(d*x+c)*sec(d*x+c)^5+5/24/d*C*b^4*tan(d*x+c)*sec(d*x+c)^3+5/16/d*C*b^
4*sec(d*x+c)*tan(d*x+c)+5/16/d*C*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02264, size = 620, normalized size = 2.

$$640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^3b+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab^3+128\left(3\tan(dx+c)^5+10\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3*b + 640*(tan(d*x + c)^3
+ 3*tan(d*x + c))*A*a*b^3 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*
tan(d*x + c))*C*a*b^3 - 5*C*b^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 +
 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1
) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*C*a^2*b^2*(2
*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1
) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*A*b^4*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*l
og(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*C*a^4*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72
0*A*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
```

```
log(sin(d*x + c) - 1)) + 480*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 1920*
A*a^3*b*tan(d*x + c))/d
```

Fricas [A] time = 0.592328, size = 716, normalized size = 2.31

$$\frac{15\left(8\left(2A+C\right)a^4+12\left(4A+3C\right)a^2b^2+\left(6A+5C\right)b^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\left(2A+C\right)a^4+12\left(4A\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(8*(2*A + C)*a^4 + 12*(4*A + 3*C)*a^2*b^2 + (6*A + 5*C)*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*(8*(2*A + C)*a^4 + 12*(4*A + 3*C)*a^2
*b^2 + (6*A + 5*C)*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(192*C*a*
b^3*cos(d*x + c) + 64*(5*(3*A + 2*C)*a^3*b + 2*(5*A + 4*C)*a*b^3)*cos(d*x +
 c)^5 + 40*C*b^4 + 15*(8*C*a^4 + 12*(4*A + 3*C)*a^2*b^2 + (6*A + 5*C)*b^4)*
cos(d*x + c)^4 + 64*(5*C*a^3*b + (5*A + 4*C)*a*b^3)*cos(d*x + c)^3 + 10*(36
*C*a^2*b^2 + (6*A + 5*C)*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)
^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+C\sec^2\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^4\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

Giac [B] time = 1.23601, size = 1485, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(16*A*a^4 + 8*C*a^4 + 48*A*a^2*b^2 + 36*C*a^2*b^2 + 6*A*b^4 + 5*C
*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(16*A*a^4 + 8*C*a^4 + 48*A*a^
2*b^2 + 36*C*a^2*b^2 + 6*A*b^4 + 5*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + 2*(120*C*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)
^11 - 960*C*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 720*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^11 + 900*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x +
1/2*c)^11 - 960*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 150*A*b^4*tan(1/2*d*x + 1
/2*c)^11 + 165*C*b^4*tan(1/2*d*x + 1/2*c)^11 - 360*C*a^4*tan(1/2*d*x + 1/2*
c)^9 + 4800*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 3520*C*a^3*b*tan(1/2*d*x + 1/2
*c)^9 - 2160*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 1260*C*a^2*b^2*tan(1/2*d*x
```

```
+ 1/2*c)^9 + 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 2240*C*a*b^3*tan(1/2*d*x
 + 1/2*c)^9 - 210*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 25*C*b^4*tan(1/2*d*x + 1/2
*c)^9 + 240*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 9600*A*a^3*b*tan(1/2*d*x + 1/2*c
)^7 - 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 1440*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^7 + 360*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x +
1/2*c)^7 - 4992*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 60*A*b^4*tan(1/2*d*x + 1/2
*c)^7 + 450*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 240*C*a^4*tan(1/2*d*x + 1/2*c)^5
 + 9600*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^
5 + 1440*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 360*C*a^2*b^2*tan(1/2*d*x + 1/2
*c)^5 + 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 4992*C*a*b^3*tan(1/2*d*x + 1/
2*c)^5 + 60*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 450*C*b^4*tan(1/2*d*x + 1/2*c)^5
 - 360*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 4800*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 -
 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2160*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^
3 - 1260*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*
c)^3 - 2240*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 210*A*b^4*tan(1/2*d*x + 1/2*c)
^3 + 25*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*C*a^4*tan(1/2*d*x + 1/2*c) + 960
*A*a^3*b*tan(1/2*d*x + 1/2*c) + 960*C*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^
2*b^2*tan(1/2*d*x + 1/2*c) + 900*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 960*A*a*b
^3*tan(1/2*d*x + 1/2*c) + 960*C*a*b^3*tan(1/2*d*x + 1/2*c) + 150*A*b^4*tan(
1/2*d*x + 1/2*c) + 165*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^6)/d
```

3.665 $\int (a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=227

$$\frac{\left(a^2b^2(85A+56C)+6a^4C+2b^4(5A+4C)\right)\tan(c+dx)}{15d}+\frac{ab\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ab(6$$

```
[Out] a^4*A*x + (a*b*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(
2*d) + ((6*a^4*C + 2*b^4*(5*A + 4*C) + a^2*b^2*(85*A + 56*C))*Tan[c + d*x])
/(15*d) + (a*b*(40*A*b^2 + 6*a^2*C + 29*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(
30*d) + ((3*a^2*C + b^2*(5*A + 4*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(
15*d) + (a*C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + (C*(a + b*Sec[c +
 d*x])^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.478201, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4057, 4056, 4048, 3770, 3767, 8}

$$\frac{\left(a^2b^2(85A+56C)+6a^4C+2b^4(5A+4C)\right)\tan(c+dx)}{15d}+\frac{ab\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ab(6$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] a^4*A*x + (a*b*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(
2*d) + ((6*a^4*C + 2*b^4*(5*A + 4*C) + a^2*b^2*(85*A + 56*C))*Tan[c + d*x])
/(15*d) + (a*b*(40*A*b^2 + 6*a^2*C + 29*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(
30*d) + ((3*a^2*C + b^2*(5*A + 4*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(
15*d) + (a*C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + (C*(a + b*Sec[c +
 d*x])^4*Tan[c + d*x])/(5*d)
```


Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4 \left(A+C\sec^2(c+dx)\right)\,dx = \frac{C(a+b\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}\int (a+b\sec(c+dx))^3\left(5aA\right.$$

$$= \frac{aC(a+b\sec(c+dx))^3\tan(c+dx)}{5d}+\frac{C(a+b\sec(c+dx))^4\tan(c+$$

$$= \frac{\left(3a^2C+b^2(5A+4C)\right)(a+b\sec(c+dx))^2\tan(c+dx)}{15d}+\frac{aC(a+b$$

$$= \frac{ab\left(40Ab^2+6a^2C+29b^2C\right)\sec(c+dx)\tan(c+dx)}{30d}+\frac{\left(3a^2C+b^2(\right.}{}$$

$$= a^4Ax+\frac{ab\left(40Ab^2+6a^2C+29b^2C\right)\sec(c+dx)\tan(c+dx)}{30d}+\frac{\left(3a^2\right.}{}$$

$$= a^4Ax+\frac{ab\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ab}{}$$

$$= a^4Ax+\frac{ab\left(4a^2(2A+C)+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(6a\right.}{}$$

Mathematica [B] time = 2.5729, size = 503, normalized size = 2.22

$$\frac{\sec^5(c+dx)\left(A\cos^2(c+dx)+C\right)\left(-120ab\left(4a^2(2A+C)+b^2(4A+3C)\right)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((C + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(150*a^4*A*(c + d*x)*Cos[c + d*x] +
75*a^4*A*(c + d*x)*Cos[3*(c + d*x)] + 15*a^4*A*c*Cos[5*(c + d*x)] + 15*a^4*
A*d*x*Cos[5*(c + d*x)] - 120*a*b*(4*a^2*(2*A + C) + b^2*(4*A + 3*C))*Cos[c
+ d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) + 180*a^2*A*b^2*Sin[c + d*x] + 40*A*b^4*Sin[c + d*x] +
30*a^4*C*Sin[c + d*x] + 240*a^2*b^2*C*Sin[c + d*x] + 80*b^4*C*Sin[c + d*x]
+ 120*a*A*b^3*Sin[2*(c + d*x)] + 120*a^3*b*C*Sin[2*(c + d*x)] + 210*a*b^3*C
*Sin[2*(c + d*x)] + 270*a^2*A*b^2*Sin[3*(c + d*x)] + 50*A*b^4*Sin[3*(c + d*
x)] + 45*a^4*C*Sin[3*(c + d*x)] + 300*a^2*b^2*C*Sin[3*(c + d*x)] + 40*b^4*C
*Sin[3*(c + d*x)] + 60*a*A*b^3*Sin[4*(c + d*x)] + 60*a^3*b*C*Sin[4*(c + d*x
)] + 45*a*b^3*C*Sin[4*(c + d*x)] + 90*a^2*A*b^2*Sin[5*(c + d*x)] + 10*A*b^4
*Sin[5*(c + d*x)] + 15*a^4*C*Sin[5*(c + d*x)] + 60*a^2*b^2*C*Sin[5*(c + d*x
)] + 8*b^4*C*Sin[5*(c + d*x)]))/(120*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.056, size = 377, normalized size = 1.7

$$a^4Ax+\frac{Aa^4c}{d}+\frac{a^4C\tan(dx+c)}{d}+4\frac{Aa^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{a^3bC\sec(dx+c)\tan(dx+c)}{d}+2\frac{a^3bC\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] a^4*A*x+1/d*A*a^4*c+1/d*a^4*C*tan(d*x+c)+4/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+
c))+2/d*a^3*b*C*sec(d*x+c)*tan(d*x+c)+2/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))
+6/d*A*a^2*b^2*tan(d*x+c)+4/d*C*a^2*b^2*tan(d*x+c)+2/d*C*a^2*b^2*tan(d*x+c)
*sec(d*x+c)^2+2/d*A*a*b^3*sec(d*x+c)*tan(d*x+c)+2/d*A*a*b^3*ln(sec(d*x+c)+t
an(d*x+c))+1/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)^3+3/2/d*C*a*b^3*sec(d*x+c)*tan
(d*x+c)+3/2/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*A*b^4*tan(d*x+c)+1/3/
d*A*b^4*tan(d*x+c)*sec(d*x+c)^2+8/15/d*C*b^4*tan(d*x+c)+1/5/d*C*b^4*tan(d*x
+c)*sec(d*x+c)^4+4/15/d*C*b^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.979258, size = 429, normalized size = 1.89

$$\frac{60\,(dx+c)Aa^4+120\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ca^2b^2+20\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ab^4+4\left(3\,\tan(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(d*x + c)*A*a^4 + 120*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b^2
+ 20*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^4 + 4*(3*tan(d*x + c)^5 + 10*tan
(d*x + c)^3 + 15*tan(d*x + c))*C*b^4 - 15*C*a*b^3*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 60*C*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^
2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*A*a*b^3*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 240*A*a^3*b*log(sec(d*x + c) + tan(d*x + c)) + 60*C*a^4*tan(d*x + c)
 + 360*A*a^2*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.582037, size = 610, normalized size = 2.69

$$\frac{60\,Aa^4dx\cos(dx+c)^5+15\left(4\,(2\,A+C)a^3b+(4\,A+3\,C)ab^3\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(4\,(2\,A+C)a^3b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(60*A*a^4*d*x*cos(d*x + c)^5 + 15*(4*(2*A + C)*a^3*b + (4*A + 3*C)*a*b
^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*(2*A + C)*a^3*b + (4*A + 3
*C)*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*C*a*b^3*cos(d*x +
c) + 6*C*b^4 + 2*(15*C*a^4 + 30*(3*A + 2*C)*a^2*b^2 + 2*(5*A + 4*C)*b^4)*co
s(d*x + c)^4 + 15*(4*C*a^3*b + (4*A + 3*C)*a*b^3)*cos(d*x + c)^3 + 2*(30*C*
```

```
a^2*b^2 + (5*A + 4*C)*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.22534, size = 1050, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/30*(30*(d*x + c)*A*a^4 + 15*(8*A*a^3*b + 4*C*a^3*b + 4*A*a*b^3 + 3*C*a*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*A*a^3*b + 4*C*a^3*b + 4*A*a*b
^3 + 3*C*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*C*a^4*tan(1/2*d*
x + 1/2*c)^9 - 60*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*A*a^2*b^2*tan(1/2*d*
x + 1/2*c)^9 + 180*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*a*b^3*tan(1/2*d*
x + 1/2*c)^9 - 75*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*A*b^4*tan(1/2*d*x + 1
/2*c)^9 + 30*C*b^4*tan(1/2*d*x + 1/2*c)^9 - 120*C*a^4*tan(1/2*d*x + 1/2*c)^
7 + 120*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)
^7 - 480*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*A*a*b^3*tan(1/2*d*x + 1/2*c
)^7 + 30*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 80*A*b^4*tan(1/2*d*x + 1/2*c)^7 -
 40*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 180*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 1080*
A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 600*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1
00*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 116*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 120*C*
a^4*tan(1/2*d*x + 1/2*c)^3 - 120*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 720*A*a^2
*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*
a*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 80*A*b^4
*tan(1/2*d*x + 1/2*c)^3 - 40*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*C*a^4*tan(1/
2*d*x + 1/2*c) + 60*C*a^3*b*tan(1/2*d*x + 1/2*c) + 180*A*a^2*b^2*tan(1/2*d*
x + 1/2*c) + 180*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a*b^3*tan(1/2*d*x +
1/2*c) + 75*C*a*b^3*tan(1/2*d*x + 1/2*c) + 30*A*b^4*tan(1/2*d*x + 1/2*c) +
30*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.666 $\int \cos(c+dx)(a+b\sec(c+dx))^4 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=229

$$-\frac{ab\left(a^2(12A-19C)-8b^2(3A+2C)\right)\tan(c+dx)}{6d}+\frac{\left(24a^2b^2(2A+C)+8a^4C+b^4(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b^2}{}$$

```
[Out] 4*a^3*A*b*x + ((8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*ArcTanh[S
in[c + d*x]])/(8*d) + (A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/d - (a*b*(a^2
*(12*A - 19*C) - 8*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*d) - (b^2*(a^2*(24*A -
 26*C) - 3*b^2*(4*A + 3*C))*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (a*b*(12*A
- 7*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - (b*(4*A - C)*(a + b*Se
c[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.492718, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4095, 4056, 4048, 3770, 3767, 8}

$$-\frac{ab\left(a^2(12A-19C)-8b^2(3A+2C)\right)\tan(c+dx)}{6d}+\frac{\left(24a^2b^2(2A+C)+8a^4C+b^4(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] 4*a^3*A*b*x + ((8*a^4*C + 24*a^2*b^2*(2*A + C) + b^4*(4*A + 3*C))*ArcTanh[S
in[c + d*x]])/(8*d) + (A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/d - (a*b*(a^2
*(12*A - 19*C) - 8*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*d) - (b^2*(a^2*(24*A -
 26*C) - 3*b^2*(4*A + 3*C))*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (a*b*(12*A
- 7*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - (b*(4*A - C)*(a + b*Se
c[c + d*x])^3*Tan[c + d*x])/(4*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
```

```
C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d} + \int (a+b\sec(c+dx$$
$$= \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d} - \frac{b(4A-C)(a+b\sec}{}$$
$$= \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d} - \frac{ab(12A-7C)(a+b}{}$$
$$= \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d} - \frac{b^2\left(a^2(24A-26C)\right.}{}$$
$$= 4a^3Abx + \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d} - \frac{b^2\left(a^2(24\right.}{}$$
$$= 4a^3Abx + \frac{\left(8a^4C+24a^2b^2(2A+C)+b^4(4A+3C)\right)\tan}{8d}$$
$$= 4a^3Abx + \frac{\left(8a^4C+24a^2b^2(2A+C)+b^4(4A+3C)\right)\tan}{8d}$$

Mathematica [B] time = 6.5741, size = 1357, normalized size = 5.93

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (8*a^3*A*b*(c + d*x)*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d
*x]^2))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((-48*a
^2*A*b^2 - 4*A*b^4 - 8*a^4*C - 24*a^2*b^2*C - 3*b^4*C)*Cos[c + d*x]^6*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d
*x]^2))/(4*d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((48*
a^2*A*b^2 + 4*A*b^4 + 8*a^4*C + 24*a^2*b^2*C + 3*b^4*C)*Cos[c + d*x]^6*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^4*(A + C*Sec[c +
d*x]^2))/(4*d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])) + (b^4
*C*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(8*d*(b +
a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^4) + ((12*A*b^4 + 72*a^2*b^2*C + 16*a*b^3*C + 9*b^4*C)*Cos[c +
d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(24*d*(b + a*Cos[c +
```

```
d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^2) + (4*a*b^3*C*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]
^2)*Sin[(c + d*x)/2])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*
d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - (b^4*C*Cos[c + d*x]^6*(a +
 b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^4*(A +
 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (4*a*
b^3*C*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*Sin[(c +
 d*x)/2])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-12*A*b^4 - 72*a^2*b^2*C - 16*a*b^3*
C - 9*b^4*C)*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2))/
(24*d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2) + (8*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C
*Sec[c + d*x]^2)*(3*a*A*b^3*Sin[(c + d*x)/2] + 3*a^3*b*C*Sin[(c + d*x)/2] +
 2*a*b^3*C*Sin[(c + d*x)/2]))/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[
2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (8*Cos[c + d*x]^6*(a
 + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2)*(3*a*A*b^3*Sin[(c + d*x)/2] + 3
*a^3*b*C*Sin[(c + d*x)/2] + 2*a*b^3*C*Sin[(c + d*x)/2]))/(3*d*(b + a*Cos[c
+ d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])) + (2*a^4*A*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2
)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 0.082, size = 316, normalized size = 1.4

$$\frac{Aa^4\sin(dx+c)}{d}+\frac{a^4C\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\,a^3Abx+4\,\frac{Aa^3bc}{d}+4\,\frac{a^3bC\tan(dx+c)}{d}+6\,\frac{Aa^2b^2\ln(\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+1/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*A*b*x+4/d*A*
a^3*b*c+4/d*a^3*b*C*tan(d*x+c)+6/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/d*
C*a^2*b^2*sec(d*x+c)*tan(d*x+c)+3/d*C*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4/d
*A*a*b^3*tan(d*x+c)+8/3/d*C*a*b^3*tan(d*x+c)+4/3/d*C*a*b^3*tan(d*x+c)*sec(d
*x+c)^2+1/2/d*A*b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^4*ln(sec(d*x+c)+tan(d*x
+c))+1/4/d*C*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*C*b^4*sec(d*x+c)*tan(d*x+c)+
3/8/d*C*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.992264, size = 413, normalized size = 1.8

$$192\,(dx+c)Aa^3b+64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Cab^3-3\,Cb^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(192*(d*x + c)*A*a^3*b + 64*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b^3
- 3*C*b^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*C*a
^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(s
in(d*x + c) - 1)) - 12*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*a^4*(log(sin(d*x + c) + 1) -
```

```
 log(sin(d*x + c) - 1)) + 144*A*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 48*A*a^4*sin(d*x + c) + 192*C*a^3*b*tan(d*x + c) + 192*A*a*b
^3*tan(d*x + c))/d
```

Fricas [A] time = 0.59003, size = 575, normalized size = 2.51

$$\frac{192\,Aa^3bdx\cos\left(dx+c\right)^4+3\left(8\,Ca^4+24\,(2\,A+C)a^2b^2+(4\,A+3\,C)b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,Ca\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(192*A*a^3*b*d*x*cos(d*x + c)^4 + 3*(8*C*a^4 + 24*(2*A + C)*a^2*b^2 +
(4*A + 3*C)*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*C*a^4 + 24*(2*
A + C)*a^2*b^2 + (4*A + 3*C)*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(24*A*a^4*cos(d*x + c)^4 + 32*C*a*b^3*cos(d*x + c) + 6*C*b^4 + 32*(3*C*a^3
*b + (3*A + 2*C)*a*b^3)*cos(d*x + c)^3 + 3*(24*C*a^2*b^2 + (4*A + 3*C)*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26404, size = 797, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(96*(d*x + c)*A*a^3*b + 48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1) + 3*(8*C*a^4 + 48*A*a^2*b^2 + 24*C*a^2*b^2 + 4*A*b^4 + 3*C*b^4
)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*C*a^4 + 48*A*a^2*b^2 + 24*C*a^2
*b^2 + 4*A*b^4 + 3*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*C*a^3*
b*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 96*A*a*b^3
*tan(1/2*d*x + 1/2*c)^7 + 96*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 12*A*b^4*tan(
1/2*d*x + 1/2*c)^7 - 15*C*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*C*a^3*b*tan(1/2*
d*x + 1/2*c)^5 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*A*a*b^3*tan(1/2*
d*x + 1/2*c)^5 - 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*A*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 9*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*C*a^3*b*tan(1/2*d*x + 1/2
*c)^3 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 288*A*a*b^3*tan(1/2*d*x + 1/2
*c)^3 + 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^4*tan(1/2*d*x + 1/2*c)^
```

```
3 - 9*C*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c) - 72*C
*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c) - 96*C*a*b^
3*tan(1/2*d*x + 1/2*c) - 12*A*b^4*tan(1/2*d*x + 1/2*c) - 15*C*b^4*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.667 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=219

$$-\frac{b^2\left(a^2(39A-34C)-2b^2(3A+2C)\right)\tan(c+dx)}{6d}+\frac{2ab\left(C\left(2a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x\left(a^2(A+\right.$$

```
[Out] (a^2*(12*A*b^2 + a^2*(A + 2*C))*x)/2 + (2*a*b*(2*A*b^2 + (2*a^2 + b^2)*C)*A
rcTanh[Sin[c + d*x]])/d + (2*A*b*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d + (
A*Cos[c + d*x]*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(2*d) - (b^2*(a^2*(39*A
 - 34*C) - 2*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*d) - (a*b^3*(9*A - 4*C)*Sec[
c + d*x]*Tan[c + d*x])/(3*d) - (b^2*(15*A - 2*C)*(a + b*Sec[c + d*x])^2*Tan
[c + d*x])/(6*d)
```

Rubi [A] time = 0.60505, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4056, 4048, 3770, 3767, 8}

$$-\frac{b^2\left(a^2(39A-34C)-2b^2(3A+2C)\right)\tan(c+dx)}{6d}+\frac{2ab\left(C\left(2a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}a^2x\left(a^2(A+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a^2*(12*A*b^2 + a^2*(A + 2*C))*x)/2 + (2*a*b*(2*A*b^2 + (2*a^2 + b^2)*C)*A
rcTanh[Sin[c + d*x]])/d + (2*A*b*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d + (
A*Cos[c + d*x]*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(2*d) - (b^2*(a^2*(39*A
 - 34*C) - 2*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*d) - (a*b^3*(9*A - 4*C)*Sec[
c + d*x]*Tan[c + d*x])/(3*d) - (b^2*(15*A - 2*C)*(a + b*Sec[c + d*x])^2*Tan
[c + d*x])/(6*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^4\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c$$
$$= \frac{2Ab(a+b\sec(c+dx))^3\sin(c+dx)}{d}+\frac{A\cos(c+dx)(a+b}{}$$
$$= \frac{2Ab(a+b\sec(c+dx))^3\sin(c+dx)}{d}+\frac{A\cos(c+dx)(a+b}{}$$
$$= \frac{2Ab(a+b\sec(c+dx))^3\sin(c+dx)}{d}+\frac{A\cos(c+dx)(a+b}{}$$
$$= \frac{1}{2}a^2\left(12Ab^2+a^2(A+2C)\right)x+\frac{2Ab(a+b\sec(c+dx))^3\sin}{d}$$
$$= \frac{1}{2}a^2\left(12Ab^2+a^2(A+2C)\right)x+\frac{2ab\left(2Ab^2+\left(2a^2+b^2\right)C\right)}{d}$$
$$= \frac{1}{2}a^2\left(12Ab^2+a^2(A+2C)\right)x+\frac{2ab\left(2Ab^2+\left(2a^2+b^2\right)C\right)}{d}$$

Mathematica [A] time = 6.11514, size = 416, normalized size = 1.9

$$6a^2(c+dx)\left(a^2(A+2C)+12Ab^2\right)+\frac{4b^2\left(2C\left(9a^2+b^2\right)+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4b^2\left(2C\left(9a^2+b^2\right)+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}-24ab\left(C\left(2a^2+b^2\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (6*a^2*(12*A*b^2 + a^2*(A + 2*C))*(c + d*x) - 24*a*b*(2*A*b^2 + (2*a^2 + b^
2)*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*A*b^2 + (2*a^2 +
 b^2)*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b)*C)/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*C*Sin[(c + d*x)/2])/(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^3 + (4*b^2*(3*A*b^2 + 2*(9*a^2 + b^2)*C)*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*b^4*C*Sin[(c + d*x)/
2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (b^3*(12*a + b)*C)/(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b^2*(3*A*b^2 + 2*(9*a^2 + b^2)*C)*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 48*a^3*A*b*Sin[c + d*x]
 + 3*a^4*A*Sin[2*(c + d*x)])/(12*d)
```

Maple [A] time = 0.085, size = 258, normalized size = 1.2

$$\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4Ax}{2}+\frac{Aa^4c}{2d}+a^4Cx+\frac{Ca^4c}{d}+4\frac{Aa^3b\sin(dx+c)}{d}+4\frac{a^3bC\ln(\sec(dx+c)+\tan(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+1/2*a^4*A*x+1/2/d*A*a^4*c+a^4*C*x+1/d*C*a
^4*c+4/d*A*a^3*b*sin(d*x+c)+4/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+6*A*a^2*b
^2*x+6/d*A*a^2*b^2*c+6/d*C*a^2*b^2*tan(d*x+c)+4/d*A*a*b^3*ln(sec(d*x+c)+tan
(d*x+c))+2/d*C*a*b^3*sec(d*x+c)*tan(d*x+c)+2/d*C*a*b^3*ln(sec(d*x+c)+tan(d*
x+c))+1/d*A*b^4*tan(d*x+c)+2/3/d*C*b^4*tan(d*x+c)+1/3/d*C*b^4*tan(d*x+c)*se
c(d*x+c)^2
```

Maxima [A] time = 0.987792, size = 298, normalized size = 1.36

$$\frac{3(2dx+2c+\sin(2dx+2c))Aa^4+12(dx+c)Ca^4+72(dx+c)Aa^2b^2+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cb^4-12\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 12*(d*x + c)*C*a^4 + 72*(d
*x + c)*A*a^2*b^2 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b^4 - 12*C*a*b^3*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)) + 24*C*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2
4*A*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^3*b*sin(
d*x + c) + 72*C*a^2*b^2*tan(d*x + c) + 12*A*b^4*tan(d*x + c))/d
```

Fricas [A] time = 0.584596, size = 509, normalized size = 2.32

$$\frac{3\left((A+2C)a^4+12Aa^2b^2\right)dx\cos(dx+c)^3+6\left(2Ca^3b+(2A+C)ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-6\left(2Ca^3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(3*((A + 2*C)*a^4 + 12*A*a^2*b^2)*d*x*cos(d*x + c)^3 + 6*(2*C*a^3*b + (
2*A + C)*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 6*(2*C*a^3*b + (2*A
+ C)*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (3*A*a^4*cos(d*x + c)^4
 + 24*A*a^3*b*cos(d*x + c)^3 + 12*C*a*b^3*cos(d*x + c) + 2*C*b^4 + 2*(18*C*
a^2*b^2 + (3*A + 2*C)*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21575, size = 536, normalized size = 2.45

$$3\left(Aa^4 + 2\,Ca^4 + 12\,Aa^2b^2\right)(dx + c) + 12\left(2\,Ca^3b + 2\,Aab^3 + Cab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 12\left(2\,Ca^3b + 2\,Aab^3 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(3*(A*a^4 + 2*C*a^4 + 12*A*a^2*b^2)*(d*x + c) + 12*(2*C*a^3*b + 2*A*a*b
^3 + C*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*(2*C*a^3*b + 2*A*a*b^
3 + C*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*(A*a^4*tan(1/2*d*x + 1/
2*c)^3 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - A*a^4*tan(1/2*d*x + 1/2*c) - 8*
A*a^3*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 4*(18*C*a^2*
b^2*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^4*tan
(1/2*d*x + 1/2*c)^5 + 3*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 36*C*a^2*b^2*tan(1/2
*d*x + 1/2*c)^3 - 6*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*C*b^4*tan(1/2*d*x + 1/
2*c)^3 + 18*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*C*a*b^3*tan(1/2*d*x + 1/2*c)
 + 3*A*b^4*tan(1/2*d*x + 1/2*c) + 3*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^3)/d
```

3.668 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=251

$$-\frac{2ab\left(a^2(2A+3C)+b^2(11A-6C)\right)\tan(c+dx)}{3d}+\frac{b^2\left(C\left(12a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(a^2(2A+3C)+\right.}{}$$

```
[Out] 2*a*b*(2*A*b^2 + a^2*(A + 2*C))*x + (b^2*(2*A*b^2 + (12*a^2 + b^2)*C)*ArcTa
nh[Sin[c + d*x]])/(2*d) + ((6*A*b^2 + a^2*(2*A + 3*C))*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(3*d) + (2*A*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c +
 d*x])/(3*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(3*d)
 - (2*a*b*(b^2*(11*A - 6*C) + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*d) - (b^2*(
3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.754271, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4095, 4094, 4048, 3770, 3767, 8}

$$-\frac{2ab\left(a^2(2A+3C)+b^2(11A-6C)\right)\tan(c+dx)}{3d}+\frac{b^2\left(C\left(12a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(a^2(2A+3C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] 2*a*b*(2*A*b^2 + a^2*(A + 2*C))*x + (b^2*(2*A*b^2 + (12*a^2 + b^2)*C)*ArcTa
nh[Sin[c + d*x]])/(2*d) + ((6*A*b^2 + a^2*(2*A + 3*C))*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(3*d) + (2*A*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c +
 d*x])/(3*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(3*d)
 - (2*a*b*(b^2*(11*A - 6*C) + a^2*(2*A + 3*C))*Tan[c + d*x])/(3*d) - (b^2*(
3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
```

```
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^4\sin(c+dx)}{3d}+\frac{1}{3}\int \cos$$

$$= \frac{2Ab\cos(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{3d}+\frac{A\cos^2(}{}$$

$$= \frac{\left(6Ab^2+a^2(2A+3C)\right)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}+$$

$$= \frac{\left(6Ab^2+a^2(2A+3C)\right)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}+$$

$$= 2ab\left(2Ab^2+a^2(A+2C)\right)x+\frac{\left(6Ab^2+a^2(2A+3C)\right)(a+}{3a}$$

$$= 2ab\left(2Ab^2+a^2(A+2C)\right)x+\frac{b^2\left(2Ab^2+\left(12a^2+b^2\right)C\right)t}{2d}$$

$$= 2ab\left(2Ab^2+a^2(A+2C)\right)x+\frac{b^2\left(2Ab^2+\left(12a^2+b^2\right)C\right)t}{2d}$$

Mathematica [A] time = 3.27344, size = 324, normalized size = 1.29

$$\frac{24ab(c+dx)\left(a^2(A+2C)+2Ab^2\right)+3a^2\left(a^2(3A+4C)+24Ab^2\right)\sin(c+dx)-6b^2\left(C\left(12a^2+b^2\right)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (24*a*b*(2*A*b^2 + a^2*(A + 2*C))*(c + d*x) - 6*b^2*(2*A*b^2 + (12*a^2 + b^
2)*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*b^2*(2*A*b^2 + (12*a^2 +
 b^2)*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*b^4*C)/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2 + (48*a*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - (3*b^4*C)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 +
 (48*a*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 3*a^
2*(24*A*b^2 + a^2*(3*A + 4*C))*Sin[c + d*x] + 12*a^3*A*b*Sin[2*(c + d*x)] +
```

```
a^4*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.076, size = 259, normalized size = 1.

$$\frac{A(\cos(dx+c))^2\sin(dx+c)\,a^4}{3d}+\frac{2\,Aa^4\sin(dx+c)}{3d}+\frac{a^4C\sin(dx+c)}{d}+2\,\frac{Aa^3b\sin(dx+c)\cos(dx+c)}{d}+2\,a^3Abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^4+2/3/d*A*a^4*sin(d*x+c)+1/d*a^4*C*sin(d*
x+c)+2/d*A*a^3*b*sin(d*x+c)*cos(d*x+c)+2*a^3*A*b*x+2/d*A*a^3*b*c+4*a^3*b*C*
x+4/d*C*a^3*b*c+6/d*A*a^2*b^2*sin(d*x+c)+6/d*C*a^2*b^2*ln(sec(d*x+c)+tan(d*
x+c))+4*A*a*b^3*x+4/d*A*a*b^3*c+4/d*C*a*b^3*tan(d*x+c)+1/d*A*b^4*ln(sec(d*x
+c)+tan(d*x+c))+1/2/d*C*b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*C*b^4*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 1.01557, size = 298, normalized size = 1.19

$$-\frac{4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^4-12\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3b-48\,(dx+c)Ca^3b-48\,(dx+c)Aab^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 12*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a^3*b - 48*(d*x + c)*C*a^3*b - 48*(d*x + c)*A*a*b^3 + 3*C*b^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) - 36*C*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
- 6*A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 12*C*a^4*sin(d*
x + c) - 72*A*a^2*b^2*sin(d*x + c) - 48*C*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.581085, size = 516, normalized size = 2.06

$$\frac{24\left((A+2\,C)a^3b+2\,Aab^3\right)dx\cos(dx+c)^2+3\left(12\,Ca^2b^2+(2\,A+C)b^4\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-3\left(12\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/12*(24*((A + 2*C)*a^3*b + 2*A*a*b^3)*d*x*cos(d*x + c)^2 + 3*(12*C*a^2*b^2
 + (2*A + C)*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(12*C*a^2*b^2 +
(2*A + C)*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*a^4*cos(d*x +
 c)^4 + 12*A*a^3*b*cos(d*x + c)^3 + 24*C*a*b^3*cos(d*x + c) + 3*C*b^4 + 2*(
(2*A + 3*C)*a^4 + 18*A*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24721, size = 537, normalized size = 2.14

$$12\left(Aa^3b+2\,Ca^3b+2\,Aab^3\right)(dx+c)+3\left(12\,Ca^2b^2+2\,Ab^4+Cb^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(12\,Ca^2b^2+2\,Ab^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(12*(A*a^3*b + 2*C*a^3*b + 2*A*a*b^3)*(d*x + c) + 3*(12*C*a^2*b^2 + 2*A
*b^4 + C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(12*C*a^2*b^2 + 2*A*b^
4 + C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*(8*C*a*b^3*tan(1/2*d*x +
1/2*c)^3 - C*b^4*tan(1/2*d*x + 1/2*c)^3 - 8*C*a*b^3*tan(1/2*d*x + 1/2*c) -
C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 4*(3*A*a^4*tan
(1/2*d*x + 1/2*c)^5 + 3*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*b*tan(1/2*d*
x + 1/2*c)^5 + 18*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*A*a^4*tan(1/2*d*x +
1/2*c)^3 + 6*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) + 3*C*a^4*tan(1/2*d*x + 1/2*c) + 6*A*a^
3*b*tan(1/2*d*x + 1/2*c) + 18*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

3.669 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=246

$$\frac{ab\left(a^2(23A+36C)+12Ab^2\right)\sin(c+dx)}{12d}-\frac{b^2\left(3a^2(3A+4C)+2b^2(13A-12C)\right)\tan(c+dx)}{24d}+\frac{\left(a^2(3A+4C)+4Ab\right.}{}$$

```
[Out] ((8*A*b^4 + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*x)/8 + (4*a*b^3*C*ArcTa
nh[Sin[c + d*x]])/d + (a*b*(12*A*b^2 + a^2*(23*A + 36*C))*Sin[c + d*x])/(12
*d) + ((4*A*b^2 + a^2*(3*A + 4*C))*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[
c + d*x])/(8*d) + (A*b*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(3*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(4*d) - (b^2
*(2*b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.846502, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4076, 4047, 8, 4045, 3770}

$$\frac{ab\left(a^2(23A+36C)+12Ab^2\right)\sin(c+dx)}{12d}-\frac{b^2\left(3a^2(3A+4C)+2b^2(13A-12C)\right)\tan(c+dx)}{24d}+\frac{\left(a^2(3A+4C)+4Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] ((8*A*b^4 + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*x)/8 + (4*a*b^3*C*ArcTa
nh[Sin[c + d*x]])/d + (a*b*(12*A*b^2 + a^2*(23*A + 36*C))*Sin[c + d*x])/(12
*d) + ((4*A*b^2 + a^2*(3*A + 4*C))*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[
c + d*x])/(8*d) + (A*b*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(3*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(4*d) - (b^2
*(2*b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Tan[c + d*x])/(24*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
```

```
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^4\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3$$

$$= \frac{Ab\cos^2(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{3d}+\frac{A\cos^3(}{}$$

$$= \frac{\left(4Ab^2+a^2(3A+4C)\right)\cos(c+dx)(a+b\sec(c+dx))^2\sin}{8d}$$

$$= \frac{\left(4Ab^2+a^2(3A+4C)\right)\cos(c+dx)(a+b\sec(c+dx))^2\sin}{8d}$$

$$= \frac{\left(4Ab^2+a^2(3A+4C)\right)\cos(c+dx)(a+b\sec(c+dx))^2\sin}{8d}$$

$$= \frac{1}{8}\left(8Ab^4+24a^2b^2(A+2C)+a^4(3A+4C)\right)x+\frac{ab\left(12Ab^2\right.}{}$$

$$= \frac{1}{8}\left(8Ab^4+24a^2b^2(A+2C)+a^4(3A+4C)\right)x+\frac{4ab^3C\tan}{}$$

Mathematica [A] time = 1.39823, size = 270, normalized size = 1.1

$$\frac{12(c+dx)\left(24a^2b^2(A+2C)+a^4(3A+4C)+8Ab^4\right)+24a^2\left(a^2(A+C)+6Ab^2\right)\sin(2(c+dx))+96ab\left(a^2(3A+4C)+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (12*(8*A*b^4 + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*(c + d*x) - 384*a*b^
3*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 384*a*b^3*C*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + (96*b^4*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]) + (96*b^4*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]) + 96*a*b*(4*A*b^2 + a^2*(3*A + 4*C))*Sin[c + d*x] + 24*a^2*(6*A*
b^2 + a^2*(A + C))*Sin[2*(c + d*x)] + 32*a^3*A*b*Sin[3*(c + d*x)] + 3*a^4*A
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.071, size = 296, normalized size = 1.2

$$\frac{Aa^4\sin(dx+c)(\cos(dx+c))^3}{4d}+\frac{3Aa^4\cos(dx+c)\sin(dx+c)}{8d}+\frac{3a^4Ax}{8}+\frac{3Aa^4c}{8d}+\frac{a^4C\sin(dx+c)\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+3/8/d*A*a^4*cos(d*x+c)*sin(d*x+c)+3/8*a
^4*A*x+3/8/d*A*a^4*c+1/2/d*a^4*C*sin(d*x+c)*cos(d*x+c)+1/2*a^4*C*x+1/2/d*C*
a^4*c+4/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3*b+8/3/d*A*a^3*b*sin(d*x+c)+4/d*a^
3*b*C*sin(d*x+c)+3/d*A*a^2*b^2*sin(d*x+c)*cos(d*x+c)+3*A*a^2*b^2*x+3/d*A*a^
2*b^2*c+6*C*a^2*b^2*x+6/d*C*a^2*b^2*c+4/d*A*a*b^3*sin(d*x+c)+4/d*C*a*b^3*ln
(sec(d*x+c)+tan(d*x+c))+A*b^4*x+1/d*A*b^4*c+1/d*C*b^4*tan(d*x+c)
```

Maxima [A] time = 1.01347, size = 275, normalized size = 1.12

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^4+24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ca^4-128\left(\sin(dx+c)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 24*
(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^4 - 128*(sin(d*x + c)^3 - 3*sin(d*x +
c))*A*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^2 + 576*(d*x + c
)*C*a^2*b^2 + 96*(d*x + c)*A*b^4 + 192*C*a*b^3*(log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 384*C*a^3*b*sin(d*x + c) + 384*A*a*b^3*sin(d*x + c) +
 96*C*b^4*tan(d*x + c))/d
```

Fricas [A] time = 0.584909, size = 504, normalized size = 2.05

$$\frac{48\,Cab^3\cos(dx+c)\log(\sin(dx+c)+1)-48\,Cab^3\cos(dx+c)\log(-\sin(dx+c)+1)+3\left((3\,A+4\,C)a^4+24\,(A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/24*(48*C*a*b^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 48*C*a*b^3*cos(d*x +
c)*log(-sin(d*x + c) + 1) + 3*((3*A + 4*C)*a^4 + 24*(A + 2*C)*a^2*b^2 + 8*A
*b^4)*d*x*cos(d*x + c) + (6*A*a^4*cos(d*x + c)^4 + 32*A*a^3*b*cos(d*x + c)^
3 + 24*C*b^4 + 3*((3*A + 4*C)*a^4 + 24*A*a^2*b^2)*cos(d*x + c)^2 + 32*((2*A
 + 3*C)*a^3*b + 3*A*a*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28809, size = 753, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/24*(96*C*a*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 96*C*a*b^3*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 48*C*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1) + 3*(3*A*a^4 + 4*C*a^4 + 24*A*a^2*b^2 + 48*C*a^2*b^2 + 8*A*b^4)*
(d*x + c) - 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^4*tan(1/2*d*x + 1/2
*c)^7 - 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c)
^7 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^
7 - 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*
A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a
^4*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*t
an(1/2*d*x + 1/2*c)^3 - 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1
/2*d*x + 1/2*c) - 12*C*a^4*tan(1/2*d*x + 1/2*c) - 96*A*a^3*b*tan(1/2*d*x +
1/2*c) - 96*C*a^3*b*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c
) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.670 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4 \left(A + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=250

$$\frac{\left(a^2b^2(56A+85C)+2a^4(4A+5C)+6Ab^4\right)\sin(c+dx)}{15d}+\frac{ab\left(a^2(29A+40C)+6Ab^2\right)\sin(c+dx)\cos(c+dx)}{30d}+\frac{(a^2(\cdots$$

```
[Out] (a*b*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/2 + (b^4*C*ArcTanh[Sin[c + d*x]
])/d + ((6*A*b^4 + 2*a^4*(4*A + 5*C) + a^2*b^2*(56*A + 85*C))*Sin[c + d*x])
/(15*d) + (a*b*(6*A*b^2 + a^2*(29*A + 40*C))*Cos[c + d*x]*Sin[c + d*x])/(30
*d) + ((3*A*b^2 + a^2*(4*A + 5*C))*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(15*d) + (A*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[c + d*x
])/(5*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.884453, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4074, 4047, 8, 4045, 3770}

$$\frac{\left(a^2b^2(56A+85C)+2a^4(4A+5C)+6Ab^4\right)\sin(c+dx)}{15d}+\frac{ab\left(a^2(29A+40C)+6Ab^2\right)\sin(c+dx)\cos(c+dx)}{30d}+\frac{(a^2(\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*b*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/2 + (b^4*C*ArcTanh[Sin[c + d*x]
])/d + ((6*A*b^4 + 2*a^4*(4*A + 5*C) + a^2*b^2*(56*A + 85*C))*Sin[c + d*x])
/(15*d) + (a*b*(6*A*b^2 + a^2*(29*A + 40*C))*Cos[c + d*x]*Sin[c + d*x])/(30
*d) + ((3*A*b^2 + a^2*(4*A + 5*C))*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(15*d) + (A*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[c + d*x
])/(5*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(5*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4$$
$$= \frac{Ab\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{5d}+\frac{A\cos^4(}{}$$
$$= \frac{\left(3Ab^2+a^2(4A+5C)\right)\cos^2(c+dx)(a+b\sec(c+dx))^2\sin}{15d}$$
$$= \frac{ab\left(6Ab^2+a^2(29A+40C)\right)\cos(c+dx)\sin(c+dx)}{30d}+\frac{(3A}{}$$
$$= \frac{ab\left(6Ab^2+a^2(29A+40C)\right)\cos(c+dx)\sin(c+dx)}{30d}+\frac{(3A}{}$$
$$= \frac{1}{2}ab\left(4b^2(A+2C)+a^2(3A+4C)\right)x+\frac{\left(6Ab^4+2a^4(4A+\right.}{}$$
$$= \frac{1}{2}ab\left(4b^2(A+2C)+a^2(3A+4C)\right)x+\frac{b^4C\tanh^{-1}(\sin(c}{d}$$

Mathematica [A] time = 0.83344, size = 223, normalized size = 0.89

$$120ab(c+dx)\left(a^2(3A+4C)+4b^2(A+2C)\right)+5a^2\left(a^2(5A+4C)+24Ab^2\right)\sin(3(c+dx))+240ab\left(a^2(A+C)+Ab^2\right)\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (120*a*b*(4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*(c + d*x) - 240*b^4*C*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] + 240*b^4*C*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + 30*(8*A*b^4 + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*Sin[c
+ d*x] + 240*a*b*(A*b^2 + a^2*(A + C))*Sin[2*(c + d*x)] + 5*a^2*(24*A*b^2 +
 a^2*(5*A + 4*C))*Sin[3*(c + d*x)] + 30*a^3*A*b*Sin[4*(c + d*x)] + 3*a^4*A*
Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.085, size = 364, normalized size = 1.5

$$\frac{8\,Aa^4\sin(dx+c)}{15\,d}+\frac{Aa^4\sin(dx+c)(\cos(dx+c))^4}{5\,d}+\frac{4\,A(\cos(dx+c))^2\sin(dx+c)\,a^4}{15\,d}+\frac{C\sin(dx+c)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 8/15/d*A*a^4*sin(d*x+c)+1/5/d*A*a^4*sin(d*x+c)*cos(d*x+c)^4+4/15/d*A*cos(d*
x+c)^2*sin(d*x+c)*a^4+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3/d*a^4*C*sin(d
*x+c)+1/d*A*a^3*b*sin(d*x+c)*cos(d*x+c)^3+3/2/d*A*a^3*b*sin(d*x+c)*cos(d*x+
c)+3/2*a^3*A*b*x+3/2/d*A*a^3*b*c+2/d*a^3*b*C*cos(d*x+c)*sin(d*x+c)+2*a^3*b*
C*x+2/d*C*a^3*b*c+2/d*A*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+4/d*A*a^2*b^2*sin(d
*x+c)+6/d*C*a^2*b^2*sin(d*x+c)+2/d*A*a*b^3*cos(d*x+c)*sin(d*x+c)+2*A*a*b^3*
x+2/d*A*a*b^3*c+4*C*a*b^3*x+4/d*C*a*b^3*c+1/d*A*b^4*sin(d*x+c)+1/d*C*b^4*ln
(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.987768, size = 323, normalized size = 1.29

$$\frac{8\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)Aa^4-40\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^4+15\,(12\,dx+12\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 - 4
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 15*(12*d*x + 12*c + sin(4*d*x +
 4*c) + 8*sin(2*d*x + 2*c))*A*a^3*b + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*
C*a^3*b - 240*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 + 120*(2*d*x + 2*
c + sin(2*d*x + 2*c))*A*a*b^3 + 480*(d*x + c)*C*a*b^3 + 60*C*b^4*(log(sin(d
*x + c) + 1) - log(sin(d*x + c) - 1)) + 720*C*a^2*b^2*sin(d*x + c) + 120*A*
b^4*sin(d*x + c))/d
```

Fricas [A] time = 0.57952, size = 473, normalized size = 1.89

$$\frac{15\,Cb^4\log(\sin(dx+c)+1)-15\,Cb^4\log(-\sin(dx+c)+1)+15\left((3\,A+4\,C)a^3b+4\,(A+2\,C)ab^3\right)dx+\left(6\,Aa^4\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/30*(15*C*b^4*log(sin(d*x + c) + 1) - 15*C*b^4*log(-sin(d*x + c) + 1) + 15
*((3*A + 4*C)*a^3*b + 4*(A + 2*C)*a*b^3)*d*x + (6*A*a^4*cos(d*x + c)^4 + 30
*A*a^3*b*cos(d*x + c)^3 + 4*(4*A + 5*C)*a^4 + 60*(2*A + 3*C)*a^2*b^2 + 30*A
*b^4 + 2*((4*A + 5*C)*a^4 + 30*A*a^2*b^2)*cos(d*x + c)^2 + 15*((3*A + 4*C)*
a^3*b + 4*A*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28982, size = 1017, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/30*(30*C*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 30*C*b^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 15*(3*A*a^3*b + 4*C*a^3*b + 4*A*a*b^3 + 8*C*a*b^3)*(
d*x + c) + 2*(30*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 30*C*a^4*tan(1/2*d*x + 1/2*
c)^9 - 75*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 60*C*a^3*b*tan(1/2*d*x + 1/2*c)^
9 + 180*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 180*C*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 60*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*A*b^4*tan(1/2*d*x + 1/2*c)^9
+ 40*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 80*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 30*A*
a^3*b*tan(1/2*d*x + 1/2*c)^7 - 120*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*A*a
^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*
A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 116*A*a
^4*tan(1/2*d*x + 1/2*c)^5 + 100*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*A*a^2*b^
2*tan(1/2*d*x + 1/2*c)^5 + 1080*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 180*A*b^
4*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 80*C*a^4*tan(1
/2*d*x + 1/2*c)^3 + 30*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 120*C*a^3*b*tan(1/2
*d*x + 1/2*c)^3 + 480*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*C*a^2*b^2*tan(
1/2*d*x + 1/2*c)^3 + 120*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*b^4*tan(1/2
*d*x + 1/2*c)^3 + 30*A*a^4*tan(1/2*d*x + 1/2*c) + 30*C*a^4*tan(1/2*d*x + 1/
2*c) + 75*A*a^3*b*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b*tan(1/2*d*x + 1/2*c) +
180*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 180*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6
0*A*a*b^3*tan(1/2*d*x + 1/2*c) + 30*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^5)/d
```

3.671 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=298

$$\frac{4ab\left(2a^2(4A+5C)+5b^2(2A+3C)\right)\sin(c+dx)}{15d}+\frac{ab\left(a^2(39A+50C)+4Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{\left(10a^2b^2(4\cdots\right.}{}$$

```
[Out] ((8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*x)/16 + (4*a*
b*(5*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Sin[c + d*x])/(15*d) + ((24*A*b^4
 + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(49*A + 66*C))*Cos[c + d*x]*Sin[c + d*x]
)/(240*d) + (a*b*(4*A*b^2 + a^2*(39*A + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])
/(60*d) + ((12*A*b^2 + 5*a^2*(5*A + 6*C))*Cos[c + d*x]^3*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(120*d) + (2*A*b*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*S
in[c + d*x])/(15*d) + (A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*Sin[c + d*x]
)/(6*d)
```

Rubi [A] time = 1.03726, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4094, 4074, 4047, 2637, 4045, 8}

$$\frac{4ab\left(2a^2(4A+5C)+5b^2(2A+3C)\right)\sin(c+dx)}{15d}+\frac{ab\left(a^2(39A+50C)+4Ab^2\right)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{\left(10a^2b^2(4\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] ((8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*x)/16 + (4*a*
b*(5*b^2*(2*A + 3*C) + 2*a^2*(4*A + 5*C))*Sin[c + d*x])/(15*d) + ((24*A*b^4
 + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(49*A + 66*C))*Cos[c + d*x]*Sin[c + d*x]
)/(240*d) + (a*b*(4*A*b^2 + a^2*(39*A + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])
/(60*d) + ((12*A*b^2 + 5*a^2*(5*A + 6*C))*Cos[c + d*x]^3*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(120*d) + (2*A*b*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*S
in[c + d*x])/(15*d) + (A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*Sin[c + d*x]
)/(6*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+b\sec(c+dx))^4\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^5$$

$$= \frac{2Ab\cos^4(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{15d}+\frac{A\cos^5}{}$$

$$= \frac{\left(12Ab^2+5a^2(5A+6C)\right)\cos^3(c+dx)(a+b\sec(c+dx))^2}{120d}$$

$$= \frac{ab\left(4Ab^2+a^2(39A+50C)\right)\cos^2(c+dx)\sin(c+dx)}{60d}+\frac{(1}{}$$

$$= \frac{ab\left(4Ab^2+a^2(39A+50C)\right)\cos^2(c+dx)\sin(c+dx)}{60d}+\frac{(1}{}$$

$$= \frac{4ab\left(5b^2(2A+3C)+2a^2(4A+5C)\right)\sin(c+dx)}{15d}+\frac{(24Ab}{}$$

$$= \frac{1}{16}\left(8b^4(A+2C)+12a^2b^2(3A+4C)+a^4(5A+6C)\right)x+$$

Mathematica [A] time = 0.906359, size = 302, normalized size = 1.01

$$\frac{480ab\left(a^2(5A+6C)+2b^2(3A+4C)\right)\sin(c+dx)+15\left(96a^2b^2(A+C)+a^4(15A+16C)+16Ab^4\right)\sin(2(c+dx))+180a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (300*a^4*A*c + 2160*a^2*A*b^2*c + 480*A*b^4*c + 360*a^4*c*C + 2880*a^2*b^2*
c*C + 960*b^4*c*C + 300*a^4*A*d*x + 2160*a^2*A*b^2*d*x + 480*A*b^4*d*x + 36
0*a^4*C*d*x + 2880*a^2*b^2*C*d*x + 960*b^4*C*d*x + 480*a*b*(2*b^2*(3*A + 4*
C) + a^2*(5*A + 6*C))*Sin[c + d*x] + 15*(16*A*b^4 + 96*a^2*b^2*(A + C) + a^
4*(15*A + 16*C))*Sin[2*(c + d*x)] + 400*a^3*A*b*Sin[3*(c + d*x)] + 320*a*A*
b^3*Sin[3*(c + d*x)] + 320*a^3*b*C*Sin[3*(c + d*x)] + 45*a^4*A*Sin[4*(c + d
*x)] + 180*a^2*A*b^2*Sin[4*(c + d*x)] + 30*a^4*C*Sin[4*(c + d*x)] + 48*a^3*
A*b*Sin[5*(c + d*x)] + 5*a^4*A*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.084, size = 294, normalized size = 1.

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4Aa^3b\sin(dx+c)}{5}\left(\frac{8}{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4/5*A*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
+6*A*a^2*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a
^4*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a*b
^3*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a^3*b*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b^
4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*C*a^2*b^2*(1/2*cos(d*x+c)*sin
(d*x+c)+1/2*d*x+1/2*c)+4*C*a*b^3*sin(d*x+c)+C*b^4*(d*x+c))
```

Maxima [A] time = 1.01971, size = 382, normalized size = 1.28

$$-\frac{5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Aa^4-30(12dx+12c+\sin(4dx+4c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^4 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*C*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c
))*A*a^3*b + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3*b - 180*(12*d*x +
 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b^2 - 1440*(2*d*x + 2*
c + sin(2*d*x + 2*c))*C*a^2*b^2 + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*
a*b^3 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 960*(d*x + c)*C*b^4 -
3840*C*a*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.567601, size = 504, normalized size = 1.69

$$\frac{15\left((5A+6C)a^4+12(3A+4C)a^2b^2+8(A+2C)b^4\right)dx+\left(40Aa^4\cos(dx+c)^5+192Aa^3b\cos(dx+c)^4+128(4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(15*((5*A + 6*C)*a^4 + 12*(3*A + 4*C)*a^2*b^2 + 8*(A + 2*C)*b^4)*d*x
+ (40*A*a^4*cos(d*x + c)^5 + 192*A*a^3*b*cos(d*x + c)^4 + 128*(4*A + 5*C)*a
^3*b + 320*(2*A + 3*C)*a*b^3 + 10*((5*A + 6*C)*a^4 + 36*A*a^2*b^2)*cos(d*x
+ c)^3 + 64*((4*A + 5*C)*a^3*b + 5*A*a*b^3)*cos(d*x + c)^2 + 15*((5*A + 6*C
)*a^4 + 12*(3*A + 4*C)*a^2*b^2 + 8*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27712, size = 1396, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 6*C*a^4 + 36*A*a^2*b^2 + 48*C*a^2*b^2 + 8*A*b^4 + 16*C
*b^4)*(d*x + c) - 2*(165*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 150*C*a^4*tan(1/2*
d*x + 1/2*c)^11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 - 960*C*a^3*b*tan(1/2
*d*x + 1/2*c)^11 + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 720*C*a^2*b^2*ta
n(1/2*d*x + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 960*C*a*b^3*t
an(1/2*d*x + 1/2*c)^11 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^11 - 25*A*a^4*tan(1
/2*d*x + 1/2*c)^9 + 210*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*A*a^3*b*tan(1/2
*d*x + 1/2*c)^9 - 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 1260*A*a^2*b^2*tan(
1/2*d*x + 1/2*c)^9 + 2160*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*A*a*b^3*t
an(1/2*d*x + 1/2*c)^9 - 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 360*A*b^4*tan
(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^4*tan(1/2*d
*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 5760*C*a^3*b*tan(1/2*
d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a^2*b^2*tan(
1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9600*C*a*b^3*tan
(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*A*a^4*tan(1/2*
d*x + 1/2*c)^5 - 60*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 4992*A*a^3*b*tan(1/2*d*x
 + 1/2*c)^5 - 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^5 - 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*A*a*b^3*tan(1/
2*d*x + 1/2*c)^5 - 9600*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*A*b^4*tan(1/2*
d*x + 1/2*c)^5 + 25*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^4*tan(1/2*d*x +
1/2*c)^3 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 3520*C*a^3*b*tan(1/2*d*x +
 1/2*c)^3 - 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2160*C*a^2*b^2*tan(1/2*
d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 4800*C*a*b^3*tan(1/2
*d*x + 1/2*c)^3 - 360*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*A*a^4*tan(1/2*d*x
```

```
+ 1/2*c) - 150*C*a^4*tan(1/2*d*x + 1/2*c) - 960*A*a^3*b*tan(1/2*d*x + 1/2*c
) - 960*C*a^3*b*tan(1/2*d*x + 1/2*c) - 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) -
 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*A*a*b^3*tan(1/2*d*x + 1/2*c) - 96
0*C*a*b^3*tan(1/2*d*x + 1/2*c) - 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 + 1)^6)/d
```

3.672 $\int \cos^7(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=339

$$-\frac{\left(3a^2b^2(50A+63C)+4a^4(6A+7C)+4Ab^4\right)\sin^3(c+dx)}{105d}+\frac{\left(3a^2b^2(162A+203C)+12a^4(6A+7C)+b^4(74A+105C\right.}{105d}$$

```
[Out] (a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*x)/4 + ((12*a^4*(6*A + 7*C) + b^
4*(74*A + 105*C) + 3*a^2*b^2*(162*A + 203*C))*Sin[c + d*x])/(105*d) + (a*b*
(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a
*b*(6*A*b^2 + a^2*(103*A + 126*C))*Cos[c + d*x]^3*Sin[c + d*x])/(210*d) + (
(2*A*b^2 + a^2*(6*A + 7*C))*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d
*x])/(35*d) + (2*A*b*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(2
1*d) + (A*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(7*d) - ((4*A
*b^4 + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A + 63*C))*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 1.15132, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4095, 4094, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{\left(3a^2b^2(50A+63C)+4a^4(6A+7C)+4Ab^4\right)\sin^3(c+dx)}{105d}+\frac{\left(3a^2b^2(162A+203C)+12a^4(6A+7C)+b^4(74A+105C\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*b*(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*x)/4 + ((12*a^4*(6*A + 7*C) + b^
4*(74*A + 105*C) + 3*a^2*b^2*(162*A + 203*C))*Sin[c + d*x])/(105*d) + (a*b*
(2*b^2*(3*A + 4*C) + a^2*(5*A + 6*C))*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a
*b*(6*A*b^2 + a^2*(103*A + 126*C))*Cos[c + d*x]^3*Sin[c + d*x])/(210*d) + (
(2*A*b^2 + a^2*(6*A + 7*C))*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d
*x])/(35*d) + (2*A*b*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(2
1*d) + (A*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(7*d) - ((4*A
*b^4 + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A + 63*C))*Sin[c + d*x]^3)/(105*d)
```

Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sec(c+dx))^4\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^6(c+dx)(a+b\sec(c+dx))^4\sin(c+dx)}{7d}+\frac{1}{7}\int \cos^6$$

$$= \frac{2Ab\cos^5(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{21d}+\frac{A\cos^6}{}$$

$$= \frac{\left(2Ab^2+a^2(6A+7C)\right)\cos^4(c+dx)(a+b\sec(c+dx))^2\sin}{35d}$$

$$= \frac{ab\left(6Ab^2+a^2(103A+126C)\right)\cos^3(c+dx)\sin(c+dx)}{210d}+$$

$$= \frac{ab\left(6Ab^2+a^2(103A+126C)\right)\cos^3(c+dx)\sin(c+dx)}{210d}+$$

$$= \frac{ab\left(2b^2(3A+4C)+a^2(5A+6C)\right)\cos(c+dx)\sin(c+dx)}{4d}$$

$$= \frac{1}{4}ab\left(2b^2(3A+4C)+a^2(5A+6C)\right)x+\frac{ab\left(2b^2(3A+4C)\right.}{}$$

$$= \frac{1}{4}ab\left(2b^2(3A+4C)+a^2(5A+6C)\right)x+\frac{\left(12a^4(6A+7C)+\right.}{}$$

Mathematica [A] time = 0.858602, size = 351, normalized size = 1.04

$$\frac{420ab\left(a^2(15A+16C)+16b^2(A+C)\right)\sin(2(c+dx))+105\left(48a^2b^2(5A+6C)+5a^4(7A+8C)+16b^4(3A+4C)\right)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sec[c + d*x])^4*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (8400*a^3*A*b*c + 10080*a*A*b^3*c + 10080*a^3*b*c*C + 13440*a*b^3*c*C + 840
0*a^3*A*b*d*x + 10080*a*A*b^3*d*x + 10080*a^3*b*C*d*x + 13440*a*b^3*C*d*x +
 105*(16*b^4*(3*A + 4*C) + 48*a^2*b^2*(5*A + 6*C) + 5*a^4*(7*A + 8*C))*Sin[
c + d*x] + 420*a*b*(16*b^2*(A + C) + a^2*(15*A + 16*C))*Sin[2*(c + d*x)] +
735*a^4*A*Sin[3*(c + d*x)] + 4200*a^2*A*b^2*Sin[3*(c + d*x)] + 560*A*b^4*Si
n[3*(c + d*x)] + 700*a^4*C*Sin[3*(c + d*x)] + 3360*a^2*b^2*C*Sin[3*(c + d*x
)] + 1260*a^3*A*b*Sin[4*(c + d*x)] + 840*a*A*b^3*Sin[4*(c + d*x)] + 840*a^3
*b*C*Sin[4*(c + d*x)] + 147*a^4*A*Sin[5*(c + d*x)] + 504*a^2*A*b^2*Sin[5*(c
 + d*x)] + 84*a^4*C*Sin[5*(c + d*x)] + 140*a^3*A*b*Sin[6*(c + d*x)] + 15*a^
4*A*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.1, size = 332, normalized size = 1.

$$\frac{1}{d}\left(\frac{Aa^4\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)+\frac{a^4C\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/7*A*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)+1/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^3*b*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c)+4*a^3*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6
/5*A*a^2*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*C*a^2*b^2*(2+
```

```
cos(d*x+c)^2)*sin(d*x+c)+4*A*a*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c)+4*C*a*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/
3*A*b^4*(2+cos(d*x+c)^2)*sin(d*x+c)+C*b^4*sin(d*x+c))
```

Maxima [A] time = 0.991636, size = 444, normalized size = 1.31

$$-\frac{48\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)Aa^4-112\left(3\sin(dx+c)^5-10\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] -1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*
sin(d*x + c))*A*a^4 - 112*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*
x + c))*C*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*
c) - 48*sin(2*d*x + 2*c))*A*a^3*b - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) +
 8*sin(2*d*x + 2*c))*C*a^3*b - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 +
15*sin(d*x + c))*A*a^2*b^2 + 3360*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b
^2 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^3 -
1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b^3 + 560*(sin(d*x + c)^3 - 3*sin
(d*x + c))*A*b^4 - 1680*C*b^4*sin(d*x + c))/d
```

Fricas [A] time = 0.581904, size = 595, normalized size = 1.76

$$\frac{105\left((5A+6C)a^3b+2(3A+4C)ab^3\right)dx+\left(60Aa^4\cos(dx+c)^6+280Aa^3b\cos(dx+c)^5+32(6A+7C)a^4+336\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/420*(105*((5*A + 6*C)*a^3*b + 2*(3*A + 4*C)*a*b^3)*d*x + (60*A*a^4*cos(d*
x + c)^6 + 280*A*a^3*b*cos(d*x + c)^5 + 32*(6*A + 7*C)*a^4 + 336*(4*A + 5*C
)*a^2*b^2 + 140*(2*A + 3*C)*b^4 + 12*((6*A + 7*C)*a^4 + 42*A*a^2*b^2)*cos(d
*x + c)^4 + 70*((5*A + 6*C)*a^3*b + 6*A*a*b^3)*cos(d*x + c)^3 + 4*(4*(6*A +
 7*C)*a^4 + 42*(4*A + 5*C)*a^2*b^2 + 35*A*b^4)*cos(d*x + c)^2 + 105*((5*A +
 6*C)*a^3*b + 2*(3*A + 4*C)*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sec(d*x+c))**4*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25626, size = 1658, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/420*(105*(5*A*a^3*b + 6*C*a^3*b + 6*A*a*b^3 + 8*C*a*b^3)*(d*x + c) + 2*(4
20*A*a^4*tan(1/2*d*x + 1/2*c)^13 + 420*C*a^4*tan(1/2*d*x + 1/2*c)^13 - 1155
*A*a^3*b*tan(1/2*d*x + 1/2*c)^13 - 1050*C*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 2
520*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 2520*C*a^2*b^2*tan(1/2*d*x + 1/2*c)
^13 - 1050*A*a*b^3*tan(1/2*d*x + 1/2*c)^13 - 840*C*a*b^3*tan(1/2*d*x + 1/2*
c)^13 + 420*A*b^4*tan(1/2*d*x + 1/2*c)^13 + 420*C*b^4*tan(1/2*d*x + 1/2*c)^
13 + 840*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 1400*C*a^4*tan(1/2*d*x + 1/2*c)^11
 - 980*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 - 2520*C*a^3*b*tan(1/2*d*x + 1/2*c)^
11 + 8400*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 11760*C*a^2*b^2*tan(1/2*d*x +
 1/2*c)^11 - 2520*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 3360*C*a*b^3*tan(1/2*d*
x + 1/2*c)^11 + 1960*A*b^4*tan(1/2*d*x + 1/2*c)^11 + 2520*C*b^4*tan(1/2*d*x
 + 1/2*c)^11 + 3612*A*a^4*tan(1/2*d*x + 1/2*c)^9 + 3164*C*a^4*tan(1/2*d*x +
 1/2*c)^9 - 2975*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1890*C*a^3*b*tan(1/2*d*x
+ 1/2*c)^9 + 18984*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 24360*C*a^2*b^2*tan(1
/2*d*x + 1/2*c)^9 - 1890*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 4200*C*a*b^3*tan(
1/2*d*x + 1/2*c)^9 + 4060*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 6300*C*b^4*tan(1/2
*d*x + 1/2*c)^9 + 2544*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 4368*C*a^4*tan(1/2*d*
x + 1/2*c)^7 + 26208*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 30240*C*a^2*b^2*tan
(1/2*d*x + 1/2*c)^7 + 5040*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 8400*C*b^4*tan(1/
2*d*x + 1/2*c)^7 + 3612*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 3164*C*a^4*tan(1/2*d
*x + 1/2*c)^5 + 2975*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 1890*C*a^3*b*tan(1/2*
d*x + 1/2*c)^5 + 18984*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24360*C*a^2*b^2*t
an(1/2*d*x + 1/2*c)^5 + 1890*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 4200*C*a*b^3*
tan(1/2*d*x + 1/2*c)^5 + 4060*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 6300*C*b^4*tan
(1/2*d*x + 1/2*c)^5 + 840*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 1400*C*a^4*tan(1/2
*d*x + 1/2*c)^3 + 980*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 2520*C*a^3*b*tan(1/2
*d*x + 1/2*c)^3 + 8400*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 11760*C*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 + 2520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 3360*C*a*b^3*
tan(1/2*d*x + 1/2*c)^3 + 1960*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 2520*C*b^4*tan
(1/2*d*x + 1/2*c)^3 + 420*A*a^4*tan(1/2*d*x + 1/2*c) + 420*C*a^4*tan(1/2*d*
x + 1/2*c) + 1155*A*a^3*b*tan(1/2*d*x + 1/2*c) + 1050*C*a^3*b*tan(1/2*d*x +
 1/2*c) + 2520*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 2520*C*a^2*b^2*tan(1/2*d*x
+ 1/2*c) + 1050*A*a*b^3*tan(1/2*d*x + 1/2*c) + 840*C*a*b^3*tan(1/2*d*x + 1/
2*c) + 420*A*b^4*tan(1/2*d*x + 1/2*c) + 420*C*b^4*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.673 $\int (a+b\sec(c+dx))^3 \left(a^2-b^2\sec^2(c+dx)\right) dx$

Optimal. Leaf size=158

$$\frac{ab^2\left(5a^2-4b^2\right)\tan(c+dx)}{2d}+\frac{b\left(-8a^2b^2+24a^4-3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3\left(2a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] a^5*x + (b*(24*a^4 - 8*a^2*b^2 - 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*b
^2*(5*a^2 - 4*b^2)*Tan[c + d*x])/(2*d) + (b^3*(2*a^2 - 3*b^2)*Sec[c + d*x]*
Tan[c + d*x])/(8*d) - (a*b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(4*d) - (
b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.28609, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {4042, 3918, 4056, 4048, 3770, 3767, 8}

$$\frac{ab^2\left(5a^2-4b^2\right)\tan(c+dx)}{2d}+\frac{b\left(-8a^2b^2+24a^4-3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3\left(2a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] a^5*x + (b*(24*a^4 - 8*a^2*b^2 - 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*b
^2*(5*a^2 - 4*b^2)*Tan[c + d*x])/(2*d) + (b^3*(2*a^2 - 3*b^2)*Sec[c + d*x]*
Tan[c + d*x])/(8*d) - (a*b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(4*d) - (
b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
```

```
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\left(a^2-b^2\sec^2(c+dx)\right)dx=-\int(-a+b\sec(c+dx))(a+b\sec(c+dx))^4\,dx$$
$$=-\frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d}-\frac{1}{4}\int(a+b\sec(c+dx))^2\left(-4a^3\right.$$
$$=-\frac{ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{4d}-\frac{b^2(a+b\sec(c+dx))^3\tan(c+}{4d}$$
$$=\frac{b^3\left(2a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{8d}-\frac{ab^2(a+b\sec(c+dx))^2\tan(c}{4d}$$
$$=a^5x+\frac{b^3\left(2a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{8d}-\frac{ab^2(a+b\sec(c+dx))^2}{4d}$$
$$=a^5x+\frac{b\left(24a^4-8a^2b^2-3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3\left(2a^2-3b^2\right)\sec}{}$$
$$=a^5x+\frac{b\left(24a^4-8a^2b^2-3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab^2\left(5a^2-4b^2\right)\tan}{2d}$$

Mathematica [B] time = 6.40467, size = 1299, normalized size = 8.22

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] (2*a^5*(c + d*x)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d
*x]^2))/(d*(b + a*Cos[c + d*x])^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])) + (
(-24*a^4*b + 8*a^2*b^3 + 3*b^5)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]]*(a + b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2))/(4*d*(b + a
*Cos[c + d*x])^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])) + ((24*a^4*b - 8*a^2
*b^3 - 3*b^5)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a +
b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2))/(4*d*(b + a*Cos[c + d*x])^3*(
a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])) - (b^5*Cos[c + d*x]^5*(a + b*Sec[c + d
*x])^3*(a^2 - b^2*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(a^2 - 2*b^2
 + a^2*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + ((-8*a^
2*b^3 - 4*a*b^4 - 3*b^5)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(a^2 - b^2*S
```

```
ec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*
d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (a*b^4*Cos[c + d*x]^5*(a +
 b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(d*(b + a*C
os[c + d*x])^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^3) + (b^5*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(a^2 - b^2*S
ec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*
d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) - (a*b^4*Cos[c + d*x]^5*(a +
 b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(d*(b + a*C
os[c + d*x])^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3) + ((8*a^2*b^3 + 4*a*b^4 + 3*b^5)*Cos[c + d*x]^5*(a + b*Se
c[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2))/(8*d*(b + a*Cos[c + d*x])^3*(a^2
- 2*b^2 + a^2*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) -
(4*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2)*(-(a^3*
b^2*Sin[(c + d*x)/2]) + a*b^4*Sin[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])^3*
(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 - (4*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(a^2 - b^2*Sec[c + d*x]^2)*(-(a
^3*b^2*Sin[(c + d*x)/2]) + a*b^4*Sin[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])
^3*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]))
```

Maple [A] time = 0.05, size = 205, normalized size = 1.3

$$a^5x + \frac{a^5c}{d} + 2\,\frac{a^3b^2\tan(dx+c)}{d} + 3\,\frac{a^4b\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{a^2b^3\sec(dx+c)\tan(dx+c)}{d} - \frac{a^2b^3\ln(\mathrm{sec}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(a^2-b^2*sec(d*x+c)^2),x)
```

```
[Out] a^5*x+1/d*a^5*c+2/d*a^3*b^2*tan(d*x+c)+3/d*a^4*b*ln(sec(d*x+c)+tan(d*x+c))-
1/d*a^2*b^3*sec(d*x+c)*tan(d*x+c)-1/d*a^2*b^3*ln(sec(d*x+c)+tan(d*x+c))-2/d
*a*b^4*tan(d*x+c)-1/d*a*b^4*tan(d*x+c)*sec(d*x+c)^2-1/4/d*b^5*tan(d*x+c)*se
c(d*x+c)^3-3/8/d*b^5*sec(d*x+c)*tan(d*x+c)-3/8/d*b^5*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 0.974706, size = 259, normalized size = 1.64

$$\frac{16(dx+c)a^5 - 16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)ab^4 + b^5\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(a^2-b^2*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(16*(d*x + c)*a^5 - 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^4 + b^5*(
2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 8*a^2*b^3*(2*sin(
d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
1)) + 48*a^4*b*log(sec(d*x + c) + tan(d*x + c)) + 32*a^3*b^2*tan(d*x + c))/
d
```

Fricas [A] time = 0.546008, size = 428, normalized size = 2.71

$$\frac{16\,a^5dx\cos(dx+c)^4 + \left(24\,a^4b - 8\,a^2b^3 - 3\,b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1) - \left(24\,a^4b - 8\,a^2b^3 - 3\,b^5\right)\cos(dx+c)^4}{16\,d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(a^2-b^2*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*d*x*cos(d*x + c)^4 + (24*a^4*b - 8*a^2*b^3 - 3*b^5)*cos(d*x +
c)^4*log(sin(d*x + c) + 1) - (24*a^4*b - 8*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) - 2*(8*a*b^4*cos(d*x + c) + 2*b^5 - 16*(a^3*b^2 - a*
b^4)*cos(d*x + c)^3 + (8*a^2*b^3 + 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(d*
cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a - b\sec(c+dx))\,(a + b\sec(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(a**2-b**2*sec(d*x+c)**2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))*(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.26838, size = 512, normalized size = 3.24

$$8\,(dx+c)a^5 + \left(24\,a^4b - 8\,a^2b^3 - 3\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(24\,a^4b - 8\,a^2b^3 - 3\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(a^2-b^2*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)*a^5 + (24*a^4*b - 8*a^2*b^3 - 3*b^5)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - (24*a^4*b - 8*a^2*b^3 - 3*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 2*(16*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*a^2*b^3*tan(1/2*d*x + 1/2
*c)^7 - 24*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 5*b^5*tan(1/2*d*x + 1/2*c)^7 - 48
*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 40*a*b
^4*tan(1/2*d*x + 1/2*c)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*a^3*b^2*tan(1
/2*d*x + 1/2*c)^3 - 8*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a*b^4*tan(1/2*d*x
 + 1/2*c)^3 + 3*b^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^3*b^2*tan(1/2*d*x + 1/2*c
) + 8*a^2*b^3*tan(1/2*d*x + 1/2*c) + 24*a*b^4*tan(1/2*d*x + 1/2*c) + 5*b^5*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.674 $\int (a+b\sec(c+dx))^2\left(a^2-b^2\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{3d}+\frac{ab\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+a^4x-\frac{ab^3\tan(c+dx)\sec(c+dx)}{3d}-\frac{b^2\tan(c+dx)(a}{3}$$

```
[Out] a^4*x + (a*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/d + (b^2*(a^2 - 2*b^2)*Ta
n[c + d*x])/(3*d) - (a*b^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (b^2*(a + b*S
ec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.172336, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4042, 3918, 4048, 3770, 3767, 8}

$$\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{3d}+\frac{ab\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+a^4x-\frac{ab^3\tan(c+dx)\sec(c+dx)}{3d}-\frac{b^2\tan(c+dx)(a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] a^4*x + (a*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/d + (b^2*(a^2 - 2*b^2)*Ta
n[c + d*x])/(3*d) - (a*b^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (b^2*(a + b*S
ec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\left(a^2-b^2\sec^2(c+dx)\right)dx &= -\int (-a+b\sec(c+dx))(a+b\sec(c+dx))^3\,dx\\ &= -\frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}-\frac{1}{3}\int (a+b\sec(c+dx))\left(-3a^3-\right.\\ &= -\frac{ab^3\sec(c+dx)\tan(c+dx)}{3d}-\frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}-\frac{1}{6}\\ &= a^4x-\frac{ab^3\sec(c+dx)\tan(c+dx)}{3d}-\frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}\\ &= a^4x+\frac{ab\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{d}-\frac{ab^3\sec(c+dx)\tan(c+dx)}{3d}\\ &= a^4x+\frac{ab\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{3d}-\end{aligned}$$

Mathematica [A] time = 0.238483, size = 86, normalized size = 0.81

$$\frac{2a^3b\tanh^{-1}(\sin(c+dx))}{d}+a^4x-\frac{ab^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{ab^3\tan(c+dx)\sec(c+dx)}{d}-\frac{b^4\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] a^4*x + (2*a^3*b*ArcTanh[Sin[c + d*x]])/d - (a*b^3*ArcTanh[Sin[c + d*x]])/d
 - (a*b^3*Sec[c + d*x]*Tan[c + d*x])/d - (b^4*(Tan[c + d*x] + Tan[c + d*x]^
3/3))/d
```

Maple [A] time = 0.043, size = 118, normalized size = 1.1

$$a^4x+\frac{a^4c}{d}+2\,\frac{a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{ab^3\sec(dx+c)\tan(dx+c)}{d}-\frac{ab^3\ln(\sec(dx+c)+\tan(dx+c))}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(a^2-b^2*sec(d*x+c)^2),x)
```

```
[Out] a^4*x+1/d*a^4*c+2/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))-a*b^3*sec(d*x+c)*tan(d*
x+c)/d-1/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))-2/3/d*b^4*tan(d*x+c)-1/3/d*b^4*t
an(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.99441, size = 142, normalized size = 1.34

$$\frac{6\,(dx+c)a^4-2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)b^4+3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(a^2-b^2*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(d*x + c)*a^4 - 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^4 + 3*a*b^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*a^3*b*log(sec(d*x + c) + tan(d*x + c)))/d
```

Fricas [A] time = 0.528129, size = 323, normalized size = 3.05

$$\frac{6\,a^4dx\cos\left(dx+c\right)^3+3\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx\right.\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(a^2-b^2*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^4*d*x*cos(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cos(d*x + c)^3*log(sin(
d*x + c) + 1) - 3*(2*a^3*b - a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) -
 2*(2*b^4*cos(d*x + c)^2 + 3*a*b^3*cos(d*x + c) + b^4)*sin(d*x + c))/(d*cos
(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a-b\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(a**2-b**2*sec(d*x+c)**2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))*(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.17904, size = 227, normalized size = 2.14

$$\frac{3\,(dx+c)a^4+3\left(2\,a^3b-ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,a^3b-ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,ab^3\tan\left(\frac{1}{2}\,d\right.\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(a^2-b^2*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 3*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 3*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a*b^3*ta
```

```
n(1/2*d*x + 1/2*c)^5 - 3*b^4*tan(1/2*d*x + 1/2*c)^5 + 2*b^4*tan(1/2*d*x + 1
/2*c)^3 - 3*a*b^3*tan(1/2*d*x + 1/2*c) - 3*b^4*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.675 $\int (a+b\sec(c+dx))\left(a^2-b^2\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=75

$$\frac{b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x-\frac{ab^2\tan(c+dx)}{2d}-\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] a^3*x + (b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (a*b^2*Tan[c + d*x]
)/(2*d) - (b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.08655, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4042, 3918, 3770, 3767, 8}

$$\frac{b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x-\frac{ab^2\tan(c+dx)}{2d}-\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] a^3*x + (b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (a*b^2*Tan[c + d*x]
)/(2*d) - (b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\left(a^2-b^2\sec^2(c+dx)\right)dx = -\int(-a+b\sec(c+dx))(a+b\sec(c+dx))^2\,dx$$
$$= -\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}\int\left(-2a^3-b\left(2a^2-b^2\right)\sec(c+\right.$$
$$= a^3x - \frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}\left(ab^2\right)\int\sec^2(c+dx)\,dx+$$
$$= a^3x + \frac{b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b^2(a+b\sec(c+dx))\tan(c+}{2d}$$
$$= a^3x + \frac{b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{ab^2\tan(c+dx)}{2d} - \frac{b^2(a+b\,\text{se}}{}$$

Mathematica [A] time = 0.0204045, size = 75, normalized size = 1.

$$\frac{a^2b\tanh^{-1}(\sin(c+dx))}{d} + a^3x - \frac{ab^2\tan(c+dx)}{d} - \frac{b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] a^3*x + (a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3*ArcTanh[Sin[c + d*x]])/(2*d)
 - (a*b^2*Tan[c + d*x])/d - (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.039, size = 94, normalized size = 1.3

$$a^3x + \frac{a^3c}{d} - \frac{ab^2\tan(dx+c)}{d} + \frac{a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{b^3\sec(dx+c)\tan(dx+c)}{2d} - \frac{b^3\ln(\sec(dx+c)+\text{t}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(a^2-b^2*sec(d*x+c)^2),x)
```

```
[Out] a^3*x+1/d*a^3*c-a*b^2*tan(d*x+c)/d+1/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-1/2/
d*b^3*sec(d*x+c)*tan(d*x+c)-1/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.970634, size = 126, normalized size = 1.68

$$\frac{4(dx+c)a^3 + b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 4a^2b\log(\sec(dx+c)+\tan(dx+c)) - 4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a^2-b^2*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*a^3 + b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*a^2*b*log(sec(d*x + c) + tan(d*x
+ c)) - 4*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.515588, size = 281, normalized size = 3.75

$$\frac{4\,a^3dx\cos\left(dx+c\right)^2+\left(2\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a^2-b^2*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + (2*a^2*b - b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (2*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*
b^2*cos(d*x + c) + b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a-b\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a**2-b**2*sec(d*x+c)**2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))*(a + b*sec(c + d*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a^2-b^2*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.676 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=186

$$\frac{\left(3a^2C+b^2(3A+2C)\right)\tan(c+dx)}{3b^3d}-\frac{a\left(C\left(2a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\ldots}{\sqrt{a+\ldots}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] -(a*(2*A*b^2 + (2*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^2*(
A*b^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^4*Sqrt[a + b]*d) + ((3*a^2*C + b^2*(3*A + 2*C))*Tan[c + d*x])/(3*b^
3*d) - (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (C*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*b*d)
```

Rubi [A] time = 0.645232, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4103, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(3a^2C+b^2(3A+2C)\right)\tan(c+dx)}{3b^3d}-\frac{a\left(C\left(2a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\ldots}{\sqrt{a+\ldots}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*(2*A*b^2 + (2*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^2*(
A*b^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^4*Sqrt[a + b]*d) + ((3*a^2*C + b^2*(3*A + 2*C))*Tan[c + d*x])/(3*b^
3*d) - (a*C*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (C*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*b*d)
```

Rule 4103

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*
(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[
e + f*x] - a*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx = \frac{C\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)\left(2aC+b(3A+2C)\sec(c+dx)-3aC\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{3b}$$

$$= -\frac{aC\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{C\sec^2(c+dx)\tan(c+dx)}{3bd} + \int \frac{\sec(c+dx)\left(-3a^2C\right.}{}$$

$$= \frac{\left(3a^2C+b^2(3A+2C)\right)\tan(c+dx)}{3b^3d} - \frac{aC\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{C\sec^2(c}{}$$

$$= \frac{\left(3a^2C+b^2(3A+2C)\right)\tan(c+dx)}{3b^3d} - \frac{aC\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{C\sec^2(c}{}$$

$$= -\frac{a\left(2Ab^2+\left(2a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(3a^2C+b^2(3A+2C)\right)\tan(}{3b^3d}$$

$$= -\frac{a\left(2Ab^2+\left(2a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(3a^2C+b^2(3A+2C)\right)\tan(}{3b^3d}$$

$$= -\frac{a\left(2Ab^2+\left(2a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^2\left(Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{\sqrt{a-b}b^4\sqrt{a+}}$$

Mathematica [C] time = 3.7097, size = 657, normalized size = 3.53

$$\frac{\cos(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{4b\sin\left(\frac{dx}{2}\right)\left(3a^2C+3Ab^2+2b^2C\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4b\sin\left(\frac{dx}{2}\right)\left(3a^2C+3Ab^2+2b\right.}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\mathrm{co}\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*(6*a*(2*A*b^2 + (
2*a^2 + b^2)*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*a*(2*A*b^2 + (
2*a^2 + b^2)*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((24*I)*a^2*(A*b
^2 + a^2*C)*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*
x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(Cos[c] - I*Sin[c]))
/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2]) + (2*b^3*C*Sin[(d*x)/2])/((C
os[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (b^2*C*((-3*
a + b)*Cos[c/2] + (3*a + b)*Sin[c/2]))/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2) + (4*b*(3*A*b^2 + 3*a^2*C + 2*b^2*C)*Sin[(d*x)/
2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*b^3*
C*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3) + (b^2*C*((3*a - b)*Cos[c/2] + (3*a + b)*Sin[c/2]))/((Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*b*(3*A*b^2 + 3*a^2*C + 2
*b^2*C)*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]))))/(6*b^4*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.087, size = 554, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A+2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C-1/3/d*C/b/(tan(1/2*d*x+1/2*c)+1)^3-1/d/b/(tan(1/2
*d*x+1/2*c)+1)*A-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2*C-1/2/d/b^2/(tan(1/2*d*
x+1/2*c)+1)*a*C-1/d/b/(tan(1/2*d*x+1/2*c)+1)*C+1/2/d*C/b^2/(tan(1/2*d*x+1/2
*c)+1)^2*a+1/2/d*C/b/(tan(1/2*d*x+1/2*c)+1)^2-1/d*a/b^2*ln(tan(1/2*d*x+1/2*
c)+1)*A-1/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)+1)*C-1/2/d*a/b^2*ln(tan(1/2*d*x+1
/2*c)+1)*C-1/3/d*C/b/(tan(1/2*d*x+1/2*c)-1)^3-1/d/b/(tan(1/2*d*x+1/2*c)-1)*
A-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a*C
-1/d/b/(tan(1/2*d*x+1/2*c)-1)*C-1/2/d*C/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/2/
d*C/b/(tan(1/2*d*x+1/2*c)-1)^2+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)*A+1/d*a^3
/b^4*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.08132, size = 1486, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [1/12*(6*(C*a^4 + A*a^2*b^2)*sqrt(a^2 - b^2)*cos(d*x + c)^3*log((2*a*b*cos(
d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c)
 + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ b^2)) - 3*(2*C*a^5 + (2*A - C)*a^3*b^2 - (2*A + C)*a*b^4)*cos(d*x + c)^3*
log(sin(d*x + c) + 1) + 3*(2*C*a^5 + (2*A - C)*a^3*b^2 - (2*A + C)*a*b^4)*c
os(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*C*a^2*b^3 - 2*C*b^5 + 2*(3*C*a^
4*b + (3*A - C)*a^2*b^3 - (3*A + 2*C)*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 -
C*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/
12*(12*(C*a^4 + A*a^2*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*C*a^5 + (2
*A - C)*a^3*b^2 - (2*A + C)*a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3
*(2*C*a^5 + (2*A - C)*a^3*b^2 - (2*A + C)*a*b^4)*cos(d*x + c)^3*log(-sin(d*
x + c) + 1) + 2*(2*C*a^2*b^3 - 2*C*b^5 + 2*(3*C*a^4*b + (3*A - C)*a^2*b^3 -
 (3*A + 2*C)*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 - C*a*b^4)*cos(d*x + c))*si
n(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.45972, size = 502, normalized size = 2.7

$$-\frac{\frac{3\left(2\,Ca^3+2\,Aab^2+Cab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\left(2\,Ca^3+2\,Aab^2+Cab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}-\frac{12\left(Ca^4+Aa^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\mathrm{arcta}\right.}{\sqrt{-a^2+b^2}b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(2*C*a^3 + 2*A*a*b^2 + C*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/
b^4 - 3*(2*C*a^3 + 2*A*a*b^2 + C*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/
b^4 - 12*(C*a^4 + A*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/(sqrt(-a^2 + b^2)*b^4) + 2*(6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*
b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2
*d*x + 1/2*c)^5 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x +
1/2*c)^3 - 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) -
3*C*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

3.677 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{\left(2a^2C+b^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{aC\tan(c+dx)}{b^2d}+\frac{C\tan(c+}{}$$

```
[Out] ((2*a^2*C + b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a*(A*b^2 +
 a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b
^3*Sqrt[a + b]*d) - (a*C*Tan[c + d*x])/(b^2*d) + (C*Sec[c + d*x]*Tan[c + d*
x])/(2*b*d)
```

Rubi [A] time = 0.380863, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4093, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(2a^2C+b^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{aC\tan(c+dx)}{b^2d}+\frac{C\tan(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2*C + b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a*(A*b^2 +
 a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b
^3*Sqrt[a + b]*d) - (a*C*Tan[c + d*x])/(b^2*d) + (C*Sec[c + d*x]*Tan[c + d*
x])/(2*b*d)
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(aC+b(2A+C)\sec(c+dx)-2aC\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2b}$$

$$= -\frac{aC\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(abC+\left(2a^2C+b^2(2A+C)\right)\ldots\right.}{a+b\sec(c+dx)}}{2b^2}$$

$$= -\frac{aC\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(a\left(Ab^2+a^2C\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+d\ldots}}{b^3}$$

$$= \frac{\left(2a^2C+b^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{aC\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\ldots}{2b\ldots}$$

$$= \frac{\left(2a^2C+b^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{aC\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\ldots}{2b\ldots}$$

$$= \frac{\left(2a^2C+b^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2a\left(Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\ldots}{\sqrt{\ldots}}\right.}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}$$

Mathematica [C] time = 1.96736, size = 428, normalized size = 3.12

$$\frac{\cos(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(-2\left(C\left(2a^2+b^2\right)+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*(-2*(2*A*b^2 + (2
*a^2 + b^2)*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 + (2*a
^2 + b^2)*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (8*a*(A*b^2 + a^2*C
)*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(S
qrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(I*Cos[c] + Sin[c]))/(Sqrt[a^2
 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2]) + (b^2*C)/(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])^2 - (4*a*b*C*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) - (b^2*C)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (4*a*b*C*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]))))/(2*b^3*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.076, size = 362, normalized size = 2.6

$$-2\,{\frac {Aa}{db\sqrt {(a+b)(a-b)}}{\rm Artanh} \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}}\right)}-2\,{\frac {{a}^{3}C}{d{b}^{3}\sqrt {(a+b)(a-b)}}{\rm Artanh} \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*A-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*C-1/2/d*C/b/(tan(1/2*d*x+1/2*c)+1)^2+1/d/b*ln(tan(1/2*
d*x+1/2*c)+1)*A+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*a^2*C+1/2/d/b*ln(tan(1/2*d
*x+1/2*c)+1)*C+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)*C+1/d/b^2/(tan(1/2*d*x+1/2*c)
+1)*a*C+1/2/d*C/b/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)*A
-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)*C+
1/2/d/b/(tan(1/2*d*x+1/2*c)-1)*C+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.02013, size = 1254, normalized size = 9.15

$$\left[\frac{2\left(Ca^{3}+Aab^{2}\right)\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)^{2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right)+\left(2\,Ca^{4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*(C*a^3 + A*a*b^2)*sqrt(a^2 - b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2)) + (2*C*a^4 + (2*A - C)*a^2*b^2 - (2*A + C)*b^4)*cos(d*x + c)^2*log(sin
(d*x + c) + 1) - (2*C*a^4 + (2*A - C)*a^2*b^2 - (2*A + C)*b^4)*cos(d*x + c)
^2*log(-sin(d*x + c) + 1) + 2*(C*a^2*b^2 - C*b^4 - 2*(C*a^3*b - C*a*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2), -1/4*(4*(C*a^
3 + A*a*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*C*a^4 + (2*A - C)*a^2*b^2
- (2*A + C)*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*C*a^4 + (2*A - C
)*a^2*b^2 - (2*A + C)*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(C*a^2
*b^2 - C*b^4 - 2*(C*a^3*b - C*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3
- b^5)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.3369, size = 327, normalized size = 2.39

$$\frac{\frac{\left(2\,Ca^2+2\,Ab^2+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}-\frac{\left(2\,Ca^2+2\,Ab^2+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^3}-\frac{4\left(Ca^3+Aab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\right.}{}\right.\right.}{\sqrt{-a^2+b^2}b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((2*C*a^2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 - (
2*C*a^2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 4*(C*a^
3 + A*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a
^2 + b^2)*b^3) + 2*(2*C*a*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)
^3 - 2*C*a*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 - 1)^2*b^2))/d
```

3.678 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aC\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}$$

```
[Out] -((a*C*ArcTanh[Sin[c + d*x]])/(b^2*d)) + (2*(A*b^2 + a^2*C)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*
Tan[c + d*x])/(b*d)
```

Rubi [A] time = 0.1929, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4083, 3998, 3770, 3831, 2659, 208}

$$\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{aC\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```


```
[Out] -((a*C*ArcTanh[Sin[c + d*x]])/(b^2*d)) + (2*(A*b^2 + a^2*C)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (C*
Tan[c + d*x])/(b*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{C\tan(c+dx)}{bd}+\frac{\int \frac{\sec(c+dx)(Ab-aC\sec(c+dx))}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{C\tan(c+dx)}{bd}-\frac{(aC)\int \sec(c+dx)\,dx}{b^2}+\left(A+\frac{a^2C}{b^2}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= -\frac{aC\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}+\frac{\left(Ab^2+a^2C\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^3}$$

$$= -\frac{aC\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}+\frac{\left(2\left(Ab^2+a^2C\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1}\right.}{b^3d}$$

$$= -\frac{aC\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{2\left(Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d}+\frac{C\tan(}{b}$$

Mathematica [C] time = 2.30183, size = 331, normalized size = 3.48

$$\frac{2\cos(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(-\frac{2i(\cos(c)-i\sin(c))\left(a^2C+Ab^2\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(a\cos(c)-b)+a\sin(c)\right)}{\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin(c))^2}}\right)}{\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin(c))^2}}+a\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*(a*C*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - a*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - ((2*I)*(A*b^2 + a^2*C)*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a
*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(Cos
[c] - I*Sin[c]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2]) + (b*C*Sin[(
d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (b
*C*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]))))/(b^2*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.072, size = 183, normalized size = 1.9

$$2\frac{A}{d\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\frac{a^2C}{db^2\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{C}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*A+2/d/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*a^2*C-1/d/b/(tan(1/2*d*x+1/2*c)+1)*C-1/d*a/b^2*ln(tan(1/2*d*x+1
/2*c)+1)*C-1/d/b/(tan(1/2*d*x+1/2*c)-1)*C+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1
)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98854, size = 967, normalized size = 10.18

$$\left[\frac{\left(Ca^2+Ab^2\right)\sqrt{a^2-b^2}\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(Ca^3-Ca\right.}{2\left(a^2b^2-b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((C*a^2 + A*b^2)*sqrt(a^2 - b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) -
 (C*a^3 - C*a*b^2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (C*a^3 - C*a*b^2)*c
os(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a^2*b - C*b^3)*sin(d*x + c))/((a^
2*b^2 - b^4)*d*cos(d*x + c)), 1/2*(2*(C*a^2 + A*b^2)*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*
x + c) - (C*a^3 - C*a*b^2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (C*a^3 - C*
a*b^2)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a^2*b - C*b^3)*sin(d*x +
c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.39042, size = 220, normalized size = 2.32

$$-\frac{\frac{Ca\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{Ca\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+\frac{2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b}+\frac{2\left(Ca^2+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -(C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - C*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/b^2 + 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b)
+ 2*(C*a^2 + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arct
an((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sq
rt(-a^2 + b^2)*b^2))/d
```

3.679 $\int \frac{A+C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}$$

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(b*d) - (2*(A*b^2 + a^2*C)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.148926, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4051, 3770, 3919, 3831, 2659, 208}

$$-\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(b*d) - (2*(A*b^2 + a^2*C)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \frac{Ab-aC\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}+\frac{C\int \sec(c+dx)\,dx}{b}\\ &= \frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}-\left(\frac{Ab}{a}+\frac{aC}{b}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx\\ &= \frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\left(\frac{Ab}{a}+\frac{aC}{b}\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}\\ &= \frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\left(2\left(\frac{Ab}{a}+\frac{aC}{b}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2\left(\frac{Ab}{a}+\frac{aC}{b}\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,bd}\end{aligned}$$

Mathematica [C] time = 0.409273, size = 239, normalized size = 2.72

$$\frac{2\left(A\cos^2(c+dx)+C\right)\left(\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin(c))^2}\left(-aC\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+aC\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{abd\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin(c))^2}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(C + A*Cos[c + d*x]^2)*(Sqrt[a^2 - b^2]*(A*b*d*x - a*C*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + a*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sqr
t[(Cos[c] - I*Sin[c])^2] + 2*(A*b^2 + a^2*C)*ArcTan[((I*Cos[c] + Sin[c])*(a
*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*
Sin[c])^2])]*(I*Cos[c] + Sin[c])))/(a*b*Sqrt[a^2 - b^2]*d*(A + 2*C + A*Cos[
2*(c + d*x)])*Sqrt[(Cos[c] - I*Sin[c])^2])
```

Maple [A] time = 0.084, size = 158, normalized size = 1.8

$$2\,\frac{A\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad}-2\,\frac{Ab}{ad\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right)-2\,\frac{aC}{db\sqrt{(a+b)(a-b)}}\text{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/a/d*A*arctan(tan(1/2*d*x+1/2*c))-2/d*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d/b*a/((a+b)*(a-b))^(1/2)*arc
```

```
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/b*ln(tan(1/2*d*x+1
/2*c)+1)*C-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99619, size = 802, normalized size = 9.11

$$\left[\frac{2\left(Aa^2b - Ab^3\right)dx + \left(Ca^2 + Ab^2\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c) - \left(a^2-2b^2\right)\cos(dx+c)^2 - 2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c) + 2a^2-b^2}{a^2\cos(dx+c)^2 + 2ab\cos(dx+c) + b^2}\right) + (\cdots}{2\left(a^3b - ab^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(A*a^2*b - A*b^3)*d*x + (C*a^2 + A*b^2)*sqrt(a^2 - b^2)*log((2*a*b*
cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
+ c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + b^2)) + (C*a^3 - C*a*b^2)*log(sin(d*x + c) + 1) - (C*a^3 - C*a*b^2)*l
og(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d), 1/2*(2*(A*a^2*b - A*b^3)*d*x -
2*(C*a^2 + A*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
 + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^3 - C*a*b^2)*log(sin(d*x + c) + 1)
 - (C*a^3 - C*a*b^2)*log(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.28436, size = 194, normalized size = 2.2

$$\frac{\frac{(dx+c)A}{a} + \frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b} - \frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b} - \frac{2\left(Ca^2+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.}{\sqrt{-a^2+b^2}}\right.\right.}{\sqrt{-a^2+b^2}\,ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A/a + C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b - C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/b - 2*(C*a^2 + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a*b))/d
```

3.680 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{Abx}{a^2}+\frac{A\sin(c+dx)}{ad}$$

```
[Out] -((A*b*x)/a^2) + (2*(A*b^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.177109, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4105, 3919, 3831, 2659, 208}

$$\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{Abx}{a^2}+\frac{A\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((A*b*x)/a^2) + (2*(A*b^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c + d*x])/(a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\sin(c+dx)}{ad} - \frac{\int \frac{Ab-aC\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$
$$= -\frac{Abx}{a^2} + \frac{A\sin(c+dx)}{ad} + \left(\frac{Ab^2}{a^2}+C\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= -\frac{Abx}{a^2} + \frac{A\sin(c+dx)}{ad} + \frac{\left(\frac{Ab^2}{a^2}+C\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}$$
$$= -\frac{Abx}{a^2} + \frac{A\sin(c+dx)}{ad} + \frac{\left(2\left(\frac{Ab^2}{a^2}+C\right)\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+\right.\right.}{bd}$$
$$= -\frac{Abx}{a^2} + \frac{2\left(\frac{Ab^2}{a^2}+C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d} + \frac{A\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.233935, size = 82, normalized size = 0.95

$$\frac{-\frac{2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + aA\sin(c+dx) - Ab(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-(A*b*(c + d*x)) - (2*(A*b^2 + a^2*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*A*Sin[c + d*x])/(a^2*d)
```

Maple [A] time = 0.103, size = 149, normalized size = 1.7

$$2\,\frac{A\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)} - 2\,\frac{Ab\arctan(\tan(1/2\,dx+c/2))}{da^2} + 2\,\frac{Ab^2}{da^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+}{\sqrt{(a+b)(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*A/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d*A/a^2*b*arctan(tan(
1/2*d*x+1/2*c))+2/d/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*A*b^2+2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.550187, size = 656, normalized size = 7.63

$$\left[-\frac{2\left(Aa^2b-Ab^3\right)dx-\left(Ca^2+Ab^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\cdots}{2\left(a^4-a^2b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/2*(2*(A*a^2*b - A*b^3)*d*x - (C*a^2 + A*b^2)*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), -((A*
a^2*b - A*b^3)*d*x - (C*a^2 + A*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (A*a^3 - A*a*b^2)*si
n(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^2(c+dx)\right)\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.2548, size = 184, normalized size = 2.14

$$-\frac{\frac{(dx+c)Ab}{a^2}-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}-\frac{2\left(Ca^2+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((d*x + c)*A*b/a^2 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*a) - 2*(C*a^2 + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/(sqrt(-a^2 + b^2)*a^2))/d
```

3.681 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{2b\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2(A+2C)+2Ab^2\right)}{2a^3}-\frac{Ab\sin(c+dx)}{a^2d}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] ((2*A*b^2 + a^2*(A + 2*C))*x)/(2*a^3) - (2*b*(A*b^2 + a^2*C)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - (A
*b*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.387222, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4105, 4104, 3919, 3831, 2659, 208}

$$-\frac{2b\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(\frac{2Ab^2}{a^2}+A+2C\right)}{2a}-\frac{Ab\sin(c+dx)}{a^2d}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((A + (2*A*b^2)/a^2 + 2*C)*x)/(2*a) - (2*b*(A*b^2 + a^2*C)*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - (A*b
*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(2Ab-a(A+2C)\sec(c+dx)-Ab\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{Ab\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int \frac{2Ab^2+a^2(A+2C)+aAb\sec(c+dx)}{a+b\sec(c+dx)}\,d\ldots}{2a^2}$$

$$= \frac{\left(2Ab^2+a^2(A+2C)\right)x}{2a^3} - \frac{Ab\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(b(A\ldots}{\ldots}$$

$$= \frac{\left(2Ab^2+a^2(A+2C)\right)x}{2a^3} - \frac{Ab\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(Ab^2\ldots}{\ldots}$$

$$= \frac{\left(2Ab^2+a^2(A+2C)\right)x}{2a^3} - \frac{Ab\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(2(\ldots}{\ldots}$$

$$= \frac{\left(2Ab^2+a^2(A+2C)\right)x}{2a^3} - \frac{2b\left(Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} - \frac{Ab\sin\ldots}{\ldots}$$

Mathematica [A] time = 0.357908, size = 115, normalized size = 0.9

$$\frac{2(c+dx)\left(a^2(A+2C)+2Ab^2\right)+\frac{8b\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a^2A\sin(2(c+dx))-4aAb\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(2*A*b^2 + a^2*(A + 2*C))*(c + d*x) + (8*b*(A*b^2 + a^2*C)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*a*A*b*Sin[c +
d*x] + a^2*A*Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] time = 0.114, size = 296, normalized size = 2.3

$$-\frac{A}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-2\frac{(\tan(1/2\,dx+c/2))^{3}Ab}{da^{2}\left(1+(\tan(1/2\,dx+c/2))^{2}\right)^{2}}+\frac{A}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*
A*tan(1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*
A*b+1/a/d*A*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A
*b^2+2/a/d*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*b/a/((a+b)*(a-b))^
(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.56873, size = 848, normalized size = 6.62

$$\left[\frac{\left((A+2\,C)a^{4}+(A-2\,C)a^{2}b^{2}-2\,Ab^{4}\right)dx+\left(Ca^{2}b+Ab^{3}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right.}{2\left(a^{5}-a^{3}b^{2}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [1/2*(((A + 2*C)*a^4 + (A - 2*C)*a^2*b^2 - 2*A*b^4)*d*x + (C*a^2*b + A*b^3)
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (2*A*a^3*b - 2*A*a*b^3 - (A*a^4 -
 A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d), 1/2*(((A + 2*C
)*a^4 + (A - 2*C)*a^2*b^2 - 2*A*b^4)*d*x - 2*(C*a^2*b + A*b^3)*sqrt(-a^2 +
b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
))) - (2*A*a^3*b - 2*A*a*b^3 - (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x +
c))/((a^5 - a^3*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\cos^2\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.2499, size = 269, normalized size = 2.1

$$\frac{\frac{\left(Aa^2+2\,Ca^2+2\,Ab^2\right)(dx+c)}{a^3} - \frac{4\left(Ca^2b+Ab^3\right)\left(\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^3} - \frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((A*a^2 + 2*C*a^2 + 2*A*b^2)*(d*x + c)/a^3 - 4*(C*a^2*b + A*b^3)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) - 2*
(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*
x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^
2))/d
```

3.682 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{\left(a^2(2A+3C)+3Ab^2\right)\sin(c+dx)}{3a^3d}+\frac{2b^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{bx\left(a^2(A+2C)+2Ab^2\right)}{2a^4}-\frac{Ab\sin}{}$$

```
[Out] -(b*(2*A*b^2 + a^2*(A + 2*C))*x)/(2*a^4) + (2*b^2*(A*b^2 + a^2*C)*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d)
 + ((3*A*b^2 + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*a^3*d) - (A*b*Cos[c + d*x]
*Sin[c + d*x])/(2*a^2*d) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.605461, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4105, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(a^2(2A+3C)+3Ab^2\right)\sin(c+dx)}{3a^3d}+\frac{2b^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{bx\left(\frac{2Ab^2}{a^2}+A+2C\right)}{2a^2}-\frac{Ab\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(b*(A + (2*A*b^2)/a^2 + 2*C)*x)/(2*a^2) + (2*b^2*(A*b^2 + a^2*C)*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d)
 + ((3*A*b^2 + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*a^3*d) - (A*b*Cos[c + d*x]
*Sin[c + d*x])/(2*a^2*d) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)\left(3Ab-a(2A+3C)\sec(c+dx)-2Ab\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3a}$$

$$= -\frac{Ab\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int \frac{\cos(c+dx)\left(2\left(3Ab^2\right.\right.}{}}{}$$

$$= \frac{\left(3Ab^2+a^2(2A+3C)\right)\sin(c+dx)}{3a^3d} - \frac{Ab\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A\cos^2(c-}{}$$

$$= -\frac{b\left(2Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{\left(3Ab^2+a^2(2A+3C)\right)\sin(c+dx)}{3a^3d} - \frac{Ab\cos(c+}{}$$

$$= -\frac{b\left(2Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{\left(3Ab^2+a^2(2A+3C)\right)\sin(c+dx)}{3a^3d} - \frac{Ab\cos(c+}{}$$

$$= -\frac{b\left(2Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{\left(3Ab^2+a^2(2A+3C)\right)\sin(c+dx)}{3a^3d} - \frac{Ab\cos(c+}{}$$

$$= -\frac{b\left(2Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{2b^2\left(Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{(3}{}$$

Mathematica [A] time = 0.500142, size = 149, normalized size = 0.85

$$\frac{-6b(c+dx)\left(a^2(A+2C)+2Ab^2\right)+3a\left(a^2(3A+4C)+4Ab^2\right)\sin(c+dx)-\frac{24b^2\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-3a^2Ab\,\mathrm{s}}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-6*b*(2*A*b^2 + a^2*(A + 2*C))*(c + d*x) - (24*b^2*(A*b^2 + a^2*C)*ArcTanh
[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(4*A*b
```

```
^2 + a^2*(3*A + 4*C))*Sin[c + d*x] - 3*a^2*A*b*Sin[2*(c + d*x)] + a^3*A*Sin
[3*(c + d*x)])/(12*a^4*d)
```

Maple [B] time = 0.118, size = 551, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+1/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^5*A*b^2+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)^5*C+4/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+4/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A*b^2+4/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)*A+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b^2+2
/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*C-1/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b-1/d*A/a^2*b*arctan(tan(1/2*d*x+1/2*c))-
2/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))*b^3-2/d/a^2*C*arctan(tan(1/2*d*x+1/2*c
))*b+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))*A+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.596147, size = 1067, normalized size = 6.1

$$\left[-\frac{3\left((A+2C)a^4b+(A-2C)a^2b^3-2Ab^5\right)dx-3\left(Ca^2b^2+Ab^4\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}(\cdots}{a^2\cos(dx+c)^2+2ab\cos(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [-1/6*(3*((A + 2*C)*a^4*b + (A - 2*C)*a^2*b^3 - 2*A*b^5)*d*x - 3*(C*a^2*b^2
 + A*b^4)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (2*(2*A + 3*C)*a^5 + 2*(
```

```
A - 3*C)*a^3*b^2 - 6*A*a*b^4 + 2*(A*a^5 - A*a^3*b^2)*cos(d*x + c)^2 - 3*(A*
a^4*b - A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d), -1/6*(3
*((A + 2*C)*a^4*b + (A - 2*C)*a^2*b^3 - 2*A*b^5)*d*x - 6*(C*a^2*b^2 + A*b^4
)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^
2)*sin(d*x + c))) - (2*(2*A + 3*C)*a^5 + 2*(A - 3*C)*a^3*b^2 - 6*A*a*b^4 +
2*(A*a^5 - A*a^3*b^2)*cos(d*x + c)^2 - 3*(A*a^4*b - A*a^2*b^3)*cos(d*x + c)
)*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21354, size = 440, normalized size = 2.51

$$-\frac{\frac{3\left(Aa^2b+2Ca^2b+2Ab^3\right)(dx+c)}{a^4}-\frac{12\left(Ca^2b^2+Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^4}-\frac{2\left(6Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6Ca\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(A*a^2*b + 2*C*a^2*b + 2*A*b^3)*(d*x + c)/a^4 - 12*(C*a^2*b^2 + A*b
^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)
*a^4) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5
+ 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2
*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/
2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x + 1/2
*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/
2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.683 $\int \frac{\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=232

$$-\frac{b\left(a^2(2A+3C)+3Ab^2\right)\sin(c+dx)}{3a^4d}+\frac{\left(a^2(3A+4C)+4Ab^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^3\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{a^5d\sqrt{a-b}\sqrt{a-}}$$

```
[Out] ((8*A*b^4 + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*x)/(8*a^5) - (2*b^3*(A*b
^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[
a - b]*Sqrt[a + b]*d) - (b*(3*A*b^2 + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*a^4
*d) + ((4*A*b^2 + a^2*(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (
A*b*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*Sin[c + d*x]
)/(4*a*d)
```

Rubi [A] time = 0.927148, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4105, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(a^2(2A+3C)+3Ab^2\right)\sin(c+dx)}{3a^4d}+\frac{\left(a^2(3A+4C)+4Ab^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^3\left(a^2C+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{a^5d\sqrt{a-b}\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*A*b^4 + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*x)/(8*a^5) - (2*b^3*(A*b
^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[
a - b]*Sqrt[a + b]*d) - (b*(3*A*b^2 + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*a^4
*d) + ((4*A*b^2 + a^2*(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (
A*b*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*Sin[c + d*x]
)/(4*a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^3(c+dx)\left(4Ab-a(3A+4C)\sec(c+dx)-3Ab\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{4a}$$

$$= -\frac{Ab\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \frac{\cos^2(c+dx)\left(3(4A\right.}{}}{}$$

$$= \frac{\left(4Ab^2+a^2(3A+4C)\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{Ab\cos^2(c+dx)\sin(c+dx)}{3a^2d}$$

$$= -\frac{b\left(3Ab^2+a^2(2A+3C)\right)\sin(c+dx)}{3a^4d} + \frac{\left(4Ab^2+a^2(3A+4C)\right)\cos(c+dx)\,\mathrm{si}}{8a^3d}$$

$$= \frac{\left(8Ab^4+4a^2b^2(A+2C)+a^4(3A+4C)\right)x}{8a^5} - \frac{b\left(3Ab^2+a^2(2A+3C)\right)\sin(c+}{3a^4d}$$

$$= \frac{\left(8Ab^4+4a^2b^2(A+2C)+a^4(3A+4C)\right)x}{8a^5} - \frac{b\left(3Ab^2+a^2(2A+3C)\right)\sin(c+}{3a^4d}$$

$$= \frac{\left(8Ab^4+4a^2b^2(A+2C)+a^4(3A+4C)\right)x}{8a^5} - \frac{b\left(3Ab^2+a^2(2A+3C)\right)\sin(c+}{3a^4d}$$

$$= \frac{\left(8Ab^4+4a^2b^2(A+2C)+a^4(3A+4C)\right)x}{8a^5} - \frac{2b^3\left(Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{ta}}{}\right.}{a^5\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.658355, size = 191, normalized size = 0.82

$$\frac{12(c+dx)\left(4a^2b^2(A+2C)+a^4(3A+4C)+8Ab^4\right)+24a^2\left(a^2(A+C)+Ab^2\right)\sin(2(c+dx))-24ab\left(a^2(3A+4C)+4Ab\right.}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (12*(8*A*b^4 + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*(c + d*x) + (192*b^3*
(A*b^2 + a^2*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] - 24*a*b*(4*A*b^2 + a^2*(3*A + 4*C))*Sin[c + d*x] + 24*a^2*(A*b^
2 + a^2*(A + C))*Sin[2*(c + d*x)] - 8*a^3*A*b*Sin[3*(c + d*x)] + 3*a^4*A*Si
n[4*(c + d*x)])/(96*a^5*d)
```

Maple [B] time = 0.125, size = 1060, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A*b^3+1/a/d*arctan
(tan(1/2*d*x+1/2*c))*C+3/4/a/d*A*arctan(tan(1/2*d*x+1/2*c))-2/d*b^5/a^5/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-10
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A*b-2/d/a^2/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*A*b-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)*A*b^3-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x
+1/2*c)^7*b*C-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b^3
-10/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*b-2/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b-6/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b*C-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)*b*C-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*
b^3-6/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b*C-1/d/a^3/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*b^2+1/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)*A*b^2+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)*C-5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A-3/
4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A+1/d/a/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*C-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)^7*C+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*C*b^2+2/d/a^5*arctan(t
an(1/2*d*x+1/2*c))*A*b^4+5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)*A-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*C+1/d/a^3*arcta
n(tan(1/2*d*x+1/2*c))*A*b^2+3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+
1/2*c)^5*A+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A*b^2-2/
d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*C-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.662604, size = 1328, normalized size = 5.72

$$\left[\frac{3\left((3A+4C)a^6+(A+4C)a^4b^2+4(A-2C)a^2b^4-8Ab^6\right)dx+12\left(Ca^2b^3+Ab^5\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos}{a^2\cos}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [1/24*(3*((3*A + 4*C)*a^6 + (A + 4*C)*a^4*b^2 + 4*(A - 2*C)*a^2*b^4 - 8*A*b
^6)*d*x + 12*(C*a^2*b^3 + A*b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (
8*(2*A + 3*C)*a^5*b + 8*(A - 3*C)*a^3*b^3 - 24*A*a*b^5 - 6*(A*a^6 - A*a^4*b
^2)*cos(d*x + c)^3 + 8*(A*a^5*b - A*a^3*b^3)*cos(d*x + c)^2 - 3*((3*A + 4*C
)*a^6 + (A - 4*C)*a^4*b^2 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7
- a^5*b^2)*d), 1/24*(3*((3*A + 4*C)*a^6 + (A + 4*C)*a^4*b^2 + 4*(A - 2*C)*a
^2*b^4 - 8*A*b^6)*d*x - 24*(C*a^2*b^3 + A*b^5)*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (8*(2*A +
3*C)*a^5*b + 8*(A - 3*C)*a^3*b^3 - 24*A*a*b^5 - 6*(A*a^6 - A*a^4*b^2)*cos(d
*x + c)^3 + 8*(A*a^5*b - A*a^3*b^3)*cos(d*x + c)^2 - 3*((3*A + 4*C)*a^6 + (
A - 4*C)*a^4*b^2 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2
)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38957, size = 775, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^4 + 4*C*a^4 + 4*A*a^2*b^2 + 8*C*a^2*b^2 + 8*A*b^4)*(d*x + c)
/a^5 - 48*(C*a^2*b^3 + A*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
+ b^2)))/(sqrt(-a^2 + b^2)*a^5) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*C
*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*C*a^2*
b*tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan
```

```
(1/2*d*x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^3*tan(1/2*d*x
 + 1/2*c)^5 + 40*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^2*b*tan(1/2*d*x +
1/2*c)^5 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c
)^5 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 40
*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*
a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^3*t
an(1/2*d*x + 1/2*c) - 12*C*a^3*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b*tan(1/2*d*
x + 1/2*c) + 24*C*a^2*b*tan(1/2*d*x + 1/2*c) - 12*A*a*b^2*tan(1/2*d*x + 1/2
*c) + 24*A*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/
d
```

3.684 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=271

$$-\frac{a\left(3a^2C+Ab^2-2b^2C\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{\left(C\left(6a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a\left(a^2Ab^2-4a^2b^2C+3a^4C-2A\right.}{b^4d(a-b)^{3/2}}$$

```
[Out] ((2*A*b^2 + (6*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a*(a^2*A
*b^2 - 2*A*b^4 + 3*a^4*C - 4*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(A*b^2 + 3*a^2*C
 - 2*b^2*C)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) + ((2*A*b^2 + 3*a^2*C - b^2*C
)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*Sec[c
 + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.86113, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4099, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(3a^2C+Ab^2-2b^2C\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{\left(C\left(6a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a\left(a^2Ab^2-4a^2b^2C+3a^4C-2A\right.}{b^4d(a-b)^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((2*A*b^2 + (6*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a*(a^2*A
*b^2 - 2*A*b^4 + 3*a^4*C - 4*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(A*b^2 + 3*a^2*C
 - 2*b^2*C)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) + ((2*A*b^2 + 3*a^2*C - b^2*C
)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*Sec[c
 + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx = -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^2(c+dx)\left(2\left(Ab^2+a^2C\right)-ab(A+C)\sec(c+d}{a+b\sec(c+}}{b\left(a^2-b\right.}$$

$$= \frac{\left(2Ab^2+3a^2C-b^2C\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan}{b\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{a\left(Ab^2+3a^2C-2b^2C\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2+3a^2C-b^2C\right)\sec(c+dx)\tan}{2b^2\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(Ab^2+3a^2C-2b^2C\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2+3a^2C-b^2C\right)\sec(c+dx)\tan}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(2Ab^2+\left(6a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(Ab^2+3a^2C-2b^2C\right)\tan(c+}{b^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(2Ab^2+\left(6a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(Ab^2+3a^2C-2b^2C\right)\tan(c+}{b^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(2Ab^2+\left(6a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a\left(a^2Ab^2-2Ab^4+3a^4C-4a\right.}{(a-b)^{3/2}b}$$

Mathematica [A] time = 3.79693, size = 461, normalized size = 1.7

$$\frac{(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{4a^2b\left(a^2C+Ab^2\right)\sin(c+dx)}{(b-a)(a+b)} - 2\left(C\left(6a^2+b^2\right)+2Ab^2\right)(a\cos(c+dx)+b)\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*((8*a*(-2*A*b^4 + a^2*b^2*(A -
 4*C) + 3*a^4*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b +
a*Cos[c + d*x]))/(a^2 - b^2)^(3/2) - 2*(2*A*b^2 + (6*a^2 + b^2)*C)*(b + a*C
os[c + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 + (6*a^2
 + b^2)*C)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
(b^2*C*(b + a*Cos[c + d*x]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - (8*a
*b*C*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]) - (b^2*C*(b + a*Cos[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2 - (8*a*b*C*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]) + (4*a^2*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((-a + b)*(a + b))
))/(2*b^4*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.104, size = 646, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)*A+2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d*a/(a+b)/
(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*A-6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*C+8/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/2/d*C/b^2/(tan(1
/2*d*x+1/2*c)+1)^2+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*A+3/d/b^4*ln(tan(1/2*d*
x+1/2*c)+1)*a^2*C+1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*C+2/d*C/b^3/(tan(1/2*d
*x+1/2*c)+1)*a+1/2/d*C/b^2/(tan(1/2*d*x+1/2*c)+1)+1/2/d*C/b^2/(tan(1/2*d*x+
1/2*c)-1)^2-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*A-3/d/b^4*ln(tan(1/2*d*x+1/2*c
)-1)*a^2*C-1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C+2/d*C/b^3/(tan(1/2*d*x+1/2*
c)-1)*a+1/2/d*C/b^2/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 29.9701, size = 2553, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/4*(2*((3*C*a^6 + (A - 4*C)*a^4*b^2 - 2*A*a^2*b^4)*cos(d*x + c)^3 + (3*C*
a^5*b + (A - 4*C)*a^3*b^3 - 2*A*a*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*log(
(2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*c
os(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*co
s(d*x + c) + b^2)) + ((6*C*a^7 + (2*A - 11*C)*a^5*b^2 - 4*(A - C)*a^3*b^4 +
 (2*A + C)*a*b^6)*cos(d*x + c)^3 + (6*C*a^6*b + (2*A - 11*C)*a^4*b^3 - 4*(A
 - C)*a^2*b^5 + (2*A + C)*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*
C*a^7 + (2*A - 11*C)*a^5*b^2 - 4*(A - C)*a^3*b^4 + (2*A + C)*a*b^6)*cos(d*x
 + c)^3 + (6*C*a^6*b + (2*A - 11*C)*a^4*b^3 - 4*(A - C)*a^2*b^5 + (2*A + C)
*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(C*a^4*b^3 - 2*C*a^2*b^5 +
 C*b^7 - 2*(3*C*a^6*b + (A - 5*C)*a^4*b^3 - (A - 2*C)*a^2*b^5)*cos(d*x + c)
^2 - 3*(C*a^5*b^2 - 2*C*a^3*b^4 + C*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d
*cos(d*x + c)^2), -1/4*(4*((3*C*a^6 + (A - 4*C)*a^4*b^2 - 2*A*a^2*b^4)*cos(
d*x + c)^3 + (3*C*a^5*b + (A - 4*C)*a^3*b^3 - 2*A*a*b^5)*cos(d*x + c)^2)*sq
rt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - ((6*C*a^7 + (2*A - 11*C)*a^5*b^2 - 4*(A - C)*a^3*b^4 + (2*A
```

```
 + C)*a*b^6)*cos(d*x + c)^3 + (6*C*a^6*b + (2*A - 11*C)*a^4*b^3 - 4*(A - C)
*a^2*b^5 + (2*A + C)*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*C*a^7
 + (2*A - 11*C)*a^5*b^2 - 4*(A - C)*a^3*b^4 + (2*A + C)*a*b^6)*cos(d*x + c)
^3 + (6*C*a^6*b + (2*A - 11*C)*a^4*b^3 - 4*(A - C)*a^2*b^5 + (2*A + C)*b^7)
*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(C*a^4*b^3 - 2*C*a^2*b^5 + C*b^
7 - 2*(3*C*a^6*b + (A - 5*C)*a^4*b^3 - (A - 2*C)*a^2*b^5)*cos(d*x + c)^2 -
3*(C*a^5*b^2 - 2*C*a^3*b^4 + C*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4
 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(
d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x
)
```

Giac [A] time = 1.39989, size = 483, normalized size = 1.78

$$-\frac{\frac{4\left(3Ca^5+Aa^3b^2-4Ca^3b^2-2Aab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{4\left(Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/2*(4*(3*C*a^5 + A*a^3*b^2 - 4*C*a^3*b^2 - 2*A*a*b^4)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/
2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - 4*(
C*a^4*tan(1/2*d*x + 1/2*c) + A*a^2*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^
5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (6*C*a^
2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (6*C*a^2 + 2*
A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(4*C*a*tan(1/2*d*
x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*tan(1/2*d*x + 1/2*c) + C*
b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

3.685 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=153

$$-\frac{2\left(3a^2b^2C-2a^4C+Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\left(a^2C+Ab^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2aC\tanh^{-1}(\sin(c+dx))}{b^3d}$$

```
[Out] (-2*a*C*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*(A*b^4 - 2*a^4*C + 3*a^2*b^2*C)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a
 + b)^(3/2)*d) + (C*Tan[c + d*x])/(b^2*d) + (a*(A*b^2 + a^2*C)*Tan[c + d*x]
)/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.484777, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4091, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{2\left(3a^2b^2C-2a^4C+Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\left(a^2C+Ab^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2aC\tanh^{-1}(\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (-2*a*C*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*(A*b^4 - 2*a^4*C + 3*a^2*b^2*C)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a
 + b)^(3/2)*d) + (C*Tan[c + d*x])/(b^2*d) + (a*(A*b^2 + a^2*C)*Tan[c + d*x]
)/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4091

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(a*(A*b^2 + a^2
*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)),
 x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*Simp[b*(m + 1)*(a^2*C + A*b^2) - a*(A*b^2*(m + 2) + C*(a^2 + b^2
*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(-b\left(Ab^2+a^2C\right)-a\left(a^2-b^2\right)C\sec(c+dx)+b\left(a^2\right.\right.}{a+b\sec(c+dx)}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(-b^2\left(Ab^2+a^2C\right)-2a\right.}{a+b\sec(c+}}{b^3\left(a^2-\right.}$$

$$= \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{(2aC)\int\sec(c+dx)\,dx}{b^3} -$$

$$= -\frac{2aC\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx)}$$

$$= -\frac{2aC\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx)}$$

$$= -\frac{2aC\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2\left(Ab^4-2a^4C+3a^2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{(a-b)^{3/2}b^3(a+b)^{3/2}d}$$

Mathematica [B] time = 2.67776, size = 336, normalized size = 2.2

$$2(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{ab\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)} + \frac{2\left(3a^2b^2C-2a^4C+Ab^4\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{bC}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*((2*(A*b^4 - 2*a^4*C + 3*a^2
*b^2*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c +
 d*x]))/(a^2 - b^2)^(3/2) + 2*a*C*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - 2*a*C*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (b*C*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]) + (b*C*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]) + (a*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((a
- b)*(a + b))))/(b^3*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^
2)
```

Maple [B] time = 0.084, size = 402, normalized size = 2.6

$$-2\,{\frac{a\tan \left(1/2\,dx+c/2 \right) A}{d \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}b-a-b \right) }}-2\,{\frac{{a}^{3}\tan \left(1/2\,dx+c/2 \right) }{d{b}^{2} \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)*A-2/d/b^2*a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d/b^3/(a+b)/(a-b)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^
4*C-6/d/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*C*a^2-1/d*C/b^2/(tan(1/2*d*x+1/2*c)+1)-2/d*a*C/b^3*ln(
tan(1/2*d*x+1/2*c)+1)+2/d*a*C/b^3*ln(tan(1/2*d*x+1/2*c)-1)-1/d*C/b^2/(tan(1
/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.49633, size = 1904, normalized size = 12.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/2*(((2*C*a^5 - 3*C*a^3*b^2 - A*a*b^4)*cos(d*x + c)^2 + (2*C*a^4*b - 3*C*
a^2*b^3 - A*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a
^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
 + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(
(C*a^6 - 2*C*a^4*b^2 + C*a^2*b^4)*cos(d*x + c)^2 + (C*a^5*b - 2*C*a^3*b^3 +
 C*a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*((C*a^6 - 2*C*a^4*b^2 + C
*a^2*b^4)*cos(d*x + c)^2 + (C*a^5*b - 2*C*a^3*b^3 + C*a*b^5)*cos(d*x + c))*
log(-sin(d*x + c) + 1) + 2*(C*a^4*b^2 - 2*C*a^2*b^4 + C*b^6 + (2*C*a^5*b +
(A - 3*C)*a^3*b^3 - (A - C)*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 -
2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x
 + c)), (((2*C*a^5 - 3*C*a^3*b^2 - A*a*b^4)*cos(d*x + c)^2 + (2*C*a^4*b - 3
*C*a^2*b^3 - A*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((C*a^6 - 2*C*a^4*b^2 +
 C*a^2*b^4)*cos(d*x + c)^2 + (C*a^5*b - 2*C*a^3*b^3 + C*a*b^5)*cos(d*x + c)
)*log(sin(d*x + c) + 1) + ((C*a^6 - 2*C*a^4*b^2 + C*a^2*b^4)*cos(d*x + c)^2
 + (C*a^5*b - 2*C*a^3*b^3 + C*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) +
 (C*a^4*b^2 - 2*C*a^2*b^4 + C*b^6 + (2*C*a^5*b + (A - 3*C)*a^3*b^3 - (A - C
)*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d
*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x
)
```

Giac [B] time = 1.28794, size = 516, normalized size = 3.37

$$2\left(\frac{\left(2\,Ca^4-3\,Ca^2b^2-Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{-a^2+b^2}}-\frac{Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}+\frac{Ca\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right.\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 2*((2*C*a^4 - 3*C*a^2*b^2 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
-a^2 + b^2)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - C*a*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/b^3 + C*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - (2*C*a^3*
tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d
*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + C*b^3*tan(1/2*d*x + 1/2*c)
^3 - 2*C*a^3*tan(1/2*d*x + 1/2*c) - C*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*
tan(1/2*d*x + 1/2*c) + C*a*b^2*tan(1/2*d*x + 1/2*c) + C*b^3*tan(1/2*d*x + 1
/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)))/d
```

3.686 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$\frac{2a\left(a^2(-C)+Ab^2+2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2C+Ab^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{C\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^2*d) + (2*a*(A*b^2 - a^2*C + 2*b^2*C)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) - ((A*b^2 + a^2*C)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
))
```

Rubi [A] time = 0.272399, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4081, 3998, 3770, 3831, 2659, 208}

$$\frac{2a\left(a^2(-C)+Ab^2+2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(a^2C+Ab^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{C\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^2*d) + (2*a*(A*b^2 - a^2*C + 2*b^2*C)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) - ((A*b^2 + a^2*C)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
))
```

Rule 4081

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A*b^2 + a^2*C)
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] +
 Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2 + a^2*C + b*(A*b + b*C)*(m + 1))*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && NeQ[a
^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab(A+C)-\left(a^2-b^2\right)C\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{C\int\sec(c+dx)\,dx}{b^2} - \frac{\left(a\left(a^2-b^2\right)C-ab^2(\ldots\right.}{b^2\left(\ldots\right.}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(a\left(a^2-b^2\right)C-ab^{\ldots}\right.}{b^3\ldots}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a\left(C-\frac{b^2(A+C)}{a^2-b^2}\right)\right)\ldots}{\ldots}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{2a\left(Ab^2-a^2C+2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \ldots$$

Mathematica [C] time = 2.3422, size = 331, normalized size = 2.45

$$\frac{2(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{b\left(a^2C+Ab^2\right)(b\sin(c)-a\sin(dx))}{a(a-b)(a+b)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)} + \frac{2a(\sin(c)+i\cos(c))\left(C\left(a^2-2b^2\right)-Ab^2\right)(a\cos(c+dx)+i\ldots}{\left(a^2-b^2\right)^{3/2}\sqrt{(c\ldots}}\ldots\right.}{b^2a\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*(-(C*(b + a*Cos[c + d*x])*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + C*(b + a*Cos[c + d*x])*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(-(A*b^2) + (a^2 - 2*b^2)*C)*ArcTan[(
(I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 -
b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b + a*Cos[c + d*x])*(I*Cos[c] + Sin[c])
)/((a^2 - b^2)^(3/2)*Sqrt[(Cos[c] - I*Sin[c])^2]) + (b*(A*b^2 + a^2*C)*(b*S
in[c] - a*Sin[d*x]))/(a*(a - b)*(a + b)*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + S
```

```
in[c/2]))))/(b^2*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.089, size = 350, normalized size = 2.6

$$2\frac{b\tan(1/2\,dx+c/2)\,A}{d\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}+2\frac{\tan(1/2\,dx+c/2)\,a^2(}{db\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)*A+2/d/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)*a^2*C+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^3/b^2/(a+b)/(a-b)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*C+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/b^2*ln(tan(1/2*d
*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.4835, size = 1513, normalized size = 11.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((C*a^3*b - (A + 2*C)*a*b^3 + (C*a^4 - (A + 2*C)*a^2*b^2)*cos(d*x + c)
)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (C*a^4*b - 2*C*a^2*b^3 + C*b^5 +
 (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) - (C*a
^4*b - 2*C*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*
log(-sin(d*x + c) + 1) - 2*(C*a^4*b + (A - C)*a^2*b^3 - A*b^5)*sin(d*x + c)
)/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*a^2*b^5 + b^
7)*d), -1/2*(2*(C*a^3*b - (A + 2*C)*a*b^3 + (C*a^4 - (A + 2*C)*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c))) - (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*
a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) + (C*a^4*b - 2*C*a^2
```

```
*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(-sin(d*x +
 c) + 1) + 2*(C*a^4*b + (A - C)*a^2*b^3 - A*b^5)*sin(d*x + c))/((a^5*b^2 -
2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*a^2*b^5 + b^7)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.27165, size = 312, normalized size = 2.31

$$\frac{\frac{2\left(Ca^3-Aab^2-2Cab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{-a^2+b^2}}+\frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(C*a^3 - A*a*b^2 - 2*C*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a
- 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + C*log(abs(tan(1/2*d*x + 1/2*
c) + 1))/b^2 - C*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*(C*a^2*tan(1/2*
d*x + 1/2*c) + A*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

3.687 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=125

$$-\frac{2b\left(2a^2A+a^2C-Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2C+Ab^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Ax}{a^2}$$

```
[Out] (A*x)/a^2 - (2*b*(2*a^2*A - A*b^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((A*b^2 + a^2*C)
*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.226777, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4061, 3919, 3831, 2659, 208}

$$-\frac{2b\left(2a^2A+a^2C-Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2C+Ab^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Ax}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (A*x)/a^2 - (2*b*(2*a^2*A - A*b^2 + a^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((A*b^2 + a^2*C)
*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-A\left(a^2-b^2\right)+ab(A+C)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$
$$= \frac{Ax}{a^2} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(b\left(Ab^2-a^2(2A+C)\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$
$$= \frac{Ax}{a^2} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(Ab^2-a^2(2A+C)\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)}$$
$$= \frac{Ax}{a^2} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\left(Ab^2-a^2(2A+C)\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x$$
$$= \frac{Ax}{a^2} - \frac{2b\left(2a^2A-Ab^2+a^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [C] time = 1.97814, size = 270, normalized size = 2.16

$$\frac{2(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{\left(a^2C+Ab^2\right)(a\sin(dx)-b\sin(c))}{d(a-b)(a+b)\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)} + \frac{2b(\sin(c)+i\cos(c))\left(a^2(2A+C)-Ab^2\right)(a\cos(c+dx)+b}{d\left(a^2-b^2\right)^{3/2}\sqrt{(c}}\right.}{a^2(a+b\sec(c+dx))^2(A\cos(2(c+dx))+A+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*(A*x*(b + a*Cos[c + d*x]) +
(2*b*(-(A*b^2) + a^2*(2*A + C))*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b
 + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*
(b + a*Cos[c + d*x])*(I*Cos[c] + Sin[c]))/((a^2 - b^2)^(3/2)*d*Sqrt[(Cos[c]
 - I*Sin[c])^2]) + ((A*b^2 + a^2*C)*(-(b*Sin[c]) + a*Sin[d*x]))/((a - b)*(a
 + b)*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))))/(a^2*(A + 2*C + A*Co
s[2*(c + d*x)])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.094, size = 328, normalized size = 2.6

$$2\,\frac{A\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2} - 2\,\frac{A\tan\left(1/2\,dx+c/2\right)b^2}{ad\left(a^2-b^2\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b-a-b\right)} - 2\,\frac{}{d\left(a^2-b^2\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*A/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A*b^2-2/d*a/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-4/d*b/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A+2/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.599719, size = 1188, normalized size = 9.5

$$\left[\frac{2\left(Aa^5 - 2\,Aa^3b^2 + Aab^4\right)dx\cos\left(dx+c\right) + 2\left(Aa^4b - 2\,Aa^2b^3 + Ab^5\right)dx - \left((2\,A+C)a^2b^2 - Ab^4 + \left((2\,A+C)a^3b - \cdots\right.\right.}{2\left(\left(a^7 - 2\,a^5b^2 + \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + 2*(A*a^4*b - 2*A
*a^2*b^3 + A*b^5)*d*x - ((2*A + C)*a^2*b^2 - A*b^4 + ((2*A + C)*a^3*b - A*a
*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^5 + (A
- C)*a^3*b^2 - A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*
x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((A*a^5 - 2*A*a^3*b^2 + A*a*b^4)
*d*x*cos(d*x + c) + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*d*x - ((2*A + C)*a^2*b^
2 - A*b^4 + ((2*A + C)*a^3*b - A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*
a^5 + (A - C)*a^3*b^2 - A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)
*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2\left(c+dx\right)}{\left(a + b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.19606, size = 277, normalized size = 2.22

$$-\frac{\frac{2\left(2\,Aa^2b+Ca^2b-Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{-a^2+b^2}}-\frac{(dx+c)A}{a^2}+\frac{2\left(Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(2*A*a^2*b + C*a^2*b - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*
a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (d*x + c)*A/a^2 + 2*(C*a^
2*tan(1/2*d*x + 1/2*c) + A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(
1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

3.688 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=171

$$-\frac{\left(2Ab^2-a^2(A-C)\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2\left(3a^2Ab^2+a^4C-2Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2C+Ab^2\right)\sin(c+d}{ad\left(a^2-b^2\right)(a+b\sec(c+}$$

```
[Out] (-2*A*b*x)/a^3 + (2*(3*a^2*A*b^2 - 2*A*b^4 + a^4*C)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*A*b
^2 - a^2*(A - C))*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.433962, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4101, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(2Ab^2-a^2(A-C)\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2\left(3a^2Ab^2+a^4C-2Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2C+Ab^2\right)\sin(c+d}{ad\left(a^2-b^2\right)(a+b\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*A*b*x)/a^3 + (2*(3*a^2*A*b^2 - 2*A*b^4 + a^4*C)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*A*b
^2 - a^2*(A - C))*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2Ab^2-a^2(A-C)+ab(A+C)\sec(c+dx)-\left(Ab^2+\right.\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(2Ab^2-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-2Ab(}{}}{}$$

$$= -\frac{2Abx}{a^3} - \frac{\left(2Ab^2-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} +$$

$$= -\frac{2Abx}{a^3} - \frac{\left(2Ab^2-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} +$$

$$= -\frac{2Abx}{a^3} - \frac{\left(2Ab^2-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} +$$

$$= -\frac{2Abx}{a^3} + \frac{2\left(3a^2Ab^2-2Ab^4+a^4C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{\left(2Ab^2-a^2(\right.}{a^2\left(\right.}$$

Mathematica [A] time = 0.95258, size = 137, normalized size = 0.8

$$\frac{-\frac{ab\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - \frac{2\left(3a^2Ab^2+a^4C-2Ab^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + aA\sin(c+dx) - 2Ab(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*A*b*(c + d*x) - (2*(3*a^2*A*b^2 - 2*A*b^4 + a^4*C)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + a*A*Sin[c + d*x] - (a
*b*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/(a
^3*d)
```

Maple [B] time = 0.123, size = 367, normalized size = 2.2

$$2\,\frac{A\tan(1/2\,dx+c/2)}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\,\frac{Ab\arctan(\tan(1/2\,dx+c/2))}{da^3}+2\,\frac{b^3\tan(1/2\,dx+c/2)\,A}{da^2\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*A/a^2*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d*A/a^3*b*arctan(ta
n(1/2*d*x+1/2*c))+2/d/a^2*b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C+6/d/a/(a+b)/(a-b)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^2-
4/d/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*A*b^4+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.630061, size = 1404, normalized size = 8.21

$$\left[-\frac{4\left(Aa^5b-2\,Aa^3b^3+Aab^5\right)dx\cos(dx+c)+4\left(Aa^4b^2-2\,Aa^2b^4+Ab^6\right)dx+\left(Ca^4b+3\,Aa^2b^3-2\,Ab^5+\left(Ca^5+3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(4*(A*a^5*b - 2*A*a^3*b^3 + A*a*b^5)*d*x*cos(d*x + c) + 4*(A*a^4*b^2
- 2*A*a^2*b^4 + A*b^6)*d*x + (C*a^4*b + 3*A*a^2*b^3 - 2*A*b^5 + (C*a^5 + 3*
A*a^3*b^2 - 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c
) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*s
in(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
```

```
- 2*((A - C)*a^5*b - (3*A - C)*a^3*b^3 + 2*A*a*b^5 + (A*a^6 - 2*A*a^4*b^2
+ A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos
(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), -(2*(A*a^5*b - 2*A*a^3*b^3 +
A*a*b^5)*d*x*cos(d*x + c) + 2*(A*a^4*b^2 - 2*A*a^2*b^4 + A*b^6)*d*x - (C*a^
4*b + 3*A*a^2*b^3 - 2*A*b^5 + (C*a^5 + 3*A*a^3*b^2 - 2*A*a*b^4)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - ((A - C)*a^5*b - (3*A - C)*a^3*b^3 + 2*A*a*b^5 + (A*a^
6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2
+ a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\cos\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.18463, size = 479, normalized size = 2.8

$$2\left(\frac{\left(Ca^4+3\,Aa^2b^2-2\,Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{-a^2+b^2}}-\frac{(dx+c)Ab}{a^3}+\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 2*((C*a^4 + 3*A*a^2*b^2 - 2*A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
-a^2 + b^2)))/((a^5 - a^3*b^2)*sqrt(-a^2 + b^2)) - (d*x + c)*A*b/a^3 + (A*a
^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + C*a^2*b*tan(1/
2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1
/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c) + C*a^2
*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*
x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan
(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)))/d
```

3.689 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=256

$$\frac{b\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}-\frac{\left(3Ab^2-a^2(A-2C)\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b\left(4a^2Ab^2-a^2b^2C+2a^4C-3\right.}{a^4d(a-b)^{3/}}$$

```
[Out] ((6*A*b^2 + a^2*(A + 2*C))*x)/(2*a^4) - (2*b*(4*a^2*A*b^2 - 3*A*b^4 + 2*a^4
*C - a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(
a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(3*A*b^2 - a^2*(2*A - C))*Sin[c + d*x])/
(a^3*(a^2 - b^2)*d) - ((3*A*b^2 - a^2*(A - 2*C))*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Cos[c + d*x]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.860479, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4101, 4104, 3919, 3831, 2659, 208}

$$\frac{b\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}-\frac{\left(3Ab^2-a^2(A-2C)\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b\left(4a^2Ab^2-a^2b^2C+2a^4C-3\right.}{a^4d(a-b)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((6*A*b^2 + a^2*(A + 2*C))*x)/(2*a^4) - (2*b*(4*a^2*A*b^2 - 3*A*b^4 + 2*a^4
*C - a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(
a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(3*A*b^2 - a^2*(2*A - C))*Sin[c + d*x])/
(a^3*(a^2 - b^2)*d) - ((3*A*b^2 - a^2*(A - 2*C))*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*Cos[c + d*x]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(3Ab^2-a^2(A-2C)+ab(A+C)\sec(c+\ldots\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(3Ab^2-a^2(A-2C)\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin\ldots}{a\left(a^2-b^2\right)d(a+b\sec(c-\ldots}$$

$$= \frac{b\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-a^2(A-2C)\right)\cos(c+dx)\sin(c-\ldots}{2a^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(6Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{b\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-a^2(A\ldots\right.}{\ldots}$$

$$= \frac{\left(6Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{b\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-a^2(A\ldots\right.}{\ldots}$$

$$= \frac{\left(6Ab^2+a^2(A+2C)\right)x}{2a^4} + \frac{b\left(3Ab^2-a^2(2A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(3Ab^2-a^2(A\ldots\right.}{\ldots}$$

$$= \frac{\left(6Ab^2+a^2(A+2C)\right)x}{2a^4} - \frac{2b\left(4a^2Ab^2-3Ab^4+2a^4C-a^2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\ldots}{\ldots}\right.}{a^4(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 0.946795, size = 176, normalized size = 0.69

$$\frac{2(c+dx)\left(a^2(A+2C)+6Ab^2\right)+\frac{4ab^2\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}-\frac{8b\left(a^2b^2(C-4A)-2a^4C+3Ab^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+a^2A\sin(2(c+dx\ldots}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(6*A*b^2 + a^2*(A + 2*C))*(c + d*x) - (8*b*(3*A*b^4 - 2*a^4*C + a^2*b^2*
(-4*A + C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) - 8*a*A*b*Sin[c + d*x] + (4*a*b^2*(A*b^2 + a^2*C)*Sin[c + d*x])/((
a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*A*Sin[2*(c + d*x)])/(4*a^4*d)
```

Maple [B] time = 0.128, size = 577, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-4/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)*A-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2
*c)*A*b+1/d*A/a^2*arctan(tan(1/2*d*x+1/2*c))+6/d/a^4*arctan(tan(1/2*d*x+1/2
*c))*A*b^2+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b^4/a^3/(a^2-b^2)*tan(1
/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-2/d*b^2
/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)*C-8/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*b^5/a^4/(a+b)/(a-b)/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-4/d*b/(a+b)/(a
-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*C+2/d*b^3/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.715402, size = 1820, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/2*(((A + 2*C)*a^7 + 4*(A - C)*a^5*b^2 - (11*A - 2*C)*a^3*b^4 + 6*A*a*b^6
)*d*x*cos(d*x + c) + ((A + 2*C)*a^6*b + 4*(A - C)*a^4*b^3 - (11*A - 2*C)*a^
2*b^5 + 6*A*b^7)*d*x + (2*C*a^4*b^2 + (4*A - C)*a^2*b^4 - 3*A*b^6 + (2*C*a^
5*b + (4*A - C)*a^3*b^3 - 3*A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d
*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) - (2*(2*A - C)*a^5*b^2 - 2*(5*A - C)*a^3*b^4 + 6*A*a*b^6 - (
A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + 3*(A*a^6*b - 2*A*a^4*b^3
+ A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos
(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*(((A + 2*C)*a^7 + 4*(A -
C)*a^5*b^2 - (11*A - 2*C)*a^3*b^4 + 6*A*a*b^6)*d*x*cos(d*x + c) + ((A + 2*C
)*a^6*b + 4*(A - C)*a^4*b^3 - (11*A - 2*C)*a^2*b^5 + 6*A*b^7)*d*x - 2*(2*C*
a^4*b^2 + (4*A - C)*a^2*b^4 - 3*A*b^6 + (2*C*a^5*b + (4*A - C)*a^3*b^3 - 3*
A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*(2*A - C)*a^5*b^2 - 2*(5*A - C)
*a^3*b^4 + 6*A*a*b^6 - (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + 3
*(A*a^6*b - 2*A*a^4*b^3 + A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*
a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25938, size = 425, normalized size = 1.66

$$-\frac{\frac{4\left(2\,Ca^4b+4\,Aa^2b^3-Ca^2b^3-3\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}+\frac{4\left(Ca^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -1/2*(4*(2*C*a^4*b + 4*A*a^2*b^3 - C*a^2*b^3 - 3*A*b^5)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/
2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2)) + 4*(
C*a^2*b^2*tan(1/2*d*x + 1/2*c) + A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^
2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (A*a^2
+ 2*C*a^2 + 6*A*b^2)*(d*x + c)/a^4 + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 4*A*b*
tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) + 4*A*b*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

3.690 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=326

$$-\frac{\left(-a^2b^2(7A-6C)+a^4(-(2A+3C))+12Ab^4\right)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}-\frac{\left(4Ab^2-a^2(A-3C)\right)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{b\left(2A\right.}{}$$

```
[Out] -((b*(4*A*b^2 + a^2*(A + 2*C))*x)/a^5) + (2*b^2*(5*a^2*A*b^2 - 4*A*b^4 + 3*
a^4*C - 2*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((12*A*b^4 - a^2*b^2*(7*A - 6*C) - a^4
*(2*A + 3*C))*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) + (b*(2*A*b^2 - a^2*(A -
C))*Cos[c + d*x]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) - ((4*A*b^2 - a^2*(A - 3
*C))*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*
Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.25529, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4101, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(-a^2b^2(7A-6C)+a^4(-(2A+3C))+12Ab^4\right)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}-\frac{\left(4Ab^2-a^2(A-3C)\right)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{b\left(2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(4*A*b^2 + a^2*(A + 2*C))*x)/a^5) + (2*b^2*(5*a^2*A*b^2 - 4*A*b^4 + 3*
a^4*C - 2*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((12*A*b^4 - a^2*b^2*(7*A - 6*C) - a^4
*(2*A + 3*C))*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) + (b*(2*A*b^2 - a^2*(A -
C))*Cos[c + d*x]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) - ((4*A*b^2 - a^2*(A - 3
*C))*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + ((A*b^2 + a^2*C)*
Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
```

```
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^3(c+dx)\left(4Ab^2-a^2(A-3C)+ab(A+C)\sec(c\cdots\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)\cdots}$$

$$= -\frac{\left(4Ab^2-a^2(A-3C)\right)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2+a^2C\right)\cos^2(c+dx)\,\mathrm{s}\cdots}{a\left(a^2-b^2\right)d(a+b\sec(c\cdots}$$

$$= \frac{b\left(2Ab^2-a^2(A-C)\right)\cos(c+dx)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} - \frac{\left(4Ab^2-a^2(A-3C)\right)\cos^2(c\cdots}{3a^2\left(a^2-b^2\right)\cdots}$$

$$= -\frac{\left(12Ab^4-a^2b^2(7A-6C)-a^4(2A+3C)\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} + \frac{b\left(2Ab^2-a^2(A-C\cdots\right.}{a^3\left(\cdots\right.}$$

$$= -\frac{b\left(4Ab^2+a^2(A+2C)\right)x}{a^5} - \frac{\left(12Ab^4-a^2b^2(7A-6C)-a^4(2A+3C)\right)\sin(c\cdots}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(4Ab^2+a^2(A+2C)\right)x}{a^5} - \frac{\left(12Ab^4-a^2b^2(7A-6C)-a^4(2A+3C)\right)\sin(c\cdots}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(4Ab^2+a^2(A+2C)\right)x}{a^5} - \frac{\left(12Ab^4-a^2b^2(7A-6C)-a^4(2A+3C)\right)\sin(c\cdots}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(4Ab^2+a^2(A+2C)\right)x}{a^5} + \frac{2b^2\left(5a^2Ab^2-4Ab^4+3a^4C-2a^2b^2C\right)\tanh^{-1}\left(\cdots\right.}{a^5(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 1.16316, size = 212, normalized size = 0.65

$$\frac{-12b(c+dx)\left(a^2(A+2C)+4Ab^2\right)+3a\left(a^2(3A+4C)+12Ab^2\right)\sin(c+dx)-\frac{12ab^3\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}+\frac{24b^2\left(a^2b^2(2C-5A)\right.}{}}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-12*b*(4*A*b^2 + a^2*(A + 2*C))*(c + d*x) + (24*b^2*(4*A*b^4 - 3*a^4*C + a
^2*b^2*(-5*A + 2*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(3/2) + 3*a*(12*A*b^2 + a^2*(3*A + 4*C))*Sin[c + d*x] - (12*a*b
^3*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) - 6
*a^2*A*b*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])/(12*a^5*d)
```

Maple [B] time = 0.128, size = 836, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)^5+2/d/a^3/(1+tan(1/
2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)^5*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)
^3*A*tan(1/2*d*x+1/2*c)^5*b^2+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*
d*x+1/2*c)^5+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+12
/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A*b^2+4/d/a^2/(1+tan
(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^3*A*tan(1/2*d*x+1/2*c)-2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1
/2*c)*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)*b^2+2/d/a^2
/(1+tan(1/2*d*x+1/2*c)^2)^3*C*tan(1/2*d*x+1/2*c)-2/d*A/a^3*b*arctan(tan(1/2
*d*x+1/2*c))-8/d/a^5*A*arctan(tan(1/2*d*x+1/2*c))*b^3-4/d/a^3*C*arctan(tan(
1/2*d*x+1/2*c))*b+2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2
*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C+10/d/a^3/(a+b)/(a
-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*A*b^4-8/d*b^6/a^5/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*b^2/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/d*b^4/a^3/(a+b)/
(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.785335, size = 2187, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/6*(6*((A + 2*C)*a^7*b + 2*(A - 2*C)*a^5*b^3 - (7*A - 2*C)*a^3*b^5 + 4*A
*a*b^7)*d*x*cos(d*x + c) + 6*((A + 2*C)*a^6*b^2 + 2*(A - 2*C)*a^4*b^4 - (7*
A - 2*C)*a^2*b^6 + 4*A*b^8)*d*x - 3*(3*C*a^4*b^3 + (5*A - 2*C)*a^2*b^5 - 4*
A*b^7 + (3*C*a^5*b^2 + (5*A - 2*C)*a^3*b^4 - 4*A*a*b^6)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((2*A + 3*C)*a^7*b + (5*A - 9*C)*a^5*
b^3 - (19*A - 6*C)*a^3*b^5 + 12*A*a*b^7 + (A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)
*cos(d*x + c)^3 - 2*(A*a^7*b - 2*A*a^5*b^3 + A*a^3*b^5)*cos(d*x + c)^2 + ((
2*A + 3*C)*a^8 + 2*(A - 3*C)*a^6*b^2 - (10*A - 3*C)*a^4*b^4 + 6*A*a^2*b^6)*
cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) +
(a^9*b - 2*a^7*b^3 + a^5*b^5)*d), -1/3*(3*((A + 2*C)*a^7*b + 2*(A - 2*C)*a^
5*b^3 - (7*A - 2*C)*a^3*b^5 + 4*A*a*b^7)*d*x*cos(d*x + c) + 3*((A + 2*C)*a^
6*b^2 + 2*(A - 2*C)*a^4*b^4 - (7*A - 2*C)*a^2*b^6 + 4*A*b^8)*d*x - 3*(3*C*a
^4*b^3 + (5*A - 2*C)*a^2*b^5 - 4*A*b^7 + (3*C*a^5*b^2 + (5*A - 2*C)*a^3*b^4
 - 4*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
s(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((2*A + 3*C)*a^7*b + (5*A - 9
*C)*a^5*b^3 - (19*A - 6*C)*a^3*b^5 + 12*A*a*b^7 + (A*a^8 - 2*A*a^6*b^2 + A*
a^4*b^4)*cos(d*x + c)^3 - 2*(A*a^7*b - 2*A*a^5*b^3 + A*a^3*b^5)*cos(d*x + c
)^2 + ((2*A + 3*C)*a^8 + 2*(A - 3*C)*a^6*b^2 - (10*A - 3*C)*a^4*b^4 + 6*A*a
^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x
 + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25442, size = 595, normalized size = 1.83

$$\frac{\frac{6\left(3\,Ca^4b^2+5\,Aa^2b^4-2\,Ca^2b^4-4\,Ab^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-a^5b^2\right)\sqrt{-a^2+b^2}}+\frac{6\left(Ca^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] 1/3*(6*(3*C*a^4*b^2 + 5*A*a^2*b^4 - 2*C*a^2*b^4 - 4*A*b^6)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2)) +
6*(C*a^2*b^3*tan(1/2*d*x + 1/2*c) + A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4
*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 3*(A
*a^2*b + 2*C*a^2*b + 4*A*b^3)*(d*x + c)/a^5 + 2*(3*A*a^2*tan(1/2*d*x + 1/2*
c)^5 + 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*
A*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*tan
(1/2*d*x + 1/2*c)^3 + 18*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^2*tan(1/2*d*x
 + 1/2*c) + 3*C*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) + 9
*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4))/d
```

3.691 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=381

$$-\frac{a\left(a^2b^2(2A-21C)+12a^4C-b^4(5A-6C)\right)\tan(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{\left(C\left(12a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2b^5d}-\frac{a\left(a^4b^2(2A-\right.}{}$$

```
[Out] ((2*A*b^2 + (12*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*b^5*d) - (a*(6*A*b^
6 + a^4*b^2*(2*A - 29*C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) -
(a*(a^2*b^2*(2*A - 21*C) - b^4*(5*A - 6*C) + 12*a^4*C)*Tan[c + d*x])/(2*b^4
*(a^2 - b^2)^2*d) + ((a^2*b^2*(A - 10*C) - b^4*(4*A - C) + 6*a^4*C)*Sec[c +
 d*x]*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*Sec[c + d*x]
^3*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3*A*b^4 - 4
*a^4*C + 7*a^2*b^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.62036, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4099, 4098, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(a^2b^2(2A-21C)+12a^4C-b^4(5A-6C)\right)\tan(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{\left(C\left(12a^2+b^2\right)+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2b^5d}-\frac{a\left(a^4b^2(2A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((2*A*b^2 + (12*a^2 + b^2)*C)*ArcTanh[Sin[c + d*x]])/(2*b^5*d) - (a*(6*A*b^
6 + a^4*b^2*(2*A - 29*C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) -
(a*(a^2*b^2*(2*A - 21*C) - b^4*(5*A - 6*C) + 12*a^4*C)*Tan[c + d*x])/(2*b^4
*(a^2 - b^2)^2*d) + ((a^2*b^2*(A - 10*C) - b^4*(4*A - C) + 6*a^4*C)*Sec[c +
 d*x]*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) - ((A*b^2 + a^2*C)*Sec[c + d*x]
^3*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3*A*b^4 - 4
*a^4*C + 7*a^2*b^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x]))
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)\left(3\left(Ab^2+a^2C\right)-2ab(A+C)\sec(c+\ldots\right.}{(a+b\sec(c+\ldots}}{2b\left(a^2-\ldots\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(3Ab^4-4a^4C+7a^2b^2C\right)\sec^2(c+\ldots}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c\ldots}$$

$$= \frac{\left(a^2b^2(A-10C)-b^4(4A-C)+6a^4C\right)\sec(c+dx)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2 d} - \frac{\left(Ab^2+a^2C\ldots\right.}{2b\left(a^2-b\ldots\right.}$$

$$= -\frac{a\left(a^2b^2(2A-21C)-b^4(5A-6C)+12a^4C\right)\tan(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^2b^2(A-10C)-\ldots\right.}{}$$

$$= -\frac{a\left(a^2b^2(2A-21C)-b^4(5A-6C)+12a^4C\right)\tan(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^2b^2(A-10C)-\ldots\right.}{}$$

$$= \frac{\left(2Ab^2+\left(12a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} - \frac{a\left(a^2b^2(2A-21C)-b^4(5A-\ldots\right.}{2b^4\left(a^2-\ldots\right.}$$

$$= \frac{\left(2Ab^2+\left(12a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} - \frac{a\left(a^2b^2(2A-21C)-b^4(5A-\ldots\right.}{2b^4\left(a^2-\ldots\right.}$$

$$= \frac{\left(2Ab^2+\left(12a^2+b^2\right)C\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} - \frac{a\left(2a^4Ab^2-5a^2Ab^4+6Ab^6+\ldots\right.}{}$$

Mathematica [A] time = 4.14997, size = 559, normalized size = 1.47

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{2a^2b^2\left(a^2C+Ab^2\right)\sin(c+dx)}{(b-a)(a+b)}+\frac{2a^2b\left(a^2b^2(9C-2A)-6a^4C+5Ab^4\right)\sin(c+dx)(a\cos(c+dx)+\ldots}{(a-b)^2(a+b)^2}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((4*a*(6*A*b^6 +
a^4*b^2*(2*A - 29*C) - 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2)
- 2*(2*A*b^2 + (12*a^2 + b^2)*C)*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 + (12*a^2 + b^2)*C)*(b + a*Cos[c + d*x])
^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^2*C*(b + a*Cos[c + d*x])^2
)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - (12*a*b*C*(b + a*Cos[c + d*x])^
2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^2*C*(b + a*C
os[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (12*a*b*C*(b + a*
Cos[c + d*x])^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (
2*a^2*b^2*(A*b^2 + a^2*C)*Sin[c + d*x])/((-a + b)*(a + b)) + (2*a^2*b*(5*A*
b^4 - 6*a^4*C + a^2*b^2*(-2*A + 9*C))*(b + a*Cos[c + d*x])*Sin[c + d*x])/((
a - b)^2*(a + b)^2)))/(2*b^5*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c
```

```
+ d*x])^3)
```

Maple [B] time = 0.109, size = 1547, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/2/d*C/b^3/(tan(1/2*d*x+1/2*c)+1)+1/2/d*C/b^3/(tan(1/2*d*x+1/2*c)-1)^2-1/d
/b^3*ln(tan(1/2*d*x+1/2*c)-1)*A-1/2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d*
C/b^3/(tan(1/2*d*x+1/2*c)-1)-1/2/d*C/b^3/(tan(1/2*d*x+1/2*c)+1)^2+1/d/b^3*l
n(tan(1/2*d*x+1/2*c)+1)*A+1/2/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*C+29/d*a^5/b^3
/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))*C-20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*a^5/b^3/(a^4-2*a^
2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-12/d*a^7/b^5/(a^4-2*a^2*b^2+b^4)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
C-6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-6/d*a*b/(a^4-2*a^2*
b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*A-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-6/d*a^6/b^4/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*
x+1/2*c)*C-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*a^5/b^3/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+2/d*
a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+10/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+6/d*a^6/b^4/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*C-1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-10/d*a^4/b^2/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*C+6/d/b^5*ln(tan(1/2*d*x+1/2*c)+1)*a^2*C+3/d*C/b^4/(tan(1/2
*d*x+1/2*c)-1)*a+3/d*C/b^4/(tan(1/2*d*x+1/2*c)+1)*a-6/d/b^5*ln(tan(1/2*d*x+
1/2*c)-1)*a^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 78.1789, size = 4635, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [1/4*(((12*C*a^9 + (2*A - 29*C)*a^7*b^2 - 5*(A - 4*C)*a^5*b^4 + 6*A*a^3*b^6
)*cos(d*x + c)^4 + 2*(12*C*a^8*b + (2*A - 29*C)*a^6*b^3 - 5*(A - 4*C)*a^4*b
^5 + 6*A*a^2*b^7)*cos(d*x + c)^3 + (12*C*a^7*b^2 + (2*A - 29*C)*a^5*b^4 - 5
*(A - 4*C)*a^3*b^6 + 6*A*a*b^8)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*
cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
+ c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + b^2)) + ((12*C*a^10 + (2*A - 35*C)*a^8*b^2 - 3*(2*A - 11*C)*a^6*b^4 +
 3*(2*A - 3*C)*a^4*b^6 - (2*A + C)*a^2*b^8)*cos(d*x + c)^4 + 2*(12*C*a^9*b
+ (2*A - 35*C)*a^7*b^3 - 3*(2*A - 11*C)*a^5*b^5 + 3*(2*A - 3*C)*a^3*b^7 - (
2*A + C)*a*b^9)*cos(d*x + c)^3 + (12*C*a^8*b^2 + (2*A - 35*C)*a^6*b^4 - 3*(
2*A - 11*C)*a^4*b^6 + 3*(2*A - 3*C)*a^2*b^8 - (2*A + C)*b^10)*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) - ((12*C*a^10 + (2*A - 35*C)*a^8*b^2 - 3*(2*A - 11
*C)*a^6*b^4 + 3*(2*A - 3*C)*a^4*b^6 - (2*A + C)*a^2*b^8)*cos(d*x + c)^4 + 2
*(12*C*a^9*b + (2*A - 35*C)*a^7*b^3 - 3*(2*A - 11*C)*a^5*b^5 + 3*(2*A - 3*C
)*a^3*b^7 - (2*A + C)*a*b^9)*cos(d*x + c)^3 + (12*C*a^8*b^2 + (2*A - 35*C)*
a^6*b^4 - 3*(2*A - 11*C)*a^4*b^6 + 3*(2*A - 3*C)*a^2*b^8 - (2*A + C)*b^10)*
cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(C*a^6*b^4 - 3*C*a^4*b^6 + 3*C*a
^2*b^8 - C*b^10 - (12*C*a^9*b + (2*A - 33*C)*a^7*b^3 - (7*A - 27*C)*a^5*b^5
 + (5*A - 6*C)*a^3*b^7)*cos(d*x + c)^3 - (18*C*a^8*b^2 + (3*A - 50*C)*a^6*b
^4 - (9*A - 43*C)*a^4*b^6 + (6*A - 11*C)*a^2*b^8)*cos(d*x + c)^2 - 4*(C*a^7
*b^3 - 3*C*a^5*b^5 + 3*C*a^3*b^7 - C*a*b^9)*cos(d*x + c))*sin(d*x + c))/((a
^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d*cos(d*x + c)^4 + 2*(a^7*b^6 -
3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 +
3*a^2*b^11 - b^13)*d*cos(d*x + c)^2), -1/4*(2*((12*C*a^9 + (2*A - 29*C)*a^7
*b^2 - 5*(A - 4*C)*a^5*b^4 + 6*A*a^3*b^6)*cos(d*x + c)^4 + 2*(12*C*a^8*b +
(2*A - 29*C)*a^6*b^3 - 5*(A - 4*C)*a^4*b^5 + 6*A*a^2*b^7)*cos(d*x + c)^3 +
(12*C*a^7*b^2 + (2*A - 29*C)*a^5*b^4 - 5*(A - 4*C)*a^3*b^6 + 6*A*a*b^8)*cos
(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c))) - ((12*C*a^10 + (2*A - 35*C)*a^8*b^2 - 3*(2*A
- 11*C)*a^6*b^4 + 3*(2*A - 3*C)*a^4*b^6 - (2*A + C)*a^2*b^8)*cos(d*x + c)^4
 + 2*(12*C*a^9*b + (2*A - 35*C)*a^7*b^3 - 3*(2*A - 11*C)*a^5*b^5 + 3*(2*A -
 3*C)*a^3*b^7 - (2*A + C)*a*b^9)*cos(d*x + c)^3 + (12*C*a^8*b^2 + (2*A - 35
*C)*a^6*b^4 - 3*(2*A - 11*C)*a^4*b^6 + 3*(2*A - 3*C)*a^2*b^8 - (2*A + C)*b^
10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((12*C*a^10 + (2*A - 35*C)*a^8*
b^2 - 3*(2*A - 11*C)*a^6*b^4 + 3*(2*A - 3*C)*a^4*b^6 - (2*A + C)*a^2*b^8)*c
os(d*x + c)^4 + 2*(12*C*a^9*b + (2*A - 35*C)*a^7*b^3 - 3*(2*A - 11*C)*a^5*b
^5 + 3*(2*A - 3*C)*a^3*b^7 - (2*A + C)*a*b^9)*cos(d*x + c)^3 + (12*C*a^8*b^
2 + (2*A - 35*C)*a^6*b^4 - 3*(2*A - 11*C)*a^4*b^6 + 3*(2*A - 3*C)*a^2*b^8 -
 (2*A + C)*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(C*a^6*b^4 - 3*
C*a^4*b^6 + 3*C*a^2*b^8 - C*b^10 - (12*C*a^9*b + (2*A - 33*C)*a^7*b^3 - (7*
A - 27*C)*a^5*b^5 + (5*A - 6*C)*a^3*b^7)*cos(d*x + c)^3 - (18*C*a^8*b^2 + (
3*A - 50*C)*a^6*b^4 - (9*A - 43*C)*a^4*b^6 + (6*A - 11*C)*a^2*b^8)*cos(d*x
+ c)^2 - 4*(C*a^7*b^3 - 3*C*a^5*b^5 + 3*C*a^3*b^7 - C*a*b^9)*cos(d*x + c))*
sin(d*x + c))/((a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d*cos(d*x + c)^
4 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c)^3 + (a^6*b
^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2(c + dx)\right)\sec^4(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x
)
```

Giac [B] time = 1.34949, size = 1602, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/2*(2*(12*C*a^7 + 2*A*a^5*b^2 - 29*C*a^5*b^2 - 5*A*a^3*b^4 + 20*C*a^3*b^4
 + 6*A*a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b
^5 - 2*a^2*b^7 + b^9)*sqrt(-a^2 + b^2)) - 2*(12*C*a^7*tan(1/2*d*x + 1/2*c)^
7 - 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 2*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7
- 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7
+ 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7
- 2*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 -
 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^7 + C
*b^7*tan(1/2*d*x + 1/2*c)^7 - 36*C*a^7*tan(1/2*d*x + 1/2*c)^5 + 18*C*a^6*b*
tan(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 67*C*a^5*b^2*
tan(1/2*d*x + 1/2*c)^5 + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*C*a^4*b^3*
tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 26*C*a^3*b^4
*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 5*C*a^2*b^5*
tan(1/2*d*x + 1/2*c)^5 + 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 3*C*b^7*tan(1/2
*d*x + 1/2*c)^5 + 36*C*a^7*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^6*b*tan(1/2*d*x
+ 1/2*c)^3 + 6*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*C*a^5*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*C*a^4*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 15*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 26*C*a^3*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*C*a^2*b^5*tan(1/2*d*x
+ 1/2*c)^3 - 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 3*C*b^7*tan(1/2*d*x + 1/2*c
)^3 - 12*C*a^7*tan(1/2*d*x + 1/2*c) - 18*C*a^6*b*tan(1/2*d*x + 1/2*c) - 2*A
*a^5*b^2*tan(1/2*d*x + 1/2*c) + 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c) - 3*A*a^4
*b^3*tan(1/2*d*x + 1/2*c) + 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c) + 5*A*a^3*b^4
*tan(1/2*d*x + 1/2*c) + 2*C*a^3*b^4*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^5*tan(
1/2*d*x + 1/2*c) - 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c) - 4*C*a*b^6*tan(1/2*d*
x + 1/2*c) + C*b^7*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*ta
n(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^
2 + a + b)^2) - (12*C*a^2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/b^5 + (12*C*a^2 + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/
b^5)/d
```

3.692 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=271

$$\frac{\left(3a^2C+Ab^2-2b^2C\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)-15a^4b^2C+6a^6C+2Ab^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2C+Ab\right.}{2bd\left(a^2\right.}$$

```
[Out] (-3*a*C*ArcTanh[Sin[c + d*x]])/(b^4*d) + ((2*A*b^6 + 6*a^6*C - 15*a^4*b^2*C
 + a^2*b^4*(A + 12*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((A*b^2 + 3*a^2*C - 2*b^2*C)*Tan[c +
 d*x])/(2*b^3*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a*(2*A*b^4 - 3*a^4*C + a^2*
b^2*(A + 6*C))*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.02213, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4099, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(3a^2C+Ab^2-2b^2C\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)-15a^4b^2C+6a^6C+2Ab^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2C+Ab\right.}{2bd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] (-3*a*C*ArcTanh[Sin[c + d*x]])/(b^4*d) + ((2*A*b^6 + 6*a^6*C - 15*a^4*b^2*C
 + a^2*b^4*(A + 12*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((A*b^2 + 3*a^2*C - 2*b^2*C)*Tan[c +
 d*x])/(2*b^3*(a^2 - b^2)*d) - ((A*b^2 + a^2*C)*Sec[c + d*x]^2*Tan[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a*(2*A*b^4 - 3*a^4*C + a^2*
b^2*(A + 6*C))*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx = -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)\left(2\left(Ab^2+a^2C\right)-2ab(A+C)\sec(c+\cdots}{(a+b\sec(c+\cdots}}{2b\left(a^2-\cdots\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\mathrm{ta}\cdots}{2b^3\left(a^2-b^2\right)^2d(a+b\sec(c+\cdots}$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \cdots$$

$$= \frac{\left(Ab^2+3a^2C-2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \cdots$$

$$= -\frac{3aC\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(Ab^2+3a^2C-2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\cdots\right.}{2b\left(a^2-\cdots\right.}$$

$$= -\frac{3aC\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(Ab^2+3a^2C-2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2+a^2C\cdots\right.}{2b\left(a^2-\cdots\right.}$$

$$= -\frac{3aC\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(a^2Ab^4+2Ab^6+6a^6C-15a^4b^2C+12a^2b^4C\right)\mathrm{t}\cdots}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

Mathematica [A] time = 2.26337, size = 421, normalized size = 1.55

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{ab^2\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)} + \frac{ab\left(-7a^2b^2C+4a^4C-3Ab^4\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} - \frac{2(a\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((-2*(2*A*b^6 + 6
*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + 6*a*C*(b
+ a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*a*C*(b + a
*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*b*C*(b + a*C
os[c + d*x])^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2
*b*C*(b + a*Cos[c + d*x])^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]) + (a*b^2*(A*b^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)) + (a*b*(-
3*A*b^4 + 4*a^4*C - 7*a^2*b^2*C)*(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b
)^2*(a + b)^2)))/(b^4*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])
^3)
```

Maple [B] time = 0.098, size = 1167, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*a^5/b^3
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*C+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+8/d/b/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^3/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*C+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2*a^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A-4/d*b/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)*A+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^5/(a+b
)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*C+1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2*a^4/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*C-8/
d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^3/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)*C+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a^2+2/d*b^2/(a^4-2*a
^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*A+6/d/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C-15/d/b^2/(a^4-2*a^2*b^2+b^4)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a
^4*C+12/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2-1/d*C/b^3/(tan(1/2*d*x+1/2*c)+1)-3/d*a*C
/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/d*C/b^3/(tan(1/2*d*x+1/2*c)-1)+3/d*a*C/b^4*
ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 28.1077, size = 3416, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [1/4*(((6*C*a^8 - 15*C*a^6*b^2 + (A + 12*C)*a^4*b^4 + 2*A*a^2*b^6)*cos(d*x
+ c)^3 + 2*(6*C*a^7*b - 15*C*a^5*b^3 + (A + 12*C)*a^3*b^5 + 2*A*a*b^7)*cos(
d*x + c)^2 + (6*C*a^6*b^2 - 15*C*a^4*b^4 + (A + 12*C)*a^2*b^6 + 2*A*b^8)*co
s(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
```

```
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*((C*a^9 - 3*C*a^7*b^
2 + 3*C*a^5*b^4 - C*a^3*b^6)*cos(d*x + c)^3 + 2*(C*a^8*b - 3*C*a^6*b^3 + 3*
C*a^4*b^5 - C*a^2*b^7)*cos(d*x + c)^2 + (C*a^7*b^2 - 3*C*a^5*b^4 + 3*C*a^3*
b^6 - C*a*b^8)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*((C*a^9 - 3*C*a^7*b^
2 + 3*C*a^5*b^4 - C*a^3*b^6)*cos(d*x + c)^3 + 2*(C*a^8*b - 3*C*a^6*b^3 + 3*
C*a^4*b^5 - C*a^2*b^7)*cos(d*x + c)^2 + (C*a^7*b^2 - 3*C*a^5*b^4 + 3*C*a^3*
b^6 - C*a*b^8)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*C*a^6*b^3 - 6*C*
a^4*b^5 + 6*C*a^2*b^7 - 2*C*b^9 + (6*C*a^8*b - 17*C*a^6*b^3 - (3*A - 13*C)*
a^4*b^5 + (3*A - 2*C)*a^2*b^7)*cos(d*x + c)^2 + (9*C*a^7*b^2 + (A - 25*C)*a
^5*b^4 - 5*(A - 4*C)*a^3*b^6 + 4*(A - C)*a*b^8)*cos(d*x + c))*sin(d*x + c))
/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^
5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8
 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)), 1/2*(((6*C*a^8 - 15*C*a^6*b^2 + (A +
 12*C)*a^4*b^4 + 2*A*a^2*b^6)*cos(d*x + c)^3 + 2*(6*C*a^7*b - 15*C*a^5*b^3
+ (A + 12*C)*a^3*b^5 + 2*A*a*b^7)*cos(d*x + c)^2 + (6*C*a^6*b^2 - 15*C*a^4*
b^4 + (A + 12*C)*a^2*b^6 + 2*A*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*((C*a
^9 - 3*C*a^7*b^2 + 3*C*a^5*b^4 - C*a^3*b^6)*cos(d*x + c)^3 + 2*(C*a^8*b - 3
*C*a^6*b^3 + 3*C*a^4*b^5 - C*a^2*b^7)*cos(d*x + c)^2 + (C*a^7*b^2 - 3*C*a^5
*b^4 + 3*C*a^3*b^6 - C*a*b^8)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*((C*a
^9 - 3*C*a^7*b^2 + 3*C*a^5*b^4 - C*a^3*b^6)*cos(d*x + c)^3 + 2*(C*a^8*b - 3
*C*a^6*b^3 + 3*C*a^4*b^5 - C*a^2*b^7)*cos(d*x + c)^2 + (C*a^7*b^2 - 3*C*a^5
*b^4 + 3*C*a^3*b^6 - C*a*b^8)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*C*a
^6*b^3 - 6*C*a^4*b^5 + 6*C*a^2*b^7 - 2*C*b^9 + (6*C*a^8*b - 17*C*a^6*b^3 -
(3*A - 13*C)*a^4*b^5 + (3*A - 2*C)*a^2*b^7)*cos(d*x + c)^2 + (9*C*a^7*b^2 +
 (A - 25*C)*a^5*b^4 - 5*(A - 4*C)*a^3*b^6 + 4*(A - C)*a*b^8)*cos(d*x + c))*
sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^
3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^
6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x
)
```

Giac [B] time = 1.31827, size = 703, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] ((6*C*a^6 - 15*C*a^4*b^2 + A*a^2*b^4 + 12*C*a^2*b^4 + 2*A*b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b
*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt
```

```
(-a^2 + b^2)) - 3*C*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + 3*C*a*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/b^4 - (4*C*a^6*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^
5*b*tan(1/2*d*x + 1/2*c)^3 - 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - A*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^2*b^4*
tan(1/2*d*x + 1/2*c)^3 + 4*A*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^6*tan(1/2
*d*x + 1/2*c) - 5*C*a^5*b*tan(1/2*d*x + 1/2*c) + 7*C*a^4*b^2*tan(1/2*d*x +
1/2*c) - A*a^3*b^3*tan(1/2*d*x + 1/2*c) + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)
+ 3*A*a^2*b^4*tan(1/2*d*x + 1/2*c) + 4*A*a*b^5*tan(1/2*d*x + 1/2*c))/((a^4*
b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
 - a - b)^2) - 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))
/d
```

3.693 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=212

$$-\frac{a\left(C\left(-5a^2b^2+2a^4+6b^4\right)+3Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2b^2(A+6C)-3a^4C+2Ab^4\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\,(a}{2b^2d\,(a^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^3*d) - (a*(3*A*b^4 + (2*a^4 - 5*a^2*b^2 + 6*b^
4)*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b
^3*(a + b)^(5/2)*d) + (a*(A*b^2 + a^2*C)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^2) + ((2*A*b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Tan[c +
 d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.594275, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4091, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(C\left(-5a^2b^2+2a^4+6b^4\right)+3Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2b^2(A+6C)-3a^4C+2Ab^4\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\,(a}{2b^2d\,(a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^3*d) - (a*(3*A*b^4 + (2*a^4 - 5*a^2*b^2 + 6*b^
4)*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b
^3*(a + b)^(5/2)*d) + (a*(A*b^2 + a^2*C)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^2) + ((2*A*b^4 - 3*a^4*C + a^2*b^2*(A + 6*C))*Tan[c +
 d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4091

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(a*(A*b^2 + a^2
*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)),
 x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*Simp[b*(m + 1)*(a^2*C + A*b^2) - a*(A*b^2*(m + 2) + C*(a^2 + b^2
*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(-2b\left(Ab^2+a^2C\right)+a\left(Ab^2-\left(a^2-2b^2\right)C\right.\right.}{(a+b\sec(c+d}}{2b^2\left(a^2-\right.}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\tan(c}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(2Ab^4-3a^4C+a^2b^2(A+6C)\right)\tan(c}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(2Ab^4-3a\right.}{2b^2\left(a^2\right.}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(2Ab^4-3a\right.}{2b^2\left(a^2\right.}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a\left(3Ab^4+2a^4C-5a^2b^2C+6b^4C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}}{}\right.}{(a-b)^{5/2}b^3(a+b)^{5/2}d}$$

Mathematica [C] time = 5.17715, size = 445, normalized size = 2.1

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{4a(\sin(c)+i\cos(c))\left(C\left(-5a^2b^2+2a^4+6b^4\right)+3Ab^4\right)(a\cos(c+dx)+b)^2\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))(\text{ta}\cdots}{\sqrt{a^2-b^2}\cdots}\right.}{\left(a^2-b^2\right)^{5/2}\sqrt{(\cos(c)-i\sin(c))^2}}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*(-4*C*(b + a*Cos[
c + d*x])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*C*(b + a*Cos[c + d
*x])^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (4*a*(3*A*b^4 + (2*a^4 -
5*a^2*b^2 + 6*b^4)*C)*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c
])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b + a*Cos
[c + d*x])^2*(I*Cos[c] + Sin[c]))/((a^2 - b^2)^(5/2)*Sqrt[(Cos[c] - I*Sin[c
])^2]) + (b*(a*Sec[c]*((4*A*b^5 - 7*a^4*b*C + a^2*b^3*(5*A + 16*C))*Sin[d*x
] + a*(a*b*(-3*A*b^2 + (a^2 - 4*b^2)*C)*Sin[2*c + d*x] + (A*b^4 - 2*a^4*C +
 a^2*b^2*(2*A + 5*C))*Sin[c + 2*d*x])) + (a^2 + 2*b^2)*(-(A*b^4) + 2*a^4*C
- a^2*b^2*(2*A + 5*C))*Tan[c]))/(a*(a^2 - b^2)^2)))/(2*b^3*d*(A + 2*C + A*C
os[2*(c + d*x)])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.097, size = 1165, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*b^2/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*A+2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d/b/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*C-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/
(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2+2/d*a^2/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*
b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1
/2*d*x+1/2*c)*A*a+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d/b/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*
x+1/2*c)*a^3*C+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C*a^2-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*a^5/b
^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*C+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-6/d*b*a/(a^4-2*a^2*b
^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*C+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1
)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 17.1034, size = 2866, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [1/4*((2*C*a^5*b^2 - 5*C*a^3*b^4 + 3*(A + 2*C)*a*b^6 + (2*C*a^7 - 5*C*a^5*b
^2 + 3*(A + 2*C)*a^3*b^4)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 + 3*(
A + 2*C)*a^2*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*
(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C
*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b
^5 - C*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*(C*a^6*b^2 - 3*C*a^4*
b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)
*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x
 + c))*log(-sin(d*x + c) + 1) - 2*(3*C*a^6*b^2 - (A + 9*C)*a^4*b^4 - (A - 6
*C)*a^2*b^6 + 2*A*b^8 + (2*C*a^7*b - (2*A + 7*C)*a^5*b^3 + (A + 5*C)*a^3*b^
5 + A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7
- a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*
d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*C*a^
5*b^2 - 5*C*a^3*b^4 + 3*(A + 2*C)*a*b^6 + (2*C*a^7 - 5*C*a^5*b^2 + 3*(A + 2
*C)*a^3*b^4)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 + 3*(A + 2*C)*a^2*
b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c
) + a)/((a^2 - b^2)*sin(d*x + c))) - (C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6
 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 +
 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x + c))*log(sin(d*
x + c) + 1) + (C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C
*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b
^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (3*C*a^6
*b^2 - (A + 9*C)*a^4*b^4 - (A - 6*C)*a^2*b^6 + 2*A*b^8 + (2*C*a^7*b - (2*A
+ 7*C)*a^5*b^3 + (A + 5*C)*a^3*b^5 + A*a*b^7)*cos(d*x + c))*sin(d*x + c))/(
(a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 -
 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*
a^2*b^9 - b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x
)
```

Giac [B] time = 1.29897, size = 687, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -((2*C*a^5 - 5*C*a^3*b^2 + 3*A*a*b^4 + 6*C*a*b^4)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a^2 + b^2))
 - C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + C*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/b^3 - (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^4*b*tan(1/2*d*x + 1/2
*c)^3 - 2*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^2*tan(1/2*d*x + 1/2*
c)^3 + A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^
3 - A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 - 2*C*a
^5*tan(1/2*d*x + 1/2*c) - 3*C*a^4*b*tan(1/2*d*x + 1/2*c) + 2*A*a^3*b^2*tan(
1/2*d*x + 1/2*c) + 5*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + A*a^2*b^3*tan(1/2*d*x
 + 1/2*c) + 6*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + A*a*b^4*tan(1/2*d*x + 1/2*c)
 + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.694 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=177

$$\frac{\left(a^2(2A+C)+b^2(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(a^2(-C)+3Ab^2+4b^2C\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{\left(a^2C+Ab^2\right)\text{ta}}{2bd\left(a^2-b^2\right)(a+}$$

```
[Out] ((a^2*(2*A + C) + b^2*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b^2 + a^2*C)*Tan[c + d*x])
/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a*(3*A*b^2 - a^2*C + 4*b^2*C
)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.321387, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4081, 4003, 12, 3831, 2659, 208}

$$\frac{\left(a^2(2A+C)+b^2(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(a^2(-C)+3Ab^2+4b^2C\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{\left(a^2C+Ab^2\right)\text{ta}}{2bd\left(a^2-b^2\right)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((a^2*(2*A + C) + b^2*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b^2 + a^2*C)*Tan[c + d*x])
/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a*(3*A*b^2 - a^2*C + 4*b^2*C
)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4081

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A*b^2 + a^2*C)
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] +
 Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2 + a^2*C + b*(A*b + b*C)*(m + 1))*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && NeQ[a
^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(-2ab(A+C)+\left(Ab^2-a^2C+2b^2C\right)\sec(c+d}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \int$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{(a}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{(a}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(3Ab^2-a^2C+4b^2C\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{(a}{}$$

$$= \frac{\left(2a^2A+Ab^2+a^2C+2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\left(Ab^2+a^2C\right)\tan}{2b\left(a^2-b^2\right)d(a+b\sec}$$

Mathematica [C] time = 3.39468, size = 342, normalized size = 1.93

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{a\sec(c)\left(\left(a^2b^2(5A+2C)+a^4C-2Ab^4\right)\sin(2c+dx)+ab\left(Ab^2-a^2(4A+3C)\right)\sin(c+2dx)+\sin(d}{\left(a^3-ab^2\right)^2}\right.}{2d(a+b\sec(c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*(((-4*I)*(a^2*(2*
A + C) + b^2*(A + 2*C))*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos
```

```
[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b + a*C
os[c + d*x])^2*(Cos[c] - I*Sin[c]))/((a^2 - b^2)^(5/2)*Sqrt[(Cos[c] - I*Sin
[c])^2]) + (a*Sec[c]*((2*A*b^4 + a^4*C - a^2*b^2*(11*A + 10*C))*Sin[d*x] +
(-2*A*b^4 + a^4*C + a^2*b^2*(5*A + 2*C))*Sin[2*c + d*x] + a*b*(A*b^2 - a^2*
(4*A + 3*C))*Sin[c + 2*d*x]) + b*(a^2 + 2*b^2)*(-(A*b^2) + a^2*(4*A + 3*C))
*Tan[c])/(a^3 - a*b^2)^2))/(2*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c
 + d*x])^3)
```

Maple [A] time = 0.093, size = 230, normalized size = 1.3

$$\frac{1}{d}\left(-2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^2}\left(-1/2\,\frac{\left(4\,Aab+Ab^2+a^2C+4\,abC\right)(\tan(1/2\,dx+c/2\right.}{(a-b)\left(a^2+2\,ab+b^2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*A*a*b+A*b^2+C*a^2+4*C*a*b)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3+1/2*(4*A*a*b-A*b^2-C*a^2+4*C*a*b)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*A*a^2
+A*b^2+C*a^2+2*C*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.647531, size = 1577, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((2*A + C)*a^2*b^2 + (A + 2*C)*b^4 + ((2*A + C)*a^4 + (A + 2*C)*a^2*b
^2)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b + (A + 2*C)*a*b^3)*cos(d*x + c))*sq
rt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sq
rt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^5 - (3*A + 5*C)*a^3*b^2 + (3*
A + 4*C)*a*b^4 - ((4*A + 3*C)*a^4*b - (5*A + 3*C)*a^2*b^3 + A*b^5)*cos(d*x
+ c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)
^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 -
```

```
3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*(((2*A + C)*a^2*b^2 + (A + 2*C)*b^4 +
((2*A + C)*a^4 + (A + 2*C)*a^2*b^2)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b + (
A + 2*C)*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^5 - (3*A + 5*C)*a^3*b^
2 + (3*A + 4*C)*a*b^4 - ((4*A + 3*C)*a^4*b - (5*A + 3*C)*a^2*b^3 + A*b^5)*c
os(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d
*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.28683, size = 501, normalized size = 2.83

$$\frac{\frac{\left(2\,Aa^2+Ca^2+Ab^2+2\,Cb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Ca^2b\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*A*a^2 + C*a^2 + A*b^2 + 2*C*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqr
t(-a^2 + b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (C*a^3*tan(1/2
*d*x + 1/2*c)^3 + 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 - 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b^2*tan(1/2*d*x + 1/2
*c)^3 - A*b^3*tan(1/2*d*x + 1/2*c)^3 + C*a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^2
*b*tan(1/2*d*x + 1/2*c) - 3*C*a^2*b*tan(1/2*d*x + 1/2*c) - 3*A*a*b^2*tan(1/
2*d*x + 1/2*c) - 4*C*a*b^2*tan(1/2*d*x + 1/2*c) + A*b^3*tan(1/2*d*x + 1/2*c
))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^2))/d
```

3.695 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{b\left(5a^2Ab^2-3a^4(2A+C)-2Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-a^2b^2(5A+2C)+a^4(-C)+2Ab^4\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{}{2a}$$

```
[Out] (A*x)/a^3 + (b*(5*a^2*A*b^2 - 2*A*b^4 - 3*a^4*(2*A + C))*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((
A*b^2 + a^2*C)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (
(2*A*b^4 - a^4*C - a^2*b^2*(5*A + 2*C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*
d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.445455, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4061, 4060, 3919, 3831, 2659, 208}

$$\frac{b\left(5a^2Ab^2-3a^4(2A+C)-2Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-a^2b^2(5A+2C)+a^4(-C)+2Ab^4\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (A*x)/a^3 + (b*(5*a^2*A*b^2 - 2*A*b^4 - 3*a^4*(2*A + C))*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((
A*b^2 + a^2*C)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (
(2*A*b^4 - a^4*C - a^2*b^2*(5*A + 2*C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*
d*(a + b*Sec[c + d*x]))
```


Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2A\left(a^2-b^2\right)+2ab(A+C)\sec(c+dx)-\left(Ab^2+a^2C\right)\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a^2b^2(5A+2C)\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\int \frac{2A\left(a^2-\right.}{}}{}$$

$$= \frac{Ax}{a^3} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a^2b^2(5A+2C)\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{(b}{}$$

$$= \frac{Ax}{a^3} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a^2b^2(5A+2C)\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{(5}{}$$

$$= \frac{Ax}{a^3} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4-a^4C-a^2b^2(5A+2C)\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{(5}{}$$

$$= \frac{Ax}{a^3} - \frac{b\left(6a^4A-5a^2Ab^2+2Ab^4+3a^4C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(Ab^2+a^2C\right)\tan}{2a\left(a^2-b^2\right)d(a+b\,\mathrm{s}}$$

Mathematica [C] time = 4.72937, size = 642, normalized size = 3.18

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{\sec(c)\left(6a^4Ab^2\sin(c+2dx)-7a^3Ab^3\sin(2c+dx)-3a^2Ab^4\sin(c+2dx)-2a^4Ab^2dx\cos(c+2dx)\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((4*b*(-5*a^2*A*b
^2 + 2*A*b^4 + 3*a^4*(2*A + C))*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b
 + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*
(b + a*Cos[c + d*x])^2*(I*Cos[c] + Sin[c]))/((a^2 - b^2)^(5/2)*Sqrt[(Cos[c]
 - I*Sin[c])^2]) + (Sec[c]*(2*A*(a^2 - b^2)^2*(a^2 + 2*b^2)*d*x*Cos[c] + 4*
a*A*b*(a^2 - b^2)^2*d*x*Cos[d*x] + 4*a^5*A*b*d*x*Cos[2*c + d*x] - 8*a^3*A*b
^3*d*x*Cos[2*c + d*x] + 4*a*A*b^5*d*x*Cos[2*c + d*x] + a^6*A*d*x*Cos[c + 2*
d*x] - 2*a^4*A*b^2*d*x*Cos[c + 2*d*x] + a^2*A*b^4*d*x*Cos[c + 2*d*x] + a^6*
A*d*x*Cos[3*c + 2*d*x] - 2*a^4*A*b^2*d*x*Cos[3*c + 2*d*x] + a^2*A*b^4*d*x*C
os[3*c + 2*d*x] - 6*a^4*A*b^2*Sin[c] - 9*a^2*A*b^4*Sin[c] + 6*A*b^6*Sin[c]
- 2*a^6*C*Sin[c] - 5*a^4*b^2*C*Sin[c] - 2*a^2*b^4*C*Sin[c] + 17*a^3*A*b^3*S
in[d*x] - 8*a*A*b^5*Sin[d*x] + 5*a^5*b*C*Sin[d*x] + 4*a^3*b^3*C*Sin[d*x] -
7*a^3*A*b^3*Sin[2*c + d*x] + 4*a*A*b^5*Sin[2*c + d*x] - 3*a^5*b*C*Sin[2*c +
 d*x] + 6*a^4*A*b^2*Sin[c + 2*d*x] - 3*a^2*A*b^4*Sin[c + 2*d*x] + 2*a^6*C*S
in[c + 2*d*x] + a^4*b^2*C*Sin[c + 2*d*x]))/(a^2 - b^2)^2))/(2*a^3*d*(A + 2*
C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.101, size = 1143, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/a/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*A*b^3+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-2/d/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*C*a^2-1/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b*C-2/d/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*C*b^2+6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^3-2/d/a^2/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)
*A*b^4+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b
)^2*tan(1/2*d*x+1/2*c)*C*a^2-1/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b*C+2/d/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C*b^2-6/
d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*A+5/d/a*b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d/a^3*b^5/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*A-3/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.712418, size = 2295, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2
+ 8*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 4*(A
*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x + (3*(2*A + C)*a^4*b^3 -
5*A*a^2*b^5 + 2*A*b^7 + (3*(2*A + C)*a^6*b - 5*A*a^4*b^3 + 2*A*a^2*b^5)*cos
(d*x + c)^2 + 2*(3*(2*A + C)*a^5*b^2 - 5*A*a^3*b^4 + 2*A*a*b^6)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^7*b + (5*A + C)*a^5*b^3
- (7*A + 2*C)*a^3*b^5 + 2*A*a*b^7 + (2*C*a^8 + (6*A - C)*a^6*b^2 - (9*A + C
)*a^4*b^4 + 3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3
*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1
/2*(2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2 +
4*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 2*(A*a
^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x - (3*(2*A + C)*a^4*b^3 - 5*
A*a^2*b^5 + 2*A*b^7 + (3*(2*A + C)*a^6*b - 5*A*a^4*b^3 + 2*A*a^2*b^5)*cos(d
*x + c)^2 + 2*(3*(2*A + C)*a^5*b^2 - 5*A*a^3*b^4 + 2*A*a*b^6)*cos(d*x + c))
*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2
)*sin(d*x + c))) + (C*a^7*b + (5*A + C)*a^5*b^3 - (7*A + 2*C)*a^3*b^5 + 2*A
*a*b^7 + (2*C*a^8 + (6*A - C)*a^6*b^2 - (9*A + C)*a^4*b^4 + 3*A*a^2*b^6)*co
s(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d
*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (
a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.24462, size = 653, normalized size = 3.23

$$-\frac{\frac{\left(6\,Aa^4b+3\,Ca^4b-5\,Aa^2b^3+2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}-\frac{(dx+c)A}{a^3}+\frac{2\,Ca^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^4b\,\mathrm{ta}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*A*a^4*b + 3*C*a^4*b - 5*A*a^2*b^3 + 2*A*b^5)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2))
 - (d*x + c)*A/a^3 + (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - C*a^4*b*tan(1/2*d*x
+ 1/2*c)^3 + 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + C*a^3*b^2*tan(1/2*d*x + 1
/2*c)^3 - 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^2*b^3*tan(1/2*d*x + 1/
2*c)^3 - 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3
- 2*C*a^5*tan(1/2*d*x + 1/2*c) - C*a^4*b*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^2
*tan(1/2*d*x + 1/2*c) - C*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*A*a^2*b^3*tan(1/
2*d*x + 1/2*c) - 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*tan(1/2*d*x +
 1/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.696 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=266

$$-\frac{\left(11a^2Ab^2+a^4(-(2A-3C))-6Ab^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-2a^6C-6Ab^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (-3*A*b*x)/a^4 - ((15*a^2*A*b^4 - 6*A*b^6 - 2*a^6*C - a^4*b^2*(12*A + C))*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a +
 b)^(5/2)*d) - ((11*a^2*A*b^2 - 6*A*b^4 - a^4*(2*A - 3*C))*Sin[c + d*x])/(2
*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(
a + b*Sec[c + d*x])^2) - ((3*A*b^4 - 2*a^4*C - a^2*b^2*(6*A + C))*Sin[c + d
*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.96297, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4101, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(11a^2Ab^2+a^4(-(2A-3C))-6Ab^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-2a^6C-6Ab^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (-3*A*b*x)/a^4 - ((15*a^2*A*b^4 - 6*A*b^6 - 2*a^6*C - a^4*b^2*(12*A + C))*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a +
 b)^(5/2)*d) - ((11*a^2*A*b^2 - 6*A*b^4 - a^4*(2*A - 3*C))*Sin[c + d*x])/(2
*a^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(
a + b*Sec[c + d*x])^2) - ((3*A*b^4 - 2*a^4*C - a^2*b^2*(6*A + C))*Sin[c + d
*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(3Ab^2-a^2(2A-C)+2ab(A+C)\sec(c+dx)-2\right.}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3Ab^4-2a^4C-a^2b^2(6A+C)\right)\sin(c+dx}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

$$= -\frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{3Abx}{a^4} - \frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\sin}{2a\left(a^2-b^2\right)d(a+b}$$

$$= -\frac{3Abx}{a^4} - \frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\sin}{2a\left(a^2-b^2\right)d(a+b}$$

$$= -\frac{3Abx}{a^4} - \frac{\left(11a^2Ab^2-6Ab^4-a^4(2A-3C)\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{\left(Ab^2+a^2C\right)\sin}{2a\left(a^2-b^2\right)d(a+b}$$

$$= -\frac{3Abx}{a^4} + \frac{\left(12a^4Ab^2-15a^2Ab^4+6Ab^6+2a^6C+a^4b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [C] time = 6.58112, size = 902, normalized size = 3.39

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(C\sec^2(c+dx)+A\right)\left(\frac{\sec(c)\left(A\sin(dx)a^7+A\sin(2c+dx)a^7+A\sin(2c+3dx)a^7+A\sin(4c+3dx)a^7-6Abdx\cos(c+2\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*(((-8*I)*(-15*a^2
*A*b^4 + 6*A*b^6 + 2*a^6*C + a^4*b^2*(12*A + C))*ArcTan[((I*Cos[c] + Sin[c]
)*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c]
- I*Sin[c])^2])]*(b + a*Cos[c + d*x])^2*(Cos[c] - I*Sin[c]))/((a^2 - b^2)^(
5/2)*Sqrt[(Cos[c] - I*Sin[c])^2]) + (Sec[c]*(-12*A*b*(a^2 - b^2)^2*(a^2 + 2
*b^2)*d*x*Cos[c] - 24*a*A*b^2*(a^2 - b^2)^2*d*x*Cos[d*x] - 24*a^5*A*b^2*d*x
*Cos[2*c + d*x] + 48*a^3*A*b^4*d*x*Cos[2*c + d*x] - 24*a*A*b^6*d*x*Cos[2*c
+ d*x] - 6*a^6*A*b*d*x*Cos[c + 2*d*x] + 12*a^4*A*b^3*d*x*Cos[c + 2*d*x] - 6
*a^2*A*b^5*d*x*Cos[c + 2*d*x] - 6*a^6*A*b*d*x*Cos[3*c + 2*d*x] + 12*a^4*A*b
^3*d*x*Cos[3*c + 2*d*x] - 6*a^2*A*b^5*d*x*Cos[3*c + 2*d*x] + 16*a^4*A*b^3*S
in[c] + 22*a^2*A*b^5*Sin[c] - 20*A*b^7*Sin[c] + 8*a^6*b*C*Sin[c] + 14*a^4*b
^3*C*Sin[c] - 4*a^2*b^5*C*Sin[c] + a^7*A*Sin[d*x] + 2*a^5*A*b^2*Sin[d*x] -
53*a^3*A*b^4*Sin[d*x] + 32*a*A*b^6*Sin[d*x] - 22*a^5*b^2*C*Sin[d*x] + 4*a^3
*b^4*C*Sin[d*x] + a^7*A*Sin[2*c + d*x] + 2*a^5*A*b^2*Sin[2*c + d*x] + 11*a^
3*A*b^4*Sin[2*c + d*x] - 8*a*A*b^6*Sin[2*c + d*x] + 10*a^5*b^2*C*Sin[2*c +
d*x] - 4*a^3*b^4*C*Sin[2*c + d*x] + 4*a^6*A*b*Sin[c + 2*d*x] - 24*a^4*A*b^3
*Sin[c + 2*d*x] + 14*a^2*A*b^5*Sin[c + 2*d*x] - 8*a^6*b*C*Sin[c + 2*d*x] +
2*a^4*b^3*C*Sin[c + 2*d*x] + 4*a^6*A*b*Sin[3*c + 2*d*x] - 8*a^4*A*b^3*Sin[3
```

```
*c + 2*d*x] + 4*a^2*A*b^5*Sin[3*c + 2*d*x] + a^7*A*Sin[2*c + 3*d*x] - 2*a^5
*A*b^2*Sin[2*c + 3*d*x] + a^3*A*b^4*Sin[2*c + 3*d*x] + a^7*A*Sin[4*c + 3*d*
x] - 2*a^5*A*b^2*Sin[4*c + 3*d*x] + a^3*A*b^4*Sin[4*c + 3*d*x]))/(a^2 - b^2
)^2))/(4*a^4*d*(A + 2*C + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.138, size = 1132, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d*A/a^4*b*arctan(ta
n(1/2*d*x+1/2*c))+8/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^3+1/d/a^2/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3*A*b^4-4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*
b^5/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*a/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*b*C+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*b^2-8/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A+1/d/a
^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a^2-2*a
*b+b^2)*tan(1/2*d*x+1/2*c)*A+4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2*b^5/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A-4/d*a/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a+b)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)*C+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
*b^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*C+12/d*b^2/(a^4-2*a^2*b^2+b^4
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*A-15/d/a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^4+6/d/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6+2/d/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*C*a^2+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.786619, size = 2654, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(12*(A*a^8*b - 3*A*a^6*b^3 + 3*A*a^4*b^5 - A*a^2*b^7)*d*x*cos(d*x + c
)^2 + 24*(A*a^7*b^2 - 3*A*a^5*b^4 + 3*A*a^3*b^6 - A*a*b^8)*d*x*cos(d*x + c)
 + 12*(A*a^6*b^3 - 3*A*a^4*b^5 + 3*A*a^2*b^7 - A*b^9)*d*x - (2*C*a^6*b^2 +
(12*A + C)*a^4*b^4 - 15*A*a^2*b^6 + 6*A*b^8 + (2*C*a^8 + (12*A + C)*a^6*b^2
 - 15*A*a^4*b^4 + 6*A*a^2*b^6)*cos(d*x + c)^2 + 2*(2*C*a^7*b + (12*A + C)*a
^5*b^3 - 15*A*a^3*b^5 + 6*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*((2*A - 3*C)*a^7*b^2 - (13*A - 3*C)*a^5*b^4 + 17*A*a^3*b^6
 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a^5*b^4 - A*a^3*b^6)*cos(d*x +
c)^2 + (4*(A - C)*a^8*b - 5*(4*A - C)*a^6*b^3 + (25*A - C)*a^4*b^5 - 9*A*a^
2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^
6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*
x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), -1/2*(6*(A*a^8*b
- 3*A*a^6*b^3 + 3*A*a^4*b^5 - A*a^2*b^7)*d*x*cos(d*x + c)^2 + 12*(A*a^7*b^2
 - 3*A*a^5*b^4 + 3*A*a^3*b^6 - A*a*b^8)*d*x*cos(d*x + c) + 6*(A*a^6*b^3 - 3
*A*a^4*b^5 + 3*A*a^2*b^7 - A*b^9)*d*x - (2*C*a^6*b^2 + (12*A + C)*a^4*b^4 -
 15*A*a^2*b^6 + 6*A*b^8 + (2*C*a^8 + (12*A + C)*a^6*b^2 - 15*A*a^4*b^4 + 6*
A*a^2*b^6)*cos(d*x + c)^2 + 2*(2*C*a^7*b + (12*A + C)*a^5*b^3 - 15*A*a^3*b^
5 + 6*A*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((2*A - 3*C)*a^7*b^2 - (13*A
 - 3*C)*a^5*b^4 + 17*A*a^3*b^6 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a
^5*b^4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*(A - C)*a^8*b - 5*(4*A - C)*a^6*b^3
 + (25*A - C)*a^4*b^5 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3
*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 +
 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 -
a^4*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29381, size = 663, normalized size = 2.49

$$\frac{\left(2\,Ca^6+12\,Aa^4b^2+Ca^4b^2-15\,Aa^2b^4+6\,Ab^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^8-2\,a^6b^2+a^4b^4\right)\sqrt{-a^2+b^2}}-\frac{3\,(dx+c)Ab}{a^4}+\frac{4\,Ca^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*C*a^6 + 12*A*a^4*b^2 + C*a^4*b^2 - 15*A*a^2*b^4 + 6*A*b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b
*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt
(-a^2 + b^2)) - 3*(d*x + c)*A*b/a^4 + (4*C*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 3
*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - C*
a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*A*a
*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^5*b*ta
n(1/2*d*x + 1/2*c) - 3*C*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b^3*tan(1/2
*d*x + 1/2*c) + C*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*A*a^2*b^4*tan(1/2*d*x +
1/2*c) + 5*A*a*b^5*tan(1/2*d*x + 1/2*c) + 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a
^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 - a - b)^2) + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^
3))/d
```

3.697 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=369

$$-\frac{b\left(-a^2b^2(21A-2C)+a^4(6A-5C)+12Ab^4\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-a^2b^2(10A-C)+a^4(A-4C)+6Ab^4\right)\sin(c+dx)\cos(c-$$

```
[Out] ((12*A*b^2 + a^2*(A + 2*C))*x)/(2*a^5) - (b*(12*A*b^6 - a^2*b^4*(29*A - 2*C
) + 5*a^4*b^2*(4*A - C) + 6*a^6*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d) - (b*(12*A*b^4 + a^4*(6*A
- 5*C) - a^2*b^2*(21*A - 2*C))*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((6*
A*b^4 + a^4*(A - 4*C) - a^2*b^2*(10*A - C))*Cos[c + d*x]*Sin[c + d*x])/(2*a
^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((7*a^2*A*b^2 - 4*A*b^4 + 3*a^4*C)*Cos[
c + d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.61451, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4101, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(-a^2b^2(21A-2C)+a^4(6A-5C)+12Ab^4\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-a^2b^2(10A-C)+a^4(A-4C)+6Ab^4\right)\sin(c+dx)\cos(c-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((12*A*b^2 + a^2*(A + 2*C))*x)/(2*a^5) - (b*(12*A*b^6 - a^2*b^4*(29*A - 2*C
) + 5*a^4*b^2*(4*A - C) + 6*a^6*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d) - (b*(12*A*b^4 + a^4*(6*A
- 5*C) - a^2*b^2*(21*A - 2*C))*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((6*
A*b^4 + a^4*(A - 4*C) - a^2*b^2*(10*A - C))*Cos[c + d*x]*Sin[c + d*x])/(2*a
^3*(a^2 - b^2)^2*d) + ((A*b^2 + a^2*C)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((7*a^2*A*b^2 - 4*A*b^4 + 3*a^4*C)*Cos[
c + d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
```

```
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(2\left(2Ab^2-a^2(A-C)\right)+2ab(A+C)\sec(}{(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right.}$$

$$= \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(7a^2Ab^2-4Ab^4+3a^4C\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= \frac{\left(6Ab^4+a^4(A-4C)-a^2b^2(10A-C)\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{\left(Ab^2+a^2C\right.}{2a\left(a^2-b^2\right.}$$

$$= -\frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2(21A-2C)\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(6Ab^4+a^4(A-\right.}{}$$

$$= \frac{\left(12Ab^2+a^2(A+2C)\right)x}{2a^5} - \frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2(21A-2C)\right)\sin(c}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(12Ab^2+a^2(A+2C)\right)x}{2a^5} - \frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2(21A-2C)\right)\sin(c}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(12Ab^2+a^2(A+2C)\right)x}{2a^5} - \frac{b\left(12Ab^4+a^4(6A-5C)-a^2b^2(21A-2C)\right)\sin(c}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(12Ab^2+a^2(A+2C)\right)x}{2a^5} - \frac{b\left(20a^4Ab^2-29a^2Ab^4+12Ab^6+6a^6C-5a^4b^2C\right.}{a^5(a-b)^{5/2}(a+b}$$

Mathematica [A] time = 2.43933, size = 256, normalized size = 0.69

$$\frac{2(c+dx)\left(a^2(A+2C)+12Ab^2\right) - \frac{2ab^3\left(a^2C+Ab^2\right)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)^2} + \frac{2ab^2\left(a^2b^2(10A-3C)+6a^4C-7Ab^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{4b\left(5a^4b^2(4A-C)+a^2b^4(2C-29A\right.}{}}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(12*A*b^2 + a^2*(A + 2*C))*(c + d*x) + (4*b*(12*A*b^6 + 5*a^4*b^2*(4*A -
 C) + 6*a^6*C + a^2*b^4*(-29*A + 2*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2]])/
Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 12*a*A*b*Sin[c + d*x] - (2*a*b^3*(A*b
^2 + a^2*C)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) + (2*a*b
^2*(-7*A*b^4 + a^2*b^2*(10*A - 3*C) + 6*a^4*C)*Sin[c + d*x])/((a - b)^2*(a
+ b)^2*(b + a*Cos[c + d*x])) + a^2*A*Sin[2*(c + d*x)])/(4*a^5*d)
```

Maple [B] time = 0.149, size = 1478, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*b^5
/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*b^6/a^4/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*A-1/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+2/d*b^4/a^2/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*C-1/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*b^3
/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(
1/2*d*x+1/2*c)*C-2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-6/d*b*a/(a^4-2*a^2*b^2+b^4)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+10/
d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*t
an(1/2*d*x+1/2*c)*A*b^4-10/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-6/d/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*C*b^2+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C*b^2-20/d/a*b^3/(a^4-2*a^2*b^2+b^4)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+29/
d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))*A-1/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2*b^5/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d/a^3*
arctan(tan(1/2*d*x+1/2*c))*C+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)*A+12/d/a^5*arctan(tan(1/2*d*x+1/2*c))*A*b^2-1/d/a^3/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)^3*A*b-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A
*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.943741, size = 3407, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [1/4*(2*((A + 2*C)*a^10 + 3*(3*A - 2*C)*a^8*b^2 - 3*(11*A - 2*C)*a^6*b^4 +
(35*A - 2*C)*a^4*b^6 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2 + 4*((A + 2*C)*a^9*
b + 3*(3*A - 2*C)*a^7*b^3 - 3*(11*A - 2*C)*a^5*b^5 + (35*A - 2*C)*a^3*b^7 -
 12*A*a*b^9)*d*x*cos(d*x + c) + 2*((A + 2*C)*a^8*b^2 + 3*(3*A - 2*C)*a^6*b^
4 - 3*(11*A - 2*C)*a^4*b^6 + (35*A - 2*C)*a^2*b^8 - 12*A*b^10)*d*x + (6*C*a
^6*b^3 + 5*(4*A - C)*a^4*b^5 - (29*A - 2*C)*a^2*b^7 + 12*A*b^9 + (6*C*a^8*b
 + 5*(4*A - C)*a^6*b^3 - (29*A - 2*C)*a^4*b^5 + 12*A*a^2*b^7)*cos(d*x + c)^
2 + 2*(6*C*a^7*b^2 + 5*(4*A - C)*a^5*b^4 - (29*A - 2*C)*a^3*b^6 + 12*A*a*b^
8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((6*A - 5*C)*a^
7*b^3 - (27*A - 7*C)*a^5*b^5 + (33*A - 2*C)*a^3*b^7 - 12*A*a*b^9 - (A*a^10
- 3*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 4*(A*a^9*b - 3*A*
a^7*b^3 + 3*A*a^5*b^5 - A*a^3*b^7)*cos(d*x + c)^2 + ((11*A - 6*C)*a^8*b^2 -
 (43*A - 9*C)*a^6*b^4 + (50*A - 3*C)*a^4*b^6 - 18*A*a^2*b^8)*cos(d*x + c))*
sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 +
 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 -
 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*(((A + 2*C)*a^10 + 3*(3*A - 2*C)*
a^8*b^2 - 3*(11*A - 2*C)*a^6*b^4 + (35*A - 2*C)*a^4*b^6 - 12*A*a^2*b^8)*d*x
*cos(d*x + c)^2 + 2*((A + 2*C)*a^9*b + 3*(3*A - 2*C)*a^7*b^3 - 3*(11*A - 2*
C)*a^5*b^5 + (35*A - 2*C)*a^3*b^7 - 12*A*a*b^9)*d*x*cos(d*x + c) + ((A + 2*
C)*a^8*b^2 + 3*(3*A - 2*C)*a^6*b^4 - 3*(11*A - 2*C)*a^4*b^6 + (35*A - 2*C)*
a^2*b^8 - 12*A*b^10)*d*x - (6*C*a^6*b^3 + 5*(4*A - C)*a^4*b^5 - (29*A - 2*C
)*a^2*b^7 + 12*A*b^9 + (6*C*a^8*b + 5*(4*A - C)*a^6*b^3 - (29*A - 2*C)*a^4*
b^5 + 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(6*C*a^7*b^2 + 5*(4*A - C)*a^5*b^4 -
 (29*A - 2*C)*a^3*b^6 + 12*A*a*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((6*A -
 5*C)*a^7*b^3 - (27*A - 7*C)*a^5*b^5 + (33*A - 2*C)*a^3*b^7 - 12*A*a*b^9 -
(A*a^10 - 3*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 4*(A*a^9*
b - 3*A*a^7*b^3 + 3*A*a^5*b^5 - A*a^3*b^7)*cos(d*x + c)^2 + ((11*A - 6*C)*a
^8*b^2 - (43*A - 9*C)*a^6*b^4 + (50*A - 3*C)*a^4*b^6 - 18*A*a^2*b^8)*cos(d*
x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x
+ c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^
11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30796, size = 1551, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] -1/2*(2*(6*C*a^6*b + 20*A*a^4*b^3 - 5*C*a^4*b^3 - 29*A*a^2*b^5 + 2*C*a^2*b^
5 + 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^9 -
 2*a^7*b^2 + a^5*b^4)*sqrt(-a^2 + b^2)) + 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 +
 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 6
*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*
C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*
C*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 2*
C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*A
*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^7*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^6*b*ta
n(1/2*d*x + 1/2*c)^5 - 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^5*b^2*tan
(1/2*d*x + 1/2*c)^5 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^4*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^3*b^4*ta
n(1/2*d*x + 1/2*c)^5 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*b^5*ta
n(1/2*d*x + 1/2*c)^5 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^7*tan(1/2
*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x +
1/2*c)^3 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*C*a^5*b^2*tan(1/2*d*x + 1
/2*c)^3 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*C*a^4*b^3*tan(1/2*d*x +
1/2*c)^3 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^3*b^4*tan(1/2*d*x +
1/2*c)^3 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^5*tan(1/2*d*x +
1/2*c)^3 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan(1/2*d*x + 1/2*c
)^3 - A*a^7*tan(1/2*d*x + 1/2*c) + 4*A*a^6*b*tan(1/2*d*x + 1/2*c) + 13*A*a^
5*b^2*tan(1/2*d*x + 1/2*c) - 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*A*a^4*b^3
*tan(1/2*d*x + 1/2*c) - 5*C*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*A*a^3*b^4*tan
(1/2*d*x + 1/2*c) + 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c) - 17*A*a^2*b^5*tan(1/2
*d*x + 1/2*c) + 2*C*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*A*a*b^6*tan(1/2*d*x +
 1/2*c) + 12*A*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*ta
n(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^2) - (A*a^2 + 2*C*a^2 + 12*A*b^2)*(d*x + c)/a^5)/d
```

3.698 $\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=378

$$-\frac{\left(5Ab^4-C\left(-23a^2b^2+12a^4+6b^4\right)\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)+28a^6b^2C-35a^4b^4C-8a^8C+2Ab^8\right)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{b^5d(a-b)^{7/2}(a+b)^{7/2}}$$

```
[Out] (-4*a*C*ArcTanh[Sin[c + d*x]])/(b^5*d) - ((2*A*b^8 - 8*a^8*C + 28*a^6*b^2*C
 - 35*a^4*b^4*C + a^2*b^6*(3*A + 20*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((5*A*b^4 - (12*a^4
 - 23*a^2*b^2 + 6*b^4)*C)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - ((A*b^2 +
 a^2*C)*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)^3) + ((3*A*b^4 - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Sec[c + d*x]^2*Tan[c + d*
x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a*(2*A*b^6 + 4*a^6*C
- 11*a^4*b^2*C + 3*a^2*b^4*(A + 4*C))*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*
(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.8112, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4099, 4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(5Ab^4-C\left(-23a^2b^2+12a^4+6b^4\right)\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)+28a^6b^2C-35a^4b^4C-8a^8C+2Ab^8\right)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{b^5d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```



```
[Out] (-4*a*C*ArcTanh[Sin[c + d*x]])/(b^5*d) - ((2*A*b^8 - 8*a^8*C + 28*a^6*b^2*C
 - 35*a^4*b^4*C + a^2*b^6*(3*A + 20*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((5*A*b^4 - (12*a^4
 - 23*a^2*b^2 + 6*b^4)*C)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - ((A*b^2 +
 a^2*C)*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)^3) + ((3*A*b^4 - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Sec[c + d*x]^2*Tan[c + d*
x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a*(2*A*b^6 + 4*a^6*C
- 11*a^4*b^2*C + 3*a^2*b^4*(A + 4*C))*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*
(a + b*Sec[c + d*x]))
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)\left(3\left(Ab^2+a^2C\right)-3ab(A+C)\sec(c+\right.}{(a+b\sec(c+}}{3b\left(a^2-\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\text{se}}{6b^2\left(a^2-b^2\right)^2d(a+b\,\text{se}}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\text{se}}{6b^2\left(a^2-b^2\right)^2d(a+b\,\text{se}}$$

$$= -\frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\,\text{t}}{3b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= -\frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{\left(Ab^2+a^2C\right)\sec^3(c+dx)\,\text{t}}{3b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= -\frac{4aC\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} -$$

$$= -\frac{4aC\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(5Ab^4-\left(12a^4-23a^2b^2+6b^4\right)C\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} -$$

$$= -\frac{4aC\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(3a^2Ab^6+2Ab^8-8a^8C+28a^6b^2C-35a^4b^4C\right.}{(a-b)^{7/2}b^5(a+}$$

Mathematica [A] time = 4.4066, size = 564, normalized size = 1.49

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(-\frac{2b\sin(c+dx)\left(6a^2b\left(a^2b^4(A+53C)-57a^4b^2C+20a^6C+3b^6(3A-2C)\right)\cos(2(c+dx))+a\left(5a^4\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2)*((-48*(-2*A*b^8
 + 8*a^8*C - 28*a^6*b^2*C + 35*a^4*b^4*C - a^2*b^6*(3*A + 20*C))*ArcTanh[((
-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x]*(b + a*Cos[c + d*x]
)^3)/(a^2 - b^2)^(7/2) + 192*a*C*Cos[c + d*x]*(b + a*Cos[c + d*x])^3*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - 192*a*C*Cos[c + d*x]*(b + a*Cos[c + d*
x])^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*b*(6*a^4*A*b^5 + 54*a^2
*A*b^7 + 120*a^8*b*C - 318*a^6*b^3*C + 246*a^4*b^5*C + 36*a^2*b^7*C - 24*b^
9*C + a*(5*a^4*b^4*(4*A - 61*C) + 72*b^8*(A - C) + 72*a^8*C - 28*a^6*b^2*C
+ a^2*b^6*(13*A + 438*C))*Cos[c + d*x] + 6*a^2*b*(3*b^6*(3*A - 2*C) + 20*a^
6*C - 57*a^4*b^2*C + a^2*b^4*(A + 53*C))*Cos[2*(c + d*x)] + 4*a^5*A*b^4*Cos
[3*(c + d*x)] + 11*a^3*A*b^6*Cos[3*(c + d*x)] + 24*a^9*C*Cos[3*(c + d*x)] -
```

```
 68*a^7*b^2*C*Cos[3*(c + d*x)] + 65*a^5*b^4*C*Cos[3*(c + d*x)] - 6*a^3*b^6*
C*Cos[3*(c + d*x)])*Sin[c + d*x])/(-a^2 + b^2)^3))/(24*b^5*d*(A + 2*C + A*C
os[2*(c + d*x)])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.11, size = 2318, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 8/d/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^8*C-3/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*A*a^2-28/d/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C+35/d/b/(a^6-3*a^4*b^2+
3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*a^4*C-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+4/3/d/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*a*b+b^2)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*a*C/b^5*ln(tan(1/2*d*x+1/2*c)+1)+40/d/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*a*b+b^2)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
A-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5*A-20/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+5/d/b/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)*C-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3*a^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-6/d/b^4/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^7/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^3*a^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5*C-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d/b^4/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)*C-116/3/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^3*a^5/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+18/d/b^2/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-5/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3*a^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5*C+3/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-3/d*b/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1
/2*d*x+1/2*c)^5*A-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+12/d*b^2/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+12/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3*a^7/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*C+18/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a+b
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-1/d*C/b^4/(tan(1/2*d*x+1/2
*c)+1)-1/d*C/b^4/(tan(1/2*d*x+1/2*c)-1)-2/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*A+4/d*a*C/b^5*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 65.1669, size = 5434, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [1/12*(3*((8*C*a^11 - 28*C*a^9*b^2 + 35*C*a^7*b^4 - (3*A + 20*C)*a^5*b^6 -
2*A*a^3*b^8)*cos(d*x + c)^4 + 3*(8*C*a^10*b - 28*C*a^8*b^3 + 35*C*a^6*b^5 -
 (3*A + 20*C)*a^4*b^7 - 2*A*a^2*b^9)*cos(d*x + c)^3 + 3*(8*C*a^9*b^2 - 28*C
*a^7*b^4 + 35*C*a^5*b^6 - (3*A + 20*C)*a^3*b^8 - 2*A*a*b^10)*cos(d*x + c)^2
 + (8*C*a^8*b^3 - 28*C*a^6*b^5 + 35*C*a^4*b^7 - (3*A + 20*C)*a^2*b^9 - 2*A*
b^11)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 24*((C*a^12 -
4*C*a^10*b^2 + 6*C*a^8*b^4 - 4*C*a^6*b^6 + C*a^4*b^8)*cos(d*x + c)^4 + 3*(C
*a^11*b - 4*C*a^9*b^3 + 6*C*a^7*b^5 - 4*C*a^5*b^7 + C*a^3*b^9)*cos(d*x + c)
^3 + 3*(C*a^10*b^2 - 4*C*a^8*b^4 + 6*C*a^6*b^6 - 4*C*a^4*b^8 + C*a^2*b^10)*
cos(d*x + c)^2 + (C*a^9*b^3 - 4*C*a^7*b^5 + 6*C*a^5*b^7 - 4*C*a^3*b^9 + C*a
*b^11)*cos(d*x + c))*log(sin(d*x + c) + 1) + 24*((C*a^12 - 4*C*a^10*b^2 + 6
*C*a^8*b^4 - 4*C*a^6*b^6 + C*a^4*b^8)*cos(d*x + c)^4 + 3*(C*a^11*b - 4*C*a^
9*b^3 + 6*C*a^7*b^5 - 4*C*a^5*b^7 + C*a^3*b^9)*cos(d*x + c)^3 + 3*(C*a^10*b
^2 - 4*C*a^8*b^4 + 6*C*a^6*b^6 - 4*C*a^4*b^8 + C*a^2*b^10)*cos(d*x + c)^2 +
 (C*a^9*b^3 - 4*C*a^7*b^5 + 6*C*a^5*b^7 - 4*C*a^3*b^9 + C*a*b^11)*cos(d*x +
 c))*log(-sin(d*x + c) + 1) + 2*(6*C*a^8*b^4 - 24*C*a^6*b^6 + 36*C*a^4*b^8
- 24*C*a^2*b^10 + 6*C*b^12 + (24*C*a^11*b - 92*C*a^9*b^3 + (4*A + 133*C)*a^
7*b^5 + (7*A - 71*C)*a^5*b^7 - (11*A - 6*C)*a^3*b^9)*cos(d*x + c)^3 + 3*(20
*C*a^10*b^2 - 77*C*a^8*b^4 + (A + 110*C)*a^6*b^6 + (8*A - 59*C)*a^4*b^8 - 3
*(3*A - 2*C)*a^2*b^10)*cos(d*x + c)^2 + (44*C*a^9*b^3 + (2*A - 169*C)*a^7*b
^5 - (7*A - 239*C)*a^5*b^7 + (23*A - 132*C)*a^3*b^9 - 18*(A - C)*a*b^11)*co
s(d*x + c))*sin(d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 +
 a^3*b^13)*d*cos(d*x + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*
b^12 + a^2*b^14)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4
*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 -
 4*a^2*b^14 + b^16)*d*cos(d*x + c)), 1/6*(3*((8*C*a^11 - 28*C*a^9*b^2 + 35*
C*a^7*b^4 - (3*A + 20*C)*a^5*b^6 - 2*A*a^3*b^8)*cos(d*x + c)^4 + 3*(8*C*a^1
0*b - 28*C*a^8*b^3 + 35*C*a^6*b^5 - (3*A + 20*C)*a^4*b^7 - 2*A*a^2*b^9)*cos
(d*x + c)^3 + 3*(8*C*a^9*b^2 - 28*C*a^7*b^4 + 35*C*a^5*b^6 - (3*A + 20*C)*a
^3*b^8 - 2*A*a*b^10)*cos(d*x + c)^2 + (8*C*a^8*b^3 - 28*C*a^6*b^5 + 35*C*a^
4*b^7 - (3*A + 20*C)*a^2*b^9 - 2*A*b^11)*cos(d*x + c))*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 12
*((C*a^12 - 4*C*a^10*b^2 + 6*C*a^8*b^4 - 4*C*a^6*b^6 + C*a^4*b^8)*cos(d*x +
```

```
c)^4 + 3*(C*a^11*b - 4*C*a^9*b^3 + 6*C*a^7*b^5 - 4*C*a^5*b^7 + C*a^3*b^9)*
cos(d*x + c)^3 + 3*(C*a^10*b^2 - 4*C*a^8*b^4 + 6*C*a^6*b^6 - 4*C*a^4*b^8 +
C*a^2*b^10)*cos(d*x + c)^2 + (C*a^9*b^3 - 4*C*a^7*b^5 + 6*C*a^5*b^7 - 4*C*a
^3*b^9 + C*a*b^11)*cos(d*x + c))*log(sin(d*x + c) + 1) + 12*((C*a^12 - 4*C*
a^10*b^2 + 6*C*a^8*b^4 - 4*C*a^6*b^6 + C*a^4*b^8)*cos(d*x + c)^4 + 3*(C*a^1
1*b - 4*C*a^9*b^3 + 6*C*a^7*b^5 - 4*C*a^5*b^7 + C*a^3*b^9)*cos(d*x + c)^3 +
 3*(C*a^10*b^2 - 4*C*a^8*b^4 + 6*C*a^6*b^6 - 4*C*a^4*b^8 + C*a^2*b^10)*cos(
d*x + c)^2 + (C*a^9*b^3 - 4*C*a^7*b^5 + 6*C*a^5*b^7 - 4*C*a^3*b^9 + C*a*b^1
1)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (6*C*a^8*b^4 - 24*C*a^6*b^6 + 36*
C*a^4*b^8 - 24*C*a^2*b^10 + 6*C*b^12 + (24*C*a^11*b - 92*C*a^9*b^3 + (4*A +
 133*C)*a^7*b^5 + (7*A - 71*C)*a^5*b^7 - (11*A - 6*C)*a^3*b^9)*cos(d*x + c)
^3 + 3*(20*C*a^10*b^2 - 77*C*a^8*b^4 + (A + 110*C)*a^6*b^6 + (8*A - 59*C)*a
^4*b^8 - 3*(3*A - 2*C)*a^2*b^10)*cos(d*x + c)^2 + (44*C*a^9*b^3 + (2*A - 16
9*C)*a^7*b^5 - (7*A - 239*C)*a^5*b^7 + (23*A - 132*C)*a^3*b^9 - 18*(A - C)*
a*b^11)*cos(d*x + c))*sin(d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*
a^5*b^11 + a^3*b^13)*d*cos(d*x + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^1
0 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^
5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*
a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^4\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x
)
```

Giac [B] time = 1.41093, size = 1185, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/3*(3*(8*C*a^8 - 28*C*a^6*b^2 + 35*C*a^4*b^4 - 3*A*a^2*b^6 - 20*C*a^2*b^6
- 2*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5
- 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(-a^2 + b^2)) - 12*C*a*log(abs(tan(1/2*
d*x + 1/2*c) + 1))/b^5 + 12*C*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 - (1
8*C*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 24*C*a
^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*
a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*A*
a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 105*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A
*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 27*
A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 36*C
*a^9*tan(1/2*d*x + 1/2*c)^3 + 152*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^
5*b^4*tan(1/2*d*x + 1/2*c)^3 - 236*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*A*
```

a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 120*C*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*
A*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^9*tan(1/2*d*x + 1/2*c) + 42*C*a^8*b
*tan(1/2*d*x + 1/2*c) - 24*C*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*C*a^6*b^3*t
an(1/2*d*x + 1/2*c) + 6*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*C*a^5*b^4*tan(1
/2*d*x + 1/2*c) + 3*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*C*a^4*b^5*tan(1/2*
d*x + 1/2*c) + 6*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b^6*tan(1/2*d*x
+ 1/2*c) + 27*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 18*A*a*b^8*tan(1/2*d*x + 1/2
*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 -
b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) - 6*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d
*x + 1/2*c)^2 - 1)*b^4))/d

3.699 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=313

$$\frac{a\left(a^2b^4(A-8C)+7a^4b^2C-2a^6C+4b^6(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(a^2C+Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^4*d) + (a*(a^2*b^4*(A - 8*C) - 2*a^6*C + 7*a^4
*b^2*C + 4*b^6*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Sec[c + d*x]^2*T
an[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a*(2*A*b^4 - 3*a
^4*C + a^2*b^2*(3*A + 8*C))*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) - ((4*A*b^6 + 9*a^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A
 + 28*C))*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.25142, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {4099, 4090, 4080, 3998, 3770, 3831, 2659, 208}

$$\frac{a\left(a^2b^4(A-8C)+7a^4b^2C-2a^6C+4b^6(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(a^2C+Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^4*d) + (a*(a^2*b^4*(A - 8*C) - 2*a^6*C + 7*a^4
*b^2*C + 4*b^6*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Sec[c + d*x]^2*T
an[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a*(2*A*b^4 - 3*a
^4*C + a^2*b^2*(3*A + 8*C))*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) - ((4*A*b^6 + 9*a^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A
 + 28*C))*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
```

```
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)\left(2\left(Ab^2+a^2C\right)-3ab(A+C)\sec\right.}{(a+b\sec(c+}}{3b\left(a^2-\right.}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8C\right.}{6b^3\left(a^2-b^2\right)^2d(a+b\sec(}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8C\right.}{6b^3\left(a^2-b^2\right)^2d(a+b\sec(}$$

$$= -\frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4-3a^4C+a^2b^2(3A+8C\right.}{6b^3\left(a^2-b^2\right)^2d(a+b\sec(}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4\right.}{6b}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4\right.}{6b}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{a\left(a^2Ab^4+4Ab^6-2a^6C+7a^4b^2C-8a^2b^4C+8\right.}{(a-b)^{7/2}b^4(a+b)^{7/}}$$

Mathematica [C] time = 7.16969, size = 1092, normalized size = 3.49

$$-\frac{2C\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)\left(C\sec^2(c+dx)+A\right)(b+a\cos(c+dx))^4}{b^4d(\cos(2c+2dx)A+A+2C)(a+b\sec(c+dx))^4} + \frac{2C\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-2*C*(b + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*S
ec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b^4*d*(A + 2*C + A*Cos[2*c + 2*d*x])
*(a + b*Sec[c + d*x])^4) + (2*C*(b + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(b^4*d*(A +
 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((a^2*A*b^4 + 4*A*b^6
- 2*a^6*C + 7*a^4*b^2*C - 8*a^2*b^4*C + 8*b^6*C)*(b + a*Cos[c + d*x])^4*Sec
[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(((2*I)*a*ArcTan[Sec[(d*x)/2]*(Cos[c]/(S
qrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*S
qrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*
Cos[c])/(b^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]]) + (2*a*ArcTan[S
ec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[
c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I
*a*Sin[c + (d*x)/2])]*Sin[c])/(b^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[
2*c]])))/((-a^2 + b^2)^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x]
)^4) - (2*(b + a*Cos[c + d*x])*Sec[c]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)
*(A*b^3*Sin[c] + a^2*b*C*Sin[c] - a*A*b^2*Sin[d*x] - a^3*C*Sin[d*x]))/(3*a*
b*(-a^2 + b^2)*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + (
(b + a*Cos[c + d*x])^2*Sec[c]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(-5*a*A
```

```
*b^3*Sin[c] + a^3*b*C*Sin[c] - 6*a*b^3*C*Sin[c] + 3*a^2*A*b^2*Sin[d*x] + 2*
A*b^4*Sin[d*x] - 3*a^4*C*Sin[d*x] + 8*a^2*b^2*C*Sin[d*x]))/(3*b^2*(-a^2 + b
^2)^2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((b + a*Co
s[c + d*x])^3*Sec[c]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(-3*a^3*A*b^3*Si
n[c] - 12*a*A*b^5*Sin[c] - 3*a^5*b*C*Sin[c] + 6*a^3*b^3*C*Sin[c] - 18*a*b^5
*C*Sin[c] + 13*a^2*A*b^4*Sin[d*x] + 2*A*b^6*Sin[d*x] + 6*a^6*C*Sin[d*x] - 1
7*a^4*b^2*C*Sin[d*x] + 26*a^2*b^4*C*Sin[d*x]))/(3*b^3*(-a^2 + b^2)^3*d*(A +
 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.113, size = 2428, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+12/d*b/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*C*a^2+44/3/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^4*C-24/d*b/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*C*a^2-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-4/d/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1
2/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2-28/3/d*b/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*A*a^2-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^6*C-6/d/b/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^3*a^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+2/
d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a-b)/(a^3+
3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*
x+1/2*c)*A-1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^
5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-6/d/b/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^
3)*tan(1/2*d*x+1/2*c)^5*C+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*b/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b
+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c
)^5*A+1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a+
b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-4/d*b^3/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A+2/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+4/d*b^2*a/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*A+7/d/b^2*a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+8/d*b^2*a
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d/b^4*a^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
```

```
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+
2/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-1/d*C/b^4*ln(tan(1/2*d*x+1/2*c)-1
)+1/d*C/b^4*ln(tan(1/2*d*x+1/2*c)+1)-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 51.2194, size = 4779, normalized size = 15.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [1/12*(3*(2*C*a^7*b^3 - 7*C*a^5*b^5 - (A - 8*C)*a^3*b^7 - 4*(A + 2*C)*a*b^9
 + (2*C*a^10 - 7*C*a^8*b^2 - (A - 8*C)*a^6*b^4 - 4*(A + 2*C)*a^4*b^6)*cos(d
*x + c)^3 + 3*(2*C*a^9*b - 7*C*a^7*b^3 - (A - 8*C)*a^5*b^5 - 4*(A + 2*C)*a^
3*b^7)*cos(d*x + c)^2 + 3*(2*C*a^8*b^2 - 7*C*a^6*b^4 - (A - 8*C)*a^4*b^6 -
4*(A + 2*C)*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 6*(C*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4*C*a^2*b^9 + C*b^11 + (C*a^11
- 4*C*a^9*b^2 + 6*C*a^7*b^4 - 4*C*a^5*b^6 + C*a^3*b^8)*cos(d*x + c)^3 + 3*(
C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 - 4*C*a^4*b^7 + C*a^2*b^9)*cos(d*x + c
)^2 + 3*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^6 - 4*C*a^3*b^8 + C*a*b^10)*co
s(d*x + c))*log(sin(d*x + c) + 1) - 6*(C*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^
7 - 4*C*a^2*b^9 + C*b^11 + (C*a^11 - 4*C*a^9*b^2 + 6*C*a^7*b^4 - 4*C*a^5*b^
6 + C*a^3*b^8)*cos(d*x + c)^3 + 3*(C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 - 4
*C*a^4*b^7 + C*a^2*b^9)*cos(d*x + c)^2 + 3*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a
^5*b^6 - 4*C*a^3*b^8 + C*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(
11*C*a^8*b^3 - (A + 43*C)*a^6*b^5 + (11*A + 68*C)*a^4*b^7 - 4*(A + 9*C)*a^2
*b^9 - 6*A*b^11 + (6*C*a^10*b - 23*C*a^8*b^3 + (13*A + 43*C)*a^6*b^5 - (11*
A + 26*C)*a^4*b^7 - 2*A*a^2*b^9)*cos(d*x + c)^2 + 3*(5*C*a^9*b^2 - (A + 20*
C)*a^7*b^4 + 5*(2*A + 7*C)*a^5*b^6 - (7*A + 20*C)*a^3*b^8 - 2*A*a*b^10)*cos
(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 +
a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^
11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^
2*b^13 + b^15)*d), -1/6*(3*(2*C*a^7*b^3 - 7*C*a^5*b^5 - (A - 8*C)*a^3*b^7 -
 4*(A + 2*C)*a*b^9 + (2*C*a^10 - 7*C*a^8*b^2 - (A - 8*C)*a^6*b^4 - 4*(A + 2
*C)*a^4*b^6)*cos(d*x + c)^3 + 3*(2*C*a^9*b - 7*C*a^7*b^3 - (A - 8*C)*a^5*b^
```

```
5 - 4*(A + 2*C)*a^3*b^7)*cos(d*x + c)^2 + 3*(2*C*a^8*b^2 - 7*C*a^6*b^4 - (A
 - 8*C)*a^4*b^6 - 4*(A + 2*C)*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(C
*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4*C*a^2*b^9 + C*b^11 + (C*a^11 - 4*C
*a^9*b^2 + 6*C*a^7*b^4 - 4*C*a^5*b^6 + C*a^3*b^8)*cos(d*x + c)^3 + 3*(C*a^1
0*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 - 4*C*a^4*b^7 + C*a^2*b^9)*cos(d*x + c)^2 +
 3*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^6 - 4*C*a^3*b^8 + C*a*b^10)*cos(d*x
 + c))*log(sin(d*x + c) + 1) + 3*(C*a^8*b^3 - 4*C*a^6*b^5 + 6*C*a^4*b^7 - 4
*C*a^2*b^9 + C*b^11 + (C*a^11 - 4*C*a^9*b^2 + 6*C*a^7*b^4 - 4*C*a^5*b^6 + C
*a^3*b^8)*cos(d*x + c)^3 + 3*(C*a^10*b - 4*C*a^8*b^3 + 6*C*a^6*b^5 - 4*C*a^
4*b^7 + C*a^2*b^9)*cos(d*x + c)^2 + 3*(C*a^9*b^2 - 4*C*a^7*b^4 + 6*C*a^5*b^
6 - 4*C*a^3*b^8 + C*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*C*a^
8*b^3 - (A + 43*C)*a^6*b^5 + (11*A + 68*C)*a^4*b^7 - 4*(A + 9*C)*a^2*b^9 -
6*A*b^11 + (6*C*a^10*b - 23*C*a^8*b^3 + (13*A + 43*C)*a^6*b^5 - (11*A + 26*
C)*a^4*b^7 - 2*A*a^2*b^9)*cos(d*x + c)^2 + 3*(5*C*a^9*b^2 - (A + 20*C)*a^7*
b^4 + 5*(2*A + 7*C)*a^5*b^6 - (7*A + 20*C)*a^3*b^8 - 2*A*a*b^10)*cos(d*x +
c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^1
2)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^
2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12
 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13
+ b^15)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x
)
```

Giac [B] time = 1.38448, size = 1183, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/3*(3*(2*C*a^7 - 7*C*a^5*b^2 - A*a^3*b^4 + 8*C*a^3*b^4 - 4*A*a*b^6 - 8*C*
a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*
a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a^2 + b^2)) - 3*C*log(abs(tan(1/2*d*x + 1
/2*c) + 1))/b^4 + 3*C*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - (6*C*a^8*tan
(1/2*d*x + 1/2*c)^5 - 15*C*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^6*b^2*tan(1
/2*d*x + 1/2*c)^5 + 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^3*tan(1
/2*d*x + 1/2*c)^5 + 12*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^4*b^4*tan(1
/2*d*x + 1/2*c)^5 - 27*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^3*b^5*tan(
1/2*d*x + 1/2*c)^5 + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b^6*tan
(1/2*d*x + 1/2*c)^5 - 6*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*d*
```

x + 1/2*c)^5 - 12*C*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 28*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^8*tan(1/2*d*x + 1/2*c) + 15*C*a^7*b*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^2*tan(1/2*d*x + 1/2*c) - 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c) - 45*C*a^5*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^4*tan(1/2*d*x + 1/2*c) + 27*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b^5*tan(1/2*d*x + 1/2*c) + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b^6*tan(1/2*d*x + 1/2*c) + 6*A*a*b^7*tan(1/2*d*x + 1/2*c) + 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d

3.700 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=261

$$-\frac{b\left(a^2(4A+3C)+b^2(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(a^2b^2(2A-5C)+2a^4C+b^4(13A+18C)\right)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+$$

```
[Out] -((b*(b^2*(A + 2*C) + a^2*(4*A + 3*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b^2 + a^2*C)*Tan[
c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((3*A*b^4 - 4*a^4*
C + a^2*b^2*(2*A + 9*C))*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c
+ d*x])^2) + (a*(a^2*b^2*(2*A - 5*C) + 2*a^4*C + b^4*(13*A + 18*C))*Tan[c +
 d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.672904, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4091, 4080, 4003, 12, 3831, 2659, 208}

$$-\frac{b\left(a^2(4A+3C)+b^2(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(a^2b^2(2A-5C)+2a^4C+b^4(13A+18C)\right)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] -((b*(b^2*(A + 2*C) + a^2*(4*A + 3*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b^2 + a^2*C)*Tan[
c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((3*A*b^4 - 4*a^4*
C + a^2*b^2*(2*A + 9*C))*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c
+ d*x])^2) + (a*(a^2*b^2*(2*A - 5*C) + 2*a^4*C + b^4*(13*A + 18*C))*Tan[c +
 d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4091

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(a*(A*b^2 + a^2
*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)),
 x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*Simp[b*(m + 1)*(a^2*C + A*b^2) - a*(A*b^2*(m + 2) + C*(a^2 + b^2
*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
```

```
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)\left(-3b\left(Ab^2+a^2C\right)+a\left(2Ab^2-\left(a^2-3b^2\right)C\right)\sec\cdots}{(a+b\sec(c+dx))^{\cdots}}}{3b^2\left(a^2-b^2\cdots\right.}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\tan(c\cdots}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\tan(c\cdots}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\tan(c\cdots}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\tan(c\cdots}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4-4a^4C+a^2b^2(2A+9C)\right)\tan(c\cdots}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= -\frac{b\left(4a^2A+Ab^2+3a^2C+2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\left(Ab^2+a^2\cdots\right.}{3b^2\left(a^2-b^2\right)d(\cdots}$$

Mathematica [A] time = 1.27293, size = 221, normalized size = 0.85

$$-\frac{\frac{2\sin(c+dx)\left(6b\left(9a^2b^2(A+C)+a^4(2A+C)-Ab^4\right)\cos(c+dx)+a\left(\left(a^2b^2(10A+11C)+a^4(6A+4C)-Ab^4\right)\cos(2(c+dx))+a^2b^2(14A+C)+a^4(6A+8C)+b^4(25A+36C)\right)\right)}{(a\cos(c+dx)+b)^3}\cdots}{24d\left(b^2-a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((24*b*(b^2*(A + 2*C) + a^2*(4*A + 3*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(6*b*(-(A*b^4) + 9*a^2*b^2*(A + C
) + a^4*(2*A + C))*Cos[c + d*x] + a*(a^2*b^2*(14*A + C) + a^4*(6*A + 8*C) +
 b^4*(25*A + 36*C) + (-(A*b^4) + a^4*(6*A + 4*C) + a^2*b^2*(10*A + 11*C))*C
os[2*(c + d*x)]))*Sin[c + d*x])/(b + a*Cos[c + d*x])^3)/(24*(-a^2 + b^2)^3*
d)
```

Maple [A] time = 0.095, size = 374, normalized size = 1.4

$$\frac{1}{d}\left(2\frac{1}{\left(\left(\tan(1/2\,dx+c/2)\right)^2 a-\left(\tan(1/2\,dx+c/2)\right)^2 b-a-b\right)^3}\left(-1/2\,\frac{\left(2\,Aa^3+2\,Aa^2b+6\,Aab^2+Ab^3+2\,a^3C+3\,a^2bC\cdots\right.}{(a-b)\left(a^3+3\,a^2b+3\,ab^2\cdots\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3+2*C*a^3+3*C*a^2*b+6*C*a*b^2
)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(3*A*a^2+7*A*b^2
+C*a^2+9*C*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*
(2*A*a^3-2*A*a^2*b+6*A*a*b^2-A*b^3+2*C*a^3-3*C*a^2*b+6*C*a*b^2)/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3-b*(4*A*a^2+A*b^2+3*C*a^2+2*C*b^2)/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.755258, size = 2475, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [-1/12*(3*((4*A + 3*C)*a^2*b^4 + (A + 2*C)*b^6 + ((4*A + 3*C)*a^5*b + (A +
2*C)*a^3*b^3)*cos(d*x + c)^3 + 3*((4*A + 3*C)*a^4*b^2 + (A + 2*C)*a^2*b^4)*
cos(d*x + c)^2 + 3*((4*A + 3*C)*a^3*b^3 + (A + 2*C)*a*b^5)*cos(d*x + c))*sq
rt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sq
rt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*C*a^7 + (2*A - 7*C)*a^5*b^2 + (
11*A + 23*C)*a^3*b^4 - (13*A + 18*C)*a*b^6 + (2*(3*A + 2*C)*a^7 + (4*A + 7*
C)*a^5*b^2 - 11*(A + C)*a^3*b^4 + A*a*b^6)*cos(d*x + c)^2 + 3*((2*A + C)*a^
6*b + (7*A + 8*C)*a^4*b^3 - (10*A + 9*C)*a^2*b^5 + A*b^7)*cos(d*x + c))*sin
(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x
+ c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x
 + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*
x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(
(4*A + 3*C)*a^2*b^4 + (A + 2*C)*b^6 + ((4*A + 3*C)*a^5*b + (A + 2*C)*a^3*b^
3)*cos(d*x + c)^3 + 3*((4*A + 3*C)*a^4*b^2 + (A + 2*C)*a^2*b^4)*cos(d*x + c
)^2 + 3*((4*A + 3*C)*a^3*b^3 + (A + 2*C)*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b
^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)
)) - (2*C*a^7 + (2*A - 7*C)*a^5*b^2 + (11*A + 23*C)*a^3*b^4 - (13*A + 18*C)
*a*b^6 + (2*(3*A + 2*C)*a^7 + (4*A + 7*C)*a^5*b^2 - 11*(A + C)*a^3*b^4 + A*
a*b^6)*cos(d*x + c)^2 + 3*((2*A + C)*a^6*b + (7*A + 8*C)*a^4*b^3 - (10*A +
9*C)*a^2*b^5 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a
^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6
*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^
4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x
)
```

Giac [B] time = 1.29461, size = 936, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/3*(3*(4*A*a^2*b + 3*C*a^2*b + A*b^3 + 2*C*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2
 + b^2)) + (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^5*tan(1/2*d*x + 1/2*c)^5
 - 6*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12
*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27
*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 1
2*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 18*C*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*
b^5*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^5*tan(
1/2*d*x + 1/2*c)^3 - 16*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*C*a^3*b^2*tan
(1/2*d*x + 1/2*c)^3 + 28*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*C*a*b^4*tan(1/
2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/2*c) + 6*C*a^5*tan(1/2*d*x + 1/2
*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) + 3*C*a^4*b*tan(1/2*d*x + 1/2*c) + 12*
A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^
2*b^3*tan(1/2*d*x + 1/2*c) + 27*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a*b^4
*tan(1/2*d*x + 1/2*c) + 18*C*a*b^4*tan(1/2*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d
*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.701 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=252

$$\frac{a\left(a^2(2A+C)+b^2(3A+4C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-a^2b^2(11A+10C)+a^4C-2b^4(2A+3C)\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

```
[Out] (a*(a^2*(2*A + C) + b^2*(3*A + 4*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Tan[c + d
*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a*(5*A*b^2 - a^2*C + 6*b
^2*C)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((a^4*C
- 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Tan[c + d*x])/(6*b*(a^2 - b^2)
^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.553413, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4081, 4003, 12, 3831, 2659, 208}

$$\frac{a\left(a^2(2A+C)+b^2(3A+4C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-a^2b^2(11A+10C)+a^4C-2b^4(2A+3C)\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (a*(a^2*(2*A + C) + b^2*(3*A + 4*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b^2 + a^2*C)*Tan[c + d
*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a*(5*A*b^2 - a^2*C + 6*b
^2*C)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((a^4*C
- 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Tan[c + d*x])/(6*b*(a^2 - b^2)
^3*d*(a + b*Sec[c + d*x]))
```


Rule 4081

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A*b^2 + a^2*C)
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] +
 Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2 + a^2*C + b*(A*b + b*C)*(m + 1))*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && NeQ[a
^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(-3ab(A+C)+\left(2Ab^2-a^2C+3b^2C\right)\sec(c+\right.}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{(a}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{(a}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{(a}{}$$

$$= -\frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(5Ab^2-a^2C+6b^2C\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{(a}{}$$

$$= \frac{a\left(2a^2A+3Ab^2+a^2C+4b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\left(Ab^2+a^2C\right)\mathrm{t}}{3b\left(a^2-b^2\right)d(a+}$$

Mathematica [C] time = 6.16409, size = 438, normalized size = 1.74

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(A+C\sec^2(c+dx)\right)\left(\frac{2b\sec(c)\left(a^2C+Ab^2\right)(b\sin(c)-a\sin(dx))}{a^5-a^3b^2} - \frac{6ia(\cos(c)-i\sin(c))\left(a^2(2A+C)+b^2(3A+4C)\right)}{\left(a^2\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(((-6*I)*a*(a^2
*(2*A + C) + b^2*(3*A + 4*C))*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b +
 a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b
 + a*Cos[c + d*x])^3*(Cos[c] - I*Sin[c]))/((a^2 - b^2)^(7/2)*Sqrt[(Cos[c] -
 I*Sin[c])^2]) + (2*b*(A*b^2 + a^2*C)*Sec[c]*(b*Sin[c] - a*Sin[d*x]))/(a^5
- a^3*b^2) + ((b + a*Cos[c + d*x])*Sec[c]*((-11*a^2*A*b^3 + 6*A*b^5 - 5*a^4
*b*C)*Sin[c] + a*(-4*A*b^4 + 3*a^4*C + a^2*b^2*(9*A + 2*C))*Sin[d*x]))/(a^3
*(a^2 - b^2)^2) + ((b + a*Cos[c + d*x])^2*Sec[c]*(3*(-6*a^2*A*b^4 + 2*A*b^6
 + a^6*C + a^4*b^2*(9*A + 4*C))*Sin[c] - a*b*(2*A*b^4 + a^2*b^2*(-5*A + 2*C
) + a^4*(18*A + 13*C))*Sin[d*x]))/(a^3 - a*b^2)^3))/(3*d*(A + 2*C + A*Cos[2
*(c + d*x)])*(a + b*Sec[c + d*x])^4)
```

Maple [A] time = 0.102, size = 373, normalized size = 1.5

$$\frac{1}{d}\left(-2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)^3}\left(-1/2\,\frac{\left(6\,Aa^2b+3\,Aab^2+2\,Ab^3+a^3C+6\,a^2bC+2\,\ldots\right.}{(a-b)\left(a^3+3\,a^2b+3\,\ldots\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3+C*a^3+6*C*a^2*b+2*C*a*b^2+2*C*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(9*A*a^2+A*b^2+
7*C*a^2+3*C*b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2
*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-C*a^3+6*C*a^2*b-2*C*a*b^2+2*C*b^3)/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^3+a*(2*A*a^2+3*A*b^2+C*a^2+4*C*b^2)/(a^6-3*a^4*b^2+3*a^2
*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.750736, size = 2473, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*((2*A + C)*a^3*b^3 + (3*A + 4*C)*a*b^5 + ((2*A + C)*a^6 + (3*A +
4*C)*a^4*b^2)*cos(d*x + c)^3 + 3*((2*A + C)*a^5*b + (3*A + 4*C)*a^3*b^3)*co
s(d*x + c)^2 + 3*((2*A + C)*a^4*b^2 + (3*A + 4*C)*a^2*b^4)*cos(d*x + c))*sq
rt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sq
rt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(C*a^6*b - 11*(A + C)*a^4*b^3 + (7
*A + 4*C)*a^2*b^5 + 2*(2*A + 3*C)*b^7 - ((18*A + 13*C)*a^6*b - (23*A + 11*C
)*a^4*b^3 + (7*A - 2*C)*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 + 3*(C*a^7 - (9*A
 + 10*C)*a^5*b^2 + (8*A + 7*C)*a^3*b^4 + (A + 2*C)*a*b^6)*cos(d*x + c))*sin
(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x
+ c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x
 + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*
x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*((
2*A + C)*a^3*b^3 + (3*A + 4*C)*a*b^5 + ((2*A + C)*a^6 + (3*A + 4*C)*a^4*b^2
)*cos(d*x + c)^3 + 3*((2*A + C)*a^5*b + (3*A + 4*C)*a^3*b^3)*cos(d*x + c)^2
 + 3*((2*A + C)*a^4*b^2 + (3*A + 4*C)*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) + (C*a^6*b - 11*(A + C)*a^4*b^3 + (7*A + 4*C)*a^2*b^5 + 2*(2*A + 3*C)*b^7
 - ((18*A + 13*C)*a^6*b - (23*A + 11*C)*a^4*b^3 + (7*A - 2*C)*a^2*b^5 - 2*A
*b^7)*cos(d*x + c)^2 + 3*(C*a^7 - (9*A + 10*C)*a^5*b^2 + (8*A + 7*C)*a^3*b^
4 + (A + 2*C)*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^
6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*
a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4
*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.31245, size = 936, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(2*A*a^3 + C*a^3 + 3*A*a*b^2 + 4*C*a*b^2)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 +
 b^2)) - (3*C*a^5*tan(1/2*d*x + 1/2*c)^5 + 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 + 12*C*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
```

```
 - 27*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
 + 12*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 -
 6*C*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^
5*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 28*C*a^4*b*t
an(1/2*d*x + 1/2*c)^3 + 32*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*C*a^2*b^3*
tan(1/2*d*x + 1/2*c)^3 + 4*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 12*C*b^5*tan(1/2*
d*x + 1/2*c)^3 - 3*C*a^5*tan(1/2*d*x + 1/2*c) + 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 12*C*a^4*b*tan(1/2*d*x + 1/2*c) + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)
+ 27*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12
*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 6*C*a*b^
4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*x + 1/2*c) + 6*C*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.702 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=292

$$-\frac{b\left(-a^4b^2(8A-C)+7a^2Ab^4+4a^6(2A+C)-2Ab^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(-13a^4b^2(2A+C)+17a^2Ab^4-2a^6C\right.}{6a^3d\left(a^2-b^2\right)^3(a+b\sec}$$

```
[Out] (A*x)/a^4 - (b*(7*a^2*A*b^4 - 2*A*b^6 - a^4*b^2*(8*A - C) + 4*a^6*(2*A + C)
)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(
a + b)^(7/2)*d) + ((A*b^2 + a^2*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^3) - ((3*A*b^4 - 2*a^4*C - a^2*b^2*(8*A + 3*C))*Tan[c + d*x])
/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((17*a^2*A*b^4 - 6*A*b^6
- 2*a^6*C - 13*a^4*b^2*(2*A + C))*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a +
 b*Sec[c + d*x]))
```

Rubi [A] time = 0.93672, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4061, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-a^4b^2(8A-C)+7a^2Ab^4+4a^6(2A+C)-2Ab^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(-13a^4b^2(2A+C)+17a^2Ab^4-2a^6C\right.}{6a^3d\left(a^2-b^2\right)^3(a+b\sec}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (A*x)/a^4 - (b*(7*a^2*A*b^4 - 2*A*b^6 - a^4*b^2*(8*A - C) + 4*a^6*(2*A + C)
)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(
a + b)^(7/2)*d) + ((A*b^2 + a^2*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^3) - ((3*A*b^4 - 2*a^4*C - a^2*b^2*(8*A + 3*C))*Tan[c + d*x])
/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((17*a^2*A*b^4 - 6*A*b^6
- 2*a^6*C - 13*a^4*b^2*(2*A + C))*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a +
 b*Sec[c + d*x]))
```


Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3A\left(a^2-b^2\right)+3ab(A+C)\sec(c+dx)-2\left(Ab^2+a^2C\right)\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{6A}{\cdots}}{\cdots}$$

$$= \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{(17a\cdots}{\cdots}$$

$$= \frac{Ax}{a^4} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} \cdots$$

$$= \frac{Ax}{a^4} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} \cdots$$

$$= \frac{Ax}{a^4} + \frac{\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4-2a^4C-a^2b^2(8A+3C)\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} \cdots$$

$$= \frac{Ax}{a^4} - \frac{b\left(8a^6A-8a^4Ab^2+7a^2Ab^4-2Ab^6+4a^6C+a^4b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \cdots$$

Mathematica [C] time = 7.21228, size = 995, normalized size = 3.41

$$\frac{2Ax\sec^2(c+dx)\left(C\sec^2(c+dx)+A\right)(b+a\cos(c+dx))^4}{a^4(\cos(2c+2dx)A+A+2C)(a+b\sec(c+dx))^4}+\frac{\left(-8Aa^6-4Ca^6+8Ab^2a^4-b^2Ca^4-7Ab^4a^2+2Ab^6\right)\sec^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*A*x*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a^4*(
A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((-8*a^6*A + 8*a^4*
A*b^2 - 7*a^2*A*b^4 + 2*A*b^6 - 4*a^6*C - a^4*b^2*C)*(b + a*Cos[c + d*x])^4
*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(((2*I)*b*ArcTan[Sec[(d*x)/2]*(Cos[c
]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^
2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2
])]*Cos[c])/(a^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]]) + (2*b*ArcT
an[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*
Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2]
 + I*a*Sin[c + (d*x)/2])]*Sin[c])/(a^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*
Sin[2*c]])))/((-a^2 + b^2)^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c +
d*x])^4) - (2*(b + a*Cos[c + d*x])*Sec[c]*Sec[c + d*x]^2*(A + C*Sec[c + d*x
]^2)*(A*b^5*Sin[c] + a^2*b^3*C*Sin[c] - a*A*b^4*Sin[d*x] - a^3*b^2*C*Sin[d*
x]))/(3*a^4*(a^2 - b^2)*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x
])^4) + ((b + a*Cos[c + d*x])^2*Sec[c]*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2
)*(14*a^2*A*b^4*Sin[c] - 9*A*b^6*Sin[c] + 8*a^4*b^2*C*Sin[c] - 3*a^2*b^4*C*
Sin[c] - 12*a^3*A*b^3*Sin[d*x] + 7*a*A*b^5*Sin[d*x] - 6*a^5*b*C*Sin[d*x] +
a^3*b^3*C*Sin[d*x]))/(3*a^4*(a^2 - b^2)^2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*
(a + b*Sec[c + d*x])^4) + ((b + a*Cos[c + d*x])^3*Sec[c]*Sec[c + d*x]^2*(A
+ C*Sec[c + d*x]^2)*(-48*a^4*A*b^3*Sin[c] + 51*a^2*A*b^5*Sin[c] - 18*A*b^7*
Sin[c] - 12*a^6*b*C*Sin[c] - 3*a^4*b^3*C*Sin[c] + 36*a^5*A*b^2*Sin[d*x] - 3
2*a^3*A*b^4*Sin[d*x] + 11*a*A*b^6*Sin[d*x] + 6*a^7*C*Sin[d*x] + 10*a^5*b^2*
C*Sin[d*x] - a^3*b^4*C*Sin[d*x]))/(3*a^4*(a^2 - b^2)^3*d*(A + 2*C + A*Cos[2
*c + 2*d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.113, size = 2407, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a^2-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*C-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3-6/d*a/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5*b^2*C-44/3/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-2/d/a^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6+6/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A
*b^4+2/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3
```

```
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2+1/d/a^2/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*A*b^5-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^6+4/d/a^3/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^6+28/3/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
*b^2*C-6/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2*C-1/d/a^2/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*A*b^5+6/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^4-7/d/a^2*b^5/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))*A+2/d/a^4*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+1/d/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*
a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
*C-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*C*a^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/
(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2-12/d*b^2/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)*A-12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+24/d*b^2/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a^2-2*a*b+b^2)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d*A/a^4*arctan(tan(1/2*d*x+1/2*c))+4/
d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5*A+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-1/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.922288, size = 3868, normalized size = 13.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*
x*cos(d*x + c)^3 + 36*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 +
 A*a^2*b^9)*d*x*cos(d*x + c)^2 + 36*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6
- 4*A*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 12*(A*a^8*b^3 - 4*A*a^6*b^5 +
6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x + 3*(4*(2*A + C)*a^6*b^4 - (8*A - C
)*a^4*b^6 + 7*A*a^2*b^8 - 2*A*b^10 + (4*(2*A + C)*a^9*b - (8*A - C)*a^7*b^3
 + 7*A*a^5*b^5 - 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(4*(2*A + C)*a^8*b^2 - (8*
A - C)*a^6*b^4 + 7*A*a^4*b^6 - 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(4*(2*A + C)
*a^7*b^3 - (8*A - C)*a^5*b^5 + 7*A*a^3*b^7 - 2*A*a*b^9)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(2*C*a^9*b^2 + (26*A + 11*C)*a^7*b^4
- (43*A + 13*C)*a^5*b^6 + 23*A*a^3*b^8 - 6*A*a*b^10 + (6*C*a^11 + 4*(9*A +
C)*a^9*b^2 - (68*A + 11*C)*a^7*b^4 + (43*A + C)*a^5*b^6 - 11*A*a^3*b^8)*cos
(d*x + c)^2 + 3*(2*C*a^10*b + (20*A + 7*C)*a^8*b^3 - 5*(7*A + 2*C)*a^6*b^5
+ (20*A + C)*a^4*b^7 - 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*
a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b -
 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*
b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12
*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), 1/6*(6*(A*a^11 -
4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*x*cos(d*x + c)^3 + 1
8*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*d*x*cos(
d*x + c)^2 + 18*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*
b^10)*d*x*cos(d*x + c) + 6*(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2
*b^9 + A*b^11)*d*x - 3*(4*(2*A + C)*a^6*b^4 - (8*A - C)*a^4*b^6 + 7*A*a^2*b
^8 - 2*A*b^10 + (4*(2*A + C)*a^9*b - (8*A - C)*a^7*b^3 + 7*A*a^5*b^5 - 2*A*
a^3*b^7)*cos(d*x + c)^3 + 3*(4*(2*A + C)*a^8*b^2 - (8*A - C)*a^6*b^4 + 7*A*
a^4*b^6 - 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(4*(2*A + C)*a^7*b^3 - (8*A - C)*
a^5*b^5 + 7*A*a^3*b^7 - 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*C*a^9
*b^2 + (26*A + 11*C)*a^7*b^4 - (43*A + 13*C)*a^5*b^6 + 23*A*a^3*b^8 - 6*A*a
*b^10 + (6*C*a^11 + 4*(9*A + C)*a^9*b^2 - (68*A + 11*C)*a^7*b^4 + (43*A + C
)*a^5*b^6 - 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(2*C*a^10*b + (20*A + 7*C)*a^8
*b^3 - 5*(7*A + 2*C)*a^6*b^5 + (20*A + C)*a^4*b^7 - 5*A*a^2*b^9)*cos(d*x +
c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d
*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9
)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5
*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^
4*b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.31741, size = 1141, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(8*A*a^6*b + 4*C*a^6*b - 8*A*a^4*b^3 + C*a^4*b^3 + 7*A*a^2*b^5 - 2*
A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8
*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) - 3*(d*x + c)*A/a^4 + (6*C*a^
8*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^7*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^6*b^2*
tan(1/2*d*x + 1/2*c)^5 + 12*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^5*b^3
*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b^4
*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*A*a^3*b^
5*tan(1/2*d*x + 1/2*c)^5 + 3*C*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^6
*tan(1/2*d*x + 1/2*c)^5 - 15*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1
/2*d*x + 1/2*c)^5 - 12*C*a^8*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^6*b^2*tan(1/2*
d*x + 1/2*c)^3 - 16*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^4*tan(1/
2*d*x + 1/2*c)^3 + 28*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 56*A*a^2*b^6*tan(1
/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^8*tan(1/2*d*x +
 1/2*c) + 6*C*a^7*b*tan(1/2*d*x + 1/2*c) + 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c
) + 12*C*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c) +
 27*C*a^5*b^3*tan(1/2*d*x + 1/2*c) - 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) + 12*
C*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c) - 3*C*a^
3*b^5*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*A*a*b^7*
tan(1/2*d*x + 1/2*c) + 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*
a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a
 - b)^3))/d
```

3.703 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=367

$$\frac{\left(-a^4b^2(65A+4C)+68a^2Ab^4+a^6(6A-11C)-24Ab^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}-\frac{\left(-a^6b^2(20A+3C)+35a^4Ab^4-28a^2Ab^6-2a^8C\right.}{a^5d(a-b)^{7/2}(a+b}$$

```
[Out] (-4*A*b*x)/a^5 - ((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 - 2*a^8*C - a^6*b^
2*(20*A + 3*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(
a - b)^(7/2)*(a + b)^(7/2)*d) + ((68*a^2*A*b^4 - 24*A*b^6 + a^6*(6*A - 11*C
) - a^4*b^2*(65*A + 4*C))*Sin[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + ((A*b^2 +
 a^2*C)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((4*A*b^
4 - 3*a^4*C - a^2*b^2*(9*A + 2*C))*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a
+ b*Sec[c + d*x])^2) - ((11*a^2*A*b^4 - 4*A*b^6 - 2*a^6*C - 3*a^4*b^2*(4*A
+ C))*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.82357, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4101, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-a^4b^2(65A+4C)+68a^2Ab^4+a^6(6A-11C)-24Ab^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}-\frac{\left(-a^6b^2(20A+3C)+35a^4Ab^4-28a^2Ab^6-2a^8C\right.}{a^5d(a-b)^{7/2}(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (-4*A*b*x)/a^5 - ((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 - 2*a^8*C - a^6*b^
2*(20*A + 3*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(
a - b)^(7/2)*(a + b)^(7/2)*d) + ((68*a^2*A*b^4 - 24*A*b^6 + a^6*(6*A - 11*C
) - a^4*b^2*(65*A + 4*C))*Sin[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + ((A*b^2 +
 a^2*C)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((4*A*b^
4 - 3*a^4*C - a^2*b^2*(9*A + 2*C))*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a
+ b*Sec[c + d*x])^2) - ((11*a^2*A*b^4 - 4*A*b^6 - 2*a^6*C - 3*a^4*b^2*(4*A
+ C))*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
```

```
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(4Ab^2-a^2(3A-C)+3ab(A+C)\sec(c+dx)-3\right.}{(a+b\sec(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(4Ab^4-3a^4C-a^2b^2(9A+2C)\right)\sin(c+d}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(Ab^2+a^2C\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(4Ab^4-3a^4C-a^2b^2(9A+2C)\right)\sin(c+d}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A+4C)\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{(Ab}{3a\left(a^2-\right.}$$

$$= -\frac{4Abx}{a^5} + \frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A+4C)\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} -$$

$$= -\frac{4Abx}{a^5} + \frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A+4C)\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} -$$

$$= -\frac{4Abx}{a^5} + \frac{\left(68a^2Ab^4-24Ab^6+a^6(6A-11C)-a^4b^2(65A+4C)\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} -$$

$$= -\frac{4Abx}{a^5} + \frac{\left(20a^6Ab^2-35a^4Ab^4+28a^2Ab^6-8Ab^8+2a^8C+3a^6b^2C\right)\tanh^{-1}}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [C] time = 7.42231, size = 1089, normalized size = 2.97

$$-\frac{8Abx\sec^2(c+dx)\left(C\sec^2(c+dx)+A\right)(b+a\cos(c+dx))^4}{a^5(\cos(2c+2dx)A+A+2C)(a+b\sec(c+dx))^4} + \frac{\left(-2Ca^8-20Ab^2a^6-3b^2Ca^6+35Ab^4a^4-28Ab^6a^2+8\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-8*A*b*x*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a^
5*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((-20*a^6*A*b^2
+ 35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 - 2*a^8*C - 3*a^6*b^2*C)*(b + a*Cos
[c + d*x])^4*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(((-2*I)*ArcTan[Sec[(d*x
)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sq
rt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[
c + (d*x)/2])]*Cos[c])/(a^5*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]])
- (2*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c
]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin
[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*Sin[c])/(a^5*Sqrt[a^2 - b^2]*d*Sqrt[Cos[
2*c] - I*Sin[2*c]])))/((-a^2 + b^2)^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b
*Sec[c + d*x])^4) + (2*(b + a*Cos[c + d*x])*Sec[c]*Sec[c + d*x]^2*(A + C*Se
c[c + d*x]^2)*(A*b^6*Sin[c] + a^2*b^4*C*Sin[c] - a*A*b^5*Sin[d*x] - a^3*b^3
*C*Sin[d*x]))/(3*a^5*(a^2 - b^2)*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Se
c[c + d*x])^4) + ((b + a*Cos[c + d*x])^2*Sec[c]*Sec[c + d*x]^2*(A + C*Sec[c
```

```
 + d*x]^2)*(-17*a^2*A*b^5*Sin[c] + 12*A*b^7*Sin[c] - 11*a^4*b^3*C*Sin[c] +
6*a^2*b^5*C*Sin[c] + 15*a^3*A*b^4*Sin[d*x] - 10*a*A*b^6*Sin[d*x] + 9*a^5*b^
2*C*Sin[d*x] - 4*a^3*b^4*C*Sin[d*x]))/(3*a^5*(a^2 - b^2)^2*d*(A + 2*C + A*C
os[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((b + a*Cos[c + d*x])^3*Sec[c]*S
ec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*(75*a^4*A*b^4*Sin[c] - 96*a^2*A*b^6*Si
n[c] + 36*A*b^8*Sin[c] + 27*a^6*b^2*C*Sin[c] - 18*a^4*b^4*C*Sin[c] + 6*a^2*
b^6*C*Sin[c] - 60*a^5*A*b^3*Sin[d*x] + 71*a^3*A*b^5*Sin[d*x] - 26*a*A*b^7*S
in[d*x] - 18*a^7*b*C*Sin[d*x] + 5*a^5*b^3*C*Sin[d*x] - 2*a^3*b^5*C*Sin[d*x]
))/(3*a^5*(a^2 - b^2)^3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x
])^4) + (2*A*(b + a*Cos[c + d*x])^4*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Tan
[c + d*x])/(a^4*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.152, size = 2283, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3+6/d/a^4/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*
x+1/2*c)*A+6/d/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^
7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+116/3/d/a^2/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a^2-2*a*b+b^2)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-12/d/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3*b^7/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*A+3/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+
3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2*C+2/d/a^3/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*A*b^6+5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^4+6/d*b/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)*C*a^2-12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^2-18
/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a
^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*A*b^6-3/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2*C-18/d/a^2/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)*A*b^5-5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^4+2/d/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*
a^2-40/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a
*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+20/d*b^3/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*A+20/d*b^2*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+3/d*b^2*a/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*C+20/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-35/d/a/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*A*b^4-8/d/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)
```

```
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^8+2
8/d/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6-4/3/d/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*C+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+2/d*A/a^4*tan(1/2*d*x+1/
2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d*A/a^5*b*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.06307, size = 4319, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(48*(A*a^11*b - 4*A*a^9*b^3 + 6*A*a^7*b^5 - 4*A*a^5*b^7 + A*a^3*b^9)
*d*x*cos(d*x + c)^3 + 144*(A*a^10*b^2 - 4*A*a^8*b^4 + 6*A*a^6*b^6 - 4*A*a^4
*b^8 + A*a^2*b^10)*d*x*cos(d*x + c)^2 + 144*(A*a^9*b^3 - 4*A*a^7*b^5 + 6*A*
a^5*b^7 - 4*A*a^3*b^9 + A*a*b^11)*d*x*cos(d*x + c) + 48*(A*a^8*b^4 - 4*A*a^
6*b^6 + 6*A*a^4*b^8 - 4*A*a^2*b^10 + A*b^12)*d*x - 3*(2*C*a^8*b^3 + (20*A +
 3*C)*a^6*b^5 - 35*A*a^4*b^7 + 28*A*a^2*b^9 - 8*A*b^11 + (2*C*a^11 + (20*A
+ 3*C)*a^9*b^2 - 35*A*a^7*b^4 + 28*A*a^5*b^6 - 8*A*a^3*b^8)*cos(d*x + c)^3
+ 3*(2*C*a^10*b + (20*A + 3*C)*a^8*b^3 - 35*A*a^6*b^5 + 28*A*a^4*b^7 - 8*A*
a^2*b^9)*cos(d*x + c)^2 + 3*(2*C*a^9*b^2 + (20*A + 3*C)*a^7*b^4 - 35*A*a^5*
b^6 + 28*A*a^3*b^8 - 8*A*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*c
os(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x +
 c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
c) + b^2)) - 2*((6*A - 11*C)*a^9*b^3 - (71*A - 7*C)*a^7*b^5 + (133*A + 4*C)
*a^5*b^7 - 92*A*a^3*b^9 + 24*A*a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2 + 6*A*a^8*
b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*cos(d*x + c)^3 + (18*(A - C)*a^11*b - (132*A
 - 23*C)*a^9*b^3 + (239*A - 7*C)*a^7*b^5 - (169*A - 2*C)*a^5*b^7 + 44*A*a^3
*b^9)*cos(d*x + c)^2 + 3*(3*(2*A - 3*C)*a^10*b^2 - (59*A - 8*C)*a^8*b^4 + (
110*A + C)*a^6*b^6 - 77*A*a^4*b^8 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x +
c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)
^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x +
 c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos
(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d),
-1/6*(24*(A*a^11*b - 4*A*a^9*b^3 + 6*A*a^7*b^5 - 4*A*a^5*b^7 + A*a^3*b^9)*d
*x*cos(d*x + c)^3 + 72*(A*a^10*b^2 - 4*A*a^8*b^4 + 6*A*a^6*b^6 - 4*A*a^4*b^
8 + A*a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(A*a^9*b^3 - 4*A*a^7*b^5 + 6*A*a^5*
b^7 - 4*A*a^3*b^9 + A*a*b^11)*d*x*cos(d*x + c) + 24*(A*a^8*b^4 - 4*A*a^6*b^
6 + 6*A*a^4*b^8 - 4*A*a^2*b^10 + A*b^12)*d*x - 3*(2*C*a^8*b^3 + (20*A + 3*C
```

```
)*a^6*b^5 - 35*A*a^4*b^7 + 28*A*a^2*b^9 - 8*A*b^11 + (2*C*a^11 + (20*A + 3*
C)*a^9*b^2 - 35*A*a^7*b^4 + 28*A*a^5*b^6 - 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*
(2*C*a^10*b + (20*A + 3*C)*a^8*b^3 - 35*A*a^6*b^5 + 28*A*a^4*b^7 - 8*A*a^2*
b^9)*cos(d*x + c)^2 + 3*(2*C*a^9*b^2 + (20*A + 3*C)*a^7*b^4 - 35*A*a^5*b^6
+ 28*A*a^3*b^8 - 8*A*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((6*A - 11*C)*
a^9*b^3 - (71*A - 7*C)*a^7*b^5 + (133*A + 4*C)*a^5*b^7 - 92*A*a^3*b^9 + 24*
A*a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2 + 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8
)*cos(d*x + c)^3 + (18*(A - C)*a^11*b - (132*A - 23*C)*a^9*b^3 + (239*A - 7
*C)*a^7*b^5 - (169*A - 2*C)*a^5*b^7 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(3*(
2*A - 3*C)*a^10*b^2 - (59*A - 8*C)*a^8*b^4 + (110*A + C)*a^6*b^6 - 77*A*a^4
*b^8 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a
^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 +
 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*
b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^1
1*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36366, size = 1143, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*C*a^8 + 20*A*a^6*b^2 + 3*C*a^6*b^2 - 35*A*a^4*b^4 + 28*A*a^2*b^6
- 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3
*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(-a^2 + b^2)) - 12*(d*x + c)*A*b/a^5 +
(18*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 +
60*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 -
105*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 -
 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 +
 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5
 - 42*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^9*tan(1/2*d*x + 1/2*c)^5 - 36
*C*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 + 32
*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 + 236*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 +
4*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 152*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^3 +
 36*A*b^9*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^8*b*tan(1/2*d*x + 1/2*c) + 27*C*a
^7*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c) + 6*C*a^6*b
^3*tan(1/2*d*x + 1/2*c) + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) + 3*C*a^5*b^4*
tan(1/2*d*x + 1/2*c) - 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*C*a^4*b^5*tan(
1/2*d*x + 1/2*c) - 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 24*A*a^2*b^7*tan(1/
```

```
2*d*x + 1/2*c) + 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 18*A*b^9*tan(1/2*d*x + 1
/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 6*A*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 + 1)*a^4))/d
```

3.704 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=513

$$\frac{b\left(a^4b^2(146A-17C)-a^2b^4(167A-6C)+a^6(-(24A-26C))+60Ab^6\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}-\frac{\left(a^4b^2(23A-2C)-a^2b^4(27A-\right.}{}$$

```
[Out] ((20*A*b^2 + a^2*(A + 2*C))*x)/(2*a^6) + ((20*A*b^9 - a^2*b^7*(69*A - 2*C)
- 8*a^6*b^3*(5*A - C) + 7*a^4*b^5*(12*A - C) - 8*a^8*b*C)*ArcTanh[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2
)^3*d) + (b*(60*A*b^6 - a^6*(24*A - 26*C) + a^4*b^2*(146*A - 17*C) - a^2*b^
4*(167*A - 6*C))*Sin[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) - ((10*A*b^6 - a^6*(
A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(27*A - C))*Cos[c + d*x]*Sin[c +
d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 + a^2*C)*Cos[c + d*x]*Sin[c + d*x])
/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*A*b^4 - 4*a^4*C - a^2*b^2
*(10*A + C))*Cos[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c
 + d*x])^2) + ((20*A*b^6 - a^2*b^4*(53*A - 2*C) + 12*a^6*C + a^4*b^2*(48*A
+ C))*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]
))
```

Rubi [A] time = 2.3451, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4101, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{b\left(a^4b^2(146A-17C)-a^2b^4(167A-6C)+a^6(-(24A-26C))+60Ab^6\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}-\frac{\left(a^4b^2(23A-2C)-a^2b^4(27A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ((20*A*b^2 + a^2*(A + 2*C))*x)/(2*a^6) + ((20*A*b^9 - a^2*b^7*(69*A - 2*C)
- 8*a^6*b^3*(5*A - C) + 7*a^4*b^5*(12*A - C) - 8*a^8*b*C)*ArcTanh[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2
)^3*d) + (b*(60*A*b^6 - a^6*(24*A - 26*C) + a^4*b^2*(146*A - 17*C) - a^2*b^
4*(167*A - 6*C))*Sin[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) - ((10*A*b^6 - a^6*(
A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(27*A - C))*Cos[c + d*x]*Sin[c +
d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 + a^2*C)*Cos[c + d*x]*Sin[c + d*x])
/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*A*b^4 - 4*a^4*C - a^2*b^2
*(10*A + C))*Cos[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c
 + d*x])^2) + ((20*A*b^6 - a^2*b^4*(53*A - 2*C) + 12*a^6*C + a^4*b^2*(48*A
+ C))*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]
))
```


Rule 4101

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*
b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/
(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a^2*(A + C)*(m + 1) - (A*b^2
+ a^2*C)*(m + n + 1) - a*b*(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(
```

```
m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(5Ab^2-a^2(3A-2C)+3ab(A+C)\ldots\right.}{(a+b\sec(c+\ldots}}{3a\left(a^2-\ldots\right.}$$

$$= \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5Ab^4-4a^4C-a^2b^2(10A+C)\right)\mathrm{co}\ldots}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec\ldots}$$

$$= \frac{\left(Ab^2+a^2C\right)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5Ab^4-4a^4C-a^2b^2(10A+C)\right)\mathrm{co}\ldots}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec\ldots}$$

$$= -\frac{\left(10Ab^6-a^6(A-6C)+a^4b^2(23A-2C)-a^2b^4(27A-C)\right)\cos(c+dx)\,\mathrm{si}\ldots}{2a^4\left(a^2-b^2\right)^3 d}$$

$$= \frac{b\left(60Ab^6-a^6(24A-26C)+a^4b^2(146A-17C)-a^2b^4(167A-6C)\right)\sin(c\ldots}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(20Ab^2+a^2(A+2C)\right)x}{2a^6} + \frac{b\left(60Ab^6-a^6(24A-26C)+a^4b^2(146A-17C\ldots\right.}{6a^5\left(a^2-b^2\right)^3\ldots}$$

$$= \frac{\left(20Ab^2+a^2(A+2C)\right)x}{2a^6} + \frac{b\left(60Ab^6-a^6(24A-26C)+a^4b^2(146A-17C\ldots\right.}{6a^5\left(a^2-b^2\right)^3\ldots}$$

$$= \frac{\left(20Ab^2+a^2(A+2C)\right)x}{2a^6} + \frac{b\left(60Ab^6-a^6(24A-26C)+a^4b^2(146A-17C\ldots\right.}{6a^5\left(a^2-b^2\right)^3\ldots}$$

$$= \frac{\left(20Ab^2+a^2(A+2C)\right)x}{2a^6} + \frac{b\left(20Ab^8-a^2b^6(69A-2C)-8a^6b^2(5A-C)\ldots\right.}{a^6\sqrt{a-b}\sqrt{\ldots}}$$

Mathematica [B] time = 5.52352, size = 1314, normalized size = 2.56

$$\frac{12Ac\cos(3(c+dx))a^{11}+24cC\cos(3(c+dx))a^{11}+12Adx\cos(3(c+dx))a^{11}+24Cdx\cos(3(c+dx))a^{11}+6A\sin(c+dx)a^{11}+9A\sin(3(c+dx))a^{11}+3A\sin(5(c+dx))a^{11}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-96*b*(20*A*b^8 + 7*a^4*b^4*(12*A - C) - 8*a^8*C + 8*a^6*b^2*(-5*A + C) +
 a^2*b^6*(-69*A + 2*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a^2 - b^2)^(7/2) + (72*a^10*A*b*c + 1272*a^8*A*b^3*c - 3288*a^6*A*b^5*c
 + 1512*a^4*A*b^7*c + 1392*a^2*A*b^9*c - 960*A*b^11*c + 144*a^10*b*c*C - 33
6*a^8*b^3*c*C + 144*a^6*b^5*c*C + 144*a^4*b^7*c*C - 96*a^2*b^9*c*C + 72*a^1
0*A*b*d*x + 1272*a^8*A*b^3*d*x - 3288*a^6*A*b^5*d*x + 1512*a^4*A*b^7*d*x +
1392*a^2*A*b^9*d*x - 960*A*b^11*d*x + 144*a^10*b*C*d*x - 336*a^8*b^3*C*d*x
+ 144*a^6*b^5*C*d*x + 144*a^4*b^7*C*d*x - 96*a^2*b^9*C*d*x + 36*a*(a^2 - b^
2)^3*(a^2 + 4*b^2)*(20*A*b^2 + a^2*(A + 2*C))*(c + d*x)*Cos[c + d*x] + 72*a
^2*b*(a^2 - b^2)^3*(20*A*b^2 + a^2*(A + 2*C))*(c + d*x)*Cos[2*(c + d*x)] +
12*a^11*A*c*Cos[3*(c + d*x)] + 204*a^9*A*b^2*c*Cos[3*(c + d*x)] - 684*a^7*A
*b^4*c*Cos[3*(c + d*x)] + 708*a^5*A*b^6*c*Cos[3*(c + d*x)] - 240*a^3*A*b^8*
```

```
c*Cos[3*(c + d*x)] + 24*a^11*c*C*Cos[3*(c + d*x)] - 72*a^9*b^2*c*C*Cos[3*(c
 + d*x)] + 72*a^7*b^4*c*C*Cos[3*(c + d*x)] - 24*a^5*b^6*c*C*Cos[3*(c + d*x)
] + 12*a^11*A*d*x*Cos[3*(c + d*x)] + 204*a^9*A*b^2*d*x*Cos[3*(c + d*x)] - 6
84*a^7*A*b^4*d*x*Cos[3*(c + d*x)] + 708*a^5*A*b^6*d*x*Cos[3*(c + d*x)] - 24
0*a^3*A*b^8*d*x*Cos[3*(c + d*x)] + 24*a^11*C*d*x*Cos[3*(c + d*x)] - 72*a^9*
b^2*C*d*x*Cos[3*(c + d*x)] + 72*a^7*b^4*C*d*x*Cos[3*(c + d*x)] - 24*a^5*b^6
*C*d*x*Cos[3*(c + d*x)] + 6*a^11*A*Sin[c + d*x] - 270*a^9*A*b^2*Sin[c + d*x
] + 750*a^7*A*b^4*Sin[c + d*x] + 1086*a^5*A*b^6*Sin[c + d*x] - 2232*a^3*A*b
^8*Sin[c + d*x] + 960*a*A*b^10*Sin[c + d*x] + 144*a^9*b^2*C*Sin[c + d*x] +
288*a^7*b^4*C*Sin[c + d*x] - 228*a^5*b^6*C*Sin[c + d*x] + 96*a^3*b^8*C*Sin[
c + d*x] - 60*a^10*A*b*Sin[2*(c + d*x)] - 372*a^8*A*b^3*Sin[2*(c + d*x)] +
2772*a^6*A*b^5*Sin[2*(c + d*x)] - 3300*a^4*A*b^7*Sin[2*(c + d*x)] + 1200*a^
2*A*b^9*Sin[2*(c + d*x)] + 480*a^8*b^3*C*Sin[2*(c + d*x)] - 360*a^6*b^5*C*S
in[2*(c + d*x)] + 120*a^4*b^7*C*Sin[2*(c + d*x)] + 9*a^11*A*Sin[3*(c + d*x)
] - 279*a^9*A*b^2*Sin[3*(c + d*x)] + 1143*a^7*A*b^4*Sin[3*(c + d*x)] - 1253
*a^5*A*b^6*Sin[3*(c + d*x)] + 440*a^3*A*b^8*Sin[3*(c + d*x)] + 144*a^9*b^2*
C*Sin[3*(c + d*x)] - 128*a^7*b^4*C*Sin[3*(c + d*x)] + 44*a^5*b^6*C*Sin[3*(c
 + d*x)] - 30*a^10*A*b*Sin[4*(c + d*x)] + 90*a^8*A*b^3*Sin[4*(c + d*x)] - 9
0*a^6*A*b^5*Sin[4*(c + d*x)] + 30*a^4*A*b^7*Sin[4*(c + d*x)] + 3*a^11*A*Sin
[5*(c + d*x)] - 9*a^9*A*b^2*Sin[5*(c + d*x)] + 9*a^7*A*b^4*Sin[5*(c + d*x)]
 - 3*a^5*A*b^6*Sin[5*(c + d*x)])/((a^2 - b^2)^3*(b + a*Cos[c + d*x])^3))/(9
6*a^6*d)
```

Maple [B] time = 0.161, size = 3023, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -44/3/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+4/d*b^6/a^3/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*C+6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+1/d*b^5/a^2/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*A+6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+24/d*b^8/a^5/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*A-2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-1/d*b^5/a^2
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
C-12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/
(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-4/d/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*C*b^3-1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+1
/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A-3/d/a^4/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)*A+3/d/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-12/d*
a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2*C+60/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(
```

```
1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*A*b^4+34/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6-30/d/a/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5*A*b^4-6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5+34/d
/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^6-212/3/d/a^3/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*A*b^6+24/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C-12/d*a/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)*b^2*C+6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5-
30/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^4+84/d/a^2*b^5/(a^6-3*a^4*b^2+3*a^
2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A-69/d/a^4*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+4/d/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*C*b^3-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+1/d*A/a^4*arc
tan(tan(1/2*d*x+1/2*c))+2/d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+20/d*b^9
/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*C+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*C-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-
b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))*A-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b-8/d/a^5/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b+8/d*b^3/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*C+20/d/a^6*arctan(tan(1/2*d*x+1/2*c))*A*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.44403, size = 5536, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [1/12*(6*((A + 2*C)*a^13 + 8*(2*A - C)*a^11*b^2 - 2*(37*A - 6*C)*a^9*b^4 +
4*(29*A - 2*C)*a^7*b^6 - (79*A - 2*C)*a^5*b^8 + 20*A*a^3*b^10)*d*x*cos(d*x
+ c)^3 + 18*((A + 2*C)*a^12*b + 8*(2*A - C)*a^10*b^3 - 2*(37*A - 6*C)*a^8*b
^5 + 4*(29*A - 2*C)*a^6*b^7 - (79*A - 2*C)*a^4*b^9 + 20*A*a^2*b^11)*d*x*cos
(d*x + c)^2 + 18*((A + 2*C)*a^11*b^2 + 8*(2*A - C)*a^9*b^4 - 2*(37*A - 6*C)
*a^7*b^6 + 4*(29*A - 2*C)*a^5*b^8 - (79*A - 2*C)*a^3*b^10 + 20*A*a*b^12)*d*
x*cos(d*x + c) + 6*((A + 2*C)*a^10*b^3 + 8*(2*A - C)*a^8*b^5 - 2*(37*A - 6*
C)*a^6*b^7 + 4*(29*A - 2*C)*a^4*b^9 - (79*A - 2*C)*a^2*b^11 + 20*A*b^13)*d*
x + 3*(8*C*a^8*b^4 + 8*(5*A - C)*a^6*b^6 - 7*(12*A - C)*a^4*b^8 + (69*A - 2
*C)*a^2*b^10 - 20*A*b^12 + (8*C*a^11*b + 8*(5*A - C)*a^9*b^3 - 7*(12*A - C)
*a^7*b^5 + (69*A - 2*C)*a^5*b^7 - 20*A*a^3*b^9)*cos(d*x + c)^3 + 3*(8*C*a^1
0*b^2 + 8*(5*A - C)*a^8*b^4 - 7*(12*A - C)*a^6*b^6 + (69*A - 2*C)*a^4*b^8 -
 20*A*a^2*b^10)*cos(d*x + c)^2 + 3*(8*C*a^9*b^3 + 8*(5*A - C)*a^7*b^5 - 7*(
12*A - C)*a^5*b^7 + (69*A - 2*C)*a^3*b^9 - 20*A*a*b^11)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*(12*A - 13*C)*a^9*b^4 - (170*A - 4
3*C)*a^7*b^6 + (313*A - 23*C)*a^5*b^8 - (227*A - 6*C)*a^3*b^10 + 60*A*a*b^1
2 - 3*(A*a^13 - 4*A*a^11*b^2 + 6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*cos(d
*x + c)^4 + 15*(A*a^12*b - 4*A*a^10*b^3 + 6*A*a^8*b^5 - 4*A*a^6*b^7 + A*a^4
*b^9)*cos(d*x + c)^3 + (9*(7*A - 4*C)*a^11*b^2 - 2*(171*A - 34*C)*a^9*b^4 +
 (590*A - 43*C)*a^7*b^6 - (421*A - 11*C)*a^5*b^8 + 110*A*a^3*b^10)*cos(d*x
+ c)^2 + 3*((23*A - 20*C)*a^10*b^3 - (146*A - 35*C)*a^8*b^5 + (263*A - 20*C
)*a^6*b^7 - 5*(38*A - C)*a^4*b^9 + 50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c
))/((a^17 - 4*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^
3 + 3*(a^16*b - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x +
 c)^2 + 3*(a^15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos
(d*x + c) + (a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d),
 1/6*(3*((A + 2*C)*a^13 + 8*(2*A - C)*a^11*b^2 - 2*(37*A - 6*C)*a^9*b^4 + 4
*(29*A - 2*C)*a^7*b^6 - (79*A - 2*C)*a^5*b^8 + 20*A*a^3*b^10)*d*x*cos(d*x +
 c)^3 + 9*((A + 2*C)*a^12*b + 8*(2*A - C)*a^10*b^3 - 2*(37*A - 6*C)*a^8*b^5
 + 4*(29*A - 2*C)*a^6*b^7 - (79*A - 2*C)*a^4*b^9 + 20*A*a^2*b^11)*d*x*cos(d
*x + c)^2 + 9*((A + 2*C)*a^11*b^2 + 8*(2*A - C)*a^9*b^4 - 2*(37*A - 6*C)*a^
7*b^6 + 4*(29*A - 2*C)*a^5*b^8 - (79*A - 2*C)*a^3*b^10 + 20*A*a*b^12)*d*x*c
os(d*x + c) + 3*((A + 2*C)*a^10*b^3 + 8*(2*A - C)*a^8*b^5 - 2*(37*A - 6*C)*
a^6*b^7 + 4*(29*A - 2*C)*a^4*b^9 - (79*A - 2*C)*a^2*b^11 + 20*A*b^13)*d*x -
 3*(8*C*a^8*b^4 + 8*(5*A - C)*a^6*b^6 - 7*(12*A - C)*a^4*b^8 + (69*A - 2*C)
*a^2*b^10 - 20*A*b^12 + (8*C*a^11*b + 8*(5*A - C)*a^9*b^3 - 7*(12*A - C)*a^
7*b^5 + (69*A - 2*C)*a^5*b^7 - 20*A*a^3*b^9)*cos(d*x + c)^3 + 3*(8*C*a^10*b
^2 + 8*(5*A - C)*a^8*b^4 - 7*(12*A - C)*a^6*b^6 + (69*A - 2*C)*a^4*b^8 - 20
*A*a^2*b^10)*cos(d*x + c)^2 + 3*(8*C*a^9*b^3 + 8*(5*A - C)*a^7*b^5 - 7*(12*
A - C)*a^5*b^7 + (69*A - 2*C)*a^3*b^9 - 20*A*a*b^11)*cos(d*x + c))*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
 + c))) - (2*(12*A - 13*C)*a^9*b^4 - (170*A - 43*C)*a^7*b^6 + (313*A - 23*C
)*a^5*b^8 - (227*A - 6*C)*a^3*b^10 + 60*A*a*b^12 - 3*(A*a^13 - 4*A*a^11*b^2
 + 6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x + c)^4 + 15*(A*a^12*b - 4
*A*a^10*b^3 + 6*A*a^8*b^5 - 4*A*a^6*b^7 + A*a^4*b^9)*cos(d*x + c)^3 + (9*(7
*A - 4*C)*a^11*b^2 - 2*(171*A - 34*C)*a^9*b^4 + (590*A - 43*C)*a^7*b^6 - (4
21*A - 11*C)*a^5*b^8 + 110*A*a^3*b^10)*cos(d*x + c)^2 + 3*((23*A - 20*C)*a^
10*b^3 - (146*A - 35*C)*a^8*b^5 + (263*A - 20*C)*a^6*b^7 - 5*(38*A - C)*a^4
*b^9 + 50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a
^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 +
 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13
*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^
12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35943, size = 1392, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] -1/6*(6*(8*C*a^8*b + 40*A*a^6*b^3 - 8*C*a^6*b^3 - 84*A*a^4*b^5 + 7*C*a^4*b^
5 + 69*A*a^2*b^7 - 2*C*a^2*b^7 - 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(-a^2
+ b^2)) + 2*(36*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^7*b^3*tan(1/2*d*x
 + 1/2*c)^5 + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^6*b^4*tan(1/2*d*x
 + 1/2*c)^5 - 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^5*tan(1/2*d
*x + 1/2*c)^5 - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^4*b^6*tan(1/2*d
*x + 1/2*c)^5 + 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^3*b^7*tan(1/2
*d*x + 1/2*c)^5 - 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*b^8*tan(1/2
*d*x + 1/2*c)^5 - 81*A*a*b^9*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^10*tan(1/2*d*x
 + 1/2*c)^5 - 72*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 - 180*A*a^6*b^4*tan(1/2*d
*x + 1/2*c)^3 + 116*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 392*A*a^4*b^6*tan(1/
2*d*x + 1/2*c)^3 - 56*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 284*A*a^2*b^8*tan(
1/2*d*x + 1/2*c)^3 + 12*C*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^10*tan(1/
2*d*x + 1/2*c)^3 + 36*C*a^8*b^2*tan(1/2*d*x + 1/2*c) + 60*C*a^7*b^3*tan(1/2
*d*x + 1/2*c) + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^4*tan(1/2*d*x
 + 1/2*c) + 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c) - 45*C*a^5*b^5*tan(1/2*d*x +
 1/2*c) - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^6*tan(1/2*d*x + 1/2
*c) - 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c) + 15*C*a^3*b^7*tan(1/2*d*x + 1/2*c
) - 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c) + 6*C*a^2*b^8*tan(1/2*d*x + 1/2*c) +
81*A*a*b^9*tan(1/2*d*x + 1/2*c) + 36*A*b^10*tan(1/2*d*x + 1/2*c))/((a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
+ 1/2*c)^2 - a - b)^3) - 3*(A*a^2 + 2*C*a^2 + 20*A*b^2)*(d*x + c)/a^6 + 6*(
A*a*tan(1/2*d*x + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x
 + 1/2*c) + 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5
))/d
```

3.705 $\int \frac{a^2-b^2\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*x - (b*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0442866, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4042, 3770}

$$ax-\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2-b^2\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int (-a+b\sec(c+dx))\,dx\\ &= ax-b\int \sec(c+dx)\,dx\\ &= ax-\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0066265, size = 17, normalized size = 1.

$$ax-\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (b*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.043, size = 31, normalized size = 1.8

$$ax - \frac{b \ln (\sec (dx + c) + \tan (dx + c))}{d} + \frac{ac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] a*x-1/d*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.497741, size = 95, normalized size = 5.59

$$\frac{2 \, adx - b \log (\sin (dx + c) + 1) + b \log (-\sin (dx + c) + 1)}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d
```

Sympy [A] time = 3.25023, size = 41, normalized size = 2.41

$$ax - b \left(\begin{cases} \frac{x\left(\tan (c) \sec (c) + \sec^2 (c)\right)}{\tan (c) + \sec (c)} & \text{for}\ d = 0 \\ \frac{\log (\tan (c + dx) + \sec (c + dx))}{d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] a*x - b*Piecewise((x*(tan(c)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), Eq(d, 0
)), (log(tan(c + d*x) + sec(c + d*x))/d, True))
```

Giac [B] time = 1.19349, size = 58, normalized size = 3.41

$$\frac{(dx+c)a - b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a - b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + b*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

3.706 $\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$x-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] x - (4*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*d)
```

Rubi [A] time = 0.130744, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4042, 3919, 3831, 2659, 208}

$$x-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x - (4*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*d)
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\int \frac{-a+b\sec(c+dx)}{a+b\sec(c+dx)}\,dx\\ &= x-(2b)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx\\ &= x-2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx\\ &= x-\frac{4\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\ &= x-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0994043, size = 56, normalized size = 1.08

$$\frac{4b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{c}{d}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] c/d + x + (4*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[
a^2 - b^2]*d)
```

Maple [A] time = 0.079, size = 61, normalized size = 1.2

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{d}-4\,\frac{b}{d\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*arctan(tan(1/2*d*x+1/2*c))-4/d*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.527682, size = 491, normalized size = 9.44

$$\left[\frac{\left(a^2-b^2\right)dx+\sqrt{a^2-b^2}\,b\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{\left(a^2-b^2\right)d},\ \frac{\left(a^2-b^2\right)dx-2\,\sqrt{-a^2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [((a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^2 - b^2)*
d), ((a^2 - b^2)*d*x - 2*sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cos
(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^2 - b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a-b\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a - b*sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.18053, size = 113, normalized size = 2.17

$$\frac{dx+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}}+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (d*x + 4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/sqrt(-a^2 +
b^2) + c)/d
```

3.707 $\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=107

$$-\frac{2b\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{3/2}(a+b)^{3/2}}+\frac{2b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x}{a}$$

```
[Out] x/a - (2*b*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d) + (2*b^2*Tan[c + d*x])/((a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.205855, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4042, 3923, 3919, 3831, 2659, 208}

$$-\frac{2b\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{3/2}(a+b)^{3/2}}+\frac{2b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] x/a - (2*b*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d) + (2*b^2*Tan[c + d*x])/((a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{-a+b\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx\\
&= \frac{2b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{\int \frac{a\left(a^2-b^2\right)-2a^2b\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{x}{a}+\frac{2b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(b\left(3a^2-b^2\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{x}{a}+\frac{2b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(3a^2-b^2\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{x}{a}+\frac{2b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(2\left(3a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c
}{a\left(a^2-b^2\right)d}\\
&= \frac{x}{a}-\frac{2b\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}d}+\frac{2b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.460015, size = 139, normalized size = 1.3

$$
\frac{\frac{b\left(\left(a^2-b^2\right)(c+dx)+2ab\sin(c+dx)\right)+a\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}-\frac{2b\left(b^2-3a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{ad(a-b)(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*b*(-3*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + b*((a^2 - b^2)*(
c + d*x) + 2*a*b*Sin[c + d*x]))/(b + a*Cos[c + d*x]))/(a*(a - b)*(a + b)*d)
```

Maple [B] time = 0.098, size = 202, normalized size = 1.9

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{ad}-4\,\frac{b^{2}\tan\left(1/2\,dx+c/2\right)}{d\left(a^{2}-b^{2}\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)}-6\,\frac{a}{d\left(a+b\right)\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a*arctan(tan(1/2*d*x+1/2*c))-4/d*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-6/d*b*a/(a+b)/(a-b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b^3/a
/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.616108, size = 1050, normalized size = 9.81

$$\left[\frac{2\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)dx\cos\left(dx+c\right)+2\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)dx+\left(3\,a^{2}b^{2}-b^{4}+\left(3\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\log\left(\right.}{2\left(\left(a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}\right)d\cos\left(dx+c\right)+\left(a^{5}b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 +
 b^5)*d*x + (3*a^2*b^2 - b^4 + (3*a^3*b - a*b^3)*cos(d*x + c))*sqrt(a^2 - b
^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b
^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + b^2)) + 4*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^6 - 2*a^
4*b^2 + a^2*b^4)*d*cos(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d), ((a^5 - 2
*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x - (3*a^2
*b^2 - b^4 + (3*a^3*b - a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(
-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + 2*(a^3*b^2 -
 a*b^4)*sin(d*x + c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*d*cos(d*x + c) + (a^5*b
- 2*a^3*b^3 + a*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a-b\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a - b*sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.26168, size = 236, normalized size = 2.21

$$-\frac{\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)\left(a^2-b^2\right)}-\frac{2\left(3\,a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^3-ab^2\right)\sqrt{-a^2+b^2}}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
/2*c)^2 - a - b)*(a^2 - b^2)) - 2*(3*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(-a^2 + b^2)))/((a^3 - a*b^2)*sqrt(-a^2 + b^2)) - (d*x + c)/a)/
d
```

3.708 $\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=162

$$-\frac{2b\left(-2a^2b^2+4a^4+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(4a^2-b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

```
[Out] x/a^2 - (2*b*(4*a^4 - 2*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tan[c + d*x])/(
(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(4*a^2 - b^2)*Tan[c + d*x])/(a
*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.339561, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {4042, 3923, 4060, 3919, 3831, 2659, 208}

$$-\frac{2b\left(-2a^2b^2+4a^4+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(4a^2-b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] x/a^2 - (2*b*(4*a^4 - 2*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tan[c + d*x])/(
(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(4*a^2 - b^2)*Tan[c + d*x])/(a
*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
```

```
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx=-\int \frac{-a+b\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$=\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{\int \frac{2a\left(a^2-b^2\right)-4a^2b\sec(c+dx)+2ab^2\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$=\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{b^2\left(4a^2-b^2\right)\tan(c+dx)}{a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}-\frac{\int \frac{-2a\left(a^2-b^2\right)^2+6a^4b\sec(}{a+b\sec(c+dx)}}{2a^2\left(a^2-b^2\right)}$$

$$=\frac{x}{a^2}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{b^2\left(4a^2-b^2\right)\tan(c+dx)}{a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}-\frac{\left(b\left(4a^4-2a^2b\right.\right.}{a}$$

$$=\frac{x}{a^2}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{b^2\left(4a^2-b^2\right)\tan(c+dx)}{a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}-\frac{\left(4a^4-2a^2b^2+\right.}{a^2(}$$

$$=\frac{x}{a^2}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{b^2\left(4a^2-b^2\right)\tan(c+dx)}{a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}-\frac{\left(2\left(4a^4-2a^2b\right.\right.}{}$$

$$=\frac{x}{a^2}-\frac{2b\left(4a^4-2a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+$$

Mathematica [A] time = 0.775569, size = 223, normalized size = 1.38

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)(a-b\sec(c+dx))\left(\frac{ab^2\left(5a^2-2b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2}+\frac{2b\left(-2a^2b^2+4a^4+b^4\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\cdots\right)}{\left(a^2-b^2\right)^{5/2}}\cdots\right.}{a^2d(a\cos(c+dx)-b)(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(a - b*Sec[c + d*x])*((c + d*x)*(b + a
*Cos[c + d*x])^2 + (2*b*(4*a^4 - 2*a^2*b^2 + b^4)*ArcTanh[((-a + b)*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*
b^3*Sin[c + d*x])/((-a + b)*(a + b)) + (a*b^2*(5*a^2 - 2*b^2)*(b + a*Cos[c
+ d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2)))/(a^2*d*(-b + a*Cos[c + d*x])*
(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.102, size = 659, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-10/d*b^2*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-2/d*b^3
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3+2/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+10/d*b^2*a/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*
d*x+1/2*c)-2/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a
+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-8/d*b*
a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))+4/d*b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d*b^5/a^2/(a^4-2*a^2*b^
2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.684094, size = 1925, normalized size = 11.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 + 4*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 2*(a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*x + (4*a^4*b^3 - 2*a^2*b^5 + b^7 + (4*a^6*b - 2*a^4
*b^3 + a^2*b^5)*cos(d*x + c)^2 + 2*(4*a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(4*a^5*b^3 - 5*a^3*b^5 + a
*b^7 + (5*a^6*b^2 - 7*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b
^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6
 - a^2*b^8)*d), ((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2
 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 -
3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x - (4*a^4*b^3 - 2*a^2*b^5 + b^7 + (4*a^6*b
- 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 + 2*(4*a^5*b^2 - 2*a^3*b^4 + a*b^6)*c
os(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c))) + (4*a^5*b^3 - 5*a^3*b^5 + a*b^7 + (5*a^6*b^2
- 7*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 3*a^8*b^2 + 3
*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a
^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - b\sec(c + dx)}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a - b*sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.30622, size = 428, normalized size = 2.64

$$\frac{\frac{2\left(4a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{-a^2+b^2}}+\frac{dx+c}{a^2}-\frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a\cdots\right.}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (2*(4*a^4*b - 2*a^2*b^3 + b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) + (d*x + c)/a^2 - 2*
```

```
(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a*
b^4*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2
*d*x + 1/2*c) - 4*a^2*b^3*tan(1/2*d*x + 1/2*c) + 2*a*b^4*tan(1/2*d*x + 1/2*
c) + b^5*tan(1/2*d*x + 1/2*c))/((a^5 - 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.709 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=467

$$\frac{2(a-b)\sqrt{a+b}\left(12a^2bC+16a^3C+6ab^2(7A+6C)+21b^3(9A+7C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellipt}}{315b^4d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*
C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(315*b^5*d) + (2*(a - b)*Sqrt[a + b]*(16*a^3*C + 12*a^2*b
*C + 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) +
(2*a*(21*A*b^2 + 8*a^2*C + 13*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(315*b^3*d) - (2*(6*a^2*C - 7*b^2*(9*A + 7*C))*Sec[c + d*x]*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*a*C*Sec[c + d*x]^2*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(63*b*d) + (2*C*Sec[c + d*x]^3*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(9*d)
```

Rubi [A] time = 1.30483, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4097, 4102, 4092, 4082, 4005, 3832, 4004}

$$-\frac{2\left(6a^2C-7b^2(9A+7C)\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}+\frac{2a\left(8a^2C+21Ab^2+13b^2C\right)\tan(c+dx)\sqrt{a}}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*
C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(315*b^5*d) + (2*(a - b)*Sqrt[a + b]*(16*a^3*C + 12*a^2*b
*C + 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) +
(2*a*(21*A*b^2 + 8*a^2*C + 13*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(315*b^3*d) - (2*(6*a^2*C - 7*b^2*(9*A + 7*C))*Sec[c + d*x]*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*a*C*Sec[c + d*x]^2*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(63*b*d) + (2*C*Sec[c + d*x]^3*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(9*d)
```


Rule 4097

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*
Simp[a*A*(m + n + 1) + a*C*n + b*(A*(m + n + 1) + C*(m + n))*Csc[e + f*x] +
 a*C*m*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&
NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{9d} + \frac{2}{9}\int$$

$$= \frac{2aC\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{63bd} + \frac{2C\,\mathrm{s}}{}$$

$$= -\frac{2\left(6a^2C-7b^2(9A+7C)\right)\sec(c+dx)\sqrt{a+b\sec(c+d}}{315b^2d}$$

$$= \frac{2a\left(21Ab^2+8a^2C+13b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+}{315b^3d}$$

$$= \frac{2a\left(21Ab^2+8a^2C+13b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+}{315b^3d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(16a^4C+6a^2b^2(7A+4C)-21b^4(9A+\right.}{}$$

Mathematica [B] time = 23.7268, size = 3518, normalized size = 7.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((4*(-42*a^
2*A*b^2 + 189*A*b^4 - 16*a^4*C - 24*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(3
15*b^4) + (4*Sec[c + d*x]^2*(63*A*b^2*Sin[c + d*x] - 6*a^2*C*Sin[c + d*x] +
 49*b^2*C*Sin[c + d*x]))/(315*b^2) + (4*Sec[c + d*x]*(21*a*A*b^2*Sin[c + d*
x] + 8*a^3*C*Sin[c + d*x] + 13*a*b^2*C*Sin[c + d*x]))/(315*b^3) + (4*a*C*Se
c[c + d*x]^2*Tan[c + d*x])/(63*b) + (4*C*Sec[c + d*x]^3*Tan[c + d*x])/9))/(
d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (4*((4*a^2*A)/(15*b*Sqrt[b + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]) - (6*A*b)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) + (32*a^4*C)/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
 + (16*a^2*C)/(105*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (14*b*C
)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*A*Sqrt[Sec[c + d*
x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*A*Sqrt[Sec[c + d*x]])/(15*b^2*S
qrt[b + a*Cos[c + d*x]]) - (8*a*C*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c
+ d*x]]) + (32*a^5*C*Sqrt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]])
 + (8*a^3*C*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (6*a*A*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*
A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos[c + d*x]]) -
(14*a*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]])
+ (32*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c
+ d*x]]) + (16*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b +
 a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]*(A + C*Sec[c + d*x]^2)*((a + b)*((16*a^4*C + 6*a^2*b^2*(7*A + 4*C) -
 21*b^4*(9*A + 7*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 b*(-16*a^3*C + 12*a^2*b*C - 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[
c + d*x] + (16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*b^4*d*
(b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3
```

```
/2)*Sec[c + d*x]^(5/2)*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c +
d*x]*((a + b)*((16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-16*a^3*C + 12*a^2*b*
C - 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (16*a^4*C + 6*
a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*b^4*(b + a*Cos[c + d*x])^(3/2)*
(Sec[(c + d*x)/2]^2)^(3/2)) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[
(c + d*x)/2]*((a + b)*((16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*
C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-16*a^3*C + 1
2*a^2*b*C - 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (16*a^
4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x]*(b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(105*b^4*Sqrt[b + a*Cos[c +
d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*((a + b)*((16*a^4*C + 6*a^2*b^2*(7*A
 + 4*C) - 21*b^4*(9*A + 7*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] + b*(-16*a^3*C + 12*a^2*b*C - 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7
*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Se
c[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b)]*Sec[c + d*x] + (16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(
Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c
 + d*x]*Tan[c + d*x]))/(315*b^4*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2
)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) + (4*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*(((16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - a*(16*a^4*C + 6*a^2
*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[
c + d*x]*Tan[(c + d*x)/2] - (16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A
 + 7*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)
/2] + 2*(16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 + (3*(a + b)*(
(16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)] + b*(-16*a^3*C + 12*a^2*b*C - 6*a*b^2*(7*
A + 6*C) + 21*b^3*(9*A + 7*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x
]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + ((a + b)*((16*a
^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + b*(-16*a^3*C + 12*a^2*b*C - 6*a*b^2*(7*A + 6
*C) + 21*b^3*(9*A + 7*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((a*Sec[(c + d*
x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)]) + (a + b)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*((b*(-16*a^3*C + 12
*a^2*b*C - 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*Sec[(c + d*x)/2]^2)/(2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
]) + ((16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sec[(c + d*x)
/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - Tan[(c +
d*x)/2]^2])) + (a + b)*((16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7
*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-16*a^3*C +
12*a^2*b*C - 6*a*b^2*(7*A + 6*C) + 21*b^3*(9*A + 7*C))*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*S
qrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c +
 d*x]))/(315*b^4*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 1.671, size = 4131, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/315/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(4*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-189*A*cos(d*x+c)^5*b^5+105*A*cos(d*x+c)^
5*a*b^4+16*C*cos(d*x+c)^5*a^4*b-26*C*cos(d*x+c)^5*a^3*b^2+24*C*cos(d*x+c)^5
*a^2*b^3+85*C*cos(d*x+c)^5*a*b^4-21*A*cos(d*x+c)^4*a^2*b^3-8*C*cos(d*x+c)^4
*a^4*b-10*C*cos(d*x+c)^4*a^2*b^3+84*A*cos(d*x+c)^3*a*b^4+2*C*cos(d*x+c)^3*a
^3*b^2+22*C*cos(d*x+c)^3*a*b^4+42*A*cos(d*x+c)^6*a^3*b^2-21*A*cos(d*x+c)^6*
a^2*b^3-189*A*cos(d*x+c)^6*a*b^4-8*C*cos(d*x+c)^6*a^4*b+24*C*cos(d*x+c)^6*a
^3*b^2-13*C*cos(d*x+c)^6*a^2*b^3-147*C*cos(d*x+c)^6*a*b^4-C*cos(d*x+c)^2*a^
2*b^3+40*C*cos(d*x+c)*a*b^4-42*A*cos(d*x+c)^5*a^3*b^2+42*A*cos(d*x+c)^5*a^2
*b^3-16*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-189*A*cos(d*x+c)^5*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5
+189*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^5-147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-16*C*cos(d*x+c)^5*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+147
*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^5-189*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+189*A*cos(d*x+c)^4*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-147*C*
cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^5-16*C*cos(d*x+c)^5*a^5-147*C*cos(d*x+c)^5*b^5+126*A*cos(d*
x+c)^4*b^5+98*C*cos(d*x+c)^4*b^5+63*A*cos(d*x+c)^2*b^5+14*C*cos(d*x+c)^2*b^
5+16*C*cos(d*x+c)^6*a^5+24*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-111*C*cos(d*x+c)^5*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
4-16*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^4*b-24*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-24*C*cos(d*x+c)^5*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b^3+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^4+42*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(c
```

```
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*A*cos(d*x+c)
^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b^4-42*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-42*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+189*A*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^4+16*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+4*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+24*C*cos(
d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*b^3-111*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-16*C*cos(d*x+c)^4*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-24*C*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b^2-24*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+147*C*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+4
2*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b^3-147*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-42*A*cos(d*x+c)^5*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
b^2-42*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b^3+189*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+16*C*cos(d*x+c)^5*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^4*b+35*C*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec(dx+c)^5 + A\sec(dx+c)^3\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2(c+dx)\right)\sqrt{a + b\sec(c+dx)}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3,
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)\sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x
)
```

3.710 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=375

$$-\frac{2(a-b)\sqrt{a+b}\left(C\left(8a^2+6ab+25b^2\right)+35Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{105b^3d}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(10
5*b^4*d) - (2*(a - b)*Sqrt[a + b]*(35*A*b^2 + (8*a^2 + 6*a*b + 25*b^2)*C)*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^3*d) + (2*(8*a^2*C + 5*b^2*(7*A + 5*C))*Sqrt[a + b*Sec[
c + d*x]]*Tan[c + d*x])/(105*b^2*d) - (8*a*C*(a + b*Sec[c + d*x])^(3/2)*Tan
[c + d*x])/(35*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(7*b*d)
```

Rubi [A] time = 0.772456, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4093, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2C+5b^2(7A+5C)\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^2d}-\frac{2(a-b)\sqrt{a+b}\left(C\left(8a^2+6ab+25b^2\right)+35Ab^2\right)\cot(c+dx)\sqrt{}}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(10
5*b^4*d) - (2*(a - b)*Sqrt[a + b]*(35*A*b^2 + (8*a^2 + 6*a*b + 25*b^2)*C)*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^3*d) + (2*(8*a^2*C + 5*b^2*(7*A + 5*C))*Sqrt[a + b*Sec[
c + d*x]]*Tan[c + d*x])/(105*b^2*d) - (8*a*C*(a + b*Sec[c + d*x])^(3/2)*Tan
[c + d*x])/(35*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(7*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
```

```
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7bd} + \frac{2\int \mathrm{s}}{}$$

$$= -\frac{8aC(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35b^2d} + \frac{2C\sec(c+dx}{}$$

$$= \frac{2\left(8a^2C+5b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx}{105b^2d}$$

$$= \frac{2\left(8a^2C+5b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx}{105b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\left(35Ab^2+8a^2C+19b^2C\right)\cot(c+dx)}{1}$$

Mathematica [A] time = 19.9581, size = 560, normalized size = 1.49

$$\frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(2b(a+b)\left(C\left(8a^2-6ab+25b^2\right)+35Ab^2\right)\sqrt{\frac{c}{cc}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + C*Se
c[c + d*x]^2)*(-2*a*(a + b)*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(35*
A*b^2 + (8*a^2 - 6*a*b + 25*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] - a*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Cos[c
+ d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*
d*(b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]
^2]*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*S
ec[c + d*x]^2)*((4*a*(35*A*b^2 + 8*a^2*C + 19*b^2*C)*Sin[c + d*x])/(105*b^3
) + (4*Sec[c + d*x]*(35*A*b^2*Sin[c + d*x] - 4*a^2*C*Sin[c + d*x] + 25*b^2*
C*Sin[c + d*x]))/(105*b^2) + (4*a*C*Sec[c + d*x]*Tan[c + d*x])/(35*b) + (4*
C*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 1., size = 2784, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(35*A*cos(d*x+c)^2*b^4+35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-8*C*cos(d*x+c)^5*
a^4+8*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^4-25*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-35*A*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+8*C
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^4-25*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+70*A*cos(d*x+c)^3*a*b^3+35*A*cos(
d*x+c)^4*a^2*b^2+35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-35*A*cos(d*x+c)^4*a*b^3-8*C*cos(d
*x+c)^4*a^3*b+20*C*cos(d*x+c)^4*a^2*b^2-19*C*cos(d*x+c)^4*a*b^3+4*C*cos(d*x
+c)^3*a^3*b+26*C*cos(d*x+c)^3*a*b^3-C*cos(d*x+c)^2*a^2*b^2+18*C*cos(d*x+c)*
a*b^3-35*A*cos(d*x+c)^5*a^2*b^2-35*A*cos(d*x+c)^5*a*b^3+4*C*cos(d*x+c)^5*a^
```

```
3*b-19*C*cos(d*x+c)^5*a^2*b^2-25*C*cos(d*x+c)^5*a*b^3-35*A*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
4-35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^3+8*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+19*C*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
^2+19*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^3-8*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-2*C*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
^2-19*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^3+35*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+35*A*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^3-35*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^3+8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+19*C*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b^2+19*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^3-8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-2*C*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b^2-19*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^3+8*C*cos(d*x+c)^4*a^4+10*C*cos(d*x+c)^2*b^4-3
5*A*cos(d*x+c)^4*b^4-25*C*cos(d*x+c)^4*b^4+15*C*b^4)/(b+a*cos(d*x+c))/cos(d
*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^4+A\sec\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(A+C\sec^2\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2,
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x
)
```

3.711 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=308

$$\frac{2(a-b)\sqrt{a+b}(2aC+15Ab+9bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^2d}+$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a^2*C - 3*b^2*(5*A + 3*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*
d) + (2*(a - b)*Sqrt[a + b]*(15*A*b + 2*a*C + 9*b*C)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) - (4*a*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (2*C*(a + b*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.505171, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4083, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\left(2a^2C-3b^2(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a^2*C - 3*b^2*(5*A + 3*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*
d) + (2*(a - b)*Sqrt[a + b]*(15*A*b + 2*a*C + 9*b*C)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) - (4*a*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (2*C*(a + b*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```

Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \sec(c+dx)\sqrt{a+}}{}$$

$$= -\frac{4aC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2C(a+b\sec(c+dx}{5bd}$$

$$= -\frac{4aC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2C(a+b\sec(c+dx}{5bd}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(2a^2C-3b^2(5A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\right.}{15b^3d}$$

Mathematica [A] time = 18.0112, size = 507, normalized size = 1.65

$$\frac{4\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*(A + C*Sec[c + d*x]^2)*((a + b)*((-15*A*b^2 + 2*a^2*C - 9*b^2*C)*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(15*A*b - 2*a*C + 9*b*C)
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
*Sec[c + d*x] - (15*A*b^2 - 2*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c +
```

```
d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(15*b^2*d*Sqrt[(1 + Cos[c + d*x
])^(-1)]*(b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)
/2]^2)^(3/2)*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]
*(A + C*Sec[c + d*x]^2)*((4*(15*A*b^2 - 2*a^2*C + 9*b^2*C)*Sin[c + d*x])/(1
5*b^2) + (4*a*C*Tan[c + d*x])/(15*b) + (4*C*Sec[c + d*x]*Tan[c + d*x])/5))/
(d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.754, size = 2453, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(15*A*cos(d*x+c)^3*b^3+2*C*cos(d*x+c)^3*a^3-15*A*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
2-15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*C*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*C*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3+9*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-15*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+15*A*cos(d*x+
c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2+2*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*C*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*b+7*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^2-15*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+15*A*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^2+2*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*C*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b+7*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
```

```
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2+15*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*C*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9
*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^3+9*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*C*cos(d*x+c)^3*b^3-15*A*cos(d*
x+c)^2*b^3-6*C*cos(d*x+c)^2*b^3-2*C*cos(d*x+c)^4*a^3+15*A*cos(d*x+c)^4*a*b^
2+C*cos(d*x+c)^4*a^2*b+9*C*cos(d*x+c)^4*a*b^2-15*A*cos(d*x+c)^3*a*b^2-2*C*c
os(d*x+c)^3*a^2*b-5*C*cos(d*x+c)^3*a*b^2+C*cos(d*x+c)^2*a^2*b-4*C*cos(d*x+c
)*a*b^2-3*C*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+C\sec^{2}\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.712 $\int \sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=355

$$\frac{2\sqrt{a+b}(3Ab-C(a-b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2A\sqrt{a+b}\,\text{co}}{}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a + b]*(3*A*b
 - (a - b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellipti
cPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/d + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.36746, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4057, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}(3Ab-C(a-b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{2A\sqrt{a+b}\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a + b]*(3*A*b
 - (a - b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellipti
cPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/d + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{1}{2}b(3A+C)\sec(c+}{\sqrt{a+b\sec}}$$

$$= \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\left(-\frac{aC}{2}+\frac{1}{2}b(3A+C\right.}{\sqrt{a+b\sec(c+}}$$

$$= -\frac{2a(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{se}}{a}}}{3b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{se}}{a}}}{3b^2d}$$

Mathematica [C] time = 11.181, size = 570, normalized size = 1.61

$$\frac{\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(\frac{4aC\sin(c+dx)}{3b}+\frac{4}{3}C\tan(c+dx)\right)}{d(A\cos(2c+2dx)+A+2C)}+\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\cos^2(c+d}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c
+ d*x]^2)*((2*I)*a*(a - b)*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
```

```
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + (2*I)*(a - b)*b*(3*A
+ C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2]], (a + b)/(a - b)] - (12*I)*a*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
b)/(a - b)] - a*Sqrt[(-a + b)/(a + b)]*C*Cos[c + d*x]*(b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*Sqrt[(-a + b)/(a + b)]*d*(b + a*
Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*Sqrt[a + b*
Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*((4*a*C*Sin[c + d*x])/(3*b) + (4*C*Tan
[c + d*x])/3))/(d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.499, size = 1510, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/b*(-1+cos(d*x+c))^2*(3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-6*A*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+c)*
cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2+C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-6*A*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+C*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b-C*cos(d*x+c)^3*a^2-C*cos(d*x+c)^3*a*b+C*cos(d*x+c)^2*a^2-C*cos
(d*x+c)^2*a*b-C*cos(d*x+c)^2*b^2+2*C*cos(d*x+c)*a*b+b^2*C)*((b+a*cos(d*x+c)
)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+C\sec^2\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a), x)
```

3.713 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=352

$$\frac{\sqrt{a+b}(2C(a-b)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{(a-b)\sqrt{a+b}}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(A - 2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A*b + 2*
(a - b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(b*d) - (A*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.379191, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4095, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(2C(a-b)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}+\frac{(a-b)\sqrt{a+b}(A-2C)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(A - 2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A*b + 2*
(a - b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(b*d) - (A*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}+\int \frac{\frac{Ab}{2}+aC\sec(c+d}{\sqrt{a}}$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}-\frac{1}{2}(b(A-2C))\int \frac{\sec(}{}$$

$$= \frac{(a-b)\sqrt{a+b}(A-2C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}(A-2C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{bd}$$

Mathematica [B] time = 18.2512, size = 727, normalized size = 2.07

$$\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{a+b\sec(c+dx)}\left(-2(Ab-C(a+b))\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right.}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[a + b*Sec[c + d*x]]*S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*Tan[(c + d*x)/2] + A*b*Tan[(c + d*x
)/2] - 2*a*C*Tan[(c + d*x)/2] - 2*b*C*Tan[(c + d*x)/2] - 2*a*A*Tan[(c + d*x
)/2]^3 + 4*a*C*Tan[(c + d*x)/2]^3 + a*A*Tan[(c + d*x)/2]^5 - A*b*Tan[(c + d
*x)/2]^5 - 2*a*C*Tan[(c + d*x)/2]^5 + 2*b*C*Tan[(c + d*x)/2]^5 - 2*A*b*Elli
pticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - 2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(A - 2*C)*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
 d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] - 2*(A*b - (a + b)*C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.508, size = 1602, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2
*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*b-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*C*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+
2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*sin(d*x+c)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b*sin(d*x+c)-2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-2*C*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-2*C*EllipticE((-1+cos(
```

```
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*C*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+2*C*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+A
*cos(d*x+c)^3*a-A*cos(d*x+c)^2*a+A*cos(d*x+c)^2*b+2*C*cos(d*x+c)^2*a-A*cos(
d*x+c)*b-2*C*cos(d*x+c)*a+2*C*cos(d*x+c)*b-2*C*b)*(cos(d*x+c)+1)^2*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*sec(d*x +
c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

3.714 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=411

$$\frac{\sqrt{a+b}(2a(A+4C)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4ad}+\frac{\sqrt{a+b}(}{}$$

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(A*b + 2*a*(A +
 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(A*b^2 - 4*a^2*(A + 2*C))*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(4*a^2*d) + (A*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.681003, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4095, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(Ab^2-4a^2(A+2C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{\sqrt{a+b}(2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(A*b + 2*a*(A +
 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(A*b^2 - 4*a^2*(A + 2*C))*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(4*a^2*d) + (A*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
```

```
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos}{}$$

$$= \frac{Ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{A\cos(c+dx)\sqrt{a+}}{}$$

$$= \frac{Ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{A\cos(c+dx)\sqrt{a+}}{}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{4ad}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{4ad}$$

<u>Mathematica</u> [C] time = 19.1701, size = 1417, normalized size = 3.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (A*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d
*x]]*(-(a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]) - A*b^2*Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2] + 2*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^3 - a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + A*b^2*Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]^5 + (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a^2
*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I
)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
 (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + I*A*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
2*I)*(a - b)*(A*b + 2*a*(A + 2*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)]))/(4*a*Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(
1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.427, size = 1834, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a*(-1+cos(d*x+c))^2*(4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*A*cos(d*x+c)^4*a^2+8*C*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-3*A*cos(d
*x+c)^3*a*b+A*cos(d*x+c)^2*a*b+2*A*cos(d*x+c)*a*b-2*A*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-8*C*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*a^2*sin(d*x+c)-8*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*A*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-A*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^2+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-16*C*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2-8*C*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c
)+4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2-A*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*
b-2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)*a*b+2*A*cos(d*x+c)^2*a^2-A*cos(d*x+c)^2*b^2+A*cos(d*x+c)*b^2)*(cos(d
*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^
5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(b*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x
)
```

3.715 $\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=502

$$\frac{\sqrt{a+b}\left(8a^2(2A+3C)+2aAb-3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{24a^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b^2 - 8*a^2*(2*A + 3*C))*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*b
*d) + (Sqrt[a + b]*(2*a*A*b - 3*A*b^2 + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[
(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(2
4*a^2*d) - (b*Sqrt[a + b]*(A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(8*a^3*d) - ((3*A*b^2 - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(24*a^2*d) + (A*b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(12*a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*d)
```

Rubi [A] time = 1.06099, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4095, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\left(3Ab^2-8a^2(2A+3C)\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24a^2d}+\frac{\sqrt{a+b}\left(8a^2(2A+3C)+2aAb-3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b^2 - 8*a^2*(2*A + 3*C))*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*b
*d) + (Sqrt[a + b]*(2*a*A*b - 3*A*b^2 + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[
(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(2
4*a^2*d) - (b*Sqrt[a + b]*(A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(8*a^3*d) - ((3*A*b^2 - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(24*a^2*d) + (A*b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(12*a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\int \frac{\cos^2(}{}$$

$$=\frac{Ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12ad}+\frac{A\cos^2(c+}{}$$

$$=-\frac{\left(3Ab^2-8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^2d}+$$

$$=-\frac{\left(3Ab^2-8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^2d}+$$

$$=\frac{(a-b)\sqrt{a+b}\left(A\left(16-\frac{3b^2}{a^2}\right)+24C\right)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{24bd}$$

$$=\frac{(a-b)\sqrt{a+b}\left(A\left(16-\frac{3b^2}{a^2}\right)+24C\right)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{24bd}$$

Mathematica [B] time = 19.3302, size = 1347, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((A*Sin[c + d*x])/12 + (A*b*Sin[2*(c + d*x)])/(24
*a) + (A*Sin[3*(c + d*x)])/12))/d + (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(1 - Tan
[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-16*a^3*A*Tan[(c + d*x)/2] - 16*a^2*A*b*T
an[(c + d*x)/2] + 3*a*A*b^2*Tan[(c + d*x)/2] + 3*A*b^3*Tan[(c + d*x)/2] - 2
4*a^3*C*Tan[(c + d*x)/2] - 24*a^2*b*C*Tan[(c + d*x)/2] + 32*a^3*A*Tan[(c +
d*x)/2]^3 - 6*a*A*b^2*Tan[(c + d*x)/2]^3 + 48*a^3*C*Tan[(c + d*x)/2]^3 - 16
*a^3*A*Tan[(c + d*x)/2]^5 + 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 3*a*A*b^2*Tan[(
c + d*x)/2]^5 - 3*A*b^3*Tan[(c + d*x)/2]^5 - 24*a^3*C*Tan[(c + d*x)/2]^5 +
24*a^2*b*C*Tan[(c + d*x)/2]^5 + 24*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A*b^3*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^2*b
*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + 24*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A*b^3*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 48*a^2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-3*A*b^2 + 8*a^2*(2
*A + 3*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b*(14*a*A - A*b + 24*a*C)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
```

```
x)/2]^2)/(a + b)]))/(24*a^2*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*S
qrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c +
d*x)/2]^2)))
```

Maple [B] time = 0.485, size = 2535, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d/a^2*(-1+cos(d*x+c))^2*(16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+24*C*cos(d*x+c)^3*a^3+6*A
*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))+24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3-24*C*cos(d*x
+c)^2*a^3+3*A*cos(d*x+c)*b^3+8*A*cos(d*x+c)^5*a^3+10*A*cos(d*x+c)^4*a^2*b+6
*A*cos(d*x+c)^2*a^2*b+3*A*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b-2*A*cos(
d*x+c)*a*b^2-24*C*cos(d*x+c)*a^2*b+16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-3*A*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3
+6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*b^3+24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+16*A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-28*A*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d
*x+c)+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^2*sin(d*x+c)+24*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-48*C*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+48*C*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+16
*A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b-3*A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-28*A*a^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d
```

```
*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b^2+24*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+24*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-48*C*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)*a^2*b+48*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-3*A*cos(d*x+c)^2*b^3-A*cos(d*x+c)^3*a
*b^2+24*C*cos(d*x+c)^2*a^2*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c)
)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^2 + A\right)\sqrt{b \sec \left(dx+c\right) + a} \cos \left(dx+c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos \left(dx+c\right)^3 \sec \left(dx+c\right)^2 + A \cos \left(dx+c\right)^3\right)\sqrt{b \sec \left(dx+c\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + A*cos(d*x + c)^3)*sqrt(b*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x
)
```

3.716 $\int \cos^4(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=587

$$-\frac{\sqrt{a+b}\left(-4a^2b(7A+12C)-24a^3(3A+4C)+10aAb^2-15Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\text{si}\right.}{192a^3d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^2 + 4*a^2*(7*A + 12*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a^3
*d) - (Sqrt[a + b]*(10*a*A*b^2 - 15*A*b^3 - 24*a^3*(3*A + 4*C) - 4*a^2*b*(7
*A + 12*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(192*a^3*d) + (Sqrt[a + b]*(5*A*b^4 + 8*a^2*b^2*(
A + 2*C) - 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^4*d) + (b*(1
5*A*b^2 + 4*a^2*(7*A + 12*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a
^3*d) - ((5*A*b^2 - 12*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(96*a^2*d) + (A*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(24*a*d) + (A*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(4*d)
```

Rubi [A] time = 1.57532, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4095, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(4a^2(7A+12C)+15Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192a^3d}-\frac{\left(5Ab^2-12a^2(3A+4C)\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec}}{96a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^2 + 4*a^2*(7*A + 12*C))*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a^3
*d) - (Sqrt[a + b]*(10*a*A*b^2 - 15*A*b^3 - 24*a^3*(3*A + 4*C) - 4*a^2*b*(7
*A + 12*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(192*a^3*d) + (Sqrt[a + b]*(5*A*b^4 + 8*a^2*b^2*(
A + 2*C) - 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^4*d) + (b*(1
5*A*b^2 + 4*a^2*(7*A + 12*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a
^3*d) - ((5*A*b^2 - 12*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(96*a^2*d) + (A*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(24*a*d) + (A*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(4*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
```

```
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}+\frac{1}{4}\int \frac{\cos^3(}{}$$

$$= \frac{Ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24ad}+\frac{A\cos^3(c}{}$$

$$= -\frac{\left(5Ab^2-12a^2(3A+4C)\right)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}}{96a^2d}$$

$$= \frac{b\left(15Ab^2+4a^2(7A+12C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{192a^3d}$$

$$= \frac{b\left(15Ab^2+4a^2(7A+12C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{192a^3d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^2+4a^2(7A+12C)\right)\cot(c+dx)E\left(\sin\right.}{192a^3}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^2+4a^2(7A+12C)\right)\cot(c+dx)E\left(\sin\right.}{192a^3}$$

Mathematica [B] time = 15.3776, size = 1845, normalized size = 3.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((A*b*Sin[c + d*x])/(96*a) + ((48*a^2*A - 5*A*b^2
 + 48*a^2*C)*Sin[2*(c + d*x)])/(192*a^2) + (A*b*Sin[3*(c + d*x)])/(96*a) +
(A*Sin[4*(c + d*x)])/32))/d + (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(28*a^3*A
*b*Tan[(c + d*x)/2] + 28*a^2*A*b^2*Tan[(c + d*x)/2] + 15*a*A*b^3*Tan[(c + d
*x)/2] + 15*A*b^4*Tan[(c + d*x)/2] + 48*a^3*b*C*Tan[(c + d*x)/2] + 48*a^2*b
^2*C*Tan[(c + d*x)/2] - 56*a^3*A*b*Tan[(c + d*x)/2]^3 - 30*a*A*b^3*Tan[(c +
 d*x)/2]^3 - 96*a^3*b*C*Tan[(c + d*x)/2]^3 + 28*a^3*A*b*Tan[(c + d*x)/2]^5
- 28*a^2*A*b^2*Tan[(c + d*x)/2]^5 + 15*a*A*b^3*Tan[(c + d*x)/2]^5 - 15*A*b^
4*Tan[(c + d*x)/2]^5 + 48*a^3*b*C*Tan[(c + d*x)/2]^5 - 48*a^2*b^2*C*Tan[(c
+ d*x)/2]^5 - 288*a^4*A*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^2*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^4*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 384*a
^4*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + 96*a^2*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*a^4*A*EllipticPi[-1, -ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a
^2*A*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
+ d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
```

```
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^4*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 384
*a^4*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] + 96*a^2*b^2*C*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + b
*(a + b)*(15*A*b^2 + 4*a^2*(7*A + 12*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(2*
a*A*b^2 + 5*A*b^3 + 24*a^3*(3*A + 4*C) - 12*a^2*b*(3*A + 4*C))*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)]))/(192*a^3*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 +
 Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)
]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.619, size = 3606, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/192/d/a^3*(-1+cos(d*x+c))^2*(-288*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-24*A*a^4*cos(d*x+c)^4+72*A*a
^4*cos(d*x+c)^2-15*A*cos(d*x+c)^2*b^4+30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-28*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-15*A*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+
c)-72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
3*b*sin(d*x+c)+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2*sin(d*x+c)+10*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+192*C*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-384*C*a^4*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-44*A*cos(
d*x+c)^3*a^3*b-5*A*cos(d*x+c)^3*a*b^3+28*A*cos(d*x+c)^2*a^3*b-30*A*cos(d*x+
c)^2*a^2*b^2+72*A*cos(d*x+c)*a^3*b+28*A*cos(d*x+c)*a^2*b^2-10*A*cos(d*x+c)*
a*b^3-48*A*a^4*cos(d*x+c)^6+15*A*cos(d*x+c)^2*a*b^3+2*A*cos(d*x+c)^4*a^2*b^
2-96*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a^3*b-48*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-48*C*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+96
```

```
*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a^2*b^2-15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+144*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+48*A*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^
2-28*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^3*b-28*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-15*A*cos(d*x+c)*b^3*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-72
*A*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b+4*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+10*A*cos(d*x+c)*b^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-144*C*c
os(d*x+c)^3*a^3*b-48*C*cos(d*x+c)^2*a^2*b^2-56*A*cos(d*x+c)^5*a^3*b+48*C*co
s(d*x+c)^2*a^3*b+96*C*cos(d*x+c)*a^3*b+48*C*cos(d*x+c)*a^2*b^2-288*A*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*a^4+30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4-15*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+144*A*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^4+48*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*a^2*b^2*sin(d*x+c)-28*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+96*C*cos(d*x+c)^2*a^4+192*C*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a^4-384*C*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)-96*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-48*C*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-48*C*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+96*C*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^
2*sin(d*x+c)-96*C*cos(d*x+c)^4*a^4+15*A*cos(d*x+c)*b^4)*(cos(d*x+c)+1)^2*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^4, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4*sec(d*x + c)^2 + A*cos(d*x + c)^4)*sqrt(b*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^4, x
)
```

3.717 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=550

$$\frac{2(a-b)\sqrt{a+b}\left(6a^2b^2(11A+8C)+12a^3bC+16a^4C+3ab^3(209A+157C)-25b^4(11A+9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{1155b^4d}$$

```
[Out] (4*a*(a - b)*Sqrt[a + b]*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 3
48*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(1155*b^5*d) + (2*(a - b)*Sqrt[a + b]*(16*a^4*C + 12*a
^3*b*C + 6*a^2*b^2*(11*A + 8*C) - 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 15
7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(1155*b^4*d) + (2*(8*a^4*C + 25*b^4*(11*A + 9*C) + a^2*
b^2*(33*A + 19*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(1155*b^3*d) + (4
*a*(132*A*b^2 - 3*a^2*C + 101*b^2*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(1155*b^2*d) + (2*(a^2*C + 3*b^2*(11*A + 9*C))*Sec[c + d*x]^2
*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(231*b*d) + (2*a*C*Sec[c + d*x]^3*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(33*d) + (2*C*Sec[c + d*x]^3*(a + b*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(11*d)
```

Rubi [A] time = 1.91918, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4097, 4096, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(a^2C+3b^2(11A+9C)\right)\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{231bd}+\frac{4a\left(-3a^2C+132Ab^2+101b^2C\right)\tan(c+dx)\sec(}{1155b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*a*(a - b)*Sqrt[a + b]*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 3
48*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(1155*b^5*d) + (2*(a - b)*Sqrt[a + b]*(16*a^4*C + 12*a
^3*b*C + 6*a^2*b^2*(11*A + 8*C) - 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 15
7*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(1155*b^4*d) + (2*(8*a^4*C + 25*b^4*(11*A + 9*C) + a^2*
b^2*(33*A + 19*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(1155*b^3*d) + (4
*a*(132*A*b^2 - 3*a^2*C + 101*b^2*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(1155*b^2*d) + (2*(a^2*C + 3*b^2*(11*A + 9*C))*Sec[c + d*x]^2
*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(231*b*d) + (2*a*C*Sec[c + d*x]^3*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(33*d) + (2*C*Sec[c + d*x]^3*(a + b*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(11*d)
```

Rule 4097

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*
Simp[a*A*(m + n + 1) + a*C*n + b*(A*(m + n + 1) + C*(m + n))*Csc[e + f*x] +
```

```
 a*C*m*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&
NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{11d}+\frac{2}{11}\int$$

$$= \frac{2aC\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{33d}+\frac{2C\,\text{se}}{}$$

$$= \frac{2\left(a^2C+3b^2(11A+9C)\right)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{231bd}$$

$$= \frac{4a\left(132Ab^2-3a^2C+101b^2C\right)\sec(c+dx)\sqrt{a+b\sec(c+}}{1155b^2d}$$

$$= \frac{2\left(8a^4C+25b^4(11A+9C)+a^2b^2(33A+19C)\right)\sqrt{a+b\,\text{s}}}{1155b^3d}$$

$$= \frac{2\left(8a^4C+25b^4(11A+9C)+a^2b^2(33A+19C)\right)\sqrt{a+b\,\text{s}}}{1155b^3d}$$

$$= \frac{4a(a-b)\sqrt{a+b}\left(8a^4C+3a^2b^2(11A+6C)-b^4(451A+\right.}{}$$

Mathematica [B] time = 26.0968, size = 3988, normalized size = 7.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((-8*a*(3
3*a^2*A*b^2 - 451*A*b^4 + 8*a^4*C + 18*a^2*b^2*C - 348*b^4*C)*Sin[c + d*x])
/(1155*b^4) + (4*Sec[c + d*x]^3*(33*A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]
 + 27*b^2*C*Sin[c + d*x]))/(231*b) + (8*Sec[c + d*x]^2*(132*a*A*b^2*Sin[c +
 d*x] - 3*a^3*C*Sin[c + d*x] + 101*a*b^2*C*Sin[c + d*x]))/(1155*b^2) + (4*S
ec[c + d*x]*(33*a^2*A*b^2*Sin[c + d*x] + 275*A*b^4*Sin[c + d*x] + 8*a^4*C*S
in[c + d*x] + 19*a^2*b^2*C*Sin[c + d*x] + 225*b^4*C*Sin[c + d*x]))/(1155*b^
3) + (16*a*C*Sec[c + d*x]^3*Tan[c + d*x])/33 + (4*b*C*Sec[c + d*x]^4*Tan[c
+ d*x])/11))/(d*(b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])) + (8*(
(4*a^3*A)/(35*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (164*a*A*b)/
(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (32*a^5*C)/(1155*b^3*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (24*a^3*C)/(385*b*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (464*a*b*C)/(385*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]) - (62*a^2*A*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c
```

+ d*x]]) + (4*a^4*A*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) + (10*A*b^2*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (26*a^2*C*Sqrt[Sec[c + d*x]])/(55*Sqrt[b + a*Cos[c + d*x]]) + (32*a^6*C*Sqrt[Sec[c + d*x]])/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]) + (64*a^4*C*Sqrt[Sec[c + d*x]])/(1155*b^2*Sqrt[b + a*Cos[c + d*x]]) + (30*b^2*C*Sqrt[Sec[c + d*x]])/(77*Sqrt[b + a*Cos[c + d*x]]) - (164*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) - (464*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(385*Sqrt[b + a*Cos[c + d*x]]) + (32*a^6*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]) + (24*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(385*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-16*a^4*C + 12*a^3*b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 157*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1155*b^4*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(7/2)*((4*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-16*a^4*C + 12*a^3*b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 157*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1155*b^4*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-16*a^4*C + 12*a^3*b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 157*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (8*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (a*(a + b)*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-16*a^4*C + 12*a^3*b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 157*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (a*(a + b)*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(-16*a^4*C + 12*a^3*b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 157*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-(

```
(a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[
c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(2*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]) - a^2*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(
451*A + 348*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/
2] - a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*(b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + a*(8*a^4*C + 3
*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-16*a^4*C + 12*a^3*
b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C) + 3*a*b^3*(209*A + 157*C
))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a*(a + b)*(8*a^4*C + 3*a^
2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)
/2]^2]))/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*
(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A + 348*C))*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a +
 b)*(-16*a^4*C + 12*a^3*b*C - 6*a^2*b^2*(11*A + 8*C) + 25*b^4*(11*A + 9*C)
+ 3*a*b^3*(209*A + 157*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(11*A + 6*C) - b^4*(451*A
+ 348*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]
^2*Sec[c + d*x]*Tan[c + d*x]))/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 2.226, size = 4696, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 2/1155/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(696*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+16*C*cos(d*x+c)^7*a^6-16*C*cos(d*x+c)
^5*a^3*b^3+584*C*cos(d*x+c)^5*a*b^5+297*A*cos(d*x+c)^4*a^2*b^4-36*C*cos(d*x
+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^3*b^3+696*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+696*C*cos(d*x+c)^6*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+66*A*c
os(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^3*b^3-561*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4-902*A*cos(d*x+c)^5*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5-
66*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
```

```
^(1/2))*sin(d*x+c)*a^4*b^2-66*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3+902*A*cos(d*x+c)^5*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^2*b^4+902*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+16*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+4*C*cos(d*x+c)^5*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a^4*b^2+36*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3-423*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4-696*C*cos(d
*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b^5-16*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b-36*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-36*C*cos
(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*a^3*b^3+696*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+66*A*cos(d*x+c)^6*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3-561
*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^2*b^4-902*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5-66*A*cos(d*x+c)^6*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b
^2-66*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^3*b^3+902*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+902*A*cos(d*x+c)^
6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^5+16*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+4*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2+36*C*cos(d*x+c)
^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^3*b^3-423*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4-696*C*cos(d*x+c)^6*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5-16*C*cos(
d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
```

```
in(d*x+c)*a^5*b-36*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-275*A*cos(d*x+c)^6*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^6-225*C*co
s(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^6-16*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^6-275*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^6-225*C*cos(d*
x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*b^6-16*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^6+66*A*cos(d*x+c)^7*a^4*b^2-33*A*cos(d*x+c
)^7*a^3*b^3-902*A*cos(d*x+c)^7*a^2*b^4-275*A*cos(d*x+c)^7*a*b^5-8*C*cos(d*x
+c)^7*a^5*b+36*C*cos(d*x+c)^7*a^4*b^2+245*C*cos(d*x+c)*a*b^5+2*C*cos(d*x+c)
^4*a^4*b^2+76*C*cos(d*x+c)^4*a^2*b^4+429*A*cos(d*x+c)^3*a*b^5-C*cos(d*x+c)^
3*a^3*b^3+92*C*cos(d*x+c)^3*a*b^5+145*C*cos(d*x+c)^2*a^2*b^4-19*C*cos(d*x+c
)^7*a^3*b^3-696*C*cos(d*x+c)^7*a^2*b^4-225*C*cos(d*x+c)^7*a*b^5-66*A*cos(d*
x+c)^6*a^4*b^2+66*A*cos(d*x+c)^6*a^3*b^3+605*A*cos(d*x+c)^6*a^2*b^4-902*A*c
os(d*x+c)^6*a*b^5+16*C*cos(d*x+c)^6*a^5*b-38*C*cos(d*x+c)^6*a^4*b^2+36*C*co
s(d*x+c)^6*a^3*b^3+475*C*cos(d*x+c)^6*a^2*b^4-696*C*cos(d*x+c)^6*a*b^5-33*A
*cos(d*x+c)^5*a^3*b^3+748*A*cos(d*x+c)^5*a*b^5-8*C*cos(d*x+c)^5*a^5*b-16*C*
cos(d*x+c)^6*a^6-275*A*cos(d*x+c)^6*b^6-225*C*cos(d*x+c)^6*b^6+110*A*cos(d*
x+c)^4*b^6+90*C*cos(d*x+c)^4*b^6+165*A*cos(d*x+c)^2*b^6+30*C*cos(d*x+c)^2*b
^6+105*C*b^6)/(b+a*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\sec(dx+c)^{6}+Ca\sec(dx+c)^{5}+Ab\sec(dx+c)^{4}+Aa\sec(dx+c)^{3}\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^6 + C*a*sec(d*x + c)^5 + A*b*sec(d*x + c)^4 + A*
a*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^{2}+A\right)\left(b \sec \left(dx+c\right)+a\right)^{\frac{3}{2}} \sec \left(dx+c\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3,
 x)
```

3.718 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=454

$$-\frac{2(a-b)\sqrt{a+b}\left(6a^2bC+8a^3C+3ab^2(21A+13C)-21b^3(9A+7C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Elliptic}}{315b^3d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 1
1*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3*C + 6*a^2*b
*C - 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A + 13*C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d)
+ (2*a*(63*A*b^2 + 8*a^2*C + 39*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(315*b^2*d) + (2*(8*a^2*C + 7*b^2*(9*A + 7*C))*(a + b*Sec[c + d*x])^(3/2
)*Tan[c + d*x])/(315*b^2*d) - (8*a*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x
])/(63*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/
(9*b*d)
```

Rubi [A] time = 1.04972, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4093, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2C+7b^2(9A+7C)\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2C+63Ab^2+39b^2C\right)\tan(c+dx)\sqrt{a+b\sec(c+dx}}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 1
1*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3*C + 6*a^2*b
*C - 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A + 13*C))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d)
+ (2*a*(63*A*b^2 + 8*a^2*C + 39*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(315*b^2*d) + (2*(8*a^2*C + 7*b^2*(9*A + 7*C))*(a + b*Sec[c + d*x])^(3/2
)*Tan[c + d*x])/(315*b^2*d) - (8*a*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x
])/(63*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/
(9*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd} + \frac{2\int \mathrm{se}}{}$$

$$= -\frac{8aC(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d} + \frac{2C\sec(c+dx)}{}$$

$$= \frac{2\left(8a^2C+7b^2(9A+7C)\right)(a+b\sec(c+dx))^{3/2}\tan(c+d}{315b^2d}$$

$$= \frac{2a\left(63Ab^2+8a^2C+39b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+}{315b^2d}$$

$$= \frac{2a\left(63Ab^2+8a^2C+39b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+}{315b^2d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(8a^4C+21b^4(9A+7C)+3a^2b^2(21A+\right.}{}$$

Mathematica [B] time = 24.1276, size = 3537, normalized size = 7.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((4*(63*a
^2*A*b^2 + 189*A*b^4 + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(3
15*b^3) + (4*Sec[c + d*x]^2*(63*A*b^2*Sin[c + d*x] + 3*a^2*C*Sin[c + d*x] +
 49*b^2*C*Sin[c + d*x]))/(315*b) + (8*Sec[c + d*x]*(63*a*A*b^2*Sin[c + d*x]
 - 2*a^3*C*Sin[c + d*x] + 44*a*b^2*C*Sin[c + d*x]))/(315*b^2) + (40*a*C*Sec
[c + d*x]^2*Tan[c + d*x])/63 + (4*b*C*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(
b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])) - (4*((-2*a^2*A)/(5*Sqr
t[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (6*A*b^2)/(5*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (22*a^2*C)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (16*a^4*C)/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) - (14*b^2*C)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a
^3*A*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (2*a*A*b*Sqrt[Sec
[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (16*a^5*C*Sqrt[Sec[c + d*x]])/(3
15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (62*a^3*C*Sqrt[Sec[c + d*x]])/(315*b*Sqr
t[b + a*Cos[c + d*x]]) + (26*a*b*C*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[
c + d*x]]) - (2*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*
Cos[c + d*x]]) - (6*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b +
a*Cos[c + d*x]]) - (16*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*b^3*
Sqrt[b + a*Cos[c + d*x]]) - (22*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
(105*b*Sqrt[b + a*Cos[c + d*x]]) - (14*a*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c +
d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((a + b)*((8*a^4*C + 21*
b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^
2*(21*A + 13*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos
[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2)/(a + b)]*Sec[c + d*x] + (8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(
21*A + 11*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c +
 d*x)/2]))/(315*b^3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(7/2)*((-2*a*Sqrt[Cos[(c + d*x)/2]
```

```
^2*Sec[c + d*x]]*Sin[c + d*x]*((a + b)*((8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a
^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
- b*(8*a^3*C - 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A + 13*C))*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[
c + d*x] + (8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*b^3*(
b + a*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((a + b)*((8*a^4*C + 21*b^4*(9*A + 7
*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] - b*(8*a^3*C - 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A + 13
*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Se
c[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b)]*Sec[c + d*x] + (8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C)
)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(
105*b^3*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) - (2*((a + b)*
((8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*C + 21*b^3*(9*A
+ 7*C) + 3*a*b^2*(21*A + 13*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (8*a^4*C + 21*b^4*(9*A + 7*C
) + 3*a^2*b^2*(21*A + 11*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2])
 + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*b^3*Sqrt[b + a*Cos[c
 + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])
- (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((8*a^4*C + 21*b^4*(9*A + 7*C)
+ 3*a^2*b^2*(21*A + 11*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^6)/2 - a*(8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Cos[c
+ d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - (8*a^4*C + 21*b^4
*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + 2*(8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^
2*b^2*(21*A + 11*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*T
an[(c + d*x)/2]^2 + (3*(a + b)*((8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(
21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a
^3*C - 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A + 13*C))*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2
]^2]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(
-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/2 + ((a + b)*((8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A
+ 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C
- 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A + 13*C))*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2
)*Sec[c + d*x]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + (a + b)*(Cos[c + d*x]*Sec[(c
+ d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*
Sec[c + d*x]*(-(b*(8*a^3*C - 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^2*(21*A
 + 13*C))*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a
- b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2
*b^2*(21*A + 11*C))*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2
)/(a + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2])) + (a + b)*((8*a^4*C + 21*b^4*
(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*C + 21*b^3*(9*A + 7*C) + 3*a*b^2*(2
1*A + 13*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c +
 d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*
(Sec[(c + d*x)/2]^2)^(3/2)))
```

Maple [B] time = 1.669, size = 4115, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(2*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+189*A*cos(d*x+c)^5*b^5+8*C*cos(d*x+c)^5
*a^4*b-34*C*cos(d*x+c)^5*a^3*b^2+33*C*cos(d*x+c)^5*a^2*b^3-10*C*cos(d*x+c)^
5*a*b^4-189*A*cos(d*x+c)^4*a^2*b^3-4*C*cos(d*x+c)^4*a^4*b-68*C*cos(d*x+c)^4
*a^2*b^3-189*A*cos(d*x+c)^3*a*b^4+C*cos(d*x+c)^3*a^3*b^2-52*C*cos(d*x+c)^3*
a*b^4+63*A*cos(d*x+c)^6*a^3*b^2+126*A*cos(d*x+c)^6*a^2*b^3+189*A*cos(d*x+c)
^6*a*b^4-4*C*cos(d*x+c)^6*a^4*b+33*C*cos(d*x+c)^6*a^3*b^2+88*C*cos(d*x+c)^6
*a^2*b^3+147*C*cos(d*x+c)^6*a*b^4-53*C*cos(d*x+c)^2*a^2*b^3-85*C*cos(d*x+c)
*a*b^4-63*A*cos(d*x+c)^5*a^3*b^2+63*A*cos(d*x+c)^5*a^2*b^3-8*C*cos(d*x+c)^4
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^5-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^5+189*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-189*A*cos(d*x+c)^5*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^5+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^5-8*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5-147*C*cos(d*x+c)^5*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+18
9*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^5-189*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*C*cos(d*x+c)^4*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-8*C*c
os(d*x+c)^5*a^5+147*C*cos(d*x+c)^5*b^5-126*A*cos(d*x+c)^4*b^5-98*C*cos(d*x+
c)^4*b^5-63*A*cos(d*x+c)^2*b^5-14*C*cos(d*x+c)^2*b^5+8*C*cos(d*x+c)^6*a^5+3
3*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b^3+186*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-8*C*cos(d*x+c)^5*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b
-33*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3*b^2-33*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*C*cos(d*x+c)^5*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^4+63*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
```

```
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b^3+252*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-63*A*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*b^2-63*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-189*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+8*C*cos(d*x
+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^4*b+2*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+33*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+186*C*cos
(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b^4-8*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-33*C*cos(d*x+c)^4*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-33*C*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2*b^3-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+63*A*cos(d*x+c)^5*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+25
2*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^4-63*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-63*A*cos(d*x+c)^5*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
^3-189*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^4+8*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-35*C*b^5)/(b+a*cos(d*x
+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{5}+Ca\sec\left(dx+c\right)^{4}+Ab\sec\left(dx+c\right)^{3}+Aa\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^5 + C*a*sec(d*x + c)^4 + A*b*sec(d*x + c)^3 + A*
a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2,
 x)
```

3.719 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=374

$$\frac{2(a-b)\sqrt{a+b}\left(6a^2C+105aAb+57abC-35Ab^2-25b^2C\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin\right.}{105b^2d}$$

```
[Out] (-4*a*(a - b)*Sqrt[a + b]*(70*A*b^2 - 3*a^2*C + 41*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(10
5*b^3*d) + (2*(a - b)*Sqrt[a + b]*(105*a*A*b - 35*A*b^2 + 6*a^2*C + 57*a*b*
C - 25*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(105*b^2*d) - (2*(6*a^2*C - 5*b^2*(7*A + 5*C))*
Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) - (4*a*C*(a + b*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c +
d*x])/(7*b*d)
```

Rubi [A] time = 0.760999, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4083, 4002, 4005, 3832, 4004}

$$-\frac{2\left(6a^2C-5b^2(7A+5C)\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2(a-b)\sqrt{a+b}\left(6a^2C+105aAb+57abC-35Ab^2-25b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-4*a*(a - b)*Sqrt[a + b]*(70*A*b^2 - 3*a^2*C + 41*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(10
5*b^3*d) + (2*(a - b)*Sqrt[a + b]*(105*a*A*b - 35*A*b^2 + 6*a^2*C + 57*a*b*
C - 25*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(105*b^2*d) - (2*(6*a^2*C - 5*b^2*(7*A + 5*C))*
Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) - (4*a*C*(a + b*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c +
d*x])/(7*b*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
```

```
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int \sec(c+dx)(a \cdots}{\cdots}$$

$$= -\frac{4aC(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd} + \frac{2C(a+b\sec(c \cdots}{\cdots}$$

$$= -\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx \cdots}{105bd}$$

$$= -\frac{2\left(6a^2C-5b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx \cdots}{105bd}$$

$$= -\frac{4a(a-b)\sqrt{a+b}\left(70Ab^2-3a^2C+41b^2C\right)\cot(c+dx)E\left(\cdots\right.}{105 \cdots}$$

Mathematica [B] time = 23.7816, size = 3214, normalized size = 8.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((-8*a*(-
70*A*b^2 + 3*a^2*C - 41*b^2*C)*Sin[c + d*x])/(105*b^2) + (4*Sec[c + d*x]*(3
5*A*b^2*Sin[c + d*x] + 3*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*
b) + (32*a*C*Sec[c + d*x]*Tan[c + d*x])/35 + (4*b*C*Sec[c + d*x]^2*Tan[c +
```

d*x])/7))/(d*(b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])) + (8*((-8
*a*A*b)/(3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^3*C)/(35*b*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (164*a*b*C)/(105*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*Sqrt[b +
 a*Cos[c + d*x]]) + (2*A*b^2*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]
]) - (62*a^2*C*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*
C*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) + (10*b^2*C*Sqrt[Se
c[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (8*a^2*A*Cos[2*(c + d*x)]*Sqrt
[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) - (164*a^2*C*Cos[2*(c + d*x)]*
Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*C*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*(2*
a*(a + b)*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-6*a^2*C + 5*b^2*(7*A
+ 5*C) + 3*a*b*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + a*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b
+ a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]
*Sec[c + d*x]^(7/2)*((4*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x
]*(2*a*(a + b)*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-6*a^2*C + 5*b^2*
(7*A + 5*C) + 3*a*b*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)] + a*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Cos[c
+ d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*
(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x
)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*a*(a + b)*(-70*A*b^2 + 3*a^2*C - 4
1*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + b*(a + b)*(-6*a^2*C + 5*b^2*(7*A + 5*C) + 3*a*b*(35*A + 19*C))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(-7
0*A*b^2 + 3*a^2*C - 41*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c +
d*x)/2]^2]) + (8*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((a*(-70*A*b^2 + 3*a
^2*C - 41*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 +
(a*(a + b)*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Co
s[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-6*a^2*C
+ 5*b^2*(7*A + 5*C) + 3*a*b*(35*A + 19*C))*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos
[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (a*(a + b)*(-70*A*
b^2 + 3*a^2*C - 41*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c
+ d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(
a + b)*(-6*a^2*C + 5*b^2*(7*A + 5*C) + 3*a*b*(35*A + 19*C))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(2*Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]) - a^2*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Cos[
c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - a*(-70*A*b^2 +
3*a^2*C - 41*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Ta
n[(c + d*x)/2] + a*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Cos[c + d*x]*(b + a*Cos

```
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-6*a^2*C + 5
*b^2*(7*A + 5*C) + 3*a*b*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]
^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]) + (a*(a + b)*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*S
ec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 -
Tan[(c + d*x)/2]^2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/
2]^2]) + (4*(2*a*(a + b)*(-70*A*b^2 + 3*a^2*C - 41*b^2*C)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-6*a^2*
C + 5*b^2*(7*A + 5*C) + 3*a*b*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(-70*A*b^2 + 3*a^2*C - 41*b^2
*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*
(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec
[c + d*x]*Tan[c + d*x]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.986, size = 2986, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-35*A*cos(d*x+c)^2*b^4+105*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-140*A*sin(d*x+
c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^2-6*C*cos(d*x+c)^5*a^4+6*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*sin(d*x+c)*co
s(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^4+35*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^4+6*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-17
5*A*cos(d*x+c)^3*a*b^3-140*A*cos(d*x+c)^4*a^2*b^2-140*A*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3
+140*A*cos(d*x+c)^4*a*b^3-6*C*cos(d*x+c)^4*a^3*b-55*C*cos(d*x+c)^4*a^2*b^2+
82*C*cos(d*x+c)^4*a*b^3+3*C*cos(d*x+c)^3*a^3*b-68*C*cos(d*x+c)^3*a*b^3-27*C
*cos(d*x+c)^2*a^2*b^2-39*C*cos(d*x+c)*a*b^3+140*A*cos(d*x+c)^5*a^2*b^2+35*A
*cos(d*x+c)^5*a*b^3+3*C*cos(d*x+c)^5*a^3*b+82*C*cos(d*x+c)^5*a^2*b^2+25*C*c
os(d*x+c)^5*a*b^3+35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+105*A*cos(d*x+c)^3*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+140*A*
```

```
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^3+6*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-82*C*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-82*C
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^3-6*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+51*C*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+82*
C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^3-140*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-140*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3
+140*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^3+6*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-82*C*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
^2-82*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^3-6*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+51*C*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
b^2+82*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^3+6*C*cos(d*x+c)^4*a^4-10*C*cos(d*x+c)^2*b^4+35*
A*cos(d*x+c)^4*b^4+25*C*cos(d*x+c)^4*b^4-15*C*b^4)/(b+a*cos(d*x+c))/cos(d*x
+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{4}+Ca\sec\left(dx+c\right)^{3}+Ab\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^4 + C*a*sec(d*x + c)^3 + A*b*sec(d*x + c)^2 + A*
a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x
)
```

3.720 $\int (a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=415

$$-\frac{2\sqrt{a+b}\left(a^2C-2ab(5A+2C)+b^2(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{5bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(a^2*C + b^2*(5*A + 3*C))*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^2*d) -
 (2*Sqrt[a + b]*(a^2*C - 2*a*b*(5*A + 2*C) + b^2*(5*A + 3*C))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(5*b*d) - (2*a*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a*C*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(5*d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[
c + d*x])/(5*d)
```

Rubi [A] time = 0.576313, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {4057, 4056, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(a^2C-2ab(5A+2C)+b^2(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(a^2*C + b^2*(5*A + 3*C))*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^2*d) -
 (2*Sqrt[a + b]*(a^2*C - 2*a*b*(5*A + 2*C) + b^2*(5*A + 3*C))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(5*b*d) - (2*a*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a*C*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(5*d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[
c + d*x])/(5*d)
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
```

```
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int\sqrt{a+b\sec(c+dx)}\Big(\frac{5}{}$$

$$=\frac{2aC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d}+\frac{2C(a+b\sec(c+dx))^{3/2}\tan}{5d}$$

$$=\frac{2aC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d}+\frac{2C(a+b\sec(c+dx))^{3/2}\tan}{5d}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(a^2C+b^2(5A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a}}\right.\right.}{5b^2d}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(a^2C+b^2(5A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a}}\right.\right.}{5b^2d}$$

Mathematica [B] time = 25.1141, size = 6143, normalized size = 14.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.773, size = 2834, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/5/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(5*A*cos(d*x+c)^3*b^3-5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+10*A*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a
^2*b-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b+10*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-C*cos(d*x+c)^3*a^
3-5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^2-5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*A*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-C*cos
(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3-3*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
```

```
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*A*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^3+10*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^2-C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*C*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
2+C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b+4*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5*A*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+10*
A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^2-C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*C*cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+C*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b+4*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-C*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3-3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3+3*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*C*cos(d*x+c)^3*b^3-5*A*
cos(d*x+c)^2*b^3-2*C*cos(d*x+c)^2*b^3+C*cos(d*x+c)^4*a^3+5*A*cos(d*x+c)^4*a
*b^2+2*C*cos(d*x+c)^4*a^2*b+3*C*cos(d*x+c)^4*a*b^2-5*A*cos(d*x+c)^3*a*b^2+C
*cos(d*x+c)^3*a^2*b-3*C*cos(d*x+c)^2*a^2*b-3*C*cos(d*x+c)*a*b^2-C*b^3)/(b+a
*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C \sec^{2}{\left(c + dx \right)}\right) \left(a + b \sec{\left(c + dx \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec\left(dx + c\right)^{2} + A\right) \left(b \sec\left(dx + c\right) + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2), x)
```

3.721 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=408

$$\frac{\sqrt{a+b}\left(6a^2C+ab(3A-8C)+2b^2(3A+C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{3bd}$$

```
[Out] (a*(a - b)*Sqrt[a + b]*(3*A - 8*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (Sqrt[a + b]*(a*
b*(3*A - 8*C) + 6*a^2*C + 2*b^2*(3*A + C))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (3*A*b*S
qrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/d - (b*(3*A - 2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.574512, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(6a^2C+ab(3A-8C)+2b^2(3A+C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(a - b)*Sqrt[a + b]*(3*A - 8*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (Sqrt[a + b]*(a*
b*(3*A - 8*C) + 6*a^2*C + 2*b^2*(3*A + C))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (3*A*b*S
qrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/d - (b*(3*A - 2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
```

```
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} + \int \sqrt{a+b\sec(c+d}$$

$$= \frac{A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3A-2C)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3A-2C)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{a(a-b)\sqrt{a+b}(3A-8C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{3bd}$$

$$= \frac{a(a-b)\sqrt{a+b}(3A-8C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{3bd}$$

Mathematica [B] time = 24.4298, size = 4024, normalized size = 9.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((16*a*C*
Sin[c + d*x])/3 + (4*b*C*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])*(A + 2*C
 + A*Cos[2*c + 2*d*x])) + (2*((4*a*A*b)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (8*a*b*C)/(3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a
^2*A*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] + (2*A*b^2*Sqrt[Sec[c + d
*x]])/Sqrt[b + a*Cos[c + d*x]] - (2*a^2*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a
*Cos[c + d*x]]) + (2*b^2*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]])
 + (a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] - (
8*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec
[c + d*x]^2)*(2*a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(3*a^2*C + b^2*(3
*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(-36*A*b*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)
/2]^2 + (3*A - 8*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Ta
n[(c + d*x)/2])))/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x]
)*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(7/2)*((a*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*Sin[c + d*x]*(2*a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(3
*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(-36*A*b*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Sec[(c + d*x)/2]^2 + (3*A - 8*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4*Tan[(c + d*x)/2])))/(3*(b + a*Cos[c + d*x])^(3/2)*(Sec[(c + d*
x)/2]^2)^(3/2)) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(
2*a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
```

]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(3*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(-36*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + (3*A - 8*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]
)))/(Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((a*(a + b)*(3*A - 8*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (2*(3*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (2*(3*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + 2*a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 4*(3*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + (2*(3*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2] + a*(((3*A - 8*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - (18*A*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (18*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 36*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - a*(3*A - 8*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*A - 8*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + 2*(3*A - 8*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 + (18*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])))))/(3*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + ((2*a*(a + b)*(3*A - 8*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(3*a^2*C + b^2*(3*A + C) + a*(-6*A*b + 4*b*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt

```
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(-36*A*b*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d
*x)/2]^2 + (3*A - 8*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4
*Tan[(c + d*x)/2]))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Co
s[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*Sqrt[b + a*Cos[c + d*x]]*(S
ec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.638, size = 2147, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/3/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(3*A*cos(d*x+c)^4*a^2+3*A*cos(d*x+c)^3*a*b-3*A*cos(d*x+c)^2*a*b+8*C*cos(
d*x+c)^3*a^2-12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+8*C*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b-8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b-12*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+18*A*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a
*b+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a*b+6*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2+6*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-8
*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-8*C*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
```

```
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2+2*C*cos(d*x+c)^3*a*b+8*C*cos(d*x+c)^2*a*b-10*C*cos(d*x+c)*a*b-3*A*cos
(d*x+c)^3*a^2+3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+6*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^2-2*b^2*C-8*C*cos(d*x+c)^2*a^2+2*C*cos(d*x+c)^2*b^2)/sin(d*x+c)^5/(b+a*cos
(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^{2}+A\right)\left(b\sec \left(dx+c\right)+a\right)^{\frac{3}{2}}\cos \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb\cos \left(dx+c\right)\sec \left(dx+c\right)^{3}+Ca\cos \left(dx+c\right)\sec \left(dx+c\right)^{2}+Ab\cos \left(dx+c\right)\sec \left(dx+c\right)+Aa\cos \left(dx+c\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)*sec(d*x + c)^3 + C*a*cos(d*x + c)*sec(d*x + c)^2
 + A*b*cos(d*x + c)*sec(d*x + c) + A*a*cos(d*x + c))*sqrt(b*sec(d*x + c) +
a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^{2}+A\right)\left(b\sec \left(dx+c\right)+a\right)^{\frac{3}{2}}\cos \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x
)
```

3.722 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=414

$$\frac{\sqrt{a+b}(2aA+16aC+5Ab-8bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4d}-$$

```
[Out] ((a - b)*Sqrt[a + b]*(5*A - 8*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a*A
+ 5*A*b + 16*a*C - 8*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(3*A*b^2 + 4*a
^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (3*A*b*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(2*d)
```

Rubi [A] time = 0.675403, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4095, 4094, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2(A+2C)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(5*A - 8*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a*A
+ 5*A*b + 16*a*C - 8*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(3*A*b^2 + 4*a
^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (3*A*b*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(2*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d} + \frac{1}{2}\int$$

$$= \frac{3Ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{A\cos(c+dx)(a}{}$$

$$= \frac{3Ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{A\cos(c+dx)(a}{}$$

$$= \frac{(a-b)\sqrt{a+b}(5A-8C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}(5A-8C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{4d}$$

Mathematica [C] time = 19.7006, size = 1618, normalized size = 3.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] ((Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(4*b*C*Sin[c + d*x] + (a*A*Sin[2*
(c + d*x)])/2))/(d*(b + a*Cos[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a
*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 5*A*b^2*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2] - 8*a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] - 8*b
^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] - 10*a*A*b*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]^3 + 16*a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^3
 + 5*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 5*A*b^2*Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]^5 - 8*a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x
)/2]^5 + 8*b^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a
+ b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)
/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] - (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c
+ d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*(5*A - 8*C)*EllipticE[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*(b*(A - 4*C) + 2*a*(A +
 2*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(2*Sqrt[(-a + b
```

```
)/(a + b)]*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c
 + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2
)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
d*x)/2]^2)]))/2
```

Maple [B] time = 0.648, size = 2617, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(-4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*A*cos(d*x+c)^4*a^2-8*C*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+
7*A*cos(d*x+c)^3*a*b-5*A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+2*A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b+16*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+16*C*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+6
*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+5*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2*sin(d*x+c)+5*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+8*C*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+5*A*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^2-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2+16*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2+16*C*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-4*A*cos(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+
```

```
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2+5*A*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+2*A*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-2*A*c
os(d*x+c)^2*a^2+5*A*cos(d*x+c)^2*b^2-5*A*cos(d*x+c)*b^2-8*A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2
+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^2+8*C*cos(d*x+c)^2*a*b-8*C*cos(d*x+c)*a*b-8*C*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-8*C*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+8*C*cos(d*x+c)*b^2-8*b^2*C)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+Ab\cos\left(dx+c\right)^2\sec\left(dx+c\right)+Aa\cos\left(dx\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + C*a*cos(d*x + c)^2*sec(d*x +
c)^2 + A*b*cos(d*x + c)^2*sec(d*x + c) + A*a*cos(d*x + c)^2)*sqrt(b*sec(d*x
 + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx + c)^2 + A\right)(b \sec (dx + c) + a)^{\frac{3}{2}} \cos (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2,
 x)
```

3.723 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=504

$$\frac{\sqrt{a+b}\left(16a^2A+24a^2C+14aAb+48abC+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{s}}}{\sqrt{}}\right.\right.}{24ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*A*b^2 + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*d)
 + (Sqrt[a + b]*(16*a^2*A + 14*a*A*b + 3*A*b^2 + 24*a^2*C + 48*a*b*C)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(24*a*d) + (b*Sqrt[a + b]*(A*b^2 - 12*a^2*(A + 2*C))*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(8*a^2*d) + ((3*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(24*a*d) + (A*b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(4*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(3*d)
```

Rubi [A] time = 1.17798, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4095, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(8a^2(2A+3C)+3Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24ad}+\frac{\sqrt{a+b}\left(16a^2A+24a^2C+14aAb+48abC+3Ab^2\right)\cot(c+}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*A*b^2 + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*d)
 + (Sqrt[a + b]*(16*a^2*A + 14*a*A*b + 3*A*b^2 + 24*a^2*C + 48*a*b*C)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(24*a*d) + (b*Sqrt[a + b]*(A*b^2 - 12*a^2*(A + 2*C))*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(8*a^2*d) + ((3*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(24*a*d) + (A*b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(4*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(3*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{1}{3}$$

$$= \frac{Ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}+\frac{A\cos}{}$$

$$= \frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+d}{24ad}$$

$$= \frac{\left(3Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+d}{24ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3Ab^2+8a^2(2A+3C)\right)\cot(c+dx)E\Big(}{2}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3Ab^2+8a^2(2A+3C)\right)\cot(c+dx)E\Big(}{2}$$

Mathematica [B] time = 18.7106, size = 1393, normalized size = 2.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((a*A*Sin
[c + d*x])/6 + (7*A*b*Sin[2*(c + d*x)])/12 + (a*A*Sin[3*(c + d*x)])/6))/(d*
(b + a*Cos[c + d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])) + ((a + b*Sec[c + d*x]
)^(3/2)*(A + C*Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a^3*
A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 3*a*A*b^2*Tan[(c + d*x)/
2] + 3*A*b^3*Tan[(c + d*x)/2] + 24*a^3*C*Tan[(c + d*x)/2] + 24*a^2*b*C*Tan[
(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 6*a*A*b^2*Tan[(c + d*x)/2]^3 -
 48*a^3*C*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan
[(c + d*x)/2]^5 + 3*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*A*b^3*Tan[(c + d*x)/2]^5
 + 24*a^3*C*Tan[(c + d*x)/2]^5 - 24*a^2*b*C*Tan[(c + d*x)/2]^5 - 72*a^2*A*b
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 144*a^2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 72*a^2*A*b*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 144*a^2*b*C*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + (a + b)*(3*A*b^2 + 8*a^2*(2*A + 3*C))*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2
)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a
*b*(26*a*A - 7*A*b + 48*a*C - 24*b*C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
```

```
a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(12*a*d*(b
 + a*Cos[c + d*x])^(3/2)*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)*
(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.477, size = 2723, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/24/d/a*(-1+cos(d*x+c))^2*(-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-24*C*cos(d*x+c)^3*a^3+6*A*b
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))
^(1/2))-24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))-48*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-8*A*cos(d*x+c)^3*a^3+16*A*cos(d*x+
c)^2*a^3+24*C*cos(d*x+c)^2*a^3+3*A*cos(d*x+c)*b^3-8*A*cos(d*x+c)^5*a^3-22*A
*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+c)^2*a^2*b+3*A*cos(d*x+c)^2*a*b^2+16*A*cos(
d*x+c)*a^2*b+14*A*cos(d*x+c)*a*b^2+24*C*cos(d*x+c)*a^2*b-16*A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a
^3-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*b^3+6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-24*C*a^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-16*A*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*
x+c)-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2*sin(d*x+c)+52*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b*sin(d*x+c)-14*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-24*C*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+96*C*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*
x+c)-144*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1
/2))*a^2*b*sin(d*x+c)-16*A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+c
```

```
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-3*A*b^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(
d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+52*A*a^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b-14*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-24*C*a^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b+96*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-144*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-3*A*cos(d*x
+c)^2*b^3-17*A*cos(d*x+c)^3*a*b^2-24*C*cos(d*x+c)^2*a^2*b-48*C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c))*(cos
(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c
)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+Aa\cos\left(dx\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3*sec(d*x + c)^3 + C*a*cos(d*x + c)^3*sec(d*x +
c)^2 + A*b*cos(d*x + c)^3*sec(d*x + c) + A*a*cos(d*x + c)^3)*sqrt(b*sec(d*x
 + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3,
 x)
```

3.724 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=583

$$\frac{\sqrt{a+b}\left(a^2(52Ab+80bC)+8a^3(3A+4C)+2aAb^2-3Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin\right.}{64a^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b^2 - 4*a^2*(13*A + 20*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2
*d) + (Sqrt[a + b]*(2*a*A*b^2 - 3*A*b^3 + 8*a^3*(3*A + 4*C) + a^2*(52*A*b +
 80*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(64*a^2*d) - (Sqrt[a + b]*(3*A*b^4 + 24*a^2*b^2*(A
+ 2*C) + 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^3*d) - (b*(3*A
*b^2 - 4*a^2*(13*A + 20*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(64*a^2*
d) + ((A*b^2 + 4*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(32*a*d) + (A*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4
*d)
```

Rubi [A] time = 1.54025, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4095, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{64a^2d}+\frac{\left(4a^2(3A+4C)+Ab^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\,\text{s}}}{32ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b^2 - 4*a^2*(13*A + 20*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2
*d) + (Sqrt[a + b]*(2*a*A*b^2 - 3*A*b^3 + 8*a^3*(3*A + 4*C) + a^2*(52*A*b +
 80*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(64*a^2*d) - (Sqrt[a + b]*(3*A*b^4 + 24*a^2*b^2*(A
+ 2*C) + 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^3*d) - (b*(3*A
*b^2 - 4*a^2*(13*A + 20*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(64*a^2*
d) + ((A*b^2 + 4*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(32*a*d) + (A*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4
*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
```

```
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
```

```
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d}+\frac{1}{4}$$

$$= \frac{Ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{8d}+\frac{A\,c}{}$$

$$= \frac{\left(Ab^2+4a^2(3A+4C)\right)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{32ad}$$

$$= -\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\sqrt{a+b\sec(c+dx)}\sin(}{64a^2d}$$

$$= -\frac{b\left(3Ab^2-4a^2(13A+20C)\right)\sqrt{a+b\sec(c+dx)}\sin(}{64a^2d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(A\left(52-\frac{3b^2}{a^2}\right)+80C\right)\cot(c+dx)E\left(\sin\right.}{6}$$

$$= \frac{(a-b)\sqrt{a+b}\left(A\left(52-\frac{3b^2}{a^2}\right)+80C\right)\cot(c+dx)E\left(\sin\right.}{6}$$

Mathematica [A] time = 14.871, size = 651, normalized size = 1.12

$$\frac{\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)\left(\frac{\left(16a^2A+16a^2C+Ab^2\right)\sin(2(c+dx))}{32a}+\frac{1}{16}aA\sin(4(c+dx))+\frac{3}{16}Ab\sin\right.}{d(a\cos(c+dx)+b)(A\cos(2c+2dx)+A+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((3*A*b*S
in[c + d*x])/16 + ((16*a^2*A + A*b^2 + 16*a^2*C)*Sin[2*(c + d*x)])/(32*a) +
 (3*A*b*Sin[3*(c + d*x)])/16 + (a*A*Sin[4*(c + d*x)])/16))/(d*(b + a*Cos[c
+ d*x])*(A + 2*C + A*Cos[2*c + 2*d*x])) - (Cos[c + d*x]^5*(a + b*Sec[c + d*
x])^(3/2)*(A + C*Sec[c + d*x]^2)*(-(a*b*(a + b)*(-3*A*b^2 + a^2*(52*A + 80*
C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(-6*a
*A*b^2 + 3*A*b^3 + 8*a^3*(3*A + 4*C) + 4*a^2*b*(7*A + 12*C))*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*A*b^4 + 24*a^2*b^2*(A + 2*C) +
16*a^4*(3*A + 4*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Se
c[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] -
a*b*(-3*A*b^2 + a^2*(52*A + 80*C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(
```

```
c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(32*a^3*d*(b + a*Cos[c
 + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(3/2))
```

Maple [B] time = 0.615, size = 3798, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/64/d/a^2*(-1+cos(d*x+c))^2*(96*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+8*A*a^4*cos(d*x+c)^4-24*A*a^4*
cos(d*x+c)^2-3*A*cos(d*x+c)^2*b^4+6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+52*A*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-3*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+24*
A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*si
n(d*x+c)-76*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^2*sin(d*x+c)+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-64*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+128*C*a^4*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+36*A*cos(d*x+c)^
3*a^3*b-A*cos(d*x+c)^3*a*b^3-52*A*cos(d*x+c)^2*a^3*b+26*A*cos(d*x+c)^2*a^2*
b^2-24*A*cos(d*x+c)*a^3*b-52*A*cos(d*x+c)*a^2*b^2-2*A*cos(d*x+c)*a*b^3+16*A
*a^4*cos(d*x+c)^6+3*A*cos(d*x+c)^2*a*b^3+26*A*cos(d*x+c)^4*a^2*b^2+32*C*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^3*b+80*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+80*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+96*C*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^2*b^2-128*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^2*sin(d*x+c)-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-48*A*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+48*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2
*b^2+52*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*b+52*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)
```

```
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-3*A*cos(d*x+c)*b^3*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+
24*A*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b-76*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*A*cos(d*x+c)*b^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+112*C
*cos(d*x+c)^3*a^3*b+80*C*cos(d*x+c)^2*a^2*b^2+40*A*cos(d*x+c)^5*a^3*b-80*C*
cos(d*x+c)^2*a^3*b-32*C*cos(d*x+c)*a^3*b-80*C*cos(d*x+c)*a^2*b^2+96*A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*a^4+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-48*A*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^4+48*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*a^2*b^2*sin(d*x+c)+52*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-32*C*cos(d*x+c)^2*a^4-128*C*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2-64*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*a^4+128*C*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)+32*C*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+80*C*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x
+c)+80*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b^2*sin(d*x+c)+96*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+32*C*cos(d*x+c)^4*a^4+3*A*cos(d*x+c)*b^
4)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/si
n(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^4,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^4\sec(dx+c)^3 + Ca\cos(dx+c)^4\sec(dx+c)^2 + Ab\cos(dx+c)^4\sec(dx+c) + Aa\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4*sec(d*x + c)^3 + C*a*cos(d*x + c)^4*sec(d*x +
c)^2 + A*b*cos(d*x + c)^4*sec(d*x + c) + A*a*cos(d*x + c)^4)*sqrt(b*sec(d*x
 + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^4,
 x)
```

3.725 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=650

$$\frac{2(a-b)\sqrt{a+b}\left(10a^3b^2(143A+94C)+15a^2b^3(1573A+1175C)+180a^4bC+240a^5C-6ab^4(2717A+2174C)+161\right.}{45045b^4d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(14
3*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(45045*b^5*d) +
(2*(a - b)*Sqrt[a + b]*(240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) +
10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) - 6*a*b^4*(2717*A
+ 2174*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(45045*b^4*d) + (2*a*(120*a^4*C + 5*a^2*b^2*(143*A
 + 79*C) + b^4*(23309*A + 18973*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/
(45045*b^3*d) - (2*(90*a^4*C - 539*b^4*(13*A + 11*C) - 15*a^2*b^2*(715*A +
543*C))*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(45045*b^2*d) +
 (2*a*(2717*A*b^2 + 15*a^2*C + 2209*b^2*C)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c
+ d*x]]*Tan[c + d*x])/(9009*b*d) + (2*(15*a^2*C + 11*b^2*(13*A + 11*C))*Sec
[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(1287*d) + (10*a*C*Sec[c
 + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(143*d) + (2*C*Sec[c + d
*x]^3*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(13*d)
```

Rubi [A] time = 2.77039, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4097, 4096, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(15a^2C+11b^2(13A+11C)\right)\tan(c+dx)\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}}{1287d}+\frac{2a\left(15a^2C+2717Ab^2+2209b^2C\right)\tan(c}{9009b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(14
3*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(45045*b^5*d) +
(2*(a - b)*Sqrt[a + b]*(240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) +
10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) - 6*a*b^4*(2717*A
+ 2174*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(45045*b^4*d) + (2*a*(120*a^4*C + 5*a^2*b^2*(143*A
 + 79*C) + b^4*(23309*A + 18973*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/
(45045*b^3*d) - (2*(90*a^4*C - 539*b^4*(13*A + 11*C) - 15*a^2*b^2*(715*A +
543*C))*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(45045*b^2*d) +
 (2*a*(2717*A*b^2 + 15*a^2*C + 2209*b^2*C)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c
+ d*x]]*Tan[c + d*x])/(9009*b*d) + (2*(15*a^2*C + 11*b^2*(13*A + 11*C))*Sec
[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(1287*d) + (10*a*C*Sec[c
 + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(143*d) + (2*C*Sec[c + d
*x]^3*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(13*d)
```


Rule 4097

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n*
Simp[a*A*(m + n + 1) + a*C*n + b*(A*(m + n + 1) + C*(m + n))*Csc[e + f*x] +
 a*C*m*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&
NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{13d}+\frac{2}{1}$$

$$= \frac{10aC\sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{143d}+$$

$$= \frac{2\left(15a^2C+11b^2(13A+11C)\right)\sec^3(c+dx)\sqrt{a+b\sec}}{1287d}$$

$$= \frac{2a\left(2717Ab^2+15a^2C+2209b^2C\right)\sec^2(c+dx)\sqrt{a+b}}{9009bd}$$

$$= -\frac{2\left(90a^4C-539b^4(13A+11C)-15a^2b^2(715A+543\right.}{45045b}$$

$$= \frac{2a\left(120a^4C+5a^2b^2(143A+79C)+b^4(23309A+18\right.}{45045b^3d}$$

$$= \frac{2a\left(120a^4C+5a^2b^2(143A+79C)+b^4(23309A+18\right.}{45045b^3d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(240a^6C-1617b^6(13A+11C)+10a^4\right.}{}$$

Mathematica [B] time = 26.4095, size = 4418, normalized size = 6.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*((4*(-143
0*a^4*A*b^2 + 39897*a^2*A*b^4 + 21021*A*b^6 - 240*a^6*C - 760*a^4*b^2*C + 3
0669*a^2*b^4*C + 17787*b^6*C)*Sin[c + d*x])/(45045*b^4) + (4*Sec[c + d*x]^4
*(143*A*b^2*Sin[c + d*x] + 159*a^2*C*Sin[c + d*x] + 121*b^2*C*Sin[c + d*x])
)/1287 + (4*Sec[c + d*x]^3*(2717*a*A*b^2*Sin[c + d*x] + 15*a^3*C*Sin[c + d*
x] + 2209*a*b^2*C*Sin[c + d*x]))/(9009*b) + (4*Sec[c + d*x]^2*(10725*a^2*A*
b^2*Sin[c + d*x] + 7007*A*b^4*Sin[c + d*x] - 90*a^4*C*Sin[c + d*x] + 8145*a
```

^2*b^2*C*Sin[c + d*x] + 5929*b^4*C*Sin[c + d*x]))/(45045*b^2) + (4*Sec[c + d*x]*(715*a^3*A*b^2*Sin[c + d*x] + 23309*a*A*b^4*Sin[c + d*x] + 120*a^5*C*Sin[c + d*x] + 395*a^3*b^2*C*Sin[c + d*x] + 18973*a*b^4*C*Sin[c + d*x]))/(45045*b^3) + (108*a*b*C*Sec[c + d*x]^4*Tan[c + d*x])/143 + (4*b^2*C*Sec[c + d*x]^5*Tan[c + d*x])/13))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])) + (4*((4*a^4*A)/(63*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (62*a^2*A*b)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (14*A*b^3)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (32*a^6*C)/(3003*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (304*a^4*C)/(9009*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (20446*a^2*b*C)/(15015*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (154*b^3*C)/(195*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (248*a^3*A*Sqrt[Sec[c + d*x]])/(315*Sqrt[b + a*Cos[c + d*x]]) + (4*a^5*A*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) + (76*a*A*b^2*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) - (27968*a^3*C*Sqrt[Sec[c + d*x]])/(45045*Sqrt[b + a*Cos[c + d*x]]) + (32*a^7*C*Sqrt[Sec[c + d*x]])/(3003*b^4*Sqrt[b + a*Cos[c + d*x]]) + (40*a^5*C*Sqrt[Sec[c + d*x]])/(1287*b^2*Sqrt[b + a*Cos[c + d*x]]) + (8696*a*b^2*C*Sqrt[Sec[c + d*x]])/(15015*Sqrt[b + a*Cos[c + d*x]]) - (62*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (4*a^5*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (14*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (20446*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15015*Sqrt[b + a*Cos[c + d*x]]) + (32*a^7*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3003*b^4*Sqrt[b + a*Cos[c + d*x]]) + (304*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(9009*b^2*Sqrt[b + a*Cos[c + d*x]]) - (154*a*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(195*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*((a + b)*((240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) - 10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(2717*A + 2174*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(45045*b^4*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(9/2)*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*((a + b)*((240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) - 10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(2717*A + 2174*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(45045*b^4*(b + a*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((a + b)*((240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) - 10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(2717*A + 2174*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(15015*b^4*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*((a + b)*((240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-2

```
40*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) - 10*a^3*b^2*(143*A + 94*C)
 + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(2717*A + 2174*C))*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(
3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] +
 (240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^
4*(13299*A + 10223*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4
*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos
[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(45045*b^4*Sqrt[b + a*Cos[c + d
*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) + (4
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((240*a^6*C - 1617*b^6*(13*A + 11*C
) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - a*(240*a^6*C - 1617*b^6*(13*A
 + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Cos[c
 + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - (240*a^6*C - 161
7*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 1022
3*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2]
 + 2*(240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^
2*b^4*(13299*A + 10223*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^4*Tan[(c + d*x)/2]^2 + (3*(a + b)*((240*a^6*C - 1617*b^6*(13*A + 11*C) +
 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-240*a^5*C + 180*a^4*b*C + 1617*
b^5*(13*A + 11*C) - 10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C
) + 6*a*b^4*(2717*A + 2174*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x
]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + ((a + b)*((240*
a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(132
99*A + 10223*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(
-240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) - 10*a^3*b^2*(143*A + 94*
C) + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(2717*A + 2174*C))*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(3/2)*Sec[c + d*x]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + (a + b)*(Cos[c + d*x]*Se
c[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b)]*Sec[c + d*x]*((b*(-240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*C) -
10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(2717*A
+ 2174*C))*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a
 - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((240*a^6*C - 1617*b^6*(13*A + 11*C)
+ 10*a^4*b^2*(143*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - Tan[(c + d
*x)/2]^2])) + (a + b)*((240*a^6*C - 1617*b^6*(13*A + 11*C) + 10*a^4*b^2*(14
3*A + 76*C) - 3*a^2*b^4*(13299*A + 10223*C))*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] + b*(-240*a^5*C + 180*a^4*b*C + 1617*b^5*(13*A + 11*
C) - 10*a^3*b^2*(143*A + 94*C) + 15*a^2*b^3*(1573*A + 1175*C) + 6*a*b^4*(27
17*A + 2174*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[
c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))/(45045*b^4*Sqrt[b + a*Cos[c + d
*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 3.911, size = 6077, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

[Out] result too large to display

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^7 + 2\,Cab\sec(dx+c)^6 + 2\,Aab\sec(dx+c)^4 + Aa^2\sec(dx+c)^3 + \left(Ca^2 + Ab^2\right)\sec(dx+c)^5\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^7 + 2*C*a*b*sec(d*x + c)^6 + 2*A*a*b*sec(d*x +
 c)^4 + A*a^2*sec(d*x + c)^3 + (C*a^2 + A*b^2)*sec(d*x + c)^5)*sqrt(b*sec(d
*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(b\sec(dx+c) + a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3,
 x)
```

3.726 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=534

$$-\frac{2(a-b)\sqrt{a+b}\left(3a^2b^2(33A+19C)+6a^3bC+8a^4C-6ab^3(132A+101C)+15b^4(11A+9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c\cdot}{a+b}}}{693b^3d}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A
 + 247*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(693*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^4*C + 6*
a^3*b*C + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A + 19*C) - 6*a*b^3*(132*A +
101*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(693*b^3*d) + (2*(8*a^4*C + 15*b^4*(11*A + 9*C) + 3*a
^2*b^2*(33*A + 19*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b^2*d) +
(2*a*(99*A*b^2 + 8*a^2*C + 67*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(693*b^2*d) + (2*(8*a^2*C + 9*b^2*(11*A + 9*C))*(a + b*Sec[c + d*x])^(5/
2)*Tan[c + d*x])/(693*b^2*d) - (8*a*C*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*
x])/(99*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])
/(11*b*d)
```

Rubi [A] time = 1.48963, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4093, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2C+9b^2(11A+9C)\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2C+99Ab^2+67b^2C\right)\tan(c+dx)(a+b\sec(c+}{693b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A
 + 247*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(693*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^4*C + 6*
a^3*b*C + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A + 19*C) - 6*a*b^3*(132*A +
101*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(693*b^3*d) + (2*(8*a^4*C + 15*b^4*(11*A + 9*C) + 3*a
^2*b^2*(33*A + 19*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b^2*d) +
(2*a*(99*A*b^2 + 8*a^2*C + 67*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(693*b^2*d) + (2*(8*a^2*C + 9*b^2*(11*A + 9*C))*(a + b*Sec[c + d*x])^(5/
2)*Tan[c + d*x])/(693*b^2*d) - (8*a*C*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*
x])/(99*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])
/(11*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
```

```
e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd} + \frac{2}{\ldots}$$

$$= -\frac{8aC(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d} + \frac{2C\sec(c+\ldots}{\ldots}$$

$$= \frac{2\left(8a^2C+9b^2(11A+9C)\right)(a+b\sec(c+dx))^{5/2}\tan(\ldots}{693b^2d}$$

$$= \frac{2a\left(99Ab^2+8a^2C+67b^2C\right)(a+b\sec(c+dx))^{3/2}\tan\ldots}{693b^2d}$$

$$= \frac{2\left(8a^4C+15b^4(11A+9C)+3a^2b^2(33A+19C)\right)\sqrt{a\ldots}}{693b^2d}$$

$$= \frac{2\left(8a^4C+15b^4(11A+9C)+3a^2b^2(33A+19C)\right)\sqrt{a\ldots}}{693b^2d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\left(8a^4C+3a^2b^2(33A+17C)+3b^4(3\ldots\right.}{\ldots}$$

Mathematica [B] time = 26.6819, size = 3989, normalized size = 7.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*((4*a*(99
*a^2*A*b^2 + 957*A*b^4 + 8*a^4*C + 51*a^2*b^2*C + 741*b^4*C)*Sin[c + d*x])/
(693*b^3) + (4*Sec[c + d*x]^3*(99*A*b^2*Sin[c + d*x] + 113*a^2*C*Sin[c + d*
x] + 81*b^2*C*Sin[c + d*x]))/693 + (4*Sec[c + d*x]^2*(297*a*A*b^2*Sin[c + d
*x] + 3*a^3*C*Sin[c + d*x] + 229*a*b^2*C*Sin[c + d*x]))/(693*b) + (4*Sec[c
+ d*x]*(297*a^2*A*b^2*Sin[c + d*x] + 165*A*b^4*Sin[c + d*x] - 4*a^4*C*Sin[c
 + d*x] + 205*a^2*b^2*C*Sin[c + d*x] + 135*b^4*C*Sin[c + d*x]))/(693*b^2) +
 (92*a*b*C*Sec[c + d*x]^3*Tan[c + d*x])/99 + (4*b^2*C*Sec[c + d*x]^4*Tan[c
+ d*x])/11))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])) - (4
*((-2*a^3*A)/(7*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (58*a*A*b^2)
/(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (34*a^3*C)/(231*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (16*a^5*C)/(693*b^2*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (494*a*b^2*C)/(231*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) - (2*a^4*A*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c +
 d*x]]) - (4*a^2*A*b*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (1
0*A*b^3*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (16*a^6*C*Sqrt[
Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (14*a^4*C*Sqrt[Sec[c +
d*x]])/(99*b*Sqrt[b + a*Cos[c + d*x]]) - (52*a^2*b*C*Sqrt[Sec[c + d*x]])/(2
31*Sqrt[b + a*Cos[c + d*x]]) + (30*b^3*C*Sqrt[Sec[c + d*x]])/(77*Sqrt[b + a
*Cos[c + d*x]]) - (2*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b
 + a*Cos[c + d*x]]) - (58*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*
Sqrt[b + a*Cos[c + d*x]]) - (16*a^6*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (34*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(231*b*Sqrt[b + a*Cos[c + d*x]]) - (494*a^2*b*C*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(2*a*(a + b
)*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*Sqrt[Cos[c +
```

```
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8
*a^4*C - 6*a^3*b*C + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A + 19*C) + 6*a*b^
3*(132*A + 101*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247
*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
)/(693*b^3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec
[(c + d*x)/2]^2]*Sec[c + d*x]^(9/2)*((-2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sin[c + d*x]*(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*
(319*A + 247*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] - 2*b*(a + b)*(8*a^4*C - 6*a^3*b*C + 15*b^4*(11*A + 9*C) + 3*
a^2*b^2*(33*A + 19*C) + 6*a*b^3*(132*A + 101*C))*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(33*
A + 17*C) + 3*b^4*(319*A + 247*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2]))/(693*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Se
c[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)
/2]*(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
 2*b*(a + b)*(8*a^4*C - 6*a^3*b*C + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A +
 19*C) + 6*a*b^3*(132*A + 101*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b
^4*(319*A + 247*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(693*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]
) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((a*(8*a^4*C + 3*a^2*b^2*(33*A
 + 17*C) + 3*b^4*(319*A + 247*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^4)/2 + (a*(a + b)*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319
*A + 247*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])
/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^4*C - 6*a^3*b*C + 15*b^4*(11*A + 9*C
) + 3*a^2*b^2*(33*A + 19*C) + 6*a*b^3*(132*A + 101*C))*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d
*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)
*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^4*C - 6*a^3*b*C + 15*b^4*(11
*A + 9*C) + 3*a^2*b^2*(33*A + 19*C) + 6*a*b^3*(132*A + 101*C))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x
])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))] - a^2*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3
*b^4*(319*A + 247*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
 d*x)/2] - a*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + a*(8*
a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*Cos[c + d*x]*(b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(8*a^4*C
 - 6*a^3*b*C + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(33*A + 19*C) + 6*a*b^3*(132
*A + 101*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a*(a + b)*(8*a^4*
C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A + 247*C))*Sqrt[Cos[c + d*x]/(1 +
```

```
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec
[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Ta
n[(c + d*x)/2]^2]))/(693*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]
^2]) - (2*(2*a*(a + b)*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C) + 3*b^4*(319*A +
247*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] - 2*b*(a + b)*(8*a^4*C - 6*a^3*b*C + 15*b^4*(11*A + 9*C) + 3*a^2*b^2*(
33*A + 19*C) + 6*a*b^3*(132*A + 101*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4*C + 3*a^2*b^2*(33*A + 17*C)
 + 3*b^4*(319*A + 247*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(693*b^3*Sqrt[b + a*Cos[c +
d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 2.224, size = 4695, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/693/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-741*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+8*C*cos(d*x+c)^7*a^6-140*C*cos(d*x+c
)^5*a^3*b^3-566*C*cos(d*x+c)^5*a*b^5-594*A*cos(d*x+c)^4*a^2*b^4-51*C*cos(d*
x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^3*b^3-741*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4-741*C*cos(d*x+c)^6*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+99*A*
cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*sin(d*x+c)*a^3*b^3+891*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+957*A*cos(d*x+c)^5*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5
-99*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^4*b^2-99*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3-957*A*cos(d*x+c)^5*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^2*b^4-957*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+8*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+2*C*cos(d*x+c)^5*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a^4*b^2+51*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
```

cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3+663*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+741*C*cos(d
x+c)^5(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b^5-8*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b-51*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d
x+c)+1))^(1/2)(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-51*C*cos(
d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^3*b^3-741*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+99*A*cos(d*x+c)^6*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3+891*
A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^2*b^4+957*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5-99*A*cos(d*x+c)^6*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^
2-99*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^3*b^3-957*A*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4-957*A*cos(d*x+c)^6
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b^5+8*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^5*b+2*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2+51*C*cos(d*x+c)^6
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^3*b^3+663*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+741*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d
x+c)+1))^(1/2)(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5-8*C*cos(d*x
+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^5*b-51*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2+165*A*cos(d*x+c)^6*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^6+135*C*cos(d
x+c)^6(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*b^6-8*C*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^6+165*A*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^6+135*C*cos(d*x+c)
^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+

```
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*b^6-8*C*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^6+99*A*cos(d*x+c)^7*a^4*b^2+297*A*cos(d*x+c)^7*
a^3*b^3+957*A*cos(d*x+c)^7*a^2*b^4+165*A*cos(d*x+c)^7*a*b^5-4*C*cos(d*x+c)^
7*a^5*b+51*C*cos(d*x+c)^7*a^4*b^2-224*C*cos(d*x+c)*a*b^5+C*cos(d*x+c)^4*a^4
*b^2-160*C*cos(d*x+c)^4*a^2*b^4-396*A*cos(d*x+c)^3*a*b^5-116*C*cos(d*x+c)^3
*a^3*b^3-86*C*cos(d*x+c)^3*a*b^5-274*C*cos(d*x+c)^2*a^2*b^4+205*C*cos(d*x+c
)^7*a^3*b^3+741*C*cos(d*x+c)^7*a^2*b^4+135*C*cos(d*x+c)^7*a*b^5-99*A*cos(d*
x+c)^6*a^4*b^2+99*A*cos(d*x+c)^6*a^3*b^3-363*A*cos(d*x+c)^6*a^2*b^4+957*A*c
os(d*x+c)^6*a*b^5+8*C*cos(d*x+c)^6*a^5*b-52*C*cos(d*x+c)^6*a^4*b^2+51*C*cos
(d*x+c)^6*a^3*b^3-307*C*cos(d*x+c)^6*a^2*b^4+741*C*cos(d*x+c)^6*a*b^5-396*A
*cos(d*x+c)^5*a^3*b^3-726*A*cos(d*x+c)^5*a*b^5-4*C*cos(d*x+c)^5*a^5*b-8*C*c
os(d*x+c)^6*a^6+165*A*cos(d*x+c)^6*b^6+135*C*cos(d*x+c)^6*b^6-66*A*cos(d*x+
c)^4*b^6-54*C*cos(d*x+c)^4*b^6-99*A*cos(d*x+c)^2*b^6-18*C*cos(d*x+c)^2*b^6-
63*C*b^6)/(b+a*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\mathrm{integral}\left(\left(Cb^2\sec\left(dx+c\right)^6+2\,Cab\sec\left(dx+c\right)^5+2\,Aab\sec\left(dx+c\right)^3+Aa^2\sec\left(dx+c\right)^2+\left(Ca^2+Ab^2\right)\sec\left(dx+\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^6 + 2*C*a*b*sec(d*x + c)^5 + 2*A*a*b*sec(d*x +
 c)^3 + A*a^2*sec(d*x + c)^2 + (C*a^2 + A*b^2)*sec(d*x + c)^4)*sqrt(b*sec(d
*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2,
 x)
```

3.727 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=454

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2b(21A+11C)+10a^3C-6ab^2(28A+19C)+21b^3(9A+7C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(}{}}}{315b^2d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A +
93*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(315*b^3*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3*C + 21*b^
3*(9*A + 7*C) + 15*a^2*b*(21*A + 11*C) - 6*a*b^2*(28*A + 19*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^2*d) + (4*a*(84*A*b^2 - 5*a^2*C + 57*b^2*C)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(315*b*d) - (2*(10*a^2*C - 7*b^2*(9*A + 7*C))*(a + b*Sec[
c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) - (4*a*C*(a + b*Sec[c + d*x])^(5/2)
*Tan[c + d*x])/(63*b*d) + (2*C*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*
b*d)
```

Rubi [A] time = 1.03208, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4083, 4002, 4005, 3832, 4004}

$$-\frac{2\left(10a^2C-7b^2(9A+7C)\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}+\frac{4a\left(-5a^2C+84Ab^2+57b^2C\right)\tan(c+dx)\sqrt{a+b\sec}}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A +
93*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(315*b^3*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3*C + 21*b^
3*(9*A + 7*C) + 15*a^2*b*(21*A + 11*C) - 6*a*b^2*(28*A + 19*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^2*d) + (4*a*(84*A*b^2 - 5*a^2*C + 57*b^2*C)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(315*b*d) - (2*(10*a^2*C - 7*b^2*(9*A + 7*C))*(a + b*Sec[
c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) - (4*a*C*(a + b*Sec[c + d*x])^(5/2)
*Tan[c + d*x])/(63*b*d) + (2*C*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*
b*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)(a \cdots}{\cdots}$$

$$= -\frac{4aC(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd} + \frac{2C(a+b\sec(c \cdots}{\cdots}$$

$$= -\frac{2\left(10a^2C-7b^2(9A+7C)\right)(a+b\sec(c+dx))^{3/2}\tan(c+ \cdots}{315bd}$$

$$= \frac{4a\left(84Ab^2-5a^2C+57b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+d \cdots}{315bd}$$

$$= \frac{4a\left(84Ab^2-5a^2C+57b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+d \cdots}{315bd}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(10a^4C-21b^4(9A+7C)-3a^2b^2(161A+ \cdots\right.}{\cdots}$$

Mathematica [A] time = 22.2411, size = 710, normalized size = 1.56

$$4\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+ \cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^
(5/2)*(A + C*Sec[c + d*x]^2)*((a + b)*((10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a
^2*b^2*(161*A + 93*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 + b*(-10*a^3*C + 21*b^3*(9*A + 7*C) + 15*a^2*b*(21*A + 11*C) + 6*a*b^2*(28
*A + 19*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)]*Sec[c + d*x] + (10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*
A + 93*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*
x)/2]))/(315*b^2*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x])^3*(A
+ 2*C + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(9/2))
+ (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*((4*(48
3*a^2*A*b^2 + 189*A*b^4 - 10*a^4*C + 279*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x
])/(315*b^2) + (4*Sec[c + d*x]^2*(63*A*b^2*Sin[c + d*x] + 75*a^2*C*Sin[c +
d*x] + 49*b^2*C*Sin[c + d*x]))/315 + (4*Sec[c + d*x]*(231*a*A*b^2*Sin[c + d
*x] + 5*a^3*C*Sin[c + d*x] + 163*a*b^2*C*Sin[c + d*x]))/(315*b) + (76*a*b*C
*Sec[c + d*x]^2*Tan[c + d*x])/63 + (4*b^2*C*Sec[c + d*x]^3*Tan[c + d*x])/9)
)/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 1.669, size = 4333, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 2/315/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-315*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-155*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-315*A*sin
(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b^2-189*A*cos(d*x+c)^5*b^5-105*A*cos(d*x+c)^5*a*b^4+10*C*cos
(d*x+c)^5*a^4*b+199*C*cos(d*x+c)^5*a^3*b^2-279*C*cos(d*x+c)^5*a^2*b^3-65*C*
cos(d*x+c)^5*a*b^4+714*A*cos(d*x+c)^4*a^2*b^3-5*C*cos(d*x+c)^4*a^4*b+272*C*
cos(d*x+c)^4*a^2*b^3+294*A*cos(d*x+c)^3*a*b^4+80*C*cos(d*x+c)^3*a^3*b^2+82*
C*cos(d*x+c)^3*a*b^4-483*A*cos(d*x+c)^6*a^3*b^2-231*A*cos(d*x+c)^6*a^2*b^3-
189*A*cos(d*x+c)^6*a*b^4-5*C*cos(d*x+c)^6*a^4*b-279*C*cos(d*x+c)^6*a^3*b^2-
163*C*cos(d*x+c)^6*a^2*b^3-147*C*cos(d*x+c)^6*a*b^4+170*C*cos(d*x+c)^2*a^2*
b^3+130*C*cos(d*x+c)*a*b^4+483*A*cos(d*x+c)^5*a^3*b^2-483*A*cos(d*x+c)^5*a^
2*b^3-10*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-189*A*cos(d*x+c)^5*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
5+189*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^5-147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*
```

x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-10*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-189*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+189*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-10*C*cos(d*x+c)^5*a^5-147*C*cos(d*x+c)^5*b^5+126*A*cos(d*x+c)^4*b^5+98*C*cos(d*x+c)^4*b^5+63*A*cos(d*x+c)^2*b^5+14*C*cos(d*x+c)^2*b^5+10*C*cos(d*x+c)^6*a^5-279*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-261*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+279*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+279*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-483*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-357*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+483*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+483*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+189*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-155*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-279*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-261*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+279*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+279*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/

```
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^4-483*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-357*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+483*A*cos(
d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*b^2+483*A*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+189*A*cos(d*x+c)^5*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*
C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^4*b+35*C*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^5 + 2\,Cab\sec(dx+c)^4 + 2\,Aab\sec(dx+c)^2 + Aa^2\sec(dx+c) + \left(Ca^2 + Ab^2\right)\sec(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^5 + 2*C*a*b*sec(d*x + c)^4 + 2*A*a*b*sec(d*x +
 c)^2 + A*a^2*sec(d*x + c) + (C*a^2 + A*b^2)*sec(d*x + c)^3)*sqrt(b*sec(d*x
 + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^2+A\right)\left(b\sec \left(dx+c\right)+a\right)^{\frac{5}{2}}\sec \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x
)
```

3.728 $\int (a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=481

$$-\frac{2\sqrt{a+b}\left(-9a^2b(7A+3C)+3a^3C+ab^2(49A+29C)-b^3(7A+5C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Elli}}{21bd}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21
*b^2*d) - (2*Sqrt[a + b]*(3*a^3*C - 9*a^2*b*(7*A + 3*C) - b^3*(7*A + 5*C) +
 a*b^2*(49*A + 29*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c +
 d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/d + (2*(3*a^2*C + b^2*(7*A + 5*C))*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(21*d) + (2*a*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(7*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.83106, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {4057, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\left(3a^2C+b^2(7A+5C)\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{21d}-\frac{2\sqrt{a+b}\left(-9a^2b(7A+3C)+3a^3C+ab^2(49A+29C)-b^3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21
*b^2*d) - (2*Sqrt[a + b]*(3*a^3*C - 9*a^2*b*(7*A + 3*C) - b^3*(7*A + 5*C) +
 a*b^2*(49*A + 29*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c +
 d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/d + (2*(3*a^2*C + b^2*(7*A + 5*C))*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(21*d) + (2*a*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(7*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```


Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx=\frac{2C(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{2}{7}\int (a+b\sec(c+dx))^{3/2}\Big(\ldots$$

$$=\frac{2aC(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2C(a+b\sec(c+dx))^{5/2}\mathrm{t}\ldots}{7d}$$

$$=\frac{2\left(3a^2C+b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d}+\frac{2aC(a\ldots}{}$$

$$=\frac{2\left(3a^2C+b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d}+\frac{2aC(a\ldots}{}$$

$$=-\frac{2a(a-b)\sqrt{a+b}\left(49Ab^2+3a^2C+29b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\ldots\right.\right.}{21b^2d}$$

$$=-\frac{2a(a-b)\sqrt{a+b}\left(49Ab^2+3a^2C+29b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\ldots\right.\right.}{21b^2d}$$

Mathematica [B] time = 25.5112, size = 4087, normalized size = 8.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*((4*a*(49
*A*b^2 + 3*a^2*C + 29*b^2*C)*Sin[c + d*x])/(21*b) + (4*Sec[c + d*x]*(7*A*b^
2*Sin[c + d*x] + 9*a^2*C*Sin[c + d*x] + 5*b^2*C*Sin[c + d*x]))/21 + (12*a*b
*C*Sec[c + d*x]*Tan[c + d*x])/7 + (4*b^2*C*Sec[c + d*x]^2*Tan[c + d*x])/7))
/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x])) + (4*((2*a^3*A)/
(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (14*a*A*b^2)/(3*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a^3*C)/(7*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (58*a*b^2*C)/(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) + (4*a^2*A*b*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) + (2*
A*b^3*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) - (2*a^4*C*Sqrt[Sec[
c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (4*a^2*b*C*Sqrt[Sec[c + d*x]])/
(21*Sqrt[b + a*Cos[c + d*x]]) + (10*b^3*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b +
a*Cos[c + d*x]]) - (14*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt
[b + a*Cos[c + d*x]]) - (2*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*
Sqrt[b + a*Cos[c + d*x]]) - (58*a^2*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]]
)/(21*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a +
 b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(-2*a*(a + b)*(49*A*b^2 + 3*a
^2*C + 29*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + 2*b*(3*a^3*(-7*A + C) + 9*a^2*b*(7*A + 3*C) + b^3*(7*A + 5*
C) + a*b^2*(49*A + 29*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] - 84*a^3*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*
Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21
*b*d*(b + a*Cos[c + d*x])^3*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*
x)/2]^2]*Sec[c + d*x]^(9/2)*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin
[c + d*x]*(-2*a*(a + b)*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(3*a^3*(-7*A + C
```

) + 9*a^2*b*(7*A + 3*C) + b^3*(7*A + 5*C) + a*b^2*(49*A + 29*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 84*a^3*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21*b*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*a*(a + b)*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(3*a^3*(-7*A + C) + 9*a^2*b*(7*A + 3*C) + b^3*(7*A + 5*C) + a*b^2*(49*A + 29*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 84*a^3*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-(a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - (a*(a + b)*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(3*a^3*(-7*A + C) + 9*a^2*b*(7*A + 3*C) + b^3*(7*A + 5*C) + a*b^2*(49*A + 29*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (42*a^3*A*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (a*(a + b)*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(3*a^3*(-7*A + C) + 9*a^2*b*(7*A + 3*C) + b^3*(7*A + 5*C) + a*b^2*(49*A + 29*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (42*a^3*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + a^2*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - a*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(3*a^3*(-7*A + C) + 9*a^2*b*(7*A + 3*C) + b^3*(7*A + 5*C) + a*b^2*(49*A + 29*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (42*a^3*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (a*(a + b)*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(21*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2

```
*(-2*a*(a + b)*(49*A*b^2 + 3*a^2*C + 29*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(3*a^3*(-7*A + C) + 9*a^2
*b*(7*A + 3*C) + b^3*(7*A + 5*C) + a*b^2*(49*A + 29*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 84*a^3*A*b*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(4
9*A*b^2 + 3*a^2*C + 29*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]
) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*b*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.008, size = 3384, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 2/21/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(7*A*cos(d*x+c)^2*b^4-42*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b+21*A*sin(d*x+c)*cos
(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b-42*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b-63*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+21*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3*b+49*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-3*C*cos(d*x+c)^5*a^4+3*C*sin(d*x+c)*
cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^4-5*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^4-7*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+3*C*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-5*
C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^4+56*A*cos(d*x+c)^3*a*b^3+49*A*cos(d*x+c)^4*a^2*b^2+49*A*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^3-49*A*cos(d*x+c)^4*a*b^3-3*C*cos(d*x+c)^4*a^3*b+11*C*cos
(d*x+c)^4*a^2*b^2-29*C*cos(d*x+c)^4*a*b^3+12*C*cos(d*x+c)^3*a^3*b+22*C*cos(
d*x+c)^3*a*b^3+18*C*cos(d*x+c)^2*a^2*b^2+12*C*cos(d*x+c)*a*b^3-49*A*cos(d*x
+c)^5*a^2*b^2-7*A*cos(d*x+c)^5*a*b^3-9*C*cos(d*x+c)^5*a^3*b-29*C*cos(d*x+c)
^5*a^2*b^2-5*C*cos(d*x+c)^5*a*b^3-7*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
```

```
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-63*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2-49*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^3+3*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+29*C*sin(d*x+c)*co
s(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*b^2+29*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-3*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-27*C*sin(d*x+c)*c
os(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b^2-29*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+49*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+49*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^3-49*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+29*C*sin(d*x+c
)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b^2+29*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-27*C*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^2-29*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+3*C*cos(d*x+c)^4*a^4+2*C*cos(d*x+c)^
2*b^4-7*A*cos(d*x+c)^4*b^4-5*C*cos(d*x+c)^4*b^4+3*C*b^4)/(b+a*cos(d*x+c))/c
os(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^4+2\,Cab\sec(dx+c)^3+2\,Aab\sec(dx+c)+Aa^2+\left(Ca^2+Ab^2\right)\sec(dx+c)^2\right)\sqrt{b\sec(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + 2*C*a*b*sec(d*x + c)^3 + 2*A*a*b*sec(d*x +
 c) + A*a^2 + (C*a^2 + A*b^2)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2), x)
```

3.729 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=478

$$\frac{\sqrt{a+b}\left(a^2b(15A-46C)+30a^3C+2ab^2(45A+17C)-6b^3(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellipti}}{15bd}$$

```
[Out] ((a - b)*Sqrt[a + b]*(a^2*(15*A - 46*C) - 6*b^2*(5*A + 3*C))*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(15*b*d) + (Sqrt[a + b]*(a^2*b*(15*A - 46*C) + 30*a^3*C - 6*b^3*(5*A + 3*C)
 + 2*a*b^2*(45*A + 17*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (5*a*A*b*Sqrt[a + b]*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/d + (A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d -
(a*b*(15*A - 16*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) - (b*(5*A
- 2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.809596, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4095, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(a^2b(15A-46C)+30a^3C+2ab^2(45A+17C)-6b^3(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-}\right.}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(a^2*(15*A - 46*C) - 6*b^2*(5*A + 3*C))*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(15*b*d) + (Sqrt[a + b]*(a^2*b*(15*A - 46*C) + 30*a^3*C - 6*b^3*(5*A + 3*C)
 + 2*a*b^2*(45*A + 17*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (5*a*A*b*Sqrt[a + b]*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/d + (A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d -
(a*b*(15*A - 16*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) - (b*(5*A
- 2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} + \int (a+b\sec(c+dx)$$

$$= \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{b(5A-2C)(a+b\,\mathrm{se}}{}$$

$$= \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{ab(15A-16C)\sqrt{a+}}{}$$

$$= \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{ab(15A-16C)\sqrt{a+}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(a^2(15A-46C)-6b^2(5A+3C)\right)\cot(c+d}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(a^2(15A-46C)-6b^2(5A+3C)\right)\cot(c+d}{}$$

Mathematica [B] time = 25.7418, size = 6811, normalized size = 14.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.977, size = 3498, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(30*A*cos(d*x+c)^3*b^3-90*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+150*A*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a^2*b-90*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b+150*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-46*C*cos(d*x+
c)^3*a^3-30*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+30*C*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
```

*a^3+15*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3+30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d
x+c)+1))^(1/2)(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-15*A*cos(d*x+c)^3*a^2*b-
30*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^3+30*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-46*C*cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-18*C*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3+18*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-30*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+90*A*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2-46*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-18*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+46*C*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b+34*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-30*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+90*A*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2-46*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-18*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+46*C*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b+34*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+15*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+15*A*sin(d*x
+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b-15*A*cos(d*x+c)^4*a^3+15*A*cos(d*x+c)^5*a^3+30*A*cos(d*x+c)^2*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3-46*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3-18*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d
x+c)+1))^(1/2)(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+18*C*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+1

```
5*A*cos(d*x+c)^4*a^2*b+18*C*cos(d*x+c)^3*b^3-30*A*cos(d*x+c)^2*b^3-12*C*cos
(d*x+c)^2*b^3+46*C*cos(d*x+c)^4*a^3+30*A*cos(d*x+c)^4*a*b^2+22*C*cos(d*x+c)
^4*a^2*b+18*C*cos(d*x+c)^4*a*b^2-30*A*cos(d*x+c)^3*a*b^2+46*C*cos(d*x+c)^3*
a^2*b+10*C*cos(d*x+c)^3*a*b^2-68*C*cos(d*x+c)^2*a^2*b-28*C*cos(d*x+c)*a*b^2
-6*C*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos(dx+c)\sec(dx+c)^4+2\,Cab\cos(dx+c)\sec(dx+c)^3+2\,Aab\cos(dx+c)\sec(dx+c)+Aa^2\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)*sec(d*x + c)^4 + 2*C*a*b*cos(d*x + c)*sec(d*x
+ c)^3 + 2*A*a*b*cos(d*x + c)*sec(d*x + c) + A*a^2*cos(d*x + c) + (C*a^2 +
A*b^2)*cos(d*x + c)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x
)
```

3.730 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=463

$$\frac{\sqrt{a+b}\left(6a^2(A+12C)+ab(27A-56C)+8b^2(3A+C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{12d}$$

```
[Out] (a*(a - b)*Sqrt[a + b]*(27*A - 56*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*d) + (Sqrt[a + b]*(a
*b*(27*A - 56*C) + 8*b^2*(3*A + C) + 6*a^2*(A + 12*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*d)
 - (Sqrt[a + b]*(15*A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d
) + (5*A*b*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]
*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d) - (b^2*(21*A - 8*C)*Sqrt[a
+ b*Sec[c + d*x]]*Tan[c + d*x])/(12*d)
```

Rubi [A] time = 0.913231, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4095, 4094, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(6a^2(A+12C)+ab(27A-56C)+8b^2(3A+C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (a*(a - b)*Sqrt[a + b]*(27*A - 56*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*d) + (Sqrt[a + b]*(a
*b*(27*A - 56*C) + 8*b^2*(3*A + C) + 6*a^2*(A + 12*C))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*d)
 - (Sqrt[a + b]*(15*A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d
) + (5*A*b*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]
*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d) - (b^2*(21*A - 8*C)*Sqrt[a
+ b*Sec[c + d*x]]*Tan[c + d*x])/(12*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{2d}+\frac{1}{2}\int$$

$$=\frac{5Ab(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d}+\frac{A\cos(c+dx}{}$$

$$=\frac{5Ab(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d}+\frac{A\cos(c+dx}{}$$

$$=\frac{5Ab(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d}+\frac{A\cos(c+dx}{}$$

$$=\frac{a(a-b)\sqrt{a+b}(27A-56C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{}\right.\right.}{12d}$$

$$=\frac{a(a-b)\sqrt{a+b}(27A-56C)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{}\right.\right.}{12d}$$

Mathematica [B] time = 25.9887, size = 4903, normalized size = 10.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] ((Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((28*a*b*C*Sin[c + d*x])/3 + (a
^2*A*Sin[2*(c + d*x)])/2 + (4*b^2*C*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x
])^2) + (((a^3*A)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (6*a*A*b^
2)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*C)/(Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) - (14*a*b^2*C)/(3*Sqrt[b + a*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) + (11*a^2*A*b*Sqrt[Sec[c + d*x]])/(4*Sqrt[b + a*Cos[c +
 d*x]]) + (2*A*b^3*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] + (4*a^2*b*
C*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) + (2*b^3*C*Sqrt[Sec[c +
d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) + (9*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[
c + d*x]])/(4*Sqrt[b + a*Cos[c + d*x]]) - (14*a^2*b*C*Cos[2*(c + d*x)]*Sqrt
[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]))*(a + b*Sec[c + d*x])^(5/2)*(-
2*a*b*(a + b)*(27*A - 56*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(4*a*b^2*(9*A - 7*C) - 4*b^
3*(3*A + C) + 6*a^3*(A + 2*C) - 3*a^2*b*(A + 12*C))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(12
*(15*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 - b*(27*A - 56*C)*Cos[c
 + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(6*d*(b
 + a*Cos[c + d*x])^3*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(5/2)*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-(Tan[(c + d*x)/2]
*(-2*a*b*(a + b)*(27*A - 56*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(4*a*b^2*(9*A - 7*C) - 4
*b^3*(3*A + C) + 6*a^3*(A + 2*C) - 3*a^2*b*(A + 12*C))*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*
(12*(15*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
```

```
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 - b*(27*A - 56*C)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(6*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)^2) + (a*Sin[c + d*x]*(-2*a*b*(a + b)*
(27*A - 56*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sec[(c + d*x)/2]^2 + 4*(4*a*b^2*(9*A - 7*C) - 4*b^3*(3*A + C) +
6*a^3*(A + 2*C) - 3*a^2*b*(A + 12*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(12*(15*A*b^2 + 4
*a^2*(A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 - b*(27*A - 56*C)*Cos[c + d*x]*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(12*(b + a*Cos[c + d*
x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(-1 + Tan[(c + d*x)/2]^2)) - (Tan[(c + d*x)/2]*(-2*a*b*(a + b)*(27*A - 56*C
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*S
ec[(c + d*x)/2]^2 + 4*(4*a*b^2*(9*A - 7*C) - 4*b^3*(3*A + C) + 6*a^3*(A + 2
*C) - 3*a^2*b*(A + 12*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*(12*(15*A*b^2 + 4*a^2*(A + 2*
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Sec[(c + d*x)/2]^2 - b*(27*A - 56*C)*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])))/(4*Sqrt[b + a*Cos[c + d*x]]*(Sec[
(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c +
d*x)/2]^2)) + (-((a*b*(a + b)*(27*A - 56*C)*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - S
in[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (
2*(4*a*b^2*(9*A - 7*C) - 4*b^3*(3*A + C) + 6*a^3*(A + 2*C) - 3*a^2*b*(A + 1
2*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*S
in[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])] - (a*b*(a + b)*(27*A - 56*C)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sec[(c + d*x)/2]^2*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + (
(b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (2*(4*a*b^2*(9*A - 7*C)
- 4*b^3*(3*A + C) + 6*a^3*(A + 2*C) - 3*a^2*b*(A + 12*C))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*S
ec[(c + d*x)/2]^2*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - 2*a*b*(a + b)*(27*A - 56*C)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 4*(4*a*b^2*(9*A - 7*C) - 4*b^3*(3*A + C)
 + 6*a^3*(A + 2*C) - 3*a^2*b*(A + 12*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] +
 (2*(4*a*b^2*(9*A - 7*C) - 4*b^3*(3*A + C) + 6*a^3*(A + 2*C) - 3*a^2*b*(A +
 12*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4)/(Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (a*b*(a + b)*(27*A - 56
*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^4*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2] + a*(-(b*(27*A - 56*C)*Cos[c +
```

```
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 + (6*(15*A*b^2 + 4*a^2*(A +
 2*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c +
 d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))
)/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (6*(15*A*b^2 + 4*a^2*(A + 2*C))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + 12*(15
*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + a*b*(27*
A - 56*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + b
*(27*A - 56*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c
+ d*x)/2] - 2*b*(27*A - 56*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^4*Tan[(c + d*x)/2]^2 - (6*(15*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*Sec[(c + d*x)/2]^4)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2
]^2)*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]))))/(6*Sqrt[b + a*Cos[c
+ d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-
1 + Tan[(c + d*x)/2]^2)) - ((-2*a*b*(a + b)*(27*A - 56*C)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 +
 4*(4*a*b^2*(9*A - 7*C) - 4*b^3*(3*A + C) + 6*a^3*(A + 2*C) - 3*a^2*b*(A +
12*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sec[(c + d*x)/2]^2 + a*(12*(15*A*b^2 + 4*a^2*(A + 2*C))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x
)/2]^2 - b*(27*A - 56*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2]))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(12*Sqrt[b + a*Cos[c + d*x]]
*(Sec[(c + d*x)/2]^2)^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-1 + T
an[(c + d*x)/2]^2)))))/2
```

Maple [B] time = 0.772, size = 3206, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/12/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(-6*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b-48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+24*C*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
3+27*A*cos(d*x+c)^3*a^2*b-90*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2-90*A*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a*b^2+56*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
```

```
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2-27*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+72*A*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2+56*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b+56*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-72*C*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b-56*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2-27*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-56*C*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b
^2-56*C*cos(d*x+c)^2*a*b^2+6*A*cos(d*x+c)^3*a^3-6*A*cos(d*x+c)^5*a^3-24*A*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3-8*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-33*A*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+
c)^2*a^2*b+27*A*cos(d*x+c)^2*a*b^2-27*A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-27*A*b^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a-6*A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b+72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+56*C*a^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-7
2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a^2*b-8*C*cos(d*x+c)^2*b^3-27*A*cos(d*x+c)^3*a*b^2-56*C*cos
(d*x+c)^3*a^2*b-8*C*cos(d*x+c)^3*a*b^2+56*C*cos(d*x+c)^2*a^2*b+64*C*cos(d*x
+c)*a*b^2+12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3-24*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-48*C*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))
^(1/2))*a^3+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3-24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-24*A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
a^3+24*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^3-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
```

```
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*C*b^3)/sin(d*x+c)^5/(b+a*cos
(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4+2\,Cab\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3+2\,Aab\cos\left(dx+c\right)^2\sec\left(dx+c\right)+Aa^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^2*sec(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^2*sec(
d*x + c)^3 + 2*A*a*b*cos(d*x + c)^2*sec(d*x + c) + A*a^2*cos(d*x + c)^2 + (
C*a^2 + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2,
 x)
```

3.731 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=507

$$\frac{\sqrt{a+b}\left(16a^2A+24a^2C+26aAb+144abC+33Ab^2-48b^2C\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-}\right.}{24d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(24*b*d) + (Sqrt[a + b]*(16*a^2*A + 26*a*A*b + 33*A*b^2 + 24*a^2*C + 144*
a*b*C - 48*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (5*b*Sqrt[a + b]*(A*b^2 + 4*a^2*(A
 + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((15*A*b^2 + 8*a^2*(2*A + 3*C)
)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (5*A*b*Cos[c + d*x]*(a +
b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c
 + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.20201, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4095, 4094, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(8a^2(2A+3C)+15Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(16a^2A+24a^2C+26aAb+144abC+33Ab^2-48b^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(24*b*d) + (Sqrt[a + b]*(16*a^2*A + 26*a*A*b + 33*A*b^2 + 24*a^2*C + 144*
a*b*C - 48*b^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (5*b*Sqrt[a + b]*(A*b^2 + 4*a^2*(A
 + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((15*A*b^2 + 8*a^2*(2*A + 3*C)
)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (5*A*b*Cos[c + d*x]*(a +
b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c
 + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \frac{1}{3}\int c$$

$$= \frac{5Ab\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{12d} + \frac{A\,\mathrm{cc}}{}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3b^2(11A-16C)+8a^2(2A+3C)\right)\cot(c+}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3b^2(11A-16C)+8a^2(2A+3C)\right)\cot(c+}{}$$

Mathematica [B] time = 19.7311, size = 1513, normalized size = 2.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*(((a^2*A
+ 24*b^2*C)*Sin[c + d*x])/6 + (13*a*A*b*Sin[2*(c + d*x)])/12 + (a^2*A*Sin[3
*(c + d*x)])/6))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + A*Cos[2*c + 2*d*x]))
+ ((a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x
)/2]^2)^(-1)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 33
*a*A*b^2*Tan[(c + d*x)/2] + 33*A*b^3*Tan[(c + d*x)/2] + 24*a^3*C*Tan[(c + d
*x)/2] + 24*a^2*b*C*Tan[(c + d*x)/2] - 48*a*b^2*C*Tan[(c + d*x)/2] - 48*b^3
*C*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 66*a*A*b^2*Tan[(c + d*x
)/2]^3 - 48*a^3*C*Tan[(c + d*x)/2]^3 + 96*a*b^2*C*Tan[(c + d*x)/2]^3 + 16*a
^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 33*a*A*b^2*Tan[(c
 + d*x)/2]^5 - 33*A*b^3*Tan[(c + d*x)/2]^5 + 24*a^3*C*Tan[(c + d*x)/2]^5 -
24*a^2*b*C*Tan[(c + d*x)/2]^5 - 48*a*b^2*C*Tan[(c + d*x)/2]^5 + 48*b^3*C*Ta
n[(c + d*x)/2]^5 - 120*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^3*EllipticPi[-1, -ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 240*a^2*b*C*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 120*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^3*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 240*a^2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(3*b^2*(11*A - 16*C) + 8*a^
2*(2*A + 3*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(24*b^2*(A - C) - a*b*(13*A +
```

```
72*C) + a^2*(38*A + 72*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(12*d*(b + a*Cos[c +
 d*x])^(5/2)*(A + 2*C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2)*(1 + Tan[(c
+ d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.886, size = 3512, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+48*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-33*A*b^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-24*C*cos(d*x+c)^3*
a^3+48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+48*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-48*C*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+48*C*b^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+48*C*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2-30*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))-24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))-144*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-48*C*cos(d*x+c)^2*a*b^
2-8*A*cos(d*x+c)^3*a^3+16*A*cos(d*x+c)^2*a^3+24*C*cos(d*x+c)^2*a^3+33*A*cos
(d*x+c)*b^3-8*A*cos(d*x+c)^5*a^3-34*A*cos(d*x+c)^4*a^2*b+18*A*cos(d*x+c)^2*
a^2*b+33*A*cos(d*x+c)^2*a*b^2+16*A*cos(d*x+c)*a^2*b+26*A*cos(d*x+c)*a*b^2+2
4*C*cos(d*x+c)*a^2*b-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-33*A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-30*A*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)*b^3-24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-33*A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+76*A*(cos(d*x+c)/
```

```
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-26*A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(
d*x+c)-120*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*a^2*b*sin(d*x+c)-24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+144*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-240*C*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-16*A*a^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b-33*A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+76*A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-26*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a*b^2-120*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-24*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+144*C*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a^2*b-240*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-33*A*cos(d*x+c)^2*b^3-59*A*cos(d*x+c)^3
*a*b^2-24*C*cos(d*x+c)^2*a^2*b+48*C*cos(d*x+c)*a*b^2+48*A*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-4
8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^3+48*C*b^3-144*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-48*C*cos(d*x+c)*b^3)*(cos(d*x+c)+
1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)\left(b\sec(dx+c)+a\right)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3,
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3,
 x)
```

3.732 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=587

$$\frac{\sqrt{a+b}\left(4a^2b(71A+108C)+24a^3(3A+4C)+2ab^2(59A+192C)+15Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{E}}{192ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a
*d) + (Sqrt[a + b]*(15*A*b^3 + 24*a^3*(3*A + 4*C) + 4*a^2*b*(71*A + 108*C)
+ 2*a*b^2*(59*A + 192*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) + (Sqrt[a + b]*(5*A*b^4 -
 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*
a^2*d) + (b*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(192*a*d) + ((5*A*b^2 + 4*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (5*A*b*Cos[c + d*x]^2*(a + b*Sec[c +
 d*x])^(3/2)*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^
(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 1.63304, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4095, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(4a^2(71A+108C)+15Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{\left(4a^2(3A+4C)+5Ab^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\text{se}}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a
*d) + (Sqrt[a + b]*(15*A*b^3 + 24*a^3*(3*A + 4*C) + 4*a^2*b*(71*A + 108*C)
+ 2*a*b^2*(59*A + 192*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) + (Sqrt[a + b]*(5*A*b^4 -
 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*
a^2*d) + (b*(15*A*b^2 + 4*a^2*(71*A + 108*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(192*a*d) + ((5*A*b^2 + 4*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(32*d) + (5*A*b*Cos[c + d*x]^2*(a + b*Sec[c +
 d*x])^(3/2)*Sin[c + d*x])/(24*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^
(5/2)*Sin[c + d*x])/(4*d)
```


Rule 4095

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
```

```
d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m
- a*(C*n + A*(n + 1))*Csc[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
```

```
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{4d}+\frac{1}{4}\int c$$

$$= \frac{5Ab\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{24d}+\frac{A c}{}$$

$$= \frac{\left(5Ab^2+4a^2(3A+4C)\right)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{32d}$$

$$= \frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+}{192ad}$$

$$= \frac{b\left(15Ab^2+4a^2(71A+108C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+}{192ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^2+4a^2(71A+108C)\right)\cot(c+dx)E}{19}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^2+4a^2(71A+108C)\right)\cot(c+dx)E}{19}$$

Mathematica [B] time = 24.139, size = 5006, normalized size = 8.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.644, size = 3986, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 1/192/d/a*(-1+cos(d*x+c))^2*(-288*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-24*A*a^4*cos(d*x+c)^4+72*A*a^4
*cos(d*x+c)^2-15*A*cos(d*x+c)^2*b^4+30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-284*A*(cos(d*x+c)/(cos(d*
```

x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+644*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-118*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+192*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-384*C*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-172*A*cos(d*x+c)^3*a^3*b-133*A*cos(d*x+c)^3*a*b^3+284*A*cos(d*x+c)^2*a^3*b-30*A*cos(d*x+c)^2*a^2*b^2+72*A*cos(d*x+c)*a^3*b+284*A*cos(d*x+c)*a^2*b^2+118*A*cos(d*x+c)*a*b^3-48*A*a^4*cos(d*x+c)^6+15*A*cos(d*x+c)^2*a*b^3-254*A*cos(d*x+c)^4*a^2*b^2-96*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-432*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-432*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-1440*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+1152*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+144*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-720*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2-284*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-284*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-15*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-72*A*cos(d*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+644*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-118*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-528*C*cos(d*x+c)^3*a^3*b-432*C*cos(d*x+c)^2*a^2*b^2-184*A*cos(d*x+c)^5*a^3*b+432*C*cos(d*x+c)^2*a^3*b+96*C*cos(d*x+c)*a^3*b+432*C*cos(d*x+c)*a^2*b^2-288*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4+30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4-15*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),

```
((a-b)/(a+b))^(1/2))*b^4+144*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-720*A*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-284*A*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x
+c)-384*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^3*sin(d*x+c)+96*C*cos(d*x+c)^2*a^4+1152*C*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-384*C*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*co
s(d*x+c)*a*b^3+192*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-384*C*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)-96*C*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-
432*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
b*sin(d*x+c)-432*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2*sin(d*x+c)-1440*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-96*C*cos(d*x+c)^4*a^4+15*A*
cos(d*x+c)*b^4)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*c
os(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{4}+2\,Cab\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+2\,Aab\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+Aa^{2}\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4*sec(d*x + c)^4 + 2*C*a*b*cos(d*x + c)^4*sec(
d*x + c)^3 + 2*A*a*b*cos(d*x + c)^4*sec(d*x + c) + A*a^2*cos(d*x + c)^4 + (
```

```
C*a^2 + A*b^2)*cos(d*x + c)^4*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4,
 x)
```

3.733 $\int (a+b\sec(c+dx))^{3/2}\left(a^2-b^2\sec^2(c+dx)\right)dx$

Optimal. Leaf size=403

$$\frac{2\sqrt{a+b}\left(-4a^2b+10a^3-4ab^2+3b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{5d}-$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(4*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*d) + (2*Sqrt[a + b
]*(10*a^3 - 4*a^2*b - 4*a*b^2 + 3*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*d) - (2*a^3*Sqrt[a
+ b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (2*a*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c
+ d*x])/(5*d) - (2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.569701, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4042, 3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(-4a^2b+10a^3-4ab^2+3b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5d}-\frac{2(a-b)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(a^2 - b^2*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(4*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*d) + (2*Sqrt[a + b
]*(10*a^3 - 4*a^2*b - 4*a*b^2 + 3*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*d) - (2*a^3*Sqrt[a
+ b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (2*a*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c
+ d*x])/(5*d) - (2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\left(a^2-b^2\sec^2(c+dx)\right)dx = -\int(-a+b\sec(c+dx))(a+b\sec(c+dx))^{5/2}dx$$
$$= -\frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} - \frac{2}{5}\int\sqrt{a+b\sec(c+dx)}\Big(-$$
$$= -\frac{2ab^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d} - \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan}{5d}$$
$$= -\frac{2ab^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d} - \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan}{5d}$$
$$= -\frac{2(a-b)\sqrt{a+b}\left(4a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5d}$$
$$= -\frac{2(a-b)\sqrt{a+b}\left(4a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5d}$$

Mathematica [B] time = 15.2999, size = 960, normalized size = 2.38

$$\frac{\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(a^2-b^2\sec^2(c+dx)\right)\left(-\frac{4}{5}\sec(c+dx)\tan(c+dx)b^3-\frac{8}{5}a\tan(c+dx)b^2-\frac{4}{5}\left(3b^2-4a^2\right.\right.}{d(b+a\cos(c+dx))\left(\cos(2c+2dx)a^2+a^2-2b^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] (-4*(a + b*Sec[c + d*x])^(3/2)*(a^2 - b^2*Sec[c + d*x]^2)*(-4*a^3*b*Tan[(c
+ d*x)/2] - 4*a^2*b^2*Tan[(c + d*x)/2] + 3*a*b^3*Tan[(c + d*x)/2] + 3*b^4*T
an[(c + d*x)/2] + 8*a^3*b*Tan[(c + d*x)/2]^3 - 6*a*b^3*Tan[(c + d*x)/2]^3 -
 4*a^3*b*Tan[(c + d*x)/2]^5 + 4*a^2*b^2*Tan[(c + d*x)/2]^5 + 3*a*b^3*Tan[(c
 + d*x)/2]^5 - 3*b^4*Tan[(c + d*x)/2]^5 - 10*a^4*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 10*a^4*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2
]^2)/(a + b)] + b*(-4*a^3 - 4*a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
 d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] - (5*a^4 - 10*a^3*b - 4*a^2*b^2 + 4*a*b^3 + 3*b^4)*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)]))/(5*d*(b + a*Cos[c + d*x])^(3/2)*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x]
)*Sec[c + d*x]^(7/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x
)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^3*(a + b*
Sec[c + d*x])^(3/2)*(a^2 - b^2*Sec[c + d*x]^2)*((-4*b*(-4*a^2 + 3*b^2)*Sin[
c + d*x])/5 - (8*a*b^2*Tan[c + d*x])/5 - (4*b^3*Sec[c + d*x]*Tan[c + d*x])/
5))/(d*(b + a*Cos[c + d*x])*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.694, size = 2169, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(a^2-b^2*sec(d*x+c)^2),x)
```

```
[Out] -2/5/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(-5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*a^4+10*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+4*cos(d*x+c)^4*a^3*b+b^4-
2*cos(d*x+c)^4*a^2*b^2-3*cos(d*x+c)^4*a*b^3-4*cos(d*x+c)^3*a^3*b+4*cos(d*x+
c)^3*a^2*b^2-2*cos(d*x+c)^2*a^2*b^2+3*cos(d*x+c)*a*b^3-4*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^
3-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^3*b-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+3*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+10*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)*a^3*b+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3-4*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3*b-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+10*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a
^3*b+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*a^2*b^2-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^4+3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+10*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^4-5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^4+3*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
b^4-3*cos(d*x+c)^3*b^4+2*cos(d*x+c)^2*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^2/si
n(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(b^2 \sec (dx+c)^2 - a^2\right)(b \sec (dx+c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(a^2-b^2*sec(d*x+c)^2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*sec(d*x + c)^2 - a^2)*(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(b^3 \sec (dx+c)^3 + ab^2 \sec (dx+c)^2 - a^2 b \sec (dx+c) - a^3\right)\sqrt{b \sec (dx+c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(a^2-b^2*sec(d*x+c)^2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^3*sec(d*x + c)^3 + a*b^2*sec(d*x + c)^2 - a^2*b*sec(d*x + c) -
 a^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a - b \sec (c+dx)) (a + b \sec (c+dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(a**2-b**2*sec(d*x+c)**2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))*(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(b^2 \sec (dx+c)^2 - a^2\right)(b \sec (dx+c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(a^2-b^2*sec(d*x+c)^2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*sec(d*x + c)^2 - a^2)*(b*sec(d*x + c) + a)^(3/2), x)
```

3.734 $\int \sqrt{a+b\sec(c+dx)}\left(a^2-b^2\sec^2(c+dx)\right)dx$

Optimal. Leaf size=353

$$\frac{2\sqrt{a+b}\left(3a^2+ab-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}}{}$$

```
[Out] (2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) + (2*Sqrt[a + b]*(3*a^2 + a*b
- b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*d) - (2*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 - (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.402935, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4042, 3918, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(3a^2+ab-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(a^2 - b^2*Sec[c + d*x]^2),x]
```



```
[Out] (2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) + (2*Sqrt[a + b]*(3*a^2 + a*b
- b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*d) - (2*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 - (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\left(a^2-b^2\sec^2(c+dx)\right)dx = -\int (-a+b\sec(c+dx))(a+b\sec(c+dx))^{3/2}\,dx$$

$$= -\frac{2b^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a^3}{2}-\frac{1}{2}b\left(3a^2-b^2\right)\sec(c$$

$$= -\frac{2b^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a^3}{2}+\left(-\frac{ab^2}{2}-\frac{1}{2}b\left(3a^2-b\right.\right.}{\sqrt{a+b\sec(c+}}$$

$$= \frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{3d}$$

$$= \frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{3d}$$

Mathematica [C] time = 12.7076, size = 598, normalized size = 1.69

$$\frac{\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(a^2-b^2\sec^2(c+dx)\right)\left(-\frac{4}{3}ab\sin(c+dx)-\frac{4}{3}b^2\tan(c+dx)\right)}{d\left(a^2\cos(2c+2dx)+a^2-2b^2\right)}-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\cos^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(a^2 - b^2*Sec[c + d*x]^2),x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(a^2 - b^2*S
ec[c + d*x]^2)*((2*I)*a*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (2*I)*(3*a^3 - 3
*a^2*b - a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + (12*I)*a^3*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c +
d*x)/2]], (a + b)/(a - b)] - a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*Sqrt[(-a + b)/(a +
b)]*d*(b + a*Cos[c + d*x])*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])) + (Cos[c +
 d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(a^2 - b^2*Sec[c + d*x]^2)*((-4*a*b*Sin[c
+ d*x])/3 - (4*b^2*Tan[c + d*x])/3))/(d*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x]
))
```

Maple [B] time = 0.505, size = 1508, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-3*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^3-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-6*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*a^3+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
```

```
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^3-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-6*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+c
os(d*x+c)^3*a^2*b+cos(d*x+c)^3*a*b^2-cos(d*x+c)^2*a^2*b+cos(d*x+c)^2*a*b^2+
cos(d*x+c)^2*b^3-2*cos(d*x+c)*a*b^2-b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(b^2\sec\left(dx+c\right)^2-a^2\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*sec(d*x + c)^2 - a^2)*sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2\sec\left(dx+c\right)^2-a^2\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*sec(d*x + c)^2 - a^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a-b\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(b^2\sec\left(dx+c\right)^2-a^2\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*sec(d*x + c)^2 - a^2)*sqrt(b*sec(d*x + c) + a), x)
```

3.735 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=393

$$\frac{2\sqrt{a+b}\left(-12a^2bC+48a^3C+2ab^2(35A+22C)+5b^3(7A+5C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{si}\right.}{105b^4d}$$

```
[Out] (4*a*(a - b)*Sqrt[a + b]*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(10
5*b^5*d) + (2*Sqrt[a + b]*(48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) + 2*a*
b^2*(35*A + 22*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d) + (2*(24*a^2*C + 5*b^2*(7*A +
5*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d) - (12*a*C*Sec[c +
d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (2*C*Sec[c + d*x]^
2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.907902, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4103, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(24a^2C+5b^2(7A+5C)\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^3d}+\frac{2\sqrt{a+b}\left(-12a^2bC+48a^3C+2ab^2(35A+22C)+5b^3(7A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (4*a*(a - b)*Sqrt[a + b]*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(10
5*b^5*d) + (2*Sqrt[a + b]*(48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) + 2*a*
b^2*(35*A + 22*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d) + (2*(24*a^2*C + 5*b^2*(7*A +
5*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d) - (12*a*C*Sec[c +
d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (2*C*Sec[c + d*x]^
2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```


Rule 4103

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*
(m + n + 1)), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[
e + f*x] - a*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2C\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7bd} + \frac{2\int \frac{\sec^2(c+dx)\left(2aC+\frac{1}{2}b(7A+5\right.}{\sqrt{a+b}}}{}$$

$$= -\frac{12aC\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35b^2d} + \frac{2C\sec^2(c+dx)\sqrt{a+}}{}$$

$$= \frac{2\left(24a^2C+5b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} - \frac{12aC\sec(c+}{}$$

$$= \frac{2\left(24a^2C+5b^2(7A+5C)\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} - \frac{12aC\sec(c+}{}$$

$$= \frac{4a(a-b)\sqrt{a+b}\left(35Ab^2+24a^2C+22b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{105b^5d}$$

Mathematica [B] time = 23.6397, size = 3255, normalized size = 8.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],
x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*((-8*a*(35*A*b^2
+ 24*a^2*C + 22*b^2*C)*Sin[c + d*x])/(105*b^4) + (4*Sec[c + d*x]*(35*A*b^2*
Sin[c + d*x] + 24*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b^3) -
(24*a*C*Sec[c + d*x]*Tan[c + d*x])/(35*b^2) + (4*C*Sec[c + d*x]^2*Tan[c + d
*x])/(7*b)))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[a + b*Sec[c + d*x]]) +
(8*((4*a*A)/(3*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (32*a^3*C)/
(35*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (88*a*C)/(105*b*Sqrt
[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[Sec[c + d*x]])/(3*Sqrt
[b + a*Cos[c + d*x]]) + (4*a^2*A*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[
c + d*x]]) + (10*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (32*
a^4*C*Sqrt[Sec[c + d*x]])/(35*b^4*Sqrt[b + a*Cos[c + d*x]]) + (64*a^2*C*Sqr
t[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]) + (4*a^2*A*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) + (32*a^4*C*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^4*Sqrt[b + a*Cos[c + d*x]]) + (88*a^2
*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*(2*a*(a + b)*
(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*
C) - 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + a*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Cos[c + d*x]*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^4*d*(A +
2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)*Sqrt[
a + b*Sec[c + d*x]]*((4*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x
]*(2*a*(a + b)*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3*C - 12*a^2*b*C +
5*b^3*(7*A + 5*C) - 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(35*A*b^2 + 24*a^2*C + 22*b^2*C
)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(
105*b^4*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*a*(a + b)*(35*A*b^2 + 24*a
^2*C + 22*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + b*(-48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) - 2*a*b^2*(35
*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] + a*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[(c + d*x)/2]^2]) + (8*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((a*
(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^4)/2 + (a*(a + b)*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[
c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(-
48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) - 2*a*b^2*(35*A + 22*C))*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2
```

```
 - Sin[c + d*x]/(1 + Cos[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]) + (a*(a + b)*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] + (b*(-48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) - 2*a*b^
2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2
)))/(2*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]) - a^2*(35*A
*b^2 + 24*a^2*C + 22*b^2*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Ta
n[(c + d*x)/2] - a*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*(b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + a*(35*A*b^2 + 24*a^2*C + 2
2*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]^2 + (b*(-48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) - 2*a*b^2*(35*A + 22
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a*(a + b)*(35*A*b^2 + 2
4*a^2*C + 22*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^4*Sqrt
[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*(2*a*(a + b)*(35*A*b^2
+ 24*a^2*C + 22*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + b*(-48*a^3*C - 12*a^2*b*C + 5*b^3*(7*A + 5*C) - 2*a*b
^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + a*(35*A*b^2 + 24*a^2*C + 22*b^2*C)*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c +
 d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(1
05*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.015, size = 2784, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(35*A*cos(d*x+c)^2*b^4-70*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+48*C*cos(d*x+c)^5
*a^4-48*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^4-25*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-35*A*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-4
8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^4-25*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-35*A*cos(d*x+c)^3*a*b^3-70*A*c
```

```
os(d*x+c)^4*a^2*b^2-70*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+70*A*cos(d*x+c)^4*a*b^3+48*C*c
os(d*x+c)^4*a^3*b-50*C*cos(d*x+c)^4*a^2*b^2+44*C*cos(d*x+c)^4*a*b^3-24*C*co
s(d*x+c)^3*a^3*b-16*C*cos(d*x+c)^3*a*b^3+6*C*cos(d*x+c)^2*a^2*b^2-3*C*cos(d
*x+c)*a*b^3+70*A*cos(d*x+c)^5*a^2*b^2-35*A*cos(d*x+c)^5*a*b^3-24*C*cos(d*x+
c)^5*a^3*b+44*C*cos(d*x+c)^5*a^2*b^2-25*C*cos(d*x+c)^5*a*b^3-35*A*sin(d*x+c
)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*b^4+70*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-48*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-44*C*sin(d*x+c)*
cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*b^2-44*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+12*C*sin(d*x+c
)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b^2+44*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-70*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-70*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^3+70*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-44*C*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2-44*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*C*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+12*C*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b^2+44*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-48*C*cos(d*x+c)^4*a^4+10*C*cos(
d*x+c)^2*b^4-35*A*cos(d*x+c)^4*b^4-25*C*cos(d*x+c)^4*b^4+15*C*b^4)/(b+a*cos
(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{3}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + A*sec(d*x + c)^3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{3}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x
)
```

3.736 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=320

$$-\frac{2\sqrt{a+b}\left(8a^2C-2abC+3b^2(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^3d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^2*C + 3*b^2*(5*A + 3*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) - (2*Sqrt[a + b]*(8*a^2*C - 2*a*b*C + 3*b^2*(5*A + 3*C))*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(15*b^3*d) - (8*a*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*
C*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.571993, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4093, 4082, 4005, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(8a^2C-2abC+3b^2(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^2*C + 3*b^2*(5*A + 3*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) - (2*Sqrt[a + b]*(8*a^2*C - 2*a*b*C + 3*b^2*(5*A + 3*C))*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(15*b^3*d) - (8*a*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*
C*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b*d)
```


Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Csc[e + f*x
]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*
(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[a*C + b*(C*(m + 2) +
 A*(m + 3))*Csc[e + f*x] - 2*a*C*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2C\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{2\int \frac{\sec(c+dx)\left(aC+\frac{1}{2}b(5A+3C)\right.}{\sqrt{a+b\sec}}}{5b}$$

$$= -\frac{8aC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2C\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$= -\frac{8aC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2C\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(15A+\left(9+\frac{8a^2}{b^2}\right)C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right|}{15b^2d}$$

Mathematica [B] time = 22.8277, size = 2993, normalized size = 9.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],
x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*((4*(15*A*b^2 + 8
*a^2*C + 9*b^2*C)*Sin[c + d*x])/(15*b^3) - (16*a*C*Tan[c + d*x])/(15*b^2) +
 (4*C*Sec[c + d*x]*Tan[c + d*x])/(5*b)))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*
Sqrt[a + b*Sec[c + d*x]]) - (4*((-2*A)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (6*C)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (16*a^2
*C)/(15*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqrt[Sec[
c + d*x]])/(b*Sqrt[b + a*Cos[c + d*x]]) - (16*a^3*C*Sqrt[Sec[c + d*x]])/(15
```

```
*b^3*Sqrt[b + a*Cos[c + d*x]]) - (14*a*C*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b +
 a*Cos[c + d*x]]) - (2*a*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*Sqrt[b +
 a*Cos[c + d*x]]) - (16*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^3*
Sqrt[b + a*Cos[c + d*x]]) - (6*a*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*
b*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*S
ec[c + d*x]^2)*(2*(a + b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(15*A*b^2 + (8*
a^2 + 2*a*b + 9*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] + (15*A*b^2 + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*d*(A + 2*C + A*C
os[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec
[c + d*x]]*((-2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a
+ b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(15*A*b^2 + (8*a^2 + 2*a*b + 9*b^2)*
C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ (15*A*b^2 + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(
c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]
*(2*(a + b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(15*A*b^2 + (8*a^2 + 2*a*b +
9*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + (15*A*b^2 + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[
Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((15*A*b^2
 + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)
/2 + ((a + b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + C
os[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(15*A*b^2 + (8*a^
2 + 2*a*b + 9*b^2)*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[
c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(15*A*b^2 + (8*a^2 + 2*a*b + 9
*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
+ ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(15*A*b^2 + 8*a^2*C
 + 9*b^2*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] -
 (15*A*b^2 + 8*a^2*C + 9*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin
[c + d*x]*Tan[(c + d*x)/2] + (15*A*b^2 + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(15*A*b^2 + (
8*a^2 + 2*a*b + 9*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a +
 b)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sq
rt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2])
)/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*(2*(a + b
)*(15*A*b^2 + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
```

```
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] - 2*b*(15*A*b^2 + (8*a^2 + 2*a*b + 9*b^2)*C)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (
15*A*b^2 + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]
) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*Sqrt[b + a*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.78, size = 2256, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(15*A*cos(d*x+c)^3*b^3-8*C*cos(d*x+c)^3*a^3-15*A*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
2-15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*C*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*C*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3+9*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-15*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*C*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b-9*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*C*cos(d*x+c)^3
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^2-15*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^2+8*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b+2*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+15*A*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
```

```
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
3-8*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3-9*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*C*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*C*cos
(d*x+c)^3*b^3-15*A*cos(d*x+c)^2*b^3-6*C*cos(d*x+c)^2*b^3+8*C*cos(d*x+c)^4*a
^3+15*A*cos(d*x+c)^4*a*b^2-4*C*cos(d*x+c)^4*a^2*b+9*C*cos(d*x+c)^4*a*b^2-15
*A*cos(d*x+c)^3*a*b^2+8*C*cos(d*x+c)^3*a^2*b-10*C*cos(d*x+c)^3*a*b^2-4*C*co
s(d*x+c)^2*a^2*b+C*cos(d*x+c)*a*b^2-3*C*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/
sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^4 + A\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x
)
```

3.737 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{2\sqrt{a+b}(C(2a+b)+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d}+\frac{4aC(a-b)}{}$$

```
[Out] (4*a*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a + b]*(3*A*b
+ (2*a + b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[
c + d*x])/(3*b*d)
```

Rubi [A] time = 0.322425, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4083, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(C(2a+b)+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{4aC(a-b)\sqrt{a+b}\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (4*a*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a + b]*(3*A*b
+ (2*a + b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[
c + d*x])/(3*b*d)
```


Rule 4083

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*
(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) - a*C*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
```

```
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}b(3A+C)-aC\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}$$

$$= \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} - \frac{(2aC)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{3b} + \frac{(3Ab}{}$$

$$= \frac{4a(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{3b^3d}$$

Mathematica [A] time = 14.7989, size = 409, normalized size = 1.62

$$\frac{8\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(b(C(b-2a)+3Ab)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\text{s}\right.\right.}{3b^2d\sqrt{\sec^2\left(\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (8*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*(2*a*(a + b
)*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 + b*(3*A*b + (-2*a + b)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + a*C*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Se
c[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (Cos[c + d
*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*((-8*a*C*Sin[c + d*x])/(3*b
^2) + (4*C*Tan[c + d*x])/(3*b)))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[a +
 b*Sec[c + d*x]])
```

Maple [B] time = 0.5, size = 1125, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/b^2*(-1+cos(d*x+c))^2*(3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*C*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+2*C*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b-2*C*cos(d*x+c)^3*a^2+C*cos(d*x+c)^3*a*b+2*C*cos(d*x+c)^2*a^2-2*C
*cos(d*x+c)^2*a*b+C*cos(d*x+c)^2*b^2+C*cos(d*x+c)*a*b-b^2*C)*((b+a*cos(d*x+
c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+
c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec\left(c + dx\right)}{\sqrt{a + b\sec\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\sqrt{b\sec\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.738 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=313

$$-\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(}{}}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*C*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.234126, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4059, 3921, 3784, 3832, 4004}

$$-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}-\frac{2C(a-b)\sqrt{a+b}\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*C*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```


Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
```

```
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = C\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{A-C\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx)}{a-b}}}{b^2d}$$

$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx)}{a-b}}}{b^2d}$$

Mathematica [C] time = 16.4139, size = 914, normalized size = 2.92

$$\frac{4C\cos(c+dx)(b+a\cos(c+dx))\left(C\sec^2(c+dx)+A\right)\sin(c+dx)}{bd(\cos(2c+2dx)A+A+2C)\sqrt{a+b\sec(c+dx)}} + \frac{4\sqrt{b+a\cos(c+dx)}\left(C\sec^2(c+dx)+A\right)\sqrt{\frac{}{1-t}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*C*Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + C*Sec[c + d*x]^2)*Sin[c + d*x])
/(b*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[a + b*Sec[c + d*x]]) + (4*Sqrt[b
+ a*Cos[c + d*x]]*(A + C*Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(-(a*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/
2]*Sqrt[1 - Tan[(c + d*x)/2]^2]) - b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)
/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] + a*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)
/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] - b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*
x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] - (2*I)*A*b*EllipticPi[-((a + b)/(a -
b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*
A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b - a*Tan[(c + d*
```

```
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*C*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + I*b*(A + C)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*Sqrt[(-a + b)/(a + b)]*d*(A + 2*
C + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(1 + Ta
n[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.467, size = 1011, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/b*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^
2*(A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+C*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b+A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*b-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*b-C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+C*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-C*cos(d*x+c)^2*a+C*cos(d*x+c)*
a-C*cos(d*x+c)*b+C*b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{\sqrt{b \sec(dx+c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{2}+A}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+C\sec^{2}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+A}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/sqrt(b*sec(d*x + c) + a), x)
```

3.739 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=352

$$\frac{\sqrt{a+b}(2aC+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd}+\frac{Ab\sqrt{a+b}\cot(c+\ldots}{\ldots}$$

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*(A*b + 2*a*C)*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a*b*d) + (A*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) +
 (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.395225, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4105, 4058, 3921, 3784, 3832, 4004}

$$\frac{Ab\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{\sqrt{a+b}(2aC+Ab)\cot(c+dx)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*(A*b + 2*a*C)*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a*b*d) + (A*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) +
 (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{Ab}{2}-aC\sec(c+dx)+\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{Ab}{2}+\left(-\frac{Ab}{2}-aC\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - \frac{(Ab)\int \frac{\sec(c}{}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{abd}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{abd}$$

Mathematica [A] time = 15.8516, size = 386, normalized size = 1.1

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(A\cos(c+dx)+C\sec(c+dx))\left(4aC\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A*Cos[c + d*x] + C*Sec[c + d*x])*
(2*A*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 4*a*C*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + 4*A*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + A*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]))/(a*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c
+ d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.462, size = 841, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(-1+cos(d*x+c))^2*(A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*b+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*sin(d*x+c)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*C*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+A*cos(d*x+c)^3*a-A*cos(d
*x+c)^2*a+A*cos(d*x+c)^2*b-A*cos(d*x+c)*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx + c\right)^{2} + A\right) \cos\left(dx + c\right)}{\sqrt{b \sec\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos(dx+c)\sec(dx+c)^2+A\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))/sqrt(b*sec(d*x +
c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+C\sec^2(c+dx))\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(C\sec(dx+c)^2+A)\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.740 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=411

$$\frac{A(2a-3b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4a^2d}-\frac{\sqrt{a+b}\left(4a^2(A+2C\right.}{}$$

```
[Out] (-3*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + (A*(2*a - 3*b)*Sqrt[a +
 b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(3*A*b^2 + 4*a^2*(A + 2*C))*Cot[
c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^3*d) - (3*A*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(4*a^2*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*
a*d)
```

Rubi [A] time = 0.640174, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4105, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2(A+2C)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}-\frac{3Ab\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-3*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + (A*(2*a - 3*b)*Sqrt[a +
 b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(3*A*b^2 + 4*a^2*(A + 2*C))*Cot[
c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^3*d) - (3*A*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(4*a^2*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*
a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(\frac{3Ab}{2}-a(A+2C)\sec(c+dx)\cdot\right.}{\sqrt{a+b\sec(c+dx)}}}{2a}$$

$$= -\frac{3Ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c}{2ad}$$

$$= -\frac{3Ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c}{2ad}$$

$$= -\frac{3A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{4a^2d}$$

$$= -\frac{3A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{4a^2d}$$

Mathematica [C] time = 15.0509, size = 1475, normalized size = 3.59

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],
x]
```

```
[Out] (A*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Se
c[c + d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a*
A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 3*A*b^2*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2] - 6*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 3*
a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*A*b^2*Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^5 + (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (
16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a^2*A*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*A*b
^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (16*
I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- (3*I)*A*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2
*I)*(-(a*A*b) + 3*A*b^2 + 2*a^2*(A + 2*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt[(-a + b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x
```

```
]]*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*
x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.414, size = 1652, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/a^2*(-1+cos(d*x+c))^2*(8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*A*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b
^2-4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2+16*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*a^2-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+8*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+6*A*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)-4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*sin(d*x+c)+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*a*b-3*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*a*b-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*A*cos(d*x+c)^4*a^2+16*C*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-
8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*si
n(d*x+c)-A*cos(d*x+c)^3*a*b-2*A*cos(d*x+c)^2*a^2+3*A*cos(d*x+c)^2*a*b-3*A*c
os(d*x+c)^2*b^2-2*A*cos(d*x+c)*a*b+3*A*cos(d*x+c)*b^2)*(cos(d*x+c)+1)^2*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)/sqrt(b*sec(d*
x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x
)
```

3.741 $\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=506

$$-\frac{\sqrt{a+b}\left(-8a^2(2A+3C)+10aAb-15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{24a^3d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^2 + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^3*b
*d) - (Sqrt[a + b]*(10*a*A*b - 15*A*b^2 - 8*a^2*(2*A + 3*C))*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(24*a^3*d) + (b*Sqrt[a + b]*(5*A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(8*a^4*d) + ((15*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(24*a^3*d) - (5*A*b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(12*a^2*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(3*a*d)
```

Rubi [A] time = 1.02765, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4105, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(8a^2(2A+3C)+15Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24a^3d}-\frac{\sqrt{a+b}\left(-8a^2(2A+3C)+10aAb-15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(}{}}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^2 + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^3*b
*d) - (Sqrt[a + b]*(10*a*A*b - 15*A*b^2 - 8*a^2*(2*A + 3*C))*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(24*a^3*d) + (b*Sqrt[a + b]*(5*A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(8*a^4*d) + ((15*A*b^2 + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(24*a^3*d) - (5*A*b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(12*a^2*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(3*a*d)
```

Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
```

```
0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)\left(\frac{5Ab}{2}-a(2A+3C)\sec\right.}{\sqrt{a+b\sec(}}}{3a}$$

$$= -\frac{5Ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12a^2d} + \frac{A\cos^2(c+dx)\sqrt{a+b\,s}}{3a}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^3d} - \frac{5Ab\cos(c+dx}{}$$

$$= \frac{\left(15Ab^2+8a^2(2A+3C)\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24a^3d} - \frac{5Ab\cos(c+dx}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^2+8a^2(2A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{24a^3bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^2+8a^2(2A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{24a^3bd}$$

Mathematica [B] time = 19.0436, size = 1363, normalized size = 2.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d*x]],
x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((A*Sin[c + d*x])/(12*a) - (5*A*b*Sin[2*
(c + d*x)])/(24*a^2) + (A*Sin[3*(c + d*x)])/(12*a)))/(d*Sqrt[a + b*Sec[c +
d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*
x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
1 + Tan[(c + d*x)/2]^2)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d
*x)/2] + 15*a*A*b^2*Tan[(c + d*x)/2] + 15*A*b^3*Tan[(c + d*x)/2] + 24*a^3*C
*Tan[(c + d*x)/2] + 24*a^2*b*C*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]
^3 - 30*a*A*b^2*Tan[(c + d*x)/2]^3 - 48*a^3*C*Tan[(c + d*x)/2]^3 + 16*a^3*A
*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 15*a*A*b^2*Tan[(c + d
*x)/2]^5 - 15*A*b^3*Tan[(c + d*x)/2]^5 + 24*a^3*C*Tan[(c + d*x)/2]^5 - 24*a
^2*b*C*Tan[(c + d*x)/2]^5 + 24*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^2*b*C*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 24*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 48*a^2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(15*A*b^2 + 8*a^2*(2*A
 + 3*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*A*b*(2*a + 5*b)*EllipticF[ArcSin[Ta
```

```
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)]))/(24*a^3*d*Sqrt[a + b*Sec[c + d*x]]*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*
(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.495, size = 2347, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/24/d/a^3*(-1+cos(d*x+c))^2*(-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-15*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-24*C*cos(d*x+c)^3*a^3+30
*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))-24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))-8*A*cos(d*x+c)^3*a^3+16*A*cos(d*x+c)^2*a^3+24*C*cos(d
*x+c)^2*a^3+15*A*cos(d*x+c)*b^3-8*A*cos(d*x+c)^5*a^3+2*A*cos(d*x+c)^4*a^2*b
-18*A*cos(d*x+c)^2*a^2*b+15*A*cos(d*x+c)^2*a*b^2+16*A*cos(d*x+c)*a^2*b-10*A
*cos(d*x+c)*a*b^2+24*C*cos(d*x+c)*a^2*b-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-15*A*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*b^3+30*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)*b^3-24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-16*A*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-15*A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c
)+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
b*sin(d*x+c)+10*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2*sin(d*x+c)+24*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+48*C*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-16*A*a
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b-15*A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+4*A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+10*A*(cos(d*x
```

```
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)*a*b^2+24*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-24*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+48*C*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*a^2*b-15*A*cos(d*x+c)^2*b^3-5*A*cos(d*x+c)^3*a*b^2-24*C*cos(d*x+c)^2*
a^2*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c)
)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x
)
```

3.742 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=460

$$-\frac{2\left(12a^2bC+16a^3C+2ab^2(5A+2C)+b^3(5A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}\cdots\right)\right)}{5b^4d\sqrt{a+b}}$$

```
[Out] (-2*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b
^5*Sqrt[a + b]*d) - (2*(16*a^3*C + 12*a^2*b*C + 2*a*b^2*(5*A + 2*C) + b^3*(
5*A + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(5*b^4*Sqrt[a + b]*d) - (2*(A*b^2 + a^2*C)*Sec[c
+ d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a*(5
*A*b^2 + 8*a^2*C - 3*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^3*(
a^2 - b^2)*d) + (2*(5*A*b^2 + 6*a^2*C - b^2*C)*Sec[c + d*x]*Sqrt[a + b*Sec[
c + d*x]]*Tan[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.08091, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4099, 4092, 4082, 4005, 3832, 4004}

$$-\frac{2\left(a^2C+Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(6a^2C+5Ab^2-b^2C\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b
^5*Sqrt[a + b]*d) - (2*(16*a^3*C + 12*a^2*b*C + 2*a*b^2*(5*A + 2*C) + b^3*(
5*A + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(5*b^4*Sqrt[a + b]*d) - (2*(A*b^2 + a^2*C)*Sec[c
+ d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a*(5
*A*b^2 + 8*a^2*C - 3*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^3*(
a^2 - b^2)*d) + (2*(5*A*b^2 + 6*a^2*C - b^2*C)*Sec[c + d*x]*Sqrt[a + b*Sec[
c + d*x]]*Tan[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(2\left(Ab^2+a^2C\right)-\frac{1}{2}ab(A+}{\sqrt{}}}{b}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(5Ab^2+6a^2C-b^2C\right)\sec(c}{5b^2(}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a\left(5Ab^2+8a^2C-3b^2C\right)\sqrt{a}}{5b^3\left(a^2-\right.}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a\left(5Ab^2+8a^2C-3b^2C\right)\sqrt{a}}{5b^3\left(a^2-\right.}$$

$$= -\frac{2\left(2a^2b^2(5A-4C)+16a^4C-b^4(5A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a-}}\right.\right.}{5b^5\sqrt{a+b}d}$$

Mathematica [B] time = 25.8587, size = 3853, normalized size = 8.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2
),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((4*(-10*a^2*A*b^2 + 5*A*b^4
 - 16*a^4*C + 8*a^2*b^2*C + 3*b^4*C)*Sin[c + d*x])/(5*b^4*(-a^2 + b^2)) + (
4*(a^2*A*b^2*Sin[c + d*x] + a^4*C*Sin[c + d*x]))/(b^3*(-a^2 + b^2)*(b + a*C
os[c + d*x])) - (12*a*C*Tan[c + d*x])/(5*b^3) + (4*C*Sec[c + d*x]*Tan[c + d
*x])/(5*b^2)))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(3/2)
) + (4*(b + a*Cos[c + d*x])*((4*a^2*A)/(b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) - (2*A*b)/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (32*a^4*C)/(5*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]) - (16*a^2*C)/(5*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (6*b*C)/(5*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (4*a*A*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Co
s[c + d*x]]) + (4*a^3*A*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Co
s[c + d*x]]) - (8*a*C*Sqrt[Sec[c + d*x]])/(5*(-a^2 + b^2)*Sqrt[b + a*Cos[c
+ d*x]]) + (32*a^5*C*Sqrt[Sec[c + d*x]])/(5*b^4*(-a^2 + b^2)*Sqrt[b + a*Cos
[c + d*x]]) - (24*a^3*C*Sqrt[Sec[c + d*x]])/(5*b^2*(-a^2 + b^2)*Sqrt[b + a*
Cos[c + d*x]]) - (2*a*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*
Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(
b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (6*a*C*Cos[2*(c + d*x)]*Sqrt[S
ec[c + d*x]])/(5*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (32*a^5*C*Cos[2*(
c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b^4*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]])
 - (16*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b^2*(-a^2 + b^2)*Sqrt[
b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*Sec[c +
d*x]^2)*(2*(a + b)*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*
(a + b)*(-16*a^3*C + 12*a^2*b*C - 2*a*b^2*(5*A + 2*C) + b^3*(5*A + 3*C))*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*
a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cos[c + d*x]*(b + a*Cos[c
```

```
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b^4*(-a^2 + b^2)*d*(A + 2
*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*(a + b
*Sec[c + d*x])^(3/2)*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*
x]*(2*(a + b)*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a +
b)*(-16*a^3*C + 12*a^2*b*C - 2*a*b^2*(5*A + 2*C) + b^3*(5*A + 3*C))*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2*b
^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b^4*(-a^2 + b^2)*(b + a*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4
*(5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] + 2*b*(a + b)*(-16*a^3*C + 12*a^2*b*C - 2*a*b^2*(5*A + 2*C) + b^
3*(5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + (2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b^4*(-a^
2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*(((2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A
+ 3*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*
(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(
-16*a^3*C + 12*a^2*b*C - 2*a*b^2*(5*A + 2*C) + b^3*(5*A + 3*C))*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (
(a + b)*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(-16*a^3*C + 12*a^2*b*C - 2*a*b^2
*(5*A + 2*C) + b^3*(5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
- a*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cos[c + d*x]*Sec[(
c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (2*a^2*b^2*(5*A - 4*C) + 16*a
^4*C - b^4*(5*A + 3*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x
]*Tan[(c + d*x)/2] + (2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b
*(a + b)*(-16*a^3*C + 12*a^2*b*C - 2*a*b^2*(5*A + 2*C) + b^3*(5*A + 3*C))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1
- ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(2*a^2*b^2*(5*A - 4*C)
+ 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1
- ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(5*
b^4*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(2
*(a + b)*(2*a^2*b^2*(5*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-
16*a^3*C + 12*a^2*b*C - 2*a*b^2*(5*A + 2*C) + b^3*(5*A + 3*C))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2*b^2*(5
```

```
*A - 4*C) + 16*a^4*C - b^4*(5*A + 3*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c
 + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(5*b^4*(-a^2 +
 b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.272, size = 4055, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/5/d/(a-b)/(a+b)/b^4*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-5*A*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^5+5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-16*C*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^5+3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^5-5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+5*A*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-3
*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^5-16*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+3*C*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+16*C*cos(
d*x+c)^4*a^5+10*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-5*A*cos(d*x+c)^3*b^5-16*C*cos(d*x+c
)^3*a^5-3*C*cos(d*x+c)^3*b^5+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*A*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3
*b^2-10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^3+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+16*C*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^4*b+4*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*b^2-8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
```

```
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+C*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^4-16*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^4*b+8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+8*C*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b^3+3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^4-5*A*cos(d*x+c)^4*a^2*b^3-8*C*cos(d*x+c)^4*a^
4*b+3*C*cos(d*x+c)^4*a^2*b^3+5*A*cos(d*x+c)^3*a*b^4+6*C*cos(d*x+c)^3*a^3*b^
2+5*C*cos(d*x+c)^3*a*b^4+6*C*cos(d*x+c)^2*a^2*b^3-2*C*cos(d*x+c)*a*b^4+5*A*
cos(d*x+c)^2*b^5+2*C*cos(d*x+c)^2*b^5-C*a^2*b^3+C*b^5+2*C*cos(d*x+c)*a^3*b^
2+10*A*cos(d*x+c)^4*a^3*b^2-5*A*cos(d*x+c)^4*a*b^4-8*C*cos(d*x+c)^4*a^3*b^2
-3*C*cos(d*x+c)^4*a*b^4-10*A*cos(d*x+c)^3*a^3*b^2+10*A*cos(d*x+c)^3*a^2*b^3
+16*C*cos(d*x+c)^3*a^4*b-8*C*cos(d*x+c)^3*a^2*b^3-5*A*cos(d*x+c)^2*a^2*b^3-
8*C*cos(d*x+c)^2*a^4*b+5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*A*sin(d*x+c)*cos(d*x+c)^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-1
0*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b^3+5*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+16*C*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+
4*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b^2-8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+C*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-1
6*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^4*b+8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+8*C*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3
+3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^4+10*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3)/(b+a*cos(d*x+c))/cos(d
*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2),
 x)
```

3.743 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=327

$$\frac{2\left(C\left(8a^2+6ab+b^2\right)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}+\frac{2a\left(a\right.}{b^2d\left(a^2\right.}$$

```
[Out] (2*a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) + (2
*(3*A*b^2 + (8*a^2 + 6*a*b + b^2)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) +
(2*a*(A*b^2 + a^2*C)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*
x]]) + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.675296, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4091, 4082, 4005, 3832, 4004}

$$\frac{2a\left(a^2C+Ab^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(C\left(8a^2+6ab+b^2\right)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a\cdots}}{}\right.\right.}{3b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) + (2
*(3*A*b^2 + (8*a^2 + 6*a*b + b^2)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) +
(2*a*(A*b^2 + a^2*C)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*
x]]) + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```


Rule 4091

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(a*(A*b^2 + a^2
*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)),
 x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*Simp[b*(m + 1)*(a^2*C + A*b^2) - a*(A*b^2*(m + 2) + C*(a^2 + b^2
*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
```

```
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}b\left(Ab^2+a^2C\right)-\frac{1}{2}a\left(Ab^2+2a^2C-b^2 \cdots\right.\right.}{\sqrt{a+b\sec(c \cdots}}}{b^2\left(a^2-\cdots\right.}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2d} + \frac{4}{\cdots}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2d} - \frac{(a \cdots}{\cdots}$$

$$= \frac{2a\left(3Ab^2+8a^2C-5b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec \cdots}{\cdots}}}{3b^4\sqrt{a+b}d}$$

Mathematica [B] time = 23.5756, size = 3312, normalized size = 10.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2
),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((4*a*(3*A*b^2 + 8*a^2*C - 5
*b^2*C)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) - (4*(a*A*b^2*Sin[c + d*x] + a^3
*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (4*C*Tan[c + d*
x])/(3*b^2)))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(3/2))
```

```
 + (4*(b + a*Cos[c + d*x])*((-2*a*A)/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) + (10*a*C)/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[c + d*x]]) - (16*a^3*C)/(3*b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) - (2*a^2*A*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)*Sqrt[b
+ a*Cos[c + d*x]]) + (2*A*b*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Co
s[c + d*x]]) - (16*a^4*C*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a
*Cos[c + d*x]]) + (14*a^2*C*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b +
a*Cos[c + d*x]]) + (2*b*C*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b + a*Co
s[c + d*x]]) - (2*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2
)*Sqrt[b + a*Cos[c + d*x]]) - (16*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]]
)/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (10*a^2*C*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*(-2*a*(a + b)*(3*A*b^2
 + 8*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)] + 2*b*(a + b)*(3*A*b^2 + (8*a^2 - 6*a*b + b^2)*C)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(3*A*
b^2 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*(A + 2*C + A*Cos[2*c + 2*d*x])*
Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((2*
a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*a*(a + b)*(3*A*b^2
 + 8*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)] + 2*b*(a + b)*(3*A*b^2 + (8*a^2 - 6*a*b + b^2)*C)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(3*A*
b^2 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^(3/2)*Sqrt[S
ec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x
)/2]*(-2*a*(a + b)*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*A*b^2 + (8*a
^2 - 6*a*b + b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] - a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*Sqrt[b
 + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/2]^2*S
ec[c + d*x]]*(-(a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^4)/2 - (a*(a + b)*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*
x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])] + (b*(a + b)*(3*A*b^2 + (8*a^2 - 6*a*b + b^2)*C)*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*
x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (a*(a + b)*
(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] +
(b*(a + b)*(3*A*b^2 + (8*a^2 - 6*a*b + b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])
/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))] + a^2*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*Sec[(c + d*
x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + a*(3*A*b^2 + 8*a^2*C - 5*b^2*C)*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - a*(3*A
*b^2 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
```

```
]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(3*A*b^2 + (8*a^2 - 6*a*b + b^2)*C)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 -
((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (a*(a + b)*(3*A*b^2 + 8*a^2*C - 5*
b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^3*(-a^2 + b^2)*Sqrt[b
 + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(-2*a*(a + b)*(3*A*b^2 +
8*a^2*C - 5*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)] + 2*b*(a + b)*(3*A*b^2 + (8*a^2 - 6*a*b + b^2)*C)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - a*(3*A*b^2
 + 8*a^2*C - 5*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[
(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
)))
```

Maple [B] time = 0.629, size = 2672, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d/(a-b)/(a+b)/b^3*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*A*co
s(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*b^4+8*C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-3*A*cos(d*x+c)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b
^4+8*C*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^4-C*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-8*C*cos(d*x+c)^3*a^4-5*C*cos(d*x+c
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a*b^3+3*A*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*A*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-3*A*cos(d*x+c)^2
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*b^3+8*C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^3*b-5*C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-5*C*cos(d*x+c)^2*
sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
```

```
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a*b^3-8*C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a^3*b-2*C*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+5*C*cos(d*x+c)^2*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a*b^3+3*A*cos(d*x+c)^3*a*b^3+3*A*cos(d*x+c)^2*a^2*b^2-3*A*cos(d*x+c)^2*a*b^
3-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^3*b+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-5*C*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+3*A*c
os(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))+3*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-3*A*cos(d*x+c)*b^3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+4*C*cos(d*x+
c)^3*a^3*b-C*cos(d*x+c)^3*a*b^3-4*C*cos(d*x+c)^2*a^2*b^2-4*C*cos(d*x+c)*a*b
^3-8*C*cos(d*x+c)^2*a^3*b+4*C*cos(d*x+c)*a^3*b+8*C*cos(d*x+c)^2*a^4-2*C*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2-C*a^2*b^2+5*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-C*cos(d*x+c)^2*b^4-3*A*cos
(d*x+c)^3*a^2*b^2+5*C*cos(d*x+c)^3*a^2*b^2+5*C*cos(d*x+c)^2*a*b^3+C*b^4)/(b
+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx + c\right)^{2} + A\right)\sec\left(dx + c\right)^{2}}{\left(b\sec\left(dx + c\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2),
 x)
```

3.744 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=279

$$\frac{2(Ab-C(2a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}-\frac{2\left(a^2C+Ab^2\right)\text{ta}}{bd\left(a^2-b^2\right)\sqrt{a+b}}$$

```
[Out] (-2*(A*b^2 + 2*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(A*b -
 (2*a + b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) - (2*(A*b^2 + a^2*C)*Tan[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.3914, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4081, 4005, 3832, 4004}

$$-\frac{2\left(a^2C+Ab^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(2a^2C+Ab^2-b^2C\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(A*b^2 + 2*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(A*b -
 (2*a + b)*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) - (2*(A*b^2 + a^2*C)*Tan[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4081

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A*b^2 + a^2*C)
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] +
 Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2 + a^2*C + b*(A*b + b*C)*(m + 1))*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && NeQ[a
^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}ab(A+C)-\frac{1}{2}\left(Ab^2+2a^2C-b^2C\right)\text{se}}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(Ab^2+2a^2C-b^2C\right)\int \frac{\sec(c+dx)(1+\sec(c+d}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2+2a^2C-b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c-}{a+b}}}{b^3\sqrt{a+b}d}$$

Mathematica [A] time = 18.757, size = 541, normalized size = 1.94

$$\frac{4\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a\cos(c+dx)+b)\left(A+C\sec^2(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((-4*(A*b^2 + 2*a^2*C - b^2*
C)*Sin[c + d*x])/(b^2*(-a^2 + b^2)) + (4*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c
+ d*x]))/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + A*Cos[2*c +
2*d*x])*(a + b*Sec[c + d*x])^(3/2)) + (4*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])^2]*(b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(C
os[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*((a + b)*((A*b
^2 + 2*a^2*C - b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ b*(-(A*b) + (-2*a + b)*C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (A*b^2 + 2*a^2*C - b^2*C)*Cos[c
+ d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(b^2*(-a^
2 + b^2)*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*(A + 2*C + A*Cos[2*c + 2*d*x])*(Se
c[(c + d*x)/2]^2)^(3/2)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.504, size = 2272, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/b^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-A*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+A*
b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^3-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2*sin(d*x+c)+C*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-C*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-C*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*C*a^3*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-
C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*a*b^2+C*cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)^2*a^3-A*cos(d*x+c)*
b^3+a^2*b*C-A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a*b^2-2*C*cos(d*x+c)*a^2*b+A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*b^3+2*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-A*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*C*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*C*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)
+2*C*cos(d*x+c)*a^3+A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+2*C*a^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*b-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)*a^2*b+A*cos(d*x+c)^2*b^3+C*cos(d*x+c)^2*a^2*b-C*cos(
d*x+c)*a*b^2-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^3+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-C*b^3-C*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
```

```
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+C*cos(d*x+c
)*b^3)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^2*
sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.745 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=381

$$-\frac{2(Ab-aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}+\frac{2\left(a^2C+Ab^2\right)\tan(}{ad\left(a^2-b^2\right)\sqrt{a+b\,\text{se}}}$$

```
[Out] (2*(A*b^2 + a^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*Sqrt[a + b]*d) - (2*(A*b - a*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a^2*d) + (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]])
```

Rubi [A] time = 0.421251, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4061, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\left(a^2C+Ab^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{ab^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(A*b^2 + a^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*Sqrt[a + b]*d) - (2*(A*b - a*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a^2*d) + (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]])
```


Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A\left(a^2-b^2\right)+\frac{1}{2}ab(A+C)\sec(c+dx)+\frac{1}{2}\left(Ab^2+a^2C\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A\left(a^2-b^2\right)+\left(\frac{1}{2}ab(A+C)+\frac{1}{2}\left(-Ab^2-a^2C\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{\left(Ab^2\right.}{}$$

$$= \frac{2\left(Ab^2+a^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab^2\sqrt{a+b}d} + -$$

$$= \frac{2\left(Ab^2+a^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab^2\sqrt{a+b}d} - 2$$

Mathematica [B] time = 18.0515, size = 1127, normalized size = 2.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((4*(A*b^2 + a^2*C)*Sin[c +
d*x])/(a*b*(-a^2 + b^2)) + (4*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(a
*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a +
 b*Sec[c + d*x])^(3/2)) - (4*(b + a*Cos[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*b^2*Tan[(c + d*x)/2] + A*b^3*T
an[(c + d*x)/2] + a^3*C*Tan[(c + d*x)/2] + a^2*b*C*Tan[(c + d*x)/2] - 2*a*A
*b^2*Tan[(c + d*x)/2]^3 - 2*a^3*C*Tan[(c + d*x)/2]^3 + a*A*b^2*Tan[(c + d*x
)/2]^5 - A*b^3*Tan[(c + d*x)/2]^5 + a^3*C*Tan[(c + d*x)/2]^5 - a^2*b*C*Tan[
(c + d*x)/2]^5 - 2*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a^2*A*b*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + 2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(A*b^2 + a^2*C)*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - a*b*(a + b)*(A + C)*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*(
-(a^2*b) + b^3)*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*
Sec[c + d*x])^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.442, size = 2043, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/b/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*b^3*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*
A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))+C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-C*cos(d*x+c)^2*a^3-A*cos(d*x+c)*b^3-A
*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a*b^2-C*cos(d*x+c)*a^2*b+A*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3
-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*b^3+C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-A*(cos(d*x+c)/(cos(d*x+
```

```
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-A*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*A*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*s
in(d*x+c)+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b*sin(d*x+c)-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b*sin(d*x+c)+C*cos(d*x+c)*a^3+A*b^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*a^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)*a*b^2+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+C*a^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*a^2*b+A*cos(d*x+c)^2*b^3+C*cos(d*x+c)^2*a^2*b-C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c))/(b+a
*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^
2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{(b\sec(dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.746 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=431

$$\frac{\left(2a^2C+aAb+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2bd\sqrt{a+b}}-\frac{b\left(3Ab^2-a^2(A\right.}{a^2d\left(a^2-b^2\right)\cdot}$$

```
[Out] -(((3*A*b^2 - a^2*(A - 2*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d)) + ((a*A*b
 + 3*A*b^2 + 2*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + (3*A*b*Sqrt[a +
 b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[c + d*x])/(a*d*Sqrt[a + b*Se
c[c + d*x]]) - (b*(3*A*b^2 - a^2*(A - 2*C))*Tan[c + d*x])/(a^2*(a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.658895, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4105, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\left(3Ab^2-a^2(A-2C)\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2C+aAb+3Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] -(((3*A*b^2 - a^2*(A - 2*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d)) + ((a*A*b
 + 3*A*b^2 + 2*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + (3*A*b*Sqrt[a +
 b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[c + d*x])/(a*d*Sqrt[a + b*Se
c[c + d*x]]) - (b*(3*A*b^2 - a^2*(A - 2*C))*Tan[c + d*x])/(a^2*(a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx = \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{3Ab}{2}-aC\sec(c+dx)-\frac{1}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{a}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{b\left(3Ab^2-a^2(A-2C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{3}{4}Ab\left(a^2-b^2\right)+}{}}{}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{b\left(3Ab^2-a^2(A-2C)\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{3}{4}Ab\left(a^2-b^2\right)+}{}}{}$$

$$= -\frac{\left(3Ab^2-a^2(A-2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+b}d}$$

$$= -\frac{\left(3Ab^2-a^2(A-2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a^2b\sqrt{a+b}d}$$

Mathematica [B] time = 18.946, size = 1259, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((4*(A*b^2 + a^2*C)*Sin[c +
d*x])/(a^2*(a^2 - b^2)) - (4*(A*b^3*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(
a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(
a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])^(3/2)*(A + C*Sec[c + d
*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*A*Tan[(c + d*x)/2
] + a^2*A*b*Tan[(c + d*x)/2] - 3*a*A*b^2*Tan[(c + d*x)/2] - 3*A*b^3*Tan[(c
+ d*x)/2] - 2*a^3*C*Tan[(c + d*x)/2] - 2*a^2*b*C*Tan[(c + d*x)/2] - 2*a^3*A
*Tan[(c + d*x)/2]^3 + 6*a*A*b^2*Tan[(c + d*x)/2]^3 + 4*a^3*C*Tan[(c + d*x)/
2]^3 + a^3*A*Tan[(c + d*x)/2]^5 - a^2*A*b*Tan[(c + d*x)/2]^5 - 3*a*A*b^2*Ta
n[(c + d*x)/2]^5 + 3*A*b^3*Tan[(c + d*x)/2]^5 - 2*a^3*C*Tan[(c + d*x)/2]^5
+ 2*a^2*b*C*Tan[(c + d*x)/2]^5 + 6*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*A*
b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/
2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-3
*A*b^2 + a^2*(A - 2*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*(a + b)*(A*b + a*C)*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)*d*(A + 2*C + A*Cos[2*c + 2*d*x]
)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/2]^2
]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.405, size = 2489, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*C*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
3+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*si
n(d*x+c)-3*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))+6*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))-2*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+A*cos(d*x+c)^3*a^3-A*cos(d*x+c)^2*a
^3+2*C*cos(d*x+c)^2*a^3+3*A*cos(d*x+c)*b^3+A*cos(d*x+c)^2*a^2*b+3*A*cos(d*x
+c)^2*a*b^2-A*cos(d*x+c)*a^2*b-2*A*cos(d*x+c)*a*b^2+2*C*cos(d*x+c)*a^2*b+A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*a^3-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-2*C*a^3*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b*sin(d*x+c)-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2*sin(d*x+c)+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-6*A*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*C*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*
C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*si
n(d*x+c)-2*C*cos(d*x+c)*a^3+A*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-3*A*b^2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+2*A*a
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b+2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-2*C*a^2*(c
```

```
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-3*A*cos(d*x+c)^2*b^3-A*cos(d*x+c)^3*a*b^2
-2*C*cos(d*x+c)^2*a^2*b+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*sec(d*x +
c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\cos\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*cos(c + d*x)/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.747 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=501

$$-\frac{\left(-2a^2(A-4C)+5aAb+15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4a^3d\sqrt{a+b}}+\frac{b^2}{}$$

```
[Out] ((15*A*b^2 - a^2*(7*A - 8*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - ((5*a*A
*b + 15*A*b^2 - 2*a^2*(A - 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqr
t[a + b]*(15*A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^4*d) -
 (5*A*b*Sin[c + d*x])/(4*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*
Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d*x]]) + (b^2*(15*A*b^2 - a^2*(7*A
- 8*C))*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.01229, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4105, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b^2\left(15Ab^2-a^2(7A-8C)\right)\tan(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-2a^2(A-4C)+5aAb+15Ab^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{s}\right.}{4a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((15*A*b^2 - a^2*(7*A - 8*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - ((5*a*A
*b + 15*A*b^2 - 2*a^2*(A - 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqr
t[a + b]*(15*A*b^2 + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^4*d) -
 (5*A*b*Sin[c + d*x])/(4*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*
Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d*x]]) + (b^2*(15*A*b^2 - a^2*(7*A
- 8*C))*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(\frac{5Ab}{2}-a(A+2C)\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{2a}$$

$$= -\frac{5Ab\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\frac{1}{4}\left(15Ab^2+4a^2(A+2C)\right)\cdots}{(a+\cdots}}{\cdots}$$

$$= -\frac{5Ab\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{b^2\left(15Ab^2-a^2(7A-\cdots}{4a^3\left(a^2-b^2\right)d\sqrt{a\cdots}}$$

$$= -\frac{5Ab\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{b^2\left(15Ab^2-a^2(7A-\cdots}{4a^3\left(a^2-b^2\right)d\sqrt{a\cdots}}$$

$$= \frac{\left(15Ab^2-a^2(7A-8C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d\cdots}{a+b}}\cdots}{4a^3\sqrt{a+b}d}$$

$$= \frac{\left(15Ab^2-a^2(7A-8C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d\cdots}{a+b}}\cdots}{4a^3\sqrt{a+b}d}$$

Mathematica [C] time = 17.4603, size = 2500, normalized size = 4.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(3/2
),x]
```

```
[Out] (((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((4*b*(A*b^2 + a^2*C)*Sin[c + d*x])
/(a^3*(-a^2 + b^2)) + (4*(A*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(a^
3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(2*a^2)))/(d*(a
+ b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x
)/2]^2)]*(-7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 7*a^2*A*b^2*
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2] + 15*A*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 8*a^
3*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] + 8*a^2*b^2*Sqrt[(-a + b)/(a
+ b)]*C*Tan[(c + d*x)/2] + 14*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^3 - 30*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 16*a^3*b*Sqrt
[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^3 - 7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]^5 + 7*a^2*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
+ 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*A*b^4*Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]^5 + 8*a^3*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c
+ d*x)/2]^5 - 8*a^2*b^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - (8*I)
*a^4*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (22*I)*a^2*A*b^2*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*A*b^4*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (16*I)*a^4*C*EllipticPi[-((a + b)/(a - b)
```

```
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] + (16*I)*a^2*b^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - (8*I)*a^4*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (22*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (30*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*Ar
cSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d
*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - (16*I)*a^4*C*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[
(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (16*I)*a^2*b^2*C*EllipticPi[-((a + b
)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*b*(-15*A*b^2 + a
^2*(7*A - 8*C))*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*
(a - b)*(10*a*A*b^2 + 15*A*b^3 + 2*a^3*(A + 2*C) + a^2*b*(A + 8*C))*Ellipti
cF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(2*a^3*Sqrt[(-a + b)/(a + b)]
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1
 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^
2) - b*(1 + Tan[(c + d*x)/2]^2))))/2
```

Maple [B] time = 0.51, size = 3529, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d/a^3/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-8*A*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d
*x+c)-2*A*a^4*cos(d*x+c)^4+2*A*a^4*cos(d*x+c)^2-15*A*cos(d*x+c)^2*b^4+30*A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*si
n(d*x+c)+7*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*b^2*sin(d*x+c)-15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+4*A*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+10*A*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+8*C*(
```

```
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x
+c)-16*C*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))+5*A*cos(d*x+c)^3*a^3*b-5*A*cos(d*x+c)^3*a*b^3-7*A*cos(d*x
+c)^2*a^3*b+5*A*cos(d*x+c)^2*a^2*b^2+2*A*cos(d*x+c)*a^3*b-7*A*cos(d*x+c)*a^
2*b^2-10*A*cos(d*x+c)*a*b^3+15*A*cos(d*x+c)^2*a*b^3+2*A*cos(d*x+c)^4*a^2*b^
2+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^3*b-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-8*C*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+16*C*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^2*b^2-15*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-22*A*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2+7*A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b+7*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-15*A*cos(d*x+c)*b^3*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*A*cos(d
*x+c)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b+4*A*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+10*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-8*C*cos(d*x+c)^
2*a^2*b^2+8*C*cos(d*x+c)^2*a^3*b-8*C*cos(d*x+c)*a^3*b+8*C*cos(d*x+c)*a^2*b^
2-8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a^4+30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4-15*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+4*A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^4-22*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+7*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+8*C*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-16*C*a^4*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*b*sin(d*x+c)-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
```

```
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+16*C*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+15*A*c
os(d*x+c)*b^4)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+A\cos\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(b*sec(d*
x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2),
 x)
```

3.748 $\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=488

$$\frac{2\left(2a^2b^2(A-8C)+12a^3bC+16a^4C+3ab^3(A-3C)-b^4(3A+C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellipti}}{3b^4d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (4*a*(a^2*b^2*(A - 14*C) - b^4*(3*A - 4*C) + 8*a^4*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^5*
Sqrt[a + b]*(a^2 - b^2)*d) + (2*(2*a^2*b^2*(A - 8*C) + 3*a*b^3*(A - 3*C) +
16*a^4*C + 12*a^3*b*C - b^4*(3*A + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*(a^
2 - b^2)*d) - (2*(A*b^2 + a^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (4*a*(2*A*b^4 - 3*a^4*C + 5*a^2*b^2*C)*
Tan[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b^2
+ 2*a^2*C - b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2
)*d)
```

Rubi [A] time = 1.30115, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4099, 4090, 4082, 4005, 3832, 4004}

$$-\frac{2\left(a^2C+Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(2a^2C+Ab^2-b^2C\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^3d\left(a^2-b^2\right)}-\frac{4a\left(5a^2b^2C-3a^4\right.}{3b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (4*a*(a^2*b^2*(A - 14*C) - b^4*(3*A - 4*C) + 8*a^4*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^5*
Sqrt[a + b]*(a^2 - b^2)*d) + (2*(2*a^2*b^2*(A - 8*C) + 3*a*b^3*(A - 3*C) +
16*a^4*C + 12*a^3*b*C - b^4*(3*A + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*(a^
2 - b^2)*d) - (2*(A*b^2 + a^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (4*a*(2*A*b^4 - 3*a^4*C + 5*a^2*b^2*C)*
Tan[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b^2
+ 2*a^2*C - b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2
)*d)
```


Rule 4099

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(d*
(A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 1))/(b*f*(a^2 - b^2)*(m + 1)), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), In
t[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1)
+ a^2*C*(n - 1) + a*b*(A + C)*(m + 1)*Csc[e + f*x] - (A*b^2*(m + n + 1) + C
*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
```

```
A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(2\left(Ab^2+a^2C\right)-\frac{3}{2}ab(A+}{(a+}}{3b}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\mathrm{ta}}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\mathrm{ta}}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(}}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{4a\left(2Ab^4-3a^4C+5a^2b^2C\right)\mathrm{ta}}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(}}$$

$$= \frac{4a\left(a^2b^2(A-14C)-b^4(3A-4C)+8a^4C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{3(a-b)b^5(a+b)^{3/2}d}$$

<u>Mathematica</u> [B] time = 26.4024, size = 4050, normalized size = 8.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2
),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((-8*a*(a^2*A*b
^2 - 3*A*b^4 + 8*a^4*C - 14*a^2*b^2*C + 4*b^4*C)*Sin[c + d*x])/(3*b^4*(a^2
- b^2)^2) - (4*(a*A*b^2*Sin[c + d*x] + a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 +
b^2)*(b + a*Cos[c + d*x])^2) - (4*(-(a^3*A*b^2*Sin[c + d*x]) + 5*a*A*b^4*Si
n[c + d*x] - 7*a^5*C*Sin[c + d*x] + 11*a^3*b^2*C*Sin[c + d*x]))/(3*b^3*(-a^
2 + b^2)^2*(b + a*Cos[c + d*x])) + (4*C*Tan[c + d*x])/(3*b^3)))/(d*(A + 2*C
 + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) + (8*(b + a*Cos[c + d*x]
)^2*((4*a^3*A)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - (4*a*A*b)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) + (32*a^5*C)/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) - (56*a^3*C)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) + (16*a*b*C)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (10*a^2*A*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Co
s[c + d*x]]) + (4*a^4*A*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b +
a*Cos[c + d*x]]) + (2*A*b^2*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*
Cos[c + d*x]]) + (10*a^2*C*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*C
os[c + d*x]]) + (32*a^6*C*Sqrt[Sec[c + d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b
+ a*Cos[c + d*x]]) - (64*a^4*C*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sq
rt[b + a*Cos[c + d*x]]) + (2*b^2*C*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sq
rt[b + a*Cos[c + d*x]]) - (4*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-
a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec
[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^2*C*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]
]) + (32*a^6*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^4*(-a^2 + b^2)^2*S
qrt[b + a*Cos[c + d*x]]) - (56*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(
3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*(2*a*(a + b)*(a^2*b^2*
(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
```

])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*b^2*(A - 8*C) + 3
*a*b^3*(A - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*(3*A + C))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(a^2*b^2*(A - 14
*C) + 8*a^4*C + b^4*(-3*A + 4*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*(A + 2*C + A*Cos[2*c
+ 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((4*a*Sqrt[Co
s[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*a*(a + b)*(a^2*b^2*(A - 14*C
) + 8*a^4*C + b^4*(-3*A + 4*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*b^2*(A - 8*C) + 3*a*b^3*(A
 - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*(3*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(a^2*b^2*(A - 14*C) + 8*a
^4*C + b^4*(-3*A + 4*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[
Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*
x)/2]*(2*a*(a + b)*(a^2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a +
b)*(-2*a^2*b^2*(A - 8*C) + 3*a*b^3*(A - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*
(3*A + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] + a*(a^2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*Cos[c + d*x]*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^
2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (8*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*((a*(a^2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C
))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (a*(a + b)*(a^
2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
 Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-2*a^2
b^2(A - 8*C) + 3*a*b^3*(A - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*(3*A + C))
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c
+ d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]) + (a*(a + b)*(a^2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b +
a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(-2*a^2*b^2*(A - 8
*C) + 3*a*b^3*(A - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*(3*A + C))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d
*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(2*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]) - a^2*(a^2*b^2*(A - 14*C) + 8*a^4*C +
b^4*(-3*A + 4*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x
)/2] - a*(a^2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + a*(a^2*b^2*(A - 14*
C) + 8*a^4*C + b^4*(-3*A + 4*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-2*a^2*b^2*(A - 8*C) + 3*a*b^3*
(A - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*(3*A + C))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(
c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d
x)/2]^2)/(a + b)]) + (a(a + b)*(a^2*b^2*(A - 14*C) + 8*a^4*C + b^4*(-3*A
+ 4*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^4*(a^2 - b^2)^2*Sqrt

```
[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*(2*a*(a + b)*(a^2*b^2*(
A - 14*C) + 8*a^4*C + b^4*(-3*A + 4*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*b^2*(A - 8*C) + 3*
a*b^3*(A - 3*C) - 16*a^4*C + 12*a^3*b*C + b^4*(3*A + C))*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(a^2*b^2*(A - 14*
C) + 8*a^4*C + b^4*(-3*A + 4*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)
/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*b^4*(a^2 - b^2)^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*S
ec[c + d*x]])))
```

Maple [B] time = 1.344, size = 7051, normalized size = 14.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2),
 x)
```

3.749 $\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=408

$$-\frac{2\left(6a^2bC+8a^3C-ab^2(A+9C)+3b^3(A-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{3b^3d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (2*(3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt
[a + b]*(a^2 - b^2)*d) - (2*(3*b^3*(A - C) + 8*a^3*C + 6*a^2*b*C - a*b^2*(A
 + 9*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b^2 + a^
2*C)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(3
*A*b^4 - 5*a^4*C + a^2*b^2*(A + 9*C))*Tan[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*
Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.821386, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4091, 4080, 4005, 3832, 4004}

$$\frac{2\left(a^2b^2(A+9C)-5a^4C+3Ab^4\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(a^2C+Ab^2\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2bC+8a^3C-ab^2(A+9C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt
[a + b]*(a^2 - b^2)*d) - (2*(3*b^3*(A - C) + 8*a^3*C + 6*a^2*b*C - a*b^2*(A
 + 9*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b^2 + a^
2*C)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(3
*A*b^4 - 5*a^4*C + a^2*b^2*(A + 9*C))*Tan[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*
Sqrt[a + b*Sec[c + d*x]])
```


Rule 4091

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(cs
c[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(a*(A*b^2 + a^2
*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)),
 x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*Simp[b*(m + 1)*(a^2*C + A*b^2) - a*(A*b^2*(m + 2) + C*(a^2 + b^2
*(m + 1)))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
```

```
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}b\left(Ab^2+a^2C\right)+\frac{1}{2}a\left(Ab^2-2a^2C+3\right.\right.}{(a+b\sec(c+}}{3b^2\left(a^2-\right.}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)\tan(}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx}}$$

$$= \frac{2a\left(Ab^2+a^2C\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4-5a^4C+a^2b^2(A+9C)\right)\tan(}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx}}$$

$$= \frac{2\left(3b^4(A-C)-8a^4C+a^2b^2(A+15C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\right.}{3(a-b)b^4(a+b)^{3/2}d}$$

Mathematica [A] time = 22.5669, size = 702, normalized size = 1.72

$$4\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\sec(c+dx)}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a\cos(c+dx)+b)^2\left(A\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2
),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((-4*(a^2*A*b^2
 + 3*A*b^4 - 8*a^4*C + 15*a^2*b^2*C - 3*b^4*C)*Sin[c + d*x])/(3*b^3*(-a^2 +
 b^2)^2) + (4*(A*b^2*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (8*(a^2*A*b^2*Sin[c + d*x] + A*b^4*Sin[c + d*x] -
 2*a^4*C*Sin[c + d*x] + 4*a^2*b^2*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b
 + a*Cos[c + d*x]))))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x]
)^(5/2)) + (4*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(b + a*Cos[c
+ d*x])^2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*(Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2)*(-((a + b)*((8*a^4*C
 + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] + b*(3*b^3*(A - C) - 8*a^3*C + 6*a^2*b*C + a*b^2*(A + 9*C
))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[
(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)]*Sec[c + d*x]) + (3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(3*b^3*(a^2
- b^2)^2*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*(A + 2*C + A*Cos[2*c + 2*d*x])*(Se
c[(c + d*x)/2]^2)^(3/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.805, size = 6135, normalized size = 15.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^2}{\left(b\sec\left(dx + c\right) + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2),
 x)
```

3.750 $\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=378

$$\frac{2\left(2a^2C+3ab(A+C)-b^2(A+3C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (-4*a*(2*A*b^2 - a^2*C + 3*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(a + b)^(3/2
)*d) + (2*(2*a^2*C + 3*a*b*(A + C) - b^2*(A + 3*C))*Cot[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*Sq
rt[a + b]*(a^2 - b^2)*d) - (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(3*b*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) - (4*a*(2*A*b^2 - a^2*C + 3*b^2*C)*Tan[c +
d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.693751, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4081, 4003, 4005, 3832, 4004}

$$-\frac{4a\left(a^2(-C)+2Ab^2+3b^2C\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\left(a^2C+Ab^2\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(2a^2C+3ab(A+C)-b^2(A+3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-4*a*(2*A*b^2 - a^2*C + 3*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(a + b)^(3/2
)*d) + (2*(2*a^2*C + 3*a*b*(A + C) - b^2*(A + 3*C))*Cot[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*Sq
rt[a + b]*(a^2 - b^2)*d) - (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(3*b*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) - (4*a*(2*A*b^2 - a^2*C + 3*b^2*C)*Tan[c +
d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4081

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[
(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[((A*b^2 + a^2*C)
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] +
 Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
 1)*Simp[a*b*(A + C)*(m + 1) - (A*b^2 + a^2*C + b*(A*b + b*C)*(m + 1))*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && LtQ[m, -1] && NeQ[a
^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
```

```
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}ab(A+C)+\frac{1}{2}\left(Ab^2-2a^2C+3b^2C\right)s\right.}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{4a\left(2Ab^2-a^2C+3b^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{4a\left(2Ab^2-a^2C+3b^2C\right)\tan(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{4a\left(2Ab^2-\left(a^2-3b^2\right)C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{3(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [B] time = 23.8041, size = 3369, normalized size = 8.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((-8*a*(-2*A*b^
2 + a^2*C - 3*b^2*C)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2) + (4*(A*b^2*Sin[c
+ d*x] + a^2*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (4
*(-5*a^2*A*b^2*Sin[c + d*x] + A*b^4*Sin[c + d*x] + a^4*C*Sin[c + d*x] - 5*a
^2*b^2*C*Sin[c + d*x]))/(3*a*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(A
 + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) + (8*(b + a*Cos[c
+ d*x])^2*((-8*a*A*b)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) + (4*a^3*C)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (4*a*b*C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c
 + d*x]]) + (2*A*b^2*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c
 + d*x]]) - (10*a^2*C*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[
c + d*x]]) + (4*a^4*C*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*
Cos[c + d*x]]) + (2*b^2*C*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Co
s[c + d*x]]) - (8*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]
])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*C*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Sec
[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*(2*
a*(a + b)*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*C + 3*a*b*(A + C) +
 b^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + a*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*d*(A + 2*C
 + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*
((4*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*a*(a + b)*(-2*A
*b^2 + (a^2 - 3*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*C + 3*a*b*(A + C) + b^2*(A + 3*C))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
a*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*(b + a*Cos[c + d*x])^(3/2
)*Sqrt[Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[
(c + d*x)/2]*(2*a*(a + b)*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*C +
 3*a*b*(A + C) + b^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + a*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x]*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b
^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (8*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*((a*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x]*(b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (a*(a + b)*(-2*A*b^2 + (a^2 - 3*b^
2)*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + C
os[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])] + (b*(a + b)*(-2*a^2*C + 3*a*b*(A + C) + b^2*(A + 3*C))*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x]
)^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]) + (a*(a + b)*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x
])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))] + (b*(a + b)*(-2*a^2*C + 3*a*b*(A + C) + b^2*(A + 3*C))*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
```

```
- b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*C
os[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(2*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]) - a^2*(-2*A*b^2 + (a^2 - 3*b^2
)*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - a*(-2*
A*b^2 + (a^2 - 3*b^2)*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*
x]*Tan[(c + d*x)/2] + a*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x]*(b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-2*a^2*C +
3*a*b*(A + C) + b^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])
+ (a*(a + b)*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)
/2]^2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/
2]^2]) + (4*(2*a*(a + b)*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a + b)*(-2*a^2*C +
3*a*b*(A + C) + b^2*(A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + a*(-2*A*b^2 + (a^2 - 3*b^2)*C)*Cos[c + d*x]*(
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2
]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c +
d*x]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]
^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.44, size = 4550, normalized size = 12.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2/b^2*4^(1/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+A*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+3*C*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+A
*cos(d*x+c)^3*b^5-2*C*cos(d*x+c)^3*a^5-4*A*cos(d*x+c)^2*a^3*b^2-4*A*cos(d*x
+c)^2*a*b^4-4*C*cos(d*x+c)^2*a^3*b^2+2*C*cos(d*x+c)^2*a^5-A*cos(d*x+c)*b^5+
7*A*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*b^3-4*A*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-2*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+C*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+6*C*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+2*C*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^4*b+2*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
```

```
b))^(1/2))*a^3*b^2-6*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b^3-6*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+5*A*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-4*A*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3*b^2-4*A*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-2*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-C*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3
*b^2+7*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b^3+9*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-4*C*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-
6*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^4+3*A*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-8*A*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^3+3*A
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b^2+4*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^4*b-12*C*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^3+A*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^4-4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-2*C*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^4*b+C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+3*C*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^4+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-6*C*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
```

```
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b^3+6*C*cos(d*x+c)^3*a^3*b^2+12*C*cos(d*x+c)^2*a^2*b^3+6*C*cos(
d*x+c)*a*b^4-6*C*cos(d*x+c)^2*a*b^4-3*A*cos(d*x+c)*a^2*b^3+4*A*cos(d*x+c)*a
*b^4+3*C*cos(d*x+c)*a^4*b-7*C*cos(d*x+c)*a^2*b^3+A*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+3*C*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^5+2*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^5+3*A*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+4*A*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-4*A*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-2*C*co
s(d*x+c)*a^3*b^2+4*A*cos(d*x+c)^3*a^3*b^2-5*A*cos(d*x+c)^3*a^2*b^3+C*cos(d*
x+c)^3*a^4*b-5*C*cos(d*x+c)^3*a^2*b^3+8*A*cos(d*x+c)^2*a^2*b^3-4*C*cos(d*x+
c)^2*a^4*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/s
in(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*
sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.751 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=517

$$-\frac{2\left(6a^2Ab+3a^2bC+a^3(-C)-aAb^2-3Ab^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{3a^2bd(a-b)(a+b)^{3/2}}$$

```
[Out] (-2*(3*A*b^4 - a^4*C - a^2*b^2*(7*A + 3*C))*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*b^2*Sqrt[a
 + b]*(a^2 - b^2)*d) - (2*(6*a^2*A*b - a*A*b^2 - 3*A*b^3 - a^3*C + 3*a^2*b*
C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*A*Sqrt[a + b]*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(a^3*d) + (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(3*a*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*(3*A*b^4 - a^4*C - a^2*b^2*(7*A +
3*C))*Tan[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.773228, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {4061, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\left(-a^2b^2(7A+3C)+a^4(-C)+3Ab^4\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2C+Ab^2\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2Ab+3a^2bC+a^3(-C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(3*A*b^4 - a^4*C - a^2*b^2*(7*A + 3*C))*Cot[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*b^2*Sqrt[a
 + b]*(a^2 - b^2)*d) - (2*(6*a^2*A*b - a*A*b^2 - 3*A*b^3 - a^3*C + 3*a^2*b*
C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*A*Sqrt[a + b]*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(a^3*d) + (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(3*a*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*(3*A*b^4 - a^4*C - a^2*b^2*(7*A +
3*C))*Tan[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}A\left(a^2-b^2\right)+\frac{3}{2}ab(A+C)\sec(c+dx)-\frac{1}{2}\left(Ab^2+a^2C\right)\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}d}{3a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int}{}$$

$$= -\frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3a^2(a-b)b^2(a+b)^{3/2}d}$$

$$= -\frac{2\left(3Ab^4-a^4C-a^2b^2(7A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3a^2(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [B] time = 20.7577, size = 1727, normalized size = 3.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((4*(-7*a^2*A*b
^2 + 3*A*b^4 - a^4*C - 3*a^2*b^2*C)*Sin[c + d*x])/(3*a^2*b*(-a^2 + b^2)^2)
- (4*(A*b^3*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) + (8*(4*a^2*A*b^2*Sin[c + d*x] - 2*A*b^4*Sin[c + d*x] + a
^4*C*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Co
s[c + d*x]))))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)
) - (4*(b + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)
*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*A*b^2*Tan[(c + d*x)/2
] + 7*a^2*A*b^3*Tan[(c + d*x)/2] - 3*a*A*b^4*Tan[(c + d*x)/2] - 3*A*b^5*Tan
[(c + d*x)/2] + a^5*C*Tan[(c + d*x)/2] + a^4*b*C*Tan[(c + d*x)/2] + 3*a^3*b
^2*C*Tan[(c + d*x)/2] + 3*a^2*b^3*C*Tan[(c + d*x)/2] - 14*a^3*A*b^2*Tan[(c
+ d*x)/2]^3 + 6*a*A*b^4*Tan[(c + d*x)/2]^3 - 2*a^5*C*Tan[(c + d*x)/2]^3 - 6
*a^3*b^2*C*Tan[(c + d*x)/2]^3 + 7*a^3*A*b^2*Tan[(c + d*x)/2]^5 - 7*a^2*A*b^
3*Tan[(c + d*x)/2]^5 - 3*a*A*b^4*Tan[(c + d*x)/2]^5 + 3*A*b^5*Tan[(c + d*x)
/2]^5 + a^5*C*Tan[(c + d*x)/2]^5 - a^4*b*C*Tan[(c + d*x)/2]^5 + 3*a^3*b^2*C
*Tan[(c + d*x)/2]^5 - 3*a^2*b^3*C*Tan[(c + d*x)/2]^5 - 6*a^4*A*b*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 12
*a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] - 6*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^4*A*b*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*
a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c
```

```
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a
+ b)*(-3*A*b^4 + a^4*C + a^2*b^2*(7*A + 3*C))*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - a*
b*(a + b)*(-2*A*b^2 + 3*a*b*(A + C) + a^2*(3*A + C))*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(3*a^2*b*(a^2 - b^2)^2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c +
 d*x])^(5/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*
(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.477, size = 6380, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C \sec^2(c + dx)}{(a + b \sec(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.752 $\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=559

$$\frac{\left(21a^2Ab^2+a^3b(3A-2C)+6a^4C-5aAb^3-15Ab^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{}\right.\right.}{3a^3bd(a-b)(a+b)^{3/2}}$$

```
[Out] -((26*a^2*A*b^2 - 15*A*b^4 - a^4*(3*A - 8*C))*Cot[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*a^3*(a - b)*
b*(a + b)^(3/2)*d) + ((21*a^2*A*b^2 - 5*a*A*b^3 - 15*A*b^4 + a^3*b*(3*A - 2
*C) + 6*a^4*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))]])/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) + (5*A*b*Sqr
t[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))]])/(a^4*d) + (A*Sin[c + d*x])/(a*d*(a + b*S
ec[c + d*x])^(3/2)) - (b*(5*A*b^2 - a^2*(3*A - 2*C))*Tan[c + d*x])/(3*a^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(26*a^2*A*b^2 - 15*A*b^4 - a^
4*(3*A - 8*C))*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]
])
```

Rubi [A] time = 1.19001, antiderivative size = 559, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4105, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\left(26a^2Ab^2+a^4(-(3A-8C))-15Ab^4\right)\tan(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{b\left(5Ab^2-a^2(3A-2C)\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{\left(21a^2Ab^2+a^3b(3A-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((26*a^2*A*b^2 - 15*A*b^4 - a^4*(3*A - 8*C))*Cot[c + d*x]*EllipticE[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*a^3*(a - b)*
b*(a + b)^(3/2)*d) + ((21*a^2*A*b^2 - 5*a*A*b^3 - 15*A*b^4 + a^3*b*(3*A - 2
*C) + 6*a^4*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))]])/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) + (5*A*b*Sqr
t[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))]])/(a^4*d) + (A*Sin[c + d*x])/(a*d*(a + b*S
ec[c + d*x])^(3/2)) - (b*(5*A*b^2 - a^2*(3*A - 2*C))*Tan[c + d*x])/(3*a^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(26*a^2*A*b^2 - 15*A*b^4 - a^
4*(3*A - 8*C))*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]
])
```


Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
```

```
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{5Ab}{2}-aC\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{a}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(5Ab^2-a^2(3A-2C)\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{15}{4}Al\cdots}{\cdots}}{\cdots}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(5Ab^2-a^2(3A-2C)\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{b\left(26a^2A\cdots\right.}{3a\cdots}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(5Ab^2-a^2(3A-2C)\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{b\left(26a^2A\cdots\right.}{3a\cdots}$$

$$= -\frac{\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+\cdots}{a-\cdots}\right.}{3a^3(a-b)b(a+b)^{3/2}d}$$

$$= -\frac{\left(26a^2Ab^2-15Ab^4-a^4(3A-8C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+\cdots}{a-\cdots}\right.}{3a^3(a-b)b(a+b)^{3/2}d}$$

Mathematica [B] time = 20.4938, size = 1714, normalized size = 3.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*((-8*(-5*a^2*A*
b^2 + 3*A*b^4 - 2*a^4*C)*Sin[c + d*x])/(3*a^3*(-a^2 + b^2)^2) + (4*(A*b^4*S
in[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*
x])^2) + (4*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*b^5*Sin[c + d*x] - 5*a^4*b*C*
Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c +
 d*x]))))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) - (
2*(b + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*(A + C*Sec[c + d*x]^2)*Sqrt
[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a^5*A*Tan[(c + d*x)/2] + 3*a^4
*A*b*Tan[(c + d*x)/2] - 26*a^3*A*b^2*Tan[(c + d*x)/2] - 26*a^2*A*b^3*Tan[(c
 + d*x)/2] + 15*a*A*b^4*Tan[(c + d*x)/2] + 15*A*b^5*Tan[(c + d*x)/2] - 8*a^
5*C*Tan[(c + d*x)/2] - 8*a^4*b*C*Tan[(c + d*x)/2] - 6*a^5*A*Tan[(c + d*x)/2
]^3 + 52*a^3*A*b^2*Tan[(c + d*x)/2]^3 - 30*a*A*b^4*Tan[(c + d*x)/2]^3 + 16*
a^5*C*Tan[(c + d*x)/2]^3 + 3*a^5*A*Tan[(c + d*x)/2]^5 - 3*a^4*A*b*Tan[(c +
d*x)/2]^5 - 26*a^3*A*b^2*Tan[(c + d*x)/2]^5 + 26*a^2*A*b^3*Tan[(c + d*x)/2]
^5 + 15*a*A*b^4*Tan[(c + d*x)/2]^5 - 15*A*b^5*Tan[(c + d*x)/2]^5 - 8*a^5*C*
Tan[(c + d*x)/2]^5 + 8*a^4*b*C*Tan[(c + d*x)/2]^5 + 30*a^4*A*b*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*a
^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] + 30*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^4*A*b*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
```

```
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*
a^2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a
 + b)*(-26*a^2*A*b^2 + 15*A*b^4 + a^4*(3*A - 8*C))*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x
)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 2*a*(a + b)*(3*a*A*b^2 - 5*A*b^3 + 3*a^3*C + a^2*b*(6*A + C))*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)]))/(3*a*(a^3 - a*b^2)^2*d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a +
 b*Sec[c + d*x])^(5/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/
2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.683, size = 6418, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*sec(d*x +
c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c)
 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.753 $\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=645

$$\frac{\left(-3a^2b^2(45A-8C)-a^3(27Ab-8bC)+6a^4(A-8C)+35aAb^3+105Ab^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{El}}{12a^4d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] -((105*A*b^4 + a^4*(33*A - 56*C) - 2*a^2*b^2*(85*A - 12*C))*Cot[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(
12*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + ((35*a*A*b^3 + 105*A*b^4 + 6*a^4*(A - 8
*C) - 3*a^2*b^2*(45*A - 8*C) - a^3*(27*A*b - 8*b*C))*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*a^4*
Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a + b]*(35*A*b^2 + 4*a^2*(A + 2*C))*Cot[
c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^5*d) - (7*A*b*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[
c + d*x])^(3/2)) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x]
)^(3/2)) + (b^2*(35*A*b^2 - a^2*(27*A - 8*C))*Tan[c + d*x])/(12*a^3*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (b^2*(105*A*b^4 + a^4*(33*A - 56*C) -
2*a^2*b^2*(85*A - 12*C))*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*S
ec[c + d*x]])
```

Rubi [A] time = 1.55869, antiderivative size = 645, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4105, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b^2\left(-2a^2b^2(85A-12C)+a^4(33A-56C)+105Ab^4\right)\tan(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b^2\left(35Ab^2-a^2(27A-8C)\right)\tan(c+dx)}{12a^3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{\left(-3a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((105*A*b^4 + a^4*(33*A - 56*C) - 2*a^2*b^2*(85*A - 12*C))*Cot[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(
12*a^4*Sqrt[a + b]*(a^2 - b^2)*d) + ((35*a*A*b^3 + 105*A*b^4 + 6*a^4*(A - 8
*C) - 3*a^2*b^2*(45*A - 8*C) - a^3*(27*A*b - 8*b*C))*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*a^4*
Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a + b]*(35*A*b^2 + 4*a^2*(A + 2*C))*Cot[
c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^5*d) - (7*A*b*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[
c + d*x])^(3/2)) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x]
)^(3/2)) + (b^2*(35*A*b^2 - a^2*(27*A - 8*C))*Tan[c + d*x])/(12*a^3*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (b^2*(105*A*b^4 + a^4*(33*A - 56*C) -
2*a^2*b^2*(85*A - 12*C))*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*S
ec[c + d*x]])
```


Rule 4105

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] +
Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[-(
A*b*(m + n + 1)) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, C, m}, x] && NeQ[a^2 - b^2,
 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(\frac{7Ab}{2}-a(A+2C)\sec(c+dx)-\frac{5}{2}Ab\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}$$

$$= -\frac{7Ab\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{1}{4}\left(35Ab^2+4a^2(A+2C\right.}{}$$

$$= -\frac{7Ab\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{b^2\left(35Ab^2-a^2(27\right.}{12a^3\left(a^2-b^2\right)d(}$$

$$= -\frac{7Ab\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{b^2\left(35Ab^2-a^2(27\right.}{12a^3\left(a^2-b^2\right)d(}$$

$$= -\frac{7Ab\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{b^2\left(35Ab^2-a^2(27\right.}{12a^3\left(a^2-b^2\right)d(}$$

$$= -\frac{\left(105Ab^4+a^4(33A-56C)-2a^2b^2(85A-12C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{s}}}{\sqrt{}}\right.\right.}{12a^4(a-b)(a+b)^{3/2}d}$$

$$= -\frac{\left(105Ab^4+a^4(33A-56C)-2a^2b^2(85A-12C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{s}}}{\sqrt{}}\right.\right.}{12a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [A] time = 14.4784, size = 801, normalized size = 1.24

$$\frac{1}{2}\left(\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{4b\left(-7Ca^4-13Ab^2a^2+3b^2Ca^2+9Ab^4\right)\sin(c+dx)}{3a^4\left(b^2-a^2\right)^2} - \frac{4\left(A\sin(c+dx)b^5+a^2C\sin(c+dx)b^3\right)}{3a^4\left(a^2-b^2\right)(b+a\cos(c+dx))^2} - \frac{8\left(5A\sin(c+dx)b^6-\right.}{}\right.}{d(a+b\sec(c+dx))^{5/2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])^(5/2
),x]
```

```
[Out] (((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((4*b*(-13*a^2*A*b^2 + 9*A*b^4 - 7*
a^4*C + 3*a^2*b^2*C)*Sin[c + d*x])/(3*a^4*(-a^2 + b^2)^2) - (4*(A*b^5*Sin[c
 + d*x] + a^2*b^3*C*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^
2) - (8*(-7*a^2*A*b^4*Sin[c + d*x] + 5*A*b^6*Sin[c + d*x] - 4*a^4*b^2*C*Sin
[c + d*x] + 2*a^2*b^4*C*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b + a*Cos[c +
d*x])) + (A*Sin[2*(c + d*x)])/(2*a^3)))/(d*(a + b*Sec[c + d*x])^(5/2)) - ((
```

```
b + a*Cos[c + d*x])^2*Sec[c + d*x]*(a*b*(a + b)*(105*A*b^4 + a^4*(33*A - 56
*C) + 2*a^2*b^2*(-85*A + 12*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] - b*(a + b)*(210*a*A*b^4 - 105*A*b^5 + 2*a^2*b^3*(29*A - 12*C) +
12*a^3*b^2*(-19*A + 4*C) - 6*a^5*(A + 12*C) + a^4*b*(39*A + 16*C))*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 3*(a - b)^2*(a + b)^2*(35*A*
b^2 + 4*a^2*(A + 2*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
 + a*b*(105*A*b^4 + a^4*(33*A - 56*C) + 2*a^2*b^2*(-85*A + 12*C))*(b + a*Co
s[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d
*x)/2]))/(6*a^5*(a^2 - b^2)^2*d*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*(a
+ b*Sec[c + d*x])^(5/2)))/2
```

Maple [B] time = 0.955, size = 9631, normalized size = 14.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+A\cos\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(b*sec(d*
x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x
```

```
+ c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2),
 x)
```

3.754 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=626

$$\frac{2\left(a^3b^2(13A+5C)+36a^2Ab^3-3a^4b(15A+8C)+3a^5C-5aAb^4-15Ab^5\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1}{a-b}}}{15a^3bd\sqrt{a+b}\left(a^2-b^2\right)^2}$$

```
[Out] (-2*(41*a^2*A*b^4 - 15*A*b^6 - 3*a^6*C - 29*a^4*b^2*(2*A + C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*a^3*b^2*Sqrt[a + b]*(a^2 - b^2)^2*d) + (2*(36*a^2*A*b^3 - 5*a*A*b^4 -
 15*A*b^5 + 3*a^5*C + a^3*b^2*(13*A + 5*C) - 3*a^4*b*(15*A + 8*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*a^3*b*Sqrt[a + b]*(a^2 - b^2)^2*d) - (2*A*Sqrt[a + b]*Cot[c + d*x
]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(a^4*d) + (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(5*a*(a^2 - b^2)*
d*(a + b*Sec[c + d*x])^(5/2)) - (2*(5*A*b^4 - 3*a^4*C - a^2*b^2*(13*A + 5*C
))*Tan[c + d*x])/(15*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2)) - (2*(
41*a^2*A*b^4 - 15*A*b^6 - 3*a^6*C - 29*a^4*b^2*(2*A + C))*Tan[c + d*x])/(15
*a^3*(a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.2757, antiderivative size = 626, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {4061, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\left(-29a^4b^2(2A+C)+41a^2Ab^4-3a^6C-15Ab^6\right)\tan(c+dx)}{15a^3d\left(a^2-b^2\right)^3\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-a^2b^2(13A+5C)-3a^4C+5Ab^4\right)\tan(c+dx)}{15a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(7/2),x]
```


```
[Out] (-2*(41*a^2*A*b^4 - 15*A*b^6 - 3*a^6*C - 29*a^4*b^2*(2*A + C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*a^3*b^2*Sqrt[a + b]*(a^2 - b^2)^2*d) + (2*(36*a^2*A*b^3 - 5*a*A*b^4 -
 15*A*b^5 + 3*a^5*C + a^3*b^2*(13*A + 5*C) - 3*a^4*b*(15*A + 8*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*a^3*b*Sqrt[a + b]*(a^2 - b^2)^2*d) - (2*A*Sqrt[a + b]*Cot[c + d*x
]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(a^4*d) + (2*(A*b^2 + a^2*C)*Tan[c + d*x])/(5*a*(a^2 - b^2)*
d*(a + b*Sec[c + d*x])^(5/2)) - (2*(5*A*b^4 - 3*a^4*C - a^2*b^2*(13*A + 5*C
))*Tan[c + d*x])/(15*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2)) - (2*(
41*a^2*A*b^4 - 15*A*b^6 - 3*a^6*C - 29*a^4*b^2*(2*A + C))*Tan[c + d*x])/(15
*a^3*(a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
```

```
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}A\left(a^2-b^2\right)+\frac{5}{2}ab(A+C)\sec(c+dx)-\frac{3}{2}\left(Ab^2+a^2C\right)\sec^2(c+d}{(a+b\sec(c+dx))^{5/2}}}{5a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\left(5Ab^4-3a^4C-a^2b^2(13A+5C)\right)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\left(5Ab^4-3a^4C-a^2b^2(13A+5C)\right)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} -$$

$$= \frac{2\left(Ab^2+a^2C\right)\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\left(5Ab^4-3a^4C-a^2b^2(13A+5C)\right)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} -$$

$$= -\frac{2\left(41a^2Ab^4-15Ab^6-3a^6C-29a^4b^2(2A+C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{15a^3(a-b)^2b^2(a+b)^{5/2}d}$$

$$= -\frac{2\left(41a^2Ab^4-15Ab^6-3a^6C-29a^4b^2(2A+C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{15a^3(a-b)^2b^2(a+b)^{5/2}d}$$

Mathematica [B] time = 22.0851, size = 2204, normalized size = 3.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + C*Sec[c + d*x]^2)*((4*(58*a^4*A
*b^2 - 41*a^2*A*b^4 + 15*A*b^6 + 3*a^6*C + 29*a^4*b^2*C)*Sin[c + d*x])/(15*
a^3*b*(-a^2 + b^2)^3) + (4*(A*b^4*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(
5*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^3) + (4*(-19*a^2*A*b^3*Sin[c + d*x]
+ 11*A*b^5*Sin[c + d*x] - 9*a^4*b*C*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x]))
/(15*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (4*(74*a^4*A*b^2*Sin[c + d
*x] - 65*a^2*A*b^4*Sin[c + d*x] + 23*A*b^6*Sin[c + d*x] + 9*a^6*C*Sin[c + d
*x] + 25*a^4*b^2*C*Sin[c + d*x] - 2*a^2*b^4*C*Sin[c + d*x]))/(15*a^3*(a^2 -
 b^2)^3*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + b*Se
c[c + d*x])^(7/2)) - (4*(b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)*(A +
C*Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(58*a^5*A*b
^2*Tan[(c + d*x)/2] + 58*a^4*A*b^3*Tan[(c + d*x)/2] - 41*a^3*A*b^4*Tan[(c +
 d*x)/2] - 41*a^2*A*b^5*Tan[(c + d*x)/2] + 15*a*A*b^6*Tan[(c + d*x)/2] + 15
*A*b^7*Tan[(c + d*x)/2] + 3*a^7*C*Tan[(c + d*x)/2] + 3*a^6*b*C*Tan[(c + d*x
)/2] + 29*a^5*b^2*C*Tan[(c + d*x)/2] + 29*a^4*b^3*C*Tan[(c + d*x)/2] - 116*
a^5*A*b^2*Tan[(c + d*x)/2]^3 + 82*a^3*A*b^4*Tan[(c + d*x)/2]^3 - 30*a*A*b^6
*Tan[(c + d*x)/2]^3 - 6*a^7*C*Tan[(c + d*x)/2]^3 - 58*a^5*b^2*C*Tan[(c + d*
x)/2]^3 + 58*a^5*A*b^2*Tan[(c + d*x)/2]^5 - 58*a^4*A*b^3*Tan[(c + d*x)/2]^5
 - 41*a^3*A*b^4*Tan[(c + d*x)/2]^5 + 41*a^2*A*b^5*Tan[(c + d*x)/2]^5 + 15*a
*A*b^6*Tan[(c + d*x)/2]^5 - 15*A*b^7*Tan[(c + d*x)/2]^5 + 3*a^7*C*Tan[(c +
d*x)/2]^5 - 3*a^6*b*C*Tan[(c + d*x)/2]^5 + 29*a^5*b^2*C*Tan[(c + d*x)/2]^5
- 29*a^4*b^3*C*Tan[(c + d*x)/2]^5 - 30*a^6*A*b*EllipticPi[-1, -ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 90*a^4*A*b^3*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
```

```
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 90*a^2*A*b^5*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + 30*A*b^7*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^6*A*b*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
+ 90*a^4*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*T
an[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 90*a^2*A*b^5*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 30*A*b^7*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-41*a^2*A*b^4 + 15*A*b^
6 + 3*a^6*C + 29*a^4*b^2*(2*A + C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - a*b*(a + b)*
(-6*a*A*b^3 + 10*A*b^4 + 3*a^4*(5*A + C) + 6*a^3*b*(5*A + 4*C) + a^2*b^2*(-
17*A + 5*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(15*a^3*b*(a^2 - b^2)^3*d*(A + 2*C
+ A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(7/2)*Sqrt[1 + Tan[(c + d*x)/2]^
2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.721, size = 11805, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{4}\sec\left(dx+c\right)^{4}+4\,ab^{3}\sec\left(dx+c\right)^{3}+6\,a^{2}b^{2}\sec\left(dx+c\right)^{2}+4\,a^{3}b\sec\left(dx+c\right)+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(b*sec(d*x + c) + a)/(b^4*sec(d*x + c)^
4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*b*sec(d*x + c
) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(7/2), x)
```

3.755 $\int \frac{a^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=303

$$\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d -
 (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rubi [A] time = 0.289166, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4042, 3916, 3784, 12, 3837, 3832, 4004}

$$\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d -
 (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3916

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
```

```
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\int(-a+b\sec(c+dx))\sqrt{a+b\sec(c+dx)}\,dx\\
&= a^2\int\frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx-\int\frac{b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\
&= -\frac{2a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}\\
&= -\frac{2a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}\\
&= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} +
\end{aligned}$$

Mathematica [C] time = 15.6402, size = 939, normalized size = 3.1

$$-\frac{4b\cos(c+dx)(b+a\cos(c+dx))\left(a^2-b^2\sec^2(c+dx)\right)\sin(c+dx)}{d\left(\cos(2c+2dx)a^2+a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{4\sqrt{b+a\cos(c+dx)}\left(a^2-b^2\sec^2(c+dx)\right)\sqrt{-\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*b*Cos[c + d*x]*(b + a*Cos[c + d*x])*(a^2 - b^2*Sec[c + d*x]^2)*Sin[c +
d*x])/(d*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])*Sqrt[a + b*Sec[c + d*x]]) - (
4*Sqrt[b + a*Cos[c + d*x]]*(a^2 - b^2*Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*
x)/2]^2)^(-1)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(-(a*b*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2]) - b^2*Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] + a*b*Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] - b^2*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + (2*I)*a^2*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
b)/(a - b)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*b*Ellipti
cE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - I*(a^2 - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a +
b)]*d*(a^2 - 2*b^2 + a^2*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.463, size = 1020, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*
(cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))+cos(d*x+c)*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)*b^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*cos(d*x+
c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))+a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))+b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+cos(d*x+c)^2*a*b-cos(d*x+c)*a*b+
```

```
cos(d*x+c)*b^2-b^2)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^2 \sec(dx+c)^2 - a^2}{\sqrt{b \sec(dx+c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((b^2*sec(d*x + c)^2 - a^2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\sqrt{b \sec(dx+c) + a}(b \sec(dx+c) - a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sec(d*x + c) + a)*(b*sec(d*x + c) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a - b \sec(c+dx)) \sqrt{a + b \sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))*sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2 \sec(dx+c)^2 - a^2}{\sqrt{b \sec(dx+c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*sec(d*x + c)^2 - a^2)/sqrt(b*sec(d*x + c) + a), x)
```

3.756 $\int \frac{a^2-b^2 \sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\text{se}}{a}}}{}$$

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/d - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a
 + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
d
```

Rubi [A] time = 0.16415, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4042, 3921, 3784, 3832}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/d - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a
 + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
d
```


Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\int \frac{-a+b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\ &= a\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx - b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\ &= -\frac{2\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - 2\sqrt{}\end{aligned}$$

Mathematica [A] time = 2.08175, size = 145, normalized size = 0.72

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left((a+b)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2a\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((a + b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.379, size = 214, normalized size = 1.1

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d\,(b+a\cos(dx+c))\,(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)\,(\cos(dx+c)+1)}}\left(\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\sec\left(dx+c\right)-a}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sec(d*x + c) - a)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a-b\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{b^{2}\sec\left(dx+c\right)^{2}-a^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*sec(d*x + c)^2 - a^2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.757 $\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=338

$$-\frac{4\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{4b^2\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+$$

```
[Out] (4*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) - (2*Sq
rt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (4*b^2*Tan[c + d*x])/((a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.403918, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4042, 3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{4b^2\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{4\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{4\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (4*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) - (2*Sq
rt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (4*b^2*Tan[c + d*x])/((a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
```

```
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{a^2 - b^2 \sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\int \frac{-a+b\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{4b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\frac{1}{2}a(a^2-b^2)-a^2b\sec(c+dx)-ab^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}$$

$$= \frac{4b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\frac{1}{2}a(a^2-b^2)+(-a^2b+ab^2)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)} - \frac{(2b^2)\int \frac{\sec(c+dx)(1+}{\sqrt{a+b\sec}}}{a^2-b^2}$$

$$= \frac{4\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} + \frac{4b^2}{(a^2-b^2)d}$$

$$= \frac{4\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} - 4\cot(c+d$$

Mathematica [C] time = 14.0087, size = 616, normalized size = 1.82

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)^2(a-b\sec(c+dx))\left(\frac{4b\sin(c+dx)}{b^2-a^2} - \frac{4b^2\sin(c+dx)}{(b^2-a^2)(a\cos(c+dx)+b)}\right)}{d(a\cos(c+dx)-b)(a+b\sec(c+dx))^{3/2}} + \frac{4\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(a - b*Sec[c + d*x])*((4*b*Sin[c + d*x
])/(-a^2 + b^2) - (4*b^2*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x])))
)/(d*(-b + a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2)) + (4*(b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*(a - b*Sec[c + d*x])*((2*I)*(a - b)*b*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)] - I*(a^2 + 2*a*b - 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + (2*I)*(a^2 -
b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - b*Sqrt[(-a + b)/(a + b)]
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(S
qrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(-b + a*Cos[c + d*x])*(a + b*Sec[c + d*
x])^(3/2)*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [B] time = 0.407, size = 1392, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d/(a-b)/(a+b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*a^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
```

```
)^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)+2*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a*b+cos(d*x+c)*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*cos(d*x+c)*b^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*c
os(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))+2*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2+a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+b^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin
(d*x+c)+2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*b^2+2*cos(d*x+c)^2*a*b-2*cos(d*x+c)^2*b^2-2*cos(d*x+c)*a*b+2
*cos(d*x+c)*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sec(dx+c)+a}(b\sec(dx+c)-a)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sec(d*x + c) + a)*(b*sec(d*x + c) - a)/(b^2*sec(d*x + c)^2
 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a - b\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a - b*sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2\sec(dx + c)^2 - a^2}{(b\sec(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*sec(d*x + c)^2 - a^2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.758 $\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=445

$$-\frac{2\left(9a^2-2ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3ad(a-b)(a+b)^{3/2}}+\frac{2b^2\left(11a^2-3b\right.}{3ad\left(a^2-b^2\right)^2\sqrt{}}$$

```
[Out] (2*(11*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(9*a^2
 - 2*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*Sqrt[a +
 b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (4*b^2*Tan[c + d*x])/(3*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(11*a^2 - 3*b^2)*Tan[c + d*x])/(3*a
*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.626379, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4042, 3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\left(11a^2-3b^2\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{4b^2\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(9a^2-2ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{3ad(a-b)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(7/2),x]
```


```
[Out] (2*(11*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(9*a^2
 - 2*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*Sqrt[a +
 b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (4*b^2*Tan[c + d*x])/(3*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(11*a^2 - 3*b^2)*Tan[c + d*x])/(3*a
*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4042

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Dist[C/b^2, Int[(a + b*Csc[e + f*x])^(m + 1)*
Simp[-a + b*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
```

```
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{a^2-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx = -\int \frac{-a+b\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{4b^2\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}a\left(a^2-b^2\right)-3a^2b\sec(c+dx)+ab^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{4b^2\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(11a^2-3b^2\right)\tan(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{-\frac{3}{4}a\left(a^2-b^2\right)^2+\frac{1}{4}a}{}}{}$$

$$= \frac{4b^2\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(11a^2-3b^2\right)\tan(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{-\frac{3}{4}a\left(a^2-b^2\right)^2+(-}{}}{}$$

$$= \frac{2\left(11a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} + \frac{}{3}$$

$$= \frac{2\left(11a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} - \frac{2}{}$$

Mathematica [C] time = 14.544, size = 1849, normalized size = 4.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - b^2*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(a - b*Sec[c + d*x])*((2*b*(-11*a^2
 + 3*b^2)*Sin[c + d*x])/(3*a*(-a^2 + b^2)^2) - (4*b^3*Sin[c + d*x])/(3*a*(a^
2 - b^2)*(b + a*Cos[c + d*x])^2) - (2*(-13*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin
[c + d*x]))/(3*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(-b + a*Cos[c + d
*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*
x]^(3/2)*(a - b*Sec[c + d*x])*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(11*a^3*b*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] + 11*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*a*b
^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*b^4*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2] - 22*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 6*a*
b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 11*a^3*b*Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]^5 - 11*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*b^4*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]^5 - (6*I)*a^4*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] + (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*a^4*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (12*I)*
a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
```

```
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - I*b*(11*a^3 - 11*a^2*b - 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + I*(3*a^4 + 9*a^3*b - 17*a^2*b^2 - a*b^3 + 6*b^4)*El
lipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a*Sqrt[(-a + b)/(a +
b)]*(a^2 - b^2)^2*d*(-b + a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2)*(-1 +
Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]
*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.433, size = 3887, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2/a*4^(1/2)*(6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^5-3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^5-11*cos(d*x+c)^3*a^4*b+22*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^2-12*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-14*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^3*b^2-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b^3+11*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^4*b+8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+6*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*si
n(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*
b-12*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*a^3*b^2-12*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^3+6*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a
*b^4+13*cos(d*x+c)^3*a^3*b^2+3*cos(d*x+c)^3*a^2*b^3-5*cos(d*x+c)^3*a*b^4+11
```

```
*cos(d*x+c)^2*a^4*b-22*cos(d*x+c)^2*a^3*b^2+8*cos(d*x+c)^2*a^2*b^3+6*cos(d*
x+c)^2*a*b^4+9*cos(d*x+c)*a^3*b^2-11*cos(d*x+c)*a^2*b^3-cos(d*x+c)*a*b^4-3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^5+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*b^5-3*cos(d*x+c)^2*b^5+3*cos(d*x+c)*b^5-3*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+6*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^5+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^5-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+11*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+11*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b^3-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^4+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a^4*b-12*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^3-3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-9*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-5*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^4+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^4-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^4+11*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+11*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*s
in(d*x+c)*a^3*b^2-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-12*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*a^3*b^2+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^4-9*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sec\left(dx+c\right)+a}(b\sec\left(dx+c\right)-a)}{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sec(d*x + c) + a)*(b*sec(d*x + c) - a)/(b^3*sec(d*x + c)^3
 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-b**2*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^2\sec\left(dx+c\right)^2-a^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-b^2*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(-(b^2*sec(d*x + c)^2 - a^2)/(b*sec(d*x + c) + a)^(7/2), x)
```

3.759 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=145

$$-\frac{2Ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a^2d(a+b)}$$

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(a^2*d) + (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rubi [A] time = 0.251407, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4107, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(a^2C+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2Ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2A\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```


```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(a^2*d) + (2*(A*b^2 + a^2*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```


Rule 4107

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))), x_Symbol] :> Dist[(A*b^2 +
 a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x]
+ Dist[1/a^2, Int[(a*A - A*b*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /;
FreeQ[{a, b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx = \frac{\int \frac{aA-Ab\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2} + \left(\frac{Ab^2}{a^2}+C\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$= \frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a} - \frac{(Ab)\int \sqrt{\sec(c+dx)}\,dx}{a^2} + \left(\left(\frac{Ab^2}{a^2}+C\right)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$

$$= \frac{2\left(\frac{Ab^2}{a^2}+C\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d} + \frac{\left(A\sqrt{\cos(c+\right.}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{a^2d}$$

Mathematica [F] time = 31.299, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),
x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),
 x]
```

Maple [A] time = 2.295, size = 259, normalized size = 1.8

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\,a^2\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(A\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2+C*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))*a^2)/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c+dx)}{(a+b\sec(c+dx))\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))),
 x)
```

3.760 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=213

$$-\frac{2Ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2C\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (-2*A*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[
c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.566429, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2Ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2C\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{a+b\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*A*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[
c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]])
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = A\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx + C\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{A\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{(Ab)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} + \frac{\left(C\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c-\right.}{\sqrt{a+b\sec(c}}$$

$$= -\frac{\left(Ab\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\left(C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{s\right.}{\sqrt{a}}$$

$$= \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{\left(Ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c\right.}{a\sqrt{a+bs}}$$

$$= -\frac{2Ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c\right.}{d\sqrt{a+bse}}$$

Mathematica [F] time = 9.0736, size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]), x]
```

Maple [C] time = 0.483, size = 1160, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)/a*(A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-A*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*a+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a-2*C*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*a+A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
```

```
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+A*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-2*C*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^2*a+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a-A*((a-b)/(a+b))^(1/
2)*cos(d*x+c)*b+A*b*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c
))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + A}{\sqrt{b \sec (dx+c) + a}\sqrt{\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c
))), x)
```

3.761 $\int (a+b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=242

$$A\text{Unintegrable}\left((a+b\sec(c+dx))^{2/3},x\right)+\frac{\sqrt{2}C(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+\right.}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*C*
AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/
(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x
]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Unintegrable[(a + b*Sec[c + d*
x])^(2/3), x]
```

Rubi [A] time = 0.310348, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*C*
AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/
(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x
]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Defer[Int][(a + b*Sec[c + d*x]
)^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx = \frac{\int (a+b\sec(c+dx))^{2/3}(Ab-aC\sec(c+dx))\,dx}{b} + \frac{C\int \sec(c+dx)(a\cdots}{\cdots}$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx - \frac{(aC)\int \sec(c+dx)(a+b\sec(c+dx))^{2\cdots}}{b}$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx + \frac{(aC\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}d\cdots\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec\cdots}}$$

$$= \frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec\cdots}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec\cdots}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] time = 56.9036, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{2/3}\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + C*Sec[c + d*x]^2), x]
```

Maple [A] time = 0.172, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{2}{3}}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(A + C\sec^2 (c + dx)\right) (a + b\sec (c + dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx + c)^2 + A\right)(b\sec (dx + c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(2/3), x)
```

3.762 $\int \sqrt[3]{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=242

$$A\text{Unintegrable}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)+\frac{\sqrt{2}C(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\right.}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*a*C*
AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/
(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x
]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Unintegrable[(a + b*Sec[c + d*
x])^(1/3), x]
```

Rubi [A] time = 0.291975, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*a*C*
AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/
(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x
]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Defer[Int][(a + b*Sec[c + d*x]
)^(1/3), x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx &= \frac{\int \sqrt[3]{a+b\sec(c+dx)}(Ab-aC\sec(c+dx))\,dx}{b}+\frac{C\int \sec(c+dx)(a+b}{b}\\
&= A\int \sqrt[3]{a+b\sec(c+dx)}\,dx-\frac{(aC)\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{b}\\
&= A\int \sqrt[3]{a+b\sec(c+dx)}\,dx+\frac{(aC\tan(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+}}\\
&= \frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(}}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}\\
&= \frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(}}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}
\end{aligned}$$

Mathematica [A] time = 46.1698, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + C*Sec[c + d*x]^2), x]
```

Maple [A] time = 0.179, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(dx+c)}\left(A+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(A+C\sec^2(c+dx)\right)\sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*(a + b*sec(c + d*x))**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(b*sec(d*x + c) + a)^(1/3), x)
```

3.763 $\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=239

$$A\text{Unintegrable}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)-\frac{\sqrt{2}aC\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+d}{a+b}\right.}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*C*AppellF1
[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Unintegrable[(a + b*Sec[c + d*x])^(-1/3
), x]
```

Rubi [A] time = 0.294659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*C*AppellF1
[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Defer[Int][(a + b*Sec[c + d*x])^(-1/3),
 x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\int \frac{Ab-aC\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b} + \frac{C\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b}$$

$$= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx - \frac{(aC)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b} - \frac{(C\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\text{se}}}$$

$$= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx + \frac{(aC\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} + A$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \sqrt{2}$$

Mathematica [A] time = 76.2612, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3), x]
```

Maple [A] time = 0.163, size = 0, normalized size = 0.

$$\int (A+C(\sec(dx+c))^2)\frac{1}{\sqrt[3]{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt[3]{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{(b\sec(dx + c) + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(1/3), x)
```

3.764 $\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=239

$$A\text{Unintegrable}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)-\frac{\sqrt{2}aC\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*a*C*AppellF1
[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Unintegrable[(a + b*Sec[c + d*x])^(-2/3
), x]
```

Rubi [A] time = 0.294517, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*a*C*AppellF1
[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Defer[Int][(a + b*Sec[c + d*x])^(-2/3),
 x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{\int \frac{Ab-aC\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{b} + \frac{C\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{b}$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx - \frac{(aC)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{b} - \frac{(C\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{}{}\right.}{bd\sqrt{1-\sec(c+dx}}$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx + \frac{(aC\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx, x, \sec(c\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} +$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} -$$

Mathematica [A] time = 58.7904, size = 0, normalized size = 0.

$$\int \frac{A+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Integrate[(A + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3), x]
```

Maple [A] time = 0.173, size = 0, normalized size = 0.

$$\int \left(A+C\left(\sec\left(dx+c\right)\right)^{2}\right)\left(a+b\sec\left(dx+c\right)\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec{(dx + c)}^2 + A}{(b\sec{(dx + c)} + a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(b*sec(d*x + c) + a)^(2/3), x)
```

3.765 $\int \sec^3(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=145

$$\frac{(5aB+4bC)\tan^3(c+dx)}{15d}+\frac{(5aB+4bC)\tan(c+dx)}{5d}+\frac{3(aC+bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(aC+bB)\tan(c+dx)\sec}{4d}$$

```
[Out] (3*(b*B + a*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((5*a*B + 4*b*C)*Tan[c + d*x]
)/(5*d) + (3*(b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((b*B + a*C)*Se
c[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
+ ((5*a*B + 4*b*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.203111, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3997, 3787, 3767, 3768, 3770}

$$\frac{(5aB+4bC)\tan^3(c+dx)}{15d}+\frac{(5aB+4bC)\tan(c+dx)}{5d}+\frac{3(aC+bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(aC+bB)\tan(c+dx)\sec}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```


```
[Out] (3*(b*B + a*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((5*a*B + 4*b*C)*Tan[c + d*x]
)/(5*d) + (3*(b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((b*B + a*C)*Se
c[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*C*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
+ ((5*a*B + 4*b*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^4(c+dx)(a+b\sec(c+dx))(B+C\sec(c+$$
$$= \frac{bC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^4(c+dx$$
$$= \frac{bC\sec^4(c+dx)\tan(c+dx)}{5d}+(bB+aC)\int \sec$$
$$= \frac{(bB+aC)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{bC\sec^4(c$$
$$= \frac{(5aB+4bC)\tan(c+dx)}{5d}+\frac{3(bB+aC)\sec(c+}{8d}$$
$$= \frac{3(bB+aC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(5aB+4bC)}{5}$$

Mathematica [A] time = 0.88028, size = 106, normalized size = 0.73

$$\frac{45(aC+bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5(aB+2bC)\tan^2(c+dx)+15(aB+bC)+3bC\tan^4(c+dx)\right)+30(a\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (45*(b*B + a*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(45*(b*B + a*C)*Sec[c
+ d*x] + 30*(b*B + a*C)*Sec[c + d*x]^3 + 8*(15*(a*B + b*C) + 5*(a*B + 2*b*C
)*Tan[c + d*x]^2 + 3*b*C*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.036, size = 213, normalized size = 1.5

$$\frac{2\,Ba\tan(dx+c)}{3\,d}+\frac{Ba\tan(dx+c)(\sec(dx+c))^2}{3\,d}+\frac{aC(\sec(dx+c))^3\tan(dx+c)}{4\,d}+\frac{3\,C\sec(dx+c)\,a\tan(dx+c)}{8\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/3/d*B*a*tan(d*x+c)+1/3/d*B*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*C*sec(d*x+c)^3
*tan(d*x+c)/d+3/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec(d*x+c)+tan(d
```

```
*x+c))+1/4/d*B*b*tan(d*x+c)*sec(d*x+c)^3+3/8/d*B*b*tan(d*x+c)*sec(d*x+c)+3/
8/d*B*b*ln(sec(d*x+c)+tan(d*x+c))+8/15*b*C*tan(d*x+c)/d+1/5*b*C*sec(d*x+c)^
4*tan(d*x+c)/d+4/15*b*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.968834, size = 270, normalized size = 1.86

$$\frac{80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Cb-15\,Ca\left(\frac{2\left(3\sin(\cdots\right.}{\sin(dx+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*C*b - 15*C*a*(2*(3*sin(d*x + c)^3 - 5*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) - 15*B*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*lo
g(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.959076, size = 397, normalized size = 2.74

$$\frac{45\,(Ca+Bb)\cos(dx+c)^5\log(\sin(dx+c)+1)-45\,(Ca+Bb)\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(16\,(5\,Ba+4\cdots\right.}{240\,d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/240*(45*(C*a + B*b)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*(C*a + B*b)
*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*B*a + 4*C*b)*cos(d*x + c)
^4 + 45*(C*a + B*b)*cos(d*x + c)^3 + 8*(5*B*a + 4*C*b)*cos(d*x + c)^2 + 24*
C*b + 30*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B+C\sec(c+dx))\,(a+b\sec(c+dx))\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [B] time = 1.27524, size = 446, normalized size = 3.08

$$45\,(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,Ca\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/120*(45*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*(C*a + B*b)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*B*a*tan(1/2*d*x + 1/2*c)^9 - 75*
C*a*tan(1/2*d*x + 1/2*c)^9 - 75*B*b*tan(1/2*d*x + 1/2*c)^9 + 120*C*b*tan(1/
2*d*x + 1/2*c)^9 - 320*B*a*tan(1/2*d*x + 1/2*c)^7 + 30*C*a*tan(1/2*d*x + 1/
2*c)^7 + 30*B*b*tan(1/2*d*x + 1/2*c)^7 - 160*C*b*tan(1/2*d*x + 1/2*c)^7 + 4
00*B*a*tan(1/2*d*x + 1/2*c)^5 + 464*C*b*tan(1/2*d*x + 1/2*c)^5 - 320*B*a*ta
n(1/2*d*x + 1/2*c)^3 - 30*C*a*tan(1/2*d*x + 1/2*c)^3 - 30*B*b*tan(1/2*d*x +
 1/2*c)^3 - 160*C*b*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*tan(1/2*d*x + 1/2*c) +
 75*C*a*tan(1/2*d*x + 1/2*c) + 75*B*b*tan(1/2*d*x + 1/2*c) + 120*C*b*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.766 $\int \sec^2(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=114

$$\frac{(aC+bB)\tan^3(c+dx)}{3d}+\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(4aB+3bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aB+3bC)\tan(c+dx)\sec(}{8d}$$

```
[Out] ((4*a*B + 3*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((b*B + a*C)*Tan[c + d*x])/
d + ((4*a*B + 3*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*C*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((b*B + a*C)*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.185197, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3997, 3787, 3768, 3770, 3767}

$$\frac{(aC+bB)\tan^3(c+dx)}{3d}+\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(4aB+3bC)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aB+3bC)\tan(c+dx)\sec(}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```



```
[Out] ((4*a*B + 3*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((b*B + a*C)*Tan[c + d*x])/
d + ((4*a*B + 3*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*C*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((b*B + a*C)*Tan[c + d*x]^3)/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+b\sec(c+dx))(B+C\sec(c+$$

$$= \frac{bC\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int \sec^3(c+dx$$

$$= \frac{bC\sec^3(c+dx)\tan(c+dx)}{4d} + (bB+aC)\int \sec$$

$$= \frac{(4aB+3bC)\sec(c+dx)\tan(c+dx)}{8d} + \frac{bC\sec^3}{}$$

$$= \frac{(4aB+3bC)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(bB+aC)t}{d}$$

Mathematica [A] time = 0.627029, size = 85, normalized size = 0.75

$$\frac{3(4aB+3bC)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(aC+bB)(\cos(2(c+dx))+2)\sec(c+dx)+12aB+6bC\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (3*(4*a*B + 3*b*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*a*B + 9*b*C + 8
*(b*B + a*C)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*b*C*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

Maple [A] time = 0.036, size = 171, normalized size = 1.5

$$\frac{Ba\sec(dx+c)\tan(dx+c)}{2d} + \frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{2aC\tan(dx+c)}{3d} + \frac{aC(\sec(dx+c))^2\tan(dx+c)}{3d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*a*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*C
*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d+2/3/d*B*b*tan(d*x+c)+1/3/d*
B*b*tan(d*x+c)*sec(d*x+c)^2+1/4*b*C*sec(d*x+c)^3*tan(d*x+c)/d+3/8*b*C*sec(d
*x+c)*tan(d*x+c)/d+3/8/d*C*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.976387, size = 220, normalized size = 1.93

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Bb-3\,Cb\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*b - 3*C*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.01197, size = 352, normalized size = 3.09

$$\frac{3\left(4\,Ba+3\,Cb\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,Ba+3\,Cb\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(Ca+\right.\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/48*(3*(4*B*a + 3*C*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*B*a + 3
*C*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(C*a + B*b)*cos(d*x + c
)^3 + 3*(4*B*a + 3*C*b)*cos(d*x + c)^2 + 6*C*b + 8*(C*a + B*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B+C\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.2163, size = 410, normalized size = 3.6

$$3\left(4\,Ba+3\,Cb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,Ba+3\,Cb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ca\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(4*B*a + 3*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*B*a + 3*C
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*B*a*tan(1/2*d*x + 1/2*c)^7 -
 24*C*a*tan(1/2*d*x + 1/2*c)^7 - 24*B*b*tan(1/2*d*x + 1/2*c)^7 + 15*C*b*tan
(1/2*d*x + 1/2*c)^7 - 12*B*a*tan(1/2*d*x + 1/2*c)^5 + 40*C*a*tan(1/2*d*x +
1/2*c)^5 + 40*B*b*tan(1/2*d*x + 1/2*c)^5 + 9*C*b*tan(1/2*d*x + 1/2*c)^5 - 1
2*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*C*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*b*tan(1
/2*d*x + 1/2*c)^3 + 9*C*b*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*tan(1/2*d*x + 1/2
*c) + 24*C*a*tan(1/2*d*x + 1/2*c) + 24*B*b*tan(1/2*d*x + 1/2*c) + 15*C*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.767 $\int \sec(c+dx)(a+b\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=93

$$\frac{(3aB+2bC)\tan(c+dx)}{3d}+\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aC+bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bC\tan(c+dx)\sec}{3d}$$

```
[Out] ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*a*B + 2*b*C)*Tan[c + d*x])/
(3*d) + ((b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*C*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.153376, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 3997, 3787, 3767, 8, 3768, 3770}

$$\frac{(3aB+2bC)\tan(c+dx)}{3d}+\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aC+bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bC\tan(c+dx)\sec}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x
]
```


```
[Out] ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*a*B + 2*b*C)*Tan[c + d*x])/
(3*d) + ((b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*C*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+b\sec(c+dx))(B+C\sec(c+$$
$$= \frac{bC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec^2(c+dx)($$
$$= \frac{bC\sec^2(c+dx)\tan(c+dx)}{3d}+(bB+aC)\int \sec^3$$
$$= \frac{(bB+aC)\sec(c+dx)\tan(c+dx)}{2d}+\frac{bC\sec^2(c+$$
$$= \frac{(bB+aC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3aB+2bC)\mathrm{ta}}{3d}$$

Mathematica [A] time = 0.290459, size = 67, normalized size = 0.72

$$\frac{3(aC+bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aC+bB)\sec(c+dx)+6aB+2bC\tan^2(c+dx)+6bC\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] (3*(b*B + a*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*B + 6*b*C + 3*(b*B
 + a*C)*Sec[c + d*x] + 2*b*C*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.034, size = 128, normalized size = 1.4

$$\frac{Ba\tan(dx+c)}{d}+\frac{C\sec(dx+c)a\tan(dx+c)}{2d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Bb\tan(dx+c)\sec(dx+c)}{2d}+\frac{Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a*tan(d*x+c)+1/2*a*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*C*ln(sec(d*x+c)+
tan(d*x+c))+1/2/d*B*b*tan(d*x+c)*sec(d*x+c)+1/2/d*B*b*ln(sec(d*x+c)+tan(d*x
```

```
+c))+2/3*b*C*tan(d*x+c)/d+1/3*b*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 0.986974, size = 171, normalized size = 1.84

$$\frac{4\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Cb - 3\,Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 3\,Bb\left(\frac{2\sin(\ldots}{\sin(dx\ldots}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b - 3*C*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*a*tan(d*x + c))/d
```

Fricas [A] time = 0.683446, size = 298, normalized size = 3.2

$$\frac{3\,(Ca+Bb)\cos(dx+c)^3\log(\sin(dx+c)+1) - 3\,(Ca+Bb)\cos(dx+c)^3\log(-\sin(dx+c)+1) + 2\left(2\,(3\,Ba+2\,Cb)\ldots\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/12*(3*(C*a + B*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(C*a + B*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*B*a + 2*C*b)*cos(d*x + c)^2 +
 2*C*b + 3*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\sec(c+dx))\,(a + b\sec(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.18757, size = 284, normalized size = 3.05

$$\frac{3\,(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(6\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] 1/6*(3*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(C*a + B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a*tan(1/2*d*x + 1/2*c)^5 - 3*C*a*tan
(1/2*d*x + 1/2*c)^5 - 3*B*b*tan(1/2*d*x + 1/2*c)^5 + 6*C*b*tan(1/2*d*x + 1/
2*c)^5 - 12*B*a*tan(1/2*d*x + 1/2*c)^3 - 4*C*b*tan(1/2*d*x + 1/2*c)^3 + 6*B
*a*tan(1/2*d*x + 1/2*c) + 3*C*a*tan(1/2*d*x + 1/2*c) + 3*B*b*tan(1/2*d*x +
1/2*c) + 6*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.768 $\int (a+b\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=61

$$\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(2aB+bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((2*a*B + b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((b*B + a*C)*Tan[c + d*x])/d
+ (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0658929, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4048, 3770, 3767, 8}

$$\frac{(aC+bB)\tan(c+dx)}{d}+\frac{(2aB+bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((2*a*B + b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((b*B + a*C)*Tan[c + d*x])/d
+ (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{bC\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left((2aB+bC)\sec(c+dx)+\right.$$
$$=\frac{bC\sec(c+dx)\tan(c+dx)}{2d}+(bB+aC)\int \sec^2(c+dx)\,dx+$$
$$=\frac{(2aB+bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\sec(c+dx)\tan(c+d}{2d}$$
$$=\frac{(2aB+bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(bB+aC)\tan(c+dx)}{d}+$$

Mathematica [A] time = 0.0270714, size = 75, normalized size = 1.23

$$\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\tan(c+dx)}{d}+\frac{bB\tan(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*B*ArcTanh[Sin[c + d*x]])/d + (b*C*ArcTanh[Sin[c + d*x]])/(2*d) + (b*B*Ta
n[c + d*x])/d + (a*C*Tan[c + d*x])/d + (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A] time = 0.03, size = 86, normalized size = 1.4

$$\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Bb\tan(dx+c)}{d}+\frac{Cb\sec(dx+c)\tan(dx+c)}{2d}+\frac{Cb\ln(\sec(dx-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+a*C*tan(d*x+c)/d+1/d*B*b*tan(d*x+c)+1/2*b
*C*sec(d*x+c)*tan(d*x+c)/d+1/2/d*C*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.962397, size = 119, normalized size = 1.95

$$-\frac{Cb\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4Ba\log(\sec(dx+c)+\tan(dx+c))-4Ca\tan(dx+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(C*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*B*a*log(sec(d*x + c) + tan(d*x + c)) - 4*C*a*tan(d
*x + c) - 4*B*b*tan(d*x + c))/d
```

Fricas [A] time = 0.52175, size = 247, normalized size = 4.05

$$\frac{(2\,Ba+Cb)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2\,Ba+Cb)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(Cb+2\,(Ca+B}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/4*((2*B*a + C*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*B*a + C*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(C*b + 2*(C*a + B*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B+C\sec(c+dx))\,(a+b\sec(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 1.18592, size = 207, normalized size = 3.39

$$\frac{(2\,Ba+Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(2\,Ba+Cb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a + C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*B*a + C*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*C*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*b*tan
(1/2*d*x + 1/2*c)^3 - C*b*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1/2*d*x + 1/2*
c) - 2*B*b*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

3.769 $\int \cos(c+dx)(a+b\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+dx\right.$

Optimal. Leaf size=35

$$\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+aBx+\frac{bC\tan(c+dx)}{d}$$

```
[Out] a*B*x + ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.0736953, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {4072, 3914, 3767, 8, 3770}

$$\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+aBx+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x
]
```

```
[Out] a*B*x + ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+b\sec(c+dx))(B+C\sec(c+dx))\,dx$$
$$= aBx+(bC)\int \sec^2(c+dx)\,dx+(bB+aC)\int$$
$$= aBx+\frac{(bB+aC)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(bC)}{}$$
$$= aBx+\frac{(bB+aC)\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\,\mathrm{t}}{}$$

Mathematica [A] time = 0.0135357, size = 43, normalized size = 1.23

$$aBx+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a*B*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])/d + (b*
C*Tan[c + d*x])/d
```

Maple [A] time = 0.046, size = 65, normalized size = 1.9

$$aBx+\frac{Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bac}{d}+\frac{Cb\tan(dx+c)}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a*B*x+1/d*B*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a*c+b*C*tan(d*x+c)/d+1/d*a*C*
ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [B] time = 0.971548, size = 99, normalized size = 2.83

$$\frac{2(dx+c)Ba+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Bb(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + C*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*C*b*tan(d*x + c))
/d
```

Fricas [B] time = 0.510689, size = 225, normalized size = 6.43

$$\frac{2\,Badx\cos(dx+c)+(Ca+Bb)\cos(dx+c)\log(\sin(dx+c)+1)-(Ca+Bb)\cos(dx+c)\log(-\sin(dx+c)+1)+2}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/2*(2*B*a*d*x*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (C*a + B*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*C*b*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\sec(c + dx))(a + b\sec(c + dx))\cos(c + dx)\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)*sec(c + d*x
), x)
```

Giac [B] time = 1.15949, size = 113, normalized size = 3.23

$$\frac{(dx+c)Ba + (Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] ((d*x + c)*B*a + (C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (C*a + B*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*C*b*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

3.770 $\int \cos^2(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+$

Optimal. Leaf size=35

$$x(aC+bB)+\frac{aB\sin(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*B + a*C)*x + (b*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rubi [A] time = 0.101763, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {4072, 3996, 3770}

$$x(aC+bB)+\frac{aB\sin(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (b*B + a*C)*x + (b*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx &= \int \cos(c+dx)(a+b\sec(c+dx))(B+C\sec(\\ &= \frac{aB\sin(c+dx)}{d}-\int(-bB-aC-bC\sec(c+\\ &= (bB+aC)x+\frac{aB\sin(c+dx)}{d}+(bC)\int\sec(c\\ &= (bB+aC)x+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB}{}\end{aligned}$$

Mathematica [A] time = 0.0274246, size = 46, normalized size = 1.31

$$\frac{aB\sin(c)\cos(dx)}{d}+\frac{aB\cos(c)\sin(dx)}{d}+aCx+bBx+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] b*B*x + a*C*x + (b*C*ArcTanh[Sin[c + d*x]])/d + (a*B*Cos[d*x]*Sin[c])/d + (
a*B*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.055, size = 56, normalized size = 1.6

$$Bbx+aCx+\frac{B\sin(dx+c)a}{d}+\frac{Bbc}{d}+\frac{Cb\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] B*b*x+a*C*x+a*B*sin(d*x+c)/d+1/d*B*b*c+1/d*C*b*ln(sec(d*x+c)+tan(d*x+c))+1/
d*C*a*c
```

Maxima [A] time = 0.969817, size = 78, normalized size = 2.23

$$\frac{2(dx+c)Ca+2(dx+c)Bb+Cb(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2Ba\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a + 2*(d*x + c)*B*b + C*b*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*B*a*sin(d*x + c))/d
```

Fricas [A] time = 0.510087, size = 142, normalized size = 4.06

$$\frac{2(Ca+Bb)dx+Cb\log(\sin(dx+c)+1)-Cb\log(-\sin(dx+c)+1)+2Ba\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a + B*b)*d*x + C*b*log(sin(d*x + c) + 1) - C*b*log(-sin(d*x + c)
+ 1) + 2*B*a*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21137, size = 107, normalized size = 3.06

$$\frac{Cb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Cb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Ca+Bb)(dx+c)+\frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] (C*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (C*a + B*b)*(d*x + c) + 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

3.771 $\int \cos^3(c+dx)(a+b\sec(c+dx)) \left(B\sec(c+dx)+C\sec^2(c+$

Optimal. Leaf size=52

$$\frac{(aC+bB)\sin(c+dx)}{d}+\frac{1}{2}x(aB+2bC)+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] ((a*B + 2*b*C)*x)/2 + ((b*B + a*C)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rubi [A] time = 0.141558, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4072, 3996, 3787, 2637, 8}

$$\frac{(aC+bB)\sin(c+dx)}{d}+\frac{1}{2}x(aB+2bC)+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] ((a*B + 2*b*C)*x)/2 + ((b*B + a*C)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+b\sec(c+dx))(B+C\sec$$
$$= \frac{aB\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx$$
$$= \frac{aB\cos(c+dx)\sin(c+dx)}{2d} - (-bB-aC)\int$$
$$= \frac{1}{2}(aB+2bC)x + \frac{(bB+aC)\sin(c+dx)}{d} + \frac{aB}{}$$

Mathematica [A] time = 0.0866127, size = 51, normalized size = 0.98

$$\frac{4(aC+bB)\sin(c+dx)+aB\sin(2(c+dx))+2aBc+2aBdx+4bCdx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (2*a*B*c + 2*a*B*d*x + 4*b*C*d*x + 4*(b*B + a*C)*Sin[c + d*x] + a*B*Sin[2*(
c + d*x)])/(4*d)
```

Maple [A] time = 0.053, size = 57, normalized size = 1.1

$$\frac{1}{d}\left(Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Bb\sin(dx+c)+aC\sin(dx+c)+Cb(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b*sin(d*x+c)+a*C*sin(d
*x+c)+C*b*(d*x+c))
```

Maxima [A] time = 0.954808, size = 74, normalized size = 1.42

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba+4\,(dx+c)Cb+4\,Ca\sin(dx+c)+4\,Bb\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 4*(d*x + c)*C*b + 4*C*a*sin(d*x
 + c) + 4*B*b*sin(d*x + c))/d
```

Fricas [A] time = 0.482725, size = 104, normalized size = 2.

$$\frac{(Ba+2\,Cb)dx+(Ba\cos(dx+c)+2\,Ca+2\,Bb)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*((B*a + 2*C*b)*d*x + (B*a*cos(d*x + c) + 2*C*a + 2*B*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17867, size = 163, normalized size = 3.13

$$\frac{(Ba + 2Cb)(dx + c) - \frac{2\left(Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*((B*a + 2*C*b)*(d*x + c) - 2*(B*a*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1/
2*d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 - B*a*tan(1/2*d*x + 1/2*c)
 - 2*C*a*tan(1/2*d*x + 1/2*c) - 2*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^2)/d
```

3.772 $\int \cos^4(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=84

$$\frac{(2aB+3bC)\sin(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aC+bB)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] ((b*B + a*C)*x)/2 + ((2*a*B + 3*b*C)*Sin[c + d*x])/(3*d) + ((b*B + a*C)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.16633, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3996, 3787, 2635, 8, 2637}

$$\frac{(2aB+3bC)\sin(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aC+bB)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] ((b*B + a*C)*x)/2 + ((2*a*B + 3*b*C)*Sin[c + d*x])/(3*d) + ((b*B + a*C)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+b\sec(c+dx))(B+C\sec(c+$$
$$= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)$$
$$= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d} - (-bB-aC)\int \cos$$
$$= \frac{(2aB+3bC)\sin(c+dx)}{3d} + \frac{(bB+aC)\cos(c+d}{2d}$$
$$= \frac{1}{2}(bB+aC)x + \frac{(2aB+3bC)\sin(c+dx)}{3d} + \frac{(bB}{}$$

Mathematica [A] time = 0.161308, size = 75, normalized size = 0.89

$$\frac{3(3aB+4bC)\sin(c+dx)+3(aC+bB)\sin(2(c+dx))+aB\sin(3(c+dx))+6acC+6aCdx+6bBc+6bBdx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (6*b*B*c + 6*a*c*C + 6*b*B*d*x + 6*a*C*d*x + 3*(3*a*B + 4*b*C)*Sin[c + d*x]
 + 3*(b*B + a*C)*Sin[2*(c + d*x)] + a*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.069, size = 85, normalized size = 1.

$$\frac{1}{d}\left(\frac{Ba\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+Bb\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aC\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+C*sin(d*x+c)*b)
```

Maxima [A] time = 0.957332, size = 107, normalized size = 1.27

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba-3(2dx+2c+\sin(2dx+2c))Ca-3(2dx+2c+\sin(2dx+2c))Bb-12Cb\sin}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*C*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b - 12*C*b*sin(d*x + c)
)/d
```

Fricas [A] time = 0.489494, size = 149, normalized size = 1.77

$$\frac{3\,(Ca+Bb)dx+\left(2\,Ba\cos\left(dx+c\right)^{2}+4\,Ba+6\,Cb+3\,(Ca+Bb)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/6*(3*(C*a + B*b)*d*x + (2*B*a*cos(d*x + c)^2 + 4*B*a + 6*C*b + 3*(C*a + B
*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17514, size = 243, normalized size = 2.89

$$\frac{3\,(Ca+Bb)(dx+c)+\frac{2\left(6\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Cb\tan\left(\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(C*a + B*b)*(d*x + c) + 2*(6*B*a*tan(1/2*d*x + 1/2*c)^5 - 3*C*a*tan(
1/2*d*x + 1/2*c)^5 - 3*B*b*tan(1/2*d*x + 1/2*c)^5 + 6*C*b*tan(1/2*d*x + 1/2
*c)^5 + 4*B*a*tan(1/2*d*x + 1/2*c)^3 + 12*C*b*tan(1/2*d*x + 1/2*c)^3 + 6*B*
a*tan(1/2*d*x + 1/2*c) + 3*C*a*tan(1/2*d*x + 1/2*c) + 3*B*b*tan(1/2*d*x + 1
/2*c) + 6*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.773 $\int \cos^5(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+a\right.$

Optimal. Leaf size=105

$$-\frac{(aC+bB)\sin^3(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)}{d}+\frac{(3aB+4bC)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3aB+4bC)+\frac{aB\sin(c+$$

```
[Out] ((3*a*B + 4*b*C)*x)/8 + ((b*B + a*C)*Sin[c + d*x])/d + ((3*a*B + 4*b*C)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(b*B + a*C)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.177823, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3996, 3787, 2633, 2635, 8}

$$-\frac{(aC+bB)\sin^3(c+dx)}{3d}+\frac{(aC+bB)\sin(c+dx)}{d}+\frac{(3aB+4bC)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3aB+4bC)+\frac{aB\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```


```
[Out] ((3*a*B + 4*b*C)*x)/8 + ((b*B + a*C)*Sin[c + d*x])/d + ((3*a*B + 4*b*C)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(b*B + a*C)*Sin[c + d*x]^3)/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+b\sec(c+dx))(B+C\sec$$
$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+$$
$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d} - (-bB-aC)\int$$
$$= \frac{(3aB+4bC)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aB\,\mathrm{cc}}{}$$
$$= \frac{1}{8}(3aB+4bC)x + \frac{(bB+aC)\sin(c+dx)}{d} + \frac{(3}{}$$

Mathematica [A] time = 0.274893, size = 91, normalized size = 0.87

$$\frac{-32(aC+bB)\sin^3(c+dx)+96(aC+bB)\sin(c+dx)+24(aB+bC)\sin(2(c+dx))+3aB\sin(4(c+dx))+36aBc+3}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (36*a*B*c + 48*b*c*C + 36*a*B*d*x + 48*b*C*d*x + 96*(b*B + a*C)*Sin[c + d*x
] - 32*(b*B + a*C)*Sin[c + d*x]^3 + 24*(a*B + b*C)*Sin[2*(c + d*x)] + 3*a*B
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.069, size = 107, normalized size = 1.

$$\frac{1}{d}\left(Ba\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Bb\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{aC\left(2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+C*b*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.982585, size = 136, normalized size = 1.3

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Ba-32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ca-32\left(\sin(dx+c)^3-3\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a - 32*(s
in(d*x + c)^3 - 3*sin(d*x + c))*C*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*
B*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*b)/d
```

Fricas [A] time = 0.495079, size = 205, normalized size = 1.95

$$\frac{3\,(3\,Ba+4\,Cb)dx+\left(6\,Ba\cos\left(dx+c\right)^{3}+8\,(Ca+Bb)\cos\left(dx+c\right)^{2}+16\,Ca+16\,Bb+3\,(3\,Ba+4\,Cb)\cos\left(dx+c\right)\right)\sin\left(d\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/24*(3*(3*B*a + 4*C*b)*d*x + (6*B*a*cos(d*x + c)^3 + 8*(C*a + B*b)*cos(d*x
 + c)^2 + 16*C*a + 16*B*b + 3*(3*B*a + 4*C*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18547, size = 367, normalized size = 3.5

$$\frac{3\,(3\,Ba+4\,Cb)(dx+c)-\frac{2\left(15\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Cb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,Ca\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a + 4*C*b)*(d*x + c) - 2*(15*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*C
*a*tan(1/2*d*x + 1/2*c)^7 - 24*B*b*tan(1/2*d*x + 1/2*c)^7 + 12*C*b*tan(1/2*
d*x + 1/2*c)^7 - 9*B*a*tan(1/2*d*x + 1/2*c)^5 - 40*C*a*tan(1/2*d*x + 1/2*c)
^5 - 40*B*b*tan(1/2*d*x + 1/2*c)^5 + 12*C*b*tan(1/2*d*x + 1/2*c)^5 + 9*B*a*
tan(1/2*d*x + 1/2*c)^3 - 40*C*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*C*b*tan(1/2*d*x + 1/2*c)^3 - 15*B*a*tan(1/2*d*x + 1/2*c) -
 24*C*a*tan(1/2*d*x + 1/2*c) - 24*B*b*tan(1/2*d*x + 1/2*c) - 12*C*b*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.774 $\int \cos^6(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=136

$$-\frac{(4aB+5bC)\sin^3(c+dx)}{15d}+\frac{(4aB+5bC)\sin(c+dx)}{5d}+\frac{(aC+bB)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3(aC+bB)\sin(c+dx}{8d}$$

```
[Out] (3*(b*B + a*C)*x)/8 + ((4*a*B + 5*b*C)*Sin[c + d*x])/(5*d) + (3*(b*B + a*C)
*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((b*B + a*C)*Cos[c + d*x]^3*Sin[c + d*x
])/(4*d) + (a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - ((4*a*B + 5*b*C)*Sin[c
 + d*x]^3)/(15*d)
```

Rubi [A] time = 0.199866, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3996, 3787, 2635, 8, 2633}

$$-\frac{(4aB+5bC)\sin^3(c+dx)}{15d}+\frac{(4aB+5bC)\sin(c+dx)}{5d}+\frac{(aC+bB)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3(aC+bB)\sin(c+dx}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] (3*(b*B + a*C)*x)/8 + ((4*a*B + 5*b*C)*Sin[c + d*x])/(5*d) + (3*(b*B + a*C)
*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((b*B + a*C)*Cos[c + d*x]^3*Sin[c + d*x
])/(4*d) + (a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - ((4*a*B + 5*b*C)*Sin[c
 + d*x]^3)/(15*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+b\sec(c+dx))\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \int \cos^5(c+dx)(a+b\sec(c+dx))(B+C\sec(c+\\ &= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d}-\frac{1}{5}\int \cos^4(c+dx)\\ &= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d}-(-bB-aC)\int \cos\\ &= \frac{(bB+aC)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aB\cos^4(c\\ &= \frac{(4aB+5bC)\sin(c+dx)}{5d}+\frac{3(bB+aC)\cos(c+}{8d}\\ &= \frac{3}{8}(bB+aC)x+\frac{(4aB+5bC)\sin(c+dx)}{5d}+\frac{3(bB}{}\end{aligned}$$

Mathematica [A] time = 0.239587, size = 88, normalized size = 0.65

$$\frac{-160(2aB+bC)\sin^3(c+dx)+480(aB+bC)\sin(c+dx)+15(aC+bB)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (480*(a*B + b*C)*Sin[c + d*x] - 160*(2*a*B + b*C)*Sin[c + d*x]^3 + 96*a*B*S
in[c + d*x]^5 + 15*(b*B + a*C)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(
c + d*x)]))/(480*d)
```

Maple [A] time = 0.067, size = 128, normalized size = 0.9

$$\frac{1}{d}\left(\frac{B\sin(dx+c)a}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Bb\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*B*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*b*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*C*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*C*b*(2+cos(d*x+c)^2)*sin(d*x+c)
```

)

Maxima [A] time = 0.97131, size = 167, normalized size = 1.23

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Ca}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a + 15*(12*d*x +
 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b - 160*(sin(d*x + c)^3 -
3*sin(d*x + c))*C*b)/d
```

Fricas [A] time = 0.50771, size = 248, normalized size = 1.82

$$\frac{45(Ca+Bb)dx+\left(24Ba\cos(dx+c)^4+30(Ca+Bb)\cos(dx+c)^3+8(4Ba+5Cb)\cos(dx+c)^2+64Ba+80Cb+\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/120*(45*(C*a + B*b)*d*x + (24*B*a*cos(d*x + c)^4 + 30*(C*a + B*b)*cos(d*x
 + c)^3 + 8*(4*B*a + 5*C*b)*cos(d*x + c)^2 + 64*B*a + 80*C*b + 45*(C*a + B*
b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17965, size = 405, normalized size = 2.98

$$45(Ca+Bb)(dx+c)+\frac{2\left(120Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-75Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-75Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+120Cb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+160Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/120*(45*(C*a + B*b)*(d*x + c) + 2*(120*B*a*tan(1/2*d*x + 1/2*c)^9 - 75*C*
a*tan(1/2*d*x + 1/2*c)^9 - 75*B*b*tan(1/2*d*x + 1/2*c)^9 + 120*C*b*tan(1/2*
d*x + 1/2*c)^9 + 160*B*a*tan(1/2*d*x + 1/2*c)^7 - 30*C*a*tan(1/2*d*x + 1/2*
c)^7 - 30*B*b*tan(1/2*d*x + 1/2*c)^7 + 320*C*b*tan(1/2*d*x + 1/2*c)^7 + 464
*B*a*tan(1/2*d*x + 1/2*c)^5 + 400*C*b*tan(1/2*d*x + 1/2*c)^5 + 160*B*a*tan(
1/2*d*x + 1/2*c)^3 + 30*C*a*tan(1/2*d*x + 1/2*c)^3 + 30*B*b*tan(1/2*d*x + 1
/2*c)^3 + 320*C*b*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*tan(1/2*d*x + 1/2*c) + 7
5*C*a*tan(1/2*d*x + 1/2*c) + 75*B*b*tan(1/2*d*x + 1/2*c) + 120*C*b*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.775 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=198

$$\frac{\left(4a^2B+6abC+3b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(4a^2B+6abC+3b^2B\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\left(5a(aC+2bB)+4b^2\right.}{15d}$$

```
[Out] ((4*a^2*B + 3*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*b^2*C + 5
*a*(2*b*B + a*C))*Tan[c + d*x])/(5*d) + ((4*a^2*B + 3*b^2*B + 6*a*b*C)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*(5*b*B + 6*a*C)*Sec[c + d*x]^3*Tan[c + d*
x])/(20*d) + (b*C*Sec[c + d*x]^3*(a + b*Sec[c + d*x])*Tan[c + d*x])/(5*d) +
 ((4*b^2*C + 5*a*(2*b*B + a*C))*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.351783, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4026, 4047, 3767, 4046, 3768, 3770}

$$\frac{\left(4a^2B+6abC+3b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(4a^2B+6abC+3b^2B\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\left(5a(aC+2bB)+4b^2\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] ((4*a^2*B + 3*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*b^2*C + 5
*a*(2*b*B + a*C))*Tan[c + d*x])/(5*d) + ((4*a^2*B + 3*b^2*B + 6*a*b*C)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*(5*b*B + 6*a*C)*Sec[c + d*x]^3*Tan[c + d*
x])/(20*d) + (b*C*Sec[c + d*x]^3*(a + b*Sec[c + d*x])*Tan[c + d*x])/(5*d) +
 ((4*b^2*C + 5*a*(2*b*B + a*C))*Tan[c + d*x]^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+b\sec(c+dx))^2(B+C\sec(c$$

$$= \frac{bC\sec^3(c+dx)(a+b\sec(c+dx))\tan(c+dx)}{5d}$$

$$= \frac{bC\sec^3(c+dx)(a+b\sec(c+dx))\tan(c+dx)}{5d}$$

$$= \frac{b(5bB+6aC)\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{bC\,\mathrm{se}}{}$$

$$= \frac{\left(4b^2C+5a(2bB+aC)\right)\tan(c+dx)}{5d} + \frac{\left(4a^2B+\right.}{}$$

$$= \frac{\left(4a^2B+3b^2B+6abC\right)\tanh^{-1}(\sin(c+dx))}{8d} +$$

Mathematica [A] time = 1.51463, size = 150, normalized size = 0.76

$$\frac{15\left(4a^2B+6abC+3b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5\left(a^2C+2abB+2b^2C\right)\tan^2(c+dx)+15\left(a^2C+2abB\right.\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (15*(4*a^2*B + 3*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*
(4*a^2*B + 3*b^2*B + 6*a*b*C)*Sec[c + d*x] + 30*b*(b*B + 2*a*C)*Sec[c + d*x
]^3 + 8*(15*(2*a*b*B + a^2*C + b^2*C) + 5*(2*a*b*B + a^2*C + 2*b^2*C)*Tan[c
 + d*x]^2 + 3*b^2*C*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.046, size = 312, normalized size = 1.6

$$\frac{Ba^2\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2a^2C\tan(dx+c)}{3d}+\frac{a^2C\tan(dx+c)(\sec(dx}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/3
/d*a^2*C*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^2+4/3/d*B*a*b*tan(d*x
+c)+2/3/d*B*a*b*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a*b*C*tan(d*x+c)*sec(d*x+c)^3
+3/4*a*b*C*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+1/
4/d*B*b^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*B*b^2*sec(d*x+c)*tan(d*x+c)+3/8/d*B
*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/15*b^2*C*tan(d*x+c)/d+1/5/d*b^2*C*tan(d*x+
c)*sec(d*x+c)^4+4/15/d*b^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.01124, size = 373, normalized size = 1.88

$$\frac{80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^2+160\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bab+16\left(3\tan(dx+c)^5+10\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 + 160*(tan(d*x + c)^3 + 3
*tan(d*x + c))*B*a*b + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*C*b^2 - 30*C*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 15*B*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.544358, size = 521, normalized size = 2.63

$$\frac{15\left(4Ba^2+6Cab+3Bb^2\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(4Ba^2+6Cab+3Bb^2\right)\cos(dx+c)^5\log(-\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1
) - 15*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1)
+ 2*(16*(5*C*a^2 + 10*B*a*b + 4*C*b^2)*cos(d*x + c)^4 + 15*(4*B*a^2 + 6*C*a
*b + 3*B*b^2)*cos(d*x + c)^3 + 24*C*b^2 + 8*(5*C*a^2 + 10*B*a*b + 4*C*b^2)*
cos(d*x + c)^2 + 30*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\sec(c + dx))(a + b\sec(c + dx))^2 \sec^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] time = 1.53993, size = 713, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(4*B*a^2 + 6*C*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(
60*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 120*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 240*B*
a*b*tan(1/2*d*x + 1/2*c)^9 + 150*C*a*b*tan(1/2*d*x + 1/2*c)^9 + 75*B*b^2*ta
n(1/2*d*x + 1/2*c)^9 - 120*C*b^2*tan(1/2*d*x + 1/2*c)^9 - 120*B*a^2*tan(1/2
*d*x + 1/2*c)^7 + 320*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 640*B*a*b*tan(1/2*d*x
+ 1/2*c)^7 - 60*C*a*b*tan(1/2*d*x + 1/2*c)^7 - 30*B*b^2*tan(1/2*d*x + 1/2*c
)^7 + 160*C*b^2*tan(1/2*d*x + 1/2*c)^7 - 400*C*a^2*tan(1/2*d*x + 1/2*c)^5 -
 800*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 464*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*
B*a^2*tan(1/2*d*x + 1/2*c)^3 + 320*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 640*B*a*b
*tan(1/2*d*x + 1/2*c)^3 + 60*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 30*B*b^2*tan(1/
2*d*x + 1/2*c)^3 + 160*C*b^2*tan(1/2*d*x + 1/2*c)^3 - 60*B*a^2*tan(1/2*d*x
+ 1/2*c) - 120*C*a^2*tan(1/2*d*x + 1/2*c) - 240*B*a*b*tan(1/2*d*x + 1/2*c)
- 150*C*a*b*tan(1/2*d*x + 1/2*c) - 75*B*b^2*tan(1/2*d*x + 1/2*c) - 120*C*b^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.776 $\int \sec(c+dx)(a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=179

$$\frac{\left(4a^2bB+a^3(-C)+8ab^2C+4b^3B\right)\tan(c+dx)}{6bd}+\frac{\left(4a^2C+8abB+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(-2a^2C+8abB+9b\right.}{}$$

```
[Out] ((8*a*b*B + 4*a^2*C + 3*b^2*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a^2*b*B +
 4*b^3*B - a^3*C + 8*a*b^2*C)*Tan[c + d*x])/(6*b*d) + ((8*a*b*B - 2*a^2*C +
 9*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*b*B - a*C)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(12*b*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(
4*b*d)
```

Rubi [A] time = 0.347556, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4072, 4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(4a^2bB+a^3(-C)+8ab^2C+4b^3B\right)\tan(c+dx)}{6bd}+\frac{\left(4a^2C+8abB+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\left(-2a^2C+8abB+9b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] ((8*a*b*B + 4*a^2*C + 3*b^2*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a^2*b*B +
 4*b^3*B - a^3*C + 8*a*b^2*C)*Tan[c + d*x])/(6*b*d) + ((8*a*b*B - 2*a^2*C +
 9*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*b*B - a*C)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(12*b*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(
4*b*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+b\sec(c+dx))^2(B+C\sec(c$$
$$= \frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4bd} + \frac{\int \sec(c+a}{}$$
$$= \frac{(4bB-aC)(a+b\sec(c+dx))^2\tan(c+dx)}{12bd} + \frac{C}{}$$
$$= \frac{\left(8abB-2a^2C+9b^2C\right)\sec(c+dx)\tan(c+dx)}{24d}$$
$$= \frac{\left(8abB-2a^2C+9b^2C\right)\sec(c+dx)\tan(c+dx)}{24d}$$
$$= \frac{\left(8abB+4a^2C+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(}{}$$
$$= \frac{\left(8abB+4a^2C+3b^2C\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(}{}$$

Mathematica [A] time = 0.761261, size = 120, normalized size = 0.67

$$\frac{3\left(4a^2C+8abB+3b^2C\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(4a^2C+8abB+3b^2C\right)\sec(c+dx)+24\left(a^2B+2abC+b^2\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (3*(8*a*b*B + 4*a^2*C + 3*b^2*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
a^2*B + b^2*B + 2*a*b*C) + 3*(8*a*b*B + 4*a^2*C + 3*b^2*C)*Sec[c + d*x] + 6
*b^2*C*Sec[c + d*x]^3 + 8*b*(b*B + 2*a*C)*Tan[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.037, size = 241, normalized size = 1.4

$$\frac{Ba^2\tan(dx+c)}{d}+\frac{a^2C\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^2C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{Bab\sec(dx+c)\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a^2*tan(d*x+c)+1/2/d*a^2*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*C*ln(sec(d
*x+c)+tan(d*x+c))+1/d*B*a*b*sec(d*x+c)*tan(d*x+c)+1/d*B*a*b*ln(sec(d*x+c)+t
an(d*x+c))+4/3/d*a*b*C*tan(d*x+c)+2/3/d*a*b*C*tan(d*x+c)*sec(d*x+c)^2+2/3/d
*B*b^2*tan(d*x+c)+1/3/d*B*b^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*b^2*C*tan(d*x+c
)*sec(d*x+c)^3+3/8/d*b^2*C*sec(d*x+c)*tan(d*x+c)+3/8/d*b^2*C*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 0.977326, size = 308, normalized size = 1.72

$$\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cab+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb^2-3Cb^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*b^2 - 3*C*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^2*tan(d*x +
 c))/d
```

Fricas [A] time = 0.534846, size = 443, normalized size = 2.47

$$\frac{3\left(4Ca^2+8Bab+3Cb^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4Ca^2+8Bab+3Cb^2\right)\cos(dx+c)^4\log(-\sin(dx-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/48*(3*(4*C*a^2 + 8*B*a*b + 3*C*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(4*C*a^2 + 8*B*a*b + 3*C*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(8*(3*B*a^2 + 4*C*a*b + 2*B*b^2)*cos(d*x + c)^3 + 6*C*b^2 + 3*(4*C*a^2 + 8
```

```
*B*a*b + 3*C*b^2)*cos(d*x + c)^2 + 8*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\sec(c + dx))(a + b\sec(c + dx))^2 \sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] time = 1.42798, size = 645, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(3*(4*C*a^2 + 8*B*a*b + 3*C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(4*C*a^2 + 8*B*a*b + 3*C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*
B*a^2*tan(1/2*d*x + 1/2*c)^7 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a*b*t
an(1/2*d*x + 1/2*c)^7 + 48*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 24*B*b^2*tan(1/2*
d*x + 1/2*c)^7 - 15*C*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*tan(1/2*d*x + 1
/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^5
 - 80*C*a*b*tan(1/2*d*x + 1/2*c)^5 - 40*B*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*C*
b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^2*tan
(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 80*C*a*b*tan(1/2*d*
x + 1/2*c)^3 + 40*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^2*tan(1/2*d*x + 1/2*
c)^3 - 24*B*a^2*tan(1/2*d*x + 1/2*c) - 12*C*a^2*tan(1/2*d*x + 1/2*c) - 24*B
*a*b*tan(1/2*d*x + 1/2*c) - 48*C*a*b*tan(1/2*d*x + 1/2*c) - 24*B*b^2*tan(1/
2*d*x + 1/2*c) - 15*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

3.777 $\int (a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=116

$$\frac{2\left(a^2C+3abB+b^2C\right)\tan(c+dx)}{3d}+\frac{\left(2a^2B+2abC+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2aC+3bB)\tan(c+dx)\sec(c+}{6d}$$

```
[Out] ((2*a^2*B + b^2*B + 2*a*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a*b*B + a
^2*C + b^2*C)*Tan[c + d*x])/(3*d) + (b*(3*b*B + 2*a*C)*Sec[c + d*x]*Tan[c +
 d*x])/(6*d) + (C*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.146461, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4056, 4048, 3770, 3767, 8}

$$\frac{2\left(a^2C+3abB+b^2C\right)\tan(c+dx)}{3d}+\frac{\left(2a^2B+2abC+b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2aC+3bB)\tan(c+dx)\sec(c+}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] ((2*a^2*B + b^2*B + 2*a*b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a*b*B + a
^2*C + b^2*C)*Tan[c + d*x])/(3*d) + (b*(3*b*B + 2*a*C)*Sec[c + d*x]*Tan[c +
 d*x])/(6*d) + (C*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\sec(c+dx))$$
$$=\frac{b(3bB+2aC)\sec(c+dx)\tan(c+dx)}{6d}+\frac{C(a+b\sec(c+dx}{3d}$$
$$=\frac{b(3bB+2aC)\sec(c+dx)\tan(c+dx)}{6d}+\frac{C(a+b\sec(c+dx}{3d}$$
$$=\frac{\left(2a^2B+b^2B+2abC\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(3bB+2aC)}{}$$
$$=\frac{\left(2a^2B+b^2B+2abC\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2\left(3abB+a^2C\right.}{}$$

Mathematica [A] time = 0.46486, size = 92, normalized size = 0.79

$$\frac{3\left(2a^2B+2abC+b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2\left(3a^2C+6abB+b^2C\tan^2(c+dx)+3b^2C\right)+3b(2aC+bB)\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(2*a^2*B + b^2*B + 2*a*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(b
*B + 2*a*C)*Sec[c + d*x] + 2*(6*a*b*B + 3*a^2*C + 3*b^2*C + b^2*C*Tan[c + d
*x]^2)))/(6*d)
```

Maple [A] time = 0.036, size = 174, normalized size = 1.5

$$\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2C\tan(dx+c)}{d}+2\frac{Bab\tan(dx+c)}{d}+\frac{abC\sec(dx+c)\tan(dx+c)}{d}+\frac{abC\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*tan(d*x+c)+2/d*B*a*b*tan(d*x+
c)+a*b*C*sec(d*x+c)*tan(d*x+c)/d+1/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*
B*b^2*sec(d*x+c)*tan(d*x+c)+1/2/d*B*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/3*b^2*C
*tan(d*x+c)/d+1/3/d*b^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.960264, size = 223, normalized size = 1.92

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cb^2-6\,Cab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Bb^2\left(\frac{2\sin(}{\sin(dx-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b^2 - 6*C*a*b*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*
b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 12*B*a^2*log(sec(d*x + c) + tan(d*x + c)) + 12*C*a^2*tan(d
*x + c) + 24*B*a*b*tan(d*x + c))/d
```

Fricas [A] time = 0.51849, size = 371, normalized size = 3.2

$$\frac{3\left(2\,Ba^2+2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,Ba^2+2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(2*B*a^2 + 2*C*a*b + B*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(2*B*a^2 + 2*C*a*b + B*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
C*b^2 + 2*(3*C*a^2 + 6*B*a*b + 2*C*b^2)*cos(d*x + c)^2 + 3*(2*C*a*b + B*b^2
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B+C\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.31842, size = 397, normalized size = 3.42

$$\frac{3\left(2\,Ba^2+2\,Cab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Ba^2+2\,Cab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Ca^2\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(2*B*a^2 + 2*C*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
2*B*a^2 + 2*C*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*C*a^2*
tan(1/2*d*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*b*tan(1/2*
d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2
*c)^5 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 -
 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) + 12*B*a*b*t
an(1/2*d*x + 1/2*c) + 6*C*a*b*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x +
1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.778 $\int \cos(c+dx)(a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=86

$$\frac{(2a^2C+4abB+b^2C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Bx+\frac{b(3aC+2bB)\tan(c+dx)}{2d}+\frac{bC\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] a^2*B*x + ((4*a*b*B + 2*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(2
*b*B + 3*a*C)*Tan[c + d*x])/(2*d) + (b*C*(a + b*Sec[c + d*x])*Tan[c + d*x])
/(2*d)
```

Rubi [A] time = 0.141975, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4072, 3918, 3770, 3767, 8}

$$\frac{(2a^2C+4abB+b^2C)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Bx+\frac{b(3aC+2bB)\tan(c+dx)}{2d}+\frac{bC\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^2*B*x + ((4*a*b*B + 2*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(2
*b*B + 3*a*C)*Tan[c + d*x])/(2*d) + (b*C*(a + b*Sec[c + d*x])*Tan[c + d*x])
/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+b\sec(c+dx))^2(B+C\sec(c+dx))\,dx$$
$$= \frac{bC(a+b\sec(c+dx))\tan(c+dx)}{2d} + \frac{1}{2}\int (2$$
$$= a^2Bx + \frac{bC(a+b\sec(c+dx))\tan(c+dx)}{2d} +$$
$$= a^2Bx + \frac{\left(4abB+2a^2C+b^2C\right)\tanh^{-1}(\sin(c$$
$$= a^2Bx + \frac{\left(4abB+2a^2C+b^2C\right)\tanh^{-1}(\sin(c$$

Mathematica [A] time = 0.26881, size = 67, normalized size = 0.78

$$\frac{\left(2a^2C+4abB+b^2C\right)\tanh^{-1}(\sin(c+dx))+2a^2Bdx+b\tan(c+dx)(4aC+2bB+bC\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (2*a^2*B*d*x + (4*a*b*B + 2*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]] + b*(2*b*B
 + 4*a*C + b*C*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.056, size = 133, normalized size = 1.6

$$a^2Bx + \frac{Ba^2c}{d} + \frac{a^2C\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\frac{Bab\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\frac{abC\tan(dx+c)}{d} + \frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^2*B*x+1/d*B*a^2*c+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b*ln(sec(d*
x+c)+tan(d*x+c))+2/d*a*b*C*tan(d*x+c)+1/d*B*b^2*tan(d*x+c)+1/2/d*b^2*C*sec(
d*x+c)*tan(d*x+c)+1/2/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.963041, size = 189, normalized size = 2.2

$$\frac{4(dx+c)Ba^2 - Cb^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 2Ca^2(\log(\sin(dx+c)+1) - \log(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 - C*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 8*C*a*b*tan(d*x + c) + 4*B*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.524719, size = 335, normalized size = 3.9

$$\frac{4\,Ba^2dx\cos\left(dx+c\right)^2+\left(2\,Ca^2+4\,Bab+Cb^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ca^2+4\,Bab+Cb^2\right)\cos\left(dx+c\right)^2\,\mathrm{l}}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/4*(4*B*a^2*d*x*cos(d*x + c)^2 + (2*C*a^2 + 4*B*a*b + C*b^2)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (2*C*a^2 + 4*B*a*b + C*b^2)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) + 2*(C*b^2 + 2*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23939, size = 259, normalized size = 3.01

$$\frac{2\,(dx+c)Ba^2+\left(2\,Ca^2+4\,Bab+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ca^2+4\,Bab+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + (2*C*a^2 + 4*B*a*b + C*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (2*C*a^2 + 4*B*a*b + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*(4*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^2*tan(1/2*d*x + 1/2*c)^3 - C
*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b*tan(1/2*d*x + 1/2*c) - 2*B*b^2*tan(1/
2*d*x + 1/2*c) - C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

3.779 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c$

Optimal. Leaf size=60

$$\frac{a^2B\sin(c+dx)}{d}+\frac{b(2aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+ax(aC+2bB)+\frac{b^2C\tan(c+dx)}{d}$$

```
[Out] a*(2*b*B + a*C)*x + (b*(b*B + 2*a*C)*ArcTanh[Sin[c + d*x]])/d + (a^2*B*Sin[
c + d*x])/d + (b^2*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.176471, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4024, 3770, 3767, 8}

$$\frac{a^2B\sin(c+dx)}{d}+\frac{b(2aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+ax(aC+2bB)+\frac{b^2C\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] a*(2*b*B + a*C)*x + (b*(b*B + 2*a*C)*ArcTanh[Sin[c + d*x]])/d + (a^2*B*Sin[
c + d*x])/d + (b^2*C*Tan[c + d*x])/d
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+b\sec(c+dx))^2(B+C\sec(c$$
$$= \frac{a^2B\sin(c+dx)}{d} - \int\left(-a(2bB+aC)+\left(-b^2B\right.\right.$$
$$= a(2bB+aC)x + \frac{a^2B\sin(c+dx)}{d} + \left(b^2C\right)\int \mathrm{se}$$
$$= a(2bB+aC)x + \frac{b(bB+2aC)\tanh^{-1}(\sin(c+d}{d}$$
$$= a(2bB+aC)x + \frac{b(bB+2aC)\tanh^{-1}(\sin(c+d}{d}$$

Mathematica [A] time = 0.49343, size = 109, normalized size = 1.82

$$\frac{a^2B\sin(c+dx)+a(c+dx)(aC+2bB)-b(2aC+bB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b(2aC+bB)\log\left(\sin\left(\frac{1}{2}(\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (a*(2*b*B + a*C)*(c + d*x) - b*(b*B + 2*a*C)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + b*(b*B + 2*a*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2
*B*Sin[c + d*x] + b^2*C*Tan[c + d*x])/d
```

Maple [A] time = 0.054, size = 104, normalized size = 1.7

$$2\,Babx + a^2Cx + \frac{Ba^2\sin(dx+c)}{d} + \frac{Bb^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\,\frac{Babc}{d} + \frac{b^2C\tan(dx+c)}{d} + 2\,\frac{abC\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2*B*a*b*x+a^2*C*x+a^2*B*sin(d*x+c)/d+1/d*B*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/
d*B*a*b*c+b^2*C*tan(d*x+c)/d+2/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*C*a^2*
c
```

Maxima [A] time = 0.972229, size = 139, normalized size = 2.32

$$\frac{2\,(dx+c)Ca^2+4\,(dx+c)Bab+2\,Cab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Bb^2(\log(\sin(dx+c)+1)-\log(\mathrm{s}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*C*a^2 + 4*(d*x + c)*B*a*b + 2*C*a*b*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
```

```
- 1)) + 2*B*a^2*sin(d*x + c) + 2*C*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.522169, size = 294, normalized size = 4.9

$$\frac{2\left(Ca^2 + 2\,Bab\right)dx\cos\left(dx + c\right) + \left(2\,Cab + Bb^2\right)\cos\left(dx + c\right)\log\left(\sin\left(dx + c\right) + 1\right) - \left(2\,Cab + Bb^2\right)\cos\left(dx + c\right)\log\left(-\right.}{2\,d\cos\left(dx + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^2 + 2*B*a*b)*d*x*cos(d*x + c) + (2*C*a*b + B*b^2)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (2*C*a*b + B*b^2)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(B*a^2*cos(d*x + c) + C*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22587, size = 208, normalized size = 3.47

$$\frac{\left(Ca^2 + 2\,Bab\right)(dx + c) + \left(2\,Cab + Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(2\,Cab + Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2(B}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] ((C*a^2 + 2*B*a*b)*(d*x + c) + (2*C*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*C*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(B*a^2*
tan(1/2*d*x + 1/2*c)^3 - C*b^2*tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x +
 1/2*c) - C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.780 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=80

$$\frac{1}{2}x\left(a^2B+4abC+2b^2B\right)+\frac{a^2B\sin(c+dx)\cos(c+dx)}{2d}+\frac{a(aC+2bB)\sin(c+dx)}{d}+\frac{b^2C\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] ((a^2*B + 2*b^2*B + 4*a*b*C)*x)/2 + (b^2*C*ArcTanh[Sin[c + d*x]])/d + (a*(2
*b*B + a*C)*Sin[c + d*x])/d + (a^2*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.251598, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4024, 4047, 8, 4045, 3770}

$$\frac{1}{2}x\left(a^2B+4abC+2b^2B\right)+\frac{a^2B\sin(c+dx)\cos(c+dx)}{2d}+\frac{a(aC+2bB)\sin(c+dx)}{d}+\frac{b^2C\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] ((a^2*B + 2*b^2*B + 4*a*b*C)*x)/2 + (b^2*C*ArcTanh[Sin[c + d*x]])/d + (a*(2
*b*B + a*C)*Sin[c + d*x])/d + (a^2*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+b\sec(c+dx))^2(B+C\sec$$
$$= \frac{a^2B\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+$$
$$= \frac{a^2B\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+$$
$$= \frac{1}{2}\left(a^2B+2b^2B+4abC\right)x + \frac{a(2bB+aC)\sin}{d}$$
$$= \frac{1}{2}\left(a^2B+2b^2B+4abC\right)x + \frac{b^2C\tanh^{-1}(\sin}{d}$$

Mathematica [A] time = 0.221225, size = 120, normalized size = 1.5

$$\frac{2(c+dx)\left(a^2B+4abC+2b^2B\right)+a^2B\sin(2(c+dx))+4a(aC+2bB)\sin(c+dx)-4b^2C\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*(a^2*B + 2*b^2*B + 4*a*b*C)*(c + d*x) - 4*b^2*C*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*b^2*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*a*(
2*b*B + a*C)*Sin[c + d*x] + a^2*B*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.058, size = 120, normalized size = 1.5

$$\frac{Ba^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2Bx}{2}+\frac{Ba^2c}{2d}+\frac{a^2C\sin(dx+c)}{d}+2\frac{Bab\sin(dx+c)}{d}+2abCx+2\frac{Cabc}{d}+Bb^2x+\frac{Bb}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2*a^2*B*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*B*x+1/2/d*B*a^2*c+1/d*a^2*C*sin(d
*x+c)+2/d*B*a*b*sin(d*x+c)+2*a*b*C*x+2/d*C*a*b*c+B*b^2*x+1/d*B*b^2*c+1/d*b^
2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.959688, size = 134, normalized size = 1.68

$$\frac{(2dx+2c+\sin(2dx+2c))Ba^2+8(dx+c)Cab+4(dx+c)Bb^2+2Cb^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 8*(d*x + c)*C*a*b + 4*(d*x +
c)*B*b^2 + 2*C*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*C*a^
2*sin(d*x + c) + 8*B*a*b*sin(d*x + c))/d
```

Fricas [A] time = 0.522297, size = 213, normalized size = 2.66

$$\frac{Cb^2 \log(\sin(dx+c)+1) - Cb^2 \log(-\sin(dx+c)+1) + (Ba^2 + 4\,Cab + 2\,Bb^2)dx + (Ba^2 \cos(dx+c) + 2\,Ca^2 + 4\,Bab)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(C*b^2*log(sin(d*x + c) + 1) - C*b^2*log(-sin(d*x + c) + 1) + (B*a^2 +
4*C*a*b + 2*B*b^2)*d*x + (B*a^2*cos(d*x + c) + 2*C*a^2 + 4*B*a*b)*sin(d*x +
 c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21033, size = 240, normalized size = 3.

$$\frac{2\,Cb^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,Cb^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + (Ba^2+4\,Cab+2\,Bb^2)(dx+c) - \frac{2\left(Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 \cdots\right)}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*C*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*C*b^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (B*a^2 + 4*C*a*b + 2*B*b^2)*(d*x + c) - 2*(B*a^2*tan(1/
2*d*x + 1/2*c)^3 - 2*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b*tan(1/2*d*x + 1
/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c) - 2*C*a^2*tan(1/2*d*x + 1/2*c) - 4*B*a
*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.781 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=107

$$\frac{\left(2a^2B+6abC+3b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(a^2C+2abB+2b^2C\right)+\frac{a^2B\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{a(aC+2bB)\sin(c+}{2d}$$

```
[Out] ((2*a*b*B + a^2*C + 2*b^2*C)*x)/2 + ((2*a^2*B + 3*b^2*B + 6*a*b*C)*Sin[c +
d*x])/(3*d) + (a*(2*b*B + a*C)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*B*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.288676, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4024, 4047, 2637, 4045, 8}

$$\frac{\left(2a^2B+6abC+3b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(a^2C+2abB+2b^2C\right)+\frac{a^2B\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{a(aC+2bB)\sin(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] ((2*a*b*B + a^2*C + 2*b^2*C)*x)/2 + ((2*a^2*B + 3*b^2*B + 6*a*b*C)*Sin[c +
d*x])/(3*d) + (a*(2*b*B + a*C)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*B*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+b\sec(c+dx))^2(B+C\sec($$
$$= \frac{a^2B\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+d$$
$$= \frac{a^2B\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+d$$
$$= \frac{\left(2a^2B+3b^2B+6abC\right)\sin(c+dx)}{3d} + \frac{a(2bB+$$
$$= \frac{1}{2}\left(2abB+a^2C+2b^2C\right)x + \frac{\left(2a^2B+3b^2B+6a\right.}{3d}$$

Mathematica [A] time = 0.244285, size = 90, normalized size = 0.84

$$\frac{6(c+dx)\left(a^2C+2abB+2b^2C\right)+3\left(3a^2B+8abC+4b^2B\right)\sin(c+dx)+a^2B\sin(3(c+dx))+3a(aC+2bB)\sin(2(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (6*(2*a*b*B + a^2*C + 2*b^2*C)*(c + d*x) + 3*(3*a^2*B + 4*b^2*B + 8*a*b*C)*
Sin[c + d*x] + 3*a*(2*b*B + a*C)*Sin[2*(c + d*x)] + a^2*B*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A] time = 0.063, size = 114, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Ba^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + 2Bab(1/2\cos(dx+c)\sin(dx+c)+1/2dx+c/2)+a^2C\left(\frac{\cos(dx+c)\sin(dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*B*a*b*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b^2*sin
(d*x+c)+2*a*b*C*sin(d*x+c)+b^2*C*(d*x+c))
```

Maxima [A] time = 0.962853, size = 146, normalized size = 1.36

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ca^{2}-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Bab-12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*C*a^2 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b - 12*(d*x + c)*
C*b^2 - 24*C*a*b*sin(d*x + c) - 12*B*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.493997, size = 201, normalized size = 1.88

$$\frac{3\left(Ca^{2}+2\,Bab+2\,Cb^{2}\right)dx+\left(2\,Ba^{2}\cos\left(dx+c\right)^{2}+4\,Ba^{2}+12\,Cab+6\,Bb^{2}+3\left(Ca^{2}+2\,Bab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(C*a^2 + 2*B*a*b + 2*C*b^2)*d*x + (2*B*a^2*cos(d*x + c)^2 + 4*B*a^2
+ 12*C*a*b + 6*B*b^2 + 3*(C*a^2 + 2*B*a*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18597, size = 343, normalized size = 3.21

$$3\left(Ca^{2}+2\,Bab+2\,Cb^{2}\right)(dx+c)+\frac{2\left(6\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ca^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Bb^{2}\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(3*(C*a^2 + 2*B*a*b + 2*C*b^2)*(d*x + c) + 2*(6*B*a^2*tan(1/2*d*x + 1/2
*c)^5 - 3*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 1
```

```
2*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^2*t
an(1/2*d*x + 1/2*c)^3 + 24*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*B*b^2*tan(1/2*
d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) + 3*C*a^2*tan(1/2*d*x + 1/2*c
) + 6*B*a*b*tan(1/2*d*x + 1/2*c) + 12*C*a*b*tan(1/2*d*x + 1/2*c) + 6*B*b^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.782 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=136

$$\frac{\left(a^2C+2abB+b^2C\right)\sin(c+dx)}{d}+\frac{\left(3a^2B+8abC+4b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2B+8abC+4b^2B\right)+\frac{a^2B}{}$$

```
[Out] ((3*a^2*B + 4*b^2*B + 8*a*b*C)*x)/8 + ((2*a*b*B + a^2*C + b^2*C)*Sin[c + d*
x])/d + ((3*a^2*B + 4*b^2*B + 8*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a^2*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(2*b*B + a*C)*Sin[c + d*x]^3)
/(3*d)
```

Rubi [A] time = 0.320546, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4024, 4047, 2635, 8, 4044, 3013}

$$\frac{\left(a^2C+2abB+b^2C\right)\sin(c+dx)}{d}+\frac{\left(3a^2B+8abC+4b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2B+8abC+4b^2B\right)+\frac{a^2B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```



```
[Out] ((3*a^2*B + 4*b^2*B + 8*a*b*C)*x)/8 + ((2*a*b*B + a^2*C + b^2*C)*Sin[c + d*
x])/d + ((3*a^2*B + 4*b^2*B + 8*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a^2*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(2*b*B + a*C)*Sin[c + d*x]^3)
/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+b\sec(c+dx))^2(B+C\sec(c$$

$$= \frac{a^2B\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+d$$

$$= \frac{a^2B\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+d$$

$$= \frac{\left(3a^2B+4b^2B+8abC\right)\cos(c+dx)\sin(c+dx)}{8d}$$

$$= \frac{1}{8}\left(3a^2B+4b^2B+8abC\right)x + \frac{\left(3a^2B+4b^2B+8\right.}{}$$

$$= \frac{1}{8}\left(3a^2B+4b^2B+8abC\right)x + \frac{\left(2abB+a^2C+b^{\cdot}\right.}{d}$$

Mathematica [A] time = 0.51283, size = 118, normalized size = 0.87

$$\frac{12(c+dx)\left(3a^2B+8abC+4b^2B\right)+24\left(3a^2C+6abB+4b^2C\right)\sin(c+dx)+24\left(a^2B+2abC+b^2B\right)\sin(2(c+dx))+3a^2}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (12*(3*a^2*B + 4*b^2*B + 8*a*b*C)*(c + d*x) + 24*(6*a*b*B + 3*a^2*C + 4*b^2
*C)*Sin[c + d*x] + 24*(a^2*B + b^2*B + 2*a*b*C)*Sin[2*(c + d*x)] + 8*a*(2*b
*B + a*C)*Sin[3*(c + d*x)] + 3*a^2*B*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.068, size = 152, normalized size = 1.1

$$\frac{1}{d}\left(Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2C\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{2Bab\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*a^2*C*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*B*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2*
a*b*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b^2*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+b^2*C*sin(d*x+c))
```

Maxima [A] time = 0.964501, size = 192, normalized size = 1.41

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Ba^2-32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ca^2-64\left(\sin\left(dx+c\right)^3-\right.}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x +
c))*B*a*b + 48*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b + 24*(2*d*x + 2*c + s
in(2*d*x + 2*c))*B*b^2 + 96*C*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.507277, size = 274, normalized size = 2.01

$$\frac{3\left(3\,Ba^2+8\,Cab+4\,Bb^2\right)dx+\left(6\,Ba^2\cos\left(dx+c\right)^3+16\,Ca^2+32\,Bab+24\,Cb^2+8\left(Ca^2+2\,Bab\right)\cos\left(dx+c\right)^2+3\left(3\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/24*(3*(3*B*a^2 + 8*C*a*b + 4*B*b^2)*d*x + (6*B*a^2*cos(d*x + c)^3 + 16*C*
a^2 + 32*B*a*b + 24*C*b^2 + 8*(C*a^2 + 2*B*a*b)*cos(d*x + c)^2 + 3*(3*B*a^2
 + 8*C*a*b + 4*B*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22382, size = 590, normalized size = 4.34

$$3\left(3\,Ba^2+8\,Cab+4\,Bb^2\right)(dx+c)-\frac{2\left(15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,Bb^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a^2 + 8*C*a*b + 4*B*b^2)*(d*x + c) - 2*(15*B*a^2*tan(1/2*d*x +
 1/2*c)^7 - 24*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*B*a*b*tan(1/2*d*x + 1/2*c)
^7 + 24*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 24
*C*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 40*C*a^2*t
an(1/2*d*x + 1/2*c)^5 - 80*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 24*C*a*b*tan(1/2*
d*x + 1/2*c)^5 + 12*B*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*C*b^2*tan(1/2*d*x + 1
/2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*C*a^2*tan(1/2*d*x + 1/2*c)^3
- 80*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*B*
b^2*tan(1/2*d*x + 1/2*c)^3 - 72*C*b^2*tan(1/2*d*x + 1/2*c)^3 - 15*B*a^2*tan
(1/2*d*x + 1/2*c) - 24*C*a^2*tan(1/2*d*x + 1/2*c) - 48*B*a*b*tan(1/2*d*x +
1/2*c) - 24*C*a*b*tan(1/2*d*x + 1/2*c) - 12*B*b^2*tan(1/2*d*x + 1/2*c) - 24
*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.783 $\int \cos^6(c+dx)(a+b\sec(c+dx))^2 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=180

$$-\frac{\left(4a^2B+10abC+5b^2B\right)\sin^3(c+dx)}{15d}+\frac{\left(4a^2B+10abC+5b^2B\right)\sin(c+dx)}{5d}+\frac{\left(3a^2C+6abB+4b^2C\right)\sin(c+dx)\operatorname{co}}{8d}$$

```
[Out] ((6*a*b*B + 3*a^2*C + 4*b^2*C)*x)/8 + ((4*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c
 + d*x])/(5*d) + ((6*a*b*B + 3*a^2*C + 4*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a*(2*b*B + a*C)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a^2*B*Cos[c
+ d*x]^4*Sin[c + d*x])/(5*d) - ((4*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x]
^3)/(15*d)
```

Rubi [A] time = 0.338106, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4024, 4047, 2633, 4045, 2635, 8}

$$-\frac{\left(4a^2B+10abC+5b^2B\right)\sin^3(c+dx)}{15d}+\frac{\left(4a^2B+10abC+5b^2B\right)\sin(c+dx)}{5d}+\frac{\left(3a^2C+6abB+4b^2C\right)\sin(c+dx)\operatorname{co}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] ((6*a*b*B + 3*a^2*C + 4*b^2*C)*x)/8 + ((4*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c
 + d*x])/(5*d) + ((6*a*b*B + 3*a^2*C + 4*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a*(2*b*B + a*C)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a^2*B*Cos[c
+ d*x]^4*Sin[c + d*x])/(5*d) - ((4*a^2*B + 5*b^2*B + 10*a*b*C)*Sin[c + d*x]
^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^2\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^5(c+dx)(a+b\sec(c+dx))^2(B+C\sec(c$$

$$= \frac{a^2B\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+d$$

$$= \frac{a^2B\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+d$$

$$= \frac{a(2bB+aC)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^2B\,\mathrm{cc}}{}$$

$$= \frac{\left(4a^2B+5b^2B+10abC\right)\sin(c+dx)}{5d} + \frac{\left(6abB+\right.}{}$$

$$= \frac{1}{8}\left(6abB+3a^2C+4b^2C\right)x + \frac{\left(4a^2B+5b^2B+1\right.}{5}$$

Mathematica [A] time = 0.473081, size = 146, normalized size = 0.81

$$\frac{60(c+dx)\left(3a^2C+6abB+4b^2C\right)+60\left(5a^2B+12abC+6b^2B\right)\sin(c+dx)+120\left(a^2C+2abB+b^2C\right)\sin(2(c+dx))+1}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^2*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (60*(6*a*b*B + 3*a^2*C + 4*b^2*C)*(c + d*x) + 60*(5*a^2*B + 6*b^2*B + 12*a*
b*C)*Sin[c + d*x] + 120*(2*a*b*B + a^2*C + b^2*C)*Sin[2*(c + d*x)] + 10*(5*
a^2*B + 4*b^2*B + 8*a*b*C)*Sin[3*(c + d*x)] + 15*a*(2*b*B + a*C)*Sin[4*(c +
 d*x)] + 6*a^2*B*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.073, size = 184, normalized size = 1.

$$\frac{1}{d}\left(\frac{Ba^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^2C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*B*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*C*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*B*a*b*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a*b*C*(2+cos(d*x+c)^2)*
sin(d*x+c)+1/3*B*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*C*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.967969, size = 238, normalized size = 1.32

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^2+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^2 + 30*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b - 320*(sin(d*x + c
)^3 - 3*sin(d*x + c))*C*a*b - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*b^2)/d
```

Fricas [A] time = 0.528613, size = 350, normalized size = 1.94

$$\frac{15\left(3Ca^2+6Bab+4Cb^2\right)dx+\left(24Ba^2\cos(dx+c)^4+30\left(Ca^2+2Bab\right)\cos(dx+c)^3+64Ba^2+160Cab+80Bb^2+\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(3*C*a^2 + 6*B*a*b + 4*C*b^2)*d*x + (24*B*a^2*cos(d*x + c)^4 + 30
*(C*a^2 + 2*B*a*b)*cos(d*x + c)^3 + 64*B*a^2 + 160*C*a*b + 80*B*b^2 + 8*(4*
B*a^2 + 10*C*a*b + 5*B*b^2)*cos(d*x + c)^2 + 15*(3*C*a^2 + 6*B*a*b + 4*C*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**2*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22813, size = 657, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^2*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(3*C*a^2 + 6*B*a*b + 4*C*b^2)*(d*x + c) + 2*(120*B*a^2*tan(1/2*d*
x + 1/2*c)^9 - 75*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 150*B*a*b*tan(1/2*d*x + 1/
2*c)^9 + 240*C*a*b*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^2*tan(1/2*d*x + 1/2*c)^
9 - 60*C*b^2*tan(1/2*d*x + 1/2*c)^9 + 160*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 30
*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 60*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 640*C*a*b
*tan(1/2*d*x + 1/2*c)^7 + 320*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*C*b^2*tan(
1/2*d*x + 1/2*c)^7 + 464*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*C*a*b*tan(1/2*d
*x + 1/2*c)^5 + 400*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 160*B*a^2*tan(1/2*d*x +
1/2*c)^3 + 30*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 60*B*a*b*tan(1/2*d*x + 1/2*c)^
3 + 640*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 320*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 1
20*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^2*tan(1/2*d*x + 1/2*c) + 75*C*a^2
*tan(1/2*d*x + 1/2*c) + 150*B*a*b*tan(1/2*d*x + 1/2*c) + 240*C*a*b*tan(1/2*
d*x + 1/2*c) + 120*B*b^2*tan(1/2*d*x + 1/2*c) + 60*C*b^2*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.784 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=278

$$\frac{\left(15a^2bB+5a^3C+12ab^2C+4b^3B\right)\tan^3(c+dx)}{15d}+\frac{\left(15a^2bB+5a^3C+12ab^2C+4b^3B\right)\tan(c+dx)}{5d}+\frac{\left(18a^2bC+8a^3\right.}{}$$

```
[Out] ((8*a^3*B + 18*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*ArcTanh[Sin[c + d*x]])/(16*d
) + ((15*a^2*b*B + 4*b^3*B + 5*a^3*C + 12*a*b^2*C)*Tan[c + d*x])/(5*d) + ((
8*a^3*B + 18*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*Sec[c + d*x]*Tan[c + d*x])/(16
*d) + (b*(18*a*b*B + 14*a^2*C + 5*b^2*C)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d
) + (b^2*(3*b*B + 4*a*C)*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (b*C*Sec[c +
 d*x]^3*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((15*a^2*b*B + 4*b^3*B
 + 5*a^3*C + 12*a*b^2*C)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.608875, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 4026, 4076, 4047, 3767, 4046, 3768, 3770}

$$\frac{\left(15a^2bB+5a^3C+12ab^2C+4b^3B\right)\tan^3(c+dx)}{15d}+\frac{\left(15a^2bB+5a^3C+12ab^2C+4b^3B\right)\tan(c+dx)}{5d}+\frac{\left(18a^2bC+8a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```



```
[Out] ((8*a^3*B + 18*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*ArcTanh[Sin[c + d*x]])/(16*d
) + ((15*a^2*b*B + 4*b^3*B + 5*a^3*C + 12*a*b^2*C)*Tan[c + d*x])/(5*d) + ((
8*a^3*B + 18*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*Sec[c + d*x]*Tan[c + d*x])/(16
*d) + (b*(18*a*b*B + 14*a^2*C + 5*b^2*C)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d
) + (b^2*(3*b*B + 4*a*C)*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (b*C*Sec[c +
 d*x]^3*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((15*a^2*b*B + 4*b^3*B
 + 5*a^3*C + 12*a*b^2*C)*Tan[c + d*x]^3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+b\sec(c+dx))^3(B+C\sec(c$$

$$= \frac{bC\sec^3(c+dx)(a+b\sec(c+dx))^2\tan(c+dx}{6d}$$

$$= \frac{b^2(3bB+4aC)\sec^4(c+dx)\tan(c+dx)}{15d} + \frac{bC\,\mathrm{s}}{}$$

$$= \frac{b^2(3bB+4aC)\sec^4(c+dx)\tan(c+dx)}{15d} + \frac{bC\,\mathrm{s}}{}$$

$$= \frac{b\left(18abB+14a^2C+5b^2C\right)\sec^3(c+dx)\tan(c}{24d}$$

$$= \frac{\left(15a^2bB+4b^3B+5a^3C+12ab^2C\right)\tan(c+dx}{5d}$$

$$= \frac{\left(8a^3B+18ab^2B+18a^2bC+5b^3C\right)\tanh^{-1}(\sin}{16d}$$

Mathematica [A] time = 2.61277, size = 214, normalized size = 0.77

$$\frac{15\left(18a^2bC+8a^3B+18ab^2B+5b^3C\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(80\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)\tan^2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (15*(8*a^3*B + 18*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*ArcTanh[Sin[c + d*x]] + T
an[c + d*x]*(240*(3*a^2*b*B + b^3*B + a^3*C + 3*a*b^2*C) + 15*(8*a^3*B + 18
*a*b^2*B + 18*a^2*b*C + 5*b^3*C)*Sec[c + d*x] + 10*b*(18*a*b*B + 18*a^2*C +
 5*b^2*C)*Sec[c + d*x]^3 + 40*b^3*C*Sec[c + d*x]^5 + 80*(3*a^2*b*B + 2*b^3*
B + a^3*C + 6*a*b^2*C)*Tan[c + d*x]^2 + 48*b^2*(b*B + 3*a*C)*Tan[c + d*x]^4
))/(240*d)
```

Maple [A] time = 0.052, size = 478, normalized size = 1.7

$$\frac{Ba^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2a^3C\tan(dx+c)}{3d}+\frac{a^3C\tan(dx+c)(\sec(dx}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/3
*a^3*C*tan(d*x+c)/d+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+2/d*B*a^2*b*tan(d*x
+c)+1/d*B*a^2*b*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)
^3+9/8/d*a^2*b*C*sec(d*x+c)*tan(d*x+c)+9/8/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+
c))+3/4/d*B*a*b^2*tan(d*x+c)*sec(d*x+c)^3+9/8/d*B*a*b^2*sec(d*x+c)*tan(d*x+
c)+9/8/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/5/d*C*a*b^2*tan(d*x+c)+3/5/d*C
*a*b^2*tan(d*x+c)*sec(d*x+c)^4+4/5/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)^2+8/15/d
*B*b^3*tan(d*x+c)+1/5/d*B*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*B*b^3*tan(d*x+
c)*sec(d*x+c)^2+1/6/d*C*b^3*tan(d*x+c)*sec(d*x+c)^5+5/24/d*C*b^3*tan(d*x+c)
*sec(d*x+c)^3+5/16/d*C*b^3*sec(d*x+c)*tan(d*x+c)+5/16/d*C*b^3*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 0.99116, size = 552, normalized size = 1.99

$$\frac{160\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^3+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2b+96\left(3\tan(dx+c)^5+10\tan(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(160*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 + 480*(tan(d*x + c)^3 +
3*tan(d*x + c))*B*a^2*b + 96*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan
(d*x + c))*C*a*b^2 + 32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
 + c))*B*b^3 - 5*C*b^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*
x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log
```

```
(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 90*C*a^2*b*(2*(3*sin(d*x +
 c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin
(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 90*B*a*b^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^3*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.579748, size = 706, normalized size = 2.54

$$\frac{15\left(8\,Ba^{3}+18\,Ca^{2}b+18\,Bab^{2}+5\,Cb^{3}\right)\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,Ba^{3}+18\,Ca^{2}b+18\,Bab^{2}+5\,Cb^{3}\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/480*(15*(8*B*a^3 + 18*C*a^2*b + 18*B*a*b^2 + 5*C*b^3)*cos(d*x + c)^6*log(
sin(d*x + c) + 1) - 15*(8*B*a^3 + 18*C*a^2*b + 18*B*a*b^2 + 5*C*b^3)*cos(d*
x + c)^6*log(-sin(d*x + c) + 1) + 2*(32*(5*C*a^3 + 15*B*a^2*b + 12*C*a*b^2
+ 4*B*b^3)*cos(d*x + c)^5 + 15*(8*B*a^3 + 18*C*a^2*b + 18*B*a*b^2 + 5*C*b^3
)*cos(d*x + c)^4 + 40*C*b^3 + 16*(5*C*a^3 + 15*B*a^2*b + 12*C*a*b^2 + 4*B*b
^3)*cos(d*x + c)^3 + 10*(18*C*a^2*b + 18*B*a*b^2 + 5*C*b^3)*cos(d*x + c)^2
+ 48*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B + C\sec\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^{3}\sec^{3}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [B] time = 1.29572, size = 1258, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/240*(15*(8*B*a^3 + 18*C*a^2*b + 18*B*a*b^2 + 5*C*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 15*(8*B*a^3 + 18*C*a^2*b + 18*B*a*b^2 + 5*C*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) + 2*(120*B*a^3*tan(1/2*d*x + 1/2*c)^11 - 240*C*a
^3*tan(1/2*d*x + 1/2*c)^11 - 720*B*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*C*a^
2*b*tan(1/2*d*x + 1/2*c)^11 + 450*B*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 720*C*a
*b^2*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*C*b^
```

3*tan(1/2*d*x + 1/2*c)^11 - 360*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 880*C*a^3*ta
n(1/2*d*x + 1/2*c)^9 + 2640*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 630*C*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 - 630*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 1680*C*a*b^2*ta
n(1/2*d*x + 1/2*c)^9 + 560*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 25*C*b^3*tan(1/2*
d*x + 1/2*c)^9 + 240*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 1440*C*a^3*tan(1/2*d*x
+ 1/2*c)^7 - 4320*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*C*a^2*b*tan(1/2*d*x
+ 1/2*c)^7 + 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 3744*C*a*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 1248*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*C*b^3*tan(1/2*d*x + 1/
2*c)^7 + 240*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1440*C*a^3*tan(1/2*d*x + 1/2*c)
^5 + 4320*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 180*C*a^2*b*tan(1/2*d*x + 1/2*c)
^5 + 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3744*C*a*b^2*tan(1/2*d*x + 1/2*c)
^5 + 1248*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 450*C*b^3*tan(1/2*d*x + 1/2*c)^5 -
360*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 880*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 2640
*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*
B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 1680*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 560*
B*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^3*
tan(1/2*d*x + 1/2*c) + 240*C*a^3*tan(1/2*d*x + 1/2*c) + 720*B*a^2*b*tan(1/2
*d*x + 1/2*c) + 450*C*a^2*b*tan(1/2*d*x + 1/2*c) + 450*B*a*b^2*tan(1/2*d*x
+ 1/2*c) + 720*C*a*b^2*tan(1/2*d*x + 1/2*c) + 240*B*b^3*tan(1/2*d*x + 1/2*c
) + 165*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

3.785 $\int \sec(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+d\right.$

Optimal. Leaf size=252

$$\frac{\left(52a^2b^2C+15a^3bB-3a^4C+60ab^3B+16b^4C\right)\tan(c+dx)}{30bd}+\frac{\left(12a^2bB+4a^3C+9ab^2C+3b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$

```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((15*a^3*b*B + 60*a*b^3*B - 3*a^4*C + 52*a^2*b^2*C + 16*b^4*C)*Tan[c + d*
x])/(30*b*d) + ((30*a^2*b*B + 45*b^3*B - 6*a^3*C + 71*a*b^2*C)*Sec[c + d*x]
*Tan[c + d*x])/(120*d) + ((15*a*b*B - 3*a^2*C + 16*b^2*C)*(a + b*Sec[c + d*
x])^2*Tan[c + d*x])/(60*b*d) + ((5*b*B - a*C)*(a + b*Sec[c + d*x])^3*Tan[c
+ d*x])/(20*b*d) + (C*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.498703, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4072, 4010, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(52a^2b^2C+15a^3bB-3a^4C+60ab^3B+16b^4C\right)\tan(c+dx)}{30bd}+\frac{\left(12a^2bB+4a^3C+9ab^2C+3b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```


```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((15*a^3*b*B + 60*a*b^3*B - 3*a^4*C + 52*a^2*b^2*C + 16*b^4*C)*Tan[c + d*
x])/(30*b*d) + ((30*a^2*b*B + 45*b^3*B - 6*a^3*C + 71*a*b^2*C)*Sec[c + d*x]
*Tan[c + d*x])/(120*d) + ((15*a*b*B - 3*a^2*C + 16*b^2*C)*(a + b*Sec[c + d*
x])^2*Tan[c + d*x])/(60*b*d) + ((5*b*B - a*C)*(a + b*Sec[c + d*x])^3*Tan[c
+ d*x])/(20*b*d) + (C*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
```

```
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \sec^2(c+dx)(a+b\sec(c+dx))^3(B+C\sec$$

$$= \frac{C(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c}{}$$

$$= \frac{(5bB-aC)(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} +$$

$$= \frac{\left(15abB-3a^2C+16b^2C\right)(a+b\sec(c+dx))^2}{60bd}$$

$$= \frac{\left(30a^2bB+45b^3B-6a^3C+71ab^2C\right)\sec(c+}{120d}$$

$$= \frac{\left(30a^2bB+45b^3B-6a^3C+71ab^2C\right)\sec(c+}{120d}$$

$$= \frac{\left(12a^2bB+3b^3B+4a^3C+9ab^2C\right)\tanh^{-1}(\mathrm{si}}{8d}$$

$$= \frac{\left(12a^2bB+3b^3B+4a^3C+9ab^2C\right)\tanh^{-1}(\mathrm{si}}{8d}$$

Mathematica [A] time = 3.29128, size = 181, normalized size = 0.72

$$\frac{15\left(12a^2bB+4a^3C+9ab^2C+3b^3B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5b\left(3a^2C+3abB+2b^2C\right)\tan^2(c+dx)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (15*(12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(12*a^2*b*B + 3*b^3*B + 4*a^3*C + 9*a*b^2*C)*Sec[c + d*x] +
30*b^2*(b*B + 3*a*C)*Sec[c + d*x]^3 + 8*(15*(a^3*B + 3*a*b^2*B + 3*a^2*b*C
+ b^3*C) + 5*b*(3*a*b*B + 3*a^2*C + 2*b^2*C)*Tan[c + d*x]^2 + 3*b^3*C*Tan[c
 + d*x]^4)))/(120*d)
```

Maple [A] time = 0.043, size = 382, normalized size = 1.5

$$\frac{Ba^3\tan(dx+c)}{d}+\frac{a^3C\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3C\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{3\,Ba^2b\sec(dx+c)\tan(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a^3*tan(d*x+c)+1/2/d*a^3*C*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*C*ln(sec(d
*x+c)+tan(d*x+c))+3/2/d*B*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+2/d*a^2*b*C*tan(d*x+c)+1/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)
^2+2/d*B*a*b^2*tan(d*x+c)+1/d*B*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/4/d*C*a*b^2
*tan(d*x+c)*sec(d*x+c)^3+9/8/d*C*a*b^2*sec(d*x+c)*tan(d*x+c)+9/8/d*C*a*b^2*
ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*B*b^3*s
ec(d*x+c)*tan(d*x+c)+3/8/d*B*b^3*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*C*b^3*tan
(d*x+c)+1/5/d*C*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*C*b^3*tan(d*x+c)*sec(d*x
+c)^2
```

Maxima [A] time = 0.984851, size = 460, normalized size = 1.83

$$\frac{240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^2b+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bab^2+16\left(3\tan(dx+c)^5+10\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b + 240*(tan(d*x + c)^3
+ 3*tan(d*x + c))*B*a*b^2 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*C*b^3 - 45*C*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 15*B*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 60*C*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) - 180*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x +
 c))/d
```

Fricas [A] time = 0.561328, size = 612, normalized size = 2.43

$$\frac{15\left(4\,Ca^3+12\,Ba^2b+9\,Cab^2+3\,Bb^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,Ca^3+12\,Ba^2b+9\,Cab^2+3\,Bb^3\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/240*(15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B*b^3)*cos(d*x
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(15*B*a^3 + 30*C*a^2*b + 30*B*a*b^2 +
8*C*b^3)*cos(d*x + c)^4 + 24*C*b^3 + 15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 +
 3*B*b^3)*cos(d*x + c)^3 + 8*(15*C*a^2*b + 15*B*a*b^2 + 4*C*b^3)*cos(d*x +
c)^2 + 30*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B + C\sec(c+dx))\,(a + b\sec(c+dx))^3 \sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] time = 1.28937, size = 975, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/120*(15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(4*C*a^3 + 12*B*a^2*b + 9*C*a*b^2 + 3*B*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 60*C*a^3*t
an(1/2*d*x + 1/2*c)^9 - 180*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*C*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*C*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 75*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*C*b^3*tan(1/2*d
*x + 1/2*c)^9 - 480*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 120*C*a^3*tan(1/2*d*x +
1/2*c)^7 + 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*C*a^2*b*tan(1/2*d*x + 1
/2*c)^7 - 960*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 90*C*a*b^2*tan(1/2*d*x + 1/2
*c)^7 + 30*B*b^3*tan(1/2*d*x + 1/2*c)^7 - 160*C*b^3*tan(1/2*d*x + 1/2*c)^7
+ 720*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 464*C*b^3*tan(1/2*d*x + 1/2*c)^5 - 48
0*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 360*B*a
^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 90*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 30*B*b^3*t
an(1/2*d*x + 1/2*c)^3 - 160*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^3*tan(1/
```

```
2*d*x + 1/2*c) + 60*C*a^3*tan(1/2*d*x + 1/2*c) + 180*B*a^2*b*tan(1/2*d*x +
1/2*c) + 360*C*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a*b^2*tan(1/2*d*x + 1/2*c
) + 225*C*a*b^2*tan(1/2*d*x + 1/2*c) + 75*B*b^3*tan(1/2*d*x + 1/2*c) + 120*
C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.786 $\int (a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=180

$$\frac{\left(16a^2bB+3a^3C+12ab^2C+4b^3B\right)\tan(c+dx)}{6d}+\frac{\left(12a^2bC+8a^3B+12ab^2B+3b^3C\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(6a^2C\right.}{}$$

```
[Out] ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((16*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*Tan[c + d*x])/(6*d) + (b*
(20*a*b*B + 6*a^2*C + 9*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*b*B
+ 3*a*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.26204, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4056, 4048, 3770, 3767, 8}

$$\frac{\left(16a^2bB+3a^3C+12ab^2C+4b^3B\right)\tan(c+dx)}{6d}+\frac{\left(12a^2bC+8a^3B+12ab^2B+3b^3C\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(6a^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((16*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*Tan[c + d*x])/(6*d) + (b*
(20*a*b*B + 6*a^2*C + 9*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*b*B
+ 3*a*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (C*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/(4*d)
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\sec(c+dx))$$
$$= \frac{(4bB+3aC)(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{C(a+b\sec(}{}$$
$$= \frac{b\left(20abB+6a^2C+9b^2C\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4bB+}{}$$
$$= \frac{b\left(20abB+6a^2C+9b^2C\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4bB+}{}$$
$$= \frac{\left(8a^3B+12ab^2B+12a^2bC+3b^3C\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$
$$= \frac{\left(8a^3B+12ab^2B+12a^2bC+3b^3C\right)\tanh^{-1}(\sin(c+dx))}{8d}+$$

Mathematica [A] time = 0.828366, size = 140, normalized size = 0.78

$$\frac{3\left(12a^2bC+8a^3B+12ab^2B+3b^3C\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(9b\left(4a^2C+4abB+b^2C\right)\sec(c+dx)+24\left(3a^2b\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (3*(8*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 3*b^3*C)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*b*B + b^3*B + a^3*C + 3*a*b^2*C) + 9*b*(4*a*b*B + 4*a
^2*C + b^2*C)*Sec[c + d*x] + 6*b^3*C*Sec[c + d*x]^3 + 8*b^2*(b*B + 3*a*C)*T
an[c + d*x]^2))/(24*d)
```

Maple [A] time = 0.043, size = 290, normalized size = 1.6

$$\frac{Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3C\tan(dx+c)}{d}+3\,\frac{Ba^2b\tan(dx+c)}{d}+\frac{3\,a^2bC\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{3\,a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*C*tan(d*x+c)/d+3/d*B*a^2*b*tan(d*x+
c)+3/2/d*a^2*b*C*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+
c))+3/2/d*B*a*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x
+c))+2/d*C*a*b^2*tan(d*x+c)+1/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)^2+2/3/d*B*b^3
*tan(d*x+c)+1/3/d*B*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*C*b^3*tan(d*x+c)*sec(
d*x+c)^3+3/8/d*C*b^3*sec(d*x+c)*tan(d*x+c)+3/8/d*C*b^3*ln(sec(d*x+c)+tan(d*
x+c))
```

Maxima [A] time = 0.987448, size = 359, normalized size = 1.99

$$\frac{48\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cab^2+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb^3-3Cb^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b^2 + 16*(tan(d*x + c)^3 + 3
*tan(d*x + c))*B*b^3 - 3*C*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*C*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^3*log
(sec(d*x + c) + tan(d*x + c)) + 48*C*a^3*tan(d*x + c) + 144*B*a^2*b*tan(d*x
 + c))/d
```

Fricas [A] time = 0.543402, size = 510, normalized size = 2.83

$$\frac{3\left(8Ba^3+12Ca^2b+12Bab^2+3Cb^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(8Ba^3+12Ca^2b+12Bab^2+3Cb^3\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 3*C*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(8*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 3*C*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*C*b^3 + 8*(3*C*a^3 + 9*B*a^2*b + 6*C*a*
b^2 + 2*B*b^3)*cos(d*x + c)^3 + 9*(4*C*a^2*b + 4*B*a*b^2 + C*b^3)*cos(d*x +
 c)^2 + 8*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (B+C\sec(c+dx))\,(a+b\sec(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] time = 1.26039, size = 791, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(8*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 3*C*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(8*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 3*C*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(24*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 72*B*a^2*b*t
an(1/2*d*x + 1/2*c)^7 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*B*a*b^2*tan(
1/2*d*x + 1/2*c)^7 + 72*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*B*b^3*tan(1/2*d
*x + 1/2*c)^7 - 15*C*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^3*tan(1/2*d*x + 1/
2*c)^5 - 216*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b*tan(1/2*d*x + 1/2*
c)^5 + 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 120*C*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 40*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*
C*a^3*tan(1/2*d*x + 1/2*c)^3 + 216*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^
2*b*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*C*a*b^
2*tan(1/2*d*x + 1/2*c)^3 + 40*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^3*tan(1/
2*d*x + 1/2*c)^3 - 24*C*a^3*tan(1/2*d*x + 1/2*c) - 72*B*a^2*b*tan(1/2*d*x +
 1/2*c) - 36*C*a^2*b*tan(1/2*d*x + 1/2*c) - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)
 - 72*C*a*b^2*tan(1/2*d*x + 1/2*c) - 24*B*b^3*tan(1/2*d*x + 1/2*c) - 15*C*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.787 $\int \cos(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=137

$$\frac{b\left(8a^2C+9abB+2b^2C\right)\tan(c+dx)}{3d}+\frac{\left(6a^2bB+2a^3C+3ab^2C+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3Bx+\frac{b^2(5aC+3bB)}{}$$

```
[Out] a^3*B*x + ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (b*(9*a*b*B + 8*a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*d) + (b^2*(3*b*B
 + 5*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*C*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(3*d)
```

Rubi [A] time = 0.240119, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3918, 4048, 3770, 3767, 8}

$$\frac{b\left(8a^2C+9abB+2b^2C\right)\tan(c+dx)}{3d}+\frac{\left(6a^2bB+2a^3C+3ab^2C+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3Bx+\frac{b^2(5aC+3bB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^3*B*x + ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (b*(9*a*b*B + 8*a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*d) + (b^2*(3*b*B
 + 5*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*C*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(3*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \int (a+b\sec(c+dx))^3(B+C\sec(c+dx))\,dx\\ &= \frac{bC(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\\ &= \frac{b^2(3bB+5aC)\sec(c+dx)\tan(c+dx)}{6d}+\frac{bC(a\\ &= a^3Bx+\frac{b^2(3bB+5aC)\sec(c+dx)\tan(c+dx)}{6d}\\ &= a^3Bx+\frac{\left(6a^2bB+b^3B+2a^3C+3ab^2C\right)\tanh^{-1}(}{2d}\\ &= a^3Bx+\frac{\left(6a^2bB+b^3B+2a^3C+3ab^2C\right)\tanh^{-1}(}{2d}\end{aligned}$$

Mathematica [A] time = 0.585876, size = 108, normalized size = 0.79

$$\frac{3\left(6a^2bB+2a^3C+3ab^2C+b^3B\right)\tanh^{-1}(\sin(c+dx))+3b\tan(c+dx)\left(6a^2C+b(3aC+bB)\sec(c+dx)+6abB+2b^2C\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (6*a^3*B*d*x + 3*(6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*ArcTanh[Sin[c +
d*x]] + 3*b*(6*a*b*B + 6*a^2*C + 2*b^2*C + b*(b*B + 3*a*C)*Sec[c + d*x])*Ta
n[c + d*x] + 2*b^3*C*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.066, size = 223, normalized size = 1.6

$$a^3Bx+\frac{Ba^3c}{d}+\frac{a^3C\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\frac{Ba^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\frac{a^2bC\tan(dx+c)}{d}+3\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*B*x+1/d*B*a^3*c+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+3/d*a^2*b*C*tan(d*x+c)+3/d*B*a*b^2*tan(d*x+c)+3/2/d*C*a*
```

```
b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*B*b
^3*sec(d*x+c)*tan(d*x+c)+1/2/d*B*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*C*b^3*
tan(d*x+c)+1/3/d*C*b^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.984015, size = 292, normalized size = 2.13

$$12(dx+c)Ba^3 + 4\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)Cb^3 - 9\,Cab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b^3 - 9*C*
a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(si
n(d*x + c) - 1)) - 3*B*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*a^3*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 18*B*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 36*C*a^2*b*tan(d*x + c) + 36*B*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.547865, size = 458, normalized size = 3.34

$$12\,Ba^3dx\cos(dx+c)^3 + 3\left(2\,Ca^3 + 6\,Ba^2b + 3\,Cab^2 + Bb^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1) - 3\left(2\,Ca^3 + 6\,Ba^2b +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(12*B*a^3*d*x*cos(d*x + c)^3 + 3*(2*C*a^3 + 6*B*a^2*b + 3*C*a*b^2 + B*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*C*a^3 + 6*B*a^2*b + 3*C*a*
b^2 + B*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*C*b^3 + 2*(9*C*a^
2*b + 9*B*a*b^2 + 2*C*b^3)*cos(d*x + c)^2 + 3*(3*C*a*b^2 + B*b^3)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23836, size = 454, normalized size = 3.31

$$6(dx+c)Ba^3 + 3\left(2Ca^3 + 6Ba^2b + 3Cab^2 + Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3\left(2Ca^3 + 6Ba^2b + 3Cab^2 + Bb^3\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*B*a^3 + 3*(2*C*a^3 + 6*B*a^2*b + 3*C*a*b^2 + B*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*C*a^3 + 6*B*a^2*b + 3*C*a*b^2 + B*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
 18*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B
*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*C*a^2*b*t
an(1/2*d*x + 1/2*c)^3 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*b^3*tan(1/2
*d*x + 1/2*c)^3 + 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a*b^2*tan(1/2*d*x
+ 1/2*c) + 9*C*a*b^2*tan(1/2*d*x + 1/2*c) + 3*B*b^3*tan(1/2*d*x + 1/2*c) +
6*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.788 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=131

$$-\frac{b\left(2a^2B-3abC-b^2B\right)\tan(c+dx)}{d}+\frac{b\left(6a^2C+6abB+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(aC+3bB)-\frac{b^2(2aB-bC)}{}$$

```
[Out] a^2*(3*b*B + a*C)*x + (b*(6*a*b*B + 6*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*B*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (b*(2*a^2*B - b^2*B
- 3*a*b*C)*Tan[c + d*x])/d - (b^2*(2*a*B - b*C)*Sec[c + d*x]*Tan[c + d*x])/
(2*d)
```

Rubi [A] time = 0.29119, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4025, 4048, 3770, 3767, 8}

$$-\frac{b\left(2a^2B-3abC-b^2B\right)\tan(c+dx)}{d}+\frac{b\left(6a^2C+6abB+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(aC+3bB)-\frac{b^2(2aB-bC)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```



```
[Out] a^2*(3*b*B + a*C)*x + (b*(6*a*b*B + 6*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*B*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (b*(2*a^2*B - b^2*B
- 3*a*b*C)*Tan[c + d*x])/d - (b^2*(2*a*B - b*C)*Sec[c + d*x]*Tan[c + d*x])/
(2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+b\sec(c+dx))^3(B+C\sec(c$$

$$= \frac{aB(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \int (a+b\,\mathrm{s}$$

$$= \frac{aB(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \frac{b^2(2aB-}{}$$

$$= a^2(3bB+aC)x + \frac{aB(a+b\sec(c+dx))^2\sin(c-}{d}$$

$$= a^2(3bB+aC)x + \frac{b\left(6abB+6a^2C+b^2C\right)\tanh}{2d}$$

$$= a^2(3bB+aC)x + \frac{b\left(6abB+6a^2C+b^2C\right)\tanh}{2d}$$

Mathematica [B] time = 2.14965, size = 277, normalized size = 2.11

$$\frac{-2b\left(6a^2C+6abB+b^2C\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\left(6a^2C+6abB+b^2C\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (4*a^2*(3*b*B + a*C)*(c + d*x) - 2*b*(6*a*b*B + 6*a^2*C + b^2*C)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 2*b*(6*a*b*B + 6*a^2*C + b^2*C)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2 + (4*b^2*(b*B + 3*a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]) - (b^3*C)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b^2*(b*B +
 3*a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*a^3*B*S
in[c + d*x])/(4*d)
```

Maple [A] time = 0.065, size = 172, normalized size = 1.3

$$\frac{Ba^3\sin(dx+c)}{d} + a^3Cx + \frac{Ca^3c}{d} + 3\,Ba^2bx + 3\,\frac{Ba^2bc}{d} + 3\,\frac{a^2bC\ln(\sec(dx+c)+\tan(dx+c))}{d} + 3\,\frac{Bab^2\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*B*sin(d*x+c)/d+a^3*C*x+1/d*C*a^3*c+3*B*a^2*b*x+3/d*B*a^2*b*c+3/d*a^2*b*
C*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/d*C*a*b
^2*tan(d*x+c)+1/d*B*b^3*tan(d*x+c)+1/2/d*C*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*
C*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.995158, size = 228, normalized size = 1.74

$$4(dx+c)Ca^3+12(dx+c)Ba^2b-Cb^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6Ca^2b(\log(\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*C*a^3 + 12*(d*x + c)*B*a^2*b - C*b^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 12*C*a*b^2*tan(
d*x + c) + 4*B*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.545468, size = 401, normalized size = 3.06

$$\frac{4\left(Ca^3+3Ba^2b\right)dx\cos(dx+c)^2+\left(6Ca^2b+6Bab^2+Cb^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-\left(6Ca^2b+6Bab^2+\right.}{4d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/4*(4*(C*a^3 + 3*B*a^2*b)*d*x*cos(d*x + c)^2 + (6*C*a^2*b + 6*B*a*b^2 + C*
b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (6*C*a^2*b + 6*B*a*b^2 + C*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*B*a^3*cos(d*x + c)^2 + C*b^3
+ 2*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24214, size = 325, normalized size = 2.48

$$\frac{\frac{4\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(Ca^3+3\,Ba^2b\right)(dx+c)+\left(6\,Ca^2b+6\,Bab^2+Cb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,Ca^2b+6\,Bab^2+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(C*a^3 +
 3*B*a^2*b)*(d*x + c) + (6*C*a^2*b + 6*B*a*b^2 + C*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (6*C*a^2*b + 6*B*a*b^2 + C*b^3)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^3*tan(1/2*d*x + 1/2
*c)^3 - C*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*C*a*b^2*tan(1/2*d*x + 1/2*c) - 2*B
*b^3*tan(1/2*d*x + 1/2*c) - C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

3.789 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c$

Optimal. Leaf size=124

$$\frac{1}{2}ax\left(a^2B+6abC+6b^2B\right)+\frac{a^2(aC+2bB)\sin(c+dx)}{d}-\frac{b^2(aB-2bC)\tan(c+dx)}{2d}+\frac{b^2(3aC+bB)\tanh^{-1}(\sin(c+d$$

```
[Out] (a*(a^2*B + 6*b^2*B + 6*a*b*C)*x)/2 + (b^2*(b*B + 3*a*C)*ArcTanh[Sin[c + d*
x]])/d + (a^2*(2*b*B + a*C)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*(a + b*Sec[
c + d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(a*B - 2*b*C)*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.404287, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4025, 4076, 4047, 8, 4045, 3770}

$$\frac{1}{2}ax\left(a^2B+6abC+6b^2B\right)+\frac{a^2(aC+2bB)\sin(c+dx)}{d}-\frac{b^2(aB-2bC)\tan(c+dx)}{2d}+\frac{b^2(3aC+bB)\tanh^{-1}(\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (a*(a^2*B + 6*b^2*B + 6*a*b*C)*x)/2 + (b^2*(b*B + 3*a*C)*ArcTanh[Sin[c + d*
x]])/d + (a^2*(2*b*B + a*C)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*(a + b*Sec[
c + d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(a*B - 2*b*C)*Tan[c + d*x])/(2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+b\sec(c+dx))^3(B+C\sec(c$$
$$= \frac{aB\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{aB\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{aB\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{1}{2}a\left(a^2B+6b^2B+6abC\right)x+\frac{a^2(2bB+aC)\sin}{d}$$
$$= \frac{1}{2}a\left(a^2B+6b^2B+6abC\right)x+\frac{b^2(bB+3aC)\tan}{d}$$

Mathematica [A] time = 0.672071, size = 217, normalized size = 1.75

$$2a(c+dx)\left(a^2B+6abC+6b^2B\right)+4a^2(aC+3bB)\sin(c+dx)+a^3B\sin(2(c+dx))-4b^2(3aC+bB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*a*(a^2*B + 6*b^2*B + 6*a*b*C)*(c + d*x) - 4*b^2*(b*B + 3*a*C)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*b^2*(b*B + 3*a*C)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*a^2*(3*b*B + a*C)*Sin[c + d*x] + a^3*B*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.059, size = 168, normalized size = 1.4

$$\frac{Ba^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{a^3Bx}{2}+\frac{Ba^3c}{2d}+\frac{a^3C\sin(dx+c)}{d}+3\frac{Ba^2b\sin(dx+c)}{d}+3a^2bCx+3\frac{Ca^2bc}{d}+3Bab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*B*a^3*sin(d*x+c)*cos(d*x+c)+1/2*a^3*B*x+1/2/d*B*a^3*c+a^3*C*sin(d*x+c
)/d+3/d*B*a^2*b*sin(d*x+c)+3*a^2*b*C*x+3/d*C*a^2*b*c+3*B*a*b^2*x+3/d*B*a*b^
2*c+3/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b^3*ln(sec(d*x+c)+tan(d*x+c
))+1/d*C*b^3*tan(d*x+c)
```

Maxima [A] time = 0.997251, size = 194, normalized size = 1.56

$$(2dx+2c+\sin(2dx+2c))Ba^3+12(dx+c)Ca^2b+12(dx+c)Bab^2+6Cab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 12*(d*x + c)*C*a^2*b + 12*(d*
x + c)*B*a*b^2 + 6*C*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*B*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*C*a^3*sin(d*x
 + c) + 12*B*a^2*b*sin(d*x + c) + 4*C*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.550354, size = 369, normalized size = 2.98

$$\frac{\left(Ba^3+6Ca^2b+6Bab^2\right)dx\cos(dx+c)+\left(3Cab^2+Bb^3\right)\cos(dx+c)\log(\sin(dx+c)+1)-\left(3Cab^2+Bb^3\right)\cos(dx+}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*((B*a^3 + 6*C*a^2*b + 6*B*a*b^2)*d*x*cos(d*x + c) + (3*C*a*b^2 + B*b^3)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (3*C*a*b^2 + B*b^3)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (B*a^3*cos(d*x + c)^2 + 2*C*b^3 + 2*(C*a^3 + 3*B*a^2*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22009, size = 316, normalized size = 2.55

$$-\frac{\frac{4\,Cb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\left(Ba^3+6\,Ca^2b+6\,Bab^2\right)(dx+c)-2\left(3\,Cab^2+Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(3\,Cab^2+Bb^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/2*(4*C*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (B*a^3 +
6*C*a^2*b + 6*B*a*b^2)*(d*x + c) - 2*(3*C*a*b^2 + B*b^3)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(3*C*a*b^2 + B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 + 2*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 6*B*a
^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c) - 2*C*a^3*tan(1/2*
d*x + 1/2*c) - 6*B*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

3.790 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=145

$$\frac{a\left(2a^2B+9abC+8b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)+\frac{a^2(3aC+5bB)\sin(c+dx)\cos(c+dx)}{6d}+$$

```
[Out] ((3*a^2*b*B + 2*b^3*B + a^3*C + 6*a*b^2*C)*x)/2 + (b^3*C*ArcTanh[Sin[c + d*
x]])/d + (a*(2*a^2*B + 8*b^2*B + 9*a*b*C)*Sin[c + d*x])/(3*d) + (a^2*(5*b*B
 + 3*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a*B*Cos[c + d*x]^2*(a + b*Sec
[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.4148, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4025, 4074, 4047, 8, 4045, 3770}

$$\frac{a\left(2a^2B+9abC+8b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)+\frac{a^2(3aC+5bB)\sin(c+dx)\cos(c+dx)}{6d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] ((3*a^2*b*B + 2*b^3*B + a^3*C + 6*a*b^2*C)*x)/2 + (b^3*C*ArcTanh[Sin[c + d*
x]])/d + (a*(2*a^2*B + 8*b^2*B + 9*a*b*C)*Sin[c + d*x])/(3*d) + (a^2*(5*b*B
 + 3*a*C)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a*B*Cos[c + d*x]^2*(a + b*Sec
[c + d*x])^2*Sin[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+b\sec(c+dx))^3(B+C\sec(c$$

$$= \frac{aB\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}$$

$$= \frac{a^2(5bB+3aC)\cos(c+dx)\sin(c+dx)}{6d} + \frac{aB\,\mathrm{co}}{}$$

$$= \frac{a^2(5bB+3aC)\cos(c+dx)\sin(c+dx)}{6d} + \frac{aB\,\mathrm{co}}{}$$

$$= \frac{1}{2}\left(3a^2bB+2b^3B+a^3C+6ab^2C\right)x + \frac{a\left(2a^2B\right.}{}$$

$$= \frac{1}{2}\left(3a^2bB+2b^3B+a^3C+6ab^2C\right)x + \frac{b^3C\tan}{}$$

Mathematica [A] time = 0.368836, size = 159, normalized size = 1.1

$$\frac{6(c+dx)\left(3a^2bB+a^3C+6ab^2C+2b^3B\right)+9a\left(a^2B+4abC+4b^2B\right)\sin(c+dx)+3a^2(aC+3bB)\sin(2(c+dx))+a^3B\sin}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (6*(3*a^2*b*B + 2*b^3*B + a^3*C + 6*a*b^2*C)*(c + d*x) - 12*b^3*C*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*C*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*a*(a^2*B + 4*b^2*B + 4*a*b*C)*Sin[c + d*x] + 3*a^2*(3*b*B + a*
C)*Sin[2*(c + d*x)] + a^3*B*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.064, size = 207, normalized size = 1.4

$$\frac{B(\cos(dx+c))^2\sin(dx+c)\,a^3}{3d}+\frac{2Ba^3\sin(dx+c)}{3d}+\frac{a^3C\sin(dx+c)\cos(dx+c)}{2d}+\frac{a^3Cx}{2}+\frac{a^3Cc}{2d}+\frac{3Ba^2b\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*B*cos(d*x+c)^2*sin(d*x+c)*a^3+2/3*a^3*B*sin(d*x+c)/d+1/2/d*a^3*C*sin(
d*x+c)*cos(d*x+c)+1/2*a^3*C*x+1/2/d*C*a^3*c+3/2/d*B*a^2*b*sin(d*x+c)*cos(d*
x+c)+3/2*B*a^2*b*x+3/2/d*B*a^2*b*c+3/d*a^2*b*C*sin(d*x+c)+3/d*B*a*b^2*sin(d
*x+c)+3*C*a*b^2*x+3/d*C*a*b^2*c+B*b^3*x+1/d*B*b^3*c+1/d*C*b^3*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 0.973428, size = 205, normalized size = 1.41

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3-3(2dx+2c+\sin(2dx+2c))Ca^3-9(2dx+2c+\sin(2dx+2c))Ba^2b-36}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*C*a^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b - 36*(d*x + c
)*C*a*b^2 - 12*(d*x + c)*B*b^3 - 6*C*b^3*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) - 36*C*a^2*b*sin(d*x + c) - 36*B*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.544111, size = 317, normalized size = 2.19

$$\frac{3Cb^3\log(\sin(dx+c)+1)-3Cb^3\log(-\sin(dx+c)+1)+3\left(Ca^3+3Ba^2b+6Cab^2+2Bb^3\right)dx+\left(2Ba^3\cos(dx+\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*C*b^3*log(sin(d*x + c) + 1) - 3*C*b^3*log(-sin(d*x + c) + 1) + 3*(C*
a^3 + 3*B*a^2*b + 6*C*a*b^2 + 2*B*b^3)*d*x + (2*B*a^3*cos(d*x + c)^2 + 4*B*
a^3 + 18*C*a^2*b + 18*B*a*b^2 + 3*(C*a^3 + 3*B*a^2*b)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27011, size = 424, normalized size = 2.92

$$6\,Cb^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Cb^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(Ca^3+3\,Ba^2b+6\,Cab^2+2\,Bb^3\right)(dx+c)+\frac{2(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(6*C*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*C*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(C*a^3 + 3*B*a^2*b + 6*C*a*b^2 + 2*B*b^3)*(d*x + c) +
 2*(6*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*B*a
^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^
2*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^2*b*tan(
1/2*d*x + 1/2*c)^3 + 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^3*tan(1/2*d*
x + 1/2*c) + 3*C*a^3*tan(1/2*d*x + 1/2*c) + 9*B*a^2*b*tan(1/2*d*x + 1/2*c)
+ 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.791 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c$

Optimal. Leaf size=179

$$\frac{\left(6a^2bB+2a^3C+9ab^2C+3b^3B\right)\sin(c+dx)}{3d}+\frac{a\left(3a^2B+12abC+10b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(12a^2bC+3a\right.$$

```
[Out] ((3*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 8*b^3*C)*x)/8 + ((6*a^2*b*B + 3*b^3*B
 + 2*a^3*C + 9*a*b^2*C)*Sin[c + d*x])/(3*d) + (a*(3*a^2*B + 10*b^2*B + 12*a
*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(3*b*B + 2*a*C)*Cos[c + d*x]^
2*Sin[c + d*x])/(6*d) + (a*B*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(4*d)
```

Rubi [A] time = 0.491216, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4025, 4074, 4047, 2637, 4045, 8}

$$\frac{\left(6a^2bB+2a^3C+9ab^2C+3b^3B\right)\sin(c+dx)}{3d}+\frac{a\left(3a^2B+12abC+10b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(12a^2bC+3a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] ((3*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 8*b^3*C)*x)/8 + ((6*a^2*b*B + 3*b^3*B
 + 2*a^3*C + 9*a*b^2*C)*Sin[c + d*x])/(3*d) + (a*(3*a^2*B + 10*b^2*B + 12*a
*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(3*b*B + 2*a*C)*Cos[c + d*x]^
2*Sin[c + d*x])/(6*d) + (a*B*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(4*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+b\sec(c+dx))^3(B+C\sec($$

$$= \frac{aB\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}$$

$$= \frac{a^2(3bB+2aC)\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{aB\,c}{}$$

$$= \frac{a^2(3bB+2aC)\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{aB\,c}{}$$

$$= \frac{\left(6a^2bB+3b^3B+2a^3C+9ab^2C\right)\sin(c+dx)}{3d} +$$

$$= \frac{1}{8}\left(3a^3B+12ab^2B+12a^2bC+8b^3C\right)x + \frac{(6a^2}{}$$

Mathematica [A] time = 0.415275, size = 140, normalized size = 0.78

$$\frac{12(c+dx)\left(12a^2bC+3a^3B+12ab^2B+8b^3C\right)+24a\left(a^2B+3abC+3b^2B\right)\sin(2(c+dx))+24\left(9a^2bB+3a^3C+12ab^2C\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (12*(3*a^3*B + 12*a*b^2*B + 12*a^2*b*C + 8*b^3*C)*(c + d*x) + 24*(9*a^2*b*B
 + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*Sin[c + d*x] + 24*a*(a^2*B + 3*b^2*B + 3
*a*b*C)*Sin[2*(c + d*x)] + 8*a^2*(3*b*B + a*C)*Sin[3*(c + d*x)] + 3*a^3*B*S
in[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.067, size = 180, normalized size = 1.

$$\frac{1}{d}\left(Ba^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+Ba^2b\left(2+(\cos(dx+c))^2\right)\sin(dx+c)+\frac{a^3C}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(B*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a
^2*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*
a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*C*(1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+B*b^3*sin(d*x+c)+3*C*a*b^2*sin(d*x+c)+C*b^3*(d*x+c
))
```

Maxima [A] time = 0.988208, size = 231, normalized size = 1.29

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ba^3-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ca^3-96\left(\sin(dx+c)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 - 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*B*a^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2*b + 72*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*B*a*b^2 + 96*(d*x + c)*C*b^3 + 288*C*a*b^2*sin(d*x + c
) + 96*B*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.520797, size = 321, normalized size = 1.79

$$\frac{3\left(3\,Ba^3+12\,Ca^2b+12\,Bab^2+8\,Cb^3\right)dx+\left(6\,Ba^3\cos(dx+c)^3+16\,Ca^3+48\,Ba^2b+72\,Cab^2+24\,Bb^3+8\left(Ca^3+3\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/24*(3*(3*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 8*C*b^3)*d*x + (6*B*a^3*cos(d*
x + c)^3 + 16*C*a^3 + 48*B*a^2*b + 72*C*a*b^2 + 24*B*b^3 + 8*(C*a^3 + 3*B*a
^2*b)*cos(d*x + c)^2 + 9*(B*a^3 + 4*C*a^2*b + 4*B*a*b^2)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24079, size = 724, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*B*a^3 + 12*C*a^2*b + 12*B*a*b^2 + 8*C*b^3)*(d*x + c) - 2*(15*B*a
^3*tan(1/2*d*x + 1/2*c)^7 - 24*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^7 + 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*B*a*b^2*tan(1
/2*d*x + 1/2*c)^7 - 72*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*b^3*tan(1/2*d*
x + 1/2*c)^7 - 9*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*C*a^3*tan(1/2*d*x + 1/2*
c)^5 - 120*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b*tan(1/2*d*x + 1/2*c)
^5 + 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 216*C*a*b^2*tan(1/2*d*x + 1/2*c)^5
 - 72*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*C*
a^3*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*C*a^2*
b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 216*C*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 72*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*B*a^3*tan(1/2
*d*x + 1/2*c) - 24*C*a^3*tan(1/2*d*x + 1/2*c) - 72*B*a^2*b*tan(1/2*d*x + 1/
2*c) - 36*C*a^2*b*tan(1/2*d*x + 1/2*c) - 36*B*a*b^2*tan(1/2*d*x + 1/2*c) -
72*C*a*b^2*tan(1/2*d*x + 1/2*c) - 24*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

3.792 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3 \left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=221

$$-\frac{a\left(4a^2B+15abC+12b^2B\right)\sin^3(c+dx)}{15d}+\frac{\left(15a^2bC+4a^3B+14ab^2B+5b^3C\right)\sin(c+dx)}{5d}+\frac{\left(9a^2bB+3a^3C+12ab^2\right.}{}$$

```
[Out] ((9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*x)/8 + ((4*a^3*B + 14*a*b^2*B
 + 15*a^2*b*C + 5*b^3*C)*Sin[c + d*x])/(5*d) + ((9*a^2*b*B + 4*b^3*B + 3*a^
3*C + 12*a*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(7*b*B + 5*a*C)*C
os[c + d*x]^3*Sin[c + d*x])/(20*d) + (a*B*Cos[c + d*x]^4*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(5*d) - (a*(4*a^2*B + 12*b^2*B + 15*a*b*C)*Sin[c + d*x]^
3)/(15*d)
```

Rubi [A] time = 0.548044, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 4025, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{a\left(4a^2B+15abC+12b^2B\right)\sin^3(c+dx)}{15d}+\frac{\left(15a^2bC+4a^3B+14ab^2B+5b^3C\right)\sin(c+dx)}{5d}+\frac{\left(9a^2bB+3a^3C+12ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] ((9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*x)/8 + ((4*a^3*B + 14*a*b^2*B
 + 15*a^2*b*C + 5*b^3*C)*Sin[c + d*x])/(5*d) + ((9*a^2*b*B + 4*b^3*B + 3*a^
3*C + 12*a*b^2*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(7*b*B + 5*a*C)*C
os[c + d*x]^3*Sin[c + d*x])/(20*d) + (a*B*Cos[c + d*x]^4*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(5*d) - (a*(4*a^2*B + 12*b^2*B + 15*a*b*C)*Sin[c + d*x]^
3)/(15*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^5(c+dx)(a+b\sec(c+dx))^3(B+C\sec($$
$$= \frac{aB\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}$$
$$= \frac{a^2(7bB+5aC)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{aB\,\text{c}}{}$$
$$= \frac{a^2(7bB+5aC)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{aB\,\text{c}}{}$$
$$= \frac{\left(9a^2bB+4b^3B+3a^3C+12ab^2C\right)\cos(c+dx)}{8d}$$
$$= \frac{1}{8}\left(9a^2bB+4b^3B+3a^3C+12ab^2C\right)x + \frac{(9a^2b}{}$$
$$= \frac{1}{8}\left(9a^2bB+4b^3B+3a^3C+12ab^2C\right)x + \frac{(4a^3B}{}$$

Mathematica [A] time = 0.667971, size = 176, normalized size = 0.8

$$\frac{60(c+dx)\left(9a^2bB+3a^3C+12ab^2C+4b^3B\right)+10a\left(5a^2B+12abC+12b^2B\right)\sin(3(c+dx))+60\left(18a^2bC+5a^3B+18ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (60*(9*a^2*b*B + 4*b^3*B + 3*a^3*C + 12*a*b^2*C)*(c + d*x) + 60*(5*a^3*B +
18*a*b^2*B + 18*a^2*b*C + 8*b^3*C)*Sin[c + d*x] + 120*(3*a^2*b*B + b^3*B +
a^3*C + 3*a*b^2*C)*Sin[2*(c + d*x)] + 10*a*(5*a^2*B + 12*b^2*B + 12*a*b*C)*
Sin[3*(c + d*x)] + 15*a^2*(3*b*B + a*C)*Sin[4*(c + d*x)] + 6*a^3*B*Sin[5*(c
 + d*x)])/(480*d)
```

Maple [A] time = 0.08, size = 227, normalized size = 1.

$$\frac{1}{d}\left(\frac{Ba^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a^3C\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*C*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*B*a^2*b*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b*C*(2+cos(d*x+c)^2)*
sin(d*x+c)+B*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*C*a*b^2*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+B*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+C*b
^3*sin(d*x+c))
```

Maxima [A] time = 0.97372, size = 293, normalized size = 1.33

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^3+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3 + 45*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b - 480*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*C*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a
*b^2 + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b^2 + 120*(2*d*x + 2*c + si
n(2*d*x + 2*c))*B*b^3 + 480*C*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.537056, size = 423, normalized size = 1.91

$$\frac{15\left(3Ca^3+9Ba^2b+12Cab^2+4Bb^3\right)dx+\left(24Ba^3\cos(dx+c)^4+64Ba^3+240Ca^2b+240Bab^2+120Cb^3+30\left(C\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*d*x + (24*B*a^3*cos(
d*x + c)^4 + 64*B*a^3 + 240*C*a^2*b + 240*B*a*b^2 + 120*C*b^3 + 30*(C*a^3 +
 3*B*a^2*b)*cos(d*x + c)^3 + 8*(4*B*a^3 + 15*C*a^2*b + 15*B*a*b^2)*cos(d*x
+ c)^2 + 15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3*(B*sec(d*x+c)+C*sec(d*x+c)**2),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27143, size = 907, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(3*C*a^3 + 9*B*a^2*b + 12*C*a*b^2 + 4*B*b^3)*(d*x + c) + 2*(120*B
*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*B*a^2*b
*tan(1/2*d*x + 1/2*c)^9 + 360*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a*b^2*
tan(1/2*d*x + 1/2*c)^9 - 180*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*B*b^3*tan(
1/2*d*x + 1/2*c)^9 + 120*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*B*a^3*tan(1/2*d
*x + 1/2*c)^7 - 30*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*B*a^2*b*tan(1/2*d*x +
1/2*c)^7 + 960*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^2*tan(1/2*d*x + 1
/2*c)^7 - 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*B*b^3*tan(1/2*d*x + 1/2*
c)^7 + 480*C*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*B*a^3*tan(1/2*d*x + 1/2*c)^5
+ 1200*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a*b^2*tan(1/2*d*x + 1/2*c)^5
 + 720*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 30
*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*C*a
^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 360*C*a*
b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*C*b^3*t
an(1/2*d*x + 1/2*c)^3 + 120*B*a^3*tan(1/2*d*x + 1/2*c) + 75*C*a^3*tan(1/2*d
*x + 1/2*c) + 225*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*C*a^2*b*tan(1/2*d*x +
1/2*c) + 360*B*a*b^2*tan(1/2*d*x + 1/2*c) + 180*C*a*b^2*tan(1/2*d*x + 1/2*c
) + 60*B*b^3*tan(1/2*d*x + 1/2*c) + 120*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.793 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(-3a^2C+3abB-2b^2C\right)\tan(c+dx)}{3b^3d}+\frac{\left(2a^2+b^2\right)(bB-aC)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+}}\right.}{b^4d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ((2*a^2 + b^2)*(b*B - a*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(b*B -
 a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]])/Sqrt[a + b]])/(Sqrt[a - b]*b^4
*Sqrt[a + b]*d) - ((3*a*b*B - 3*a^2*C - 2*b^2*C)*Tan[c + d*x])/(3*b^3*d) +
((b*B - a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (C*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*b*d)
```

Rubi [A] time = 0.716545, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {4072, 4033, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-3a^2C+3abB-2b^2C\right)\tan(c+dx)}{3b^3d}+\frac{\left(2a^2+b^2\right)(bB-aC)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+}}\right.}{b^4d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
]),x]
```

```
[Out] ((2*a^2 + b^2)*(b*B - a*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(b*B -
 a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]])/Sqrt[a + b]])/(Sqrt[a - b]*b^4
*Sqrt[a + b]*d) - ((3*a*b*B - 3*a^2*C - 2*b^2*C)*Tan[c + d*x])/(3*b^3*d) +
((b*B - a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (C*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*b*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$= \frac{C\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)\left(2aC+2bC\sec(c+dx)+3(bB-a \ldots\right.}{a+b\sec(c+dx)}}{3b} \ldots$$

$$= \frac{(bB-aC)\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{C\sec^2(c+dx)\tan(c+dx \ldots}{3bd} \ldots$$

$$= -\frac{\left(3abB-3a^2C-2b^2C\right)\tan(c+dx)}{3b^3d} + \frac{(bB-aC)\sec(c+dx)\,t \ldots}{2b^2d} \ldots$$

$$= -\frac{\left(3abB-3a^2C-2b^2C\right)\tan(c+dx)}{3b^3d} + \frac{(bB-aC)\sec(c+dx)\,t \ldots}{2b^2d} \ldots$$

$$= \frac{\left(2a^2+b^2\right)(bB-aC)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{\left(3abB-3a^2C-2 \ldots\right.}{3b^3 \ldots} \ldots$$

$$= \frac{\left(2a^2+b^2\right)(bB-aC)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{\left(3abB-3a^2C-2 \ldots\right.}{3b^3 \ldots} \ldots$$

$$= \frac{\left(2a^2+b^2\right)(bB-aC)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3(bB-aC)\tanh \ldots}{\sqrt{a-b \ldots}} \ldots$$

<u>Mathematica</u> [B] time = 2.34835, size = 422, normalized size = 2.26

$$\frac{\frac{4b\left(3a^2C-3abB+2b^2C\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4b\left(3a^2C-3abB+2b^2C\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} + \frac{24a^3(bB-aC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 6\left(2a^2+b^2\right)(aC-bB \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x]),x]
```

```
[Out] ((24*a^3*(b*B - a*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + 6*(2*a^2 + b^2)*(-(b*B) + a*C)*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 6*(2*a^2 + b^2)*(-(b*B) + a*C)*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + (b^2*(-3*a*C + b*(3*B + C)))/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2 + (2*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^3 + (4*b*(-3*a*b*B + 3*a^2*C + 2*b^2*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]) + (2*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3 - (b^2*(-3*a*C + b*(3*B + C)))/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (4*b*(-3*a*b*B + 3*a^2*C + 2*b^2*C)*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]))/(12*b^4*d)
```

<u>Maple</u> [B] time = 0.095, size = 688, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*B+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*C-1/2/
d/b*ln(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/b*ln(
tan(1/2*d*x+1/2*c)+1)*B+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)*B-1/3/d*C/b/(tan(1/2
*d*x+1/2*c)-1)^3+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2*B-1/2/d/b/(tan(1/2*d*x+1/
2*c)-1)^2*C-1/3/d*C/b/(tan(1/2*d*x+1/2*c)+1)^3-2/d*a^3/b^3/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-1/d/b/(tan(1/2*
d*x+1/2*c)+1)*C-1/d/b/(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/b^2/(tan(1/2*d*x+1/2*c
)+1)^2*a*C+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*B*a^2-1/2/d*a/b^2*ln(tan(1/2*d*
x+1/2*c)+1)*C+1/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C+2/d*a^4/b^4/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/d/b^3*l
n(tan(1/2*d*x+1/2*c)-1)*B*a^2+1/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*a^3*C+1/d/b^
2/(tan(1/2*d*x+1/2*c)-1)*B*a-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/2/d/b^2
/(tan(1/2*d*x+1/2*c)-1)*a*C-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2*C-1/2/d/b^2/
(tan(1/2*d*x+1/2*c)+1)*a*C-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a*C-1/d/b^4*l
n(tan(1/2*d*x+1/2*c)+1)*a^3*C+1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29042, size = 1650, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [-1/12*(6*(C*a^4 - B*a^3*b)*sqrt(a^2 - b^2)*cos(d*x + c)^3*log((2*a*b*cos(d
*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c)
+ a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 b^2)) + 3*(2*C*a^5 - 2*B*a^4*b - C*a^3*b^2 + B*a^2*b^3 - C*a*b^4 + B*b^5)*
cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*C*a^5 - 2*B*a^4*b - C*a^3*b^2 +
 B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*
C*a^2*b^3 - 2*C*b^5 + 2*(3*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + 3*B*a*b^4 -
2*C*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 - B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d
*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*(C*a^4
- B*a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*C*a^5 - 2*B*a^4*b - C*a^3*
b^2 + B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3
*(2*C*a^5 - 2*B*a^4*b - C*a^3*b^2 + B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x +
c)^3*log(-sin(d*x + c) + 1) + 2*(2*C*a^2*b^3 - 2*C*b^5 + 2*(3*C*a^4*b - 3*B
*a^3*b^2 - C*a^2*b^3 + 3*B*a*b^4 - 2*C*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 -
 B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*
```

```
d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^4(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.23036, size = 556, normalized size = 2.97

$$-\frac{\frac{3\left(2Ca^3-2Ba^2b+Cab^2-Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2Ca^3-2Ba^2b+Cab^2-Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}-\frac{12\left(Ca^4-Ba^3b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/6*(3*(2*C*a^3 - 2*B*a^2*b + C*a*b^2 - B*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/b^4 - 3*(2*C*a^3 - 2*B*a^2*b + C*a*b^2 - B*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^4 - 12*(C*a^4 - B*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) + 2*(6*C*a^2*tan(1/2*d*x + 1/2
*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 - 3
*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*C*a^2*t
an(1/2*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 4*C*b^2*tan(1/2*d
*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c)
 - 3*C*a*b*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*ta
n(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

3.794 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{\left(-2a^2C+2abB-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{2a^2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\,\mathrm{ta}}{}$$

```
[Out] -((2*a*b*B - 2*a^2*C - b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + (2*a^2*(b*
B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
b^3*Sqrt[a + b]*d) + ((b*B - a*C)*Tan[c + d*x])/(b^2*d) + (C*Sec[c + d*x]*T
an[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.450333, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 4033, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-2a^2C+2abB-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{2a^2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
]),x]
```


```
[Out] -((2*a*b*B - 2*a^2*C - b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + (2*a^2*(b*
B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
b^3*Sqrt[a + b]*d) + ((b*B - a*C)*Tan[c + d*x])/(b^2*d) + (C*Sec[c + d*x]*T
an[c + d*x])/(2*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
```

```
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$= \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(aC+bC\sec(c+dx)+2(bB-aC)\,\mathrm{se}\right.}{a+b\sec(c+dx)}}{2b}$$

$$= \frac{(bB-aC)\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+d}{}}{}$$

$$= \frac{(bB-aC)\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\left(a^2(bB-\right.}{}$$

$$= -\frac{\left(2abB-2a^2C-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(bB-aC)\tan(c}{b^2d}$$

$$= -\frac{\left(2abB-2a^2C-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(bB-aC)\tan(c}{b^2d}$$

$$= -\frac{\left(2abB-2a^2C-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{2a^2(bB-aC)\,\mathrm{ta}}{\sqrt{a-}}$$

Mathematica [B] time = 1.73591, size = 300, normalized size = 2.1

$$\frac{\frac{8a^2(aC-bB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-2\left(2a^2C-2abB+b^2C\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(2a^2C-2abB+b^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x]),x]
```

```
[Out] ((8*a^2*(-(b*B) + a*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] - 2*(-2*a*b*B + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*(-2*a*b*B + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (b^2*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b*(b
*B - a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^2*C)
/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b*(b*B - a*C)*Sin[(c + d*x)/2
])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*b^3*d)
```

Maple [B] time = 0.08, size = 410, normalized size = 2.9

$$2\frac{Ba^2}{db^2\sqrt{(a+b)(a-b)}}\operatorname{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-2\frac{a^3C}{db^3\sqrt{(a+b)(a-b)}}\operatorname{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*B-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*C-1/d/b/(tan(1/2
*d*x+1/2*c)+1)*B+1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a*C+1/2/d/b/(tan(1/2*d*x+1/
2*c)+1)*C-1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*B*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c
)+1)*a^2*C+1/2/d/b*ln(tan(1/2*d*x+1/2*c)+1)*C+1/2/d/b/(tan(1/2*d*x+1/2*c)-1
)^2*C-1/d/b/(tan(1/2*d*x+1/2*c)-1)*B+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a*C+1/2
/d/b/(tan(1/2*d*x+1/2*c)-1)*C+1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*B*a-1/d/b^3*
ln(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.3992, size = 1353, normalized size = 9.46

$$\left[-\frac{2\left(Ca^3 - Ba^2b\right)\sqrt{a^2 - b^2}\cos\left(dx + c\right)^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right) - \left(2\,Ca\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [-1/4*(2*(C*a^3 - B*a^2*b)*sqrt(a^2 - b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2)) - (2*C*a^4 - 2*B*a^3*b - C*a^2*b^2 + 2*B*a*b^3 - C*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (2*C*a^4 - 2*B*a^3*b - C*a^2*b^2 + 2*B*a*b^3 - C*
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(C*a^2*b^2 - C*b^4 - 2*(C*a^
3*b - B*a^2*b^2 - C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d*cos(d*x + c)^2), -1/4*(4*(C*a^3 - B*a^2*b)*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x +
 c)^2 - (2*C*a^4 - 2*B*a^3*b - C*a^2*b^2 + 2*B*a*b^3 - C*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (2*C*a^4 - 2*B*a^3*b - C*a^2*b^2 + 2*B*a*b^3 - C*
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(C*a^2*b^2 - C*b^4 - 2*(C*a^
3*b - B*a^2*b^2 - C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^3(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.21725, size = 363, normalized size = 2.54

$$\frac{\frac{\left(2\,Ca^2-2\,Bab+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3} - \frac{\left(2\,Ca^2-2\,Bab+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^3} - \frac{4\left(Ca^3-Ba^2b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan(}{}\right.\right.}{\sqrt{-a^2+b^2}b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((2*C*a^2 - 2*B*a*b + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 - (
2*C*a^2 - 2*B*a*b + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 4*(C*a^
3 - B*a^2*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
```

```
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a
^2 + b^2)*b^3) + 2*(2*C*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*
c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(
1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^
2*b^2))/d
```

3.795 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{C\tan(c+dx)}{bd}$$

```
[Out] ((b*B - a*C)*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*a*(b*B - a*C)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) +
(C*Tan[c + d*x])/(b*d)
```

Rubi [A] time = 0.265515, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4072, 4010, 12, 3789, 3770, 3831, 2659, 208}

$$\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{C\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
,x]
```

```
[Out] ((b*B - a*C)*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*a*(b*B - a*C)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) +
(C*Tan[c + d*x])/(b*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
```

```
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$= \frac{C\tan(c+dx)}{bd} + \frac{\int \frac{(bB-aC)\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{C\tan(c+dx)}{bd} + \frac{(bB-aC)\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{C\tan(c+dx)}{bd} + \frac{(bB-aC)\int \sec(c+dx)\,dx}{b^2} - \frac{(a(bB-aC))\int \frac{\sec(}{a+b\,\mathrm{se}}}{b^2}$$

$$= \frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{C\tan(c+dx)}{bd} - \frac{(a(bB-aC))\int \frac{}{1}}{b^3}$$

$$= \frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{C\tan(c+dx)}{bd} - \frac{(2a(bB-aC))\,\mathrm{Su}}{}$$

$$= \frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{\sqrt{a-b}b^2\sqrt{a+b}d}$$

Mathematica [A] time = 0.578953, size = 130, normalized size = 1.33

$$\frac{-\frac{2a(aC-bB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - (bB-aC)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```

```
[Out] ((-2*a*(-(b*B) + a*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] - (b*B - a*C)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b*C*Tan[c + d*x])/(b^2*d)
```

Maple [B] time = 0.064, size = 228, normalized size = 2.3

$$-2\,{\frac {Ba}{db\sqrt {(a+b)(a-b)}}{\rm Artanh} \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}}\right)}+2\,{\frac {{a}^{2}C}{d{b}^{2}\sqrt {(a+b)(a-b)}}{\rm Artanh} \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*B+2/d/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*a^2*C-1/d/b/(tan(1/2*d*x+1/2*c)+1)*C+1/d/b*ln(tan(1/2*d*x+
1/2*c)+1)*B-1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/b/(tan(1/2*d*x+1/2*c)-
1)*C-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)*B+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.889201, size = 1065, normalized size = 10.87

$$\left[-\frac{\left(Ca^{2}-Bab\right)\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)\log\left(\frac{2\,ab\cos\left(dx+c\right)-\left(a^{2}-2\,b^{2}\right)\cos\left(dx+c\right)^{2}-2\,\sqrt{a^{2}-b^{2}}\left(b\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^{2}-b^{2}}{a^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+b^{2}}\right)+\left(Ca^{3}-B\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] [-1/2*((C*a^2 - B*a*b)*sqrt(a^2 - b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
 - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*si
n(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
+ (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) -
(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2
*(C*a^2*b - C*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c)), 1/2*(2*(
```

```
C*a^2 - B*a*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (C*a^3 - B*a^2*b - C*a*b^2 +
B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (C*a^3 - B*a^2*b - C*a*b^2 + B*
b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a^2*b - C*b^3)*sin(d*x + c)
)/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^2(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.23684, size = 236, normalized size = 2.41

$$-\frac{\frac{(Ca-Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2} - \frac{(Ca-Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2} + \frac{2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b} + \frac{2\left(Ca^2-Bab\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\right.}{}\right.\right.}{\sqrt{-a^2+b^2}b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] -((C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - (C*a - B*b)*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*b) + 2*(C*a^2 - B*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/d
```

3.796 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b*d) + (2*(b*B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.119925, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4050, 3770, 12, 3831, 2659, 208}

$$\frac{2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b*d) + (2*(b*B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 4050

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e
 + f*x], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Csc[e + f*x])/(a + b*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \frac{(bB-aC)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}+\frac{C\int \sec(c+dx)\,dx}{b}\\
&= \frac{C\tanh^{-1}(\sin(c+dx))}{bd}+\frac{(bB-aC)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}\\
&= \frac{C\tanh^{-1}(\sin(c+dx))}{bd}+\frac{(bB-aC)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^2}\\
&= \frac{C\tanh^{-1}(\sin(c+dx))}{bd}+\frac{(2(bB-aC))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx\right.\right.}{b^2d}\\
&= \frac{C\tanh^{-1}(\sin(c+dx))}{bd}+\frac{2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}
\end{aligned}
$$

Mathematica [A] time = 0.167688, size = 112, normalized size = 1.47

$$
\frac{\frac{2(aC-bB)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+C\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(-(b*B) + a*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + C*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]))/(b*d)
```

Maple [A] time = 0.069, size = 135, normalized size = 1.8

$$
2\frac{B}{d\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-2\frac{aC}{db\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+\frac{C}{db}\,\mathrm{l}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*B/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))-2/d/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*a*C+1/d/b*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/b*ln(tan(1/2*d*x+1/2*c)-
1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9653, size = 707, normalized size = 9.3

$$\left[-\frac{(Ca-Bb)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(Ca^2-Cb^2\right)\log\left(\sin\left(dx\right.\right.}{2\left(a^2b-b^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/2*((C*a - B*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (C*a^2 - C*b^2)
*log(sin(d*x + c) + 1) + (C*a^2 - C*b^2)*log(-sin(d*x + c) + 1))/((a^2*b -
b^3)*d), -1/2*(2*(C*a - B*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (C*a^2 - C*b^2)*log(sin(d*x
+ c) + 1) + (C*a^2 - C*b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B+C\sec(c+dx))\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.22444, size = 173, normalized size = 2.28

$$\frac{\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Ca-Bb)}{\sqrt{-a^2+b^2}\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] (C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b - C*log(abs(tan(1/2*d*x + 1/2*c) -
1))/b - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(C*a - B*b)
/(sqrt(-a^2 + b^2)*b))/d
```

3.797 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{Bx}{a}-\frac{2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (B*x)/a - (2*(b*B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.163537, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {4072, 3919, 3831, 2659, 208}

$$\frac{Bx}{a}-\frac{2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
,x]
```

```
[Out] (B*x)/a - (2*(b*B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx &= \int \frac{B+C\sec(c+dx)}{a+b\sec(c+dx)}\,dx \\
&= \frac{Bx}{a}-\frac{(bB-aC)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a} \\
&= \frac{Bx}{a}-\frac{(bB-aC)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{ab} \\
&= \frac{Bx}{a}-\frac{(2(bB-aC))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd} \\
&= \frac{Bx}{a}-\frac{2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] time = 0.116234, size = 68, normalized size = 1.01

$$\frac{\frac{2(bB-aC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+B(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```

```
[Out] (B*(c + d*x) + (2*(b*B - a*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/Sqrt[a^2 - b^2])/(a*d)
```

Maple [A] time = 0.093, size = 113, normalized size = 1.7

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)B}{ad}-2\,\frac{Bb}{ad\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{C}{d\sqrt{(a+b)(a-b)}}\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/a/d*arctan(tan(1/2*d*x+1/2*c))*B-2/d/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b+2/d/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.535513, size = 540, normalized size = 8.06

$$\left[\frac{2\left(Ba^2-Bb^2\right)dx-(Ca-Bb)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\left(a^3-ab^2\right)d}, \frac{\left(Ba^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] [1/2*(2*(B*a^2 - B*b^2)*d*x - (C*a - B*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2)))/((a^3 - a*b^2)*d), ((B*a^2 - B*b^2)*d*x + (C*a - B*b)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B+C\sec(c+dx))\cos(c+dx)\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a + b*sec(c + d*x)
), x)
```

Giac [A] time = 1.27133, size = 136, normalized size = 2.03

$$\frac{\frac{(dx+c)B}{a}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Ca-Bb)}{\sqrt{-a^2+b^2}a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] ((d*x + c)*B/a + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arct
an(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(C
*a - B*b)/(sqrt(-a^2 + b^2)*a))/d
```

3.798 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{2b(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(bB-aC)}{a^2}+\frac{B\sin(c+dx)}{ad}$$

```
[Out] -(((b*B - a*C)*x)/a^2) + (2*b*(b*B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(a*d
)
```

Rubi [A] time = 0.225905, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4034, 12, 3783, 2659, 208}

$$\frac{2b(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(bB-aC)}{a^2}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
]),x]
```

```
[Out] -(((b*B - a*C)*x)/a^2) + (2*b*(b*B - a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(a*d
)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$= \frac{B\sin(c+dx)}{ad} - \frac{\int \frac{bB-aC}{a+b\sec(c+dx)}\,dx}{a}$$

$$= \frac{B\sin(c+dx)}{ad} - \frac{(bB-aC)\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}$$

$$= -\frac{(bB-aC)x}{a^2} + \frac{B\sin(c+dx)}{ad} + \frac{(bB-aC)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}$$

$$= -\frac{(bB-aC)x}{a^2} + \frac{B\sin(c+dx)}{ad} + \frac{(2(bB-aC))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-$$

$$= -\frac{(bB-aC)x}{a^2} + \frac{2b(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+bd}} + \frac{B\sin$$

Mathematica [A] time = 0.203042, size = 85, normalized size = 0.94

$$\frac{-\frac{2b(bB-aC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + (c+dx)(aC-bB) + aB\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x]),x]
```

```
[Out] ((-(b*B) + a*C)*(c + d*x) - (2*b*(b*B - a*C)*ArcTanh[((-a + b)*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*B*Sin[c + d*x])/(a^2*d)
```

Maple [B] time = 0.114, size = 172, normalized size = 1.9

$$2\,\frac{B\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)} - 2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,Bb}{da^2} + 2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,C}{ad} + 2\,\frac{Bb^2}{da^2\sqrt{(a+b)}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/a^2*arctan(tan(1/2*
d*x+1/2*c))*B*b+2/a/d*arctan(tan(1/2*d*x+1/2*c))*C+2/d*B/a^2/((a+b)*(a-b))^
(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2-2/d*b/a/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.555837, size = 702, normalized size = 7.8

$$\left[\frac{2\left(Ca^3 - Ba^2b - Cab^2 + Bb^3\right)dx - \left(Cab - Bb^2\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{a^2 - b^2}(b\cos(dx+c) + a)\sin(dx+c) + 2}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right.}{2\left(a^4 - a^2b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*(2*(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x - (C*a*b - B*b^2)*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a
^2*b^2)*d), ((C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x - (C*a*b - B*b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\cos^2(c + dx)\sec(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)/(a + b*sec(c + d
*x)), x)
```

Giac [A] time = 1.19928, size = 190, normalized size = 2.11

$$\frac{\frac{(Ca-Bb)(dx+c)}{a^2}+\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}-\frac{2\left(Cab-Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] ((C*a - B*b)*(d*x + c)/a^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*a) - 2*(C*a*b - B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^
2 + b^2)))/(sqrt(-a^2 + b^2)*a^2))/d
```

3.799 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=134

$$-\frac{2b^2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2B-2abC+2b^2B\right)}{2a^3}-\frac{(bB-aC)\sin(c+dx)}{a^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] ((a^2*B + 2*b^2*B - 2*a*b*C)*x)/(2*a^3) - (2*b^2*(b*B - a*C)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - ((
b*B - a*C)*Sin[c + d*x])/(a^2*d) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.478919, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4034, 4104, 3919, 3831, 2659, 208}

$$-\frac{2b^2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2B-2abC+2b^2B\right)}{2a^3}-\frac{(bB-aC)\sin(c+dx)}{a^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
]),x]
```

```
[Out] ((a^2*B + 2*b^2*B - 2*a*b*C)*x)/(2*a^3) - (2*b^2*(b*B - a*C)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - ((
b*B - a*C)*Sin[c + d*x])/(a^2*d) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
```

```
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$= \frac{B\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(2(bB-aC)-aB\sec(c+dx)-bB\sec\cdots\right.}{a+b\sec(c+dx)}}{2a}\cdots$$

$$= -\frac{(bB-aC)\sin(c+dx)}{a^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int \frac{a^2B+2b\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(a^2B+2b^2B-2abC\right)x}{2a^3} - \frac{(bB-aC)\sin(c+dx)}{a^2d} + \frac{B\cos(c+d\cdots}{2\cdots}$$

$$= \frac{\left(a^2B+2b^2B-2abC\right)x}{2a^3} - \frac{(bB-aC)\sin(c+dx)}{a^2d} + \frac{B\cos(c+d\cdots}{2\cdots}$$

$$= \frac{\left(a^2B+2b^2B-2abC\right)x}{2a^3} - \frac{(bB-aC)\sin(c+dx)}{a^2d} + \frac{B\cos(c+d\cdots}{2\cdots}$$

$$= \frac{\left(a^2B+2b^2B-2abC\right)x}{2a^3} - \frac{2b^2(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx\cdots\right.}{\sqrt{a+b}}\right.}{a^3\sqrt{a-b}\sqrt{a+b}d}\cdots$$

Mathematica [A] time = 0.32288, size = 121, normalized size = 0.9

$$\frac{2(c+dx)\left(a^2B-2abC+2b^2B\right)+\frac{8b^2(bB-aC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a^2B\sin(2(c+dx))+4a(aC-bB)\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x]),x]
```

```
[Out] (2*(a^2*B + 2*b^2*B - 2*a*b*C)*(c + d*x) + (8*b^2*(b*B - a*C)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*(-(b*B) + a*
C)*Sin[c + d*x] + a^2*B*Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] time = 0.116, size = 367, normalized size = 2.7

$$-\frac{B}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-2\,\frac{(\tan(1/2\,dx+c/2))^{3}Bb}{da^{2}\left(1+(\tan(1/2\,dx+c/2))^{2}\right)^{2}}+2\,\frac{C\,(\tan(1/2\,dx+c/2))^{3}}{ad\left(1+(\tan(1/2\,dx+c/2))^{2}\right)^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*b+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)^3*C+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c
)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*b+2/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*C+1/a/d*arctan(tan(1/2*d*x+1/2*c))*B+2
/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B*b^2-2/d/a^2*C*arctan(tan(1/2*d*x+1/2*c)
)*b-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*B+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.574448, size = 934, normalized size = 6.97

$$\left[\frac{\left(Ba^{4}-2\,Ca^{3}b+Ba^{2}b^{2}+2\,Cab^{3}-2\,Bb^{4}\right)dx-\left(Cab^{2}-Bb^{3}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+c)}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right.}{2\left(a^{5}-a^{3}b^{2}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*((B*a^4 - 2*C*a^3*b + B*a^2*b^2 + 2*C*a*b^3 - 2*B*b^4)*d*x - (C*a*b^2
- B*b^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*C*a^4 - 2*B*a^3*b - 2*
C*a^2*b^2 + 2*B*a*b^3 + (B*a^4 - B*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a
^5 - a^3*b^2)*d), 1/2*((B*a^4 - 2*C*a^3*b + B*a^2*b^2 + 2*C*a*b^3 - 2*B*b^4
)*d*x + 2*(C*a*b^2 - B*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
s(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*C*a^4 - 2*B*a^3*b - 2*C*a^
2*b^2 + 2*B*a*b^3 + (B*a^4 - B*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 -
 a^3*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22528, size = 306, normalized size = 2.28

$$\frac{\frac{(Ba^2-2Cab+2Bb^2)(dx+c)}{a^3}+\frac{4(Cab^2-Bb^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^3}-\frac{2\left(Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ca\tan\left(\frac{1}{2}\right.\right.}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*((B*a^2 - 2*C*a*b + 2*B*b^2)*(d*x + c)/a^3 + 4*(C*a*b^2 - B*b^3)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) - 2*
(B*a*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*b*tan(1/2*
d*x + 1/2*c)^3 - B*a*tan(1/2*d*x + 1/2*c) - 2*C*a*tan(1/2*d*x + 1/2*c) + 2*
B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.800 $\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{\left(2a^2B-3abC+3b^2B\right)\sin(c+dx)}{3a^3d}+\frac{2b^3(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(a^2+2b^2\right)(bB-aC)}{2a^4}-\frac{(bB-aC)\sin(c}{2}$$

```
[Out] -((a^2 + 2*b^2)*(b*B - a*C)*x)/(2*a^4) + (2*b^3*(b*B - a*C)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2
*a^2*B + 3*b^2*B - 3*a*b*C)*Sin[c + d*x])/(3*a^3*d) - ((b*B - a*C)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.704363, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4034, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(2a^2B-3abC+3b^2B\right)\sin(c+dx)}{3a^3d}+\frac{2b^3(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(a^2+2b^2\right)(bB-aC)}{2a^4}-\frac{(bB-aC)\sin(c}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
]),x]
```

```
[Out] -((a^2 + 2*b^2)*(b*B - a*C)*x)/(2*a^4) + (2*b^3*(b*B - a*C)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2
*a^2*B + 3*b^2*B - 3*a*b*C)*Sin[c + d*x])/(3*a^3*d) - ((b*B - a*C)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
```

```
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx=\int \frac{\cos^3(c+dx)(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$=\frac{B\cos^2(c+dx)\sin(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(3(bB-aC)-2aB\sec(c+dx)-2b\right.}{a+b\sec(c+dx)}}{3a}$$

$$=-\frac{(bB-aC)\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{B\cos^2(c+dx)\sin(c+d}{3ad}$$

$$=\frac{\left(2a^2B+3b^2B-3abC\right)\sin(c+dx)}{3a^3d}-\frac{(bB-aC)\cos(c+dx)\sin}{2a^2d}$$

$$=-\frac{\left(a^2+2b^2\right)(bB-aC)x}{2a^4}+\frac{\left(2a^2B+3b^2B-3abC\right)\sin(c+dx)}{3a^3d}-$$

$$=-\frac{\left(a^2+2b^2\right)(bB-aC)x}{2a^4}+\frac{\left(2a^2B+3b^2B-3abC\right)\sin(c+dx)}{3a^3d}-$$

$$=-\frac{\left(a^2+2b^2\right)(bB-aC)x}{2a^4}+\frac{\left(2a^2B+3b^2B-3abC\right)\sin(c+dx)}{3a^3d}-$$

$$=-\frac{\left(a^2+2b^2\right)(bB-aC)x}{2a^4}+\frac{2b^3(bB-aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{a^4\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.490599, size = 152, normalized size = 0.85

$$\frac{6\left(a^2+2b^2\right)(c+dx)(aC-bB)+3a\left(3a^2B-4abC+4b^2B\right)\sin(c+dx)-\frac{24b^3(bB-aC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+3a^2(aC-bB)}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x]),x]
```

```
[Out] (6*(a^2 + 2*b^2)*(-(b*B) + a*C)*(c + d*x) - (24*b^3*(b*B - a*C)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2*B +
 4*b^2*B - 4*a*b*C)*Sin[c + d*x] + 3*a^2*(-(b*B) + a*C)*Sin[2*(c + d*x)] +
a^3*B*Sin[3*(c + d*x)])/(12*a^4*d)
```

Maple [B] time = 0.12, size = 641, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B+1/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^5*B*b^2-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)^5*C-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*b*C+4/3/d/
a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B+4/d/a^3/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B*b^2-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^3*b*C+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)
*B+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*B*b^2-2/d/a^2/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*b*C-1/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)*B*b+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)*C-1/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B*b-2/d/a^4*arctan(tan(1/2*d*x+1
/2*c))*B*b^3+1/a/d*arctan(tan(1/2*d*x+1/2*c))*C+2/d/a^3*arctan(tan(1/2*d*x+
1/2*c))*C*b^2+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*B-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.636973, size = 1177, normalized size = 6.61

$$\left[\frac{3\left(Ca^5 - Ba^4b + Ca^3b^2 - Ba^2b^3 - 2\,Cab^4 + 2\,Bb^5\right)dx - 3\left(Cab^3 - Bb^4\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{\cdots}}{a^2\cos(dx+c)^2 + 2\,\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/6*(3*(C*a^5 - B*a^4*b + C*a^3*b^2 - B*a^2*b^3 - 2*C*a*b^4 + 2*B*b^5)*d*x
 - 3*(C*a*b^3 - B*b^4)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (4*B*a^5 -
6*C*a^4*b + 2*B*a^3*b^2 + 6*C*a^2*b^3 - 6*B*a*b^4 + 2*(B*a^5 - B*a^3*b^2)*c
os(d*x + c)^2 + 3*(C*a^5 - B*a^4*b - C*a^3*b^2 + B*a^2*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^6 - a^4*b^2)*d), 1/6*(3*(C*a^5 - B*a^4*b + C*a^3*b^2 - B*a
^2*b^3 - 2*C*a*b^4 + 2*B*b^5)*d*x - 6*(C*a*b^3 - B*b^4)*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (
4*B*a^5 - 6*C*a^4*b + 2*B*a^3*b^2 + 6*C*a^2*b^3 - 6*B*a*b^4 + 2*(B*a^5 - B*
a^3*b^2)*cos(d*x + c)^2 + 3*(C*a^5 - B*a^4*b - C*a^3*b^2 + B*a^2*b^3)*cos(d
*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22358, size = 486, normalized size = 2.73

$$\frac{\frac{3\left(Ca^3 - Ba^2b + 2\,Cab^2 - 2\,Bb^3\right)(dx+c)}{a^4} - \frac{12\left(Cab^3 - Bb^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2}\,a^4} + \frac{2\left(6\,Ba^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 \cdots\right)}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(C*a^3 - B*a^2*b + 2*C*a*b^2 - 2*B*b^3)*(d*x + c)/a^4 - 12*(C*a*b^3
- B*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 +
 b^2)*a^4) + 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^2*tan(1/2*d*x + 1/2*
c)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*
B*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*b*ta
```

```
n(1/2*d*x + 1/2*c)^3 + 12*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*
x + 1/2*c) + 3*C*a^2*tan(1/2*d*x + 1/2*c) - 3*B*a*b*tan(1/2*d*x + 1/2*c) -
6*C*a*b*tan(1/2*d*x + 1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^3*a^3))/d
```

3.801 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=272

$$\frac{\left(2a^2bB-3a^3C+2ab^2C-b^3B\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-6a^2C+4abB-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(2a^2bB-3a^3C+4a\right.}{b^4d}$$

```
[Out] -((4*a*b*B - 6*a^2*C - b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^2*(2*
a^2*b*B - 3*b^3*B - 3*a^3*C + 4*a*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*b*B - b^3*
B - 3*a^3*C + 2*a*b^2*C)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*b*B - 3*
a^2*C + b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(b*B -
 a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.884221, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {4072, 4029, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(2a^2bB-3a^3C+2ab^2C-b^3B\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-6a^2C+4abB-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(2a^2bB-3a^3C+4a\right.}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^2,x]
```


```
[Out] -((4*a*b*B - 6*a^2*C - b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^2*(2*
a^2*b*B - 3*b^3*B - 3*a^3*C + 4*a*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*b*B - b^3*
B - 3*a^3*C + 2*a*b^2*C)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*b*B - 3*
a^2*C + b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(b*B -
 a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \int \frac{\sec^2(c+dx)\left(2a(bB-aC)-b(bB\right.}{}$$

$$= -\frac{\left(2abB-3a^2C+b^2C\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} + \frac{a(bB-aC)\,\mathrm{s}}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2bB-b^3B-3a^3C+2ab^2C\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2abB-3a^2C+\right.}{2}$$

$$= \frac{\left(2a^2bB-b^3B-3a^3C+2ab^2C\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2abB-3a^2C+\right.}{2}$$

$$= -\frac{\left(4abB-6a^2C-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(2a^2bB-b^3B-\right.}{b}$$

$$= -\frac{\left(4abB-6a^2C-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(2a^2bB-b^3B-\right.}{b}$$

$$= -\frac{\left(4abB-6a^2C-b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^2\left(2a^2bB-3b\right.}{}$$

Mathematica [A] time = 6.27324, size = 438, normalized size = 1.61

$$\frac{a^4C\sin(c+dx)-a^3bB\sin(c+dx)}{b^3d(b-a)(a+b)(a\cos(c+dx)+b)} - \frac{2a^2\left(-2a^2bB+3a^3C-4ab^2C+3b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}\left(b^2-a^2\right)} + \frac{\left(-6a^2C+4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^2,x]
```

```
[Out] (-2*a^2*(-2*a^2*b*B + 3*b^3*B + 3*a^3*C - 4*a*b^2*C)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)*d) + ((4*
a*b*B - 6*a^2*C - b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*b^4*d
) + ((-4*a*b*B + 6*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/(2*b^4*d) + C/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - C/(4*b^2
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (b*B*Sin[(c + d*x)/2] - 2*a*C
*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (b*B*Sin
[(c + d*x)/2] - 2*a*C*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (-(a^3*b*B*Sin[c + d*x]) + a^4*C*Sin[c + d*x])/(b^3*(-a + b)*(
a + b)*d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.11, size = 698, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)*B+2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*a^2/b
/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*B-6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+8/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/2/d*C/b^2
/(tan(1/2*d*x+1/2*c)+1)^2-1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*B+2/d/b^3/(tan(1/2
*d*x+1/2*c)+1)*a*C+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*C-2/d/b^3*ln(tan(1/2*d*
x+1/2*c)+1)*B*a+3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a^2*C+1/2/d/b^2*ln(tan(1/2
*d*x+1/2*c)+1)*C+1/2/d*C/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^2/(tan(1/2*d*x+
1/2*c)-1)*B+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a*C+1/2/d/b^2/(tan(1/2*d*x+1/2*c
)-1)*C+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B*a-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1
)*a^2*C-1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 50.2189, size = 2969, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/4*(2*((3*C*a^6 - 2*B*a^5*b - 4*C*a^4*b^2 + 3*B*a^3*b^3)*cos(d*x + c)^3 +
 (3*C*a^5*b - 2*B*a^4*b^2 - 4*C*a^3*b^3 + 3*B*a^2*b^4)*cos(d*x + c)^2)*sqrt
(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
 c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*C*a^7 - 4*B*a^6*b - 11*C*a^5*b^2 +
 8*B*a^4*b^3 + 4*C*a^3*b^4 - 4*B*a^2*b^5 + C*a*b^6)*cos(d*x + c)^3 + (6*C*a
^6*b - 4*B*a^5*b^2 - 11*C*a^4*b^3 + 8*B*a^3*b^4 + 4*C*a^2*b^5 - 4*B*a*b^6 +
 C*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*C*a^7 - 4*B*a^6*b - 11*
C*a^5*b^2 + 8*B*a^4*b^3 + 4*C*a^3*b^4 - 4*B*a^2*b^5 + C*a*b^6)*cos(d*x + c)
^3 + (6*C*a^6*b - 4*B*a^5*b^2 - 11*C*a^4*b^3 + 8*B*a^3*b^4 + 4*C*a^2*b^5 -
4*B*a*b^6 + C*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(C*a^4*b^3 -
2*C*a^2*b^5 + C*b^7 - 2*(3*C*a^6*b - 2*B*a^5*b^2 - 5*C*a^4*b^3 + 3*B*a^3*b^
4 + 2*C*a^2*b^5 - B*a*b^6)*cos(d*x + c)^2 - (3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*
C*a^3*b^4 + 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*cos(d*x + c))*sin(d*x + c))/
```

```
((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^
9)*d*cos(d*x + c)^2), -1/4*(4*((3*C*a^6 - 2*B*a^5*b - 4*C*a^4*b^2 + 3*B*a^3
*b^3)*cos(d*x + c)^3 + (3*C*a^5*b - 2*B*a^4*b^2 - 4*C*a^3*b^3 + 3*B*a^2*b^4
)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
 + a)/((a^2 - b^2)*sin(d*x + c))) - ((6*C*a^7 - 4*B*a^6*b - 11*C*a^5*b^2 +
8*B*a^4*b^3 + 4*C*a^3*b^4 - 4*B*a^2*b^5 + C*a*b^6)*cos(d*x + c)^3 + (6*C*a^
6*b - 4*B*a^5*b^2 - 11*C*a^4*b^3 + 8*B*a^3*b^4 + 4*C*a^2*b^5 - 4*B*a*b^6 +
C*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*C*a^7 - 4*B*a^6*b - 11*C
*a^5*b^2 + 8*B*a^4*b^3 + 4*C*a^3*b^4 - 4*B*a^2*b^5 + C*a*b^6)*cos(d*x + c)^
3 + (6*C*a^6*b - 4*B*a^5*b^2 - 11*C*a^4*b^3 + 8*B*a^3*b^4 + 4*C*a^2*b^5 - 4
*B*a*b^6 + C*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(C*a^4*b^3 - 2
*C*a^2*b^5 + C*b^7 - 2*(3*C*a^6*b - 2*B*a^5*b^2 - 5*C*a^4*b^3 + 3*B*a^3*b^4
 + 2*C*a^2*b^5 - B*a*b^6)*cos(d*x + c)^2 - (3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*C
*a^3*b^4 + 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*cos(d*x + c))*sin(d*x + c))/(
(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9
)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,
x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.25696, size = 518, normalized size = 1.9

$$-\frac{\frac{4\left(3Ca^5-2Ba^4b-4Ca^3b^2+3Ba^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{4\left(Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ba^3b\tan\left(\frac{1}{2}dx\right.\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -1/2*(4*(3*C*a^5 - 2*B*a^4*b - 4*C*a^3*b^2 + 3*B*a^2*b^3)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - 4
*(C*a^4*tan(1/2*d*x + 1/2*c) - B*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^
5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (6*C*a^
2 - 4*B*a*b + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (6*C*a^2 - 4*
B*a*b + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(4*C*a*tan(1/2*d*
x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 -
4*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

3.802 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$-\frac{2a\left(a^2bB-2a^3C+3ab^2C-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(bB-aC)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(bB-2aC)\tanh^{-1}(s}{b^3d}$$

```
[Out] ((b*B - 2*a*C)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(a^2*b*B - 2*b^3*B - 2
*a^3*C + 3*a*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (C*Tan[c + d*x])/(b^2*d) - (a^2*(b*B -
a*C)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.622821, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 4028, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{2a\left(a^2bB-2a^3C+3ab^2C-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(bB-aC)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(bB-2aC)\tanh^{-1}(s}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^2,x]
```


```
[Out] ((b*B - 2*a*C)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(a^2*b*B - 2*b^3*B - 2
*a^3*C + 3*a*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (C*Tan[c + d*x])/(b^2*d) - (a^2*(b*B -
a*C)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
```

```
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2(bB-aC)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab(bB-aC)-\left(a^2-b^2\right)(bB\right.}{a+b\,\mathrm{s}}}{b^2\,(}$$

$$= \frac{C\tan(c+dx)}{b^2d} - \frac{a^2(bB-aC)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \sec(c+dx)\left(-ab^2\right.}{}$$

$$= \frac{C\tan(c+dx)}{b^2d} - \frac{a^2(bB-aC)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(bB-2aC)\int}{b}$$

$$= \frac{(bB-2aC)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{C\tan(c+dx)}{b^2d} - \frac{a^2(bB-aC}{b^2\left(a^2-b^2\right)d(}$$

$$= \frac{(bB-2aC)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{C\tan(c+dx)}{b^2d} - \frac{a^2(bB-aC}{b^2\left(a^2-b^2\right)d(}$$

$$= \frac{(bB-2aC)\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a\left(a^2bB-2b^3B-2a^3C+3ab^2\right.}{(a-b)^{3/2}b^3(}$$

Mathematica [A] time = 2.05006, size = 240, normalized size = 1.46

$$-\frac{2a\left(-a^2bB+2a^3C-3ab^2C+2b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a^2b(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} + 2aC\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^2,x]
```

```
[Out] ((-2*a*(-(a^2*b*B) + 2*b^3*B + 2*a^3*C - 3*a*b^2*C)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*B*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 2*a*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*B
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*a*C*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (a^2*b*(-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(b +
 a*Cos[c + d*x])) + b*C*Tan[c + d*x])/(b^3*d)
```

Maple [B] time = 0.095, size = 510, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)*B-2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2
```

```
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4/d*a/(a+b)/
(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*B+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*C-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/d/b^2/(tan(1/2*d*x
+1/2*c)+1)*C+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*B-2/d/b^3*ln(tan(1/2*d*x+1/2*
c)+1)*a*C-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*C-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)
*B+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 31.158, size = 2433, normalized size = 14.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*(((2*C*a^5 - B*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^3)*cos(d*x + c)^2 + (2*
C*a^4*b - B*a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) - ((2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3
+ 2*C*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + (2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*
b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ ((2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 + 2*C*a^2*b^4 - B*a*b^5)*
cos(d*x + c)^2 + (2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a
*b^5 - B*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(C*a^4*b^2 - 2*C*a^2
*b^4 + C*b^6 + (2*C*a^5*b - B*a^4*b^2 - 3*C*a^3*b^3 + B*a^2*b^4 + C*a*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)), 1/2*(2*((2*C*a^5 - B*a^4*b
- 3*C*a^3*b^2 + 2*B*a^2*b^3)*cos(d*x + c)^2 + (2*C*a^4*b - B*a^3*b^2 - 3*C*
a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((2*C*a^6 - B*a^5*b -
4*C*a^4*b^2 + 2*B*a^3*b^3 + 2*C*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + (2*C*a^
5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*cos(d*x +
c))*log(sin(d*x + c) + 1) + ((2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3
 + 2*C*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + (2*C*a^5*b - B*a^4*b^2 - 4*C*a^3
*b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1
) + 2*(C*a^4*b^2 - 2*C*a^2*b^4 + C*b^6 + (2*C*a^5*b - B*a^4*b^2 - 3*C*a^3*b
^3 + B*a^2*b^4 + C*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5
 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^3(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,
x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.23889, size = 545, normalized size = 3.32

$$\frac{\frac{2\left(2Ca^4-Ba^3b-3Ca^2b^2+2Bab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{-a^2+b^2}}-\frac{2\left(2Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-C}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] (2*(2*C*a^4 - B*a^3*b - 3*C*a^2*b^2 + 2*B*a*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - 2*(2*C*a^3
*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*tan(1/2*
d*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + C*b^3*tan(1/2*d*x + 1/2*c
)^3 - 2*C*a^3*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c) - C*a^2*b
*tan(1/2*d*x + 1/2*c) + C*a*b^2*tan(1/2*d*x + 1/2*c) + C*b^3*tan(1/2*d*x +
1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) - (2*C*a - B*b)*log(abs(tan(1/2*d
*x + 1/2*c) + 1))/b^3 + (2*C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
3)/d
```

3.803 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$-\frac{2\left(a^3C-2ab^2C+b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(bB-aC)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{C\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*(b^3*B + a^3*C - 2*a*b^2*C)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) + (a*(b*B - a*C)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.319354, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {4072, 4009, 3998, 3770, 3831, 2659, 208}

$$-\frac{2\left(a^3C-2ab^2C+b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(bB-aC)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{C\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^2,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*(b^3*B + a^3*C - 2*a*b^2*C)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) + (a*(b*B - a*C)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{a(bB-aC)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(-b(bB-aC)+\left(a^2-b^2\right)C\sec(c+\right.}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= \frac{a(bB-aC)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{C\int\sec(c+dx)\,dx}{b^2} - \frac{\left(b^3B+a^3C\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a(bB-aC)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b^3B+\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a(bB-aC)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(b^3B\right.\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2\left(b^3B+a^3C-2ab^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+}}\right.}{(a-b)^{3/2}b^2(a+b)^{3/2}d}$$

Mathematica [A] time = 0.661351, size = 191, normalized size = 1.46

$$\frac{\cos(c+dx)(B+C\sec(c+dx))\left(\frac{2\left(a^3C-2ab^2C+b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab(aC-bB)\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)} - C\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \mathrm{s}\right.\right.}{b^2d(B\cos(c+dx)+C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(B + C*Sec[c + d*x])*((2*(b^3*B + a^3*C - 2*a*b^2*C)*ArcTanh[
((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - C*Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] + C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + (a*b*(-(b*B) + a*C)*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*
x]))))/(b^2*d*(C + B*Cos[c + d*x]))
```

Maple [B] time = 0.087, size = 350, normalized size = 2.7

$$-2\,\frac{a\tan(1/2\,dx+c/2)\,B}{d\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}+2\,\frac{a^2\tan(1/2\,dx+c/2}{db\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)*B+2/d/b*a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*a^3/b^2/(a+b)/(a-b)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
C+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*C+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/b^2*ln(tan(1/2*
d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.64177, size = 1551, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] [1/2*((C*a^3*b - 2*C*a*b^3 + B*b^4 + (C*a^4 - 2*C*a^2*b^2 + B*a*b^3)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (C*a^4*b - 2*C*a^2*b^3 + C
*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1)
- (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(C*a^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4
)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 -
```

```
2*a^2*b^5 + b^7)*d), -1/2*(2*(C*a^3*b - 2*C*a*b^3 + B*b^4 + (C*a^4 - 2*C*a^
2*b^2 + B*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (C*a^4*b - 2*C*a^2*b^3 + C
*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) + 2*(C*a^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4
)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 -
2*a^2*b^5 + b^7)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.23299, size = 309, normalized size = 2.36

$$\frac{\frac{2\left(Ca^3-2Cab^2+Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{-a^2+b^2}}+\frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+\frac{}{(a^2b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] (2*(C*a^3 - 2*C*a*b^2 + B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + C*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/b^2 - C*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*(C*a^2*tan(1/2*d*
x + 1/2*c) - B*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/
2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

3.804 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aB-bC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bB-aC)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

```
[Out] (2*(a*B - b*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*d) - ((b*B - a*C)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b
*Sec[c + d*x]))
```

Rubi [A] time = 0.125723, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4060, 12, 3831, 2659, 208}

$$\frac{2(aB-bC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bB-aC)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a*B - b*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*d) - ((b*B - a*C)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b
*Sec[c + d*x]))
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{\int \frac{a(aB-bC)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{(aB-bC)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{(aB-bC)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{(2(aB-bC))\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\right.\right.}{b\left(a^2-b^2\right)d}$$

$$= \frac{2(aB-bC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.338846, size = 97, normalized size = 0.97

$$\frac{\frac{(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}-\frac{2(aB-bC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*(a*B - b*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c
 + d*x])))/d
```

Maple [A] time = 0.085, size = 132, normalized size = 1.3

$$\frac{1}{d}\left(2\frac{(Bb-aC)\tan(1/2\,dx+c/2)}{\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)}+2\frac{Ba-Cb}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\operatorname{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(2*(B*b-C*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)+2*(B*a-C*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh(
```

```
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.542636, size = 861, normalized size = 8.61

$$\left[\frac{\left(Bab - Cb^2 + \left(Ba^2 - Cab\right)\cos\left(dx + c\right)\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{a^2-b^2}\left(b\cos(dx+c) + a\right)\sin(dx+c) + 2\,a^2 - b}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right.}{2\left(\left(a^5 - 2\,a^3b^2 + ab^4\right)d\cos\left(dx + c\right) + \left(a^4b - 2\,a^2b^3 + b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((B*a*b - C*b^2 + (B*a^2 - C*a*b)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2
*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + b^2)) + 2*(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*sin(d*x + c))/((a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d), ((B*a
*b - C*b^2 + (B*a^2 - C*a*b)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^3 - B*a^2
*b - C*a*b^2 + B*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c) + (a^4*b - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.17938, size = 235, normalized size = 2.35

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Ba-Cb)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}} - \frac{Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(B*a - C*b)/((a^2
- b^2)*sqrt(-a^2 + b^2)) - (C*a*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 -
 b^2)))/d
```

3.805 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{2\left(2a^2bB+a^3(-C)-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(bB-aC)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Bx}{a^2}$$

```
[Out] (B*x)/a^2 - (2*(2*a^2*b*B - b^3*B - a^3*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(b*B - a*C)*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.273872, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4072, 3923, 3919, 3831, 2659, 208}

$$-\frac{2\left(2a^2bB+a^3(-C)-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(bB-aC)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Bx}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^2,x]
```

```
[Out] (B*x)/a^2 - (2*(2*a^2*b*B - b^3*B - a^3*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(b*B - a*C)*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{B+C\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\left(a^2-b^2\right)B+a(bB-aC)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{Bx}{a^2} + \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2bB-b^3B-a^3C\right)\int \frac{\text{se}}{a+b}}{a^2\left(a^2-b^2\right)}$$

$$= \frac{Bx}{a^2} + \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2bB-b^3B-a^3C\right)\int \frac{}{1+\frac{a}{}}}{a^2b\left(a^2-b^2\right)}$$

$$= \frac{Bx}{a^2} + \frac{b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(2a^2bB-b^3B-a^3C\right)\right)\text{Su}}{}$$

$$= \frac{Bx}{a^2} - \frac{2\left(2a^2bB-b^3B-a^3C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b(bB}{a\left(a^2-b^2\right.}$$

Mathematica [A] time = 0.550004, size = 119, normalized size = 0.96

$$\frac{-\frac{2\left(-2a^2bB+a^3C+b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab(bB-aC)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} + B(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```

```
[Out] (B*(c + d*x) - (2*(-2*a^2*b*B + b^3*B + a^3*C)*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (a*b*(b*B - a*C)*Sin[c + d*x]
)/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/(a^2*d)
```

Maple [B] time = 0.111, size = 328, normalized size = 2.7

$$2\,\frac{B\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^{2}}-2\,\frac{b^{2}\tan\left(1/2\,dx+c/2\right)B}{ad\left(a^{2}-b^{2}\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)}+2\,\frac{}{d\left(a^{2}-b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*B/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+2/d*b/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-4/d*b/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*B+2/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b^3+2/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.601166, size = 1226, normalized size = 9.89

$$\left[\frac{2\left(Ba^{5}-2\,Ba^{3}b^{2}+Bab^{4}\right)dx\cos\left(dx+c\right)+2\left(Ba^{4}b-2\,Ba^{2}b^{3}+Bb^{5}\right)dx-\left(Ca^{3}b-2\,Ba^{2}b^{2}+Bb^{4}+\left(Ca^{4}-2\,Ba^{3}b+B\right.\right.}{2\left(\left(a^{7}-2\,a^{5}b^{2}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] [1/2*(2*(B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*d*x*cos(d*x + c) + 2*(B*a^4*b - 2*B
*a^2*b^3 + B*b^5)*d*x - (C*a^3*b - 2*B*a^2*b^2 + B*b^4 + (C*a^4 - 2*B*a^3*b
 + B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
 + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(C*a^4
*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((B*a^5 - 2*B*a^3*
b^2 + B*a*b^4)*d*x*cos(d*x + c) + (B*a^4*b - 2*B*a^2*b^3 + B*b^5)*d*x + (C*
a^3*b - 2*B*a^2*b^2 + B*b^4 + (C*a^4 - 2*B*a^3*b + B*a*b^3)*cos(d*x + c))*s
qrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*
sin(d*x + c))) - (C*a^4*b - B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*sin(d*x + c))/
((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)
```

```
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\cos(c + dx)\sec(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a + b*sec(c + d*x)
)**2, x)
```

Giac [A] time = 1.21217, size = 271, normalized size = 2.19

$$\frac{\frac{2\left(Ca^3-2\,Ba^2b+Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{-a^2+b^2}}+\frac{(dx+c)B}{a^2}+\frac{2\left(Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] (2*(C*a^3 - 2*B*a^2*b + B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) + (d*x + c)*B/a^2 + 2*(C*a*b*t
an(1/2*d*x + 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

3.806 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=180

$$\frac{\left(a^2B+abC-2b^2B\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2b\left(3a^2bB-2a^3C+ab^2C-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(bB-aC)\sin}{ad\left(a^2-b^2\right)(a+b}$$

```
[Out] -(((2*b*B - a*C)*x)/a^3) + (2*b*(3*a^2*b*B - 2*b^3*B - 2*a^3*C + a*b^2*C)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a +
 b)^(3/2)*d) + ((a^2*B - 2*b^2*B + a*b*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.632915, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4030, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(a^2B+abC-2b^2B\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2b\left(3a^2bB-2a^3C+ab^2C-2b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(bB-aC)\sin}{ad\left(a^2-b^2\right)(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^2,x]
```


```
[Out] -(((2*b*B - a*C)*x)/a^3) + (2*b*(3*a^2*b*B - 2*b^3*B - 2*a^3*C + a*b^2*C)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a +
 b)^(3/2)*d) + ((a^2*B - 2*b^2*B + a*b*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-a^2B+2b^2B-abC+a(bB-aC)\right.}{a+b\sec(c}}{a\left(a^2-\right.}$$

$$= \frac{\left(a^2B-2b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(bB-aC)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

$$= -\frac{(2bB-aC)x}{a^3} + \frac{\left(a^2B-2b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(bB-aC}{a\left(a^2-b^2\right)d(a}$$

$$= -\frac{(2bB-aC)x}{a^3} + \frac{\left(a^2B-2b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(bB-aC}{a\left(a^2-b^2\right)d(a}$$

$$= -\frac{(2bB-aC)x}{a^3} + \frac{\left(a^2B-2b^2B+abC\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(bB-aC}{a\left(a^2-b^2\right)d(a}$$

$$= -\frac{(2bB-aC)x}{a^3} + \frac{2b\left(3a^2bB-2b^3B-2a^3C+ab^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{t}}{}\right.}{a^3(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 0.778907, size = 147, normalized size = 0.82

$$\frac{\frac{2b\left(-3a^2bB+2a^3C-ab^2C+2b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{ab^2(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}+(c+dx)(aC-2bB)+aB\sin(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^2,x]
```

```
[Out] ((-2*b*B + a*C)*(c + d*x) + (2*b*(-3*a^2*b*B + 2*b^3*B + 2*a^3*C - a*b^2*C)
*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) +
a*B*Sin[c + d*x] + (a*b^2*(-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(b
+ a*Cos[c + d*x]))))/(a^3*d)
```

Maple [B] time = 0.121, size = 453, normalized size = 2.5

$$2\frac{B\tan(1/2\,dx+c/2)}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\frac{B\arctan(\tan(1/2\,dx+c/2))\,b}{da^3}+2\frac{C\arctan(\tan(1/2\,dx+c/2))}{da^2}+2\frac{}{da^2\left(a^2-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^3*B*arctan(tan(
1/2*d*x+1/2*c))*b+2/d/a^2*C*arctan(tan(1/2*d*x+1/2*c))+2/d/a^2*b^3/(a^2-b^2
)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-
2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)*C+6/d/a*b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d/a^3*b^4/(a+b)/(a-b)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d*b/(a+
b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*C+2/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.680988, size = 1715, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*(2*(C*a^6 - 2*B*a^5*b - 2*C*a^4*b^2 + 4*B*a^3*b^3 + C*a^2*b^4 - 2*B*a*
b^5)*d*x*cos(d*x + c) + 2*(C*a^5*b - 2*B*a^4*b^2 - 2*C*a^3*b^3 + 4*B*a^2*b^
4 + C*a*b^5 - 2*B*b^6)*d*x + (2*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4 + 2*B*b^5
 + (2*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^5*b + C*a^4*b^2 - 3*B*a^3*b^3 - C*a
^2*b^4 + 2*B*a*b^5 + (B*a^6 - 2*B*a^4*b^2 + B*a^2*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 +
a^3*b^5)*d), ((C*a^6 - 2*B*a^5*b - 2*C*a^4*b^2 + 4*B*a^3*b^3 + C*a^2*b^4 -
2*B*a*b^5)*d*x*cos(d*x + c) + (C*a^5*b - 2*B*a^4*b^2 - 2*C*a^3*b^3 + 4*B*a^
2*b^4 + C*a*b^5 - 2*B*b^6)*d*x - (2*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4 + 2*B
*b^5 + (2*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + 2*B*a*b^4)*cos(d*x + c))*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (B*a^5*b + C*a^4*b^2 - 3*B*a^3*b^3 - C*a^2*b^4 + 2*B*a*b^5 +
(B*a^6 - 2*B*a^4*b^2 + B*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6
*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22539, size = 505, normalized size = 2.81

$$-\frac{\frac{2\left(2\,Ca^3b-3\,Ba^2b^2-Cab^3+2\,Bb^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{-a^2+b^2}}-\frac{2\left(Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-B\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] -(2*(2*C*a^3*b - 3*B*a^2*b^2 - C*a*b^3 + 2*B*b^4)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/((a^5 - a^3*b^2)*sqrt(-a^2 + b^2)) - 2*(B*a^3*
tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d
*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^3*tan(1/2*d*x + 1/2*
c)^3 - B*a^3*tan(1/2*d*x + 1/2*c) - B*a^2*b*tan(1/2*d*x + 1/2*c) + B*a*b^2*
tan(1/2*d*x + 1/2*c) - C*a*b^2*tan(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x +
```

```
1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/
2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) - (C*a - 2*B*b)*(d*x + c)/a^3)/d
```

3.807 $\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(2a^2bB+a^3(-C)+2ab^2C-3b^3B\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2B+2abC-3b^2B\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b^2\left(4a^2bB-3a^3C+\right.}{a}$$

```
[Out] ((a^2*B + 6*b^2*B - 4*a*b*C)*x)/(2*a^4) - (2*b^2*(4*a^2*b*B - 3*b^3*B - 3*a
^3*C + 2*a*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*a^2*b*B - 3*b^3*B - a^3*C + 2*a*b^2*C
)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*B - 3*b^2*B + 2*a*b*C)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(b*B - a*C)*Cos[c + d*x]*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.930986, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 4030, 4104, 3919, 3831, 2659, 208}

$$-\frac{\left(2a^2bB+a^3(-C)+2ab^2C-3b^3B\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2B+2abC-3b^2B\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b^2\left(4a^2bB-3a^3C+\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^2,x]
```


```
[Out] ((a^2*B + 6*b^2*B - 4*a*b*C)*x)/(2*a^4) - (2*b^2*(4*a^2*b*B - 3*b^3*B - 3*a
^3*C + 2*a*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*a^2*b*B - 3*b^3*B - a^3*C + 2*a*b^2*C
)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*B - 3*b^2*B + 2*a*b*C)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(b*B - a*C)*Cos[c + d*x]*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\cos^2(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$$

$$=\frac{b(bB-aC)\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\int \frac{\cos^2(c+dx)\left(-a^2B+3b^2B-2abC+a(\ldots\right.}{a+\ldots}}{a\ldots}$$

$$=\frac{\left(a^2B-3b^2B+2abC\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d}+\frac{b(bB-aC)\cos(c\ldots}{a\left(a^2-b^2\right)d(a+\ldots}$$

$$=-\frac{\left(2a^2bB-3b^3B-a^3C+2ab^2C\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d}+\frac{\left(a^2B-3b^2B+2ab\ldots\right.}{2a^2\ldots}$$

$$=\frac{\left(a^2B+6b^2B-4abC\right)x}{2a^4}-\frac{\left(2a^2bB-3b^3B-a^3C+2ab^2C\right)\sin(c+\ldots}{a^3\left(a^2-b^2\right)d}$$

$$=\frac{\left(a^2B+6b^2B-4abC\right)x}{2a^4}-\frac{\left(2a^2bB-3b^3B-a^3C+2ab^2C\right)\sin(c+\ldots}{a^3\left(a^2-b^2\right)d}$$

$$=\frac{\left(a^2B+6b^2B-4abC\right)x}{2a^4}-\frac{\left(2a^2bB-3b^3B-a^3C+2ab^2C\right)\sin(c+\ldots}{a^3\left(a^2-b^2\right)d}$$

$$=\frac{\left(a^2B+6b^2B-4abC\right)x}{2a^4}-\frac{2b^2\left(4a^2bB-3b^3B-3a^3C+2ab^2C\right)\tan\ldots}{a^4(a-b)^{3/2}(a+b)^{3/\ldots}}$$

Mathematica [A] time = 1.01909, size = 184, normalized size = 0.7

$$\frac{2(c+dx)\left(a^2B-4abC+6b^2B\right)-\frac{8b^2\left(-4a^2bB+3a^3C-2ab^2C+3b^3B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+a^2B\sin(2(c+dx))-\frac{4ab^3(aC-bB)\sin(c+\ldots}{(a-b)(a+b)(a\cos(c+d\ldots}}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^2,x]
```

```
[Out] (2*(a^2*B + 6*b^2*B - 4*a*b*C)*(c + d*x) - (8*b^2*(-4*a^2*b*B + 3*b^3*B + 3
*a^3*C - 2*a*b^2*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(3/2) + 4*a*(-2*b*B + a*C)*Sin[c + d*x] - (4*a*b^3*(-(b*B) + a*C
)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*B*Sin[2*(c + d
*x)])/(4*a^4*d)
```

Maple [B] time = 0.121, size = 651, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-4/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*b+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)^3*C+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)*B-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*b+2/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*C+1/d*B/a^2*arctan(tan(1/2*d*x
+1/2*c))+6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B*b^2-4/d/a^3*C*arctan(tan(1/2*
d*x+1/2*c))*b-2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-8/d/a^2/(a+b)/(a-b)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*
b^3+6/d*b^5/a^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*B+6/d*b^2/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-4/d*b^4/a^3/(a+b)/(a-b)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*
C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.755832, size = 2136, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*((B*a^7 - 4*C*a^6*b + 4*B*a^5*b^2 + 8*C*a^4*b^3 - 11*B*a^3*b^4 - 4*C*a
^2*b^5 + 6*B*a*b^6)*d*x*cos(d*x + c) + (B*a^6*b - 4*C*a^5*b^2 + 4*B*a^4*b^3
 + 8*C*a^3*b^4 - 11*B*a^2*b^5 - 4*C*a*b^6 + 6*B*b^7)*d*x + (3*C*a^3*b^3 - 4
*B*a^2*b^4 - 2*C*a*b^5 + 3*B*b^6 + (3*C*a^4*b^2 - 4*B*a^3*b^3 - 2*C*a^2*b^4
 + 3*B*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*C*a
^6*b - 4*B*a^5*b^2 - 6*C*a^4*b^3 + 10*B*a^3*b^4 + 4*C*a^2*b^5 - 6*B*a*b^6 +
 (B*a^7 - 2*B*a^5*b^2 + B*a^3*b^4)*cos(d*x + c)^2 + (2*C*a^7 - 3*B*a^6*b -
4*C*a^5*b^2 + 6*B*a^4*b^3 + 2*C*a^3*b^4 - 3*B*a^2*b^5)*cos(d*x + c))*sin(d*
x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 +
a^4*b^5)*d), 1/2*((B*a^7 - 4*C*a^6*b + 4*B*a^5*b^2 + 8*C*a^4*b^3 - 11*B*a^3
*b^4 - 4*C*a^2*b^5 + 6*B*a*b^6)*d*x*cos(d*x + c) + (B*a^6*b - 4*C*a^5*b^2 +
 4*B*a^4*b^3 + 8*C*a^3*b^4 - 11*B*a^2*b^5 - 4*C*a*b^6 + 6*B*b^7)*d*x + 2*(3
*C*a^3*b^3 - 4*B*a^2*b^4 - 2*C*a*b^5 + 3*B*b^6 + (3*C*a^4*b^2 - 4*B*a^3*b^3
 - 2*C*a^2*b^4 + 3*B*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*C*a^6*b - 4*
B*a^5*b^2 - 6*C*a^4*b^3 + 10*B*a^3*b^4 + 4*C*a^2*b^5 - 6*B*a*b^6 + (B*a^7 -
 2*B*a^5*b^2 + B*a^3*b^4)*cos(d*x + c)^2 + (2*C*a^7 - 3*B*a^6*b - 4*C*a^5*b
```

```
^2 + 6*B*a^4*b^3 + 2*C*a^3*b^4 - 3*B*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
(a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*
d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19554, size = 459, normalized size = 1.76

$$\frac{\frac{4\left(3\,Ca^3b^2-4\,Ba^2b^3-2\,Cab^4+3\,Bb^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}+\frac{4\left(Cab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/2*(4*(3*C*a^3*b^2 - 4*B*a^2*b^3 - 2*C*a*b^4 + 3*B*b^5)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2)) + 4*
(C*a*b^3*tan(1/2*d*x + 1/2*c) - B*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2
)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + (B*a^2 -
 4*C*a*b + 6*B*b^2)*(d*x + c)/a^4 - 2*(B*a*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*t
an(1/2*d*x + 1/2*c)^3 + 4*B*b*tan(1/2*d*x + 1/2*c)^3 - B*a*tan(1/2*d*x + 1/
2*c) - 2*C*a*tan(1/2*d*x + 1/2*c) + 4*B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^2*a^3))/d
```

3.808 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$-\frac{\left(-3a^2C+abB+2b^2C\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2b^3B+15a^3b^2C+2a^4bB-6a^5C-12ab^4C+6b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(\right.}{\sqrt{a+b}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((b*B - 3*a*C)*ArcTanh[Sin[c + d*x]])/(b^4*d) - (a*(2*a^4*b*B - 5*a^2*b^3*B
 + 6*b^5*B - 6*a^5*C + 15*a^3*b^2*C - 12*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*b*B -
 3*a^2*C + 2*b^2*C)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(b*B - a*C)*Se
c[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^
2*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*a*b^2*C)*Tan[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.42482, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {4072, 4029, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-3a^2C+abB+2b^2C\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2b^3B+15a^3b^2C+2a^4bB-6a^5C-12ab^4C+6b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(\right.}{\sqrt{a+b}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^3,x]
```


```
[Out] ((b*B - 3*a*C)*ArcTanh[Sin[c + d*x]])/(b^4*d) - (a*(2*a^4*b*B - 5*a^2*b^3*B
 + 6*b^5*B - 6*a^5*C + 15*a^3*b^2*C - 12*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*b*B -
 3*a^2*C + 2*b^2*C)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(b*B - a*C)*Se
c[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^
2*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*a*b^2*C)*Tan[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
```

```
A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \int \frac{\sec^2(c+dx)\left(2a(bB-aC)-2b(\right.}{}$$

$$= \frac{a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(a^2bB-4b^3B-3a^3C\right.}{2b^3\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{\left(abB-3a^2C+2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(bB-aC)\sec^2(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec}$$

$$= -\frac{\left(abB-3a^2C+2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(bB-aC)\sec^2(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec}$$

$$= \frac{(bB-3aC)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(abB-3a^2C+2b^2C\right)\tan(c}{2b^3\left(a^2-b^2\right)d}$$

$$= \frac{(bB-3aC)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(abB-3a^2C+2b^2C\right)\tan(c}{2b^3\left(a^2-b^2\right)d}$$

$$= \frac{(bB-3aC)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a\left(2a^4bB-5a^2b^3B+6b^5B-\right.}{}$$

Mathematica [A] time = 6.44998, size = 418, normalized size = 1.45

$$\frac{a^2bB\sin(c+dx)-a^3C\sin(c+dx)}{2b^2d(b-a)(a+b)(a\cos(c+dx)+b)^2} + \frac{5a^2b^3B\sin(c+dx)-7a^3b^2C\sin(c+dx)-2a^4bB\sin(c+dx)+4a^5C\sin(c+}{2b^3d(b-a)^2(a+b)^2(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^3,x]
```

```
[Out] (a*(2*a^4*b*B - 5*a^2*b^3*B + 6*b^5*B - 6*a^5*C + 15*a^3*b^2*C - 12*a*b^4*C
)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2
]*(-a^2 + b^2)^2*d) + ((-(b*B) + 3*a*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]])/(b^4*d) + ((b*B - 3*a*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(
b^4*d) + (C*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + (C*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^
2*b*B*Sin[c + d*x] - a^3*C*Sin[c + d*x])/(2*b^2*(-a + b)*(a + b)*d*(b + a*C
os[c + d*x])^2) + (-2*a^4*b*B*Sin[c + d*x] + 5*a^2*b^3*B*Sin[c + d*x] + 4*a
^5*C*Sin[c + d*x] - 7*a^3*b^2*C*Sin[c + d*x])/(2*b^3*(-a + b)^2*(a + b)^2*d
*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.101, size = 1406, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-6/d*a
^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*B-4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d*a^4/b^2/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*C+8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d*a^4/b^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+
1/2*c)*B-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+4/d*a^5/b^3/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*C+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-2/d*a^5/b^3/(a^4-2*
a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*B+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+6
/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*C-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+12/d*a^2/
(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*C-1/d*C/b^3/(tan(1/2*d*x+1/2*c)+1)+1/d/b^3*ln(tan(1/2*d*
x+1/2*c)+1)*B-3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a*C-1/d*C/b^3/(tan(1/2*d*x+1
/2*c)-1)-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B+3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1
)*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 114.551, size = 4591, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [-1/4*(((6*C*a^8 - 2*B*a^7*b - 15*C*a^6*b^2 + 5*B*a^5*b^3 + 12*C*a^4*b^4 -
6*B*a^3*b^5)*cos(d*x + c)^3 + 2*(6*C*a^7*b - 2*B*a^6*b^2 - 15*C*a^5*b^3 + 5
*B*a^4*b^4 + 12*C*a^3*b^5 - 6*B*a^2*b^6)*cos(d*x + c)^2 + (6*C*a^6*b^2 - 2*
B*a^5*b^3 - 15*C*a^4*b^4 + 5*B*a^3*b^5 + 12*C*a^2*b^6 - 6*B*a*b^7)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*((3*C*a^9 - B*a^8*b - 9*C*
a^7*b^2 + 3*B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^3*b^6 + B*a^2*b^7
)*cos(d*x + c)^3 + 2*(3*C*a^8*b - B*a^7*b^2 - 9*C*a^6*b^3 + 3*B*a^5*b^4 + 9
*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^7 + B*a*b^8)*cos(d*x + c)^2 + (3*C*a^7
*b^2 - B*a^6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 -
3*C*a*b^8 + B*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((3*C*a^9 - B*a^
8*b - 9*C*a^7*b^2 + 3*B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^3*b^6 +
 B*a^2*b^7)*cos(d*x + c)^3 + 2*(3*C*a^8*b - B*a^7*b^2 - 9*C*a^6*b^3 + 3*B*a
^5*b^4 + 9*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^7 + B*a*b^8)*cos(d*x + c)^2
+ (3*C*a^7*b^2 - B*a^6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*
a^2*b^7 - 3*C*a*b^8 + B*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*C*
a^6*b^3 - 6*C*a^4*b^5 + 6*C*a^2*b^7 - 2*C*b^9 + (6*C*a^8*b - 2*B*a^7*b^2 -
17*C*a^6*b^3 + 7*B*a^5*b^4 + 13*C*a^4*b^5 - 5*B*a^3*b^6 - 2*C*a^2*b^7)*cos(
d*x + c)^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 - 25*C*a^5*b^4 + 9*B*a^4*b^5 + 20*C
*a^3*b^6 - 6*B*a^2*b^7 - 4*C*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 -
 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^
7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^1
0 - b^12)*d*cos(d*x + c)), 1/2*(((6*C*a^8 - 2*B*a^7*b - 15*C*a^6*b^2 + 5*B*
a^5*b^3 + 12*C*a^4*b^4 - 6*B*a^3*b^5)*cos(d*x + c)^3 + 2*(6*C*a^7*b - 2*B*a
^6*b^2 - 15*C*a^5*b^3 + 5*B*a^4*b^4 + 12*C*a^3*b^5 - 6*B*a^2*b^6)*cos(d*x +
 c)^2 + (6*C*a^6*b^2 - 2*B*a^5*b^3 - 15*C*a^4*b^4 + 5*B*a^3*b^5 + 12*C*a^2*
b^6 - 6*B*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((3*C*a^9 - B*a^8*b - 9*C*
a^7*b^2 + 3*B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^3*b^6 + B*a^2*b^7
)*cos(d*x + c)^3 + 2*(3*C*a^8*b - B*a^7*b^2 - 9*C*a^6*b^3 + 3*B*a^5*b^4 + 9
*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^7 + B*a*b^8)*cos(d*x + c)^2 + (3*C*a^7
*b^2 - B*a^6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 -
3*C*a*b^8 + B*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((3*C*a^9 - B*a^8*
b - 9*C*a^7*b^2 + 3*B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - 3*C*a^3*b^6 + B
*a^2*b^7)*cos(d*x + c)^3 + 2*(3*C*a^8*b - B*a^7*b^2 - 9*C*a^6*b^3 + 3*B*a^5
*b^4 + 9*C*a^4*b^5 - 3*B*a^3*b^6 - 3*C*a^2*b^7 + B*a*b^8)*cos(d*x + c)^2 +
(3*C*a^7*b^2 - B*a^6*b^3 - 9*C*a^5*b^4 + 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^
2*b^7 - 3*C*a*b^8 + B*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*C*a^6*
b^3 - 6*C*a^4*b^5 + 6*C*a^2*b^7 - 2*C*b^9 + (6*C*a^8*b - 2*B*a^7*b^2 - 17*C
*a^6*b^3 + 7*B*a^5*b^4 + 13*C*a^4*b^5 - 5*B*a^3*b^6 - 2*C*a^2*b^7)*cos(d*x
+ c)^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 - 25*C*a^5*b^4 + 9*B*a^4*b^5 + 20*C*a^3
*b^6 - 6*B*a^2*b^7 - 4*C*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a
^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 +
3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 -
b^12)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,
x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.37067, size = 784, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] ((6*C*a^6 - 2*B*a^5*b - 15*C*a^4*b^2 + 5*B*a^3*b^3 + 12*C*a^2*b^4 - 6*B*a*b
^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(-a^2 + b^2)) - (4*C*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^5*b
*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*B*a^4*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*b^3*tan
(1/2*d*x + 1/2*c)^3 + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*B*a^2*b^4*tan(
1/2*d*x + 1/2*c)^3 - 4*C*a^6*tan(1/2*d*x + 1/2*c) + 2*B*a^5*b*tan(1/2*d*x +
 1/2*c) - 5*C*a^5*b*tan(1/2*d*x + 1/2*c) + 3*B*a^4*b^2*tan(1/2*d*x + 1/2*c)
 + 7*C*a^4*b^2*tan(1/2*d*x + 1/2*c) - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) + 8*
C*a^3*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^
3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 -
 a - b)^2) - (3*C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (3*C*a
- B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*C*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

3.809 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=220

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-6ab^4C+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(bB-aC)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2bB\right.}{2b^2}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3
*b^2*C - 6*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*(b*B - a*C)*Tan[c + d*x])/(2*b^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*
a*b^2*C)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.750799, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 4028, 4080, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-6ab^4C+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(bB-aC)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2bB\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^3,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3
*b^2*C - 6*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*(b*B - a*C)*Tan[c + d*x])/(2*b^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2*b*B - 4*b^3*B - 3*a^3*C + 6*
a*b^2*C)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2(bB-aC)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\sec(c+dx)\left(-2ab(bB-aC)-\left(a^2\right.\right.}{}$$

$$= -\frac{a^2(bB-aC)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2bB-4b^3B-3a^3C\right.}{2b^2\left(a^2-b^2\right)^2d(a}$$

$$= -\frac{a^2(bB-aC)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2bB-4b^3B-3a^3C\right.}{2b^2\left(a^2-b^2\right)^2d(a}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(bB-aC)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(bB-aC)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{\left(a^2b^3B+2b^5B-2a^5C+5a^3b^2C-6a\right.}{(a-b)^{5/2}b^3(a}$$

Mathematica [A] time = 1.88124, size = 270, normalized size = 1.23

$$\frac{\cos(c+dx)(B+C\sec(c+dx))\left(\frac{ab\left(-2a^3C+5ab^2C-3b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{2\left(-a^2b^3B-5a^3b^2C+2a^5C+6ab^4C-2b^5B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \right.}{2b^3d(B\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(B + C*Sec[c + d*x])*((2*(-(a^2*b^3*B) - 2*b^5*B + 2*a^5*C -
5*a^3*b^2*C + 6*a*b^4*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(5/2) - 2*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*C*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(-(b*B) + a*C)*Sin[c + d*
x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])^2) + (a*b*(-3*b^3*B - 2*a^3*C +
5*a*b^2*C)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/(2*b^
3*d*(C + B*Cos[c + d*x]))
```

Maple [B] time = 0.098, size = 1085, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d*a^4/b^2
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*C-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-6/d/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*C+1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*a^4/b^2/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*
x+1/2*c)*C-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/
(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2*a^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+1/d/(a^4-2*a^2*b^2
+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*B*a^2+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/
d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))*C-6/d*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a+1/d/b^3*ln(tan(1/
2*d*x+1/2*c)+1)*C-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 43.407, size = 3051, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [-1/4*((2*C*a^5*b^2 - 5*C*a^3*b^4 - B*a^2*b^5 + 6*C*a*b^6 - 2*B*b^7 + (2*C*
a^7 - 5*C*a^5*b^2 - B*a^4*b^3 + 6*C*a^3*b^4 - 2*B*a^2*b^5)*cos(d*x + c)^2 +
 2*(2*C*a^6*b - 5*C*a^4*b^3 - B*a^3*b^4 + 6*C*a^2*b^5 - 2*B*a*b^6)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(C*a^6*b^2 - 3*C*a^4*b^4 +
 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*cos(
d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x + c)
)*log(sin(d*x + c) + 1) + 2*(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8
+ (C*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7
*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x + c))*log(-sin(d*x + c) +
 1) + 2*(3*C*a^6*b^2 - B*a^5*b^3 - 9*C*a^4*b^4 + 5*B*a^3*b^5 + 6*C*a^2*b^6
```

```
- 4*B*a*b^7 + (2*C*a^7*b - 7*C*a^5*b^3 + 3*B*a^4*b^4 + 5*C*a^3*b^5 - 3*B*a^
2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*
b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(
d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*C*a^5*b^2
- 5*C*a^3*b^4 - B*a^2*b^5 + 6*C*a*b^6 - 2*B*b^7 + (2*C*a^7 - 5*C*a^5*b^2 -
B*a^4*b^3 + 6*C*a^3*b^4 - 2*B*a^2*b^5)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*
a^4*b^3 - B*a^3*b^4 + 6*C*a^2*b^5 - 2*B*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) - (C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 +
 3*C*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a
^3*b^5 - C*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + (C*a^6*b^2 - 3*C*a^
4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^
6)*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d
*x + c))*log(-sin(d*x + c) + 1) + (3*C*a^6*b^2 - B*a^5*b^3 - 9*C*a^4*b^4 +
5*B*a^3*b^5 + 6*C*a^2*b^6 - 4*B*a*b^7 + (2*C*a^7*b - 7*C*a^5*b^3 + 3*B*a^4*
b^4 + 5*C*a^3*b^5 - 3*B*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*
a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 +
3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^1
1)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,
x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.46216, size = 656, normalized size = 2.98

$$-\frac{\frac{\left(2\,Ca^5-5\,Ca^3b^2-Ba^2b^3+6\,Cab^4-2\,Bb^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{-a^2+b^2}}-\frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}+\frac{C\log\left(\left|\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -((2*C*a^5 - 5*C*a^3*b^2 - B*a^2*b^3 + 6*C*a*b^4 - 2*B*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a
^2 + b^2)) - C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + C*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/b^3 - (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^4*b*tan(1/2
*d*x + 1/2*c)^3 + B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^2*tan(1/2*d*
x + 1/2*c)^3 + 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^3 - 4*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^5*tan(1/2*d*x + 1/2*
c) - 3*C*a^4*b*tan(1/2*d*x + 1/2*c) + B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*C*
a^3*b^2*tan(1/2*d*x + 1/2*c) - 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*C*a^2*b
```

```
^3*tan(1/2*d*x + 1/2*c) - 4*B*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2
*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2
))/d
```

3.810 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(a^2(-C)+3abB-2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2bB+a^3C-4ab^2C+2b^3B\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a(bB-aC)}{2bd\left(a^2-b^2\right)(a}$$

```
[Out] -(((3*a*b*B - a^2*C - 2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(b*B - a*C)*Tan[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*b*B + 2*b^3*B + a^3*C - 4*
a*b^2*C)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.370616, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {4072, 4009, 4003, 12, 3831, 2659, 208}

$$-\frac{\left(a^2(-C)+3abB-2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2bB+a^3C-4ab^2C+2b^3B\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a(bB-aC)}{2bd\left(a^2-b^2\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^3,x]
```

```
[Out] -(((3*a*b*B - a^2*C - 2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(b*B - a*C)*Tan[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*b*B + 2*b^3*B + a^3*C - 4*
a*b^2*C)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
```

```
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(-2b(bB-aC)+\left(abB+a^2C-2\right.\right.}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(bB-aC)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= \frac{a(bB-aC)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= \frac{a(bB-aC)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= \frac{a(bB-aC)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C-4ab^2C\right)}{2b\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= -\frac{\left(3abB-a^2C-2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(bB-aC}{2b\left(a^2-b^2\right)d(}$$

Mathematica [A] time = 0.664668, size = 157, normalized size = 0.87

$$\frac{\frac{(2a^2B-3abC+b^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)}-\frac{2(a^2C-3abB+2b^2C)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^3,x]
```

```
[Out] ((-2*(-3*a*b*B + a^2*C + 2*b^2*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) + ((-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a
+ b)*(b + a*Cos[c + d*x])^2) + ((2*a^2*B + b^2*B - 3*a*b*C)*Sin[c + d*x])/(
(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*d)
```

Maple [A] time = 0.087, size = 238, normalized size = 1.3

$$\frac{1}{d}\left(2\frac{1}{\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)^2}\left(-1/2\frac{\left(2\,Ba^2+Bab+2\,Bb^2-a^2C-4\,abC\right)(\tan(1/2\,dx\right.}{(a-b)\left(a^2+2\,ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(2*(-1/2*(2*B*a^2+B*a*b+2*B*b^2-C*a^2-4*C*a*b)/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3+1/2*(2*B*a^2-B*a*b+2*B*b^2+C*a^2-4*C*a*b)/(a+b)/(a^2-2*a
*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^2-(3*B*a*b-C*a^2-2*C*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.648945, size = 1631, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] [1/4*((C*a^2*b^2 - 3*B*a*b^3 + 2*C*b^4 + (C*a^4 - 3*B*a^3*b + 2*C*a^2*b^2)*
cos(d*x + c)^2 + 2*(C*a^3*b - 3*B*a^2*b^2 + 2*C*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^5 + B*a^4*b - 5*C*a^3*b^2 + B*a^2
*b^3 + 4*C*a*b^4 - 2*B*b^5 + (2*B*a^5 - 3*C*a^4*b - B*a^3*b^2 + 3*C*a^2*b^3
 - B*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((C*a^2*b^2 - 3*B*
a*b^3 + 2*C*b^4 + (C*a^4 - 3*B*a^3*b + 2*C*a^2*b^2)*cos(d*x + c)^2 + 2*(C*a
^3*b - 3*B*a^2*b^2 + 2*C*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^5 + B*
a^4*b - 5*C*a^3*b^2 + B*a^2*b^3 + 4*C*a*b^4 - 2*B*b^5 + (2*B*a^5 - 3*C*a^4*
b - B*a^3*b^2 + 3*C*a^2*b^3 - B*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.40995, size = 540, normalized size = 3.

$$\frac{\frac{(Ca^2-3Bab+2Cb^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{2Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\cdots}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] ((C*a^2 - 3*B*a*b + 2*C*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*B*a^3*tan(1/2*d*x +
 1/2*c)^3 - C*a^3*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
 3*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*C*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1
/2*d*x + 1/2*c) - C*a^3*tan(1/2*d*x + 1/2*c) - B*a^2*b*tan(1/2*d*x + 1/2*c)
 + 3*C*a^2*b*tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) + 4*C*a*b^
2*tan(1/2*d*x + 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 +
b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.811 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{\left(2a^2B-3abC+b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-C)+3abB-2b^2C\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(bB-aC)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+}$$

```
[Out] ((2*a^2*B + b^2*B - 3*a*b*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((b*B - a*C)*Tan[c + d*x])/(2*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((3*a*b*B - a^2*C - 2*b^2*C)*Tan[c + d*
x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.264986, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4060, 12, 3831, 2659, 208}

$$\frac{\left(2a^2B-3abC+b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-C)+3abB-2b^2C\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(bB-aC)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a^2*B + b^2*B - 3*a*b*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((b*B - a*C)*Tan[c + d*x])/(2*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((3*a*b*B - a^2*C - 2*b^2*C)*Tan[c + d*
x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{(bB-aC)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a(aB-bC)\sec(c+dx)+a(bB-aC)\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3abB-a^2C-2b^2C\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{a^2(2a^2}{}}{}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3abB-a^2C-2b^2C\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(2a^2B+\right.}{}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3abB-a^2C-2b^2C\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(2a^2B+\right.}{}$$

$$= -\frac{(bB-aC)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3abB-a^2C-2b^2C\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(2a^2B+\right.}{}$$

$$= \frac{\left(2a^2B+b^2B-3abC\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(bB-aC)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx)}$$

Mathematica [A] time = 0.840085, size = 172, normalized size = 1.05

$$\frac{\frac{\left(-4a^2bB+2a^3C+ab^2C+b^3B\right)\sin(c+dx)}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{2\left(2a^2B-3abC+b^2B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b(bB-aC)\sin(c+dx)}{a(a-b)(a+b)(a\cos(c+dx)+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*B + b^2*B - 3*a*b*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*(b*B - a*C)*Sin[c + d*x])/(a*(a - b)*(a
+ b)*(b + a*Cos[c + d*x])^2) + ((-4*a^2*b*B + b^3*B + 2*a^3*C + a*b^2*C)*Si
n[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*d)
```

Maple [A] time = 0.091, size = 236, normalized size = 1.4

$$\frac{1}{d}\left(-2\frac{1}{\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b\right)^2}\left(-1/2\frac{\left(4\,Bab+Bb^2-2\,a^2C-abC-2\,b^2C\right)(\tan(1/2\,dx}{(a-b)\left(a^2+2\,ab+b^2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*B*a*b+B*b^2-2*C*a^2-C*a*b-2*C*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*(4*B*a*b-B*b^2-2*C*a^2+C*a*b-2*C*b^2)/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2+(2*B*a^2+B*b^2-3*C*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.651, size = 1631, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*((2*B*a^2*b^2 - 3*C*a*b^3 + B*b^4 + (2*B*a^4 - 3*C*a^3*b + B*a^2*b^2)*
cos(d*x + c)^2 + 2*(2*B*a^3*b - 3*C*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^4*b - 3*B*a^3*b^2 + C*a^2*b^3 + 3
*B*a*b^4 - 2*C*b^5 + (2*C*a^5 - 4*B*a^4*b - C*a^3*b^2 + 5*B*a^2*b^3 - C*a*b
^4 - B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((2*B*a^2*b^2 - 3*
C*a*b^3 + B*b^4 + (2*B*a^4 - 3*C*a^3*b + B*a^2*b^2)*cos(d*x + c)^2 + 2*(2*B
*a^3*b - 3*C*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^4*b -
3*B*a^3*b^2 + C*a^2*b^3 + 3*B*a*b^4 - 2*C*b^5 + (2*C*a^5 - 4*B*a^4*b - C*a^
3*b^2 + 5*B*a^2*b^3 - C*a*b^4 - B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.42175, size = 539, normalized size = 3.29

$$\frac{\frac{\left(2\,Ba^2-3\,Cab+Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{2\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ca^2b\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*B*a^2 - 3*C*a*b + B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*C*a^3*tan(1/2*d*x +
 1/2*c)^3 - 4*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 3*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*
b^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^3*tan(1
/2*d*x + 1/2*c) + 4*B*a^2*b*tan(1/2*d*x + 1/2*c) - C*a^2*b*tan(1/2*d*x + 1/
2*c) + 3*B*a*b^2*tan(1/2*d*x + 1/2*c) - C*a*b^2*tan(1/2*d*x + 1/2*c) - B*b^
3*tan(1/2*d*x + 1/2*c) - 2*C*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 +
b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.812 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=205

$$-\frac{\left(-5a^2b^3B-a^3b^2C+6a^4bB-2a^5C+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2bB-3a^3C-2b^3B\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{}{2a}$$

```
[Out] (B*x)/a^3 - ((6*a^4*b*B - 5*a^2*b^3*B + 2*b^5*B - 2*a^5*C - a^3*b^2*C)*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)
^(5/2)*d) + (b*(b*B - a*C)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^2) + (b*(5*a^2*b*B - 2*b^3*B - 3*a^3*C)*Tan[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.580986, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {4072, 3923, 4060, 3919, 3831, 2659, 208}

$$-\frac{\left(-5a^2b^3B-a^3b^2C+6a^4bB-2a^5C+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2bB-3a^3C-2b^3B\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^3,x]
```

```
[Out] (B*x)/a^3 - ((6*a^4*b*B - 5*a^2*b^3*B + 2*b^5*B - 2*a^5*C - a^3*b^2*C)*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)
^(5/2)*d) + (b*(b*B - a*C)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^2) + (b*(5*a^2*b*B - 2*b^3*B - 3*a^3*C)*Tan[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{B+C\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2\left(a^2-b^2\right)B+2a(bB-aC)\sec(c+d}{(a+b\sec(c+d}}{2a\left(a^2-\right.}$$

$$= \frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2bB-2b^3B-3a^3C\right)\text{ta}}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(}$$

$$= \frac{Bx}{a^3} + \frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2bB-2b^3B-3a\right.}{2a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{Bx}{a^3} + \frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2bB-2b^3B-3a\right.}{2a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{Bx}{a^3} + \frac{b(bB-aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2bB-2b^3B-3a\right.}{2a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{Bx}{a^3} - \frac{\left(6a^4bB-5a^2b^3B+2b^5B-2a^5C-a^3b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\text{ta}}{\sqrt{}}\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 1.38194, size = 203, normalized size = 0.99

$$\frac{\frac{ab\left(6a^2bB-4a^3C+ab^2C-3b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{2\left(5a^2b^3B+a^3b^2C-6a^4bB+2a^5C-2b^5B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^2(aC-bB)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)^2} + 2B(c+}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^3,x]
```

```
[Out] (2*B*(c + d*x) - (2*(-6*a^4*b*B + 5*a^2*b^3*B - 2*b^5*B + 2*a^5*C + a^3*b^2
*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (a*b^2*(-(b*B) + a*C)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x]
)^2) + (a*b*(6*a^2*b*B - 3*b^3*B - 4*a^3*C + a*b^2*C)*Sin[c + d*x])/((a - b
)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*a^3*d)
```

Maple [B] time = 0.125, size = 1063, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*B/a^3*arctan(tan(1/2*d*x+1/2*c))-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/a/
```

```
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d*a/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*C+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2*b^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+6/d/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-
1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a-b)
^2*tan(1/2*d*x+1/2*c)*B-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-4/d*a/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+1/d
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*ta
n(1/2*d*x+1/2*c)*C-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+5/d/a/(a^4-2*a^2*b^2+b^4)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*
b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b^5+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*b^2*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.751762, size = 2479, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] [1/4*(4*(B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x*cos(d*x + c)^2
+ 8*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 4*(B
*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x - (2*C*a^5*b^2 - 6*B*a^4*
b^3 + C*a^3*b^4 + 5*B*a^2*b^5 - 2*B*b^7 + (2*C*a^7 - 6*B*a^6*b + C*a^5*b^2
+ 5*B*a^4*b^3 - 2*B*a^2*b^5)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 6*B*a^5*b^2 +
C*a^4*b^3 + 5*B*a^3*b^4 - 2*B*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d
*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) - 2*(3*C*a^6*b^2 - 5*B*a^5*b^3 - 3*C*a^4*b^4 + 7*B*a^3*b^5 -
 2*B*a*b^7 + (4*C*a^7*b - 6*B*a^6*b^2 - 5*C*a^5*b^3 + 9*B*a^4*b^4 + C*a^3*b
^5 - 3*B*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^
4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7
)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(
B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x*cos(d*x + c)^2 + 4*(B*a^
```

```
7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 2*(B*a^6*b^2
- 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x + (2*C*a^5*b^2 - 6*B*a^4*b^3 + C*a
^3*b^4 + 5*B*a^2*b^5 - 2*B*b^7 + (2*C*a^7 - 6*B*a^6*b + C*a^5*b^2 + 5*B*a^4
*b^3 - 2*B*a^2*b^5)*cos(d*x + c)^2 + 2*(2*C*a^6*b - 6*B*a^5*b^2 + C*a^4*b^3
 + 5*B*a^3*b^4 - 2*B*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*C*a^6*b^2 -
5*B*a^5*b^3 - 3*C*a^4*b^4 + 7*B*a^3*b^5 - 2*B*a*b^7 + (4*C*a^7*b - 6*B*a^6*
b^2 - 5*C*a^5*b^3 + 9*B*a^4*b^4 + C*a^3*b^5 - 3*B*a^2*b^6)*cos(d*x + c))*si
n(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*
(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^
7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61489, size = 617, normalized size = 3.01

$$\frac{\left(2\,Ca^5-6\,Ba^4b+Ca^3b^2+5\,Ba^2b^3-2\,Bb^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}+\frac{(dx+c)B}{a^3}+\frac{4\,Ca^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,B\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] ((2*C*a^5 - 6*B*a^4*b + C*a^3*b^2 + 5*B*a^2*b^3 - 2*B*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^
2 + b^2)) + (d*x + c)*B/a^3 + (4*C*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*B*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^2*b^
3*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*B*a*b^4*tan
(1/2*d*x + 1/2*c)^3 - 2*B*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^4*b*tan(1/2*d*
x + 1/2*c) + 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 3*C*a^3*b^2*tan(1/2*d*x + 1
/2*c) + 5*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + C*a^2*b^3*tan(1/2*d*x + 1/2*c) -
 3*B*a*b^4*tan(1/2*d*x + 1/2*c) - 2*B*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a
^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a
- b)^2))/d
```

3.813 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=290

$$\frac{\left(-11a^2b^2B+5a^3bC+2a^4B-2ab^3C+6b^4B\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-15a^2b^3B+5a^3b^2C+12a^4bB-6a^5C-2ab^4C+6b^5B\right)\text{ta}}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] -(((3*b*B - a*C)*x)/a^4) + (b*(12*a^4*b*B - 15*a^2*b^3*B + 6*b^5*B - 6*a^5*
C + 5*a^3*b^2*C - 2*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4*B - 11*a^2*b^2*B + 6*b
^4*B + 5*a^3*b*C - 2*a*b^3*C)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(b
*B - a*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b*(6*
a^2*b*B - 3*b^3*B - 4*a^3*C + a*b^2*C)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.53299, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 4030, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-11a^2b^2B+5a^3bC+2a^4B-2ab^3C+6b^4B\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-15a^2b^3B+5a^3b^2C+12a^4bB-6a^5C-2ab^4C+6b^5B\right)\text{ta}}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^3,x]
```


```
[Out] -(((3*b*B - a*C)*x)/a^4) + (b*(12*a^4*b*B - 15*a^2*b^3*B + 6*b^5*B - 6*a^5*
C + 5*a^3*b^2*C - 2*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4*B - 11*a^2*b^2*B + 6*b
^4*B + 5*a^3*b*C - 2*a*b^3*C)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(b
*B - a*C)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b*(6*
a^2*b*B - 3*b^3*B - 4*a^3*C + a*b^2*C)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
```

```
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a^2B+3b^2B-abC+2a(bB\right.}{(a+b\,\mathrm{s}}}{2a\,(}$$

$$= \frac{b(bB-aC)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(6a^2bB-3b^3B-4a^3C+ab^2\right.}{2a^2\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}}$$

$$= \frac{\left(2a^4B-11a^2b^2B+6b^4B+5a^3bC-2ab^3C\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b(}{2a\left(a^2-\right.}$$

$$= -\frac{(3bB-aC)x}{a^4} + \frac{\left(2a^4B-11a^2b^2B+6b^4B+5a^3bC-2ab^3C\right)\sin(c}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(3bB-aC)x}{a^4} + \frac{\left(2a^4B-11a^2b^2B+6b^4B+5a^3bC-2ab^3C\right)\sin(c}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(3bB-aC)x}{a^4} + \frac{\left(2a^4B-11a^2b^2B+6b^4B+5a^3bC-2ab^3C\right)\sin(c}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(3bB-aC)x}{a^4} + \frac{b\left(12a^4bB-15a^2b^3B+6b^5B-6a^5C+5a^3b^2C-\right.}{a^4(a-b)^{5/2}(a+b)^{5}}$$

Mathematica [A] time = 2.04294, size = 232, normalized size = 0.8

$$\frac{\frac{ab^2\left(-8a^2bB+6a^3C-3ab^2C+5b^3B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} - \frac{2b\left(-15a^2b^3B+5a^3b^2C+12a^4bB-6a^5C-2ab^4C+6b^5B\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^3(bB-aC)\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^3,x]
```

```
[Out] (2*(-3*b*B + a*C)*(c + d*x) - (2*b*(12*a^4*b*B - 15*a^2*b^3*B + 6*b^5*B - 6
*a^5*C + 5*a^3*b^2*C - 2*a*b^4*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) + 2*a*B*Sin[c + d*x] + (a*b^3*(b*B - a*C)*Si
n[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) + (a*b^2*(-8*a^2*b*B +
 5*b^3*B + 6*a^3*C - 3*a*b^2*C)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*C
os[c + d*x])))/(2*a^4*d)
```

Maple [B] time = 0.138, size = 1349, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^4*B*arctan(tan(
1/2*d*x+1/2*c))*b+2/d/a^3*C*arctan(tan(1/2*d*x+1/2*c))+8/d/a/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*B+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d*b^5/a^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*B-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^2/(a
-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*
C+2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-8/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d/a^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*
d*x+1/2*c)*B+4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d*b^3/a/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x
+1/2*c)*C-2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-15/d*b^4/
a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*B+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*b/(a^4-2*a^2*
b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*C*a+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.896501, size = 3394, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [1/4*(4*(C*a^9 - 3*B*a^8*b - 3*C*a^7*b^2 + 9*B*a^6*b^3 + 3*C*a^5*b^4 - 9*B*
a^4*b^5 - C*a^3*b^6 + 3*B*a^2*b^7)*d*x*cos(d*x + c)^2 + 8*(C*a^8*b - 3*B*a^
7*b^2 - 3*C*a^6*b^3 + 9*B*a^5*b^4 + 3*C*a^4*b^5 - 9*B*a^3*b^6 - C*a^2*b^7 +
 3*B*a*b^8)*d*x*cos(d*x + c) + 4*(C*a^7*b^2 - 3*B*a^6*b^3 - 3*C*a^5*b^4 + 9
```

```
*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 - C*a*b^8 + 3*B*b^9)*d*x - (6*C*a^5*
b^3 - 12*B*a^4*b^4 - 5*C*a^3*b^5 + 15*B*a^2*b^6 + 2*C*a*b^7 - 6*B*b^8 + (6*
C*a^7*b - 12*B*a^6*b^2 - 5*C*a^5*b^3 + 15*B*a^4*b^4 + 2*C*a^3*b^5 - 6*B*a^2
*b^6)*cos(d*x + c)^2 + 2*(6*C*a^6*b^2 - 12*B*a^5*b^3 - 5*C*a^4*b^4 + 15*B*a
^3*b^5 + 2*C*a^2*b^6 - 6*B*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*
cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
+ c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + b^2)) + 2*(2*B*a^7*b^2 + 5*C*a^6*b^3 - 13*B*a^5*b^4 - 7*C*a^4*b^5 + 1
7*B*a^3*b^6 + 2*C*a^2*b^7 - 6*B*a*b^8 + 2*(B*a^9 - 3*B*a^7*b^2 + 3*B*a^5*b^
4 - B*a^3*b^6)*cos(d*x + c)^2 + (4*B*a^8*b + 6*C*a^7*b^2 - 20*B*a^6*b^3 - 9
*C*a^5*b^4 + 25*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7)*cos(d*x + c))*sin(d*
x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^
11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*
b^4 + 3*a^6*b^6 - a^4*b^8)*d), 1/2*(2*(C*a^9 - 3*B*a^8*b - 3*C*a^7*b^2 + 9*
B*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 - C*a^3*b^6 + 3*B*a^2*b^7)*d*x*cos(d*
x + c)^2 + 4*(C*a^8*b - 3*B*a^7*b^2 - 3*C*a^6*b^3 + 9*B*a^5*b^4 + 3*C*a^4*b
^5 - 9*B*a^3*b^6 - C*a^2*b^7 + 3*B*a*b^8)*d*x*cos(d*x + c) + 2*(C*a^7*b^2 -
 3*B*a^6*b^3 - 3*C*a^5*b^4 + 9*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 - C*a*
b^8 + 3*B*b^9)*d*x - (6*C*a^5*b^3 - 12*B*a^4*b^4 - 5*C*a^3*b^5 + 15*B*a^2*b
^6 + 2*C*a*b^7 - 6*B*b^8 + (6*C*a^7*b - 12*B*a^6*b^2 - 5*C*a^5*b^3 + 15*B*a
^4*b^4 + 2*C*a^3*b^5 - 6*B*a^2*b^6)*cos(d*x + c)^2 + 2*(6*C*a^6*b^2 - 12*B*
a^5*b^3 - 5*C*a^4*b^4 + 15*B*a^3*b^5 + 2*C*a^2*b^6 - 6*B*a*b^7)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + (2*B*a^7*b^2 + 5*C*a^6*b^3 - 13*B*a^5*b^4 - 7*C*a^4*b^
5 + 17*B*a^3*b^6 + 2*C*a^2*b^7 - 6*B*a*b^8 + 2*(B*a^9 - 3*B*a^7*b^2 + 3*B*a
^5*b^4 - B*a^3*b^6)*cos(d*x + c)^2 + (4*B*a^8*b + 6*C*a^7*b^2 - 20*B*a^6*b^
3 - 9*C*a^5*b^4 + 25*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7)*cos(d*x + c))*s
in(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 +
2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3
*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,
x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40077, size = 737, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -((6*C*a^5*b - 12*B*a^4*b^2 - 5*C*a^3*b^3 + 15*B*a^2*b^4 + 2*C*a*b^5 - 6*B*
b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 2*a^6*b^
```

2 + a^4*b^4)*sqrt(-a^2 + b^2)) + (6*C*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*B*
a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*B*a
^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*B*a*
b^5*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^6*tan
(1/2*d*x + 1/2*c)^3 - 6*C*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*B*a^3*b^3*tan(1/
2*d*x + 1/2*c) - 5*C*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*B*a^2*b^4*tan(1/2*d*x
 + 1/2*c) + 3*C*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*B*a*b^5*tan(1/2*d*x + 1/2*
c) + 2*C*a*b^5*tan(1/2*d*x + 1/2*c) - 4*B*b^6*tan(1/2*d*x + 1/2*c))/((a^7 -
 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2) - (C*a - 3*B*b)*(d*x + c)/a^4 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(
1/2*d*x + 1/2*c)^2 + 1)*a^3))/d

3.814 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=485

$$-\frac{2(a-b)\sqrt{a+b}\left(12a^2b(2B-C)-16a^3C+18ab^2(B-2C)+3b^3(25B-49C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{315b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 24*a^2*b^2*C
+ 147*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(315*b^5*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*
B - 49*C) + 18*a*b^2*(B - 2*C) + 12*a^2*b*(2*B - C) - 16*a^3*C)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^4*d) - (2*(12*a^2*b*B - 75*b^3*B - 8*a^3*C - 13*a*b^2*C)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^3*d) + (2*(9*a*b*B - 6*a^2*C + 49*b^2
*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9
*b*B + a*C)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d)
+ (2*C*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```

Rubi [A] time = 1.45366, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4031, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(-6a^2C+9abB+49b^2C\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2\left(12a^2bB-8a^3C-13ab^2C-75b^3B\right)\tan(c+d}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 24*a^2*b^2*C
+ 147*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(315*b^5*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*
B - 49*C) + 18*a*b^2*(B - 2*C) + 12*a^2*b*(2*B - C) - 16*a^3*C)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^4*d) - (2*(12*a^2*b*B - 75*b^3*B - 8*a^3*C - 13*a*b^2*C)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^3*d) + (2*(9*a*b*B - 6*a^2*C + 49*b^2
*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9
*b*B + a*C)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d)
+ (2*C*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
```

```
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^4(c+dx)\sqrt{a+b\sec(c+dx)}(B+C\sec(c-$$

$$= \frac{2C\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{9d}$$

$$= \frac{2(9bB+aC)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan}{63bd}$$

$$= \frac{2\left(9abB-6a^2C+49b^2C\right)\sec(c+dx)\sqrt{a+b\sec}}{315b^2d}$$

$$= -\frac{2\left(12a^2bB-75b^3B-8a^3C-13ab^2C\right)\sqrt{a+b\,s}}{315b^3d}$$

$$= -\frac{2\left(12a^2bB-75b^3B-8a^3C-13ab^2C\right)\sqrt{a+b\,s}}{315b^3d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(24a^3bB+57ab^3B-16a^4C-2\right.}{}$$

Mathematica [B] time = 25.5508, size = 3734, normalized size = 7.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 24*a^2*
b^2*C + 147*b^4*C)*Sin[c + d*x])/(315*b^4) + (2*Sec[c + d*x]^3*(9*b*B*Sin[c
 + d*x] + a*C*Sin[c + d*x]))/(63*b) + (2*Sec[c + d*x]^2*(9*a*b*B*Sin[c + d*
x] - 6*a^2*C*Sin[c + d*x] + 49*b^2*C*Sin[c + d*x]))/(315*b^2) + (2*Sec[c +
d*x]*(-12*a^2*b*B*Sin[c + d*x] + 75*b^3*B*Sin[c + d*x] + 8*a^3*C*Sin[c + d*
x] + 13*a*b^2*C*Sin[c + d*x]))/(315*b^3) + (2*C*Sec[c + d*x]^3*Tan[c + d*x]
)/9))/d + (2*((-19*a*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (8*a^3*B)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^4*
C)/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a^2*C)/(105*b
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*b*C)/(15*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*B*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[
b + a*Cos[c + d*x]]) - (17*a^2*B*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[
c + d*x]]) + (5*b*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (4*
a*C*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*C*Sqrt[Sec[
c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*C*Sqrt[Sec[c + d*x]]
)/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^2*B*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) - (7*a*C*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*C*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (8*a^3*C*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[
```

Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(-24*a
^3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*
a^3*C + 12*a^2*b*(2*B + C) - 18*a*b^2*(B + 2*C) + 3*b^3*(25*B + 49*C))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*
a^3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^4*d*(b + a*
Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*((a*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B
 + 16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*
B + C) - 18*a*b^2*(B + 2*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*b*B - 57*a*b^3*B
 + 16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^4*(b + a*Cos[c + d*x])^(3/2)*Sq
rt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d
x)/2](2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 147
*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + 2*b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 18*a*b^2*(B + 2*C) + 3
b^3(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + (-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 14
7*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(315*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C +
24*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^4)/2 + ((a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 147
*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 +
 Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])] + (b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 18*a*b^2*(B
+ 2*C) + 3*b^3*(25*B + 49*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d
*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-24*a^3*b*B - 57*a*b^3*B
+ 16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
x]))] + (b(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 18*a*b^2*(B + 2*C) +
3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x
])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-24*a^
3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*Sec[
(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-24*a^3*b*B - 57*a*b^3*B +
16*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 24
*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 18*a*b^
2*(B + 2*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
) + ((a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 24*a^2*b^2*C - 147*b^4*

```
C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*b^4*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*
a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C
) - 18*a*b^2*(B + 2*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*b*B - 57*a*b^3*B + 16
*a^4*C + 24*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*
x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*b^4*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]])))
```

Maple [B] time = 1.737, size = 4395, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/315/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-75*B*cos(d*x+c)^5*b^5+4*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-24*B*sin(d*x+c)*c
os(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3*b^2+16*C*cos(d*x+c)^5*a^4*b-26*C*cos(d*x+c)^5*a^3*b^2+24*C*cos(d*x+c)
^5*a^2*b^3+85*C*cos(d*x+c)^5*a*b^4-8*C*cos(d*x+c)^4*a^4*b-10*C*cos(d*x+c)^4
*a^2*b^3+2*C*cos(d*x+c)^3*a^3*b^2+22*C*cos(d*x+c)^3*a*b^4-8*C*cos(d*x+c)^6*
a^4*b+24*C*cos(d*x+c)^6*a^3*b^2-13*C*cos(d*x+c)^6*a^2*b^3-147*C*cos(d*x+c)^
6*a*b^4-C*cos(d*x+c)^2*a^2*b^3+40*C*cos(d*x+c)*a*b^4+24*B*sin(d*x+c)*cos(d*
x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4
*b+24*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*b^2+57*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+57*B*sin(d*x+c)*co
s(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^4-6*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b^3-57*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+24*B*sin(d*x+c)*c
os(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^4*b+24*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-16*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
```

os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^5-147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^5-16*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x+c)^5*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^5-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^5-16*C*cos(d*x+c)^5*a^5-147*C*cos(d*x+c)^5*b^5+98*
C*cos(d*x+c)^4*b^5+14*C*cos(d*x+c)^2*b^5+16*C*cos(d*x+c)^6*a^5+57*B*sin(d*x
+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b^3+57*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-24*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-6*B*sin(
d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*b^3-57*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+24*C*cos(d*x+c)^5*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-111*C
*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^4-16*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-24*C*cos(d*x+c)^5*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-24
*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*b^3+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+16*C*cos(d*x+c)^4*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b
+4*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^3*b^2+24*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-111*C*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^4-16*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^4*b-24*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-24*C*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*b^3+147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+16*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+35*C*b^5+30*B*c

```
os(d*x+c)^3*b^5+45*B*cos(d*x+c)*b^5+12*B*cos(d*x+c)^4*a^3*b^2+78*B*cos(d*x+
c)^4*a*b^4-3*B*cos(d*x+c)^3*a^2*b^3+54*B*cos(d*x+c)^2*a*b^4-24*B*cos(d*x+c)
^6*a^4*b+12*B*cos(d*x+c)^6*a^3*b^2-57*B*cos(d*x+c)^6*a^2*b^3-75*B*cos(d*x+c
)^6*a*b^4+24*B*cos(d*x+c)^5*a^4*b-24*B*cos(d*x+c)^5*a^3*b^2+60*B*cos(d*x+c)
^5*a^2*b^3-57*B*cos(d*x+c)^5*a*b^4-75*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-75*B*sin(d*x+c)*c
os(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B+C\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c)^3, x)
```

3.815 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c-\right.$

Optimal. Leaf size=397

$$\frac{2(a-b)\sqrt{a+b}\left(-8a^2C+2ab(7B-3C)+b^2(63B-25C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a}}{}\right.\right.}{105b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - 19*a*b^2*C)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(105*b^4*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*B - 25*C) + 2*a*b*(7
*B - 3*C) - 8*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(7*a*b*B - 4*a^2*C + 25
*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^2*d) + (2*(7*b*B + a*
C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b*d) + (2*C*Sec[
c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 1.00261, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4031, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(-4a^2C+7abB+25b^2C\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^2d}+\frac{2(a-b)\sqrt{a+b}\left(-8a^2C+2ab(7B-3C)+b^2(63B-25C)\right)\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - 19*a*b^2*C)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(105*b^4*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*B - 25*C) + 2*a*b*(7
*B - 3*C) - 8*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(7*a*b*B - 4*a^2*C + 25
*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^2*d) + (2*(7*b*B + a*
C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b*d) + (2*C*Sec[
c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
```

```
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}(B+C\sec(c$$

$$= \frac{2C\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7d}$$

$$= \frac{2(7bB+aC)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan}{35bd}$$

$$= \frac{2\left(7abB-4a^2C+25b^2C\right)\sqrt{a+b\sec(c+dx)}\text{ta}}{105b^2d}$$

$$= \frac{2\left(7abB-4a^2C+25b^2C\right)\sqrt{a+b\sec(c+dx)}\text{ta}}{105b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(14a^2bB-63b^3B-8a^3C-19ab\right.}{}$$

Mathematica [B] time = 24.4625, size = 3330, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*
C)*Sin[c + d*x])/(105*b^3) + (2*Sec[c + d*x]^2*(7*b*B*Sin[c + d*x] + a*C*Si
n[c + d*x]))/(35*b) + (2*Sec[c + d*x]*(7*a*b*B*Sin[c + d*x] - 4*a^2*C*Sin[c
 + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b^2) + (2*C*Sec[c + d*x]^2*Tan[c + d
*x])/7))/d - (2*((2*a^2*B)/(15*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (3*b*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (19*a*C)/(10
5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*C)/(105*b^2*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*B*Sqrt[Sec[c + d*x]])/(15*Sqrt
[b + a*Cos[c + d*x]]) + (2*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos
[c + d*x]]) - (8*a^4*C*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (17*a^2*C*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) + (5*b*
C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (3*a*B*Cos[2*(c + d*x
)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*B*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*C*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^
2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(
-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*C - 2*a*b*
(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^
2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
)/(105*b^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x
]]*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-14*
a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*C - 2*a*b*(7*B
 + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
```

```
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-14*a^2*b*B + 63*b^3
*B + 8*a^3*C + 19*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(6
3*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*(((-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-14*a^2*b*B + 63*b^3*B
 + 8*a^3*C + 19*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*S
in[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^2*C - 2*a*b*(7*B + 3*C)
+ b^2*(63*B + 25*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c
 + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-14*a^2*b*B + 63*b^3*B + 8*a^3*C +
 19*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^2
*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((
a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))] - a*(-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Cos[c + d*x
]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-14*a^2*b*B + 63*b^3*
B + 8*a^3*C + 19*a*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d
*x]*Tan[(c + d*x)/2] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a
 + b)*(8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)]) + ((a + b)*(-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*
b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^3*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-14*a^2*b*B + 63*b^3*B + 8
*a^3*C + 19*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B +
25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
 + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/
(105*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.141, size = 3439, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-63*B*cos(d*x+c)^4*b^4+42*B*cos(d*x+c)^3*b^4+21*B*cos(d*x+c)*b^4-1
4*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^3*b-8*C*cos(d*x+c)^5*a^4+8*C*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-25*C*sin
(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^4+8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-25*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-7*B*cos(d*x+c)^
3*a^2*b^2+28*B*cos(d*x+c)^2*a*b^3+63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-63*B*cos(d*x+c)^4*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*b^4+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*b^4-63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-14*B*cos(d*x+c)^4*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b
^2+63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b^3+14*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-49*B*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
*b^3-14*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a^3*b-14*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+63*B*cos(d*x+c)^3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b^3+14*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-49*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-14*B*cos(d*x+c)^
4*a^3*b+14*B*cos(d*x+c)^4*a^2*b^2+35*B*cos(d*x+c)^4*a*b^3+14*B*cos(d*x+c)^5
*a^3*b-7*B*cos(d*x+c)^5*a^2*b^2-63*B*cos(d*x+c)^5*a*b^3-8*C*cos(d*x+c)^4*a^
3*b+20*C*cos(d*x+c)^4*a^2*b^2-19*C*cos(d*x+c)^4*a*b^3+4*C*cos(d*x+c)^3*a^3*
b+26*C*cos(d*x+c)^3*a*b^3-C*cos(d*x+c)^2*a^2*b^2+18*C*cos(d*x+c)*a*b^3+4*C*
cos(d*x+c)^5*a^3*b-19*C*cos(d*x+c)^5*a^2*b^2-25*C*cos(d*x+c)^5*a*b^3+8*C*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^3*b+19*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+19*C*sin(d*x+c)*cos(d*x+c)^4*(
```

```
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*C*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b-2*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-19*C*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*C*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b+19*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+19*C*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*C*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b-2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-19*C*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*C*
cos(d*x+c)^4*a^4+10*C*cos(d*x+c)^2*b^4-25*C*cos(d*x+c)^4*b^4+15*C*b^4)/(b+a
*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^4+B\sec\left(dx+c\right)^3\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")

[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B+C\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c)^2, x)
```

3.816 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=314

$$-\frac{2(a-b)\sqrt{a+b}(-2aC+5bB-9bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(5*a*b*B - 2*a^2*C + 9*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3
*d) - (2*(a - b)*Sqrt[a + b]*(5*b*B - 2*a*C - 9*b*C)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) + (2*(5*b*B - 2*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (
2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.601844, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4010, 4002, 4005, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\left(-2a^2C+5abB+9b^2C\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-2$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(5*a*b*B - 2*a^2*C + 9*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3
*d) - (2*(a - b)*Sqrt[a + b]*(5*b*B - 2*a*C - 9*b*C)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) + (2*(5*b*B - 2*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (
2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}(B+C\sec(c+$$

$$= \frac{2C(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \sec(c}{}$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2}{}$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(5abB-2a^2C+9b^2C\right)\cot(c+}{}$$

Mathematica [A] time = 18.4604, size = 434, normalized size = 1.38

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2b(a+b)(-2aC+5bB+9bC)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{Ellipti}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b
)*(-5*a*b*B + 2*a^2*C - 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(5*b*B - 2*a*C + 9*b*C)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-5*a*b*
B + 2*a^2*C - 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))/(15*b^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]
*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Sec[c + d*x]]*((2*(5*a*b*B - 2*a^2*C + 9
*b^2*C)*Sin[c + d*x])/(15*b^2) + (2*Sec[c + d*x]*(5*b*B*Sin[c + d*x] + a*C*
Sin[c + d*x]))/(15*b) + (2*C*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.781, size = 2498, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(2*C*cos(d*x+c)^3*a^3-5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3
+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^3-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*C*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*B*cos(
d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^3+5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*B*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*B*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^2+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*C*sin(d*x+c)*cos(d*
```

```
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b+7*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-
2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b+7*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+C*cos(d*x+c)^4*a^2*b+9*C*co
s(d*x+c)^4*a*b^2-2*C*cos(d*x+c)^3*a^2*b-5*C*cos(d*x+c)^3*a*b^2-4*C*cos(d*x+
c)*a*b^2+5*B*cos(d*x+c)^3*a*b^2-10*B*cos(d*x+c)^2*a*b^2+5*B*cos(d*x+c)^4*a^
2*b+5*B*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)^3*a^2*b+C*cos(d*x+c)^2*a^2*b-2*C*
cos(d*x+c)^4*a^3+9*C*cos(d*x+c)^3*b^3-6*C*cos(d*x+c)^2*b^3+5*B*cos(d*x+c)^3
*b^3-5*B*cos(d*x+c)*b^3+2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*C*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-3*C*b^3
)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^3+B\sec\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B+C\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c), x)
```

3.817 $\int \sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=256

$$\frac{2(a-b)\sqrt{a+b}(3B-C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2(a-b)\sqrt{a+}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*(a - b)*
Sqrt[a + b]*(3*B - C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*C*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.288772, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4056, 4058, 12, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}(aC+3bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2(a-b)\sqrt{a+b}(3B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*(a - b)*
Sqrt[a + b]*(3*B - C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*C*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(3*d)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}(3aB+bC)\,\mathrm{s}}{}$$

$$=\frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int\frac{\left(\frac{1}{2}(-3bB-aC\right.}{\sqrt{}}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bB+aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{3b^2d}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bB+aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{3b^2d}$$

Mathematica [A] time = 14.9254, size = 358, normalized size = 1.4

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2b(a+b)(3B+C)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(-2*(a +
b)*(3*b*B + a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + 2*b*(a + b)*(3*B + C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] - (3*b*B + a*C)*Cos[c + d*x]*(b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*d*(b + a*Cos[c + d*x])
*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Sec[c + d*x]]*(
(2*(3*b*B + a*C)*Sin[c + d*x])/(3*b) + (2*C*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.524, size = 1752, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/b*(-1+cos(d*x+c))^2*(3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b+C*cos
(d*x+c)^3*a*b+C*cos(d*x+c)^2*a*b-2*C*cos(d*x+c)*a*b-3*B*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+
3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)^2*b^2+3*B*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-C*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+C*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^2-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*B*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^2-b^2*C-3*B*cos(d*x+c)*b^2+C*cos(d*x+c)^3*a^2-C*cos(d*x+c)^
2*a^2+C*cos(d*x+c)^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1
)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B+C\sec\left(c+dx\right)\right)\sqrt{a+b\sec\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

3.818 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=320

$$\frac{2\sqrt{a+b}(aC+b(B-C))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2B\sqrt{a+b}\cot}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(b*(B - C) +
 a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rubi [A] time = 0.361109, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 3916, 3784, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(aC+b(B-C))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2B\sqrt{a+b}\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(b*(B - C) +
 a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3916

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sqrt{a+b\sec(c+dx)}(B+C\sec(c+dx))\,dx$$
$$= (aB)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+d\ldots}{\ldots}$$
$$= -\frac{2\sqrt{a+b}B\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec\ldots}}{\sqrt{a+\ldots}}\right.\right.}{a\ldots}$$
$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\ldots}}{\sqrt{\ldots}}\right.\right.}{b\ldots}$$

Mathematica [C] time = 17.5319, size = 863, normalized size = 2.7

$$\frac{2C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}\left(a\sqrt{\frac{b-a}{a+b}}C\tan^5\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{b-a}{a+b}}C\tan^5\left(\frac{1}{2}(c+dx)\right)-2a\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*Sqrt[a + b*Sec[c + d*x]]
*(a*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] + b*Sqrt[(-a + b)/(a + b)]*C*
Tan[(c + d*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^3 + a*Sqrt
[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c
 + d*x)/2]^5 + (2*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (
2*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 - I*(a - b)*C*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b
)*(B - C)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(Sqrt[(-a +
b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c
+ d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)])
```

Maple [B] time = 0.45, size = 1372, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*
(B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*a-B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)*b-2*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-C*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-C*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*b+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*a+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+B*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-B*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-2*B*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-C*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-C*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+
C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
```

```
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*a+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)*b-C*cos(d*x+c)^2*a+C*cos(d*x+c)*a-C*cos(d*x+c)*b+C*b)/sin(d*x+c)^5/
(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*cos(
d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)\sec(dx+c)^2 + B\cos(dx+c)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c))*sqrt
(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*cos(
d*x + c), x)
```

3.819 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=344

$$\frac{\sqrt{a+b}(B+2C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(2aC+bB)}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(B + 2*C)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (Sqrt[a + b]*(b*B + 2*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.46364, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4032, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(B+2C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}(2aC+bB)\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(B + 2*C)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (Sqrt[a + b]*(b*B + 2*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \cos(c+dx)\sqrt{a+b\sec(c+dx)}(B+C\sec(c+$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{\frac{1}{2}(bB+2$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}(bB)\int \frac{\sec$$

$$= \frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{bd}$$

Mathematica [C] time = 18.0138, size = 1107, normalized size = 3.22

$$\frac{\sqrt{a+b\sec(c+dx)}\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(a\sqrt{\frac{b-a}{a+b}}B\tan^5\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{b-a}{a+b}}B\tan^5\left(\frac{1}{2}(c+dx)\right)-2a\sqrt{\frac{b-}{a-}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*
x)/2] + b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 2*a*Sqrt[(-a + b)/(a
+ b)]*B*Tan[(c + d*x)/2]^3 + a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5
- b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (2*I)*b*B*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*C*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] - (4*I)*a*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - I*(a - b)*B*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + (2*I)*(a - b)*C*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)]))/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b - b*Tan[(c +
d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2))
```

Maple [B] time = 0.423, size = 1386, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(-2*B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*B*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*b+B*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a+B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
```

```
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*b-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+2*C*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+4*C*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a-2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*b+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*b+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b-2*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+2*C*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+4*C*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+B*cos(d*x+c)^3*a-
B*a*cos(d*x+c)^2+B*cos(d*x+c)^2*b-B*b*cos(d*x+c))*(cos(d*x+c)+1)^2*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*cos(
d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)^2\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c))*
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*cos(
d*x + c)^2, x)
```

3.820 $\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=429

$$\frac{\sqrt{a+b}(2a(B+2C)+bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4ad}-\frac{\sqrt{a+b}\left(4a^2\right.}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(b*B + 4*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
b*B + 2*a*(B + 2*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2*B - b^2*B
+ 4*a*b*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((b*B + 4*a*C)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (B*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.805279, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4032, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2B+4abC-b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{(4aC+bB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(b*B + 4*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
b*B + 2*a*(B + 2*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2*B - b^2*B
+ 4*a*b*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((b*B + 4*a*C)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (B*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4032

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}(B+C\sec(c$$

$$= \frac{B\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} +$$

$$= \frac{(bB+4aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{B}{}$$

$$= \frac{(bB+4aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{B}{}$$

$$= \frac{(a-b)\sqrt{a+b}(bB+4aC)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{4}$$

$$= \frac{(a-b)\sqrt{a+b}(bB+4aC)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{4}$$

Mathematica [B] time = 18.6505, size = 1161, normalized size = 2.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (B*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d
*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*b*B*Tan[(c + d*x)/2] + b^2*B*Ta
n[(c + d*x)/2] + 4*a^2*C*Tan[(c + d*x)/2] + 4*a*b*C*Tan[(c + d*x)/2] - 2*a*
b*B*Tan[(c + d*x)/2]^3 - 8*a^2*C*Tan[(c + d*x)/2]^3 + a*b*B*Tan[(c + d*x)/2
]^5 - b^2*B*Tan[(c + d*x)/2]^5 + 4*a^2*C*Tan[(c + d*x)/2]^5 - 4*a*b*C*Tan[(
c + d*x)/2]^5 - 8*a^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*b*C*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a^2*B*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 2*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*a*b*C*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] + (a + b)*(b*B + 4*a*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(2*
a*B - b*B + 4*b*C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*d*Sqrt[b + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.403, size = 2065, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/a*(-1+cos(d*x+c))^2*(3*B*cos(d*x+c)^3*a*b-B*cos(d*x+c)^2*a*b+4*C*cos
(d*x+c)^2*a*b-4*C*cos(d*x+c)*a*b+8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*C*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*C*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*B*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+B*cos(d*x+c)^
2*b^2+2*B*a^2*cos(d*x+c)^4+8*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-4*B*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+4*C*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+B*b^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
-2*B*cos(d*x+c)*a*b-2*B*cos(d*x+c)^2*a^2+4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^2-B*cos(d*x+c)*b^2+4*C*cos(d*x+c)^3*a^2-4*C*cos(d*x+c)^2*a^2+2*B*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+B*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
a*b+4*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*a*b-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b*sin(d*x+c)+8*B*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-2*B*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2-4*B*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
cos(d*x+c)*a^2+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a*b*sin(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*cos(
d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)^3\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c))*
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*cos(
d*x + c)^3, x)
```

3.821 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=573

$$-\frac{2(a-b)\sqrt{a+b}\left(6a^2b^2(11B-24C)+4a^3b(22B-9C)-48a^4C+3ab^3(143B-471C)-3b^4(539B-225C)\right)\cot(c+d}{3465b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(88*a^4*b*B + 363*a^2*b^3*B + 1617*b^5*B - 48*a^5*C
 - 108*a^3*b^2*C + 2088*a*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^5*d) - (2*(a - b)*Sq
rt[a + b]*(3*a*b^3*(143*B - 471*C) - 3*b^4*(539*B - 225*C) + 6*a^2*b^2*(11*
B - 24*C) + 4*a^3*b*(22*B - 9*C) - 48*a^4*C)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) + (
2*(88*a^3*b*B + 429*a*b^3*B - 48*a^4*C - 144*a^2*b^2*C + 675*b^4*C)*Sqrt[a
+ b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^3*d) + (2*(88*a^2*b*B + 539*b^3*B -
 48*a^3*C - 204*a*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(3465*b^3
*d) - (2*(44*a*b*B - 24*a^2*C - 81*b^2*C)*(a + b*Sec[c + d*x])^(5/2)*Tan[c
+ d*x])/(693*b^3*d) + (2*(11*b*B - 6*a*C)*Sec[c + d*x]*(a + b*Sec[c + d*x])
^(5/2)*Tan[c + d*x])/(99*b^2*d) + (2*C*Sec[c + d*x]^2*(a + b*Sec[c + d*x])^
(5/2)*Tan[c + d*x])/(11*b*d)
```

Rubi [A] time = 1.98108, antiderivative size = 573, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4033, 4092, 4082, 4002, 4005, 3832, 4004}

$$-\frac{2\left(-24a^2C+44abB-81b^2C\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^3d}+\frac{2\left(88a^2bB-48a^3C-204ab^2C+539b^3B\right)\tan(c+}{3465b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(88*a^4*b*B + 363*a^2*b^3*B + 1617*b^5*B - 48*a^5*C
 - 108*a^3*b^2*C + 2088*a*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^5*d) - (2*(a - b)*Sq
rt[a + b]*(3*a*b^3*(143*B - 471*C) - 3*b^4*(539*B - 225*C) + 6*a^2*b^2*(11*
B - 24*C) + 4*a^3*b*(22*B - 9*C) - 48*a^4*C)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) + (
2*(88*a^3*b*B + 429*a*b^3*B - 48*a^4*C - 144*a^2*b^2*C + 675*b^4*C)*Sqrt[a
+ b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^3*d) + (2*(88*a^2*b*B + 539*b^3*B -
 48*a^3*C - 204*a*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(3465*b^3
*d) - (2*(44*a*b*B - 24*a^2*C - 81*b^2*C)*(a + b*Sec[c + d*x])^(5/2)*Tan[c
+ d*x])/(693*b^3*d) + (2*(11*b*B - 6*a*C)*Sec[c + d*x]*(a + b*Sec[c + d*x])
^(5/2)*Tan[c + d*x])/(99*b^2*d) + (2*C*Sec[c + d*x]^2*(a + b*Sec[c + d*x])^
(5/2)*Tan[c + d*x])/(11*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
```

```
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}(B+C$$

$$= \frac{2C\sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\tan(}{11bd}$$

$$= \frac{2(11bB-6aC)\sec(c+dx)(a+b\sec(c+d}{99b^2d}$$

$$= -\frac{2\left(44abB-24a^2C-81b^2C\right)(a+b\sec(c}{693b^3d}$$

$$= \frac{2\left(88a^2bB+539b^3B-48a^3C-204ab^2C\right)}{3465b^3d}$$

$$= \frac{2\left(88a^3bB+429ab^3B-48a^4C-144a^2b^2C\right.}{346}$$

$$= \frac{2\left(88a^3bB+429ab^3B-48a^4C-144a^2b^2C\right.}{346}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(88a^4bB+363a^2b^3B+16\right.}{}$$

Mathematica [B] time = 26.5058, size = 4220, normalized size = 7.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*(-88*a^4*b*B - 363*a^2*b^3*B
- 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Sin[c + d*x])/(3465
*b^4) + (2*Sec[c + d*x]^4*(11*b*B*Sin[c + d*x] + 12*a*C*Sin[c + d*x]))/99 +
 (2*Sec[c + d*x]^3*(110*a*b*B*Sin[c + d*x] + 3*a^2*C*Sin[c + d*x] + 81*b^2*
C*Sin[c + d*x]))/(693*b) + (2*Sec[c + d*x]^2*(33*a^2*b*B*Sin[c + d*x] + 539
*b^3*B*Sin[c + d*x] - 18*a^3*C*Sin[c + d*x] + 606*a*b^2*C*Sin[c + d*x]))/(3
465*b^2) + (2*Sec[c + d*x]*(-44*a^3*b*B*Sin[c + d*x] + 968*a*b^3*B*Sin[c +
d*x] + 24*a^4*C*Sin[c + d*x] + 57*a^2*b^2*C*Sin[c + d*x] + 675*b^4*C*Sin[c
+ d*x]))/(3465*b^3) + (2*b*C*Sec[c + d*x]^4*Tan[c + d*x])/11))/(d*(b + a*Co
s[c + d*x])) + (2*((-11*a^2*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]) - (8*a^4*B)/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (
7*b^2*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^5*C)/(115
5*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (12*a^3*C)/(385*b*Sqrt
[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (232*a*b*C)/(385*Sqrt[b + a*Cos[
```

```
c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*B*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[
b + a*Cos[c + d*x]]) - (31*a^3*B*Sqrt[Sec[c + d*x]])/(315*b*Sqrt[b + a*Cos[
c + d*x]]) + (13*a*b*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) -
 (13*a^2*C*Sqrt[Sec[c + d*x]])/(55*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*C*Sq
rt[Sec[c + d*x]])/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]) + (32*a^4*C*Sqrt[Sec[
c + d*x]])/(1155*b^2*Sqrt[b + a*Cos[c + d*x]]) + (15*b^2*C*Sqrt[Sec[c + d*x
]])/(77*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c +
d*x]])/(315*b^3*Sqrt[b + a*Cos[c + d*x]]) - (11*a^3*B*Cos[2*(c + d*x)]*Sqrt
[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) - (7*a*b*B*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (232*a^2*C*Cos[2*(c +
 d*x)]*Sqrt[Sec[c + d*x]])/(385*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*C*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(1155*b^4*Sqrt[b + a*Cos[c + d*x]]) + (12*a
^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(385*b^2*Sqrt[b + a*Cos[c + d*x]]
))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a +
 b)*(-88*a^4*b*B - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C -
2088*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] + 2*b*(a + b)*(-48*a^4*C + 4*a^3*b*(22*B + 9*C) - 6*a^2*b^2*(11*
B + 24*C) + 3*b^4*(539*B + 225*C) + 3*a*b^3*(143*B + 471*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-88*a^4*b*B -
363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Cos[c
 + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^
4*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)*((a*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-88*a^4*b*B
 - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*
(a + b)*(-48*a^4*C + 4*a^3*b*(22*B + 9*C) - 6*a^2*b^2*(11*B + 24*C) + 3*b^4
*(539*B + 225*C) + 3*a*b^3*(143*B + 471*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-88*a^4*b*B - 363*a^2*b^3*B - 1
617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Cos[c + d*x]*(b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^4*(b + a*Cos[c +
d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-88*a^4*b*B - 363*a^2*b^3*B - 1617*b^5*B +
 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-48*a^4*C + 4*a^3*b*(
22*B + 9*C) - 6*a^2*b^2*(11*B + 24*C) + 3*b^4*(539*B + 225*C) + 3*a*b^3*(14
3*B + 471*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] + (-88*a^4*b*B - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3
*b^2*C - 2088*a*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))/(3465*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2
]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-88*a^4*b*B - 363*a^2*b
^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Cos[c + d*x]*(
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-88*a^4*b*B - 363*a^2
*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - S
in[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b
*(a + b)*(-48*a^4*C + 4*a^3*b*(22*B + 9*C) - 6*a^2*b^2*(11*B + 24*C) + 3*b^
4*(539*B + 225*C) + 3*a*b^3*(143*B + 471*C))*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + C
os[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-88*a^4*b*
B - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
```

```
 - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*
Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(-48*a^4*C + 4*a^3*b
*(22*B + 9*C) - 6*a^2*b^2*(11*B + 24*C) + 3*b^4*(539*B + 225*C) + 3*a*b^3*(
143*B + 471*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)
))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-88*a^4*b*B
 - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-88*a^4*b*B
- 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-88*a
^4*b*B - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4
*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2
 + (b*(a + b)*(-48*a^4*C + 4*a^3*b*(22*B + 9*C) - 6*a^2*b^2*(11*B + 24*C) +
 3*b^4*(539*B + 225*C) + 3*a*b^3*(143*B + 471*C))*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c
 + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]) + ((a + b)*(-88*a^4*b*B - 363*a^2*b^3*B - 1617*b^5*B + 48*
a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2
*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^
2]))/(3465*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a
+ b)*(-88*a^4*b*B - 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C -
 2088*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + 2*b*(a + b)*(-48*a^4*C + 4*a^3*b*(22*B + 9*C) - 6*a^2*b^2*(11
*B + 24*C) + 3*b^4*(539*B + 225*C) + 3*a*b^3*(143*B + 471*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-88*a^4*b*B -
 363*a^2*b^3*B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(
c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x
]*Tan[c + d*x]))/(3465*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2
]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 2.543, size = 5368, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{6}+Ba\sec\left(dx+c\right)^{4}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{5}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^6 + B*a*sec(d*x + c)^4 + (C*a + B*b)*sec(d*x + c
)^5)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*se
c(d*x + c)^3, x)
```

3.822 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=475

$$-\frac{2(a-b)\sqrt{a+b}\left(-6a^2b(3B-C)+8a^3C-3ab^2(57B-13C)+3b^3(25B-49C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+d}{a-b}}}{315b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 33*a^2*b^2*C -
 147*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*B
 - 49*C) - 3*a*b^2*(57*B - 13*C) - 6*a^2*b*(3*B - C) + 8*a^3*C)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^3*d) - (2*(18*a^2*b*B - 75*b^3*B - 8*a^3*C - 39*a*b^2*C)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) - (2*(18*a*b*B - 8*a^2*C - 49*b^
2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*b*B - 4*a
*C)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*C*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 1.22388, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4033, 4082, 4002, 4005, 3832, 4004}

$$-\frac{2\left(-8a^2C+18abB-49b^2C\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2bB-8a^3C-39ab^2C-75b^3B\right)\tan(c+dx)\sqrt{}}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 33*a^2*b^2*C -
 147*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*B
 - 49*C) - 3*a*b^2*(57*B - 13*C) - 6*a^2*b*(3*B - C) + 8*a^3*C)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^3*d) - (2*(18*a^2*b*B - 75*b^3*B - 8*a^3*C - 39*a*b^2*C)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) - (2*(18*a*b*B - 8*a^2*C - 49*b^
2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*b*B - 4*a
*C)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*C*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}(B+C$$

$$= \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c}{9bd}$$

$$= \frac{2(9bB-4aC)(a+b\sec(c+dx))^{5/2}\tan(c+}{63b^2d}$$

$$= -\frac{2\left(18abB-8a^2C-49b^2C\right)(a+b\sec(c+}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-39ab^2C\right)\sqrt{a}}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-39ab^2C\right)\sqrt{a}}{315b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(18a^3bB-246ab^3B-8a^4C\right.}{}$$

Mathematica [B] time = 25.9137, size = 3766, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(-18*a^3*b*B + 246*a*b^3*B + 8
*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x
]^3*(9*b*B*Sin[c + d*x] + 10*a*C*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(72*
a*b*B*Sin[c + d*x] + 3*a^2*C*Sin[c + d*x] + 49*b^2*C*Sin[c + d*x]))/(315*b)
 + (2*Sec[c + d*x]*(9*a^2*b*B*Sin[c + d*x] + 75*b^3*B*Sin[c + d*x] - 4*a^3*
C*Sin[c + d*x] + 88*a*b^2*C*Sin[c + d*x]))/(315*b^2) + (2*b*C*Sec[c + d*x]^
3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])) - (2*((2*a^3*B)/(35*b*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82*a*b*B)/(105*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) - (11*a^2*C)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (8*a^4*C)/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (7*b^2*C)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a^2*
B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c
+ d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) + (5*b^2*B*Sqrt[Sec[c + d*x]])/(
21*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*C*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b
 + a*Cos[c + d*x]]) - (31*a^3*C*Sqrt[Sec[c + d*x]])/(315*b*Sqrt[b + a*Cos[c
 + d*x]]) + (13*a*b*C*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) -
(82*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]
]) + (2*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c
 + d*x]]) - (8*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b +
 a*Cos[c + d*x]]) - (11*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*S
qrt[b + a*Cos[c + d*x]]) - (7*a*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(1
5*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*S
ec[c + d*x])^(3/2)*(2*(a + b)*(-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2
*b^2*C + 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(57*B
 + 13*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
```

+ d*x)/2]], (a - b)/(a + b)] + (-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^
2*b^2*C + 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(315*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2
]*Sec[c + d*x]^(3/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x
]*(2*(a + b)*(-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*
C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
- 2*b*(a + b)*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(57*B + 13*C) + 3*b^3*
(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + (-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4
*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))
/(315*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-18*a^3*b*B + 246
*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*C - 6*a^2*
b*(3*B + C) + 3*a*b^2*(57*B + 13*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-18*a^3*b*B + 24
6*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^3*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((
(-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Cos[c + d*
x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-18*a^3*b*B + 246
*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
 Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^3*
C - 6*a^2*b*(3*B + C) + 3*a*b^2*(57*B + 13*C) + 3*b^3*(25*B + 49*C))*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
] + ((a + b)*(-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*
C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b
+ a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^3*C - 6*a^2
b(3*B + C) + 3*a*b^2*(57*B + 13*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))] - a*(-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a
^2*b^2*C + 147*b^4*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
 d*x)/2] - (-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-
18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(8
*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(57*B + 13*C) + 3*b^3*(25*B + 49*C))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1
- ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-18*a^3*b*B + 246*a*b^
3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)
/2]^2]))/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*
(a + b)*(-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b

```
*(a + b)*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(57*B + 13*C) + 3*b^3*(25*B
 + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + (-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 33*a^2*b^2*C + 147*b^4*C)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Co
s[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c +
d*x]*Tan[c + d*x]))/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]
^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.678, size = 4395, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/315/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-75*B*cos(d*x+c)^5*b^5-2*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+18*B*sin(d*x+c)*c
os(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3*b^2-8*C*cos(d*x+c)^5*a^4*b+34*C*cos(d*x+c)^5*a^3*b^2-33*C*cos(d*x+c)^
5*a^2*b^3+10*C*cos(d*x+c)^5*a*b^4+4*C*cos(d*x+c)^4*a^4*b+68*C*cos(d*x+c)^4*
a^2*b^3-C*cos(d*x+c)^3*a^3*b^2+52*C*cos(d*x+c)^3*a*b^4+4*C*cos(d*x+c)^6*a^4
*b-33*C*cos(d*x+c)^6*a^3*b^2-88*C*cos(d*x+c)^6*a^2*b^3-147*C*cos(d*x+c)^6*a
*b^4+53*C*cos(d*x+c)^2*a^2*b^3+85*C*cos(d*x+c)*a*b^4-18*B*sin(d*x+c)*cos(d*
x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4
*b-18*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*b^2+246*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+246*B*sin(d*x+c)*
cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^4-153*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-246*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-18*B*sin(d*x
+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^4*b-18*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+8*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x
+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^5-147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^5+8*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5+147*C*cos(d*x+c)^5
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
```

s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^5-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^5+8*C*cos(d*x+c)^5*a^5-147*C*cos(d*x+c)^5*b^5+9
8*C*cos(d*x+c)^4*b^5+14*C*cos(d*x+c)^2*b^5-8*C*cos(d*x+c)^6*a^5+246*B*sin(d
*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^3+246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+18*B*sin(d*x+c)*cos(d*x+c)^4*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-153*B
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*b^3-246*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-33*C*cos(d*x+c)^5*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3
-186*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^4+8*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+33*C*cos(d*x+c)^5*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b
^2+33*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^3+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-8*C*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^4*b-2*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*b^2-33*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-186*C*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^4+8*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^4*b+33*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+33*C*cos(d*x+c)
^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^3+147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-8*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+35*C*b^5+30*B
*cos(d*x+c)^3*b^5+45*B*cos(d*x+c)*b^5-9*B*cos(d*x+c)^4*a^3*b^2+204*B*cos(d*
x+c)^4*a*b^4+81*B*cos(d*x+c)^3*a^2*b^3+117*B*cos(d*x+c)^2*a*b^4+18*B*cos(d*
x+c)^6*a^4*b-9*B*cos(d*x+c)^6*a^3*b^2-246*B*cos(d*x+c)^6*a^2*b^3-75*B*cos(d
*x+c)^6*a*b^4-18*B*cos(d*x+c)^5*a^4*b+18*B*cos(d*x+c)^5*a^3*b^2+165*B*cos(d
*x+c)^5*a^2*b^3-246*B*cos(d*x+c)^5*a*b^4-75*B*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-75*B*sin(d*

```
x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{5}+Ba\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^5 + B*a*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c
)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*se
c(d*x + c)^2, x)
```

3.823 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=387

$$-\frac{2(a-b)\sqrt{a+b}\left(-6a^2C+ab(21B-57C)-b^2(63B-25C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{105b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 82*a*b^2*C)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(105*b^3*d) - (2*(a - b)*Sqrt[a + b]*(a*b*(21*B - 57*C) - b^2*(63
*B - 25*C) - 6*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) + (2*(21*a*b*B - 6*a^2*C +
25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) + (2*(7*b*B - 2*
a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*C*(a + b*Sec[c
+ d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.853035, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4010, 4002, 4005, 3832, 4004}

$$\frac{2\left(-6a^2C+21abB+25b^2C\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}-\frac{2(a-b)\sqrt{a+b}\left(-6a^2C+ab(21B-57C)-b^2(63B-25C)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 82*a*b^2*C)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(105*b^3*d) - (2*(a - b)*Sqrt[a + b]*(a*b*(21*B - 57*C) - b^2*(63
*B - 25*C) - 6*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) + (2*(21*a*b*B - 6*a^2*C +
25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) + (2*(7*b*B - 2*
a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*C*(a + b*Sec[c
+ d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
```

```
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}(B+C$$

$$= \frac{2C(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int}{}$$

$$= \frac{2(7bB-2aC)(a+b\sec(c+dx))^{3/2}\tan(c+}{35bd}$$

$$= \frac{2\left(21abB-6a^2C+25b^2C\right)\sqrt{a+b\sec(c+}}{105bd}$$

$$= \frac{2\left(21abB-6a^2C+25b^2C\right)\sqrt{a+b\sec(c+}}{105bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(21a^2bB+63b^3B-6a^3C-\right.}{}$$

Mathematica [B] time = 24.3403, size = 3342, normalized size = 8.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*(-21*a^2*b*B - 63*b^3*B + 6*a
^3*C - 82*a*b^2*C)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]^2*(7*b*B*Sin[c
 + d*x] + 8*a*C*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(42*a*b*B*Sin[c + d*x]
+ 3*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b) + (2*b*C*Sec[c + d
*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])) + (2*(-(a^2*B)/(5*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*b^2*B)/(5*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]) + (2*a^3*C)/(35*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (82*a*b*C)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(a^3*B*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (a*b*B*Sqrt[Sec
[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (31*a^2*C*Sqrt[Sec[c + d*x]])/(1
05*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*C*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b
+ a*Cos[c + d*x]]) + (5*b^2*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*
x]]) - (a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c +
d*x]]) - (3*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c
+ d*x]]) - (82*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*C
os[c + d*x]]) + (2*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[
b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)*(2*(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(
a + b)*(-6*a^2*C + 3*a*b*(7*B + 19*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*b*B - 63
*b^3*B + 6*a^3*C - 82*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c +
 d*x)/2]^2]*Sec[c + d*x]^(3/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Si
n[c + d*x]*(2*(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a
 + b)*(-6*a^2*C + 3*a*b*(7*B + 19*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*b*B - 63*
b^3*B + 6*a^3*C - 82*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c
+ d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*
(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^2*
C + 3*a*b*(7*B + 19*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*b*B - 63*b^3*B + 6*a^3*
C - 82*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) +
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-21*a^2*b*B - 63*b^3*B + 6*a^3*
C - 82*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + (
(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b
)*(-6*a^2*C + 3*a*b*(7*B + 19*C) + b^2*(63*B + 25*C))*Sqrt[(b + a*Cos[c + d
```

```
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*
x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-
21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))] + (b*(a + b)*(-6*a^2*C + 3*a*b*(7*B + 19*C) + b^2*(63*B + 25*
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b
 + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-21*a^2*b*B - 63*b^3*B
+ 6*a^3*C - 82*a*b^2*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c
 + d*x)/2] - (-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-21*a^2*b*B - 63
*b^3*B + 6*a^3*C - 82*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-6*a^2*C + 3*a*b*(7*B + 19*C) + b
^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-21
*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x
)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*
x)/2]^2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((
2*(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^
2*C + 3*a*b*(7*B + 19*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*b*B - 63*b^3*B + 6*a^
3*C - 82*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
 d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.112, size = 3424, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(63*B*cos(d*x+c)^4*b^4-42*B*cos(d*x+c)^3*b^4-21*B*cos(d*x+c)*b^4-2
1*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^3*b-6*C*cos(d*x+c)^5*a^4+6*C*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*sin
(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^4+6*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-63*B*cos(d*x+c)
```

```
^3*a^2*b^2-63*B*cos(d*x+c)^2*a*b^3-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)^4
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*b^4-63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-21*B*cos(d*x+c)^4*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*
b^2-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b^3+21*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+84*B*cos(d*x+c)^4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a*b^3-21*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^3*b-21*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b^3+21*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+84*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-21*B*cos(d*x+c)
^4*a^3*b+21*B*cos(d*x+c)^4*a^2*b^2+21*B*cos(d*x+c)^5*a^3*b+42*B*cos(d*x+c)^
5*a^2*b^2+63*B*cos(d*x+c)^5*a*b^3-6*C*cos(d*x+c)^4*a^3*b-55*C*cos(d*x+c)^4*
a^2*b^2+82*C*cos(d*x+c)^4*a*b^3+3*C*cos(d*x+c)^3*a^3*b-68*C*cos(d*x+c)^3*a*
b^3-27*C*cos(d*x+c)^2*a^2*b^2-39*C*cos(d*x+c)*a*b^3+3*C*cos(d*x+c)^5*a^3*b+
82*C*cos(d*x+c)^5*a^2*b^2+25*C*cos(d*x+c)^5*a*b^3+6*C*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-8
2*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b^2-82*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*C*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+
51*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b^2+82*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*C*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b
-82*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b^2-82*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*C*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
```

```
b+51*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b^2+82*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*C*cos(d*x+c)^4*a^4-
10*C*cos(d*x+c)^2*b^4+25*C*cos(d*x+c)^4*b^4-15*C*b^4)/(b+a*cos(d*x+c))/cos(
d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{4}+Ba\sec\left(dx+c\right)^{2}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^4 + B*a*sec(d*x + c)^2 + (C*a + B*b)*sec(d*x + c
)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*se
c(d*x + c), x)
```

3.824 $\int (a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=312

$$\frac{2(a-b)\sqrt{a+b}(15aB-3aC-5bB+9bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{15bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(20*a*b*B + 3*a^2*C + 9*b^2*C)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^
2*d) + (2*(a - b)*Sqrt[a + b]*(15*a*B - 5*b*B - 3*a*C + 9*b*C)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*b*d) + (2*(5*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*
d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.467749, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4056, 4058, 12, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\left(3a^2C+20abB+9b^2C\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^2d}+\frac{2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(20*a*b*B + 3*a^2*C + 9*b^2*C)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^
2*d) + (2*(a - b)*Sqrt[a + b]*(15*a*B - 5*b*B - 3*a*C + 9*b*C)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*b*d) + (2*(5*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*
d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int\sqrt{a+b\sec(c\ldots}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2C(a+b\,\mathrm{s}\ldots}{}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2C(a+b\,\mathrm{s}\ldots}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(20abB+3a^2C+9b^2C\right)\cot(c+dx)E\left(\mathrm{si}\ldots\right.}{15b\ldots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(20abB+3a^2C+9b^2C\right)\cot(c+dx)E\left(\mathrm{si}\ldots\right.}{15b\ldots}$$

Mathematica [A] time = 18.6249, size = 456, normalized size = 1.46

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2\left(3a^2C+20abB+9b^2C\right)\sin(c+dx)}{15b}+\frac{2}{15}\sec(c+dx)(6aC\sin(c+dx)+5bB\sin(c+dx))+\frac{2}{5}bC\,\mathrm{t}\ldots\right.}{d(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a
+ b)*(20*a*b*B + 3*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*(5*B + C) + b*(5*B + 9*
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 + (20*a*b*B + 3*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c +
```

```
d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*
((2*(20*a*b*B + 3*a^2*C + 9*b^2*C)*Sin[c + d*x])/(15*b) + (2*Sec[c + d*x]*(
5*b*B*Sin[c + d*x] + 6*a*C*Sin[c + d*x]))/15 + (2*b*C*Sec[c + d*x]*Tan[c +
d*x])/5))/(d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.732, size = 2683, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/15/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(3*C*cos(d*x+c)^3*a^3+20*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-15*B*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
b-9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^3+3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+9*C*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*C*sin
(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^3-5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+20*B*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2-20*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+20*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+20*B*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2-20*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2-3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b-12*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
```

```
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2-3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-12*C*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^2-15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b-6*C*cos(d*x+c)^4*a^2*b-9*C*cos(d*x+c)^4*a*b^2-3
*C*cos(d*x+c)^3*a^2*b+9*C*cos(d*x+c)*a*b^2-20*B*cos(d*x+c)^3*a*b^2+25*B*cos
(d*x+c)^2*a*b^2-20*B*cos(d*x+c)^4*a^2*b-5*B*cos(d*x+c)^4*a*b^2+20*B*cos(d*x
+c)^3*a^2*b+9*C*cos(d*x+c)^2*a^2*b-3*C*cos(d*x+c)^4*a^3-9*C*cos(d*x+c)^3*b^
3+6*C*cos(d*x+c)^2*b^3-5*B*cos(d*x+c)^3*b^3+5*B*cos(d*x+c)*b^3+3*C*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*C*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(
d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec(dx+c)^3 + Ba\sec(dx+c) + (Ca+Bb)\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + B*a*sec(d*x + c) + (C*a + B*b)*sec(d*x + c)^
2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2), x
)
```

3.825 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=380

$$\frac{2\sqrt{a+b}\left(3a^2C+ab(6B-4C)-b^2(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{3bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + 4*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a +
 b]*(a*b*(6*B - 4*C) - b^2*(3*B - C) + 3*a^2*C)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*
a*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*C*Sqrt[a + b*Sec[c + d*x
]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.529385, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 3918, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(3a^2C+ab(6B-4C)-b^2(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + 4*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a +
 b]*(a*b*(6*B - 4*C) - b^2*(3*B - C) + 3*a^2*C)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*
a*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*C*Sqrt[a + b*Sec[c + d*x
]]*Tan[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
```

```
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+b\sec(c+dx))^{3/2}(B+C\sec(c+dx))\,dx$$

$$= \frac{2bC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{3a^2}{2}$$

$$= \frac{2bC\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{3a^2}{2}$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bB+4aC)\cot(c+dx)E\left(\mathrm{si}\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bB+4aC)\cot(c+dx)E\left(\mathrm{si}\right.}{}$$

Mathematica [B] time = 24.0928, size = 6047, normalized size = 15.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.509, size = 2340, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(-3*B*cos(d*x+c)^3*a*b+3*B*cos(d*x+c)^2*a*b-C*cos(d
*x+c)^3*a*b-4*C*cos(d*x+c)^2*a*b+5*C*cos(d*x+c)*a*b+3*B*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-4
*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b+4*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-4*C*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*C*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*B*cos(d*x+c)^2*b^2-6*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
```

```
+b))^(1/2))*a*b+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2-3*C*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2-3*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-C*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*C*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+3*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2
-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+b^2*C+3*B*cos(d*x+c)*b^2-4*C*c
os(d*x+c)^3*a^2+4*C*cos(d*x+c)^2*a^2-C*cos(d*x+c)^2*b^2-6*B*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)
*a^2+3*B*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)*a^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1
)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos(dx+c)\sec(dx+c)^3 + Ba\cos(dx+c)\sec(dx+c) + (Ca+Bb)\cos(dx+c)\sec(dx+c)^2\right)\sqrt{b\sec(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)*sec(d*x + c)^3 + B*a*cos(d*x + c)*sec(d*x + c) +
 (C*a + B*b)*cos(d*x + c)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^{2} + B \sec \left(dx+c\right)\right)\left(b \sec \left(dx+c\right) + a\right)^{\frac{3}{2}} \cos \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c), x)
```

3.826 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=361

$$\frac{\sqrt{a+b}(a(B+4C)+2b(B-C))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{(a}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(a*B - 2*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(2*b*
(B - C) + a*(B + 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (Sqrt[a + b]*(3*b*B + 2*a*C)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d + (a*B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.548051, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4025, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(a(B+4C)+2b(B-C))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(a*B - 2*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(2*b*
(B - C) + a*(B + 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (Sqrt[a + b]*(3*b*B + 2*a*C)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d + (a*B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+b\sec(c+dx))^{3/2}(B+C$$

$$= \frac{aB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \int \frac{-\frac{1}{2}}{}$$

$$= \frac{aB\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}(b(a$$

$$= \frac{(a-b)\sqrt{a+b}(aB-2bC)\cot(c+dx)E\left(\sin\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}(aB-2bC)\cot(c+dx)E\left(\sin\right.}{}$$

Mathematica [B] time = 18.3257, size = 979, normalized size = 2.71

$$\frac{2bC\cos(c+dx)\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{d(b+a\cos(c+dx))} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(a^2B\tan^5\left(\frac{1}{2}(c+dx)\right)-abB\tan^5\left(\frac{1}{2}(c+dx)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*b*C*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[
c + d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)
]*(a^2*B*Tan[(c + d*x)/2] + a*b*B*Tan[(c + d*x)/2] - 2*a*b*C*Tan[(c + d*x)/
2] - 2*b^2*C*Tan[(c + d*x)/2] - 2*a^2*B*Tan[(c + d*x)/2]^3 + 4*a*b*C*Tan[(c
 + d*x)/2]^3 + a^2*B*Tan[(c + d*x)/2]^5 - a*b*B*Tan[(c + d*x)/2]^5 - 2*a*b*
C*Tan[(c + d*x)/2]^5 + 2*b^2*C*Tan[(c + d*x)/2]^5 - 6*a*b*B*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^2*C
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - 6*a*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^2*C*EllipticPi[-1, -ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (a + b)*(a*B - 2*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a*b*(B - C) + a^2
*C - b^2*(B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(b + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.492, size = 2199, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*B*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-B*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-B*cos(d*x+c)^
2*a*b-2*C*cos(d*x+c)^2*a*b+2*C*cos(d*x+c)*a*b+2*C*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+
2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*C*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-B*cos(d*
x+c)^3*a^2+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2-6*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+B*cos(d*x+c)*a*b+B*c
os(d*x+c)^2*a^2-2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*C*cos(d*x+c)*b^2+
2*b^2*C+4*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b*sin(d*x+c)-B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*a*b+2*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-6*B*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+2*
C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}+Ba\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + B*a*cos(d*x + c)^2*sec(d*x +
c) + (C*a + B*b)*cos(d*x + c)^2*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c)^2, x)
```

3.827 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=428

$$\frac{\sqrt{a+b}(2aB+4aC+5bB+8bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4d}-\sqrt{a+}$$

```
[Out] ((a - b)*Sqrt[a + b]*(5*b*B + 4*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[a + b]*(
2*a*B + 5*b*B + 4*a*C + 8*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2*B
+ 3*b^2*B + 12*a*b*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + ((5*b*B + 4*a*C)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Cos[c + d*x]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.874381, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4025, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2B+12abC+3b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{(4aC+5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(5*b*B + 4*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[a + b]*(
2*a*B + 5*b*B + 4*a*C + 8*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2*B
+ 3*b^2*B + 12*a*b*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + ((5*b*B + 4*a*C)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Cos[c + d*x]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(B+C\sec$$

$$= \frac{aB\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}$$

$$= \frac{(5bB+4aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} +$$

$$= \frac{(5bB+4aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} +$$

$$= \frac{(a-b)\sqrt{a+b}(5bB+4aC)\cot(c+dx)E\left(\sin^{-}\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}(5bB+4aC)\cot(c+dx)E\left(\sin^{-}\right.}{}$$

Mathematica [C] time = 19.2847, size = 1580, normalized size = 3.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (a*B*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) - (Sqrt[a + b*Sec[c +
 d*x]]*(5*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 5*b^2*Sqrt[(-a +
b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*
x)/2] + 4*a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] - 10*a*b*Sqrt[(-a +
 b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - 8*a^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c +
 d*x)/2]^3 + 5*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 5*b^2*Sqrt
[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*C*Ta
n[(c + d*x)/2]^5 - 4*a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - (8*I
)*a^2*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*B*Ellipt
icPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (24*I)*a*b*C*EllipticPi[-((a + b
)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^2*B*EllipticPi[-((a + b)/(a - b)), I*
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*B*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (24*I)*a*b*C*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(5*b*B + 4*a*C)*Ell
ipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*(2*a*B + b*
```

```
(B + 4*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a
 + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[
(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3
/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
+ d*x)/2]^2)])
```

Maple [B] time = 0.416, size = 2439, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+7*B*cos(d*x+c)^3*a*b-5*B*cos(d*
x+c)^2*a*b+4*C*cos(d*x+c)^2*a*b-4*C*cos(d*x+c)*a*b+8*B*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-16*C*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b+4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+5*B*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+6*B*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin
(d*x+c)+5*B*cos(d*x+c)^2*b^2+2*B*a^2*cos(d*x+c)^4+24*C*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-
4*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*si
n(d*x+c)+4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*sin(d*x+c)+5*B*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))-2*B*cos(d*x+c)*a*b-2*B*cos(d*x+c)^2*a^2-8*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^2+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+5*B*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-5*B
*cos(d*x+c)*b^2+4*C*cos(d*x+c)^3*a^2-4*C*cos(d*x+c)^2*a^2+2*B*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+5*B*Elli
```

```
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b
+4*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*a*b-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b*sin(d*x+c)+8*B*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+6*B*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2-4*B*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^2+24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*a*b*sin(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3\sec(dx+c)^3 + Ba\cos(dx+c)^3\sec(dx+c) + (Ca+Bb)\cos(dx+c)^3\sec(dx+c)^2\right)\sqrt{b\sec(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3*sec(d*x + c)^3 + B*a*cos(d*x + c)^3*sec(d*x +
c) + (C*a + B*b)*cos(d*x + c)^3*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c)^3, x)
```

3.828 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=520

$$\frac{\sqrt{a+b}\left(16a^2B+12a^2C+14abB+30abC+3b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{24ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*B + 3*b^2*B + 30*a*b*C)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*
d) + (Sqrt[a + b]*(16*a^2*B + 14*a*b*B + 3*b^2*B + 12*a^2*C + 30*a*b*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(24*a*d) - (Sqrt[a + b]*(12*a^2*b*B - b^3*B + 8*a^3*C + 6*a*b^2*
C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((16*a^2*B + 3*b^2*B + 30*a*b*C)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a*d) + ((7*b*B + 6*a*C)*Cos[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (a*B*Cos[c + d*x]^2*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.32453, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4025, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2B+30abC+3b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24ad}+\frac{\sqrt{a+b}\left(16a^2B+12a^2C+14abB+30abC+3b^2B\right)\cot(c+dx}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*B + 3*b^2*B + 30*a*b*C)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*
d) + (Sqrt[a + b]*(16*a^2*B + 14*a*b*B + 3*b^2*B + 12*a^2*C + 30*a*b*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(24*a*d) - (Sqrt[a + b]*(12*a^2*b*B - b^3*B + 8*a^3*C + 6*a*b^2*
C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((16*a^2*B + 3*b^2*B + 30*a*b*C)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a*d) + ((7*b*B + 6*a*C)*Cos[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (a*B*Cos[c + d*x]^2*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{3/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}(B+C\,\mathrm{se}$$

$$= \frac{aB\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{3d}$$

$$= \frac{(7bB+6aC)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{12d}$$

$$= \frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}}{24ad}$$

$$= \frac{\left(16a^2B+3b^2B+30abC\right)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}}{24ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2B+3b^2B+30abC\right)\cot(c}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2B+3b^2B+30abC\right)\cot(c}{}$$

Mathematica [B] time = 18.9626, size = 1532, normalized size = 2.95

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((a*B*Sin[c + d*x])/12 + ((7*b*B + 6*a*C)*Sin[2*(
c + d*x)])/24 + (a*B*Sin[3*(c + d*x)])/12))/d + (Sqrt[a + b*Sec[c + d*x]]*S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a^3*B*Tan[(c + d*x)/2] + 16*a^2*b*B*
Tan[(c + d*x)/2] + 3*a*b^2*B*Tan[(c + d*x)/2] + 3*b^3*B*Tan[(c + d*x)/2] +
30*a^2*b*C*Tan[(c + d*x)/2] + 30*a*b^2*C*Tan[(c + d*x)/2] - 32*a^3*B*Tan[(c
 + d*x)/2]^3 - 6*a*b^2*B*Tan[(c + d*x)/2]^3 - 60*a^2*b*C*Tan[(c + d*x)/2]^3
 + 16*a^3*B*Tan[(c + d*x)/2]^5 - 16*a^2*b*B*Tan[(c + d*x)/2]^5 + 3*a*b^2*B*
Tan[(c + d*x)/2]^5 - 3*b^3*B*Tan[(c + d*x)/2]^5 + 30*a^2*b*C*Tan[(c + d*x)/
2]^5 - 30*a*b^2*C*Tan[(c + d*x)/2]^5 - 72*a^2*b*B*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^3*B*EllipticP
i[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4
8*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2
]^2)/(a + b)] - 36*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - 72*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*
b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d
*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d
```

```
*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 36*a*b
^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*B + 3*b^2*B + 30*a*b*C)*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - 2*a*(a*b*(26*B - 6*C) + 12*a^2*C + b^2*(-7*B + 24*C
))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)]))/(24*a*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.459, size = 3142, normalized size = 6.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*(-1+cos(d*x+c))^2*(16*B*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+3*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*C*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*C*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-6*B
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*s
in(d*x+c)+72*B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-52*B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+14*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^2+30*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^2+30*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+12*C*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*
b+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+36*C*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2-4
8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a*b^2+42*C*cos(d*x+c)^3*a^2*b-30*C*cos(d*x+c)*a*b^2+17*B*co
```

```
s(d*x+c)^3*a*b^2-3*B*cos(d*x+c)^2*a*b^2+22*B*cos(d*x+c)^4*a^2*b-30*C*cos(d*
x+c)^2*a^2*b+8*B*cos(d*x+c)^5*a^3+8*B*cos(d*x+c)^3*a^3-16*B*cos(d*x+c)^2*a^
3+3*B*cos(d*x+c)^2*b^3-12*C*cos(d*x+c)^2*a^3+12*C*cos(d*x+c)^4*a^3-3*B*cos(
d*x+c)*b^3-6*B*cos(d*x+c)^2*a^2*b-16*B*cos(d*x+c)*a^2*b-14*B*cos(d*x+c)*a*b
^2-12*C*cos(d*x+c)*a^2*b+30*C*cos(d*x+c)^2*a*b^2-48*C*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+30*C*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+30
*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*s
in(d*x+c)+12*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b*sin(d*x+c)-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+16*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+3*B*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^3+48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-24*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+72*B*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-5
2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*
sin(d*x+c)+14*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)+16*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+3*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+36*C*b^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a)*(cos(d*x+c)+
1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+Ba\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+\left(Ca+Bb\right)\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4*sec(d*x + c)^3 + B*a*cos(d*x + c)^4*sec(d*x +
c) + (C*a + B*b)*cos(d*x + c)^4*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(3/2)*co
s(d*x + c)^4, x)
```

3.829 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=565

$$-\frac{2(a-b)\sqrt{a+b}\left(-15a^2b^2(121B-19C)-a^3b(110B-30C)+40a^4C+6ab^3(209B-505C)-3b^4(539B-225C)\right)\cot(c+}{3465b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*
C - 255*a^3*b^2*C - 3705*a*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(a - b)*S
qrt[a + b]*(6*a*b^3*(209*B - 505*C) - 3*b^4*(539*B - 225*C) - a^3*b*(110*B
- 30*C) - 15*a^2*b^2*(121*B - 19*C) + 40*a^4*C)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d)
- (2*(110*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 285*a^2*b^2*C - 675*b^4*C)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*b*B - 539*b
^3*B - 40*a^3*C - 335*a*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(34
65*b^2*d) - (2*(22*a*b*B - 8*a^2*C - 81*b^2*C)*(a + b*Sec[c + d*x])^(5/2)*T
an[c + d*x])/(693*b^2*d) + (2*(11*b*B - 4*a*C)*(a + b*Sec[c + d*x])^(7/2)*T
an[c + d*x])/(99*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Tan[
c + d*x])/(11*b*d)
```

Rubi [A] time = 1.81123, antiderivative size = 565, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4033, 4082, 4002, 4005, 3832, 4004}

$$-\frac{2\left(-8a^2C+22abB-81b^2C\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2bB-40a^3C-335ab^2C-539b^3B\right)\tan(c+dx)}{3465b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*
C - 255*a^3*b^2*C - 3705*a*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(a - b)*S
qrt[a + b]*(6*a*b^3*(209*B - 505*C) - 3*b^4*(539*B - 225*C) - a^3*b*(110*B
- 30*C) - 15*a^2*b^2*(121*B - 19*C) + 40*a^4*C)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d)
- (2*(110*a^3*b*B - 1254*a*b^3*B - 40*a^4*C - 285*a^2*b^2*C - 675*b^4*C)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*b*B - 539*b
^3*B - 40*a^3*C - 335*a*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(34
65*b^2*d) - (2*(22*a*b*B - 8*a^2*C - 81*b^2*C)*(a + b*Sec[c + d*x])^(5/2)*T
an[c + d*x])/(693*b^2*d) + (2*(11*b*B - 4*a*C)*(a + b*Sec[c + d*x])^(7/2)*T
an[c + d*x])/(99*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Tan[
c + d*x])/(11*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
```

```
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}(B+C\,\mathrm{se}$$

$$= \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+}{11bd}$$

$$= \frac{2(11bB-4aC)(a+b\sec(c+dx))^{7/2}\tan(c+}{99b^2d}$$

$$= -\frac{2\left(22abB-8a^2C-81b^2C\right)(a+b\sec(c+dx}{693b^2d}$$

$$= -\frac{2\left(110a^2bB-539b^3B-40a^3C-335ab^2C\right)(}{3465b^2d}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-285a^2b^2}{3465}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-285a^2b^2}{3465}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(110a^4bB-3069a^2b^3B-1617\right.}{}$$

Mathematica [B] time = 26.5292, size = 4227, normalized size = 7.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-110*a^4*b*B + 3069*a^2*b^3
*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Sin[c + d*x])/(3
465*b^3) + (2*Sec[c + d*x]^4*(11*b^2*B*Sin[c + d*x] + 23*a*b*C*Sin[c + d*x]
))/99 + (2*Sec[c + d*x]^3*(209*a*b*B*Sin[c + d*x] + 113*a^2*C*Sin[c + d*x]
+ 81*b^2*C*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(825*a^2*b*B*Sin[c + d*x]
 + 539*b^3*B*Sin[c + d*x] + 15*a^3*C*Sin[c + d*x] + 1145*a*b^2*C*Sin[c + d*
x]))/(3465*b) + (2*Sec[c + d*x]*(55*a^3*b*B*Sin[c + d*x] + 1793*a*b^3*B*Sin
[c + d*x] - 20*a^4*C*Sin[c + d*x] + 1025*a^2*b^2*C*Sin[c + d*x] + 675*b^4*C
*Sin[c + d*x]))/(3465*b^2) + (2*b^2*C*Sec[c + d*x]^4*Tan[c + d*x])/11))/(d*
(b + a*Cos[c + d*x])^2) - (2*((2*a^4*B)/(63*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (31*a^2*b*B)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - (7*b^3*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (17*a^3
*C)/(231*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*C)/(693*b^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (247*a*b^2*C)/(231*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (124*a^3*B*Sqrt[Sec[c + d*x]])/(315*S
qrt[b + a*Cos[c + d*x]]) + (2*a^5*B*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*
Cos[c + d*x]]) + (38*a*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*
x]]) - (8*a^6*C*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (7
*a^4*C*Sqrt[Sec[c + d*x]])/(99*b*Sqrt[b + a*Cos[c + d*x]]) - (26*a^2*b*C*Sq
rt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]) + (15*b^3*C*Sqrt[Sec[c + d
*x]])/(77*Sqrt[b + a*Cos[c + d*x]]) - (31*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c
 + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (2*a^5*B*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (7*a*b^2*B*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (8*a^6*C*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (17*a^4*C*Cos
```

[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*b*Sqrt[b + a*Cos[c + d*x]]) - (247*a
^2*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]))
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(a + b
)*(-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C +
3705*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - 2*b*(a + b)*(40*a^4*C - 10*a^3*b*(11*B + 3*C) + 15*a^2*b^2*(12
1*B + 19*C) + 3*b^4*(539*B + 225*C) + 6*a*b^3*(209*B + 505*C))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*b*B
 + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(346
5*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*
(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-110*a^
4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4
*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(a + b)*(40*a^4*C - 10*a^3*b*(11*B + 3*C) + 15*a^2*b^2*(121*B + 19*C
) + 3*b^4*(539*B + 225*C) + 6*a*b^3*(209*B + 505*C))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*b*B + 3069*a^
2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^3*(b +
 a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-110*a^4*b*B + 3069*a^2*b^3*B +
 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*C
 - 10*a^3*b*(11*B + 3*C) + 15*a^2*b^2*(121*B + 19*C) + 3*b^4*(539*B + 225*C
) + 6*a*b^3*(209*B + 505*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + (-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 4
0*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-110*a
^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^
4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-1
10*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*
a*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1
+ Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])] - (b*(a + b)*(40*a^4*C - 10*a^3*b*(11*B + 3*C) + 15*a^2*b^
2*(121*B + 19*C) + 3*b^4*(539*B + 225*C) + 6*a*b^3*(209*B + 505*C))*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^
2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
 + ((a + b)*(-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^
3*b^2*C + 3705*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a
+ b)*(40*a^4*C - 10*a^3*b*(11*B + 3*C) + 15*a^2*b^2*(121*B + 19*C) + 3*b^4*
(539*B + 225*C) + 6*a*b^3*(209*B + 505*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a
 + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))] - a*(-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255
*a^3*b^2*C + 3705*a*b^4*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan
[(c + d*x)/2] - (-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 25

```
5*a^3*b^2*C + 3705*a*b^4*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c +
 d*x]*Tan[(c + d*x)/2] + (-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a
^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(40*a^4*C - 10*a^3*b*(11*B +
 3*C) + 15*a^2*b^2*(121*B + 19*C) + 3*b^4*(539*B + 225*C) + 6*a*b^3*(209*B
+ 505*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-110*a^4*b*
B + 3069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)
/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-110*a^4*b*B + 3069*a^2*b^3*B +
1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4*C
- 10*a^3*b*(11*B + 3*C) + 15*a^2*b^2*(121*B + 19*C) + 3*b^4*(539*B + 225*C)
 + 6*a*b^3*(209*B + 505*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + (-110*a^4*b*B + 3069*a^2*b^3*B + 1617*b^5*B + 40
*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c
+ d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3465*b^3*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]])))
```

Maple [B] time = 2.526, size = 5368, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\sec\left(dx+c\right)^6 + Ba^2\sec\left(dx+c\right)^3 + \left(2\,Cab + Bb^2\right)\sec\left(dx+c\right)^5 + \left(Ca^2 + 2\,Bab\right)\sec\left(dx+c\right)^4\right)\sqrt{b\sec\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^6 + B*a^2*sec(d*x + c)^3 + (2*C*a*b + B*b^2)*s
ec(d*x + c)^5 + (C*a^2 + 2*B*a*b)*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec\left(dx+c\right)^{2} + B \sec\left(dx+c\right)\right)\left(b \sec\left(dx+c\right)+a\right)^{\frac{5}{2}} \sec\left(dx+c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*se
c(d*x + c)^2, x)
```

3.830 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=469

$$-\frac{2(a-b)\sqrt{a+b}\left(15a^2b(3B-11C)-10a^3C-6ab^2(60B-19C)+3b^3(25B-49C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+d}{a-b}}}{315b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 279*a^2*b^2*
C + 147*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(2
5*B - 49*C) - 6*a*b^2*(60*B - 19*C) + 15*a^2*b*(3*B - 11*C) - 10*a^3*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(315*b^2*d) + (2*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 114*a*b^2*C
)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b*d) + (2*(45*a*b*B - 10*a^2*
C + 49*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*b*
B - 2*a*C)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*C*(a + b*
Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 1.19562, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4010, 4002, 4005, 3832, 4004}

$$\frac{2\left(-10a^2C+45abB+49b^2C\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2bB-10a^3C+114ab^2C+75b^3B\right)\tan(c+dx)\sqrt{}}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 279*a^2*b^2*
C + 147*b^4*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(2
5*B - 49*C) - 6*a*b^2*(60*B - 19*C) + 15*a^2*b*(3*B - 11*C) - 10*a^3*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(315*b^2*d) + (2*(45*a^2*b*B + 75*b^3*B - 10*a^3*C + 114*a*b^2*C
)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b*d) + (2*(45*a*b*B - 10*a^2*
C + 49*b^2*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*b*
B - 2*a*C)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*C*(a + b*
Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}(B+C\sec$$

$$= \frac{2C(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \sec}{}$$

$$= \frac{2(9bB-2aC)(a+b\sec(c+dx))^{5/2}\tan(c+dx}{63bd}$$

$$= \frac{2\left(45abB-10a^2C+49b^2C\right)(a+b\sec(c+dx)}{315bd}$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+114ab^2C\right)\sqrt{a+}}{315bd}$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+114ab^2C\right)\sqrt{a+}}{315bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(45a^3bB+435ab^3B-10a^4C\right.}{}$$

Mathematica [B] time = 26.0134, size = 3781, normalized size = 8.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(45*a^3*b*B + 435*a*b^3*B -
10*a^4*C + 279*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(315*b^2) + (2*Sec[c +
d*x]^3*(9*b^2*B*Sin[c + d*x] + 19*a*b*C*Sin[c + d*x]))/63 + (2*Sec[c + d*x]
^2*(135*a*b*B*Sin[c + d*x] + 75*a^2*C*Sin[c + d*x] + 49*b^2*C*Sin[c + d*x])
)/315 + (2*Sec[c + d*x]*(135*a^2*b*B*Sin[c + d*x] + 75*b^3*B*Sin[c + d*x] +
 5*a^3*C*Sin[c + d*x] + 163*a*b^2*C*Sin[c + d*x]))/(315*b) + (2*b^2*C*Sec[c
 + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^2) + (2*(-(a^3*B)/(7*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*b^2*B)/(21*Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^4*C)/(63*b*Sqrt[b + a*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) - (31*a^2*b*C)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) - (7*b^3*C)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^
4*B*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*b*B*Sqrt[Se
c[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5*b^3*B*Sqrt[Sec[c + d*x]])/(
21*Sqrt[b + a*Cos[c + d*x]]) - (124*a^3*C*Sqrt[Sec[c + d*x]])/(315*Sqrt[b +
 a*Cos[c + d*x]]) + (2*a^5*C*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c +
 d*x]]) + (38*a*b^2*C*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) -
(a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]])
- (29*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d
*x]]) - (31*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c
 + d*x]]) + (2*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b +
a*Cos[c + d*x]]) - (7*a*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt
[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c +
 d*x])^(5/2)*(2*(a + b)*(-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2
*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 2*b*(a + b)*(-10*a^3*C + 15*a^2*b*(3*B + 11*C) + 6*a*b^2*(60
*B + 19*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
```

c + d*x)/2]], (a - b)/(a + b)] + (-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 27
9*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/
2]^2]*Sec[c + d*x]^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c +
d*x]*(2*(a + b)*(-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147
*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + 2*b*(a + b)*(-10*a^3*C + 15*a^2*b*(3*B + 11*C) + 6*a*b^2*(60*B + 19*
C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] + (-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^
2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(315*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2])
 - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-45*
a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-
10*a^3*C + 15*a^2*b*(3*B + 11*C) + 6*a*b^2*(60*B + 19*C) + 3*b^3*(25*B + 49
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^
2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x
)/2]^2*Sec[c + d*x]]*(((-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*
C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((
a + b)*(-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c +
d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])] + (b*(a + b)*(-10*a^3*C + 15*a^2*b*(3*B + 11*C) + 6*a*b^2*(60*B + 19
*C) + 3*b^3*(25*B + 49*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]
*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-45*a^3*b*B - 435*a*b^3*B +
10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))] + (b*(a + b)*(-10*a^3*C + 15*a^2*b*(3*B + 11*C) + 6*a*b^2*(60*B + 19*
C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c
+ d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-
45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-45*a^3*b*B - 435*a*
b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-45*a^3*b*B - 435*a*b^3*B + 10*
a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-10*a^3*C + 15*a^2*b*(3*B + 1
1*C) + 6*a*b^2*(60*B + 19*C) + 3*b^3*(25*B + 49*C))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[
(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)]) + ((a + b)*(-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a
^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(315*b^2*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-45*a^3*b*B
 - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E

```
llipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-10*a^3*
C + 15*a^2*b*(3*B + 11*C) + 6*a*b^2*(60*B + 19*C) + 3*b^3*(25*B + 49*C))*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-4
5*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 279*a^2*b^2*C - 147*b^4*C)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x
)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c
 + d*x]))/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.703, size = 4395, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(75*B*cos(d*x+c)^5*b^5+155*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+45*B*sin(d*x+c)
*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^3*b^2-10*C*cos(d*x+c)^5*a^4*b-199*C*cos(d*x+c)^5*a^3*b^2+279*C*cos(d*
x+c)^5*a^2*b^3+65*C*cos(d*x+c)^5*a*b^4+5*C*cos(d*x+c)^4*a^4*b-272*C*cos(d*x
+c)^4*a^2*b^3-80*C*cos(d*x+c)^3*a^3*b^2-82*C*cos(d*x+c)^3*a*b^4+5*C*cos(d*x
+c)^6*a^4*b+279*C*cos(d*x+c)^6*a^3*b^2+163*C*cos(d*x+c)^6*a^2*b^3+147*C*cos
(d*x+c)^6*a*b^4-170*C*cos(d*x+c)^2*a^2*b^3-130*C*cos(d*x+c)*a*b^4-45*B*sin(
d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^4*b-45*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-435*B*sin(d*x+c)*cos(d*x+c)^5*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-435*
B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^4+405*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+435*B*sin(d*x+c)*cos(d*x+
c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4
-45*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^4*b-45*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+10*C*cos(d*x+c)^4*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5
-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^5+147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+10*C*cos(d*x+c)^5*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5-147
```

*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+10*C*cos(d*x+c)^5*a^5+147*C*cos(d*x+c)^5*b^5-98*C*cos(d*x+c)^4*b^5-14*C*cos(d*x+c)^2*b^5-10*C*cos(d*x+c)^6*a^5-435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+45*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+405*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+435*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+279*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+261*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-279*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-279*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+155*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+279*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+261*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-279*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-279*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*C*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-35*C*b^5-30*B*cos(d*x+c)^3*b^5-45*B*cos(d*x+c)*b^5-180*B*cos(d*x+c)^4*a^3*b^2-330*B*cos(d*x+c)^4*a*b^4-270*B*cos(d*x+c)^3*a^2*b^3-180*B*cos(d*x+c)^2*a*b^4+45*B*cos(d*x+c)^6*a^4*b+135*B*cos(d*x+c)^6*a^3*b^2+435*B*cos(d*x+c)^6*a^2*b^3+75*B*cos(d*x+c)^6*a*b^4-45*B*cos(d*x+c)^5*a^4*b+45*B*cos(d*x+c)^5*a^3*b^2-165*B*cos(d*x+c)^5*a^2*b^3+435*B*cos(d*x+c)^5*a*b^4+75*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+

```
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^5+75*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^
4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^5 + Ba^2\sec(dx+c)^2 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^4 + \left(Ca^2 + 2\,Bab\right)\sec(dx+c)^3\right)\sqrt{b\sec(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^5 + B*a^2*sec(d*x + c)^2 + (2*C*a*b + B*b^2)*s
ec(d*x + c)^4 + (C*a^2 + 2*B*a*b)*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*se
c(d*x + c), x)
```

3.831 $\int (a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=384

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2(7B-C)-8ab(7B-15C)+b^2(63B-25C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{si}\right.}{105bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^2*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*B - 25*C) - 8*a*
b*(7*B - 15*C) + 15*a^2*(7*B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b*d) + (2*(56*a*b*B +
 15*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(
7*b*B + 5*a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a +
b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.677614, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4056, 4058, 12, 3832, 4004}

$$\frac{2\left(15a^2C+56abB+25b^2C\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\left(15a^2(7B-C)-8ab(7B-15C)+b^2(63B-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^2*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*B - 25*C) - 8*a*
b*(7*B - 15*C) + 15*a^2*(7*B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b*d) + (2*(56*a*b*B +
 15*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(
7*b*B + 5*a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a +
b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{2}{7}\int (a+b\sec($$

$$= \frac{2(7bB+5aC)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35d}+ \frac{2C(a}{}$$

$$= \frac{2\left(56abB+15a^2C+25b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c}{105d}$$

$$= \frac{2\left(56abB+15a^2C+25b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c}{105d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(161a^2bB+63b^3B+15a^3C+145ab^2C\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(161a^2bB+63b^3B+15a^3C+145ab^2C\right.}{}$$

Mathematica [B] time = 22.9373, size = 2913, normalized size = 7.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(161*a^2*b*B + 63*b^3*B + 15
*a^3*C + 145*a*b^2*C)*Sin[c + d*x])/(105*b) + (2*Sec[c + d*x]^2*(7*b^2*B*Si
n[c + d*x] + 15*a*b*C*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(77*a*b*B*Sin[c +
 d*x] + 45*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/105 + (2*b^2*C*Sec[
c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2) + (2*((-23*a^2*b*B)
```

```
/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*b^3*B)/(5*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^3*C)/(7*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[c + d*x]]) - (29*a*b^2*C)/(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) - (8*a^3*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (8*a*
b^2*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (a^4*C*Sqrt[Sec[c
 + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*b*C*Sqrt[Sec[c + d*x]])/(
21*Sqrt[b + a*Cos[c + d*x]]) + (5*b^3*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*
Cos[c + d*x]]) - (23*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b
+ a*Cos[c + d*x]]) - (3*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqr
t[b + a*Cos[c + d*x]]) - (a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*S
qrt[b + a*Cos[c + d*x]]) - (29*a^2*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])
/(21*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a +
b*Sec[c + d*x])^(5/2)*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^
2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - b*(15*a^2*(7*B + C) + 8*a*b*(7*B + 15*C) + b^2*(63*B
 + 25*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x
])/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*b*B +
 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]
^2)))/(105*b*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]
^(5/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*((-2*(Cos[c
+ d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*
a*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(
7*B + C) + 8*a*b*(7*B + 15*C) + b^2*(63*B + 25*C))*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))] + (161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C
)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)))/(105*b*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x
)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x])
)^(3/2)*((161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*B + C) + 8*a*b*(7*B + 1
5*C) + b^2*(63*B + 25*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)])*Sec[c + d*x])/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
+ (161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*Tan[(c + d*x)/2]*(-1 +
Tan[(c + d*x)/2]^2)))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[b + a*Cos[c
+ d*x]]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*b*B + 63*b^3
*B + 15*a^3*C + 145*a*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] - b*(15*a^2*(7*B + C) + 8*a*b*(7*B + 15*C) + b^2*(63*B + 25*C))*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*b*B + 63*b^3*B + 15
*a^3*C + 145*a*b^2*C)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2))*(-(Cos[(c
 + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]
*Tan[c + d*x]))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]) + (2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*((-3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((161*a^2*b*B + 63*b^3*B +
15*a^3*C + 145*a*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] - b*(15*a^2*(7*B + C) + 8*a*b*(7*B + 15*C) + b^2*(63*B + 25*C))*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x]*((Cos[c + d*x]*Si
n[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((Cos[c + d*x]/(1 + Cos
[c + d*x]))^(3/2)*((161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*B + C) + 8*a*
b*(7*B + 15*C) + b^2*(63*B + 25*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)])*Sec[c + d*x]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/(
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])))^(3/2) + (161*a^2*b*B + 6
3*b^3*B + 15*a^3*C + 145*a*b^2*C)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (
(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*Sec[(c + d*x)/2]^2*(-1 +
Tan[(c + d*x)/2]^2))/2 - (2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sec[c +
 d*x]*(-(b*(15*a^2*(7*B + C) + 8*a*b*(7*B + 15*C) + b^2*(63*B + 25*C))*Sec[
```

```
(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + ((161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145*a*b^2*C)*S
ec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1
 - Tan[(c + d*x)/2]^2])))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] - (2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*b*B + 63*b^3*
B + 15*a^3*C + 145*a*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] - b*(15*a^2*(7*B + C) + 8*a*b*(7*B + 15*C) + b^2*(63*B + 25*C))*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x]*Tan[c + d*x])
/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]))/(105*b*Sqrt[Sec[
(c + d*x)/2]^2])))
```

Maple [B] time = 1.117, size = 3637, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x
+c))^2*(63*B*cos(d*x+c)^4*b^4+105*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+105*B*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^3*b-42*B*cos(d*x+c)^3*b^4-21*B*cos(d*x+c)*b^4-161*B*cos(d*x+c)^4*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+
15*C*cos(d*x+c)^5*a^4-15*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-15*C*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^4+25*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-238*B*cos(d*x+c)^3*a^2*b^2-98*B*cos(
d*x+c)^2*a*b^3-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-63*B*cos(d*x+c
)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*b^4+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*b^4-161*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)
^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b^3+161*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+119*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-161*B*cos(d
```

```
*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a^3*b-161*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)^3*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+161*B*c
os(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b^2+119*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-161*B*cos(d*x+c)^4*a^3*b+161
*B*cos(d*x+c)^4*a^2*b^2+35*B*cos(d*x+c)^4*a*b^3+161*B*cos(d*x+c)^5*a^3*b+77
*B*cos(d*x+c)^5*a^2*b^2+63*B*cos(d*x+c)^5*a*b^3+15*C*cos(d*x+c)^4*a^3*b-55*
C*cos(d*x+c)^4*a^2*b^2+145*C*cos(d*x+c)^4*a*b^3-60*C*cos(d*x+c)^3*a^3*b-110
*C*cos(d*x+c)^3*a*b^3-90*C*cos(d*x+c)^2*a^2*b^2-60*C*cos(d*x+c)*a*b^3+45*C*
cos(d*x+c)^5*a^3*b+145*C*cos(d*x+c)^5*a^2*b^2+25*C*cos(d*x+c)^5*a*b^3-15*C*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b-145*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-145*C*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+1
5*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b+135*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+145*C*sin(d*x+c)*cos(d*
x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*b-145*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-145*C*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^3+15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*b+135*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+145*C*sin(d*x+c
)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^3-15*C*cos(d*x+c)^4*a^4-10*C*cos(d*x+c)^2*b^4+25*C*cos(d*x+c)^4*b^
4-15*C*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^4 + Ba^2\sec(dx+c) + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + \left(Ca^2 + 2\,Bab\right)\sec(dx+c)^2\right)\sqrt{b\sec(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + B*a^2*sec(d*x + c) + (2*C*a*b + B*b^2)*sec
(d*x + c)^3 + (C*a^2 + 2*B*a*b)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2), x
)
```

3.832 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+\right.$

Optimal. Leaf size=442

$$\frac{2\sqrt{a+b}\left(a^2b(45B-23C)+15a^3C-ab^2(35B-17C)+b^3(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}}{15bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*B + 23*a^2*C + 9*b^2*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b
*d) + (2*Sqrt[a + b]*(a^2*b*(45*B - 23*C) - a*b^2*(35*B - 17*C) + b^3*(5*B
- 9*C) + 15*a^3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a^2*Sqrt[a + b]*B*Cot[c + d*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d + (2*b*(5*b*B + 8*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(15*d) + (2*b*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.715349, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(a^2b(45B-23C)+15a^3C-ab^2(35B-17C)+b^3(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*B + 23*a^2*C + 9*b^2*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b
*d) + (2*Sqrt[a + b]*(a^2*b*(45*B - 23*C) - a*b^2*(35*B - 17*C) + b^3*(5*B
- 9*C) + 15*a^3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a^2*Sqrt[a + b]*B*Cot[c + d*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d + (2*b*(5*b*B + 8*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(15*d) + (2*b*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
```

```
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int (a+b\sec(c+dx))^{5/2}(B+C\sec(c+dx))\,dx$$
$$= \frac{2bC(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int$$
$$= \frac{2b(5bB+8aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}$$
$$= \frac{2b(5bB+8aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}$$
$$= -\frac{2(a-b)\sqrt{a+b}\left(35abB+23a^2C+9b^2C\right)\cot}{}$$
$$= -\frac{2(a-b)\sqrt{a+b}\left(35abB+23a^2C+9b^2C\right)\cot}{}$$

Mathematica [B] time = 25.0387, size = 7124, normalized size = 16.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.781, size = 3285, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(23*C*cos(d*x+c)^3*a^3+35*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-45*B*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
-9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^3+23*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+9*C*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*C*sin
(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^3-5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
```

)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+35*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-35*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+35*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+35*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-35*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+23*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-23*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-17*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+23*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-23*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-17*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-45*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-11*C*cos(d*x+c)^4*a^2*b-9*C*cos(d*x+c)^4*a*b^2-23*C*cos(d*x+c)^3*a^2*b-5*C*cos(d*x+c)^3*a*b^2+14*C*cos(d*x+c)*a*b^2-35*B*cos(d*x+c)^3*a*b^2+40*B*cos(d*x+c)^2*a*b^2-35*B*cos(d*x+c)^4*a^2*b-5*B*cos(d*x+c)^4*a*b^2+35*B*cos(d*x+c)^3*a^2*b+34*C*cos(d*x+c)^2*a^2*b+15*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-30*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-30*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-23*C*cos(d*x+c)^4*a^3-9*C*cos(d*x+c)^3*b^3+6*C*cos(d*x+c)^2*b^3-5*B*cos(d*x+c)^3*b^3+5*B*cos(d*x+c)*b^3+23*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*C*b^3-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)^{4}+Ba^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+\left(Ca^{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)*sec(d*x + c)^4 + B*a^2*cos(d*x + c)*sec(d*x +
c) + (2*C*a*b + B*b^2)*cos(d*x + c)*sec(d*x + c)^3 + (C*a^2 + 2*B*a*b)*cos(
d*x + c)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c), x)
```

3.833 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2} \left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=433

$$\frac{\sqrt{a+b}\left(3a^2(B+6C)+2ab(9B-7C)-2b^2(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{}\right.\right.}{3d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*B - 6*b^2*B - 14*a*b*C)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) +
 (Sqrt[a + b]*(2*a*b*(9*B - 7*C) - 2*b^2*(3*B - C) + 3*a^2*(B + 6*C))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*d) - (a*Sqrt[a + b]*(5*b*B + 2*a*C)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 + (a*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*a*B - 2*b*C)*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.777233, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4025, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(3a^2(B+6C)+2ab(9B-7C)-2b^2(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*B - 6*b^2*B - 14*a*b*C)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) +
 (Sqrt[a + b]*(2*a*b*(9*B - 7*C) - 2*b^2*(3*B - C) + 3*a^2*(B + 6*C))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*d) - (a*Sqrt[a + b]*(5*b*B + 2*a*C)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 + (a*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*a*B - 2*b*C)*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos(c+dx)(a+b\sec(c+dx))^{5/2}(B+C$$

$$= \frac{aB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \int$$

$$= \frac{aB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3}{}$$

$$= \frac{aB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3a^2B-6b^2B-14abC\right)\cot(}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3a^2B-6b^2B-14abC\right)\cot(}{}$$

Mathematica [B] time = 19.2026, size = 1146, normalized size = 2.65

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(3*a^3*B*Ta
n[(c + d*x)/2] + 3*a^2*b*B*Tan[(c + d*x)/2] - 6*a*b^2*B*Tan[(c + d*x)/2] -
6*b^3*B*Tan[(c + d*x)/2] - 14*a^2*b*C*Tan[(c + d*x)/2] - 14*a*b^2*C*Tan[(c
+ d*x)/2] - 6*a^3*B*Tan[(c + d*x)/2]^3 + 12*a*b^2*B*Tan[(c + d*x)/2]^3 + 28
*a^2*b*C*Tan[(c + d*x)/2]^3 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B*Tan[(c
 + d*x)/2]^5 - 6*a*b^2*B*Tan[(c + d*x)/2]^5 + 6*b^3*B*Tan[(c + d*x)/2]^5 -
14*a^2*b*C*Tan[(c + d*x)/2]^5 + 14*a*b^2*C*Tan[(c + d*x)/2]^5 - 30*a^2*b*B*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] - 12*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 30*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^3*C*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (a + b)*(3*a^2*B - 6*b^2*B - 14*a*b*C)*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 2*(9*a^2*b*(B - C) + 3*a^3*C - b^3*(3*B + C) - a*b^2*(9*B +
7*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*
x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^2*(a + b
*Sec[c + d*x])^(5/2)*((2*b*(3*b*B + 7*a*C)*Sin[c + d*x])/3 + (2*b^2*C*Tan[c
 + d*x])/3))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.658, size = 3215, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/3/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^3+6*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*B*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-6*B
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2+18*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-14*C*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-14*C
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2+18*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+14*C*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+30*B
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*a^2*b-18*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+30*B*cos(d*x+c)*a^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b-18*B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b+18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-14*C*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-14*C
*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b+18*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-6*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2+14*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+14*C*cos(d*x+c)^3*a^2*b+2*C*cos(d*x+c)^3*
a*b^2-16*C*cos(d*x+c)*a*b^2+6*B*cos(d*x+c)^3*a*b^2-6*B*cos(d*x+c)^2*a*b^2+3
*B*cos(d*x+c)^3*a^2*b-14*C*cos(d*x+c)^2*a^2*b-3*B*cos(d*x+c)^3*a^3+6*B*cos(
d*x+c)^2*b^3+2*C*cos(d*x+c)^2*b^3-6*B*cos(d*x+c)*b^3+3*B*cos(d*x+c)^4*a^3+1
```

```
2*C*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a^3+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*B*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-6*B*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^3+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^3-3*B*cos(d*x+c)^2*a^2*b+14*C*cos(d*x+c)^2
*a*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+12*C*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-6*C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3-2*C*b^3-6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3)/sin(d*x+c)^5/(b+a*cos(d*x+c
))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{4}+Ba^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+\left(2\,Cab+Bb^{2}\right)\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^2*sec(d*x + c)^4 + B*a^2*cos(d*x + c)^2*sec(d*
x + c) + (2*C*a*b + B*b^2)*cos(d*x + c)^2*sec(d*x + c)^3 + (C*a^2 + 2*B*a*b
)*cos(d*x + c)^2*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^2, x)
```

3.834 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=450

$$\frac{\sqrt{a+b}\left(2a^2(B+2C)+3ab(3B+8C)+8b^2(B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{4d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(9*a*b*B + 4*a^2*C - 8*b^2*C)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) +
(Sqrt[a + b]*(8*b^2*(B - C) + 2*a^2*(B + 2*C) + 3*a*b*(3*B + 8*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*d) - (Sqrt[a + b]*(4*a^2*B + 15*b^2*B + 20*a*b*C)*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(4*d) + (a*(7*b*B + 4*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(4*d) + (a*B*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.89977, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4025, 4094, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(2a^2(B+2C)+3ab(3B+8C)+8b^2(B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(9*a*b*B + 4*a^2*C - 8*b^2*C)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) +
(Sqrt[a + b]*(8*b^2*(B - C) + 2*a^2*(B + 2*C) + 3*a*b*(3*B + 8*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*d) - (Sqrt[a + b]*(4*a^2*B + 15*b^2*B + 20*a*b*C)*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(4*d) + (a*(7*b*B + 4*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(4*d) + (a*B*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}(B+C$$

$$= \frac{aB\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c}{2d}$$

$$= \frac{a(7bB+4aC)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4d}$$

$$= \frac{a(7bB+4aC)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(9abB+4a^2C-8b^2C\right)\cot(}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(9abB+4a^2C-8b^2C\right)\cot(}{}$$

Mathematica [B] time = 19.2177, size = 1338, normalized size = 2.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(2*b^2*C*Sin[c + d*x] + (a^2*B*S
in[2*(c + d*x)])/4))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/
2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(9*a^2*b*B*Tan[(c + d*x)/2] + 9*a*b^
2*B*Tan[(c + d*x)/2] + 4*a^3*C*Tan[(c + d*x)/2] + 4*a^2*b*C*Tan[(c + d*x)/2
] - 8*a*b^2*C*Tan[(c + d*x)/2] - 8*b^3*C*Tan[(c + d*x)/2] - 18*a^2*b*B*Tan[
(c + d*x)/2]^3 - 8*a^3*C*Tan[(c + d*x)/2]^3 + 16*a*b^2*C*Tan[(c + d*x)/2]^3
 + 9*a^2*b*B*Tan[(c + d*x)/2]^5 - 9*a*b^2*B*Tan[(c + d*x)/2]^5 + 4*a^3*C*Ta
n[(c + d*x)/2]^5 - 4*a^2*b*C*Tan[(c + d*x)/2]^5 - 8*a*b^2*C*Tan[(c + d*x)/2
]^5 + 8*b^3*C*Tan[(c + d*x)/2]^5 - 8*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*b^2*B*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 40*a^
2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 8*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a*b^2*B*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - 40*a^2*b*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(9*a*b*B + 4*a^2*
C - 8*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a^3*B - a^2*b*(B - 12*C) + 12*
a*b^2*(B - C) - 4*b^3*(B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*d*(b + a*Cos[c
 + d*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a +
```

```
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.652, size = 3271, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(4*C*cos(d*x+c)^3*a^3-4*B*a^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-4*B*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3
+30*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
a*b^2*sin(d*x+c)+40*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+4*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-8*C*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*
B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a^3+4*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2+8*B*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*B*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*
x+c)-8*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^3+8*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b^3+40*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+2*B*cos(d*x+c)*a^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-24*
B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^2-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+4*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-24*C*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*a^2*b+9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
```

```
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+24*C*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*
b^2-8*C*cos(d*x+c)*a*b^2+9*B*cos(d*x+c)^2*a*b^2+11*B*cos(d*x+c)^3*a^2*b+4*C
*cos(d*x+c)^2*a^2*b-2*B*cos(d*x+c)^2*a^3-4*C*cos(d*x+c)^2*a^3+2*B*cos(d*x+c
)^4*a^3+8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*B*cos(d*x+c)^2*a^2*b-2*B*
cos(d*x+c)*a^2*b-9*B*cos(d*x+c)*a*b^2-4*C*cos(d*x+c)*a^2*b+8*C*cos(d*x+c)^2
*a*b^2+24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^2*sin(d*x+c)-8*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2+4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-24*B*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)
+9*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
*sin(d*x+c)+9*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)-8*C*b^3+8*C*cos(d*x+c)*b^3)*(cos(d*x+c)+1)^2*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^3\sec(dx+c)^4 + Ba^2\cos(dx+c)^3\sec(dx+c) + \left(2\,Cab + Bb^2\right)\cos(dx+c)^3\sec(dx+c)^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3*sec(d*x + c)^4 + B*a^2*cos(d*x + c)^3*sec(d*
x + c) + (2*C*a*b + B*b^2)*cos(d*x + c)^3*sec(d*x + c)^3 + (C*a^2 + 2*B*a*b
```

```
)*cos(d*x + c)^3*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + B \sec (dx+c)\right)(b \sec (dx+c)+a)^{\frac{5}{2}} \cos (dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^3, x)
```

3.835 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=518

$$\frac{\sqrt{a+b}\left(4a^2(4B+3C)+ab(26B+54C)+3b^2(11B+16C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\right.}{24d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*B + 33*b^2*B + 54*a*b*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d
) + (Sqrt[a + b]*(4*a^2*(4*B + 3*C) + 3*b^2*(11*B + 16*C) + a*b*(26*B + 54*
C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(24*d) - (Sqrt[a + b]*(20*a^2*b*B + 5*b^3*B + 8*a^3*C + 3
0*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2*B + 33*b^2*B + 54*a
*b*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(3*b*B + 2*a*C)*Co
s[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Cos[c + d*x]
^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.27429, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4072, 4025, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2B+54abC+33b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(4a^2(4B+3C)+ab(26B+54C)+3b^2(11B+16C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*B + 33*b^2*B + 54*a*b*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d
) + (Sqrt[a + b]*(4*a^2*(4*B + 3*C) + 3*b^2*(11*B + 16*C) + a*b*(26*B + 54*
C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(24*d) - (Sqrt[a + b]*(20*a^2*b*B + 5*b^3*B + 8*a^3*C + 3
0*a*b^2*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2*B + 33*b^2*B + 54*a
*b*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(3*b*B + 2*a*C)*Co
s[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Cos[c + d*x]
^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}(B+C$$

$$= \frac{aB\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c}{3d}$$

$$= \frac{a(3bB+2aC)\cos(c+dx)\sqrt{a+b\sec(c+d}}{4d}$$

$$= \frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\sec(c+}}{24d}$$

$$= \frac{\left(16a^2B+33b^2B+54abC\right)\sqrt{a+b\sec(c+}}{24d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2B+33b^2B+54abC\right)c}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2B+33b^2B+54abC\right)c}{}$$

Mathematica [B] time = 19.3402, size = 1546, normalized size = 2.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((a^2*B*Sin[c + d*x])/12 + (a*(13*b*B + 6*a*C)*Si
n[2*(c + d*x)])/24 + (a^2*B*Sin[3*(c + d*x)])/12))/d + (Sqrt[a + b*Sec[c +
d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a^3*B*Tan[(c + d*x)/2] + 16*a
^2*b*B*Tan[(c + d*x)/2] + 33*a*b^2*B*Tan[(c + d*x)/2] + 33*b^3*B*Tan[(c + d
*x)/2] + 54*a^2*b*C*Tan[(c + d*x)/2] + 54*a*b^2*C*Tan[(c + d*x)/2] - 32*a^3
*B*Tan[(c + d*x)/2]^3 - 66*a*b^2*B*Tan[(c + d*x)/2]^3 - 108*a^2*b*C*Tan[(c
+ d*x)/2]^3 + 16*a^3*B*Tan[(c + d*x)/2]^5 - 16*a^2*b*B*Tan[(c + d*x)/2]^5 +
 33*a*b^2*B*Tan[(c + d*x)/2]^5 - 33*b^3*B*Tan[(c + d*x)/2]^5 + 54*a^2*b*C*T
an[(c + d*x)/2]^5 - 54*a*b^2*C*Tan[(c + d*x)/2]^5 - 120*a^2*b*B*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*
b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 48*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
```

```
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - 180*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 120*a^2*b*B*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 30*b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 48*a^3*C*EllipticPi[-1,
-ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - 180*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*B + 33*b
^2*B + 54*a*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(a^2*b*(38*B - 6*C) + 24*b^3*
(B - C) + 12*a^3*C + a*b^2*(-13*B + 72*C))*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*d
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)])
```

Maple [B] time = 0.496, size = 3511, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^2*(16*B*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+33*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-48*B*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*C*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+48*C*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d
*x+c)-24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^3*sin(d*x+c)+30*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*b^3*sin(d*x+c)+120*B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-76*B*cos(d*x+c)*a
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b+26*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2+54*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+54*C*a^2*(cos(d*x+c)/(c
```

```
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+
12*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*a^2*b+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+33*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+180*C*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*a*b^2-144*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+66*C*cos(d*x+c)^3*a^2*b-54*C*
cos(d*x+c)*a*b^2+59*B*cos(d*x+c)^3*a*b^2-33*B*cos(d*x+c)^2*a*b^2+34*B*cos(d
*x+c)^4*a^2*b-54*C*cos(d*x+c)^2*a^2*b+8*B*cos(d*x+c)^5*a^3+8*B*cos(d*x+c)^3
*a^3-16*B*cos(d*x+c)^2*a^3+33*B*cos(d*x+c)^2*b^3-12*C*cos(d*x+c)^2*a^3+12*C
*cos(d*x+c)^4*a^3-33*B*cos(d*x+c)*b^3-48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+48*C*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3-18*B*cos(d*x+c)^2*a^2*b-16*B*cos(d*x+c)*a^2*b-26*B*cos(d*x+c)*a*b^2-12
*C*cos(d*x+c)*a^2*b+54*C*cos(d*x+c)^2*a*b^2-144*C*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+54*C*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+54*C*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d
*x+c)+12*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*b*sin(d*x+c)+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+16*B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+33*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^3+48*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-24*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+120*B*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-76
*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*s
in(d*x+c)+26*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)+16*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+33*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+180*C*b^2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a)*(cos(d*x+c)
+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^4+Ba^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4*sec(d*x + c)^4 + B*a^2*cos(d*x + c)^4*sec(d*
x + c) + (2*C*a*b + B*b^2)*cos(d*x + c)^4*sec(d*x + c)^3 + (C*a^2 + 2*B*a*b
)*cos(d*x + c)^4*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^4, x)
```

3.836 $\int \cos^5(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=617

$$\frac{\sqrt{a+b}\left(4a^2b(71B+52C)+8a^3(9B+16C)+2ab^2(59B+132C)+15b^3B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\mathrm{E}}{192ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2*b*B + 15*b^3*B + 128*a^3*C + 264*a*b^2*C)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(192*a*b*d) + (Sqrt[a + b]*(15*b^3*B + 8*a^3*(9*B + 16*C) + 4*a^
2*b*(71*B + 52*C) + 2*a*b^2*(59*B + 132*C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sqrt
[a + b]*(48*a^4*B + 120*a^2*b^2*B - 5*b^4*B + 160*a^3*b*C + 40*a*b^3*C)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(64*a^2*d) + ((284*a^2*b*B + 15*b^3*B + 128*a^3*C +
264*a*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((36*a^2*B
+ 59*b^2*B + 104*a*b*C)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(96*d) + (a*(11*b*B + 8*a*C)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(24*d) + (a*B*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(4*d)
```

Rubi [A] time = 1.82782, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4072, 4025, 4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(284a^2bB+128a^3C+264ab^2C+15b^3B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{\left(36a^2B+104abC+59b^2B\right)\sin(c+dx)\cos}{96d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2*b*B + 15*b^3*B + 128*a^3*C + 264*a*b^2*C)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(192*a*b*d) + (Sqrt[a + b]*(15*b^3*B + 8*a^3*(9*B + 16*C) + 4*a^
2*b*(71*B + 52*C) + 2*a*b^2*(59*B + 132*C))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sqrt
[a + b]*(48*a^4*B + 120*a^2*b^2*B - 5*b^4*B + 160*a^3*b*C + 40*a*b^3*C)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(64*a^2*d) + ((284*a^2*b*B + 15*b^3*B + 128*a^3*C +
264*a*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((36*a^2*B
+ 59*b^2*B + 104*a*b*C)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(96*d) + (a*(11*b*B + 8*a*C)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(24*d) + (a*B*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(4*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^{5/2}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}(B+C$$

$$= \frac{aB\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c}{4d}$$

$$= \frac{a(11bB+8aC)\cos^2(c+dx)\sqrt{a+b\sec(c+}}{24d}$$

$$= \frac{\left(36a^2B+59b^2B+104abC\right)\cos(c+dx)\sqrt{a}}{96d}$$

$$= \frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)}{192ad}$$

$$= \frac{\left(284a^2bB+15b^3B+128a^3C+264ab^2C\right)}{192ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2bB+15b^3B+128a^3C\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2bB+15b^3B+128a^3C\right.}{}$$

Mathematica [B] time = 24.0517, size = 5186, normalized size = 8.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^(5/2)*(B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.582, size = 4231, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/192/d/a*(-1+cos(d*x+c))^2*(15*B*cos(d*x+c)^2*b^4+24*B*a^4*cos(d*x+c)^4+6
4*C*cos(d*x+c)^3*a^4+288*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-72*B*cos(d*x+c)^2*a^4-15*B*cos(d*x+c)*b
^4-30*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*b^4*sin(d*x+c)+48*B*cos(d*x+c)^6*a^4+64*C*cos(d*x+c)^5*a^4+15*B*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+128*
C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(
d*x+c)-144*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^4*sin(d*x+c)+284*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*b*sin(d*x+c)+284*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+15*B*b^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+72*B*a^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-644*B
*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))-284*B*cos(d*x+c)^2*a^3*b-15*B*cos(d*x+c)^2*a*b^3-72*B*cos(d*x+c)*
a^3*b-284*B*cos(d*x+c)*a^2*b^2+30*B*cos(d*x+c)^2*a^2*b^2-118*B*cos(d*x+c)*a
*b^3+128*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a^4+118*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+720*B*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+960*C*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)
+240*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a*b^3*sin(d*x+c)+264*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^3*sin(d*x+c)-608*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+128*C*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
3*b+264*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*a^2*b^2+264*C*cos(d*x+c)^2*a*b^3+172*B*cos(d*x+c)^3*a
^3*b+133*B*cos(d*x+c)^3*a*b^3+472*C*cos(d*x+c)^3*a^2*b^2+208*C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+254
*B*cos(d*x+c)^4*a^2*b^2+184*B*cos(d*x+c)^5*a^3*b+272*C*cos(d*x+c)^4*a^3*b-2
```

```
64*C*cos(d*x+c)^2*a^2*b^2-264*C*cos(d*x+c)*a*b^3-144*C*cos(d*x+c)^2*a^3*b-1
28*C*cos(d*x+c)*a^3*b-208*C*cos(d*x+c)*a^2*b^2+284*B*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+284*
B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^2+15*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+72*B*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-644*B*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b^2-384*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^3*sin(d*x+c)-128*C*cos(d*x+c)^2*a^4+118*B*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*
b^3+720*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+960*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+240*C*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^3+264*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+208*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-384*C*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)*a*b^3-608*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*b*sin(d*x+c)+128*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+264*C*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-144*B*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
4+288*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a^4-30*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4+15*B*cos(d*x+c)*b^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)))*
(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
```

```
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^5\sec(dx+c)^4 + Ba^2\cos(dx+c)^5\sec(dx+c) + \left(2\,Cab + Bb^2\right)\cos(dx+c)^5\sec(dx+c)^3 + \left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5*sec(d*x + c)^4 + B*a^2*cos(d*x + c)^5*sec(d*
x + c) + (2*C*a*b + B*b^2)*cos(d*x + c)^5*sec(d*x + c)^3 + (C*a^2 + 2*B*a*b
)*cos(d*x + c)^5*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(5/2)*co
s(d*x + c)^5, x)
```

3.837 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=411

$$-\frac{2\sqrt{a+b}\left(4a^2b(14B+3C)-48a^3C-2ab^2(7B+22C)+b^3(63B-25C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\mathrm{E}}{105b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(56*a^2*b*B + 63*b^3*B - 48*a^3*C - 44*a*b^2*C)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(105*b^5*d) - (2*Sqrt[a + b]*(b^3*(63*B - 25*C) - 48*a^3*C + 4*a
^2*b*(14*B + 3*C) - 2*a*b^2*(7*B + 22*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d) - (2*(2
8*a*b*B - 24*a^2*C - 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*
b^3*d) + (2*(7*b*B - 6*a*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(35*b^2*d) + (2*C*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(7*b*d)
```

Rubi [A] time = 1.04485, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4033, 4092, 4082, 4005, 3832, 4004}

$$-\frac{2\left(-24a^2C+28abB-25b^2C\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^3d}-\frac{2\sqrt{a+b}\left(4a^2b(14B+3C)-48a^3C-2ab^2(7B+22C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c +
 d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(56*a^2*b*B + 63*b^3*B - 48*a^3*C - 44*a*b^2*C)*Cot
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(105*b^5*d) - (2*Sqrt[a + b]*(b^3*(63*B - 25*C) - 48*a^3*C + 4*a
^2*b*(14*B + 3*C) - 2*a*b^2*(7*B + 22*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d) - (2*(2
8*a*b*B - 24*a^2*C - 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*
b^3*d) + (2*(7*b*B - 6*a*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(35*b^2*d) + (2*C*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(7*b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
```

```
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2C\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7bd} + \frac{2\int \frac{\sec^2(c+dx)(}{}}{}$$

$$= \frac{2(7bB-6aC)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35b^2d} + \frac{2C}{}$$

$$= -\frac{2\left(28abB-24a^2C-25b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} +$$

$$= -\frac{2\left(28abB-24a^2C-25b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} +$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(56a^2bB+63b^3B-48a^3C-44ab^2C\right)\cot(c+}{10}$$

Mathematica [B] time = 24.6313, size = 3426, normalized size = 8.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*S
ec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(56*a^2*b*B + 63*b^3*B - 48*a^3*C -
44*a*b^2*C)*Sin[c + d*x])/(105*b^4) + (2*Sec[c + d*x]^2*(7*b*B*Sin[c + d*x]
 - 6*a*C*Sin[c + d*x]))/(35*b^2) + (2*Sec[c + d*x]*(-28*a*b*B*Sin[c + d*x]
+ 24*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b^3) + (2*C*Sec[c +
d*x]^2*Tan[c + d*x])/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*((-3*B)/(5*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^2*B)/(15*b^2*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^3*C)/(35*b^3*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) + (44*a*C)/(105*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (8*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (7*a*B*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) + (5*C*Sqrt
[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (16*a^4*C*Sqrt[Sec[c + d*x]
])/(35*b^4*Sqrt[b + a*Cos[c + d*x]]) + (32*a^2*C*Sqrt[Sec[c + d*x]])/(105*b
^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]]
)/(15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (3*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c +
d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (16*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec
[c + d*x]])/(35*b^4*Sqrt[b + a*Cos[c + d*x]]) + (44*a^2*C*Cos[2*(c + d*x)]*
Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Sec[c + d*x]]*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-56*a^2*b*B - 63*b^3*B +
48*a^3*C + 44*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + 2*b*(2*a*b^2*(7*B - 22*C) + 4*a^2*b*(14*B - 3*C) - 48
*a^3*C + b^3*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*b^2*C)
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*b^4*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]*((a*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-56*a^2*b*B - 63*b^3*B +
 48*a^3*C + 44*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
```

```
]], (a - b)/(a + b)] + 2*b*(2*a*b^2*(7*B - 22*C) + 4*a^2*b*(14*B - 3*C) - 4
8*a^3*C + b^3*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*b^2*C
)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(
105*b^4*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-56*a^2*b*B - 63*b^
3*B + 48*a^3*C + 44*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + 2*b*(2*a*b^2*(7*B - 22*C) + 4*a^2*b*(14*B - 3*C
) - 48*a^3*C + b^3*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*
b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*(((-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a
*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*
(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1
 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(2*a*b^2*(7*B
 - 22*C) + 4*a^2*b*(14*B - 3*C) - 48*a^3*C + b^3*(63*B + 25*C))*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (
(a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*b^2*C)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-
((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin
[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))] + (b*(2*a*b^2*(7*B - 22*C) + 4*a^2*b*(14*B - 3*C)
- 48*a^3*C + b^3*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
- a*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*b^2*C)*Cos[c + d*x]*Sec[(c +
d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-56*a^2*b*B - 63*b^3*B + 48*a^3*
C + 44*a*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c
 + d*x)/2] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 44*a*b^2*C)*Cos[c + d*x]*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(2*a*b^2*(7
*B - 22*C) + 4*a^2*b*(14*B - 3*C) - 48*a^3*C + b^3*(63*B + 25*C))*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^
3*C + 44*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^4*Sqrt[b
 + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-56*a^2*b*B - 6
3*b^3*B + 48*a^3*C + 44*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + 2*b*(2*a*b^2*(7*B - 22*C) + 4*a^2*b*(14*B -
 3*C) - 48*a^3*C + b^3*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 4
4*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2
]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 1.115, size = 3439, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(63*B*cos(d*x+c)^4*b^4-42*B*cos(d*x+c)^3*b^4-21*B*cos(d*x+c)*b^4-5
6*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^3*b-48*C*cos(d*x+c)^5*a^4+48*C*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^4+48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-28*B*cos(d*x
+c)^3*a^2*b^2+7*B*cos(d*x+c)^2*a*b^3-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)
^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*b^4-63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-56*B*cos(d*x+c)^4*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
2*b^2-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b^3+56*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+14*B*cos(d*x+c)^4
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b^3-56*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-56*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b^3+56*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+14*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-56*B*cos(d*x+
c)^4*a^3*b+56*B*cos(d*x+c)^4*a^2*b^2-70*B*cos(d*x+c)^4*a*b^3+56*B*cos(d*x+c
)^5*a^3*b-28*B*cos(d*x+c)^5*a^2*b^2+63*B*cos(d*x+c)^5*a*b^3-48*C*cos(d*x+c)
^4*a^3*b+50*C*cos(d*x+c)^4*a^2*b^2-44*C*cos(d*x+c)^4*a*b^3+24*C*cos(d*x+c)^
3*a^3*b+16*C*cos(d*x+c)^3*a*b^3-6*C*cos(d*x+c)^2*a^2*b^2+3*C*cos(d*x+c)*a*b
^3+24*C*cos(d*x+c)^5*a^3*b-44*C*cos(d*x+c)^5*a^2*b^2+25*C*cos(d*x+c)^5*a*b^
3+48*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
```

```
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^3*b+44*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+44*C*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b^3-48*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*b-12*C*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-44*C*sin(d*x+c)*cos
(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^3+48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^3*b+44*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+44*C*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^3-48*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3*b-12*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-44*C*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b^3+48*C*cos(d*x+c)^4*a^4-10*C*cos(d*x+c)^2*b^4+25*C*cos(d*x+c)^4*b^4
-15*C*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^4(c + dx)}{\sqrt{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx + c)^2 + B\sec(dx + c)\right)\sec(dx + c)^3}{\sqrt{b\sec(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^3/sqrt(b*sec(d*x
 + c) + a), x)
```

3.838 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=329

$$\frac{2\sqrt{a+b}\left(-8a^2C+2ab(5B+C)+b^2(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{15b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a*b*B - 8*a^2*C - 9*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) + (2*Sqrt[a + b]*(b^2*(5*B - 9*C) - 8*a^2*C + 2*a*b*(5*B + C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*b^3*d) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(15*b^2*d) + (2*C*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b*d)
```

Rubi [A] time = 0.68511, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4033, 4082, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(-8a^2C+2ab(5B+C)+b^2(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c +
 d*x]],x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a*b*B - 8*a^2*C - 9*b^2*C)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) + (2*Sqrt[a + b]*(b^2*(5*B - 9*C) - 8*a^2*C + 2*a*b*(5*B + C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*b^3*d) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(15*b^2*d) + (2*C*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
```

```
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2C\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{2\int \frac{\sec(c+dx)\left(aC\right.}{}}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2C\sec(c+dx)}{}$$

$$= \frac{2(5bB-4aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2C\sec(c+dx)}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(10abB-8a^2C-9b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{}\right.\right.}{15b^4d}$$

Mathematica [B] time = 22.667, size = 3000, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*S
ec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(-10*a*b*B + 8*a^2*C + 9*b^2*C)*Sin[
c + d*x])/(15*b^3) + (2*Sec[c + d*x]*(5*b*B*Sin[c + d*x] - 4*a*C*Sin[c + d*
x]))/(15*b^2) + (2*C*Sec[c + d*x]*Tan[c + d*x])/(5*b)))/(d*Sqrt[a + b*Sec[c
 + d*x]]) - (2*((2*a*B)/(3*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (3*C)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^2*C)/(15*b^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]])/(3*Sq
rt[b + a*Cos[c + d*x]]) + (2*a^2*B*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Co
s[c + d*x]]) - (8*a^3*C*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (7*a*C*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*B*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*
a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]
) - (3*a*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x
]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-
10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (-10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a
+ b*Sec[c + d*x]]*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(
2*(a + b)*(-10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*C + 2*a*b*(-5*B + C) +
b^2*(5*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + (-10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*(b + a*Cos[c + d*x])^(
3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan
[(c + d*x)/2]*(2*(a + b)*(-10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*C + 2*a*
b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + (-10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(((-10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x]
)^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)] - (b*(8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d
*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(
-10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
 (b*(8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Si
n[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d
*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
```

```
 + Cos[c + d*x]))] - a*(-10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-10*a*b*B + 8*a^2*C + 9*b^2*C)
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-
10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-10*a*b*B + 8*a^2
*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(15*b^3*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-10*a*b*B + 8*a^2*C
+ 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] - 2*b*(8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*b*B + 8*
a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.761, size = 2499, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-8*C*cos(d*x+c)^3*a^3+10*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(
d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^3-8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^3-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*C*sin(d*x+c)*cos(d*x+c)^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*B*co
s(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^3+5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+10*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*B*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2+10*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+10*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*B*cos(d*x
```

```
+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2-8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*C*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2+8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b+2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-4*C*cos(d*x+c)^4*a^2
*b+9*C*cos(d*x+c)^4*a*b^2+8*C*cos(d*x+c)^3*a^2*b-10*C*cos(d*x+c)^3*a*b^2+C*
cos(d*x+c)*a*b^2-10*B*cos(d*x+c)^3*a*b^2+5*B*cos(d*x+c)^2*a*b^2-10*B*cos(d*
x+c)^4*a^2*b+5*B*cos(d*x+c)^4*a*b^2+10*B*cos(d*x+c)^3*a^2*b-4*C*cos(d*x+c)^
2*a^2*b+8*C*cos(d*x+c)^4*a^3+9*C*cos(d*x+c)^3*b^3-6*C*cos(d*x+c)^2*b^3+5*B*
cos(d*x+c)^3*b^3-5*B*cos(d*x+c)*b^3-8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*C*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3-3*C*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c + dx))\sec^3(c + dx)}{\sqrt{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx + c)^2 + B\sec(dx + c)\right)\sec(dx + c)^2}{\sqrt{b\sec(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^2/sqrt(b*sec(d*x
 + c) + a), x)
```

3.839 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=261

$$-\frac{2\sqrt{a+b}(-2aC+3bB-bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2(a-b)}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B - 2*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) - (2*Sqrt[a
 + b]*(3*b*B - 2*a*C - b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*C*Sqrt[a + b*Sec[c
 + d*x]]*Tan[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.436528, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4010, 4005, 3832, 4004}

$$-\frac{2\sqrt{a+b}(-2aC+3bB-bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2(a-b)\sqrt{a+b}(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d
*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B - 2*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) - (2*Sqrt[a
 + b]*(3*b*B - 2*a*C - b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*C*Sqrt[a + b*Sec[c
 + d*x]]*Tan[c + d*x])/(3*b*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$=\frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd}+\frac{2\int \frac{\sec(c+dx)\left(\frac{bC}{2}+\frac{1}{2}(3bB-2aC)\,\mathrm{s}\right.}{\sqrt{a+b\sec(c+dx)}}}{3b}$$

$$=\frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd}+\frac{(3bB-2aC)\int \frac{\sec(c+dx)(1+\mathrm{s}}{\sqrt{a+b\sec(c}}}{3b}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bB-2aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{3b^3d}$$

Mathematica [A] time = 16.0296, size = 372, normalized size = 1.43

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2b(b(3B+C)-2aC)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\mathrm{t}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec
[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-3*
b*B + 2*a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 2*b*(-2*a*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*b*B - 2*a*C)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^2*d*Sqrt[Sec[(c + d
```

```
*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((
2*(3*b*B - 2*a*C)*Sin[c + d*x])/(3*b^2) + (2*C*Tan[c + d*x])/(3*b)))/(d*Sqr
t[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.509, size = 1563, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/b^2*(-1+cos(d*x+c))^2*(3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b+C*c
os(d*x+c)^3*a*b-2*C*cos(d*x+c)^2*a*b+C*cos(d*x+c)*a*b-3*B*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
-2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)^2*b^2+3*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+C*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-b^2*C-3*B*
cos(d*x+c)*b^2-2*C*cos(d*x+c)^3*a^2+2*C*cos(d*x+c)^2*a^2+C*cos(d*x+c)^2*b^2
)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos
(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)/sqrt(b*sec(d*x +
 c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)/sqrt(b*sec(d*x +
 c) + a), x)
```

3.840 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=210

$$\frac{2\sqrt{a+b}(B-C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2C(a-b)\sqrt{a+b}\cot(c}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(B - C)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d)
```

Rubi [A] time = 0.173793, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4058, 12, 3832, 4004}

$$\frac{2\sqrt{a+b}(B-C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2C(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(B - C)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d)
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = C\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{(B-C)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{b^2d}$$

$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{b^2d}$$

Mathematica [A] time = 14.2852, size = 312, normalized size = 1.49

$$\frac{2C\tan(c+dx)(a\cos(c+dx)+b)}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2b(B+C)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(}{(a+b)(co}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*C*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*
b*(B + C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + C*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]))/(b*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + (2*C*(b +
 a*Cos[c + d*x])*Tan[c + d*x])/(b*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.418, size = 829, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/b*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^2*(B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*b+C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
```

```
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-C*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+B*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+C*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*b-C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*a-C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*b+C*cos(d*x+c)^2*a-C*cos(d*x+c)*a+C*cos(d*x+c)*b-C*b)/sin(d*x+c)
^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.841 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-}{}}}{}$$

```
[Out] (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*d)
```

Rubi [A] time = 0.208505, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4072, 3921, 3784, 3832}

$$\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c + d
*x]],x]
```

```
[Out] (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx &= \int \frac{B+C\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\ &= B\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + C\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\ &= \frac{2\sqrt{a+b}C\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{bd}\end{aligned}$$

Mathematica [A] time = 2.19066, size = 147, normalized size = 0.71

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left((B-C)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2B}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec
[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((B - C)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.365, size = 215, normalized size = 1.

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d\,(b+a\cos(dx+c))(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\Big(B\text{Ellipti}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*
x+c))*(B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*B*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-C*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/sqrt(b*sec(d*x +
 c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c))/sqrt
(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B+C\sec\left(c+dx\right)\right)\cos\left(c+dx\right)\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x)/sqrt(a + b*sec(c +
d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/sqrt(b*sec(d*x +
 c) + a), x)
```

3.842 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=348

$$\frac{B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{\sqrt{a+b}(bB-2aC)\cot(c+$$

```
[Out] ((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*B*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (Sqrt[a + b]*(b*B - 2*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.499379, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4034, 4059, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(bB-2aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{B\sin(c+dx)\sqrt{a+b}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c +
 d*x]],x]
```


```
[Out] ((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*B*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (Sqrt[a + b]*(b*B - 2*a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
```

```
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(bB-2aC)+\frac{1}{2}bB\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(bB-2aC)-\frac{1}{2}bB\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - \frac{(b\ldots}{\ldots}$$

$$= \frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c\ldots}{a+b\ldots}}\ldots}{abd}$$

$$= \frac{(a-b)\sqrt{a+b}B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c\ldots}{a+b\ldots}}\ldots}{abd}$$

Mathematica [C] time = 16.7446, size = 1027, normalized size = 2.95

$$\frac{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(-a\sqrt{\frac{b-a}{a+b}}B\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\tan^3\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*S
ec[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^
(-1)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x
)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] + b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x
)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] - a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x
)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d
*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + (2*I)*b*B*EllipticPi[-((a + b)/(a -
 b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)
*a*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] + (2*I)*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2
]^2*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
4*I)*a*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*B*EllipticE[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - (2*I)*(b*B - a*C)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/(a + b)
]*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.385, size = 1025, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(-1+cos(d*x+c))^2*(B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a+B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b-2*B*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+
c)*b+4*C*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
```

```
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+B*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*B*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+4*C*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*sin(d*x+c
)-2*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)*a+B*cos(d*x+c)^3*a-B*a*cos(d*x+c)^2+B*cos(d*x+c)^2*b-B*b*cos(d*x+c))
*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/sqrt(b*sec(d*x
 + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)^2\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c))/
sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/sqrt(b*sec(d*x
 + c) + a), x)
```

3.843 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=471

$$\frac{2\left(4a^2b(10B-9C)-48a^3C+6ab^2(5B-2C)+b^3(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{15b^4d\sqrt{a+b}}$$

```
[Out] (2*(40*a^3*b*B - 25*a*b^3*B - 48*a^4*C + 24*a^2*b^2*C + 9*b^4*C)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(15*b^5*Sqrt[a + b]*d) + (2*(b^3*(5*B - 9*C) + 4*a^2*b*(10*B - 9*C) + 6
*a*b^2*(5*B - 2*C) - 48*a^3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*Sqrt[a + b]*d) + (2*a*
(b*B - a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]]) + (2*(20*a^2*b*B - 5*b^3*B - 24*a^3*C + 9*a*b^2*C)*Sqrt[a + b*Sec[
c + d*x]]*Tan[c + d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*b*B - 6*a^2*C + b^
2*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^2*(a^2 - b^2)
*d)
```

Rubi [A] time = 1.27295, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4029, 4092, 4082, 4005, 3832, 4004}

$$\frac{2a(bB-aC)\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-6a^2C+5abB+b^2C\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2\left(20a^2bB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^(3/2),x]
```


```
[Out] (2*(40*a^3*b*B - 25*a*b^3*B - 48*a^4*C + 24*a^2*b^2*C + 9*b^4*C)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(15*b^5*Sqrt[a + b]*d) + (2*(b^3*(5*B - 9*C) + 4*a^2*b*(10*B - 9*C) + 6
*a*b^2*(5*B - 2*C) - 48*a^3*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*Sqrt[a + b]*d) + (2*a*
(b*B - a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]]) + (2*(20*a^2*b*B - 5*b^3*B - 24*a^3*C + 9*a*b^2*C)*Sqrt[a + b*Sec[
c + d*x]]*Tan[c + d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*b*B - 6*a^2*C + b^
2*C)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^2*(a^2 - b^2)
*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec^2(c+dx)\left(2a(bB-aC)-\frac{1}{2}b(\ldots}{\ldots}}{\ldots}$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(5abB-6a^2C+b^2C\right)\mathrm{se}\ldots}{\ldots}$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(20a^2bB-5b^3B-24a^3\ldots\right.}{\ldots}$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(20a^2bB-5b^3B-24a^3\ldots\right.}{\ldots}$$

$$= \frac{2\left(40a^3bB-25ab^3B-48a^4C+24a^2b^2C+9b^4C\right)\cot(c+dx)E\left(\mathrm{si}\ldots\right.}{15b^5\sqrt{a+b\ldots}}$$

Mathematica [B] time = 25.8178, size = 3953, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(40*a^3*b*B - 25*a*b^3*B - 48*a^
4*C + 24*a^2*b^2*C + 9*b^4*C)*Sin[c + d*x])/(15*b^4*(-a^2 + b^2)) + (2*Sec[
c + d*x]*(5*b*B*Sin[c + d*x] - 9*a*C*Sin[c + d*x]))/(15*b^3) - (2*(a^3*b*B*
Sin[c + d*x] - a^4*C*Sin[c + d*x]))/(b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x]))
 + (2*C*Sec[c + d*x]*Tan[c + d*x])/(5*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2))
 + (2*(b + a*Cos[c + d*x])*((5*a*B)/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(3*b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) + (16*a^4*C)/(5*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^2*C)/(5*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (3*b*C)/(5*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (8*a^4*B*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*S
qrt[b + a*Cos[c + d*x]]) + (7*a^2*B*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*S
qrt[b + a*Cos[c + d*x]]) + (b*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b
+ a*Cos[c + d*x]]) - (4*a*C*Sqrt[Sec[c + d*x]])/(5*(-a^2 + b^2)*Sqrt[b + a*
Cos[c + d*x]]) + (16*a^5*C*Sqrt[Sec[c + d*x]])/(5*b^4*(-a^2 + b^2)*Sqrt[b +
 a*Cos[c + d*x]]) - (12*a^3*C*Sqrt[Sec[c + d*x]])/(5*b^2*(-a^2 + b^2)*Sqrt[
b + a*Cos[c + d*x]]) - (8*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3
*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (5*a^2*B*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (3*a*C*Cos[2*(c
+ d*x)]*Sqrt[Sec[c + d*x]])/(5*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (16
*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b^4*(-a^2 + b^2)*Sqrt[b + a*
Cos[c + d*x]]) - (8*a^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b^2*(-a^2
 + b^2)*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*(2*(a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b
^2*C - 9*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 2*b*(a + b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*
```

C) + 4*a^2*b*(10*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^4*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(3/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*C) + 4*a^2*b*(10*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^4*(-a^2 + b^2)*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*C) + 4*a^2*b*(10*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^4*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*C) + 4*a^2*b*(10*B + 9*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*C) + 4*a^2*b*(10*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*C) + 4*a^2*b*(10*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(15*b^4*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Sqrt[C

```
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a
+ b)*(-48*a^3*C - 6*a*b^2*(5*B + 2*C) + b^3*(5*B + 9*C) + 4*a^2*b*(10*B + 9
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (-40*a^3*b*B + 25*a*b^3*B + 48*a^4*C - 24*a^2*b^2*C - 9*b^4*C)*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c +
d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Ta
n[c + d*x]))/(15*b^4*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 1.248, size = 4320, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/15/d/(a-b)/(a+b)/b^4*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-40*B*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^4*b+3*C*a^2*b^3+24*C*cos(d*x+c)^4*a^4*b-9*C*cos(d*x+c)^4*a^2
*b^3-18*C*cos(d*x+c)^3*a^3*b^2-15*C*cos(d*x+c)^3*a*b^4-18*C*cos(d*x+c)^2*a^
2*b^3+6*C*cos(d*x+c)*a*b^4+48*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-9*C*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5+9*C
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*b^5+5*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5+48*C*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-9*C*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*b^5+9*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*b^5+5*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5-6*C*cos(d*x+c)^2*b^5
-3*C*b^5-24*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-24*C*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-9*C*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a*b^4-48*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-12*C*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+24*C*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
```

)^(1/2)*a^2*b^3-3*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+5*B*cos(d*x+c)^3*b^5-5*B*cos(d*x+c)
*b^5-40*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a^3*b^2+25*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+25*B*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a*b^4+40*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+10*B*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-25*B*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a*b^4+48*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-24*C*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-24*C*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^2*b^3-9*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4-48*C*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-12*C*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a^3*b^2+24*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-3*C*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4-40*B*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d
x+c)(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^4*b-40*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+25*B*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+25
*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a*b^4+40*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+10*B*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^
3-25*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a*b^4+48*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-25*B*cos(d*x+c)^3*a*b^4
-48*C*cos(d*x+c)^3*a^4*b+24*C*cos(d*x+c)^3*a^2*b^3-20*B*cos(d*x+c)^2*a^3*b^
2+24*C*cos(d*x+c)^2*a^4*b+5*B*cos(d*x+c)*a^2*b^3-6*C*cos(d*x+c)*a^3*b^2+40*
B*cos(d*x+c)^4*a^4*b-25*B*cos(d*x+c)^4*a^2*b^3+24*C*cos(d*x+c)^4*a^3*b^2+9*
C*cos(d*x+c)^4*a*b^4-40*B*cos(d*x+c)^3*a^4*b+40*B*cos(d*x+c)^3*a^3*b^2-20*B

```
*cos(d*x+c)^4*a^3*b^2+5*B*cos(d*x+c)^4*a*b^4+20*B*cos(d*x+c)^3*a^2*b^3+20*B
*cos(d*x+c)^2*a*b^4-48*C*cos(d*x+c)^4*a^5+48*C*cos(d*x+c)^3*a^5+9*C*cos(d*x
+c)^3*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{4}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^3/(b*sec(d*x + c
) + a)^(3/2), x)
```

3.844 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(2a+b)(-4aC+3bB-bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}-\frac{2a^2(}{b^2d\left(a^2\right.}$$

```
[Out] (-2*(6*a^2*b*B - 3*b^3*B - 8*a^3*C + 5*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a
 + b]*d) - (2*(2*a + b)*(3*b*B - 4*a*C - b*C)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a +
 b]*d) - (2*a^2*(b*B - a*C)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.810245, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4028, 4082, 4005, 3832, 4004}

$$-\frac{2a^2(bB-aC)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(6a^2bB-8a^3C+5ab^2C-3b^3B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\right.}{3b^4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^(3/2),x]
```


```
[Out] (-2*(6*a^2*b*B - 3*b^3*B - 8*a^3*C + 5*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a
 + b]*d) - (2*(2*a + b)*(3*b*B - 4*a*C - b*C)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a +
 b]*d) - (2*a^2*(b*B - a*C)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2(bB-aC)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}ab(bB-aC)-\frac{1}{2}(\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2a^2(bB-aC)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{a+b\sec(c+dx)}\tan\cdots}{3b^2d}$$

$$= -\frac{2a^2(bB-aC)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{a+b\sec(c+dx)}\tan\cdots}{3b^2d}$$

$$= -\frac{2\left(6a^2bB-3b^3B-8a^3C+5ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cdots}}{\sqrt{\cdots}}\right.\right.}{3b^4\sqrt{a+b}d}$$

Mathematica [B] time = 24.5688, size = 3460, normalized size = 10.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-6*a^2*b*B + 3*b^3*B + 8*a^3*C
- 5*a*b^2*C)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) + (2*(a^2*b*B*Sin[c + d*x]
- a^3*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*C*Tan[c
 + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])
*((2*a^2*B)/(b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(b*B)/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*C)/
(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*C)/(3
*b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*B*Sqr
t[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*B*Sqrt[Se
c[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*C*Sqrt[Se
c[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (7*a^2*C*Sqrt[
Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (b*C*Sqrt[Sec[
c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (a*B*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*B*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x
]]) - (8*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqr
t[b + a*Cos[c + d*x]]) + (5*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b
*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2
*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(-2*a^2 - a*b + b^2)*(-4*a*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*b*B + 3*b^3*B + 8
*a^3*C - 5*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[
c + d*x])^(3/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*
(a + b)*(-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2
)*(-4*a*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)] + (-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Cos[c
+ d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-
a^2 + b^2)*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-6*a^2*b*B + 3*b
^3*B + 8*a^3*C - 5*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(-4*a*C + b*(3*B + C))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (
-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*(((-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*b*B + 3*b^3*B + 8*a^3*C
 - 5*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x]
)/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] - (b*(-2*a^2 - a*b + b^2)*(-4*a*C + b*(3*B + C))*Sqrt
```

```
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x
])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])] + ((a + b)*(-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))] - (b*(-2*a^2 - a*b + b^2)*(-4*a*C + b*(3*B + C)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-6*a^2*b*B + 3*b^3*B + 8*
a^3*C - 5*a*b^2*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*
x)/2] - (-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*a^2*b*B + 3*b^3*B + 8
*a^3*C - 5*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]^2 - (b*(-2*a^2 - a*b + b^2)*(-4*a*C + b*(3*B + C))*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-6*a^2*b*B + 3*b^3*B + 8*a^3*C
 - 5*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^3*(-a^2 + b^2)
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-6*a^2*b
*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(-4*a*C + b*(3*
B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + (-6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c +
 d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3
*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Co
s[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.732, size = 3333, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d/(a-b)/(a+b)/b^3*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*B*co
s(d*x+c)^2*b^4-8*C*cos(d*x+c)^3*a^4+3*B*cos(d*x+c)*b^4+6*B*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b^3-6*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-6*B*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2
*b^2+3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a*b^3-8*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
```

)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-2*C*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b^2+5*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a*b^3+8*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-5*C*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
^2-5*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a*b^3-3*B*cos(d*x+c)^3*a^2*b^2-6*B*cos(d*x+c)^2*a^3*b+
3*B*cos(d*x+c)^2*a*b^3-3*B*cos(d*x+c)*a^2*b^2+6*B*cos(d*x+c)^2*a^2*b^2+8*C*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a^4-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-5*C*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*a^2*b^2+5*C*cos(d*x+c)^2*a*b^3+6*B*cos(d*x+c)^3*a^3*b-3*B*cos(d*x+c)^3*a*b
^3+5*C*cos(d*x+c)^3*a^2*b^2+4*C*cos(d*x+c)^3*a^3*b-C*cos(d*x+c)^3*a*b^3-4*C
*cos(d*x+c)^2*a^2*b^2-4*C*cos(d*x+c)*a*b^3-8*C*cos(d*x+c)^2*a^3*b+4*C*cos(d
*x+c)*a^3*b-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+3*B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b^2-C*a^2*b^2+8*C*cos(d*x+c)^2*a^4+3*B*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*a*b^3-5*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a*b^3-2*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*C*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-
C*cos(d*x+c)^2*b^4+C*b^4-3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+3*B*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-C*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*b^4+8*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-C*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/

```
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+3
*B*cos(d*x+c)*b^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^2/(b*sec(d*x + c
) + a)^(3/2), x)
```

3.845 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{2(b(B-C)-2aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\frac{2a(bB-aC)}{bd\left(a^2-b^2\right)\sqrt{a}}$$

```
[Out] (2*(a*b*B - 2*a^2*C + b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(b*(B -
 C) - 2*a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*a*(b*B - a*C)*Tan[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.504907, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4072, 4009, 4005, 3832, 4004}

$$\frac{2a(bB-aC)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(-2a^2C+abB+b^2C\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^(3/2),x]
```


```
[Out] (2*(a*b*B - 2*a^2*C + b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(b*(B -
 C) - 2*a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*a*(b*B - a*C)*Tan[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(bB-aC)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}b(bB-aC)-\frac{1}{2}\left(abB-2a^2C\right.\right.}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(bB-aC)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{((a-b)(b(B-C)-2aC))\int \frac{\sec}{\sqrt{a+b}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2\left(abB-2a^2C+b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{b^3\sqrt{a+b}d}$$

Mathematica [A] time = 18.4937, size = 466, normalized size = 1.69

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)\left(2b(a+b)(b(B+C)-2aC)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(a*b*B - 2*a^2*C + b^2*C)*Sin[c
+ d*x])/(b^2*(-a^2 + b^2)) - (2*(a*b*B*Sin[c + d*x] - a^2*C*Sin[c + d*x]))/
(b*(-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2
*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
```

```
x]]*(2*(a + b)*(-(a*b*B) + 2*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-2*a*C + b*(B + C))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
(-(a*b*B) + 2*a^2*C - b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
 b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.48, size = 2275, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/b^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*C*cos(
d*x+c)*a^3-2*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-2*C*a^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-B*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+C*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-C*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^3-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^2-a^2*b*C+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*C*a^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2
*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^2+C*cos(d*x+c)*a*b^2+B*cos(d*x+c)^2*a*b^2-C*cos(d*x+c)^2*a^2*b+2*C*cos(
d*x+c)^2*a^3-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^3-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-B*cos(d*x+c)^2*a^2*b+B*cos
(d*x+c)*a^2*b-B*cos(d*x+c)*a*b^2+2*C*cos(d*x+c)*a^2*b-C*cos(d*x+c)^2*a*b^2+
C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*si
n(d*x+c)+C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
```

```
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^2-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2*b*sin(d*x+c)+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+C*b^3-C*
cos(d*x+c)*b^3)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)/(b*sec(d*x + c)
+ a)^(3/2), x)
```

3.846 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=254

$$\frac{2(B+C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2(bB-aC)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx}}$$

```
[Out] (-2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*(B + C)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*Sqrt[a + b]*d) - (2*(b*B - a*C)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rubi [A] time = 0.304376, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4060, 4058, 12, 3832, 4004}

$$-\frac{2(bB-aC)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(bB-aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*(B + C)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*Sqrt[a + b]*d) - (2*(b*B - a*C)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}a(aB-bC)\sec(c+dx)-\frac{1}{2}a(bB-aC)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2(bB-aC)\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\left(\frac{1}{2}a(bB-aC)-\frac{1}{2}a(aB-bC)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} + \frac{(bB-}{}$$

$$= -\frac{2(bB-aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+se}{a}}}{b^2\sqrt{a+b}d}$$

$$= -\frac{2(bB-aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+se}{a}}}{b^2\sqrt{a+b}d}$$

Mathematica [A] time = 15.3451, size = 426, normalized size = 1.68

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(\frac{2(bB\sin(c+dx)-aC\sin(c+dx))}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}-\frac{2(bB-aC)\sin(c+dx)}{b\left(b^2-a^2\right)}\right)}{d(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*(b*B - a*C)*Sin[c + d*x])/(b*(-
a^2 + b^2)) + (2*(b*B*Sin[c + d*x] - a*C*Sin[c + d*x]))/((-a^2 + b^2)*(b +
a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])
*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-(b*B
) + a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + 2*b*(a + b)*(B - C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
```

```
x)/2]], (a - b)/(a + b)] - (b*B - a*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/((-(a^2*b) + b^3)*d*Sqrt[Sec[(c + d*x)/
2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.392, size = 1633, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/b/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-C*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+B*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+B*b^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-B*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+
c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*s
in(d*x+c)-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*sin(d*x+c)-C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*a*b+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b*sin(d*x+c)+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-B*cos(d*x+c)^2*a*b+B*cos(d*x+c)^2*b^2
+C*cos(d*x+c)^2*a^2-C*cos(d*x+c)^2*a*b+B*cos(d*x+c)*a*b-B*cos(d*x+c)*b^2-C*
cos(d*x+c)*a^2+C*cos(d*x+c)*a*b)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec(dx+c)^2+B\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^2*
sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B+C\sec(c+dx))\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec(dx+c)^2+B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x
)
```

3.847 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{2(bB-aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}+\frac{2b(bB-aC)\tan(c+}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c}$$

```
[Out] (2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*(b*B - a*C)*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a*b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*
d) + (2*b*(b*B - a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]])
```

Rubi [A] time = 0.518813, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {4072, 3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(bB-aC)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^(3/2),x]
```


```
[Out] (2*(b*B - a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*(b*B - a*C)*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a*b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*
d) + (2*b*(b*B - a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
```

```
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{B+C\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}\left(a^2-b^2\right)B+\frac{1}{2}a(bB-aC)\sec(c+dx)+}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(bB-aC)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}\left(a^2-b^2\right)B+\left(\frac{1}{2}a(bB-aC)-\frac{1}{2}b(bB-aC\right.}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2(bB-aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{ab\sqrt{a+b}d}$$

$$= \frac{2(bB-aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{ab\sqrt{a+b}d}$$

Mathematica [C] time = 14.4943, size = 1445, normalized size = 3.84

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(-(b*B) + a*C)*Sin[c + d*x])/(a*(a^2
 - b^2)) - (2*(-(b^2*B*Sin[c + d*x]) + a*b*C*Sin[c + d*x]))/(a*(a^2 - b^2)*
(b + a*Cos[c + d*x]))))/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c +
 d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - a^2*Sqrt[(-a + b
)/(a + b)]*C*Tan[(c + d*x)/2] - a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/
2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 2*a^2*Sqrt[(-a + b
)/(a + b)]*C*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x
)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - a^2*Sqrt[(-a + b
)/(a + b)]*C*Tan[(c + d*x)/2]^5 + a*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x
)/2]^5 - (2*I)*a^2*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)
*b^2*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*B*Ellipti
cPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*B*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)
*(-(b*B) + a*C)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a -
 b)*(2*b*B + a*(B - C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)
/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)])
)/(a*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Ta
```

```
n[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(
a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.392, size = 2009, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*cos(d*x+c)
^2*a*b+C*cos(d*x+c)^2*a*b-C*cos(d*x+c)*a*b-2*B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-C*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*B*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-C*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-B*
cos(d*x+c)^2*b^2-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-B*b^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-B*cos(d*x+c)*a*b+C*
cos(d*x+c)*a^2+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+B*cos(d*x+c)*b^2-C*cos(d
*x+c)^2*a^2+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b*sin(d*x+c)-B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*a*b+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*B*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+2*B*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*co
s(d*x+c)*b^2+B*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
```

```
)/(a+b))^(1/2))*cos(d*x+c)*a^2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/(b*sec(d*x + c)
+ a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B+C\sec\left(c+dx\right)\right)\cos\left(c+dx\right)\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*cos(c + d*x)*sec(c + d*x)/(a + b*sec(c + d*x)
)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/(b*sec(d*x + c)
+ a)^(3/2), x)
```

3.848 $\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=427

$$\frac{(a(B-2C)+3bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}+\frac{b\left(a^2B+2abC-3b\ldots\right.}{a^2d\left(a^2-b^2\right)\sqrt{a-\ldots}}$$

```
[Out] ((a^2*B - 3*b^2*B + 2*a*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((3*b*B
+ a*(B - 2*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*Sqrt[a + b]*d) + (Sqrt[a + b]*(3*b*B - 2*
a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (B*Sin[c + d*x])/(a*d*Sqrt[a + b*S
ec[c + d*x]]) + (b*(a^2*B - 3*b^2*B + 2*a*b*C)*Tan[c + d*x])/(a^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.79253, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4072, 4034, 4061, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(a^2B+2abC-3b^2B\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2B+2abC-3b^2B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c\ldots}}{\sqrt{a+b}}\right.\right.}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^(3/2),x]
```


```
[Out] ((a^2*B - 3*b^2*B + 2*a*b*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((3*b*B
+ a*(B - 2*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*Sqrt[a + b]*d) + (Sqrt[a + b]*(3*b*B - 2*
a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (B*Sin[c + d*x])/(a*d*Sqrt[a + b*S
ec[c + d*x]]) + (b*(a^2*B - 3*b^2*B + 2*a*b*C)*Tan[c + d*x])/(a^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{B\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{2}(3bB-2aC)-\frac{1}{2}bB\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{B\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2B-3b^2B+2abC\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \cdots}{\cdots}$$

$$= \frac{B\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2B-3b^2B+2abC\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \cdots}{\cdots}$$

$$= \frac{\left(a^2B-3b^2B+2abC\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b\cdots}{\cdots}}\cdots}{a^2b\sqrt{a+b}d}$$

$$= \frac{\left(a^2B-3b^2B+2abC\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b\cdots}{\cdots}}\cdots}{a^2b\sqrt{a+b}d}$$

Mathematica [B] time = 19.4106, size = 1613, normalized size = 3.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b*(b*B - a*C)*Sin[c + d*x])/(a^
2*(-a^2 + b^2)) + (2*(-(b^3*B*Sin[c + d*x]) + a*b^2*C*Sin[c + d*x]))/(a^2*(
a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - ((b + a
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]*(a^3*B*Tan[(c + d*x)/2] + a^2*b*B*Tan[(c + d*x)/2] - 3*a*b^2*B*T
an[(c + d*x)/2] - 3*b^3*B*Tan[(c + d*x)/2] + 2*a^2*b*C*Tan[(c + d*x)/2] + 2
*a*b^2*C*Tan[(c + d*x)/2] - 2*a^3*B*Tan[(c + d*x)/2]^3 + 6*a*b^2*B*Tan[(c +
 d*x)/2]^3 - 4*a^2*b*C*Tan[(c + d*x)/2]^3 + a^3*B*Tan[(c + d*x)/2]^5 - a^2*
b*B*Tan[(c + d*x)/2]^5 - 3*a*b^2*B*Tan[(c + d*x)/2]^5 + 3*b^3*B*Tan[(c + d*
x)/2]^5 + 2*a^2*b*C*Tan[(c + d*x)/2]^5 - 2*a*b^2*C*Tan[(c + d*x)/2]^5 + 6*a
^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 6*b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^3*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*b^2*C*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*b*B*
EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 6*b^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^3*C*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
```

```
d*x)/2]^2)/(a + b)] + 4*a*b^2*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(a^2*B -
 3*b^2*B + 2*a*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(a + b)*(-(b*B) + a*C)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*Sqr
t[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*
x)/2]^2)))
```

Maple [B] time = 0.388, size = 2871, normalized size = 6.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*a^3*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-
3*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*si
n(d*x+c)+4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*a^3*sin(d*x+c)-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^3*sin(d*x+c)+6*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*B*cos(d*x+c)*a^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+2*B*c
os(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*C*a^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-4*C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a*b^2-2*C*cos(d*x+c)*a*b^2-B*cos(d*x+c)^3*a*b^2+3*B*cos(d*
x+c)^2*a*b^2-2*C*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^3*a^3-B*cos(d*x+c)^2*a^3-3
*B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*b^3+B*cos(d*x+c)^2*a^2*b-B*cos(d*x+c)*a^
2*b-2*B*cos(d*x+c)*a*b^2+2*C*cos(d*x+c)*a^2*b+2*C*cos(d*x+c)^2*a*b^2+2*C*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
```

```
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b*sin(d*x+c)-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*b*sin(d*x+c)+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-3*B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^3+4*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-6*B*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*
B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*si
n(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2*b*sin(d*x+c)-3*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*C*b^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a)/(b+a*cos(d*x+c))/si
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/(b*sec(d*x + c
) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c))*
sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^2 + B \sec\left(dx+c\right)\right) \cos\left(dx+c\right)^2}{\left(b \sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^2/(b*sec(d*x + c
) + a)^(3/2), x)
```

3.849 $\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=509

$$-\frac{2\left(2a^2b^2(3B+8C)+a^3b(8B-12C)-16a^4C-9ab^3(B-C)-b^4(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\mathrm{Ell}}{3b^4d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (-2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 16*a^5*C + 28*a^3*b^2*C - 8*a*b^4
*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)*d) - (2*(a^3*b*(8*B - 12*C)
- 9*a*b^3*(B - C) - b^4*(3*B - C) - 16*a^4*C + 2*a^2*b^2*(3*B + 8*C))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(b*B - a*C)*Sec[c + d*x]^
2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(3*
a^2*b*B - 7*b^3*B - 6*a^3*C + 10*a*b^2*C)*Tan[c + d*x])/(3*b^3*(a^2 - b^2)^
2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(a*b*B - 2*a^2*C + b^2*C)*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 1.57479, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4072, 4029, 4090, 4082, 4005, 3832, 4004}

$$\frac{2a(bB-aC)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2bB-6a^3C+10ab^2C-7b^3B\right)\tan(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-2a^2C+abB+b^2C\right)\tan}{3b^3d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^(5/2),x]
```

```
[Out] (-2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 16*a^5*C + 28*a^3*b^2*C - 8*a*b^4
*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)*d) - (2*(a^3*b*(8*B - 12*C)
- 9*a*b^3*(B - C) - b^4*(3*B - C) - 16*a^4*C + 2*a^2*b^2*(3*B + 8*C))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(b*B - a*C)*Sec[c + d*x]^
2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(3*
a^2*b*B - 7*b^3*B - 6*a^3*C + 10*a*b^2*C)*Tan[c + d*x])/(3*b^3*(a^2 - b^2)^
2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(a*b*B - 2*a^2*C + b^2*C)*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec^2(c+dx)\left(2a(bB-aC)-\frac{3}{2}b(\right.}{}}{}$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3C\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3C\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= \frac{2a(bB-aC)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2bB-7b^3B-6a^3C\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= -\frac{2\left(8a^4bB-15a^2b^3B+3b^5B-16a^5C+28a^3b^2C-8ab^4C\right)\cot(c}{3(a-b)b^5}$$

Mathematica [B] time = 26.4609, size = 4342, normalized size = 8.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^
5*B - 16*a^5*C + 28*a^3*b^2*C - 8*a*b^4*C)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2
)^2) + (2*(a^2*b*B*Sin[c + d*x] - a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (2*(-4*a^4*b*B*Sin[c + d*x] + 8*a^2*b^3*B*Sin[c +
 d*x] + 7*a^5*C*Sin[c + d*x] - 11*a^3*b^2*C*Sin[c + d*x]))/(3*b^3*(-a^2 + b
^2)^2*(b + a*Cos[c + d*x])) + (2*C*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c
+ d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*((5*a^2*B)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*B)/(3*b^2*(-a^2 + b^2)^2*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*B)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^5*C)/(3*b^3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (28*a^3*C)/(3*b*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a*b*C)/(3*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*B*Sqrt[Sec[c + d*x
]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a^3*B*Sqrt[Sec[c
+ d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3*a*b*B*Sqrt[Sec[
c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^2*C*Sqrt[Sec[c
+ d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*C*Sqrt[Sec[c +
 d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (32*a^4*C*Sqrt[Se
c[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*C*Sqrt[
Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*
x]]) + (5*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt
[b + a*Cos[c + d*x]]) - (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2
+ b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (8*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c +
 d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*C*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) -
```

(28*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(2*(a + b)*(-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2
*C + 8*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
x])/((a + b)(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + 2*b*(a + b)*(-16*a^4*C - 9*a*b^3*(B + C) + b^4*(3*B + C) + 4
*a^3*b*(2*B + 3*C) + 2*a^2*b^2*(-3*B + 8*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*b*B + 15*a^2*b^3*B - 3*
b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(
c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*Sin[c + d*x]*(2*(a + b)*(-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*
a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*C - 9*a*b^3*(B + C) +
b^4*(3*B + C) + 4*a^3*b*(2*B + 3*C) + 2*a^2*b^2*(-3*B + 8*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*b*B +
15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b
^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-8*a^4*b*B + 15*a^2*b
^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*C
- 9*a*b^3*(B + C) + b^4*(3*B + C) + 4*a^3*b*(2*B + 3*C) + 2*a^2*b^2*(-3*B
+ 8*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + (-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a
*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]
) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-8*a^4*b*B + 15*a^2*b^3*B -
3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Cos[c + d*x]*(b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*
B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
+ d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-16*a^4*C -
9*a*b^3*(B + C) + b^4*(3*B + C) + 4*a^3*b*(2*B + 3*C) + 2*a^2*b^2*(-3*B + 8
*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Co
s[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])] + ((a + b)*(-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 2
8*a^3*b^2*C + 8*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a
+ b)*(-16*a^4*C - 9*a*b^3*(B + C) + b^4*(3*B + C) + 4*a^3*b*(2*B + 3*C) +
2*a^2*b^2*(-3*B + 8*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-8*
a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Cos
[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-8*a^4*b*B +
15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*(b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-8*a^4*b*B + 15
*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Cos[c + d*x]*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-16*

```
a^4*C - 9*a*b^3*(B + C) + b^4*(3*B + C) + 4*a^3*b*(2*B + 3*C) + 2*a^2*b^2*(
-3*B + 8*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-8*a^4*b
*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-8*a^4*b*B + 15*a^2*b^3*B - 3
*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*C - 9*a*b
^3*(B + C) + b^4*(3*B + C) + 4*a^3*b*(2*B + 3*C) + 2*a^2*b^2*(-3*B + 8*C))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (
-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*C)*
Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(C
os[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c +
 d*x]*Tan[c + d*x]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.589, size = 8046, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^3/(b*sec(d*x + c
) + a)^(5/2), x)
```

3.850 $\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{2\left(2a^2b(B-3C)-8a^3C+3ab^2(B+3C)-3b^3(B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{}}\right.\right.}{3b^3d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (2*(2*a^3*b*B - 6*a*b^3*B - 8*a^4*C + 15*a^2*b^2*C - 3*b^4*C)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*(a - b)*b^4*(a + b)^(3/2)*d) + (2*(2*a^2*b*(B - 3*C) - 3*b^3*(B - C) -
8*a^3*C + 3*a*b^2*(B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d
) - (2*a^2*(b*B - a*C)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^(3/2)) + (2*a*(2*a^2*b*B - 6*b^3*B - 5*a^3*C + 9*a*b^2*C)*Tan[c + d*x])
/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.00497, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4028, 4080, 4005, 3832, 4004}

$$-\frac{2a^2(bB-aC)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(2a^2bB-5a^3C+9ab^2C-6b^3B\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2b(B-3C)-8a^3C+3al\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x
])^(5/2),x]
```


```
[Out] (2*(2*a^3*b*B - 6*a*b^3*B - 8*a^4*C + 15*a^2*b^2*C - 3*b^4*C)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*(a - b)*b^4*(a + b)^(3/2)*d) + (2*(2*a^2*b*(B - 3*C) - 3*b^3*(B - C) -
8*a^3*C + 3*a*b^2*(B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d
) - (2*a^2*(b*B - a*C)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^(3/2)) + (2*a*(2*a^2*b*B - 6*b^3*B - 5*a^3*C + 9*a*b^2*C)*Tan[c + d*x])
/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4028

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(a^2*(A*b - a*B)*
```

```
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x]
+ Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc[e
 + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = \int \frac{\sec^3(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}dx$$

$$= -\frac{2a^2(bB-aC)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}ab(bB-aC)-\frac{1}{2}(\right.}{}}{}$$

$$= -\frac{2a^2(bB-aC)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3C\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{a}}$$

$$= -\frac{2a^2(bB-aC)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(2a^2bB-6b^3B-5a^3C\right.}{3b^2\left(a^2-b^2\right)^2d\sqrt{a}}$$

$$= \frac{2\left(2a^3bB-6ab^3B-8a^4C+15a^2b^2C-3b^4C\right)\cot(c+dx)E\left(\sin^{-1}\right.}{3(a-b)b^4(a+b)^{3}}$$

Mathematica [B] time = 26.0989, size = 3920, normalized size = 9.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c
 + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*
C - 15*a^2*b^2*C + 3*b^4*C)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)^2) - (2*(a*b*
B*Sin[c + d*x] - a^2*C*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos[c + d*x]
)^2) - (2*(-(a^3*b*B*Sin[c + d*x]) + 5*a*b^3*B*Sin[c + d*x] + 4*a^4*C*Sin[c
 + d*x] - 8*a^2*b^2*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d
*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) - (2*(b + a*Cos[c + d*x])^2*((2*a^3*
B)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*
b*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*
C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*C)
/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*
C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (5*a^2*B*
Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*
Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*
B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*C*
Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a
^3*C*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3
*a*b*C*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a
^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]) + (2*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]
])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^3*C*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*b*
C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d
*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*
(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a
*b^2*(B - 3*C) + 8*a^3*C + 3*b^3*(B + C) - 2*a^2*b*(B + 3*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*b*B +
```

```
6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[
(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a
^2*b^2*C + 3*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(B - 3*C) + 8*a^3*C + 3*b^3*(B + C)
 - 2*a^2*b*(B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)] + (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b
^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))/(3*b^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]
) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-2*
a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2
*(B - 3*C) + 8*a^3*C + 3*b^3*(B + C) - 2*a^2*b*(B + 3*C))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*b*B + 6*a*
b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(((-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-2*a^3*b*B + 6*a
*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos
[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(3*a*b^2*(B
 - 3*C) + 8*a^3*C + 3*b^3*(B + C) - 2*a^2*b*(B + 3*C))*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d
*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(
-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(3*a*b^2*(B - 3*C) + 8*a^3*C + 3*b^
3*(B + C) - 2*a^2*b*(B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
 a*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Cos[c + d*x]
*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-2*a^3*b*B + 6*a*b^3*B
 + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^
2*b^2*C + 3*b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]^2 - (b*(a + b)*(3*a*b^2*(B - 3*C) + 8*a^3*C + 3*b^3*(B + C) -
 2*a^2*b*(B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-
2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*b^4*C)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1
 - Tan[(c + d*x)/2]^2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*
a^2*b^2*C + 3*b^4*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(B - 3*C) + 8*a^3*C + 3*b^3*(B + C
) - 2*a^2*b*(B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
```

```
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C - 15*a^2*b^2*C + 3*
b^4*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*
Sec[c + d*x]*Tan[c + d*x]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.802, size = 6455, normalized size = 15.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5
/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)^2/(b*sec(d*x + c
) + a)^(5/2), x)
```

3.851 $\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=387

$$\frac{2\left(2a^2C+ab(B+3C)-3b^2(B+C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d\sqrt{a+b}\left(a^2-b^2\right)}+$$

```
[Out] (2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) + (2*(2*a^2*C - 3*b^2*(B + C) + a*b*(B + 3*C))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(b*B - a*C)*Tan[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2*b*B + 3*b^3*B + 2*a^3*C
- 6*a*b^2*C)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.734576, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4072, 4009, 4003, 4005, 3832, 4004}

$$\frac{2\left(a^2bB+2a^3C-6ab^2C+3b^3B\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a(bB-aC)\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(2a^2C+ab(B+3C)-3b^2(B+C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^(5/2),x]
```

```
[Out] (2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) + (2*(2*a^2*C - 3*b^2*(B + C) + a*b*(B + 3*C))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(b*B - a*C)*Tan[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2*b*B + 3*b^3*B + 2*a^3*C
- 6*a*b^2*C)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4009

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*(A*b - a*B)*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)(B+C\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(bB-aC)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}b(bB-aC)+\frac{1}{2}(a \cdots\right.}{(a+b\sec(\cdots}}{3b\left(a^2 \cdots\right.}$$

$$= \frac{2a(bB-aC)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-\cdots\right.}{3b\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2a(bB-aC)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a^3C-\cdots\right.}{3b\left(a^2-b^2\right)^2d\sqrt{a+\cdots}}$$

$$= \frac{2\left(a^2bB+3b^3B+2a^3C-6ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c\cdots}}{\sqrt{a+b}}\cdots\right.\right.}{3(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [B] time = 24.4565, size = 3514, normalized size = 9.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2*b*B + 3*b^3*B + 2*a^3*C -
6*a*b^2*C)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) + (2*(b*B*Sin[c + d*x] - a*
C*Sin[c + d*x]))/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(2*a^2*b*B*Si
n[c + d*x] + 2*b^3*B*Sin[c + d*x] + a^3*C*Sin[c + d*x] - 5*a*b^2*C*Sin[c +
d*x]))/(3*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^
(5/2)) + (2*(b + a*Cos[c + d*x])^2*((a^2*B)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*C)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*b*C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (a^3*B*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^
2*Sqrt[b + a*Cos[c + d*x]]) - (a*b*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) - (5*a^2*C*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*C*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*C*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*
b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[S
ec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*C*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2
*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b +
a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(B - 3
*C) + 3*b^2*(B - C) + 2*a^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + (a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(
a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(a^2*b*B + 3*b
^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(B - 3*C) + 3*b^2*(B - C) + 2*a
^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + (a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*(b + a*Co
s[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^
2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] - 2*b*(a + b)*(a*b*(B - 3*C) + 3*b^2*(B - C) + 2*a^2*C)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*b*B + 3*b^3*B
 + 2*a^3*C - 6*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((a^2*b*B +
3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^4)/2 + ((a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
```

```
 Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] -
(b*(a + b)*(a*b*(B - 3*C) + 3*b^2*(B - C) + 2*a^2*C)*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(a^
2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] - (b*(a + b)*(a*b*(B - 3*C) + 3*b^2*(B - C) + 2*a^2*C)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x]
)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))] - a*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)
*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (a^2*b*B +
 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin
[c + d*x]*Tan[(c + d*x)/2] + (a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a
 + b)*(a*b*(B - 3*C) + 3*b^2*(B - C) + 2*a^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c +
 d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)]) + ((a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C
 - 6*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - 2*b*(a + b)*(a*b*(B - 3*C) + 3*b^2*(B - C) + 2*a^2*C)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*b*B
+ 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)
/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*(-(a^2*b) + b^3)^2
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]])))
```

Maple [B] time = 0.444, size = 5170, normalized size = 13.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B+C\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sec(d*x + c)/(b*sec(d*x + c)
+ a)^(5/2), x)
```

3.852 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=353

$$\frac{2(3aB+aC-bB-3bC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}-\frac{2\left(a^2(-C)\right.}{3d\left(a^2\right.}$$

```
[Out] (-2*(4*a*b*B - a^2*C - 3*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*
d) + (2*(3*a*B - b*B + a*C - 3*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2
)*d) - (2*(b*B - a*C)*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(
3/2)) - (2*(4*a*b*B - a^2*C - 3*b^2*C)*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.521092, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4060, 4058, 12, 3832, 4004}

$$-\frac{2\left(a^2(-C)+4abB-3b^2C\right)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2(bB-aC)\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(a^2(-C)+4abB-3b^2C\right)\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(4*a*b*B - a^2*C - 3*b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*
d) + (2*(3*a*B - b*B + a*C - 3*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2
)*d) - (2*(b*B - a*C)*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(
3/2)) - (2*(4*a*b*B - a^2*C - 3*b^2*C)*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
```

```
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(bB-aC)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}a(aB-bC)\sec(c+dx)+\frac{1}{2}a(bB-aC)\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3a\left(a^2-b^2\right)}$$

$$= -\frac{2(bB-aC)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4abB-a^2C-3b^2C\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2(bB-aC)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4abB-a^2C-3b^2C\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2\left(4abB-a^2C-3b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\cdots}{3(a-b)b^2(a+b)^{3/2}d}$$

$$= -\frac{2\left(4abB-a^2C-3b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\cdots}{3(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [A] time = 19.088, size = 559, normalized size = 1.58

$$\frac{2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)^2\left(2b(a+b)(3aB-aC+bB-3bC)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(\cdots}{(a+b)(\cdots}}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(-4*a*b*B + a^2*C + 3*b^2*C)*Si
n[c + d*x])/(3*b*(-a^2 + b^2)^2) + (2*(b^2*B*Sin[c + d*x] - a*b*C*Sin[c + d
*x]))/(3*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (2*(-5*a^2*b*B*Sin[c + d*x
] + b^3*B*Sin[c + d*x] + 2*a^3*C*Sin[c + d*x] + 2*a*b^2*C*Sin[c + d*x]))/(3
*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (
2*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*(2*(a + b)*(-4*a*b*B + a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*a*B + b*B - a*
C - 3*b*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] + (-4*a*b*B + a^2*C + 3*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)
/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.383, size = 4213, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2/b*4^(1/2)*(-C*cos(d*x+c)^3*a^4-3*C*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+3*C*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+B*c
os(d*x+c)^3*b^4-B*cos(d*x+c)*b^4+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+4*B*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a^2*b^2+B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b^3-4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-4*B*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2
*b^2-C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^3*b-4*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-3*C*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^
3+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^3*b+3*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*C*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-4
*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2
*sin(d*x+c)-4*B*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
```

+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a+3*B*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-5*B*cos(d*x+c)^3*a^2*b^2-4*B*cos(d*x+c)
^2*a^3*b-4*B*cos(d*x+c)^2*a*b^3-3*B*cos(d*x+c)*a^2*b^2+8*B*cos(d*x+c)^2*a^2
*b^2+4*B*cos(d*x+c)*a*b^3+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+4*B*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+3*C*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-C*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a^3*b+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-6*C*cos(d*x+c)^2*a
*b^3+4*B*cos(d*x+c)^3*a^3*b-3*C*cos(d*x+c)^3*a^2*b^2-C*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+2*C*cos(d*x
+c)^3*a^3*b+2*C*cos(d*x+c)^3*a*b^3+4*C*cos(d*x+c)^2*a^2*b^2+4*C*cos(d*x+c)*
a*b^3-2*C*cos(d*x+c)^2*a^3*b-C*cos(d*x+c)*a^2*b^2+3*C*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-4*B*c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b-8*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+3*B*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^3*b+7*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-4*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-3*C*cos(d*x+c)*b^4+B*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+C*co
s(d*x+c)^2*a^4+5*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+6*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-5*C*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2-7*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*cos(d*x+c)*a*b^3+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+3*C*cos(d*x+c)^2*b^4+C*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^4+B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))

```
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-3*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4)*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*
sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x
)
```

3.853 $\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$-\frac{2\left(6a^2bB+a^2bC-3a^3C-ab^2B-3b^3B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{3a^2bd(a-b)(a+b)^{3/2}}$$

```
[Out] (2*(7*a^2*b*B - 3*b^3*B - 4*a^3*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*d) - (2*(6*a^2*b*B - a*b^2*B - 3*b^3*B - 3*a^3*C + a^2*b*C)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(a^3*d) + (2*b*(b*B - a*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^(3/2)) + (2*b*(7*a^2*b*B - 3*b^3*B - 4*a^3*C)*Tan[c + d*x])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.856809, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4072, 3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b\left(7a^2bB-4a^3C-3b^3B\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b(bB-aC)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2bB+a^2bC-3a^3C-ab^2B-3b^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d*x])
^(5/2),x]
```


```
[Out] (2*(7*a^2*b*B - 3*b^3*B - 4*a^3*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*d) - (2*(6*a^2*b*B - a*b^2*B - 3*b^3*B - 3*a^3*C + a^2*b*C)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(a^3*d) + (2*b*(b*B - a*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^(3/2)) + (2*b*(7*a^2*b*B - 3*b^3*B - 4*a^3*C)*Tan[c + d*x])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{B+C\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$=\frac{2b(bB-aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}-\frac{2\int \frac{-\frac{3}{2}\left(a^2-b^2\right)B+\frac{3}{2}a(bB-aC)\sec}{(a+b\sec}}{3a\left(a^2\right.}$$

$$=\frac{2b(bB-aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{2b\left(7a^2bB-3b^3B-4a^3C\right)}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\,s}}$$

$$=\frac{2b(bB-aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{2b\left(7a^2bB-3b^3B-4a^3C\right)}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\,s}}$$

$$=\frac{2\left(7a^2bB-3b^3B-4a^3C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{3a^2(a-b)b(a+b)^{3/2}d}$$

$$=\frac{2\left(7a^2bB-3b^3B-4a^3C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{3a^2(a-b)b(a+b)^{3/2}d}$$

Mathematica [C] time = 16.2126, size = 2039, normalized size = 4.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-7*a^2*b*B + 3*b^3*B + 4*a^3*C)
*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*B*Sin[c + d*x] - a*b^2*C*Sin
[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) - (2*(-8*a^2*b^2*B*S
in[c + d*x] + 4*b^4*B*Sin[c + d*x] + 5*a^3*b*C*Sin[c + d*x] - a*b^3*C*Sin[c
 + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*
x])^(5/2)) + (2*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*
a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(
a + b)]*B*Tan[(c + d*x)/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)
/2] - 3*b^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 4*a^4*Sqrt[(-a + b)
/(a + b)]*C*Tan[(c + d*x)/2] - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*
x)/2] - 14*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 6*a*b^3*Sqrt
[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 8*a^4*Sqrt[(-a + b)/(a + b)]*C*Ta
n[(c + d*x)/2]^3 + 7*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 7*
a^2*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)
/(a + b)]*B*Tan[(c + d*x)/2]^5 + 3*b^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*
x)/2]^5 - 4*a^4*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 + 4*a^3*b*Sqrt[
(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 - (6*I)*a^4*B*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (12*I)*a^2*b^2*B*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (6*I)*b^4*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
```

```
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - (6*I)*a^4*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + (12*I)*a^2*b^2*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] - (6*I)*b^4*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + I*(a - b)*(-7*a^2*b*B + 3*b^3*B + 4*a^3*C)*Elliptic
E[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(-4*a*b^2*B - 6*b^3*
B + 3*a^3*(B - C) + a^2*b*(9*B + C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)]))/(3*a^2*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c +
 d*x])^(5/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - T
an[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^
2)))
```

Maple [B] time = 0.426, size = 5710, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/(b*sec(d*x + c)
+ a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^2 + B\cos\left(dx+c\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^3\sec\left(dx+c\right)^3 + 3\,ab^2\sec\left(dx+c\right)^2 + 3\,a^2b\sec\left(dx+c\right) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c))*sqrt
(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b
*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)/(b*sec(d*x + c)
+ a)^(5/2), x)
```

3.854 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=446

$$\frac{2\left(3a^2(5B+C)-8ab(B+3C)+b^2(9B+5C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{15bd\sqrt{a+b}\left(a^2-b^2\right)^2}$$

```
[Out] (-2*(23*a^2*b*B + 9*b^3*B - 3*a^3*C - 29*a*b^2*C)*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*(a - b)
^2*b^2*(a + b)^(5/2)*d) + (2*(3*a^2*(5*B + C) - 8*a*b*(B + 3*C) + b^2*(9*B
+ 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(15*b*Sqrt[a + b]*(a^2 - b^2)^2*d) - (2*(b*B - a*C)*T
an[c + d*x])/(5*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(5/2)) - (2*(8*a*b*B - 3
*a^2*C - 5*b^2*C)*Tan[c + d*x])/(15*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3
/2)) - (2*(23*a^2*b*B + 9*b^3*B - 3*a^3*C - 29*a*b^2*C)*Tan[c + d*x])/(15*(
a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.828185, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4060, 4058, 12, 3832, 4004}

$$-\frac{2\left(23a^2bB-3a^3C-29ab^2C+9b^3B\right)\tan(c+dx)}{15d\left(a^2-b^2\right)^3\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-3a^2C+8abB-5b^2C\right)\tan(c+dx)}{15d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^{3/2}}-\frac{2(bB-aC)\tan(c+dx)}{5d\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(7/2),x]
```


```
[Out] (-2*(23*a^2*b*B + 9*b^3*B - 3*a^3*C - 29*a*b^2*C)*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*(a - b)
^2*b^2*(a + b)^(5/2)*d) + (2*(3*a^2*(5*B + C) - 8*a*b*(B + 3*C) + b^2*(9*B
+ 5*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(15*b*Sqrt[a + b]*(a^2 - b^2)^2*d) - (2*(b*B - a*C)*T
an[c + d*x])/(5*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(5/2)) - (2*(8*a*b*B - 3
*a^2*C - 5*b^2*C)*Tan[c + d*x])/(15*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3
/2)) - (2*(23*a^2*b*B + 9*b^3*B - 3*a^3*C - 29*a*b^2*C)*Tan[c + d*x])/(15*(
a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx = -\frac{2(bB-aC)\tan(c+dx)}{5\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}a(aB-bC)\sec(c+dx)+\frac{3}{2}a(bB-aC)\sec^2(c+dx}{(a+b\sec(c+dx))^{5/2}}}{5a\left(a^2-b^2\right)}$$

$$= -\frac{2(bB-aC)\tan(c+dx)}{5\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\left(8abB-3a^2C-5b^2C\right)\tan(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} +$$

$$= -\frac{2(bB-aC)\tan(c+dx)}{5\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\left(8abB-3a^2C-5b^2C\right)\tan(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} -$$

$$= -\frac{2(bB-aC)\tan(c+dx)}{5\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\left(8abB-3a^2C-5b^2C\right)\tan(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} -$$

$$= -\frac{2\left(23a^2bB+9b^3B-3a^3C-29ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{15(a-b)^2b^2(a+b)^{5/2}d}$$

$$= -\frac{2\left(23a^2bB+9b^3B-3a^3C-29ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right.}{15(a-b)^2b^2(a+b)^{5/2}d}$$

Mathematica [B] time = 24.7313, size = 3729, normalized size = 8.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^4*Sec[c + d*x]^4*((-2*(23*a^2*b*B + 9*b^3*B - 3*a^3*C
 - 29*a*b^2*C)*Sin[c + d*x])/(15*b*(-a^2 + b^2)^3) - (2*(b^3*B*Sin[c + d*x]
 - a*b^2*C*Sin[c + d*x]))/(5*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^3) - (2*(
-14*a^2*b^2*B*Sin[c + d*x] + 6*b^4*B*Sin[c + d*x] + 9*a^3*b*C*Sin[c + d*x]
- a*b^3*C*Sin[c + d*x]))/(15*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (2
*(-34*a^4*b*B*Sin[c + d*x] + 5*a^2*b^3*B*Sin[c + d*x] - 3*b^5*B*Sin[c + d*x
] + 9*a^5*C*Sin[c + d*x] + 25*a^3*b^2*C*Sin[c + d*x] - 2*a*b^4*C*Sin[c + d*
x]))/(15*a^2*(a^2 - b^2)^3*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^
(7/2)) - (2*(b + a*Cos[c + d*x])^3*((23*a^2*b*B)/(15*(-a^2 + b^2)^3*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (3*b^3*B)/(5*(-a^2 + b^2)^3*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^3*C)/(5*(-a^2 + b^2)^3*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*b^2*C)/(15*(-a^2 + b^2)^3*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a^3*B*Sqrt[Sec[c + d*x]])/(15*
(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]) - (8*a*b^2*B*Sqrt[Sec[c + d*x]])/(
15*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]) - (a^4*C*Sqrt[Sec[c + d*x]])/(5
*b*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*b*C*Sqrt[Sec[c + d*x]]
)/(15*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]) + (b^3*C*Sqrt[Sec[c + d*x]])
/(3*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]) + (23*a^3*B*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(15*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]) + (3*a*b^2*
B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c +
 d*x]]) - (a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*(-a^2 + b^2)^3*S
qrt[b + a*Cos[c + d*x]]) - (29*a^2*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])
/(15*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(7/2)*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-23*a^2*b*B - 9*b^3*B + 3*a^3*C +
29*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] + 2*b*(a + b)*(b^2*(9*B - 5*C) + 8*a*b*(B - 3*C) + 3*a^2*(5*B - C)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 (-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*(-a^2 + b^2)^3*d*Sqrt[Se
c[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(7/2)*(-(a*Sqrt[Cos[(c + d*x)/2]^2*S
ec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*
a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + 2*b*(a + b)*(b^2*(9*B - 5*C) + 8*a*b*(B - 3*C) + 3*a^2*(5*B - C))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-
23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*(-a^2 + b^2)^3*(b + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*
b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + 2*b*(a + b)*(b^2*(9*B - 5*C) + 8*a*b*(B - 3*C) + 3*a^2*(5*B - C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-23
*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*(-a^2 + b^2)^3*Sqrt[b + a*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*(((-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-23*a^2*b*B - 9*b^3*B + 3*a^
3*C + 29*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(b^2*(9*B - 5*C) + 8*a*b*(B - 3*C) +
 3*a^2*(5*B - C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
```

```
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*
a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
+ ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(b^2*(9*B
- 5*C) + 8*a*b*(B - 3*C) + 3*a^2*(5*B - C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((
a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))] - a*(-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c + d*x]
*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-23*a^2*b*B - 9*b^3*B
+ 3*a^3*C + 29*a*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x
]*Tan[(c + d*x)/2] + (-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a +
b)*(b^2*(9*B - 5*C) + 8*a*b*(B - 3*C) + 3*a^2*(5*B - C))*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c
 + d*x)/2]^2)/(a + b)]) + ((a + b)*(-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*
b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(15*b*(-a^2 + b^2)^3*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-23*a^2*b*B -
9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(b^2*(9*B - 5*C) + 8*a*b*(B - 3*
C) + 3*a^2*(5*B - C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)] + (-23*a^2*b*B - 9*b^3*B + 3*a^3*C + 29*a*b^2*C)*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c +
 d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*T
an[c + d*x]))/(15*b*(-a^2 + b^2)^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d
*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.577, size = 7695, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{4}\sec\left(dx+c\right)^{4}+4\,ab^{3}\sec\left(dx+c\right)^{3}+6\,a^{2}b^{2}\sec\left(dx+c\right)^{2}+4\,a^{3}b\sec\left(dx+c\right)+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^4*
sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*
b*sec(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(7/2), x
)
```

3.855 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=101

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d)
```

Rubi [A] time = 0.28374, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4072, 4038, 3771, 2641, 3849, 2805}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
d*x])),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d)
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx=\int \frac{\sqrt{\sec(c+dx)}(B+C\sec(c+dx))}{a+b\sec(c+dx)}\,dx$$

$$=\frac{B\int\sqrt{\sec(c+dx)}\,dx}{a}-\frac{(bB-aC)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$

$$=\frac{\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a}-\frac{\left((bB-aC)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$=\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2(bB-aC)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+l}\right.}{a(a+}$$

Mathematica [A] time = 0.669167, size = 78, normalized size = 0.77

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(aC\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+(aC-bB)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right.}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x]*(a*C*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(b*B) +
a*C)*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x
]^2])/(a*b*d)
```

Maple [A] time = 2.124, size = 217, normalized size = 2.2

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{a\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(B\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(B*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+B*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-C*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/((b*sec(d*x + c) + a)*sqrt(se
c(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sqrt{\sec(c+dx)}}{a + b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))/sec(d*x+c)**(1/2)
,x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/((b*sec(d*x + c) + a)*sqrt(se
c(d*x + c))), x)
```

3.856 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2C\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.514283, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4072, 4036, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2C\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]),x]
```

```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4036

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (
A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx &= \int \frac{\sqrt{\sec(c+dx)}(B+C\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx\\
&= B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + C\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\\
&= \frac{\left(B\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(C\sqrt{b+a\cos(c+dx)}\right.}{\sqrt{a}}\\
&= \frac{\left(B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+\right.}{\sqrt{a+b\sec}}\\
&= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(}}
\end{aligned}$$

Mathematica [A] time = 0.26701, size = 91, normalized size = 0.66

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left(B\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+C\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(B*EllipticF[(c + d*x)/2, (2*a)/(a +
b)] + C*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/(d*S
qrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.394, size = 277, normalized size = 2.

$$-2\frac{\left(\left(\cos\left(dx+c\right)+1\right)^{-1}\right)^{3/2}\left(\sin\left(dx+c\right)\right)^{4}}{d\left(-1+\cos\left(dx+c\right)\right)^{2}\left(b+a\cos\left(dx+c\right)\right)\sqrt{\left(\cos\left(dx+c\right)\right)^{-1}}}\sqrt{\frac{b+a\cos\left(dx+c\right)}{\left(a+b\right)\left(\cos\left(dx+c\right)+1\right)}}\left(B\mathrm{EllipticF}\left(\frac{-1+\cos\left(dx\right.}{\sin\left(dx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x
)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(B
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))-C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))+2*C*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/(c
os(d*x+c)+1))^(3/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^2/(b+a*cos(d*x+c))/(1/cos(
d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(sqrt(b*sec(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(sqrt(b*sec(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

3.857 $\int (a+b\sec(c+dx))^{2/3}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}C(a+b)\tan(c+$$

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rubi [A] time = 0.267442, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4062, 12, 3834, 139, 138}

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}C(a+b)\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rule 4062

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Dist[1/b, Int[
(a + b*Csc[e + f*x])^m*(A*b + (b*B - a*C)*Csc[e + f*x]), x], x] + Dist[C/b,
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\int (bB-aC)\sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b}+\frac{C\int \sec$$

$$=\frac{(bB-aC)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b}-\frac{(C\tan(}{}$$

$$=-\frac{((bB-aC)\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(c+}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$=\frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+}{a+b}}$$

$$=\frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+}{a+b}}$$

Mathematica [B] time = 26.87, size = 21744, normalized size = 94.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{2}{3}}\left(B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x
)
```

3.858 $\int \sqrt[3]{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+\frac{\sqrt{2}C(a+b)\tan(c+dx)}{}$$

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rubi [A] time = 0.246931, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4062, 12, 3834, 139, 138}

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+\frac{\sqrt{2}C(a+b)\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```


Rule 4062

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Dist[1/b, Int[
(a + b*Csc[e + f*x])^m*(A*b + (b*B - a*C)*Csc[e + f*x]), x], x] + Dist[C/b,
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{\int (bB-aC)\sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{b}+\frac{C\int \sec(c+$$

$$=\frac{(bB-aC)\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{b}-\frac{(C\tan(c+}{bd}$$

$$=-\frac{((bB-aC)\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(c+}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$=\frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+}{a+b}}}$$

$$=\frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+}{a+b}}}$$

Mathematica [B] time = 26.6249, size = 21684, normalized size = 94.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(dx+c)}\left(B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B + C\sec\left(c+dx\right)\right)\sqrt[3]{a + b\sec\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*(a + b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right)\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x
)
```

3.859 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{\sqrt{2}C\tan(c+dx)(a+b\sec(c+d}{bd\sqrt{\sec}}$$

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.240461, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4062, 12, 3834, 139, 138}

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{\sqrt{2}C\tan(c+dx)(a+b\sec(c+d}{bd\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3),x]
```


```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```


Rule 4062

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Dist[1/b, Int[
(a + b*Csc[e + f*x])^m*(A*b + (b*B - a*C)*Csc[e + f*x]), x], x] + Dist[C/b,
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\int \frac{(bB-aC)\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b} + \frac{C\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b}$$

$$= \frac{(bB-aC)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b} - \frac{(C\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{((bB-aC)\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{(C(a}{}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] time = 27.0034, size = 12792, normalized size = 56.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+C(\sec(dx+c))^2)\frac{1}{\sqrt[3]{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(1/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(1/3), x
)
```

3.860 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}+\frac{\sqrt{2}C\tan(c+dx)\sqrt[3]{a+b\sec(\cdots}{bd\sqrt{\cdots}}$$

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.243971, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {4062, 12, 3834, 139, 138}

$$\frac{\sqrt{2}(bB-aC)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}+\frac{\sqrt{2}C\tan(c+dx)\sqrt[3]{a+b\sec(\cdots}{bd\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 4062

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Dist[1/b, Int[
(a + b*Csc[e + f*x])^m*(A*b + (b*B - a*C)*Csc[e + f*x]), x], x] + Dist[C/b,
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{\int \frac{(bB-aC)\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{b} + \frac{C\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{(bB-aC)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{b} - \frac{(C\tan(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx}}$$

$$= -\frac{((bB-aC)\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{(C\sqrt[3]{}}{}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [B] time = 27.0486, size = 12774, normalized size = 56.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (B\sec(dx+c)+C(\sec(dx+c))^2)(a+b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(2/3), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/(b*sec(d*x + c) + a)^(2/3), x
)
```

3.861 $\int \sec^3(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=165

$$\frac{\tan^3(c+dx)(5aB+5Ab+4bC)}{15d}+\frac{\tan(c+dx)(5aB+5Ab+4bC)}{5d}+\frac{(4aA+3aC+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c-}{}$$

```
[Out] ((4*a*A + 3*b*B + 3*a*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((5*A*b + 5*a*B + 4
*b*C)*Tan[c + d*x])/(5*d) + ((4*a*A + 3*b*B + 3*a*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((b*B + a*C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*C*Sec[c +
 d*x]^4*Tan[c + d*x])/(5*d) + ((5*A*b + 5*a*B + 4*b*C)*Tan[c + d*x]^3)/(15*
d)
```

Rubi [A] time = 0.232319, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4076, 4047, 3767, 4046, 3768, 3770}

$$\frac{\tan^3(c+dx)(5aB+5Ab+4bC)}{15d}+\frac{\tan(c+dx)(5aB+5Ab+4bC)}{5d}+\frac{(4aA+3aC+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((4*a*A + 3*b*B + 3*a*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((5*A*b + 5*a*B + 4
*b*C)*Tan[c + d*x])/(5*d) + ((4*a*A + 3*b*B + 3*a*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((b*B + a*C)*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*C*Sec[c +
 d*x]^4*Tan[c + d*x])/(5*d) + ((5*A*b + 5*a*B + 4*b*C)*Tan[c + d*x]^3)/(15*
d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{bC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3$$
$$= \frac{bC\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^3$$
$$= \frac{(bB+aC)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{bC}{}$$
$$= \frac{(5Ab+5aB+4bC)\tan(c+dx)}{5d}+\frac{(4aA}{}$$
$$= \frac{(4aA+3bB+3aC)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [A] time = 1.21854, size = 124, normalized size = 0.75

$$\frac{15(4aA+3aC+3bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(5\tan^2(c+dx)(aB+Ab+2bC)+15(aB+Ab+bC)+3\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (15*(4*a*A + 3*b*B + 3*a*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(4*a*A
 + 3*b*B + 3*a*C)*Sec[c + d*x] + 30*(b*B + a*C)*Sec[c + d*x]^3 + 8*(15*(A*b
 + a*B + b*C) + 5*(A*b + a*B + 2*b*C)*Tan[c + d*x]^2 + 3*b*C*Tan[c + d*x]^4
)))/(120*d)
```

Maple [A] time = 0.043, size = 287, normalized size = 1.7

$$\frac{Aa\sec(dx+c)\tan(dx+c)}{2d}+\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2Ba\tan(dx+c)}{3d}+\frac{Ba\tan(dx+c)(\sec(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B
*a*tan(d*x+c)+1/3/d*B*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*C*sec(d*x+c)^3*tan(d*
x+c)/d+3/8*a*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+
2/3/d*A*b*tan(d*x+c)+1/3/d*A*b*tan(d*x+c)*sec(d*x+c)^2+1/4/d*B*b*tan(d*x+c)
*sec(d*x+c)^3+3/8/d*B*b*tan(d*x+c)*sec(d*x+c)+3/8/d*B*b*ln(sec(d*x+c)+tan(d
*x+c))+8/15*b*C*tan(d*x+c)/d+1/5*b*C*sec(d*x+c)^4*tan(d*x+c)/d+4/15*b*C*sec
(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.05941, size = 359, normalized size = 2.18

$$\frac{80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 80*(tan(d*x + c)^3 + 3*ta
n(d*x + c))*A*b + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c
))*C*b - 15*C*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
15*B*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
 c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*A*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)))/d
```

Fricas [A] time = 0.555424, size = 467, normalized size = 2.83

$$\frac{15\left((4A+3C)a+3Bb\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left((4A+3C)a+3Bb\right)\cos(dx+c)^5\log(-\sin(dx+c)+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*((4*A + 3*C)*a + 3*B*b)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15
*((4*A + 3*C)*a + 3*B*b)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*B
*a + (5*A + 4*C)*b)*cos(d*x + c)^4 + 15*((4*A + 3*C)*a + 3*B*b)*cos(d*x + c
)^3 + 8*(5*B*a + (5*A + 4*C)*b)*cos(d*x + c)^2 + 24*C*b + 30*(C*a + B*b)*co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(
c + d*x)**3, x)
```

Giac [B] time = 1.43949, size = 639, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(4*A*a + 3*C*a + 3*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(
4*A*a + 3*C*a + 3*B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*A*a*tan(1
/2*d*x + 1/2*c)^9 - 120*B*a*tan(1/2*d*x + 1/2*c)^9 + 75*C*a*tan(1/2*d*x + 1
/2*c)^9 - 120*A*b*tan(1/2*d*x + 1/2*c)^9 + 75*B*b*tan(1/2*d*x + 1/2*c)^9 -
120*C*b*tan(1/2*d*x + 1/2*c)^9 - 120*A*a*tan(1/2*d*x + 1/2*c)^7 + 320*B*a*t
an(1/2*d*x + 1/2*c)^7 - 30*C*a*tan(1/2*d*x + 1/2*c)^7 + 320*A*b*tan(1/2*d*x
 + 1/2*c)^7 - 30*B*b*tan(1/2*d*x + 1/2*c)^7 + 160*C*b*tan(1/2*d*x + 1/2*c)^
7 - 400*B*a*tan(1/2*d*x + 1/2*c)^5 - 400*A*b*tan(1/2*d*x + 1/2*c)^5 - 464*C
*b*tan(1/2*d*x + 1/2*c)^5 + 120*A*a*tan(1/2*d*x + 1/2*c)^3 + 320*B*a*tan(1/
2*d*x + 1/2*c)^3 + 30*C*a*tan(1/2*d*x + 1/2*c)^3 + 320*A*b*tan(1/2*d*x + 1/
2*c)^3 + 30*B*b*tan(1/2*d*x + 1/2*c)^3 + 160*C*b*tan(1/2*d*x + 1/2*c)^3 - 6
0*A*a*tan(1/2*d*x + 1/2*c) - 120*B*a*tan(1/2*d*x + 1/2*c) - 75*C*a*tan(1/2*
d*x + 1/2*c) - 120*A*b*tan(1/2*d*x + 1/2*c) - 75*B*b*tan(1/2*d*x + 1/2*c) -
 120*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.862 $\int \sec^2(c+dx)(a+b\sec(c+dx)) \left(A + B\sec(c+dx) + C\sec^2(\right.$

Optimal. Leaf size=137

$$\frac{\tan(c+dx)(3aA+2aC+2bB)}{3d} + \frac{(4aB+4Ab+3bC)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\tan(c+dx)\sec(c+dx)(4aB+4Ab+3bC)}{8d}$$

```
[Out] ((4*A*b + 4*a*B + 3*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((3*a*A + 2*b*B + 2
*a*C)*Tan[c + d*x])/(3*d) + ((4*A*b + 4*a*B + 3*b*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((b*B + a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (b*C*Sec[c +
 d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.204339, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4076, 4047, 3768, 3770, 4046, 3767, 8}

$$\frac{\tan(c+dx)(3aA+2aC+2bB)}{3d} + \frac{(4aB+4Ab+3bC)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\tan(c+dx)\sec(c+dx)(4aB+4Ab+3bC)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((4*A*b + 4*a*B + 3*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((3*a*A + 2*b*B + 2
*a*C)*Tan[c + d*x])/(3*d) + ((4*A*b + 4*a*B + 3*b*C)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + ((b*B + a*C)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (b*C*Sec[c +
 d*x]^3*Tan[c + d*x])/(4*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{bC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^2$$
$$= \frac{bC\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^2$$
$$= \frac{(4Ab+4aB+3bC)\sec(c+dx)\tan(c+}{8d}$$
$$= \frac{(4Ab+4aB+3bC)\tanh^{-1}(\sin(c+dx))}{8d}$$
$$= \frac{(4Ab+4aB+3bC)\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [A] time = 0.664224, size = 100, normalized size = 0.73

$$\frac{3(4aB+4Ab+3bC)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8\left(3a(A+C)+(aC+bB)\tan^2(c+dx)+3bB\right)+3\sec(c+d\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (3*(4*A*b + 4*a*B + 3*b*C)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*A*b +
 4*a*B + 3*b*C)*Sec[c + d*x] + 6*b*C*Sec[c + d*x]^3 + 8*(3*b*B + 3*a*(A + C
) + (b*B + a*C)*Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.041, size = 223, normalized size = 1.6

$$\frac{Aa\tan(dx+c)}{d}+\frac{B\sec(dx+c)\,a\tan(dx+c)}{2\,d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{2\,aC\tan(dx+c)}{3\,d}+\frac{C(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*tan(d*x+c)+1/2/d*B*a*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a*ln(sec(d*x+c)+
tan(d*x+c))+2/3*a*C*tan(d*x+c)/d+1/3*a*C*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*A*
b*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*b*tan(d
*x+c)+1/3/d*B*b*tan(d*x+c)*sec(d*x+c)^2+1/4*b*C*sec(d*x+c)^3*tan(d*x+c)/d+3
/8*b*C*sec(d*x+c)*tan(d*x+c)/d+3/8/d*C*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.0246, size = 294, normalized size = 2.15

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ca+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Bb-3\,Cb\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*b - 3*C*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)) - 12*A*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a*tan(d*x + c))/d
```

Fricas [A] time = 0.534688, size = 400, normalized size = 2.92

$$\frac{3\,(4\,Ba+(4\,A+3\,C)b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,Ba+(4\,A+3\,C)b)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)}{48\,d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(4*B*a + (4*A + 3*C)*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4
*B*a + (4*A + 3*C)*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*((3*A +
2*C)*a + 2*B*b)*cos(d*x + c)^3 + 3*(4*B*a + (4*A + 3*C)*b)*cos(d*x + c)^2 +
 6*C*b + 8*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^{2}(c+dx)\right)\sec^{2}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(
c + d*x)**2, x)
```

Giac [B] time = 1.31941, size = 578, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(4*B*a + 4*A*b + 3*C*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*B
*a + 4*A*b + 3*C*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*A*a*tan(1/2*
d*x + 1/2*c)^7 - 12*B*a*tan(1/2*d*x + 1/2*c)^7 + 24*C*a*tan(1/2*d*x + 1/2*c
)^7 - 12*A*b*tan(1/2*d*x + 1/2*c)^7 + 24*B*b*tan(1/2*d*x + 1/2*c)^7 - 15*C*
b*tan(1/2*d*x + 1/2*c)^7 - 72*A*a*tan(1/2*d*x + 1/2*c)^5 + 12*B*a*tan(1/2*d
*x + 1/2*c)^5 - 40*C*a*tan(1/2*d*x + 1/2*c)^5 + 12*A*b*tan(1/2*d*x + 1/2*c)
^5 - 40*B*b*tan(1/2*d*x + 1/2*c)^5 - 9*C*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*a*
tan(1/2*d*x + 1/2*c)^3 + 12*B*a*tan(1/2*d*x + 1/2*c)^3 + 40*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 12*A*b*tan(1/2*d*x + 1/2*c)^3 + 40*B*b*tan(1/2*d*x + 1/2*c)^3
 - 9*C*b*tan(1/2*d*x + 1/2*c)^3 - 24*A*a*tan(1/2*d*x + 1/2*c) - 12*B*a*tan(
1/2*d*x + 1/2*c) - 24*C*a*tan(1/2*d*x + 1/2*c) - 12*A*b*tan(1/2*d*x + 1/2*c
) - 24*B*b*tan(1/2*d*x + 1/2*c) - 15*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^4)/d
```

3.863 $\int \sec(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c\right.$

Optimal. Leaf size=101

$$\frac{\tan(c+dx)(3aB+3Ab+2bC)}{3d}+\frac{(a(2A+C)+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aC+bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bC\tan}{}$$

```
[Out] ((b*B + a*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*A*b + 3*a*B + 2*b*C
)*Tan[c + d*x])/(3*d) + ((b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*
C*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.13822, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4076, 4047, 3767, 8, 4046, 3770}

$$\frac{\tan(c+dx)(3aB+3Ab+2bC)}{3d}+\frac{(a(2A+C)+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aC+bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bC\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```



```
[Out] ((b*B + a*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*A*b + 3*a*B + 2*b*C
)*Tan[c + d*x])/(3*d) + ((b*B + a*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*
C*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{bC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c$$
$$= \frac{bC\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c$$
$$= \frac{(bB+aC)\sec(c+dx)\tan(c+dx)}{2d}+\frac{bC\,s}{}$$
$$= \frac{(bB+a(2A+C))\tanh^{-1}(\sin(c+dx))}{2d}+$$

Mathematica [A] time = 0.370176, size = 75, normalized size = 0.74

$$\frac{3(a(2A+C)+bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aC+bB)\sec(c+dx)+6aB+6Ab+2bC\tan^2(c+dx)+6b\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (3*(b*B + a*(2*A + C))*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*A*b + 6*a*B
+ 6*b*C + 3*(b*B + a*C)*Sec[c + d*x] + 2*b*C*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.038, size = 160, normalized size = 1.6

$$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba\tan(dx+c)}{d}+\frac{aC\sec(dx+c)\tan(dx+c)}{2d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a*tan(d*x+c)+1/2*a*C*sec(d*x+c)*tan
(d*x+c)/d+1/2/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b*tan(d*x+c)+1/2/d*B*b*
tan(d*x+c)*sec(d*x+c)+1/2/d*B*b*ln(sec(d*x+c)+tan(d*x+c))+2/3*b*C*tan(d*x+c
)/d+1/3*b*C*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.05017, size = 209, normalized size = 2.07

$$4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cb-3\,Ca\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Bb\left(\frac{2\sin(}{\sin(dx\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b - 3*C*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*A*a*log(sec(d*x + c) + tan(d*x + c)) + 12*B*a*tan(d*x + c) +
12*A*b*tan(d*x + c))/d
```

Fricas [A] time = 0.597705, size = 331, normalized size = 3.28

$$\frac{3\left((2A+C)a+Bb\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left((2A+C)a+Bb\right)\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2(\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(3*((2*A + C)*a + B*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*((2*A
+ C)*a + B*b)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*B*a + (3*A +
2*C)*b)*cos(d*x + c)^2 + 2*C*b + 3*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(
c + d*x), x)
```

Giac [B] time = 1.25643, size = 352, normalized size = 3.49

$$3(2Aa+Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3(2Aa+Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(6Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(3*(2*A*a + C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a +
C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a*tan(1/2*d*x + 1/2*
```

```
c)^5 - 3*C*a*tan(1/2*d*x + 1/2*c)^5 + 6*A*b*tan(1/2*d*x + 1/2*c)^5 - 3*B*b*
tan(1/2*d*x + 1/2*c)^5 + 6*C*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*tan(1/2*d*x
+ 1/2*c)^3 - 12*A*b*tan(1/2*d*x + 1/2*c)^3 - 4*C*b*tan(1/2*d*x + 1/2*c)^3 +
 6*B*a*tan(1/2*d*x + 1/2*c) + 3*C*a*tan(1/2*d*x + 1/2*c) + 6*A*b*tan(1/2*d*
x + 1/2*c) + 3*B*b*tan(1/2*d*x + 1/2*c) + 6*C*b*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.864 $\int (a+b\sec(c+dx)) \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=69

$$\frac{(2aB+2Ab+bC)\tanh^{-1}(\sin(c+dx))}{2d}+aAx+\frac{(aC+bB)\tan(c+dx)}{d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] a*A*x + ((2*A*b + 2*a*B + b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((b*B + a*C)*
Tan[c + d*x])/d + (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0706864, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4048, 3770, 3767, 8}

$$\frac{(2aB+2Ab+bC)\tanh^{-1}(\sin(c+dx))}{2d}+aAx+\frac{(aC+bB)\tan(c+dx)}{d}+\frac{bC\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + ((2*A*b + 2*a*B + b*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((b*B + a*C)*
Tan[c + d*x])/d + (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{bC\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2aA+(2Ab+2a\right.$$
$$= aAx+\frac{bC\sec(c+dx)\tan(c+dx)}{2d}+(bB+aC)\int \mathrm{se}$$
$$= aAx+\frac{(2Ab+2aB+bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bC}{}$$
$$= aAx+\frac{(2Ab+2aB+bC)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(bB}{}$$

Mathematica [A] time = 0.021202, size = 92, normalized size = 1.33

$$aAx+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aC\tan(c+dx)}{d}+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}+\frac{bC\tanh^{-1}(\sin(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a*A*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*
C*ArcTanh[Sin[c + d*x]])/(2*d) + (b*B*Tan[c + d*x])/d + (a*C*Tan[c + d*x])/
d + (b*C*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.038, size = 117, normalized size = 1.7

$$aAx+\frac{Aac}{d}+\frac{Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aC\tan(dx+c)}{d}+\frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bb\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a*A*x+1/d*A*a*c+1/d*B*a*ln(sec(d*x+c)+tan(d*x+c))+a*C*tan(d*x+c)/d+1/d*A*b*
ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b*tan(d*x+c)+1/2*b*C*sec(d*x+c)*tan(d*x+c)/
d+1/2/d*C*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.15323, size = 157, normalized size = 2.28

$$\frac{4(dx+c)Aa-Cb\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Ba\log(\sec(dx+c)+\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a - C*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a*log(sec(d*x + c) + tan(d*x +
c)) + 4*A*b*log(sec(d*x + c) + tan(d*x + c)) + 4*C*a*tan(d*x + c) + 4*B*b*t
an(d*x + c))/d
```

Fricas [A] time = 0.592188, size = 305, normalized size = 4.42

$$\frac{4\,Aadx\cos\left(dx+c\right)^{2}+\left(2\,Ba+\left(2\,A+C\right)b\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ba+\left(2\,A+C\right)b\right)\cos\left(dx+c\right)^{2}\log\left(-s\right.}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*A*a*d*x*cos(d*x + c)^2 + (2*B*a + (2*A + C)*b)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - (2*B*a + (2*A + C)*b)*cos(d*x + c)^2*log(-sin(d*x + c) +
1) + 2*(C*b + 2*(C*a + B*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2), x)
```

Giac [B] time = 1.23104, size = 230, normalized size = 3.33

$$\frac{2\left(dx+c\right)Aa+\left(2\,Ba+2\,Ab+Cb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ba+2\,Ab+Cb\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Ca\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + (2*B*a + 2*A*b + C*b)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (2*B*a + 2*A*b + C*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*C*a*
tan(1/2*d*x + 1/2*c)^3 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 - C*b*tan(1/2*d*x + 1
/2*c)^3 - 2*C*a*tan(1/2*d*x + 1/2*c) - 2*B*b*tan(1/2*d*x + 1/2*c) - C*b*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.865 $\int \cos(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=52

$$x(aB+Ab)+\frac{aA\sin(c+dx)}{d}+\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

```
[Out] (A*b + a*B)*x + ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/
d + (b*C*Tan[c + d*x])/d
```

Rubi [A] time = 0.121883, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4076, 4047, 8, 4045, 3770}

$$x(aB+Ab)+\frac{aA\sin(c+dx)}{d}+\frac{(aC+bB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2),x]
```

```
[Out] (A*b + a*B)*x + ((b*B + a*C)*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/
d + (b*C*Tan[c + d*x])/d
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{bC\tan(c+dx)}{d}+\int \cos(c+dx)\left(aA+(Ab\right.$$
$$=\frac{bC\tan(c+dx)}{d}+(Ab+aB)\int 1\,dx+\int \text{co}$$
$$=(Ab+aB)x+\frac{aA\sin(c+dx)}{d}+\frac{bC\tan(c+}{d}$$
$$=(Ab+aB)x+\frac{(bB+aC)\tanh^{-1}(\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0219333, size = 71, normalized size = 1.37

$$\frac{aA\sin(c)\cos(dx)}{d}+\frac{aA\cos(c)\sin(dx)}{d}+aBx+\frac{aC\tanh^{-1}(\sin(c+dx))}{d}+Abx+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{bC\tan(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] A*b*x + a*B*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*C*ArcTanh[Sin[c + d*x]])
/d + (a*A*Cos[d*x]*Sin[c])/d + (a*A*Cos[c]*Sin[d*x])/d + (b*C*Tan[c + d*x])
/d
```

Maple [A] time = 0.057, size = 88, normalized size = 1.7

$$Abx+aBx+\frac{A\sin(dx+c)\,a}{d}+\frac{Abc}{d}+\frac{Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bac}{d}+\frac{aC\ln(\sec(dx+c)+\tan(dx+c))}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] A*b*x+a*B*x+a*A*sin(d*x+c)/d+1/d*A*b*c+1/d*B*b*ln(sec(d*x+c)+tan(d*x+c))+1/
d*B*a*c+1/d*a*C*ln(sec(d*x+c)+tan(d*x+c))+b*C*tan(d*x+c)/d
```

Maxima [A] time = 0.985311, size = 124, normalized size = 2.38

$$\frac{2(dx+c)Ba+2(dx+c)Ab+Ca(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Bb(\log(\sin(dx+c)+1)-\log(\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + 2*(d*x + c)*A*b + C*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2
*A*a*sin(d*x + c) + 2*C*b*tan(d*x + c))/d
```

Fricas [A] time = 0.551275, size = 265, normalized size = 5.1

$$\frac{2\,(Ba + Ab)dx\cos\left(dx + c\right) + (Ca + Bb)\cos\left(dx + c\right)\log\left(\sin\left(dx + c\right) + 1\right) - (Ca + Bb)\cos\left(dx + c\right)\log\left(-\sin\left(dx + c\right) \cdots\right.}{2\,d\cos\left(dx + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(B*a + A*b)*d*x*cos(d*x + c) + (C*a + B*b)*cos(d*x + c)*log(sin(d*x
+ c) + 1) - (C*a + B*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a*cos(d*
x + c) + C*b)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec\left(c + dx\right))\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\cos\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(
c + d*x), x)
```

Giac [B] time = 1.17684, size = 181, normalized size = 3.48

$$\frac{(Ba + Ab)(dx + c) + (Ca + Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (Ca + Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(Aa\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c \cdots\right.\right.}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] ((B*a + A*b)*(d*x + c) + (C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (
C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)
^3 - C*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - C*b*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.866 $\int \cos^2(c+dx)(a+b\sec(c+dx))\left(A + B\sec(c + dx) + C\sec^2(\right.$

Optimal. Leaf size=69

$$\frac{(aB + Ab)\sin(c + dx)}{d} + \frac{1}{2}x(a(A + 2C) + 2bB) + \frac{aA\sin(c + dx)\cos(c + dx)}{2d} + \frac{bC\tanh^{-1}(\sin(c + dx))}{d}$$

```
[Out] ((2*b*B + a*(A + 2*C))*x)/2 + (b*C*ArcTanh[Sin[c + d*x]])/d + ((A*b + a*B)*
Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.15622, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4074, 4047, 8, 4045, 3770}

$$\frac{(aB + Ab)\sin(c + dx)}{d} + \frac{1}{2}x(a(A + 2C) + 2bB) + \frac{aA\sin(c + dx)\cos(c + dx)}{2d} + \frac{bC\tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] ((2*b*B + a*(A + 2*C))*x)/2 + (b*C*ArcTanh[Sin[c + d*x]])/d + ((A*b + a*B)*
Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c$$
$$= \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c$$
$$= \frac{1}{2}(2bB+a(A+2C))x + \frac{(Ab+aB)\sin(c}{d}$$
$$= \frac{1}{2}(2bB+a(A+2C))x + \frac{bC\tanh^{-1}(\sin}{d}$$

Mathematica [A] time = 0.122287, size = 68, normalized size = 0.99

$$\frac{4(aB+Ab)\sin(c+dx)+aA\sin(2(c+dx))+2aAc+2aAdx+4aCdx+4bBdx+4bC\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a*A*c + 2*a*A*d*x + 4*b*B*d*x + 4*a*C*d*x + 4*b*C*ArcTanh[Sin[c + d*x]]
+ 4*(A*b + a*B)*Sin[c + d*x] + a*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.064, size = 100, normalized size = 1.5

$$\frac{Aa\cos(dx+c)\sin(dx+c)}{2d} + \frac{aAx}{2} + \frac{Aac}{2d} + \frac{Ba\sin(dx+c)}{d} + aCx + \frac{Cac}{d} + \frac{Ab\sin(dx+c)}{d} + Bbx + \frac{Bbc}{d} + \frac{Cb\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2*a*A*cos(d*x+c)*sin(d*x+c)/d+1/2*a*A*x+1/2/d*A*a*c+a*B*sin(d*x+c)/d+a*C*
x+1/d*C*a*c+A*b*sin(d*x+c)/d+B*b*x+1/d*B*b*c+1/d*C*b*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.03043, size = 120, normalized size = 1.74

$$\frac{(2dx+2c+\sin(2dx+2c))Aa+4(dx+c)Ca+4(dx+c)Bb+2Cb(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*C*a + 4*(d*x + c)*B
*b + 2*C*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a*sin(d*x
+ c) + 4*A*b*sin(d*x + c))/d
```

Fricas [A] time = 0.541492, size = 192, normalized size = 2.78

$$\frac{((A+2C)a+2Bb)dx + Cb\log(\sin(dx+c)+1) - Cb\log(-\sin(dx+c)+1) + (Aa\cos(dx+c)+2Ba+2Ab)\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(((A + 2*C)*a + 2*B*b)*d*x + C*b*log(sin(d*x + c) + 1) - C*b*log(-sin(d
*x + c) + 1) + (A*a*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.19254, size = 215, normalized size = 3.12

$$\frac{2Cb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 2Cb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + (Aa+2Ca+2Bb)(dx+c) - \frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2B\right.}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*C*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*C*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (A*a + 2*C*a + 2*B*b)*(d*x + c) - 2*(A*a*tan(1/2*d*x + 1/2*
c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*ta
n(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.867 $\int \cos^3(c+dx)(a+b\sec(c+dx)) \left(A + B\sec(c+dx) + C\sec\right.$

Optimal. Leaf size=92

$$\frac{\sin(c+dx)(2aA+3aC+3bB)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab+2bC)+\frac{aA\sin(c+dx)\cos^2(c+}{3d}$$

```
[Out] ((A*b + a*B + 2*b*C)*x)/2 + ((2*a*A + 3*b*B + 3*a*C)*Sin[c + d*x])/(3*d) +
((A*b + a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d)
```

Rubi [A] time = 0.179562, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4074, 4047, 2637, 4045, 8}

$$\frac{\sin(c+dx)(2aA+3aC+3bB)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab+2bC)+\frac{aA\sin(c+dx)\cos^2(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((A*b + a*B + 2*b*C)*x)/2 + ((2*a*A + 3*b*B + 3*a*C)*Sin[c + d*x])/(3*d) +
((A*b + a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d)
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c$$
$$= \frac{(2aA+3bB+3aC)\sin(c+dx)}{3d} + \frac{(Ab+a}{}$$
$$= \frac{1}{2}(Ab+aB+2bC)x + \frac{(2aA+3bB+3aC)}{3d}$$

Mathematica [A] time = 0.183066, size = 85, normalized size = 0.92

$$\frac{3\sin(c+dx)(3aA+4aC+4bB)+3(aB+Ab)\sin(2(c+dx))+aA\sin(3(c+dx))+6aBc+6aBdx+6Abc+6Abdx+12}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 12*b*C*d*x + 3*(3*a*A + 4*b*B
+ 4*a*C)*Sin[c + d*x] + 3*(A*b + a*B)*Sin[2*(c + d*x)] + a*A*Sin[3*(c + d*x
)])/(12*d)
```

Maple [A] time = 0.061, size = 102, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Aa\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3} + Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b*sin(d*x+c)+a*
C*sin(d*x+c)+C*b*(d*x+c))
```

Maxima [A] time = 1.0218, size = 132, normalized size = 1.43

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-3(2dx+2c+\sin(2dx+2c))Ba-3(2dx+2c+\sin(2dx+2c))Ab-12(dx+}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b - 12*(d*x + c)*C*b -
12*C*a*sin(d*x + c) - 12*B*b*sin(d*x + c))/d
```

Fricas [A] time = 0.509012, size = 173, normalized size = 1.88

$$\frac{3\,(Ba + (A + 2\,C)b)dx + \left(2\,Aa\cos\left(dx + c\right)^2 + 2\,(2\,A + 3\,C)a + 6\,Bb + 3\,(Ba + Ab)\cos\left(dx + c\right)\right)\sin\left(dx + c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/6*(3*(B*a + (A + 2*C)*b)*d*x + (2*A*a*cos(d*x + c)^2 + 2*(2*A + 3*C)*a +
6*B*b + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.17735, size = 306, normalized size = 3.33

$$3\,(Ba + Ab + 2\,Cb)(dx + c) + \frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 3\,Ba\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 6\,Ca\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 3\,Ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 6\,Bb\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(3*(B*a + A*b + 2*C*b)*(d*x + c) + 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*
B*a*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*tan(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d
*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3
 + 12*C*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*a*ta
n(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 6*C*a*tan(1/2*d*x + 1/2*c
) + 3*A*b*tan(1/2*d*x + 1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 + 1)^3)/d
```

3.868 $\int \cos^4(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=116

$$\frac{\sin(c+dx)(aB+Ab+bC)}{d}+\frac{\sin(c+dx)\cos(c+dx)(3aA+4aC+4bB)}{8d}-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{1}{8}x(3aA+4aC+4$$

```
[Out] ((3*a*A + 4*b*B + 4*a*C)*x)/8 + ((A*b + a*B + b*C)*Sin[c + d*x])/d + ((3*a*
A + 4*b*B + 4*a*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*S
in[c + d*x])/(4*d) - ((A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.214684, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4074, 4047, 2635, 8, 4044, 3013}

$$\frac{\sin(c+dx)(aB+Ab+bC)}{d}+\frac{\sin(c+dx)\cos(c+dx)(3aA+4aC+4bB)}{8d}-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{1}{8}x(3aA+4aC+4$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((3*a*A + 4*b*B + 4*a*C)*x)/8 + ((A*b + a*B + b*C)*Sin[c + d*x])/d + ((3*a*
A + 4*b*B + 4*a*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*S
in[c + d*x])/(4*d) - ((A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3$$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3$$
$$= \frac{(3aA+4bB+4aC)\cos(c+dx)\sin(c+}{8d}$$
$$= \frac{1}{8}(3aA+4bB+4aC)x + \frac{(3aA+4bB+}{}$$
$$= \frac{1}{8}(3aA+4bB+4aC)x + \frac{(Ab+aB+bC}{d}$$

Mathematica [A] time = 0.315989, size = 117, normalized size = 1.01

$$\frac{24\sin(c+dx)(3aB+3Ab+4bC)+24\sin(2(c+dx))(a(A+C)+bB)+3aA\sin(4(c+dx))+36aAc+36aAdx+8aB}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (36*a*A*c + 48*b*B*c + 48*a*c*C + 36*a*A*d*x + 48*b*B*d*x + 48*a*C*d*x + 24
*(3*A*b + 3*a*B + 4*b*C)*Sin[c + d*x] + 24*(b*B + a*(A + C))*Sin[2*(c + d*x
)] + 8*A*b*Sin[3*(c + d*x)] + 8*a*B*Sin[3*(c + d*x)] + 3*a*A*Sin[4*(c + d*x
)])/(96*d)
```

Maple [A] time = 0.073, size = 141, normalized size = 1.2

$$\frac{1}{d}\left(Aa\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{Ba\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*
x+1/2*c)+C*sin(d*x+c)*b)
```

Maxima [A] time = 1.03443, size = 178, normalized size = 1.53

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa-32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba+24(2dx+2c+\sin(2d}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(s
in(d*x + c)^3 - 3*sin(d*x + c))*B*a + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*C
*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b + 24*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*b + 96*C*b*sin(d*x + c))/d
```

Fricas [A] time = 0.518022, size = 239, normalized size = 2.06

$$\frac{3((3A+4C)a+4Bb)dx+\left(6Aa\cos(dx+c)^3+8(Ba+Ab)\cos(dx+c)^2+16Ba+8(2A+3C)b+3((3A+4C)a+\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/24*(3*((3*A + 4*C)*a + 4*B*b)*d*x + (6*A*a*cos(d*x + c)^3 + 8*(B*a + A*b)
*cos(d*x + c)^2 + 16*B*a + 8*(2*A + 3*C)*b + 3*((3*A + 4*C)*a + 4*B*b)*cos(
d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.16558, size = 529, normalized size = 4.56

$$\frac{3(3Aa+4Ca+4Bb)(dx+c)-\frac{2\left(15Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a + 4*C*a + 4*B*b)*(d*x + c) - 2*(15*A*a*tan(1/2*d*x + 1/2*c)^
7 - 24*B*a*tan(1/2*d*x + 1/2*c)^7 + 12*C*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*
tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan(1/2*d*x + 1/2*c)^7 - 24*C*b*tan(1/2*d*x
 + 1/2*c)^7 - 9*A*a*tan(1/2*d*x + 1/2*c)^5 - 40*B*a*tan(1/2*d*x + 1/2*c)^5
+ 12*C*a*tan(1/2*d*x + 1/2*c)^5 - 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 12*B*b*ta
n(1/2*d*x + 1/2*c)^5 - 72*C*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a*tan(1/2*d*x +
1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 12*C*a*tan(1/2*d*x + 1/2*c)^3 -
40*A*b*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 - 72*C*b*tan(
1/2*d*x + 1/2*c)^3 - 15*A*a*tan(1/2*d*x + 1/2*c) - 24*B*a*tan(1/2*d*x + 1/2
*c) - 12*C*a*tan(1/2*d*x + 1/2*c) - 24*A*b*tan(1/2*d*x + 1/2*c) - 12*B*b*ta
n(1/2*d*x + 1/2*c) - 24*C*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

3.869 $\int \cos^5(c+dx)(a+b\sec(c+dx)) \left(A + B\sec(c+dx) + C\sec^2(\right.$

Optimal. Leaf size=156

$$-\frac{\sin^3(c+dx)(4aA+5aC+5bB)}{15d}+\frac{\sin(c+dx)(4aA+5aC+5bB)}{5d}+\frac{\sin(c+dx)\cos(c+dx)(3aB+3Ab+4bC)}{8d}+\frac{(aB}{}$$

```
[Out] ((3*A*b + 3*a*B + 4*b*C)*x)/8 + ((4*a*A + 5*b*B + 5*a*C)*Sin[c + d*x])/(5*d
) + ((3*A*b + 3*a*B + 4*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((A*b + a*B
)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5
*d) - ((4*a*A + 5*b*B + 5*a*C)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.234685, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4074, 4047, 2633, 4045, 2635, 8}

$$-\frac{\sin^3(c+dx)(4aA+5aC+5bB)}{15d}+\frac{\sin(c+dx)(4aA+5aC+5bB)}{5d}+\frac{\sin(c+dx)\cos(c+dx)(3aB+3Ab+4bC)}{8d}+\frac{(aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```



```
[Out] ((3*A*b + 3*a*B + 4*b*C)*x)/8 + ((4*a*A + 5*b*B + 5*a*C)*Sin[c + d*x])/(5*d
) + ((3*A*b + 3*a*B + 4*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((A*b + a*B
)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5
*d) - ((4*a*A + 5*b*B + 5*a*C)*Sin[c + d*x]^3)/(15*d)
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d}-\frac{1}{5}\int \cos^4$$
$$=\frac{aA\cos^4(c+dx)\sin(c+dx)}{5d}-\frac{1}{5}\int \cos^4$$
$$=\frac{(Ab+aB)\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{aA}{}$$
$$=\frac{(4aA+5bB+5aC)\sin(c+dx)}{5d}+\frac{(3Ab}{}$$
$$=\frac{1}{8}(3Ab+3aB+4bC)x+\frac{(4aA+5bB+}{}$$

Mathematica [A] time = 0.456157, size = 117, normalized size = 0.75

$$\frac{-160\sin^3(c+dx)(a(2A+C)+bB)+480\sin(c+dx)(a(A+C)+bB)+15(4(c+dx)(3aB+3Ab+4bC)+8\sin(2(c+}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (480*(b*B + a*(A + C))*Sin[c + d*x] - 160*(b*B + a*(2*A + C))*Sin[c + d*x]^
3 + 96*a*A*Sin[c + d*x]^5 + 15*(4*(3*A*b + 3*a*B + 4*b*C)*(c + d*x) + 8*(A*
b + a*B + b*C)*Sin[2*(c + d*x)] + (A*b + a*B)*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.073, size = 173, normalized size = 1.1

$$\frac{1}{d}\left(\frac{A\sin(dx+c)a}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ab\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*b*(2+cos(d*x+c)^2)*sin(d*x+c)
+1/3*a*C*(2+cos(d*x+c)^2)*sin(d*x+c)+C*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
```

```
+1/2*c))
```

Maxima [A] time = 1.03971, size = 224, normalized size = 1.44

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a - 160*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*C*a + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*si
n(2*d*x + 2*c))*A*b - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b + 120*(2*d*
x + 2*c + sin(2*d*x + 2*c))*C*b)/d
```

Fricas [A] time = 0.526962, size = 305, normalized size = 1.96

$$\frac{15\left(3Ba+(3A+4C)b\right)dx+\left(24Aa\cos(dx+c)^4+30(Ba+Ab)\cos(dx+c)^3+8\left((4A+5C)a+5Bb\right)\cos(dx+c)^2+\cdots\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/120*(15*(3*B*a + (3*A + 4*C)*b)*d*x + (24*A*a*cos(d*x + c)^4 + 30*(B*a +
A*b)*cos(d*x + c)^3 + 8*((4*A + 5*C)*a + 5*B*b)*cos(d*x + c)^2 + 16*(4*A +
5*C)*a + 80*B*b + 15*(3*B*a + (3*A + 4*C)*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.14913, size = 590, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(3*B*a + 3*A*b + 4*C*b)*(d*x + c) + 2*(120*A*a*tan(1/2*d*x + 1/2*
c)^9 - 75*B*a*tan(1/2*d*x + 1/2*c)^9 + 120*C*a*tan(1/2*d*x + 1/2*c)^9 - 75*
A*b*tan(1/2*d*x + 1/2*c)^9 + 120*B*b*tan(1/2*d*x + 1/2*c)^9 - 60*C*b*tan(1/
2*d*x + 1/2*c)^9 + 160*A*a*tan(1/2*d*x + 1/2*c)^7 - 30*B*a*tan(1/2*d*x + 1/
2*c)^7 + 320*C*a*tan(1/2*d*x + 1/2*c)^7 - 30*A*b*tan(1/2*d*x + 1/2*c)^7 + 3
20*B*b*tan(1/2*d*x + 1/2*c)^7 - 120*C*b*tan(1/2*d*x + 1/2*c)^7 + 464*A*a*ta
n(1/2*d*x + 1/2*c)^5 + 400*C*a*tan(1/2*d*x + 1/2*c)^5 + 400*B*b*tan(1/2*d*x
 + 1/2*c)^5 + 160*A*a*tan(1/2*d*x + 1/2*c)^3 + 30*B*a*tan(1/2*d*x + 1/2*c)^
3 + 320*C*a*tan(1/2*d*x + 1/2*c)^3 + 30*A*b*tan(1/2*d*x + 1/2*c)^3 + 320*B*
b*tan(1/2*d*x + 1/2*c)^3 + 120*C*b*tan(1/2*d*x + 1/2*c)^3 + 120*A*a*tan(1/2
*d*x + 1/2*c) + 75*B*a*tan(1/2*d*x + 1/2*c) + 120*C*a*tan(1/2*d*x + 1/2*c)
+ 75*A*b*tan(1/2*d*x + 1/2*c) + 120*B*b*tan(1/2*d*x + 1/2*c) + 60*C*b*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.870 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=233

$$\frac{\tan(c+dx)\left(5a^2(3A+2C)+20abB+2b^2(5A+4C)\right)}{15d}+\frac{\left(4a^2B+8aAb+6abC+3b^2B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\tan(c+$$

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((2
0*a*b*B + 5*a^2*(3*A + 2*C) + 2*b^2*(5*A + 4*C))*Tan[c + d*x])/(15*d) + ((8
*a*A*b + 4*a^2*B + 3*b^2*B + 6*a*b*C)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((
5*A*b^2 + 10*a*b*B + 2*a^2*C + 4*b^2*C)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
 + (b*(5*b*B + 2*a*C)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (C*Sec[c + d*x]
^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.59159, antiderivative size = 281, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4092, 4082, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{\tan(c+dx)\left(-4a^2b^2(5A+3C)+5a^3bB-2a^4C-40ab^3B-4b^4(5A+4C)\right)}{30b^2d}+\frac{\left(4a^2B+8aAb+6abC+3b^2B\right)\tanh^{-1}(\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B + 6*a*b*C)*ArcTanh[Sin[c + d*x]])/(8*d) - ((5
*a^3*b*B - 40*a*b^3*B - 2*a^4*C - 4*a^2*b^2*(5*A + 3*C) - 4*b^4*(5*A + 4*C)
)*Tan[c + d*x])/(30*b^2*d) - ((10*a^2*b*B - 45*b^3*B - 4*a^3*C - 2*a*b^2*(2
0*A + 13*C))*Sec[c + d*x]*Tan[c + d*x])/(120*b*d) + ((20*A*b^2 - 5*a*b*B +
2*a^2*C + 16*b^2*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b^2*d) + ((5*b
*B - 2*a*C)*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b^2*d) + (C*Sec[c + d*
x]*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*b*d)
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec(c+dx)(a+b\sec(c+dx))^3\tan(c}{5bd}$$

$$= \frac{(5bB-2aC)(a+b\sec(c+dx))^3\tan(c}{20b^2d}$$

$$= \frac{\left(20Ab^2-5abB+2a^2C+16b^2C\right)(a+}{60b^2d}$$

$$= -\frac{\left(10a^2bB-45b^3B-4a^3C-2ab^2(20A}{120bd}$$

$$= -\frac{\left(10a^2bB-45b^3B-4a^3C-2ab^2(20A}{120bd}$$

$$= \frac{\left(8aAb+4a^2B+3b^2B+6abC\right)\tanh^{-1}}{8d}$$

$$= \frac{\left(8aAb+4a^2B+3b^2B+6abC\right)\tanh^{-1}}{8d}$$

Mathematica [A] time = 2.1341, size = 371, normalized size = 1.59

$$\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\Big(2\sin(c+dx)\Big(15\cos(c+dx)\left(12a^2B+24aAb+34abC+17b^2B\right)+48$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] ((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(-120*(8*a*A*b + 4*
a^2*B + 3*b^2*B + 6*a*b*C)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(180*a^2*A + 200*A
*b^2 + 400*a*b*B + 200*a^2*C + 256*b^2*C + 15*(24*a*A*b + 12*a^2*B + 17*b^2
*B + 34*a*b*C)*Cos[c + d*x] + 48*(10*a*b*B + 5*a^2*(A + C) + b^2*(5*A + 4*C
))*Cos[2*(c + d*x)] + 120*a*A*b*Cos[3*(c + d*x)] + 60*a^2*B*Cos[3*(c + d*x)
] + 45*b^2*B*Cos[3*(c + d*x)] + 90*a*b*C*Cos[3*(c + d*x)] + 60*a^2*A*Cos[4*
(c + d*x)] + 40*A*b^2*Cos[4*(c + d*x)] + 80*a*b*B*Cos[4*(c + d*x)] + 40*a^2
*C*Cos[4*(c + d*x)] + 32*b^2*C*Cos[4*(c + d*x)])*Sin[c + d*x]))/(480*d*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.05, size = 404, normalized size = 1.7

$$\frac{a^2A\tan(dx+c)}{d}+\frac{Ba^2\sec(dx+c)\tan(dx+c)}{2d}+\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2a^2C\tan(dx+c)}{3d}+\frac{a^2C\tan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*tan(d*x+c)+1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*B*a^2*ln(sec(d
*x+c)+tan(d*x+c))+2/3/d*a^2*C*tan(d*x+c)+1/3/d*a^2*C*tan(d*x+c)*sec(d*x+c)^
2+1/d*A*a*b*sec(d*x+c)*tan(d*x+c)+1/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+4/3/d
*B*a*b*tan(d*x+c)+2/3/d*B*a*b*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a*b*C*tan(d*x+c
)*sec(d*x+c)^3+3/4*a*b*C*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a*b*C*ln(sec(d*x+c)+
tan(d*x+c))+2/3/d*A*b^2*tan(d*x+c)+1/3/d*A*b^2*tan(d*x+c)*sec(d*x+c)^2+1/4/
d*B*b^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*B*b^2*sec(d*x+c)*tan(d*x+c)+3/8/d*B*b
^2*ln(sec(d*x+c)+tan(d*x+c))+8/15*b^2*C*tan(d*x+c)/d+1/5/d*b^2*C*tan(d*x+c)
*sec(d*x+c)^4+4/15/d*b^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.06861, size = 482, normalized size = 2.07

$$80\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^2+160\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bab+80\left(\tan(dx+c)^3+3\tan(dx+c)\right)A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2 + 160*(tan(d*x + c)^3 + 3
*tan(d*x + c))*B*a*b + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^2 + 16*(3*t
an(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*b^2 - 30*C*a*b*(2*(3
```

```
*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 15*B*b^2*(2*(3*sin(d*
x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(
sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*B*a^2*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*A*
a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 240*A*a^2*tan(d*x + c))/d
```

Fricas [A] time = 0.560782, size = 598, normalized size = 2.57

$$\frac{15\left(4Ba^2+2(4A+3C)ab+3Bb^2\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(4Ba^2+2(4A+3C)ab+3Bb^2\right)\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*B*a^2 + 2*(4*A + 3*C)*a*b + 3*B*b^2)*cos(d*x + c)^5*log(sin(d*
x + c) + 1) - 15*(4*B*a^2 + 2*(4*A + 3*C)*a*b + 3*B*b^2)*cos(d*x + c)^5*log
(-sin(d*x + c) + 1) + 2*(8*(5*(3*A + 2*C)*a^2 + 20*B*a*b + 2*(5*A + 4*C)*b^
2)*cos(d*x + c)^4 + 15*(4*B*a^2 + 2*(4*A + 3*C)*a*b + 3*B*b^2)*cos(d*x + c)
^3 + 24*C*b^2 + 8*(5*C*a^2 + 10*B*a*b + (5*A + 4*C)*b^2)*cos(d*x + c)^2 + 3
0*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*s
ec(c + d*x)**2, x)
```

Giac [B] time = 1.25892, size = 1034, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(15*(4*B*a^2 + 8*A*a*b + 6*C*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 15*(4*B*a^2 + 8*A*a*b + 6*C*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(120*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 60*B*a^2*tan(1/2*d*x
 + 1/2*c)^9 + 120*C*a^2*tan(1/2*d*x + 1/2*c)^9 - 120*A*a*b*tan(1/2*d*x + 1/
2*c)^9 + 240*B*a*b*tan(1/2*d*x + 1/2*c)^9 - 150*C*a*b*tan(1/2*d*x + 1/2*c)^
```

```
9 + 120*A*b^2*tan(1/2*d*x + 1/2*c)^9 - 75*B*b^2*tan(1/2*d*x + 1/2*c)^9 + 12
0*C*b^2*tan(1/2*d*x + 1/2*c)^9 - 480*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 120*B*a
^2*tan(1/2*d*x + 1/2*c)^7 - 320*C*a^2*tan(1/2*d*x + 1/2*c)^7 + 240*A*a*b*ta
n(1/2*d*x + 1/2*c)^7 - 640*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 60*C*a*b*tan(1/2*
d*x + 1/2*c)^7 - 320*A*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*B*b^2*tan(1/2*d*x +
1/2*c)^7 - 160*C*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*A*a^2*tan(1/2*d*x + 1/2*c
)^5 + 400*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*a*b*tan(1/2*d*x + 1/2*c)^5 +
 400*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 464*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 480*
A*a^2*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 320*C*a^2
*tan(1/2*d*x + 1/2*c)^3 - 240*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 640*B*a*b*tan(
1/2*d*x + 1/2*c)^3 - 60*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 320*A*b^2*tan(1/2*d*
x + 1/2*c)^3 - 30*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 160*C*b^2*tan(1/2*d*x + 1/
2*c)^3 + 120*A*a^2*tan(1/2*d*x + 1/2*c) + 60*B*a^2*tan(1/2*d*x + 1/2*c) + 1
20*C*a^2*tan(1/2*d*x + 1/2*c) + 120*A*a*b*tan(1/2*d*x + 1/2*c) + 240*B*a*b*
tan(1/2*d*x + 1/2*c) + 150*C*a*b*tan(1/2*d*x + 1/2*c) + 120*A*b^2*tan(1/2*d
*x + 1/2*c) + 75*B*b^2*tan(1/2*d*x + 1/2*c) + 120*C*b^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.871 $\int \sec(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=200

$$\frac{\tan(c+dx)\left(4a^2bB+a^3(-C)+4ab^2(3A+2C)+4b^3B\right)}{6bd}+\frac{\left(4a^2(2A+C)+8abB+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] ((8*a*b*B + 4*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((4*a^2*b*B + 4*b^3*B - a^3*C + 4*a*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*b*
d) + ((12*A*b^2 + 8*a*b*B - 2*a^2*C + 9*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(
24*d) + ((4*b*B - a*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*b*d) + (C*(
a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.348008, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4082, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(4a^2bB+a^3(-C)+4ab^2(3A+2C)+4b^3B\right)}{6bd}+\frac{\left(4a^2(2A+C)+8abB+b^2(4A+3C)\right)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] ((8*a*b*B + 4*a^2*(2*A + C) + b^2*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((4*a^2*b*B + 4*b^3*B - a^3*C + 4*a*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*b*
d) + ((12*A*b^2 + 8*a*b*B - 2*a^2*C + 9*b^2*C)*Sec[c + d*x]*Tan[c + d*x])/(
24*d) + ((4*b*B - a*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*b*d) + (C*(
a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*b*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
```

```
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4bd}+\frac{\int \sec(}{}$$
$$= \frac{(4bB-aC)(a+b\sec(c+dx))^2\tan(c+dx)}{12bd}$$
$$= \frac{\left(12Ab^2+8abB-2a^2C+9b^2C\right)\sec(c+dx}{24d}$$
$$= \frac{\left(12Ab^2+8abB-2a^2C+9b^2C\right)\sec(c+dx}{24d}$$
$$= \frac{\left(8abB+4a^2(2A+C)+b^2(4A+3C)\right)\tanh}{8d}$$
$$= \frac{\left(8abB+4a^2(2A+C)+b^2(4A+3C)\right)\tanh}{8d}$$

Mathematica [A] time = 1.55599, size = 300, normalized size = 1.5

$$-\frac{\sec^4(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(12\cos^4(c+dx)\left(4a^2(2A+C)+8abB+b^2(4A+3C)\right)\left(\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^4*(12*(8*a*b*B + 4*a
^2*(2*A + C) + b^2*(4*A + 3*C))*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2*(12*A*b^2 + 2
4*a*b*B + 12*a^2*C + 21*b^2*C + 4*(9*a^2*B + 10*b^2*B + 2*a*b*(9*A + 10*C))
*Cos[c + d*x] + 3*(4*A*b^2 + 8*a*b*B + 4*a^2*C + 3*b^2*C)*Cos[2*(c + d*x)]
+ 24*a*A*b*Cos[3*(c + d*x)] + 12*a^2*B*Cos[3*(c + d*x)] + 8*b^2*B*Cos[3*(c
```

```
+ d*x)] + 16*a*b*C*Cos[3*(c + d*x)])*Sin[c + d*x]))/(48*d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.05, size = 321, normalized size = 1.6

$$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{B a^2 \tan(dx+c)}{d}+\frac{a^2 C \sec(dx+c)\tan(dx+c)}{2d}+\frac{a^2 C \ln(\sec(dx+c)+\tan(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^2*tan(d*x+c)+1/2/d*a^2*C*sec(d*
x+c)*tan(d*x+c)+1/2/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a*b*tan(d*x+c)+
1/d*B*a*b*sec(d*x+c)*tan(d*x+c)+1/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*a
*b*C*tan(d*x+c)+2/3/d*a*b*C*tan(d*x+c)*sec(d*x+c)^2+1/2/d*A*b^2*sec(d*x+c)*
tan(d*x+c)+1/2/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*b^2*tan(d*x+c)+1/3
/d*B*b^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*b^2*C*tan(d*x+c)*sec(d*x+c)^3+3/8/d*
b^2*C*sec(d*x+c)*tan(d*x+c)+3/8/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.02902, size = 413, normalized size = 2.06

$$\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cab+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb^2-3Cb^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*b^2 - 3*C*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*C*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(2*sin(d*
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)
) + 48*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 48*B*a^2*tan(d*x + c) + 96*
A*a*b*tan(d*x + c))/d
```

Fricas [A] time = 0.559991, size = 510, normalized size = 2.55

$$\frac{3\left(4(2A+C)a^2+8Bab+(4A+3C)b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4(2A+C)a^2+8Bab+(4A+3C)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/48*(3*(4*(2*A + C)*a^2 + 8*B*a*b + (4*A + 3*C)*b^2)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(4*(2*A + C)*a^2 + 8*B*a*b + (4*A + 3*C)*b^2)*cos(d*x +
```

```
c)^4*log(-sin(d*x + c) + 1) + 2*(8*(3*B*a^2 + 2*(3*A + 2*C)*a*b + 2*B*b^2)
*cos(d*x + c)^3 + 6*C*b^2 + 3*(4*C*a^2 + 8*B*a*b + (4*A + 3*C)*b^2)*cos(d*x
 + c)^2 + 8*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2 \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*s
ec(c + d*x), x)
```

Giac [B] time = 1.21748, size = 851, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(8*A*a^2 + 4*C*a^2 + 8*B*a*b + 4*A*b^2 + 3*C*b^2)*log(abs(tan(1/2*d
*x + 1/2*c) + 1)) - 3*(8*A*a^2 + 4*C*a^2 + 8*B*a*b + 4*A*b^2 + 3*C*b^2)*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 12*C
*a^2*tan(1/2*d*x + 1/2*c)^7 + 48*A*a*b*tan(1/2*d*x + 1/2*c)^7 - 24*B*a*b*ta
n(1/2*d*x + 1/2*c)^7 + 48*C*a*b*tan(1/2*d*x + 1/2*c)^7 - 12*A*b^2*tan(1/2*d
*x + 1/2*c)^7 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 15*C*b^2*tan(1/2*d*x + 1/
2*c)^7 - 72*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^5
- 144*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 80*C
*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 40*B*b^2*ta
n(1/2*d*x + 1/2*c)^5 - 9*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*tan(1/2*d*
x + 1/2*c)^3 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 144*A*a*b*tan(1/2*d*x + 1/
2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 80*C*a*b*tan(1/2*d*x + 1/2*c)^3
+ 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 40*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*C*b
^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a^2*tan(1/2*d*x + 1/2*c) - 12*C*a^2*tan(1/
2*d*x + 1/2*c) - 48*A*a*b*tan(1/2*d*x + 1/2*c) - 24*B*a*b*tan(1/2*d*x + 1/2
*c) - 48*C*a*b*tan(1/2*d*x + 1/2*c) - 12*A*b^2*tan(1/2*d*x + 1/2*c) - 24*B*
b^2*tan(1/2*d*x + 1/2*c) - 15*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

3.872 $\int (a+b\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)$

Optimal. Leaf size=134

$$\frac{\tan(c+dx)\left(2a^2C + 6abB + 3Ab^2 + 2b^2C\right)}{3d} + \frac{\left(2a^2B + 2ab(2A+C) + b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d} + a^2Ax + \frac{b(2aC+3}{}$$

```
[Out] a^2*A*x + ((2*a^2*B + b^2*B + 2*a*b*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*d)
 + ((3*A*b^2 + 6*a*b*B + 2*a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*d) + (b*(3*b*B
 + 2*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (C*(a + b*Sec[c + d*x])^2*Tan[
c + d*x])/(3*d)
```

Rubi [A] time = 0.172216, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4056, 4048, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(2a^2C + 6abB + 3Ab^2 + 2b^2C\right)}{3d} + \frac{\left(2a^2B + 2ab(2A+C) + b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d} + a^2Ax + \frac{b(2aC+3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a^2*A*x + ((2*a^2*B + b^2*B + 2*a*b*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*d)
 + ((3*A*b^2 + 6*a*b*B + 2*a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*d) + (b*(3*b*B
 + 2*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (C*(a + b*Sec[c + d*x])^2*Tan[
c + d*x])/(3*d)
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\sec(c+$$
$$= \frac{b(3bB+2aC)\sec(c+dx)\tan(c+dx)}{6d}+\frac{C(a+b\sec(c$$
$$= a^2Ax+\frac{b(3bB+2aC)\sec(c+dx)\tan(c+dx)}{6d}+\frac{C(a+$$
$$= a^2Ax+\frac{\left(2a^2B+b^2B+2ab(2A+C)\right)\tanh^{-1}(\sin(c+d}{2d}$$
$$= a^2Ax+\frac{\left(2a^2B+b^2B+2ab(2A+C)\right)\tanh^{-1}(\sin(c+d}{2d}$$

Mathematica [B] time = 1.74006, size = 322, normalized size = 2.4

$$\sec^3(c+dx)\left(4\sin(c+dx)\left(\cos(2(c+dx))\left(3a^2C+6abB+3Ab^2+2b^2C\right)+3a^2C+3b(2aC+bB)\cos(c+dx)+6abB+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sec[c + d*x]^3*(9*Cos[c + d*x]*(2*a^2*A*(c + d*x) - (2*a^2*B + b^2*B + 2*a
*b*(2*A + C))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + (2*a^2*B + b^2*B +
 2*a*b*(2*A + C))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Cos[3*(c +
d*x)]*(2*a^2*A*(c + d*x) - (2*a^2*B + b^2*B + 2*a*b*(2*A + C))*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + (2*a^2*B + b^2*B + 2*a*b*(2*A + C))*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*(3*A*b^2 + 6*a*b*B + 3*a^2*C + 4*b^2*
C + 3*b*(b*B + 2*a*C)*Cos[c + d*x] + (3*A*b^2 + 6*a*b*B + 3*a^2*C + 2*b^2*C
)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(24*d)
```

Maple [A] time = 0.047, size = 225, normalized size = 1.7

$$a^2Ax+\frac{Aa^2c}{d}+\frac{Ba^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2C\tan(dx+c)}{d}+2\frac{Aab\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{Bab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^2*A*x+1/d*A*a^2*c+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*C*tan(d*x+c
)+2/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b*tan(d*x+c)+a*b*C*sec(d*x+c)
*tan(d*x+c)/d+1/d*a*b*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b^2*tan(d*x+c)+1/2/
d*B*b^2*sec(d*x+c)*tan(d*x+c)+1/2/d*B*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/3*b^2
*C*tan(d*x+c)/d+1/3/d*b^2*C*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.0433, size = 279, normalized size = 2.08

$$12\,(dx+c)Aa^2+4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Cb^2-6\,Cab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^2 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*b^2 - 6*C*
a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 3*B*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 12*B*a^2*log(sec(d*x + c) + tan(d*x +
 c)) + 24*A*a*b*log(sec(d*x + c) + tan(d*x + c)) + 12*C*a^2*tan(d*x + c) +
24*B*a*b*tan(d*x + c) + 12*A*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.552142, size = 444, normalized size = 3.31

$$12\,Aa^2dx\cos\left(dx+c\right)^3+3\left(2\,Ba^2+2\,(2\,A+C)ab+Bb^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,Ba^2+2\,(2\,A+C)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/12*(12*A*a^2*d*x*cos(d*x + c)^3 + 3*(2*B*a^2 + 2*(2*A + C)*a*b + B*b^2)*c
os(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*B*a^2 + 2*(2*A + C)*a*b + B*b^2)
*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*C*b^2 + 2*(3*C*a^2 + 6*B*a*b
+ (3*A + 2*C)*b^2)*cos(d*x + c)^2 + 3*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*(A + B*sec(c + d*x) + C*sec(c + d*x)**2),
x)
```

Giac [B] time = 1.23147, size = 491, normalized size = 3.66

$$6\,(dx+c)Aa^2+3\left(2\,Ba^2+4\,Aab+2\,Cab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Ba^2+4\,Aab+2\,Cab+Bb^2\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^2 + 3*(2*B*a^2 + 4*A*a*b + 2*C*a*b + B*b^2)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 3*(2*B*a^2 + 4*A*a*b + 2*C*a*b + B*b^2)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 12*B*a*b*t
an(1/2*d*x + 1/2*c)^5 - 6*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*
x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c
)^5 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 1
2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*t
an(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*c) + 6*C*a*b*tan(1/2*d*x +
 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C
*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.873 $\int \cos(c+dx)(a+b\sec(c+dx))^2 \left(A + B\sec(c + dx) + C\sec\right.$

Optimal. Leaf size=126

$$\frac{(2a^2C + 4abB + 2Ab^2 + b^2C)\tanh^{-1}(\sin(c + dx))}{2d} - \frac{b\tan(c + dx)(2aA - 2aC - bB)}{d} + ax(aB + 2Ab) + \frac{A\sin(c + dx}{}$$

```
[Out] a*(2*A*b + a*B)*x + ((2*A*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*ArcTanh[Sin[c +
d*x]])/(2*d) + (A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (b*(2*a*A - b*B
- 2*a*C)*Tan[c + d*x])/d - (b^2*(2*A - C)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.202626, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4094, 4048, 3770, 3767, 8}

$$\frac{(2a^2C + 4abB + 2Ab^2 + b^2C)\tanh^{-1}(\sin(c + dx))}{2d} - \frac{b\tan(c + dx)(2aA - 2aC - bB)}{d} + ax(aB + 2Ab) + \frac{A\sin(c + dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a*(2*A*b + a*B)*x + ((2*A*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*ArcTanh[Sin[c +
d*x]])/(2*d) + (A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (b*(2*a*A - b*B
- 2*a*C)*Tan[c + d*x])/d - (b^2*(2*A - C)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A(a+b\sec(c+dx))^2\sin(c+dx)}{d}+\int(a+$$
$$= \frac{A(a+b\sec(c+dx))^2\sin(c+dx)}{d}-\frac{b^2(2A$$
$$= a(2Ab+aB)x+\frac{A(a+b\sec(c+dx))^2\sin($$
$$= a(2Ab+aB)x+\frac{\left(2Ab^2+4abB+2a^2C+b\right.}{2d}$$
$$= a(2Ab+aB)x+\frac{\left(2Ab^2+4abB+2a^2C+b\right.}{2d}$$

Mathematica [B] time = 1.40957, size = 453, normalized size = 3.6

$$\sec^2(c+dx)\left(\cos(2(c+dx))\left(-\left(2a^2C+4abB+2Ab^2+b^2C\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+\left(2a^2C+4abB+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (Sec[c + d*x]^2*(4*a*A*b*c + 2*a^2*B*c + 4*a*A*b*d*x + 2*a^2*B*d*x - 2*A*b^
2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*a*b*B*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - 2*a^2*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b^2
*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*b^2*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] + 4*a*b*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*
a^2*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2*C*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(2*a*(2*A*b + a*B)*(c + d*x) - (2*A
*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ (2*A*b^2 + 4*a*b*B + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) + (a^2*A + 2*b^2*C)*Sin[c + d*x] + 2*b^2*B*Sin[2*(c + d*x)] + 4*a*b*
C*Sin[2*(c + d*x)] + a^2*A*Sin[3*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.069, size = 184, normalized size = 1.5

$$\frac{a^2A\sin(dx+c)}{d}+a^2Bx+\frac{Ba^2c}{d}+\frac{a^2C\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\,aAbx+2\,\frac{Aabc}{d}+2\,\frac{Bab\ln(\sec(dx+c)+\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*a^2*A*sin(d*x+c)+a^2*B*x+1/d*B*a^2*c+1/d*a^2*C*ln(sec(d*x+c)+tan(d*x+c)
)+2*a*A*b*x+2/d*A*a*b*c+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*C*tan(d
*x+c)+1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b^2*tan(d*x+c)+1/2/d*b^2*C*
sec(d*x+c)*tan(d*x+c)+1/2/d*b^2*C*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.07345, size = 255, normalized size = 2.02

$$\frac{4\,(dx+c)Ba^2+8\,(dx+c)Aab-Cb^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+2\,Ca^2(\log\left(\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 + 8*(d*x + c)*A*a*b - C*b^2*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*C*a^2*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 4*A*a^2*sin(d*x + c) + 8*C*a*b*tan(d*x + c) + 4*B*b^2*tan(d*x
+ c))/d
```

Fricas [A] time = 0.555586, size = 406, normalized size = 3.22

$$\frac{4\left(Ba^2+2\,Aab\right)dx\cos\left(dx+c\right)^2+\left(2\,Ca^2+4\,Bab+(2\,A+C)b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ca^2+4\,Bab\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/4*(4*(B*a^2 + 2*A*a*b)*d*x*cos(d*x + c)^2 + (2*C*a^2 + 4*B*a*b + (2*A + C
)*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*C*a^2 + 4*B*a*b + (2*A + C
)*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*a^2*cos(d*x + c)^2 +
C*b^2 + 2*(2*C*a*b + B*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.22505, size = 325, normalized size = 2.58

$$\frac{\frac{4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(Ba^2+2\,Aab\right)(dx+c)+\left(2\,Ca^2+4\,Bab+2\,Ab^2+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ca^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(4*A*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(B*a^2 +
 2*A*a*b)*(d*x + c) + (2*C*a^2 + 4*B*a*b + 2*A*b^2 + C*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (2*C*a^2 + 4*B*a*b + 2*A*b^2 + C*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(4*C*a*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^2*tan(1/2*d
*x + 1/2*c)^3 - C*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b*tan(1/2*d*x + 1/2*c)
 - 2*B*b^2*tan(1/2*d*x + 1/2*c) - C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^2)/d
```

3.874 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\text{se}\right.$

Optimal. Leaf size=118

$$\frac{1}{2}x\left(a^2(A+2C)+4abB+2Ab^2\right)+\frac{a(aB+Ab)\sin(c+dx)}{d}+\frac{A\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{b(2aC-$$

```
[Out] ((2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*x)/2 + (b*(b*B + 2*a*C)*ArcTanh[Sin[c
+ d*x]])/d + (a*(A*b + a*B)*Sin[c + d*x])/d + (A*Cos[c + d*x]*(a + b*Sec[c
+ d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(A - 2*C)*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.317219, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4076, 4047, 8, 4045, 3770}

$$\frac{1}{2}x\left(a^2(A+2C)+4abB+2Ab^2\right)+\frac{a(aB+Ab)\sin(c+dx)}{d}+\frac{A\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{b(2aC-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*x)/2 + (b*(b*B + 2*a*C)*ArcTanh[Sin[c
+ d*x]])/d + (a*(A*b + a*B)*Sin[c + d*x])/d + (A*Cos[c + d*x]*(a + b*Sec[c
+ d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(A - 2*C)*Tan[c + d*x])/(2*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+
}{2d}$$
$$= \frac{A\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+
}{2d}$$
$$= \frac{A\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+
}{2d}$$
$$= \frac{1}{2}\left(2Ab^2+4abB+a^2(A+2C)\right)x+\frac{a(Ab}{}$$
$$= \frac{1}{2}\left(2Ab^2+4abB+a^2(A+2C)\right)x+\frac{b(bB}{}$$

Mathematica [A] time = 1.16415, size = 153, normalized size = 1.3

$$\frac{2(c+dx)\left(a^2(A+2C)+4abB+2Ab^2\right)+\tan(c+dx)\left(a^2A\cos(2(c+dx))+a^2A+4a(aB+2Ab)\cos(c+dx)+4b^2C\right)-}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*(2*A*b^2 + 4*a*b*B + a^2*(A + 2*C))*(c + d*x) - 4*b*(b*B + 2*a*C)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*b*(b*B + 2*a*C)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + (a^2*A + 4*b^2*C + 4*a*(2*A*b + a*B)*Cos[c + d*x] +
a^2*A*Cos[2*(c + d*x)])*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.065, size = 171, normalized size = 1.5

$$\frac{a^2A\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2Ax}{2}+\frac{a^2Ac}{2d}+\frac{Ba^2\sin(dx+c)}{d}+a^2Cx+\frac{Ca^2c}{d}+2\frac{Aab\sin(dx+c)}{d}+2Babx+2\frac{Babc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*a^2*A*cos(d*x+c)*sin(d*x+c)+1/2*a^2*A*x+1/2/d*A*a^2*c+a^2*B*sin(d*x+c
)/d+a^2*C*x+1/d*C*a^2*c+2*a*A*b*sin(d*x+c)/d+2*B*a*b*x+2/d*B*a*b*c+2/d*a*b*
```

```
C*ln(sec(d*x+c)+tan(d*x+c))+A*b^2*x+1/d*A*b^2*c+1/d*B*b^2*ln(sec(d*x+c)+tan
(d*x+c))+b^2*C*tan(d*x+c)/d
```

Maxima [A] time = 1.06307, size = 200, normalized size = 1.69

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^2 + 4\,(dx + c)Ca^2 + 8\,(dx + c)Bab + 4\,(dx + c)Ab^2 + 4\,Cab(\log(\sin(dx + c) + 1) - \log\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*C*a^2 + 8*(d*x +
c)*B*a*b + 4*(d*x + c)*A*b^2 + 4*C*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) + 2*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B
*a^2*sin(d*x + c) + 8*A*a*b*sin(d*x + c) + 4*C*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.548423, size = 366, normalized size = 3.1

$$\frac{\left((A + 2\,C)a^2 + 4\,Bab + 2\,Ab^2\right)dx\cos(dx + c) + \left(2\,Cab + Bb^2\right)\cos(dx + c)\log(\sin(dx + c) + 1) - \left(2\,Cab + Bb^2\right)\cos\cdots}{2\,d\cos(dx + c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/2*(((A + 2*C)*a^2 + 4*B*a*b + 2*A*b^2)*d*x*cos(d*x + c) + (2*C*a*b + B*b^
2)*cos(d*x + c)*log(sin(d*x + c) + 1) - (2*C*a*b + B*b^2)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (A*a^2*cos(d*x + c)^2 + 2*C*b^2 + 2*(B*a^2 + 2*A*a*b)*
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25563, size = 309, normalized size = 2.62

$$-\frac{\frac{4\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \left(Aa^2 + 2\,Ca^2 + 4\,Bab + 2\,Ab^2\right)(dx + c) - 2\left(2\,Cab + Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 2\left(2\,Cab\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -1/2*(4*C*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (A*a^2 +
2*C*a^2 + 4*B*a*b + 2*A*b^2)*(d*x + c) - 2*(2*C*a*b + B*b^2)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) + 2*(2*C*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A
*a*b*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x + 1/2*c) - 2*B*a^2*tan(1/2*
d*x + 1/2*c) - 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2
)/d
```

3.875 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=141

$$\frac{\sin(c+dx)\left(a^2(2A+3C)+6abB+2Ab^2\right)}{3d}+\frac{1}{2}x\left(a^2B+2ab(A+2C)+2b^2B\right)+\frac{a(3aB+2Ab)\sin(c+dx)\cos(c+dx}{6d}$$

```
[Out] ((a^2*B + 2*b^2*B + 2*a*b*(A + 2*C))*x)/2 + (b^2*C*ArcTanh[Sin[c + d*x]])/d
 + ((2*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*d) + (a*(2*A*b +
 3*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c +
 d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.365023, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4074, 4047, 8, 4045, 3770}

$$\frac{\sin(c+dx)\left(a^2(2A+3C)+6abB+2Ab^2\right)}{3d}+\frac{1}{2}x\left(a^2B+2ab(A+2C)+2b^2B\right)+\frac{a(3aB+2Ab)\sin(c+dx)\cos(c+dx}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((a^2*B + 2*b^2*B + 2*a*b*(A + 2*C))*x)/2 + (b^2*C*ArcTanh[Sin[c + d*x]])/d
 + ((2*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Sin[c + d*x])/(3*d) + (a*(2*A*b +
 3*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c +
 d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+}{3d}$$
$$= \frac{a(2Ab+3aB)\cos(c+dx)\sin(c+dx)}{6d} + \frac{A}{}$$
$$= \frac{a(2Ab+3aB)\cos(c+dx)\sin(c+dx)}{6d} + \frac{A}{}$$
$$= \frac{1}{2}\left(a^2B+2b^2B+2ab(A+2C)\right)x + \frac{(2Ab^2}{}$$
$$= \frac{1}{2}\left(a^2B+2b^2B+2ab(A+2C)\right)x + \frac{b^2C\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.521084, size = 157, normalized size = 1.11

$$\frac{6(c+dx)\left(a^2B+2ab(A+2C)+2b^2B\right)+3\sin(c+dx)\left(a^2(3A+4C)+8abB+4Ab^2\right)+a^2A\sin(3(c+dx))+3a(aB+2A}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (6*(a^2*B + 2*b^2*B + 2*a*b*(A + 2*C))*(c + d*x) - 12*b^2*C*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] + 12*b^2*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 3*(4*A*b^2 + 8*a*b*B + a^2*(3*A + 4*C))*Sin[c + d*x] + 3*a*(2*A*b + a*
B)*Sin[2*(c + d*x)] + a^2*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.077, size = 204, normalized size = 1.5

$$\frac{A\sin(dx+c)(\cos(dx+c))^2a^2}{3d}+\frac{2a^2A\sin(dx+c)}{3d}+\frac{Ba^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2Bx}{2}+\frac{Ba^2c}{2d}+\frac{a^2C\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^2+2/3/d*a^2*A*sin(d*x+c)+1/2*a^2*B*cos(d*
x+c)*sin(d*x+c)/d+1/2*a^2*B*x+1/2/d*B*a^2*c+1/d*a^2*C*sin(d*x+c)+a*A*b*cos(
```

```
d*x+c)*sin(d*x+c)/d+a*A*b*x+1/d*A*a*b*c+2/d*B*a*b*sin(d*x+c)+2*a*b*C*x+2/d*
C*a*b*c+1/d*A*b^2*sin(d*x+c)+B*b^2*x+1/d*B*b^2*c+1/d*b^2*C*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 1.02046, size = 212, normalized size = 1.5

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^{2}-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aab-24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^2 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b - 24*(d*x + c)*
C*a*b - 12*(d*x + c)*B*b^2 - 6*C*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) - 12*C*a^2*sin(d*x + c) - 24*B*a*b*sin(d*x + c) - 12*A*b^2*sin(d*
x + c))/d
```

Fricas [A] time = 0.548668, size = 313, normalized size = 2.22

$$\frac{3\,Cb^{2}\log\left(\sin\left(dx+c\right)+1\right)-3\,Cb^{2}\log\left(-\sin\left(dx+c\right)+1\right)+3\left(Ba^{2}+2\left(A+2\,C\right)ab+2\,Bb^{2}\right)dx+\left(2\,Aa^{2}\cos\left(dx+c\right)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/6*(3*C*b^2*log(sin(d*x + c) + 1) - 3*C*b^2*log(-sin(d*x + c) + 1) + 3*(B*
a^2 + 2*(A + 2*C)*a*b + 2*B*b^2)*d*x + (2*A*a^2*cos(d*x + c)^2 + 2*(2*A + 3
*C)*a^2 + 12*B*a*b + 6*A*b^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20832, size = 467, normalized size = 3.31

$$6\,Cb^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Cb^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(Ba^{2}+2\,Aab+4\,Cab+2\,Bb^{2}\right)\left(dx+c\right)+\frac{2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/6*(6*C*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*C*b^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(B*a^2 + 2*A*a*b + 4*C*a*b + 2*B*b^2)*(d*x + c) + 2*(
6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*t
an(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d
*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*
c)^3 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 +
12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*ta
n(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2*d*x + 1/2*c) + 6*A*a*b*tan(1/2*d*x + 1
/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.876 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\text{se}\right.$

Optimal. Leaf size=175

$$\frac{\sin(c+dx)\left(2a^2B+4aAb+6abC+3b^2B\right)}{3d}+\frac{\sin(c+dx)\cos(c+dx)\left(a^2(3A+4C)+8abB+2Ab^2\right)}{8d}+\frac{1}{8}x\left(a^2(3A+4\right.$$

```
[Out] ((8*a*b*B + 4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/8 + ((4*a*A*b + 2*a^2*B +
 3*b^2*B + 6*a*b*C)*Sin[c + d*x])/(3*d) + ((2*A*b^2 + 8*a*b*B + a^2*(3*A +
4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(A*b + 2*a*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(6*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/
(4*d)
```

Rubi [A] time = 0.44836, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4074, 4047, 2637, 4045, 8}

$$\frac{\sin(c+dx)\left(2a^2B+4aAb+6abC+3b^2B\right)}{3d}+\frac{\sin(c+dx)\cos(c+dx)\left(a^2(3A+4C)+8abB+2Ab^2\right)}{8d}+\frac{1}{8}x\left(a^2(3A+4\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((8*a*b*B + 4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*x)/8 + ((4*a*A*b + 2*a^2*B +
 3*b^2*B + 6*a*b*C)*Sin[c + d*x])/(3*d) + ((2*A*b^2 + 8*a*b*B + a^2*(3*A +
4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(A*b + 2*a*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(6*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/
(4*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+}{4d}$$
$$= \frac{a(Ab+2aB)\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{A}{}$$
$$= \frac{a(Ab+2aB)\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{A}{}$$
$$= \frac{\left(4aAb+2a^2B+3b^2B+6abC\right)\sin(c+dx}{3d}$$
$$= \frac{1}{8}\left(8abB+4b^2(A+2C)+a^2(3A+4C)\right)x$$

Mathematica [A] time = 0.730957, size = 134, normalized size = 0.77

$$\frac{12(c+dx)\left(a^2(3A+4C)+8abB+4b^2(A+2C)\right)+24\sin(c+dx)\left(3a^2B+6aAb+8abC+4b^2B\right)+24\sin(2(c+dx))\left(a^2\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (12*(8*a*b*B + 4*b^2*(A + 2*C) + a^2*(3*A + 4*C))*(c + d*x) + 24*(6*a*A*b +
 3*a^2*B + 4*b^2*B + 8*a*b*C)*Sin[c + d*x] + 24*(A*b^2 + 2*a*b*B + a^2*(A +
 C))*Sin[2*(c + d*x)] + 8*a*(2*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^2*A*Sin[4*
(c + d*x)])/(96*d)
```

Maple [A] time = 0.072, size = 200, normalized size = 1.1

$$\frac{1}{d}\left(a^2A\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2Aab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+\frac{Ba^2(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3
*A*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+A*
b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*B*a*b*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+a^2*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b^2*
sin(d*x+c)+2*a*b*C*sin(d*x+c)+b^2*C*(d*x+c))
```

Maxima [A] time = 1.01152, size = 252, normalized size = 1.44

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Aa^2-32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^2+24\,(2\,dx+2\,c+\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c
))*C*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b + 48*(2*d*x + 2*c + s
in(2*d*x + 2*c))*B*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 + 96*(d*
x + c)*C*b^2 + 192*C*a*b*sin(d*x + c) + 96*B*b^2*sin(d*x + c))/d
```

Fricas [A] time = 0.519028, size = 320, normalized size = 1.83

$$\frac{3\left((3\,A+4\,C)a^2+8\,Bab+4\,(A+2\,C)b^2\right)dx+\left(6\,Aa^2\cos\left(dx+c\right)^3+16\,Ba^2+16\,(2\,A+3\,C)ab+24\,Bb^2+8\left(Ba^2+2\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/24*(3*((3*A + 4*C)*a^2 + 8*B*a*b + 4*(A + 2*C)*b^2)*d*x + (6*A*a^2*cos(d*
x + c)^3 + 16*B*a^2 + 16*(2*A + 3*C)*a*b + 24*B*b^2 + 8*(B*a^2 + 2*A*a*b)*c
os(d*x + c)^2 + 3*((3*A + 4*C)*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c))*sin(d
*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21662, size = 779, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 4*C*a^2 + 8*B*a*b + 4*A*b^2 + 8*C*b^2)*(d*x + c) - 2*(15
*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^2*
tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2
*d*x + 1/2*c)^7 - 48*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x +
1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^2*tan(1/2*d*x + 1/2*c)^5
 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 80*A
*a*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 144*C*a*b*t
an(1/2*d*x + 1/2*c)^5 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*b^2*tan(1/2*
d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(1/2*d*x + 1/
2*c)^3 - 12*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3
- 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 144*C*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A
*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^2*ta
n(1/2*d*x + 1/2*c) - 24*B*a^2*tan(1/2*d*x + 1/2*c) - 12*C*a^2*tan(1/2*d*x +
 1/2*c) - 48*A*a*b*tan(1/2*d*x + 1/2*c) - 24*B*a*b*tan(1/2*d*x + 1/2*c) - 4
8*C*a*b*tan(1/2*d*x + 1/2*c) - 12*A*b^2*tan(1/2*d*x + 1/2*c) - 24*B*b^2*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.877 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2 \left(A + B\sec(c+dx) + C\sec\right.$

Optimal. Leaf size=215

$$-\frac{\sin^3(c+dx)\left(a^2(4A+5C)+10abB+2Ab^2\right)}{15d}+\frac{\sin(c+dx)\left(a^2(4A+5C)+10abB+b^2(4A+5C)\right)}{5d}+\frac{\sin(c+dx)\,\mathrm{co}}{}$$

```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*x)/8 + ((10*a*b*B + a^2*(4*A + 5*C
) + b^2*(4*A + 5*C))*Sin[c + d*x])/(5*d) + ((6*a*A*b + 3*a^2*B + 4*b^2*B +
8*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(2*A*b + 5*a*B)*Cos[c + d*x]
^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(5*d) - ((2*A*b^2 + 10*a*b*B + a^2*(4*A + 5*C))*Sin[c + d*x]^3)/(15*d
)
```

Rubi [A] time = 0.513512, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4094, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{\sin^3(c+dx)\left(a^2(4A+5C)+10abB+2Ab^2\right)}{15d}+\frac{\sin(c+dx)\left(a^2(4A+5C)+10abB+b^2(4A+5C)\right)}{5d}+\frac{\sin(c+dx)\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*x)/8 + ((10*a*b*B + a^2*(4*A + 5*C
) + b^2*(4*A + 5*C))*Sin[c + d*x])/(5*d) + ((6*a*A*b + 3*a^2*B + 4*b^2*B +
8*a*b*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(2*A*b + 5*a*B)*Cos[c + d*x]
^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(5*d) - ((2*A*b^2 + 10*a*b*B + a^2*(4*A + 5*C))*Sin[c + d*x]^3)/(15*d
)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
```

```
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+$$
$$= \frac{a(2Ab+5aB)\cos^3(c+dx)\sin(c+dx)}{20d}+$$
$$= \frac{a(2Ab+5aB)\cos^3(c+dx)\sin(c+dx)}{20d}+$$
$$= \frac{\left(6aAb+3a^2B+4b^2B+8abC\right)\cos(c+dx}{8d}$$
$$= \frac{1}{8}\left(6aAb+3a^2B+4b^2B+8abC\right)x+\frac{(6a}{}$$
$$= \frac{1}{8}\left(6aAb+3a^2B+4b^2B+8abC\right)x+\frac{(10a}{}$$

Mathematica [A] time = 0.802154, size = 169, normalized size = 0.79

$$\frac{60(c+dx)\left(3a^2B+6aAb+8abC+4b^2B\right)+60\sin(c+dx)\left(a^2(5A+6C)+12abB+2b^2(3A+4C)\right)+120\sin(2(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (60*(6*a*A*b + 3*a^2*B + 4*b^2*B + 8*a*b*C)*(c + d*x) + 60*(12*a*b*B + 2*b^
2*(3*A + 4*C) + a^2*(5*A + 6*C))*Sin[c + d*x] + 120*(a^2*B + b^2*B + 2*a*b*
(A + C))*Sin[2*(c + d*x)] + 10*(4*A*b^2 + 8*a*b*B + a^2*(5*A + 4*C))*Sin[3*
(c + d*x)] + 15*a*(2*A*b + a*B)*Sin[4*(c + d*x)] + 6*a^2*A*Sin[5*(c + d*x)]
```

```
)/(480*d)
```

Maple [A] time = 0.081, size = 244, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2A\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+Ba^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*C*(2+cos(d*x+
c)^2)*sin(d*x+c)+2*A*a*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*
d*x+3/8*c)+2/3*B*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*C*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b^2*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^2*C*sin(d*x+c))
```

Maxima [A] time = 1.0328, size = 315, normalized size = 1.47

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 160*(sin
(d*x + c)^3 - 3*sin(d*x + c))*C*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c)
+ 8*sin(2*d*x + 2*c))*A*a*b - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b +
 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b - 160*(sin(d*x + c)^3 - 3*sin(d
*x + c))*A*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^2 + 480*C*b^2*sin
(d*x + c))/d
```

Fricas [A] time = 0.535981, size = 414, normalized size = 1.93

$$\frac{15\left(3Ba^2+2(3A+4C)ab+4Bb^2\right)dx+\left(24Aa^2\cos(dx+c)^4+30\left(Ba^2+2Aab\right)\cos(dx+c)^3+16(4A+5C)a^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/120*(15*(3*B*a^2 + 2*(3*A + 4*C)*a*b + 4*B*b^2)*d*x + (24*A*a^2*cos(d*x +
 c)^4 + 30*(B*a^2 + 2*A*a*b)*cos(d*x + c)^3 + 16*(4*A + 5*C)*a^2 + 160*B*a*
b + 40*(2*A + 3*C)*b^2 + 8*((4*A + 5*C)*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x +
 c)^2 + 15*(3*B*a^2 + 2*(3*A + 4*C)*a*b + 4*B*b^2)*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20855, size = 972, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 8*C*a*b + 4*B*b^2)*(d*x + c) + 2*(120*A*a^2*
tan(1/2*d*x + 1/2*c)^9 - 75*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 120*C*a^2*tan(1/
2*d*x + 1/2*c)^9 - 150*A*a*b*tan(1/2*d*x + 1/2*c)^9 + 240*B*a*b*tan(1/2*d*x
 + 1/2*c)^9 - 120*C*a*b*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^2*tan(1/2*d*x + 1/
2*c)^9 - 60*B*b^2*tan(1/2*d*x + 1/2*c)^9 + 120*C*b^2*tan(1/2*d*x + 1/2*c)^9
 + 160*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 320
*C*a^2*tan(1/2*d*x + 1/2*c)^7 - 60*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 640*B*a*b
*tan(1/2*d*x + 1/2*c)^7 - 240*C*a*b*tan(1/2*d*x + 1/2*c)^7 + 320*A*b^2*tan(
1/2*d*x + 1/2*c)^7 - 120*B*b^2*tan(1/2*d*x + 1/2*c)^7 + 480*C*b^2*tan(1/2*d
*x + 1/2*c)^7 + 464*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 400*C*a^2*tan(1/2*d*x +
1/2*c)^5 + 800*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 400*A*b^2*tan(1/2*d*x + 1/2*c
)^5 + 720*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^2*tan(1/2*d*x + 1/2*c)^3 +
 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 320*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 60*A*
a*b*tan(1/2*d*x + 1/2*c)^3 + 640*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 240*C*a*b*t
an(1/2*d*x + 1/2*c)^3 + 320*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*b^2*tan(1/
2*d*x + 1/2*c)^3 + 480*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^2*tan(1/2*d*x
 + 1/2*c) + 75*B*a^2*tan(1/2*d*x + 1/2*c) + 120*C*a^2*tan(1/2*d*x + 1/2*c)
+ 150*A*a*b*tan(1/2*d*x + 1/2*c) + 240*B*a*b*tan(1/2*d*x + 1/2*c) + 120*C*a
*b*tan(1/2*d*x + 1/2*c) + 120*A*b^2*tan(1/2*d*x + 1/2*c) + 60*B*b^2*tan(1/2
*d*x + 1/2*c) + 120*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1
)^5)/d
```

3.878 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\,\mathrm{se}\right.$

Optimal. Leaf size=381

$$-\frac{\tan(c+dx)\left(-a^3b^2(30A+17C)-104a^2b^3B+6a^4bB-2a^5C-24ab^4(5A+4C)-32b^5B\right)}{60b^2d}+\frac{\left(6a^2b(4A+3C)+8a^3\right.}{}$$

```
[Out] ((8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*ArcTanh[Sin
[c + d*x]])/(16*d) - ((6*a^4*b*B - 104*a^2*b^3*B - 32*b^5*B - 2*a^5*C - 24*
a*b^4*(5*A + 4*C) - a^3*b^2*(30*A + 17*C))*Tan[c + d*x])/(60*b^2*d) - ((12*
a^3*b*B - 142*a*b^3*B - 4*a^4*C - 12*a^2*b^2*(5*A + 3*C) - 15*b^4*(6*A + 5*
C))*Sec[c + d*x]*Tan[c + d*x])/(240*b*d) - ((6*a^2*b*B - 32*b^3*B - 2*a^3*C
 - 3*a*b^2*(10*A + 7*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(120*b^2*d) +
 ((30*A*b^2 - 6*a*b*B + 2*a^2*C + 25*b^2*C)*(a + b*Sec[c + d*x])^3*Tan[c +
d*x])/(120*b^2*d) + ((3*b*B - a*C)*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(15
*b^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.869532, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4092, 4082, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{\tan(c+dx)\left(-a^3b^2(30A+17C)-104a^2b^3B+6a^4bB-2a^5C-24ab^4(5A+4C)-32b^5B\right)}{60b^2d}+\frac{\left(6a^2b(4A+3C)+8a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*ArcTanh[Sin
[c + d*x]])/(16*d) - ((6*a^4*b*B - 104*a^2*b^3*B - 32*b^5*B - 2*a^5*C - 24*
a*b^4*(5*A + 4*C) - a^3*b^2*(30*A + 17*C))*Tan[c + d*x])/(60*b^2*d) - ((12*
a^3*b*B - 142*a*b^3*B - 4*a^4*C - 12*a^2*b^2*(5*A + 3*C) - 15*b^4*(6*A + 5*
C))*Sec[c + d*x]*Tan[c + d*x])/(240*b*d) - ((6*a^2*b*B - 32*b^3*B - 2*a^3*C
 - 3*a*b^2*(10*A + 7*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(120*b^2*d) +
 ((30*A*b^2 - 6*a*b*B + 2*a^2*C + 25*b^2*C)*(a + b*Sec[c + d*x])^3*Tan[c +
d*x])/(120*b^2*d) + ((3*b*B - a*C)*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(15
*b^2*d) + (C*Sec[c + d*x]*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(6*b*d)
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
```

```
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec(c+dx)(a+b\sec(c+dx))^4\tan(c}{6bd}$$
$$= \frac{(3bB-aC)(a+b\sec(c+dx))^4\tan(c+}{15b^2d}$$
$$= \frac{\left(30Ab^2-6abB+2a^2C+25b^2C\right)(a+}{120b^2d}$$
$$= -\frac{\left(6a^2bB-32b^3B-2a^3C-3ab^2(10A\right.}{120}$$
$$= -\frac{\left(12a^3bB-142ab^3B-4a^4C-12a^2b^2(\right.}{}$$
$$= -\frac{\left(12a^3bB-142ab^3B-4a^4C-12a^2b^2(\right.}{}$$
$$= \frac{\left(8a^3B+18ab^2B+6a^2b(4A+3C)+b^3\right.}{16d}$$
$$= \frac{\left(8a^3B+18ab^2B+6a^2b(4A+3C)+b^3\right.}{16d}$$

Mathematica [A] time = 3.17543, size = 384, normalized size = 1.01

$$-\frac{\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(-b\left(5\sin(2(c+dx))\left(18a^2C+18abB+6Ab^2+5b^2C\right)+48b(3aC\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(15*(8*a^3*B + 18*
a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A + 5*C))*Cos[c + d*x]^5*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
- 16*(15*a^2*b*B + 4*b^3*B + 5*a^3*C + 3*a*b^2*(5*A + 4*C))*Cos[c + d*x]^2*
Sin[c + d*x] - 15*(8*a^3*B + 18*a*b^2*B + 6*a^2*b*(4*A + 3*C) + b^3*(6*A +
5*C))*Cos[c + d*x]^3*Sin[c + d*x] - 16*(30*a^2*b*B + 8*b^3*B + 5*a^3*(3*A +
 2*C) + 6*a*b^2*(5*A + 4*C))*Cos[c + d*x]^4*Sin[c + d*x] - b*(48*b*(b*B + 3
*a*C)*Sin[c + d*x] + 5*(6*A*b^2 + 18*a*b*B + 18*a^2*C + 5*b^2*C)*Sin[2*(c +
 d*x)] + 40*b^2*C*Tan[c + d*x])))/(120*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Co
s[2*(c + d*x)]))
```

Maple [A] time = 0.062, size = 644, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/8/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+5/16/d*C*b^3*ln(sec(d*x+c)+tan(d*x+c)
)+9/8/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+8/5/d*C*a*b^2*tan(d*x+c)+5/16/d*C
*b^3*sec(d*x+c)*tan(d*x+c)+1/d*A*a^3*tan(d*x+c)+1/2/d*B*a^3*ln(sec(d*x+c)+t
an(d*x+c))+8/15/d*B*b^3*tan(d*x+c)+1/3/d*a^3*C*tan(d*x+c)*sec(d*x+c)^2+1/5/
d*B*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*B*b^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*
```

```
B*a^3*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+9/8/d*B
*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*A*b^3*tan(d*x+c)*sec(d*x+c)^3+3/2/d*
A*a^2*b*sec(d*x+c)*tan(d*x+c)+3/5/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)^4+4/5/d*C
*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)^3+2/3*a^
3*C*tan(d*x+c)/d+3/8/d*A*b^3*sec(d*x+c)*tan(d*x+c)+9/8/d*a^2*b*C*sec(d*x+c)
*tan(d*x+c)+3/4/d*B*a*b^2*tan(d*x+c)*sec(d*x+c)^3+9/8/d*B*a*b^2*sec(d*x+c)*
tan(d*x+c)+1/d*B*a^2*b*tan(d*x+c)*sec(d*x+c)^2+1/d*A*a*b^2*tan(d*x+c)*sec(d
*x+c)^2+1/6/d*C*b^3*tan(d*x+c)*sec(d*x+c)^5+2/d*B*a^2*b*tan(d*x+c)+2/d*A*a*
b^2*tan(d*x+c)+5/24/d*C*b^3*tan(d*x+c)*sec(d*x+c)^3
```

Maxima [A] time = 1.10175, size = 763, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(160*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3 + 480*(tan(d*x + c)^3 +
3*tan(d*x + c))*B*a^2*b + 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a*b^2 + 9
6*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a*b^2 + 32*(3*
tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*b^3 - 5*C*b^3*(2*(1
5*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3
*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log
(sin(d*x + c) - 1)) - 90*C*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(si
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) - 90*B*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
 + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
 c) - 1)) - 30*A*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4
 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1
)) - 120*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
 + log(sin(d*x + c) - 1)) - 360*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 480*A*a^3*tan(d*x + c)
)/d
```

Fricas [A] time = 0.612898, size = 824, normalized size = 2.16

$$15\left(8Ba^3 + 6(4A + 3C)a^2b + 18Bab^2 + (6A + 5C)b^3\right)\cos(dx+c)^6\log(\sin(dx+c)+1) - 15\left(8Ba^3 + 6(4A + 3C)a^2 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*B*a^3 + 6*(4*A + 3*C)*a^2*b + 18*B*a*b^2 + (6*A + 5*C)*b^3)*co
s(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(8*B*a^3 + 6*(4*A + 3*C)*a^2*b + 18
*B*a*b^2 + (6*A + 5*C)*b^3)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(
5*(3*A + 2*C)*a^3 + 30*B*a^2*b + 6*(5*A + 4*C)*a*b^2 + 8*B*b^3)*cos(d*x + c
)^5 + 15*(8*B*a^3 + 6*(4*A + 3*C)*a^2*b + 18*B*a*b^2 + (6*A + 5*C)*b^3)*cos
(d*x + c)^4 + 40*C*b^3 + 16*(5*C*a^3 + 15*B*a^2*b + 3*(5*A + 4*C)*a*b^2 + 4
*B*b^3)*cos(d*x + c)^3 + 10*(18*C*a^2*b + 18*B*a*b^2 + (6*A + 5*C)*b^3)*cos
(d*x + c)^2 + 48*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
```

```
+ c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*s
ec(c + d*x)**2, x)
```

Giac [B] time = 1.26334, size = 1850, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(8*B*a^3 + 24*A*a^2*b + 18*C*a^2*b + 18*B*a*b^2 + 6*A*b^3 + 5*C*b
^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*B*a^3 + 24*A*a^2*b + 18*C*a^
2*b + 18*B*a*b^2 + 6*A*b^3 + 5*C*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) -
2*(240*A*a^3*tan(1/2*d*x + 1/2*c)^11 - 120*B*a^3*tan(1/2*d*x + 1/2*c)^11 +
240*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 360*A*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 7
20*B*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 450*C*a^2*b*tan(1/2*d*x + 1/2*c)^11 +
720*A*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 450*B*a*b^2*tan(1/2*d*x + 1/2*c)^11 +
 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 150*A*b^3*tan(1/2*d*x + 1/2*c)^11 +
240*B*b^3*tan(1/2*d*x + 1/2*c)^11 - 165*C*b^3*tan(1/2*d*x + 1/2*c)^11 - 120
0*A*a^3*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 880*C*a
^3*tan(1/2*d*x + 1/2*c)^9 + 1080*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 2640*B*a^
2*b*tan(1/2*d*x + 1/2*c)^9 + 630*C*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 2640*A*a*
b^2*tan(1/2*d*x + 1/2*c)^9 + 630*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 1680*C*a*
b^2*tan(1/2*d*x + 1/2*c)^9 + 210*A*b^3*tan(1/2*d*x + 1/2*c)^9 - 560*B*b^3*t
an(1/2*d*x + 1/2*c)^9 - 25*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 2400*A*a^3*tan(1/
2*d*x + 1/2*c)^7 - 240*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a^3*tan(1/2*d*
x + 1/2*c)^7 - 720*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4320*B*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 180*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4320*A*a*b^2*tan(1/2*d*
x + 1/2*c)^7 - 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 3744*C*a*b^2*tan(1/2*d*
x + 1/2*c)^7 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 1248*B*b^3*tan(1/2*d*x + 1
/2*c)^7 - 450*C*b^3*tan(1/2*d*x + 1/2*c)^7 - 2400*A*a^3*tan(1/2*d*x + 1/2*c
)^5 - 240*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1440*C*a^3*tan(1/2*d*x + 1/2*c)^5
- 720*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 4320*B*a^2*b*tan(1/2*d*x + 1/2*c)^5
- 180*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 4320*A*a*b^2*tan(1/2*d*x + 1/2*c)^5
- 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3744*C*a*b^2*tan(1/2*d*x + 1/2*c)^5
- 60*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 1248*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 450
*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 360*B*a
^3*tan(1/2*d*x + 1/2*c)^3 + 880*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 1080*A*a^2*b
*tan(1/2*d*x + 1/2*c)^3 + 2640*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 630*C*a^2*b
*tan(1/2*d*x + 1/2*c)^3 + 2640*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 630*B*a*b^2
*tan(1/2*d*x + 1/2*c)^3 + 1680*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 210*A*b^3*t
```

```
an(1/2*d*x + 1/2*c)^3 + 560*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 25*C*b^3*tan(1/2
*d*x + 1/2*c)^3 - 240*A*a^3*tan(1/2*d*x + 1/2*c) - 120*B*a^3*tan(1/2*d*x +
1/2*c) - 240*C*a^3*tan(1/2*d*x + 1/2*c) - 360*A*a^2*b*tan(1/2*d*x + 1/2*c)
- 720*B*a^2*b*tan(1/2*d*x + 1/2*c) - 450*C*a^2*b*tan(1/2*d*x + 1/2*c) - 720
*A*a*b^2*tan(1/2*d*x + 1/2*c) - 450*B*a*b^2*tan(1/2*d*x + 1/2*c) - 720*C*a*
b^2*tan(1/2*d*x + 1/2*c) - 150*A*b^3*tan(1/2*d*x + 1/2*c) - 240*B*b^3*tan(1
/2*d*x + 1/2*c) - 165*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^6)/d
```

3.879 $\int \sec(c+dx)(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=286

$$\frac{\tan(c+dx)\left(4a^2b^2(20A+13C)+15a^3bB-3a^4C+60ab^3B+4b^4(5A+4C)\right)}{30bd}+\frac{\left(4a^3(2A+C)+12a^2bB+3ab^2(4A+\right.}{8d}$$

```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*(2*A + C) + 3*a*b^2*(4*A + 3*C))*ArcTanh[Sin
[c + d*x]])/(8*d) + ((15*a^3*b*B + 60*a*b^3*B - 3*a^4*C + 4*b^4*(5*A + 4*C)
 + 4*a^2*b^2*(20*A + 13*C))*Tan[c + d*x])/(30*b*d) + ((30*a^2*b*B + 45*b^3*
B - 6*a^3*C + a*b^2*(100*A + 71*C))*Sec[c + d*x]*Tan[c + d*x])/(120*d) + ((
4*b^2*(5*A + 4*C) + 3*a*(5*b*B - a*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])
/(60*b*d) + ((5*b*B - a*C)*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) +
(C*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.587281, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4082, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(4a^2b^2(20A+13C)+15a^3bB-3a^4C+60ab^3B+4b^4(5A+4C)\right)}{30bd}+\frac{\left(4a^3(2A+C)+12a^2bB+3ab^2(4A+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] ((12*a^2*b*B + 3*b^3*B + 4*a^3*(2*A + C) + 3*a*b^2*(4*A + 3*C))*ArcTanh[Sin
[c + d*x]])/(8*d) + ((15*a^3*b*B + 60*a*b^3*B - 3*a^4*C + 4*b^4*(5*A + 4*C)
 + 4*a^2*b^2*(20*A + 13*C))*Tan[c + d*x])/(30*b*d) + ((30*a^2*b*B + 45*b^3*
B - 6*a^3*C + a*b^2*(100*A + 71*C))*Sec[c + d*x]*Tan[c + d*x])/(120*d) + ((
4*b^2*(5*A + 4*C) + 3*a*(5*b*B - a*C))*(a + b*Sec[c + d*x])^2*Tan[c + d*x])
/(60*b*d) + ((5*b*B - a*C)*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) +
(C*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^4\tan(c+dx)}{5bd}+\frac{\int \sec(}{}$$

$$= \frac{(5bB-aC)(a+b\sec(c+dx))^3\tan(c+dx)}{20bd}$$

$$= \frac{\left(4b^2(5A+4C)+3a(5bB-aC)\right)(a+b\sec(}{60bd}$$

$$= \frac{\left(30a^2bB+45b^3B-6a^3C+ab^2(100A+71\right.}{120d}$$

$$= \frac{\left(30a^2bB+45b^3B-6a^3C+ab^2(100A+71\right.}{120d}$$

$$= \frac{\left(12a^2bB+3b^3B+4a^3(2A+C)+3ab^2(4A\right.}{8d}$$

$$= \frac{\left(12a^2bB+3b^3B+4a^3(2A+C)+3ab^2(4A\right.}{8d}$$

Mathematica [A] time = 2.93432, size = 451, normalized size = 1.58

$$-\frac{\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(120\cos^5(c+dx)\left(4a^3(2A+C)+12a^2bB+3ab^2(4A+3C)+3b^3B\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(120*(12*a^2*b*B +
 3*b^3*B + 4*a^3*(2*A + C) + 3*a*b^2*(4*A + 3*C))*Cos[c + d*x]^5*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 - 2*(540*a^2*A*b + 200*A*b^3 + 180*a^3*B + 600*a*b^2*B + 600*a^2*b*C + 256
*b^3*C + 15*(36*a^2*b*B + 17*b^3*B + 12*a^3*C + 3*a*b^2*(12*A + 17*C))*Cos[
c + d*x] + 48*(5*a^3*B + 15*a*b^2*B + 15*a^2*b*(A + C) + b^3*(5*A + 4*C))*C
os[2*(c + d*x)] + 180*a*A*b^2*Cos[3*(c + d*x)] + 180*a^2*b*B*Cos[3*(c + d*x
)] + 45*b^3*B*Cos[3*(c + d*x)] + 60*a^3*C*Cos[3*(c + d*x)] + 135*a*b^2*C*Co
s[3*(c + d*x)] + 180*a^2*A*b*Cos[4*(c + d*x)] + 40*A*b^3*Cos[4*(c + d*x)] +
 60*a^3*B*Cos[4*(c + d*x)] + 120*a*b^2*B*Cos[4*(c + d*x)] + 120*a^2*b*C*Cos
[4*(c + d*x)] + 32*b^3*C*Cos[4*(c + d*x)])*Sin[c + d*x]))/(480*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.061, size = 504, normalized size = 1.8

$$\frac{Aa^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Ba^3\tan(dx+c)}{d}+\frac{a^3C\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3C\ln(\sec(dx+c)+\tan(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^3*tan(d*x+c)+1/2/d*a^3*C*sec(d*
x+c)*tan(d*x+c)+1/2/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^2*b*tan(d*x+c
)+3/2/d*B*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a^2*b*ln(sec(d*x+c)+tan(d*x+c
))+2/d*a^2*b*C*tan(d*x+c)+1/d*a^2*b*C*tan(d*x+c)*sec(d*x+c)^2+3/2/d*A*a*b^2
*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b^2*
tan(d*x+c)+1/d*B*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/4/d*C*a*b^2*tan(d*x+c)*sec
(d*x+c)^3+9/8/d*C*a*b^2*sec(d*x+c)*tan(d*x+c)+9/8/d*C*a*b^2*ln(sec(d*x+c)+t
an(d*x+c))+2/3/d*A*b^3*tan(d*x+c)+1/3/d*A*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d
*B*b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*B*b^3*sec(d*x+c)*tan(d*x+c)+3/8/d*B*b^
3*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*C*b^3*tan(d*x+c)+1/5/d*C*b^3*tan(d*x+c)*
sec(d*x+c)^4+4/15/d*C*b^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.05533, size = 601, normalized size = 2.1

$$240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ca^2b+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bab^2+80\left(\tan(dx+c)^3+3\tan(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b + 240*(tan(d*x + c)^3
+ 3*tan(d*x + c))*B*a*b^2 + 80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^3 + 16
*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*b^3 - 45*C*a*b^
2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 15*B*b^3*(2*(3
*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*C*a^3*(2*sin(d*x +
 c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) -
 180*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 180*A*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
```

```
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*log(sec(d*x +
c) + tan(d*x + c)) + 240*B*a^3*tan(d*x + c) + 720*A*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 0.589616, size = 711, normalized size = 2.49

$$\frac{15\left(4\,(2\,A+C)a^3+12\,Ba^2b+3\,(4\,A+3\,C)ab^2+3\,Bb^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,(2\,A+C)a^3+12\,Ba^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/240*(15*(4*(2*A + C)*a^3 + 12*B*a^2*b + 3*(4*A + 3*C)*a*b^2 + 3*B*b^3)*co
s(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*(2*A + C)*a^3 + 12*B*a^2*b + 3*(
4*A + 3*C)*a*b^2 + 3*B*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(1
5*B*a^3 + 15*(3*A + 2*C)*a^2*b + 30*B*a*b^2 + 2*(5*A + 4*C)*b^3)*cos(d*x +
c)^4 + 24*C*b^3 + 15*(4*C*a^3 + 12*B*a^2*b + 3*(4*A + 3*C)*a*b^2 + 3*B*b^3)
*cos(d*x + c)^3 + 8*(15*C*a^2*b + 15*B*a*b^2 + (5*A + 4*C)*b^3)*cos(d*x + c
)^2 + 30*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*s
ec(c + d*x), x)
```

Giac [B] time = 1.26946, size = 1335, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/120*(15*(8*A*a^3 + 4*C*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 9*C*a*b^2 + 3*B*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*A*a^3 + 4*C*a^3 + 12*B*a^2*b
+ 12*A*a*b^2 + 9*C*a*b^2 + 3*B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*
(120*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 60*C*a^3*tan(1/2*d*x + 1/2*c)^9 + 360*A
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 180*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*C*
a^2*b*tan(1/2*d*x + 1/2*c)^9 - 180*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 360*B*a
*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^
3*tan(1/2*d*x + 1/2*c)^9 - 75*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*C*b^3*tan(
1/2*d*x + 1/2*c)^9 - 480*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 120*C*a^3*tan(1/2*d
```

```
*x + 1/2*c)^7 - 1440*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 360*B*a^2*b*tan(1/2*d
*x + 1/2*c)^7 - 960*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a*b^2*tan(1/2*d*
x + 1/2*c)^7 - 960*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 90*C*a*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 320*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 30*B*b^3*tan(1/2*d*x + 1/2*
c)^7 - 160*C*b^3*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^3*tan(1/2*d*x + 1/2*c)^5
+ 2160*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1200*C*a^2*b*tan(1/2*d*x + 1/2*c)^5
 + 1200*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 400*A*b^3*tan(1/2*d*x + 1/2*c)^5 +
 464*C*b^3*tan(1/2*d*x + 1/2*c)^5 - 480*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*
C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1440*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 360*B*
a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 360*A*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - 960*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 90*C*a*b
^2*tan(1/2*d*x + 1/2*c)^3 - 320*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*B*b^3*tan
(1/2*d*x + 1/2*c)^3 - 160*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^3*tan(1/2*
d*x + 1/2*c) + 60*C*a^3*tan(1/2*d*x + 1/2*c) + 360*A*a^2*b*tan(1/2*d*x + 1/
2*c) + 180*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*C*a^2*b*tan(1/2*d*x + 1/2*c)
+ 180*A*a*b^2*tan(1/2*d*x + 1/2*c) + 360*B*a*b^2*tan(1/2*d*x + 1/2*c) + 225
*C*a*b^2*tan(1/2*d*x + 1/2*c) + 120*A*b^3*tan(1/2*d*x + 1/2*c) + 75*B*b^3*t
an(1/2*d*x + 1/2*c) + 120*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 - 1)^5)/d
```

3.880 $\int (a+b\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) d$

Optimal. Leaf size=207

$$\frac{\tan(c+dx)\left(16a^2bB + 3a^3C + 6ab^2(3A+2C) + 4b^3B\right)}{6d} + \frac{\left(12a^2b(2A+C) + 8a^3B + 12ab^2B + b^3(4A+3C)\right)\tanh^{-1}(\sin}{8d}$$

```
[Out] a^3*A*x + ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^3*(4*A + 3*C))*Ar
cTanh[Sin[c + d*x]])/(8*d) + ((16*a^2*b*B + 4*b^3*B + 3*a^3*C + 6*a*b^2*(3*
A + 2*C))*Tan[c + d*x])/(6*d) + (b*(12*A*b^2 + 20*a*b*B + 6*a^2*C + 9*b^2*C
)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*b*B + 3*a*C)*(a + b*Sec[c + d*x])
^2*Tan[c + d*x])/(12*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.339917, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4056, 4048, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(16a^2bB + 3a^3C + 6ab^2(3A+2C) + 4b^3B\right)}{6d} + \frac{\left(12a^2b(2A+C) + 8a^3B + 12ab^2B + b^3(4A+3C)\right)\tanh^{-1}(\sin}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] a^3*A*x + ((8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^3*(4*A + 3*C))*Ar
cTanh[Sin[c + d*x]])/(8*d) + ((16*a^2*b*B + 4*b^3*B + 3*a^3*C + 6*a*b^2*(3*
A + 2*C))*Tan[c + d*x])/(6*d) + (b*(12*A*b^2 + 20*a*b*B + 6*a^2*C + 9*b^2*C
)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*b*B + 3*a*C)*(a + b*Sec[c + d*x])
^2*Tan[c + d*x])/(12*d) + (C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\sec($$
$$= \frac{(4bB+3aC)(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{C(a}{}$$
$$= \frac{b\left(12Ab^2+20abB+6a^2C+9b^2C\right)\sec(c+dx)\tan(}{24d}$$
$$= a^3Ax+\frac{b\left(12Ab^2+20abB+6a^2C+9b^2C\right)\sec(c+}{24d}$$
$$= a^3Ax+\frac{\left(8a^3B+12ab^2B+12a^2b(2A+C)+b^3(4A\right.}{8d}$$
$$= a^3Ax+\frac{\left(8a^3B+12ab^2B+12a^2b(2A+C)+b^3(4A\right.}{8d}$$

Mathematica [B] time = 5.51976, size = 525, normalized size = 2.54

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-12\cos^4(c+dx)\left(12a^2b(2A+C)+8a^3B+12a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*(36*a^3*A*(c + d*x) + 48*a^3*A*(c + d*x)*Cos[2*(c + d*x)] + 12*a^3*A*(c +
 d*x)*Cos[4*(c + d*x)] - 12*(8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^
3*(4*A + 3*C))*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12
*(8*a^3*B + 12*a*b^2*B + 12*a^2*b*(2*A + C) + b^3*(4*A + 3*C))*Cos[c + d*x]
^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*A*b^3*Sin[c + d*x] + 36*a*
b^2*B*Sin[c + d*x] + 36*a^2*b*C*Sin[c + d*x] + 33*b^3*C*Sin[c + d*x] + 72*a
*A*b^2*Sin[2*(c + d*x)] + 72*a^2*b*B*Sin[2*(c + d*x)] + 32*b^3*B*Sin[2*(c +
 d*x)] + 24*a^3*C*Sin[2*(c + d*x)] + 96*a*b^2*C*Sin[2*(c + d*x)] + 12*A*b^3
*Sin[3*(c + d*x)] + 36*a*b^2*B*Sin[3*(c + d*x)] + 36*a^2*b*C*Sin[3*(c + d*x
)] + 9*b^3*C*Sin[3*(c + d*x)] + 36*a*A*b^2*Sin[4*(c + d*x)] + 36*a^2*b*B*Si
n[4*(c + d*x)] + 8*b^3*B*Sin[4*(c + d*x)] + 12*a^3*C*Sin[4*(c + d*x)] + 24*
a*b^2*C*Sin[4*(c + d*x)]))/(48*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.057, size = 389, normalized size = 1.9

$$a^3Ax+\frac{Aa^3c}{d}+\frac{Ba^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3C\tan(dx+c)}{d}+3\frac{Aa^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*x+1/d*A*a^3*c+1/d*B*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*C*tan(d*x+c)/d+
3/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a^2*b*tan(d*x+c)+3/2/d*a^2*b*C*
sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a*b^2*t
an(d*x+c)+3/2/d*B*a*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*B*a*b^2*ln(sec(d*x+c)+t
an(d*x+c))+2/d*C*a*b^2*tan(d*x+c)+1/d*C*a*b^2*tan(d*x+c)*sec(d*x+c)^2+1/2/d
*A*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*
b^3*tan(d*x+c)+1/3/d*B*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*C*b^3*tan(d*x+c)*s
ec(d*x+c)^3+3/8/d*C*b^3*sec(d*x+c)*tan(d*x+c)+3/8/d*C*b^3*ln(sec(d*x+c)+tan
(d*x+c))
```

Maxima [A] time = 1.14788, size = 483, normalized size = 2.33

$$48\,(dx+c)Aa^3 + 48\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Cab^2 + 16\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)Bb^3 - 3\,Cb^3\left(\frac{2\left(3\,\sin(dx+c)^3 \cdots}{\sin(dx+c)^4-2\,\mathrm{s}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/48*(48*(d*x + c)*A*a^3 + 48*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a*b^2 + 1
6*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^3 - 3*C*b^3*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 36*C*a^2*b*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a*b^2*(2*s
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) - 12*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) + 48*B*a^3*log(sec(d*x + c) + tan(d*x + c)) +
144*A*a^2*b*log(sec(d*x + c) + tan(d*x + c)) + 48*C*a^3*tan(d*x + c) + 144*
B*a^2*b*tan(d*x + c) + 144*A*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.594853, size = 624, normalized size = 3.01

$$48\,Aa^3dx\cos\left(dx+c\right)^4 + 3\left(8\,Ba^3 + 12\,(2\,A+C)a^2b + 12\,Bab^2 + (4\,A+3\,C)b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right) - 3\left(8 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/48*(48*A*a^3*d*x*cos(d*x + c)^4 + 3*(8*B*a^3 + 12*(2*A + C)*a^2*b + 12*B*
a*b^2 + (4*A + 3*C)*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*B*a^3
+ 12*(2*A + C)*a^2*b + 12*B*a*b^2 + (4*A + 3*C)*b^3)*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(6*C*b^3 + 8*(3*C*a^3 + 9*B*a^2*b + 3*(3*A + 2*C)*a*b^2
 + 2*B*b^3)*cos(d*x + c)^3 + 3*(12*C*a^2*b + 12*B*a*b^2 + (4*A + 3*C)*b^3)*
cos(d*x + c)^2 + 8*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*(A + B*sec(c + d*x) + C*sec(c + d*x)**2),
x)
```

Giac [B] time = 1.34382, size = 1025, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/24*(24*(d*x + c)*A*a^3 + 3*(8*B*a^3 + 24*A*a^2*b + 12*C*a^2*b + 12*B*a*b^
2 + 4*A*b^3 + 3*C*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^3 + 24
*A*a^2*b + 12*C*a^2*b + 12*B*a*b^2 + 4*A*b^3 + 3*C*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(24*C*a^3*tan(1/2*d*x + 1/2*c)^7 + 72*B*a^2*b*tan(1/2*d
*x + 1/2*c)^7 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*a*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 72*C*a*b^2*tan(1/2*d*x + 1
/2*c)^7 - 12*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*B*b^3*tan(1/2*d*x + 1/2*c)^7
 - 15*C*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 216*
B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 216*A*
a*b^2*tan(1/2*d*x + 1/2*c)^5 + 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 120*C*a*
b^2*tan(1/2*d*x + 1/2*c)^5 + 12*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*b^3*tan
(1/2*d*x + 1/2*c)^5 - 9*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 216*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^2*b*tan(1/2*d*x +
 1/2*c)^3 + 216*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^2*tan(1/2*d*x + 1
/2*c)^3 + 120*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^3*tan(1/2*d*x + 1/2*c
)^3 + 40*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^3*tan(1/2*d*x + 1/2*c)^3 - 24
*C*a^3*tan(1/2*d*x + 1/2*c) - 72*B*a^2*b*tan(1/2*d*x + 1/2*c) - 36*C*a^2*b*
tan(1/2*d*x + 1/2*c) - 72*A*a*b^2*tan(1/2*d*x + 1/2*c) - 36*B*a*b^2*tan(1/2
*d*x + 1/2*c) - 72*C*a*b^2*tan(1/2*d*x + 1/2*c) - 12*A*b^3*tan(1/2*d*x + 1/
2*c) - 24*B*b^3*tan(1/2*d*x + 1/2*c) - 15*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.881 $\int \cos(c+dx)(a+b\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\sec^2(\right.$

Optimal. Leaf size=192

$$\frac{b\tan(c+dx)\left(a^2(-(6A-8C))+9abB+b^2(3A+2C)\right)}{3d}+\frac{\left(6a^2bB+2a^3C+3ab^2(2A+C)+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+$$

```
[Out] a^2*(3*A*b + a*B)*x + ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*(2*A + C))*Ar
cTanh[Sin[c + d*x]])/(2*d) + (A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d + (b
*(9*a*b*B - a^2*(6*A - 8*C) + b^2*(3*A + 2*C))*Tan[c + d*x])/(3*d) - (b^2*(
6*a*A - 3*b*B - 5*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(3*A - C)*(a +
 b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.371263, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4094, 4056, 4048, 3770, 3767, 8}

$$\frac{b\tan(c+dx)\left(a^2(-(6A-8C))+9abB+b^2(3A+2C)\right)}{3d}+\frac{\left(6a^2bB+2a^3C+3ab^2(2A+C)+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a^2*(3*A*b + a*B)*x + ((6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*(2*A + C))*Ar
cTanh[Sin[c + d*x]])/(2*d) + (A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d + (b
*(9*a*b*B - a^2*(6*A - 8*C) + b^2*(3*A + 2*C))*Tan[c + d*x])/(3*d) - (b^2*(
6*a*A - 3*b*B - 5*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(3*A - C)*(a +
 b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} + \int (\ldots$$
$$= \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b(3\ldots}{}$$
$$= \frac{A(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b^2(6\ldots}{}$$
$$= a^2(3Ab+aB)x + \frac{A(a+b\sec(c+dx))^3\ldots}{d}$$
$$= a^2(3Ab+aB)x + \frac{\left(6a^2bB+b^3B+2a^3C\ldots\right.}{}$$
$$= a^2(3Ab+aB)x + \frac{\left(6a^2bB+b^3B+2a^3C\ldots\right.}{}$$

Mathematica [B] time = 6.58619, size = 1335, normalized size = 6.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (2*a^2*(3*A*b + a*B)*(c + d*x)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((-6*a*A*b^2 - 6*a^2*b*B - b^3*B - 2
*a^3*C - 3*a*b^2*C)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(d*(b + a*
Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((6*a*
A*b^2 + 6*a^2*b*B + b^3*B + 2*a^3*C + 3*a*b^2*C)*Cos[c + d*x]^5*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2))/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])) + ((3*b^3*B + 9*a*b^2*C + b^3*C)*Cos[c + d*x]^5*(a + b
*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(6*d*(b + a*Cos[c
 + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])^2) + (b^3*C*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(3*d*(b + a*Cos[c
+ d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/
```

```
2] - Sin[(c + d*x)/2])^3) + (b^3*C*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin[(c + d*x)/2])/(3*d*(b + a*Cos[c +
 d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^3) + ((-3*b^3*B - 9*a*b^2*C - b^3*C)*Cos[c + d*x]^5*(
a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(6*d*(b + a*
Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^2) + (2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3*A*b^3*Sin[(c + d*x)/2] + 9*a*b^2
*B*Sin[(c + d*x)/2] + 9*a^2*b*C*Sin[(c + d*x)/2] + 2*b^3*C*Sin[(c + d*x)/2]
))/(3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*Cos[c + d*x]^5*(a + b*Sec
[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3*A*b^3*Sin[(c + d*x)
/2] + 9*a*b^2*B*Sin[(c + d*x)/2] + 9*a^2*b*C*Sin[(c + d*x)/2] + 2*b^3*C*Sin
[(c + d*x)/2]))/(3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (2*a^3*A*Cos[c
+ d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sin
[c + d*x])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x]))
```

Maple [A] time = 0.078, size = 294, normalized size = 1.5

$$\frac{Aa^3\sin(dx+c)}{d}+a^3Bx+\frac{Ba^3c}{d}+\frac{a^3C\ln(\sec(dx+c)+\tan(dx+c))}{d}+3a^2Abx+3\frac{Aa^2bc}{d}+3\frac{Ba^2b\ln(\sec(dx+c)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^3*A*sin(d*x+c)/d+a^3*B*x+1/d*B*a^3*c+1/d*a^3*C*ln(sec(d*x+c)+tan(d*x+c))+
3*a^2*A*b*x+3/d*A*a^2*b*c+3/d*B*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*C
*tan(d*x+c)+3/d*A*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3/d*B*a*b^2*tan(d*x+c)+3/
2/d*C*a*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1
/d*A*b^3*tan(d*x+c)+1/2/d*B*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*B*b^3*ln(sec(d*
x+c)+tan(d*x+c))+2/3/d*C*b^3*tan(d*x+c)+1/3/d*C*b^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.03056, size = 378, normalized size = 1.97

$$\frac{12(dx+c)Ba^3+36(dx+c)Aa^2b+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cb^3-9Cab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^3 + 36*(d*x + c)*A*a^2*b + 4*(tan(d*x + c)^3 + 3*tan
(d*x + c))*C*b^3 - 9*C*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*b^3*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*a^3*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*B*a^2*b*(log(sin(d*x + c)
+ 1) - log(sin(d*x + c) - 1)) + 18*A*a*b^2*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) + 12*A*a^3*sin(d*x + c) + 36*C*a^2*b*tan(d*x + c) + 36*B*a*
b^2*tan(d*x + c) + 12*A*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.578599, size = 543, normalized size = 2.83

$$\frac{12\left(Ba^3+3\,Aa^2b\right)dx\cos\left(dx+c\right)^3+3\left(2\,Ca^3+6\,Ba^2b+3\,(2\,A+C)ab^2+Bb^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/12*(12*(B*a^3 + 3*A*a^2*b)*d*x*cos(d*x + c)^3 + 3*(2*C*a^3 + 6*B*a^2*b +
3*(2*A + C)*a*b^2 + B*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*C*a^
3 + 6*B*a^2*b + 3*(2*A + C)*a*b^2 + B*b^3)*cos(d*x + c)^3*log(-sin(d*x + c)
 + 1) + 2*(6*A*a^3*cos(d*x + c)^3 + 2*C*b^3 + 2*(9*C*a^2*b + 9*B*a*b^2 + (3
*A + 2*C)*b^3)*cos(d*x + c)^2 + 3*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.37315, size = 591, normalized size = 3.08

$$\frac{\frac{12\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(Ba^3+3\,Aa^2b\right)(dx+c)+3\left(2\,Ca^3+6\,Ba^2b+6\,Aab^2+3\,Cab^2+Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/6*(12*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(B*a^3
+ 3*A*a^2*b)*(d*x + c) + 3*(2*C*a^3 + 6*B*a^2*b + 6*A*a*b^2 + 3*C*a*b^2 + B
*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*C*a^3 + 6*B*a^2*b + 6*A*a*b
^2 + 3*C*a*b^2 + B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*C*a^2*b*
tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*C*a*b^2*tan(
1/2*d*x + 1/2*c)^5 + 6*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^3*tan(1/2*d*x +
 1/2*c)^5 + 6*C*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*C*a^2*b*tan(1/2*d*x + 1/2*c
)^3 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^3*tan(1/2*d*x + 1/2*c)^3 -
 4*C*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a*
b^2*tan(1/2*d*x + 1/2*c) + 9*C*a*b^2*tan(1/2*d*x + 1/2*c) + 6*A*b^3*tan(1/2
*d*x + 1/2*c) + 3*B*b^3*tan(1/2*d*x + 1/2*c) + 6*C*b^3*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.882 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\sec^2\right.$

Optimal. Leaf size=204

$$-\frac{b\tan(c+dx)\left(4a^2B+9aAb-6abC-2b^2B\right)}{2d}+\frac{b\left(6a^2C+6abB+2Ab^2+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2(A+2C\right.$$

```
[Out] (a*(6*A*b^2 + 6*a*b*B + a^2*(A + 2*C))*x)/2 + (b*(2*A*b^2 + 6*a*b*B + 6*a^2
*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*A*b + 2*a*B)*(a + b*Sec[c +
d*x])^2*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c
+ d*x])/(2*d) - (b*(9*a*A*b + 4*a^2*B - 2*b^2*B - 6*a*b*C)*Tan[c + d*x])/(2
*d) - (b^2*(4*A*b + 2*a*B - b*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.455644, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4094, 4048, 3770, 3767, 8}

$$-\frac{b\tan(c+dx)\left(4a^2B+9aAb-6abC-2b^2B\right)}{2d}+\frac{b\left(6a^2C+6abB+2Ab^2+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2(A+2C\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a*(6*A*b^2 + 6*a*b*B + a^2*(A + 2*C))*x)/2 + (b*(2*A*b^2 + 6*a*b*B + 6*a^2
*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*A*b + 2*a*B)*(a + b*Sec[c +
d*x])^2*Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c
+ d*x])/(2*d) - (b*(9*a*A*b + 4*a^2*B - 2*b^2*B - 6*a*b*C)*Tan[c + d*x])/(2
*d) - (b^2*(4*A*b + 2*a*B - b*C)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^3\sin(c}{2d}$$
$$= \frac{(3Ab+2aB)(a+b\sec(c+dx))^2\sin(c}{2d}$$
$$= \frac{(3Ab+2aB)(a+b\sec(c+dx))^2\sin(c}{2d}$$
$$= \frac{1}{2}a\left(6Ab^2+6abB+a^2(A+2C)\right)x+$$
$$= \frac{1}{2}a\left(6Ab^2+6abB+a^2(A+2C)\right)x+\frac{b}{}$$
$$= \frac{1}{2}a\left(6Ab^2+6abB+a^2(A+2C)\right)x+\frac{b}{}$$

Mathematica [A] time = 3.03943, size = 320, normalized size = 1.57

$$\frac{2a(c+dx)\left(a^2(A+2C)+6abB+6Ab^2\right)-2b\left(6a^2C+6abB+2Ab^2+b^2C\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*a*(6*A*b^2 + 6*a*b*B + a^2*(A + 2*C))*(c + d*x) - 2*b*(2*A*b^2 + 6*a*b*B
 + 6*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*b*(2*A*b^2
 + 6*a*b*B + 6*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b
^3*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b^2*(b*B + 3*a*C)*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^3*C)/(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 + (4*b^2*(b*B + 3*a*C)*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) + 4*a^2*(3*A*b + a*B)*Sin[c + d*x] + a^3*A*Sin
[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.081, size = 267, normalized size = 1.3

$$\frac{Aa^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{a^3Ax}{2}+\frac{Aa^3c}{2d}+\frac{Ba^3\sin(dx+c)}{d}+a^3Cx+\frac{Ca^3c}{d}+3\frac{Aa^2b\sin(dx+c)}{d}+3Ba^2bx+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^3*sin(d*x+c)*cos(d*x+c)+1/2*a^3*A*x+1/2/d*A*a^3*c+a^3*B*sin(d*x+c
)/d+a^3*C*x+1/d*C*a^3*c+3/d*A*a^2*b*sin(d*x+c)+3*B*a^2*b*x+3/d*B*a^2*b*c+3/
d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+3*A*a*b^2*x+3/d*A*a*b^2*c+3/d*B*a*b^2*l
n(sec(d*x+c)+tan(d*x+c))+3/d*C*a*b^2*tan(d*x+c)+1/d*A*b^3*ln(sec(d*x+c)+tan
(d*x+c))+1/d*B*b^3*tan(d*x+c)+1/2/d*C*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*C*b^3
*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.0552, size = 328, normalized size = 1.61

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3+4\,(dx+c)Ca^3+12\,(dx+c)Ba^2b+12\,(dx+c)Aab^2-Cb^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 4*(d*x + c)*C*a^3 + 12*(d*x +
 c)*B*a^2*b + 12*(d*x + c)*A*a*b^2 - C*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*C*a^2*b*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 4*B*a^3*sin(d*x + c) + 12*A*a^2*b*sin(d*x + c) + 12*C*a*b^2*tan(d*
x + c) + 4*B*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.57597, size = 500, normalized size = 2.45

$$\frac{2\left((A+2\,C)a^3+6\,Ba^2b+6\,Aab^2\right)dx\cos(dx+c)^2+\left(6\,Ca^2b+6\,Bab^2+(2\,A+C)b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/4*(2*((A + 2*C)*a^3 + 6*B*a^2*b + 6*A*a*b^2)*d*x*cos(d*x + c)^2 + (6*C*a^
2*b + 6*B*a*b^2 + (2*A + C)*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (6*
C*a^2*b + 6*B*a*b^2 + (2*A + C)*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(A*a^3*cos(d*x + c)^3 + C*b^3 + 2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^2 +
2*(3*C*a*b^2 + B*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33571, size = 729, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/2*((A*a^3 + 2*C*a^3 + 6*B*a^2*b + 6*A*a*b^2)*(d*x + c) + (6*C*a^2*b + 6*B
*a*b^2 + 2*A*b^3 + C*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (6*C*a^2*b +
 6*B*a*b^2 + 2*A*b^3 + C*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^3
*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 6*A*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 6*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*b^3*tan(1/2*d*x +
 1/2*c)^7 - C*b^3*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
 2*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*
b^2*tan(1/2*d*x + 1/2*c)^5 + 2*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*C*b^3*tan(1
/2*d*x + 1/2*c)^5 + 3*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x +
1/2*c)^3 + 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 6*C*a*b^2*tan(1/2*d*x + 1/2*c
)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*b^3*tan(1/2*d*x + 1/2*c)^3 - A*a
^3*tan(1/2*d*x + 1/2*c) - 2*B*a^3*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b*tan(1/2*
d*x + 1/2*c) - 6*C*a*b^2*tan(1/2*d*x + 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/2*c
) - C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.883 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=196

$$\frac{a\sin(c+dx)\left(a^2(2A+3C)+6abB+3Ab^2\right)}{3d}+\frac{1}{2}x\left(3a^2b(A+2C)+a^3B+6ab^2B+2Ab^3\right)-\frac{b^2\tan(c+dx)(3aB+5Ab-}{6d}$$

```
[Out] ((2*A*b^3 + a^3*B + 6*a*b^2*B + 3*a^2*b*(A + 2*C))*x)/2 + (b^2*(b*B + 3*a*C
)*ArcTanh[Sin[c + d*x]])/d + (a*(3*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Sin[c
 + d*x])/(3*d) + ((A*b + a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d
*x])/(2*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) -
 (b^2*(5*A*b + 3*a*B - 6*b*C)*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.60091, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4076, 4047, 8, 4045, 3770}

$$\frac{a\sin(c+dx)\left(a^2(2A+3C)+6abB+3Ab^2\right)}{3d}+\frac{1}{2}x\left(3a^2b(A+2C)+a^3B+6ab^2B+2Ab^3\right)-\frac{b^2\tan(c+dx)(3aB+5Ab-}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((2*A*b^3 + a^3*B + 6*a*b^2*B + 3*a^2*b*(A + 2*C))*x)/2 + (b^2*(b*B + 3*a*C
)*ArcTanh[Sin[c + d*x]])/d + (a*(3*A*b^2 + 6*a*b*B + a^2*(2*A + 3*C))*Sin[c
 + d*x])/(3*d) + ((A*b + a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d
*x])/(2*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) -
 (b^2*(5*A*b + 3*a*B - 6*b*C)*Tan[c + d*x])/(6*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
```

```
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^3\sin(}{3d}$$
$$= \frac{(Ab+aB)\cos(c+dx)(a+b\sec(c+dx}{2d}$$
$$= \frac{(Ab+aB)\cos(c+dx)(a+b\sec(c+dx}{2d}$$
$$= \frac{(Ab+aB)\cos(c+dx)(a+b\sec(c+dx}{2d}$$
$$= \frac{1}{2}\left(2Ab^3+a^3B+6ab^2B+3a^2b(A+2C\right.$$
$$= \frac{1}{2}\left(2Ab^3+a^3B+6ab^2B+3a^2b(A+2C\right.$$

Mathematica [A] time = 1.35415, size = 263, normalized size = 1.34

$$6(c+dx)\left(3a^2b(A+2C)+a^3B+6ab^2B+2Ab^3\right)+3a\sin(c+dx)\left(a^2(3A+4C)+12abB+12Ab^2\right)+3a^2(aB+3Ab)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (6*(2*A*b^3 + a^3*B + 6*a*b^2*B + 3*a^2*b*(A + 2*C))*(c + d*x) - 12*b^2*(b*
B + 3*a*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^2*(b*B + 3*a*C)*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*b^3*C*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]) + (12*b^3*C*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 3*a*(12*A*b^2 + 12*a*b*B + a^2*(3*A + 4*C))*Si
n[c + d*x] + 3*a^2*(3*A*b + a*B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A] time = 0.076, size = 278, normalized size = 1.4

$$\frac{A(\cos(dx+c))^2\sin(dx+c)\,a^3}{3d}+\frac{2\,Aa^3\sin(dx+c)}{3d}+\frac{Ba^3\sin(dx+c)\cos(dx+c)}{2d}+\frac{a^3Bx}{2}+\frac{Ba^3c}{2d}+\frac{a^3C\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)^2*sin(d*x+c)*a^3+2/3*a^3*A*sin(d*x+c)/d+1/2/d*B*a^3*sin(
d*x+c)*cos(d*x+c)+1/2*a^3*B*x+1/2/d*B*a^3*c+a^3*C*sin(d*x+c)/d+3/2/d*A*a^2*
b*sin(d*x+c)*cos(d*x+c)+3/2*a^2*A*b*x+3/2/d*A*a^2*b*c+3/d*B*a^2*b*sin(d*x+c
)+3*a^2*b*C*x+3/d*C*a^2*b*c+3/d*A*a*b^2*sin(d*x+c)+3*B*a*b^2*x+3/d*B*a*b^2*
c+3/d*C*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+A*b^3*x+1/d*A*b^3*c+1/d*B*b^3*ln(se
c(d*x+c)+tan(d*x+c))+1/d*C*b^3*tan(d*x+c)
```

Maxima [A] time = 1.0333, size = 292, normalized size = 1.49

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^3-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3-9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b-36\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b - 36*(d*x + c
)*C*a^2*b - 36*(d*x + c)*B*a*b^2 - 12*(d*x + c)*A*b^3 - 18*C*a*b^2*(log(sin
(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 6*B*b^3*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) - 12*C*a^3*sin(d*x + c) - 36*B*a^2*b*sin(d*x + c) -
36*A*a*b^2*sin(d*x + c) - 12*C*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.572554, size = 486, normalized size = 2.48

$$\frac{3\left(Ba^3+3\,(A+2\,C)a^2b+6\,Bab^2+2\,Ab^3\right)dx\cos(dx+c)+3\left(3\,Cab^2+Bb^3\right)\cos(dx+c)\log(\sin(dx+c)+1)-3\left(3\,Ca\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/6*(3*(B*a^3 + 3*(A + 2*C)*a^2*b + 6*B*a*b^2 + 2*A*b^3)*d*x*cos(d*x + c) +
 3*(3*C*a*b^2 + B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(3*C*a*b^2 +
B*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*A*a^3*cos(d*x + c)^3 + 6*C*
b^3 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^2 + 2*((2*A + 3*C)*a^3 + 9*B*a^2*b
 + 9*A*a*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3365, size = 564, normalized size = 2.88

$$-\frac{\frac{12\,Cb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\left(Ba^3+3\,Aa^2b+6\,Ca^2b+6\,Bab^2+2\,Ab^3\right)(dx+c)-6\left(3\,Cab^2+Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -1/6*(12*C*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(B*a^3
 + 3*A*a^2*b + 6*C*a^2*b + 6*B*a*b^2 + 2*A*b^3)*(d*x + c) - 6*(3*C*a*b^2 +
B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(3*C*a*b^2 + B*b^3)*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^3*tan
(1/2*d*x + 1/2*c)^5 + 6*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*
x + 1/2*c)^5 + 18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 4*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^
3 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 +
 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3*B*a^3*tan(1/2*d*x + 1/2*c) + 6*C*a^3*tan(
1/2*d*x + 1/2*c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a^2*b*tan(1/2*d*x
+ 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

3.884 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=223

$$\frac{\sin(c+dx)\left(6a^2b(2A+3C)+4a^3B+16ab^2B+3Ab^3\right)}{6d}+\frac{a\sin(c+dx)\cos(c+dx)\left(3a^2(3A+4C)+20abB+6Ab^2\right)}{24d}+\frac{1}{8}$$

```
[Out] ((12*a^2*b*B + 8*b^3*B + 12*a*b^2*(A + 2*C) + a^3*(3*A + 4*C))*x)/8 + (b^3*
C*ArcTanh[Sin[c + d*x]])/d + ((3*A*b^3 + 4*a^3*B + 16*a*b^2*B + 6*a^2*b*(2*
A + 3*C))*Sin[c + d*x])/(6*d) + (a*(6*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 4*C))
*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((3*A*b + 4*a*B)*Cos[c + d*x]^2*(a + b
*Sec[c + d*x])^2*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.656655, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4074, 4047, 8, 4045, 3770}

$$\frac{\sin(c+dx)\left(6a^2b(2A+3C)+4a^3B+16ab^2B+3Ab^3\right)}{6d}+\frac{a\sin(c+dx)\cos(c+dx)\left(3a^2(3A+4C)+20abB+6Ab^2\right)}{24d}+\frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((12*a^2*b*B + 8*b^3*B + 12*a*b^2*(A + 2*C) + a^3*(3*A + 4*C))*x)/8 + (b^3*
C*ArcTanh[Sin[c + d*x]])/d + ((3*A*b^3 + 4*a^3*B + 16*a*b^2*B + 6*a^2*b*(2*
A + 3*C))*Sin[c + d*x])/(6*d) + (a*(6*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 4*C))
*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((3*A*b + 4*a*B)*Cos[c + d*x]^2*(a + b
*Sec[c + d*x])^2*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
```

```
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(}{4d}$$

$$= \frac{(3Ab+4aB)\cos^2(c+dx)(a+b\sec(c+}{12d}$$

$$= \frac{a\left(6Ab^2+20abB+3a^2(3A+4C)\right)\cos}{24d}$$

$$= \frac{a\left(6Ab^2+20abB+3a^2(3A+4C)\right)\cos}{24d}$$

$$= \frac{1}{8}\left(12a^2bB+8b^3B+12ab^2(A+2C)+\right.$$

$$= \frac{1}{8}\left(12a^2bB+8b^3B+12ab^2(A+2C)+\right.$$

Mathematica [A] time = 0.937164, size = 215, normalized size = 0.96

$$\frac{12(c+dx)\left(a^3(3A+4C)+12a^2bB+12ab^2(A+2C)+8b^3B\right)+24a\sin(2(c+dx))\left(a^2(A+C)+3abB+3Ab^2\right)+24\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (12*(12*a^2*b*B + 8*b^3*B + 12*a*b^2*(A + 2*C) + a^3*(3*A + 4*C))*(c + d*x)
 - 96*b^3*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*b^3*C*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 24*(4*A*b^3 + 3*a^3*B + 12*a*b^2*B + 3*a^2*
b*(3*A + 4*C))*Sin[c + d*x] + 24*a*(3*A*b^2 + 3*a*b*B + a^2*(A + C))*Sin[2*
(c + d*x)] + 8*a^2*(3*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^3*A*Sin[4*(c + d*x)
])/(96*d)
```

Maple [A] time = 0.08, size = 362, normalized size = 1.6

$$3\frac{Bab^2\sin(dx+c)}{d}+\frac{B\sin(dx+c)(\cos(dx+c))^2a^3}{3d}+\frac{3Ba^2bx}{2}+\frac{a^3Cc}{2d}+\frac{3a^3Ax}{8}+\frac{Bb^3c}{d}+\frac{Ab^3\sin(dx+c)}{d}+3Cab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/d*B*a*b^2*sin(d*x+c)+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^3+3/2*B*a^2*b*x+1/
2/d*C*a^3*c+3/8*a^3*A*x+1/d*B*b^3*c+1/d*A*b^3*sin(d*x+c)+3*C*a*b^2*x+1/d*C*
b^3*ln(sec(d*x+c)+tan(d*x+c))+B*b^3*x+3/8/d*A*a^3*c+3/2*A*a*b^2*x+2/3*a^3*B
*sin(d*x+c)/d+3/8/d*A*a^3*sin(d*x+c)*cos(d*x+c)+2/d*A*a^2*b*sin(d*x+c)+3/2/
d*B*a^2*b*c+1/2*a^3*C*x+3/2/d*A*a*b^2*c+1/4/d*A*a^3*sin(d*x+c)*cos(d*x+c)^3
+1/2/d*a^3*C*sin(d*x+c)*cos(d*x+c)+3/d*a^2*b*C*sin(d*x+c)+3/d*C*a*b^2*c+3/2
/d*B*a^2*b*sin(d*x+c)*cos(d*x+c)+3/2/d*A*a*b^2*sin(d*x+c)*cos(d*x+c)+1/d*A*
sin(d*x+c)*cos(d*x+c)^2*a^2*b
```

Maxima [A] time = 0.991735, size = 332, normalized size = 1.49

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Aa^3-32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3+24(2dx+2c+\sin(2\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c
))*C*a^3 - 96*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b + 72*(2*d*x + 2*c +
 sin(2*d*x + 2*c))*B*a^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 +
288*(d*x + c)*C*a*b^2 + 96*(d*x + c)*B*b^3 + 48*C*b^3*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 288*C*a^2*b*sin(d*x + c) + 288*B*a*b^2*sin(d*x
 + c) + 96*A*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.587165, size = 463, normalized size = 2.08

$$\frac{12Cb^3\log(\sin(dx+c)+1)-12Cb^3\log(-\sin(dx+c)+1)+3\left((3A+4C)a^3+12Ba^2b+12(A+2C)ab^2+8Bb^3\right)dx\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/24*(12*C*b^3*log(sin(d*x + c) + 1) - 12*C*b^3*log(-sin(d*x + c) + 1) + 3*
((3*A + 4*C)*a^3 + 12*B*a^2*b + 12*(A + 2*C)*a*b^2 + 8*B*b^3)*d*x + (6*A*a^
3*cos(d*x + c)^3 + 16*B*a^3 + 24*(2*A + 3*C)*a^2*b + 72*B*a*b^2 + 24*A*b^3
+ 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^2 + 3*((3*A + 4*C)*a^3 + 12*B*a^2*b +
12*A*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35594, size = 976, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/24*(24*C*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*C*b^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(3*A*a^3 + 4*C*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 24*
C*a*b^2 + 8*B*b^3)*(d*x + c) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^
3*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan
(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d
*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x + 1/2
*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^5 -
 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 2
16*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 216
*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^3
*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^3*tan(1/
2*d*x + 1/2*c)^3 - 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*
d*x + 1/2*c)^3 - 216*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2*d*
x + 1/2*c)^3 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*A*a^3*tan(1/2*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x + 1/2*c) -
12*C*a^3*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b*tan(1/2*d*x + 1/2*c) - 36*B*a^2*
b*tan(1/2*d*x + 1/2*c) - 72*C*a^2*b*tan(1/2*d*x + 1/2*c) - 36*A*a*b^2*tan(1
/2*d*x + 1/2*c) - 72*B*a*b^2*tan(1/2*d*x + 1/2*c) - 24*A*b^3*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.885 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=269

$$\frac{\sin(c+dx)\left(2a^3(4A+5C)+30a^2bB+15ab^2(2A+3C)+15b^3B\right)}{15d}+\frac{a\sin(c+dx)\cos^2(c+dx)\left(2a^2(4A+5C)+15abB+\right.}{30d}$$

```
[Out] ((3*a^3*B + 12*a*b^2*B + 4*b^3*(A + 2*C) + 3*a^2*b*(3*A + 4*C))*x)/8 + ((30
*a^2*b*B + 15*b^3*B + 15*a*b^2*(2*A + 3*C) + 2*a^3*(4*A + 5*C))*Sin[c + d*x
])/(15*d) + ((6*A*b^3 + 15*a^3*B + 50*a*b^2*B + 15*a^2*b*(3*A + 4*C))*Cos[c
 + d*x]*Sin[c + d*x])/(40*d) + (a*(3*A*b^2 + 15*a*b*B + 2*a^2*(4*A + 5*C))*
Cos[c + d*x]^2*Sin[c + d*x])/(30*d) + ((3*A*b + 5*a*B)*Cos[c + d*x]^3*(a +
b*Sec[c + d*x])^2*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d
*x])^3*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.751157, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4074, 4047, 2637, 4045, 8}

$$\frac{\sin(c+dx)\left(2a^3(4A+5C)+30a^2bB+15ab^2(2A+3C)+15b^3B\right)}{15d}+\frac{a\sin(c+dx)\cos^2(c+dx)\left(2a^2(4A+5C)+15abB+\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((3*a^3*B + 12*a*b^2*B + 4*b^3*(A + 2*C) + 3*a^2*b*(3*A + 4*C))*x)/8 + ((30
*a^2*b*B + 15*b^3*B + 15*a*b^2*(2*A + 3*C) + 2*a^3*(4*A + 5*C))*Sin[c + d*x
])/(15*d) + ((6*A*b^3 + 15*a^3*B + 50*a*b^2*B + 15*a^2*b*(3*A + 4*C))*Cos[c
 + d*x]*Sin[c + d*x])/(40*d) + (a*(3*A*b^2 + 15*a*b*B + 2*a^2*(4*A + 5*C))*
Cos[c + d*x]^2*Sin[c + d*x])/(30*d) + ((3*A*b + 5*a*B)*Cos[c + d*x]^3*(a +
b*Sec[c + d*x])^2*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d
*x])^3*Sin[c + d*x])/(5*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^3\sin(}{5d}$$
$$= \frac{(3Ab+5aB)\cos^3(c+dx)(a+b\sec(c+}{20d}$$
$$= \frac{a\left(3Ab^2+15abB+2a^2(4A+5C)\right)\cos}{30d}$$
$$= \frac{a\left(3Ab^2+15abB+2a^2(4A+5C)\right)\cos}{30d}$$
$$= \frac{\left(30a^2bB+15b^3B+15ab^2(2A+3C)+\right.}{15d}$$
$$= \frac{1}{8}\left(3a^3B+12ab^2B+4b^3(A+2C)+3a\right.$$

Mathematica [A] time = 1.0612, size = 288, normalized size = 1.07

$$\frac{60\sin(c+dx)\left(a^3(5A+6C)+18a^2bB+6ab^2(3A+4C)+8b^3B\right)+120\sin(2(c+dx))\left(3a^2b(A+C)+a^3B+3ab^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (540*a^2*A*b*c + 240*A*b^3*c + 180*a^3*B*c + 720*a*b^2*B*c + 720*a^2*b*c*C
+ 480*b^3*c*C + 540*a^2*A*b*d*x + 240*A*b^3*d*x + 180*a^3*B*d*x + 720*a*b^2
*B*d*x + 720*a^2*b*C*d*x + 480*b^3*C*d*x + 60*(18*a^2*b*B + 8*b^3*B + 6*a*b
^2*(3*A + 4*C) + a^3*(5*A + 6*C))*Sin[c + d*x] + 120*(A*b^3 + a^3*B + 3*a*b
^2*B + 3*a^2*b*(A + C))*Sin[2*(c + d*x)] + 50*a^3*A*Sin[3*(c + d*x)] + 120*
a*A*b^2*Sin[3*(c + d*x)] + 120*a^2*b*B*Sin[3*(c + d*x)] + 40*a^3*C*Sin[3*(c
 + d*x)] + 45*a^2*A*b*Sin[4*(c + d*x)] + 15*a^3*B*Sin[4*(c + d*x)] + 6*a^3*
A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.08, size = 301, normalized size = 1.1

$$\frac{1}{d}\left(\frac{Aa^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3Aa^2b\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*A*a^2*b*(1/
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a^3*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a*b^2*(2+cos(d*x+c)^2)*
sin(d*x+c)+B*a^2*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*C*(2+cos(d*x+c)^2)*s
in(d*x+c)+A*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*B*a*b^2*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*C*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*b^3*sin(d*x+c)+3*C*a*b^2*sin(d*x+c)+C*b^3*(d*x+c))
```

Maxima [A] time = 1.06811, size = 389, normalized size = 1.45

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))Ba^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 160*(sin
(d*x + c)^3 - 3*sin(d*x + c))*C*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c)
+ 8*sin(2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2
*b + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2*b - 480*(sin(d*x + c)^3 - 3
*sin(d*x + c))*A*a*b^2 + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^2 + 120
*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^3 + 480*(d*x + c)*C*b^3 + 1440*C*a*b^
2*sin(d*x + c) + 480*B*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.554683, size = 498, normalized size = 1.85

$$\frac{15\left(3Ba^3+3(3A+4C)a^2b+12Bab^2+4(A+2C)b^3\right)dx+\left(24Aa^3\cos(dx+c)^4+16(4A+5C)a^3+240Ba^2b+120\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/120*(15*(3*B*a^3 + 3*(3*A + 4*C)*a^2*b + 12*B*a*b^2 + 4*(A + 2*C)*b^3)*d*
x + (24*A*a^3*cos(d*x + c)^4 + 16*(4*A + 5*C)*a^3 + 240*B*a^2*b + 120*(2*A
+ 3*C)*a*b^2 + 120*B*b^3 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^3 + 8*((4*A
+ 5*C)*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x + c)^2 + 15*(3*B*a^3 + 3*(3*A
 + 4*C)*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39036, size = 1250, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*C*a^2*b + 12*B*a*b^2 + 4*A*b^3 + 8*C*b^
3)*(d*x + c) + 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*tan(1/2*d*x +
 1/2*c)^9 + 120*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/
2*c)^9 + 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 180*C*a^2*b*tan(1/2*d*x + 1/2
*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan(1/2*d*x + 1/2*
c)^9 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9
 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 30
*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 320*C*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*A*a^2
*b*tan(1/2*d*x + 1/2*c)^7 + 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 360*C*a^2*
b*tan(1/2*d*x + 1/2*c)^7 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 360*B*a*b^2
*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*A*b^3*t
an(1/2*d*x + 1/2*c)^7 + 480*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^3*tan(1/
2*d*x + 1/2*c)^5 + 400*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*
d*x + 1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 2160*C*a*b^2*tan(1/2
*d*x + 1/2*c)^5 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*tan(1/2*d*x
+ 1/2*c)^3 + 30*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 320*C*a^3*tan(1/2*d*x + 1/2*
c)^3 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*B*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 360*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^
3 + 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 1440*C*a*b^2*tan(1/2*d*x + 1/2*c)^
3 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
20*A*a^3*tan(1/2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 120*C*a^3*t
an(1/2*d*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/
2*d*x + 1/2*c) + 180*C*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x
 + 1/2*c) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c) + 360*C*a*b^2*tan(1/2*d*x + 1/
2*c) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.886 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\sec^2\right.$

Optimal. Leaf size=320

$$-\frac{\sin^3(c+dx)\left(3a^2b(4A+5C)+4a^3B+12ab^2B+Ab^3\right)}{15d}+\frac{\sin(c+dx)\left(9a^2b(4A+5C)+12a^3B+42ab^2B+b^3(11A+15\right.}{15d}$$

```
[Out] ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))*x)/16 + ((1
2*a^3*B + 42*a*b^2*B + 9*a^2*b*(4*A + 5*C) + b^3*(11*A + 15*C))*Sin[c + d*x
])/(15*d) + ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))
*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A*b^2 + 42*a*b*B + 5*a^2*(5*A +
6*C))*Cos[c + d*x]^3*Sin[c + d*x])/(120*d) + ((A*b + 2*a*B)*Cos[c + d*x]^4*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(10*d) + (A*Cos[c + d*x]^5*(a + b*Sec[
c + d*x])^3*Sin[c + d*x])/(6*d) - ((A*b^3 + 4*a^3*B + 12*a*b^2*B + 3*a^2*b*
(4*A + 5*C))*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.895196, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4094, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{\sin^3(c+dx)\left(3a^2b(4A+5C)+4a^3B+12ab^2B+Ab^3\right)}{15d}+\frac{\sin(c+dx)\left(9a^2b(4A+5C)+12a^3B+42ab^2B+b^3(11A+15\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```



```
[Out] ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))*x)/16 + ((1
2*a^3*B + 42*a*b^2*B + 9*a^2*b*(4*A + 5*C) + b^3*(11*A + 15*C))*Sin[c + d*x
])/(15*d) + ((18*a^2*b*B + 8*b^3*B + 6*a*b^2*(3*A + 4*C) + a^3*(5*A + 6*C))
*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A*b^2 + 42*a*b*B + 5*a^2*(5*A +
6*C))*Cos[c + d*x]^3*Sin[c + d*x])/(120*d) + ((A*b + 2*a*B)*Cos[c + d*x]^4*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(10*d) + (A*Cos[c + d*x]^5*(a + b*Sec[
c + d*x])^3*Sin[c + d*x])/(6*d) - ((A*b^3 + 4*a^3*B + 12*a*b^2*B + 3*a^2*b*
(4*A + 5*C))*Sin[c + d*x]^3)/(15*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+b\sec(c+dx))^3\sin(}{6d}$$
$$= \frac{(Ab+2aB)\cos^4(c+dx)(a+b\sec(c+}{10d}$$
$$= \frac{a\left(6Ab^2+42abB+5a^2(5A+6C)\right)\cos}{120d}$$
$$= \frac{a\left(6Ab^2+42abB+5a^2(5A+6C)\right)\cos}{120d}$$
$$= \frac{\left(18a^2bB+8b^3B+6ab^2(3A+4C)+a^3\right.}{16d}$$
$$= \frac{1}{16}\left(18a^2bB+8b^3B+6ab^2(3A+4C)+\right.$$
$$= \frac{1}{16}\left(18a^2bB+8b^3B+6ab^2(3A+4C)+\right.$$

Mathematica [A] time = 1.18818, size = 369, normalized size = 1.15

$$\frac{120\sin(c+dx)\left(3a^2b(5A+6C)+5a^3B+18ab^2B+2b^3(3A+4C)\right)+15\sin(2(c+dx))\left(a^3(15A+16C)+48a^2bB+4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (300*a^3*A*c + 1080*a*A*b^2*c + 1080*a^2*b*B*c + 480*b^3*B*c + 360*a^3*c*C
+ 1440*a*b^2*c*C + 300*a^3*A*d*x + 1080*a*A*b^2*d*x + 1080*a^2*b*B*d*x + 48
0*b^3*B*d*x + 360*a^3*C*d*x + 1440*a*b^2*C*d*x + 120*(5*a^3*B + 18*a*b^2*B
+ 2*b^3*(3*A + 4*C) + 3*a^2*b*(5*A + 6*C))*Sin[c + d*x] + 15*(48*a^2*b*B +
16*b^3*B + 48*a*b^2*(A + C) + a^3*(15*A + 16*C))*Sin[2*(c + d*x)] + 300*a^2
*A*b*Sin[3*(c + d*x)] + 80*A*b^3*Sin[3*(c + d*x)] + 100*a^3*B*Sin[3*(c + d*
x)] + 240*a*b^2*B*Sin[3*(c + d*x)] + 240*a^2*b*C*Sin[3*(c + d*x)] + 45*a^3*
A*Sin[4*(c + d*x)] + 90*a*A*b^2*Sin[4*(c + d*x)] + 90*a^2*b*B*Sin[4*(c + d*
x)] + 30*a^3*C*Sin[4*(c + d*x)] + 36*a^2*A*b*Sin[5*(c + d*x)] + 12*a^3*B*Si
n[5*(c + d*x)] + 5*a^3*A*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.093, size = 370, normalized size = 1.2

$$\frac{1}{d}\left(Aa^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{Ba^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*B*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a
^3*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*A*a^2
*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*B*a^2*b*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b*C*(2+cos(d*x+c)^2)*sin(
d*x+c)+3*A*a*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*
c)+B*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*C*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+1/3*A*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b^3*(1/2*cos(d*x+c)
*sin(d*x+c)+1/2*d*x+1/2*c)+C*b^3*sin(d*x+c))
```

Maxima [A] time = 1.05398, size = 486, normalized size = 1.52

$$-\frac{5\left(4\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)Aa^3-64\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^3 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*B*a^3 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*C*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^
2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b +
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^2*b - 90*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^2 + 960*(sin(d*x + c)^3 - 3*sin(d*x +
 c))*B*a*b^2 - 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a*b^2 + 320*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*A*b^3 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^3
 - 960*C*b^3*sin(d*x + c))/d
```

Fricas [A] time = 0.577018, size = 610, normalized size = 1.91

$$\frac{15\left((5A+6C)a^3+18Ba^2b+6(3A+4C)ab^2+8Bb^3\right)dx+\left(40Aa^3\cos(dx+c)^5+48\left(Ba^3+3Aa^2b\right)\cos(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*((5*A + 6*C)*a^3 + 18*B*a^2*b + 6*(3*A + 4*C)*a*b^2 + 8*B*b^3)*d*
x + (40*A*a^3*cos(d*x + c)^5 + 48*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^4 + 128*
B*a^3 + 96*(4*A + 5*C)*a^2*b + 480*B*a*b^2 + 80*(2*A + 3*C)*b^3 + 10*((5*A
+ 6*C)*a^3 + 18*B*a^2*b + 18*A*a*b^2)*cos(d*x + c)^3 + 16*(4*B*a^3 + 3*(4*A
 + 5*C)*a^2*b + 15*B*a*b^2 + 5*A*b^3)*cos(d*x + c)^2 + 15*((5*A + 6*C)*a^3
+ 18*B*a^2*b + 6*(3*A + 4*C)*a*b^2 + 8*B*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3343, size = 1764, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(5*A*a^3 + 6*C*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 24*C*a*b^2 + 8*B*b
^3)*(d*x + c) - 2*(165*A*a^3*tan(1/2*d*x + 1/2*c)^11 - 240*B*a^3*tan(1/2*d*
x + 1/2*c)^11 + 150*C*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*A*a^2*b*tan(1/2*d*x
 + 1/2*c)^11 + 450*B*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 720*C*a^2*b*tan(1/2*d*
x + 1/2*c)^11 + 450*A*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 720*B*a*b^2*tan(1/2*d
*x + 1/2*c)^11 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 240*A*b^3*tan(1/2*d*
x + 1/2*c)^11 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^11 - 240*C*b^3*tan(1/2*d*x +
 1/2*c)^11 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 560*B*a^3*tan(1/2*d*x + 1/2*
c)^9 + 210*C*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*A*a^2*b*tan(1/2*d*x + 1/2*c)
^9 + 630*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 2640*C*a^2*b*tan(1/2*d*x + 1/2*c)
^9 + 630*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 2640*B*a*b^2*tan(1/2*d*x + 1/2*c)
^9 + 1080*C*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*A*b^3*tan(1/2*d*x + 1/2*c)^9
 + 360*B*b^3*tan(1/2*d*x + 1/2*c)^9 - 1200*C*b^3*tan(1/2*d*x + 1/2*c)^9 + 4
50*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 1248*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*C*
a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*B*a^
2*b*tan(1/2*d*x + 1/2*c)^7 - 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 180*A*a*
b^2*tan(1/2*d*x + 1/2*c)^7 - 4320*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*C*a*
```

b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*B*b^3*tan(1/2*d*x + 1/2*c)^7 - 2400*C*b^3*tan(1/2*d*x + 1/2*c)^7 - 450*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 1248*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 180*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 4320*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 180*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4320*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 720*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 2400*C*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 560*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2640*C*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2640*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 1080*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 880*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 1200*C*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*A*a^3*tan(1/2*d*x + 1/2*c) - 240*B*a^3*tan(1/2*d*x + 1/2*c) - 150*C*a^3*tan(1/2*d*x + 1/2*c) - 720*A*a^2*b*tan(1/2*d*x + 1/2*c) - 450*B*a^2*b*tan(1/2*d*x + 1/2*c) - 720*C*a^2*b*tan(1/2*d*x + 1/2*c) - 450*A*a*b^2*tan(1/2*d*x + 1/2*c) - 720*B*a*b^2*tan(1/2*d*x + 1/2*c) - 360*C*a*b^2*tan(1/2*d*x + 1/2*c) - 240*A*b^3*tan(1/2*d*x + 1/2*c) - 120*B*b^3*tan(1/2*d*x + 1/2*c) - 240*C*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d

3.887 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\text{se}\right.$

Optimal. Leaf size=491

$$-\frac{\tan(c+dx)\left(-4a^4b^2(42A+23C)-32a^2b^4(49A+39C)-847a^3b^3B+28a^5bB-8a^6C-896ab^5B-32b^6(7A+6C)\right)}{420b^2d}$$

```
[Out] ((8*a^4*B + 36*a^2*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5
*C))*ArcTanh[Sin[c + d*x]])/(16*d) - ((28*a^5*b*B - 847*a^3*b^3*B - 896*a*b
^5*B - 8*a^6*C - 32*b^6*(7*A + 6*C) - 4*a^4*b^2*(42*A + 23*C) - 32*a^2*b^4*
(49*A + 39*C))*Tan[c + d*x])/(420*b^2*d) - ((56*a^4*b*B - 1246*a^2*b^3*B -
525*b^5*B - 16*a^5*C - 48*a^3*b^2*(7*A + 4*C) - 4*a*b^4*(406*A + 333*C))*Se
c[c + d*x]*Tan[c + d*x])/(1680*b*d) - ((28*a^3*b*B - 371*a*b^3*B - 8*a^4*C
- 32*b^4*(7*A + 6*C) - 12*a^2*b^2*(14*A + 9*C))*(a + b*Sec[c + d*x])^2*Tan[
c + d*x])/(840*b^2*d) - ((28*a^2*b*B - 175*b^3*B - 8*a^3*C - 4*a*b^2*(42*A
+ 31*C))*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(840*b^2*d) + ((42*A*b^2 - 7*
a*b*B + 2*a^2*C + 36*b^2*C)*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(210*b^2*d
) + ((7*b*B - 2*a*C)*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(42*b^2*d) + (C*S
ec[c + d*x]*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 1.23404, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4092, 4082, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{\tan(c+dx)\left(-4a^4b^2(42A+23C)-32a^2b^4(49A+39C)-847a^3b^3B+28a^5bB-8a^6C-896ab^5B-32b^6(7A+6C)\right)}{420b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((8*a^4*B + 36*a^2*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5
*C))*ArcTanh[Sin[c + d*x]])/(16*d) - ((28*a^5*b*B - 847*a^3*b^3*B - 896*a*b
^5*B - 8*a^6*C - 32*b^6*(7*A + 6*C) - 4*a^4*b^2*(42*A + 23*C) - 32*a^2*b^4*
(49*A + 39*C))*Tan[c + d*x])/(420*b^2*d) - ((56*a^4*b*B - 1246*a^2*b^3*B -
525*b^5*B - 16*a^5*C - 48*a^3*b^2*(7*A + 4*C) - 4*a*b^4*(406*A + 333*C))*Se
c[c + d*x]*Tan[c + d*x])/(1680*b*d) - ((28*a^3*b*B - 371*a*b^3*B - 8*a^4*C
- 32*b^4*(7*A + 6*C) - 12*a^2*b^2*(14*A + 9*C))*(a + b*Sec[c + d*x])^2*Tan[
c + d*x])/(840*b^2*d) - ((28*a^2*b*B - 175*b^3*B - 8*a^3*C - 4*a*b^2*(42*A
+ 31*C))*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(840*b^2*d) + ((42*A*b^2 - 7*
a*b*B + 2*a^2*C + 36*b^2*C)*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(210*b^2*d
) + ((7*b*B - 2*a*C)*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(42*b^2*d) + (C*S
ec[c + d*x]*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(7*b*d)
```


Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec(c+dx)(a+b\sec(c+dx))^5\tan(c}{7bd}$$
$$= \frac{(7bB-2aC)(a+b\sec(c+dx))^5\tan(c}{42b^2d}$$
$$= \frac{\left(42Ab^2-7abB+2a^2C+36b^2C\right)(a+}{210b^2d}$$
$$= -\frac{\left(28a^2bB-175b^3B-8a^3C-4ab^2(42\right.}{8}$$
$$= -\frac{\left(28a^3bB-371ab^3B-8a^4C-32b^4(7\right.}{}$$
$$= -\frac{\left(56a^4bB-1246a^2b^3B-525b^5B-16\right.}{}$$
$$= -\frac{\left(56a^4bB-1246a^2b^3B-525b^5B-16\right.}{}$$
$$= \frac{\left(8a^4B+36a^2b^2B+5b^4B+8a^3b(4A+\right.}{}$$
$$= \frac{\left(8a^4B+36a^2b^2B+5b^4B+8a^3b(4A+\right.}{}$$

Mathematica [A] time = 4.61019, size = 486, normalized size = 0.99

$$-\frac{\sec^6(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(-8b^2\left(3\sin(2(c+dx))\left(42a^2C+28abB+7Ab^2+6b^2C\right)+35b(4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^6*(105*(8*a^4*B + 36
*a^2*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*A + 5*C))*Cos[c + d
*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) - 70*b*(36*a^2*b*B + 5*b^3*B + 24*a^3*C + 4*a*b^2*(6*A + 5
*C))*Cos[c + d*x]^2*Sin[c + d*x] - 16*(140*a^3*b*B + 112*a*b^3*B + 35*a^4*C
 + 42*a^2*b^2*(5*A + 4*C) + 4*b^4*(7*A + 6*C))*Cos[c + d*x]^3*Sin[c + d*x]
- 105*(8*a^4*B + 36*a^2*b^2*B + 5*b^4*B + 8*a^3*b*(4*A + 3*C) + 4*a*b^3*(6*
A + 5*C))*Cos[c + d*x]^4*Sin[c + d*x] - 16*(280*a^3*b*B + 224*a*b^3*B + 35*
a^4*(3*A + 2*C) + 84*a^2*b^2*(5*A + 4*C) + 8*b^4*(7*A + 6*C))*Cos[c + d*x]^
5*Sin[c + d*x] - 8*b^2*(35*b*(b*B + 4*a*C)*Sin[c + d*x] + 3*(7*A*b^2 + 28*a
*b*B + 42*a^2*C + 6*b^2*C)*Sin[2*(c + d*x)] + 30*b^2*C*Tan[c + d*x])))/(840
*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)]))
```

Maple [A] time = 0.077, size = 905, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*a^4*C*tan(d*x+c)*sec(d*x+c)^2+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+3
/2/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+16/5/d*C*a^2*b^2*tan(d*x+c)+1/5/d*A*
b^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*b^4*tan(d*x+c)*sec(d*x+c)^2+1/7/d*C*b^
4*tan(d*x+c)*sec(d*x+c)^6+6/35/d*C*b^4*tan(d*x+c)*sec(d*x+c)^4+8/35/d*C*b^4
*tan(d*x+c)*sec(d*x+c)^2+32/15/d*a*b^3*B*tan(d*x+c)+8/3/d*B*a^3*b*tan(d*x+c
)+4/d*A*a^2*b^2*tan(d*x+c)+5/4/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/6/d*B*
b^4*tan(d*x+c)*sec(d*x+c)^5+5/24/d*B*b^4*tan(d*x+c)*sec(d*x+c)^3+5/16/d*B*b
^4*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^4*
C*tan(d*x+c)+1/d*A*a^4*tan(d*x+c)+1/2/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+16/
35/d*C*b^4*tan(d*x+c)+8/15/d*A*b^4*tan(d*x+c)+5/16/d*B*b^4*ln(sec(d*x+c)+ta
n(d*x+c))+1/2/d*B*a^4*sec(d*x+c)*tan(d*x+c)+4/5/d*a*b^3*B*tan(d*x+c)*sec(d*
x+c)^4+16/15/d*a*b^3*B*tan(d*x+c)*sec(d*x+c)^2+4/3/d*B*a^3*b*tan(d*x+c)*sec
(d*x+c)^2+2/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+2/3/d*C*a*b^3*tan(d*x+c)*se
c(d*x+c)^5+5/6/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)^3+5/4/d*C*a*b^3*sec(d*x+c)*t
an(d*x+c)+9/4/d*a^2*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a^3*b*sec(d*x+c)*
tan(d*x+c)+1/d*a^3*b*C*tan(d*x+c)*sec(d*x+c)^3+3/2/d*a^3*b*C*sec(d*x+c)*tan
(d*x+c)+3/2/d*a^2*b^2*B*tan(d*x+c)*sec(d*x+c)^3+9/4/d*a^2*b^2*B*sec(d*x+c)*
tan(d*x+c)+1/d*A*a*b^3*tan(d*x+c)*sec(d*x+c)^3+3/2/d*A*a*b^3*sec(d*x+c)*tan
(d*x+c)+6/5/d*C*a^2*b^2*tan(d*x+c)*sec(d*x+c)^4+8/5/d*C*a^2*b^2*tan(d*x+c)*
sec(d*x+c)^2
```

Maxima [A] time = 1.07927, size = 1007, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/3360*(1120*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^4 + 4480*(tan(d*x + c)^3
 + 3*tan(d*x + c))*B*a^3*b + 6720*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2*b
^2 + 1344*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a^2*b^
2 + 896*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a*b^3 +
224*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*b^4 + 96*(5*
tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*C
*b^4 - 140*C*a*b^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
 c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(si
n(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 35*B*b^4*(2*(15*sin(d*x + c)^
5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4
 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) -
 1)) - 840*C*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 1260*B*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 840*A*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 84
0*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 3360*A*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 3360*A*a^4*tan(d*x + c))/d
```

Fricas [A] time = 0.676581, size = 1085, normalized size = 2.21

$$105\left(8\,Ba^4 + 8\,(4\,A + 3\,C)a^3b + 36\,Ba^2b^2 + 4\,(6\,A + 5\,C)ab^3 + 5\,Bb^4\right)\cos\left(dx + c\right)^7\log\left(\sin\left(dx + c\right) + 1\right) - 105\left(8\,Ba^4 + 8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/3360*(105*(8*B*a^4 + 8*(4*A + 3*C)*a^3*b + 36*B*a^2*b^2 + 4*(6*A + 5*C)*a
*b^3 + 5*B*b^4)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(8*B*a^4 + 8*(4*
A + 3*C)*a^3*b + 36*B*a^2*b^2 + 4*(6*A + 5*C)*a*b^3 + 5*B*b^4)*cos(d*x + c)
^7*log(-sin(d*x + c) + 1) + 2*(16*(35*(3*A + 2*C)*a^4 + 280*B*a^3*b + 84*(5
*A + 4*C)*a^2*b^2 + 224*B*a*b^3 + 8*(7*A + 6*C)*b^4)*cos(d*x + c)^6 + 105*(
8*B*a^4 + 8*(4*A + 3*C)*a^3*b + 36*B*a^2*b^2 + 4*(6*A + 5*C)*a*b^3 + 5*B*b^
4)*cos(d*x + c)^5 + 240*C*b^4 + 16*(35*C*a^4 + 140*B*a^3*b + 42*(5*A + 4*C)
*a^2*b^2 + 112*B*a*b^3 + 4*(7*A + 6*C)*b^4)*cos(d*x + c)^4 + 70*(24*C*a^3*b
 + 36*B*a^2*b^2 + 4*(6*A + 5*C)*a*b^3 + 5*B*b^4)*cos(d*x + c)^3 + 48*(42*C*
a^2*b^2 + 28*B*a*b^3 + (7*A + 6*C)*b^4)*cos(d*x + c)^2 + 280*(4*C*a*b^3 + B
*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^4 \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*s
ec(c + d*x)**2, x)
```

Giac [B] time = 1.46661, size = 2549, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(8*B*a^4 + 32*A*a^3*b + 24*C*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3
+ 20*C*a*b^3 + 5*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*B*a^4 +
 32*A*a^3*b + 24*C*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 20*C*a*b^3 + 5*B*b^4
)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(1680*A*a^4*tan(1/2*d*x + 1/2*c)^1
3 - 840*B*a^4*tan(1/2*d*x + 1/2*c)^13 + 1680*C*a^4*tan(1/2*d*x + 1/2*c)^13
- 3360*A*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 6720*B*a^3*b*tan(1/2*d*x + 1/2*c)^
13 - 4200*C*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 10080*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^13 - 6300*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 10080*C*a^2*b^2*tan(1/2
*d*x + 1/2*c)^13 - 4200*A*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 6720*B*a*b^3*tan(
1/2*d*x + 1/2*c)^13 - 4620*C*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 1680*A*b^4*tan
(1/2*d*x + 1/2*c)^13 - 1155*B*b^4*tan(1/2*d*x + 1/2*c)^13 + 1680*C*b^4*tan(
1/2*d*x + 1/2*c)^13 - 10080*A*a^4*tan(1/2*d*x + 1/2*c)^11 + 3360*B*a^4*tan(
1/2*d*x + 1/2*c)^11 - 7840*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 13440*A*a^3*b*ta
n(1/2*d*x + 1/2*c)^11 - 31360*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 10080*C*a^3
*b*tan(1/2*d*x + 1/2*c)^11 - 47040*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 1512
```

0*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 33600*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^
11 + 10080*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 22400*B*a*b^3*tan(1/2*d*x + 1/
2*c)^11 + 3920*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 - 5600*A*b^4*tan(1/2*d*x + 1
/2*c)^11 + 980*B*b^4*tan(1/2*d*x + 1/2*c)^11 - 3360*C*b^4*tan(1/2*d*x + 1/2
*c)^11 + 25200*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 4200*B*a^4*tan(1/2*d*x + 1/2*
c)^9 + 16240*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 16800*A*a^3*b*tan(1/2*d*x + 1/2
*c)^9 + 64960*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 7560*C*a^3*b*tan(1/2*d*x + 1
/2*c)^9 + 97440*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 11340*B*a^2*b^2*tan(1/2*
d*x + 1/2*c)^9 + 75936*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 7560*A*a*b^3*tan(
1/2*d*x + 1/2*c)^9 + 50624*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 11900*C*a*b^3*t
an(1/2*d*x + 1/2*c)^9 + 12656*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 2975*B*b^4*tan
(1/2*d*x + 1/2*c)^9 + 14448*C*b^4*tan(1/2*d*x + 1/2*c)^9 - 33600*A*a^4*tan(
1/2*d*x + 1/2*c)^7 - 20160*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 80640*B*a^3*b*tan
(1/2*d*x + 1/2*c)^7 - 120960*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 104832*C*a^
2*b^2*tan(1/2*d*x + 1/2*c)^7 - 69888*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 17472
*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 10176*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 25200*
A*a^4*tan(1/2*d*x + 1/2*c)^5 + 4200*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 16240*C*
a^4*tan(1/2*d*x + 1/2*c)^5 + 16800*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 64960*B
*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 7560*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 97440
*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 11340*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5
+ 75936*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 7560*A*a*b^3*tan(1/2*d*x + 1/2*c
)^5 + 50624*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 11900*C*a*b^3*tan(1/2*d*x + 1/
2*c)^5 + 12656*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 2975*B*b^4*tan(1/2*d*x + 1/2*
c)^5 + 14448*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 10080*A*a^4*tan(1/2*d*x + 1/2*c
)^3 - 3360*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 7840*C*a^4*tan(1/2*d*x + 1/2*c)^3
 - 13440*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 31360*B*a^3*b*tan(1/2*d*x + 1/2*c
)^3 - 10080*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 47040*A*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 - 15120*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 33600*C*a^2*b^2*tan(1/2
*d*x + 1/2*c)^3 - 10080*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 22400*B*a*b^3*tan(
1/2*d*x + 1/2*c)^3 - 3920*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 5600*A*b^4*tan(1
/2*d*x + 1/2*c)^3 - 980*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 3360*C*b^4*tan(1/2*d
*x + 1/2*c)^3 + 1680*A*a^4*tan(1/2*d*x + 1/2*c) + 840*B*a^4*tan(1/2*d*x + 1
/2*c) + 1680*C*a^4*tan(1/2*d*x + 1/2*c) + 3360*A*a^3*b*tan(1/2*d*x + 1/2*c)
 + 6720*B*a^3*b*tan(1/2*d*x + 1/2*c) + 4200*C*a^3*b*tan(1/2*d*x + 1/2*c) +
10080*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6300*B*a^2*b^2*tan(1/2*d*x + 1/2*c)
+ 10080*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 4200*A*a*b^3*tan(1/2*d*x + 1/2*c)
+ 6720*B*a*b^3*tan(1/2*d*x + 1/2*c) + 4620*C*a*b^3*tan(1/2*d*x + 1/2*c) + 1
680*A*b^4*tan(1/2*d*x + 1/2*c) + 1155*B*b^4*tan(1/2*d*x + 1/2*c) + 1680*C*b
^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d

3.888 $\int \sec(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec\right.$

Optimal. Leaf size=384

$$\frac{\tan(c+dx)\left(a^3b^2(190A+121C)+224a^2b^3B+24a^4bB-4a^5C+32ab^4(5A+4C)+32b^5B\right)}{60bd}+\frac{\left(12a^2b^2(4A+3C)+8\right.}{}$$

```
[Out] ((32*a^3*b*B + 24*a*b^3*B + 8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*
(6*A + 5*C))*ArcTanh[Sin[c + d*x]])/(16*d) + ((24*a^4*b*B + 224*a^2*b^3*B +
 32*b^5*B - 4*a^5*C + 32*a*b^4*(5*A + 4*C) + a^3*b^2*(190*A + 121*C))*Tan[c
 + d*x])/(60*b*d) + ((48*a^3*b*B + 232*a*b^3*B - 8*a^4*C + 15*b^4*(6*A + 5*
C) + 2*a^2*b^2*(130*A + 89*C))*Sec[c + d*x]*Tan[c + d*x])/(240*d) + ((24*a^
2*b*B + 32*b^3*B - 4*a^3*C + a*b^2*(70*A + 53*C))*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(120*b*d) + ((5*b^2*(6*A + 5*C) + 4*a*(6*b*B - a*C))*(a + b*Sec
[c + d*x])^3*Tan[c + d*x])/(120*b*d) + ((6*b*B - a*C)*(a + b*Sec[c + d*x])^
4*Tan[c + d*x])/(30*b*d) + (C*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.875709, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {4082, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\tan(c+dx)\left(a^3b^2(190A+121C)+224a^2b^3B+24a^4bB-4a^5C+32ab^4(5A+4C)+32b^5B\right)}{60bd}+\frac{\left(12a^2b^2(4A+3C)+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```


```
[Out] ((32*a^3*b*B + 24*a*b^3*B + 8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*
(6*A + 5*C))*ArcTanh[Sin[c + d*x]])/(16*d) + ((24*a^4*b*B + 224*a^2*b^3*B +
 32*b^5*B - 4*a^5*C + 32*a*b^4*(5*A + 4*C) + a^3*b^2*(190*A + 121*C))*Tan[c
 + d*x])/(60*b*d) + ((48*a^3*b*B + 232*a*b^3*B - 8*a^4*C + 15*b^4*(6*A + 5*
C) + 2*a^2*b^2*(130*A + 89*C))*Sec[c + d*x]*Tan[c + d*x])/(240*d) + ((24*a^
2*b*B + 32*b^3*B - 4*a^3*C + a*b^2*(70*A + 53*C))*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(120*b*d) + ((5*b^2*(6*A + 5*C) + 4*a*(6*b*B - a*C))*(a + b*Sec
[c + d*x])^3*Tan[c + d*x])/(120*b*d) + ((6*b*B - a*C)*(a + b*Sec[c + d*x])^
4*Tan[c + d*x])/(30*b*d) + (C*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{C(a+b\sec(c+dx))^5\tan(c+dx)}{6bd}+\frac{\int \sec(}{}$$

$$=\frac{(6bB-aC)(a+b\sec(c+dx))^4\tan(c+dx)}{30bd}$$

$$=\frac{\left(5b^2(6A+5C)+4a(6bB-aC)\right)(a+b\sec(}{120bd}$$

$$=\frac{\left(24a^2bB+32b^3B-4a^3C+ab^2(70A+53C\right.}{120bd}$$

$$=\frac{\left(48a^3bB+232ab^3B-8a^4C+15b^4(6A+5\right.}{}$$

$$=\frac{\left(48a^3bB+232ab^3B-8a^4C+15b^4(6A+5\right.}{}$$

$$=\frac{\left(32a^3bB+24ab^3B+8a^4(2A+C)+12a^2b^2\right.}{}$$

$$=\frac{\left(32a^3bB+24ab^3B+8a^4(2A+C)+12a^2b^2\right.}{}$$

Mathematica [A] time = 3.68672, size = 424, normalized size = 1.1

$$-\frac{\sec^6(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(-16\sin(c+dx)\cos^5(c+dx)\left(20a^3b(3A+2C)+60a^2b^2B+15a^4B+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] -((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^6*(15*(32*a^3*b*B +
24*a*b^3*B + 8*a^4*(2*A + C) + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*Co
s[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]) - 10*b^2*(6*A*b^2 + 24*a*b*B + 36*a^2*C + 5*b^2*C)*
Cos[c + d*x]^2*Sin[c + d*x] - 32*b*(15*a^2*b*B + 2*b^3*B + 10*a^3*C + 2*a*b
^2*(5*A + 4*C))*Cos[c + d*x]^3*Sin[c + d*x] - 15*(32*a^3*b*B + 24*a*b^3*B +
 8*a^4*C + 12*a^2*b^2*(4*A + 3*C) + b^4*(6*A + 5*C))*Cos[c + d*x]^4*Sin[c +
 d*x] - 16*(15*a^4*B + 60*a^2*b^2*B + 8*b^4*B + 20*a^3*b*(3*A + 2*C) + 8*a*
b^3*(5*A + 4*C))*Cos[c + d*x]^5*Sin[c + d*x] - 8*b^3*(5*b*C + 6*(b*B + 4*a*
C)*Cos[c + d*x])*Sin[c + d*x]))/(120*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*(c + d*x)]))
```

Maple [B] time = 0.069, size = 745, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/8/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+5/16/d*C*b^4*ln(sec(d*x+c)+tan(d*x+c)
)+8/15/d*B*b^4*tan(d*x+c)+1/d*B*a^4*tan(d*x+c)+1/2/d*a^4*C*ln(sec(d*x+c)+ta
n(d*x+c))+1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^3*b*tan(d*x+c)+8/3/d*
a^3*b*C*tan(d*x+c)+3/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+9/4/d*C*a^2*b^2*
ln(sec(d*x+c)+tan(d*x+c))+8/3/d*A*a*b^3*tan(d*x+c)+32/15/d*C*a*b^3*tan(d*x+
c)+1/4/d*A*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*b^4*sec(d*x+c)*tan(d*x+c)+1/
6/d*C*b^4*tan(d*x+c)*sec(d*x+c)^5+5/24/d*C*b^4*tan(d*x+c)*sec(d*x+c)^3+5/16
/d*C*b^4*sec(d*x+c)*tan(d*x+c)+1/5/d*B*b^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*B
*b^4*tan(d*x+c)*sec(d*x+c)^2+3/d*A*a^2*b^2*sec(d*x+c)*tan(d*x+c)+3/2/d*C*a^
2*b^2*tan(d*x+c)*sec(d*x+c)^3+9/4/d*C*a^2*b^2*sec(d*x+c)*tan(d*x+c)+4/3/d*A
*a*b^3*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a*b^3*B*sec(d*x+c)*tan(d*x+c)+2/d*a^2*
b^2*B*tan(d*x+c)*sec(d*x+c)^2+2/d*B*a^3*b*sec(d*x+c)*tan(d*x+c)+1/d*a*b^3*B
*tan(d*x+c)*sec(d*x+c)^3+4/5/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)^4+16/15/d*C*a*
b^3*tan(d*x+c)*sec(d*x+c)^2+4/3/d*a^3*b*C*tan(d*x+c)*sec(d*x+c)^2+2/d*B*a^3
*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*b^2*B*tan(d*x+c)+3/2/d*a*b^3*B*ln(sec(
d*x+c)+tan(d*x+c))+1/2/d*a^4*C*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.05854, size = 882, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^3*b + 960*(tan(d*x + c)^3
+ 3*tan(d*x + c))*B*a^2*b^2 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a*b^3
 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*C*a*b^3 + 3
2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*b^4 - 5*C*b^4*
(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^
```

```
6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 1
5*log(sin(d*x + c) - 1)) - 180*C*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x +
 c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*
log(sin(d*x + c) - 1)) - 120*B*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))
/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(
sin(d*x + c) - 1)) - 30*A*b^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 120*C*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
+ c) + 1) + log(sin(d*x + c) - 1)) - 480*B*a^3*b*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 720*A*a^2*b^2
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) + 480*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 480*B*a^4*tan(d*
x + c) + 1920*A*a^3*b*tan(d*x + c))/d
```

Fricas [A] time = 0.659256, size = 941, normalized size = 2.45

$$\frac{15\left(8(2A+C)a^4+32Ba^3b+12(4A+3C)a^2b^2+24Bab^3+(6A+5C)b^4\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left(8(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/480*(15*(8*(2*A + C)*a^4 + 32*B*a^3*b + 12*(4*A + 3*C)*a^2*b^2 + 24*B*a*b
^3 + (6*A + 5*C)*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(8*(2*A + C
)*a^4 + 32*B*a^3*b + 12*(4*A + 3*C)*a^2*b^2 + 24*B*a*b^3 + (6*A + 5*C)*b^4)
*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(15*B*a^4 + 20*(3*A + 2*C)*a
^3*b + 60*B*a^2*b^2 + 8*(5*A + 4*C)*a*b^3 + 8*B*b^4)*cos(d*x + c)^5 + 40*C*
b^4 + 15*(8*C*a^4 + 32*B*a^3*b + 12*(4*A + 3*C)*a^2*b^2 + 24*B*a*b^3 + (6*A
 + 5*C)*b^4)*cos(d*x + c)^4 + 32*(10*C*a^3*b + 15*B*a^2*b^2 + 2*(5*A + 4*C)
*a*b^3 + 2*B*b^4)*cos(d*x + c)^3 + 10*(36*C*a^2*b^2 + 24*B*a*b^3 + (6*A + 5
*C)*b^4)*cos(d*x + c)^2 + 48*(4*C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*s
ec(c + d*x), x)
```

Giac [B] time = 1.40354, size = 2238, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/240*(15*(16*A*a^4 + 8*C*a^4 + 32*B*a^3*b + 48*A*a^2*b^2 + 36*C*a^2*b^2 +
24*B*a*b^3 + 6*A*b^4 + 5*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(16
*A*a^4 + 8*C*a^4 + 32*B*a^3*b + 48*A*a^2*b^2 + 36*C*a^2*b^2 + 24*B*a*b^3 +
6*A*b^4 + 5*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(240*B*a^4*tan(1/
2*d*x + 1/2*c)^11 - 120*C*a^4*tan(1/2*d*x + 1/2*c)^11 + 960*A*a^3*b*tan(1/2
*d*x + 1/2*c)^11 - 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 960*C*a^3*b*tan(1/
2*d*x + 1/2*c)^11 - 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 1440*B*a^2*b^2*
tan(1/2*d*x + 1/2*c)^11 - 900*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 960*A*a*b
^3*tan(1/2*d*x + 1/2*c)^11 - 600*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 960*C*a*
b^3*tan(1/2*d*x + 1/2*c)^11 - 150*A*b^4*tan(1/2*d*x + 1/2*c)^11 + 240*B*b^4
*tan(1/2*d*x + 1/2*c)^11 - 165*C*b^4*tan(1/2*d*x + 1/2*c)^11 - 1200*B*a^4*t
an(1/2*d*x + 1/2*c)^9 + 360*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 4800*A*a^3*b*tan
(1/2*d*x + 1/2*c)^9 + 1440*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 3520*C*a^3*b*ta
n(1/2*d*x + 1/2*c)^9 + 2160*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 5280*B*a^2*b
^2*tan(1/2*d*x + 1/2*c)^9 + 1260*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*A*
a*b^3*tan(1/2*d*x + 1/2*c)^9 + 840*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 2240*C*
a*b^3*tan(1/2*d*x + 1/2*c)^9 + 210*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 560*B*b^4
*tan(1/2*d*x + 1/2*c)^9 - 25*C*b^4*tan(1/2*d*x + 1/2*c)^9 + 2400*B*a^4*tan(
1/2*d*x + 1/2*c)^7 - 240*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 9600*A*a^3*b*tan(1/
2*d*x + 1/2*c)^7 - 960*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 5760*C*a^3*b*tan(1/
2*d*x + 1/2*c)^7 - 1440*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 8640*B*a^2*b^2*t
an(1/2*d*x + 1/2*c)^7 - 360*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 5760*A*a*b^3
*tan(1/2*d*x + 1/2*c)^7 - 240*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 4992*C*a*b^3
*tan(1/2*d*x + 1/2*c)^7 - 60*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 1248*B*b^4*tan(
1/2*d*x + 1/2*c)^7 - 450*C*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*a^4*tan(1/2*
d*x + 1/2*c)^5 - 240*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 9600*A*a^3*b*tan(1/2*d*
x + 1/2*c)^5 - 960*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 5760*C*a^3*b*tan(1/2*d*
x + 1/2*c)^5 - 1440*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 8640*B*a^2*b^2*tan(1
/2*d*x + 1/2*c)^5 - 360*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*A*a*b^3*tan
(1/2*d*x + 1/2*c)^5 - 240*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 4992*C*a*b^3*tan
(1/2*d*x + 1/2*c)^5 - 60*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 1248*B*b^4*tan(1/2*
d*x + 1/2*c)^5 - 450*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 360*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 4800*A*a^3*b*tan(1/2*d*x +
1/2*c)^3 + 1440*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 3520*C*a^3*b*tan(1/2*d*x +
 1/2*c)^3 + 2160*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 5280*B*a^2*b^2*tan(1/2*
d*x + 1/2*c)^3 + 1260*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3520*A*a*b^3*tan(1
/2*d*x + 1/2*c)^3 + 840*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2240*C*a*b^3*tan(1
/2*d*x + 1/2*c)^3 + 210*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 560*B*b^4*tan(1/2*d*
x + 1/2*c)^3 - 25*C*b^4*tan(1/2*d*x + 1/2*c)^3 - 240*B*a^4*tan(1/2*d*x + 1/
2*c) - 120*C*a^4*tan(1/2*d*x + 1/2*c) - 960*A*a^3*b*tan(1/2*d*x + 1/2*c) -
480*B*a^3*b*tan(1/2*d*x + 1/2*c) - 960*C*a^3*b*tan(1/2*d*x + 1/2*c) - 720*A
*a^2*b^2*tan(1/2*d*x + 1/2*c) - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 900*C
*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*A*a*b^3*tan(1/2*d*x + 1/2*c) - 600*B*a*
b^3*tan(1/2*d*x + 1/2*c) - 960*C*a*b^3*tan(1/2*d*x + 1/2*c) - 150*A*b^4*tan
(1/2*d*x + 1/2*c) - 240*B*b^4*tan(1/2*d*x + 1/2*c) - 165*C*b^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.889 $\int (a+b\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\sec^2(c + dx)\right) d$

Optimal. Leaf size=290

$$\frac{\tan(c + dx)\left(2a^2b^2(85A + 56C) + 95a^3bB + 12a^4C + 80ab^3B + 4b^4(5A + 4C)\right)}{30d} + \frac{\left(16a^3b(2A + C) + 24a^2b^2B + 8a^4B + 4\right.}{}$$

```
[Out] a^4*A*x + ((8*a^4*B + 24*a^2*b^2*B + 3*b^4*B + 16*a^3*b*(2*A + C) + 4*a*b^3
*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) + ((95*a^3*b*B + 80*a*b^3*B + 12
*a^4*C + 4*b^4*(5*A + 4*C) + 2*a^2*b^2*(85*A + 56*C))*Tan[c + d*x])/(30*d)
+ (b*(130*a^2*b*B + 45*b^3*B + 24*a^3*C + 4*a*b^2*(40*A + 29*C))*Sec[c + d*
x]*Tan[c + d*x])/(120*d) + ((20*A*b^2 + 35*a*b*B + 12*a^2*C + 16*b^2*C)*(a
+ b*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((5*b*B + 4*a*C)*(a + b*Sec[c +
d*x])^3*Tan[c + d*x])/(20*d) + (C*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*d
)
```

Rubi [A] time = 0.542844, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4056, 4048, 3770, 3767, 8}

$$\frac{\tan(c + dx)\left(2a^2b^2(85A + 56C) + 95a^3bB + 12a^4C + 80ab^3B + 4b^4(5A + 4C)\right)}{30d} + \frac{\left(16a^3b(2A + C) + 24a^2b^2B + 8a^4B + 4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] a^4*A*x + ((8*a^4*B + 24*a^2*b^2*B + 3*b^4*B + 16*a^3*b*(2*A + C) + 4*a*b^3
*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) + ((95*a^3*b*B + 80*a*b^3*B + 12
*a^4*C + 4*b^4*(5*A + 4*C) + 2*a^2*b^2*(85*A + 56*C))*Tan[c + d*x])/(30*d)
+ (b*(130*a^2*b*B + 45*b^3*B + 24*a^3*C + 4*a*b^2*(40*A + 29*C))*Sec[c + d*
x]*Tan[c + d*x])/(120*d) + ((20*A*b^2 + 35*a*b*B + 12*a^2*C + 16*b^2*C)*(a
+ b*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((5*b*B + 4*a*C)*(a + b*Sec[c +
d*x])^3*Tan[c + d*x])/(20*d) + (C*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*d
)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C(a+b\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}\int (a+b\sec($$

$$= \frac{(5bB+4aC)(a+b\sec(c+dx))^3\tan(c+dx)}{20d}+\frac{C(a}{}$$

$$= \frac{\left(20Ab^2+35abB+12a^2C+16b^2C\right)(a+b\sec(c+d}{60d}$$

$$= \frac{b\left(130a^2bB+45b^3B+24a^3C+4ab^2(40A+29C)\right)s}{120d}$$

$$= a^4Ax+\frac{b\left(130a^2bB+45b^3B+24a^3C+4ab^2(40A+\right.}{120d}$$

$$= a^4Ax+\frac{\left(8a^4B+24a^2b^2B+3b^4B+16a^3b(2A+C)\right.}{8d}$$

$$= a^4Ax+\frac{\left(8a^4B+24a^2b^2B+3b^4B+16a^3b(2A+C)\right.}{8d}$$

Mathematica [B] time = 4.10403, size = 690, normalized size = 2.38

$$\frac{\sec^5(c+dx)\left(A\cos^2(c+dx)+B\cos(c+dx)+C\right)\left(-120\cos^5(c+dx)\left(16a^3b(2A+C)+24a^2b^2B+8a^4B+4ab^3(4A\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] ((C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*Sec[c + d*x]^5*(600*a^4*A*(c + d*x
)*Cos[c + d*x] + 300*a^4*A*(c + d*x)*Cos[3*(c + d*x)] + 60*a^4*A*c*Cos[5*(c
 + d*x)] + 60*a^4*A*d*x*Cos[5*(c + d*x)] - 120*(8*a^4*B + 24*a^2*b^2*B + 3*
b^4*B + 16*a^3*b*(2*A + C) + 4*a*b^3*(4*A + 3*C))*Cos[c + d*x]^5*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 + 720*a^2*A*b^2*Sin[c + d*x] + 160*A*b^4*Sin[c + d*x] + 480*a^3*b*B*Sin[c
+ d*x] + 640*a*b^3*B*Sin[c + d*x] + 120*a^4*C*Sin[c + d*x] + 960*a^2*b^2*C*
Sin[c + d*x] + 320*b^4*C*Sin[c + d*x] + 480*a*A*b^3*Sin[2*(c + d*x)] + 720*
a^2*b^2*B*Sin[2*(c + d*x)] + 210*b^4*B*Sin[2*(c + d*x)] + 480*a^3*b*C*Sin[2
*(c + d*x)] + 840*a*b^3*C*Sin[2*(c + d*x)] + 1080*a^2*A*b^2*Sin[3*(c + d*x)
] + 200*A*b^4*Sin[3*(c + d*x)] + 720*a^3*b*B*Sin[3*(c + d*x)] + 800*a*b^3*B
*Sin[3*(c + d*x)] + 180*a^4*C*Sin[3*(c + d*x)] + 1200*a^2*b^2*C*Sin[3*(c +
d*x)] + 160*b^4*C*Sin[3*(c + d*x)] + 240*a*A*b^3*Sin[4*(c + d*x)] + 360*a^2
*b^2*B*Sin[4*(c + d*x)] + 45*b^4*B*Sin[4*(c + d*x)] + 240*a^3*b*C*Sin[4*(c
+ d*x)] + 180*a*b^3*C*Sin[4*(c + d*x)] + 360*a^2*A*b^2*Sin[5*(c + d*x)] + 4
0*A*b^4*Sin[5*(c + d*x)] + 240*a^3*b*B*Sin[5*(c + d*x)] + 160*a*b^3*B*Sin[5
*(c + d*x)] + 60*a^4*C*Sin[5*(c + d*x)] + 240*a^2*b^2*C*Sin[5*(c + d*x)] +
32*b^4*C*Sin[5*(c + d*x)]))/(480*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c
```

```
 + d*x)]))
```

Maple [B] time = 0.066, size = 572, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] a^4*A*x+1/d*A*a^4*c+4/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^3*b*C*ln(se
c(d*x+c)+tan(d*x+c))+4/d*C*a^2*b^2*tan(d*x+c)+1/3/d*A*b^4*tan(d*x+c)*sec(d*
x+c)^2+1/5/d*C*b^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*C*b^4*tan(d*x+c)*sec(d*x+
c)^2+8/3/d*a*b^3*B*tan(d*x+c)+4/d*B*a^3*b*tan(d*x+c)+6/d*A*a^2*b^2*tan(d*x+
c)+3/2/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*B*b^4*tan(d*x+c)*sec(d*x+c
)^3+3/8/d*B*b^4*sec(d*x+c)*tan(d*x+c)+2/d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))
+1/d*a^4*C*tan(d*x+c)+1/d*B*a^4*ln(sec(d*x+c)+tan(d*x+c))+8/15/d*C*b^4*tan(
d*x+c)+2/3/d*A*b^4*tan(d*x+c)+3/8/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*a
*b^3*B*tan(d*x+c)*sec(d*x+c)^2+1/d*C*a*b^3*tan(d*x+c)*sec(d*x+c)^3+3/2/d*C*
a*b^3*sec(d*x+c)*tan(d*x+c)+3/d*a^2*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^3
*b*C*sec(d*x+c)*tan(d*x+c)+3/d*a^2*b^2*B*sec(d*x+c)*tan(d*x+c)+2/d*A*a*b^3*
sec(d*x+c)*tan(d*x+c)+2/d*C*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.05134, size = 671, normalized size = 2.31

$$240\,(dx+c)Aa^4+480\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Ca^2b^2+320\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Bab^3+80\left(\tan\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] 1/240*(240*(d*x + c)*A*a^4 + 480*(tan(d*x + c)^3 + 3*tan(d*x + c))*C*a^2*b^
2 + 320*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b^3 + 80*(tan(d*x + c)^3 + 3*
tan(d*x + c))*A*b^4 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
 + c))*C*b^4 - 60*C*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*b^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 240*C*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
 + log(sin(d*x + c) - 1)) - 360*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 -
 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 240*A*a*b^3*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 240*B*a^4*log(sec(d*x + c) + tan(d*x + c)) + 960*A*a^3*b*log(sec(d*x +
 c) + tan(d*x + c)) + 240*C*a^4*tan(d*x + c) + 960*B*a^3*b*tan(d*x + c) + 1
440*A*a^2*b^2*tan(d*x + c))/d
```

Fricas [A] time = 0.625407, size = 824, normalized size = 2.84

$$240\,Aa^4dx\cos\left(dx+c\right)^5+15\left(8\,Ba^4+16\,(2\,A+C)a^3b+24\,Ba^2b^2+4\,(4\,A+3\,C)ab^3+3\,Bb^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/240*(240*A*a^4*d*x*cos(d*x + c)^5 + 15*(8*B*a^4 + 16*(2*A + C)*a^3*b + 24
*B*a^2*b^2 + 4*(4*A + 3*C)*a*b^3 + 3*B*b^4)*cos(d*x + c)^5*log(sin(d*x + c)
 + 1) - 15*(8*B*a^4 + 16*(2*A + C)*a^3*b + 24*B*a^2*b^2 + 4*(4*A + 3*C)*a*b
^3 + 3*B*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*C*b^4 + 8*(15*C
*a^4 + 60*B*a^3*b + 30*(3*A + 2*C)*a^2*b^2 + 40*B*a*b^3 + 2*(5*A + 4*C)*b^4
)*cos(d*x + c)^4 + 15*(16*C*a^3*b + 24*B*a^2*b^2 + 4*(4*A + 3*C)*a*b^3 + 3*
B*b^4)*cos(d*x + c)^3 + 8*(30*C*a^2*b^2 + 20*B*a*b^3 + (5*A + 4*C)*b^4)*cos
(d*x + c)^2 + 30*(4*C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*(A + B*sec(c + d*x) + C*sec(c + d*x)**2),
x)
```

Giac [B] time = 1.41148, size = 1539, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] 1/120*(120*(d*x + c)*A*a^4 + 15*(8*B*a^4 + 32*A*a^3*b + 16*C*a^3*b + 24*B*a
^2*b^2 + 16*A*a*b^3 + 12*C*a*b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 15*(8*B*a^4 + 32*A*a^3*b + 16*C*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 1
2*C*a*b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*C*a^4*tan(
1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 240*C*a^3*b*tan(1
/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*ta
n(1/2*d*x + 1/2*c)^9 + 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3*t
an(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 300*C*a*b^3*ta
n(1/2*d*x + 1/2*c)^9 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 75*B*b^4*tan(1/2*
d*x + 1/2*c)^9 + 120*C*b^4*tan(1/2*d*x + 1/2*c)^9 - 480*C*a^4*tan(1/2*d*x +
 1/2*c)^7 - 1920*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*C*a^3*b*tan(1/2*d*x +
 1/2*c)^7 - 2880*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 1920*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 480*A*a*b^3*tan(1/2
*d*x + 1/2*c)^7 - 1280*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*C*a*b^3*tan(1/2
*d*x + 1/2*c)^7 - 320*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 30*B*b^4*tan(1/2*d*x +
 1/2*c)^7 - 160*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 720*C*a^4*tan(1/2*d*x + 1/2*
c)^5 + 2880*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 4320*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^5 + 2400*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 1600*B*a*b^3*tan(1/2*d*x
```

```
+ 1/2*c)^5 + 400*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 464*C*b^4*tan(1/2*d*x + 1/2
*c)^5 - 480*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 1920*B*a^3*b*tan(1/2*d*x + 1/2*c
)^3 - 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2880*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^3 - 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 1920*C*a^2*b^2*tan(1/2*d*x +
 1/2*c)^3 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1280*B*a*b^3*tan(1/2*d*x +
 1/2*c)^3 - 120*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 320*A*b^4*tan(1/2*d*x + 1/
2*c)^3 - 30*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 160*C*b^4*tan(1/2*d*x + 1/2*c)^3
 + 120*C*a^4*tan(1/2*d*x + 1/2*c) + 480*B*a^3*b*tan(1/2*d*x + 1/2*c) + 240*
C*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 360*B*a
^2*b^2*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*
b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x + 1/2*c) + 300*C*a*b^3*t
an(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c) + 75*B*b^4*tan(1/2*d*x
 + 1/2*c) + 120*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)
/d
```

3.890 $\int \cos(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec\right.$

Optimal. Leaf size=273

$$\frac{b\tan(c+dx)\left(a^3(-(12A-19C))+34a^2bB+8ab^2(3A+2C)+4b^3B\right)}{6d}+\frac{\left(24a^2b^2(2A+C)+32a^3bB+8a^4C+16ab^3B\right.}{8d}$$

```
[Out] a^3*(4*A*b + a*B)*x + ((32*a^3*b*B + 16*a*b^3*B + 8*a^4*C + 24*a^2*b^2*(2*A
 + C) + b^4*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (A*(a + b*Sec[c + d
*x])^4*Sin[c + d*x])/d + (b*(34*a^2*b*B + 4*b^3*B - a^3*(12*A - 19*C) + 8*a
*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*d) + (b^2*(32*a*b*B - a^2*(24*A - 26*C)
+ 3*b^2*(4*A + 3*C))*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (b*(12*a*A - 4*b*B
 - 7*a*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - (b*(4*A - C)*(a + b
*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.582644, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4094, 4056, 4048, 3770, 3767, 8}

$$\frac{b\tan(c+dx)\left(a^3(-(12A-19C))+34a^2bB+8ab^2(3A+2C)+4b^3B\right)}{6d}+\frac{\left(24a^2b^2(2A+C)+32a^3bB+8a^4C+16ab^3B\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2),x]
```

```
[Out] a^3*(4*A*b + a*B)*x + ((32*a^3*b*B + 16*a*b^3*B + 8*a^4*C + 24*a^2*b^2*(2*A
 + C) + b^4*(4*A + 3*C))*ArcTanh[Sin[c + d*x]])/(8*d) + (A*(a + b*Sec[c + d
*x])^4*Sin[c + d*x])/d + (b*(34*a^2*b*B + 4*b^3*B - a^3*(12*A - 19*C) + 8*a
*b^2*(3*A + 2*C))*Tan[c + d*x])/(6*d) + (b^2*(32*a*b*B - a^2*(24*A - 26*C)
+ 3*b^2*(4*A + 3*C))*Sec[c + d*x]*Tan[c + d*x])/(24*d) - (b*(12*a*A - 4*b*B
 - 7*a*C)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) - (b*(4*A - C)*(a + b
*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d}+\int(a+$$
$$= \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d}-\frac{b(4A-}{}$$
$$= \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d}-\frac{b(12aA}{}$$
$$= \frac{A(a+b\sec(c+dx))^4\sin(c+dx)}{d}+\frac{b^2\left(32a\right.}{}$$
$$= a^3(4Ab+aB)x+\frac{A(a+b\sec(c+dx))^4\sin}{d}$$
$$= a^3(4Ab+aB)x+\frac{\left(32a^3bB+16ab^3B+8a^4C\right.}{}$$
$$= a^3(4Ab+aB)x+\frac{\left(32a^3bB+16ab^3B+8a^4C\right.}{}$$

Mathematica [B] time = 6.89277, size = 813, normalized size = 2.98

$$\frac{\left(-8Ca^4-32bBa^3-48Ab^2a^2-24b^2Ca^2-16b^3Ba-4Ab^4-3b^4C\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+}{4d(b+a\cos(c+dx))^4(\cos(2c+2dx)A+A+2C+2B\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] ((-48*a^2*A*b^2 - 4*A*b^4 - 32*a^3*b*B - 16*a*b^3*B - 8*a^4*C - 24*a^2*b^2*
C - 3*b^4*C)*Cos[c + d*x]^6*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b
*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(4*d*(b + a*Cos[c
 + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((48*a^2*A*
b^2 + 4*A*b^4 + 32*a^3*b*B + 16*a*b^3*B + 8*a^4*C + 24*a^2*b^2*C + 3*b^4*C)
*Cos[c + d*x]^6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x
```

```
])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(4*d*(b + a*Cos[c + d*x])^4*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^2*(a + b*
Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(144*a^3*A*b*(c + d
*x) + 36*a^4*B*(c + d*x) + 192*a^3*A*b*(c + d*x)*Cos[2*(c + d*x)] + 48*a^4*
B*(c + d*x)*Cos[2*(c + d*x)] + 48*a^3*A*b*(c + d*x)*Cos[4*(c + d*x)] + 12*a
^4*B*(c + d*x)*Cos[4*(c + d*x)] + 12*a^4*A*Sin[c + d*x] + 12*A*b^4*Sin[c +
d*x] + 48*a*b^3*B*Sin[c + d*x] + 72*a^2*b^2*C*Sin[c + d*x] + 33*b^4*C*Sin[c
 + d*x] + 96*a*A*b^3*Sin[2*(c + d*x)] + 144*a^2*b^2*B*Sin[2*(c + d*x)] + 32
*b^4*B*Sin[2*(c + d*x)] + 96*a^3*b*C*Sin[2*(c + d*x)] + 128*a*b^3*C*Sin[2*(
c + d*x)] + 18*a^4*A*Sin[3*(c + d*x)] + 12*A*b^4*Sin[3*(c + d*x)] + 48*a*b^
3*B*Sin[3*(c + d*x)] + 72*a^2*b^2*C*Sin[3*(c + d*x)] + 9*b^4*C*Sin[3*(c + d
*x)] + 48*a*A*b^3*Sin[4*(c + d*x)] + 72*a^2*b^2*B*Sin[4*(c + d*x)] + 8*b^4*
B*Sin[4*(c + d*x)] + 48*a^3*b*C*Sin[4*(c + d*x)] + 32*a*b^3*C*Sin[4*(c + d*
x)] + 6*a^4*A*Sin[5*(c + d*x)]))/(48*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 0.09, size = 457, normalized size = 1.7

$$\frac{Aa^4\sin(dx+c)}{d}+Ba^4x+\frac{Ba^4c}{d}+\frac{a^4C\ln(\sec(dx+c)+\tan(dx+c))}{d}+4a^3Abx+4\frac{Aa^3bc}{d}+4\frac{Ba^3b\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+B*a^4*x+1/d*B*a^4*c+1/d*a^4*C*ln(sec(d*x+c)+tan(d*x+c)
)+4*a^3*A*b*x+4/d*A*a^3*b*c+4/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b
*C*tan(d*x+c)+6/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*B*tan(d*x
+c)+3/d*C*a^2*b^2*sec(d*x+c)*tan(d*x+c)+3/d*C*a^2*b^2*ln(sec(d*x+c)+tan(d*x
+c))+4/d*A*a*b^3*tan(d*x+c)+2/d*a*b^3*B*sec(d*x+c)*tan(d*x+c)+2/d*a*b^3*B*l
n(sec(d*x+c)+tan(d*x+c))+8/3/d*C*a*b^3*tan(d*x+c)+4/3/d*C*a*b^3*tan(d*x+c)*
sec(d*x+c)^2+1/2/d*A*b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*b^4*ln(sec(d*x+c)+ta
n(d*x+c))+2/3/d*B*b^4*tan(d*x+c)+1/3/d*B*b^4*tan(d*x+c)*sec(d*x+c)^2+1/4/d*
C*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*C*b^4*sec(d*x+c)*tan(d*x+c)+3/8/d*C*b^4
*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.08799, size = 582, normalized size = 2.13

$$\frac{48(dx+c)Ba^4+192(dx+c)Aa^3b+64\left(\tan(dx+c)^3+3\tan(dx+c)\right)Cab^3+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/48*(48*(d*x + c)*B*a^4 + 192*(d*x + c)*A*a^3*b + 64*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a*b^3 + 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^4 - 3*C*b^4
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*C*a^2*b^2*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) - 48*B*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*a^4*(log(sin(
```

```
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*B*a^3*b*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 144*A*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 48*A*a^4*sin(d*x + c) + 192*C*a^3*b*tan(d*x + c) + 288*B*a^
2*b^2*tan(d*x + c) + 192*A*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.629816, size = 725, normalized size = 2.66

$$\frac{48\left(Ba^4+4\,Aa^3b\right)dx\cos\left(dx+c\right)^4+3\left(8\,Ca^4+32\,Ba^3b+24\,(2\,A+C)a^2b^2+16\,Bab^3+(4\,A+3\,C)b^4\right)\cos\left(dx+c\right)^4\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/48*(48*(B*a^4 + 4*A*a^3*b)*d*x*cos(d*x + c)^4 + 3*(8*C*a^4 + 32*B*a^3*b +
 24*(2*A + C)*a^2*b^2 + 16*B*a*b^3 + (4*A + 3*C)*b^4)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(8*C*a^4 + 32*B*a^3*b + 24*(2*A + C)*a^2*b^2 + 16*B*a*b
^3 + (4*A + 3*C)*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(24*A*a^4*c
os(d*x + c)^4 + 6*C*b^4 + 16*(6*C*a^3*b + 9*B*a^2*b^2 + 2*(3*A + 2*C)*a*b^3
 + B*b^4)*cos(d*x + c)^3 + 3*(24*C*a^2*b^2 + 16*B*a*b^3 + (4*A + 3*C)*b^4)*
cos(d*x + c)^2 + 8*(4*C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.45465, size = 1134, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(48*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 24*(B*a^
4 + 4*A*a^3*b)*(d*x + c) + 3*(8*C*a^4 + 32*B*a^3*b + 48*A*a^2*b^2 + 24*C*a^
2*b^2 + 16*B*a*b^3 + 4*A*b^4 + 3*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 3*(8*C*a^4 + 32*B*a^3*b + 48*A*a^2*b^2 + 24*C*a^2*b^2 + 16*B*a*b^3 + 4*A*
b^4 + 3*C*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*C*a^3*b*tan(1/2*d
*x + 1/2*c)^7 + 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*C*a^2*b^2*tan(1/2
*d*x + 1/2*c)^7 + 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 48*B*a*b^3*tan(1/2*d*
x + 1/2*c)^7 + 96*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 12*A*b^4*tan(1/2*d*x + 1
```

/2*c)^7 + 24*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*C*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 48*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 40*B*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 48*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 160*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 40*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*C*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c) - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c) - 48*B*a*b^3*tan(1/2*d*x + 1/2*c) - 96*C*a*b^3*tan(1/2*d*x + 1/2*c) - 12*A*b^4*tan(1/2*d*x + 1/2*c) - 24*B*b^4*tan(1/2*d*x + 1/2*c) - 15*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d

3.891 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=274

$$-\frac{b\tan(c+dx)\left(a^2b(39A-34C)+12a^3B-24ab^2B-2b^3(3A+2C)\right)}{6d}+\frac{b\left(12a^2bB+8a^3C+4ab^2(2A+C)+b^3B\right)\tanh^{-}}{2d}$$

```
[Out] (a^2*(12*A*b^2 + 8*a*b*B + a^2*(A + 2*C))*x)/2 + (b*(12*a^2*b*B + b^3*B + 8
*a^3*C + 4*a*b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*A*b + a*B)*(
a + b*Sec[c + d*x])^3*Sin[c + d*x])/d + (A*Cos[c + d*x]*(a + b*Sec[c + d*x]
)^4*Sin[c + d*x])/(2*d) - (b*(12*a^3*B - 24*a*b^2*B + a^2*b*(39*A - 34*C) -
 2*b^3*(3*A + 2*C))*Tan[c + d*x])/(6*d) - (b^2*(6*a^2*B - 3*b^2*B + 2*a*b*(
9*A - 4*C))*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(15*A*b + 6*a*B - 2*b*C)*
(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.677395, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4056, 4048, 3770, 3767, 8}

$$-\frac{b\tan(c+dx)\left(a^2b(39A-34C)+12a^3B-24ab^2B-2b^3(3A+2C)\right)}{6d}+\frac{b\left(12a^2bB+8a^3C+4ab^2(2A+C)+b^3B\right)\tanh^{-}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] (a^2*(12*A*b^2 + 8*a*b*B + a^2*(A + 2*C))*x)/2 + (b*(12*a^2*b*B + b^3*B + 8
*a^3*C + 4*a*b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*A*b + a*B)*(
a + b*Sec[c + d*x])^3*Sin[c + d*x])/d + (A*Cos[c + d*x]*(a + b*Sec[c + d*x]
)^4*Sin[c + d*x])/(2*d) - (b*(12*a^3*B - 24*a*b^2*B + a^2*b*(39*A - 34*C) -
 2*b^3*(3*A + 2*C))*Tan[c + d*x])/(6*d) - (b^2*(6*a^2*B - 3*b^2*B + 2*a*b*(
9*A - 4*C))*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(15*A*b + 6*a*B - 2*b*C)*
(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(6*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^4\sin(c}{2d}$$
$$= \frac{(2Ab+aB)(a+b\sec(c+dx))^3\sin(c+}{d}$$
$$= \frac{(2Ab+aB)(a+b\sec(c+dx))^3\sin(c+}{d}$$
$$= \frac{(2Ab+aB)(a+b\sec(c+dx))^3\sin(c+}{d}$$
$$= \frac{1}{2}a^2\left(12Ab^2+8abB+a^2(A+2C)\right)x+$$
$$= \frac{1}{2}a^2\left(12Ab^2+8abB+a^2(A+2C)\right)x+$$
$$= \frac{1}{2}a^2\left(12Ab^2+8abB+a^2(A+2C)\right)x+$$

Mathematica [A] time = 2.45191, size = 348, normalized size = 1.27

$$\frac{\sec^3(c+dx)\left(36a^2(c+dx)\cos(c+dx)\left(a^2(A+2C)+8abB+12Ab^2\right)+12a^2(c+dx)\cos(3(c+dx))\left(a^2(A+2C)+8a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Sec[c + d*x]^3*(36*a^2*(12*A*b^2 + 8*a*b*B + a^2*(A + 2*C))*(c + d*x)*Cos[
c + d*x] + 12*a^2*(12*A*b^2 + 8*a*b*B + a^2*(A + 2*C))*(c + d*x)*Cos[3*(c +
 d*x)] - 48*b*(12*a^2*b*B + b^3*B + 8*a^3*C + 4*a*b^2*(2*A + C))*Cos[c + d*
x]^3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) + 2*(9*a^4*A + 24*A*b^4 + 96*a*b^3*B + 144*a^2*b^2*C + 32*b
^4*C + 12*(12*a^3*A*b + 3*a^4*B + 2*b^4*B + 8*a*b^3*C)*Cos[c + d*x] + 4*(3*
```

```
a^4*A + 6*A*b^4 + 24*a*b^3*B + 36*a^2*b^2*C + 4*b^4*C)*Cos[2*(c + d*x)] + 4
8*a^3*A*b*Cos[3*(c + d*x)] + 12*a^4*B*Cos[3*(c + d*x)] + 3*a^4*A*Cos[4*(c +
 d*x)])*Sin[c + d*x]))/(96*d)
```

Maple [A] time = 0.091, size = 377, normalized size = 1.4

$$\frac{Aa^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4Ax}{2}+\frac{Aa^4c}{2d}+\frac{Ba^4\sin(dx+c)}{d}+a^4Cx+\frac{Ca^4c}{d}+4\,\frac{Aa^3b\sin(dx+c)}{d}+4\,Ba^3bx+4\,\frac{Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+1/2*a^4*A*x+1/2/d*A*a^4*c+1/d*B*a^4*sin(d
*x+c)+a^4*C*x+1/d*C*a^4*c+4/d*A*a^3*b*sin(d*x+c)+4*B*a^3*b*x+4/d*B*a^3*b*c+
4/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+6*A*a^2*b^2*x+6/d*A*a^2*b^2*c+6/d*a^2
*b^2*B*ln(sec(d*x+c)+tan(d*x+c))+6/d*C*a^2*b^2*tan(d*x+c)+4/d*A*a*b^3*ln(se
c(d*x+c)+tan(d*x+c))+4/d*a*b^3*B*tan(d*x+c)+2/d*C*a*b^3*sec(d*x+c)*tan(d*x+
c)+2/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b^4*tan(d*x+c)+1/2/d*B*b^4*s
ec(d*x+c)*tan(d*x+c)+1/2/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*C*b^4*tan(
d*x+c)+1/3/d*C*b^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.0535, size = 452, normalized size = 1.65

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+12\,(dx+c)Ca^4+48\,(dx+c)Ba^3b+72\,(dx+c)Aa^2b^2+4\left(\tan(dx+c)^3+3\,\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 12*(d*x + c)*C*a^4 + 48*(d
*x + c)*B*a^3*b + 72*(d*x + c)*A*a^2*b^2 + 4*(tan(d*x + c)^3 + 3*tan(d*x +
c))*C*b^4 - 12*C*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
 c) + 1) + log(sin(d*x + c) - 1)) - 3*B*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*a^3*b*(log(si
n(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*B*a^2*b^2*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 24*A*a*b^3*(log(sin(d*x + c) + 1) - log(sin(
d*x + c) - 1)) + 12*B*a^4*sin(d*x + c) + 48*A*a^3*b*sin(d*x + c) + 72*C*a^2
*b^2*tan(d*x + c) + 48*B*a*b^3*tan(d*x + c) + 12*A*b^4*tan(d*x + c))/d
```

Fricas [A] time = 0.615278, size = 644, normalized size = 2.35

$$\frac{6\left((A+2\,C)a^4+8\,Ba^3b+12\,Aa^2b^2\right)dx\cos(dx+c)^3+3\left(8\,Ca^3b+12\,Ba^2b^2+4\,(2\,A+C)ab^3+Bb^4\right)\cos(dx+c)^3\log(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/12*(6*((A + 2*C)*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*d*x*cos(d*x + c)^3 + 3*(
8*C*a^3*b + 12*B*a^2*b^2 + 4*(2*A + C)*a*b^3 + B*b^4)*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(8*C*a^3*b + 12*B*a^2*b^2 + 4*(2*A + C)*a*b^3 + B*b^4)*
cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(3*A*a^4*cos(d*x + c)^4 + 2*C*b^4
 + 6*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^3 + 2*(18*C*a^2*b^2 + 12*B*a*b^3 + (3
*A + 2*C)*b^4)*cos(d*x + c)^2 + 3*(4*C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40329, size = 743, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/6*(3*(A*a^4 + 2*C*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*(d*x + c) + 3*(8*C*a^3*
b + 12*B*a^2*b^2 + 8*A*a*b^3 + 4*C*a*b^3 + B*b^4)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 3*(8*C*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3 + 4*C*a*b^3 + B*b^4)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*(A*a^4*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^
4*tan(1/2*d*x + 1/2*c)^3 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - A*a^4*tan(1/2
*d*x + 1/2*c) - 2*B*a^4*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - 2*(36*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5
 + 24*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 +
6*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^4*t
an(1/2*d*x + 1/2*c)^5 - 72*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 48*B*a*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 12*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*C*b^4*tan(1/2*d*
x + 1/2*c)^3 + 36*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*B*a*b^3*tan(1/2*d*x +
 1/2*c) + 12*C*a*b^3*tan(1/2*d*x + 1/2*c) + 6*A*b^4*tan(1/2*d*x + 1/2*c) +
3*B*b^4*tan(1/2*d*x + 1/2*c) + 6*C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^3)/d
```

3.892 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=303

$$-\frac{b\tan(c+dx)\left(4a^3(2A+3C)+39a^2bB+4ab^2(11A-6C)-6b^3B\right)}{6d}+\frac{b^2\left(12a^2C+8abB+2Ab^2+b^2C\right)\tanh^{-1}(\sin(c+}{2d}$$

```
[Out] (a*(8*A*b^3 + a^3*B + 12*a*b^2*B + 4*a^2*b*(A + 2*C))*x)/2 + (b^2*(2*A*b^2
+ 8*a*b*B + 12*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((12*A*b^2 + 1
5*a*b*B + a^2*(4*A + 6*C))*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + ((4
*A*b + 3*a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + (A*
Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(3*d) - (b*(39*a^2*b*B
- 6*b^3*B + 4*a*b^2*(11*A - 6*C) + 4*a^3*(2*A + 3*C))*Tan[c + d*x])/(6*d) -
 (b^2*(18*a*b*B + 3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Sec[c + d*x]*Tan[c + d
*x])/(6*d)
```

Rubi [A] time = 0.850054, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4094, 4048, 3770, 3767, 8}

$$-\frac{b\tan(c+dx)\left(4a^3(2A+3C)+39a^2bB+4ab^2(11A-6C)-6b^3B\right)}{6d}+\frac{b^2\left(12a^2C+8abB+2Ab^2+b^2C\right)\tanh^{-1}(\sin(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (a*(8*A*b^3 + a^3*B + 12*a*b^2*B + 4*a^2*b*(A + 2*C))*x)/2 + (b^2*(2*A*b^2
+ 8*a*b*B + 12*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + ((12*A*b^2 + 1
5*a*b*B + a^2*(4*A + 6*C))*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + ((4
*A*b + 3*a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(6*d) + (A*
Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(3*d) - (b*(39*a^2*b*B
- 6*b^3*B + 4*a*b^2*(11*A - 6*C) + 4*a^3*(2*A + 3*C))*Tan[c + d*x])/(6*d) -
 (b^2*(18*a*b*B + 3*b^2*(6*A - C) + a^2*(4*A + 6*C))*Sec[c + d*x]*Tan[c + d
*x])/(6*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^4\sin(}{3d}$$
$$= \frac{(4Ab+3aB)\cos(c+dx)(a+b\sec(c+}{6d}$$
$$= \frac{\left(12Ab^2+15abB+a^2(4A+6C)\right)(a+b}{6d}$$
$$= \frac{\left(12Ab^2+15abB+a^2(4A+6C)\right)(a+b}{6d}$$
$$= \frac{1}{2}a\left(8Ab^3+a^3B+12ab^2B+4a^2b(A+\right.$$
$$= \frac{1}{2}a\left(8Ab^3+a^3B+12ab^2B+4a^2b(A+\right.$$
$$= \frac{1}{2}a\left(8Ab^3+a^3B+12ab^2B+4a^2b(A+\right.$$

Mathematica [A] time = 5.2081, size = 370, normalized size = 1.22

$$6a(c+dx)\left(4a^2b(A+2C)+a^3B+12ab^2B+8Ab^3\right)+3a^2\sin(c+dx)\left(a^2(3A+4C)+16abB+24Ab^2\right)-6b^2\left(12a^2C-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (6*a*(8*A*b^3 + a^3*B + 12*a*b^2*B + 4*a^2*b*(A + 2*C))*(c + d*x) - 6*b^2*(
2*A*b^2 + 8*a*b*B + 12*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + 6*b^2*(2*A*b^2 + 8*a*b*B + 12*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + (3*b^4*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*
b^3*(b*B + 4*a*C)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) -
 (3*b^4*C)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (12*b^3*(b*B + 4*a*C)*
Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 3*a^2*(24*A*b^2 +
 16*a*b*B + a^2*(3*A + 4*C))*Sin[c + d*x] + 3*a^3*(4*A*b + a*B)*Sin[2*(c +
d*x)] + a^4*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.09, size = 374, normalized size = 1.2

$$\frac{A\sin(dx+c)(\cos(dx+c))^2 a^4}{3d}+\frac{2Aa^4\sin(dx+c)}{3d}+\frac{Ba^4\sin(dx+c)\cos(dx+c)}{2d}+\frac{Ba^4x}{2}+\frac{Ba^4c}{2d}+\frac{a^4C\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3/d*A*a^4*sin(d*x+c)+1/2/d*B*a^4*sin(
d*x+c)*cos(d*x+c)+1/2*B*a^4*x+1/2/d*B*a^4*c+1/d*a^4*C*sin(d*x+c)+2/d*A*a^3*
b*sin(d*x+c)*cos(d*x+c)+2*a^3*A*b*x+2/d*A*a^3*b*c+4/d*B*a^3*b*sin(d*x+c)+4*
a^3*b*C*x+4/d*C*a^3*b*c+6/d*A*a^2*b^2*sin(d*x+c)+6*a^2*b^2*B*x+6/d*B*a^2*b^
2*c+6/d*C*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*A*a*b^3*x+4/d*A*a*b^3*c+4/d*a
*b^3*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*C*a*b^3*tan(d*x+c)+1/d*A*b^4*ln(sec(d*
x+c)+tan(d*x+c))+1/d*B*b^4*tan(d*x+c)+1/2/d*C*b^4*sec(d*x+c)*tan(d*x+c)+1/2
/d*C*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.03411, size = 420, normalized size = 1.39

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-3(2dx+2c+\sin(2dx+2c))Ba^4-12(2dx+2c+\sin(2dx+2c))Aa^3b-48(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^4 - 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3*b - 48*(d*x +
c)*C*a^3*b - 72*(d*x + c)*B*a^2*b^2 - 48*(d*x + c)*A*a*b^3 + 3*C*b^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) - 36*C*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 24*B*
a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 6*A*b^4*(log(sin(d*
x + c) + 1) - log(sin(d*x + c) - 1)) - 12*C*a^4*sin(d*x + c) - 48*B*a^3*b*s
in(d*x + c) - 72*A*a^2*b^2*sin(d*x + c) - 48*C*a*b^3*tan(d*x + c) - 12*B*b^
4*tan(d*x + c))/d
```

Fricas [A] time = 0.617009, size = 628, normalized size = 2.07

$$\frac{6\left(Ba^4+4(A+2C)a^3b+12Ba^2b^2+8Aab^3\right)dx\cos(dx+c)^2+3\left(12Ca^2b^2+8Bab^3+(2A+C)b^4\right)\cos(dx+c)^2\log(si}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/12*(6*(B*a^4 + 4*(A + 2*C)*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*d*x*cos(d*x
+ c)^2 + 3*(12*C*a^2*b^2 + 8*B*a*b^3 + (2*A + C)*b^4)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - 3*(12*C*a^2*b^2 + 8*B*a*b^3 + (2*A + C)*b^4)*cos(d*x + c)
^2*log(-sin(d*x + c) + 1) + 2*(2*A*a^4*cos(d*x + c)^4 + 3*C*b^4 + 3*(B*a^4
+ 4*A*a^3*b)*cos(d*x + c)^3 + 2*((2*A + 3*C)*a^4 + 12*B*a^3*b + 18*A*a^2*b^
```

```
2)*cos(d*x + c)^2 + 6*(4*C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39076, size = 733, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/6*(3*(B*a^4 + 4*A*a^3*b + 8*C*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*(d*x + c)
 + 3*(12*C*a^2*b^2 + 8*B*a*b^3 + 2*A*b^4 + C*b^4)*log(abs(tan(1/2*d*x + 1/2
*c) + 1)) - 3*(12*C*a^2*b^2 + 8*B*a*b^3 + 2*A*b^4 + C*b^4)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 6*(8*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^4*tan(1/2*
d*x + 1/2*c)^3 - C*b^4*tan(1/2*d*x + 1/2*c)^3 - 8*C*a*b^3*tan(1/2*d*x + 1/2
*c) - 2*B*b^4*tan(1/2*d*x + 1/2*c) - C*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 - 1)^2 + 2*(6*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*tan(1/2*
d*x + 1/2*c)^5 + 6*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x +
1/2*c)^5 + 24*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^5 + 4*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^4*tan(1/2*d*x + 1/2*c)^3
+ 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 +
 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan(1/2*d*x + 1/2*c) + 6*C*a^4*tan(
1/2*d*x + 1/2*c) + 12*A*a^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*tan(1/2*d*x
 + 1/2*c) + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^3)/d
```

3.893 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=293

$$\frac{a\sin(c+dx)\left(a^2b(23A+36C)+8a^3B+36ab^2B+12Ab^3\right)}{12d}-\frac{b^2\tan(c+dx)\left(3a^2(3A+4C)+32abB+2b^2(13A-12C)\right)}{24d}$$

```
[Out] ((8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C
))*x)/8 + (b^3*(b*B + 4*a*C)*ArcTanh[Sin[c + d*x]])/d + (a*(12*A*b^3 + 8*a^
3*B + 36*a*b^2*B + a^2*b*(23*A + 36*C))*Sin[c + d*x])/(12*d) + ((4*A*b^2 +
8*a*b*B + a^2*(3*A + 4*C))*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x]
)/(8*d) + ((A*b + a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(3*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(4*d) - (b^2
*(32*a*b*B + 2*b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Tan[c + d*x])/(24*d)
```

Rubi [A] time = 0.97243, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4076, 4047, 8, 4045, 3770}

$$\frac{a\sin(c+dx)\left(a^2b(23A+36C)+8a^3B+36ab^2B+12Ab^3\right)}{12d}-\frac{b^2\tan(c+dx)\left(3a^2(3A+4C)+32abB+2b^2(13A-12C)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```

```
[Out] ((8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B + 24*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C
))*x)/8 + (b^3*(b*B + 4*a*C)*ArcTanh[Sin[c + d*x]])/d + (a*(12*A*b^3 + 8*a^
3*B + 36*a*b^2*B + a^2*b*(23*A + 36*C))*Sin[c + d*x])/(12*d) + ((4*A*b^2 +
8*a*b*B + a^2*(3*A + 4*C))*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x]
)/(8*d) + ((A*b + a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(3*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(4*d) - (b^2
*(32*a*b*B + 2*b^2*(13*A - 12*C) + 3*a^2*(3*A + 4*C))*Tan[c + d*x])/(24*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^4\sin(}{4d}$$

$$= \frac{(Ab+aB)\cos^2(c+dx)(a+b\sec(c+d}{3d}$$

$$= \frac{\left(4Ab^2+8abB+a^2(3A+4C)\right)\cos(c+}{8d}$$

$$= \frac{\left(4Ab^2+8abB+a^2(3A+4C)\right)\cos(c+}{8d}$$

$$= \frac{\left(4Ab^2+8abB+a^2(3A+4C)\right)\cos(c+}{8d}$$

$$= \frac{1}{8}\left(8Ab^4+16a^3bB+32ab^3B+24a^2b^2(\right.$$

$$= \frac{1}{8}\left(8Ab^4+16a^3bB+32ab^3B+24a^2b^2(\right.$$

Mathematica [A] time = 3.97338, size = 382, normalized size = 1.3

$$\frac{32a\sin(c+dx)\left(4a^2b(5A+6C)+5a^3B+36ab^2B+24Ab^3\right)+a^2\sec(c+dx)\left(3\sin(3(c+dx))\left(a^2(9A+8C)+32abB\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (32*a*(24*A*b^3 + 5*a^3*B + 36*a*b^2*B + 4*a^2*b*(5*A + 6*C))*Sin[c + d*x]
+ a^2*Sec[c + d*x]*(3*(48*A*b^2 + 32*a*b*B + a^2*(9*A + 8*C))*Sin[3*(c + d*
x)] + a*(8*(4*A*b + a*B)*Sin[4*(c + d*x)] + 3*a*A*Sin[5*(c + d*x)])) + 24*(
3*a^4*A*c + 24*a^2*A*b^2*c + 8*A*b^4*c + 16*a^3*b*B*c + 32*a*b^3*B*c + 4*a^
4*c*C + 48*a^2*b^2*c*C + 3*a^4*A*d*x + 24*a^2*A*b^2*d*x + 8*A*b^4*d*x + 16*
a^3*b*B*d*x + 32*a*b^3*B*d*x + 4*a^4*C*d*x + 48*a^2*b^2*C*d*x - 8*b^3*(b*B
```

```
+ 4*a*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 8*b^4*B*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 32*a*b^3*C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] + (6*a^2*A*b^2 + 4*a^3*b*B + 8*b^4*C + a^4*(A + C))*Tan[c + d*x]))/(19
2*d)
```

Maple [A] time = 0.082, size = 434, normalized size = 1.5

$$\frac{3\,a^4Ax}{8}+6\,\frac{Ca^2b^2c}{d}+4\,\frac{Bab^3c}{d}+\frac{3\,Aa^4c}{8\,d}+\frac{8\,Aa^3b\sin(dx+c)}{3\,d}+\frac{a^4Cc}{2\,d}+\frac{2\,Ba^4\sin(dx+c)}{3\,d}+2\,Ba^3bx+3\,Aa^2b^2x+4\,\frac{A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 3/8*a^4*A*x+6/d*C*a^2*b^2*c+4/d*B*a*b^3*c+3/8/d*A*a^4*c+8/3/d*A*a^3*b*sin(d
*x+c)+1/2/d*C*a^4*c+2/3/d*B*a^4*sin(d*x+c)+2*B*a^3*b*x+3*A*a^2*b^2*x+4/d*A*
a*b^3*sin(d*x+c)+A*b^4*x+6/d*a^2*b^2*B*sin(d*x+c)+4/d*C*a*b^3*ln(sec(d*x+c)
+tan(d*x+c))+1/d*C*b^4*tan(d*x+c)+1/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))+3/8/d
*A*a^4*cos(d*x+c)*sin(d*x+c)+2/d*B*a^3*b*c+3/d*A*a^2*b^2*c+1/3/d*B*sin(d*x+
c)*cos(d*x+c)^2*a^4+1/4/d*A*a^4*sin(d*x+c)*cos(d*x+c)^3+1/2/d*a^4*C*sin(d*x
+c)*cos(d*x+c)+4/d*a^3*b*C*sin(d*x+c)+4/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^3*b
+2/d*B*a^3*b*sin(d*x+c)*cos(d*x+c)+3/d*A*a^2*b^2*sin(d*x+c)*cos(d*x+c)+1/d*
A*b^4*c+4*a*b^3*B*x+6*C*a^2*b^2*x+1/2*a^4*C*x
```

Maxima [A] time = 1.044, size = 412, normalized size = 1.41

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^4-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^4+24\,(2\,dx+2\,c+\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 24*(2*d*x + 2*c + sin(2*d*x + 2*c
))*C*a^4 - 128*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3*b + 96*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*B*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^
2 + 576*(d*x + c)*C*a^2*b^2 + 384*(d*x + c)*B*a*b^3 + 96*(d*x + c)*A*b^4 +
192*C*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*B*b^4*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 384*C*a^3*b*sin(d*x + c) + 57
6*B*a^2*b^2*sin(d*x + c) + 384*A*a*b^3*sin(d*x + c) + 96*C*b^4*tan(d*x + c)
)/d
```

Fricas [A] time = 0.615794, size = 636, normalized size = 2.17

$$\frac{3\left((3\,A+4\,C)a^4+16\,Ba^3b+24\,(A+2\,C)a^2b^2+32\,Bab^3+8\,Ab^4\right)dx\cos(dx+c)+12\left(4\,Cab^3+Bb^4\right)\cos(dx+c)\log(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/24*(3*((3*A + 4*C)*a^4 + 16*B*a^3*b + 24*(A + 2*C)*a^2*b^2 + 32*B*a*b^3 +
 8*A*b^4)*d*x*cos(d*x + c) + 12*(4*C*a*b^3 + B*b^4)*cos(d*x + c)*log(sin(d*
x + c) + 1) - 12*(4*C*a*b^3 + B*b^4)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
(6*A*a^4*cos(d*x + c)^4 + 24*C*b^4 + 8*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^3 +
 3*((3*A + 4*C)*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*cos(d*x + c)^2 + 16*(B*a^4
 + 2*(2*A + 3*C)*a^3*b + 9*B*a^2*b^2 + 6*A*a*b^3)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35701, size = 1083, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] -1/24*(48*C*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(3*A*
a^4 + 4*C*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 48*C*a^2*b^2 + 32*B*a*b^3 + 8*A
*b^4)*(d*x + c) - 24*(4*C*a*b^3 + B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 + 24*(4*C*a*b^3 + B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^4*
tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^4*tan(1/2
*d*x + 1/2*c)^7 - 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*
x + 1/2*c)^7 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x
 + 1/2*c)^7 - 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^4*tan(1/2*d*x + 1/2*c
)^5 + 12*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5
+ 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 288*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5
- 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 40*
B*a^4*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 288*C*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^
2*tan(1/2*d*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*ta
n(1/2*d*x + 1/2*c) - 24*B*a^4*tan(1/2*d*x + 1/2*c) - 12*C*a^4*tan(1/2*d*x +
 1/2*c) - 96*A*a^3*b*tan(1/2*d*x + 1/2*c) - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)
 - 96*C*a^3*b*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 14
4*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.894 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=314

$$\frac{\sin(c+dx)\left(2a^2b^2(56A+85C)+4a^4(4A+5C)+80a^3bB+95ab^3B+12Ab^4\right)}{30d}+\frac{a\sin(c+dx)\cos(c+dx)\left(4a^2b(29A+\right.}{120d}$$

```
[Out] ((3*a^4*B + 24*a^2*b^2*B + 8*b^4*B + 16*a*b^3*(A + 2*C) + 4*a^3*b*(3*A + 4*
C))*x)/8 + (b^4*C*ArcTanh[Sin[c + d*x]])/d + ((12*A*b^4 + 80*a^3*b*B + 95*a
*b^3*B + 4*a^4*(4*A + 5*C) + 2*a^2*b^2*(56*A + 85*C))*Sin[c + d*x])/(30*d)
+ (a*(24*A*b^3 + 45*a^3*B + 130*a*b^2*B + 4*a^2*b*(29*A + 40*C))*Cos[c + d*
x]*Sin[c + d*x])/(120*d) + ((12*A*b^2 + 35*a*b*B + 4*a^2*(4*A + 5*C))*Cos[c
 + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(60*d) + ((4*A*b + 5*a*B)*Co
s[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^
4*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.04871, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4074, 4047, 8, 4045, 3770}

$$\frac{\sin(c+dx)\left(2a^2b^2(56A+85C)+4a^4(4A+5C)+80a^3bB+95ab^3B+12Ab^4\right)}{30d}+\frac{a\sin(c+dx)\cos(c+dx)\left(4a^2b(29A+\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((3*a^4*B + 24*a^2*b^2*B + 8*b^4*B + 16*a*b^3*(A + 2*C) + 4*a^3*b*(3*A + 4*
C))*x)/8 + (b^4*C*ArcTanh[Sin[c + d*x]])/d + ((12*A*b^4 + 80*a^3*b*B + 95*a
*b^3*B + 4*a^4*(4*A + 5*C) + 2*a^2*b^2*(56*A + 85*C))*Sin[c + d*x])/(30*d)
+ (a*(24*A*b^3 + 45*a^3*B + 130*a*b^2*B + 4*a^2*b*(29*A + 40*C))*Cos[c + d*
x]*Sin[c + d*x])/(120*d) + ((12*A*b^2 + 35*a*b*B + 4*a^2*(4*A + 5*C))*Cos[c
 + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(60*d) + ((4*A*b + 5*a*B)*Co
s[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^
4*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(5*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^4\sin(}{5d}$$

$$= \frac{(4Ab+5aB)\cos^3(c+dx)(a+b\sec(c+}{20d}$$

$$= \frac{\left(12Ab^2+35abB+4a^2(4A+5C)\right)\cos^2}{6}$$

$$= \frac{a\left(24Ab^3+45a^3B+130ab^2B+4a^2b(2\right.}{120}$$

$$= \frac{a\left(24Ab^3+45a^3B+130ab^2B+4a^2b(2\right.}{120}$$

$$= \frac{1}{8}\left(3a^4B+24a^2b^2B+8b^4B+16ab^3(A\right.$$

$$= \frac{1}{8}\left(3a^4B+24a^2b^2B+8b^4B+16ab^3(A\right.$$

Mathematica [A] time = 1.24771, size = 382, normalized size = 1.22

$$\frac{120a\sin(2(c+dx))\left(4a^2b(A+C)+a^3B+6ab^2B+4Ab^3\right)+60\sin(c+dx)\left(12a^2b^2(3A+4C)+a^4(5A+6C)+24a^3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (720*a^3*A*b*c + 960*a*A*b^3*c + 180*a^4*B*c + 1440*a^2*b^2*B*c + 480*b^4*B
*c + 960*a^3*b*c*C + 1920*a*b^3*c*C + 720*a^3*A*b*d*x + 960*a*A*b^3*d*x + 1
80*a^4*B*d*x + 1440*a^2*b^2*B*d*x + 480*b^4*B*d*x + 960*a^3*b*C*d*x + 1920*
a*b^3*C*d*x - 480*b^4*C*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 480*b^4*
C*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*(8*A*b^4 + 24*a^3*b*B + 32*
```

```
a*b^3*B + 12*a^2*b^2*(3*A + 4*C) + a^4*(5*A + 6*C))*Sin[c + d*x] + 120*a*(4
*A*b^3 + a^3*B + 6*a*b^2*B + 4*a^2*b*(A + C))*Sin[2*(c + d*x)] + 50*a^4*A*S
in[3*(c + d*x)] + 240*a^2*A*b^2*Sin[3*(c + d*x)] + 160*a^3*b*B*Sin[3*(c + d
*x)] + 40*a^4*C*Sin[3*(c + d*x)] + 60*a^3*A*b*Sin[4*(c + d*x)] + 15*a^4*B*S
in[4*(c + d*x)] + 6*a^4*A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.092, size = 543, normalized size = 1.7

$$\frac{2\,a^4C\sin(dx+c)}{3\,d}+2\,a^3bCx+3\,a^2b^2Bx+2\,Aab^3x+\frac{Cb^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3\,Aa^3b\sin(dx+c)\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/3/d*a^4*C*sin(d*x+c)+2*a^3*b*C*x+3*a^2*b^2*B*x+2*A*a*b^3*x+1/d*C*b^4*ln(s
ec(d*x+c)+tan(d*x+c))+3/2/d*A*a^3*b*sin(d*x+c)*cos(d*x+c)+2/d*a^3*b*C*cos(d
*x+c)*sin(d*x+c)+4/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a^3*b+2/d*A*sin(d*x+c)*cos
(d*x+c)^2*a^2*b^2+3/d*a^2*b^2*B*cos(d*x+c)*sin(d*x+c)+2/d*A*a*b^3*c+2/d*C*a
^3*b*c+3/d*B*a^2*b^2*c+1/d*A*b^4*sin(d*x+c)+4/d*C*a*b^3*c+B*b^4*x+2/d*A*a*b
^3*cos(d*x+c)*sin(d*x+c)+1/d*A*a^3*b*sin(d*x+c)*cos(d*x+c)^3+3/2/d*A*a^3*b*
c+4/15/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+3/8/d*B*a^4*sin(d*x+c)*cos(d*x+c)+8/
3/d*B*a^3*b*sin(d*x+c)+4/d*A*a^2*b^2*sin(d*x+c)+3/8/d*B*a^4*c+8/15/d*A*a^4*
sin(d*x+c)+1/d*B*b^4*c+4*C*a*b^3*x+1/3/d*C*sin(d*x+c)*cos(d*x+c)^2*a^4+1/4/
d*B*a^4*sin(d*x+c)*cos(d*x+c)^3+1/5/d*A*a^4*sin(d*x+c)*cos(d*x+c)^4+6/d*C*a
^2*b^2*sin(d*x+c)+4/d*a*b^3*B*sin(d*x+c)+3/8*B*a^4*x+3/2*a^3*A*b*x
```

Maxima [A] time = 1.06217, size = 468, normalized size = 1.49

$$\frac{32\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)Aa^4+15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ba^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 160*(sin
(d*x + c)^3 - 3*sin(d*x + c))*C*a^4 + 60*(12*d*x + 12*c + sin(4*d*x + 4*c)
+ 8*sin(2*d*x + 2*c))*A*a^3*b - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3
*b + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^3*b - 960*(sin(d*x + c)^3 - 3
*sin(d*x + c))*A*a^2*b^2 + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b^2 +
 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^3 + 1920*(d*x + c)*C*a*b^3 + 48
0*(d*x + c)*B*b^4 + 240*C*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 2880*C*a^2*b^2*sin(d*x + c) + 1920*B*a*b^3*sin(d*x + c) + 480*A*b^4*si
n(d*x + c))/d
```

Fricas [A] time = 0.612538, size = 640, normalized size = 2.04

$$\frac{60\,Cb^4\log(\sin(dx+c)+1)-60\,Cb^4\log(-\sin(dx+c)+1)+15\left(3\,Ba^4+4\,(3\,A+4\,C)a^3b+24\,Ba^2b^2+16\,(A+2\,C)ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/120*(60*C*b^4*log(sin(d*x + c) + 1) - 60*C*b^4*log(-sin(d*x + c) + 1) + 1
5*(3*B*a^4 + 4*(3*A + 4*C)*a^3*b + 24*B*a^2*b^2 + 16*(A + 2*C)*a*b^3 + 8*B*
b^4)*d*x + (24*A*a^4*cos(d*x + c)^4 + 16*(4*A + 5*C)*a^4 + 320*B*a^3*b + 24
0*(2*A + 3*C)*a^2*b^2 + 480*B*a*b^3 + 120*A*b^4 + 30*(B*a^4 + 4*A*a^3*b)*co
s(d*x + c)^3 + 8*((4*A + 5*C)*a^4 + 20*B*a^3*b + 30*A*a^2*b^2)*cos(d*x + c)
^2 + 15*(3*B*a^4 + 4*(3*A + 4*C)*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41457, size = 1477, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/120*(120*C*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 120*C*b^4*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 15*(3*B*a^4 + 12*A*a^3*b + 16*C*a^3*b + 24*B*a^2*
b^2 + 16*A*a*b^3 + 32*C*a*b^3 + 8*B*b^4)*(d*x + c) + 2*(120*A*a^4*tan(1/2*d
*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 120*C*a^4*tan(1/2*d*x + 1
/2*c)^9 - 300*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/
2*c)^9 - 240*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 720*C*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^9 - 240*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x +
 1/2*c)^9 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^4*tan(1/2*d*x + 1/2*
c)^7 - 30*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 320*C*a^4*tan(1/2*d*x + 1/2*c)^7 -
 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 1280*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 -
 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7
 - 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 2880*C*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1920*B*a*b^3*tan(1/2*d*x + 1/2*
c)^7 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*tan(1/2*d*x + 1/2*c)^5
+ 400*C*a^4*tan(1/2*d*x + 1/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
2400*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4320*C*a^2*b^2*tan(1/2*d*x + 1/2*c)
^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*tan(1/2*d*x + 1/2*c)^5
 + 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 320
*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 1280*B
*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 480*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 1920*A
```

*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 28
80*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 +
1920*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 12
0*A*a^4*tan(1/2*d*x + 1/2*c) + 75*B*a^4*tan(1/2*d*x + 1/2*c) + 120*C*a^4*ta
n(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1/2*d*x + 1/2*c) + 480*B*a^3*b*tan(1/2
*d*x + 1/2*c) + 240*C*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*d*
x + 1/2*c) + 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*C*a^2*b^2*tan(1/2*d*x
 + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x + 1/
2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d

3.895 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\text{se}\right.$

Optimal. Leaf size=372

$$\frac{\sin(c+dx)\left(8a^3b(4A+5C)+60a^2b^2B+8a^4B+20ab^3(2A+3C)+15b^4B\right)}{15d}+\frac{a\sin(c+dx)\cos^2(c+dx)\left(a^2b(39A+\right.}{60d}$$

```
[Out] ((24*a^3*b*B + 32*a*b^3*B + 8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*
(5*A + 6*C))*x)/16 + ((8*a^4*B + 60*a^2*b^2*B + 15*b^4*B + 20*a*b^3*(2*A +
3*C) + 8*a^3*b*(4*A + 5*C))*Sin[c + d*x])/(15*d) + ((24*A*b^4 + 360*a^3*b*B
 + 336*a*b^3*B + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(49*A + 66*C))*Cos[c + d*x
]*Sin[c + d*x])/(240*d) + (a*(4*A*b^3 + 16*a^3*B + 36*a*b^2*B + a^2*b*(39*A
 + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])/(60*d) + ((12*A*b^2 + 48*a*b*B + 5*a
^2*(5*A + 6*C))*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(120*d)
 + ((2*A*b + 3*a*B)*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(15
*d) + (A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 1.2084, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4074, 4047, 2637, 4045, 8}

$$\frac{\sin(c+dx)\left(8a^3b(4A+5C)+60a^2b^2B+8a^4B+20ab^3(2A+3C)+15b^4B\right)}{15d}+\frac{a\sin(c+dx)\cos^2(c+dx)\left(a^2b(39A+\right.}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((24*a^3*b*B + 32*a*b^3*B + 8*b^4*(A + 2*C) + 12*a^2*b^2*(3*A + 4*C) + a^4*
(5*A + 6*C))*x)/16 + ((8*a^4*B + 60*a^2*b^2*B + 15*b^4*B + 20*a*b^3*(2*A +
3*C) + 8*a^3*b*(4*A + 5*C))*Sin[c + d*x])/(15*d) + ((24*A*b^4 + 360*a^3*b*B
 + 336*a*b^3*B + 15*a^4*(5*A + 6*C) + 10*a^2*b^2*(49*A + 66*C))*Cos[c + d*x
]*Sin[c + d*x])/(240*d) + (a*(4*A*b^3 + 16*a^3*B + 36*a*b^2*B + a^2*b*(39*A
 + 50*C))*Cos[c + d*x]^2*Sin[c + d*x])/(60*d) + ((12*A*b^2 + 48*a*b*B + 5*a
^2*(5*A + 6*C))*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(120*d)
 + ((2*A*b + 3*a*B)*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(15
*d) + (A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(6*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^5(c+dx)(a+b\sec(c+dx))^4\sin(c+}{6d}$$
$$= \frac{(2Ab+3aB)\cos^4(c+dx)(a+b\sec(c+dx}{15d}$$
$$= \frac{\left(12Ab^2+48abB+5a^2(5A+6C)\right)\cos^3(c}{120d}$$
$$= \frac{a\left(4Ab^3+16a^3B+36ab^2B+a^2b(39A+5\right.}{60d}$$
$$= \frac{a\left(4Ab^3+16a^3B+36ab^2B+a^2b(39A+5\right.}{60d}$$
$$= \frac{\left(8a^4B+60a^2b^2B+15b^4B+20ab^3(2A+3\right.}{15d}$$
$$= \frac{1}{16}\left(24a^3bB+32ab^3B+8b^4(A+2C)+12\right.$$

Mathematica [A] time = 1.65, size = 432, normalized size = 1.16

$$\frac{120\sin(c+dx)\left(4a^3b(5A+6C)+36a^2b^2B+5a^4B+8ab^3(3A+4C)+8b^4B\right)+15\sin(2(c+dx))\left(96a^2b^2(A+C)+a^4(15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (300*a^4*A*c + 2160*a^2*A*b^2*c + 480*A*b^4*c + 1440*a^3*b*B*c + 1920*a*b^3
*B*c + 360*a^4*c*C + 2880*a^2*b^2*c*C + 960*b^4*c*C + 300*a^4*A*d*x + 2160*
a^2*A*b^2*d*x + 480*A*b^4*d*x + 1440*a^3*b*B*d*x + 1920*a*b^3*B*d*x + 360*a
```

```
^4*C*d*x + 2880*a^2*b^2*C*d*x + 960*b^4*C*d*x + 120*(5*a^4*B + 36*a^2*b^2*B
 + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6*C))*Sin[c + d*x] + 15*(
16*A*b^4 + 64*a^3*b*B + 64*a*b^3*B + 96*a^2*b^2*(A + C) + a^4*(15*A + 16*C)
)*Sin[2*(c + d*x)] + 400*a^3*A*b*Sin[3*(c + d*x)] + 320*a*A*b^3*Sin[3*(c +
d*x)] + 100*a^4*B*Sin[3*(c + d*x)] + 480*a^2*b^2*B*Sin[3*(c + d*x)] + 320*a
^3*b*C*Sin[3*(c + d*x)] + 45*a^4*A*Sin[4*(c + d*x)] + 180*a^2*A*b^2*Sin[4*(
c + d*x)] + 120*a^3*b*B*Sin[4*(c + d*x)] + 30*a^4*C*Sin[4*(c + d*x)] + 48*a
^3*A*b*Sin[5*(c + d*x)] + 12*a^4*B*Sin[5*(c + d*x)] + 5*a^4*A*Sin[6*(c + d*
x)])/(960*d)
```

Maple [A] time = 0.09, size = 431, normalized size = 1.2

$$\frac{1}{d}\left(Aa^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{4\,Aa^3b\sin(dx+c)}{5}\left(\frac{8}{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+4/5*A*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
+1/5*B*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*A*a^2*b^2*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*B*a^3*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^4*C*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a*b^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+2*a^2*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*a^3*b*C*(2+cos(d*x+c)^
2)*sin(d*x+c)+A*b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a*b^3*B*(1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*C*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+B*b^4*sin(d*x+c)+4*C*a*b^3*sin(d*x+c)+C*b^4*(d*x+c))
```

Maxima [A] time = 1.10912, size = 560, normalized size = 1.51

$$-\frac{5\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)Aa^4-64\left(3\,\sin(dx+c)^5-10\,\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^4 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*B*a^4 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*C*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^
3*b - 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3*b +
 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*C*a^3*b - 180*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b^2 + 1920*(sin(d*x + c)^3 - 3*sin
(d*x + c))*B*a^2*b^2 - 1440*(2*d*x + 2*c + sin(2*d*x + 2*c))*C*a^2*b^2 + 12
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^3 - 960*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a*b^3 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 960*(d*x +
c)*C*b^4 - 3840*C*a*b^3*sin(d*x + c) - 960*B*b^4*sin(d*x + c))/d
```

Fricas [A] time = 0.59473, size = 698, normalized size = 1.88

$$\frac{15\left((5\,A+6\,C)a^4+24\,Ba^3b+12\,(3\,A+4\,C)a^2b^2+32\,Bab^3+8\,(A+2\,C)b^4\right)dx+\left(40\,Aa^4\cos\left(dx+c\right)^5+128\,Ba^4+128\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/240*(15*((5*A + 6*C)*a^4 + 24*B*a^3*b + 12*(3*A + 4*C)*a^2*b^2 + 32*B*a*b
^3 + 8*(A + 2*C)*b^4)*d*x + (40*A*a^4*cos(d*x + c)^5 + 128*B*a^4 + 128*(4*A
 + 5*C)*a^3*b + 960*B*a^2*b^2 + 320*(2*A + 3*C)*a*b^3 + 240*B*b^4 + 48*(B*a
^4 + 4*A*a^3*b)*cos(d*x + c)^4 + 10*((5*A + 6*C)*a^4 + 24*B*a^3*b + 36*A*a^
2*b^2)*cos(d*x + c)^3 + 32*(2*B*a^4 + 2*(4*A + 5*C)*a^3*b + 15*B*a^2*b^2 +
10*A*a*b^3)*cos(d*x + c)^2 + 15*((5*A + 6*C)*a^4 + 24*B*a^3*b + 12*(3*A + 4
*C)*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46198, size = 2130, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 6*C*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 48*C*a^2*b^2 + 3
2*B*a*b^3 + 8*A*b^4 + 16*C*b^4)*(d*x + c) - 2*(165*A*a^4*tan(1/2*d*x + 1/2*
c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 150*C*a^4*tan(1/2*d*x + 1/2*c)^
11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2*c)
^11 - 960*C*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 720*C*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*
x + 1/2*c)^11 - 960*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*A*b^4*tan(1/2*d*x
 + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 - 25*A*a^4*tan(1/2*d*x + 1
/2*c)^9 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 210*C*a^4*tan(1/2*d*x + 1/2*c)
^9 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 840*B*a^3*b*tan(1/2*d*x + 1/2*c)
^9 - 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 1260*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^9 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 2160*C*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^9 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1440*B*a*b^3*tan(1/2*d*x
 + 1/2*c)^9 - 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 360*A*b^4*tan(1/2*d*x +
 1/2*c)^9 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2
```

*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 60*C*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9600*C*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^4*tan(1/2*d*x + 1/2*c)^5 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 5760*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9600*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 210*C*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 3520*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2160*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 4800*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*A*a^4*tan(1/2*d*x + 1/2*c) - 240*B*a^4*tan(1/2*d*x + 1/2*c) - 150*C*a^4*tan(1/2*d*x + 1/2*c) - 960*A*a^3*b*tan(1/2*d*x + 1/2*c) - 600*B*a^3*b*tan(1/2*d*x + 1/2*c) - 960*C*a^3*b*tan(1/2*d*x + 1/2*c) - 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 720*C*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*A*a*b^3*tan(1/2*d*x + 1/2*c) - 480*B*a*b^3*tan(1/2*d*x + 1/2*c) - 960*C*a*b^3*tan(1/2*d*x + 1/2*c) - 120*A*b^4*tan(1/2*d*x + 1/2*c) - 240*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d

3.896 $\int \cos^7(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=438

$$-\frac{\sin^3(c+dx)\left(3a^2b^2(50A+63C)+4a^4(6A+7C)+112a^3bB+91ab^3B+4Ab^4\right)}{105d}+\frac{\sin(c+dx)\left(3a^2b^2(162A+203C)+\right.}{}$$

```
[Out] ((5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6
*C))*x)/16 + ((336*a^3*b*B + 371*a*b^3*B + 12*a^4*(6*A + 7*C) + b^4*(74*A +
 105*C) + 3*a^2*b^2*(162*A + 203*C))*Sin[c + d*x])/(105*d) + ((5*a^4*B + 36
*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6*C))*Cos[c + d
*x]*Sin[c + d*x])/(16*d) + (a*(24*A*b^3 + 175*a^3*B + 336*a*b^2*B + a^2*(41
2*A*b + 504*b*C))*Cos[c + d*x]^3*Sin[c + d*x])/(840*d) + ((4*A*b^2 + 21*a*b
*B + 2*a^2*(6*A + 7*C))*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d*x])
/(70*d) + ((4*A*b + 7*a*B)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*Sin[c + d*
x])/(42*d) + (A*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(7*d) -
 ((4*A*b^4 + 112*a^3*b*B + 91*a*b^3*B + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A
 + 63*C))*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 1.38427, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4094, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{\sin^3(c+dx)\left(3a^2b^2(50A+63C)+4a^4(6A+7C)+112a^3bB+91ab^3B+4Ab^4\right)}{105d}+\frac{\sin(c+dx)\left(3a^2b^2(162A+203C)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((5*a^4*B + 36*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6
*C))*x)/16 + ((336*a^3*b*B + 371*a*b^3*B + 12*a^4*(6*A + 7*C) + b^4*(74*A +
 105*C) + 3*a^2*b^2*(162*A + 203*C))*Sin[c + d*x])/(105*d) + ((5*a^4*B + 36
*a^2*b^2*B + 8*b^4*B + 8*a*b^3*(3*A + 4*C) + 4*a^3*b*(5*A + 6*C))*Cos[c + d
*x]*Sin[c + d*x])/(16*d) + (a*(24*A*b^3 + 175*a^3*B + 336*a*b^2*B + a^2*(41
2*A*b + 504*b*C))*Cos[c + d*x]^3*Sin[c + d*x])/(840*d) + ((4*A*b^2 + 21*a*b
*B + 2*a^2*(6*A + 7*C))*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d*x])
/(70*d) + ((4*A*b + 7*a*B)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*Sin[c + d*
x])/(42*d) + (A*Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(7*d) -
 ((4*A*b^4 + 112*a^3*b*B + 91*a*b^3*B + 4*a^4*(6*A + 7*C) + 3*a^2*b^2*(50*A
 + 63*C))*Sin[c + d*x]^3)/(105*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^6(c+dx)(a+b\sec(c+dx))^4\sin(}{7d}$$

$$= \frac{(4Ab+7aB)\cos^5(c+dx)(a+b\sec(c+}{42d}$$

$$= \frac{\left(4Ab^2+21abB+2a^2(6A+7C)\right)\cos^4(}{70}$$

$$= \frac{a\left(24Ab^3+175a^3B+336ab^2B+a^2(41}{84}$$

$$= \frac{a\left(24Ab^3+175a^3B+336ab^2B+a^2(41}{84}$$

$$= \frac{\left(5a^4B+36a^2b^2B+8b^4B+8ab^3(3A+\right.}{}$$

$$= \frac{1}{16}\left(5a^4B+36a^2b^2B+8b^4B+8ab^3(3A\right.$$

$$= \frac{1}{16}\left(5a^4B+36a^2b^2B+8b^4B+8ab^3(3A\right.$$

Mathematica [A] time = 1.59563, size = 528, normalized size = 1.21

$$\frac{105\sin(c+dx)\left(48a^2b^2(5A+6C)+5a^4(7A+8C)+160a^3bB+192ab^3B+16b^4(3A+4C)\right)+105\sin(2(c+dx))\left(a^3(60A\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (8400*a^3*A*b*c + 10080*a*A*b^3*c + 2100*a^4*B*c + 15120*a^2*b^2*B*c + 3360
*b^4*B*c + 10080*a^3*b*c*C + 13440*a*b^3*c*C + 8400*a^3*A*b*d*x + 10080*a*A
*b^3*d*x + 2100*a^4*B*d*x + 15120*a^2*b^2*B*d*x + 3360*b^4*B*d*x + 10080*a^
3*b*C*d*x + 13440*a*b^3*C*d*x + 105*(160*a^3*b*B + 192*a*b^3*B + 16*b^4*(3*
A + 4*C) + 48*a^2*b^2*(5*A + 6*C) + 5*a^4*(7*A + 8*C))*Sin[c + d*x] + 105*(
15*a^4*B + 96*a^2*b^2*B + 16*b^4*B + 64*a*b^3*(A + C) + a^3*(60*A*b + 64*b*
C))*Sin[2*(c + d*x)] + 735*a^4*A*Sin[3*(c + d*x)] + 4200*a^2*A*b^2*Sin[3*(c
 + d*x)] + 560*A*b^4*Sin[3*(c + d*x)] + 2800*a^3*b*B*Sin[3*(c + d*x)] + 224
0*a*b^3*B*Sin[3*(c + d*x)] + 700*a^4*C*Sin[3*(c + d*x)] + 3360*a^2*b^2*C*Si
n[3*(c + d*x)] + 1260*a^3*A*b*Sin[4*(c + d*x)] + 840*a*A*b^3*Sin[4*(c + d*x
)] + 315*a^4*B*Sin[4*(c + d*x)] + 1260*a^2*b^2*B*Sin[4*(c + d*x)] + 840*a^3
*b*C*Sin[4*(c + d*x)] + 147*a^4*A*Sin[5*(c + d*x)] + 504*a^2*A*b^2*Sin[5*(c
 + d*x)] + 336*a^3*b*B*Sin[5*(c + d*x)] + 84*a^4*C*Sin[5*(c + d*x)] + 140*a
^3*A*b*Sin[6*(c + d*x)] + 35*a^4*B*Sin[6*(c + d*x)] + 15*a^4*A*Sin[7*(c + d
*x)])/(6720*d)
```

Maple [A] time = 0.097, size = 505, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/d*(1/7*A*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)+B*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*a^4*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
*A*a^3*b*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*B*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
*a^3*b*C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*A
*a^2*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^2*b^2*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*C*a^2*b^2*(2+cos(d*
x+c)^2)*sin(d*x+c)+4*A*a*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/8*d*x+3/8*c)+4/3*a*b^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4*C*a*b^3*(1/2*cos(d
*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*A*b^4*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b^4
*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+C*b^4*sin(d*x+c))
```

Maxima [A] time = 1.07809, size = 672, normalized size = 1.53

$$-\frac{192\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)Aa^4+35\left(4\sin(2dx+2c)^3-60dx-60c\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] -1/6720*(192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*A*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d
*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 - 448*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*C*a^4 + 140*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*
c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3*b - 1792*(3*sin(d*x + c
)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3*b - 840*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*C*a^3*b - 2688*(3*sin(d*x + c)^5 - 10
*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2*b^2 - 1260*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b^2 + 13440*(sin(d*x + c)^3 - 3*sin(
d*x + c))*C*a^2*b^2 - 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
 2*c))*A*a*b^3 + 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^3 - 6720*(2*d
*x + 2*c + sin(2*d*x + 2*c))*C*a*b^3 + 2240*(sin(d*x + c)^3 - 3*sin(d*x + c
))*A*b^4 - 1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^4 - 6720*C*b^4*sin(d*x
 + c))/d
```

Fricas [A] time = 0.644114, size = 852, normalized size = 1.95

$$\frac{105\left(5\,Ba^4+4\,(5\,A+6\,C)a^3b+36\,Ba^2b^2+8\,(3\,A+4\,C)ab^3+8\,Bb^4\right)dx+\left(240\,Aa^4\cos\left(dx+c\right)^6+280\left(Ba^4+4\,Aa^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/1680*(105*(5*B*a^4 + 4*(5*A + 6*C)*a^3*b + 36*B*a^2*b^2 + 8*(3*A + 4*C)*a
*b^3 + 8*B*b^4)*d*x + (240*A*a^4*cos(d*x + c)^6 + 280*(B*a^4 + 4*A*a^3*b)*c
os(d*x + c)^5 + 128*(6*A + 7*C)*a^4 + 3584*B*a^3*b + 1344*(4*A + 5*C)*a^2*b
^2 + 4480*B*a*b^3 + 560*(2*A + 3*C)*b^4 + 48*((6*A + 7*C)*a^4 + 28*B*a^3*b
+ 42*A*a^2*b^2)*cos(d*x + c)^4 + 70*(5*B*a^4 + 4*(5*A + 6*C)*a^3*b + 36*B*a
^2*b^2 + 24*A*a*b^3)*cos(d*x + c)^3 + 16*(4*(6*A + 7*C)*a^4 + 112*B*a^3*b +
 42*(4*A + 5*C)*a^2*b^2 + 140*B*a*b^3 + 35*A*b^4)*cos(d*x + c)^2 + 105*(5*B
*a^4 + 4*(5*A + 6*C)*a^3*b + 36*B*a^2*b^2 + 8*(3*A + 4*C)*a*b^3 + 8*B*b^4)*
cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50933, size = 2450, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] 1/1680*(105*(5*B*a^4 + 20*A*a^3*b + 24*C*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3
+ 32*C*a*b^3 + 8*B*b^4)*(d*x + c) + 2*(1680*A*a^4*tan(1/2*d*x + 1/2*c)^13 -
 1155*B*a^4*tan(1/2*d*x + 1/2*c)^13 + 1680*C*a^4*tan(1/2*d*x + 1/2*c)^13 -
4620*A*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 6720*B*a^3*b*tan(1/2*d*x + 1/2*c)^13
 - 4200*C*a^3*b*tan(1/2*d*x + 1/2*c)^13 + 10080*A*a^2*b^2*tan(1/2*d*x + 1/2
*c)^13 - 6300*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 10080*C*a^2*b^2*tan(1/2*d
*x + 1/2*c)^13 - 4200*A*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 6720*B*a*b^3*tan(1/
2*d*x + 1/2*c)^13 - 3360*C*a*b^3*tan(1/2*d*x + 1/2*c)^13 + 1680*A*b^4*tan(1
/2*d*x + 1/2*c)^13 - 840*B*b^4*tan(1/2*d*x + 1/2*c)^13 + 1680*C*b^4*tan(1/2
*d*x + 1/2*c)^13 + 3360*A*a^4*tan(1/2*d*x + 1/2*c)^11 - 980*B*a^4*tan(1/2*d
*x + 1/2*c)^11 + 5600*C*a^4*tan(1/2*d*x + 1/2*c)^11 - 3920*A*a^3*b*tan(1/2*
d*x + 1/2*c)^11 + 22400*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 - 10080*C*a^3*b*tan
(1/2*d*x + 1/2*c)^11 + 33600*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 15120*B*a^
2*b^2*tan(1/2*d*x + 1/2*c)^11 + 47040*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 1
0080*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 31360*B*a*b^3*tan(1/2*d*x + 1/2*c)^1
1 - 13440*C*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 7840*A*b^4*tan(1/2*d*x + 1/2*c)
^11 - 3360*B*b^4*tan(1/2*d*x + 1/2*c)^11 + 10080*C*b^4*tan(1/2*d*x + 1/2*c)
^11 + 14448*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 2975*B*a^4*tan(1/2*d*x + 1/2*c)^
9 + 12656*C*a^4*tan(1/2*d*x + 1/2*c)^9 - 11900*A*a^3*b*tan(1/2*d*x + 1/2*c)
^9 + 50624*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 7560*C*a^3*b*tan(1/2*d*x + 1/2*
c)^9 + 75936*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 11340*B*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^9 + 97440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 7560*A*a*b^3*tan(1/2
*d*x + 1/2*c)^9 + 64960*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 16800*C*a*b^3*tan(
1/2*d*x + 1/2*c)^9 + 16240*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 4200*B*b^4*tan(1/
2*d*x + 1/2*c)^9 + 25200*C*b^4*tan(1/2*d*x + 1/2*c)^9 + 10176*A*a^4*tan(1/2
*d*x + 1/2*c)^7 + 17472*C*a^4*tan(1/2*d*x + 1/2*c)^7 + 69888*B*a^3*b*tan(1/
2*d*x + 1/2*c)^7 + 104832*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 120960*C*a^2*b
^2*tan(1/2*d*x + 1/2*c)^7 + 80640*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 20160*A*
b^4*tan(1/2*d*x + 1/2*c)^7 + 33600*C*b^4*tan(1/2*d*x + 1/2*c)^7 + 14448*A*a
^4*tan(1/2*d*x + 1/2*c)^5 + 2975*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 12656*C*a^4
*tan(1/2*d*x + 1/2*c)^5 + 11900*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 50624*B*a^
3*b*tan(1/2*d*x + 1/2*c)^5 + 7560*C*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 75936*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 11340*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 9
7440*C*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 7560*A*a*b^3*tan(1/2*d*x + 1/2*c)^5
 + 64960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 16800*C*a*b^3*tan(1/2*d*x + 1/2*c
)^5 + 16240*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 4200*B*b^4*tan(1/2*d*x + 1/2*c)^
5 + 25200*C*b^4*tan(1/2*d*x + 1/2*c)^5 + 3360*A*a^4*tan(1/2*d*x + 1/2*c)^3
+ 980*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 5600*C*a^4*tan(1/2*d*x + 1/2*c)^3 + 39
20*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 22400*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 +
10080*C*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 33600*A*a^2*b^2*tan(1/2*d*x + 1/2*c)
^3 + 15120*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 47040*C*a^2*b^2*tan(1/2*d*x +
 1/2*c)^3 + 10080*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 31360*B*a*b^3*tan(1/2*d*
x + 1/2*c)^3 + 13440*C*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 7840*A*b^4*tan(1/2*d*
x + 1/2*c)^3 + 3360*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 10080*C*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 1680*A*a^4*tan(1/2*d*x + 1/2*c) + 1155*B*a^4*tan(1/2*d*x + 1/2
*c) + 1680*C*a^4*tan(1/2*d*x + 1/2*c) + 4620*A*a^3*b*tan(1/2*d*x + 1/2*c) +
 6720*B*a^3*b*tan(1/2*d*x + 1/2*c) + 4200*C*a^3*b*tan(1/2*d*x + 1/2*c) + 10
080*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6300*B*a^2*b^2*tan(1/2*d*x + 1/2*c) +
10080*C*a^2*b^2*tan(1/2*d*x + 1/2*c) + 4200*A*a*b^3*tan(1/2*d*x + 1/2*c) +
6720*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3360*C*a*b^3*tan(1/2*d*x + 1/2*c) + 168
0*A*b^4*tan(1/2*d*x + 1/2*c) + 840*B*b^4*tan(1/2*d*x + 1/2*c) + 1680*C*b^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.897 $\int(a+b\sec(c+dx))^3\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\mathrm{s}\right.$

Optimal. Leaf size=214

$$\frac{b^2\left(34a^2bB-15a^3C+12ab^2C+4b^3B\right)\tan(c+dx)}{6d}+\frac{b\left(8a^2b^2C+32a^3bB-24a^4C+16ab^3B+3b^4C\right)\tanh^{-1}(\sin(c+$$

```
[Out] a^4*(b*B - a*C)*x + (b*(32*a^3*b*B + 16*a*b^3*B - 24*a^4*C + 8*a^2*b^2*C +
3*b^4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (b^2*(34*a^2*b*B + 4*b^3*B - 15*a^3
*C + 12*a*b^2*C)*Tan[c + d*x])/(6*d) + (b^3*(32*a*b*B - 6*a^2*C + 9*b^2*C)*
Sec[c + d*x]*Tan[c + d*x])/(24*d) + (b^2*(4*b*B + 3*a*C)*(a + b*Sec[c + d*x
])^2*Tan[c + d*x])/(12*d) + (b^2*C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*
d)
```

Rubi [A] time = 0.476074, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4041, 3918, 4056, 4048, 3770, 3767, 8}

$$\frac{b^2\left(34a^2bB-15a^3C+12ab^2C+4b^3B\right)\tan(c+dx)}{6d}+\frac{b\left(8a^2b^2C+32a^3bB-24a^4C+16ab^3B+3b^4C\right)\tanh^{-1}(\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[
c + d*x]^2),x]
```


```
[Out] a^4*(b*B - a*C)*x + (b*(32*a^3*b*B + 16*a*b^3*B - 24*a^4*C + 8*a^2*b^2*C +
3*b^4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + (b^2*(34*a^2*b*B + 4*b^3*B - 15*a^3
*C + 12*a*b^2*C)*Tan[c + d*x])/(6*d) + (b^3*(32*a*b*B - 6*a^2*C + 9*b^2*C)*
Sec[c + d*x]*Tan[c + d*x])/(24*d) + (b^2*(4*b*B + 3*a*C)*(a + b*Sec[c + d*x
])^2*Tan[c + d*x])/(12*d) + (b^2*C*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*
d)
```


Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right)dx=\frac{\int (a+b\sec(c+dx))^4\left(b^2(bB-aC)+b^3C\right.}{b^2}$$
$$=\frac{b^2C(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\int(\ldots$$
$$=\frac{b^2(4bB+3aC)(a+b\sec(c+dx))^2\tan(c+\ldots}{12d}$$
$$=\frac{b^3\left(32abB-6a^2C+9b^2C\right)\sec(c+dx)\tan\ldots}{24d}$$
$$=a^4(bB-aC)x+\frac{b^3\left(32abB-6a^2C+9b^2C\ldots\right.}{24\ldots}$$
$$=a^4(bB-aC)x+\frac{b\left(32a^3bB+16ab^3B-24a\ldots\right.}{\ldots}$$
$$=a^4(bB-aC)x+\frac{b\left(32a^3bB+16ab^3B-24a\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.27378, size = 170, normalized size = 0.79

$$\frac{3b\left(8a^2b^2C+32a^3bB-24a^4C+16ab^3B+3b^4C\right)\tanh^{-1}(\sin(c+dx))+3b^2\tan(c+dx)\left(b\left(8a^2C+16abB+3b^2C\right)\sec(c\ldots\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*
C*Sec[c + d*x]^2),x]
```

```
[Out] (24*a^4*(b*B - a*C)*d*x + 3*b*(32*a^3*b*B + 16*a*b^3*B - 24*a^4*C + 8*a^2*b
^2*C + 3*b^4*C)*ArcTanh[Sin[c + d*x]] + 3*b^2*(8*(6*a^2*b*B + b^3*B - 2*a^3
```

```
*C + 3*a*b^2*C) + b*(16*a*b*B + 8*a^2*C + 3*b^2*C)*Sec[c + d*x] + 2*b^3*C*S
ec[c + d*x]^3)*Tan[c + d*x] + 8*b^4*(b*B + 3*a*C)*Tan[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.059, size = 360, normalized size = 1.7

$$Ba^4bx + \frac{Ba^4bc}{d} - a^5Cx - \frac{Ca^5c}{d} + 4\,\frac{Ba^3b^2\ln(\sec(dx+c)+\tan(dx+c))}{d} - 2\,\frac{a^3b^2C\tan(dx+c)}{d} - 3\,\frac{a^4bC\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2),x)
```

```
[Out] B*a^4*b*x+1/d*B*a^4*b*c-a^5*C*x-1/d*C*a^5*c+4/d*B*a^3*b^2*ln(sec(d*x+c)+tan
(d*x+c))-2/d*a^3*b^2*C*tan(d*x+c)-3/d*a^4*b*C*ln(sec(d*x+c)+tan(d*x+c))+6/d
*B*a^2*b^3*tan(d*x+c)+1/d*C*a^2*b^3*sec(d*x+c)*tan(d*x+c)+1/d*C*a^2*b^3*ln(
sec(d*x+c)+tan(d*x+c))+2/d*B*a*b^4*sec(d*x+c)*tan(d*x+c)+2/d*B*a*b^4*ln(sec
(d*x+c)+tan(d*x+c))+2/d*b^4*C*a*tan(d*x+c)+1/d*b^4*C*a*tan(d*x+c)*sec(d*x+c
)^2+2/3/d*B*b^5*tan(d*x+c)+1/3/d*B*b^5*tan(d*x+c)*sec(d*x+c)^2+1/4/d*C*b^5*
tan(d*x+c)*sec(d*x+c)^3+3/8/d*C*b^5*sec(d*x+c)*tan(d*x+c)+3/8/d*C*b^5*ln(se
c(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.10925, size = 432, normalized size = 2.02

$$-\frac{48\,(dx+c)Ca^5 - 48\,(dx+c)Ba^4b - 48\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Cab^4 - 16\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] -1/48*(48*(d*x + c)*C*a^5 - 48*(d*x + c)*B*a^4*b - 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*C*a*b^4 - 16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^5 + 3*C*b^5
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 24*C*a^2*b^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 48*B*a*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) + 144*C*a^4*b*log(sec(d*x + c) + tan(d*x +
 c)) - 192*B*a^3*b^2*log(sec(d*x + c) + tan(d*x + c)) + 96*C*a^3*b^2*tan(d*
x + c) - 288*B*a^2*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.59161, size = 633, normalized size = 2.96

$$-\frac{48\left(Ca^5 - Ba^4b\right)dx\cos(dx+c)^4 + 3\left(24\,Ca^4b - 32\,Ba^3b^2 - 8\,Ca^2b^3 - 16\,Bab^4 - 3\,Cb^5\right)\cos(dx+c)^4\log(\sin(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] -1/48*(48*(C*a^5 - B*a^4*b)*d*x*cos(d*x + c)^4 + 3*(24*C*a^4*b - 32*B*a^3*b
^2 - 8*C*a^2*b^3 - 16*B*a*b^4 - 3*C*b^5)*cos(d*x + c)^4*log(sin(d*x + c) +
1) - 3*(24*C*a^4*b - 32*B*a^3*b^2 - 8*C*a^2*b^3 - 16*B*a*b^4 - 3*C*b^5)*cos
(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(6*C*b^5 - 16*(3*C*a^3*b^2 - 9*B*a^2
*b^3 - 3*C*a*b^4 - B*b^5)*cos(d*x + c)^3 + 3*(8*C*a^2*b^3 + 16*B*a*b^4 + 3*
C*b^5)*cos(d*x + c)^2 + 8*(3*C*a*b^4 + B*b^5)*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int Ca^5\,dx - \int -Ba^4b\,dx - \int -Bb^5\sec^4(c+dx)\,dx - \int -Cb^5\sec^5(c+dx)\,dx - \int -4Bab^4\sec^3(c+dx)\,dx - \int -6Ba^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*
x+c)**2),x)
```

```
[Out] -Integral(C*a**5, x) - Integral(-B*a**4*b, x) - Integral(-B*b**5*sec(c + d*
x)**4, x) - Integral(-C*b**5*sec(c + d*x)**5, x) - Integral(-4*B*a*b**4*sec
(c + d*x)**3, x) - Integral(-6*B*a**2*b**3*sec(c + d*x)**2, x) - Integral(-
4*B*a**3*b**2*sec(c + d*x), x) - Integral(-3*C*a*b**4*sec(c + d*x)**4, x) -
 Integral(-2*C*a**2*b**3*sec(c + d*x)**3, x) - Integral(2*C*a**3*b**2*sec(c
 + d*x)**2, x) - Integral(3*C*a**4*b*sec(c + d*x), x)
```

Giac [B] time = 1.37162, size = 888, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] -1/24*(24*(C*a^5 - B*a^4*b)*(d*x + c) + 3*(24*C*a^4*b - 32*B*a^3*b^2 - 8*C*
a^2*b^3 - 16*B*a*b^4 - 3*C*b^5)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(24*
C*a^4*b - 32*B*a^3*b^2 - 8*C*a^2*b^3 - 16*B*a*b^4 - 3*C*b^5)*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(48*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b
^3*tan(1/2*d*x + 1/2*c)^7 + 24*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 48*B*a*b^
4*tan(1/2*d*x + 1/2*c)^7 - 72*C*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*b^5*tan
(1/2*d*x + 1/2*c)^7 + 15*C*b^5*tan(1/2*d*x + 1/2*c)^7 - 144*C*a^3*b^2*tan(1
/2*d*x + 1/2*c)^5 + 432*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*C*a^2*b^3*tan
(1/2*d*x + 1/2*c)^5 - 48*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 120*C*a*b^4*tan(1
/2*d*x + 1/2*c)^5 + 40*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 9*C*b^5*tan(1/2*d*x +
 1/2*c)^5 + 144*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^3*tan(1/2*d*
x + 1/2*c)^3 - 24*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*B*a*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 120*C*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 40*B*b^5*tan(1/2*d*x + 1
/2*c)^3 + 9*C*b^5*tan(1/2*d*x + 1/2*c)^3 - 48*C*a^3*b^2*tan(1/2*d*x + 1/2*c
) + 144*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 24*C*a^2*b^3*tan(1/2*d*x + 1/2*c)
+ 48*B*a*b^4*tan(1/2*d*x + 1/2*c) + 72*C*a*b^4*tan(1/2*d*x + 1/2*c) + 24*B*
b^5*tan(1/2*d*x + 1/2*c) + 15*C*b^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

3.898 $\int (a+b\sec(c+dx))^2 \left(abB - a^2C + b^2B\sec(c+dx) + b^2C\,\mathrm{s}\right.$

Optimal. Leaf size=149

$$\frac{b^2\left(a^2(-C)+9abB+2b^2C\right)\tan(c+dx)}{3d}+\frac{b\left(6a^2bB-4a^3C+2ab^2C+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x(bB-aC)+\frac{b^3}{}$$

```
[Out] a^3*(b*B - a*C)*x + (b*(6*a^2*b*B + b^3*B - 4*a^3*C + 2*a*b^2*C)*ArcTanh[Si
n[c + d*x]])/(2*d) + (b^2*(9*a*b*B - a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*d) +
 (b^3*(3*b*B + 2*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b^2*C*(a + b*Sec[
c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.308815, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4041, 3918, 4048, 3770, 3767, 8}

$$\frac{b^2\left(a^2(-C)+9abB+2b^2C\right)\tan(c+dx)}{3d}+\frac{b\left(6a^2bB-4a^3C+2ab^2C+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x(bB-aC)+\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[
c + d*x]^2),x]
```



```
[Out] a^3*(b*B - a*C)*x + (b*(6*a^2*b*B + b^3*B - 4*a^3*C + 2*a*b^2*C)*ArcTanh[Si
n[c + d*x]])/(2*d) + (b^2*(9*a*b*B - a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*d) +
 (b^3*(3*b*B + 2*a*C)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b^2*C*(a + b*Sec[
c + d*x])^2*Tan[c + d*x])/(3*d)
```


Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right)dx=\frac{\int (a+b\sec(c+dx))^3\left(b^2(bB-aC)+b^3C\right.}{b^2}$$

$$=\frac{b^2C(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{\int (a}{}$$

$$=\frac{b^3(3bB+2aC)\sec(c+dx)\tan(c+dx)}{6d}+$$

$$=a^3(bB-aC)x+\frac{b^3(3bB+2aC)\sec(c+dx}{6d}$$

$$=a^3(bB-aC)x+\frac{b\left(6a^2bB+b^3B-4a^3C+\right.}{}$$

$$=a^3(bB-aC)x+\frac{b\left(6a^2bB+b^3B-4a^3C+\right.}{}$$

Mathematica [A] time = 0.919261, size = 114, normalized size = 0.77

$$\frac{3b\left(6a^2bB-4a^3C+2ab^2C+b^3B\right)\tanh^{-1}(\sin(c+dx))+6a^3dx(bB-aC)+3b^3\tan(c+dx)\sec(c+dx)(2(3aB+bC)\cos}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*
C*Sec[c + d*x]^2),x]
```

```
[Out] (6*a^3*(b*B - a*C)*d*x + 3*b*(6*a^2*b*B + b^3*B - 4*a^3*C + 2*a*b^2*C)*ArcT
anh[Sin[c + d*x]] + 3*b^3*(b*B + 2*a*C + 2*(3*a*B + b*C)*Cos[c + d*x])*Sec[
c + d*x]*Tan[c + d*x] + 2*b^4*C*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.052, size = 228, normalized size = 1.5

$$Ba^3bx+\frac{Ba^3bc}{d}-a^4Cx-\frac{Ca^4c}{d}+3\,\frac{a^2b^2B\ln(\sec(dx+c)+\tan(dx+c))}{d}-2\,\frac{a^3bC\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\,\frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2),x)
```

```
[Out] B*a^3*b*x+1/d*B*a^3*b*c-a^4*C*x-1/d*C*a^4*c+3/d*a^2*b^2*B*ln(sec(d*x+c)+tan
(d*x+c))-2/d*a^3*b*C*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^3*B*tan(d*x+c)+1/d*C
```

```
*a*b^3*sec(d*x+c)*tan(d*x+c)+1/d*C*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*B*
b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*C*b^4
*tan(d*x+c)+1/3/d*C*b^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.0294, size = 275, normalized size = 1.85

$$-\frac{12\,(dx+c)Ca^4-12\,(dx+c)Ba^3b-4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Cb^4+6\,Cab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(d*x + c)*C*a^4 - 12*(d*x + c)*B*a^3*b - 4*(tan(d*x + c)^3 + 3*ta
n(d*x + c))*C*b^4 + 6*C*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 3*B*b^4*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*C*a^3*b*
log(sec(d*x + c) + tan(d*x + c)) - 36*B*a^2*b^2*log(sec(d*x + c) + tan(d*x
+ c)) - 36*B*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.556101, size = 470, normalized size = 3.15

$$-\frac{12\left(Ca^4-Ba^3b\right)dx\cos\left(dx+c\right)^3+3\left(4\,Ca^3b-6\,Ba^2b^2-2\,Cab^3-Bb^4\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,Ca^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*(C*a^4 - B*a^3*b)*d*x*cos(d*x + c)^3 + 3*(4*C*a^3*b - 6*B*a^2*b^2
 - 2*C*a*b^3 - B*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(4*C*a^3*b -
 6*B*a^2*b^2 - 2*C*a*b^3 - B*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2
*(2*C*b^4 + 2*(9*B*a*b^3 + 2*C*b^4)*cos(d*x + c)^2 + 3*(2*C*a*b^3 + B*b^4)*
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int Ca^4\,dx-\int -Ba^3b\,dx-\int -Bb^4\sec^3\left(c+dx\right)dx-\int -Cb^4\sec^4\left(c+dx\right)dx-\int -3Bab^3\sec^2\left(c+dx\right)dx-\int -3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*
x+c)**2),x)
```

```
[Out] -Integral(C*a**4, x) - Integral(-B*a**3*b, x) - Integral(-B*b**4*sec(c + d*
x)**3, x) - Integral(-C*b**4*sec(c + d*x)**4, x) - Integral(-3*B*a*b**3*sec
(c + d*x)**2, x) - Integral(-3*B*a**2*b**2*sec(c + d*x), x) - Integral(-2*C
*a*b**3*sec(c + d*x)**3, x) - Integral(2*C*a**3*b*sec(c + d*x), x)
```

Giac [B] time = 1.26227, size = 406, normalized size = 2.72

$$-\frac{6\left(Ca^4 - Ba^3b\right)(dx + c) + 3\left(4\,Ca^3b - 6\,Ba^2b^2 - 2\,Cab^3 - Bb^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(4\,Ca^3b - 6\,Ba^2b^2 - 2\,Ca\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] -1/6*(6*(C*a^4 - B*a^3*b)*(d*x + c) + 3*(4*C*a^3*b - 6*B*a^2*b^2 - 2*C*a*b^
3 - B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*C*a^3*b - 6*B*a^2*b^2
- 2*C*a*b^3 - B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(18*B*a*b^3*tan
(1/2*d*x + 1/2*c)^5 - 6*C*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^4*tan(1/2*d*
x + 1/2*c)^5 + 6*C*b^4*tan(1/2*d*x + 1/2*c)^5 - 36*B*a*b^3*tan(1/2*d*x + 1/
2*c)^3 - 4*C*b^4*tan(1/2*d*x + 1/2*c)^3 + 18*B*a*b^3*tan(1/2*d*x + 1/2*c) +
 6*C*a*b^3*tan(1/2*d*x + 1/2*c) + 3*B*b^4*tan(1/2*d*x + 1/2*c) + 6*C*b^4*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.899 $\int (a+b\sec(c+dx))\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec\right.$

Optimal. Leaf size=97

$$\frac{b\left(-2a^2C+4abB+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(bB-aC)+\frac{b^2(aC+2bB)\tan(c+dx)}{2d}+\frac{b^2C\tan(c+dx)(a+b\sec}{2d}$$

```
[Out] a^2*(b*B - a*C)*x + (b*(4*a*b*B - 2*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(
2*d) + (b^2*(2*b*B + a*C)*Tan[c + d*x])/(2*d) + (b^2*C*(a + b*Sec[c + d*x])
*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.16879, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {4041, 3918, 3770, 3767, 8}

$$\frac{b\left(-2a^2C+4abB+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(bB-aC)+\frac{b^2(aC+2bB)\tan(c+dx)}{2d}+\frac{b^2C\tan(c+dx)(a+b\sec}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c
+ d*x]^2),x]
```

```
[Out] a^2*(b*B - a*C)*x + (b*(4*a*b*B - 2*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(
2*d) + (b^2*(2*b*B + a*C)*Tan[c + d*x])/(2*d) + (b^2*C*(a + b*Sec[c + d*x])
*Tan[c + d*x])/(2*d)
```

Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right)dx=\frac{\int (a+b\sec(c+dx))^2\left(b^2(bB-aC)+b^3C\,\mathrm{s}\right.}{b^2}$$
$$=\frac{b^2C(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{\int (2a}{}$$
$$=a^2(bB-aC)x+\frac{b^2C(a+b\sec(c+dx))\tan}{2d}$$
$$=a^2(bB-aC)x+\frac{b\left(4abB-2a^2C+b^2C\right)\tan}{2d}$$
$$=a^2(bB-aC)x+\frac{b\left(4abB-2a^2C+b^2C\right)\tan}{2d}$$

Mathematica [A] time = 0.562134, size = 77, normalized size = 0.79

$$\frac{b\left(-2a^2C+4abB+b^2C\right)\tanh^{-1}(\sin(c+dx))+2a^2dx(bB-aC)+b^2\tan(c+dx)(2aC+2bB+bC\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*
Sec[c + d*x]^2),x]
```

```
[Out] (2*a^2*(b*B - a*C)*d*x + b*(4*a*b*B - 2*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]
] + b^2*(2*b*B + 2*a*C + b*C*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.042, size = 157, normalized size = 1.6

$$Ba^2bx+\frac{Ba^2bc}{d}-a^3Cx-\frac{Ca^3c}{d}+2\,\frac{Bab^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Cab^2\tan(dx+c)}{d}-\frac{a^2bC\ln(\sec(dx+c)+\mathrm{t}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2),x)
```

```
[Out] B*a^2*b*x+1/d*B*a^2*b*c-a^3*C*x-1/d*C*a^3*c+2/d*B*a*b^2*ln(sec(d*x+c)+tan(d
*x+c))+1/d*C*a*b^2*tan(d*x+c)-1/d*a^2*b*C*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b
^3*tan(d*x+c)+1/2/d*C*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*C*b^3*ln(sec(d*x+c)+t
an(d*x+c))
```

Maxima [A] time = 1.02709, size = 192, normalized size = 1.98

$$-\frac{4(dx+c)Ca^3-4(dx+c)Ba^2b+Cb^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+4Ca^2b\log(\sec(dx}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2
),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c)*C*a^3 - 4*(d*x + c)*B*a^2*b + C*b^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*C*a^2*
b*log(sec(d*x + c) + tan(d*x + c)) - 8*B*a*b^2*log(sec(d*x + c) + tan(d*x +
 c)) - 4*C*a*b^2*tan(d*x + c) - 4*B*b^3*tan(d*x + c))/d
```

Fricas [A] time = 0.551722, size = 363, normalized size = 3.74

$$-\frac{4\left(Ca^3-Ba^2b\right)dx\cos\left(dx+c\right)^2+\left(2\,Ca^2b-4\,Bab^2-Cb^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ca^2b-4\,Bab^2-C\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2
),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(C*a^3 - B*a^2*b)*d*x*cos(d*x + c)^2 + (2*C*a^2*b - 4*B*a*b^2 - C*b
^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*C*a^2*b - 4*B*a*b^2 - C*b^3)*
cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(C*b^3 + 2*(C*a*b^2 + B*b^3)*cos(
d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int Ca^3\,dx-\int -Ba^2b\,dx-\int -Bb^3\sec^2\left(c+dx\right)dx-\int -Cb^3\sec^3\left(c+dx\right)dx-\int -2Bab^2\sec\left(c+dx\right)dx-\int -C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c
)**2),x)
```

```
[Out] -Integral(C*a**3, x) - Integral(-B*a**2*b, x) - Integral(-B*b**3*sec(c + d*
x)**2, x) - Integral(-C*b**3*sec(c + d*x)**3, x) - Integral(-2*B*a*b**2*sec
(c + d*x), x) - Integral(-C*a*b**2*sec(c + d*x)**2, x) - Integral(C*a**2*b*
sec(c + d*x), x)
```

Giac [B] time = 1.21328, size = 288, normalized size = 2.97

$$-\frac{2\left(Ca^3-Ba^2b\right)(dx+c)+\left(2\,Ca^2b-4\,Bab^2-Cb^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ca^2b-4\,Bab^2-Cb^3\right)\log\left(\left|\tan\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2
),x, algorithm="giac")
```

```
[Out] -1/2*(2*(C*a^3 - B*a^2*b)*(d*x + c) + (2*C*a^2*b - 4*B*a*b^2 - C*b^3)*log(a
bs(tan(1/2*d*x + 1/2*c) + 1)) - (2*C*a^2*b - 4*B*a*b^2 - C*b^3)*log(abs(tan
```

```
(1/2*d*x + 1/2*c) - 1)) + 2*(2*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^3*tan
(1/2*d*x + 1/2*c)^3 - C*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*b^2*tan(1/2*d*x
+ 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/2*c) - C*b^3*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.900 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=215

$$\frac{\tan(c+dx)\left(3a^2C-3abB+3Ab^2+2b^2C\right)}{3b^3d}+\frac{\left(2a^2bB-2a^3C-ab^2(2A+C)+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(Ab^2\right.}{}$$

```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C - a*b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*
b^4*d) + (2*a^2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3*A*b^2 - 3*a*b*B + 3
*a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*b^3*d) + ((b*B - a*C)*Sec[c + d*x]*Tan[c
 + d*x])/(2*b^2*d) + (C*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.740864, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4102, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\tan(c+dx)\left(3a^2C-3abB+3Ab^2+2b^2C\right)}{3b^3d}+\frac{\left(2a^2bB-2a^3C-ab^2(2A+C)+b^3B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2\left(Ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```


```
[Out] ((2*a^2*b*B + b^3*B - 2*a^3*C - a*b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(2*
b^4*d) + (2*a^2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3*A*b^2 - 3*a*b*B + 3
*a^2*C + 2*b^2*C)*Tan[c + d*x])/(3*b^3*d) + ((b*B - a*C)*Sec[c + d*x]*Tan[c
 + d*x])/(2*b^2*d) + (C*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)
```


Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{C\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)\left(2aC+b(3A+2C)\sec(c\right.}{a+b\sec(c+}}{3b}$$

$$= \frac{(bB-aC)\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{C\sec^2(c+dx)\tan(c}{3bd}$$

$$= \frac{\left(3Ab^2-3abB+3a^2C+2b^2C\right)\tan(c+dx)}{3b^3d} + \frac{(bB-aC)\,\mathrm{se}}{}$$

$$= \frac{\left(3Ab^2-3abB+3a^2C+2b^2C\right)\tan(c+dx)}{3b^3d} + \frac{(bB-aC)\,\mathrm{se}}{}$$

$$= \frac{\left(2a^2bB+b^3B-2a^3C-ab^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}$$

$$= \frac{\left(2a^2bB+b^3B-2a^3C-ab^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}$$

$$= \frac{\left(2a^2bB+b^3B-2a^3C-ab^2(2A+C)\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}$$

Mathematica [C] time = 3.58884, size = 512, normalized size = 2.38

$$\sec^2(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(b\sec(c)\left(-6\sin(2c+dx)\left(a^2C-abB+Ab^2\right)+1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*(12*(-2*a^2*b*B - b^3*B + 2*a^3*C + a*b^2*(2*A + C))*Cos[c + d*x]^3*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - 12*(-2*a^2*b*B - b^3*B + 2*a^3*C + a*
b^2*(2*A + C))*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((
48*I)*a^2*(A*b^2 + a*(-(b*B) + a*C))*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c]
+ (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^
2])]*Cos[c + d*x]^3*(Cos[c] - I*Sin[c]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*
Sin[c])^2]) + b*Sec[c]*(12*(A*b^2 - a*b*B + a^2*C + b^2*C)*Sin[d*x] - 6*(A*
b^2 - a*b*B + a^2*C)*Sin[2*c + d*x] + 3*b^2*B*Sin[c + 2*d*x] - 3*a*b*C*Sin[
c + 2*d*x] + 3*b^2*B*Sin[3*c + 2*d*x] - 3*a*b*C*Sin[3*c + 2*d*x] + 6*A*b^2*
Sin[2*c + 3*d*x] - 6*a*b*B*Sin[2*c + 3*d*x] + 6*a^2*C*Sin[2*c + 3*d*x] + 4*
b^2*C*Sin[2*c + 3*d*x])))/(12*b^4*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(
c + d*x)])*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.091, size = 825, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/b/(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/b*
ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/b/(tan(1/2*d*x+1/2*c)+1)*C-1/d/b/(tan(1/2*d*
x+1/2*c)+1)*A+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)*B-1/3/d*C/b/(tan(1/2*d*x+1/2*c
)-1)^3-1/3/d*C/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*
B+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*C+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2*B-1/2
/d/b/(tan(1/2*d*x+1/2*c)-1)^2*C-1/d/b/(tan(1/2*d*x+1/2*c)-1)*A+1/2/d/b/(tan
(1/2*d*x+1/2*c)-1)*B+1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*A*a-1/d/b^2*ln(tan(1/
2*d*x+1/2*c)+1)*A*a-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/2/d/b^2/(tan(1/2
*d*x+1/2*c)-1)*a*C+1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C*a+1/2/d/b^2/(tan(1/
2*d*x+1/2*c)+1)^2*a*C-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2*C+1/d/b^4*ln(tan(1
/2*d*x+1/2*c)-1)*a^3*C-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B*a^2+1/d/b^2/(tan(
1/2*d*x+1/2*c)-1)*B*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*B*a^2-1/d/b^4*ln(tan
(1/2*d*x+1/2*c)+1)*a^3*C-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a*C+1/d/b^2/(ta
n(1/2*d*x+1/2*c)+1)*B*a-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/2/d/b^2*ln(tan(1/2*d*x+
1/2*c)+1)*C*a-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a*C+2/d*a^2/b^2/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 54.4325, size = 1810, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [1/12*(6*(C*a^4 - B*a^3*b + A*a^2*b^2)*sqrt(a^2 - b^2)*cos(d*x + c)^3*log((
2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*co
s(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + b^2)) - 3*(2*C*a^5 - 2*B*a^4*b + (2*A - C)*a^3*b^2 + B*a^2*b^3
- (2*A + C)*a*b^4 + B*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*C*a^
5 - 2*B*a^4*b + (2*A - C)*a^3*b^2 + B*a^2*b^3 - (2*A + C)*a*b^4 + B*b^5)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*C*a^2*b^3 - 2*C*b^5 + 2*(3*C*a^4
*b - 3*B*a^3*b^2 + (3*A - C)*a^2*b^3 + 3*B*a*b^4 - (3*A + 2*C)*b^5)*cos(d*x
 + c)^2 - 3*(C*a^3*b^2 - B*a^2*b^3 - C*a*b^4 + B*b^5)*cos(d*x + c))*sin(d*x
 + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*(C*a^4 - B*a^3*b + A*a^
2*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*C*a^5 - 2*B*a^4*b + (2*A - C)*
a^3*b^2 + B*a^2*b^3 - (2*A + C)*a*b^4 + B*b^5)*cos(d*x + c)^3*log(sin(d*x +
```

```
 c) + 1) + 3*(2*C*a^5 - 2*B*a^4*b + (2*A - C)*a^3*b^2 + B*a^2*b^3 - (2*A +
C)*a*b^4 + B*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*C*a^2*b^3 -
2*C*b^5 + 2*(3*C*a^4*b - 3*B*a^3*b^2 + (3*A - C)*a^2*b^3 + 3*B*a*b^4 - (3*A
 + 2*C)*b^5)*cos(d*x + c)^2 - 3*(C*a^3*b^2 - B*a^2*b^3 - C*a*b^4 + B*b^5)*c
os(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*se
c(c + d*x)), x)
```

Giac [B] time = 1.31899, size = 652, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] -1/6*(3*(2*C*a^3 - 2*B*a^2*b + 2*A*a*b^2 + C*a*b^2 - B*b^3)*log(abs(tan(1/2
*d*x + 1/2*c) + 1))/b^4 - 3*(2*C*a^3 - 2*B*a^2*b + 2*A*a*b^2 + C*a*b^2 - B*
b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 12*(C*a^4 - B*a^3*b + A*a^2*b
^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)
*b^4) + 2*(6*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5
+ 3*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^2
*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*C*a^2*tan(1/2
*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x +
1/2*c)^3 - 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*tan(1/2*d*x + 1/2*c) -
6*B*a*b*tan(1/2*d*x + 1/2*c) - 3*C*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1
/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

3.901 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{\left(b^2(2A+C)-2a(bB-aC)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(bB-aC)\tan(c}{b^2d}$$

```
[Out] ((b^2*(2*A + C) - 2*a*(b*B - a*C))*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a*
(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((b*B - a*C)*Tan[c + d*x])/(b^2*d) + (C
*Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.453871, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(b^2(2A+C)-2a(bB-aC)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{(bB-aC)\tan(c}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```


```
[Out] ((b^2*(2*A + C) - 2*a*(b*B - a*C))*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a*
(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((b*B - a*C)*Tan[c + d*x])/(b^2*d) + (C
*Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```


Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(aC+b(2A+C)\sec(c+dx)\cdots\right.}{a+b\sec(c+dx}}{2b}$$

$$= \frac{(bB-aC)\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \int \cdots$$

$$= \frac{(bB-aC)\tan(c+dx)}{b^2d} + \frac{C\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\left(b^2(\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2(2A+C)-2a(bB-aC)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(bB-\cdots}{\cdots}$$

$$= \frac{\left(b^2(2A+C)-2a(bB-aC)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(bB-\cdots}{\cdots}$$

$$= \frac{\left(b^2(2A+C)-2a(bB-aC)\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2a\left(A\cdots\right.}{\cdots}$$

Mathematica [C] time = 2.5958, size = 472, normalized size = 3.08

$$\frac{\cos(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-2\left(2a^2C-2abB+2Ab^2+b^2C\right)\log\left(\cos\left(\frac{1}{2}(c+\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
(-2*(2*A*b^2 - 2*a*b*B + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 2*(2*A*b^2 - 2*a*b*B + 2*a^2*C + b^2*C)*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (8*a*(A*b^2 + a*(-(b*B) + a*C))*ArcTan[((I*Cos[c] + Sin[c
])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c]
 - I*Sin[c])^2])]*(I*Cos[c] + Sin[c]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Si
n[c])^2]) + (b^2*C)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b*(b*B - a
*C)*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])) - (b^2*C)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b*(b*B - a*C)*S
in[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
)/(2*b^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a + b*Sec[c +
 d*x]))
```

Maple [B] time = 0.083, size = 499, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*A+2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*B-2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2*C-1/d/
b/(tan(1/2*d*x+1/2*c)+1)*B+1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a*C+1/2/d/b/(tan(
1/2*d*x+1/2*c)+1)*C+1/d/b*ln(tan(1/2*d*x+1/2*c)+1)*A-1/d/b^2*ln(tan(1/2*d*x
+1/2*c)+1)*B*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*a^2*C+1/2/d/b*ln(tan(1/2*d*
x+1/2*c)+1)*C+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2*C-1/d/b/(tan(1/2*d*x+1/2*c)-
1)*B+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a*C+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)*C-1/
d/b*ln(tan(1/2*d*x+1/2*c)-1)*A+1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*B*a-1/d/b^3
*ln(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 37.0882, size = 1465, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [1/4*(2*(C*a^3 - B*a^2*b + A*a*b^2)*sqrt(a^2 - b^2)*cos(d*x + c)^2*log((2*a
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d
*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) + (2*C*a^4 - 2*B*a^3*b + (2*A - C)*a^2*b^2 + 2*B*a*b^3 - (2*
A + C)*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*C*a^4 - 2*B*a^3*b + (
2*A - C)*a^2*b^2 + 2*B*a*b^3 - (2*A + C)*b^4)*cos(d*x + c)^2*log(-sin(d*x +
 c) + 1) + 2*(C*a^2*b^2 - C*b^4 - 2*(C*a^3*b - B*a^2*b^2 - C*a*b^3 + B*b^4)
*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2), -1/4*(4*(C
*a^3 - B*a^2*b + A*a*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(
d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*C*a^4 - 2*B*a
^3*b + (2*A - C)*a^2*b^2 + 2*B*a*b^3 - (2*A + C)*b^4)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) + (2*C*a^4 - 2*B*a^3*b + (2*A - C)*a^2*b^2 + 2*B*a*b^3 - (2
*A + C)*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(C*a^2*b^2 - C*b^4 -
 2*(C*a^3*b - B*a^2*b^2 - C*a*b^3 + B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^3 - b^5)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*se
c(c + d*x)), x)
```

Giac [B] time = 1.28547, size = 387, normalized size = 2.53

$$\frac{\left(2\,Ca^2-2\,Bab+2\,Ab^2+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}-\frac{\left(2\,Ca^2-2\,Bab+2\,Ab^2+Cb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^3}-\frac{4\left(Ca^3-Ba^2b+Aab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+\right.}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/2*((2*C*a^2 - 2*B*a*b + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/b^3 - (2*C*a^2 - 2*B*a*b + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^3 - 4*(C*a^3 - B*a^2*b + A*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^3) + 2*(2*C*a*tan(1/2*d*x + 1/2*c
)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3 - 2*C*a*tan
(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x + 1/2*c))/
((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

3.902 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}$$

```
[Out] ((b*B - a*C)*ArcTanh[Sin[c + d*x]])/(b^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a
+ b]*d) + (C*Tan[c + d*x])/(b*d)
```

Rubi [A] time = 0.223764, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4082, 3998, 3770, 3831, 2659, 208}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}+\frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x]),x]
```

```
[Out] ((b*B - a*C)*ArcTanh[Sin[c + d*x]])/(b^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a
+ b]*d) + (C*Tan[c + d*x])/(b*d)
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx = \frac{C\tan(c+dx)}{bd}+\frac{\int \frac{\sec(c+dx)(Ab+(bB-aC)\sec(c+dx))}{a+b\sec(c+dx)}dx}{b}$$

$$= \frac{C\tan(c+dx)}{bd}+\frac{(bB-aC)\int \sec(c+dx)\,dx}{b^2}+\left(A-\frac{a(bB-}{b^2}\right.$$

$$= \frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}+\frac{\left(A-\frac{a(b}{}\right.}{}$$

$$= \frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{C\tan(c+dx)}{bd}+\frac{\left(2\left(Ab^2\right.\right.}{}$$

$$= \frac{(bB-aC)\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{2\left(Ab^2-a(bB-aC)\right)\tan}{\sqrt{a-b}b^2\sqrt{}}$$

Mathematica [C] time = 2.43875, size = 365, normalized size = 3.44

$$\frac{2\cos(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-\frac{2i(\cos(c)-i\sin(c))\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))(\text{ta}}{\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin(c))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```

```
[Out] (2*Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*(-((b*B - a*C)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + (b*B - a*C)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((2*I)*(A*b^2 + a*(-(b*B) + a*C))*
ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqr
t[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(Cos[c] - I*Sin[c]))/(Sqrt[a^2 -
 b^2]*Sqrt[(Cos[c] - I*Sin[c])^2]) + (b*C*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/
2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (b*C*Sin[(d*x)/2])/((Cos[c/2]
+ Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(b^2*d*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x]))
```

Maple [B] time = 0.075, size = 272, normalized size = 2.6

$$2\frac{A}{d\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-2\frac{Ba}{db\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))*A-2/d*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*B+2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*C-1/d/b/(tan(1/2*d*x+1/2*c)+1)*C+1/d/b*ln(tan(1/2*
d*x+1/2*c)+1)*B-1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*C*a-1/d/b/(tan(1/2*d*x+1/2
*c)-1)*C-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)*B+1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*
C*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.3571, size = 1085, normalized size = 10.24

$$\left[\frac{\left(Ca^2-Bab+Ab^2\right)\sqrt{a^2-b^2}\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(Ca^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*((C*a^2 - B*a*b + A*b^2)*sqrt(a^2 - b^2)*cos(d*x + c)*log((2*a*b*cos(d
*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c)
+ a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 b^2)) - (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(sin(d*x + c)
+ 1) + (C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(-sin(d*x + c) +
 1) + 2*(C*a^2*b - C*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c)), 1
/2*(2*(C*a^2 - B*a*b + A*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (C*a^3 - B*a^2
*b - C*a*b^2 + B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (C*a^3 - B*a^2*b
 - C*a*b^2 + B*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(C*a^2*b - C*b^
3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c
 + d*x)), x)
```

Giac [A] time = 1.32987, size = 243, normalized size = 2.29

$$-\frac{\frac{(Ca-Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2} - \frac{(Ca-Bb)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2} + \frac{2C\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b} + \frac{2\left(Ca^2-Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\right.}{\sqrt{-a^2+b^2}b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -((C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - (C*a - B*b)*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*b) + 2*(C*a^2 - B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/d
```

3.903 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}$$

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(b*d) - (2*(A*b^2 - a*(b*B - a*C))*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a +
b]*d)
```

Rubi [A] time = 0.172202, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4050, 3770, 3919, 3831, 2659, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{Ax}{a}+\frac{C\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a + (C*ArcTanh[Sin[c + d*x]])/(b*d) - (2*(A*b^2 - a*(b*B - a*C))*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a +
b]*d)
```

Rule 4050

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e
 + f*x], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Csc[e + f*x])/(a + b*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\int \frac{Ab+(bB-aC)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b} + \frac{C\int \sec(c+dx)\,dx}{b}$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{bd} - \left(\frac{Ab}{a} - B + \frac{aC}{b}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(\frac{Ab}{a} - B + \frac{aC}{b}\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\left(2\left(\frac{Ab}{a} - B + \frac{aC}{b}\right)\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\right.}{bd}$$
$$= \frac{Ax}{a} + \frac{C\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2\left(\frac{Ab}{a} - B + \frac{aC}{b}\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [C] time = 0.52008, size = 261, normalized size = 2.78

$$\frac{2\left(A\cos^2(c+dx) + B\cos(c+dx) + C\right)\left(2(\sin(c)+i\cos(c))\left(a(aC-bB)+Ab^2\right)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\tan\left(\frac{dx}{2}\right)(a\cos(c)-b)+\right.}{\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin(c))^2}}\right.\right.}{abd\sqrt{a^2-b^2}\sqrt{(\cos(c)-i\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(C + B*Cos[c + d*x] + A*Cos[c + d*x]^2)*(Sqrt[a^2 - b^2]*(A*b*d*x - a*C*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + a*C*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]])*Sqrt[(Cos[c] - I*Sin[c])^2] + 2*(A*b^2 + a*(-(b*B) + a*C))*Ar
cTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[
a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(I*Cos[c] + Sin[c])))/(a*b*Sqrt[a^
2 - b^2]*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sqrt[(Cos[c] -
 I*Sin[c])^2])
```

Maple [B] time = 0.081, size = 202, normalized size = 2.2

$$2\,\frac{A\arctan(\tan(1/2\,dx+c/2))}{ad} - 2\,\frac{Ab}{ad\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right) + 2\,\frac{B}{d\sqrt{(a+b)(a-b)}}\mathrm{Arta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/a/d*A*arctan(tan(1/2*d*x+1/2*c))-2/d*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*B/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d/b*a/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/b*ln(tan(1/2*
d*x+1/2*c)+1)*C-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.41009, size = 824, normalized size = 8.77

$$\left[\frac{2\left(Aa^2b - Ab^3\right)dx + \left(Ca^2 - Bab + Ab^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2-2\,b^2\right)\cos(dx+c)^2 - 2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right) \cdot}{2\left(a^3b - ab^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] [1/2*(2*(A*a^2*b - A*b^3)*d*x + (C*a^2 - B*a*b + A*b^2)*sqrt(a^2 - b^2)*log
((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*
cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + b^2)) + (C*a^3 - C*a*b^2)*log(sin(d*x + c) + 1) - (C*a^3 - C*
a*b^2)*log(-sin(d*x + c) + 1))/((a^3*b - a*b^3)*d), 1/2*(2*(A*a^2*b - A*b^3
)*d*x - 2*(C*a^2 - B*a*b + A*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^3 - C*a*b^2)*log(s
in(d*x + c) + 1) - (C*a^3 - C*a*b^2)*log(-sin(d*x + c) + 1))/((a^3*b - a*b^
3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)}{a + b\sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.33198, size = 201, normalized size = 2.14

$$\frac{\frac{(dx+c)A}{a} + \frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b} - \frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b} - \frac{2\left(Ca^2-Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,\cdots\right.}{\sqrt{-a^2+b^2}}\right.\right.}{\sqrt{-a^2+b^2}ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] ((d*x + c)*A/a + C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b - C*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/b - 2*(C*a^2 - B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a*b))/d
```

3.904 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(Ab-aB)}{a^2}+\frac{A\sin(c+dx)}{ad}$$

```
[Out] -(((A*b - a*B)*x)/a^2) + (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c +
d*x])/(a*d)
```

Rubi [A] time = 0.216397, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4104, 3919, 3831, 2659, 208}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(Ab-aB)}{a^2}+\frac{A\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x]),x]
```

```
[Out] -(((A*b - a*B)*x)/a^2) + (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c +
d*x])/(a*d)
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\sin(c+dx)}{ad}-\frac{\int \frac{Ab-aB-aC\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$
$$= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\left(\frac{b(Ab-aB)}{a^2}+C\right)\int \frac{\text{se}}{a+\text{b}}$$
$$= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\frac{\left(\frac{b(Ab-aB)}{a^2}+C\right)\int \frac{1}{1+\frac{a\cos(c+d}{b}}}{b}$$
$$= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\frac{\left(2\left(\frac{b(Ab-aB)}{a^2}+C\right)\right)\text{Subst}\,(}{}$$
$$= -\frac{(Ab-aB)x}{a^2}+\frac{2\left(\frac{b(Ab-aB)}{a^2}+C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.232455, size = 92, normalized size = 0.94

$$\frac{-\frac{2\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+(c+dx)(aB-Ab)+aA\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```

```
[Out] ((-(A*b) + a*B)*(c + d*x) - (2*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((-a + b)
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*A*Sin[c + d*x])/(a
^2*d)
```

Maple [B] time = 0.11, size = 216, normalized size = 2.2

$$2\,\frac{A\tan(1/2\,dx+c/2)}{ad\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{Ab\arctan(\tan(1/2\,dx+c/2))}{da^2}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,B}{ad}+2\,\frac{Ab^2}{da^2\sqrt{(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*A/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d*A/a^2*b*arctan(tan(
1/2*d*x+1/2*c))+2/a/d*arctan(tan(1/2*d*x+1/2*c))*B+2/d/a^2/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^2-2/d/a/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b+2/
d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.56765, size = 724, normalized size = 7.39

$$\left[\frac{2\left(Ba^3 - Aa^2b - Bab^2 + Ab^3\right)dx + \left(Ca^2 - Bab + Ab^2\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 + 2\,\sqrt{a^2 - b^2}\left(b\cos(dx+c) + a\right)\sin}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right.}{2\left(a^4 - a^2b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] [1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x + (C*a^2 - B*a*b + A*b^2)*s
qrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*s
qrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/(
(a^4 - a^2*b^2)*d), ((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x + (C*a^2 - B*a
*b + A*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/
((a^2 - b^2)*sin(d*x + c))) + (A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b
^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec{\left(c + dx \right)} + C\sec^{2}{\left(c + dx \right)}\right)\cos{\left(c + dx \right)}}{a + b\sec{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)/(a + b*sec(c
 + d*x)), x)
```

Giac [A] time = 1.24374, size = 197, normalized size = 2.01

$$\frac{\frac{(Ba-Ab)(dx+c)}{a^2}+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}+\frac{2\left(Ca^2-Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x, al
gorithm="giac")
```

```
[Out] ((B*a - A*b)*(d*x + c)/a^2 + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*a) + 2*(C*a^2 - B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2))/d
```

3.905 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{2b\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2(A+2C)-2abB+2Ab^2\right)}{2a^3}-\frac{(Ab-aB)\sin(c+dx)}{a^2d}+\frac{A\sin(c$$

```
[Out] ((2*A*b^2 - 2*a*b*B + a^2*(A + 2*C))*x)/(2*a^3) - (2*b*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*S
qrt[a + b]*d) - ((A*b - a*B)*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c
+ d*x])/(2*a*d)
```

Rubi [A] time = 0.452222, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4104, 3919, 3831, 2659, 208}

$$-\frac{2b\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2(A+2C)-2abB+2Ab^2\right)}{2a^3}-\frac{(Ab-aB)\sin(c+dx)}{a^2d}+\frac{A\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```

```
[Out] ((2*A*b^2 - 2*a*b*B + a^2*(A + 2*C))*x)/(2*a^3) - (2*b*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*S
qrt[a + b]*d) - ((A*b - a*B)*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c
+ d*x])/(2*a*d)
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(2(Ab-aB)-a(A+2C)\sec(\right.}{a+b\sec(c+dx}}{2a}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int 2}{}$$

$$= \frac{\left(2Ab^2-2abB+a^2(A+2C)\right)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} +$$

$$= \frac{\left(2Ab^2-2abB+a^2(A+2C)\right)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} +$$

$$= \frac{\left(2Ab^2-2abB+a^2(A+2C)\right)x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} +$$

$$= \frac{\left(2Ab^2-2abB+a^2(A+2C)\right)x}{2a^3} - \frac{2b\left(Ab^2-a(bB-aC)\right)\mathrm{t}}{a^3\sqrt{a-b}}$$

Mathematica [A] time = 0.436987, size = 131, normalized size = 0.9

$$\frac{2(c+dx)\left(a^2(A+2C)-2abB+2Ab^2\right)+\frac{8b\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a^2A\sin(2(c+dx))+4a(aB-Ab)\sin}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x]),x]
```

```
[Out] (2*(2*A*b^2 - 2*a*b*B + a^2*(A + 2*C))*(c + d*x) + (8*b*(A*b^2 + a*(-(b*B)
+ a*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] + 4*a*(-(A*b) + a*B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] time = 0.123, size = 434, normalized size = 3.

$$-\frac{A}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-2\,\frac{\left(\tan(1/2\,dx+c/2)\right)^3Ab}{da^2\left(1+\left(\tan(1/2\,dx+c/2)\right)^2\right)^2}+2\,\frac{\left(\tan(1/2\,dx+c/2)\right)^3B}{ad\left(1+\left(\tan(1/2\,dx+c/2)\right)^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)^3*B+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c
)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b+2/a/d/(1+tan(1/
2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+1/a/d*A*arctan(tan(1/2*d*x+1/2*c))+2
/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A*b^2-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*
B*b+2/a/d*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d*B/a^2/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^2-2/d*b/a/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.591216, size = 1004, normalized size = 6.92

$$\left[\frac{\left((A+2C)a^4-2Ba^3b+(A-2C)a^2b^2+2Bab^3-2Ab^4\right)dx+\left(Ca^2b-Bab^2+Ab^3\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos\cdots}{a^2\cdots}\right.}{2\left(a^5\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [1/2*(((A + 2*C)*a^4 - 2*B*a^3*b + (A - 2*C)*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)
*d*x + (C*a^2*b - B*a*b^2 + A*b^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 (2*B*a^4 - 2*A*a^3*b - 2*B*a^2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d
*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d), 1/2*(((A + 2*C)*a^4 - 2*B*a^3*b
 + (A - 2*C)*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*d*x - 2*(C*a^2*b - B*a*b^2 + A*
b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
 b^2)*sin(d*x + c))) + (2*B*a^4 - 2*A*a^3*b - 2*B*a^2*b^2 + 2*A*a*b^3 + (A*
a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)**2/(a + b*se
c(c + d*x)), x)
```

Giac [A] time = 1.22908, size = 323, normalized size = 2.23

$$\frac{\frac{(Aa^2+2Ca^2-2Bab+2Ab^2)(dx+c)}{a^3}-\frac{4(Ca^2b-Bab^2+Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^3}-2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/2*((A*a^2 + 2*C*a^2 - 2*B*a*b + 2*A*b^2)*(d*x + c)/a^3 - 4*(C*a^2*b - B*a
*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-
a^2 + b^2)*a^3) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c
)^3 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1
/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)
^2*a^2))/d
```

3.906 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{\sin(c+dx)\left(a^2(2A+3C)-3abB+3Ab^2\right)}{3a^3d}+\frac{2b^2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(a^2b(A+2C)+a^3(-\right.}{2a^4}$$

```
[Out] -((2*A*b^3 - a^3*B - 2*a*b^2*B + a^2*b*(A + 2*C))*x)/(2*a^4) + (2*b^2*(A*b^
2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
4*Sqrt[a - b]*Sqrt[a + b]*d) + ((3*A*b^2 - 3*a*b*B + a^2*(2*A + 3*C))*Sin[c
 + d*x])/(3*a^3*d) - ((A*b - a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (A
*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.740061, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4104, 3919, 3831, 2659, 208}

$$\frac{\sin(c+dx)\left(a^2(2A+3C)-3abB+3Ab^2\right)}{3a^3d}+\frac{2b^2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{x\left(a^2b(A+2C)+a^3(-\right.}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```

```
[Out] -((2*A*b^3 - a^3*B - 2*a*b^2*B + a^2*b*(A + 2*C))*x)/(2*a^4) + (2*b^2*(A*b^
2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
4*Sqrt[a - b]*Sqrt[a + b]*d) + ((3*A*b^2 - 3*a*b*B + a^2*(2*A + 3*C))*Sin[c
 + d*x])/(3*a^3*d) - ((A*b - a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (A
*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)\left(3(Ab-aB)-a(2A+3C)\ldots\right.}{a+b\sec(c+\ldots}}{3a}$$

$$= -\frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A\cos^2(c+dx)\sin(\ldots}{3ad}$$

$$= \frac{\left(3Ab^2-3abB+a^2(2A+3C)\right)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos\ldots}{}$$

$$= -\frac{\left(2Ab^3-a^3B-2ab^2B+a^2b(A+2C)\right)x}{2a^4} + \frac{\left(3Ab^2-3abB\ldots\right.}{}$$

$$= -\frac{\left(2Ab^3-a^3B-2ab^2B+a^2b(A+2C)\right)x}{2a^4} + \frac{\left(3Ab^2-3abB\ldots\right.}{}$$

$$= -\frac{\left(2Ab^3-a^3B-2ab^2B+a^2b(A+2C)\right)x}{2a^4} + \frac{\left(3Ab^2-3abB\ldots\right.}{}$$

$$= -\frac{\left(2Ab^3-a^3B-2ab^2B+a^2b(A+2C)\right)x}{2a^4} + \frac{2b^2\left(Ab^2-a(\ldots\right.}{}$$

Mathematica [A] time = 0.602023, size = 178, normalized size = 0.87

$$\frac{6(c+dx)\left(-a^2b(A+2C)+a^3B+2ab^2B-2Ab^3\right)+3a\sin(c+dx)\left(a^2(3A+4C)-4abB+4Ab^2\right)-\frac{24b^2\left(a(aC-bB)+Ab^2\right)\tan\ldots}{\sqrt{a\ldots}}}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x]),x]
```

```
[Out] (6*(-2*A*b^3 + a^3*B + 2*a*b^2*B - a^2*b*(A + 2*C))*(c + d*x) - (24*b^2*(A*
b^2 + a*(-(b*B) + a*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + 3*a*(4*A*b^2 - 4*a*b*B + a^2*(3*A + 4*C))*Sin[c + d*x]
 + 3*a^2*(-(A*b) + a*B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])/(12*a^4*
d)
```

Maple [B] time = 0.137, size = 814, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+1/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^5*A*b^2-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)^5*B-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B*b+2/d/a/
(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*C+4/3/d/a/(1+tan(1/2*d*x+1/
2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)^3*A*b^2-4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3
*B*b+4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*C+2/d/a/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*
tan(1/2*d*x+1/2*c)*A*b^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)*B*b+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*C-1/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)*B-1/d*A/a^2*b*arctan(tan(1/2*d*x+1/2*c))-2/d/a^4*arct
an(tan(1/2*d*x+1/2*c))*A*b^3+1/a/d*arctan(tan(1/2*d*x+1/2*c))*B+2/d/a^3*arc
tan(tan(1/2*d*x+1/2*c))*B*b^2-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*b*C+2/d*b^
4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*A-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*B+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.624831, size = 1301, normalized size = 6.35

$$\left[\frac{3\left(Ba^5-(A+2C)a^4b+Ba^3b^2-(A-2C)a^2b^3-2Bab^4+2Ab^5\right)dx+3\left(Ca^2b^2-Bab^3+Ab^4\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [1/6*(3*(B*a^5 - (A + 2*C)*a^4*b + B*a^3*b^2 - (A - 2*C)*a^2*b^3 - 2*B*a*b^
4 + 2*A*b^5)*d*x + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(a^2 - b^2)*log((2*a
```

```
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d
*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) + (2*(2*A + 3*C)*a^5 - 6*B*a^4*b + 2*(A - 3*C)*a^3*b^2 + 6*B
*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*a^3*b^2)*cos(d*x + c)^2 + 3*(B*a^5 - A*
a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)
*d), 1/6*(3*(B*a^5 - (A + 2*C)*a^4*b + B*a^3*b^2 - (A - 2*C)*a^2*b^3 - 2*B*
a*b^4 + 2*A*b^5)*d*x + 6*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2
*(2*A + 3*C)*a^5 - 6*B*a^4*b + 2*(A - 3*C)*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^
4 + 2*(A*a^5 - A*a^3*b^2)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 +
 A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.29894, size = 572, normalized size = 2.79

$$\frac{3\left(Ba^3 - Aa^2b - 2\,Ca^2b + 2\,Bab^2 - 2\,Ab^3\right)(dx+c)}{a^4} + \frac{12\left(Ca^2b^2 - Bab^3 + Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^4} + \frac{2\left(6\,Aa^2 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b - 2*C*a^2*b + 2*B*a*b^2 - 2*A*b^3)*(d*x + c)/a^4 +
12*(C*a^2*b^2 - B*a*b^3 + A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a
^2 + b^2)))/(sqrt(-a^2 + b^2)*a^4) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*
B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan
(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^2*tan(1/2*d*x + 1/2*c)
^3 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*
A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) + 6*C*a^2*tan(1/2
*d*x + 1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c)
 + 6*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.907 $\int \frac{\cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=276

$$-\frac{\sin(c+dx)\left(a^2b(2A+3C)-2a^3B-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{\sin(c+dx)\cos(c+dx)\left(a^2(3A+4C)-4abB+4Ab^2\right)}{8a^3d}-\frac{2b^3(A\cdots}{\cdots}$$

```
[Out] ((8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*
x)/(8*a^5) - (2*b^3*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) - ((3*A*b^3 - 2*a^3*B
 - 3*a*b^2*B + a^2*b*(2*A + 3*C))*Sin[c + d*x])/(3*a^4*d) + ((4*A*b^2 - 4*a
*b*B + a^2*(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - ((A*b - a*B)
*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*Sin[c + d*x])/(
4*a*d)
```

Rubi [A] time = 1.10384, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4104, 3919, 3831, 2659, 208}

$$-\frac{\sin(c+dx)\left(a^2b(2A+3C)-2a^3B-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{\sin(c+dx)\cos(c+dx)\left(a^2(3A+4C)-4abB+4Ab^2\right)}{8a^3d}-\frac{2b^3(A\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x]),x]
```

```
[Out] ((8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C))*
x)/(8*a^5) - (2*b^3*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) - ((3*A*b^3 - 2*a^3*B
 - 3*a*b^2*B + a^2*b*(2*A + 3*C))*Sin[c + d*x])/(3*a^4*d) + ((4*A*b^2 - 4*a
*b*B + a^2*(3*A + 4*C))*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - ((A*b - a*B)
*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*Sin[c + d*x])/(
4*a*d)
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^3(c+dx)\left(4(Ab-aB)-a(3A+4C)\right.}{a+b\sec(c+}}{4a}$$

$$= -\frac{(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{A\cos^3(c+dx)\sin}{4ad}$$

$$= \frac{\left(4Ab^2-4abB+a^2(3A+4C)\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - $$

$$= -\frac{\left(3Ab^3-2a^3B-3ab^2B+a^2b(2A+3C)\right)\sin(c+dx)}{3a^4d} + $$

$$= \frac{\left(8Ab^4-4a^3bB-8ab^3B+4a^2b^2(A+2C)+a^4(3A+4C)\right)}{8a^5}$$

$$= \frac{\left(8Ab^4-4a^3bB-8ab^3B+4a^2b^2(A+2C)+a^4(3A+4C)\right)}{8a^5}$$

$$= \frac{\left(8Ab^4-4a^3bB-8ab^3B+4a^2b^2(A+2C)+a^4(3A+4C)\right)}{8a^5}$$

$$= \frac{\left(8Ab^4-4a^3bB-8ab^3B+4a^2b^2(A+2C)+a^4(3A+4C)\right)}{8a^5}$$

Mathematica [A] time = 0.831756, size = 235, normalized size = 0.85

$$\frac{12(c+dx)\left(4a^2b^2(A+2C)+a^4(3A+4C)-4a^3bB-8ab^3B+8Ab^4\right)+24a^2\sin(2(c+dx))\left(a^2(A+C)-abB+Ab^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x]),x]
```

```
[Out] (12*(8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B + 4*a^2*b^2*(A + 2*C) + a^4*(3*A + 4*C
))*(c + d*x) + (192*b^3*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[((-a + b)*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 24*a*(-4*A*b^3 + 3*a^3*B +
4*a*b^2*B - a^2*b*(3*A + 4*C))*Sin[c + d*x] + 24*a^2*(A*b^2 - a*b*B + a^2*(
A + C))*Sin[2*(c + d*x)] + 8*a^3*(-(A*b) + a*B)*Sin[3*(c + d*x)] + 3*a^4*A*
Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [B] time = 0.134, size = 1580, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/a/d*arctan(tan(1/2*d*x+1/2*c))*C+3/4/a/d*A*arctan(tan(1/2*d*x+1/2*c))-1/d
/a^2*arctan(tan(1/2*d*x+1/2*c))*B*b-10/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*t
an(1/2*d*x+1/2*c)^3*A*b-10/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^5*A*b-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*B*b+6/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^2*B-2/d/a^4/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*A*b^3-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^5*A*b^3+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x
+1/2*c)^5*B*b-2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^3*B+2/d/a^3*arctan(tan(1
/2*d*x+1/2*c))*C*b^2-5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^
7*A+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*B-1/d/a/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*C+10/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^5*B+3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)^5*A-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*C-3/4/d/a/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A+1/d/a/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^3*C+10/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x
+1/2*c)^3*B+5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*A+1/d/a/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*C+2/d/a/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)*B+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*A*b^4-6/d/a^4/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A*b^3-6/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b*C+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*t
an(1/2*d*x+1/2*c)^3*A*b^2-6/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^3*b*C+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B-2/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1
/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^3*B*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
*b^2*B-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b*C+1/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*B*b+1/d/a^3/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)*A*b^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)*A*b+1/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A*b^2-2/d*b^5/a^5/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-1/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b^2-2/d/a^4/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b^3+6/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^4*tan(1/2*d*x+1/2*c)^3*b^2*B+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)^7*b^2*B-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7
*b*C-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A*b^2-2/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*A*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.703641, size = 1690, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="fricas")
```

```
[Out] [1/24*(3*((3*A + 4*C)*a^6 - 4*B*a^5*b + (A + 4*C)*a^4*b^2 - 4*B*a^3*b^3 + 4
*(A - 2*C)*a^2*b^4 + 8*B*a*b^5 - 8*A*b^6)*d*x + 12*(C*a^2*b^3 - B*a*b^4 + A
*b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (16*B*a^6 - 8*(2*A + 3*C)*a^
5*b + 8*B*a^4*b^2 - 8*(A - 3*C)*a^3*b^3 - 24*B*a^2*b^4 + 24*A*a*b^5 + 6*(A*
a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b - B*a^4*b^2 + A*a^3*b^
3)*cos(d*x + c)^2 + 3*((3*A + 4*C)*a^6 - 4*B*a^5*b + (A - 4*C)*a^4*b^2 + 4*
B*a^3*b^3 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), 1
/24*(3*((3*A + 4*C)*a^6 - 4*B*a^5*b + (A + 4*C)*a^4*b^2 - 4*B*a^3*b^3 + 4*(
A - 2*C)*a^2*b^4 + 8*B*a*b^5 - 8*A*b^6)*d*x - 24*(C*a^2*b^3 - B*a*b^4 + A*b
^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) + (16*B*a^6 - 8*(2*A + 3*C)*a^5*b + 8*B*a^4*b^2 - 8*(A
- 3*C)*a^3*b^3 - 24*B*a^2*b^4 + 24*A*a*b^5 + 6*(A*a^6 - A*a^4*b^2)*cos(d*x
+ c)^3 + 8*(B*a^6 - A*a^5*b - B*a^4*b^2 + A*a^3*b^3)*cos(d*x + c)^2 + 3*((3
*A + 4*C)*a^6 - 4*B*a^5*b + (A - 4*C)*a^4*b^2 + 4*B*a^3*b^3 - 4*A*a^2*b^4)*
cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.31537, size = 1081, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x,
algorithm="giac")
```

```
[Out] 1/24*(3*(3*A*a^4 + 4*C*a^4 - 4*B*a^3*b + 4*A*a^2*b^2 + 8*C*a^2*b^2 - 8*B*a*
b^3 + 8*A*b^4)*(d*x + c)/a^5 - 48*(C*a^2*b^3 - B*a*b^4 + A*b^5)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^5) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*C*a^3*ta
n(1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*b*tan(1
/2*d*x + 1/2*c)^7 + 24*C*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*tan(1/2*
d*x + 1/2*c)^7 - 24*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan(1/2*d*x +
 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^
5 + 12*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1
2*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 72*C*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*A
*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^
3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/
2*d*x + 1/2*c)^3 - 12*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^2*b*tan(1/2*d*x
 + 1/2*c)^3 + 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b*tan(1/2*d*x +
1/2*c)^3 - 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*a*b^2*tan(1/2*d*x + 1/2
*c)^3 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^3*tan(1/2*d*x + 1/2*c) - 2
4*B*a^3*tan(1/2*d*x + 1/2*c) - 12*C*a^3*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b*t
an(1/2*d*x + 1/2*c) + 12*B*a^2*b*tan(1/2*d*x + 1/2*c) + 24*C*a^2*b*tan(1/2*
d*x + 1/2*c) - 12*A*a*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a*b^2*tan(1/2*d*x + 1
/2*c) + 24*A*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4)
)/d
```

3.908 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=407

$$-\frac{\tan(c+dx)\left(-a^2b^2(6A-7C)+9a^3bB-12a^4C-6ab^3B+b^4(3A+2C)\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(6a^2bB-8a^3C-2ab^2(2A+C)+b^3B\right)\text{t}}{2b^5d}$$

```
[Out] ((6*a^2*b*B + b^3*B - 8*a^3*C - 2*a*b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(
2*b^5*d) + (2*a^2*(2*a^2*A*b^2 - 3*A*b^4 - 3*a^3*b*B + 4*a*b^3*B + 4*a^4*C
- 5*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*b^5*(a + b)^(3/2)*d) - ((9*a^3*b*B - 6*a*b^3*B - a^2*b^2*(6*A - 7*C
) - 12*a^4*C + b^4*(3*A + 2*C))*Tan[c + d*x])/(3*b^4*(a^2 - b^2)*d) + ((3*a
^2*b*B - b^3*B - 2*a*b^2*(A - C) - 4*a^3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b
^3*(a^2 - b^2)*d) + ((3*A*b^2 - 3*a*b*B + 4*a^2*C - b^2*C)*Sec[c + d*x]^2*T
an[c + d*x])/(3*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^
3*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.73952, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {4098, 4102, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\tan(c+dx)\left(-a^2b^2(6A-7C)+9a^3bB-12a^4C-6ab^3B+b^4(3A+2C)\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(6a^2bB-8a^3C-2ab^2(2A+C)+b^3B\right)\text{t}}{2b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```

```
[Out] ((6*a^2*b*B + b^3*B - 8*a^3*C - 2*a*b^2*(2*A + C))*ArcTanh[Sin[c + d*x]])/(
2*b^5*d) + (2*a^2*(2*a^2*A*b^2 - 3*A*b^4 - 3*a^3*b*B + 4*a*b^3*B + 4*a^4*C
- 5*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*b^5*(a + b)^(3/2)*d) - ((9*a^3*b*B - 6*a*b^3*B - a^2*b^2*(6*A - 7*C
) - 12*a^4*C + b^4*(3*A + 2*C))*Tan[c + d*x])/(3*b^4*(a^2 - b^2)*d) + ((3*a
^2*b*B - b^3*B - 2*a*b^2*(A - C) - 4*a^3*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b
^3*(a^2 - b^2)*d) + ((3*A*b^2 - 3*a*b*B + 4*a^2*C - b^2*C)*Sec[c + d*x]^2*T
an[c + d*x])/(3*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^
3*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^3(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\sec^3(c+}{}$$

$$= \frac{\left(3Ab^2-3abB+4a^2C-b^2C\right)\sec^2(c+dx)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d} - \Big($$

$$= \frac{\left(3a^2bB-b^3B-2ab^2(A-C)-4a^3C\right)\sec(c+dx)\tan(c+}{2b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(9a^3bB-6ab^3B-a^2b^2(6A-7C)-12a^4C+b^4(3A+2C\right.}{3b^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(9a^3bB-6ab^3B-a^2b^2(6A-7C)-12a^4C+b^4(3A+2C\right.}{3b^4\left(a^2-b^2\right)d}$$

$$= \frac{\left(6a^2bB+b^3B-8a^3C-2ab^2(2A+C)\right)\tanh^{-1}(\sin(c+dx}{2b^5d}$$

$$= \frac{\left(6a^2bB+b^3B-8a^3C-2ab^2(2A+C)\right)\tanh^{-1}(\sin(c+dx}{2b^5d}$$

$$= \frac{\left(6a^2bB+b^3B-8a^3C-2ab^2(2A+C)\right)\tanh^{-1}(\sin(c+dx}{2b^5d}$$

Mathematica [A] time = 3.81893, size = 605, normalized size = 1.49

$$\frac{(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(6\left(-6a^2bB+8a^3C+2ab^2(2A+C)-b^3B\right)(a\cos(c+dx)+b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-24*a^2*(-3
*A*b^4 - 3*a^3*b*B + 4*a*b^3*B + a^2*b^2*(2*A - 5*C) + 4*a^4*C)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)
^(3/2) + 6*(-6*a^2*b*B - b^3*B + 8*a^3*C + 2*a*b^2*(2*A + C))*(b + a*Cos[c
+ d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(6*a^2*b*B + b^3*B - 8
*a^3*C - 2*a*b^2*(2*A + C))*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (b*(-6*a^2*A*b^3 + 6*A*b^5 + 9*a^3*b^2*B - 9*a*b^4*B - 12*
a^4*b*C + 4*a^2*b^3*C + 8*b^5*C + (27*a^4*b*B - 24*a^2*b^3*B + 6*b^5*B + a*
b^4*(9*A - 2*C) - 36*a^5*C + a^3*b^2*(-18*A + 29*C))*Cos[c + d*x] + b*(-a^2
 + b^2)*(6*A*b^2 - 9*a*b*B + 12*a^2*C + 4*b^2*C)*Cos[2*(c + d*x)] - 6*a^3*A
*b^2*Cos[3*(c + d*x)] + 3*a*A*b^4*Cos[3*(c + d*x)] + 9*a^4*b*B*Cos[3*(c + d
*x)] - 6*a^2*b^3*B*Cos[3*(c + d*x)] - 12*a^5*C*Cos[3*(c + d*x)] + 7*a^3*b^2
*C*Cos[3*(c + d*x)] + 2*a*b^4*C*Cos[3*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*
x])/(-a^2 + b^2)))/(6*b^5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)
])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.117, size = 1254, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*A+8/d*a^6/b^5/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-10/d*a^4/b^3/(a+b)/(a-b)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+
1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*B+8
/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-1/3/d*C/b^2/(tan(1/2*d*x+1
/2*c)-1)^3+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*B-1/2/d/b^2/(tan(1/2*d*x+1/2*
c)-1)^2*C-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*A+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)
*B-1/3/d*C/b^2/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2*
B+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2*C-6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-1/d/b^3*ln(
tan(1/2*d*x+1/2*c)+1)*a*C+1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a*C-1/d/b^2/(tan
(1/2*d*x+1/2*c)+1)*A+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*B-1/d/b^2/(tan(1/2*d*
x+1/2*c)-1)*C-1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*C+2/d*a^4/b^3/(a^2-b^2)*tan(1/
2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-1/d/b^3/
(tan(1/2*d*x+1/2*c)+1)*a*C-2/d*a^5/b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-3/d/b^4/(tan(1/2*d*x+1/2*c)+
1)*a^2*C-4/d/b^5*ln(tan(1/2*d*x+1/2*c)+1)*a^3*C-3/d/b^4/(tan(1/2*d*x+1/2*c)
-1)*a^2*C-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*B*a^2+4/d/b^5*ln(tan(1/2*d*x+1/2
*c)-1)*a^3*C+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*A*a-1/d/b^3/(tan(1/2*d*x+1/2*
c)-1)*a*C+1/d/b^3/(tan(1/2*d*x+1/2*c)+1)^2*a*C+2/d/b^3/(tan(1/2*d*x+1/2*c)-
1)*B*a-2/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*A*a+3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1
)*B*a^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)^2*a*C+2/d/b^3/(tan(1/2*d*x+1/2*c)+1)
*B*a-2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sec^4\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
2,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a + b*se
c(c + d*x))**2, x)
```

Giac [A] time = 1.4407, size = 846, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] 1/6*(12*(4*C*a^6 - 3*B*a^5*b + 2*A*a^4*b^2 - 5*C*a^4*b^2 + 4*B*a^3*b^3 - 3*
A*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^5
- b^7)*sqrt(-a^2 + b^2)) - 12*(C*a^5*tan(1/2*d*x + 1/2*c) - B*a^4*b*tan(1/2
*d*x + 1/2*c) + A*a^3*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^4 - b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 3*(8*C*a^3 - 6*B*a^2
*b + 4*A*a*b^2 + 2*C*a*b^2 - B*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5
+ 3*(8*C*a^3 - 6*B*a^2*b + 4*A*a*b^2 + 2*C*a*b^2 - B*b^3)*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/b^5 - 2*(18*C*a^2*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b*tan(1
/2*d*x + 1/2*c)^5 + 6*C*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x +
1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5
- 36*C*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*
b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^2*tan(
1/2*d*x + 1/2*c) - 12*B*a*b*tan(1/2*d*x + 1/2*c) - 6*C*a*b*tan(1/2*d*x + 1/
2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^
2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^4))/d
```

3.909 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=312

$$\frac{\tan(c+dx)\left(2a^2bB-3a^3C-ab^2(A-2C)-b^3B\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(6a^2C-4abB+2Ab^2+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a\left(a^2Ab^2-4a\right.}{}$$

```
[Out] ((2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (
2*a*(a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 4*a^2*b^2*C)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a +
 b)^(3/2)*d) + ((2*a^2*b*B - b^3*B - a*b^2*(A - 2*C) - 3*a^3*C)*Tan[c + d*x
])/(b^3*(a^2 - b^2)*d) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Sec[c + d*x
]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*
x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.2334, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4098, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\tan(c+dx)\left(2a^2bB-3a^3C-ab^2(A-2C)-b^3B\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(6a^2C-4abB+2Ab^2+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a\left(a^2Ab^2-4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```



```
[Out] ((2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (
2*a*(a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 4*a^2*b^2*C)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a +
 b)^(3/2)*d) + ((2*a^2*b*B - b^3*B - a*b^2*(A - 2*C) - 3*a^3*C)*Tan[c + d*x
])/(b^3*(a^2 - b^2)*d) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)*Sec[c + d*x
]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*
x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\sec^2(c+dx)(2}{}$$

$$= \frac{\left(2Ab^2-2abB+3a^2C-b^2C\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2\right.}{}$$

$$= \frac{\left(2a^2bB-b^3B-ab^2(A-2C)-3a^3C\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2-\right.}{}$$

$$= \frac{\left(2a^2bB-b^3B-ab^2(A-2C)-3a^3C\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(2Ab^2-\right.}{}$$

$$= \frac{\left(2Ab^2-4abB+6a^2C+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(2a^2bB\right.}{}$$

$$= \frac{\left(2Ab^2-4abB+6a^2C+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(2a^2bB\right.}{}$$

$$= \frac{\left(2Ab^2-4abB+6a^2C+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a\left(a^2A\right.}{}$$

Mathematica [A] time = 2.85377, size = 519, normalized size = 1.66

$$\frac{(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4a^2b\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{(b-a)(a+b)} - 2\left(6a^2C-4abB+2Ab^2+b^2C\right)(a\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*a*(-2*A*b
^4 - 2*a^3*b*B + 3*a*b^3*B + a^2*b^2*(A - 4*C) + 3*a^4*C)*ArcTanh[((-a + b)
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)^(3/2)
 - 2*(2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*(b + a*Cos[c + d*x])*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 2*(2*A*b^2 - 4*a*b*B + 6*a^2*C + b^2*C)*(b
+ a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^2*C*(b + a*
Cos[c + d*x]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b*(b*B - 2*a*C)
*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]) - (b^2*C*(b + a*Cos[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 +
 (4*b*(b*B - 2*a*C)*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + (4*a^2*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/(
(-a + b)*(a + b))))/(2*b^4*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x
)])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.105, size = 926, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -6/d*a^5/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-6/d*a^2/b/(a+b)/(a-b)/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2
/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*A+8/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/2/d/b^2/(tan(1/2*d*x+1/2
*c)-1)^2*C-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1
)^2*C+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)*A+1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)
*C-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*A-1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C-
1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*B+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/b
^2/(tan(1/2*d*x+1/2*c)+1)*C+2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a*C-2/d/b^3*ln(t
an(1/2*d*x+1/2*c)+1)*B*a+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a*C+3/d/b^4*ln(tan(
1/2*d*x+1/2*c)+1)*a^2*C+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B*a-3/d/b^4*ln(tan
(1/2*d*x+1/2*c)-1)*a^2*C+2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*
x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+4/d*a/(a+b)/
(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*A+2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 160.142, size = 3341, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*C*a^6 - 2*B*a^5*b + (A - 4*C)*a^4*b^2 + 3*B*a^3*b^3 - 2*A*a^2*b
^4)*cos(d*x + c)^3 + (3*C*a^5*b - 2*B*a^4*b^2 + (A - 4*C)*a^3*b^3 + 3*B*a^2
*b^4 - 2*A*a*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
 (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(
d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
((6*C*a^7 - 4*B*a^6*b + (2*A - 11*C)*a^5*b^2 + 8*B*a^4*b^3 - 4*(A - C)*a^3*
b^4 - 4*B*a^2*b^5 + (2*A + C)*a*b^6)*cos(d*x + c)^3 + (6*C*a^6*b - 4*B*a^5*
b^2 + (2*A - 11*C)*a^4*b^3 + 8*B*a^3*b^4 - 4*(A - C)*a^2*b^5 - 4*B*a*b^6 +
(2*A + C)*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*C*a^7 - 4*B*a^6*
```

```
b + (2*A - 11*C)*a^5*b^2 + 8*B*a^4*b^3 - 4*(A - C)*a^3*b^4 - 4*B*a^2*b^5 +
(2*A + C)*a*b^6)*cos(d*x + c)^3 + (6*C*a^6*b - 4*B*a^5*b^2 + (2*A - 11*C)*a
^4*b^3 + 8*B*a^3*b^4 - 4*(A - C)*a^2*b^5 - 4*B*a*b^6 + (2*A + C)*b^7)*cos(d
*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7 - 2*
(3*C*a^6*b - 2*B*a^5*b^2 + (A - 5*C)*a^4*b^3 + 3*B*a^3*b^4 - (A - 2*C)*a^2*
b^5 - B*a*b^6)*cos(d*x + c)^2 - (3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*C*a^3*b^4 +
4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 -
2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x
 + c)^2), -1/4*(4*((3*C*a^6 - 2*B*a^5*b + (A - 4*C)*a^4*b^2 + 3*B*a^3*b^3 -
 2*A*a^2*b^4)*cos(d*x + c)^3 + (3*C*a^5*b - 2*B*a^4*b^2 + (A - 4*C)*a^3*b^3
 + 3*B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-
a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((6*C*a^7 - 4
*B*a^6*b + (2*A - 11*C)*a^5*b^2 + 8*B*a^4*b^3 - 4*(A - C)*a^3*b^4 - 4*B*a^2
*b^5 + (2*A + C)*a*b^6)*cos(d*x + c)^3 + (6*C*a^6*b - 4*B*a^5*b^2 + (2*A -
11*C)*a^4*b^3 + 8*B*a^3*b^4 - 4*(A - C)*a^2*b^5 - 4*B*a*b^6 + (2*A + C)*b^7
)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*C*a^7 - 4*B*a^6*b + (2*A - 11
*C)*a^5*b^2 + 8*B*a^4*b^3 - 4*(A - C)*a^3*b^4 - 4*B*a^2*b^5 + (2*A + C)*a*b
^6)*cos(d*x + c)^3 + (6*C*a^6*b - 4*B*a^5*b^2 + (2*A - 11*C)*a^4*b^3 + 8*B*
a^3*b^4 - 4*(A - C)*a^2*b^5 - 4*B*a*b^6 + (2*A + C)*b^7)*cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1) - 2*(C*a^4*b^3 - 2*C*a^2*b^5 + C*b^7 - 2*(3*C*a^6*b -
2*B*a^5*b^2 + (A - 5*C)*a^4*b^3 + 3*B*a^3*b^4 - (A - 2*C)*a^2*b^5 - B*a*b^6
)*cos(d*x + c)^2 - (3*C*a^5*b^2 - 2*B*a^4*b^3 - 6*C*a^3*b^4 + 4*B*a^2*b^5 +
 3*C*a*b^6 - 2*B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a
*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
2,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*se
c(c + d*x))**2, x)
```

Giac [A] time = 1.38434, size = 578, normalized size = 1.85

$$-\frac{\frac{4\left(3Ca^5-2Ba^4b+Aa^3b^2-4Ca^3b^2+3Ba^2b^3-2Aab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{4\left(Ca^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ba^3b\,\mathrm{ta}\ldots\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}dx+\ldots\right.\right.}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/2*(4*(3*C*a^5 - 2*B*a^4*b + A*a^3*b^2 - 4*C*a^3*b^2 + 3*B*a^2*b^3 - 2*A*
a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^
```

```
6)*sqrt(-a^2 + b^2)) - 4*(C*a^4*tan(1/2*d*x + 1/2*c) - B*a^3*b*tan(1/2*d*x
+ 1/2*c) + A*a^2*b^2*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*(a*tan(1/2*d*x
+ 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (6*C*a^2 - 4*B*a*b + 2*A*
b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (6*C*a^2 - 4*B*a*b +
2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(4*C*a*tan(1/2*
d*x + 1/2*c)^3 - 2*B*b*tan(1/2*d*x + 1/2*c)^3 + C*b*tan(1/2*d*x + 1/2*c)^3
- 4*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + C*b*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

3.910 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=177

$$-\frac{2\left(3a^2b^2C+a^3bB-2a^4C-2ab^3B+Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(bB-2aC)}{}$$

```
[Out] ((b*B - 2*a*C)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*(A*b^4 + a^3*b*B - 2*a*b
^3*B - 2*a^4*C + 3*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (C*Tan[c + d*x])/(b^2*d) + (a
*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))
```

Rubi [A] time = 0.64743, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4090, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{2\left(3a^2b^2C+a^3bB-2a^4C-2ab^3B+Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{(bB-2aC)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```


```
[Out] ((b*B - 2*a*C)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*(A*b^4 + a^3*b*B - 2*a*b
^3*B - 2*a^4*C + 3*a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (C*Tan[c + d*x])/(b^2*d) + (a
*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))
```


Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(-b\left(Ab^2-a(bB-\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \cdots}{\cdots}$$

$$= \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(bB \cdots}{\cdots}$$

$$= \frac{(bB-2aC)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2 \cdots\right.}{b^2\left(a^2 \cdots\right.}$$

$$= \frac{(bB-2aC)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{C\tan(c+dx)}{b^2d} + \frac{a\left(Ab^2 \cdots\right.}{b^2\left(a^2 \cdots\right.}$$

$$= \frac{(bB-2aC)\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2\left(Ab^4+a^3bB-2ab^3B \cdots\right.}{\cdots}$$

Mathematica [B] time = 3.00144, size = 382, normalized size = 2.16

$$\frac{2(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2\left(a\left(a^2bB-2a^3C+3ab^2C-2b^3B\right)+Ab^4\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^2,x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(A*b^4
+ a*(a^2*b*B - 2*b^3*B - 2*a^3*C + 3*a*b^2*C))*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)^(3/2) - (b*B - 2
*a*C)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + (b*B
- 2*a*C)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b
*C*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]) + (b*C*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]) + (a*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a - b)*(a +
 b))))/(b^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a + b*Sec[
c + d*x])^2)
```

Maple [B] time = 0.092, size = 630, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)*A+2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B-2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-2/d*b/(a+b)/(a-b)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d
*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))*B+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-6/d*a^2/b
/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*C-1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*C+1/d/b^2*ln(tan(1/2*d*x+1/2*c
)+1)*B-2/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*a*C-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*
C-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*B+2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 63.7846, size = 2514, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(((2*C*a^5 - B*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^3 - A*a*b^4)*cos(d*x +
c)^2 + (2*C*a^4*b - B*a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4 - A*b^5)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
+ 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - ((2*C*a^6 - B*a^5*b - 4*C*a^4*
b^2 + 2*B*a^3*b^3 + 2*C*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + (2*C*a^5*b - B*
a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*cos(d*x + c))*log(
sin(d*x + c) + 1) + ((2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 + 2*C*a
^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + (2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2
*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(C
*a^4*b^2 - 2*C*a^2*b^4 + C*b^6 + (2*C*a^5*b - B*a^4*b^2 + (A - 3*C)*a^3*b^3
 + B*a^2*b^4 - (A - C)*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3
*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)
), 1/2*(2*((2*C*a^5 - B*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^3 - A*a*b^4)*cos(d*
x + c)^2 + (2*C*a^4*b - B*a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4 - A*b^5)*cos(d*
x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c))) - ((2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 +
 2*C*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + (2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b
^3 + 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1) +
 ((2*C*a^6 - B*a^5*b - 4*C*a^4*b^2 + 2*B*a^3*b^3 + 2*C*a^2*b^4 - B*a*b^5)*c
os(d*x + c)^2 + (2*C*a^5*b - B*a^4*b^2 - 4*C*a^3*b^3 + 2*B*a^2*b^4 + 2*C*a*
b^5 - B*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(C*a^4*b^2 - 2*C*a^2*
b^4 + C*b^6 + (2*C*a^5*b - B*a^4*b^2 + (A - 3*C)*a^3*b^3 + B*a^2*b^4 - (A -
 C)*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos
(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
2,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*se
c(c + d*x))**2, x)
```

Giac [B] time = 1.38399, size = 598, normalized size = 3.38

$$\frac{\frac{2\left(2Ca^4-Ba^3b-3Ca^2b^2+2Bab^3-Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{-a^2+b^2}}-\frac{2\left(2Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] (2*(2*C*a^4 - B*a^3*b - 3*C*a^2*b^2 + 2*B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - 2*
(2*C*a^3*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - C*a^2*b*
tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - C*a*b^2*tan(1/2*d
*x + 1/2*c)^3 + C*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^3*tan(1/2*d*x + 1/2*c)
 + B*a^2*b*tan(1/2*d*x + 1/2*c) - C*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*ta
n(1/2*d*x + 1/2*c) + C*a*b^2*tan(1/2*d*x + 1/2*c) + C*b^3*tan(1/2*d*x + 1/2
*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*
x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) - (2*C*a - B*b)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/b^3 + (2*C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3)/
d
```

3.911 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{2\left(a^3(-C)+aAb^2+2ab^2C-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{C\tanh^{-1}(\sin(c+}{b^2d}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^2*d) + (2*(a*A*b^2 - b^3*B - a^3*C + 2*a*b^2*C
)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(
a + b)^(3/2)*d) - ((A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(b*(a^2 - b^2)*d*(
a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.303745, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4080, 3998, 3770, 3831, 2659, 208}

$$\frac{2\left(a^3(-C)+aAb^2+2ab^2C-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{C\tanh^{-1}(\sin(c+}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^2,x]
```

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^2*d) + (2*(a*A*b^2 - b^3*B - a^3*C + 2*a*b^2*C
)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(
a + b)^(3/2)*d) - ((A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(b*(a^2 - b^2)*d*(
a + b*Sec[c + d*x]))
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(b(bB-a(A+C))-\left(a^2-\right.\right.}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{C\int \sec(c+dx)\,dx}{b^2} - \frac{\left(b^3B\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{2\left(aAb^2-b^3B-a^3C+2ab^2C\right)\tanh}{(a-b)^{3/2}b^2(a+b)^{3/}}$$

Mathematica [C] time = 3.0895, size = 356, normalized size = 2.41

$$\frac{2(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2(\sin(c)+i\cos(c))\left(a^3C-ab^2(A+2C)+b^3B\right)(a\cos(c+dx)+b)\tan^{-1}\left(\frac{(\sin(c)+i\cos(c))\left(\mathrm{t}\right.}{\sqrt{a^2-b^2}}\right.}{\left(a^2-b^2\right)^{3/2}\sqrt{(\cos(c)-i\sin(c))^2}}\right.}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-(C*(b + a
*Cos[c + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + C*(b + a*Cos[c +
 d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*(b^3*B + a^3*C - a*b^2
*(A + 2*C))*ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*
x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b + a*Cos[c + d*x])
```

```
*(I*Cos[c] + Sin[c]))/((a^2 - b^2)^(3/2)*Sqrt[(Cos[c] - I*Sin[c])^2]) + (b*
(A*b^2 + a*(-(b*B) + a*C))*(b*Sin[c] - a*Sin[d*x]))/(a*(a - b)*(a + b)*(Cos
[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))))/(b^2*d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.095, size = 470, normalized size = 3.2

$$2\,{\frac{b\tan \left(1/2\,dx+c/2 \right) A}{d \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}b-a-b \right) }}-2\,{\frac{\tan \left(1/2\,dx+c/2 \right) Ba}{d \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)*A-2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)*B*a+2/d/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*a^2*C+2/d*a/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-2/d*b/(a+b
)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*B-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+1/d/b^2*ln(tan(1/2*d*x
+1/2*c)+1)*C-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 18.0891, size = 1621, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*((C*a^3*b - (A + 2*C)*a*b^3 + B*b^4 + (C*a^4 - (A + 2*C)*a^2*b^2 + B*a
*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (C*a^4*b - 2*C
*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(sin(d*
x + c) + 1) - (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b
^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(C*a^4*b - B*a^3*b^2 + (A - C)
```

```
*a^2*b^3 + B*a*b^4 - A*b^5)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*
cos(d*x + c) + (a^4*b^3 - 2*a^2*b^5 + b^7)*d), -1/2*(2*(C*a^3*b - (A + 2*C)
*a*b^3 + B*b^4 + (C*a^4 - (A + 2*C)*a^2*b^2 + B*a*b^3)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d
*x + c))) - (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + (C*a^5 - 2*C*a^3*b^2 + C*a*b^4
)*cos(d*x + c))*log(sin(d*x + c) + 1) + (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + (C
*a^5 - 2*C*a^3*b^2 + C*a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(C*a
^4*b - B*a^3*b^2 + (A - C)*a^2*b^3 + B*a*b^4 - A*b^5)*sin(d*x + c))/((a^5*b
^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*a^2*b^5 + b^7)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c
 + d*x))**2, x)
```

Giac [A] time = 1.29616, size = 338, normalized size = 2.28

$$\frac{\frac{2\left(Ca^3 - Aab^2 - 2\,Cab^2 + Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\left(a^2b^2 - b^4\right)\sqrt{-a^2 + b^2}} + \frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right)}{b^2} - \frac{C\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{b^2} -}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] (2*(C*a^3 - A*a*b^2 - 2*C*a*b^2 + B*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + C*log(abs(tan(1/2*d*
x + 1/2*c) + 1))/b^2 - C*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*(C*a^2*
tan(1/2*d*x + 1/2*c) - B*a*b*tan(1/2*d*x + 1/2*c) + A*b^2*tan(1/2*d*x + 1/2
*c))/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 -
a - b)))/d
```

3.912 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2\left(2a^2Ab+a^2bC+a^3(-B)-Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Ax}{a^2}$$

```
[Out] (A*x)/a^2 - (2*(2*a^2*A*b - A*b^3 - a^3*B + a^2*b*C)*ArcTanh[(Sqrt[a - b]*T
an[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((A*b^
2 - a*(b*B - a*C))*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.252392, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4060, 3919, 3831, 2659, 208}

$$-\frac{2\left(2a^2Ab+a^2bC+a^3(-B)-Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{Ax}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```



```
[Out] (A*x)/a^2 - (2*(2*a^2*A*b - A*b^3 - a^3*B + a^2*b*C)*ArcTanh[(Sqrt[a - b]*T
an[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((A*b^
2 - a*(b*B - a*C))*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-A\left(a^2-b^2\right)+a(Ab-aB+bC)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{Ax}{a^2} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(Ab^3+a^3B-a^2b(2A+C)\right)\int \frac{\mathrm{sec}}{a+b\,\mathrm{s}}}{a^2\left(a^2-b^2\right)}$$

$$= \frac{Ax}{a^2} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(Ab^3+a^3B-a^2b(2A+C)\right)\int \frac{}{1+\frac{a\,c}{}}}{a^2b\left(a^2-b^2\right)}$$

$$= \frac{Ax}{a^2} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\left(Ab^3+a^3B-a^2b(2A+C)\right)\right)\mathrm{Su}}{a^2b}$$

$$= \frac{Ax}{a^2} - \frac{2\left(2a^2Ab-Ab^3-a^3B+a^2bC\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(Ab^2-a(\right.}{a\left(a^2-b^{\cdot}\right.}$$

Mathematica [C] time = 2.20879, size = 299, normalized size = 2.17

$$\frac{2(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(-\frac{2i(\cos(c)-i\sin(c))\left(-a^2b(2A+C)+a^3B+Ab^3\right)(a\cos(c+dx)+b)\tan^{-1}\left(\frac{(\sin(c)+i\cos(}{\sqrt{a^{2}}}\right.}{d\left(a^2-b^2\right)^{3/2}\sqrt{(\cos(c)-i\sin(c))^2}}\right.}{a^2(a+b\sec(c+dx))^2(A\cos(2(c+dx))+A+2B\,c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*x*(b + a
*Cos[c + d*x]) - ((2*I)*(A*b^3 + a^3*B - a^2*b*(2*A + C))*ArcTan[((I*Cos[c]
 + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt
[(Cos[c] - I*Sin[c])^2]]]*(b + a*Cos[c + d*x])*(Cos[c] - I*Sin[c]))/((a^2 -
 b^2)^(3/2)*d*Sqrt[(Cos[c] - I*Sin[c])^2]) + ((A*b^2 + a*(-(b*B) + a*C))*(-
(b*Sin[c]) + a*Sin[d*x]))/((a - b)*(a + b)*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2
] + Sin[c/2])))))/(a^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a
+ b*Sec[c + d*x])^2)
```

Maple [B] time = 0.097, size = 448, normalized size = 3.3

$$2\,\frac{A\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2} - 2\,\frac{A\tan\left(1/2\,dx+c/2\right)b^2}{ad\left(a^2-b^2\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b-a-b\right)} + 2\,\frac{}{d\left(a^2-b^2\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*A/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A*b^2+2/d/(a^2-b^2)*tan(1/
2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B*b-2/d*a/
(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)*C-4/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*A+2/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^3+2/d*a/(a+b)/(a-b)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*b*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.626144, size = 1296, normalized size = 9.39

$$\left[\frac{2\left(Aa^5-2\,Aa^3b^2+Aab^4\right)dx\cos\left(dx+c\right)+2\left(Aa^4b-2\,Aa^2b^3+Ab^5\right)dx+\left(Ba^3b-\left(2\,A+C\right)a^2b^2+Ab^4+\left(Ba^4-(2\right.\right.}{2(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [1/2*(2*(A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + 2*(A*a^4*b - 2*A
*a^2*b^3 + A*b^5)*d*x + (B*a^3*b - (2*A + C)*a^2*b^2 + A*b^4 + (B*a^4 - (2*
A + C)*a^3*b + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x +
c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*
sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)
) + 2*(C*a^5 - B*a^4*b + (A - C)*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*sin(d*x + c
))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b
^5)*d), ((A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + (A*a^4*b - 2*A*
a^2*b^3 + A*b^5)*d*x + (B*a^3*b - (2*A + C)*a^2*b^2 + A*b^4 + (B*a^4 - (2*A
 + C)*a^3*b + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^5 - B*a^4*b + (
A - C)*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(a + b\sec(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**2,
x)
```

Giac [A] time = 1.24265, size = 300, normalized size = 2.17

$$\frac{\frac{2\,(Ba^3 - 2\,Aa^2b - Ca^2b + Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{(a^4 - a^2b^2)\sqrt{-a^2 + b^2}} + \frac{(dx+c)A}{a^2} - \frac{2\left(Ca^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - Bab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \cdots\right.}{(a^3 - ab^2)\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - b\tan\left(\frac{1}{2}\right.\cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b - C*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))
/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) + (d*x + c)*A/a^2 -
2*(C*a^2*tan(1/2*d*x + 1/2*c) - B*a*b*tan(1/2*d*x + 1/2*c) + A*b^2*tan(1/2*
d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/
2*c)^2 - a - b)))/d
```

3.913 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\sin(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{2\left(3a^2Ab^2-2a^3bB+a^4C+ab^3B-2Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+$$

```
[Out] -(((2*A*b - a*B)*x)/a^3) + (2*(3*a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + a*b^3*B
+ a^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^
(3/2)*(a + b)^(3/2)*d) - ((2*A*b^2 - a*b*B - a^2*(A - C))*Sin[c + d*x])/(a^
2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.644187, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\sin(c+dx)\left(a^2(-(A-C))-abB+2Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{2\left(3a^2Ab^2-2a^3bB+a^4C+ab^3B-2Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^2,x]
```

```
[Out] -(((2*A*b - a*B)*x)/a^3) + (2*(3*a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + a*b^3*B
+ a^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^
(3/2)*(a + b)^(3/2)*d) - ((2*A*b^2 - a*b*B - a^2*(A - C))*Sin[c + d*x])/(a^
2*(a^2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\cos(c+dx)\left(2Ab^2-abB-a^2(A-C)+\ldots\right.}{\ldots}$$

$$= -\frac{\left(2Ab^2-abB-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2-a(bB-aC)\right)\ldots}{a\left(a^2-b^2\right)d(a+b\,\mathrm{s}\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} - \frac{\left(2Ab^2-abB-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{(Ab\ldots}{a(a\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} - \frac{\left(2Ab^2-abB-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{(Ab\ldots}{a(a\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} - \frac{\left(2Ab^2-abB-a^2(A-C)\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{(Ab\ldots}{a(a\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{2\left(3a^2Ab^2-2Ab^4-2a^3bB+ab^3B+a^4C\right)\mathrm{ta}\ldots}{a^3(a-b)^{3/2}(a+b)^{3/2}d\ldots}$$

Mathematica [A] time = 0.955774, size = 160, normalized size = 0.79

$$\frac{-\frac{2\left(3a^2Ab^2-2a^3bB+a^4C+ab^3B-2Ab^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{ab\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{(a-b)(a+b)(a\cos(c+dx)+b)}+(c+dx)(aB-2Ab)+aA\sin(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```

```
[Out] ((-2*A*b + a*B)*(c + d*x) - (2*(3*a^2*A*b^2 - 2*A*b^4 - 2*a^3*b*B + a*b^3*B
 + a^4*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(3/2) + a*A*Sin[c + d*x] - (a*b*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/(
(a - b)*(a + b)*(b + a*Cos[c + d*x])))/(a^3*d)
```

Maple [B] time = 0.131, size = 573, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*A/a^2*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d*A/a^3*b*arctan(ta
n(1/2*d*x+1/2*c))+2/d*B/a^2*arctan(tan(1/2*d*x+1/2*c))+2/d/a^2*b^3/(a^2-b^2
)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-
2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)*B+2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C+6/d/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^2-4/d/a^3/(a+b)/(a-b)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A
*b^4-4/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*B+2/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b^3+2/d*a/(a+b)/(a-b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.726212, size = 1796, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] [1/2*(2*(B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*
b^5)*d*x*cos(d*x + c) + 2*(B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^
```

```
4 + B*a*b^5 - 2*A*b^6)*d*x - (C*a^4*b - 2*B*a^3*b^2 + 3*A*a^2*b^3 + B*a*b^4
 - 2*A*b^5 + (C*a^5 - 2*B*a^4*b + 3*A*a^3*b^2 + B*a^2*b^3 - 2*A*a*b^4)*cos(
d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*((A - C)*a^5*b + B*a^4
*b^2 - (3*A - C)*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5 + (A*a^6 - 2*A*a^4*b^2 + A
*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*
x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), ((B*a^6 - 2*A*a^5*b - 2*B*a^4*b^
2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*d*x*cos(d*x + c) + (B*a^5*b - 2*A*
a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*d*x + (C*a^4*b - 2
*B*a^3*b^2 + 3*A*a^2*b^3 + B*a*b^4 - 2*A*b^5 + (C*a^5 - 2*B*a^4*b + 3*A*a^3
*b^2 + B*a^2*b^3 - 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-
a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + ((A - C)*a^5*
b + B*a^4*b^2 - (3*A - C)*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5 + (A*a^6 - 2*A*a^
4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)
*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\cos\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2,x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)/(a + b*sec(c
 + d*x))**2, x)
```

Giac [B] time = 1.29528, size = 551, normalized size = 2.73

$$\frac{\frac{2\left(Ca^4-2Ba^3b+3Aa^2b^2+Bab^3-2Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{-a^2+b^2}}+\frac{2\left(Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] (2*(C*a^4 - 2*B*a^3*b + 3*A*a^2*b^2 + B*a*b^3 - 2*A*b^4)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^5 - a^3*b^2)*sqrt(-a^2 + b^2)) + 2*
(A*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + C*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c) -
 A*a^2*b*tan(1/2*d*x + 1/2*c) + C*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(
1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x + 1/2
*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*
x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + (B*a - 2*A*b)*(d*x + c)/a^3)/d
```

3.914 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=298

$$\frac{\sin(c+dx)\left(-a^2b(2A-C)+a^3B-2ab^2B+3Ab^3\right)}{a^3d\left(a^2-b^2\right)}-\frac{\sin(c+dx)\cos(c+dx)\left(a^2(-(A-2C))-2abB+3Ab^2\right)}{2a^2d\left(a^2-b^2\right)}-\frac{2b(4\ldots}{\ldots}$$

```
[Out] ((6*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*x)/(2*a^4) - (2*b*(4*a^2*A*b^2 - 3*A*b
^4 - 3*a^3*b*B + 2*a*b^3*B + 2*a^4*C - a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((3*A*b^3
 + a^3*B - 2*a*b^2*B - a^2*b*(2*A - C))*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) -
 ((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a^
2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]*Sin[c + d*x])/(a*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.23513, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\sin(c+dx)\left(-a^2b(2A-C)+a^3B-2ab^2B+3Ab^3\right)}{a^3d\left(a^2-b^2\right)}-\frac{\sin(c+dx)\cos(c+dx)\left(a^2(-(A-2C))-2abB+3Ab^2\right)}{2a^2d\left(a^2-b^2\right)}-\frac{2b(4\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```

```
[Out] ((6*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*x)/(2*a^4) - (2*b*(4*a^2*A*b^2 - 3*A*b
^4 - 3*a^3*b*B + 2*a*b^3*B + 2*a^4*C - a^2*b^2*C)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((3*A*b^3
 + a^3*B - 2*a*b^2*B - a^2*b*(2*A - C))*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) -
 ((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a^
2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]*Sin[c + d*x])/(a*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
```

```
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx=\frac{\left(Ab^2-a(bB-aC)\right)\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\int \frac{\cos^2(c+dx)\left(3Ab\right.}{}}{}$$

$$=-\frac{\left(3Ab^2-2abB-a^2(A-2C)\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d}+\frac{\left(Ab\right.}{}$$

$$=\frac{\left(3Ab^3+a^3B-2ab^2B-a^2b(2A-C)\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d}-\frac{\left(3Ab^2-\right.}{}$$

$$=\frac{\left(6Ab^2-4abB+a^2(A+2C)\right)x}{2a^4}+\frac{\left(3Ab^3+a^3B-2ab^2B-a^2b\right.}{a^3\left(a^2-b\right.}$$

$$=\frac{\left(6Ab^2-4abB+a^2(A+2C)\right)x}{2a^4}+\frac{\left(3Ab^3+a^3B-2ab^2B-a^2b\right.}{a^3\left(a^2-b\right.}$$

$$=\frac{\left(6Ab^2-4abB+a^2(A+2C)\right)x}{2a^4}+\frac{\left(3Ab^3+a^3B-2ab^2B-a^2b\right.}{a^3\left(a^2-b\right.}$$

$$=\frac{\left(6Ab^2-4abB+a^2(A+2C)\right)x}{2a^4}-\frac{2b\left(4a^2Ab^2-3Ab^4-3a^3bB\right.}{}$$

Mathematica [A] time = 1.40944, size = 206, normalized size = 0.69

$$\frac{2(c+dx)\left(a^2(A+2C)-4abB+6Ab^2\right)-\frac{8b\left(a^2b^2(C-4A)+3a^3bB-2a^4C-2ab^3B+3Ab^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+a^2A\sin(2(c+dx)}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^2,x]
```

```
[Out] (2*(6*A*b^2 - 4*a*b*B + a^2*(A + 2*C))*(c + d*x) - (8*b*(3*A*b^4 + 3*a^3*b*
B - 2*a*b^3*B - 2*a^4*C + a^2*b^2*(-4*A + C))*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 4*a*(-2*A*b + a*B)*Sin[c + d*x
] + (4*a*b^2*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a - b)*(a + b)*(b +
 a*Cos[c + d*x])) + a^2*A*Sin[2*(c + d*x)])/(4*a^4*d)
```

Maple [B] time = 0.138, size = 857, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-4/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)^3*B+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)*A-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b+2/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B+1/d*A/a^2*arctan(tan(1/2*d*x
+1/2*c))+6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*A*b^2-4/d/a^3*arctan(tan(1/2*d*
x+1/2*c))*B*b+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b^4/a^3/(a^2-b^2)*ta
n(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A+2/d*
b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)*B-2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-8/d/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^3+6/d*b^5/a^4/(a+b
)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^
(1/2))*A+6/d*b^2/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d*b^4/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d/(a+b)/(a-b)/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b*C
+2/d*b^3/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.819258, size = 2398, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(((A + 2*C)*a^7 - 4*B*a^6*b + 4*(A - C)*a^5*b^2 + 8*B*a^4*b^3 - (11*A
- 2*C)*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + ((A + 2*C)*a^6
*b - 4*B*a^5*b^2 + 4*(A - C)*a^4*b^3 + 8*B*a^3*b^4 - (11*A - 2*C)*a^2*b^5 -
 4*B*a*b^6 + 6*A*b^7)*d*x + (2*C*a^4*b^2 - 3*B*a^3*b^3 + (4*A - C)*a^2*b^4
+ 2*B*a*b^5 - 3*A*b^6 + (2*C*a^5*b - 3*B*a^4*b^2 + (4*A - C)*a^3*b^3 + 2*B*
a^2*b^4 - 3*A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 (2*B*a^6*b - 2*(2*A - C)*a^5*b^2 - 6*B*a^4*b^3 + 2*(5*A - C)*a^3*b^4 + 4*B
*a^2*b^5 - 6*A*a*b^6 + (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (
2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5
)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) +
 (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*(((A + 2*C)*a^7 - 4*B*a^6*b + 4*(A -
 C)*a^5*b^2 + 8*B*a^4*b^3 - (11*A - 2*C)*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)
*d*x*cos(d*x + c) + ((A + 2*C)*a^6*b - 4*B*a^5*b^2 + 4*(A - C)*a^4*b^3 + 8*
B*a^3*b^4 - (11*A - 2*C)*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x - 2*(2*C*a^4*b^
2 - 3*B*a^3*b^3 + (4*A - C)*a^2*b^4 + 2*B*a*b^5 - 3*A*b^6 + (2*C*a^5*b - 3*
B*a^4*b^2 + (4*A - C)*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5)*cos(d*x + c))*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (2*B*a^6*b - 2*(2*A - C)*a^5*b^2 - 6*B*a^4*b^3 + 2*(5*A - C)*
a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 + (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d
*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4
- 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*c
os(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27657, size = 513, normalized size = 1.72

$$-\frac{\frac{4\left(2\,Ca^4b-3\,Ba^3b^2+4\,Aa^2b^3-Ca^2b^3+2\,Bab^4-3\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}+\frac{4\left(Ca^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bab^3}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="giac")
```

```
[Out] -1/2*(4*(2*C*a^4*b - 3*B*a^3*b^2 + 4*A*a^2*b^3 - C*a^2*b^3 + 2*B*a*b^4 - 3*
A*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^
2)*sqrt(-a^2 + b^2)) + 4*(C*a^2*b^2*tan(1/2*d*x + 1/2*c) - B*a*b^3*tan(1/2*
d*x + 1/2*c) + A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)*(a*tan(1/2*d*x
+ 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (A*a^2 + 2*C*a^2 - 4*B*a*
b + 6*A*b^2)*(d*x + c)/a^4 + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*
d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) -
2*B*a*tan(1/2*d*x + 1/2*c) + 4*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^2*a^3))/d
```

3.915 $\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=396

$$-\frac{\sin(c+dx)\left(-a^2b^2(7A-6C)+a^4(-(2A+3C))+6a^3bB-9ab^3B+12Ab^4\right)}{3a^4d\left(a^2-b^2\right)}-\frac{\sin(c+dx)\cos^2(c+dx)\left(a^2(-(A-3C))\right.}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] -((8*A*b^3 - a^3*B - 6*a*b^2*B + 2*a^2*b*(A + 2*C))*x)/(2*a^5) + (2*b^2*(5*
a^2*A*b^2 - 4*A*b^4 - 4*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 2*a^2*b^2*C)*ArcTan
h[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(
3/2)*d) - ((12*A*b^4 + 6*a^3*b*B - 9*a*b^3*B - a^2*b^2*(7*A - 6*C) - a^4*(2
*A + 3*C))*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) + ((4*A*b^3 + a^3*B - 3*a*b^
2*B - 2*a^2*b*(A - C))*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) - (
(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*(a^
2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2
 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.75885, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\sin(c+dx)\left(-a^2b^2(7A-6C)+a^4(-(2A+3C))+6a^3bB-9ab^3B+12Ab^4\right)}{3a^4d\left(a^2-b^2\right)}-\frac{\sin(c+dx)\cos^2(c+dx)\left(a^2(-(A-3C))\right.}{3a^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^2,x]
```


```
[Out] -((8*A*b^3 - a^3*B - 6*a*b^2*B + 2*a^2*b*(A + 2*C))*x)/(2*a^5) + (2*b^2*(5*
a^2*A*b^2 - 4*A*b^4 - 4*a^3*b*B + 3*a*b^3*B + 3*a^4*C - 2*a^2*b^2*C)*ArcTan
h[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(
3/2)*d) - ((12*A*b^4 + 6*a^3*b*B - 9*a*b^3*B - a^2*b^2*(7*A - 6*C) - a^4*(2
*A + 3*C))*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) + ((4*A*b^3 + a^3*B - 3*a*b^
2*B - 2*a^2*b*(A - C))*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) - (
(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*(a^
2 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2
 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\cos^3(c+dx)\left(4A\right.}{}$$

$$= -\frac{\left(4Ab^2-3abB-a^2(A-3C)\right)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{(A}{}$$

$$= \frac{\left(4Ab^3+a^3B-3ab^2B-2a^2b(A-C)\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(12Ab^4+6a^3bB-9ab^3B-a^2b^2(7A-6C)-a^4(2A+3C)\right)}{3a^4\left(a^2-b^2\right)d}$$

$$= -\frac{\left(8Ab^3-a^3B-6ab^2B+2a^2b(A+2C)\right)x}{2a^5} - \frac{\left(12Ab^4+6a^3bB\right.}{}$$

$$= -\frac{\left(8Ab^3-a^3B-6ab^2B+2a^2b(A+2C)\right)x}{2a^5} - \frac{\left(12Ab^4+6a^3bB\right.}{}$$

$$= -\frac{\left(8Ab^3-a^3B-6ab^2B+2a^2b(A+2C)\right)x}{2a^5} - \frac{\left(12Ab^4+6a^3bB\right.}{}$$

$$= -\frac{\left(8Ab^3-a^3B-6ab^2B+2a^2b(A+2C)\right)x}{2a^5} + \frac{2b^2\left(5a^2Ab^2-4\right.}{}$$

Mathematica [A] time = 1.74879, size = 255, normalized size = 0.64

$$\frac{6(c+dx)\left(-2a^2b(A+2C)+a^3B+6ab^2B-8Ab^3\right)+3a\sin(c+dx)\left(a^2(3A+4C)-8abB+12Ab^2\right)+\frac{24b^2\left(a^2b^2(2C-5A)+4a^3b\right.}{}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^2,x]
```

```
[Out] (6*(-8*A*b^3 + a^3*B + 6*a*b^2*B - 2*a^2*b*(A + 2*C))*(c + d*x) + (24*b^2*(
4*A*b^4 + 4*a^3*b*B - 3*a*b^3*B - 3*a^4*C + a^2*b^2*(-5*A + 2*C))*ArcTanh[(
(-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 3*a*(12*A*
b^2 - 8*a*b*B + a^2*(3*A + 4*C))*Sin[c + d*x] - (12*a*b^3*(A*b^2 + a*(-(b*B
) + a*C))*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + 3*a^2*(-2*
A*b + a*B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])/(12*a^5*d)
```

Maple [B] time = 0.16, size = 1241, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -4/d/a^3*C*arctan(tan(1/2*d*x+1/2*c))*b-8/d*b^6/a^5/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*b^2/a/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*C-4/d*b^4/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2
*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-8/d/a^2/(a+b)/(a-
b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*B*b^3+10/d/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*A*b^4+6/d*b^5/a^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/
2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+1/d*B/a^2*arctan
(tan(1/2*d*x+1/2*c))-2/d*A/a^3*b*arctan(tan(1/2*d*x+1/2*c))+6/d/a^4*arctan(
tan(1/2*d*x+1/2*c))*B*b^2-2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*B+2/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^5*A*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d
*x+1/2*c)^5*A*b^2-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B
*b+12/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A*b^2+2/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d*b^5/a^4/(a^2-b^2)*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*A-1/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B+2/d/a^2/(1+tan(1/2*d*x+
1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*C+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)^3*A+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3
*C+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A+2/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*C+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*
tan(1/2*d*x+1/2*c)*B-8/d/a^5*arctan(tan(1/2*d*x+1/2*c))*A*b^3-8/d/a^3/(1+ta
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c
)^2)^3*tan(1/2*d*x+1/2*c)*A*b^2-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)*B*b-2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.930769, size = 3000, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*a^8 - 2*(A + 2*C)*a^7*b + 4*B*a^6*b^2 - 4*(A - 2*C)*a^5*b^3 - 11
*B*a^4*b^4 + 2*(7*A - 2*C)*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*cos(d*x +
 c) + 3*(B*a^7*b - 2*(A + 2*C)*a^6*b^2 + 4*B*a^5*b^3 - 4*(A - 2*C)*a^4*b^4
- 11*B*a^3*b^5 + 2*(7*A - 2*C)*a^2*b^6 + 6*B*a*b^7 - 8*A*b^8)*d*x + 3*(3*C*
a^4*b^3 - 4*B*a^3*b^4 + (5*A - 2*C)*a^2*b^5 + 3*B*a*b^6 - 4*A*b^7 + (3*C*a^
5*b^2 - 4*B*a^4*b^3 + (5*A - 2*C)*a^3*b^4 + 3*B*a^2*b^5 - 4*A*a*b^6)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
```

```
)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*(2*A + 3*C)*a^7*b - 12*
B*a^6*b^2 + 2*(5*A - 9*C)*a^5*b^3 + 30*B*a^4*b^4 - 2*(19*A - 6*C)*a^3*b^5 -
 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)*cos(d*x +
c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b^2 + 8*A*a^5*b^3 + 3*B*a^4*b^4 - 4*A
*a^3*b^5)*cos(d*x + c)^2 + (2*(2*A + 3*C)*a^8 - 9*B*a^7*b + 4*(A - 3*C)*a^6
*b^2 + 18*B*a^5*b^3 - 2*(10*A - 3*C)*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a^2*b^6)*
cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) +
(a^9*b - 2*a^7*b^3 + a^5*b^5)*d), 1/6*(3*(B*a^8 - 2*(A + 2*C)*a^7*b + 4*B*a
^6*b^2 - 4*(A - 2*C)*a^5*b^3 - 11*B*a^4*b^4 + 2*(7*A - 2*C)*a^3*b^5 + 6*B*a
^2*b^6 - 8*A*a*b^7)*d*x*cos(d*x + c) + 3*(B*a^7*b - 2*(A + 2*C)*a^6*b^2 + 4
*B*a^5*b^3 - 4*(A - 2*C)*a^4*b^4 - 11*B*a^3*b^5 + 2*(7*A - 2*C)*a^2*b^6 + 6
*B*a*b^7 - 8*A*b^8)*d*x + 6*(3*C*a^4*b^3 - 4*B*a^3*b^4 + (5*A - 2*C)*a^2*b^
5 + 3*B*a*b^6 - 4*A*b^7 + (3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A - 2*C)*a^3*b^4
+ 3*B*a^2*b^5 - 4*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*(2*A + 3*C)*a
^7*b - 12*B*a^6*b^2 + 2*(5*A - 9*C)*a^5*b^3 + 30*B*a^4*b^4 - 2*(19*A - 6*C)
*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)*
cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b^2 + 8*A*a^5*b^3 + 3*B*a^4
*b^4 - 4*A*a^3*b^5)*cos(d*x + c)^2 + (2*(2*A + 3*C)*a^8 - 9*B*a^7*b + 4*(A
- 3*C)*a^6*b^2 + 18*B*a^5*b^3 - 2*(10*A - 3*C)*a^4*b^4 - 9*B*a^3*b^5 + 12*A
*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d
*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
2,x)

[Out] Timed out
```

Giac [A] time = 1.29786, size = 761, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x
, algorithm="giac")

[Out] 1/6*(12*(3*C*a^4*b^2 - 4*B*a^3*b^3 + 5*A*a^2*b^4 - 2*C*a^2*b^4 + 3*B*a*b^5
- 4*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^
5*b^2)*sqrt(-a^2 + b^2)) + 12*(C*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a*b^4*tan
(1/2*d*x + 1/2*c) + A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 3*(B*a^3 - 2*A*a^2*b
 - 4*C*a^2*b + 6*B*a*b^2 - 8*A*b^3)*(d*x + c)/a^5 + 2*(6*A*a^2*tan(1/2*d*x
+ 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*tan(1/2*d*x + 1/2*c)^
5 + 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 18*A
*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^2*tan
```

```
(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^2*tan(1/2*d*
x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c)
+ 6*C*a^2*tan(1/2*d*x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 12*B*a*b*ta
n(1/2*d*x + 1/2*c) + 18*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^
2 + 1)^3*a^4))/d
```

3.916 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=465

$$\frac{\tan(c+dx)\left(-a^3b^2(2A-21C)-11a^2b^3B+6a^4bB-12a^5C+ab^4(5A-6C)+2b^5B\right)}{2b^4d\left(a^2-b^2\right)^2}+\frac{\left(12a^2C-6abB+2Ab^2+b^2C\right)\text{ta}}{2b^5d}$$

```
[Out] ((2*A*b^2 - 6*a*b*B + 12*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^5*d) -
(a*(6*A*b^6 - 6*a^5*b*B + 15*a^3*b^3*B - 12*a*b^5*B + a^4*b^2*(2*A - 29*C)
- 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*b*B - 11*a^2*b^3*
B + 2*b^5*B - a^3*b^2*(2*A - 21*C) + a*b^4*(5*A - 6*C) - 12*a^5*C)*Tan[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*b*B - 6*a*b^3*B - a^2*b^2*(A - 10*C
) + b^4*(4*A - C) - 6*a^4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^
2*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^3*Tan[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^2) + ((3*A*b^4 + a*(2*a^2*b*B - 5*b^3*B - 4*a^3*C
 + 7*a*b^2*C))*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x]))
```

Rubi [A] time = 4.74502, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4098, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\tan(c+dx)\left(-a^3b^2(2A-21C)-11a^2b^3B+6a^4bB-12a^5C+ab^4(5A-6C)+2b^5B\right)}{2b^4d\left(a^2-b^2\right)^2}+\frac{\left(12a^2C-6abB+2Ab^2+b^2C\right)\text{ta}}{2b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^3,x]
```


```
[Out] ((2*A*b^2 - 6*a*b*B + 12*a^2*C + b^2*C)*ArcTanh[Sin[c + d*x]])/(2*b^5*d) -
(a*(6*A*b^6 - 6*a^5*b*B + 15*a^3*b^3*B - 12*a*b^5*B + a^4*b^2*(2*A - 29*C)
- 5*a^2*b^4*(A - 4*C) + 12*a^6*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*b*B - 11*a^2*b^3*
B + 2*b^5*B - a^3*b^2*(2*A - 21*C) + a*b^4*(5*A - 6*C) - 12*a^5*C)*Tan[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*b*B - 6*a*b^3*B - a^2*b^2*(A - 10*C
) + b^4*(4*A - C) - 6*a^4*C)*Sec[c + d*x]*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^
2*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^3*Tan[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^2) + ((3*A*b^4 + a*(2*a^2*b*B - 5*b^3*B - 4*a^3*C
 + 7*a*b^2*C))*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
```

```
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^3(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\sec^3(c+dx)(3}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^3(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(3Ab^4+a\left(\right.\right.}{}$$

$$= -\frac{\left(3a^3bB-6ab^3B-a^2b^2(A-10C)+b^4(4A-C)-6a^4C\right)\sec}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(6a^4bB-11a^2b^3B+2b^5B-a^3b^2(2A-21C)+ab^4(5A-6C)\right.}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(6a^4bB-11a^2b^3B+2b^5B-a^3b^2(2A-21C)+ab^4(5A-6C)\right.}{2b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(2Ab^2-6abB+12a^2C+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{\left(6a^4bB\right.}{}$$

$$= \frac{\left(2Ab^2-6abB+12a^2C+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{\left(6a^4bB\right.}{}$$

$$= \frac{\left(2Ab^2-6abB+12a^2C+b^2C\right)\tanh^{-1}(\sin(c+dx))}{2b^5d} - \frac{a\left(2a^4\right.}{}$$

Mathematica [B] time = 6.47949, size = 1124, normalized size = 2.42

$$\frac{\left(-12Ca^2+6bBa-2Ab^2-b^2C\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)}{b^5d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^3,x]
```

```
[Out] (2*a*(2*a^4*A*b^2 - 5*a^2*A*b^4 + 6*A*b^6 - 6*a^5*b*B + 15*a^3*b^3*B - 12*a
*b^5*B + 12*a^6*C - 29*a^4*b^2*C + 20*a^2*b^4*C)*ArcTanh[((-a + b)*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2))/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3) + ((-2*A*
b^2 + 6*a*b*B - 12*a^2*C - b^2*C)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/(b^5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d
*x])^3) + ((2*A*b^2 - 6*a*b*B + 12*a^2*C + b^2*C)*(b + a*Cos[c + d*x])^3*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2))/(b^5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*(-6*a^4*A*b^3*Sin[c + d*x] + 12*a^2*A*b^5*Sin[c
 + d*x] + 18*a^5*b^2*B*Sin[c + d*x] - 32*a^3*b^4*B*Sin[c + d*x] + 8*a*b^6*B
```

```
*Sin[c + d*x] - 36*a^6*b*C*Sin[c + d*x] + 72*a^4*b^3*C*Sin[c + d*x] - 38*a^
2*b^5*C*Sin[c + d*x] + 8*b^7*C*Sin[c + d*x] - 4*a^5*A*b^2*Sin[2*(c + d*x)]
+ 10*a^3*A*b^4*Sin[2*(c + d*x)] + 12*a^6*b*B*Sin[2*(c + d*x)] - 14*a^4*b^3*
B*Sin[2*(c + d*x)] - 12*a^2*b^5*B*Sin[2*(c + d*x)] + 8*b^7*B*Sin[2*(c + d*x
)] - 24*a^7*C*Sin[2*(c + d*x)] + 26*a^5*b^2*C*Sin[2*(c + d*x)] + 20*a^3*b^4
*C*Sin[2*(c + d*x)] - 16*a*b^6*C*Sin[2*(c + d*x)] - 6*a^4*A*b^3*Sin[3*(c +
d*x)] + 12*a^2*A*b^5*Sin[3*(c + d*x)] + 18*a^5*b^2*B*Sin[3*(c + d*x)] - 32*
a^3*b^4*B*Sin[3*(c + d*x)] + 8*a*b^6*B*Sin[3*(c + d*x)] - 36*a^6*b*C*Sin[3*
(c + d*x)] + 64*a^4*b^3*C*Sin[3*(c + d*x)] - 22*a^2*b^5*C*Sin[3*(c + d*x)]
- 2*a^5*A*b^2*Sin[4*(c + d*x)] + 5*a^3*A*b^4*Sin[4*(c + d*x)] + 6*a^6*b*B*S
in[4*(c + d*x)] - 11*a^4*b^3*B*Sin[4*(c + d*x)] + 2*a^2*b^5*B*Sin[4*(c + d*
x)] - 12*a^7*C*Sin[4*(c + d*x)] + 21*a^5*b^2*C*Sin[4*(c + d*x)] - 6*a^3*b^4
*C*Sin[4*(c + d*x)]))/(8*b^4*(-a^2 + b^2)^2*d*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.123, size = 2275, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 12/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*B*a^2+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*A+1/2/d*C/b^
3/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/b^3/(tan(
1/2*d*x+1/2*c)-1)*C-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*A-1/2/d*C/b^3/(tan(1/2
*d*x+1/2*c)+1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*B+1/2/d/b^3/(tan(1/2*d*x+1/
2*c)+1)*C-1/2/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*C+1/2/d/b^3*ln(tan(1/2*d*x+1/2
*c)+1)*C+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+
1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*a^6/b^4/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*
x+1/2*c)*C+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+8/d*a^3/b/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*B+2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^4/b^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+
1/2*c)*A-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+10/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-10/d*a^4/b^2/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*C+3/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a*C+3/d/b^4*ln(tan(1/2
*d*x+1/2*c)-1)*B*a-6/d/b^5*ln(tan(1/2*d*x+1/2*c)-1)*a^2*C-1/d*a^3/b/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A-4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*a^6/b^4/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*C-1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d*a^5/b^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C
+3/d/b^4/(tan(1/2*d*x+1/2*c)+1)*a*C-12/d*a^7/b^5/(a^4-2*a^2*b^2+b^4)/((a+b)
```

```
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/d*a
^3/b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*A-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+6/d*a^2/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*
A-6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-3/d/b^4*
ln(tan(1/2*d*x+1/2*c)+1)*B*a+6/d/b^5*ln(tan(1/2*d*x+1/2*c)+1)*a^2*C+6/d*a^6
/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))*B-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec^{4}\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
3,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a + b*se
c(c + d*x))**3, x)
```

Giac [B] time = 1.47309, size = 2349, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -1/2*(2*(12*C*a^7 - 6*B*a^6*b + 2*A*a^5*b^2 - 29*C*a^5*b^2 + 15*B*a^4*b^3 -
 5*A*a^3*b^4 + 20*C*a^3*b^4 - 12*B*a^2*b^5 + 6*A*a*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/
2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(-a^2 +
 b^2)) - 2*(12*C*a^7*tan(1/2*d*x + 1/2*c)^7 - 6*B*a^6*b*tan(1/2*d*x + 1/2*c
)^7 - 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 2*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^
7 + 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^
7 - 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7
 + 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7
 - 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^7
 + 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7
- 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 +
4*C*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*B*b^7*tan(1/2*d*x + 1/2*c)^7 + C*b^7*t
an(1/2*d*x + 1/2*c)^7 - 36*C*a^7*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^6*b*tan(1/
2*d*x + 1/2*c)^5 + 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^2*tan(1/2*
d*x + 1/2*c)^5 - 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 67*C*a^5*b^2*tan(1/2*
d*x + 1/2*c)^5 + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 35*B*a^4*b^3*tan(1/2*
d*x + 1/2*c)^5 - 29*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*b^4*tan(1/2
*d*x + 1/2*c)^5 + 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 26*C*a^3*b^4*tan(1/
2*d*x + 1/2*c)^5 - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 10*B*a^2*b^5*tan(1/
2*d*x + 1/2*c)^5 + 5*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 4*B*a*b^6*tan(1/2*d
*x + 1/2*c)^5 + 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 2*B*b^7*tan(1/2*d*x + 1/
2*c)^5 + 3*C*b^7*tan(1/2*d*x + 1/2*c)^5 + 36*C*a^7*tan(1/2*d*x + 1/2*c)^3 -
 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 6*
A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*
C*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*
B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 15
*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 2
6*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 1
0*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4
*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*C*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^
7*tan(1/2*d*x + 1/2*c)^3 + 3*C*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*C*a^7*tan(1/
2*d*x + 1/2*c) + 6*B*a^6*b*tan(1/2*d*x + 1/2*c) - 18*C*a^6*b*tan(1/2*d*x +
1/2*c) - 2*A*a^5*b^2*tan(1/2*d*x + 1/2*c) + 9*B*a^5*b^2*tan(1/2*d*x + 1/2*c
) + 17*C*a^5*b^2*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^3*tan(1/2*d*x + 1/2*c) -
9*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*C*a^4*b^3*tan(1/2*d*x + 1/2*c) + 5*A*
a^3*b^4*tan(1/2*d*x + 1/2*c) - 16*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 2*C*a^3*
b^4*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^5*tan(1/2*d*x + 1/2*c) - 2*B*a^2*b^5*t
an(1/2*d*x + 1/2*c) - 13*C*a^2*b^5*tan(1/2*d*x + 1/2*c) + 4*B*a*b^6*tan(1/2
*d*x + 1/2*c) - 4*C*a*b^6*tan(1/2*d*x + 1/2*c) + 2*B*b^7*tan(1/2*d*x + 1/2*
c) + C*b^7*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*
x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
b)^2) - (12*C*a^2 - 6*B*a*b + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/b^5 + (12*C*a^2 - 6*B*a*b + 2*A*b^2 + C*b^2)*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/b^5)/d
```

3.917 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=323

$$\frac{\tan(c+dx)\left(3a^2C-abB+Ab^2-2b^2C\right)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)+5a^3b^3B-15a^4b^2C-2a^5bB+6a^6C-6ab^5B+2Ab^6\right)\tanh}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] ((b*B - 3*a*C)*ArcTanh[Sin[c + d*x]])/(b^4*d) + ((2*A*b^6 - 2*a^5*b*B + 5*a
^3*b^3*B - 6*a*b^5*B + 6*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*ArcTanh
[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5
/2)*d) + ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Tan[c + d*x])/(2*b^3*(a^2 - b
^2)*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^2) - (a*(2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*
C + a^2*b^2*(A + 6*C))*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c +
d*x]))
```

Rubi [A] time = 2.98692, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\tan(c+dx)\left(3a^2C-abB+Ab^2-2b^2C\right)}{2b^3d\left(a^2-b^2\right)}+\frac{\left(a^2b^4(A+12C)+5a^3b^3B-15a^4b^2C-2a^5bB+6a^6C-6ab^5B+2Ab^6\right)\tanh}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^3,x]
```


```
[Out] ((b*B - 3*a*C)*ArcTanh[Sin[c + d*x]])/(b^4*d) + ((2*A*b^6 - 2*a^5*b*B + 5*a
^3*b^3*B - 6*a*b^5*B + 6*a^6*C - 15*a^4*b^2*C + a^2*b^4*(A + 12*C))*ArcTanh
[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5
/2)*d) + ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Tan[c + d*x])/(2*b^3*(a^2 - b
^2)*d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^2) - (a*(2*A*b^4 + a^3*b*B - 4*a*b^3*B - 3*a^4*
C + a^2*b^2*(A + 6*C))*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c +
d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\sec^2(c+dx)(2}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(2Ab^4+a\right.}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a(bB-aC\right.}{2b\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a(bB-aC\right.}{2b\left(a^2-b^2\right)}$$

$$= \frac{(bB-3aC)\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(Ab^2-abB+3a^2C-2b^2C\right.}{2b^3\left(a^2-b^2\right)}$$

$$= \frac{(bB-3aC)\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(Ab^2-abB+3a^2C-2b^2C\right.}{2b^3\left(a^2-b^2\right)}$$

$$= \frac{(bB-3aC)\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(a^2Ab^4+2Ab^6-2a^5bB+\right.}{}$$

Mathematica [A] time = 2.9103, size = 492, normalized size = 1.52

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2b\sin(c+dx)\left(a^2\cos(2(c+dx))\left(-11a^2b^2C-2a^3bB+6a^4C+5ab^3B-3A\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-8*(2*A*b^6 - 2*a^5*b*B + 5*a^3*b^3*B - 6*a*b^5*B + 6*a^6*C - 15*a^4*b^
2*C + a^2*b^4*(A + 12*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]]*Cos[c + d*x]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) - 8*(b*B - 3*a*C
)*Cos[c + d*x]*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + 8*(b*B - 3*a*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + (2*b*(-3*a^2*A*b^4 - 2*a^5*b*B + 5*a^3*b^3*B + 6*
a^6*C - 7*a^4*b^2*C - 6*a^2*b^4*C + 4*b^6*C + 2*a*b*(-3*a^3*b*B + 6*a*b^3*B
 + a^2*b^2*(A - 16*C) + 9*a^4*C + 4*b^4*(-A + C))*Cos[c + d*x] + a^2*(-3*A*
b^4 - 2*a^3*b*B + 5*a*b^3*B + 6*a^4*C - 11*a^2*b^2*C + 2*b^4*C)*Cos[2*(c +
d*x)])*Sin[c + d*x])/(a^2 - b^2)^2))/(4*b^4*d*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.106, size = 1813, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*C-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*C+4/d*b/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*A-4/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-8/d*a^3/b/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+2/d*a
^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*a^4/b^2/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2
*c)*B-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+1/d*a^4/b
^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*C-4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+4/d*a^5/b^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2
*d*x+1/2*c)*C+8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+3/d/b^4*ln(tan(1/2*d*x+1/2*
c)-1)*a*C-3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a*C+1/d/b^3*ln(tan(1/2*d*x+1/2*c
)+1)*B-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*B+6/d/b^4/(a^4-2*a^2*b^2+b^4)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^6*C-
15/d/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*a^4*C-2/d/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^5+5/d/b/(a
^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*B*a^3-6/d*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a*B-6/d*a^2/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3*B+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b
)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*a^2/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1
/2*c)^3*A+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a^2+12/d/(a
^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))*C*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
3,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*se
c(c + d*x))**3, x)
```

Giac [B] time = 1.53882, size = 952, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] ((6*C*a^6 - 2*B*a^5*b - 15*C*a^4*b^2 + 5*B*a^3*b^3 + A*a^2*b^4 + 12*C*a^2*b
^4 - 6*B*a*b^5 + 2*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(-a^2 + b^2)) - (4*C*a^6*tan(1/2*d*x
+ 1/2*c)^3 - 2*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^5*b*tan(1/2*d*x + 1/2
*c)^3 + 3*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 7*C*a^4*b^2*tan(1/2*d*x + 1/2*
c)^3 - A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^
3 + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*A*a^2*b^4*tan(1/2*d*x + 1/2*c)^3
 - 6*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*a*b^5*tan(1/2*d*x + 1/2*c)^3 -
4*C*a^6*tan(1/2*d*x + 1/2*c) + 2*B*a^5*b*tan(1/2*d*x + 1/2*c) - 5*C*a^5*b*t
an(1/2*d*x + 1/2*c) + 3*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 7*C*a^4*b^2*tan(1/
2*d*x + 1/2*c) - A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 5*B*a^3*b^3*tan(1/2*d*x +
 1/2*c) + 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a^2*b^4*tan(1/2*d*x + 1/2*
c) - 6*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 4*A*a*b^5*tan(1/2*d*x + 1/2*c))/((a
^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 - a - b)^2) - (3*C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (3
*C*a - B*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*C*tan(1/2*d*x + 1/2*
c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

3.918 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=242

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-3ab^4(A+2C)+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\tan(c+dx)\left(a^2b^2(A+6C)+a^3bB-3\right.}{2b^2d\left(a^2-b^2\right)^2(a+b\sec\right.}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3
*b^2*C - 3*a*b^4*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) + (a*(A*b^2 - a*(b*B - a*C))*Tan[
c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((2*A*b^4 + a^3*b*
B - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 6*C))*Tan[c + d*x])/(2*b^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.918053, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4090, 4080, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(a^2b^3B+5a^3b^2C-2a^5C-3ab^4(A+2C)+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\tan(c+dx)\left(a^2b^2(A+6C)+a^3bB-3\right.}{2b^2d\left(a^2-b^2\right)^2(a+b\sec\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^3,x]
```


```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*b^3*B + 2*b^5*B - 2*a^5*C + 5*a^3
*b^2*C - 3*a*b^4*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) + (a*(A*b^2 - a*(b*B - a*C))*Tan[
c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((2*A*b^4 + a^3*b*
B - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 6*C))*Tan[c + d*x])/(2*b^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
```

```
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(-2b\left(Ab^2-a(bB-a\right.\right.}{}}{}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(2Ab^4+a^3bB-4ab^3B-\right.}{2b^2\left(a^2-b\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(2Ab^4+a^3bB-4ab^3B-\right.}{2b^2\left(a^2-b\right.}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{\left(3aAb^4-a^2b^3B-2b^5B+2a^5C-5a\right.}{(a-b)^{5/}}$$

Mathematica [C] time = 6.00229, size = 514, normalized size = 2.12

$$\sec(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4(\sin(c)+i\cos(c))\left(-a^2b^3B-5a^3b^2C+2a^5C+3ab^4(A+2C)-2b^5B\right)(}{\left(a^2-b^2\right)^{5/2}\sqrt{(}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
(-4*C*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*C
*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (4*(-(a^
2*b^3*B) - 2*b^5*B + 2*a^5*C - 5*a^3*b^2*C + 3*a*b^4*(A + 2*C))*ArcTan[((I*
Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2
]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b + a*Cos[c + d*x])^2*(I*Cos[c] + Sin[c]))
/((a^2 - b^2)^(5/2)*Sqrt[(Cos[c] - I*Sin[c])^2]) + (b*(a*Sec[c]*(b*(4*A*b^4
 + a^3*b*B - 10*a*b^3*B - 7*a^4*C + a^2*b^2*(5*A + 16*C))*Sin[d*x] + a*(b*(
a^2*b*B + 2*b^3*B + a^3*C - a*b^2*(3*A + 4*C))*Sin[2*c + d*x] + (A*b^4 - 3*
a*b^3*B - 2*a^4*C + a^2*b^2*(2*A + 5*C))*Sin[c + 2*d*x])) + (a^2 + 2*b^2)*(
-(A*b^4) + 3*a*b^3*B + 2*a^4*C - a^2*b^2*(2*A + 5*C))*Tan[c]))/(a*(a^2 - b^
2)^2)))/(2*b^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a + b*S
ec[c + d*x])^3)
```

Maple [B] time = 0.11, size = 1572, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*b^2/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*A+1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d*b/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*B+2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-1/d*a^3/b/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*C-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+2/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*b/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/
2*c)*A+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/
(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-4/d*b/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/
d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)
^2*tan(1/2*d*x+1/2*c)*C-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+6/d/(tan(1/2*d*x+1/2*c)^2*a
```

```
-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-3/d*a
*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))*A+1/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^2+2/d*b^2/(a^4-2*a^2*b^2
+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))*B-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-6/d*
b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*C*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*C-1/d/b^3*ln(tan(
1/2*d*x+1/2*c)-1)*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 78.2604, size = 3240, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] [-1/4*((2*C*a^5*b^2 - 5*C*a^3*b^4 - B*a^2*b^5 + 3*(A + 2*C)*a*b^6 - 2*B*b^7
 + (2*C*a^7 - 5*C*a^5*b^2 - B*a^4*b^3 + 3*(A + 2*C)*a^3*b^4 - 2*B*a^2*b^5)*
cos(d*x + c)^2 + 2*(2*C*a^6*b - 5*C*a^4*b^3 - B*a^3*b^4 + 3*(A + 2*C)*a^2*b
^5 - 2*B*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^
2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(C
*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C*a
^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5
 - C*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(C*a^6*b^2 - 3*C*a^4*b^
4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*c
os(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x +
 c))*log(-sin(d*x + c) + 1) + 2*(3*C*a^6*b^2 - B*a^5*b^3 - (A + 9*C)*a^4*b^
4 + 5*B*a^3*b^5 - (A - 6*C)*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 + (2*C*a^7*b - (2
*A + 7*C)*a^5*b^3 + 3*B*a^4*b^4 + (A + 5*C)*a^3*b^5 - 3*B*a^2*b^6 + A*a*b^7
)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*
d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x +
 c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*C*a^5*b^2 - 5*C
*a^3*b^4 - B*a^2*b^5 + 3*(A + 2*C)*a*b^6 - 2*B*b^7 + (2*C*a^7 - 5*C*a^5*b^2
 - B*a^4*b^3 + 3*(A + 2*C)*a^3*b^4 - 2*B*a^2*b^5)*cos(d*x + c)^2 + 2*(2*C*a
^6*b - 5*C*a^4*b^3 - B*a^3*b^4 + 3*(A + 2*C)*a^2*b^5 - 2*B*a*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
 b^2)*sin(d*x + c))) - (C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*
a^8 - 3*C*a^6*b^2 + 3*C*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7*b -
```

```
3*C*a^5*b^3 + 3*C*a^3*b^5 - C*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) +
(C*a^6*b^2 - 3*C*a^4*b^4 + 3*C*a^2*b^6 - C*b^8 + (C*a^8 - 3*C*a^6*b^2 + 3*C
*a^4*b^4 - C*a^2*b^6)*cos(d*x + c)^2 + 2*(C*a^7*b - 3*C*a^5*b^3 + 3*C*a^3*b
^5 - C*a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (3*C*a^6*b^2 - B*a^5*b
^3 - (A + 9*C)*a^4*b^4 + 5*B*a^3*b^5 - (A - 6*C)*a^2*b^6 - 4*B*a*b^7 + 2*A*
b^8 + (2*C*a^7*b - (2*A + 7*C)*a^5*b^3 + 3*B*a^4*b^4 + (A + 5*C)*a^3*b^5 -
3*B*a^2*b^6 + A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 +
3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8
- a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
3,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*se
c(c + d*x))**3, x)
```

Giac [B] time = 1.42163, size = 853, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -((2*C*a^5 - 5*C*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4 + 6*C*a*b^4 - 2*B*b^5)*(pi
*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1
/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^3 - 2*a^2*b^5 +
b^7)*sqrt(-a^2 + b^2)) - C*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + C*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*C*a
^4*b*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a^3*b^
2*tan(1/2*d*x + 1/2*c)^3 - 5*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*b^3*t
an(1/2*d*x + 1/2*c)^3 + 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 - A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b^4*tan(1/2*d
*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^5*tan(1/2*d*x + 1/2*
c) - 3*C*a^4*b*tan(1/2*d*x + 1/2*c) + 2*A*a^3*b^2*tan(1/2*d*x + 1/2*c) + B*
a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + A*a^2*b^3
*tan(1/2*d*x + 1/2*c) - 3*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*C*a^2*b^3*tan(
1/2*d*x + 1/2*c) + A*a*b^4*tan(1/2*d*x + 1/2*c) - 4*B*a*b^4*tan(1/2*d*x + 1
/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.919 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{\left(a^2(-(2A+C))+3abB-b^2(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\tan(c+dx)\left(a^2bB+a^3C-ab^2(3A+4C)+2b^3B\right)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}\cdot$$

```
[Out] -(((3*a*b*B - a^2*(2*A + C) - b^2*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) - ((A*b^2 - a*(b*B
- a*C))*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*b*
B + 2*b^3*B + a^3*C - a*b^2*(3*A + 4*C))*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.421351, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4080, 4003, 12, 3831, 2659, 208}

$$-\frac{\left(a^2(-(2A+C))+3abB-b^2(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\tan(c+dx)\left(a^2bB+a^3C-ab^2(3A+4C)+2b^3B\right)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^3,x]
```



```
[Out] -(((3*a*b*B - a^2*(2*A + C) - b^2*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) - ((A*b^2 - a*(b*B
- a*C))*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*b*
B + 2*b^3*B + a^3*C - a*b^2*(3*A + 4*C))*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```


Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(2b(bB-a(A+C\cdots\right.}{(a\cdots}}{2\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C\cdots\right.}{2b\left(a^2-b^2\right)^2\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C\cdots\right.}{2b\left(a^2-b^2\right)^2\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C\cdots\right.}{2b\left(a^2-b^2\right)^2\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2bB+2b^3B+a^3C\cdots\right.}{2b\left(a^2-b^2\right)^2\cdots}$$

$$= \frac{\left(2a^2A+Ab^2-3abB+a^2C+2b^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\cdots\right.}{\sqrt{a+b}}\cdots\right.}{(a-b)^{5/2}(a+b)^{5/2}d}\cdots$$

Mathematica [C] time = 4.25346, size = 410, normalized size = 2.03

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{a\sec(c)\left(\sin(2c+dx)\left(a^2b^2(5A+2C)-3a^3bB+a^4C-2Ab^4\right)+a\sin(c+\cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
(((-4*I)*(-3*a*b*B + a^2*(2*A + C) + b^2*(A + 2*C))*ArcTan[((I*Cos[c] + Sin
[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqrt[a^2 - b^2]*Sqrt[(Cos[
c] - I*Sin[c])^2])]*(b + a*Cos[c + d*x])^2*(Cos[c] - I*Sin[c]))/((a^2 - b^2
)^(5/2)*Sqrt[(Cos[c] - I*Sin[c])^2]) + (a*Sec[c]*((2*A*b^4 + 5*a^3*b*B + 4*
a*b^3*B + a^4*C - a^2*b^2*(11*A + 10*C))*Sin[d*x] + (-2*A*b^4 - 3*a^3*b*B +
 a^4*C + a^2*b^2*(5*A + 2*C))*Sin[2*c + d*x] + a*(A*b^3 + 2*a^3*B + a*b^2*B
 - a^2*b*(4*A + 3*C))*Sin[c + 2*d*x]) - (a^2 + 2*b^2)*(A*b^3 + 2*a^3*B + a*
b^2*B - a^2*b*(4*A + 3*C))*Tan[c])/(a^3 - a*b^2)^2))/(2*d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*(c + d*x)])*(a + b*Sec[c + d*x])^3)
```

Maple [A] time = 0.092, size = 268, normalized size = 1.3

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)^{2}}\left(-1/2\,\frac{\left(4\,Aab+Ab^{2}-2\,Ba^{2}-Bab-2\,Bb^{2}+a^{2}C+4\,ab C\right)}{\left(a-b\right)\left(a^{2}+2\,ab+b^{2}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2+C*a^2+4*C*a*b)/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2-
C*a^2+4*C*a*b)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b+C*a^2+2*C*b^2)/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.686306, size = 1852, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [1/4*(((2*A + C)*a^2*b^2 - 3*B*a*b^3 + (A + 2*C)*b^4 + ((2*A + C)*a^4 - 3*B
*a^3*b + (A + 2*C)*a^2*b^2)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b - 3*B*a^2*b
^2 + (A + 2*C)*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
 - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*si
n(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
+ 2*(C*a^5 + B*a^4*b - (3*A + 5*C)*a^3*b^2 + B*a^2*b^3 + (3*A + 4*C)*a*b^4
- 2*B*b^5 + (2*B*a^5 - (4*A + 3*C)*a^4*b - B*a^3*b^2 + (5*A + 3*C)*a^2*b^3
- B*a*b^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^
4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d
*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*(((2*A + C)
*a^2*b^2 - 3*B*a*b^3 + (A + 2*C)*b^4 + ((2*A + C)*a^4 - 3*B*a^3*b + (A + 2*
C)*a^2*b^2)*cos(d*x + c)^2 + 2*((2*A + C)*a^3*b - 3*B*a^2*b^2 + (A + 2*C)*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
c) + a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^5 + B*a^4*b - (3*A + 5*C)*a^3*b^
2 + B*a^2*b^3 + (3*A + 4*C)*a*b^4 - 2*B*b^5 + (2*B*a^5 - (4*A + 3*C)*a^4*b
- B*a^3*b^2 + (5*A + 3*C)*a^2*b^3 - B*a*b^4 - A*b^5)*cos(d*x + c))*sin(d*x
+ c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*
a^2*b^6 - b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c
 + d*x))**3, x)
```

Giac [B] time = 1.38032, size = 689, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] ((2*A*a^2 + C*a^2 - 3*B*a*b + A*b^2 + 2*C*b^2)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*B
*a^3*tan(1/2*d*x + 1/2*c)^3 - C*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^2*b*tan(
1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*b*tan(1/2*d*x
 + 1/2*c)^3 + 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*
c)^3 + 4*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*
B*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c) - C*a^3*tan(1/2
*d*x + 1/2*c) + 4*A*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^2*b*tan(1/2*d*x + 1/2*
c) + 3*C*a^2*b*tan(1/2*d*x + 1/2*c) + 3*A*a*b^2*tan(1/2*d*x + 1/2*c) - B*a*
b^2*tan(1/2*d*x + 1/2*c) + 4*C*a*b^2*tan(1/2*d*x + 1/2*c) - A*b^3*tan(1/2*d
*x + 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.920 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$\frac{\left(5a^2Ab^3-3a^4b(2A+C)+a^3b^2B+2a^5B-2Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\tan(c+dx)\left(-a^2b^2(5A+2C)+3a^3bB+\right.}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+}$$

```
[Out] (A*x)/a^3 + ((5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B - 3*a^4*b*(2*A +
C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)
*(a + b)^(5/2)*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) - ((2*A*b^4 + 3*a^3*b*B - a^4*C - a^2*b^2*(5*A +
 2*C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.759451, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4060, 3919, 3831, 2659, 208}

$$\frac{\left(5a^2Ab^3-3a^4b(2A+C)+a^3b^2B+2a^5B-2Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\tan(c+dx)\left(-a^2b^2(5A+2C)+3a^3bB+\right.}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (A*x)/a^3 + ((5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B - 3*a^4*b*(2*A +
C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)
*(a + b)^(5/2)*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) - ((2*A*b^4 + 3*a^3*b*B - a^4*C - a^2*b^2*(5*A +
 2*C))*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2A\left(a^2-b^2\right)+2a(Ab-aB+bC)\sec(c+dx)-\left(Ab^2\right.}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4+3a^3bB-a^4C-a^2b^2(5A+2\right.}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c}$$

$$= \frac{Ax}{a^3} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4+3a^3bB-a^4C-a^2b^2(5\right.}{2a^2\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{Ax}{a^3} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4+3a^3bB-a^4C-a^2b^2(5\right.}{2a^2\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{Ax}{a^3} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(2Ab^4+3a^3bB-a^4C-a^2b^2(5\right.}{2a^2\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{Ax}{a^3} - \frac{\left(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B+3a^4bC\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{t}}{}\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [C] time = 6.19554, size = 793, normalized size = 3.46

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{\sec(c)\left(6a^4Ab^2\sin(c+2dx)-7a^3Ab^3\sin(2c+dx)-3a^2Ab^4\sin(c+2\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
(((-4*I)*(5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a^3*b^2*B - 3*a^4*b*(2*A + C))*
ArcTan[((I*Cos[c] + Sin[c])*(a*Sin[c] + (-b + a*Cos[c])*Tan[(d*x)/2]))/(Sqr
t[a^2 - b^2]*Sqrt[(Cos[c] - I*Sin[c])^2])]*(b + a*Cos[c + d*x])^2*(Cos[c] -
 I*Sin[c]))/((a^2 - b^2)^(5/2)*Sqrt[(Cos[c] - I*Sin[c])^2]) + (Sec[c]*(2*A*
```

```
(a^2 - b^2)^2*(a^2 + 2*b^2)*d*x*Cos[c] + 4*a*A*b*(a^2 - b^2)^2*d*x*Cos[d*x]
 + 4*a^5*A*b*d*x*Cos[2*c + d*x] - 8*a^3*A*b^3*d*x*Cos[2*c + d*x] + 4*a*A*b^
5*d*x*Cos[2*c + d*x] + a^6*A*d*x*Cos[c + 2*d*x] - 2*a^4*A*b^2*d*x*Cos[c + 2
*d*x] + a^2*A*b^4*d*x*Cos[c + 2*d*x] + a^6*A*d*x*Cos[3*c + 2*d*x] - 2*a^4*A
*b^2*d*x*Cos[3*c + 2*d*x] + a^2*A*b^4*d*x*Cos[3*c + 2*d*x] - 6*a^4*A*b^2*Si
n[c] - 9*a^2*A*b^4*Sin[c] + 6*A*b^6*Sin[c] + 4*a^5*b*B*Sin[c] + 7*a^3*b^3*B
*Sin[c] - 2*a*b^5*B*Sin[c] - 2*a^6*C*Sin[c] - 5*a^4*b^2*C*Sin[c] - 2*a^2*b^
4*C*Sin[c] + 17*a^3*A*b^3*Sin[d*x] - 8*a*A*b^5*Sin[d*x] - 11*a^4*b^2*B*Sin[
d*x] + 2*a^2*b^4*B*Sin[d*x] + 5*a^5*b*C*Sin[d*x] + 4*a^3*b^3*C*Sin[d*x] - 7
*a^3*A*b^3*Sin[2*c + d*x] + 4*a*A*b^5*Sin[2*c + d*x] + 5*a^4*b^2*B*Sin[2*c
+ d*x] - 2*a^2*b^4*B*Sin[2*c + d*x] - 3*a^5*b*C*Sin[2*c + d*x] + 6*a^4*A*b^
2*Sin[c + 2*d*x] - 3*a^2*A*b^4*Sin[c + 2*d*x] - 4*a^5*b*B*Sin[c + 2*d*x] +
a^3*b^3*B*Sin[c + 2*d*x] + 2*a^6*C*Sin[c + 2*d*x] + a^4*b^2*C*Sin[c + 2*d*x
]))/(a^2 - b^2)^2))/(2*a^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x
)])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.107, size = 1550, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/a/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*A*b^3+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+4/d*b/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*B+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B-2/d/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1
/2*c)^3*C-1/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)
/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b*C-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*b^2+6
/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*
tan(1/2*d*x+1/2*c)*A-1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^3-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4-4/d*
b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*a/(a+b)/(a-b)^2*tan
(1/2*d*x+1/2*c)*B+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b^2*B+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*a^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-1/d*a/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/
2*c)*b*C+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a
-b)^2*tan(1/2*d*x+1/2*c)*C*b^2-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+5/d/a/(a^4-2*a^
2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))*A*b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^5+2/d/(a^4-2*a^2*b^2+b^4)/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a
^2+1/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B-3/d*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/
2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.7853, size = 2668, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
fricas")
```

```
[Out] [1/4*(4*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2
+ 8*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 4*(A
*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x - (2*B*a^5*b^2 - 3*(2*A +
 C)*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 3*(2*A + C)*a^
6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b
- 3*(2*A + C)*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*
sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*
sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d
*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^7*b - 3*B*a^6*b^2 + (5*A +
C)*a^5*b^3 + 3*B*a^4*b^4 - (7*A + 2*C)*a^3*b^5 + 2*A*a*b^7 + (2*C*a^8 - 4*B
*a^7*b + (6*A - C)*a^6*b^2 + 5*B*a^5*b^3 - (9*A + C)*a^4*b^4 - B*a^3*b^5 +
3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a
^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*c
os(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(A*a^8
 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2 + 4*(A*a^7*b -
 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 2*(A*a^6*b^2 - 3*A
*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x + (2*B*a^5*b^2 - 3*(2*A + C)*a^4*b^3 +
B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 3*(2*A + C)*a^6*b + B*a^5*b^
2 + 5*A*a^4*b^3 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 3*(2*A + C)*
a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) + (C*a^7*b - 3*B*a^6*b^2 + (5*A + C)*a^5*b^3 + 3*B*a^4*b^4 - (7*A + 2*C)*
a^3*b^5 + 2*A*a*b^7 + (2*C*a^8 - 4*B*a^7*b + (6*A - C)*a^6*b^2 + 5*B*a^5*b^
3 - (9*A + C)*a^4*b^4 - B*a^3*b^5 + 3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c)
)/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b -
3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*
a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**3,
x)
```

Giac [B] time = 1.39053, size = 818, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x, algorithm="
giac")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b - 3*C*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*
floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/
2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b
^4)*sqrt(-a^2 + b^2)) + (d*x + c)*A/a^3 - (2*C*a^5*tan(1/2*d*x + 1/2*c)^3 -
 4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - C*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^
3*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + C*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + B*a^2*b^3*
tan(1/2*d*x + 1/2*c)^3 - 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*A*a*b^4*tan
(1/2*d*x + 1/2*c)^3 + 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 - 2*C*a^5*tan(1/2*d*x
+ 1/2*c) + 4*B*a^4*b*tan(1/2*d*x + 1/2*c) - C*a^4*b*tan(1/2*d*x + 1/2*c) -
6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - C*a^3
*b^2*tan(1/2*d*x + 1/2*c) - 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a^2*b^3*ta
n(1/2*d*x + 1/2*c) - 2*C*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*tan(1/2*d
*x + 1/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a
*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.921 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=330

$$-\frac{\sin(c+dx)\left(11a^2Ab^2+a^4(-(2A-3C))-5a^3bB+2ab^3B-6Ab^4\right)}{2a^3d\left(a^2-b^2\right)^2}-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-5a^3b^3B+6a^5bB\right.}{a^4d(a-}$$

```
[Out] -(((3*A*b - a*B)*x)/a^4) - ((15*a^2*A*b^4 - 6*A*b^6 + 6*a^5*b*B - 5*a^3*b^3
*B + 2*a*b^5*B - 2*a^6*C - a^4*b^2*(12*A + C))*ArcTanh[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) - ((11*a^2*A*b
^2 - 6*A*b^4 - 5*a^3*b*B + 2*a*b^3*B - a^4*(2*A - 3*C))*Sin[c + d*x])/(2*a^
3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^2) - ((3*A*b^4 + 4*a^3*b*B - a*b^3*B - 2*a^4*C - a
^2*b^2*(6*A + C))*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])
)
```

Rubi [A] time = 3.14895, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\sin(c+dx)\left(11a^2Ab^2+a^4(-(2A-3C))-5a^3bB+2ab^3B-6Ab^4\right)}{2a^3d\left(a^2-b^2\right)^2}-\frac{\left(-a^4b^2(12A+C)+15a^2Ab^4-5a^3b^3B+6a^5bB\right.}{a^4d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^3,x]
```


```
[Out] -(((3*A*b - a*B)*x)/a^4) - ((15*a^2*A*b^4 - 6*A*b^6 + 6*a^5*b*B - 5*a^3*b^3
*B + 2*a*b^5*B - 2*a^6*C - a^4*b^2*(12*A + C))*ArcTanh[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) - ((11*a^2*A*b
^2 - 6*A*b^4 - 5*a^3*b*B + 2*a*b^3*B - a^4*(2*A - 3*C))*Sin[c + d*x])/(2*a^
3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^2) - ((3*A*b^4 + 4*a^3*b*B - a*b^3*B - 2*a^4*C - a
^2*b^2*(6*A + C))*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])
)
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(3Ab^2-abB-a^2(2\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3Ab^4+4a^3bB-ab^3B\cdots\right.}{2a^2\left(a^2-\cdots\right.}$$

$$= -\frac{\left(11a^2Ab^2-6Ab^4-5a^3bB+2ab^3B-a^4(2A-3C)\right)\sin(c\cdots}{2a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{(3Ab-aB)x}{a^4} - \frac{\left(11a^2Ab^2-6Ab^4-5a^3bB+2ab^3B-a^4(2\cdots\right.}{2a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{(3Ab-aB)x}{a^4} - \frac{\left(11a^2Ab^2-6Ab^4-5a^3bB+2ab^3B-a^4(2\cdots\right.}{2a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{(3Ab-aB)x}{a^4} - \frac{\left(11a^2Ab^2-6Ab^4-5a^3bB+2ab^3B-a^4(2\cdots\right.}{2a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(12a^4Ab^2-15a^2Ab^4+6Ab^6-6a^5bB+5\cdots\right.}{a^4\cdots}$$

Mathematica [C] time = 7.12364, size = 1015, normalized size = 3.08

$$-\frac{2(3Ab-aB)x\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)(b+a\cos(c+dx))^3}{a^4(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b\sec(c+dx))^3} + \frac{\left(2Ca^6-6bBa^5+12Ab^2a^4+\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```

```
[Out] (-2*(3*A*b - a*B)*x*(b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2))/(a^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(a + b*Sec[c + d*x])^3) + ((12*a^4*A*b^2 - 15*a^2*A*b^4 + 6*A*b^6 - 6*a^5
*b*B + 5*a^3*b^3*B - 2*a*b^5*B + 2*a^6*C + a^4*b^2*C)*(b + a*Cos[c + d*x])^
3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-2*I)*ArcTan[Sec[
(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])
/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*
Sin[c + (d*x)/2])]*Cos[c])/(a^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c
]]) - (2*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin
[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b
*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*Sin[c])/(a^4*Sqrt[a^2 - b^2]*d*Sqrt[
Cos[2*c] - I*Sin[2*c]])))/((-a^2 + b^2)^2*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])*Sec[c]*Sec
[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-(A*b^5*Sin[c]) + a*b^4*
B*Sin[c] - a^2*b^3*C*Sin[c] + a*A*b^4*Sin[d*x] - a^2*b^3*B*Sin[d*x] + a^3*b
^2*C*Sin[d*x]))/(a^4*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^2*Sec[c]*Sec[c +
```

```
d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(9*a^2*A*b^4*Sin[c] - 6*A*b^6*
Sin[c] - 7*a^3*b^3*B*Sin[c] + 4*a*b^5*B*Sin[c] + 5*a^4*b^2*C*Sin[c] - 2*a^2
*b^4*C*Sin[c] - 8*a^3*A*b^3*Sin[d*x] + 5*a*A*b^5*Sin[d*x] + 6*a^4*b^2*B*Sin
[d*x] - 3*a^2*b^4*B*Sin[d*x] - 4*a^5*b*C*Sin[d*x] + a^3*b^3*C*Sin[d*x]))/(a
^4*(a^2 - b^2)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b
*Sec[c + d*x])^3) + (2*A*(b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2)*Tan[c + d*x])/(a^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.139, size = 1756, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -4/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*
tan(1/2*d*x+1/2*c)*b*C-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B+4/d*a/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1
/2*c)^3*b*C-8/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^3+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4+8/d/a/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A*b^3+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-2/d/a^2/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1
/2*c)*B-1/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^2*b^5/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/a/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*B+2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d/a^3/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^5/(a+b)/(a-b)^2*tan(1/2
*d*x+1/2*c)*A+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+
b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C*b^2-6/d*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a*B+1/d/(a^4-2
*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*b^2*C+2/d/a^3*B*arctan(tan(1/2*d*x+1/2*c))+5/d/a/(a^4-2*a^2*b^
2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*B*b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*b^5*B-15/d/a^2/(a^4-2*a^2*b^2+b^4)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*
b^4+6/d/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6+2/d/a^3*A*tan(1/2*d*x+1/2*c)/(1+tan(1/
2*d*x+1/2*c)^2)-6/d/a^4*A*arctan(tan(1/2*d*x+1/2*c))*b+6/d/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*b^2*
B+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+1/d/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.961969, size = 3634, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] [1/4*(4*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*
a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*cos(d*x + c)^2 + 8*(B*a^8*b - 3*A*a^
7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 +
 3*A*a*b^8)*d*x*cos(d*x + c) + 4*(B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9
*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*d*x + (2*C*a^6*
b^2 - 6*B*a^5*b^3 + (12*A + C)*a^4*b^4 + 5*B*a^3*b^5 - 15*A*a^2*b^6 - 2*B*a
*b^7 + 6*A*b^8 + (2*C*a^8 - 6*B*a^7*b + (12*A + C)*a^6*b^2 + 5*B*a^5*b^3 -
15*A*a^4*b^4 - 2*B*a^3*b^5 + 6*A*a^2*b^6)*cos(d*x + c)^2 + 2*(2*C*a^7*b - 6
*B*a^6*b^2 + (12*A + C)*a^5*b^3 + 5*B*a^4*b^4 - 15*A*a^3*b^5 - 2*B*a^2*b^6
+ 6*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*((2*A
 - 3*C)*a^7*b^2 + 5*B*a^6*b^3 - (13*A - 3*C)*a^5*b^4 - 7*B*a^4*b^5 + 17*A*a
^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a^5*b^4 - A
*a^3*b^6)*cos(d*x + c)^2 + (4*(A - C)*a^8*b + 6*B*a^7*b^2 - 5*(4*A - C)*a^6
*b^3 - 9*B*a^5*b^4 + (25*A - C)*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*
x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x
+ c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^1
0*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), 1/2*(2*(B*a^9 - 3*A*a^8*b - 3*
B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b
^7)*d*x*cos(d*x + c)^2 + 4*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b
^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*d*x*cos(d*x + c) +
2*(B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*
a^2*b^7 - B*a*b^8 + 3*A*b^9)*d*x + (2*C*a^6*b^2 - 6*B*a^5*b^3 + (12*A + C)*
a^4*b^4 + 5*B*a^3*b^5 - 15*A*a^2*b^6 - 2*B*a*b^7 + 6*A*b^8 + (2*C*a^8 - 6*B
*a^7*b + (12*A + C)*a^6*b^2 + 5*B*a^5*b^3 - 15*A*a^4*b^4 - 2*B*a^3*b^5 + 6*
A*a^2*b^6)*cos(d*x + c)^2 + 2*(2*C*a^7*b - 6*B*a^6*b^2 + (12*A + C)*a^5*b^3
 + 5*B*a^4*b^4 - 15*A*a^3*b^5 - 2*B*a^2*b^6 + 6*A*a*b^7)*cos(d*x + c))*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + ((2*A - 3*C)*a^7*b^2 + 5*B*a^6*b^3 - (13*A - 3*C)*a^5*b^4 - 7
*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^
2 + 3*A*a^5*b^4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*(A - C)*a^8*b + 6*B*a^7*b^
2 - 5*(4*A - C)*a^6*b^3 - 9*B*a^5*b^4 + (25*A - C)*a^4*b^5 + 3*B*a^3*b^6 -
9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 -
a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*
cos(d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.52182, size = 900, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] ((2*C*a^6 - 6*B*a^5*b + 12*A*a^4*b^2 + C*a^4*b^2 + 5*B*a^3*b^3 - 15*A*a^2*b
^4 - 2*B*a*b^5 + 6*A*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) + (4*C*a^5*b*tan(1/2*d*
x + 1/2*c)^3 - 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^3*b^3*tan(1/2*d*x
+ 1/2*c)^3 - C*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^2*b^4*tan(1/2*d*x + 1
/2*c)^3 + 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*A*a*b^5*tan(1/2*d*x + 1/2*
c)^3 - 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^6*tan(1/2*d*x + 1/2*c)^3 -
4*C*a^5*b*tan(1/2*d*x + 1/2*c) + 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 3*C*a^4
*b^2*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b^3*tan(1/2*d*x + 1/2*c) + 5*B*a^3*b^3*
tan(1/2*d*x + 1/2*c) + C*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*A*a^2*b^4*tan(1/2
*d*x + 1/2*c) - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*A*a*b^5*tan(1/2*d*x +
1/2*c) - 2*B*a*b^5*tan(1/2*d*x + 1/2*c) + 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a
^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 - a - b)^2) + (B*a - 3*A*b)*(d*x + c)/a^4 + 2*A*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

3.922 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=453

$$-\frac{\sin(c+dx)\left(-a^2b^3(21A-2C)+a^4b(6A-5C)+11a^3b^2B-2a^5B-6ab^4B+12Ab^5\right)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\sin(c+dx)\cos(c+dx)\left(-a^2b\right.}{}$$

```
[Out] ((12*A*b^2 - 6*a*b*B + a^2*(A + 2*C))*x)/(2*a^5) - (b*(12*A*b^6 - 12*a^5*b*
B + 15*a^3*b^3*B - 6*a*b^5*B - a^2*b^4*(29*A - 2*C) + 5*a^4*b^2*(4*A - C) +
 6*a^6*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)
^(5/2)*(a + b)^(5/2)*d) - ((12*A*b^5 - 2*a^5*B + 11*a^3*b^2*B - 6*a*b^4*B +
 a^4*b*(6*A - 5*C) - a^2*b^3*(21*A - 2*C))*Sin[c + d*x])/(2*a^4*(a^2 - b^2)
^2*d) + ((6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B + a^4*(A - 4*C) - a^2*b^2*(10*A -
 C))*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B
- a*C))*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + ((7*a^2*A*b^2 - 4*A*b^4 - 5*a^3*b*B + 2*a*b^3*B + 3*a^4*C)*Cos[c + d*x
]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 4.82448, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{\sin(c+dx)\left(-a^2b^3(21A-2C)+a^4b(6A-5C)+11a^3b^2B-2a^5B-6ab^4B+12Ab^5\right)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\sin(c+dx)\cos(c+dx)\left(-a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^3,x]
```


```
[Out] ((12*A*b^2 - 6*a*b*B + a^2*(A + 2*C))*x)/(2*a^5) - (b*(12*A*b^6 - 12*a^5*b*
B + 15*a^3*b^3*B - 6*a*b^5*B - a^2*b^4*(29*A - 2*C) + 5*a^4*b^2*(4*A - C) +
 6*a^6*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)
^(5/2)*(a + b)^(5/2)*d) - ((12*A*b^5 - 2*a^5*B + 11*a^3*b^2*B - 6*a*b^4*B +
 a^4*b*(6*A - 5*C) - a^2*b^3*(21*A - 2*C))*Sin[c + d*x])/(2*a^4*(a^2 - b^2)
^2*d) + ((6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B + a^4*(A - 4*C) - a^2*b^2*(10*A -
 C))*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B
- a*C))*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + ((7*a^2*A*b^2 - 4*A*b^4 - 5*a^3*b*B + 2*a*b^3*B + 3*a^4*C)*Cos[c + d*x
]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
```

```
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(}{}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(7a^2Ab^2-\right.}{}$$

$$= \frac{\left(6Ab^4+6a^3bB-3ab^3B+a^4(A-4C)-a^2b^2(10A-C)\right)c}{2a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(12Ab^5-2a^5B+11a^3b^2B-6ab^4B+a^4b(6A-5C)-a^2b\right.}{2a^4\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(12Ab^2-6abB+a^2(A+2C)\right)x}{2a^5} - \frac{\left(12Ab^5-2a^5B+11a^3b\right.}{}$$

$$= \frac{\left(12Ab^2-6abB+a^2(A+2C)\right)x}{2a^5} - \frac{\left(12Ab^5-2a^5B+11a^3b\right.}{}$$

$$= \frac{\left(12Ab^2-6abB+a^2(A+2C)\right)x}{2a^5} - \frac{\left(12Ab^5-2a^5B+11a^3b\right.}{}$$

$$= \frac{\left(12Ab^2-6abB+a^2(A+2C)\right)x}{2a^5} - \frac{b\left(20a^4Ab^2-29a^2Ab^4\right.}{}$$

Mathematica [A] time = 5.22424, size = 881, normalized size = 1.94

$$\frac{\frac{16b\left(6Ca^6-12bBa^5+5b^2(4A-C)a^4+15b^3Ba^3+b^4(2C-29A)a^2-6b^5Ba+12Ab^6\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{4Aca^8+8cCa^8+4Adxa^8+8Cdxa^8+4B\sin(c+dx)}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^3,x]
```

```
[Out] ((16*b*(12*A*b^6 - 12*a^5*b*B + 15*a^3*b^3*B - 6*a*b^5*B + 5*a^4*b^2*(4*A -
 C) + 6*a^6*C + a^2*b^4*(-29*A + 2*C))*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (4*a^8*A*c + 48*a^6*A*b^2*c - 12*a^4*
A*b^4*c - 136*a^2*A*b^6*c + 96*A*b^8*c - 24*a^7*b*B*c + 72*a^3*b^5*B*c - 48
*a*b^7*B*c + 8*a^8*c*C - 24*a^4*b^4*c*C + 16*a^2*b^6*c*C + 4*a^8*A*d*x + 48
*a^6*A*b^2*d*x - 12*a^4*A*b^4*d*x - 136*a^2*A*b^6*d*x + 96*A*b^8*d*x - 24*a
^7*b*B*d*x + 72*a^3*b^5*B*d*x - 48*a*b^7*B*d*x + 8*a^8*C*d*x - 24*a^4*b^4*C
*d*x + 16*a^2*b^6*C*d*x + 16*a*b*(a^2 - b^2)^2*(12*A*b^2 - 6*a*b*B + a^2*(A
 + 2*C))*(c + d*x)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*(12*A*b^2 - 6*a*b*B + a
^2*(A + 2*C))*(c + d*x)*Cos[2*(c + d*x)] - 8*a^7*A*b*Sin[c + d*x] - 32*a^5*
A*b^3*Sin[c + d*x] + 160*a^3*A*b^5*Sin[c + d*x] - 96*a*A*b^7*Sin[c + d*x] +
 4*a^8*B*Sin[c + d*x] + 8*a^6*b^2*B*Sin[c + d*x] - 84*a^4*b^4*B*Sin[c + d*x
] + 48*a^2*b^6*B*Sin[c + d*x] + 40*a^5*b^3*C*Sin[c + d*x] - 16*a^3*b^5*C*Si
n[c + d*x] + 2*a^8*A*Sin[2*(c + d*x)] - 48*a^6*A*b^2*Sin[2*(c + d*x)] + 130
```

```
*a^4*A*b^4*Sin[2*(c + d*x)] - 72*a^2*A*b^6*Sin[2*(c + d*x)] + 16*a^7*b*B*Si
n[2*(c + d*x)] - 64*a^5*b^3*B*Sin[2*(c + d*x)] + 36*a^3*b^5*B*Sin[2*(c + d*
x)] + 24*a^6*b^2*C*Sin[2*(c + d*x)] - 12*a^4*b^4*C*Sin[2*(c + d*x)] - 8*a^7
*A*b*Sin[3*(c + d*x)] + 16*a^5*A*b^3*Sin[3*(c + d*x)] - 8*a^3*A*b^5*Sin[3*(
c + d*x)] + 4*a^8*B*Sin[3*(c + d*x)] - 8*a^6*b^2*B*Sin[3*(c + d*x)] + 4*a^4
*b^4*B*Sin[3*(c + d*x)] + a^8*A*Sin[4*(c + d*x)] - 2*a^6*A*b^2*Sin[4*(c + d
*x)] + a^4*A*b^4*Sin[4*(c + d*x)])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/
(16*a^5*d)
```

Maple [B] time = 0.164, size = 2206, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d*
A/a^3*arctan(tan(1/2*d*x+1/2*c))+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-15/d*b^4/a^2
/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))*B+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+5/d*b^3/a/(a^4-2*a^2
*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))*C-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+10/d/a^2/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A*b^4-10/d
/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b
+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d/a/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1
/2*c)*B-1/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^5/(
a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+8/d/a/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
*B+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*b^4/(a-b)/
(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2*b^5/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-6/d/a^4/(1
+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b-6/d/a^4/(1+tan(1/2*d*x+1/
2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b-2/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+2/d*b^4/a^2/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*C-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d*b^3/a/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C+4/d*b
^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*
tan(1/2*d*x+1/2*c)*B+6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C*
b^2-20/d/a/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^3+29/d/a^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^5+2/d/a
^3*arctan(tan(1/2*d*x+1/2*c))*C+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*
d*x+1/2*c)^3*B+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A+2/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B+12/d/a^5*arctan(tan(1/2
*d*x+1/2*c))*A*b^2-6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B*b-12/d*b^7/a^5/(a^4
```

```
-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))*A-6/d*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a-1/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^2*tan(1/2*d*x+1/2*c)^3*A-6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.17964, size = 4668, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="fricas")
```

```
[Out] [1/4*(2*((A + 2*C)*a^10 - 6*B*a^9*b + 3*(3*A - 2*C)*a^8*b^2 + 18*B*a^7*b^3
- 3*(11*A - 2*C)*a^6*b^4 - 18*B*a^5*b^5 + (35*A - 2*C)*a^4*b^6 + 6*B*a^3*b^
7 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2 + 4*((A + 2*C)*a^9*b - 6*B*a^8*b^2 + 3
*(3*A - 2*C)*a^7*b^3 + 18*B*a^6*b^4 - 3*(11*A - 2*C)*a^5*b^5 - 18*B*a^4*b^6
 + (35*A - 2*C)*a^3*b^7 + 6*B*a^2*b^8 - 12*A*a*b^9)*d*x*cos(d*x + c) + 2*((
A + 2*C)*a^8*b^2 - 6*B*a^7*b^3 + 3*(3*A - 2*C)*a^6*b^4 + 18*B*a^5*b^5 - 3*(
11*A - 2*C)*a^4*b^6 - 18*B*a^3*b^7 + (35*A - 2*C)*a^2*b^8 + 6*B*a*b^9 - 12*
A*b^10)*d*x + (6*C*a^6*b^3 - 12*B*a^5*b^4 + 5*(4*A - C)*a^4*b^5 + 15*B*a^3*
b^6 - (29*A - 2*C)*a^2*b^7 - 6*B*a*b^8 + 12*A*b^9 + (6*C*a^8*b - 12*B*a^7*b
^2 + 5*(4*A - C)*a^6*b^3 + 15*B*a^5*b^4 - (29*A - 2*C)*a^4*b^5 - 6*B*a^3*b^
6 + 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(6*C*a^7*b^2 - 12*B*a^6*b^3 + 5*(4*A -
 C)*a^5*b^4 + 15*B*a^4*b^5 - (29*A - 2*C)*a^3*b^6 - 6*B*a^2*b^7 + 12*A*a*b^
8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(2*B*a^8*b^2 -
(6*A - 5*C)*a^7*b^3 - 13*B*a^6*b^4 + (27*A - 7*C)*a^5*b^5 + 17*B*a^4*b^6 -
(33*A - 2*C)*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9 + (A*a^10 - 3*A*a^8*b^2 + 3
*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 2*(B*a^10 - 2*A*a^9*b - 3*B*a^8*b^
2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B*a^4*b^6 + 2*A*a^3*b^7)*cos(
d*x + c)^2 + (4*B*a^9*b - (11*A - 6*C)*a^8*b^2 - 20*B*a^7*b^3 + (43*A - 9*C
)*a^6*b^4 + 25*B*a^5*b^5 - (50*A - 3*C)*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^
8)*cos(d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d
*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x +
 c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*(((A + 2*C)*a^10
 - 6*B*a^9*b + 3*(3*A - 2*C)*a^8*b^2 + 18*B*a^7*b^3 - 3*(11*A - 2*C)*a^6*b^
4 - 18*B*a^5*b^5 + (35*A - 2*C)*a^4*b^6 + 6*B*a^3*b^7 - 12*A*a^2*b^8)*d*x*c
os(d*x + c)^2 + 2*((A + 2*C)*a^9*b - 6*B*a^8*b^2 + 3*(3*A - 2*C)*a^7*b^3 +
18*B*a^6*b^4 - 3*(11*A - 2*C)*a^5*b^5 - 18*B*a^4*b^6 + (35*A - 2*C)*a^3*b^7
 + 6*B*a^2*b^8 - 12*A*a*b^9)*d*x*cos(d*x + c) + ((A + 2*C)*a^8*b^2 - 6*B*a^
```

```
7*b^3 + 3*(3*A - 2*C)*a^6*b^4 + 18*B*a^5*b^5 - 3*(11*A - 2*C)*a^4*b^6 - 18*
B*a^3*b^7 + (35*A - 2*C)*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*d*x - (6*C*a^6*b^
3 - 12*B*a^5*b^4 + 5*(4*A - C)*a^4*b^5 + 15*B*a^3*b^6 - (29*A - 2*C)*a^2*b^
7 - 6*B*a*b^8 + 12*A*b^9 + (6*C*a^8*b - 12*B*a^7*b^2 + 5*(4*A - C)*a^6*b^3
+ 15*B*a^5*b^4 - (29*A - 2*C)*a^4*b^5 - 6*B*a^3*b^6 + 12*A*a^2*b^7)*cos(d*x
 + c)^2 + 2*(6*C*a^7*b^2 - 12*B*a^6*b^3 + 5*(4*A - C)*a^5*b^4 + 15*B*a^4*b^
5 - (29*A - 2*C)*a^3*b^6 - 6*B*a^2*b^7 + 12*A*a*b^8)*cos(d*x + c))*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
 + c))) + (2*B*a^8*b^2 - (6*A - 5*C)*a^7*b^3 - 13*B*a^6*b^4 + (27*A - 7*C)*
a^5*b^5 + 17*B*a^4*b^6 - (33*A - 2*C)*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9 +
(A*a^10 - 3*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 2*(B*a^10
 - 2*A*a^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B*a^
4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^2 + (4*B*a^9*b - (11*A - 6*C)*a^8*b^2 - 2
0*B*a^7*b^3 + (43*A - 9*C)*a^6*b^4 + 25*B*a^5*b^5 - (50*A - 3*C)*a^4*b^6 -
9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2
 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b
^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)
*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52141, size = 2291, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x
, algorithm="giac")
```

```
[Out] -1/2*(2*(6*C*a^6*b - 12*B*a^5*b^2 + 20*A*a^4*b^3 - 5*C*a^4*b^3 + 15*B*a^3*b
^4 - 29*A*a^2*b^5 + 2*C*a^2*b^5 - 6*B*a*b^6 + 12*A*b^7)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/
2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(-a^2 +
 b^2)) + 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 +
 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A
*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 6*C*
a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*
a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 5*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*
a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*C*a
^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a
^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 2*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a
*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*t
an(1/2*d*x + 1/2*c)^7 - 3*A*a^7*tan(1/2*d*x + 1/2*c)^5 + 2*B*a^7*tan(1/2*d*
x + 1/2*c)^5 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^6*b*tan(1/2*d*x + 1
```

/2*c)^5 - 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*C*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^3 - A*a^7*tan(1/2*d*x + 1/2*c) - 2*B*a^7*tan(1/2*d*x + 1/2*c) + 4*A*a^6*b*tan(1/2*d*x + 1/2*c) - 4*B*a^6*b*tan(1/2*d*x + 1/2*c) + 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c) - 6*C*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) - 5*C*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c) + 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 3*C*a^3*b^4*tan(1/2*d*x + 1/2*c) - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c) - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) + 2*C*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*A*a*b^6*tan(1/2*d*x + 1/2*c) - 6*B*a*b^6*tan(1/2*d*x + 1/2*c) + 12*A*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (A*a^2 + 2*C*a^2 - 6*B*a*b + 12*A*b^2)*(d*x + c)/a^5)/d

3.923 $\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=470

$$-\frac{\tan(c+dx)\left(23a^2b^2C+3a^3bB-12a^4C-8ab^3B+5Ab^4-6b^4C\right)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)-7a^5b^3B+8a^3b^5B+28a^6b^2C-\ldots\right.}{b^5\ldots}$$

```
[Out] ((b*B - 4*a*C)*ArcTanh[Sin[c + d*x]])/(b^5*d) - ((2*A*b^8 + 2*a^7*b*B - 7*a
^5*b^3*B + 8*a^3*b^5*B - 8*a*b^7*B - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C
+ a^2*b^6*(3*A + 20*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((5*A*b^4 + 3*a^3*b*B - 8*a*b^3*B -
 12*a^4*C + 23*a^2*b^2*C - 6*b^4*C)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) -
 ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(
a + b*Sec[c + d*x])^3) + ((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^
2*(2*A + 9*C))*Sec[c + d*x]^2*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x])^2) + (a*(2*A*b^6 - a^5*b*B + 2*a^3*b^3*B - 6*a*b^5*B + 4*a^6*C
 - 11*a^4*b^2*C + 3*a^2*b^4*(A + 4*C))*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d
*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 9.91341, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{\tan(c+dx)\left(23a^2b^2C+3a^3bB-12a^4C-8ab^3B+5Ab^4-6b^4C\right)}{6b^4d\left(a^2-b^2\right)^2}-\frac{\left(a^2b^6(3A+20C)-7a^5b^3B+8a^3b^5B+28a^6b^2C-\ldots\right.}{b^5\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^4,x]
```


```
[Out] ((b*B - 4*a*C)*ArcTanh[Sin[c + d*x]])/(b^5*d) - ((2*A*b^8 + 2*a^7*b*B - 7*a
^5*b^3*B + 8*a^3*b^5*B - 8*a*b^7*B - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C
+ a^2*b^6*(3*A + 20*C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((5*A*b^4 + 3*a^3*b*B - 8*a*b^3*B -
 12*a^4*C + 23*a^2*b^2*C - 6*b^4*C)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) -
 ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(
a + b*Sec[c + d*x])^3) + ((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^
2*(2*A + 9*C))*Sec[c + d*x]^2*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x])^2) + (a*(2*A*b^6 - a^5*b*B + 2*a^3*b^3*B - 6*a*b^5*B + 4*a^6*C
 - 11*a^4*b^2*C + 3*a^2*b^4*(A + 4*C))*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d
*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
```

```
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)(3}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3b\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3b\right.}{}$$

$$= -\frac{\left(5Ab^4+3a^3bB-8ab^3B-12a^4C+23a^2b^2C-6b^4C\right)\tan(c}{6b^4\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(5Ab^4+3a^3bB-8ab^3B-12a^4C+23a^2b^2C-6b^4C\right)\tan(c}{6b^4\left(a^2-b^2\right)^2d}$$

$$= \frac{(bB-4aC)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(5Ab^4+3a^3bB-8ab^3B-\right.}{6b}$$

$$= \frac{(bB-4aC)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(5Ab^4+3a^3bB-8ab^3B-\right.}{6b}$$

$$= \frac{(bB-4aC)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{\left(3a^2Ab^6+2Ab^8+2a^7bB\right.}{}$$

Mathematica [B] time = 6.48735, size = 1197, normalized size = 2.55

$$-\frac{2(bB-4aC)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)(b+a\cos(c+dx}{b^5d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^4,x]
```

```
[Out] (-2*(3*a^2*A*b^6 + 2*A*b^8 + 2*a^7*b*B - 7*a^5*b^3*B + 8*a^3*b^5*B - 8*a*b^
7*B - 8*a^8*C + 28*a^6*b^2*C - 35*a^4*b^4*C + 20*a^2*b^6*C)*ArcTanh[((-a +
b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)
^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])
^4) - (2*(b*B - 4*a*C)*(b + a*Cos[c + d*x])^4*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b^5*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) +
 (2*(b*B - 4*a*C)*(b + a*Cos[c + d*x])^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b^5*d*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((b
+ a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-
6*a^4*A*b^5*Sin[c + d*x] - 54*a^2*A*b^7*Sin[c + d*x] + 30*a^7*b^2*B*Sin[c +
 d*x] - 90*a^5*b^4*B*Sin[c + d*x] + 120*a^3*b^6*B*Sin[c + d*x] - 120*a^8*b*
```

```
C*Sin[c + d*x] + 294*a^6*b^3*C*Sin[c + d*x] - 174*a^4*b^5*C*Sin[c + d*x] -
108*a^2*b^7*C*Sin[c + d*x] + 48*b^9*C*Sin[c + d*x] - 16*a^5*A*b^4*Sin[2*(c
+ d*x)] - 2*a^3*A*b^6*Sin[2*(c + d*x)] - 72*a*A*b^8*Sin[2*(c + d*x)] + 12*a
^8*b*B*Sin[2*(c + d*x)] + 10*a^6*b^3*B*Sin[2*(c + d*x)] - 76*a^4*b^5*B*Sin[
2*(c + d*x)] + 144*a^2*b^7*B*Sin[2*(c + d*x)] - 48*a^9*C*Sin[2*(c + d*x)] -
 40*a^7*b^2*C*Sin[2*(c + d*x)] + 370*a^5*b^4*C*Sin[2*(c + d*x)] - 444*a^3*b
^6*C*Sin[2*(c + d*x)] + 72*a*b^8*C*Sin[2*(c + d*x)] - 6*a^4*A*b^5*Sin[3*(c
+ d*x)] - 54*a^2*A*b^7*Sin[3*(c + d*x)] + 30*a^7*b^2*B*Sin[3*(c + d*x)] - 9
0*a^5*b^4*B*Sin[3*(c + d*x)] + 120*a^3*b^6*B*Sin[3*(c + d*x)] - 120*a^8*b*C
*Sin[3*(c + d*x)] + 342*a^6*b^3*C*Sin[3*(c + d*x)] - 318*a^4*b^5*C*Sin[3*(c
 + d*x)] + 36*a^2*b^7*C*Sin[3*(c + d*x)] - 4*a^5*A*b^4*Sin[4*(c + d*x)] - 1
1*a^3*A*b^6*Sin[4*(c + d*x)] + 6*a^8*b*B*Sin[4*(c + d*x)] - 17*a^6*b^3*B*Si
n[4*(c + d*x)] + 26*a^4*b^5*B*Sin[4*(c + d*x)] - 24*a^9*C*Sin[4*(c + d*x)]
+ 68*a^7*b^2*C*Sin[4*(c + d*x)] - 65*a^5*b^4*C*Sin[4*(c + d*x)] + 6*a^3*b^6
*C*Sin[4*(c + d*x)]))/(24*b^4*(-a^2 + b^2)^3*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.117, size = 3764, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 8/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a-28/d/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-
b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))*a^6*C+35/d/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^4*C+12/d*b/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5*B-3/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*b^2/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^3*a^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c
)^5*B+44/3/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a
^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-24/d*b/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*B+12/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3*a^7/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+1/d/b
^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+18/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*C-5/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-1/d/b^2/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^
3)*tan(1/2*d*x+1/2*c)^5*B-6/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3*a^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/d/b
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*C+18/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+5/d/b/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)*C+12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b^2/(tan(1
```

```
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c)*A+3/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d/b
^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^7/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-116/3/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3*C-6/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
*a^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d/b^3/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a-b)/(a^3+3*a^2*b+3*a*b
^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3*a^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+
12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a^2-2*a*b
+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*B+8/d/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^8*C-3/d*b/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*A*a^2-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+4/d/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-
20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+4/3/d/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*A-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-1/d/b^4*ln(tan(1/2*d*x+
1/2*c)-1)*B-1/d*C/b^4/(tan(1/2*d*x+1/2*c)+1)+1/d/b^4*ln(tan(1/2*d*x+1/2*c)+
1)*B-1/d*C/b^4/(tan(1/2*d*x+1/2*c)-1)-20/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
*a^2+40/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*A-2/d/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^7+7/d/b^2/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))*B*a^5-4/d/b^5*ln(tan(1/2*d*x+1/2*c)+1)*a*C-2/d*b^3/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*A-8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a^3+4/d/b^5*ln(
tan(1/2*d*x+1/2*c)-1)*a*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^4\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
4,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**4/(a + b*se
c(c + d*x))**4, x)
```

Giac [B] time = 1.53547, size = 1706, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/3*(3*(8*C*a^8 - 2*B*a^7*b - 28*C*a^6*b^2 + 7*B*a^5*b^3 + 35*C*a^4*b^4 - 8
*B*a^3*b^5 - 3*A*a^2*b^6 - 20*C*a^2*b^6 + 8*B*a*b^7 - 2*A*b^8)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b
*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9
 - b^11)*sqrt(-a^2 + b^2)) - (18*C*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^8*b*t
an(1/2*d*x + 1/2*c)^5 - 42*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^7*b^2*ta
n(1/2*d*x + 1/2*c)^5 - 24*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^6*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 117*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^5*b^4*t
an(1/2*d*x + 1/2*c)^5 - 45*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*C*a^5*b^4*
tan(1/2*d*x + 1/2*c)^5 - 3*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^4*b^5*t
an(1/2*d*x + 1/2*c)^5 - 105*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^3*b^6*
tan(1/2*d*x + 1/2*c)^5 + 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 60*C*a^3*b^6
*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^2*b^
7*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 36*C*a^9*tan
(1/2*d*x + 1/2*c)^3 + 12*B*a^8*b*tan(1/2*d*x + 1/2*c)^3 + 152*C*a^7*b^2*tan
(1/2*d*x + 1/2*c)^3 - 56*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^5*b^4*tan
(1/2*d*x + 1/2*c)^3 - 236*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 116*B*a^4*b^5*
tan(1/2*d*x + 1/2*c)^3 - 32*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 120*C*a^3*b^
6*tan(1/2*d*x + 1/2*c)^3 - 72*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^8
*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^9*tan(1/2*d*x + 1/2*c) - 6*B*a^8*b*tan(1/2
*d*x + 1/2*c) + 42*C*a^8*b*tan(1/2*d*x + 1/2*c) - 15*B*a^7*b^2*tan(1/2*d*x
+ 1/2*c) - 24*C*a^7*b^2*tan(1/2*d*x + 1/2*c) + 6*B*a^6*b^3*tan(1/2*d*x + 1/
2*c) - 117*C*a^6*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^5*b^4*tan(1/2*d*x + 1/2*c
) + 45*B*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*C*a^5*b^4*tan(1/2*d*x + 1/2*c) +
 3*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*
```

```
C*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 60*B*a^
3*b^6*tan(1/2*d*x + 1/2*c) + 60*C*a^3*b^6*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b
^7*tan(1/2*d*x + 1/2*c) - 36*B*a^2*b^7*tan(1/2*d*x + 1/2*c) + 18*A*a*b^8*ta
n(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) - 3*(4*C*a - B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) + 1))/b^5 + 3*(4*C*a - B*b)*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/b^5 - 6*C*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^
4))/d
```

3.924 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=358

$$-\frac{\left(-a^3b^4(A-8C)+3a^2b^5B-7a^5b^2C+2a^7C-4ab^6(A+2C)+2b^7B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\tan(c+dx)\sec^2(c}{3bd\left(a^2-b^2\right.}$$

```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^4*d) - ((3*a^2*b^5*B + 2*b^7*B - a^3*b^4*(A -
8*C) + 2*a^7*C - 7*a^5*b^2*C - 4*a*b^6*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - ((A*b^2 -
 a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x])^3) - (a*(2*A*b^4 - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 8*C))*Tan
[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((4*A*b^6 + a^3
*b^3*B - 16*a*b^5*B + 9*a^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A + 28*
C))*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 2.50854, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {4098, 4090, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{\left(-a^3b^4(A-8C)+3a^2b^5B-7a^5b^2C+2a^7C-4ab^6(A+2C)+2b^7B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\tan(c+dx)\sec^2(c}{3bd\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^4,x]
```


```
[Out] (C*ArcTanh[Sin[c + d*x]])/(b^4*d) - ((3*a^2*b^5*B + 2*b^7*B - a^3*b^4*(A -
8*C) + 2*a^7*C - 7*a^5*b^2*C - 4*a*b^6*(A + 2*C))*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - ((A*b^2 -
 a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x])^3) - (a*(2*A*b^4 - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 8*C))*Tan
[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((4*A*b^6 + a^3
*b^3*B - 16*a*b^5*B + 9*a^6*C + 2*a^2*b^4*(7*A + 17*C) - a^4*b^2*(3*A + 28*
C))*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \int \frac{\sec^2(c+d}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a\left(2Ab^4\right.}{}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= \frac{C\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(a^3Ab^4+4aAb^6-3a^2b^5B-2b^7\right.}{}$$

Mathematica [C] time = 7.3465, size = 1302, normalized size = 3.64

$$-\frac{2C\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)(b+a\cos(c+dx))^4}{b^4d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b\sec(c+dx))^4} + \frac{2C\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^4,x]
```

```
[Out] (-2*C*(b + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*S
ec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b^4*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + (2*C*(b + a*
Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(b^4*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((-(a^3*A*b^4) - 4*a*A*b^
6 + 3*a^2*b^5*B + 2*b^7*B + 2*a^7*C - 7*a^5*b^2*C + 8*a^3*b^4*C - 8*a*b^6*C
)*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*(((-2*I)*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] -
I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((
-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*Cos[c])/(b^4*Sqrt[a^2 - b^2]*d*
Sqrt[Cos[2*c] - I*Sin[2*c]]) - (2*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b
^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c
] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*Sin[c])/(b^
4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]])))/((-a^2 + b^2)^3*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) - (2*(b +
 a*Cos[c + d*x])*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
```

```
^2)*(A*b^3*Sin[c] - a*b^2*B*Sin[c] + a^2*b*C*Sin[c] - a*A*b^2*Sin[d*x] + a^
2*b*B*Sin[d*x] - a^3*C*Sin[d*x]))/(3*a*b*(-a^2 + b^2)*d*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((b + a*Cos[c + d*
x])^2*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-5*a*A
*b^3*Sin[c] + 2*a^2*b^2*B*Sin[c] + 3*b^4*B*Sin[c] + a^3*b*C*Sin[c] - 6*a*b^
3*C*Sin[c] + 3*a^2*A*b^2*Sin[d*x] + 2*A*b^4*Sin[d*x] - 5*a*b^3*B*Sin[d*x] -
 3*a^4*C*Sin[d*x] + 8*a^2*b^2*C*Sin[d*x]))/(3*b^2*(-a^2 + b^2)^2*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((b + a
*Cos[c + d*x])^3*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*(-3*a^3*A*b^3*Sin[c] - 12*a*A*b^5*Sin[c] + 9*a^2*b^4*B*Sin[c] + 6*b^6*B
*Sin[c] - 3*a^5*b*C*Sin[c] + 6*a^3*b^3*C*Sin[c] - 18*a*b^5*C*Sin[c] + 13*a^
2*A*b^4*Sin[d*x] + 2*A*b^6*Sin[d*x] - 4*a^3*b^3*B*Sin[d*x] - 11*a*b^5*B*Sin
[d*x] + 6*a^6*C*Sin[d*x] - 17*a^4*b^2*C*Sin[d*x] + 26*a^2*b^4*C*Sin[d*x]))/
(3*b^3*(-a^2 + b^2)^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(
a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.125, size = 3244, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*C/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/d*C/b^4*ln(tan(1/2*d*x+1/2*c)-1)-3/d*b
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*A+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a-
b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-1/d/b^2/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*C-6/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3*a^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+2/d/b^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^6/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+1/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3*a^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*C-6/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^4/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+3/d*b/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*B-2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*b/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3
)*tan(1/2*d*x+1/2*c)*A+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3*a^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+1/d/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5
*B-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5*C-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+
b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d/b^3/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*a^6*C-28/3/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*a^2+12/d*b^
2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a*B-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1
```

```
/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*a*B+44/3/d/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*a^4*C-24/d*b/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3*C*a^2+12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+12/d*b/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*
a*B+2/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*A-4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2
-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*b^3/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*B+1/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a^3-8/d/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))*C*a^3+8/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a-3/d*b/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*B*a^2+7/d/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^5*C+4/3/d/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*a^3-2/d/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a
^7*C+4/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a+2/d*b^3/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^2\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
4,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*se
c(c + d*x))**4, x)
```

Giac [B] time = 1.57917, size = 1532, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] -1/3*(3*(2*C*a^7 - 7*C*a^5*b^2 - A*a^3*b^4 + 8*C*a^3*b^4 + 3*B*a^2*b^5 - 4*
A*a*b^6 - 8*C*a*b^6 + 2*B*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a^2 + b^2)) - 3*C
*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + 3*C*log(abs(tan(1/2*d*x + 1/2*c)
- 1))/b^4 - (6*C*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^7*b*tan(1/2*d*x + 1/2*
c)^5 - 6*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c
)^5 - 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^3*tan(1/2*d*x + 1/2*c
)^5 + 12*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c
)^5 - 6*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^3*b^5*tan(1/2*d*x + 1/2*c
)^5 - 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*C*a^3*b^5*tan(1/2*d*x + 1/2*c
)^5 + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^6*tan(1/2*d*x + 1/2*
c)^5 + 36*C*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b^7*tan(1/2*d*x + 1/2*c)
^5 - 18*B*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 - 1
2*C*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*
a^5*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*
C*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 32*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 + 16
*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 72*C*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 3
6*B*a*b^7*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*C*a^
8*tan(1/2*d*x + 1/2*c) + 15*C*a^7*b*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^2*tan(
1/2*d*x + 1/2*c) - 3*A*a^5*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^5*b^3*tan(1/2*d
*x + 1/2*c) - 45*C*a^5*b^3*tan(1/2*d*x + 1/2*c) + 12*A*a^4*b^4*tan(1/2*d*x
+ 1/2*c) - 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^4*b^4*tan(1/2*d*x + 1/2
*c) + 27*A*a^3*b^5*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c)
+ 60*C*a^3*b^5*tan(1/2*d*x + 1/2*c) + 12*A*a^2*b^6*tan(1/2*d*x + 1/2*c) - 2
7*B*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b^6*tan(1/2*d*x + 1/2*c) + 6*A*
a*b^7*tan(1/2*d*x + 1/2*c) - 18*B*a*b^7*tan(1/2*d*x + 1/2*c) + 6*A*b^8*tan(
1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x +
 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.925 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=314

$$\frac{\left(-a^2b(4A+3C)+a^3B+4ab^2B-b^3(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\tan(c+dx)\left(a^3b^2(2A-5C)-10a^2b^3B+\right.}{6b^2d\left(a^2-b^2\right)^3(a}$$

```
[Out] ((a^3*B + 4*a*b^2*B - b^3*(A + 2*C) - a^2*b*(4*A + 3*C))*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) + (a*(A*
b^2 - a*(b*B - a*C))*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)^3) + ((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Tan
[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((a^4*b*B - 10*
a^2*b^3*B - 6*b^5*B + a^3*b^2*(2*A - 5*C) + 2*a^5*C + a*b^4*(13*A + 18*C))*
Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.03922, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4090, 4080, 4003, 12, 3831, 2659, 208}

$$\frac{\left(-a^2b(4A+3C)+a^3B+4ab^2B-b^3(A+2C)\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\tan(c+dx)\left(a^3b^2(2A-5C)-10a^2b^3B+\right.}{6b^2d\left(a^2-b^2\right)^3(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^4,x]
```



```
[Out] ((a^3*B + 4*a*b^2*B - b^3*(A + 2*C) - a^2*b*(4*A + 3*C))*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) + (a*(A*
b^2 - a*(b*B - a*C))*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)^3) + ((3*A*b^4 + a^3*b*B - 6*a*b^3*B - 4*a^4*C + a^2*b^2*(2*A + 9*C))*Tan
[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((a^4*b*B - 10*
a^2*b^3*B - 6*b^5*B + a^3*b^2*(2*A - 5*C) + 2*a^5*C + a*b^4*(13*A + 18*C))*
Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
```

```
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)\left(-3b\left(Ab^2-a(b\right.\right.}{}}{}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3\right.}{6b^2\left(a^2\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3\right.}{6b^2\left(a^2\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3\right.}{6b^2\left(a^2\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3\right.}{6b^2\left(a^2\right.}$$

$$= \frac{a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(3Ab^4+a^3bB-6ab^3\right.}{6b^2\left(a^2\right.}$$

$$= -\frac{\left(4a^2Ab+Ab^3-a^3B-4ab^2B+3a^2bC+2b^3C\right)\tanh^{-1}\left(\right.}{(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 1.47225, size = 299, normalized size = 0.95

$$\frac{\frac{24\left(-a^2b(4A+3C)+a^3B+4ab^2B-b^3(A+2C)\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{2\sin(c+dx)\left(a\cos(2(c+dx))\left(a^2b^2(10A+11C)+a^4(6A+4C)-13a^3bB-2ab^3B-Ab^4\right)+\right.}{}}{24d\left(b^2-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^4,x]
```

```
[Out] ((24*(a^3*B + 4*a*b^2*B - b^3*(A + 2*C) - a^2*b*(4*A + 3*C))*ArcTanh[((-a +
 b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (2*(6*a^5*A + 14*
a^3*A*b^2 + 25*a*A*b^4 - 11*a^4*b*B - 22*a^2*b^3*B - 12*b^5*B + 8*a^5*C + a
^3*b^2*C + 36*a*b^4*C + 6*(-(A*b^5) + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B + 9*a
^2*b^3*(A + C) + a^4*b*(2*A + C))*Cos[c + d*x] + a*(-(A*b^4) - 13*a^3*b*B -
 2*a*b^3*B + a^4*(6*A + 4*C) + a^2*b^2*(10*A + 11*C))*Cos[2*(c + d*x)])*Sin
[c + d*x])/(b + a*Cos[c + d*x])^3)/(24*(-a^2 + b^2)^3*d)
```

Maple [A] time = 0.101, size = 453, normalized size = 1.4

$$\frac{1}{d}\left(2\frac{1}{\left(\left(\tan(1/2\,dx+c/2)\right)^2a-\left(\tan(1/2\,dx+c/2)\right)^2b-a-b\right)^3}\left(-1/2\,\frac{\left(2\,Aa^3+2\,Aa^2b+6\,Aab^2+Ab^3-Ba^3-6\,Ba^2b\right.}{(a-b)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3-B*a^3-6*B*a^2*b-2*B*a*b^2-2
*B*b^3+2*C*a^3+3*C*a^2*b+6*C*a*b^2)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+2/3*(3*A*a^3+7*A*a*b^2-7*B*a^2*b-3*B*b^3+C*a^3+9*C*a*b^2)/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*A*a^3-2*A*a^2*b+6*
A*a*b^2-A*b^3+B*a^3-6*B*a^2*b+2*B*a*b^2-2*B*b^3+2*C*a^3-3*C*a^2*b+6*C*a*b^2
)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2+3*C*a^2*b
+2*C*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.40935, size = 3109, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] [1/12*(3*(B*a^3*b^3 - (4*A + 3*C)*a^2*b^4 + 4*B*a*b^5 - (A + 2*C)*b^6 + (B*
a^6 - (4*A + 3*C)*a^5*b + 4*B*a^4*b^2 - (A + 2*C)*a^3*b^3)*cos(d*x + c)^3 +
 3*(B*a^5*b - (4*A + 3*C)*a^4*b^2 + 4*B*a^3*b^3 - (A + 2*C)*a^2*b^4)*cos(d*
x + c)^2 + 3*(B*a^4*b^2 - (4*A + 3*C)*a^3*b^3 + 4*B*a^2*b^4 - (A + 2*C)*a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(2*C*a^7 + B*a
^6*b + (2*A - 7*C)*a^5*b^2 - 11*B*a^4*b^3 + (11*A + 23*C)*a^3*b^4 + 4*B*a^2
*b^5 - (13*A + 18*C)*a*b^6 + 6*B*b^7 + (2*(3*A + 2*C)*a^7 - 13*B*a^6*b + (4
*A + 7*C)*a^5*b^2 + 11*B*a^4*b^3 - 11*(A + C)*a^3*b^4 + 2*B*a^2*b^5 + A*a*b
^6)*cos(d*x + c)^2 + 3*(B*a^7 + (2*A + C)*a^6*b - 10*B*a^5*b^2 + (7*A + 8*C
)*a^4*b^3 + 7*B*a^3*b^4 - (10*A + 9*C)*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x
 + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*
d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)
*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10
)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d),
 1/6*(3*(B*a^3*b^3 - (4*A + 3*C)*a^2*b^4 + 4*B*a*b^5 - (A + 2*C)*b^6 + (B*a
^6 - (4*A + 3*C)*a^5*b + 4*B*a^4*b^2 - (A + 2*C)*a^3*b^3)*cos(d*x + c)^3 +
3*(B*a^5*b - (4*A + 3*C)*a^4*b^2 + 4*B*a^3*b^3 - (A + 2*C)*a^2*b^4)*cos(d*x
 + c)^2 + 3*(B*a^4*b^2 - (4*A + 3*C)*a^3*b^3 + 4*B*a^2*b^4 - (A + 2*C)*a*b^
5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) + (2*C*a^7 + B*a^6*b + (2*A - 7*C)*a^5*b^2
```

```
 - 11*B*a^4*b^3 + (11*A + 23*C)*a^3*b^4 + 4*B*a^2*b^5 - (13*A + 18*C)*a*b^6
 + 6*B*b^7 + (2*(3*A + 2*C)*a^7 - 13*B*a^6*b + (4*A + 7*C)*a^5*b^2 + 11*B*a
^4*b^3 - 11*(A + C)*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6)*cos(d*x + c)^2 + 3*(B*
a^7 + (2*A + C)*a^6*b - 10*B*a^5*b^2 + (7*A + 8*C)*a^4*b^3 + 7*B*a^3*b^4 -
(10*A + 9*C)*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^1
1 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10
*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9
*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^
3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec^{2}\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
4,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*se
c(c + d*x))**4, x)
```

Giac [B] time = 1.48136, size = 1310, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] 1/3*(3*(B*a^3 - 4*A*a^2*b - 3*C*a^2*b + 4*B*a*b^2 - A*b^3 - 2*C*b^3)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*sqrt(-a^2 + b^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan
(1/2*d*x + 1/2*c)^5 + 6*C*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b*tan(1/2*d*
x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^4*b*tan(1/2*d*x +
1/2*c)^5 + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A*a^2*b^3*tan(1/2*d*x +
 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^5 + 12*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/
2*c)^5 + 18*C*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5*tan(1/2*d*x + 1/2*c)^5
 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*C*a
^5*tan(1/2*d*x + 1/2*c)^3 + 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^3*b^
2*tan(1/2*d*x + 1/2*c)^3 - 32*C*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*B*a^2*b
^3*tan(1/2*d*x + 1/2*c)^3 + 28*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*C*a*b^4*
tan(1/2*d*x + 1/2*c)^3 - 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*
d*x + 1/2*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*C*a^5*tan(1/2*d*x + 1/2*c)
+ 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B*a^4*b*tan(1/2*d*x + 1/2*c) + 3*C*a^
4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*b^2
*tan(1/2*d*x + 1/2*c) + 6*C*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan
(1/2*d*x + 1/2*c) - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*C*a^2*b^3*tan(1/
2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2*d*x +
```

```
1/2*c) + 18*C*a*b^4*tan(1/2*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6
*B*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/
2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.926 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\left(a^3(-(2A+C))+4a^2bB-ab^2(3A+4C)+b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\tan(c+dx)\left(-a^2b^2(11A+10C)+2a^3\right.}{6bd\left(a^2-b^2\right)^3(a}$$

```
[Out] -(((4*a^2*b*B + b^3*B - a^3*(2*A + C) - a*b^2*(3*A + 4*C))*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) - ((A
*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^3) + ((2*a^2*b*B + 3*b^3*B + a^3*C - a*b^2*(5*A + 6*C))*Tan[c + d*x])/(6*b
*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((2*a^3*b*B + 13*a*b^3*B + a^4*C
 - 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Tan[c + d*x])/(6*b*(a^2 - b^2
)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.855812, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4080, 4003, 12, 3831, 2659, 208}

$$-\frac{\left(a^3(-(2A+C))+4a^2bB-ab^2(3A+4C)+b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\tan(c+dx)\left(-a^2b^2(11A+10C)+2a^3\right.}{6bd\left(a^2-b^2\right)^3(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^4,x]
```


```
[Out] -(((4*a^2*b*B + b^3*B - a^3*(2*A + C) - a*b^2*(3*A + 4*C))*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) - ((A
*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^3) + ((2*a^2*b*B + 3*b^3*B + a^3*C - a*b^2*(5*A + 6*C))*Tan[c + d*x])/(6*b
*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((2*a^3*b*B + 13*a*b^3*B + a^4*C
 - 2*b^4*(2*A + 3*C) - a^2*b^2*(11*A + 10*C))*Tan[c + d*x])/(6*b*(a^2 - b^2
)^3*d*(a + b*Sec[c + d*x]))
```


Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
```

```
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(3b(bB-a(A+C))+\right.}{(a+b}}{3b}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-\right.}{6b\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-\right.}{6b\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-\right.}{6b\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-\right.}{6b\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2bB+3b^3B+a^3C-\right.}{6b\left(a^2-b^2\right)^2 d(}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B+a^3C+4ab^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{t}}{}\right.}{(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [C] time = 7.55916, size = 1069, normalized size = 3.58

$$\frac{\left(-2Aa^3-Ca^3+4bBa^2-3Ab^2a-4b^2Ca+b^3B\right)\sec^2(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2i\tan^{-1}\left(\sec\left(\frac{dx}{2}\right)\left(\cdots\right.\right.}{\cdots}\right.}{\left(b^2-a^2\right)^3(\cos\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^4,x]
```

```
[Out] ((-2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B + b^3*B - a^3*C - 4*a*b^2*C)*(b + a*Cos[
c + d*x])^4*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-2*I)
*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]])
- (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*
x)/2] + I*a*Sin[c + (d*x)/2])]*Cos[c])/(Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I
*Sin[2*c]]) - (2*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c]
 - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))
*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*Sin[c])/(Sqrt[a^2 - b^2]*d*S
qrt[Cos[2*c] - I*Sin[2*c]])))/((-a^2 + b^2)^3*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + (2*(b + a*Cos[c + d*x])*Sec[
c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*b^4*Sin[c] - a
*b^3*B*Sin[c] + a^2*b^2*C*Sin[c] - a*A*b^3*Sin[d*x] + a^2*b^2*B*Sin[d*x] -
a^3*b*C*Sin[d*x]))/(3*a^3*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + ((b + a*Cos[c + d*x])^2*Sec[c]*Sec
[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-11*a^2*A*b^3*Sin[c] +
 6*A*b^5*Sin[c] + 8*a^3*b^2*B*Sin[c] - 3*a*b^4*B*Sin[c] - 5*a^4*b*C*Sin[c]
+ 9*a^3*A*b^2*Sin[d*x] - 4*a*A*b^4*Sin[d*x] - 6*a^4*b*B*Sin[d*x] + a^2*b^3*
B*Sin[d*x] + 3*a^5*C*Sin[d*x] + 2*a^3*b^2*C*Sin[d*x]))/(3*a^3*(a^2 - b^2)^2
*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4
) + ((b + a*Cos[c + d*x])^3*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*(27*a^4*A*b^2*Sin[c] - 18*a^2*A*b^4*Sin[c] + 6*A*b^6*Sin[c]
- 12*a^5*b*B*Sin[c] - 3*a^3*b^3*B*Sin[c] + 3*a^6*C*Sin[c] + 12*a^4*b^2*C*Si
n[c] - 18*a^5*A*b*Sin[d*x] + 5*a^3*A*b^3*Sin[d*x] - 2*a*A*b^5*Sin[d*x] + 6*
a^6*B*Sin[d*x] + 10*a^4*b^2*B*Sin[d*x] - a^2*b^4*B*Sin[d*x] - 13*a^5*b*C*Si
n[d*x] - 2*a^3*b^3*C*Sin[d*x]))/(3*a^3*(a^2 - b^2)^3*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [A] time = 0.103, size = 452, normalized size = 1.5

$$\frac{1}{d}\left(-2\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b-a-b\right)^3}\left(-1/2\frac{\left(6\,Aa^2b+3\,Aab^2+2\,Ab^3-2\,Ba^3-2\,Ba^2b-6\cdots\right.}{(a-b\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3+C*a^3+6*C*a^2*b+2*C*a*b^2+2*C*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*B*a*b^2+7*C*a^2*b+3*C*b^3)/(a
^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(6*A*a^2*b-3*A*a*b^2
+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3-C*a^3+6*C*a^2*b-2*C*a*b^2+2*C*b^
3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3+C*a^3+
```

```
4*C*a*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x,
algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.30993, size = 3109, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x,
algorithm="fricas")

[Out] [1/12*(3*((2*A + C)*a^3*b^3 - 4*B*a^2*b^4 + (3*A + 4*C)*a*b^5 - B*b^6 + ((2
*A + C)*a^6 - 4*B*a^5*b + (3*A + 4*C)*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 +
 3*((2*A + C)*a^5*b - 4*B*a^4*b^2 + (3*A + 4*C)*a^3*b^3 - B*a^2*b^4)*cos(d*
x + c)^2 + 3*((2*A + C)*a^4*b^2 - 4*B*a^3*b^3 + (3*A + 4*C)*a^2*b^4 - B*a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(C*a^6*b + 2*B
*a^5*b^2 - 11*(A + C)*a^4*b^3 + 11*B*a^3*b^4 + (7*A + 4*C)*a^2*b^5 - 13*B*a
*b^6 + 2*(2*A + 3*C)*b^7 + (6*B*a^7 - (18*A + 13*C)*a^6*b + 4*B*a^5*b^2 + (
23*A + 11*C)*a^4*b^3 - 11*B*a^3*b^4 - (7*A - 2*C)*a^2*b^5 + B*a*b^6 + 2*A*b
^7)*cos(d*x + c)^2 + 3*(C*a^7 + 2*B*a^6*b - (9*A + 10*C)*a^5*b^2 + 7*B*a^4*
b^3 + (8*A + 7*C)*a^3*b^4 - 10*B*a^2*b^5 + (A + 2*C)*a*b^6 + B*b^7)*cos(d*x
 + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*
d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)
*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10
)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d),
 1/6*(3*((2*A + C)*a^3*b^3 - 4*B*a^2*b^4 + (3*A + 4*C)*a*b^5 - B*b^6 + ((2*
A + C)*a^6 - 4*B*a^5*b + (3*A + 4*C)*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 +
3*((2*A + C)*a^5*b - 4*B*a^4*b^2 + (3*A + 4*C)*a^3*b^3 - B*a^2*b^4)*cos(d*x
 + c)^2 + 3*((2*A + C)*a^4*b^2 - 4*B*a^3*b^3 + (3*A + 4*C)*a^2*b^4 - B*a*b^
5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) + (C*a^6*b + 2*B*a^5*b^2 - 11*(A + C)*a^4*
b^3 + 11*B*a^3*b^4 + (7*A + 4*C)*a^2*b^5 - 13*B*a*b^6 + 2*(2*A + 3*C)*b^7 +
 (6*B*a^7 - (18*A + 13*C)*a^6*b + 4*B*a^5*b^2 + (23*A + 11*C)*a^4*b^3 - 11*
B*a^3*b^4 - (7*A - 2*C)*a^2*b^5 + B*a*b^6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(C*
a^7 + 2*B*a^6*b - (9*A + 10*C)*a^5*b^2 + 7*B*a^4*b^3 + (8*A + 7*C)*a^3*b^4
- 10*B*a^2*b^5 + (A + 2*C)*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^1
1 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10
*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9
*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^
3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x
)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c
 + d*x))**4, x)
```

Giac [B] time = 1.51201, size = 1307, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] -1/3*(3*(2*A*a^3 + C*a^3 - 4*B*a^2*b + 3*A*a*b^2 + 4*C*a*b^2 - B*b^3)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c
) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*sqrt(-a^2 + b^2)) + (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^5*tan(
1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b*tan(1/2*
d*x + 1/2*c)^5 - 12*C*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d
*x + 1/2*c)^5 + 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*C*a^3*b^2*tan(1/2*
d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^2*b^3*tan(1/2*
d*x + 1/2*c)^5 - 12*C*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*
x + 1/2*c)^5 + 12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*b^4*tan(1/2*d*x +
1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^5*tan(1/2*d*x + 1/2*c)^5
- 6*C*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 36*A*a
^4*b*tan(1/2*d*x + 1/2*c)^3 + 28*C*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*B*a^3*
b^2*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 16*C*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 + 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^5*t
an(1/2*d*x + 1/2*c)^3 - 12*C*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d
*x + 1/2*c) + 3*C*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/2*c
) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 12*C*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A
*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*C*a^
3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*B*a^2*b^
3*tan(1/2*d*x + 1/2*c) - 12*C*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(
1/2*d*x + 1/2*c) + 12*B*a*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a*b^4*tan(1/2*d*x
+ 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x + 1/2*c) - 6*
C*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.927 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=336

$$-\frac{\left(-a^4b^3(8A-C)+7a^2Ab^5+4a^6b(2A+C)-3a^5b^2B-2a^7B-2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\tan(c+dx)\left(-13a^4b^2\cdots\right.}{\cdots}$$

```
[Out] (A*x)/a^4 - ((7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B - a^4*b^3*(8*A
 - C) + 4*a^6*b*(2*A + C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c +
d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((3*A*b^4 + 5*a^3*b*B -
2*a^4*C - a^2*b^2*(8*A + 3*C))*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*
Sec[c + d*x])^2) - ((17*a^2*A*b^4 - 6*A*b^6 + 11*a^5*b*B + 4*a^3*b^3*B - 2*
a^6*C - 13*a^4*b^2*(2*A + C))*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*S
ec[c + d*x]))
```

Rubi [A] time = 2.13695, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4060, 3919, 3831, 2659, 208}

$$-\frac{\left(-a^4b^3(8A-C)+7a^2Ab^5+4a^6b(2A+C)-3a^5b^2B-2a^7B-2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\tan(c+dx)\left(-13a^4b^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (A*x)/a^4 - ((7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B - a^4*b^3*(8*A
 - C) + 4*a^6*b*(2*A + C))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + ((A*b^2 - a*(b*B - a*C))*Tan[c +
d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((3*A*b^4 + 5*a^3*b*B -
2*a^4*C - a^2*b^2*(8*A + 3*C))*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*
Sec[c + d*x])^2) - ((17*a^2*A*b^4 - 6*A*b^6 + 11*a^5*b*B + 4*a^3*b^3*B - 2*
a^6*C - 13*a^4*b^2*(2*A + C))*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*S
ec[c + d*x]))
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3A\left(a^2-b^2\right)+3a(Ab-aB+bC)\sec(c+dx)-2(A}{(a+b\sec(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C-a^2b^2(8A+\right.}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C-a^2b^2(8A+\right.}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c}$$

$$= \frac{Ax}{a^4} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C-a^2b^2\right.}{6a^2\left(a^2-b^2\right)^2d(a+b}$$

$$= \frac{Ax}{a^4} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C-a^2b^2\right.}{6a^2\left(a^2-b^2\right)^2d(a+b}$$

$$= \frac{Ax}{a^4} + \frac{\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(3Ab^4+5a^3bB-2a^4C-a^2b^2\right.}{6a^2\left(a^2-b^2\right)^2d(a+b}$$

$$= \frac{Ax}{a^4} - \frac{\left(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B+4a^6bC+a\right.}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [C] time = 7.90422, size = 1230, normalized size = 3.66

$$\frac{2Ax\sec^2(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)(b+a\cos(c+dx))^4}{a^4(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b\sec(c+dx))^4} + \frac{\left(2Ba^7-8Aba^6-4bCa^6+3b^2Ba^5+8A\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*A*x*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2))/(a^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Se
c[c + d*x])^4) + ((-8*a^6*A*b + 8*a^4*A*b^3 - 7*a^2*A*b^5 + 2*A*b^7 + 2*a^7
*B + 3*a^5*b^2*B - 4*a^6*b*C - a^4*b^3*C)*(b + a*Cos[c + d*x])^4*Sec[c + d*
x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((2*I)*ArcTan[Sec[(d*x)/2]*(C
os[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2
- b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*
x)/2])]*Cos[c])/(a^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]]) + (2*Ar
cTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (
I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/
2] + I*a*Sin[c + (d*x)/2])]*Sin[c])/(a^4*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] -
I*Sin[2*c]])))/((-a^2 + b^2)^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*(a + b*Sec[c + d*x])^4) - (2*(b + a*Cos[c + d*x])*Sec[c]*Sec[c + d*x]
^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(A*b^5*Sin[c] - a*b^4*B*Sin[c] +
 a^2*b^3*C*Sin[c] - a*A*b^4*Sin[d*x] + a^2*b^3*B*Sin[d*x] - a^3*b^2*C*Sin[d
*x]))/(3*a^4*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(a + b*Sec[c + d*x])^4) + ((b + a*Cos[c + d*x])^2*Sec[c]*Sec[c + d*x]^2*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(14*a^2*A*b^4*Sin[c] - 9*A*b^6*Sin[c
] - 11*a^3*b^3*B*Sin[c] + 6*a*b^5*B*Sin[c] + 8*a^4*b^2*C*Sin[c] - 3*a^2*b^4
*C*Sin[c] - 12*a^3*A*b^3*Sin[d*x] + 7*a*A*b^5*Sin[d*x] + 9*a^4*b^2*B*Sin[d*
x] - 4*a^2*b^4*B*Sin[d*x] - 6*a^5*b*C*Sin[d*x] + a^3*b^3*C*Sin[d*x]))/(3*a^
4*(a^2 - b^2)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*
Sec[c + d*x])^4) + ((b + a*Cos[c + d*x])^3*Sec[c]*Sec[c + d*x]^2*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-48*a^4*A*b^3*Sin[c] + 51*a^2*A*b^5*Sin[c] -
 18*A*b^7*Sin[c] + 27*a^5*b^2*B*Sin[c] - 18*a^3*b^4*B*Sin[c] + 6*a*b^6*B*Si
n[c] - 12*a^6*b*C*Sin[c] - 3*a^4*b^3*C*Sin[c] + 36*a^5*A*b^2*Sin[d*x] - 32*
a^3*A*b^4*Sin[d*x] + 11*a*A*b^6*Sin[d*x] - 18*a^6*b*B*Sin[d*x] + 5*a^4*b^3*
B*Sin[d*x] - 2*a^2*b^5*B*Sin[d*x] + 6*a^7*C*Sin[d*x] + 10*a^5*b^2*C*Sin[d*x
] - a^3*b^4*C*Sin[d*x]))/(3*a^4*(a^2 - b^2)^3*d*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.125, size = 3223, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 3/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*a+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5*B-12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-12/d*b/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^2/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-12/d*b^2
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+24/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)
^3*A-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a^2-2/d/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*
x+1/2*c)*C-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C+6/d/a/(tan(1/2*d*x+1/2
```

```
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1
/2*d*x+1/2*c)^5*A*b^4+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5-2/d/a^3
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^6+4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*A*b^6+28/3/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C-6/d*a/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)*b^2*C-44/3/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-6/d*
a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2*C-3/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*a*B-1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5+6/d/a/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*A*b^4-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6-2/d*b/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b
^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+2/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*a^2+3/
d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a*B+4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*A+2/d*A/a^4*arctan(tan(1/2*d*x+1/2*c))-1/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*C-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2+4/d/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*a^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3*C+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))*B*a^3-7/d/a^2*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^
(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A+2/d/a^4*b^7/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))*A-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3+1/d
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*
b^3+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*b^3-4/3/d/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3*B*b^3-4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78504, size = 4535, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
fricas")
```

```
[Out] [1/12*(12*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*
x*cos(d*x + c)^3 + 36*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 +
 A*a^2*b^9)*d*x*cos(d*x + c)^2 + 36*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6
- 4*A*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 12*(A*a^8*b^3 - 4*A*a^6*b^5 +
6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x - 3*(2*B*a^7*b^3 - 4*(2*A + C)*a^6*
b^4 + 3*B*a^5*b^5 + (8*A - C)*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10
- 4*(2*A + C)*a^9*b + 3*B*a^8*b^2 + (8*A - C)*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a
^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^9*b - 4*(2*A + C)*a^8*b^2 + 3*B*a^7*b^3 +
 (8*A - C)*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8
*b^2 - 4*(2*A + C)*a^7*b^3 + 3*B*a^6*b^4 + (8*A - C)*a^5*b^5 - 7*A*a^3*b^7
+ 2*A*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(2*C*
a^9*b^2 - 11*B*a^8*b^3 + (26*A + 11*C)*a^7*b^4 + 7*B*a^6*b^5 - (43*A + 13*C
)*a^5*b^6 + 4*B*a^4*b^7 + 23*A*a^3*b^8 - 6*A*a*b^10 + (6*C*a^11 - 18*B*a^10
*b + 4*(9*A + C)*a^9*b^2 + 23*B*a^8*b^3 - (68*A + 11*C)*a^7*b^4 - 7*B*a^6*b
^5 + (43*A + C)*a^5*b^6 + 2*B*a^4*b^7 - 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(2
*C*a^10*b - 9*B*a^9*b^2 + (20*A + 7*C)*a^8*b^3 + 8*B*a^7*b^4 - 5*(7*A + 2*C
)*a^6*b^5 + B*a^5*b^6 + (20*A + C)*a^4*b^7 - 5*A*a^2*b^9)*cos(d*x + c))*sin
(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*
x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos
(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*
d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)
*d), 1/6*(6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*
d*x*cos(d*x + c)^3 + 18*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7
 + A*a^2*b^9)*d*x*cos(d*x + c)^2 + 18*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^
6 - 4*A*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 6*(A*a^8*b^3 - 4*A*a^6*b^5 +
 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x + 3*(2*B*a^7*b^3 - 4*(2*A + C)*a^6
*b^4 + 3*B*a^5*b^5 + (8*A - C)*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10
 - 4*(2*A + C)*a^9*b + 3*B*a^8*b^2 + (8*A - C)*a^7*b^3 - 7*A*a^5*b^5 + 2*A*
a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^9*b - 4*(2*A + C)*a^8*b^2 + 3*B*a^7*b^3
+ (8*A - C)*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(2*B*a^
8*b^2 - 4*(2*A + C)*a^7*b^3 + 3*B*a^6*b^4 + (8*A - C)*a^5*b^5 - 7*A*a^3*b^7
 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
s(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*C*a^9*b^2 - 11*B*a^8*b^3 +
 (26*A + 11*C)*a^7*b^4 + 7*B*a^6*b^5 - (43*A + 13*C)*a^5*b^6 + 4*B*a^4*b^7
+ 23*A*a^3*b^8 - 6*A*a*b^10 + (6*C*a^11 - 18*B*a^10*b + 4*(9*A + C)*a^9*b^2
 + 23*B*a^8*b^3 - (68*A + 11*C)*a^7*b^4 - 7*B*a^6*b^5 + (43*A + C)*a^5*b^6
+ 2*B*a^4*b^7 - 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(2*C*a^10*b - 9*B*a^9*b^2
+ (20*A + 7*C)*a^8*b^3 + 8*B*a^7*b^4 - 5*(7*A + 2*C)*a^6*b^5 + B*a^5*b^6 +
(20*A + C)*a^4*b^7 - 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^
13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4
*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^
2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b
^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**4,
x)
```

Giac [B] time = 1.44537, size = 1493, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x, algorithm="
giac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b - 4*C*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - C*a^4
*b^3 - 7*A*a^2*b^5 + 2*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
+ b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) + 3*(d
*x + c)*A/a^4 - (6*C*a^8*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^7*b*tan(1/2*d*x +
1/2*c)^5 - 6*C*a^7*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^6*b^2*tan(1/2*d*x + 1/
2*c)^5 + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^6*b^2*tan(1/2*d*x + 1
/2*c)^5 - 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*b^3*tan(1/2*d*x + 1
/2*c)^5 - 27*C*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^4*b^4*tan(1/2*d*x + 1
/2*c)^5 + 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 12*C*a^4*b^4*tan(1/2*d*x + 1
/2*c)^5 + 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^5*tan(1/2*d*x + 1
/2*c)^5 + 3*C*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^6*tan(1/2*d*x + 1/
2*c)^5 - 15*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^8*tan(1/2*d*x + 1/2*c)^5
 - 12*C*a^8*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^7*b*tan(1/2*d*x + 1/2*c)^3 - 72
*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*C*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 3
2*B*a^5*b^3*tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 +
 28*C*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 -
 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x + 1/2*c)^3 + 6*
C*a^8*tan(1/2*d*x + 1/2*c) - 18*B*a^7*b*tan(1/2*d*x + 1/2*c) + 6*C*a^7*b*ta
n(1/2*d*x + 1/2*c) + 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^6*b^2*tan(1
/2*d*x + 1/2*c) + 12*C*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^5*b^3*tan(1/2*
d*x + 1/2*c) - 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 27*C*a^5*b^3*tan(1/2*d*x
+ 1/2*c) - 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b^4*tan(1/2*d*x + 1/2
*c) + 12*C*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)
 - 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c) - 3*C*a^3*b^5*tan(1/2*d*x + 1/2*c) - 6*
A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*A*a*b^7*tan(1/2*d*x + 1/2*c) + 6*A*b^8*
tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d
*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.928 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=471

$$\frac{\sin(c+dx)\left(-a^4b^2(65A+4C)+68a^2Ab^4+a^6(6A-11C)-17a^3b^3B+26a^5bB+6ab^5B-24Ab^6\right)}{6a^4d\left(a^2-b^2\right)^3}-\frac{\left(-a^6b^2(20A+3C)\right.}{}$$

```
[Out] -(((4*A*b - a*B)*x)/a^5) - ((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 + 8*a^7*
b*B - 8*a^5*b^3*B + 7*a^3*b^5*B - 2*a*b^7*B - 2*a^8*C - a^6*b^2*(20*A + 3*C
))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*
(a + b)^(7/2)*d) + ((68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3*B +
6*a*b^5*B + a^6*(6*A - 11*C) - a^4*b^2*(65*A + 4*C))*Sin[c + d*x])/(6*a^4*(
a^2 - b^2)^3*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^3) - ((4*A*b^4 + 6*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*
b^2*(9*A + 2*C))*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^
2) - ((11*a^2*A*b^4 - 4*A*b^6 + 6*a^5*b*B - 2*a^3*b^3*B + a*b^5*B - 2*a^6*C
 - 3*a^4*b^2*(4*A + C))*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c +
 d*x]))
```

Rubi [A] time = 10.1006, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\sin(c+dx)\left(-a^4b^2(65A+4C)+68a^2Ab^4+a^6(6A-11C)-17a^3b^3B+26a^5bB+6ab^5B-24Ab^6\right)}{6a^4d\left(a^2-b^2\right)^3}-\frac{\left(-a^6b^2(20A+3C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^4,x]
```


```
[Out] -(((4*A*b - a*B)*x)/a^5) - ((35*a^4*A*b^4 - 28*a^2*A*b^6 + 8*A*b^8 + 8*a^7*
b*B - 8*a^5*b^3*B + 7*a^3*b^5*B - 2*a*b^7*B - 2*a^8*C - a^6*b^2*(20*A + 3*C
))*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*
(a + b)^(7/2)*d) + ((68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3*B +
6*a*b^5*B + a^6*(6*A - 11*C) - a^4*b^2*(65*A + 4*C))*Sin[c + d*x])/(6*a^4*(
a^2 - b^2)^3*d) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^3) - ((4*A*b^4 + 6*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*
b^2*(9*A + 2*C))*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^
2) - ((11*a^2*A*b^4 - 4*A*b^6 + 6*a^5*b*B - 2*a^3*b^3*B + a*b^5*B - 2*a^6*C
 - 3*a^4*b^2*(4*A + C))*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c +
 d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(4Ab^2-abB-a^2(3A-C\right.}{}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(4Ab^4+6a^3bB-ab^3B-3\right.}{6a^2\left(a^2-b^2\right)^2}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(4Ab^4+6a^3bB-ab^3B-3\right.}{6a^2\left(a^2-b^2\right)^2}$$

$$= \frac{\left(68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B+a^6(6A-11\right.}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6a\right.}{6a^4\left(\right.}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6a\right.}{6a^4\left(\right.}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6a\right.}{6a^4\left(\right.}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{\left(20a^6Ab^2-35a^4Ab^4+28a^2Ab^6-8Ab^8-8a\right.}{}$$

Mathematica [C] time = 8.21854, size = 1367, normalized size = 2.9

$$-\frac{2(4Ab-aB)x\sec^2(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)(b+a\cos(c+dx))^4}{a^5(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b\sec(c+dx))^4} + \frac{\left(-2Ca^8+8bBa^7-20Ab^2a^6-3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^4,x]
```

```
[Out] (-2*(4*A*b - a*B)*x*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2))/(a^5*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*(a + b*Sec[c + d*x])^4) + ((-20*a^6*A*b^2 + 35*a^4*A*b^4 - 28*a^2*A*b^6
 + 8*A*b^8 + 8*a^7*b*B - 8*a^5*b^3*B + 7*a^3*b^5*B - 2*a*b^7*B - 2*a^8*C -
3*a^6*b^2*C)*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*(((-2*I)*ArcTan[Sec[(d*x)/2]*(Cos[c]/(Sqrt[a^2 - b^2]*Sqrt[
Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin
[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*Cos[c])/(a^5*Sqrt[a^
2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (2*ArcTan[Sec[(d*x)/2]*(Cos[c]/(S
qrt[a^2 - b^2]*Sqrt[Cos[2*c] - I*Sin[2*c]]) - (I*Sin[c])/(Sqrt[a^2 - b^2]*S
qrt[Cos[2*c] - I*Sin[2*c]]))*((-I)*b*Sin[(d*x)/2] + I*a*Sin[c + (d*x)/2])]*
Sin[c])/(a^5*Sqrt[a^2 - b^2]*d*Sqrt[Cos[2*c] - I*Sin[2*c]])))/((-a^2 + b^2)
^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4
) + (2*(b + a*Cos[c + d*x])*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*S
```

```
ec[c + d*x]^2)*(A*b^6*Sin[c] - a*b^5*B*Sin[c] + a^2*b^4*C*Sin[c] - a*A*b^5*
Sin[d*x] + a^2*b^4*B*Sin[d*x] - a^3*b^3*C*Sin[d*x]))/(3*a^5*(a^2 - b^2)*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) +
((b + a*Cos[c + d*x])^2*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*(-17*a^2*A*b^5*Sin[c] + 12*A*b^7*Sin[c] + 14*a^3*b^4*B*Sin[c] -
9*a*b^6*B*Sin[c] - 11*a^4*b^3*C*Sin[c] + 6*a^2*b^5*C*Sin[c] + 15*a^3*A*b^4*
Sin[d*x] - 10*a*A*b^6*Sin[d*x] - 12*a^4*b^3*B*Sin[d*x] + 7*a^2*b^5*B*Sin[d*
x] + 9*a^5*b^2*C*Sin[d*x] - 4*a^3*b^4*C*Sin[d*x]))/(3*a^5*(a^2 - b^2)^2*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) +
((b + a*Cos[c + d*x])^3*Sec[c]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*(75*a^4*A*b^4*Sin[c] - 96*a^2*A*b^6*Sin[c] + 36*A*b^8*Sin[c] - 4
8*a^5*b^3*B*Sin[c] + 51*a^3*b^5*B*Sin[c] - 18*a*b^7*B*Sin[c] + 27*a^6*b^2*C
*Sin[c] - 18*a^4*b^4*C*Sin[c] + 6*a^2*b^6*C*Sin[c] - 60*a^5*A*b^3*Sin[d*x]
+ 71*a^3*A*b^5*Sin[d*x] - 26*a*A*b^7*Sin[d*x] + 36*a^6*b^2*B*Sin[d*x] - 32*
a^4*b^4*B*Sin[d*x] + 11*a^2*b^6*B*Sin[d*x] - 18*a^7*b*C*Sin[d*x] + 5*a^5*b^
3*C*Sin[d*x] - 2*a^3*b^5*C*Sin[d*x]))/(3*a^5*(a^2 - b^2)^3*d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^4) + (2*A*(b + a*C
os[c + d*x])^4*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Tan[c +
 d*x])/(a^4*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[
c + d*x])^4)
```

Maple [B] time = 0.159, size = 3707, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -8/d/a^5*A*arctan(tan(1/2*d*x+1/2*c))*b+5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5*A*b^4-18/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-
b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5-2/d/a^3/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*A*b^6-3/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2*C+3/d*a/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2*C+24/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*a*B-12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*a*B-18/d/a^2/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*A*b^5-5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^4+2/d/a^3/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6-12/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C*a^
2+6/d*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*a^2+6/d*b/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*C*a^2-12/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*a*B+6/d/a^4/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)*A+6/d/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+20
/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d/a^4*A*tan(1/2*d*x+1/2*c)/(1+tan(
```

```
1/2*d*x+1/2*c)^2)-40/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+8/d*b^3/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*B+2/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^3+2/d/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b
^3)*tan(1/2*d*x+1/2*c)^5*C*b^3+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b^3-4/d/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b^3+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*b
^3+2/d/a^4*B*arctan(tan(1/2*d*x+1/2*c))+6/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*B+1/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-44/3/d/a/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a^2-2*a*b+b^2)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*B+4/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3*b^6/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+11
6/3/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/a^2/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*B-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-12/d/a^4/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^7/(a^2-2*a*b+b^2)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-
2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-4/3/d/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*
d*x+1/2*c)^3*C-8/d/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^8-35/d/a/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))*A*b^4+28/d/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*b^6-7/d/a^
2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))*B*b^5+2/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B
*b^7+3/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))*B*a^2+20/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A*a+20/d*b^3/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x,
algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.40032, size = 6179, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x,
algorithm="fricas")
```

```
[Out] [1/12*(12*(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4
- 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x*
cos(d*x + c)^3 + 36*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 +
 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*
a^2*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 +
 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B
*a^2*b^10 - 4*A*a*b^11)*d*x*cos(d*x + c) + 12*(B*a^9*b^3 - 4*A*a^8*b^4 - 4*
B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*
a^2*b^10 + B*a*b^11 - 4*A*b^12)*d*x + 3*(2*C*a^8*b^3 - 8*B*a^7*b^4 + (20*A
+ 3*C)*a^6*b^5 + 8*B*a^5*b^6 - 35*A*a^4*b^7 - 7*B*a^3*b^8 + 28*A*a^2*b^9 +
2*B*a*b^10 - 8*A*b^11 + (2*C*a^11 - 8*B*a^10*b + (20*A + 3*C)*a^9*b^2 + 8*B
*a^8*b^3 - 35*A*a^7*b^4 - 7*B*a^6*b^5 + 28*A*a^5*b^6 + 2*B*a^4*b^7 - 8*A*a^
3*b^8)*cos(d*x + c)^3 + 3*(2*C*a^10*b - 8*B*a^9*b^2 + (20*A + 3*C)*a^8*b^3
+ 8*B*a^7*b^4 - 35*A*a^6*b^5 - 7*B*a^5*b^6 + 28*A*a^4*b^7 + 2*B*a^3*b^8 - 8
*A*a^2*b^9)*cos(d*x + c)^2 + 3*(2*C*a^9*b^2 - 8*B*a^8*b^3 + (20*A + 3*C)*a^
7*b^4 + 8*B*a^6*b^5 - 35*A*a^5*b^6 - 7*B*a^4*b^7 + 28*A*a^3*b^8 + 2*B*a^2*b
^9 - 8*A*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a
^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
 + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(
(6*A - 11*C)*a^9*b^3 + 26*B*a^8*b^4 - (71*A - 7*C)*a^7*b^5 - 43*B*a^6*b^6 +
 (133*A + 4*C)*a^5*b^7 + 23*B*a^4*b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*
a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2 + 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*
cos(d*x + c)^3 + (18*(A - C)*a^11*b + 36*B*a^10*b^2 - (132*A - 23*C)*a^9*b^
3 - 68*B*a^8*b^4 + (239*A - 7*C)*a^7*b^5 + 43*B*a^6*b^6 - (169*A - 2*C)*a^5
*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(3*(2*A - 3*C)*a^10*
b^2 + 20*B*a^9*b^3 - (59*A - 8*C)*a^8*b^4 - 35*B*a^7*b^5 + (110*A + C)*a^6*
b^6 + 20*B*a^5*b^7 - 77*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x +
c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*
d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^
9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a
^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 +
a^5*b^11)*d), 1/6*(6*(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6
*B*a^8*b^4 - 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^
3*b^9)*d*x*cos(d*x + c)^3 + 18*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*
A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3
*b^9 - 4*A*a^2*b^10)*d*x*cos(d*x + c)^2 + 18*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*
B*a^8*b^4 + 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*
a^3*b^9 + B*a^2*b^10 - 4*A*a*b^11)*d*x*cos(d*x + c) + 6*(B*a^9*b^3 - 4*A*a^
8*b^4 - 4*B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b
^9 + 16*A*a^2*b^10 + B*a*b^11 - 4*A*b^12)*d*x + 3*(2*C*a^8*b^3 - 8*B*a^7*b^
4 + (20*A + 3*C)*a^6*b^5 + 8*B*a^5*b^6 - 35*A*a^4*b^7 - 7*B*a^3*b^8 + 28*A*
a^2*b^9 + 2*B*a*b^10 - 8*A*b^11 + (2*C*a^11 - 8*B*a^10*b + (20*A + 3*C)*a^9
*b^2 + 8*B*a^8*b^3 - 35*A*a^7*b^4 - 7*B*a^6*b^5 + 28*A*a^5*b^6 + 2*B*a^4*b^
7 - 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(2*C*a^10*b - 8*B*a^9*b^2 + (20*A + 3*C
)*a^8*b^3 + 8*B*a^7*b^4 - 35*A*a^6*b^5 - 7*B*a^5*b^6 + 28*A*a^4*b^7 + 2*B*a
^3*b^8 - 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(2*C*a^9*b^2 - 8*B*a^8*b^3 + (20*A
 + 3*C)*a^7*b^4 + 8*B*a^6*b^5 - 35*A*a^5*b^6 - 7*B*a^4*b^7 + 28*A*a^3*b^8 +
 2*B*a^2*b^9 - 8*A*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + ((6*A - 11*C)*a^
9*b^3 + 26*B*a^8*b^4 - (71*A - 7*C)*a^7*b^5 - 43*B*a^6*b^6 + (133*A + 4*C)*
```

```
a^5*b^7 + 23*B*a^4*b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*a*b^11 + 6*(A*a
^12 - 4*A*a^10*b^2 + 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*cos(d*x + c)^3
+ (18*(A - C)*a^11*b + 36*B*a^10*b^2 - (132*A - 23*C)*a^9*b^3 - 68*B*a^8*b^
4 + (239*A - 7*C)*a^7*b^5 + 43*B*a^6*b^6 - (169*A - 2*C)*a^5*b^7 - 11*B*a^4
*b^8 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(3*(2*A - 3*C)*a^10*b^2 + 20*B*a^9*
b^3 - (59*A - 8*C)*a^8*b^4 - 35*B*a^7*b^5 + (110*A + C)*a^6*b^6 + 20*B*a^5*
b^7 - 77*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x + c
))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^
3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x +
c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(
d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**4,x
)
```

```
[Out] Timed out
```

Giac [B] time = 1.49459, size = 1654, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x,
algorithm="giac")
```

```
[Out] 1/3*(3*(2*C*a^8 - 8*B*a^7*b + 20*A*a^6*b^2 + 3*C*a^6*b^2 + 8*B*a^5*b^3 - 35
*A*a^4*b^4 - 7*B*a^3*b^5 + 28*A*a^2*b^6 + 2*B*a*b^7 - 8*A*b^8)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b
*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 -
a^5*b^6)*sqrt(-a^2 + b^2)) + (18*C*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^7*
b^2*tan(1/2*d*x + 1/2*c)^5 - 27*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^6
*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^6
*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^
5*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^
4*b^5*tan(1/2*d*x + 1/2*c)^5 - 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^
4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 6*B*a
^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 15*B*
a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*A*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*
b^8*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*C*a^8*b*t
an(1/2*d*x + 1/2*c)^3 + 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^6*b^3
*tan(1/2*d*x + 1/2*c)^3 + 32*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 - 116*B*a^5*b
^4*tan(1/2*d*x + 1/2*c)^3 + 236*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*C*a^4*
b^5*tan(1/2*d*x + 1/2*c)^3 + 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*A*a^
2*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^9
*tan(1/2*d*x + 1/2*c)^3 + 18*C*a^8*b*tan(1/2*d*x + 1/2*c) - 36*B*a^7*b^2*ta
n(1/2*d*x + 1/2*c) + 27*C*a^7*b^2*tan(1/2*d*x + 1/2*c) + 60*A*a^6*b^3*tan(1
/2*d*x + 1/2*c) - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) + 6*C*a^6*b^3*tan(1/2*d
```

```
*x + 1/2*c) + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) + 6*B*a^5*b^4*tan(1/2*d*x
+ 1/2*c) + 3*C*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*A*a^4*b^5*tan(1/2*d*x + 1/
2*c) + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 6*C*a^4*b^5*tan(1/2*d*x + 1/2*c)
 - 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c) -
24*A*a^2*b^7*tan(1/2*d*x + 1/2*c) - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*
A*a*b^8*tan(1/2*d*x + 1/2*c) - 6*B*a*b^8*tan(1/2*d*x + 1/2*c) + 18*A*b^9*ta
n(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(B*a - 4*A*b)*(d*x
+ c)/a^5 + 6*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^4))/d
```

3.929 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=648

$$\frac{\sin(c+dx)\left(a^4b^3(146A-17C)-a^2b^5(167A-6C)-a^6(24Ab-26bC)-65a^5b^2B+68a^3b^4B+6a^7B-24ab^6B+60Ab^7\right)}{6a^5d\left(a^2-b^2\right)^3}$$

```
[Out] ((20*A*b^2 - 8*a*b*B + a^2*(A + 2*C))*x)/(2*a^6) + (b*(20*A*b^8 + 20*a^7*b*
B - 35*a^5*b^3*B + 28*a^3*b^5*B - 8*a*b^7*B - a^2*b^6*(69*A - 2*C) - 8*a^6*
b^2*(5*A - C) + 7*a^4*b^4*(12*A - C) - 8*a^8*C)*ArcTanh[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(a^6*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^3*d) + (
(60*A*b^7 + 6*a^7*B - 65*a^5*b^2*B + 68*a^3*b^4*B - 24*a*b^6*B + a^4*b^3*(1
46*A - 17*C) - a^2*b^5*(167*A - 6*C) - a^6*(24*A*b - 26*b*C))*Sin[c + d*x])
/(6*a^5*(a^2 - b^2)^3*d) - ((10*A*b^6 - 12*a^5*b*B + 11*a^3*b^3*B - 4*a*b^5
*B - a^6*(A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(27*A - C))*Cos[c + d*x
]*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c +
d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*A*b^4 +
 7*a^3*b*B - 2*a*b^3*B - 4*a^4*C - a^2*b^2*(10*A + C))*Cos[c + d*x]*Sin[c +
 d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((20*A*b^6 - 27*a^5
*b*B + 20*a^3*b^3*B - 8*a*b^5*B - a^2*b^4*(53*A - 2*C) + 12*a^6*C + a^4*b^2
*(48*A + C))*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c
 + d*x]))
```

Rubi [A] time = 12.5141, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\sin(c+dx)\left(a^4b^3(146A-17C)-a^2b^5(167A-6C)-a^6(24Ab-26bC)-65a^5b^2B+68a^3b^4B+6a^7B-24ab^6B+60Ab^7\right)}{6a^5d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^4,x]
```


```
[Out] ((20*A*b^2 - 8*a*b*B + a^2*(A + 2*C))*x)/(2*a^6) + (b*(20*A*b^8 + 20*a^7*b*
B - 35*a^5*b^3*B + 28*a^3*b^5*B - 8*a*b^7*B - a^2*b^6*(69*A - 2*C) - 8*a^6*
b^2*(5*A - C) + 7*a^4*b^4*(12*A - C) - 8*a^8*C)*ArcTanh[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(a^6*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^3*d) + (
(60*A*b^7 + 6*a^7*B - 65*a^5*b^2*B + 68*a^3*b^4*B - 24*a*b^6*B + a^4*b^3*(1
46*A - 17*C) - a^2*b^5*(167*A - 6*C) - a^6*(24*A*b - 26*b*C))*Sin[c + d*x])
/(6*a^5*(a^2 - b^2)^3*d) - ((10*A*b^6 - 12*a^5*b*B + 11*a^3*b^3*B - 4*a*b^5
*B - a^6*(A - 6*C) + a^4*b^2*(23*A - 2*C) - a^2*b^4*(27*A - C))*Cos[c + d*x
]*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c +
d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*A*b^4 +
 7*a^3*b*B - 2*a*b^3*B - 4*a^4*C - a^2*b^2*(10*A + C))*Cos[c + d*x]*Sin[c +
 d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + ((20*A*b^6 - 27*a^5
*b*B + 20*a^3*b^3*B - 8*a*b^5*B - a^2*b^4*(53*A - 2*C) + 12*a^6*C + a^4*b^2
*(48*A + C))*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c
 + d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^4}dx = \frac{\left(Ab^2-a(bB-aC)\right)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \int \frac{\cos^2(c+dx)\left(5Ab\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5Ab^4+7a^3bB\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\left(5Ab^4+7a^3bB\right.}{}$$

$$= -\frac{\left(10Ab^6-12a^5bB+11a^3b^3B-4ab^5B-a^6(A-6C)+a^4b^2(2\right.}{2a^4\left(a^2-b^2\right.}$$

$$= \frac{\left(60Ab^7+6a^7B-65a^5b^2B+68a^3b^4B-24ab^6B+a^4b^3(146A\right.}{6a^5\left(a^2-\right.}$$

$$= \frac{\left(20Ab^2-8abB+a^2(A+2C)\right)x}{2a^6} + \frac{\left(60Ab^7+6a^7B-65a^5b^2B\right.}{}$$

$$= \frac{\left(20Ab^2-8abB+a^2(A+2C)\right)x}{2a^6} + \frac{\left(60Ab^7+6a^7B-65a^5b^2B\right.}{}$$

$$= \frac{\left(20Ab^2-8abB+a^2(A+2C)\right)x}{2a^6} + \frac{\left(60Ab^7+6a^7B-65a^5b^2B\right.}{}$$

$$= \frac{\left(20Ab^2-8abB+a^2(A+2C)\right)x}{2a^6} + \frac{b\left(20Ab^8+20a^7bB-35a\right.}{}$$

<u>Mathematica</u> [C] time = 6.99992, size = 658, normalized size = 1.02

$$\frac{(c+dx)\left(a^2A+2a^2C-8abB+20Ab^2\right)}{2a^6d} + \frac{a^2b^4C\sin(c+dx)-ab^5B\sin(c+dx)+Ab^6\sin(c+dx)}{3a^5d\left(a^2-b^2\right)(a\cos(c+dx)+b)^3} + \frac{-18a^2Ab^5\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^4,x]
```

```
[Out] ((a^2*A + 20*A*b^2 - 8*a*b*B + 2*a^2*C)*(c + d*x))/(2*a^6*d) + (b*(-40*a^6*
A*b^2 + 84*a^4*A*b^4 - 69*a^2*A*b^6 + 20*A*b^8 + 20*a^7*b*B - 35*a^5*b^3*B
+ 28*a^3*b^5*B - 8*a*b^7*B - 8*a^8*C + 8*a^6*b^2*C - 7*a^4*b^4*C + 2*a^2*b^
6*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 -
b^2]*(-a^2 + b^2)^3*d) + ((4*A*b - a*B)*(((-I/2)*Cos[c + d*x])/a^5 - Sin[c
+ d*x]/(2*a^5)))/d + ((4*A*b - a*B)*(((I/2)*Cos[c + d*x])/a^5 - Sin[c + d*x
]/(2*a^5)))/d + (A*b^6*Sin[c + d*x] - a*b^5*B*Sin[c + d*x] + a^2*b^4*C*Sin[
c + d*x])/(3*a^5*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^3) + (-18*a^2*A*b^5*Sin
[c + d*x] + 13*A*b^7*Sin[c + d*x] + 15*a^3*b^4*B*Sin[c + d*x] - 10*a*b^6*B*
Sin[c + d*x] - 12*a^4*b^3*C*Sin[c + d*x] + 7*a^2*b^5*C*Sin[c + d*x])/(6*a^5
*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (90*a^4*A*b^4*Sin[c + d*x] - 122
```

```
*a^2*A*b^6*Sin[c + d*x] + 47*A*b^8*Sin[c + d*x] - 60*a^5*b^3*B*Sin[c + d*x]
 + 71*a^3*b^5*B*Sin[c + d*x] - 26*a*b^7*B*Sin[c + d*x] + 36*a^6*b^2*C*Sin[c
 + d*x] - 32*a^4*b^4*C*Sin[c + d*x] + 11*a^2*b^6*C*Sin[c + d*x]])/(6*a^5*(a^
2 - b^2)^3*d*(b + a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(4*a^4*d)
```

Maple [B] time = 0.162, size = 4523, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+20/d*b^2/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*B*a-30/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^4-6/d/a
^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b
+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^5+34/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*A*b^6-212/3/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^6+24/d*a/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b
+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*C-12/d*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2*C
+60/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^
2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^4-12/d*a/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*b^2*C+6/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^5-30/d/a/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)*A*b^4+34/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^6+2/
d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*A-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-3/d/a^4/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^7/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)*A+3/d/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3*b^7/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-8/d/a^5
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A*b-8/d/a^5/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A*b-44/3/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3*C+4/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(
a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+6/d*b^4/a/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*C+1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-12/d*b
^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*C+24/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*b^6/a^3/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*C-1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-2/d*b^6
```

```
/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*A+1/d*A/a^4*arctan(tan(1/2*d*x+1/2*c))+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))*C-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C*a^2-40/d*b^3/(a^6-3*a^4*b^2+
3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))*A-12/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*b^7/a^4/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*B+116/3/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+28/d*b^6/a^3/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2))*B-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-35/d*b^4/a
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*
d*x+1/2*c)*B-8/d/a^5*arctan(tan(1/2*d*x+1/2*c))*B*b+20/d/a^6*arctan(tan(1/2
*d*x+1/2*c))*A*b^2-1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*
A+1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A+84/d/a^2*b^5/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))*A-69/d/a^4*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*A-4/d/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C*b^3+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C*b
^3+20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*b^3+20/d/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1
/2*c)*B*b^3-40/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2
-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*b^3+5/d/a/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*B-18/d/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-18/d
/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^5/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x
+1/2*c)*B-5/d/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^4/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d/a^3/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*B+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.80355, size = 7804, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="fricas")
```

```
[Out] [1/12*(6*((A + 2*C)*a^13 - 8*B*a^12*b + 8*(2*A - C)*a^11*b^2 + 32*B*a^10*b^
3 - 2*(37*A - 6*C)*a^9*b^4 - 48*B*a^8*b^5 + 4*(29*A - 2*C)*a^7*b^6 + 32*B*a
^6*b^7 - (79*A - 2*C)*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^10)*d*x*cos(d*x +
c)^3 + 18*((A + 2*C)*a^12*b - 8*B*a^11*b^2 + 8*(2*A - C)*a^10*b^3 + 32*B*a^
9*b^4 - 2*(37*A - 6*C)*a^8*b^5 - 48*B*a^7*b^6 + 4*(29*A - 2*C)*a^6*b^7 + 32
*B*a^5*b^8 - (79*A - 2*C)*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^11)*d*x*cos(d
*x + c)^2 + 18*((A + 2*C)*a^11*b^2 - 8*B*a^10*b^3 + 8*(2*A - C)*a^9*b^4 + 3
2*B*a^8*b^5 - 2*(37*A - 6*C)*a^7*b^6 - 48*B*a^6*b^7 + 4*(29*A - 2*C)*a^5*b^
8 + 32*B*a^4*b^9 - (79*A - 2*C)*a^3*b^10 - 8*B*a^2*b^11 + 20*A*a*b^12)*d*x*
cos(d*x + c) + 6*((A + 2*C)*a^10*b^3 - 8*B*a^9*b^4 + 8*(2*A - C)*a^8*b^5 +
32*B*a^7*b^6 - 2*(37*A - 6*C)*a^6*b^7 - 48*B*a^5*b^8 + 4*(29*A - 2*C)*a^4*b
^9 + 32*B*a^3*b^10 - (79*A - 2*C)*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*d*x +
3*(8*C*a^8*b^4 - 20*B*a^7*b^5 + 8*(5*A - C)*a^6*b^6 + 35*B*a^5*b^7 - 7*(12*
A - C)*a^4*b^8 - 28*B*a^3*b^9 + (69*A - 2*C)*a^2*b^10 + 8*B*a*b^11 - 20*A*b
^12 + (8*C*a^11*b - 20*B*a^10*b^2 + 8*(5*A - C)*a^9*b^3 + 35*B*a^8*b^4 - 7*
(12*A - C)*a^7*b^5 - 28*B*a^6*b^6 + (69*A - 2*C)*a^5*b^7 + 8*B*a^4*b^8 - 20
*A*a^3*b^9)*cos(d*x + c)^3 + 3*(8*C*a^10*b^2 - 20*B*a^9*b^3 + 8*(5*A - C)*a
^8*b^4 + 35*B*a^7*b^5 - 7*(12*A - C)*a^6*b^6 - 28*B*a^5*b^7 + (69*A - 2*C)*
a^4*b^8 + 8*B*a^3*b^9 - 20*A*a^2*b^10)*cos(d*x + c)^2 + 3*(8*C*a^9*b^3 - 20
*B*a^8*b^4 + 8*(5*A - C)*a^7*b^5 + 35*B*a^6*b^6 - 7*(12*A - C)*a^5*b^7 - 28
*B*a^4*b^8 + (69*A - 2*C)*a^3*b^9 + 8*B*a^2*b^10 - 20*A*a*b^11)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(6*B*a^10*b^3 - 2*(12*A - 13*
C)*a^9*b^4 - 71*B*a^8*b^5 + (170*A - 43*C)*a^7*b^6 + 133*B*a^6*b^7 - (313*A
 - 23*C)*a^5*b^8 - 92*B*a^4*b^9 + (227*A - 6*C)*a^3*b^10 + 24*B*a^2*b^11 -
60*A*a*b^12 + 3*(A*a^13 - 4*A*a^11*b^2 + 6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*
b^8)*cos(d*x + c)^4 + 3*(2*B*a^13 - 5*A*a^12*b - 8*B*a^11*b^2 + 20*A*a^10*b
^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8
 - 5*A*a^4*b^9)*cos(d*x + c)^3 + (18*B*a^12*b - 9*(7*A - 4*C)*a^11*b^2 - 13
2*B*a^10*b^3 + 2*(171*A - 34*C)*a^9*b^4 + 239*B*a^8*b^5 - (590*A - 43*C)*a^
7*b^6 - 169*B*a^6*b^7 + (421*A - 11*C)*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b
^10)*cos(d*x + c)^2 + 3*(6*B*a^11*b^2 - (23*A - 20*C)*a^10*b^3 - 59*B*a^9*b
^4 + (146*A - 35*C)*a^8*b^5 + 110*B*a^7*b^6 - (263*A - 20*C)*a^6*b^7 - 77*B
*a^5*b^8 + 5*(38*A - C)*a^4*b^9 + 20*B*a^3*b^10 - 50*A*a^2*b^11)*cos(d*x +
c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*
d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b
^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 +
a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9
+ a^6*b^11)*d), 1/6*(3*((A + 2*C)*a^13 - 8*B*a^12*b + 8*(2*A - C)*a^11*b^2
+ 32*B*a^10*b^3 - 2*(37*A - 6*C)*a^9*b^4 - 48*B*a^8*b^5 + 4*(29*A - 2*C)*a^
7*b^6 + 32*B*a^6*b^7 - (79*A - 2*C)*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^10)*
d*x*cos(d*x + c)^3 + 9*((A + 2*C)*a^12*b - 8*B*a^11*b^2 + 8*(2*A - C)*a^10*
b^3 + 32*B*a^9*b^4 - 2*(37*A - 6*C)*a^8*b^5 - 48*B*a^7*b^6 + 4*(29*A - 2*C)
*a^6*b^7 + 32*B*a^5*b^8 - (79*A - 2*C)*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^
11)*d*x*cos(d*x + c)^2 + 9*((A + 2*C)*a^11*b^2 - 8*B*a^10*b^3 + 8*(2*A - C)
*a^9*b^4 + 32*B*a^8*b^5 - 2*(37*A - 6*C)*a^7*b^6 - 48*B*a^6*b^7 + 4*(29*A -
 2*C)*a^5*b^8 + 32*B*a^4*b^9 - (79*A - 2*C)*a^3*b^10 - 8*B*a^2*b^11 + 20*A*
a*b^12)*d*x*cos(d*x + c) + 3*((A + 2*C)*a^10*b^3 - 8*B*a^9*b^4 + 8*(2*A - C
)*a^8*b^5 + 32*B*a^7*b^6 - 2*(37*A - 6*C)*a^6*b^7 - 48*B*a^5*b^8 + 4*(29*A
```

```
- 2*C)*a^4*b^9 + 32*B*a^3*b^10 - (79*A - 2*C)*a^2*b^11 - 8*B*a*b^12 + 20*A*
b^13)*d*x - 3*(8*C*a^8*b^4 - 20*B*a^7*b^5 + 8*(5*A - C)*a^6*b^6 + 35*B*a^5*
b^7 - 7*(12*A - C)*a^4*b^8 - 28*B*a^3*b^9 + (69*A - 2*C)*a^2*b^10 + 8*B*a*b
^11 - 20*A*b^12 + (8*C*a^11*b - 20*B*a^10*b^2 + 8*(5*A - C)*a^9*b^3 + 35*B*
a^8*b^4 - 7*(12*A - C)*a^7*b^5 - 28*B*a^6*b^6 + (69*A - 2*C)*a^5*b^7 + 8*B*
a^4*b^8 - 20*A*a^3*b^9)*cos(d*x + c)^3 + 3*(8*C*a^10*b^2 - 20*B*a^9*b^3 + 8
*(5*A - C)*a^8*b^4 + 35*B*a^7*b^5 - 7*(12*A - C)*a^6*b^6 - 28*B*a^5*b^7 + (
69*A - 2*C)*a^4*b^8 + 8*B*a^3*b^9 - 20*A*a^2*b^10)*cos(d*x + c)^2 + 3*(8*C*
a^9*b^3 - 20*B*a^8*b^4 + 8*(5*A - C)*a^7*b^5 + 35*B*a^6*b^6 - 7*(12*A - C)*
a^5*b^7 - 28*B*a^4*b^8 + (69*A - 2*C)*a^3*b^9 + 8*B*a^2*b^10 - 20*A*a*b^11)
*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
a)/((a^2 - b^2)*sin(d*x + c))) + (6*B*a^10*b^3 - 2*(12*A - 13*C)*a^9*b^4 -
71*B*a^8*b^5 + (170*A - 43*C)*a^7*b^6 + 133*B*a^6*b^7 - (313*A - 23*C)*a^5*
b^8 - 92*B*a^4*b^9 + (227*A - 6*C)*a^3*b^10 + 24*B*a^2*b^11 - 60*A*a*b^12 +
 3*(A*a^13 - 4*A*a^11*b^2 + 6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x
+ c)^4 + 3*(2*B*a^13 - 5*A*a^12*b - 8*B*a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9
*b^4 - 30*A*a^8*b^5 - 8*B*a^7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^
9)*cos(d*x + c)^3 + (18*B*a^12*b - 9*(7*A - 4*C)*a^11*b^2 - 132*B*a^10*b^3
+ 2*(171*A - 34*C)*a^9*b^4 + 239*B*a^8*b^5 - (590*A - 43*C)*a^7*b^6 - 169*B
*a^6*b^7 + (421*A - 11*C)*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)*cos(d*x
+ c)^2 + 3*(6*B*a^11*b^2 - (23*A - 20*C)*a^10*b^3 - 59*B*a^9*b^4 + (146*A -
 35*C)*a^8*b^5 + 110*B*a^7*b^6 - (263*A - 20*C)*a^6*b^7 - 77*B*a^5*b^8 + 5*
(38*A - C)*a^4*b^9 + 20*B*a^3*b^10 - 50*A*a^2*b^11)*cos(d*x + c))*sin(d*x +
 c))/((a^17 - 4*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c
)^3 + 3*(a^16*b - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x
 + c)^2 + 3*(a^15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*c
os(d*x + c) + (a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52875, size = 1941, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x
, algorithm="giac")
```

```
[Out] -1/6*(6*(8*C*a^8*b - 20*B*a^7*b^2 + 40*A*a^6*b^3 - 8*C*a^6*b^3 + 35*B*a^5*b
^4 - 84*A*a^4*b^5 + 7*C*a^4*b^5 - 28*B*a^3*b^6 + 69*A*a^2*b^7 - 2*C*a^2*b^7
 + 8*B*a*b^8 - 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(-a^2 + b^2)) + 2*(36*C*
```

a^8*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 60*C *a^7*b^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 105 *B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 16 2*A*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 45*C*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 - 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 6*C*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^ 5 - 15*C*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c) ^5 + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 6*C*a^2*b^8*tan(1/2*d*x + 1/2*c) ^5 - 81*A*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^10*tan(1/2*d*x + 1/2*c)^5 - 72*C*a^8*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 - 180*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 116*C*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 - 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 + 392*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 56*C*a^4*b^6*tan(1/2*d*x + 1/2 *c)^3 + 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 - 284*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*C*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^9*tan(1/2*d*x + 1 /2*c)^3 + 72*A*b^10*tan(1/2*d*x + 1/2*c)^3 + 36*C*a^8*b^2*tan(1/2*d*x + 1/2 *c) - 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) + 60*C*a^7*b^3*tan(1/2*d*x + 1/2*c) + 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c) - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 6*C*a^6*b^4*tan(1/2*d*x + 1/2*c) + 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c) + 24 *B*a^5*b^5*tan(1/2*d*x + 1/2*c) - 45*C*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*A* a^4*b^6*tan(1/2*d*x + 1/2*c) + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) - 6*C*a^4 *b^6*tan(1/2*d*x + 1/2*c) - 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b ^7*tan(1/2*d*x + 1/2*c) + 15*C*a^3*b^7*tan(1/2*d*x + 1/2*c) - 48*A*a^2*b^8* tan(1/2*d*x + 1/2*c) - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) + 6*C*a^2*b^8*tan(1/2*d*x + 1/2*c) + 81*A*a*b^9*tan(1/2*d*x + 1/2*c) - 18*B*a*b^9*tan(1/2*d*x + 1/2*c) + 36*A*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) - 3*(A*a^2 + 2*C*a^2 - 8*B*a*b + 20*A*b^2)*(d*x + c)/a^6 + 6*(A*a*tan(1/2* d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^ 3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 8*A*b*tan(1/2*d *x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d

3.930 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=24

$$x(bB-aC)+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*B - a*C)*x + (b*C*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0232022, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {24, 3770}

$$x(bB-aC)+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x]),x]
```

```
[Out] (b*B - a*C)*x + (b*C*ArcTanh[Sin[c + d*x]])/d
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \left(b^2(bB-aC)+b^3C\sec(c+dx)\right)dx}{b^2}\\ &= (bB-aC)x+(bC)\int \sec(c+dx)\,dx\\ &= (bB-aC)x+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0110044, size = 23, normalized size = 0.96

$$-aCx+bBx+\frac{bC\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x]),x]
```

```
[Out] b*B*x - a*C*x + (b*C*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.046, size = 46, normalized size = 1.9

$$Bbx - aCx + \frac{Bbc}{d} + \frac{Cb\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{Cac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] B*b*x-a*C*x+1/d*B*b*c+1/d*C*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*C*a*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.49816, size = 115, normalized size = 4.79

$$-\frac{2(Ca - Bb)dx - Cb\log(\sin(dx+c)+1) + Cb\log(-\sin(dx+c)+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a - B*b)*d*x - C*b*log(sin(d*x + c) + 1) + C*b*log(-sin(d*x + c)
 + 1))/d
```

Sympy [A] time = 3.51186, size = 75, normalized size = 3.12

$$\begin{cases} -\frac{-Bb(c+dx)+Ca(c+dx)-Cb\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{for}\ d \neq 0 \\ \frac{x\left(Bab+Bb^2\sec(c)-Ca^2+Cb^2\sec^2(c)\right)}{a+b\sec(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c)),x)
```

```
[Out] Piecewise((-(-B*b*(c + d*x) + C*a*(c + d*x) - C*b*log(tan(c + d*x) + sec(c
+ d*x)))/d, Ne(d, 0)), (x*(B*a*b + B*b**2*sec(c) - C*a**2 + C*b**2*sec(c)**
```

```
2)/(a + b*sec(c)), True))
```

Giac [B] time = 1.25493, size = 72, normalized size = 3.

$$\frac{Cb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-Cb\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-(Ca-Bb)(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
),x, algorithm="giac")
```

```
[Out] (C*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - C*b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - (C*a - B*b)*(d*x + c))/d
```

3.931 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{x(bB-aC)}{a}-\frac{2b(bB-2aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ((b*B - a*C)*x)/a - (2*b*(b*B - 2*a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.150927, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {24, 3919, 3831, 2659, 208}

$$\frac{x(bB-aC)}{a}-\frac{2b(bB-2aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^2,x]
```

```
[Out] ((b*B - a*C)*x)/a - (2*b*(b*B - 2*a*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\sec(c+dx) + b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^2}$$
$$= \frac{(bB-aC)x}{a} - \frac{(b(bB-2aC))\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$
$$= \frac{(bB-aC)x}{a} - \frac{(bB-2aC)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a}$$
$$= \frac{(bB-aC)x}{a} - \frac{(2(bB-2aC))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}\right.\right.}{ad}$$
$$= \frac{(bB-aC)x}{a} - \frac{2b(bB-2aC)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

Mathematica [A] time = 0.206372, size = 76, normalized size = 1.01

$$\frac{\frac{2b(bB-2aC)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + (c+dx)(bB-aC)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^2,x]
```

```
[Out] ((b*B - a*C)*(c + d*x) + (2*b*(b*B - 2*a*C)*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2])/(a*d)
```

Maple [A] time = 0.089, size = 133, normalized size = 1.8

$$2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,Bb}{ad} - 2\,\frac{\arctan(\tan(1/2\,dx+c/2))\,C}{d} - 2\,\frac{Bb^2}{ad\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c}{\sqrt{(a+b)(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a*arctan(tan(1/2*d*x+1/2*c))*B*b-2/d*arctan(tan(1/2*d*x+1/2*c))*C-2/d*b
^2/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))*B+4/d*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.533034, size = 613, normalized size = 8.17

$$\left[-\frac{2\left(Ca^3 - Ba^2b - Cab^2 + Bb^3\right)dx + \left(2\,Cab - Bb^2\right)\sqrt{a^2 - b^2}\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2 - 2\,b^2\right)\cos(dx+c)^2 - 2\,\sqrt{a^2 - b^2}(b\cos(dx+c) + a)\sin(\cdots}{a^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + b^2}\right.}{2\left(a^3 - ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(C*a^3 - B*a^2*b - C*a*b^2 + B*b^3)*d*x + (2*C*a*b - B*b^2)*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^3 - a*b^2)*d), -((C*a^3 - B*a^2*b - C
*a*b^2 + B*b^3)*d*x - (2*C*a*b - B*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{Bb}{a + b\sec(c + dx)}\,dx - \int \frac{Ca}{a + b\sec(c + dx)}\,dx - \int -\frac{Cb\sec(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**2,x)
```

```
[Out] -Integral(-B*b/(a + b*sec(c + d*x)), x) - Integral(C*a/(a + b*sec(c + d*x))
, x) - Integral(-C*b*sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.29049, size = 153, normalized size = 2.04

$$-\frac{\frac{(Ca-Bb)(dx+c)}{a} - \frac{2\left(2\,Cab - Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^2,x, algorithm="giac")
```

```
[Out] -((C*a - B*b)*(d*x + c)/a - 2*(2*C*a*b - B*b^2)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a))/d
```

3.932 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=140

$$-\frac{2b\left(2a^2bB-3a^3C+ab^2C-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2(bB-2aC)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x(bB-aC)}{a^2}$$

```
[Out] ((b*B - a*C)*x)/a^2 - (2*b*(2*a^2*b*B - b^3*B - 3*a^3*C + a*b^2*C)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/
2)*d) + (b^2*(b*B - 2*a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))
```

Rubi [A] time = 0.389699, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {24, 3923, 3919, 3831, 2659, 208}

$$-\frac{2b\left(2a^2bB-3a^3C+ab^2C-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2(bB-2aC)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x(bB-aC)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^3,x]
```


```
[Out] ((b*B - a*C)*x)/a^2 - (2*b*(2*a^2*b*B - b^3*B - 3*a^3*C + a*b^2*C)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/
2)*d) + (b^2*(b*B - 2*a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx=\frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{b^2}$$

$$=\frac{b^2(bB-2aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\int \frac{-b^2\left(a^2-b^2\right)(bB-aC)+ab^3(bB-2aC)\,\mathrm{s}}{a+b\sec(c+dx)}}{ab^2\left(a^2-b^2\right)}$$

$$=\frac{(bB-aC)x}{a^2}+\frac{b^2(bB-2aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(b\left(2a^2bB-b^3\right.\right.}{}$$

$$=\frac{(bB-aC)x}{a^2}+\frac{b^2(bB-2aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(2a^2bB-b^3B-\right.}{}$$

$$=\frac{(bB-aC)x}{a^2}+\frac{b^2(bB-2aC)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(2\left(2a^2bB-b^3\right.\right.}{}$$

$$=\frac{(bB-aC)x}{a^2}-\frac{2b\left(2a^2bB-b^3B-3a^3C+ab^2C\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan}{\sqrt{}}\right.}{a^2(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A] time = 0.827663, size = 211, normalized size = 1.51

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)(-aC+bB+bC\sec(c+dx))\left(-\frac{2b\left(-2a^2bB+3a^3C-ab^2C+b^3B\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\right.}{a^2d(a+b\sec(c+dx))^2((bB-aC)\cos(c+dx)+bC)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*(b*B - a*C + b*C*Sec[c + d*x])*((b*B - a
*C)*(c + d*x)*(b + a*Cos[c + d*x]) - (2*b*(-2*a^2*b*B + b^3*B + 3*a^3*C - a
*b^2*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c +
```

```
d*x]))/(a^2 - b^2)^(3/2) + (a*b^2*(b*B - 2*a*C)*Sin[c + d*x])/((a - b)*(a
+ b))))/(a^2*d*(b*C + (b*B - a*C)*Cos[c + d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 0.105, size = 415, normalized size = 3.

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)Bb}{da^{2}}-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)C}{ad}-2\,\frac{b^{3}\tan\left(1/2\,dx+c/2\right)B}{ad\left(a^{2}-b^{2}\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B*b-2/a/d*arctan(tan(1/2*d*x+1/2*c))*C-2
/d*b^3/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)*B+4/d*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)*C-4/d*b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*b^4/a^2/(a+b)/(a-
b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*B+6/d*b*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))*C-2/d*b^3/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.64219, size = 1526, normalized size = 10.9

$$\left[-\frac{2\left(Ca^{6}-Ba^{5}b-2\,Ca^{4}b^{2}+2\,Ba^{3}b^{3}+Ca^{2}b^{4}-Bab^{5}\right)dx\cos\left(dx+c\right)+2\left(Ca^{5}b-Ba^{4}b^{2}-2\,Ca^{3}b^{3}+2\,Ba^{2}b^{4}+Cab^{5}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(C*a^6 - B*a^5*b - 2*C*a^4*b^2 + 2*B*a^3*b^3 + C*a^2*b^4 - B*a*b^5
)*d*x*cos(d*x + c) + 2*(C*a^5*b - B*a^4*b^2 - 2*C*a^3*b^3 + 2*B*a^2*b^4 + C
*a*b^5 - B*b^6)*d*x - (3*C*a^3*b^2 - 2*B*a^2*b^3 - C*a*b^4 + B*b^5 + (3*C*a
^4*b - 2*B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log
((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*
cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*c
```

```
os(d*x + c) + b^2)) + 2*(2*C*a^4*b^2 - B*a^3*b^3 - 2*C*a^2*b^4 + B*a*b^5)*s
in(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b
^3 + a^2*b^5)*d), -((C*a^6 - B*a^5*b - 2*C*a^4*b^2 + 2*B*a^3*b^3 + C*a^2*b^
4 - B*a*b^5)*d*x*cos(d*x + c) + (C*a^5*b - B*a^4*b^2 - 2*C*a^3*b^3 + 2*B*a^
2*b^4 + C*a*b^5 - B*b^6)*d*x - (3*C*a^3*b^2 - 2*B*a^2*b^3 - C*a*b^4 + B*b^5
 + (3*C*a^4*b - 2*B*a^3*b^2 - C*a^2*b^3 + B*a*b^4)*cos(d*x + c))*sqrt(-a^2
+ b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x +
 c))) + (2*C*a^4*b^2 - B*a^3*b^3 - 2*C*a^2*b^4 + B*a*b^5)*sin(d*x + c))/((a
^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{Bb}{a^2+2ab\sec(c+dx)+b^2\sec^2(c+dx)}\,dx-\int \frac{Ca}{a^2+2ab\sec(c+dx)+b^2\sec^2(c+dx)}\,dx-\int -\frac{Cb\,\mathrm{se}}{a^2+2ab\sec(c+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**3,x)
```

```
[Out] -Integral(-B*b/(a**2 + 2*a*b*sec(c + d*x) + b**2*sec(c + d*x)**2), x) - Int
egral(C*a/(a**2 + 2*a*b*sec(c + d*x) + b**2*sec(c + d*x)**2), x) - Integral
(-C*b*sec(c + d*x)/(a**2 + 2*a*b*sec(c + d*x) + b**2*sec(c + d*x)**2), x)
```

Giac [A] time = 1.35741, size = 301, normalized size = 2.15

$$\frac{\frac{2\left(3Ca^3b-2Ba^2b^2-Cab^3+Bb^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{-a^2+b^2}}-\frac{(Ca-Bb)(dx+c)}{a^2}+\frac{2\left(2Cab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Bb^3\right.}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^3,x, algorithm="giac")
```

```
[Out] (2*(3*C*a^3*b - 2*B*a^2*b^2 - C*a*b^3 + B*b^4)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (C*a - B*b)*
(d*x + c)/a^2 + 2*(2*C*a*b^2*tan(1/2*d*x + 1/2*c) - B*b^3*tan(1/2*d*x + 1/2
*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 -
a - b)))/d
```

3.933 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=231

$$-\frac{b\left(-5a^2b^3B+4a^3b^2C+6a^4bB-8a^5C-2ab^4C+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2bB-8a^3C+2ab^2C-2b\right.}{2a^2d\left(a^2-b^2\right)^2(a+b\sec\right.}$$

```
[Out] ((b*B - a*C)*x)/a^3 - (b*(6*a^4*b*B - 5*a^2*b^3*B + 2*b^5*B - 8*a^5*C + 4*a
^3*b^2*C - 2*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*(b*B - 2*a*C)*Tan[c + d*x])/(2*a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(5*a^2*b*B - 2*b^3*B - 8*a^3*
C + 2*a*b^2*C)*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.10007, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {24, 3923, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-5a^2b^3B+4a^3b^2C+6a^4bB-8a^5C-2ab^4C+2b^5B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2bB-8a^3C+2ab^2C-2b\right.}{2a^2d\left(a^2-b^2\right)^2(a+b\sec\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^4,x]
```


```
[Out] ((b*B - a*C)*x)/a^3 - (b*(6*a^4*b*B - 5*a^2*b^3*B + 2*b^5*B - 8*a^5*C + 4*a
^3*b^2*C - 2*a*b^4*C)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*(b*B - 2*a*C)*Tan[c + d*x])/(2*a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(5*a^2*b*B - 2*b^3*B - 8*a^3*
C + 2*a*b^2*C)*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
```

```
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\sec(c+dx) + b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx}{b^2}$$

$$= \frac{b^2(bB-2aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2b^2\left(a^2-b^2\right)(bB-aC)+2ab^3(bB-2}{(a+b}}{2ab^2}$$

$$= \frac{b^2(bB-2aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2bB-2b^3B-8a^3C+\right.}{2a^2\left(a^2-b^2\right)^2d(a+}$$

$$= \frac{(bB-aC)x}{a^3} + \frac{b^2(bB-2aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2bB-\right.}{2a^2\left(a\right.}$$

$$= \frac{(bB-aC)x}{a^3} + \frac{b^2(bB-2aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2bB-\right.}{2a^2\left(a\right.}$$

$$= \frac{(bB-aC)x}{a^3} + \frac{b^2(bB-2aC)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2bB-\right.}{2a^2\left(a\right.}$$

$$= \frac{(bB-aC)x}{a^3} - \frac{b\left(6a^4bB-5a^2b^3B+2b^5B-8a^5C+4a^3b^2C-2a\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}}$$

Mathematica [A] time = 1.85421, size = 302, normalized size = 1.31

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)(-aC+bB+bC\sec(c+dx))\left(-\frac{ab^2\left(-6a^2bB+10a^3C-4ab^2C+3b^3B\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2}+\frac{2b(-5a\cdots}{\cdots}\right.}{2a^3d(a+b\sec(c+d\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(b*B - a*C + b*C*Sec[c + d*x])*(2*(b*B
 - a*C)*(c + d*x)*(b + a*Cos[c + d*x])^2 + (2*b*(6*a^4*b*B - 5*a^2*b^3*B +
2*b^5*B - 8*a^5*C + 4*a^3*b^2*C - 2*a*b^4*C)*ArcTanh[((-a + b)*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*(
-(b*B) + 2*a*C)*Sin[c + d*x])/((a - b)*(a + b)) - (a*b^2*(-6*a^2*b*B + 3*b^
3*B + 10*a^3*C - 4*a*b^2*C)*(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(
a + b)^2)))/(2*a^3*d*(b*C + (b*B - a*C)*Cos[c + d*x])*(a + b*Sec[c + d*x])^
3)
```

Maple [B] time = 0.114, size = 1308, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B*b-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*C
-6/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d/a*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+2/d/a^2*
b^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*B+10/d*a*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+2/d*b^3/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*C-2/d/a*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C+6/d*b^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B
-1/d/a*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b
)^2*tan(1/2*d*x+1/2*c)*B-2/d/a^2*b^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-10/d*a*b^2/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C
+2/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a-b)^
2*tan(1/2*d*x+1/2*c)*C+2/d/a*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*C-6/d*a*b^2/(a^4-2*a^2*b^2+b^4
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
*B+5/d/a*b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-2/d/a^3*b^6/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+8/d*a^2*
b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))*C-4/d*b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+2/d/a^2*b^5/(a^4-2*a^2*
b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
```

```
^(1/2))*C
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 0.80603, size = 3089, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^4,x, algorithm="fricas")

[Out] [-1/4*(4*(C*a^9 - B*a^8*b - 3*C*a^7*b^2 + 3*B*a^6*b^3 + 3*C*a^5*b^4 - 3*B*a
^4*b^5 - C*a^3*b^6 + B*a^2*b^7)*d*x*cos(d*x + c)^2 + 8*(C*a^8*b - B*a^7*b^2
 - 3*C*a^6*b^3 + 3*B*a^5*b^4 + 3*C*a^4*b^5 - 3*B*a^3*b^6 - C*a^2*b^7 + B*a*
b^8)*d*x*cos(d*x + c) + 4*(C*a^7*b^2 - B*a^6*b^3 - 3*C*a^5*b^4 + 3*B*a^4*b^
5 + 3*C*a^3*b^6 - 3*B*a^2*b^7 - C*a*b^8 + B*b^9)*d*x + (8*C*a^5*b^3 - 6*B*a
^4*b^4 - 4*C*a^3*b^5 + 5*B*a^2*b^6 + 2*C*a*b^7 - 2*B*b^8 + (8*C*a^7*b - 6*B
*a^6*b^2 - 4*C*a^5*b^3 + 5*B*a^4*b^4 + 2*C*a^3*b^5 - 2*B*a^2*b^6)*cos(d*x +
 c)^2 + 2*(8*C*a^6*b^2 - 6*B*a^5*b^3 - 4*C*a^4*b^4 + 5*B*a^3*b^5 + 2*C*a^2*
b^6 - 2*B*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a
^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
 + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(
8*C*a^6*b^3 - 5*B*a^5*b^4 - 10*C*a^4*b^5 + 7*B*a^3*b^6 + 2*C*a^2*b^7 - 2*B*
a*b^8 + (10*C*a^7*b^2 - 6*B*a^6*b^3 - 14*C*a^5*b^4 + 9*B*a^4*b^5 + 4*C*a^3*
b^6 - 3*B*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b
^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^
7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), -1/2*(2
*(C*a^9 - B*a^8*b - 3*C*a^7*b^2 + 3*B*a^6*b^3 + 3*C*a^5*b^4 - 3*B*a^4*b^5 -
 C*a^3*b^6 + B*a^2*b^7)*d*x*cos(d*x + c)^2 + 4*(C*a^8*b - B*a^7*b^2 - 3*C*a
^6*b^3 + 3*B*a^5*b^4 + 3*C*a^4*b^5 - 3*B*a^3*b^6 - C*a^2*b^7 + B*a*b^8)*d*x
*cos(d*x + c) + 2*(C*a^7*b^2 - B*a^6*b^3 - 3*C*a^5*b^4 + 3*B*a^4*b^5 + 3*C*
a^3*b^6 - 3*B*a^2*b^7 - C*a*b^8 + B*b^9)*d*x - (8*C*a^5*b^3 - 6*B*a^4*b^4 -
 4*C*a^3*b^5 + 5*B*a^2*b^6 + 2*C*a*b^7 - 2*B*b^8 + (8*C*a^7*b - 6*B*a^6*b^2
 - 4*C*a^5*b^3 + 5*B*a^4*b^4 + 2*C*a^3*b^5 - 2*B*a^2*b^6)*cos(d*x + c)^2 +
2*(8*C*a^6*b^2 - 6*B*a^5*b^3 - 4*C*a^4*b^4 + 5*B*a^3*b^5 + 2*C*a^2*b^6 - 2*
B*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (8*C*a^6*b^3 - 5*B*a^5*b^4 - 10*C*
a^4*b^5 + 7*B*a^3*b^6 + 2*C*a^2*b^7 - 2*B*a*b^8 + (10*C*a^7*b^2 - 6*B*a^6*b
^3 - 14*C*a^5*b^4 + 9*B*a^4*b^5 + 4*C*a^3*b^6 - 3*B*a^2*b^7)*cos(d*x + c))*
sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 +
2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*
a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{Bb}{a^3 + 3a^2b\sec(c+dx) + 3ab^2\sec^2(c+dx) + b^3\sec^3(c+dx)}\,dx - \int \frac{Ca}{a^3 + 3a^2b\sec(c+dx) + 3ab^2\sec^2(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**4,x)
```

```
[Out] -Integral(-B*b/(a**3 + 3*a**2*b*sec(c + d*x) + 3*a*b**2*sec(c + d*x)**2 + b
**3*sec(c + d*x)**3), x) - Integral(C*a/(a**3 + 3*a**2*b*sec(c + d*x) + 3*a
*b**2*sec(c + d*x)**2 + b**3*sec(c + d*x)**3), x) - Integral(-C*b*sec(c + d
*x)/(a**3 + 3*a**2*b*sec(c + d*x) + 3*a*b**2*sec(c + d*x)**2 + b**3*sec(c +
 d*x)**3), x)
```

Giac [B] time = 1.42383, size = 695, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^4,x, algorithm="giac")
```

```
[Out] ((8*C*a^5*b - 6*B*a^4*b^2 - 4*C*a^3*b^3 + 5*B*a^2*b^4 + 2*C*a*b^5 - 2*B*b^6
)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 +
 a^3*b^4)*sqrt(-a^2 + b^2)) - (C*a - B*b)*(d*x + c)/a^3 + (10*C*a^4*b^2*tan
(1/2*d*x + 1/2*c)^3 - 6*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*C*a^3*b^3*tan(
1/2*d*x + 1/2*c)^3 + 5*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*C*a^2*b^4*tan(1
/2*d*x + 1/2*c)^3 + 3*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*C*a*b^5*tan(1/2*d*
x + 1/2*c)^3 - 2*B*b^6*tan(1/2*d*x + 1/2*c)^3 - 10*C*a^4*b^2*tan(1/2*d*x +
1/2*c) + 6*B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*C*a^3*b^3*tan(1/2*d*x + 1/2*c
) + 5*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 4*C*a^2*b^4*tan(1/2*d*x + 1/2*c) - 3
*B*a*b^5*tan(1/2*d*x + 1/2*c) + 2*C*a*b^5*tan(1/2*d*x + 1/2*c) - 2*B*b^6*ta
n(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.934 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^5}\,dx$

Optimal. Leaf size=336

$$-\frac{b\left(-8a^4b^3B+7a^2b^5B+5a^5b^2C-7a^3b^4C+8a^6bB-10a^7C+2ab^6C-2b^7B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^3B\right.}{}$$

```
[Out] ((b*B - a*C)*x)/a^4 - (b*(8*a^6*b*B - 8*a^4*b^3*B + 7*a^2*b^5*B - 2*b^7*B -
 10*a^7*C + 5*a^5*b^2*C - 7*a^3*b^4*C + 2*a*b^6*C)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + (b^2*(b*
B - 2*a*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b^2*
(8*a^2*b*B - 3*b^3*B - 13*a^3*C + 3*a*b^2*C)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x])^2) + (b^2*(26*a^4*b*B - 17*a^2*b^3*B + 6*b^5*B
- 37*a^5*C + 13*a^3*b^2*C - 6*a*b^4*C)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d
*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 4.67127, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {24, 3923, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-8a^4b^3B+7a^2b^5B+5a^5b^2C-7a^3b^4C+8a^6bB-10a^7C+2ab^6C-2b^7B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^5,x]
```



```
[Out] ((b*B - a*C)*x)/a^4 - (b*(8*a^6*b*B - 8*a^4*b^3*B + 7*a^2*b^5*B - 2*b^7*B -
 10*a^7*C + 5*a^5*b^2*C - 7*a^3*b^4*C + 2*a*b^6*C)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + (b^2*(b*
B - 2*a*C)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b^2*
(8*a^2*b*B - 3*b^3*B - 13*a^3*C + 3*a*b^2*C)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x])^2) + (b^2*(26*a^4*b*B - 17*a^2*b^3*B + 6*b^5*B
- 37*a^5*C + 13*a^3*b^2*C - 6*a*b^4*C)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d
*(a + b*Sec[c + d*x]))
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
```

```
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\sec(c+dx) + b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^5}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx}{b^2}$$

$$= \frac{b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3b^2\left(a^2-b^2\right)(bB-aC)+3ab^3(bB-2}{(a+b}}{3ab}$$

$$= \frac{b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2bB-3b^3B-13a^3C\right.}{6a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2bB-3b^3B-13a^3C\right.}{6a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{(bB-aC)x}{a^4} + \frac{b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2bB-\right.}{6a^2\left(a\right.}$$

$$= \frac{(bB-aC)x}{a^4} + \frac{b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2bB-\right.}{6a^2\left(a\right.}$$

$$= \frac{(bB-aC)x}{a^4} + \frac{b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2bB-\right.}{6a^2\left(a\right.}$$

$$= \frac{(bB-aC)x}{a^4} - \frac{b\left(8a^6bB-8a^4b^3B+7a^2b^5B-2b^7B-10a^7C+5\right.}{a^4(a-b)^7}$$

Mathematica [B] time = 5.59439, size = 1097, normalized size = 3.26

$$\frac{(b+a\cos(c+dx))\sec^3(c+dx)(bB-aC+bC\sec(c+dx))\left(\frac{24b\left(-10Ca^7+8bBa^6+5b^2Ca^5-8b^3Ba^4-7b^4Ca^3+7b^5Ba^2+2b^6Ca-2b^7B\right)\tanh^{-1}\left(\right.}{\left(a^2-b^2\right)^{7/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^5,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(b*B - a*C + b*C*Sec[c + d*x])*((24*b*
(8*a^6*b*B - 8*a^4*b^3*B + 7*a^2*b^5*B - 2*b^7*B - 10*a^7*C + 5*a^5*b^2*C -
 7*a^3*b^4*C + 2*a*b^6*C)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (36*a^8*b^2*B*c - 84*a^6*b^
4*B*c + 36*a^4*b^6*B*c + 36*a^2*b^8*B*c - 24*b^10*B*c - 36*a^9*b*c*C + 84*a
^7*b^3*c*C - 36*a^5*b^5*c*C - 36*a^3*b^7*c*C + 24*a*b^9*c*C + 36*a^8*b^2*B*
d*x - 84*a^6*b^4*B*d*x + 36*a^4*b^6*B*d*x + 36*a^2*b^8*B*d*x - 24*b^10*B*d*
x - 36*a^9*b*C*d*x + 84*a^7*b^3*C*d*x - 36*a^5*b^5*C*d*x - 36*a^3*b^7*C*d*x
 + 24*a*b^9*C*d*x - 18*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(-(b*B) + a*C)*(c + d*
x)*Cos[c + d*x] - 36*a^2*b*(a^2 - b^2)^3*(-(b*B) + a*C)*(c + d*x)*Cos[2*(c
+ d*x)] + 6*a^9*b*B*c*Cos[3*(c + d*x)] - 18*a^7*b^3*B*c*Cos[3*(c + d*x)] +
18*a^5*b^5*B*c*Cos[3*(c + d*x)] - 6*a^3*b^7*B*c*Cos[3*(c + d*x)] - 6*a^10*c
*C*Cos[3*(c + d*x)] + 18*a^8*b^2*c*C*Cos[3*(c + d*x)] - 18*a^6*b^4*c*C*Cos[
3*(c + d*x)] + 6*a^4*b^6*c*C*Cos[3*(c + d*x)] + 6*a^9*b*B*d*x*Cos[3*(c + d*
```

```
x)] - 18*a^7*b^3*B*d*x*Cos[3*(c + d*x)] + 18*a^5*b^5*B*d*x*Cos[3*(c + d*x)]
 - 6*a^3*b^7*B*d*x*Cos[3*(c + d*x)] - 6*a^10*C*d*x*Cos[3*(c + d*x)] + 18*a^
8*b^2*C*d*x*Cos[3*(c + d*x)] - 18*a^6*b^4*C*d*x*Cos[3*(c + d*x)] + 6*a^4*b^
6*C*d*x*Cos[3*(c + d*x)] + 36*a^7*b^3*B*Sin[c + d*x] + 72*a^5*b^5*B*Sin[c +
 d*x] - 57*a^3*b^7*B*Sin[c + d*x] + 24*a*b^9*B*Sin[c + d*x] - 54*a^8*b^2*C*
Sin[c + d*x] - 111*a^6*b^4*C*Sin[c + d*x] + 39*a^4*b^6*C*Sin[c + d*x] - 24*
a^2*b^8*C*Sin[c + d*x] + 120*a^6*b^4*B*Sin[2*(c + d*x)] - 90*a^4*b^6*B*Sin[
2*(c + d*x)] + 30*a^2*b^8*B*Sin[2*(c + d*x)] - 174*a^7*b^3*C*Sin[2*(c + d*x
)] + 84*a^5*b^5*C*Sin[2*(c + d*x)] - 30*a^3*b^7*C*Sin[2*(c + d*x)] + 36*a^7
*b^3*B*Sin[3*(c + d*x)] - 32*a^5*b^5*B*Sin[3*(c + d*x)] + 11*a^3*b^7*B*Sin[
3*(c + d*x)] - 54*a^8*b^2*C*Sin[3*(c + d*x)] + 37*a^6*b^4*C*Sin[3*(c + d*x)
] - 13*a^4*b^6*C*Sin[3*(c + d*x)])/(a^2 - b^2)^3))/(24*a^4*d*(b*C + (b*B -
a*C)*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.127, size = 2853, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^5,x)
```

```
[Out] 24/d*b^3*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b
+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+18/d*b^2*a^2/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5*C-8/d*b^2*a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B+2/d*b^8/a^4/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))*B+10/d*b*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+7/d*b
^5/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C-2/d*b^7/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)*C-7/d*b^6/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d*b^4/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*C-4/d*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C+40/3/d*b^4/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*C+4/d*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d*b^4/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b
^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-5/d*b^3*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*C+6/d
*b^5/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+1/d*b^5/a/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c)*C+2/d*b^6/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+
b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-2/d/a^3*arctan(tan(1/2*d*
x+1/2*c))*C+6/d*b^5/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-36/d*b^2*a^2/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*C+8/d*b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*B-4/d*b^6/
a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*C-2/d*b^7/a^3/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
```

```
2*c)^5*B+2/d*b^6/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-12/d*b^3*a/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5*B-1/d*b^5/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*C-1/d*b^
6/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+4/d*b^7/a^3/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3*B-2/d*b^7/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/
(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-7/d*b^3*a/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)*C+18/d*b^2*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*C-44/3/d*b^5
/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2-2*a*b+b^2)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-12/d*b^3*a/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*B+1/d*b^6/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+7/d*b^3*a/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan
(1/2*d*x+1/2*c)^5*C+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.12316, size = 5277, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^5,x, algorithm="fricas")
```

```
[Out] [-1/12*(12*(C*a^12 - B*a^11*b - 4*C*a^10*b^2 + 4*B*a^9*b^3 + 6*C*a^8*b^4 -
6*B*a^7*b^5 - 4*C*a^6*b^6 + 4*B*a^5*b^7 + C*a^4*b^8 - B*a^3*b^9)*d*x*cos(d*
x + c)^3 + 36*(C*a^11*b - B*a^10*b^2 - 4*C*a^9*b^3 + 4*B*a^8*b^4 + 6*C*a^7*
b^5 - 6*B*a^6*b^6 - 4*C*a^5*b^7 + 4*B*a^4*b^8 + C*a^3*b^9 - B*a^2*b^10)*d*x
*cos(d*x + c)^2 + 36*(C*a^10*b^2 - B*a^9*b^3 - 4*C*a^8*b^4 + 4*B*a^7*b^5 +
6*C*a^6*b^6 - 6*B*a^5*b^7 - 4*C*a^4*b^8 + 4*B*a^3*b^9 + C*a^2*b^10 - B*a*b^
11)*d*x*cos(d*x + c) + 12*(C*a^9*b^3 - B*a^8*b^4 - 4*C*a^7*b^5 + 4*B*a^6*b^
6 + 6*C*a^5*b^7 - 6*B*a^4*b^8 - 4*C*a^3*b^9 + 4*B*a^2*b^10 + C*a*b^11 - B*b
^12)*d*x + 3*(10*C*a^7*b^4 - 8*B*a^6*b^5 - 5*C*a^5*b^6 + 8*B*a^4*b^7 + 7*C*
a^3*b^8 - 7*B*a^2*b^9 - 2*C*a*b^10 + 2*B*b^11 + (10*C*a^10*b - 8*B*a^9*b^2
- 5*C*a^8*b^3 + 8*B*a^7*b^4 + 7*C*a^6*b^5 - 7*B*a^5*b^6 - 2*C*a^4*b^7 + 2*B
*a^3*b^8)*cos(d*x + c)^3 + 3*(10*C*a^9*b^2 - 8*B*a^8*b^3 - 5*C*a^7*b^4 + 8*
B*a^6*b^5 + 7*C*a^5*b^6 - 7*B*a^4*b^7 - 2*C*a^3*b^8 + 2*B*a^2*b^9)*cos(d*x
+ c)^2 + 3*(10*C*a^8*b^3 - 8*B*a^7*b^4 - 5*C*a^6*b^5 + 8*B*a^5*b^6 + 7*C*a^
```

```
4*b^7 - 7*B*a^3*b^8 - 2*C*a^2*b^9 + 2*B*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^
2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^
2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2
*a*b*cos(d*x + c) + b^2)) + 2*(37*C*a^8*b^4 - 26*B*a^7*b^5 - 50*C*a^6*b^6 +
 43*B*a^5*b^7 + 19*C*a^4*b^8 - 23*B*a^3*b^9 - 6*C*a^2*b^10 + 6*B*a*b^11 + (
54*C*a^10*b^2 - 36*B*a^9*b^3 - 91*C*a^8*b^4 + 68*B*a^7*b^5 + 50*C*a^6*b^6 -
 43*B*a^5*b^7 - 13*C*a^4*b^8 + 11*B*a^3*b^9)*cos(d*x + c)^2 + 3*(29*C*a^9*b
^3 - 20*B*a^8*b^4 - 43*C*a^7*b^5 + 35*B*a^6*b^6 + 19*C*a^5*b^7 - 20*B*a^4*b
^8 - 5*C*a^3*b^9 + 5*B*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^1
3*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*
a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2
 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^
3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), -1/6*(6*(C*a^12 - B*
a^11*b - 4*C*a^10*b^2 + 4*B*a^9*b^3 + 6*C*a^8*b^4 - 6*B*a^7*b^5 - 4*C*a^6*b
^6 + 4*B*a^5*b^7 + C*a^4*b^8 - B*a^3*b^9)*d*x*cos(d*x + c)^3 + 18*(C*a^11*b
 - B*a^10*b^2 - 4*C*a^9*b^3 + 4*B*a^8*b^4 + 6*C*a^7*b^5 - 6*B*a^6*b^6 - 4*C
*a^5*b^7 + 4*B*a^4*b^8 + C*a^3*b^9 - B*a^2*b^10)*d*x*cos(d*x + c)^2 + 18*(C
*a^10*b^2 - B*a^9*b^3 - 4*C*a^8*b^4 + 4*B*a^7*b^5 + 6*C*a^6*b^6 - 6*B*a^5*b
^7 - 4*C*a^4*b^8 + 4*B*a^3*b^9 + C*a^2*b^10 - B*a*b^11)*d*x*cos(d*x + c) +
6*(C*a^9*b^3 - B*a^8*b^4 - 4*C*a^7*b^5 + 4*B*a^6*b^6 + 6*C*a^5*b^7 - 6*B*a^
4*b^8 - 4*C*a^3*b^9 + 4*B*a^2*b^10 + C*a*b^11 - B*b^12)*d*x - 3*(10*C*a^7*b
^4 - 8*B*a^6*b^5 - 5*C*a^5*b^6 + 8*B*a^4*b^7 + 7*C*a^3*b^8 - 7*B*a^2*b^9 -
2*C*a*b^10 + 2*B*b^11 + (10*C*a^10*b - 8*B*a^9*b^2 - 5*C*a^8*b^3 + 8*B*a^7*
b^4 + 7*C*a^6*b^5 - 7*B*a^5*b^6 - 2*C*a^4*b^7 + 2*B*a^3*b^8)*cos(d*x + c)^3
 + 3*(10*C*a^9*b^2 - 8*B*a^8*b^3 - 5*C*a^7*b^4 + 8*B*a^6*b^5 + 7*C*a^5*b^6
- 7*B*a^4*b^7 - 2*C*a^3*b^8 + 2*B*a^2*b^9)*cos(d*x + c)^2 + 3*(10*C*a^8*b^3
 - 8*B*a^7*b^4 - 5*C*a^6*b^5 + 8*B*a^5*b^6 + 7*C*a^4*b^7 - 7*B*a^3*b^8 - 2*
C*a^2*b^9 + 2*B*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (37*C*a^8*b^4 - 26*
B*a^7*b^5 - 50*C*a^6*b^6 + 43*B*a^5*b^7 + 19*C*a^4*b^8 - 23*B*a^3*b^9 - 6*C
*a^2*b^10 + 6*B*a*b^11 + (54*C*a^10*b^2 - 36*B*a^9*b^3 - 91*C*a^8*b^4 + 68*
B*a^7*b^5 + 50*C*a^6*b^6 - 43*B*a^5*b^7 - 13*C*a^4*b^8 + 11*B*a^3*b^9)*cos(
d*x + c)^2 + 3*(29*C*a^9*b^3 - 20*B*a^8*b^4 - 43*C*a^7*b^5 + 35*B*a^6*b^6 +
 19*C*a^5*b^7 - 20*B*a^4*b^8 - 5*C*a^3*b^9 + 5*B*a^2*b^10)*cos(d*x + c))*si
n(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d
*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*co
s(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)
*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11
)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{Bb}{a^4 + 4a^3b\sec(c+dx) + 6a^2b^2\sec^2(c+dx) + 4ab^3\sec^3(c+dx) + b^4\sec^4(c+dx)}\,dx - \int \frac{}{a^4 + 4a^3b\sec(c+dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**5,x)
```

```
[Out] -Integral(-B*b/(a**4 + 4*a**3*b*sec(c + d*x) + 6*a**2*b**2*sec(c + d*x)**2
+ 4*a*b**3*sec(c + d*x)**3 + b**4*sec(c + d*x)**4), x) - Integral(C*a/(a**4
 + 4*a**3*b*sec(c + d*x) + 6*a**2*b**2*sec(c + d*x)**2 + 4*a*b**3*sec(c + d
*x)**3 + b**4*sec(c + d*x)**4), x) - Integral(-C*b*sec(c + d*x)/(a**4 + 4*a
**3*b*sec(c + d*x) + 6*a**2*b**2*sec(c + d*x)**2 + 4*a*b**3*sec(c + d*x)**3
 + b**4*sec(c + d*x)**4), x)
```

Giac [B] time = 1.61446, size = 1161, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(10*C*a^7*b - 8*B*a^6*b^2 - 5*C*a^5*b^3 + 8*B*a^4*b^4 + 7*C*a^3*b^5
- 7*B*a^2*b^6 - 2*C*a*b^7 + 2*B*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqr
t(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2))
 - 3*(C*a - B*b)*(d*x + c)/a^4 + (54*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 36*
B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 87*C*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 60
*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 42
*C*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 45*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 6
*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 1
5*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 + 6*C*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 6*B*b
^9*tan(1/2*d*x + 1/2*c)^5 - 108*C*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*B*a^6
*b^3*tan(1/2*d*x + 1/2*c)^3 + 148*C*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*B*
a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 52*C*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 56*B
*a^2*b^7*tan(1/2*d*x + 1/2*c)^3 + 12*C*a*b^8*tan(1/2*d*x + 1/2*c)^3 - 12*B*
b^9*tan(1/2*d*x + 1/2*c)^3 + 54*C*a^7*b^2*tan(1/2*d*x + 1/2*c) - 36*B*a^6*b
^3*tan(1/2*d*x + 1/2*c) + 87*C*a^6*b^3*tan(1/2*d*x + 1/2*c) - 60*B*a^5*b^4*
tan(1/2*d*x + 1/2*c) + 6*B*a^4*b^5*tan(1/2*d*x + 1/2*c) - 42*C*a^4*b^5*tan(
1/2*d*x + 1/2*c) + 45*B*a^3*b^6*tan(1/2*d*x + 1/2*c) + 6*B*a^2*b^7*tan(1/2*
d*x + 1/2*c) + 15*C*a^2*b^7*tan(1/2*d*x + 1/2*c) - 15*B*a*b^8*tan(1/2*d*x +
 1/2*c) + 6*C*a*b^8*tan(1/2*d*x + 1/2*c) - 6*B*b^9*tan(1/2*d*x + 1/2*c))/((
a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/
2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.935 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=517

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(12a^2b(2B-C)-16a^3C-6ab^2(7A-3B+6C)-3b^3(63A-25B+49C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{315b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 6*a^2*b^2*(7*
A + 4*C) + 21*b^4*(9*A + 7*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^5*d) - (2*(a - b)*Sqrt[
a + b]*(12*a^2*b*(2*B - C) - 16*a^3*C - 6*a*b^2*(7*A - 3*B + 6*C) - 3*b^3*(
63*A - 25*B + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) - (2*(12*a^2*b*B - 75*b^3*B
- 8*a^3*C - a*b^2*(21*A + 13*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(31
5*b^3*d) + (2*(63*A*b^2 + 9*a*b*B - 6*a^2*C + 49*b^2*C)*Sec[c + d*x]*Sqrt[a
 + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9*b*B + a*C)*Sec[c + d*x
]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d) + (2*C*Sec[c + d*x]^3*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```

Rubi [A] time = 1.5568, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4096, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\sec(c+dx)\left(-6a^2C+9abB+63Ab^2+49b^2C\right)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2\tan(c+dx)\left(12a^2bB-8a^3C-ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*b*B + 57*a*b^3*B - 16*a^4*C - 6*a^2*b^2*(7*
A + 4*C) + 21*b^4*(9*A + 7*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^5*d) - (2*(a - b)*Sqrt[
a + b]*(12*a^2*b*(2*B - C) - 16*a^3*C - 6*a*b^2*(7*A - 3*B + 6*C) - 3*b^3*(
63*A - 25*B + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) - (2*(12*a^2*b*B - 75*b^3*B
- 8*a^3*C - a*b^2*(21*A + 13*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(31
5*b^3*d) + (2*(63*A*b^2 + 9*a*b*B - 6*a^2*C + 49*b^2*C)*Sec[c + d*x]*Sqrt[a
 + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9*b*B + a*C)*Sec[c + d*x
]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d) + (2*C*Sec[c + d*x]^3*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
```

```
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\tan(}{9d}$$

$$= \frac{2(9bB+aC)\sec^2(c+dx)\sqrt{a+b\sec(c+}}{63bd}$$

$$= \frac{2\left(63Ab^2+9abB-6a^2C+49b^2C\right)\sec(}{315b}$$

$$= -\frac{2\left(12a^2bB-75b^3B-8a^3C-ab^2(21A\right.}{315}$$

$$= -\frac{2\left(12a^2bB-75b^3B-8a^3C-ab^2(21A\right.}{315}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(24a^3bB+57ab^3B-16\right.}{}$$

Mathematica [B] time = 27.7395, size = 4780, normalized size = 9.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((4*(-42*a^2*A*b^2 + 189*A*b^4 + 24*a^3*b*B + 57*a*b^3*B - 16*a^4*C -
 24*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(315*b^4) + (4*Sec[c + d*x]^3*(9*b
*B*Sin[c + d*x] + a*C*Sin[c + d*x]))/(63*b) + (4*Sec[c + d*x]^2*(63*A*b^2*S
in[c + d*x] + 9*a*b*B*Sin[c + d*x] - 6*a^2*C*Sin[c + d*x] + 49*b^2*C*Sin[c
+ d*x]))/(315*b^2) + (4*Sec[c + d*x]*(21*a*A*b^2*Sin[c + d*x] - 12*a^2*b*B*
Sin[c + d*x] + 75*b^3*B*Sin[c + d*x] + 8*a^3*C*Sin[c + d*x] + 13*a*b^2*C*Si
n[c + d*x]))/(315*b^3) + (4*C*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*((4*a^2*A)/(15*b*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (6*A*b)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (38*a*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (16*a^3*B)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (32
*a^4*C)/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^2*C)/
(105*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (14*b*C)/(15*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*A*Sqrt[Sec[c + d*x]])/(15*Sqrt[
b + a*Cos[c + d*x]]) + (4*a^3*A*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos[
c + d*x]]) - (16*a^4*B*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (34*a^2*B*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) + (10*b
*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (8*a*C*Sqrt[Sec[c +
d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (32*a^5*C*Sqrt[Sec[c + d*x]])/(315*b
^4*Sqrt[b + a*Cos[c + d*x]]) + (8*a^3*C*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b
+ a*Cos[c + d*x]]) - (6*a*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b
+ a*Cos[c + d*x]]) + (4*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*
Sqrt[b + a*Cos[c + d*x]]) - (16*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/
(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (38*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) - (14*a*C*Cos[2*(c + d*x)]*Sqrt[S
ec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (32*a^5*C*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (16*a^3*C*Cos[2*(c +
```

d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*(2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2
*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(a + b)*(-16*a^3*C +
12*a^2*b*(2*B + C) - 6*a*b^2*(7*A + 3*B + 6*C) + 3*b^3*(63*A + 25*B + 49*C
))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
]*Sec[c + d*x] + (-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*
C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2]))/(315*b^4*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(5/2)*
((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-24*a^
3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Se
c[(c + d*x)/2]^2 + b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(7*A
 + 3*B + 6*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-24*a^3*b*B - 57
*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d
x](b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*b^4*(b
+ a*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) - (2*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B +
16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + b
(a + b)(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(7*A + 3*B + 6*C) + 3*b^
3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C
+ 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]
]*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*(2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B + 1
6*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2 + b*(
a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(7*A + 3*B + 6*C) + 3*b^3*
(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C +
6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Si
n[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*b^4*S
qrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-24*a^3*b*B -
57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 + ((a + b)*(-24*a^3*b*B -
57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] + ((a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2
b^2(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(
-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Si
n[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))] + 2*(a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C

```
+ 6*a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2] - a*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b
^4*(9*A + 7*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/
2] - (-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) - 21*b^4*
(9*A + 7*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c +
d*x)/2] + 2*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*a^2*b^2*(7*A + 4*C) -
21*b^4*(9*A + 7*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Ta
n[(c + d*x)/2]^2 + (3*b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(
7*A + 3*B + 6*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2
]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 +
(b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(7*A + 3*B + 6*C) + 3*
b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((a*Sec[(c + d*x
)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)]) + (b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(7*A + 3*
B + 6*C) + 3*b^3*(63*A + 25*B + 49*C))*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)]*Sec[c + d*x])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)]) + ((a + b)*(-24*a^3*b*B - 57*a*b^3*B + 16*a^4*C + 6*
a^2*b^2*(7*A + 4*C) - 21*b^4*(9*A + 7*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^4*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)
/2]^2] + b*(a + b)*(-16*a^3*C + 12*a^2*b*(2*B + C) - 6*a*b^2*(7*A + 3*B + 6
*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))/(315*b^4*Sqrt[
b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 2.016, size = 5961, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*se
c(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec\left(c+dx\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*
sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sec(d*x + c)^3, x)
```

3.936 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx)+C\,\mathrm{s}$

Optimal. Leaf size=413

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(8a^2C-a(14bB-6bC)+35Ab^2-b^2(63B-25C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellipti}}{105b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - a*b^2*(35*A + 19*
C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(105*b^4*d) - (2*(a - b)*Sqrt[a + b]*(35*A*b^2 - b^2*(63*
B - 25*C) + 8*a^2*C - a*(14*b*B - 6*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(3
5*A*b^2 - 14*a*b*B + 8*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(105*b^2*d) + (2*(7*b*B - 4*a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*
x])/(35*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])
/(7*b*d)
```

Rubi [A] time = 0.920382, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4092, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(8a^2C-14abB+35Ab^2+25b^2C\right)\sqrt{a+b\sec(c+dx)}}{105b^2d}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(8a^2C-a(14bB-6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*b*B - 63*b^3*B - 8*a^3*C - a*b^2*(35*A + 19*
C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(105*b^4*d) - (2*(a - b)*Sqrt[a + b]*(35*A*b^2 - b^2*(63*
B - 25*C) + 8*a^2*C - a*(14*b*B - 6*b*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(3
5*A*b^2 - 14*a*b*B + 8*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(105*b^2*d) + (2*(7*b*B - 4*a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*
x])/(35*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])
/(7*b*d)
```


Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{3/2}\tan(c}{7bd}$$

$$= \frac{2(7bB-4aC)(a+b\sec(c+dx))^{3/2}\tan(c+}{35b^2d}$$

$$= \frac{2\left(35Ab^2-14abB+8a^2C+25b^2C\right)\sqrt{a+}}{105b^2d}$$

$$= \frac{2\left(35Ab^2-14abB+8a^2C+25b^2C\right)\sqrt{a+}}{105b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(14a^2bB-63b^3B-8a^3C-\right.}{}$$

Mathematica [B] time = 26.1731, size = 3706, normalized size = 8.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((4*(35*a*A*b^2 - 14*a^2*b*B + 63*b^3*B + 8*a^3*C + 19*a*b^2*C)*Sin[c
 + d*x])/(105*b^3) + (4*Sec[c + d*x]^2*(7*b*B*Sin[c + d*x] + a*C*Sin[c + d*
x]))/(35*b) + (4*Sec[c + d*x]*(35*A*b^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]
 - 4*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b^2) + (4*C*Sec[c +
d*x]^2*Tan[c + d*x])/7))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])) - (4*((-2*a*A)/(3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^2
*B)/(15*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (6*b*B)/(5*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (38*a*C)/(105*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) - (16*a^3*C)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*b*Sqrt[b + a*Cos[c + d*x]
]) + (2*A*b*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) - (4*a*B*Sqrt[
Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*B*Sqrt[Sec[c + d*x]])
/(15*b^2*Sqrt[b + a*Cos[c + d*x]]) - (16*a^4*C*Sqrt[Sec[c + d*x]])/(105*b^3
*Sqrt[b + a*Cos[c + d*x]]) - (34*a^2*C*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b +
a*Cos[c + d*x]]) + (10*b*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]
) - (2*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*Sqrt[b + a*Cos[c + d
*x]]) - (6*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d
*x]]) + (4*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Co
s[c + d*x]]) - (16*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt
[b + a*Cos[c + d*x]]) - (38*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105
*b*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(2*(a + b)*(-14*a
^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(35*A*b^2 + 8
*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B + 8*
a^3*C + a*b^2*(35*A + 19*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2
)*((-2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-14
*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(35*A*b^2 +
 8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B +
8*a^3*C + a*b^2*(35*A + 19*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c
 + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*
(2*(a + b)*(-14*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a +
 b)*(35*A*b^2 + 8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*b*
B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*Sqrt[b + a*Cos[c + d*x]
```

```
]*Sqrt[Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-
14*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Cos[c + d*x]*(b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-14*a^2*b*B + 63*b^3*B + 8*
a^3*C + a*b^2*(35*A + 19*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*
x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(35*A*b^2 + 8*a^2*C - 2*a
*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos
[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*
x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-14*a^2*b*B + 63*b^
3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*
(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))] - (b*(a + b)*(35*A*b^2 + 8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*
C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b
 + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-14*a^2*b*B + 63*b^3*B
+ 8*a^3*C + a*b^2*(35*A + 19*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*
x]*Tan[(c + d*x)/2] - (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*
C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] +
 (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Cos[c + d*x]*(b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(a + b)*(35*A*b
^2 + 8*a^2*C - 2*a*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Se
c[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + ((a + b)*(-14*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^2*(
35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^3*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*(2*(a + b)*(-14*a^2*b*B + 63*
b^3*B + 8*a^3*C + a*b^2*(35*A + 19*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(35*A*b^2 + 8*a^2*C - 2*a
*b*(7*B + 3*C) + b^2*(63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*b*B + 63*b^3*B + 8*a^3*C + a*b^
2*(35*A + 19*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.204, size = 4339, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-35*A*cos(d*x+c)^2*b^4-21*B*cos(d*x+c)*b^
4+8*C*cos(d*x+c)^5*a^4-70*A*cos(d*x+c)^3*a*b^3+7*B*cos(d*x+c)^3*a^2*b^2-28*
```

B*cos(d*x+c)^2*a*b^3+63*B*cos(d*x+c)^4*b^4-42*B*cos(d*x+c)^3*b^4+35*A*cos(d
*x+c)^4*b^4+25*C*cos(d*x+c)^4*b^4-35*A*cos(d*x+c)^4*a^2*b^2+35*A*cos(d*x+c)
^5*a^2*b^2+35*A*cos(d*x+c)^5*a*b^3+35*A*cos(d*x+c)^4*a*b^3+14*B*cos(d*x+c)^
4*a^3*b-35*B*cos(d*x+c)^4*a*b^3+7*B*cos(d*x+c)^5*a^2*b^2+63*B*cos(d*x+c)^5*
a*b^3-4*C*cos(d*x+c)^5*a^3*b+19*C*cos(d*x+c)^5*a^2*b^2+25*C*cos(d*x+c)^5*a*
b^3-20*C*cos(d*x+c)^4*a^2*b^2+19*C*cos(d*x+c)^4*a*b^3-35*A*sin(d*x+c)*cos(d
x+c)^4(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b^2-35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^3+35*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-35*A*sin(d*x+c)*cos
(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2+14*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+14*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)
^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b^3-14*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+49*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+14*B*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^3*b+14*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
x+c)+1))^(1/2)(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-14*B*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^2*b^2+49*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+8*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+2*C*cos(d*
x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b^2+19*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-19*C*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2-19*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*C*cos(d*x+c)^3*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-35*A*sin(
d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*b^3+2*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)

```
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+19*C*cos(d*x+c)^3*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*C*cos(
d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*b-19*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-19*C*cos(d*x+c)^3*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-14*B*co
s(d*x+c)^4*a^2*b^2-14*B*cos(d*x+c)^5*a^3*b+8*C*cos(d*x+c)^4*a^3*b-4*C*cos(d
*x+c)^3*a^3*b-26*C*cos(d*x+c)^3*a*b^3+C*cos(d*x+c)^2*a^2*b^2-18*C*cos(d*x+c
)*a*b^3-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-63*B*cos(d*x+c)^3*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^
4+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*b^4+35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+35*A*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+25*C
*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^4-8*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+25*C*cos(d*x+c)^3*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-8*C*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^4-8*C*cos(d*x+c)^4*a^4-10*C*cos(d*x+c)^2*b^4-15*C*b^4)/(b+a*cos(d*
x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*se
c(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec{(c + dx)}} \left(A + B \sec{(c + dx)} + C \sec^{2}{(c + dx)}\right) \sec^{2}{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*
sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec\left(dx + c\right)^{2} + B \sec\left(dx + c\right) + A\right)\sqrt{b \sec\left(dx + c\right) + a} \sec\left(dx + c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sec(d*x + c)^2, x)
```

3.937 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=324

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)(2aC+15Ab-5bB+9bC)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{15b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b^2*(5*A + 3*C) + a*(5*b*B - 2*a*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(15*b^3*d) + (2*(a - b)*Sqrt[a + b]*(15*A*b - 5*b*B + 2*a*C + 9*b*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(15*b^2*d) + (2*(5*b*B - 2*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c +
 d*x])/(15*b*d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.592077, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4082, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)(2aC+15Ab-5bB+9bC)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^2d}-\frac{2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b^2*(5*A + 3*C) + a*(5*b*B - 2*a*C))*Cot[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(15*b^3*d) + (2*(a - b)*Sqrt[a + b]*(15*A*b - 5*b*B + 2*a*C + 9*b*C)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(15*b^2*d) + (2*(5*b*B - 2*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c +
 d*x])/(15*b*d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \cdots$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\sec(c+dx)}\tan(c+\cdots}{15bd}$$

$$= \frac{2(5bB-2aC)\sqrt{a+b\sec(c+dx)}\tan(c+\cdots}{15bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(3b^2(5A+3C)+a(5bB\cdots\right.}{\cdots}$$

Mathematica [A] time = 20.5021, size = 579, normalized size = 1.79

$$\frac{4\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt[Cos[c + d*x]*Sec[(c
 + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((a + b)*(-((15*A*b^2 + 5*a*b*B
 - 2*a^2*C + 9*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])
```

```
 + b*(15*A*b + 5*b*B - 2*a*C + 9*b*C)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] - (15*A*b^2 + 5*a*b*B -
2*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan
[(c + d*x)/2]))/(15*b^2*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x]
)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3
/2)*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((4*(15*A*b^2 + 5*a*b*B - 2*a^2*C + 9*b^2*C
)*Sin[c + d*x])/(15*b^2) + (4*Sec[c + d*x]*(5*b*B*Sin[c + d*x] + a*C*Sin[c
+ d*x]))/(15*b) + (4*C*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.821, size = 3344, normalized size = 10.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(2*C*cos(d*x+c)^3*a^3+15*A*cos(d*x+c)^4*a*b^2-15*A*cos(d*x+c)^3*a*b
^2-5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*C*sin(d*x+c)*cos(d*x+c)^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*C*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^3+9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*B*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*b^3-5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5*B*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2+5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*C*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^2-2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+7*C*sin(d*x+c)*cos(d*x+c
```

```
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+
2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*C*sin(d*x+c)*cos(d*x+c)^3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+7*C
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2+15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*A*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*A
*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a*b^2+C*cos(d*x+c)^4*a^2*b+9*C*cos(d*x+c)^4*a*b^2-2*C*cos(d*
x+c)^3*a^2*b-5*C*cos(d*x+c)^3*a*b^2-4*C*cos(d*x+c)*a*b^2+5*B*cos(d*x+c)^3*a
*b^2-10*B*cos(d*x+c)^2*a*b^2+5*B*cos(d*x+c)^4*a^2*b+5*B*cos(d*x+c)^4*a*b^2-
5*B*cos(d*x+c)^3*a^2*b+C*cos(d*x+c)^2*a^2*b-2*C*cos(d*x+c)^4*a^3+9*C*cos(d*
x+c)^3*b^3-6*C*cos(d*x+c)^2*b^3+5*B*cos(d*x+c)^3*b^3-5*B*cos(d*x+c)*b^3+2*C
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3-9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+15*A*cos(d*x+c)^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*A*cos(
d*x+c)^2*b^3+15*A*cos(d*x+c)^3*b^3+15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-15*A*cos(d*x+c)^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*b^3+15*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^3-15*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-3*C*b^3)/(b+a*cos(d*x+c
))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec(dx+c)^3 + B\sec(dx+c)^2 + A\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(
d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec(c+dx)}\left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/
2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)*
sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sec(d*x + c), x)
```

3.938 $\int \sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx\right.$

Optimal. Leaf size=366

$$\frac{2\sqrt{a+b}\cot(c+dx)((a-b)(3B-C)+3Ab)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a +
 b]*(3*A*b + (a - b)*(3*B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*A*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/d + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*
d)
```

Rubi [A] time = 0.406283, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\cot(c+dx)((a-b)(3B-C)+3Ab)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{2A\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a +
 b]*(3*A*b + (a - b)*(3*B - C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*A*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/d + (2*C*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*
d)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{1}{2}(3Ab$$

$$= \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\left(\frac{1}{2}(-3$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bB+aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{s}}}{\sqrt{}}\right.\right.}{3b^2d}$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bB+aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{s}}}{\sqrt{}}\right.\right.}{3b^2d}$$

Mathematica [B] time = 24.4541, size = 5313, normalized size = 14.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),
x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.51, size = 2334, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/b*(-1+cos(d*x+c))^2*(-3*B*cos(d*x+c)^3*a*b+3*B*cos(d*x+c)^2*a*b-C*cos
(d*x+c)^3*a*b-C*cos(d*x+c)^2*a*b+2*C*cos(d*x+c)*a*b+3*B*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-C
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b+C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-
3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-3*B*cos(d*x+c)^2*b^2-6*A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*a*b-3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-6*A*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-C*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^2+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+3*B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*B*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
```

```
in(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*A*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2+b^2*C+3*B*cos(d*x+c)*b^2-C*cos(d*x+c)^3*a^2+C*cos(d*x+c)^2*a^2-C*cos(d*x+
c)^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x
+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec\left(c+dx\right)}\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a),
 x)
```

3.939 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=362

$$\frac{\sqrt{a+b}\cot(c+dx)(2aC+Ab+2bB-2bC)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\sqrt{a+}$$

```
[Out] ((a - b)*Sqrt[a + b]*(A - 2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A*b + 2*
b*B + 2*a*C - 2*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (Sqrt[a + b]*(A*b + 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(a*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.411707, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4094, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\cot(c+dx)(2aC+Ab+2bB-2bC)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{\sqrt{a+b}(2aB+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(A - 2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A*b + 2*
b*B + 2*a*C - 2*b*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (Sqrt[a + b]*(A*b + 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(a*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{1}{2}($$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}(b($$

$$= \frac{(a-b)\sqrt{a+b}(A-2C)\cot(c+dx)E\left(\sin\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}(A-2C)\cot(c+dx)E\left(\sin\right.}{}$$

Mathematica [B] time = 18.7006, size = 930, normalized size = 2.57

$$\frac{2C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{\sqrt{a+b\sec(c+dx)}\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(aA\tan^5\left(\frac{1}{2}(c+dx)\right)-Ab\tan^5\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[a + b*Sec[c + d*x]]*S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*Tan[(c + d*x)/2] + A*b*Tan[(c + d*x
)/2] - 2*a*C*Tan[(c + d*x)/2] - 2*b*C*Tan[(c + d*x)/2] - 2*a*A*Tan[(c + d*x
)/2]^3 + 4*a*C*Tan[(c + d*x)/2]^3 + a*A*Tan[(c + d*x)/2]^5 - A*b*Tan[(c + d
*x)/2]^5 - 2*a*C*Tan[(c + d*x)/2]^5 + 2*b*C*Tan[(c + d*x)/2]^5 - 2*A*b*Elli
pticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - 4*a*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - 2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*B*EllipticPi[-1, -Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + (a + b)*(A - 2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(A*b + a*(B - C) -
b*(B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.49, size = 2153, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(-A*cos(d*x+c)^3*a-A*cos(d*x+c)^2*b+2*A*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*C*b-2*C*
cos(d*x+c)^2*a-2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*b-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b*sin(d*x+c)-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-4*B*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-2*C*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+2*C*cos
(d*x+c)*a-2*C*cos(d*x+c)*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d
```

```
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-2*C*cos(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*a+A*cos(d*x+c)^2*a+A*cos(d*x+c)*b-2*C*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+2*C*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+2*C*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-2*B*cos(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*b-2*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*b+2*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+2*C*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b)*(cos(d*x+c)+1)^2*
((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c), x)
```

3.940 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=435

$$\frac{\sqrt{a+b}\cot(c+dx)(2a(A+2B+4C)+Ab)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4ad}+\ldots$$

```
[Out] ((a - b)*Sqrt[a + b]*(A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
A*b + 2*a*(A + 2*B + 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(A*b^2 - 4*
a*b*B - 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((A*b + 4*a*
B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.773208, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\cot(c+dx)\left(-4a^2(A+2C)-4abB+Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
A*b + 2*a*(A + 2*B + 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(A*b^2 - 4*
a*b*B - 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((A*b + 4*a*
B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{2d}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4ad}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4ad}$$

$$= \frac{(a-b)\sqrt{a+b}(Ab+4aB)\cot(c+dx)E\Big(}{}$$

$$= \frac{(a-b)\sqrt{a+b}(Ab+4aB)\cot(c+dx)E\Big(}{}$$

Mathematica [C] time = 20.025, size = 1842, normalized size = 4.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (A*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d
*x]]*(-(a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]) - A*b^2*Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2] - 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2] - 4*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 2*a*A*b*Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]^3 + 8*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x
)/2]^3 - a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + A*b^2*Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]^5 - 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d
*x)/2]^5 + 4*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + (8*I)*a^2*A*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*A*b^2*EllipticPi[-((
a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] + (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] - (2*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)] + (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/
2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(A*b + 4*a*B)*EllipticE[I*ArcSinh[Sq
```

```
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(a - b)*(A*b + 2*a*(A + 2*C))*Elliptic
F[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*Sqrt[(-a + b)/(a + b)]*d*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(
-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.417, size = 2626, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a*(-1+cos(d*x+c))^2*(-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*A*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*si
n(d*x+c)-4*B*cos(d*x+c)^2*a*b+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*B*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B*cos(d*x+c)^3*a^2+8*C*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b-8*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)
*a*b-3*A*cos(d*x+c)^3*a*b+A*cos(d*x+c)^2*a*b+2*A*cos(d*x+c)*a*b+4*B*cos(d*x
+c)^2*a^2-2*A*cos(d*x+c)^4*a^2-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*A*cos(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*co
s(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*
```

```
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2-A*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-2*A*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+2*A*cos(d*x+c)^2*
a^2-A*cos(d*x+c)^2*b^2+A*cos(d*x+c)*b^2+8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-4*B*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-8*C*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-4*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2-8*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*a*b*sin(d*x+c)-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2)*(cos(d*x+c)+1)^2*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)^2\sec(dx+c)^2 + B\cos(dx+c)^2\sec(dx+c) + A\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^2, x)
```

3.941 $\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=538

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(-4a^2(4A+3B+6C)-2ab(A+3B)+3Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\right.\right.}{24a^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b^2 - 6*a*b*B - 8*a^2*(2*A + 3*C))*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(24*a^2*b*d) - (Sqrt[a + b]*(3*A*b^2 - 2*a*b*(A + 3*B) - 4*a^2*(4*A + 3*B
+ 6*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(24*a^2*d) - (Sqrt[a + b]*(A*b^3 + 8*a^3*B - 2*a*b^2*
B + 4*a^2*b*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^3*d) - ((3*A*b^2 - 6*
a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a^2*d
) + ((A*b + 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*
a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.23094, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sin(c+dx)\left(-8a^2(2A+3C)-6abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}}{24a^2d}-\frac{\sqrt{a+b}\cot(c+dx)\left(-4a^2(4A+3B+6C)-2ab(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b^2 - 6*a*b*B - 8*a^2*(2*A + 3*C))*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(24*a^2*b*d) - (Sqrt[a + b]*(3*A*b^2 - 2*a*b*(A + 3*B) - 4*a^2*(4*A + 3*B
+ 6*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(24*a^2*d) - (Sqrt[a + b]*(A*b^3 + 8*a^3*B - 2*a*b^2*
B + 4*a^2*b*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^3*d) - ((3*A*b^2 - 6*
a*b*B - 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a^2*d
) + ((A*b + 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*
a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c}{3d}$$

$$= \frac{(Ab+6aB)\cos(c+dx)\sqrt{a+b\sec(c+}}{12ad}$$

$$= -\frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+}}{24a^2d}$$

$$= -\frac{\left(3Ab^2-6abB-8a^2(2A+3C)\right)\sqrt{a+}}{24a^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(3Ab^2-6abB-8a^2(2A\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(3Ab^2-6abB-8a^2(2A\right.}{}$$

Mathematica [A] time = 14.475, size = 544, normalized size = 1.01

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{(6aB+Ab)\sin(2(c+dx))}{24a}+\frac{1}{12}A\sin(c+dx)+\frac{1}{12}A\sin(3(c+dx))\right)}{d} - \frac{\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\left(b(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((A*Sin[c + d*x])/12 + ((A*b + 6*a*B)*Sin[2*(c +
d*x)])/(24*a) + (A*Sin[3*(c + d*x)])/12))/d - (Cos[c + d*x]^2*Sqrt[a + b*Se
c[c + d*x]]*(-(a*(a + b)*(-3*A*b^2 + 6*a*b*B + 8*a^2*(2*A + 3*C))*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(3*A*b^2 - 6*a*b*(
A + B) + 4*a^2*(4*A + 3*B + 6*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + 3*(A*b^3 + 8*a^3*B - 2*a*b^2*B + 4*a^2*b*(A + 2*C))*((a - b)*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(-3*A*b^2 + 6*a*b*B + 8*a^2
*(2*A + 3*C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*
Sec[c + d*x]*Tan[(c + d*x)/2]))/(24*a^3*d*(b + a*Cos[c + d*x])*(Cos[c + d*x
]*Sec[(c + d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.478, size = 3761, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/24/d/a^2*(-1+cos(d*x+c))^2*(-24*C*cos(d*x+c)^3*a^3+24*B*a^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-6*A*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)
+A*cos(d*x+c)^3*a*b^2-16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*sin(d*x+c)+24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+12*B*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-48*C*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*s
in(d*x+c)-24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))-48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-24*C*a^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+12*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*a*b^2-48*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a^3*sin(d*x+c)-48*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-3*A*cos(d*x+c)*b
^3-8*A*cos(d*x+c)^5*a^3-8*A*cos(d*x+c)^3*a^3+16*A*cos(d*x+c)^2*a^3-12*B*cos
(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b-24*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+48*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-6*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2-6*B*cos(d*x+c)^2*a*b^2-18*B*cos(d*x+c)^3*a^2*
b-24*C*cos(d*x+c)^2*a^2*b+28*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b-24*A*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b
-16*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*cos(d*x+c)*b+3*A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*A*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+12*B*co
s(d*x+c)^2*a^3+24*C*cos(d*x+c)^2*a^3-12*B*cos(d*x+c)^4*a^3-3*A*cos(d*x+c)^2
*a*b^2+2*A*cos(d*x+c)*a*b^2-6*A*cos(d*x+c)^2*a^2*b+16*A*cos(d*x+c)*a^2*b-10
*A*cos(d*x+c)^4*a^2*b+3*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))+3*A*cos(d*x+c)^2*b^3+6*B*cos(d*x+c)^2*a^2*b+
12*B*cos(d*x+c)*a^2*b+6*B*cos(d*x+c)*a*b^2+24*C*cos(d*x+c)*a^2*b-6*A*Ellipt
```

```
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*cos(d*x+c)-16*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+28*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-24*A*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-16*A*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+
3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*b^3+3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-24*C*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+48*C*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-12*B*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+
c)-6*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b*sin(d*x+c)-6*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2*sin(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^3, x)
```

3.942 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2} \left(A + B\sec(c + dx) + C s\right.$

Optimal. Leaf size=628

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(-6a^2b^2(33A-11B+24C)+4a^3b(22B-9C)-48a^4C-3ab^3(627A-143B+471C)+3\right.}{3465b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(88*a^4*b*B + 363*a^2*b^3*B + 1617*b^5*B - 48*a^5*C
 - 18*a^3*b^2*(11*A + 6*C) + 6*a*b^4*(451*A + 348*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b
^5*d) - (2*(a - b)*Sqrt[a + b]*(4*a^3*b*(22*B - 9*C) - 48*a^4*C - 6*a^2*b^2
*(33*A - 11*B + 24*C) + 3*b^4*(275*A - 539*B + 225*C) - 3*a*b^3*(627*A - 14
3*B + 471*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(44*a^3*b*B - 968*a*b^3*B - 2
4*a^4*C - 75*b^4*(11*A + 9*C) - 3*a^2*b^2*(33*A + 19*C))*Sqrt[a + b*Sec[c +
 d*x]]*Tan[c + d*x])/(3465*b^3*d) + (2*(33*a^2*b*B + 539*b^3*B - 18*a^3*C +
 6*a*b^2*(132*A + 101*C))*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(3465*b^2*d) + (2*(99*A*b^2 + 110*a*b*B + 3*a^2*C + 81*b^2*C)*Sec[c + d*
x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b*d) + (2*(11*b*B + 3*a*C)
*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(99*d) + (2*C*Sec[c
+ d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(11*d)
```

Rubi [A] time = 2.62743, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4096, 4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\sec^2(c+dx)\left(3a^2C+110abB+99Ab^2+81b^2C\right)\sqrt{a+b\sec(c+dx)}}{693bd}+\frac{2\tan(c+dx)\sec(c+dx)\left(33a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(88*a^4*b*B + 363*a^2*b^3*B + 1617*b^5*B - 48*a^5*C
 - 18*a^3*b^2*(11*A + 6*C) + 6*a*b^4*(451*A + 348*C))*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b
^5*d) - (2*(a - b)*Sqrt[a + b]*(4*a^3*b*(22*B - 9*C) - 48*a^4*C - 6*a^2*b^2
*(33*A - 11*B + 24*C) + 3*b^4*(275*A - 539*B + 225*C) - 3*a*b^3*(627*A - 14
3*B + 471*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(44*a^3*b*B - 968*a*b^3*B - 2
4*a^4*C - 75*b^4*(11*A + 9*C) - 3*a^2*b^2*(33*A + 19*C))*Sqrt[a + b*Sec[c +
 d*x]]*Tan[c + d*x])/(3465*b^3*d) + (2*(33*a^2*b*B + 539*b^3*B - 18*a^3*C +
 6*a*b^2*(132*A + 101*C))*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(3465*b^2*d) + (2*(99*A*b^2 + 110*a*b*B + 3*a^2*C + 81*b^2*C)*Sec[c + d*
x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b*d) + (2*(11*b*B + 3*a*C)
*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(99*d) + (2*C*Sec[c
+ d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(11*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
```

```
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\frac{2C\sec^3(c+dx)(a+b\sec(c+dx))^{3/2}}{11d}$$

$$=\frac{2(11bB+3aC)\sec^3(c+dx)\sqrt{a+b\sec}}{99d}$$

$$=\frac{2\left(99Ab^2+110abB+3a^2C+81b^2C\right)}{}$$

$$=\frac{2\left(33a^2bB+539b^3B-18a^3C+6ab^2(\right.}{}$$

$$=-\frac{2\left(44a^3bB-968ab^3B-24a^4C-75b\right.}{}$$

$$=-\frac{2\left(44a^3bB-968ab^3B-24a^4C-75b\right.}{}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(88a^4bB+363a^2b^3B\right.}{}$$

Mathematica [A] time = 21.6226, size = 1087, normalized size = 1.73

$$\frac{(a+b\sec(c+dx))^{3/2}\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{4}{99}(11bB\sin(c+dx)+12aC\sin(c+dx))\sec^4(c+dx)+\frac{4}{11}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (4*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*((a + b)*(-88*a^4*b*B - 363*a^2*b^3*B - 1617
*b^5*B + 48*a^5*C + 18*a^3*b^2*(11*A + 6*C) - 6*a*b^4*(451*A + 348*C))*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)] + b*(a + b)*(-48*a^4*C + 4*a^3*b*(22*B + 9*C) - 6*a^2*
b^2*(33*A + 11*B + 24*C) + 3*b^4*(275*A + 539*B + 225*C) + 3*a*b^3*(627*A +
 143*B + 471*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (-88*a^4*b*B - 363*a^2*b^3*B -
1617*b^5*B + 48*a^5*C + 18*a^3*b^2*(11*A + 6*C) - 6*a*b^4*(451*A + 348*C))*
Tan[(c + d*x)/2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2)
))/(3465*b^4*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*Sec[c + d*x]^(7/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
```

```
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)
]) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2)*((-4*(198*a^3*A*b^2 - 2706*a*A*b^4 - 88*a^4*b*B - 363*a^2*b^3*
B - 1617*b^5*B + 48*a^5*C + 108*a^3*b^2*C - 2088*a*b^4*C)*Sin[c + d*x])/(34
65*b^4) + (4*Sec[c + d*x]^4*(11*b*B*Sin[c + d*x] + 12*a*C*Sin[c + d*x]))/99
 + (4*Sec[c + d*x]^3*(99*A*b^2*Sin[c + d*x] + 110*a*b*B*Sin[c + d*x] + 3*a^
2*C*Sin[c + d*x] + 81*b^2*C*Sin[c + d*x]))/(693*b) + (4*Sec[c + d*x]^2*(792
*a*A*b^2*Sin[c + d*x] + 33*a^2*b*B*Sin[c + d*x] + 539*b^3*B*Sin[c + d*x] -
18*a^3*C*Sin[c + d*x] + 606*a*b^2*C*Sin[c + d*x]))/(3465*b^2) + (4*Sec[c +
d*x]*(99*a^2*A*b^2*Sin[c + d*x] + 825*A*b^4*Sin[c + d*x] - 44*a^3*b*B*Sin[c
 + d*x] + 968*a*b^3*B*Sin[c + d*x] + 24*a^4*C*Sin[c + d*x] + 57*a^2*b^2*C*S
in[c + d*x] + 675*b^4*C*Sin[c + d*x]))/(3465*b^3) + (4*b*C*Sec[c + d*x]^4*T
an[c + d*x])/11))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x]))
```

Maple [B] time = 3.062, size = 7208, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{6}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{5}+Aa\sec\left(dx+c\right)^{3}+\left(Ba+Ab\right)\sec\left(dx+c\right)^{4}\right)\sqrt{b\sec\left(dx+c\right)+a},x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^6 + (C*a + B*b)*sec(d*x + c)^5 + A*a*sec(d*x + c
)^3 + (B*a + A*b)*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sec(d*x + c)^3, x)
```

3.943 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=505

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(6a^2b(3B-C)-8a^3C-3ab^2(21A-57B+13C)+3b^3(63A-25B+49C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{315b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 21*b^4*(9*A +
7*C) - 3*a^2*b^2*(21*A + 11*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) + (2*(a - b)*Sqrt
[a + b]*(6*a^2*b*(3*B - C) - 8*a^3*C - 3*a*b^2*(21*A - 57*B + 13*C) + 3*b^3
*(63*A - 25*B + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(18*a^2*b*B - 75*b^3*
B - 8*a^3*C - 3*a*b^2*(21*A + 13*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(315*b^2*d) + (2*(63*A*b^2 - 18*a*b*B + 8*a^2*C + 49*b^2*C)*(a + b*Sec[c +
 d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*b*B - 4*a*C)*(a + b*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x
])^(5/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 1.32093, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4092, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(8a^2C-18abB+63Ab^2+49b^2C\right)(a+b\sec(c+dx))^{3/2}}{315b^2d}-\frac{2\tan(c+dx)\left(18a^2bB-8a^3C-3ab^2(21A+13C\right.}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*b*B - 246*a*b^3*B - 8*a^4*C - 21*b^4*(9*A +
7*C) - 3*a^2*b^2*(21*A + 11*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) + (2*(a - b)*Sqrt
[a + b]*(6*a^2*b*(3*B - C) - 8*a^3*C - 3*a*b^2*(21*A - 57*B + 13*C) + 3*b^3
*(63*A - 25*B + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(18*a^2*b*B - 75*b^3*
B - 8*a^3*C - 3*a*b^2*(21*A + 13*C))*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(315*b^2*d) + (2*(63*A*b^2 - 18*a*b*B + 8*a^2*C + 49*b^2*C)*(a + b*Sec[c +
 d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*b*B - 4*a*C)*(a + b*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*x
])^(5/2)*Tan[c + d*x])/(9*b*d)
```


Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
```

```
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(}{9bd}$$

$$= \frac{2(9bB-4aC)(a+b\sec(c+dx))^{5/2}\tan(c}{63b^2d}$$

$$= \frac{2\left(63Ab^2-18abB+8a^2C+49b^2C\right)(a+}{315b^2d}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-3ab^2(21A\right.}{315}$$

$$= -\frac{2\left(18a^2bB-75b^3B-8a^3C-3ab^2(21A\right.}{315}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(18a^3bB-246ab^3B-8a^4\right.}{}$$

Mathematica [B] time = 26.4849, size = 4186, normalized size = 8.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((4*(63*a^2*A*b^2 + 189*A*b^4 - 18*a^3*b*B + 246*a*b^3*B + 8*a^4*C
+ 33*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(315*b^3) + (4*Sec[c + d*x]^3*(9*
b*B*Sin[c + d*x] + 10*a*C*Sin[c + d*x]))/63 + (4*Sec[c + d*x]^2*(63*A*b^2*S
in[c + d*x] + 72*a*b*B*Sin[c + d*x] + 3*a^2*C*Sin[c + d*x] + 49*b^2*C*Sin[c
 + d*x]))/(315*b) + (4*Sec[c + d*x]*(126*a*A*b^2*Sin[c + d*x] + 9*a^2*b*B*S
in[c + d*x] + 75*b^3*B*Sin[c + d*x] - 4*a^3*C*Sin[c + d*x] + 88*a*b^2*C*Sin
[c + d*x]))/(315*b^2) + (4*b*C*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*C
os[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*((-2*a
^2*A)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (6*A*b^2)/(5*Sqrt[b
 + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^3*B)/(35*b*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (164*a*b*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (22*a^2*C)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (16*a^4*C)/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (1
4*b^2*C)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a^3*A*Sqrt[S
ec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (2*a*A*b*Sqrt[Sec[c + d*x]])
/(5*Sqrt[b + a*Cos[c + d*x]]) - (62*a^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b +
 a*Cos[c + d*x]]) + (4*a^4*B*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c +
 d*x]]) + (10*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (16
*a^5*C*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b + a*Cos[c + d*x]]) - (62*a^3*C*S
qrt[Sec[c + d*x]])/(315*b*Sqrt[b + a*Cos[c + d*x]]) + (26*a*b*C*Sqrt[Sec[c
+ d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) - (2*a^3*A*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) - (6*a*A*b*Cos[2*(c + d*x)]*Sqr
t[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (164*a^2*B*Cos[2*(c + d*x)]
*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*B*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) - (16*a^5*C*Cos[
2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b + a*Cos[c + d*x]]) - (22*a
^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]])
 - (14*a*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x
]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A +
```

B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((a + b)*((-18*a^3*b*B + 246*a*b^3*B + 8
*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2
*(21*A + 57*B + 13*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt
[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-18*a^3
*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C)
)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(
315*b^3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(7/2)*((-2*a*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*((a + b)*((-18*a^3*b*B + 246*a*b^3*B +
 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b
^2*(21*A + 57*B + 13*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-18*a
^3*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*
C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))
/(315*b^3*(b + a*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((a + b)*((-18*a^3*b*B +
246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*(3*B
 + C) + 3*a*b^2*(21*A + 57*B + 13*C) + 3*b^3*(63*A + 25*B + 49*C))*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c +
d*x] + (-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^
2*(21*A + 11*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(
c + d*x)/2]))/(105*b^3*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))
 - (2*((a + b)*((-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) +
 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] - b*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(21*A + 57*B + 13*C) + 3*b^3
*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C
+ 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x
]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*b
^3*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-18*a^3*b*
B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - a*(-18*a^3*b*B + 2
46*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Cos[c
+ d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - (-18*a^3*b*B + 24
6*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*(b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + 2*(-18*a^3
*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C)
)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]^2 +
 (3*(a + b)*((-18*a^3*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*
a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - b*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(21*A + 57*B + 13*C) + 3*b^3*(6
3*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*S
qrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c
+ d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + ((a + b)*((-18*a^3*b*B + 2
46*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*(3*B
+ C) + 3*a*b^2*(21*A + 57*B + 13*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(3/2)*Sec[c + d*x]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((

```
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + (a + b)*(Cos[c + d*x]
*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)]*Sec[c + d*x]*(-(b*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(21*A + 57
*B + 13*C) + 3*b^3*(63*A + 25*B + 49*C))*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((-18*a
^3*b*B + 246*a*b^3*B + 8*a^4*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*
C))*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*S
qrt[1 - Tan[(c + d*x)/2]^2])) + (a + b)*((-18*a^3*b*B + 246*a*b^3*B + 8*a^4
*C + 21*b^4*(9*A + 7*C) + 3*a^2*b^2*(21*A + 11*C))*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] - b*(8*a^3*C - 6*a^2*b*(3*B + C) + 3*a*b^2*(21
*A + 57*B + 13*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))
/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)))
```

Maple [B] time = 2.063, size = 5945, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{5}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{4}+Aa\sec\left(dx+c\right)^{2}+\left(Ba+Ab\right)\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^5 + (C*a + B*b)*sec(d*x + c)^4 + A*a*sec(d*x + c
)^2 + (B*a + A*b)*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sec(d*x + c)^2, x)
```

3.944 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=406

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(6a^2C+3ab(35A-7B+19C)-b^2(35A-63B+25C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellipti}}{105b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 2*a*b^2*(70*A +
41*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(105*b^3*d) + (2*(a - b)*Sqrt[a + b]*(6*a^2*C + 3*a*b*
(35*A - 7*B + 19*C) - b^2*(35*A - 63*B + 25*C))*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) +
 (2*(35*A*b^2 + 21*a*b*B - 6*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan
[c + d*x])/(105*b*d) + (2*(7*b*B - 2*a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(35*b*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.830225, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4082, 4002, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(-6a^2C+21abB+35Ab^2+25b^2C\right)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(6a^2C+3ab(35A-7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*b*B + 63*b^3*B - 6*a^3*C + 2*a*b^2*(70*A +
41*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(105*b^3*d) + (2*(a - b)*Sqrt[a + b]*(6*a^2*C + 3*a*b*
(35*A - 7*B + 19*C) - b^2*(35*A - 63*B + 25*C))*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) +
 (2*(35*A*b^2 + 21*a*b*B - 6*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec[c + d*x]]*Tan
[c + d*x])/(105*b*d) + (2*(7*b*B - 2*a*C)*(a + b*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(35*b*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{2C(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd}+$$

$$=\frac{2(7bB-2aC)(a+b\sec(c+dx))^{3/2}\tan}{35bd}$$

$$=\frac{2\left(35Ab^2+21abB-6a^2C+25b^2C\right)\sqrt{}}{105bd}$$

$$=\frac{2\left(35Ab^2+21abB-6a^2C+25b^2C\right)\sqrt{}}{105bd}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(21a^2bB+63b^3B-6a\right.}{}$$

Mathematica [B] time = 26.0178, size = 3724, normalized size = 9.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((-4*(-140*a*A*b^2 - 21*a^2*b*B - 63*b^3*B + 6*a^3*C - 82*a*b^2*C)*
```

Sin[c + d*x])/(105*b^2) + (4*Sec[c + d*x]^2*(7*b*B*Sin[c + d*x] + 8*a*C*Sin
[c + d*x]))/35 + (4*Sec[c + d*x]*(35*A*b^2*Sin[c + d*x] + 42*a*b*B*Sin[c +
d*x] + 3*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b) + (4*b*C*Sec[
c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])) + (4*((-8*a*A*b)/(3*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (2*a^2*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (6*b^2*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^3*C)
/(35*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (164*a*b*C)/(105*Sqrt
[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*
Sqrt[b + a*Cos[c + d*x]]) + (2*A*b^2*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[
c + d*x]]) - (2*a^3*B*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) +
(2*a*b*B*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (62*a^2*C*Sqrt[
Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*C*Sqrt[Sec[c + d*x]]
)/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) + (10*b^2*C*Sqrt[Sec[c + d*x]])/(21*Sqr
t[b + a*Cos[c + d*x]]) - (8*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*S
qrt[b + a*Cos[c + d*x]]) - (2*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5
*b*Sqrt[b + a*Cos[c + d*x]]) - (6*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]]
)/(5*Sqrt[b + a*Cos[c + d*x]]) - (164*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d
*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*C*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
(2*(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C))*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a
+ b)*(-6*a^2*C + 3*a*b*(35*A + 7*B + 19*C) + b^2*(35*A + 63*B + 25*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21
*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C))*Cos[c + d*x]*(b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c
+ d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c +
d*x)/2]^2]*Sec[c + d*x]^(7/2)*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*S
in[c + d*x]*(2*(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A +
41*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + 2*b*(a + b)*(-6*a^2*C + 3*a*b*(35*A + 7*B + 19*C) + b^2*(35*A + 63*B
+ 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + (-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C))*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*(b
+ a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-21*a^2*b*B - 63*b^3*B + 6*
a^3*C - 2*a*b^2*(70*A + 41*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^2*C + 3*a*b*(35*A + 7*B + 1
9*C) + b^2*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + (-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2
*(70*A + 41*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) +
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-21*a^2*b*B - 63*b^3*B + 6*a^3*
C - 2*a*b^2*(70*A + 41*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^4)/2 + ((a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C
))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[
c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])] + (b*(a + b)*(-6*a^2*C + 3*a*b*(35*A + 7*B + 19*C) + b^2*(35*A +
63*B + 25*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x
])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*

```
x]/(1 + Cos[c + d*x])] + ((a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b
^2*(70*A + 41*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^
2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(
-6*a^2*C + 3*a*b*(35*A + 7*B + 19*C) + b^2*(35*A + 63*B + 25*C))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d
*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))] - a*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*
a*b^2*(70*A + 41*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
d*x)/2] - (-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C))*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-21*a^2*
b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C))*Cos[c + d*x]*(b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-6*a^2*C + 3*a
*b*(35*A + 7*B + 19*C) + b^2*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(
c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)]) + ((a + b)*(-21*a^2*b*B - 63*b^3*B + 6*a^3*C - 2*a*b^2*(7
0*A + 41*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c
 + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^2*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(2*(a + b)*(-21*a^2*b*B - 63*b
^3*B + 6*a^3*C - 2*a*b^2*(70*A + 41*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^2*C + 3*a*b*(35*A
+ 7*B + 19*C) + b^2*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*b*B - 63*b^3*B + 6*a^3*C
- 2*a*b^2*(70*A + 41*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + C
os[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^2*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.217, size = 4527, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/105/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-140*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-105*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-105*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2+35*A*cos(d*x+c)^2*b^4+21*B*cos(d*x+c)*b^4+6*C*cos(d*x+c)^5
*a^4+175*A*cos(d*x+c)^3*a*b^3+63*B*cos(d*x+c)^3*a^2*b^2+63*B*cos(d*x+c)^2*a
*b^3-63*B*cos(d*x+c)^4*b^4+42*B*cos(d*x+c)^3*b^4-35*A*cos(d*x+c)^4*b^4-25*C
*cos(d*x+c)^4*b^4+140*A*cos(d*x+c)^4*a^2*b^2-140*A*cos(d*x+c)^5*a^2*b^2-35*
A*cos(d*x+c)^5*a*b^3-140*A*cos(d*x+c)^4*a*b^3+21*B*cos(d*x+c)^4*a^3*b-42*B*
cos(d*x+c)^5*a^2*b^2-63*B*cos(d*x+c)^5*a*b^3-3*C*cos(d*x+c)^5*a^3*b-82*C*co
s(d*x+c)^5*a^2*b^2-25*C*cos(d*x+c)^5*a*b^3+55*C*cos(d*x+c)^4*a^2*b^2-82*C*c
```

os(d*x+c)^4*a*b^3+140*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+140*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-140*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+140*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+21*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+21*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-21*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-84*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+21*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+21*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-21*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-84*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+6*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-51*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-82*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+82*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+82*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+140*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-51*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-82*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+82*C*cos(d*x+

```
c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^2+82*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-21*B*cos(d*x+c)^4*a^2*b^2-21*B*cos(d
*x+c)^5*a^3*b+6*C*cos(d*x+c)^4*a^3*b-3*C*cos(d*x+c)^3*a^3*b+68*C*cos(d*x+c)
^3*a*b^3+27*C*cos(d*x+c)^2*a^2*b^2+39*C*cos(d*x+c)*a*b^3+63*B*cos(d*x+c)^4*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*b^4-63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-63*B*cos(d*x+c)^3*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-3
5*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^4-35*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-25*C*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-6*C*cos
(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^4-25*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-6*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-6*C*cos(d*x+c)^
4*a^4+10*C*cos(d*x+c)^2*b^4+15*C*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x
+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^4+\left(Ca+Bb\right)\sec\left(dx+c\right)^3+Aa\sec\left(dx+c\right)+\left(Ba+Ab\right)\sec\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a},$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^4 + (C*a + B*b)*sec(d*x + c)^3 + A*a*sec(d*x + c
) + (B*a + A*b)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sec(d*x + c), x)
```

3.945 $\int (a+b\sec(c+dx))^{3/2} \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=443

$$\frac{2\sqrt{a+b}\cot(c+dx)\left(3a^2(5B-C)+2ab(15A-10B+6C)-b^2(15A-5B+9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellip}}{15bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(15*A*b^2 + 20*a*b*B + 3*a^2*C + 9*b^2*C)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(15*b^2*d) + (2*Sqrt[a + b]*(3*a^2*(5*B - C) + 2*a*b*(15*A - 10*B + 6*
C) - b^2*(15*A - 5*B + 9*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a*A*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/d + (2*(5*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan
[c + d*x])/(15*d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.660718, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\cot(c+dx)\left(3a^2(5B-C)+2ab(15A-10B+6C)-b^2(15A-5B+9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin\right.}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(15*A*b^2 + 20*a*b*B + 3*a^2*C + 9*b^2*C)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(15*b^2*d) + (2*Sqrt[a + b]*(3*a^2*(5*B - C) + 2*a*b*(15*A - 10*B + 6*
C) - b^2*(15*A - 5*B + 9*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a*A*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/d + (2*(5*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Tan
[c + d*x])/(15*d) + (2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int\sqrt{a+b\,\mathrm{se}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2C(a}{}$$

$$= \frac{2(5bB+3aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2C(a}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(15Ab^2+20abB+3a^2C+9b^2C\right)\mathrm{co}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(15Ab^2+20abB+3a^2C+9b^2C\right)\mathrm{co}}{}$$

Mathematica [B] time = 26.1719, size = 6972, normalized size = 15.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.812, size = 3927, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/15/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(3*C*cos(d*x+c)^3*a^3-15*A*cos(d*x+c)^4*a*b^2+15*A*cos(d*x+c)^3*a*b^2-
15*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b+20*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*C
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3+9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*B*cos(d*x+
c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^3-5*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b^3+20*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-20*B*cos(d*x+c)^3*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2+20*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b+20*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-20*B*cos(d*x+c)^2*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2+3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-3*C*sin(d*x+c)*cos(d
*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b-12*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2+3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
```

```
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
2-3*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b-12*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-30*A*cos(d*x+c)^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2
+15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b^2+15*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*B*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b-30*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+15*A*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b-30*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-6*C*cos(d*x+c)^4*a^2*b-9*C*cos(d*x+c)^4*a
*b^2-3*C*cos(d*x+c)^3*a^2*b+9*C*cos(d*x+c)*a*b^2-20*B*cos(d*x+c)^3*a*b^2+25
*B*cos(d*x+c)^2*a*b^2-20*B*cos(d*x+c)^4*a^2*b-5*B*cos(d*x+c)^4*a*b^2+20*B*c
os(d*x+c)^3*a^2*b+9*C*cos(d*x+c)^2*a^2*b-3*C*cos(d*x+c)^4*a^3-9*C*cos(d*x+c
)^3*b^3+6*C*cos(d*x+c)^2*b^3-5*B*cos(d*x+c)^3*b^3+5*B*cos(d*x+c)*b^3+3*C*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^3+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-30*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+15*A*cos(d*x
+c)^2*b^3-15*A*cos(d*x+c)^3*b^3-15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+15*A*cos(d*x+c)^2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b
^3-15*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*b^3+15*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+3*C*b^3)/(b+a*cos(d*x+c))/
cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
```

```
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*(A + B*sec(c + d*x) + C*sec(c + d*x)**
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
), x)
```

3.946 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=426

$$\frac{\sqrt{a+b}\cot(c+dx)\left(6a^2C+ab(3A+12B-8C)+2b^2(3A-3B+C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a}}{}\right.\right.}{3bd}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*a*A - 6*b*B - 8*a*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (Sqrt[
a + b]*(a*b*(3*A + 12*B - 8*C) + 6*a^2*C + 2*b^2*(3*A - 3*B + C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(3*b*d) - (Sqrt[a + b]*(3*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (
A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*A - 2*C)*Sqrt[a + b*Se
c[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.664433, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4094, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\cot(c+dx)\left(6a^2C+ab(3A+12B-8C)+2b^2(3A-3B+C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(3*a*A - 6*b*B - 8*a*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (Sqrt[
a + b]*(a*b*(3*A + 12*B - 8*C) + 6*a^2*C + 2*b^2*(3*A - 3*B + C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(3*b*d) - (Sqrt[a + b]*(3*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (
A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*A - 2*C)*Sqrt[a + b*Se
c[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} + \int \sqrt{}$$

$$= \frac{A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3A}{}$$

$$= \frac{A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3A}{}$$

$$= \frac{(a-b)\sqrt{a+b}(3aA-6bB-8aC)\cot(c+}{}$$

$$= \frac{(a-b)\sqrt{a+b}(3aA-6bB-8aC)\cot(c+}{}$$

Mathematica [B] time = 26.1361, size = 7722, normalized size = 18.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.644, size = 3361, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/3/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(6*B*cos(d*x+c)^3*a*b-6*B*cos(d*x+c)^2*a*b+2*C*cos(d*x+c)^3*a*b+8*C*cos(
d*x+c)^2*a*b-10*C*cos(d*x+c)*a*b-3*A*cos(d*x+c)^3*a^2-6*B*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
+8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b-8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-8*C*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+6*B*cos(d*x+c)^2*b^2+6*C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
```

d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2+18*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+12*B*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-12*A*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b+18*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b-12*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b+3*A*cos(d*x+c)^3*a*b-3*A*cos(d*x+c)^2*a*b+3*A*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*A*c
os(d*x+c)^4*a^2+6*C*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+3*A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-6*B*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^2+3*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^2-6*B*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+12*B*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+1
2*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^2+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-8*C*sin(d
*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2+2*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b^2-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-6*B*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+6*
B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+6*A*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*b^2-2*b^2*C-6*B*cos(d*x+c)*b^2+8*C*cos(d*x+c)^3*a^2-8*C*cos(d*x+c)^2*a
^2+2*C*cos(d*x+c)^2*b^2)/sin(d*x+c)^5/(b+a*cos(d*x+c))/cos(d*x+c)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right)(b \sec(dx+c) + a)^{\frac{3}{2}} \cos(dx+c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c), x)
```

3.947 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=442

$$\frac{\sqrt{a+b}\cot(c+dx)(2a(A+2B+8C)+b(5A+8B-8C))\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{4d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(5*A*b + 4*a*B - 8*b*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[
a + b]*(b*(5*A + 8*B - 8*C) + 2*a*(A + 2*B + 8*C))*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (S
qrt[a + b]*(3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(4*a*d) + ((3*A*b + 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) +
(A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.810519, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4094, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2(A+2C)+12abB+3Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(5*A*b + 4*a*B - 8*b*C)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[
a + b]*(b*(5*A + 8*B - 8*C) + 2*a*(A + 2*B + 8*C))*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (S
qrt[a + b]*(3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(4*a*d) + ((3*A*b + 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) +
(A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c}{2d}$$

$$= \frac{(3Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+a}{4d}$$

$$= \frac{(3Ab+4aB)\sqrt{a+b\sec(c+dx)}\sin(c+a}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}(5Ab+4aB-8bC)\cot(c+}{}$$

$$= \frac{(a-b)\sqrt{a+b}(5Ab+4aB-8bC)\cot(c+}{}$$

Mathematica [B] time = 23.9665, size = 4520, normalized size = 10.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(4*b*C*Sin[c + d*x] + (a*A*Sin[2*
(c + d*x)])/2))/(d*(b + a*Cos[c + d*x])) - (Sqrt[Cos[c + d*x]*Sec[(c + d*x)
/2]^2]*((a^2*A)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^2)/(
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a*b*B)/(Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*C)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (2*b^2*C)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (7*a
*A*b*Sqrt[Sec[c + d*x]])/(4*Sqrt[b + a*Cos[c + d*x]]) + (a^2*B*Sqrt[Sec[c +
 d*x]])/Sqrt[b + a*Cos[c + d*x]] + (2*b^2*B*Sqrt[Sec[c + d*x]])/Sqrt[b + a*
Cos[c + d*x]] + (2*a*b*C*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] + (5*
a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(4*Sqrt[b + a*Cos[c + d*x]]) + (
a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] - (2*a*
b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]])*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(-(a*(a + b)*(5*A*
b + 4*a*B - 8*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec
[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) +
b*(a + b)*(3*A*b + 2*a*(A + 2*B - 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)] + (3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*((a - b)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(5*A*b + 4*a*B - 8*b*C)*(b + a*Cos
[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*
x)/2]))/(2*a*d*(b + a*Cos[c + d*x])^2*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*
x]^(3/2)*(-(Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*S
ec[c + d*x]]*Sin[c + d*x]*(-(a*(a + b)*(5*A*b + 4*a*B - 8*b*C)*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(3*A*b + 2*a*(A + 2*B
 - 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*A*b^2 +
12*a*b*B + 4*a^2*(A + 2*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)] - a*(5*A*b + 4*a*B - 8*b*C)*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(4*(b + a*Cos[c + d*x])
^(3/2)*(Sec[(c + d*x)/2]^2)^(3/2)) + (3*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^
2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-(a*(a + b)*(5*A
*b + 4*a*B - 8*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Se
c[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) +
 b*(a + b)*(3*A*b + 2*a*(A + 2*B - 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] + (3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*((a - b)*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(5*A*b + 4*a*B - 8*b*C)*(b + a*Co
s[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d
*x)/2]))/(4*a*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) - (Sqrt[
Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-
(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])*(-(a*(a + b)*(5*A*b + 4*a*B - 8*b*C)*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(3*A*b + 2*a*(A + 2*B - 4*C))*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*A*b^2 + 12*a*b*B + 4*a^
2*(A + 2*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
```

```
+ 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(5*A*
b + 4*a*B - 8*b*C)*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(
3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(4*a*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c
+ d*x)/2]^2)^(3/2)) - (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-(a*(a + b)*(
5*A*b + 4*a*B - 8*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
) + b*(a + b)*(3*A*b + 2*a*(A + 2*B - 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] + (3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*((a - b)*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(5*A*b + 4*a*B - 8*b*C)*(b + a
*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c
+ d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*
x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(4*a*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c +
 d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) - (Sqrt[Cos[c + d*
x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-(a*(5*A*b +
4*a*B - 8*b*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(3/2)*Sec[c + d*x])/2 - a*(a + b)*(5*A*b + 4*a*B - 8*b*C)*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[(
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + b*(a +
 b)*(3*A*b + 2*a*(A + 2*B - 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)]*Tan[(c + d*x)/2] + (3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*((a -
 b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - (3*a*(5*
A*b + 4*a*B - 8*b*C)*(b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2
]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 - (a*(a + b)*(5*A*b + 4*
a*B - 8*b*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c +
d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + (b*(a + b)*(3*A*b + 2*a*(A + 2*B -
 4*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2
]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)]) + ((3*A*b^2 + 12*a*b*B + 4*a^2*(A + 2*C))*
((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*(-(
(a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)]) + (b*(a + b)*(3*A*b + 2*a*(A + 2*B - 4*C))*Sec[(c
 + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])
- (a*(a + b)*(5*A*b + 4*a*B - 8*b*C)*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/
(a + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]) + (3*A*b^2 + 12*a*b*B + 4*a^2*(A
 + 2*C))*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]*(((a - b)*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (a*Sec[(c + d*x)/2]^2)/(Sqrt[
1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((a - b)*Tan[(c +
 d*x)/2]^2)/(a + b)])) + a^2*(5*A*b + 4*a*B - 8*b*C)*(Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]*Tan[c + d*x] - a*(5*A*b + 4*a*B - 8*b*C)
*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*
Tan[(c + d*x)/2]*Tan[c + d*x]))/(2*a*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x
)/2]^2)^(3/2)))))/2
```

Maple [B] time = 0.651, size = 3595, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(-8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*B*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*B*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*
x+c)-6*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-5*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^2*sin(d*x+c)-4*B*cos(d*x+c)^2*a*b-8*C*cos(d*x+c)^2*a*b+8*C*co
s(d*x+c)*a*b+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^2*sin(d*x+c)+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-16*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*C*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b+16*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*C*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B*cos(d*x+c)^3*a^
2+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-24*B*cos(d*x+c)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-5*A*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b+4*B*cos(d*x+c)*a*b-7*A*cos(d*x+c)^3*a*b+5*A*cos(d*x+c)^2*a*b+2*
A*cos(d*x+c)*a*b+4*B*cos(d*x+c)^2*a^2-2*A*cos(d*x+c)^4*a^2-8*B*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b^2-8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2-5*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
```

```
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*A*cos(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*cos(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)-6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2-5*A*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-2*A*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-8*C*cos(d*x+c)*b^2+8
*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^2+8*b^2*C+2*A*cos(d*x+c)^2*a^2-5*A*cos(d*x+c)^2*b^2+5*A*co
s(d*x+c)*b^2+16*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b*sin(d*x+c)-4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+8*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-4*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-24*B
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*s
in(d*x+c)+8*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos
(d*x+c)*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2)*(cos(d*x+c)+1)^2*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^2\sec(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2\sec(dx+c)^2 + Aa\cos(dx+c)^2 + (Ba+Ab)\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2*se
c(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c)^2*sec(d*x + c)
)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^2, x)
```

3.948 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=540

$$\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2(4A+3B+6C)+2ab(7A+15B+24C)+3Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\right.}{24ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(3*A*b^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(24*a*b*d) + (Sqrt[a + b]*(3*A*b^2 + 4*a^2*(4*A + 3*B + 6*C) + 2*a*b*(7*A
+ 15*B + 24*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(24*a*d) + (Sqrt[a + b]*(A*b^3 - 8*a^3*B - 6*
a*b^2*B - 12*a^2*b*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((3*A*b
^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
24*a*d) + ((A*b + 2*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(4*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.31337, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sin(c+dx)\left(8a^2(2A+3C)+30abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}}{24ad}+\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2(4A+3B+6C)+2ab(7A+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(3*A*b^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(24*a*b*d) + (Sqrt[a + b]*(3*A*b^2 + 4*a^2*(4*A + 3*B + 6*C) + 2*a*b*(7*A
+ 15*B + 24*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(24*a*d) + (Sqrt[a + b]*(A*b^3 - 8*a^3*B - 6*
a*b^2*B - 12*a^2*b*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((3*A*b
^2 + 30*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
24*a*d) + ((A*b + 2*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(4*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\sin(}{3d}$$

$$= \frac{(Ab+2aB)\cos(c+dx)\sqrt{a+b\sec(c+d}}{4d}$$

$$= \frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a+b}}{24ad}$$

$$= \frac{\left(3Ab^2+30abB+8a^2(2A+3C)\right)\sqrt{a+b}}{24ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3Ab^2+30abB+8a^2(2A\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(3Ab^2+30abB+8a^2(2A\right.}{}$$

Mathematica [B] time = 24.6843, size = 5054, normalized size = 9.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.501, size = 4138, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*(-1+cos(d*x+c))^2*(24*C*cos(d*x+c)^3*a^3-24*B*a^3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-6*A*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+1
7*A*cos(d*x+c)^3*a*b^2+16*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+36*B*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+144*C
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b
*sin(d*x+c)+24*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))
```

```
/sin(d*x+c),((a-b)/(a+b))^(1/2))+48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+24*C*a^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+36*B*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*a*b^2+48*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+144*C*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-3*A*cos(d*x+c
)*b^3+8*A*cos(d*x+c)^5*a^3+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+12*B*
cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b-48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+24*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-96*C*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*a^2*b+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+48*C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^2+30*B*cos(d*x+c)^2*a*b^2+42*B*cos(d*x+c)^3*a^2*b+24*C*cos(d*x+c)^2*a^2
*b-52*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*cos(d*x+c)*b+72*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b+16*A*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b
+3*A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a+14*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-12*B*cos(d*x+c)^2*a^3-24*C*co
s(d*x+c)^2*a^3+12*B*cos(d*x+c)^4*a^3-3*A*cos(d*x+c)^2*a*b^2-14*A*cos(d*x+c)
*a*b^2-6*A*cos(d*x+c)^2*a^2*b-16*A*cos(d*x+c)*a^2*b+22*A*cos(d*x+c)^4*a^2*b
+3*A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))+3*A*cos(d*x+c)^2*b^3-30*B*cos(d*x+c)^2*a^2*b-12*B*cos(d*x+c)*a^2*
b-30*B*cos(d*x+c)*a*b^2-24*C*cos(d*x+c)*a^2*b-6*A*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+16*A*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)-52*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*b+72*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+16*A*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+3*A*(cos(d*x+c)/(co
```

```
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3
+3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2
*sin(d*x+c)+14*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2*sin(d*x+c)+48*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+24*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-96*C*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+12*B*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-48*
B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*si
n(d*x+c)+30*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b*sin(d*x+c)+30*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^2*sin(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*
x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + B \sec (dx+c) + A\right)(b \sec (dx+c) + a)^{\frac{3}{2}} \cos (dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb \cos (dx+c)^3 \sec (dx+c)^3 + (Ca+Bb) \cos (dx+c)^3 \sec (dx+c)^2 + Aa \cos (dx+c)^3 + (Ba+Ab) \cos (dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^3*se
c(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A*b)*cos(d*x + c)^3*sec(d*x + c)
)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^3, x)
```

3.949 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=650

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(-4a^2b(39A+28B+60C)-8a^3(9A+16B+12C)-6ab^2(A+4B)+9Ab^3\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(}{}}}{192a^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A +
20*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(192*a^2*b*d) - (Sqrt[a + b]*(9*A*b^3 - 6*a*b^2*(A + 4
*B) - 8*a^3*(9*A + 16*B + 12*C) - 4*a^2*b*(39*A + 28*B + 60*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(192*a^2*d) - (Sqrt[a + b]*(3*A*b^4 + 96*a^3*b*B - 8*a*b^3*B + 24*a^2*b^
2*(A + 2*C) + 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^3*d) - ((
9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A + 20*C))*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(192*a^2*d) + ((3*A*b^2 + 56*a*b*B + 12*a^2*(3*A + 4
*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*a*d) + ((3*A*b
 + 8*a*B)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (A
*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 1.90629, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sin(c+dx)\left(-12a^2b(13A+20C)-128a^3B-24ab^2B+9Ab^3\right)\sqrt{a+b\sec(c+dx)}}{192a^2d}+\frac{\sin(c+dx)\cos(c+dx)\left(12a^2(3A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A +
20*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(192*a^2*b*d) - (Sqrt[a + b]*(9*A*b^3 - 6*a*b^2*(A + 4
*B) - 8*a^3*(9*A + 16*B + 12*C) - 4*a^2*b*(39*A + 28*B + 60*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(192*a^2*d) - (Sqrt[a + b]*(3*A*b^4 + 96*a^3*b*B - 8*a*b^3*B + 24*a^2*b^
2*(A + 2*C) + 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^3*d) - ((
9*A*b^3 - 128*a^3*B - 24*a*b^2*B - 12*a^2*b*(13*A + 20*C))*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(192*a^2*d) + ((3*A*b^2 + 56*a*b*B + 12*a^2*(3*A + 4
*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*a*d) + ((3*A*b
 + 8*a*B)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (A
*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\sin(}{4d}$$

$$= \frac{(3Ab+8aB)\cos^2(c+dx)\sqrt{a+b\sec(c+}}{24d}$$

$$= \frac{\left(3Ab^2+56abB+12a^2(3A+4C)\right)\cos(c}{96ad}$$

$$= -\frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13A\right.}{192}$$

$$= -\frac{\left(9Ab^3-128a^3B-24ab^2B-12a^2b(13A\right.}{192}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(9Ab^3-128a^3B-24ab^2\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(9Ab^3-128a^3B-24ab^2\right.}{}$$

Mathematica [A] time = 16.9781, size = 761, normalized size = 1.17

$$\frac{\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{\sin(2(c+dx))\left(48a^2A+48a^2C+56abB+3Ab^2\right)}{96a}+\frac{1}{48}(8aB+9A\right.}{d(a\cos(c+dx)+b)(A\cos(2c+2dx)+A+2B\cos(c+dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*(((9*A*b + 8*a*B)*Sin[c + d*x])/48 + ((48*a^2*A + 3*A*b^2 + 56*a*b*
B + 48*a^2*C)*Sin[2*(c + d*x)])/(96*a) + ((9*A*b + 8*a*B)*Sin[3*(c + d*x)])
/48 + (a*A*Sin[4*(c + d*x)])/16))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])) - (Cos[c + d*x]^5*(a + b*Sec[c + d*x])^(3
/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-(a*(a + b)*(-9*A*b^3 + 128*a^
3*B + 24*a*b^2*B + 12*a^2*b*(13*A + 20*C))*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)]) + b*(a + b)*(9*A*b^3 - 6*a*b^2*(3*A + 4*B) + 8*a^3*(9
*A + 16*B + 12*C) + 12*a^2*b*(7*A + 4*(B + 3*C)))*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)] + 3*(3*A*b^4 + 96*a^3*b*B - 8*a*b^3*B + 24*a^2*
b^2*(A + 2*C) + 16*a^4*(3*A + 4*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] - a*(-9*A*b^3 + 128*a^3*B + 24*a*b^2*B + 12*a^2*b*(13*A + 20*
C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*
x]*Tan[(c + d*x)/2]))/(96*a^3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.738, size = 5474, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Cb\cos(dx+c)^4\sec(dx+c)^3 + (Ca+Bb)\cos(dx+c)^4\sec(dx+c)^2 + Aa\cos(dx+c)^4 + (Ba+Ab)\cos(d\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^4*se
c(d*x + c)^2 + A*a*cos(d*x + c)^4 + (B*a + A*b)*cos(d*x + c)^4*sec(d*x + c)
)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^4, x)
```

3.950 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=610

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(-15a^2b^2(33A-121B+19C)+10a^3b(11B-3C)-40a^4C+6ab^3(660A-209B+505C)\right.}{3465b^3d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*
C - 15*a^3*b^2*(33*A + 17*C) - 15*a*b^4*(319*A + 247*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(346
5*b^4*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3*b*(11*B - 3*C) - 40*a^4*C - 15*a^
2*b^2*(33*A - 121*B + 19*C) - 3*b^4*(275*A - 539*B + 225*C) + 6*a*b^3*(660*
A - 209*B + 505*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d) - (2*(110*a^3*b*B - 1254*a*b
^3*B - 40*a^4*C - 75*b^4*(11*A + 9*C) - 15*a^2*b^2*(33*A + 19*C))*Sqrt[a +
b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*b*B - 539*b^3*B -
40*a^3*C - 5*a*b^2*(99*A + 67*C))*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(3465*b^2*d) + (2*(99*A*b^2 - 22*a*b*B + 8*a^2*C + 81*b^2*C)*(a + b*Sec[c +
 d*x])^(5/2)*Tan[c + d*x])/(693*b^2*d) + (2*(11*b*B - 4*a*C)*(a + b*Sec[c +
 d*x])^(7/2)*Tan[c + d*x])/(99*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*
x])^(7/2)*Tan[c + d*x])/(11*b*d)
```

Rubi [A] time = 2.10959, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4092, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(8a^2C-22abB+99Ab^2+81b^2C\right)(a+b\sec(c+dx))^{5/2}}{693b^2d}-\frac{2\tan(c+dx)\left(110a^2bB-40a^3C-5ab^2(99A\right.}{3465b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*b*B - 3069*a^2*b^3*B - 1617*b^5*B - 40*a^5*
C - 15*a^3*b^2*(33*A + 17*C) - 15*a*b^4*(319*A + 247*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(346
5*b^4*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3*b*(11*B - 3*C) - 40*a^4*C - 15*a^
2*b^2*(33*A - 121*B + 19*C) - 3*b^4*(275*A - 539*B + 225*C) + 6*a*b^3*(660*
A - 209*B + 505*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d) - (2*(110*a^3*b*B - 1254*a*b
^3*B - 40*a^4*C - 75*b^4*(11*A + 9*C) - 15*a^2*b^2*(33*A + 19*C))*Sqrt[a +
b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*b*B - 539*b^3*B -
40*a^3*C - 5*a*b^2*(99*A + 67*C))*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(3465*b^2*d) + (2*(99*A*b^2 - 22*a*b*B + 8*a^2*C + 81*b^2*C)*(a + b*Sec[c +
 d*x])^(5/2)*Tan[c + d*x])/(693*b^2*d) + (2*(11*b*B - 4*a*C)*(a + b*Sec[c +
 d*x])^(7/2)*Tan[c + d*x])/(99*b^2*d) + (2*C*Sec[c + d*x]*(a + b*Sec[c + d*
x])^(7/2)*Tan[c + d*x])/(11*b*d)
```


Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec(c+dx)(a+b\sec(c+dx))^{7/2}\text{t}}{11bd}$$

$$= \frac{2(11bB-4aC)(a+b\sec(c+dx))^{7/2}\text{t}}{99b^2d}$$

$$= \frac{2\left(99Ab^2-22abB+8a^2C+81b^2C\right)(}{693b^2}$$

$$= -\frac{2\left(110a^2bB-539b^3B-40a^3C-5ab\right.}{}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-7\right.}{}$$

$$= -\frac{2\left(110a^3bB-1254ab^3B-40a^4C-7\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(110a^4bB-3069a^2b^3B\right.}{}$$

Mathematica [A] time = 21.8605, size = 1090, normalized size = 1.79

$$\frac{\cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{4}{99}\left(11B\sin(c+dx)b^2+23aC\sin(c+dx)b\right)\text{se}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (-4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt
[(1 - Tan[(c + d*x)/2]^2)^(-1)]*((a + b)*(-110*a^4*b*B + 3069*a^2*b^3*B + 1
617*b^5*B + 40*a^5*C + 15*a^3*b^2*(33*A + 17*C) + 15*a*b^4*(319*A + 247*C))
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] - b*(a + b)*(40*a^4*C - 10*a^3*b*(11*B + 3*C) + 1
5*a^2*b^2*(33*A + 121*B + 19*C) + 3*b^4*(275*A + 539*B + 225*C) + 6*a*b^3*(
660*A + 209*B + 505*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (-110*a^4*b*B + 3069*a^2
*b^3*B + 1617*b^5*B + 40*a^5*C + 15*a^3*b^2*(33*A + 17*C) + 15*a*b^4*(319*A
 + 247*C))*Tan[(c + d*x)/2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*
x)/2]^2)^2)))/(3465*b^3*d*(b + a*Cos[c + d*x])^(5/2)*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2)*(1 + Tan[(c + d*x)/2]^2)^(3/
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2)]) + (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*((4*(495*a^3*A*b^2 + 4785*a*A*b^4 - 110*a^4*b*B + 3
069*a^2*b^3*B + 1617*b^5*B + 40*a^5*C + 255*a^3*b^2*C + 3705*a*b^4*C)*Sin[c
 + d*x])/(3465*b^3) + (4*Sec[c + d*x]^4*(11*b^2*B*Sin[c + d*x] + 23*a*b*C*S
in[c + d*x]))/99 + (4*Sec[c + d*x]^3*(99*A*b^2*Sin[c + d*x] + 209*a*b*B*Sin
[c + d*x] + 113*a^2*C*Sin[c + d*x] + 81*b^2*C*Sin[c + d*x]))/693 + (4*Sec[c
 + d*x]^2*(1485*a*A*b^2*Sin[c + d*x] + 825*a^2*b*B*Sin[c + d*x] + 539*b^3*B
```

```
*Sin[c + d*x] + 15*a^3*C*Sin[c + d*x] + 1145*a*b^2*C*Sin[c + d*x]))/(3465*b
) + (4*Sec[c + d*x]*(1485*a^2*A*b^2*Sin[c + d*x] + 825*A*b^4*Sin[c + d*x] +
 55*a^3*b*B*Sin[c + d*x] + 1793*a*b^3*B*Sin[c + d*x] - 20*a^4*C*Sin[c + d*x
] + 1025*a^2*b^2*C*Sin[c + d*x] + 675*b^4*C*Sin[c + d*x]))/(3465*b^2) + (4*
b^2*C*Sec[c + d*x]^4*Tan[c + d*x])/11))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 3.07, size = 7208, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \sec(dx+c)^6 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^5 + Aa^2\sec(dx+c)^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^4 + \left(B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^6 + (2*C*a*b + B*b^2)*sec(d*x + c)^5 + A*a^2*s
ec(d*x + c)^2 + (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^4 + (B*a^2 + 2*A*a*b
)*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*sec(d*x + c)^2, x)
```

3.951 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(A + B\sec(c+dx) + C\sec^2\right.$

Optimal. Leaf size=502

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\left(15a^2b(21A-3B+11C)+10a^3C-6ab^2(28A-60B+19C)+3b^3(63A-25B+49C)\right)\sqrt{\frac{b(1-}{}}}{315b^2d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 21*b^4*(9*A
+ 7*C) + 3*a^2*b^2*(161*A + 93*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) + (2*(a - b)*S
qrt[a + b]*(10*a^3*C + 15*a^2*b*(21*A - 3*B + 11*C) - 6*a*b^2*(28*A - 60*B
+ 19*C) + 3*b^3*(63*A - 25*B + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^2*d) + (2*(45*a^
2*b*B + 75*b^3*B - 10*a^3*C + 6*a*b^2*(28*A + 19*C))*Sqrt[a + b*Sec[c + d*x
]]*Tan[c + d*x])/(315*b*d) + (2*(63*A*b^2 + 45*a*b*B - 10*a^2*C + 49*b^2*C)
*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*b*B - 2*a*C)*(a
 + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*C*(a + b*Sec[c + d*x])
^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 1.25594, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4082, 4002, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(-10a^2C+45abB+63Ab^2+49b^2C\right)(a+b\sec(c+dx))^{3/2}}{315bd}+\frac{2\tan(c+dx)\left(45a^2bB-10a^3C+6ab^2(28A\right.}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*b*B + 435*a*b^3*B - 10*a^4*C + 21*b^4*(9*A
+ 7*C) + 3*a^2*b^2*(161*A + 93*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) + (2*(a - b)*S
qrt[a + b]*(10*a^3*C + 15*a^2*b*(21*A - 3*B + 11*C) - 6*a*b^2*(28*A - 60*B
+ 19*C) + 3*b^3*(63*A - 25*B + 49*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^2*d) + (2*(45*a^
2*b*B + 75*b^3*B - 10*a^3*C + 6*a*b^2*(28*A + 19*C))*Sqrt[a + b*Sec[c + d*x
]]*Tan[c + d*x])/(315*b*d) + (2*(63*A*b^2 + 45*a*b*B - 10*a^2*C + 49*b^2*C)
*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*b*B - 2*a*C)*(a
 + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*C*(a + b*Sec[c + d*x])
^(7/2)*Tan[c + d*x])/(9*b*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \cdots$$

$$= \frac{2(9bB-2aC)(a+b\sec(c+dx))^{5/2}\tan\cdots}{63bd}$$

$$= \frac{2\left(63Ab^2+45abB-10a^2C+49b^2C\right)(\cdots}{315bd\cdots}$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+6ab^2(28\cdots\right.}{3\cdots}$$

$$= \frac{2\left(45a^2bB+75b^3B-10a^3C+6ab^2(28\cdots\right.}{3\cdots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(45a^3bB+435ab^3B-\cdots\right.}{\cdots}$$

Mathematica [B] time = 26.6775, size = 4220, normalized size = 8.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((4*(483*a^2*A*b^2 + 189*A*b^4 + 45*a^3*b*B + 435*a*b^3*B - 10*a^4*
C + 279*a^2*b^2*C + 147*b^4*C)*Sin[c + d*x])/(315*b^2) + (4*Sec[c + d*x]^3*
(9*b^2*B*Sin[c + d*x] + 19*a*b*C*Sin[c + d*x]))/63 + (4*Sec[c + d*x]^2*(63*
A*b^2*Sin[c + d*x] + 135*a*b*B*Sin[c + d*x] + 75*a^2*C*Sin[c + d*x] + 49*b^
2*C*Sin[c + d*x]))/315 + (4*Sec[c + d*x]*(231*a*A*b^2*Sin[c + d*x] + 135*a^
2*b*B*Sin[c + d*x] + 75*b^3*B*Sin[c + d*x] + 5*a^3*C*Sin[c + d*x] + 163*a*b
^2*C*Sin[c + d*x]))/(315*b) + (4*b^2*C*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*
(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
 (4*((-46*a^2*A*b)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (6*A*
b^3)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a^3*B)/(7*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (58*a*b^2*B)/(21*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^4*C)/(63*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (62*a^2*b*C)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (14*b^3*C)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (16*a^3
*A*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (16*a*A*b^2*Sqrt[Sec
[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (2*a^4*B*Sqrt[Sec[c + d*x]])/(7
*b*Sqrt[b + a*Cos[c + d*x]]) - (4*a^2*b*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b +
a*Cos[c + d*x]]) + (10*b^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x
]]) - (248*a^3*C*Sqrt[Sec[c + d*x]])/(315*Sqrt[b + a*Cos[c + d*x]]) + (4*a^
5*C*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) + (76*a*b^2*C*Sqr
t[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) - (46*a^3*A*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (6*a*A*b^2*Cos[2*(c +
 d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (2*a^4*B*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (58*a^2*b*B*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (62*a^3
*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (4*
a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]
) - (14*a*b^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c +
 d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((a + b)*((-45*a^3*b*B - 435*a*b^3*B
 + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*EllipticE[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-10*a^3*C + 15*a^2*b*(21*A + 3*
B + 11*C) + 6*a*b^2*(28*A + 60*B + 19*C) + 3*b^3*(63*A + 25*B + 49*C))*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[
c + d*x] + (-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a
^2*b^2*(161*A + 93*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4
*Tan[(c + d*x)/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(9/2)*
((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*((a + b)*((-45*a^3
*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*
C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-10*a^3*C + 1
5*a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A + 60*B + 19*C) + 3*b^3*(63*A +
25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos[c
+ d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)]*Sec[c + d*x] + (-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4
*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(315*b^2*(b + a*Cos[c + d*x])^(3/2)*(
Sec[(c + d*x)/2]^2)^(3/2)) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(
c + d*x)/2]*((a + b)*((-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A +
 7*C) - 3*a^2*b^2*(161*A + 93*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + b*(-10*a^3*C + 15*a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A +
 60*B + 19*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/
```

```
2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (-45*a^3*b*B - 4
35*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(105*b^
2*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (2*((a + b)*((-45*
a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A +
93*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-10*a^3*C
+ 15*a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A + 60*B + 19*C) + 3*b^3*(63*A
 + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*(Cos
[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2)/(a + b)]*Sec[c + d*x] + (-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*
b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Si
n[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*b^2*S
qrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-45*a^3*b*B -
435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^6)/2 - a*(-45*a^3*b*B - 435
*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Cos[c
+ d*x]*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] - (-45*a^3*b*B - 43
5*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*(b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sin[c + d*x]*Tan[(c + d*x)/2] + 2*(-45*a
^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 9
3*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]
^2 + (3*(a + b)*((-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C)
 - 3*a^2*b^2*(161*A + 93*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + b*(-10*a^3*C + 15*a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A + 60*B
 + 19*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*(-(Sec[(c + d*x)/2]^2*Sin
[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/2 + ((a + b
)*((-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(
161*A + 93*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-1
0*a^3*C + 15*a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A + 60*B + 19*C) + 3*b
^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*(-((a*Sec[(c + d*x
)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)]) + (a + b)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*((b*(-10*a^3*C + 15*
a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A + 60*B + 19*C) + 3*b^3*(63*A + 25
*B + 49*C))*Sec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((
a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((-45*a^3*b*B - 435*a*b^3*B + 10*a^4
*C - 21*b^4*(9*A + 7*C) - 3*a^2*b^2*(161*A + 93*C))*Sec[(c + d*x)/2]^2*Sqrt
[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]
)) + (a + b)*((-45*a^3*b*B - 435*a*b^3*B + 10*a^4*C - 21*b^4*(9*A + 7*C) -
3*a^2*b^2*(161*A + 93*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + b*(-10*a^3*C + 15*a^2*b*(21*A + 3*B + 11*C) + 6*a*b^2*(28*A + 60*B +
19*C) + 3*b^3*(63*A + 25*B + 49*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x]*Tan[c + d*x]))/(315*b^2*Sq
rt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2))))
```

Maple [B] time = 2.056, size = 6163, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec\left(dx+c\right)^5+\left(2\,Cab+Bb^2\right)\sec\left(dx+c\right)^4+Aa^2\sec\left(dx+c\right)+\left(Ca^2+2\,Bab+Ab^2\right)\sec\left(dx+c\right)^3+\left(Ba\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^5 + (2*C*a*b + B*b^2)*sec(d*x + c)^4 + A*a^2*s
ec(d*x + c) + (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^3 + (B*a^2 + 2*A*a*b)*
sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*sec(d*x + c), x)
```

3.952 $\int (a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right.$

Optimal. Leaf size=521

$$\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2b(315A-161B+135C)+15a^3(7B-C)-ab^2(245A-119B+145C)+b^3(35A-63B+25C)\right.}{105bd}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 5*a*b^2*(49*A
+ 29*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(105*b^2*d) + (2*Sqrt[a + b]*(15*a^3*(7*B - C) + b^3
*(35*A - 63*B + 25*C) + a^2*b*(315*A - 161*B + 135*C) - a*b^2*(245*A - 119*
B + 145*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(105*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/d + (2*(35*A*b^2 + 56*a*b*B + 15*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*b*B + 5*a*C)*(a + b*Sec[c + d*x])^
(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/
(7*d)
```

Rubi [A] time = 0.971456, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\tan(c+dx)\left(15a^2C+56abB+35Ab^2+25b^2C\right)\sqrt{a+b\sec(c+dx)}}{105d}+\frac{2\sqrt{a+b}\cot(c+dx)\left(a^2b(315A-161B+135\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 5*a*b^2*(49*A
+ 29*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(105*b^2*d) + (2*Sqrt[a + b]*(15*a^3*(7*B - C) + b^3
*(35*A - 63*B + 25*C) + a^2*b*(315*A - 161*B + 135*C) - a*b^2*(245*A - 119*
B + 145*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(105*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/d + (2*(35*A*b^2 + 56*a*b*B + 15*a^2*C + 25*b^2*C)*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*b*B + 5*a*C)*(a + b*Sec[c + d*x])^
(3/2)*Tan[c + d*x])/(35*d) + (2*C*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/
(7*d)
```


Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{2}{7}\int (a+b$$

$$=\frac{2(7bB+5aC)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35d}+$$

$$=\frac{2\left(35Ab^2+56abB+15a^2C+25b^2C\right)\sqrt{a+b\sec(c}}{105d}$$

$$=\frac{2\left(35Ab^2+56abB+15a^2C+25b^2C\right)\sqrt{a+b\sec(c}}{105d}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(161a^2bB+63b^3B+15a^3C+5ab\right.}{}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(161a^2bB+63b^3B+15a^3C+5ab\right.}{}$$

Mathematica [B] time = 21.2756, size = 1405, normalized size = 2.7

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] (-4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt
[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(245*a^2*A*b^2*Tan[(c + d*x)/2] + 245*a*A*b
^3*Tan[(c + d*x)/2] + 161*a^3*b*B*Tan[(c + d*x)/2] + 161*a^2*b^2*B*Tan[(c +
 d*x)/2] + 63*a*b^3*B*Tan[(c + d*x)/2] + 63*b^4*B*Tan[(c + d*x)/2] + 15*a^4
*C*Tan[(c + d*x)/2] + 15*a^3*b*C*Tan[(c + d*x)/2] + 145*a^2*b^2*C*Tan[(c +
d*x)/2] + 145*a*b^3*C*Tan[(c + d*x)/2] - 490*a^2*A*b^2*Tan[(c + d*x)/2]^3 -
 322*a^3*b*B*Tan[(c + d*x)/2]^3 - 126*a*b^3*B*Tan[(c + d*x)/2]^3 - 30*a^4*C
*Tan[(c + d*x)/2]^3 - 290*a^2*b^2*C*Tan[(c + d*x)/2]^3 + 245*a^2*A*b^2*Tan[
(c + d*x)/2]^5 - 245*a*A*b^3*Tan[(c + d*x)/2]^5 + 161*a^3*b*B*Tan[(c + d*x)
/2]^5 - 161*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 63*a*b^3*B*Tan[(c + d*x)/2]^5 -
63*b^4*B*Tan[(c + d*x)/2]^5 + 15*a^4*C*Tan[(c + d*x)/2]^5 - 15*a^3*b*C*Tan[
(c + d*x)/2]^5 + 145*a^2*b^2*C*Tan[(c + d*x)/2]^5 - 145*a*b^3*C*Tan[(c + d*
x)/2]^5 + 210*a^3*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] + 210*a^3*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b
)*(161*a^2*b*B + 63*b^3*B + 15*a^3*C + 5*a*b^2*(49*A + 29*C))*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - b*(-15*a^3*(7*A - 7*B - C) + b^3*(35*A + 63*B + 25*C) + a^2*b
*(315*A + 161*B + 135*C) + a*b^2*(245*A + 119*B + 145*C))*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(
c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)]))/(105*b*d*(b + a*Cos[c + d*x])^(5/2)*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2
]^2)]) + (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*((4*(245*a*A*b^2 + 161*a^2*b*B + 63*b^3*B + 15*a^3*C + 145
*a*b^2*C)*Sin[c + d*x])/(105*b) + (4*Sec[c + d*x]^2*(7*b^2*B*Sin[c + d*x] +
```

```
 15*a*b*C*Sin[c + d*x]))/35 + (4*Sec[c + d*x]*(35*A*b^2*Sin[c + d*x] + 77*a
*b*B*Sin[c + d*x] + 45*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/105 + (
4*b^2*C*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 1.259, size = 5138, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\text{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
), x)
```

3.953 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=505

$$\frac{\sqrt{a+b}\cot(c+dx)\left(a^2b(15A+90B-46C)+30a^3C+2ab^2(45A-35B+17C)-2b^3(15A-5B+9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{15bd}$$

```
[Out] -((a - b)*Sqrt[a + b]*(70*a*b*B - a^2*(15*A - 46*C) + 6*b^2*(5*A + 3*C))*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(15*b*d) + (Sqrt[a + b]*(a^2*b*(15*A + 90*B - 46*C) + 30*a^3*C
- 2*b^3*(15*A - 5*B + 9*C) + 2*a*b^2*(45*A - 35*B + 17*C))*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[
(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(1
5*b*d) - (a*Sqrt[a + b]*(5*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (A*(a
 + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d - (b*(15*a*A - 10*b*B - 16*a*C)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) - (b*(5*A - 2*C)*(a + b*Sec[c +
 d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.965178, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4094, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\cot(c+dx)\left(a^2b(15A+90B-46C)+30a^3C+2ab^2(45A-35B+17C)-2b^3(15A-5B+9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(70*a*b*B - a^2*(15*A - 46*C) + 6*b^2*(5*A + 3*C))*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(15*b*d) + (Sqrt[a + b]*(a^2*b*(15*A + 90*B - 46*C) + 30*a^3*C
- 2*b^3*(15*A - 5*B + 9*C) + 2*a*b^2*(45*A - 35*B + 17*C))*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[
(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(1
5*b*d) - (a*Sqrt[a + b]*(5*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (A*(a
 + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/d - (b*(15*a*A - 10*b*B - 16*a*C)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) - (b*(5*A - 2*C)*(a + b*Sec[c +
 d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} + \int (\ldots$$

$$= \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{b(5\ldots}{\ldots}$$

$$= \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{b(15\ldots}{\ldots}$$

$$= \frac{A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d} - \frac{b(15\ldots}{\ldots}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(70abB-a^2(15A-46C)\ldots\right.}{\ldots}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(70abB-a^2(15A-46C)\ldots\right.}{\ldots}$$

Mathematica [B] time = 20.9614, size = 1498, normalized size = 2.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(15*a^3*A*Tan[(c + d*x)/2] + 15*a^2*A*b*Tan[
(c + d*x)/2] - 30*a*A*b^2*Tan[(c + d*x)/2] - 30*A*b^3*Tan[(c + d*x)/2] - 70
*a^2*b*B*Tan[(c + d*x)/2] - 70*a*b^2*B*Tan[(c + d*x)/2] - 46*a^3*C*Tan[(c +
 d*x)/2] - 46*a^2*b*C*Tan[(c + d*x)/2] - 18*a*b^2*C*Tan[(c + d*x)/2] - 18*b
^3*C*Tan[(c + d*x)/2] - 30*a^3*A*Tan[(c + d*x)/2]^3 + 60*a*A*b^2*Tan[(c + d
*x)/2]^3 + 140*a^2*b*B*Tan[(c + d*x)/2]^3 + 92*a^3*C*Tan[(c + d*x)/2]^3 + 3
6*a*b^2*C*Tan[(c + d*x)/2]^3 + 15*a^3*A*Tan[(c + d*x)/2]^5 - 15*a^2*A*b*Tan
[(c + d*x)/2]^5 - 30*a*A*b^2*Tan[(c + d*x)/2]^5 + 30*A*b^3*Tan[(c + d*x)/2]
^5 - 70*a^2*b*B*Tan[(c + d*x)/2]^5 + 70*a*b^2*B*Tan[(c + d*x)/2]^5 - 46*a^3
*C*Tan[(c + d*x)/2]^5 + 46*a^2*b*C*Tan[(c + d*x)/2]^5 - 18*a*b^2*C*Tan[(c +
 d*x)/2]^5 + 18*b^3*C*Tan[(c + d*x)/2]^5 - 150*a^2*A*b*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*a^3*B*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] - 150*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*a^3*B*EllipticPi[-1, -ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] + (a + b)*(-70*a*b*B + a^2*(15*A - 46*C) - 6*b^2*(5*A + 3*C))*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 2*(a^2*b*(45*A - 45*B - 23*C) + 15*a^3*(B - C) - b^3*(15*A
 + 5*B + 9*C) - a*b^2*(45*A + 35*B + 17*C))*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(15*
d*(b + a*Cos[c + d*x])^(5/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*Sec[c + d*x]^(9/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(
```

```
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c +
 d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((4*(15*A*b^2 + 35*a*b*B + 23*a^2*C + 9*b^2*C)*Sin[c + d*x])/15 + (4*Sec[c
+ d*x]*(5*b^2*B*Sin[c + d*x] + 11*a*b*C*Sin[c + d*x]))/15 + (4*b^2*C*Sec[c
+ d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 1.099, size = 4981, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-46*C*cos(d*x+c)^3*a^3+30*A*cos(d*x+c)^4*a*b^2-30*A*cos(d*x+c)^3*a*b^2
+90*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b-70*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+18*C*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-4
6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3-18*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+18*C*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+10*B*co
s(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^3+10*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-70*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+70*B*cos(d*
x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2-70*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-70*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+70*B*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2-46*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-18*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+46*C*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b+34*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-46*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
```

```
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-18*C*sin(d*
x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2+46*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+34*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+90*A*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a*b^2-30*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-30*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+15*A*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^3+15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-30*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+60*B*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*a^3+30*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3-30*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+60*B*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*a^3+30*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^3+15*A*cos(d*x+c)^5*a^3+90*B*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b-90*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b+150*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-90*A*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b+150*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+22*C*cos(d*x+c)^4*a^2*b+18*C*cos(d*x+
c)^4*a*b^2+46*C*cos(d*x+c)^3*a^2*b+10*C*cos(d*x+c)^3*a*b^2-28*C*cos(d*x+c)*
a*b^2+70*B*cos(d*x+c)^3*a*b^2-80*B*cos(d*x+c)^2*a*b^2+70*B*cos(d*x+c)^4*a^2
*b+10*B*cos(d*x+c)^4*a*b^2-70*B*cos(d*x+c)^3*a^2*b-68*C*cos(d*x+c)^2*a^2*b+
46*C*cos(d*x+c)^4*a^3+18*C*cos(d*x+c)^3*b^3-12*C*cos(d*x+c)^2*b^3+10*B*cos(
d*x+c)^3*b^3-10*B*cos(d*x+c)*b^3-15*A*cos(d*x+c)^3*a^2*b-46*C*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^3-18*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3+15*A*cos(d*x+c)^4*a^2*b+15*A*cos(d*x+c)^3*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*
```

```
b+15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^2*b-15*A*cos(d*x+c)^4*a^3+90*A*cos(d*x+c)^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-3
0*A*cos(d*x+c)^2*b^3+30*A*cos(d*x+c)^3*b^3+30*A*cos(d*x+c)^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-30*A*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^3+30*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-30*A*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*C*b^3)/(b+a*c
os(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^2\cos\left(dx+c\right)\sec\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)\sec\left(dx+c\right)^3+Aa^2\cos\left(dx+c\right)+\left(Ca^2+2\,Bab\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c
)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x +
 c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(d*x + c))*sqrt(b*se
c(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c), x)
```

3.954 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A + B\sec(c + dx) + C\,\mathrm{s}\right.$

Optimal. Leaf size=507

$$\frac{\sqrt{a+b}\cot(c+dx)\left(6a^2(A+2(B+6C))+ab(27A+72B-56C)+8b^2(3A-3B+C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{12d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(12*b*d) + (Sqrt[a + b]*(a*b*(27*A + 72*B - 56*C) + 8*b^2*(3*A - 3*B + C)
 + 6*a^2*(A + 2*(B + 6*C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*d) - (Sqrt[a + b]*(15*A*b^2 +
 20*a*b*B + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + ((5*A*b + 4*
a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]*(a +
b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d) - (b*(21*A*b + 12*a*B - 8*b*C)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(12*d)
```

Rubi [A] time = 1.06138, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\cot(c+dx)\left(6a^2(A+2(B+6C))+ab(27A+72B-56C)+8b^2(3A-3B+C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*Cot[c + d*x]
*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(12*b*d) + (Sqrt[a + b]*(a*b*(27*A + 72*B - 56*C) + 8*b^2*(3*A - 3*B + C)
 + 6*a^2*(A + 2*(B + 6*C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*d) - (Sqrt[a + b]*(15*A*b^2 +
 20*a*b*B + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + ((5*A*b + 4*
a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d) + (A*Cos[c + d*x]*(a +
b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(2*d) - (b*(21*A*b + 12*a*B - 8*b*C)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(12*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos(c+dx)(a+b\sec(c+dx))^{5/2}\sin}{2d}$$

$$= \frac{(5Ab+4aB)(a+b\sec(c+dx))^{3/2}\sin}{4d}$$

$$= \frac{(5Ab+4aB)(a+b\sec(c+dx))^{3/2}\sin}{4d}$$

$$= \frac{(5Ab+4aB)(a+b\sec(c+dx))^{3/2}\sin}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(12a^2B-24b^2B+ab(2\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(12a^2B-24b^2B+ab(2\right.}{}$$

Mathematica [B] time = 25.4083, size = 4902, normalized size = 9.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((4*b*(3*b*B + 7*a*C)*Sin[c + d
*x])/3 + (a^2*A*Sin[2*(c + d*x)])/2 + (4*b^2*C*Tan[c + d*x])/3))/(d*(b + a*
Cos[c + d*x])^2) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*((a^3*A)/(Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (6*a*A*b^2)/(Sqrt[b + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) + (6*a^2*b*B)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) - (2*b^3*B)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*
C)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (14*a*b^2*C)/(3*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (11*a^2*A*b*Sqrt[Sec[c + d*x]])/(4*S
qrt[b + a*Cos[c + d*x]]) + (2*A*b^3*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c +
d*x]] + (a^3*B*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] + (4*a*b^2*B*Sq
rt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]] + (4*a^2*b*C*Sqrt[Sec[c + d*x]])
/(3*Sqrt[b + a*Cos[c + d*x]]) + (2*b^3*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*
Cos[c + d*x]]) + (9*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(4*Sqrt[b
+ a*Cos[c + d*x]]) + (a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/Sqrt[b + a
*Cos[c + d*x]] - (2*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/Sqrt[b + a
*Cos[c + d*x]] - (14*a^2*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[b
 + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d
*x])^(5/2)*((a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(a + b)*(21*A*b + 6*a*(A + 2*B
- 8*C) - 8*b*(3*B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
] - 3*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*((a - b)*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2)/(a + b)] + (12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*(b + a*
Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c +
 d*x)/2]))/(6*d*(b + a*Cos[c + d*x])^3*(Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d
*x]^(5/2)*((a*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*Sin[c + d*x]*((a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*
```

C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(a + b)*(21*A*
b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B + C))*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
x)/2]^2)/(a + b)] - 3(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*((a - b)*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (12*a^2*B - 24*b^2*B + a*b*(27*A
 - 56*C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[
c + d*x]*Tan[(c + d*x)/2]))/(12*(b + a*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2
]^2)^(3/2)) - (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A
- 56*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(a + b)*(
21*A*b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B + C))*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2)/(a + b)] - 3*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*((a - b
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (12*a^2*B - 24*b^2*B + a*b*
(27*A - 56*C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)
*Sec[c + d*x]*Tan[(c + d*x)/2]))/(4*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)
/2]^2)^(3/2)) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^(3/2)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2])*((a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A
- 56*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(a + b)*(
21*A*b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B + C))*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2)/(a + b)] - 3*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*((a - b
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (12*a^2*B - 24*b^2*B + a*b*
(27*A - 56*C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)
*Sec[c + d*x]*Tan[(c + d*x)/2]))/(12*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x
)/2]^2)^(3/2)) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*((a + b)*(12*a^2*B
- 24*b^2*B + a*b*(27*A - 56*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] - b*(a + b)*(21*A*b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B + C))*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 3*(15*A*b^2 + 20*a*b*B + 4
a^2(A + 2*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c
 + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (12*
a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Se
c[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*
Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*
x]))/(12*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*(((12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/
2)*Sec[c + d*x])/2 + (a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - b*(a +
b)*(21*A*b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B + C))*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - 3*(15*A*b^2 + 20*a*b*B + 4*a
^2*(A + 2*C))*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c +

```
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c +
 d*x)/2] + (3*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*(b + a*Cos[c + d*x]
)*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]*(-(Se
c[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/2 + ((a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c +
 d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)]) - (b*(a + b)*(21*A*b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B + C))*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-(
(a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)]) - (3*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*((a
- b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*(-((a*S
ec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)]) - (b*(a + b)*(21*A*b + 6*a*(A + 2*B - 8*C) - 8*b*(3*B
 + C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^
2)/(a + b)]) + ((a + b)*(12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*Sec[(c +
d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 -
((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]) - 3
*(15*A*b^2 + 20*a*b*B + 4*a^2*(A + 2*C))*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*(((a - b)*Sec[(c + d*x)/2]^2)/(2*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])
 - (a*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/
2]^2)*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])) - a*(12*a^2*B - 24*b
^2*B + a*b*(27*A - 56*C))*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c +
d*x)/2]*Tan[c + d*x] + (12*a^2*B - 24*b^2*B + a*b*(27*A - 56*C))*(b + a*Cos
[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*
x)/2]*Tan[c + d*x]))/(6*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)
))))/2
```

Maple [B] time = 0.94, size = 4884, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -1/12/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(27*A*cos(d*x+c)^3*a*b^2+8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+24*B*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+1
2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b-24*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+72*B*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5
6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
```

```
-b)/(a+b))^(1/2))*a^2*b-56*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+72*C*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+5
6*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^2+90*A*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+120*B*cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
2*b+90*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+120*B*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-72*A*cos(d*x
+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a*b^2+27*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-24*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+6*A*cos(d*x+c)
^5*a^3-6*A*cos(d*x+c)^3*a^3-72*B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+72*B*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-1
2*B*cos(d*x+c)^3*a^3+24*B*cos(d*x+c)^2*b^3+12*B*cos(d*x+c)^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-24*B*cos(
d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^3+48*C*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-12*A*cos(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+24*A*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^3+12*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-24*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-24*C*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+
48*C*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^3+24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-56*C*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-56*C*a
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b+72*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
```

```
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-24*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2+56*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-72*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+6*A*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b+56*C*cos(d*x+c)^3*a^2*b+8*C*cos(d*x+c)^3*a*b^2-64*C*cos(d*x+c)*a*b^
2+24*B*cos(d*x+c)^3*a*b^2-24*B*cos(d*x+c)^2*a*b^2+12*B*cos(d*x+c)^3*a^2*b-5
6*C*cos(d*x+c)^2*a^2*b+6*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b+27*A*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b+27*A*b^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a-72*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+8*C*cos(d*x+c)^2*b^3-24*B*cos(d*x+c)
*b^3-27*A*cos(d*x+c)^3*a^2*b+12*B*cos(d*x+c)^4*a^3+24*B*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*C
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^3-27*A*cos(d*x+c)^2*a*b^2-6*A*cos(d*x+c)^2*a^2*b+33*A*cos(d*
x+c)^4*a^2*b+27*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-12*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+24*A*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a^3+24*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-12*B*cos(d*x+c)^2*a^2*b+56*C*cos(d*x+c)
^2*a*b^2-8*C*b^3)/sin(d*x+c)^5/(b+a*cos(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^2\sec(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^2\sec(dx+c)^3+Aa^2\cos(dx+c)^2+\left(Ca^2+2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^2, x)
```

3.955 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\text{s}\right.$

Optimal. Leaf size=549

$$\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2(4A+3B+6C)+2ab(13A+27B+72C)+3b^2(11A+16(B-C))\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+d}{a-b}}}{24d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(54*a*b*B + 3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(24*b*d) + (Sqrt[a + b]*(3*b^2*(11*A + 16*(B - C)) + 4*a^2*(4*
A + 3*B + 6*C) + 2*a*b*(13*A + 27*B + 72*C))*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (Sqrt[a
 + b]*(5*A*b^3 + 8*a^3*B + 30*a*b^2*B + 20*a^2*b*(A + 2*C))*Cot[c + d*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(8*a*d) + ((15*A*b^2 + 42*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(24*d) + ((5*A*b + 6*a*B)*Cos[c + d*x]*(a + b*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c +
d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.27867, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4094, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sin(c+dx)\left(8a^2(2A+3C)+42abB+15Ab^2\right)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2(4A+3B+6C)+2ab(13\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(54*a*b*B + 3*b^2*(11*A - 16*C) + 8*a^2*(2*A + 3*C))*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(24*b*d) + (Sqrt[a + b]*(3*b^2*(11*A + 16*(B - C)) + 4*a^2*(4*
A + 3*B + 6*C) + 2*a*b*(13*A + 27*B + 72*C))*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (Sqrt[a
 + b]*(5*A*b^3 + 8*a^3*B + 30*a*b^2*B + 20*a^2*b*(A + 2*C))*Cot[c + d*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(8*a*d) + ((15*A*b^2 + 42*a*b*B + 8*a^2*(2*A + 3*C))*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(24*d) + ((5*A*b + 6*a*B)*Cos[c + d*x]*(a + b*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d) + (A*Cos[c + d*x]^2*(a + b*Sec[c +
d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
```

```
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,\text{s}}{3d}$$

$$= \frac{(5Ab+6aB)\cos(c+dx)(a+b\sec(c}{12d}$$

$$= \frac{\left(15Ab^2+42abB+8a^2(2A+3C)\right)\sqrt{a}}{24d}$$

$$= \frac{\left(15Ab^2+42abB+8a^2(2A+3C)\right)\sqrt{a}}{24d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(54abB+3b^2(11A-16\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(54abB+3b^2(11A-16\right.}{}$$

Mathematica [B] time = 25.8241, size = 5361, normalized size = 9.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.967, size = 5113, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+Aa^2\cos\left(dx+c\right)^3+\left(Ca^2+2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^3, x)
```

3.956 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=652

$$\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2b(71A+52B+108C)+8a^3(9A+16B+12C)+2ab^2(59A+132B+192C)+15Ab^3\right)\sqrt{\frac{b(1-\sec}{a}}}{192ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*b*(71*A +
108*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))]])/(192*a*b*d) + (Sqrt[a + b]*(15*A*b^3 + 8*a^3*(9*A + 1
6*B + 12*C) + 4*a^2*b*(71*A + 52*B + 108*C) + 2*a*b^2*(59*A + 132*B + 192*C
))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))]])/(192*a*d) + (Sqrt[a + b]*(5*A*b^4 - 160*a^3*b*B - 40*a*b^3
*B - 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a
 + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/
(64*a^2*d) + ((15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*b*(71*A + 108*C))
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((5*A*b^2 + 24*a*b*B +
4*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(32*
d) + ((5*A*b + 8*a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x
])/(24*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*d
)
```

Rubi [A] time = 2.01955, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sin(c+dx)\left(4a^2b(71A+108C)+128a^3B+264ab^2B+15Ab^3\right)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{\sin(c+dx)\cos(c+dx)\left(4a^2(3A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```



```
[Out] ((a - b)*Sqrt[a + b]*(15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*b*(71*A +
108*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))]])/(192*a*b*d) + (Sqrt[a + b]*(15*A*b^3 + 8*a^3*(9*A + 1
6*B + 12*C) + 4*a^2*b*(71*A + 52*B + 108*C) + 2*a*b^2*(59*A + 132*B + 192*C
))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))]])/(192*a*d) + (Sqrt[a + b]*(5*A*b^4 - 160*a^3*b*B - 40*a*b^3
*B - 120*a^2*b^2*(A + 2*C) - 16*a^4*(3*A + 4*C))*Cot[c + d*x]*EllipticPi[(a
 + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/
(64*a^2*d) + ((15*A*b^3 + 128*a^3*B + 264*a*b^2*B + 4*a^2*b*(71*A + 108*C))
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((5*A*b^2 + 24*a*b*B +
4*a^2*(3*A + 4*C))*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(32*
d) + ((5*A*b + 8*a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x
])/(24*d) + (A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*d
)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,\mathrm{s}}{4d}$$

$$= \frac{(5Ab+8aB)\cos^2(c+dx)(a+b\sec(c}{24d}$$

$$= \frac{\left(5Ab^2+24abB+4a^2(3A+4C)\right)\cos}{3}$$

$$= \frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(\right.}{}$$

$$= \frac{\left(15Ab^3+128a^3B+264ab^2B+4a^2b(\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^3+128a^3B+26\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(15Ab^3+128a^3B+26\right.}{}$$

Mathematica [B] time = 26.1058, size = 5681, normalized size = 8.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.706, size = 5850, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^4\sec(dx+c)^3 + Aa^2\cos(dx+c)^4 + \left(Ca^2 + 2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^4*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^4, x)
```

3.957 $\int \cos^5(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=774

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(-4a^2b^2(423A+295B+660C)-8a^3b(193A+355B+260C)-16a^4(64A+45B+80C)-30ab\right.}{1920a^2d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*(4*A
 + 5*C) - 12*a^2*b^2*(141*A + 220*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(1920*a^2*b*d) - (Sqrt[
a + b]*(45*A*b^4 - 30*a*b^3*(A + 5*B) - 16*a^4*(64*A + 45*B + 80*C) - 8*a^3
*b*(193*A + 355*B + 260*C) - 4*a^2*b^2*(423*A + 295*B + 660*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(1920*a^2*d) - (Sqrt[a + b]*(3*A*b^5 + 96*a^5*B + 240*a^3*b^2*B - 10*a*b
^4*B + 40*a^2*b^3*(A + 2*C) + 80*a^4*b*(3*A + 4*C))*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(128*a^3*d) - ((45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*(4*A + 5
*C) - 12*a^2*b^2*(141*A + 220*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(1
920*a^2*d) + ((15*A*b^3 + 360*a^3*B + 590*a*b^2*B + 4*a^2*b*(193*A + 260*C)
)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(960*a*d) + ((15*A*b^
2 + 110*a*b*B + 16*a^2*(4*A + 5*C))*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(240*d) + ((A*b + 2*a*B)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(5*d)
```

Rubi [A] time = 3.20325, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sin(c+dx)\left(-12a^2b^2(141A+220C)-256a^4(4A+5C)-2840a^3bB-150ab^3B+45Ab^4\right)\sqrt{a+b\sec(c+dx)}}{1920a^2d}+\frac{\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*(4*A
 + 5*C) - 12*a^2*b^2*(141*A + 220*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(1920*a^2*b*d) - (Sqrt[
a + b]*(45*A*b^4 - 30*a*b^3*(A + 5*B) - 16*a^4*(64*A + 45*B + 80*C) - 8*a^3
*b*(193*A + 355*B + 260*C) - 4*a^2*b^2*(423*A + 295*B + 660*C))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(1920*a^2*d) - (Sqrt[a + b]*(3*A*b^5 + 96*a^5*B + 240*a^3*b^2*B - 10*a*b
^4*B + 40*a^2*b^3*(A + 2*C) + 80*a^4*b*(3*A + 4*C))*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(128*a^3*d) - ((45*A*b^4 - 2840*a^3*b*B - 150*a*b^3*B - 256*a^4*(4*A + 5
*C) - 12*a^2*b^2*(141*A + 220*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(1
920*a^2*d) + ((15*A*b^3 + 360*a^3*B + 590*a*b^2*B + 4*a^2*b*(193*A + 260*C)
)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(960*a*d) + ((15*A*b^
```

```
2 + 110*a*b*B + 16*a^2*(4*A + 5*C))*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(240*d) + ((A*b + 2*a*B)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(5*d)
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
```

```
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{A\cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\text{s}}{5d}$$

$$= \frac{(Ab+2aB)\cos^3(c+dx)(a+b\sec(c}{8d}$$

$$= \frac{\left(15Ab^2+110abB+16a^2(4A+5C)\right)}{}$$

$$= \frac{\left(15Ab^3+360a^3B+590ab^2B+4a^2b(\right.}{}$$

$$= -\frac{\left(45Ab^4-2840a^3bB-150ab^3B-25\right.}{}$$

$$= -\frac{\left(45Ab^4-2840a^3bB-150ab^3B-25\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(45Ab^4-2840a^3bB-\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(45Ab^4-2840a^3bB-\right.}{}$$

Mathematica [C] time = 20.6554, size = 800, normalized size = 1.03

$$\frac{\cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{1}{40}A\sin(5(c+dx))a^2+\frac{1}{160}(21Ab+10aB)\sin\right.}{d(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*(((88*a^2*A + 93*A*b^2 + 170*a*b*B + 80*a^2*C)*Sin[c + d*x])/480 +
((1024*a^2*A*b + 15*A*b^3 + 480*a^3*B + 590*a*b^2*B + 1040*a^2*b*C)*Sin[2*(
c + d*x)])/(960*a) + ((100*a^2*A + 93*A*b^2 + 170*a*b*B + 80*a^2*C)*Sin[3*(
c + d*x)])/480 + (a*(21*A*b + 10*a*B)*Sin[4*(c + d*x)])/160 + (a^2*A*Sin[5*
(c + d*x)])/40))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])) - (Cos[c + d*x]^5*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((I*((a - b)*(-45*A*b^4 + 2840*a^3*b*B + 150*a
*b^3*B + 256*a^4*(4*A + 5*C) + 12*a^2*b^2*(141*A + 220*C))*EllipticE[I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - 2*(a - b)*
(-45*A*b^4 - 30*a*b^3*(A - 5*B) + 720*a^4*B + 4*a^2*b^2*(129*A + 185*B + 18
0*C) + 8*a^3*b*(161*A + 45*B + 220*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a
```

```
 + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + 30*(3*A*b^5 + 96*a^5*B + 240*a
^3*b^2*B - 10*a*b^4*B + 40*a^2*b^3*(A + 2*C) + 80*a^4*b*(3*A + 4*C))*Ellipt
icPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)])*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
)/(Sqrt[(-a + b)/(a + b)]*(b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d
*x)/2]^2]) - (-45*A*b^4 + 2840*a^3*b*B + 150*a*b^3*B + 256*a^4*(4*A + 5*C)
+ 12*a^2*b^2*(141*A + 220*C))*Tan[(c + d*x)/2]))/(960*a^2*d*(b + a*Cos[c +
d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 0.938, size = 7029, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right) + a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^5, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^5, x)
```

3.958 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=429

$$\frac{2\sqrt{a+b}\cot(c+dx)\left(-4a^2b(14B+3C)+48a^3C+2ab^2(35A+7B+22C)+b^3(35A-63B+25C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(s}{}}}{105b^4d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(56*a^2*b*B + 63*b^3*B - 48*a^3*C - 2*a*b^2*(35*A +
 22*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(105*b^5*d) + (2*Sqrt[a + b]*(48*a^3*C - 4*a^2*b*(14*
B + 3*C) + 2*a*b^2*(35*A + 7*B + 22*C) + b^3*(35*A - 63*B + 25*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(105*b^4*d) + (2*(35*A*b^2 - 28*a*b*B + 24*a^2*C + 25*b^2*C)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d) + (2*(7*b*B - 6*a*C)*Sec[c + d*x
]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (2*C*Sec[c + d*x]^2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 1.0573, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4102, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(24a^2C-28abB+35Ab^2+25b^2C\right)\sqrt{a+b\sec(c+dx)}}{105b^3d}+\frac{2\sqrt{a+b}\cot(c+dx)\left(-4a^2b(14B+3C)+48a^3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec
[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(56*a^2*b*B + 63*b^3*B - 48*a^3*C - 2*a*b^2*(35*A +
 22*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(105*b^5*d) + (2*Sqrt[a + b]*(48*a^3*C - 4*a^2*b*(14*
B + 3*C) + 2*a*b^2*(35*A + 7*B + 22*C) + b^3*(35*A - 63*B + 25*C))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(105*b^4*d) + (2*(35*A*b^2 - 28*a*b*B + 24*a^2*C + 25*b^2*C)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d) + (2*(7*b*B - 6*a*C)*Sec[c + d*x
]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (2*C*Sec[c + d*x]^2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```


Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \frac{2C\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7bd} + \frac{2\int \frac{\sec^2(c+dx}{}}{}$$

$$= \frac{2(7bB-6aC)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35b^2d} + 2($$

$$= \frac{2\left(35Ab^2-28abB+24a^2C+25b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(}{105b^3d}$$

$$= \frac{2\left(35Ab^2-28abB+24a^2C+25b^2C\right)\sqrt{a+b\sec(c+dx)}\tan(}{105b^3d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(56a^2bB+63b^3B-48a^3C-2ab^2(35A+22(\right.}{}$$

Mathematica [B] time = 25.9581, size = 3811, normalized size = 8.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 b*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((4*(-70*a*A*b^2 + 56*a^2*b*B + 63*b^3*B - 48*a^3*C - 44*a*b^2*C)*Sin[c + d
*x])/(105*b^4) + (4*Sec[c + d*x]^2*(7*b*B*Sin[c + d*x] - 6*a*C*Sin[c + d*x]
))/(35*b^2) + (4*Sec[c + d*x]*(35*A*b^2*Sin[c + d*x] - 28*a*b*B*Sin[c + d*x
] + 24*a^2*C*Sin[c + d*x] + 25*b^2*C*Sin[c + d*x]))/(105*b^3) + (4*C*Sec[c
+ d*x]^2*Tan[c + d*x])/(7*b)))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*Sqrt[a + b*Sec[c + d*x]]) + (4*((4*a*A)/(3*b*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) - (6*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - (16*a^2*B)/(15*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (3
2*a^3*C)/(35*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (88*a*C)/(1
05*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[Sec[c + d*x]]
)/(3*Sqrt[b + a*Cos[c + d*x]]) + (4*a^2*A*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b
 + a*Cos[c + d*x]]) - (16*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[
c + d*x]]) - (14*a*B*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) +
(10*C*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (32*a^4*C*Sqrt[Se
c[c + d*x]])/(35*b^4*Sqrt[b + a*Cos[c + d*x]]) + (64*a^2*C*Sqrt[Sec[c + d*x
]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]) + (4*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec
[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (16*a^3*B*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (6*a*B*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (32*a^4*C*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^4*Sqrt[b + a*Cos[c + d*x]]) + (88*a^2*C*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*(2*(a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*
(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A
+ 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*
Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(10
```

5*b^4*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^4*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A + 63*B + 25*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(2*(a + b)*(-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35*A + 22*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(4*a^2*b*(14*B - 3*C) - 48*a^3*C - 2*a*b^2*(35*A - 7*B + 22*C) + b^3*(35*A + 63*B + 25*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +

```
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + (-56*a^2*b*B - 63*b^3*B + 48*a^3*C + 2*a*b^2*(35
*A + 22*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/
2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 1.293, size = 4340, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(70*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+35*A*cos(d*x+c)^2*b^4+21*B*cos(d*x+c)*b^4
+48*C*cos(d*x+c)^5*a^4-35*A*cos(d*x+c)^3*a*b^3+28*B*cos(d*x+c)^3*a^2*b^2-7*
B*cos(d*x+c)^2*a*b^3-63*B*cos(d*x+c)^4*b^4+42*B*cos(d*x+c)^3*b^4-35*A*cos(d
*x+c)^4*b^4-25*C*cos(d*x+c)^4*b^4-70*A*cos(d*x+c)^4*a^2*b^2+70*A*cos(d*x+c)
^5*a^2*b^2-35*A*cos(d*x+c)^5*a*b^3+70*A*cos(d*x+c)^4*a*b^3+56*B*cos(d*x+c)^
4*a^3*b+70*B*cos(d*x+c)^4*a*b^3+28*B*cos(d*x+c)^5*a^2*b^2-63*B*cos(d*x+c)^5
*a*b^3-24*C*cos(d*x+c)^5*a^3*b+44*C*cos(d*x+c)^5*a^2*b^2-25*C*cos(d*x+c)^5*
a*b^3-50*C*cos(d*x+c)^4*a^2*b^2+44*C*cos(d*x+c)^4*a*b^3-70*A*sin(d*x+c)*cos
(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2-70*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^3+70*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-70*A*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^2*b^2+56*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+56*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+63*B*cos(d*x+
c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a*b^3-56*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-14*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+56*B*cos(d*
x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^3*b+56*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-56*B*cos(
d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^2*b^2-14*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
```

+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+48*C*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+12*C*co
s(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2*b^2+44*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-48*C*cos(d*x+c)^4*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-44*C*
cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^2-44*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*C*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-70*
A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^3+12*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+44*C*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-4
8*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b-44*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-44*C*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3
-56*B*cos(d*x+c)^4*a^2*b^2-56*B*cos(d*x+c)^5*a^3*b+48*C*cos(d*x+c)^4*a^3*b-
24*C*cos(d*x+c)^3*a^3*b-16*C*cos(d*x+c)^3*a*b^3+6*C*cos(d*x+c)^2*a^2*b^2-3*
C*cos(d*x+c)*a*b^3+63*B*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-63*B*cos(d*x+c)^4*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+63*B*cos(d
x+c)^3(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*b^4-63*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-35*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-35*A*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^4-25*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^4-48*C*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-25*C*cos(d*x+c)^3*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4
-48*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^4-48*C*cos(d*x+c)^4*a^4+10*C*cos(d*x+c)^2*b^4+15*C*b^
4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^5+B\sec(dx+c)^4+A\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4 + A*sec(d*x + c)^3)/sqrt(b*se
c(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/sqrt(a +
b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*sec
(d*x + c) + a), x)
```

3.959 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=342

$$-\frac{2\sqrt{a+b}\cot(c+dx)\left(8a^2C-2ab(5B+C)+15Ab^2-b^2(5B-9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\cdot\right.\right.}{15b^3d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(15*b^4*d) - (2*Sqrt[a + b]*(15*A*b^2 - b^2*(5*B - 9*C) + 8*a^2*C - 2*
a*b*(5*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*C*Sec[c + d*x]*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.664079, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4092, 4082, 4005, 3832, 4004}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\left(8a^2C-2ab(5B+C)+15Ab^2-b^2(5B-9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{15b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec
[c + d*x]],x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Cot[c + d
*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(15*b^4*d) - (2*Sqrt[a + b]*(15*A*b^2 - b^2*(5*B - 9*C) + 8*a^2*C - 2*
a*b*(5*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) + (2*(5*b*B - 4*a*C)*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*C*Sec[c + d*x]*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(5*b*d)
```


Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
```

```
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2C\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}+\frac{2\int\frac{\sec(c+dx)(}{}}{}$$

$$=\frac{2(5bB-4aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2C\sec(c+dx}{}$$

$$=\frac{2(5bB-4aC)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2C\sec(c+dx}{}$$

$$=-\frac{2(a-b)\sqrt{a+b}\left(15Ab^2-10abB+8a^2C+9b^2C\right)\cot(c+dx)}{15b}$$

Mathematica [B] time = 24.8959, size = 3332, normalized size = 9.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 b*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((4*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Sin[c + d*x])/(15*b^3) + (4*S
ec[c + d*x]*(5*b*B*Sin[c + d*x] - 4*a*C*Sin[c + d*x]))/(15*b^2) + (4*C*Sec[
c + d*x]*Tan[c + d*x])/(5*b)))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
```

2*d*x])*Sqrt[a + b*Sec[c + d*x]]) + (4*((-2*A)/(Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (4*a*B)/(3*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (6*C)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (16*a^2*C)/(1
5*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqrt[Sec[c + d*
x]])/(b*Sqrt[b + a*Cos[c + d*x]]) + (2*B*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*
Cos[c + d*x]]) + (4*a^2*B*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x
]]) - (16*a^3*C*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (14
*a*C*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a*A*Cos[2*(c
+ d*x)]*Sqrt[Sec[c + d*x]])/(b*Sqrt[b + a*Cos[c + d*x]]) + (4*a^2*B*Cos[2*(
c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (16*a^3*C*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]) - (6
*a*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]))*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*(-2*(a + b)*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(15*A*b^2 + 8*a^
2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (15*A*b^2 - 10*a*b*B + 8*a^2*C + 9
*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]))/(15*b^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(
c + d*x)/2]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((2*a*Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(15*A*b^2 - 10*a*b*B +
8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + 2*b*(15*A*b^2 + 8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B +
9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] - (15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*(b + a*Cos[c + d*x])^(3
/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Ta
n[(c + d*x)/2]*(-2*(a + b)*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(15
*A*b^2 + 8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (15*A*b^2 - 10*a*b*B
+ 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]
) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-((15*A*b^2 - 10*a*b*B + 8*a^
2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((
a + b)*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1
+ Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(15*A*b^2 + 8
*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + C
os[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(15*A*b^2 -
10*a*b*B + 8*a^2*C + 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] +
(b*(15*A*b^2 + 8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x]
)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))] + a*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)
*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (15*A*b^2

```
- 10*a*b*B + 8*a^2*C + 9*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin
[c + d*x]*Tan[(c + d*x)/2] - (15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(1
5*A*b^2 + 8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)]) - ((a + b)*(15*A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2
)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(-2*(a + b)*(15*A*b^2 - 10*a*b*B + 8*a^2*C
+ 9*b^2*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] + 2*b*(15*A*b^2 + 8*a^2*C + 2*a*b*(-5*B + C) + b^2*(5*B + 9*C))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (15*
A*b^2 - 10*a*b*B + 8*a^2*C + 9*b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c +
 d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]))
```

Maple [B] time = 0.871, size = 3147, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-8*C*cos(d*x+c)^3*a^3+15*A*cos(d*x+c)^4*a*b^2-15*A*cos(d*x+c)^3*a*
b^2+10*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*C*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*
C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^3+9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*B*cos(d*x+c)^3*sin(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*B*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^3+10*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+10*B*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b+10*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
```

```
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-8*C*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*C*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2-8*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b-9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*C*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b+2*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^2-15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-15*A*cos(d*x+c)^3*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b
^2-4*C*cos(d*x+c)^4*a^2*b+9*C*cos(d*x+c)^4*a*b^2+8*C*cos(d*x+c)^3*a^2*b-10*
C*cos(d*x+c)^3*a*b^2+C*cos(d*x+c)*a*b^2-10*B*cos(d*x+c)^3*a*b^2+5*B*cos(d*x
+c)^2*a*b^2-10*B*cos(d*x+c)^4*a^2*b+5*B*cos(d*x+c)^4*a*b^2+10*B*cos(d*x+c)^
3*a^2*b-4*C*cos(d*x+c)^2*a^2*b+8*C*cos(d*x+c)^4*a^3+9*C*cos(d*x+c)^3*b^3-6*
C*cos(d*x+c)^2*b^3+5*B*cos(d*x+c)^3*b^3-5*B*cos(d*x+c)*b^3-8*C*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3-9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3-15*A*cos(d*x+c)^2*b^3+15*A*cos(d*x+c)^3*b^3+15*
A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*b^3-15*A*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+15*A*cos(d*x+c)^3*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-15*A*cos(
d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^3-3*C*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)/sqrt(b*se
c(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/sqrt(a +
b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec
(d*x + c) + a), x)
```

3.960 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=267

$$\frac{2\sqrt{a+b}\cot(c+dx)(2aC+3Ab-b(3B-C))\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d}-$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B - 2*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a
 + b]*(3*A*b - b*(3*B - C) + 2*a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*C*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.364579, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4082, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}\cot(c+dx)(2aC+3Ab-b(3B-C))\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2(a-b)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c
 + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B - 2*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a
 + b]*(3*A*b - b*(3*B - C) + 2*a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*C*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d)
```


Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd}+\frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}b(3A+C)+\frac{1}{2}(3b}{\sqrt{a+b\sec(c+}}}{3b}$$

$$= \frac{2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd}+\frac{(3bB-2aC)\int \frac{\sec(c+dx)(1+}{\sqrt{a+b\sec}}}{3b}$$

$$= -\frac{2(a-b)\sqrt{a+b}(3bB-2aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{3b^3d}$$

Mathematica [B] time = 21.8344, size = 2741, normalized size = 10.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((4*(3*b*B - 2*a*C)*Sin[c + d*x])/(3*b^2) + (4*C*Tan[c + d*x])/(3*b)))/(d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[a + b*Sec[c + d*x]])
+ (4*((-2*B)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a*C)/(3*b*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[Sec[c + d*x]])/Sqrt
[b + a*Cos[c + d*x]] - (2*a*B*Sqrt[Sec[c + d*x]])/(b*Sqrt[b + a*Cos[c + d*x
]]) + (2*C*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) + (4*a^2*C*Sqrt
[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a*B*Cos[2*(c + d*x)]*
Sqrt[Sec[c + d*x]])/(b*Sqrt[b + a*Cos[c + d*x]]) + (4*a^2*C*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(2*(a + b)*(-3*b
*B + 2*a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] + 2*b*(3*A*b - 2*a*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*b*B - 2*a*C)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^2*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sin[c + d*x]*(2*(a + b)*(-3*b*B + 2*a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
```

```
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(3*A*b - 2*a*C + b*(3*B + C
))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
- (3*b*B - 2*a*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(3*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2])
- (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-3*
b*B + 2*a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 2*b*(3*A*b - 2*a*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*b*B - 2*a*C)*Cos[c + d*x]*(
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^2*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*(-((3*b*B - 2*a*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^4)/2 + ((a + b)*(-3*b*B + 2*a*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Co
s[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d
*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(3*A*b - 2*a*C + b*(3*B +
 C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Co
s[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])] + ((a + b)*(-3*b*B + 2*a*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a +
 b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))] + (b*(3*A*b - 2*a*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((
a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))] + a*(3*b*B - 2*a*C)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x
]*Tan[(c + d*x)/2] + (3*b*B - 2*a*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*b*B - 2*a*C)*Cos[c + d*x]*(b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(3*A*b - 2*a*C + b*(3*B
 + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-3*b*B + 2*a*C
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^
2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^2*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[(c + d*x)/2]^2]) + (2*(2*(a + b)*(-3*b*B + 2*a*C)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(3*A*b - 2*a
*C + b*(3*B + C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] - (3*b*B - 2*a*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)
/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*b^2*Sqrt[b + a*Cos
[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
)
```

Maple [B] time = 0.508, size = 1757, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/b^2*(-1+cos(d*x+c))^2*(3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b+C*c
os(d*x+c)^3*a*b-2*C*cos(d*x+c)^2*a*b+C*cos(d*x+c)*a*b-3*B*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
-2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-2*C*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*C*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b+3*B*cos(d*x+c)^2*b^2+3*B*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+C*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+2*C*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+3*A*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b^2-b^2*C-3*B*cos(d*x+c)*b^2-2*C*cos(d*x+c)^3*a
^2+2*C*cos(d*x+c)^2*a^2+C*cos(d*x+c)^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(
1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))/sqrt(b*sec(
d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/sqrt(a + b*s
ec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(b*sec(d
*x + c) + a), x)
```

3.961 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=317

$$\frac{2\sqrt{a+b}(B-C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(B - C)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.248938, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4058, 3921, 3784, 3832, 4004}

$$-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}-\frac{2C(a-b)\sqrt{a+b}\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(B - C)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```


Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
```

```
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = C\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{A+(B-C)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx)}{a+b}}}{b^2d}$$
$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx)}{a+b}}}{b^2d}$$

Mathematica [B] time = 18.3383, size = 762, normalized size = 2.4

$$\frac{4C\sin(c+dx)\cos(c+dx)(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{bd\sqrt{a+b\sec(c+dx)}(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)} - \frac{4\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{a\cos(c+dx}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[a + b*Sec[c + d*x]],
x]
```

```
[Out] (4*C*Cos[c + d*x]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sin[c + d*x])/(b*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sq
rt[a + b*Sec[c + d*x]]) - (4*Sqrt[b + a*Cos[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*C*Tan[(c + d*x)/2
] + b*C*Tan[(c + d*x)/2] - 2*a*C*Tan[(c + d*x)/2]^3 + a*C*Tan[(c + d*x)/2]^
5 - b*C*Tan[(c + d*x)/2]^5 + 2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*A*b*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
```

```
 (a + b)*C*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + b*(A - B - C)*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*
x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
]))/(b*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/
2)*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.446, size = 1193, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/b*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^
2*(A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-C*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+C*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*a+C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*c
os(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*b+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-B*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-C*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+C*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+C*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-C*cos(d*x
+c)^2*a+C*cos(d*x+c)*a-C*cos(d*x+c)*b+C*b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{\sqrt{b \sec (dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sqrt(a + b*sec(c + d*x)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a),
 x)
```

3.962 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=358

$$\frac{\sqrt{a+b}(2aC+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd}+\frac{\sqrt{a+b}(Ab-2aB)}{}$$

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*(A*b + 2*a*C)*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a*b*d) + (Sqrt[a + b]*(A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a^2*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.424398, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}(Ab-2aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{\sqrt{a+b}(2aC+Ab)\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec[c
 + d*x]],x]
```


```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*(A*b + 2*a*C)*C
ot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a*b*d) + (Sqrt[a + b]*(A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a^2*d) + (A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)-aC\sec(c+dx)+}{\sqrt{a+b\sec(c+d}}}{a}$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)+\left(-\frac{Ab}{2}-aC\right)\sec}{\sqrt{a+b\sec(c+dx)}}}{a}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{abd}$$

$$= \frac{A(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{abd}$$

Mathematica [B] time = 18.0118, size = 861, normalized size = 2.41

$$2\sqrt{b+a\cos(c+dx)}(B+A\cos(c+dx)+C\sec(c+dx))\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(aA\tan^5\left(\frac{1}{2}(c+dx)\right)-Ab\tan^5\left(\frac{1}{2}(c+dx)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[b + a*Cos[c + d*x]]*(B + A*Cos[c + d*x] + C*Sec[c + d*x])*Sqrt[(1 -
 Tan[(c + d*x)/2]^2)^(-1)]*(a*A*Tan[(c + d*x)/2] + A*b*Tan[(c + d*x)/2] - 2
*a*A*Tan[(c + d*x)/2]^3 + a*A*Tan[(c + d*x)/2]^5 - A*b*Tan[(c + d*x)/2]^5 +
 2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 4*a*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] + 2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*B*Ellipti
cPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] + A*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(B - C)*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)]))/(a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)])
```

Maple [B] time = 0.423, size = 1210, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(-1+cos(d*x+c))^2*(2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-2*C*cos(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+2*A*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+
c)-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin
(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
```

```
)*a*sin(d*x+c)-4*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a*sin(d*x+c)-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*cos(d*x+c)^3*a+A*cos(d*x+c)^2*a-A*cos(d*
x+c)^2*b+A*cos(d*x+c)*b)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d
*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\cos\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)/sqrt(a + b*s
ec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d
*x + c) + a), x)
```

3.963 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=439

$$-\frac{\sqrt{a+b}(3Ab-2a(A+2B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4a^2d}-\frac{\sqrt{a+}}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b - 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*b*d) - (Sqrt[a +
 b]*(3*A*b - 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(3*A*b^2
 - 4*a*b*B + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*d) - ((3*A*b
 - 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (A*Cos[c + d*x
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.749037, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\cot(c+dx)\left(4a^2(A+2C)-4abB+3Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b*Sec
[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*A*b - 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*b*d) - (Sqrt[a +
 b]*(3*A*b - 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(3*A*b^2
 - 4*a*b*B + 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*d) - ((3*A*b
 - 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (A*Cos[c + d*x
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad} - \int \frac{\cos(c+dx)\left(\frac{1}{2}(3\ldots\right.}{\ldots}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\ldots}{\ldots}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\ldots}{\ldots}$$

$$= -\frac{(a-b)\sqrt{a+b}(3Ab-4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx\ldots}}{\sqrt{a+b}}\ldots\right)\right.}{4a^2bd}$$

$$= -\frac{(a-b)\sqrt{a+b}(3Ab-4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx\ldots}}{\sqrt{a+b}}\ldots\right)\right.}{4a^2bd}$$

Mathematica [C] time = 16.1449, size = 1905, normalized size = 4.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a +
 b*Sec[c + d*x]],x]
```

```
[Out] (A*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Se
c[c + d*x]]) + (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a
*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*A*b^2*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2] + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a
*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 6*a*A*b*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]^3 - 8*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
 3*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*A*b^2*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2]^5 - 4*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] - (16*I)*a^2*C*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - I*(a - b)*(-3*A*b + 4*a*B)*EllipticE[I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] + (2*I)*(3*A*b^2 - a*b*(A + 4*B) + 2*a^2*(A + 2
*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt[(-a +
 b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 +
 Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2)
 - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.408, size = 2259, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a^2*(-1+cos(d*x+c))^2*(-16*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-6*A*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*sin(d*x+c)-4*B*cos(d*x+c)^2*a*b+4*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*C*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*B
*cos(d*x+c)^3*a^2+8*C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*B*cos(d*x+c)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*a*b+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b+4*B*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*A*cos(d*
x+c)^2*a*b+2*A*cos(d*x+c)*a*b+4*B*cos(d*x+c)^2*a^2-2*A*cos(d*x+c)^4*a^2+3*A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b^2+4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*b^2+3*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*a*b-2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*a*b+2*A*cos(d*x+c)^2*a^2+3*A*cos(d*x+c)^2*b^2-3*A*c
os(d*x+c)*b^2-4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*a*b-4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+8*B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-16*C*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^2)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c)
)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)^2\sec\left(dx+c\right)+A\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec
(d*x + c) + a), x)
```

3.964 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=510

$$\frac{2\cot(c+dx)\left(a^2b(40B-36C)-48a^3C-6ab^2(5A-5B+2C)-b^3(15A-5B+9C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellip}}{15b^4d\sqrt{a+b}}$$

```
[Out] (2*(40*a^3*b*B - 25*a*b^3*B - 6*a^2*b^2*(5*A - 4*C) - 48*a^4*C + 3*b^4*(5*A
 + 3*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(15*b^5*Sqrt[a + b]*d) + (2*(a^2*b*(40*B - 36*C) - 4
8*a^3*C - 6*a*b^2*(5*A - 5*B + 2*C) - b^3*(15*A - 5*B + 9*C))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(15*b^4*Sqrt[a + b]*d) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(20*a^2*b*B - 5*b^3*
B - 3*a*b^2*(5*A - 3*C) - 24*a^3*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/
(15*b^3*(a^2 - b^2)*d) + (2*(5*A*b^2 - 5*a*b*B + 6*a^2*C - b^2*C)*Sec[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.34519, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4098, 4092, 4082, 4005, 3832, 4004}

$$-\frac{2\tan(c+dx)\sec^2(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\tan(c+dx)\sec(c+dx)\left(6a^2C-5abB+5Ab^2-b^2C\right)\sqrt{a+b\sec(c+}}{5b^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(3/2),x]
```


```
[Out] (2*(40*a^3*b*B - 25*a*b^3*B - 6*a^2*b^2*(5*A - 4*C) - 48*a^4*C + 3*b^4*(5*A
 + 3*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(15*b^5*Sqrt[a + b]*d) + (2*(a^2*b*(40*B - 36*C) - 4
8*a^3*C - 6*a*b^2*(5*A - 5*B + 2*C) - b^3*(15*A - 5*B + 9*C))*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(15*b^4*Sqrt[a + b]*d) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(20*a^2*b*B - 5*b^3*
B - 3*a*b^2*(5*A - 3*C) - 24*a^3*C)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/
(15*b^3*(a^2 - b^2)*d) + (2*(5*A*b^2 - 5*a*b*B + 6*a^2*C - b^2*C)*Sec[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
```

```
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec^2(c+}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(5Ab^2-\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(20a^2bB\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(20a^2bB\right.}{}$$

$$= \frac{2\left(40a^3bB-25ab^3B-6a^2b^2(5A-4C)-48a^4C+3b^4(5A+3\right.}{}$$

Mathematica [A] time = 20.8391, size = 874, normalized size = 1.71

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{4\left(-48Ca^4+40bBa^3-30Ab^2a^2+24b^2Ca^2-25b^3Ba+15Ab^4+9b^4C\right)\sin(c+dx)}{15b^4\left(b^2-a^2\right)}+\frac{4\sec(c+dx)(5bB\sin(c+dx)-9a}{15b^3}\right.}{d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^(3/2),x]
```

```
[Out] (4*(b + a*Cos[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*((a + b)*(-40*a^3*b*B + 25*a*b^3*B + 6*a^2*b
^2*(5*A - 4*C) + 48*a^4*C - 3*b^4*(5*A + 3*C))*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + b
*(a + b)*(-48*a^3*C - 6*a*b^2*(5*A + 5*B + 2*C) + b^3*(15*A + 5*B + 9*C) +
4*a^2*b*(10*B + 9*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (-40*a^3*b*B + 25*a*b^3*B
+ 6*a^2*b^2*(5*A - 4*C) + 48*a^4*C - 3*b^4*(5*A + 3*C))*Tan[(c + d*x)/2]*(b
 - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2)))/(15*b^4*(-a^2 +
b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]
*(a + b*Sec[c + d*x])^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + ((b
 + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(-30*a^2*A
*b^2 + 15*A*b^4 + 40*a^3*b*B - 25*a*b^3*B - 48*a^4*C + 24*a^2*b^2*C + 9*b^4
*C)*Sin[c + d*x])/(15*b^4*(-a^2 + b^2)) + (4*Sec[c + d*x]*(5*b*B*Sin[c + d*
x] - 9*a*C*Sin[c + d*x]))/(15*b^3) + (4*(a^2*A*b^2*Sin[c + d*x] - a^3*b*B*S
in[c + d*x] + a^4*C*Sin[c + d*x]))/(b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x]))
+ (4*C*Sec[c + d*x]*Tan[c + d*x])/(5*b^2)))/(d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 1.534, size = 5857, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*se
c(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*se
c(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x
 + c) + a)^(3/2), x)
```

3.965 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=352

$$\frac{2\cot(c+dx)\left(3Ab^2-(2a+b)(b(3B-C)-4aC)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}$$

```
[Out] (-2*(6*a^2*b*B - 3*b^3*B - a*b^2*(3*A - 5*C) - 8*a^3*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^
4*Sqrt[a + b]*d) + (2*(3*A*b^2 - (2*a + b)*(b*(3*B - C) - 4*a*C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(3*b^3*Sqrt[a + b]*d) + (2*a*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(b^
2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[a + b*Sec[c + d*x]]*T
an[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.801251, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4090, 4082, 4005, 3832, 4004}

$$\frac{2a\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\cot(c+dx)\left(6a^2bB-8a^3C-ab^2(3A-5C)-3b^3B\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+d}{a-b}}}{3b^4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(3/2),x]
```


```
[Out] (-2*(6*a^2*b*B - 3*b^3*B - a*b^2*(3*A - 5*C) - 8*a^3*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^
4*Sqrt[a + b]*d) + (2*(3*A*b^2 - (2*a + b)*(b*(3*B - C) - 4*a*C))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(3*b^3*Sqrt[a + b]*d) + (2*a*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(b^
2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[a + b*Sec[c + d*x]]*T
an[c + d*x])/(3*b^2*d)
```


Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
```

```
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}b\left(Ab^2-a(bB\right.\right.}{}}{}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{a+b\sec(c+dx)}\,\mathrm{t}}{3b^2d}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{a+b\sec(c+dx)}\,\mathrm{t}}{3b^2d}$$

$$= -\frac{2\left(6a^2bB-3b^3B-ab^2(3A-5C)-8a^3C\right)\cot(c+dx)E\left(\sin\right.}{3b^4\sqrt{a+}}$$

Mathematica [B] time = 26.0625, size = 3856, normalized size = 10.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^(3/2),x]
```

[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(3*a*A*
b^2 - 6*a^2*b*B + 3*b^3*B + 8*a^3*C - 5*a*b^2*C)*Sin[c + d*x])/(3*b^3*(-a^2
 + b^2)) - (4*(a*A*b^2*Sin[c + d*x] - a^2*b*B*Sin[c + d*x] + a^3*C*Sin[c +
d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (4*C*Tan[c + d*x])/(3*b^2)
))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x]
)^(3/2)) - (4*(b + a*Cos[c + d*x])*((-2*a*A)/((-a^2 + b^2)*Sqrt[b + a*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^2*B)/(b*(-a^2 + b^2)*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (2*b*B)/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) + (10*a*C)/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[c + d*x]]) - (16*a^3*C)/(3*b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) - (2*a^2*A*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)*Sqrt[b
+ a*Cos[c + d*x]]) + (2*A*b*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Co
s[c + d*x]]) - (4*a*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c +
d*x]]) + (4*a^3*B*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c +
d*x]]) - (16*a^4*C*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c
 + d*x]]) + (14*a^2*C*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Cos[
c + d*x]]) + (2*b*C*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c +
d*x]]) - (2*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)*Sqrt
[b + a*Cos[c + d*x]]) - (2*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2
+ b^2)*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d
x]])/(b^2(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (16*a^4*C*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (10
*a^2*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Co
s[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*(2*(a + b)*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a
^3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] - 2*b*(a + b)*(3*A*b^2 + (2*a - b)*(4*a*C - b*(3*B + C)))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*b*B + 3
*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]
*(a + b*Sec[c + d*x])^(3/2)*((-2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Si
n[c + d*x]*(2*(a + b)*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2
b(a + b)*(3*A*b^2 + (2*a - b)*(4*a*C - b*(3*B + C)))*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*b*B + 3*b^3*B
 + a*b^2*(3*A - 5*C) + 8*a^3*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^(3/2)
*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(
c + d*x)/2]*(2*(a + b)*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
2*b*(a + b)*(3*A*b^2 + (2*a - b)*(4*a*C - b*(3*B + C)))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*b*B + 3*b^3*
B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*
Sqrt[Sec[(c + d*x)/2]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-6*
a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Cos[c + d*x]*(b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A
 - 5*C) + 8*a^3*C)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c +
 d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(3*A*b^2 + (2*a - b)*(4*a*C - b*(3*
B + C)))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[

```
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1
+ Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])] + ((a + b)*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a
^3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + (
(b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(3*A*b^2 + (2
*a - b)*(4*a*C - b*(3*B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
 a*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Cos[c + d*x]*Sec[(c
 + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-6*a^2*b*B + 3*b^3*B + a*b^2*
(3*A - 5*C) + 8*a^3*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]
*Tan[(c + d*x)/2] + (-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*
(a + b)*(3*A*b^2 + (2*a - b)*(4*a*C - b*(3*B + C)))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[
(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)]) + ((a + b)*(-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8
*a^3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^3*(-a^2 + b^2)*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*(2*(a + b)*(-6*a^2*b*B +
 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*A*b^2 + (2*a - b)*(4*
a*C - b*(3*B + C)))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + (-6*a^2*b*B + 3*b^3*B + a*b^2*(3*A - 5*C) + 8*a^3*C)*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[
(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*
x]*Tan[c + d*x]))/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c
+ d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.745, size = 4183, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d/(a-b)/(a+b)/b^3*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*B*co
s(d*x+c)^2*b^4-8*C*cos(d*x+c)^3*a^4+3*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^2*b^2+3*B*cos(d*x+c
)*b^4+6*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-6*B*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^3*b-6*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
```

```
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-8*C*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
3*b-2*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^2*b^2+5*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+8*C*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3
*b-5*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^2*b^2-5*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-3*A*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*b^4-
3*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)*b^4+3*A*cos(d*x+c)^3*a*b^3-3*B*cos(d*x+c)^3*a^2*b^2+3*A*cos
(d*x+c)^2*a^2*b^2-6*B*cos(d*x+c)^2*a^3*b+3*B*cos(d*x+c)^2*a*b^3-3*B*cos(d*x
+c)*a^2*b^2+3*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)^2*a*b^3-3*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a*b^3-3*A*cos(d*x+c)^
2*a*b^3+6*B*cos(d*x+c)^2*a^2*b^2+8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-8*C*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+
8*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3*b-5*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+5*C*cos(d*x+c)^2*a*b^3+6*B*co
s(d*x+c)^3*a^3*b-3*B*cos(d*x+c)^3*a*b^3+5*C*cos(d*x+c)^3*a^2*b^2+3*A*cos(d*
x+c)*a^2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))+3*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-3*A*cos(d*x+c)*b^3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+4*C*cos(d*x+c)^3*
a^3*b-C*cos(d*x+c)^3*a*b^3-4*C*cos(d*x+c)^2*a^2*b^2-4*C*cos(d*x+c)*a*b^3-8*
C*cos(d*x+c)^2*a^3*b+4*C*cos(d*x+c)*a^3*b-6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-6*B*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b^2+3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-C*a^2*b^2+8*C*co
s(d*x+c)^2*a^4+3*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-5*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
```

```
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-2*C*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2+5*C*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*cos(d*x+c)*a*b^3-3*A*cos(d*x+c)^3*a^2*b^2-C*cos(d*x+c)^2*b^4+C
*b^4-3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*b^4+3*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-C*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+8*C*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^4-3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+3*B*cos(d*x+c)*b^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)))/
(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*se
c(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*se
c(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right) \sec \left(dx+c\right)^{2}}{\left(b \sec \left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c) + a)^(3/2), x)
```

3.966 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2\cot(c+dx)(-2aC+Ab+b(B-C))\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}-\frac{2\tan(c+d}{bd\left(a^2-b^2\right.}$$

```
[Out] (-2*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (
2*(A*b + b*(B - C) - 2*a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) - (2*(A*b^2 -
 a*(b*B - a*C))*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.464037, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {4080, 4005, 3832, 4004}

$$-\frac{2\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\cot(c+dx)\left(2a^2C-abB+Ab^2-b^2C\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\right.\right.}{b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^(3/2),x]
```


```
[Out] (-2*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (
2*(A*b + b*(B - C) - 2*a*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) - (2*(A*b^2 -
 a*(b*B - a*C))*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}b(bB-a(A\right.}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{((a-b)(Ab+b(B-}{b}$$

$$= -\frac{2\left(Ab^2-abB+2a^2C-b^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{b^3\sqrt{a+bd}}$$

Mathematica [B] time = 20.9415, size = 603, normalized size = 2.06

$$\frac{4\sqrt{2}\sqrt{\frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a\cos(c+dx)+b)\left(A+B\sec(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(A*b^2
 - a*b*B + 2*a^2*C - b^2*C)*Sin[c + d*x])/(b^2*(-a^2 + b^2)) + (4*(A*b^2*Si
n[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(b*(-a^2 + b^2)*(b +
 a*Cos[c + d*x]))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + b*Sec[c + d*x])^(3/2)) + (4*Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
^2]*(b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*
x)/2]^2*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((a + b
)*((A*b^2 - a*b*B + 2*a^2*C - b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + b*(-(A*b) - 2*a*C + b*(B + C))*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sec[c + d*x] + (A*b^2 - a*b
*B + 2*a^2*C - b^2*C)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4*
Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[(1 + Cos[c + d*x])^(-1)]*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt
```

```
[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.507, size = 3071, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/b^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*C*a^3*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^3-2*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+A*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-B*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+C*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-C*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+A*cos(d*
x+c)*b^3-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2-a^2*b*C+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+C*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-
2*C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a*b^2+C*cos(d*x+c)*a*b^2+B*cos(d*x+c)^2*a*b^2-C*cos(
d*x+c)^2*a^2*b-A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+2*C*cos(d*x+c)^2*a^3
-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^3-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+A*cos(d*x+c)^2*a*b^2-A*cos(d*x+c)*a*b^
2-2*C*cos(d*x+c)*a^3-A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(
```

```
d*x+c),((a-b)/(a+b))^(1/2))-A*cos(d*x+c)^2*b^3-B*cos(d*x+c)^2*a^2*b+B*cos(d
*x+c)*a^2*b-B*cos(d*x+c)*a*b^2+2*C*cos(d*x+c)*a^2*b-C*cos(d*x+c)^2*a*b^2-A*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*b^3-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^2*sin(d*x+c)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+C*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*C*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*C*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-B
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin
(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^2*sin(d*x+c)+C*b^3-C*cos(d*x+c)*b^3)/(b+a*cos(d*x+c))/sin(d*x+
c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(
d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c
 + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A\right)\sec\left(dx + c\right)}{\left(b\sec\left(dx + c\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c) + a)^(3/2), x)
```

3.967 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=395

$$-\frac{2\cot(c+dx)(Ab-a(B+C))\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}+\frac{2\tan(c+dx)\,(\ldots}{ad\left(a^2-b^2\right)\sqrt{\ldots}}$$

```
[Out] (2*(A*b^2 - a*(b*B - a*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*Sqrt[a + b]*d) - (2*(A*b -
a*(B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x
]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(a^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(a*(a^2 -
 b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.488739, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\ldots\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(A*b^2 - a*(b*B - a*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*Sqrt[a + b]*d) - (2*(A*b -
a*(B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x
]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(a^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(a*(a^2 -
 b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A\left(a^2-b^2\right)+\frac{1}{2}a(Ab-aB+bC)\sec(c+dx)+\frac{1}{2}(A}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A\left(a^2-b^2\right)+\left(\frac{1}{2}a(Ab-aB+bC)+\frac{1}{2}\left(-Ab^2+a(bB\right.\right.}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx)}{a+b}}}{ab^2\sqrt{a+b}d}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx)}{a+b}}}{ab^2\sqrt{a+b}d}$$

Mathematica [B] time = 18.8224, size = 1275, normalized size = 3.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(3/2
),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(A*b^2
 - a*b*B + a^2*C)*Sin[c + d*x])/(a*b*(-a^2 + b^2)) + (4*(A*b^2*Sin[c + d*x]
 - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(a*(a^2 - b^2)*(b + a*Cos[c + d
*x]))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c +
 d*x])^(3/2)) - (4*(b + a*Cos[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*b^2*Tan[(c + d*x)/2] +
A*b^3*Tan[(c + d*x)/2] - a^2*b*B*Tan[(c + d*x)/2] - a*b^2*B*Tan[(c + d*x)/2
] + a^3*C*Tan[(c + d*x)/2] + a^2*b*C*Tan[(c + d*x)/2] - 2*a*A*b^2*Tan[(c +
d*x)/2]^3 + 2*a^2*b*B*Tan[(c + d*x)/2]^3 - 2*a^3*C*Tan[(c + d*x)/2]^3 + a*A
*b^2*Tan[(c + d*x)/2]^5 - A*b^3*Tan[(c + d*x)/2]^5 - a^2*b*B*Tan[(c + d*x)/
2]^5 + a*b^2*B*Tan[(c + d*x)/2]^5 + a^3*C*Tan[(c + d*x)/2]^5 - a^2*b*C*Tan[
(c + d*x)/2]^5 - 2*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a^2*A*b*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + 2*A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(A*b^2 + a*(-(b*B)
+ a*C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - a*b*(a + b)*(A - B + C)*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[
(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)]))/(a*(-(a^2*b) + b^3)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(1 + Tan[(c + d*x)/2]
^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + T
an[(c + d*x)/2]^2)])
```

Maple [B] time = 0.358, size = 2844, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/b/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+
c)-C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))-C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))+A*cos(d*x+c)*b^3-B*cos(d*x+c)*a^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b^2-C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
```

```
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^2+C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+B*cos(d*x+c)^2*a*b^2-C*cos(d*x+c)^2*a^2*
b+A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*cos(d*x+c)*b-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b-A*b^2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+A*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b^2+C*cos(d*x+c)^2*a^3+A*cos(d*x+c)^2*a*b^2-A*cos(d*x+c)*a*b^2-C*c
os(d*x+c)*a^3-A*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))-A*cos(d*x+c)^2*b^3-B*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)*a
^2*b-B*cos(d*x+c)*a*b^2+C*cos(d*x+c)*a^2*b+2*A*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+A*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b
-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*b-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+A*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+C*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+C*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x
+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
b*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/(
b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(3/
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(3/2
), x)
```

3.968 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=451

$$\frac{\cot(c+dx)\left(2a^2C+ab(A-2B)+3Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2bd\sqrt{a+b}}-\frac{b\tan(c}{a}$$

```
[Out] -(((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d))
 + ((3*A*b^2 + a*b*(A - 2*B) + 2*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d)
 + (Sqrt[a + b]*(3*A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[
c + d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]]) - (b*(3*A*b^2 - 2*a*b*B - a^2*(A -
 2*C))*Tan[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.750707, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\tan(c+dx)\left(a^2(-(A-2C))-2abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\cot(c+dx)\left(2a^2C+ab(A-2B)+3Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+}{a-b}}}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^(3/2),x]
```


```
[Out] -(((3*A*b^2 - 2*a*b*B - a^2*(A - 2*C))*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d))
 + ((3*A*b^2 + a*b*(A - 2*B) + 2*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d)
 + (Sqrt[a + b]*(3*A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[
c + d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]]) - (b*(3*A*b^2 - 2*a*b*B - a^2*(A -
 2*C))*Tan[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{2}(3Ab-2aB)-aC\sec(c+dx)-\frac{1}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,d}{a}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{b\left(3Ab^2-2abB-a^2(A-2C)\right)\tan(c+}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{b\left(3Ab^2-2abB-a^2(A-2C)\right)\tan(c+}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{\left(3Ab^2-2abB-a^2(A-2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{a^2b\sqrt{a+b}d}$$

$$= -\frac{\left(3Ab^2-2abB-a^2(A-2C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+}}{\sqrt{a+b}}\right.\right.}{a^2b\sqrt{a+b}d}$$

Mathematica [B] time = 20.8853, size = 1814, normalized size = 4.02

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(A*b^2
 - a*b*B + a^2*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)) - (4*(A*b^3*Sin[c + d*x] -
 a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(b + a*Cos[
c + d*x]))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Se
c[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*A*Tan[(c
 + d*x)/2] + a^2*A*b*Tan[(c + d*x)/2] - 3*a*A*b^2*Tan[(c + d*x)/2] - 3*A*b^
3*Tan[(c + d*x)/2] + 2*a^2*b*B*Tan[(c + d*x)/2] + 2*a*b^2*B*Tan[(c + d*x)/2
] - 2*a^3*C*Tan[(c + d*x)/2] - 2*a^2*b*C*Tan[(c + d*x)/2] - 2*a^3*A*Tan[(c
+ d*x)/2]^3 + 6*a*A*b^2*Tan[(c + d*x)/2]^3 - 4*a^2*b*B*Tan[(c + d*x)/2]^3 +
 4*a^3*C*Tan[(c + d*x)/2]^3 + a^3*A*Tan[(c + d*x)/2]^5 - a^2*A*b*Tan[(c + d
*x)/2]^5 - 3*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*A*b^3*Tan[(c + d*x)/2]^5 + 2*a^
2*b*B*Tan[(c + d*x)/2]^5 - 2*a*b^2*B*Tan[(c + d*x)/2]^5 - 2*a^3*C*Tan[(c +
d*x)/2]^5 + 2*a^2*b*C*Tan[(c + d*x)/2]^5 + 6*a^2*A*b*EllipticPi[-1, -ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*Ellipt
icPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- 4*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] + 4*a*b^2*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*A*b*EllipticPi[-1, -ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*
A*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d
*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^3*B*EllipticPi[-1, -ArcSin[Tan[(c + d*
```

```
x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*b^2
*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)
/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-3*A*b^2 + 2*a*b*B + a^2*(A - 2*C))*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 2*a*(a + b)*(-(A*b) + a*(B - C))*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)]))/(a^2*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/
2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.408, size = 3673, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*B*a^
3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))+6*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*b^3*sin(d*x+c)+2*C*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^3-A*cos(d*x+c)^3*a*b^2+A*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-2*B*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-4*
B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^
2*sin(d*x+c)-2*C*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-2*C*a^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-4*B*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*a*b^2+4*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*a^3*sin(d*x+c)+3*A*cos(d*x+c)*b^3+A*cos(d*x+c)^3*a^3-A*cos(d
*x+c)^2*a^3-2*B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*C*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-2*C*a^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
```

```
*x+c)*cos(d*x+c)*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*B*cos(
d*x+c)^2*a*b^2-2*C*cos(d*x+c)^2*a^2*b+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*b-6*A*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*co
s(d*x+c)*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*cos(d*x+c)*b-3*A*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+2*A*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+
2*C*cos(d*x+c)^2*a^3+3*A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a*b^2+A*cos(d*x+
c)^2*a^2*b-A*cos(d*x+c)*a^2*b-2*C*cos(d*x+c)*a^3-3*A*b^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-3*A*cos(d*x+c)^2*
b^3-2*B*cos(d*x+c)^2*a^2*b+2*B*cos(d*x+c)*a^2*b-2*B*cos(d*x+c)*a*b^2+2*C*co
s(d*x+c)*a^2*b+6*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*b^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+A*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+2*A*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-6*A*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*b-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-3*A*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*A*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*C*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d
*x+c)-2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b*sin(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b*sin(d*x+c)+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x +
 c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\cos\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*cos(c + d*x)/(a + b*sec(c
 + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x +
 c) + a)^(3/2), x)
```

3.969 $\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=552

$$-\frac{\cot(c+dx)\left(-2a^2(A+2B-4C)+ab(5A-12B)+15Ab^2\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{4a^3d\sqrt{a+b}}$$

```
[Out] ((15*A*b^3 + 4*a^3*B - 12*a*b^2*B - a^2*(7*A*b - 8*b*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a
^3*b*Sqrt[a + b]*d) - ((15*A*b^2 + a*b*(5*A - 12*B) - 2*a^2*(A + 2*B - 4*C)
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(15*A*b^2 - 12*a*b*B +
 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^4*d) - ((5*A*b - 4*a*B)*Si
n[c + d*x])/(4*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(15*A*b^3 + 4*a^3*B - 12*a*b^2*B
- a^2*(7*A*b - 8*b*C))*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]])
```

Rubi [A] time = 1.24644, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\tan(c+dx)\left(-a^2(7Ab-8bC)+4a^3B-12ab^2B+15Ab^3\right)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\cot(c+dx)\left(-2a^2(A+2B-4C)+ab(5A-12B)+15Ab^2\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(3/2),x]
```


```
[Out] ((15*A*b^3 + 4*a^3*B - 12*a*b^2*B - a^2*(7*A*b - 8*b*C))*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a
^3*b*Sqrt[a + b]*d) - ((15*A*b^2 + a*b*(5*A - 12*B) - 2*a^2*(A + 2*B - 4*C)
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(15*A*b^2 - 12*a*b*B +
 4*a^2*(A + 2*C))*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^4*d) - ((5*A*b - 4*a*B)*Si
n[c + d*x])/(4*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(15*A*b^3 + 4*a^3*B - 12*a*b^2*B
- a^2*(7*A*b - 8*b*C))*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
```

*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rule 4060

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(5Ab-4aB)-a(A+2C)\sec(c\right.}{(a+b\sec(c+dx))^{3/}}}{2a}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{1}{4}}{}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{b\,(1}{}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{b\,(1}{}$$

$$= \frac{\left(15Ab^3+4a^3B-12ab^2B-a^2(7Ab-8bC)\right)\cot(c+dx)E\,(\sin}{4a^3b\sqrt{a+}}$$

$$= \frac{\left(15Ab^3+4a^3B-12ab^2B-a^2(7Ab-8bC)\right)\cot(c+dx)E\,(\sin}{4a^3b\sqrt{a+}}$$

Mathematica [C] time = 17.0449, size = 490, normalized size = 0.89

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{a\sin(2(c+dx))\left(b\left(a^2(A-4C)+4abB-5Ab^2\right)+aA\left(a^2-b^2\right)\cos(c+dx)\right)}{a\cos(c+dx)+b}+\cos(c+dx)\left(\tan\left(\frac{1}{2}(c+dx)\right)\left(a^2(8bC-7Ab)+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((a*(b*(-5*A*b^2 + 4*a*b*B + a^2*(A - 4*C)) + a*A
*(a^2 - b^2)*Cos[c + d*x])*Sin[2*(c + d*x)])/(b + a*Cos[c + d*x]) + Cos[c +
 d*x]*((I*(a - b)*((-15*A*b^3 - 4*a^3*B + 12*a*b^2*B + a^2*b*(7*A - 8*C))*E
llipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)] + 2*(15*A*b^3 + 2*a*b^2*(5*A - 6*B) + 2*a^3*(A + 2*C) + a^2*b*(A - 8*B +
 8*C))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)] - 2*(a + b)*(15*A*b^2 - 12*a*b*B + 4*a^2*(A + 2*C))*EllipticPi[-
((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
 b)/(a - b)])*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(Sqr
t[(-a + b)/(a + b)]*(b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]
^2]) + (15*A*b^3 + 4*a^3*B - 12*a*b^2*B + a^2*(-7*A*b + 8*b*C))*Tan[(c + d*
x)/2])))/(4*a^3*(a^2 - b^2)*d)
```

Maple [B] time = 0.56, size = 5176, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x
 + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec
(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x
 + c) + a)^(3/2), x)
```

3.970 $\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=549

$$-\frac{2\cot(c+dx)\left(-2a^2b^2(A-3B-8C)+a^3b(8B-12C)-16a^4C-3ab^3(A+3B-3C)+b^4(3A-3B+C)\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{3b^4d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (-2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 2*a^3*b^2*(A - 14*C) + 2*a*b^4*(3
*A - 4*C) - 16*a^5*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^5*Sqrt[a + b]*(a^2 - b^2)*d) - (2*
(a^3*b*(8*B - 12*C) - 2*a^2*b^2*(A - 3*B - 8*C) - 3*a*b^3*(A + 3*B - 3*C) -
 16*a^4*C + b^4*(3*A - 3*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^
2)*d) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^4 + a*(3*a^2*b*B - 7*b^3*
B - 6*a^3*C + 10*a*b^2*C))*Tan[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 1.85034, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4098, 4090, 4082, 4005, 3832, 4004}

$$-\frac{2\tan(c+dx)\sec^2(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\tan(c+dx)\left(2a^2C-abB+Ab^2-b^2C\right)\sqrt{a+b\sec(c+dx)}}{3b^3d\left(a^2-b^2\right)}-\frac{2a\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(5/2),x]
```


```
[Out] (-2*(8*a^4*b*B - 15*a^2*b^3*B + 3*b^5*B - 2*a^3*b^2*(A - 14*C) + 2*a*b^4*(3
*A - 4*C) - 16*a^5*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^5*Sqrt[a + b]*(a^2 - b^2)*d) - (2*
(a^3*b*(8*B - 12*C) - 2*a^2*b^2*(A - 3*B - 8*C) - 3*a*b^3*(A + 3*B - 3*C) -
 16*a^4*C + b^4*(3*A - 3*B + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^
2)*d) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^4 + a*(3*a^2*b*B - 7*b^3*
B - 6*a^3*C + 10*a*b^2*C))*Tan[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
```

```
x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a\left(4A\right.}{}$$

$$= -\frac{2\left(8a^4bB-15a^2b^3B+3b^5B-2a^3b^2(A-14C)+2ab^4(3A\right.}{}$$

Mathematica [A] time = 21.58, size = 989, normalized size = 1.8

$$\frac{\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{4\left(16Ca^5-8bBa^4+2Ab^2a^3-28b^2Ca^3+15b^3Ba^2-6Ab^4a+8b^4Ca-3b^5B\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)^2}-\frac{4(C}{}\right.}{d(\cos(2c+2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^(5/2),x]
```

```
[Out] (4*(b + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*((a + b)*(-8*a^4*b*B + 15
*a^2*b^3*B - 3*b^5*B + 2*a^3*b^2*(A - 14*C) + 16*a^5*C + 2*a*b^4*(-3*A + 4*
C))*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + b*(a + b)*(-2*a^2*b^2*(A + 3*B - 8*C) - 16*a
^4*C + b^4*(3*A + 3*B + C) + 4*a^3*b*(2*B + 3*C) + 3*a*b^3*(A - 3*(B + C)))
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + (-8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 2*a^3*b^
2*(A - 14*C) + 16*a^5*C + 2*a*b^4*(-3*A + 4*C))*Tan[(c + d*x)/2]*(b - b*Tan
[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2)))/(3*b^4*(a^2 - b^2)^2*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)
*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + ((b + a*Cos[c + d*x])^3*Sec[c
+ d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(2*a^3*A*b^2 - 6*a*A*b^
4 - 8*a^4*b*B + 15*a^2*b^3*B - 3*b^5*B + 16*a^5*C - 28*a^3*b^2*C + 8*a*b^4*
C)*Sin[c + d*x])/(3*b^4*(a^2 - b^2)^2) - (4*(a*A*b^2*Sin[c + d*x] - a^2*b*B
*Sin[c + d*x] + a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x
])^2) - (4*(-(a^3*A*b^2*Sin[c + d*x]) + 5*a*A*b^4*Sin[c + d*x] + 4*a^4*b*B*
Sin[c + d*x] - 8*a^2*b^3*B*Sin[c + d*x] - 7*a^5*C*Sin[c + d*x] + 11*a^3*b^2
*C*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (4*C*Tan[c
+ d*x])/(3*b^3)))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a +
```

```
b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 1.746, size = 10856, normalized size = 19.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{5}+B\sec\left(dx+c\right)^{4}+A\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^5 + B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*se
c(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(
d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**3/(a + b*se
c(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x
 + c) + a)^(5/2), x)
```

3.971 $\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=449

$$\frac{2\cot(c+dx)\left(2a^2b(B-3C)-8a^3C+ab^2(A+3B+9C)-3b^3(A+B-C)\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\Big(\sin}{3b^3d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (2*(2*a^3*b*B - 6*a*b^3*B + 3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))]])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(2*a^2*b*(B - 3*C) - 3*
b^3*(A + B - C) - 8*a^3*C + a*b^2*(A + 3*B + 9*C))*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*b^3*Sqr
t[a + b]*(a^2 - b^2)*d) + (2*a*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(3*b^2
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^3*b*B - 6*a*
b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Tan[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*S
qrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.04373, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4090, 4080, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+3Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\cot(c+dx)\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c +
 d*x])^(5/2),x]
```


```
[Out] (2*(2*a^3*b*B - 6*a*b^3*B + 3*b^4*(A - C) - 8*a^4*C + a^2*b^2*(A + 15*C))*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))]])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*(2*a^2*b*(B - 3*C) - 3*
b^3*(A + B - C) - 8*a^3*C + a*b^2*(A + 3*B + 9*C))*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*b^3*Sqr
t[a + b]*(a^2 - b^2)*d) + (2*a*(A*b^2 - a*(b*B - a*C))*Tan[c + d*x])/(3*b^2
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^3*b*B - 6*a*
b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Tan[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*S
qrt[a + b*Sec[c + d*x]])
```


Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}b\left(Ab^2\right.\right.}{}}{}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+2a^3bB-\right.}{3b^2\left(\right.}$$

$$= \frac{2a\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+2a^3bB-\right.}{3b^2\left(\right.}$$

$$= \frac{2\left(2a^3bB-6ab^3B+3b^4(A-C)-8a^4C+a^2b^2(A+15C)\right)}{3(a}$$

Mathematica [B] time = 27.6009, size = 4504, normalized size = 10.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*S
ec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-4*(a^2*A*b^2 + 3*A*b^4 + 2*a^3*b*B - 6*a*b^3*B - 8*a^4*C + 15*a^2*b^2*
C - 3*b^4*C)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)^2) + (4*(A*b^2*Sin[c + d*x]
- a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos[c
+ d*x])^2) + (4*(2*a^2*A*b^2*Sin[c + d*x] + 2*A*b^4*Sin[c + d*x] + a^3*b*B*
Sin[c + d*x] - 5*a*b^3*B*Sin[c + d*x] - 4*a^4*C*Sin[c + d*x] + 8*a^2*b^2*C*
Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) - (4*(b
+ a*Cos[c + d*x])^2*((2*a^2*A)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (2*A*b^2)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (4*a^3*B)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) - (4*a*b*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (10*a^2*C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (16*a^4*C)/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) - (2*b^2*C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (2*a^3*A*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]) - (2*a*A*b*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]) - (10*a^2*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt
[b + a*Cos[c + d*x]]) + (4*a^4*B*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (2*b^2*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sq
rt[b + a*Cos[c + d*x]]) - (16*a^5*C*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]) + (34*a^3*C*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b
^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (6*a*b*C*Sqrt[Sec[c + d*x]])/((-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]
])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a*A*b*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*a^2*B*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x
]]) + (4*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*S
qrt[b + a*Cos[c + d*x]]) - (16*a^5*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(
3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (10*a^3*C*Cos[2*(c + d*x)]
*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a*b*C
*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*
x]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*(2*(a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C +
3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*b^3*(A - B - C) - 8*a^3*
C + 2*a^2*b*(B + 3*C) + a*b^2*(A - 3*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*b*B + 6*a*b^3*B + 8*a
^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[
c + d*x])^(5/2)*((-2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(
2*(a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A +
 15*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + 2*b*(a + b)*(3*b^3*(A - B - C) - 8*a^3*C + 2*a^2*b*(B + 3*C) + a*b^2
*(A - 3*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b
^2*(A + 15*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2]))/(3*b^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d
*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(
a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
```

```
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + 2*b*(a + b)*(3*b^3*(A - B - C) - 8*a^3*C + 2*a^2*b*(B + 3*C) + a*b^2*(A
 - 3*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*
(A + 15*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2
]^2]) - (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-2*a^3*b*B + 6*a*b^3*B
+ 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Cos[c + d*x]*(b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C
+ 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((
Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c +
 d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(3*b^3*(A - B -
 C) - 8*a^3*C + 2*a^2*b*(B + 3*C) + a*b^2*(A - 3*B + 9*C))*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c
 + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a +
b)*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b +
a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*(a + b)*(3*b^3*(A - B - C)
 - 8*a^3*C + 2*a^2*b*(B + 3*C) + a*b^2*(A - 3*B + 9*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-(
(a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[
c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))] - a*(-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A +
 C) - a^2*b^2*(A + 15*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[
(c + d*x)/2] - (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2
*(A + 15*C))*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
d*x)/2] + (-2*a^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A +
 15*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]^2 + (b*(a + b)*(3*b^3*(A - B - C) - 8*a^3*C + 2*a^2*b*(B + 3*C) + a*b^2*
(A - 3*B + 9*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-2*a
^3*b*B + 6*a*b^3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c +
d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*(2*(a + b)*(-2*a^3*b*B + 6*a*b^3*B + 8
*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*b^3*(A - B - C)
 - 8*a^3*C + 2*a^2*b*(B + 3*C) + a*b^2*(A - 3*B + 9*C))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*b*B + 6*a*b^
3*B + 8*a^4*C + 3*b^4*(-A + C) - a^2*b^2*(A + 15*C))*Cos[c + d*x]*(b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c
+ d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(
3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

Maple [B] time = 0.845, size = 8858, normalized size = 19.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{4}+B\sec\left(dx+c\right)^{3}+A\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^4 + B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*se
c(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(
d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**
(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)**2/(a + b*se
c(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x
 + c) + a)^(5/2), x)
```

3.972 $\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=416

$$\frac{2\cot(c+dx)\left(2a^2C+ab(3A+B+3C)-b^2(A+3(B+C))\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{3b^2d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (-2*(4*a*A*b^2 - a^2*b*B - 3*b^3*B - 2*a^3*C + 6*a*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*
(a - b)*b^3*(a + b)^(3/2)*d) + (2*(2*a^2*C + a*b*(3*A + B + 3*C) - b^2*(A +
 3*(B + C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*(A*b^2 -
a*(b*B - a*C))*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
 + (2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Tan[c + d*x])/(3*
b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.793927, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {4080, 4003, 4005, 3832, 4004}

$$\frac{2\tan(c+dx)\left(a^2bB+2a^3C-2ab^2(2A+3C)+3b^3B\right)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\cot(c+dx)\left(2a^2C+ab(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^(5/2),x]
```


```
[Out] (-2*(4*a*A*b^2 - a^2*b*B - 3*b^3*B - 2*a^3*C + 6*a*b^2*C)*Cot[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*
(a - b)*b^3*(a + b)^(3/2)*d) + (2*(2*a^2*C + a*b*(3*A + B + 3*C) - b^2*(A +
 3*(B + C)))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*(A*b^2 -
a*(b*B - a*C))*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
 + (2*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Tan[c + d*x])/(3*
b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(\frac{3}{2}b(bB-a(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a\ldots\right.}{3b\left(a^2-b\ldots\right.}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2bB+3b^3B+2a\ldots\right.}{3b\left(a^2-b\ldots\right.}$$

$$= \frac{2\left(a^2bB+3b^3B+2a^3C-2ab^2(2A+3C)\right)\cot(c+dx)E\left(\sin\ldots\right.}{3(a-b)b^3(a\ldots}$$

Mathematica [B] time = 26.0426, size = 3980, normalized size = 9.57

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]

[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-4*(-4*a*A*b^2 + a^2*b*B + 3*b^3*B + 2*a^3*C - 6*a*b^2*C)*Sin[c + d*x])
/(3*b^2*(a^2 - b^2)^2) + (4*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*
C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (4*(-5*a^2*A*b^
2*Sin[c + d*x] + A*b^4*Sin[c + d*x] + 2*a^3*b*B*Sin[c + d*x] + 2*a*b^3*B*Si
n[c + d*x] + a^4*C*Sin[c + d*x] - 5*a^2*b^2*C*Sin[c + d*x]))/(3*a*b*(-a^2 +
 b^2)^2*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) + (4*(b + a*Cos[c + d*x])^2*((-8*a*A*
b)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*
B)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*
B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a^3*C)
/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*b*
C)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a^2*A*
Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*A*b^2*
Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*B*
Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a*b*
B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (10*a^2
*C*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4
*C*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2
*b^2*C*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a
^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[
c + d*x]]) + (2*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]
])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*a^2*C*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*a^4*C*Cos[
2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d
*x]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*(2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*
b^2*(2*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + 2*b*(a + b)*(-2*a^2*C + b^2*(A - 3*B + 3*C) + a*b*(3*A - B
+ 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + (a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x]*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)
^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[(c + d*x)/2
]^2]*(a + b*Sec[c + d*x])^(5/2)*((2*a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 2*b*(a + b)*(-2*a^2*C + b^2*(A - 3*B + 3*C) + a*b*(3*A - B + 3*C))*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*b*B
 + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*(b + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2
*(2*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] + 2*b*(a + b)*(-2*a^2*C + b^2*(A - 3*B + 3*C) + a*b*(3*A - B + 3
*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x]*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (4*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(((a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Co

```
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(a^2*b*B +
 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[
c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-2*a^2*C +
b^2*(A - 3*B + 3*C) + a*b*(3*A - B + 3*C))*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos
[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(a^2*b*B + 3*
b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + (b*(a + b)*(-2*a^2*C + b^2*(A - 3*B + 3*C) + a*b*(3*A - B + 3*C))*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a
*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(a^2*b*B + 3*b^3*B + 2*a^3*C
 - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c
 + d*x)/2] - (a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (a^2*b*B + 3*
b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x]*(b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-2*a^2*C + b^2*(A - 3*B +
 3*C) + a*b*(3*A - B + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((
a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[
1 - Tan[(c + d*x)/2]^2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[(c + d*x)/2]^2]) + (2*(2*(a + b)*(a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*
b^2*(2*A + 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + 2*b*(a + b)*(-2*a^2*C + b^2*(A - 3*B + 3*C) + a*b*(3*A - B
+ 3*C))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + (a^2*b*B + 3*b^3*B + 2*a^3*C - 2*a*b^2*(2*A + 3*C))*Cos[c + d*x]*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*
Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*
x]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2
]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

Maple [B] time = 0.483, size = 6953, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{3}+B\sec\left(dx+c\right)^{2}+A\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^3 + B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(
d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*
x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sec(c + d*x)/(a + b*sec(c
 + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x +
 c) + a)^(5/2), x)
```

3.973 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=541

$$\frac{2\cot(c+dx)\left(-a^2b(6A+B+3C)+a^3(3B+C)+aAb^2+3Ab^3\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+}}{}\right.\right.}{3a^2bd\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (2*(7*a^2*A*b^2 - 3*A*b^4 - 4*a^3*b*B + a^4*C + 3*a^2*b^2*C)*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/
(3*a^2*(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*A*b^2 + 3*A*b^3 + a^3*(3*B + C)
 - a^2*b*(6*A + B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*a^2*b*Sqrt[a + b]*(a^2 - b^2)*d)
 - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(a^3*d) + (2*(A*b^2 - a*(b*B
- a*C))*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*(
3*A*b^4 + 4*a^3*b*B - a^4*C - a^2*b^2*(7*A + 3*C))*Tan[c + d*x])/(3*a^2*(a^
2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.910364, antiderivative size = 541, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\tan(c+dx)\left(-a^2b^2(7A+3C)+4a^3bB+a^4(-C)+3Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\tan(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\cot(c+dx)\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(7*a^2*A*b^2 - 3*A*b^4 - 4*a^3*b*B + a^4*C + 3*a^2*b^2*C)*Cot[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/
(3*a^2*(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*A*b^2 + 3*A*b^3 + a^3*(3*B + C)
 - a^2*b*(6*A + B + 3*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*a^2*b*Sqrt[a + b]*(a^2 - b^2)*d)
 - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(a^3*d) + (2*(A*b^2 - a*(b*B
- a*C))*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*(
3*A*b^4 + 4*a^3*b*B - a^4*C - a^2*b^2*(7*A + 3*C))*Tan[c + d*x])/(3*a^2*(a^
2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2 - a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}A\left(a^2-b^2\right)+\frac{3}{2}a(Ab-aB+bC)\sec(c+dx)\cdots}{(a+b\sec(c+dx\cdots}}{3a\left(a^2-b^{\cdots}\right.}$$

$$= \frac{2\left(Ab^2 - a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(3Ab^4 + 4a^3bB - a^4C - a^2b^2(7A \cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec\cdots}}$$

$$= \frac{2\left(Ab^2 - a(bB-aC)\right)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(3Ab^4 + 4a^3bB - a^4C - a^2b^2(7A \cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec\cdots}}$$

$$= -\frac{2\left(3Ab^4 + 4a^3bB - a^4C - a^2b^2(7A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(\cdots}}{\sqrt{a+\cdots}}\right.\right.}{3a^2(a-b)b^2(a+b)^{3/2}d}$$

$$= -\frac{2\left(3Ab^4 + 4a^3bB - a^4C - a^2b^2(7A+3C)\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(\cdots}}{\sqrt{a+\cdots}}\right.\right.}{3a^2(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [B] time = 28.012, size = 11444, normalized size = 21.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(5/2
),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.465, size = 8177, normalized size = 15.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/(
b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(5/
2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2
), x)
```

3.974 $\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=618

$$-\frac{\cot(c+dx)\left(-a^2b^2(21A+2B)-a^3b(3A-2(6B+C))-6a^4C+ab^3(5A-6B)+15Ab^4\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)\cdot}{a-b}}}{3a^3bd\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] -((26*a^2*A*b^2 - 15*A*b^4 - 14*a^3*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) - ((15*A*b^4 + a*b^3*(5*A - 6*B
) - a^2*b^2*(21*A + 2*B) - 6*a^4*C - a^3*b*(3*A - 2*(6*B + C)))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(3*a^3*b*Sqrt[a + b]*(a^2 - b^2)*d) + (Sqrt[a + b]*(5*A*b - 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(a^4*d) + (A*Sin[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(
3/2)) - (b*(5*A*b^2 - 2*a*b*B - a^2*(3*A - 2*C))*Tan[c + d*x])/(3*a^2*(a^2
- b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(26*a^2*A*b^2 - 15*A*b^4 - 14*a^3
*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.48122, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b\tan(c+dx)\left(26a^2Ab^2+a^4(-(3A-8C))-14a^3bB+6ab^3B-15Ab^4\right)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{b\tan(c+dx)\left(a^2(-(3A-2C))-2abB+5\right.}{3a^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec[c + d
*x])^(5/2),x]
```


```
[Out] -((26*a^2*A*b^2 - 15*A*b^4 - 14*a^3*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) - ((15*A*b^4 + a*b^3*(5*A - 6*B
) - a^2*b^2*(21*A + 2*B) - 6*a^4*C - a^3*b*(3*A - 2*(6*B + C)))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(3*a^3*b*Sqrt[a + b]*(a^2 - b^2)*d) + (Sqrt[a + b]*(5*A*b - 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(a^4*d) + (A*Sin[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(
3/2)) - (b*(5*A*b^2 - 2*a*b*B - a^2*(3*A - 2*C))*Tan[c + d*x])/(3*a^2*(a^2
- b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(26*a^2*A*b^2 - 15*A*b^4 - 14*a^3
*b*B + 6*a*b^3*B - a^4*(3*A - 8*C))*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{2}(5Ab-2aB)-aC\sec(c+dx)-\frac{3}{2}Ab\sec^2(c}{(a+b\sec(c+dx))^{5/2}}}{a}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{b\left(5Ab^2-2abB-a^2(3A-2C)\right)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{\left(26a^2Ab^2-15Ab^4-14a^3bB+6ab^3B-a^4(3A-8C)\right)\cot}{3a^3(a}$$

$$= -\frac{\left(26a^2Ab^2-15Ab^4-14a^3bB+6ab^3B-a^4(3A-8C)\right)\cot}{3a^3(a}$$

Mathematica [B] time = 28.8693, size = 20207, normalized size = 32.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.654, size = 10319, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x
 + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x +
 c) + a)^(5/2), x)
```

3.975 $\int (a+b\sec(c+dx))^{3/2}\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\right.$

Optimal. Leaf size=448

$$-\frac{2\sqrt{a+b}\left(-3a^2b(15B+4C)+30a^3C+ab^2(35B-12C)-b^3(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ell}}{15d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*B - 12*a^2*C + 9*b^2*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d
) - (2*Sqrt[a + b]*(a*b^2*(35*B - 12*C) - b^3*(5*B - 9*C) + 30*a^3*C - 3*a^
2*b*(15*B + 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(15*d) - (2*a^2*Sqrt[a + b]*(b*B - a*C)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/d + (2*b^2*(5*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(15*d) + (2*b^2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5
*d)
```

Rubi [A] time = 0.892883, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4041, 3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(-3a^2b(15B+4C)+30a^3C+ab^2(35B-12C)-b^3(5B-9C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*
Sec[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*b*B - 12*a^2*C + 9*b^2*C)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d
) - (2*Sqrt[a + b]*(a*b^2*(35*B - 12*C) - b^3*(5*B - 9*C) + 30*a^3*C - 3*a^
2*b*(15*B + 4*C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(15*d) - (2*a^2*Sqrt[a + b]*(b*B - a*C)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/d + (2*b^2*(5*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(15*d) + (2*b^2*C*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5
*d)
```


Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
```

```
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right)dx = \frac{\int (a+b\sec(c+dx))^{5/2}\left(b^2(bB-aC)\right.}{b^2}$$

$$= \frac{2b^2C(a+b\sec(c+dx))^{3/2}\tan(c+dx}{5d}$$

$$= \frac{2b^2(5bB+3aC)\sqrt{a+b\sec(c+dx)}\,\mathrm{ta}}{15d}$$

$$= \frac{2b^2(5bB+3aC)\sqrt{a+b\sec(c+dx)}\,\mathrm{ta}}{15d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(35abB-12a^2C+9\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(35abB-12a^2C+9\right.}{}$$

Mathematica [B] time = 23.6362, size = 4778, normalized size = 10.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] +
b^2*C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(b*B - a*C + b*C*Sec[c + d*x])*(
(2*b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*Sin[c + d*x])/15 + (2*Sec[c + d*x]*(5*
b^3*B*Sin[c + d*x] + 6*a*b^2*C*Sin[c + d*x]))/15 + (2*b^3*C*Sec[c + d*x]*Ta
n[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2*(b*C + b*B*Cos[c + d*x] - a*C*Cos
[c + d*x])) + (2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*((a^3*b*B)/(Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*a*b^3*B)/(3*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) - (a^4*C)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) + (4*a^2*b^2*C)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*b
^4*C)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*b^2*B*Sqrt[S
ec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) + (b^4*B*Sqrt[Sec[c + d*x]])/(3*
Sqrt[b + a*Cos[c + d*x]]) - (6*a^3*b*C*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Co
s[c + d*x]]) + (a*b^3*C*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) -
(7*a^2*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x
]]) + (4*a^3*b*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c +
 d*x]]) - (3*a*b^3*C*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos
[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/
2)*(b*B - a*C + b*C*Sec[c + d*x])*(-(b*(a + b)*(35*a*b*B - 12*a^2*C + 9*b^2
*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(-15*
a^2*C + 3*a*b*(10*B + C) + b^2*(5*B + 9*C))*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] - 15*a^2*(b*B - a*C)*((a - b)*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] - b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*(b + a*Cos[c + d*x
])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/
(15*d*(b + a*Cos[c + d*x])^3*(b*C + b*B*Cos[c + d*x] - a*C*Cos[c + d*x])*(S
ec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(7/2)*((a*Sqrt[Cos[c + d*x]*Sec[(c +
d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-(b*(a + b)*
```

(35*a*b*B - 12*a^2*C + 9*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)]) + b*(a + b)*(-15*a^2*C + 3*a*b*(10*B + C) + b^2*(5*B + 9*C))*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[(
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 15*a^2*(b*B - a*C)*((a
- b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[
-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(35*a*b*B - 12*a^2*C +
 9*b^2*C)*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[
c + d*x]*Tan[(c + d*x)/2]))/(15*(b + a*Cos[c + d*x])^(3/2)*(Sec[(c + d*x)/2
]^2)^(3/2)) - (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-(b*(a + b)*(35*a*b*B - 12*a^2*C + 9*b^2*
C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(-15*a
^2*C + 3*a*b*(10*B + C) + b^2*(5*B + 9*C))*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] - 15*a^2*(b*B - a*C)*((a - b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] - b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*(b + a*Cos[c + d*x]
)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(
5*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)) + (Sqrt[Cos[c + d*x]
*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-(Sec[(c + d*
x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(
-(b*(a + b)*(35*a*b*B - 12*a^2*C + 9*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)]) + b*(a + b)*(-15*a^2*C + 3*a*b*(10*B + C) + b^2*(5*B
 + 9*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 15*a^2*(b*B
 - a*C)*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/
2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(35*a*b*B
- 12*a^2*C + 9*b^2*C)*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(15*Sqrt[b + a*Cos[c + d*x]]*(Sec[(
c + d*x)/2]^2)^(3/2)) + (Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-(b*(a + b)
*(35*a*b*B - 12*a^2*C + 9*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)]) + b*(a + b)*(-15*a^2*C + 3*a*b*(10*B + C) + b^2*(5*B + 9*C))*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 15*a^2*(b*B - a*C)*((a
 - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[(
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - b*(35*a*b*B - 12*a^2*C
+ 9*b^2*C)*(b + a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec
[c + d*x]*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/
2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(15*Sqrt[b + a*Cos[c +
 d*x]]*(Sec[(c + d*x)/2]^2)^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]) +
(2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(-(b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*(b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x])/2 - b*(a + b)*(
35*a*b*B - 12*a^2*C + 9*b^2*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]*Tan[(c + d*x)/2] + b*(a + b)*(-15*a^2*C + 3*a*b*(10*B + C) + b^2*(
5*B + 9*C))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d
*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d
*x)/2] - 15*a^2*(b*B - a*C)*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)]*Tan[(c + d*x)/2] - (3*b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*(b + a*Cos[c

```
+ d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]
*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/2 - (b*(a + b)*(35*a*b*B - 12*a^2*C + 9*b^2*C)*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d
*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)]) + (b*(a + b)*(-15*a^2*C + 3*a*b*(10*B + C) + b^2*(5*B + 9*C))*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*(-(
(a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)]) - (15*a^2*(b*B - a*C)*((a - b)*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c
 + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(a + b)))/(2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) +
 (b*(a + b)*(-15*a^2*C + 3*a*b*(10*B + C) + b^2*(5*B + 9*C))*Sec[(c + d*x)/
2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (b*(a
+ b)*(35*a*b*B - 12*a^2*C + 9*b^2*C)*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/
(a + b)])/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]) - 15*a^2*(b*B - a*C)*Sec[(c + d*
x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*(((a - b)*S
ec[(c + d*x)/2]^2)/(2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c
 + d*x)/2]^2)/(a + b)]) - (a*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
)) + a*b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^
(3/2)*Tan[(c + d*x)/2]*Tan[c + d*x] - b*(35*a*b*B - 12*a^2*C + 9*b^2*C)*(b
+ a*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[
(c + d*x)/2]*Tan[c + d*x]))/(15*Sqrt[b + a*Cos[c + d*x]]*(Sec[(c + d*x)/2]^
2)^(3/2))))
```

Maple [B] time = 0.799, size = 3700, normalized size = 8.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2
),x)
```

```
[Out] 2/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(5*B*cos(d*x+c)*b^4-45*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-35*B*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+
35*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^2*b^2+35*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+30*C*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*
b+12*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^2*b^2-12*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
```

```
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-12*C*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
3*b-12*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^2*b^2+9*C*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+35*B*cos(d*x+c)^3*a^
2*b^2+40*B*cos(d*x+c)^2*a*b^3-5*B*cos(d*x+c)^3*b^4-35*B*cos(d*x+c)^3*a*b^3+
12*C*cos(d*x+c)^3*a^2*b^2-5*B*cos(d*x+c)^4*a*b^3-6*C*cos(d*x+c)^4*a^2*b^2-9
*C*cos(d*x+c)^4*a*b^3+35*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+35*B*cos(d*x+c)^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-4
5*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^2*b^2-35*B*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+30*C*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b
+12*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b^2-12*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-12*C*cos(d*x+c)^3*sin(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3
*b-12*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^2+9*C*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-35*B*cos(d*x+c)^4*a^2
*b^2+12*C*cos(d*x+c)^4*a^3*b-12*C*cos(d*x+c)^3*a^3*b-6*C*cos(d*x+c)^2*a^2*b
^2+9*C*cos(d*x+c)*a*b^3-9*C*cos(d*x+c)^3*b^4-5*B*cos(d*x+c)^3*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-9*C*cos(
d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^4-15*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4+9*C*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+30*C*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*a^4-15*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^4+9*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+30*C*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*a^4-30*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b+15*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+15*B*sin(d*x+
```

```
c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3*b-30*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*b+6*C*cos(d*x+c)^2*b^4+3*C*b^4-5*B*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*b^4-9*C*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d
*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int Ca^3\sqrt{a+b\sec(c+dx)}\,dx-\int -Ba^2b\sqrt{a+b\sec(c+dx)}\,dx-\int -Bb^3\sqrt{a+b\sec(c+dx)}\sec^2(c+dx)\,dx-\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*se
c(d*x+c)**2),x)
```

```
[Out] -Integral(C*a**3*sqrt(a + b*sec(c + d*x)), x) - Integral(-B*a**2*b*sqrt(a +
 b*sec(c + d*x)), x) - Integral(-B*b**3*sqrt(a + b*sec(c + d*x))*sec(c + d*
x)**2, x) - Integral(-C*b**3*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x) -
 Integral(-2*B*a*b**2*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x) - Integral(
-C*a*b**2*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x) - Integral(C*a**2*b*
sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^2 \sec \left(dx+c\right)^2 + Bb^2 \sec \left(dx+c\right) - Ca^2 + Bab\right)\left(b \sec \left(dx+c\right) + a\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*(b*se
c(d*x + c) + a)^(3/2), x)
```

3.976 $\int \sqrt{a+b\sec(c+dx)}\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\right.$

Optimal. Leaf size=382

$$-\frac{2\sqrt{a+b}\left(3a^2C-ab(6B-C)+b^2(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{3d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) - (2*Sqrt[a + b]*
(b^2*(3*B - C) - a*b*(6*B - C) + 3*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) - (2*a*Sqrt[a
 + b]*(b*B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b^2*C*Sqrt[a + b*Sec[c +
 d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.623483, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4041, 3918, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(3a^2C-ab(6B-C)+b^2(3B-C)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Se
c[c + d*x]^2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*b*B + a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) - (2*Sqrt[a + b]*
(b^2*(3*B - C) - a*b*(6*B - C) + 3*a^2*C)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) - (2*a*Sqrt[a
 + b]*(b*B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b^2*C*Sqrt[a + b*Sec[c +
 d*x]]*Tan[c + d*x])/(3*d)
```


Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3918

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +
(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m -
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right)dx=\frac{\int(a+b\sec(c+dx))^{3/2}\left(b^2(bB-aC)+\right.}{b^2}$$

$$=\frac{2b^2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+$$

$$=\frac{2b^2C\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bB+aC)\cot(c+dx}{}$$

$$=-\frac{2(a-b)\sqrt{a+b}(3bB+aC)\cot(c+dx}{}$$

Mathematica [B] time = 18.5345, size = 1139, normalized size = 2.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^
2*C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(3/2)*(b*B - a*C + b*C*Sec[c + d*x])*(3*a*b^2*B*Tan
[(c + d*x)/2] + 3*b^3*B*Tan[(c + d*x)/2] + a^2*b*C*Tan[(c + d*x)/2] + a*b^2
*C*Tan[(c + d*x)/2] - 6*a*b^2*B*Tan[(c + d*x)/2]^3 - 2*a^2*b*C*Tan[(c + d*x
)/2]^3 + 3*a*b^2*B*Tan[(c + d*x)/2]^5 - 3*b^3*B*Tan[(c + d*x)/2]^5 + a^2*b*
C*Tan[(c + d*x)/2]^5 - a*b^2*C*Tan[(c + d*x)/2]^5 + 6*a^2*b*B*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3
*C*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + 6*a^2*b*B*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*C*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + b*(a + b)*(3*b*B + a*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*a^3*C - 3*a^2
*b*(B + C) + b^3*(3*B + C) + a*b^2*(6*B + C))*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3
*d*(b + a*Cos[c + d*x])^(3/2)*(b*C + b*B*Cos[c + d*x] - a*C*Cos[c + d*x])*S
ec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2
]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^2*(a + b*Sec
[c + d*x])^(3/2)*(b*B - a*C + b*C*Sec[c + d*x])*((2*b*(3*b*B + a*C)*Sin[c +
 d*x])/3 + (2*b^2*C*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])*(b*C + b*B*Co
s[c + d*x] - a*C*Cos[c + d*x]))
```

Maple [B] time = 0.529, size = 2761, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2
),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(-C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-3*B*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+3*B*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b^2-6*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+C*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b^2+3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-6*B*cos(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a^2*b-6*B*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*C*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+3*B*cos(d*x+c)*a^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^2-3*B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+6*C*co
s(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^3+3*B*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-3*C*cos(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+6*C*cos(d*x+c)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3+C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2+C*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+3*C*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+3*B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^2-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
```

```
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-C*cos(d*x+c)^
3*a^2*b-C*cos(d*x+c)^3*a*b^2+2*C*cos(d*x+c)*a*b^2-3*B*cos(d*x+c)^3*a*b^2+3*
B*cos(d*x+c)^2*a*b^2+C*cos(d*x+c)^2*a^2*b-C*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)
*b^3-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-C*cos(d*x+c)^2*a*b^2+C*b^3)*((b+
a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c
)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^2\sec(dx+c)^2 + Bb^2\sec(dx+c) - Ca^2 + Bab\right)\sqrt{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)*(a+b*sec(d*x+c)
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*sqrt(
b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^2 + Bb^2\sec(dx+c) - Ca^2 + Bab\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)*(a+b*sec(d*x+c)
)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*sqrt(b
*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int Ca^2\sqrt{a+b\sec(c+dx)}\,dx - \int -Bab\sqrt{a+b\sec(c+dx)}\,dx - \int -Bb^2\sqrt{a+b\sec(c+dx)}\sec(c+dx)\,dx - \int -C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)*(a+b*sec(d*
x+c))**(1/2),x)
```

```
[Out] -Integral(C*a**2*sqrt(a + b*sec(c + d*x)), x) - Integral(-B*a*b*sqrt(a + b*
sec(c + d*x)), x) - Integral(-B*b**2*sqrt(a + b*sec(c + d*x))*sec(c + d*x),
 x) - Integral(-C*b**2*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^2 \sec(dx+c)^2 + Bb^2 \sec(dx+c) - Ca^2 + Bab\right)\sqrt{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)*(a+b*sec(d*x+c)
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*sqrt(
b*sec(d*x + c) + a), x)
```

3.977 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=316

$$\frac{2b\sqrt{a+b}(B-C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2\sqrt{a+b}(bB-aC}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*Sqrt[a + b]*(B - C)*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/d - (2*Sqrt[a + b]*(b*B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rubi [A] time = 0.410386, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4041, 3916, 3784, 4005, 3832, 4004}

$$\frac{2b\sqrt{a+b}(B-C)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2\sqrt{a+b}(bB-aC)\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/Sqrt[a + b*
Sec[c + d*x]],x]
```



```
[Out] (-2*(a - b)*Sqrt[a + b]*C*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*Sqrt[a + b]*(B - C)*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/d - (2*Sqrt[a + b]*(b*B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```


Rule 4041

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Dist[1/b^2, I
nt[(a + b*Csc[e + f*x])^(m + 1)*Simp[b*B - a*C + b*C*Csc[e + f*x], x], x],
x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 3916

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\int \sqrt{a+b\sec(c+dx)}\left(b^2(bB-aC)+b^3C\sec(c+dx)\right)dx}{b^2}$$

$$= \frac{\int \frac{\sec(c+dx)\left(ab^3C+b^3(bB-aC)+b^4C\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b^2} + (a(bB-aC))\int \frac{}{\sqrt{a\ }}$$

$$= -\frac{2\sqrt{a+b}(bB-aC)\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{d}$$

$$= -\frac{2(a-b)\sqrt{a+b}C\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{d}$$

Mathematica [C] time = 18.093, size = 1232, normalized size = 3.9

$$\frac{2bC\cos(c+dx)\sqrt{a+b\sec(c+dx)}(bB-aC+bC\sec(c+dx))\sin(c+dx)}{d(bC-a\cos(c+dx)C+bB\cos(c+dx))} + \frac{2\sqrt{a+b\sec(c+dx)}(bB-aC+bC\sec(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/Sqrt[
a + b*Sec[c + d*x]],x]
```

```
[Out] (2*b*C*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(b*B - a*C + b*C*Sec[c + d*x])
*Sin[c + d*x])/(d*(b*C + b*B*Cos[c + d*x] - a*C*Cos[c + d*x])) + (2*Sqrt[a
+ b*Sec[c + d*x]]*(b*B - a*C + b*C*Sec[c + d*x])*(a*b*Sqrt[(-a + b)/(a + b)
]*C*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] - 2*a*
b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*
C*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^5 + (2
*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*C*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*b*B*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*C*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*C*Ellipt
icE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(a*C + b*(-B + C))
*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b
)]*d*Sqrt[b + a*Cos[c + d*x]]*(b*C + b*B*Cos[c + d*x] - a*C*Cos[c + d*x])*S
ec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2
]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.447, size = 1588, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2
),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*
(B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b-B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-C*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2-C*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^2+2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+C*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
```

```
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+C*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2*sin(d*x+c)-2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*C*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+C*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b+C*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-
C*cos(d*x+c)^2*a*b+C*cos(d*x+c)*a*b-C*cos(d*x+c)*b^2+b^2*C)/sin(d*x+c)^5/(b
+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \sec(dx+c)^2 + Bb^2 \sec(dx+c) - Ca^2 + Bab}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)/sqrt(
b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((Cb\sec(dx+c) - Ca + Bb)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c) - C*a + B*b)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -Bb\sqrt{a+b\sec(c+dx)}\,dx - \int Ca\sqrt{a+b\sec(c+dx)}\,dx - \int -Cb\sqrt{a+b\sec(c+dx)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**(1/2),x)
```

```
[Out] -Integral(-B*b*sqrt(a + b*sec(c + d*x)), x) - Integral(C*a*sqrt(a + b*sec(c
 + d*x)), x) - Integral(-C*b*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2 \sec (dx+c)^2 + Bb^2 \sec (dx+c) - Ca^2 + Bab}{\sqrt{b \sec (dx+c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)/sqrt(
b*sec(d*x + c) + a), x)
```

3.978 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2\sqrt{a+b}(bB-aC)\cot(c+d$$

```
[Out] (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (2*Sqrt[a + b]*(b*B - a*C)*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a*d)
```

Rubi [A] time = 0.164173, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {24, 3921, 3784, 3832}

$$\frac{2C\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2\sqrt{a+b}(bB-aC)\cot(c+dx)\sqrt{\frac{b(1-s}{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a + b]*C*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (2*Sqrt[a + b]*(b*B - a*C)*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a*d)
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b^2}$$
$$=(bC)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx+(bB-aC)\int \frac{1}{\sqrt{a+b\sec(c+}$$
$$=\frac{2\sqrt{a+b}C\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{d}$$

Mathematica [A] time = 2.85622, size = 160, normalized size = 0.75

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left((aC+b(C-B))\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-}{a+}\right.\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((a*C + b*(-B + C))*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*(-(b*B) + a*C)*EllipticPi[-1, -A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[
c + d*x]])
```

Maple [A] time = 0.359, size = 289, normalized size = 1.4

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d\,(b+a\cos(dx+c))(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(B\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2
),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*
x+c))*(B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*B*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-C*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-C*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*C*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*a)/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{Bb}{\sqrt{a+b\sec{(c+dx)}}}\,dx-\int \frac{Ca}{\sqrt{a+b\sec{(c+dx)}}}\,dx-\int -\frac{Cb\sec{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**(3/2),x)
```

```
[Out] -Integral(-B*b/sqrt(a + b*sec(c + d*x)), x) - Integral(C*a/sqrt(a + b*sec(c
 + d*x)), x) - Integral(-C*b*sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2\sec{(dx+c)}^2+Bb^2\sec{(dx+c)}-Ca^2+Bab}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)/(b*se
c(d*x + c) + a)^(3/2), x)
```

3.979 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=379

$$-\frac{2(bB-2aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{2b^2(bB-2aC)\tan}{ad\left(a^2-b^2\right)\sqrt{a+b\,\text{s}}}$$

```
[Out] (2*(b*B - 2*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*(b*B - 2*a*C)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*Sqrt[a + b]*d) - (2*Sqrt[a + b]*(b*B - a*C)*Cot[c + d*x]*Elliptic
Pi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a^2*d) + (2*b^2*(b*B - 2*a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rubi [A] time = 0.537011, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {24, 3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2(bB-2aC)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}(bB-aC)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+b}}\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^(5/2),x]
```

```
[Out] (2*(b*B - 2*a*C)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*(b*B - 2*a*C)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*Sqrt[a + b]*d) - (2*Sqrt[a + b]*(b*B - a*C)*Cot[c + d*x]*Elliptic
Pi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a^2*d) + (2*b^2*(b*B - 2*a*C)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
```

```
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{abB - a^2C + b^2B\sec(c+dx) + b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{b^2}$$

$$= \frac{2b^2(bB-2aC)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}b^2\left(a^2-b^2\right)(bB-aC)+\frac{1}{2}ab^3(}{}}{}$$

$$= \frac{2b^2(bB-2aC)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}b^2\left(a^2-b^2\right)(bB-aC)+\left(\frac{1}{2}ab^3\right.}{\sqrt{a+}}}{ab^2}$$

$$= \frac{2(bB-2aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s}{}}}{a\sqrt{a+b}d}$$

$$= \frac{2(bB-2aC)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s}{}}}{a\sqrt{a+b}d}$$

Mathematica [C] time = 15.2224, size = 2090, normalized size = 5.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(b*B - a*C + b*C*Sec[c + d*x])*((2*b*(
b*B - 2*a*C)*Sin[c + d*x])/(a*(-a^2 + b^2)) - (2*(-(b^3*B*Sin[c + d*x]) + 2
*a*b^2*C*Sin[c + d*x]))/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(b*C + b*
B*Cos[c + d*x] - a*C*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*
Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*(b*B - a*C + b*C*Sec[c + d*x])*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(-(a*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]) - b^3*Sqrt[(-a + b
)/(a + b)]*B*Tan[(c + d*x)/2] + 2*a^2*b*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d
*x)/2] + 2*a*b^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2] + 2*a*b^2*Sqrt[(
-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - 4*a^2*b*Sqrt[(-a + b)/(a + b)]*C*Ta
n[(c + d*x)/2]^3 - a*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + b^3*
Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + 2*a^2*b*Sqrt[(-a + b)/(a + b)
]*C*Tan[(c + d*x)/2]^5 - 2*a*b^2*Sqrt[(-a + b)/(a + b)]*C*Tan[(c + d*x)/2]^
5 + (2*I)*a^2*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*b^
3*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^3*C*EllipticPi
[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*b^2*C*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (2*I)*a^2*b*B*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*b^3*B*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[
(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
```

```
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^3*C*EllipticPi[-((a + b)/(a
- b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*b^2*C*EllipticPi[-((a +
 b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(
a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*(-(b*B) + 2*
a*C)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(-2*b^2
*B - a*b*(B - 3*C) + a^2*C)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)]))/(a*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(b*C + b*B*Cos[c + d*x] - a*C
*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1
 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^
2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.413, size = 2613, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2
),x)
```

```
[Out] 1/d/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-C*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+2*C*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^3+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^3-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2-2*B*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^2*b+B*cos(d*x+c)*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-B*cos(d*x+c)^2*b^3-B*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*b^3-C*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^3+2*C*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+2*C*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*C
*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
```

```
b)/(a+b))^(1/2))*b-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-C*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^2-2*C*cos(d*x+c)*a*b^2+B*cos(d*x+c)^2*a*b^2-2*C*cos(d*x+c)^2*a^2*b+B*co
s(d*x+c)*b^3-2*B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^2*b-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+2*B*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3-B*cos(d*
x+c)*a*b^2+2*C*cos(d*x+c)*a^2*b+2*C*cos(d*x+c)^2*a*b^2-C*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*C*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+
2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*
sin(d*x+c)-2*C*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-B*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+
c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cb\sec(dx+c) - Ca + Bb)\sqrt{b\sec(dx+c) + a}}{b^2\sec(dx+c)^2 + 2\,ab\sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c) - C*a + B*b)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d
*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{Bb}{a\sqrt{a+b\sec\left(c+dx\right)}+b\sqrt{a+b\sec\left(c+dx\right)}\sec\left(c+dx\right)}\,dx-\int \frac{Ca}{a\sqrt{a+b\sec\left(c+dx\right)}+b\sqrt{a+b\sec\left(c+dx\right)}\sec\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**(5/2),x)
```

```
[Out] -Integral(-B*b/(a*sqrt(a + b*sec(c + d*x)) + b*sqrt(a + b*sec(c + d*x))*sec
(c + d*x)), x) - Integral(C*a/(a*sqrt(a + b*sec(c + d*x)) + b*sqrt(a + b*se
c(c + d*x))*sec(c + d*x)), x) - Integral(-C*b*sec(c + d*x)/(a*sqrt(a + b*se
c(c + d*x)) + b*sqrt(a + b*sec(c + d*x))*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2\sec\left(dx+c\right)^2+Bb^2\sec\left(dx+c\right)-Ca^2+Bab}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)/(b*se
c(d*x + c) + a)^(5/2), x)
```

3.980 $\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=519

$$\frac{2\left(-2a^2b(3B+C)+9a^3C+ab^2(B-3C)+3b^3B\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+b}}\right.\right.}{3a^2d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] (2*(7*a^2*b*B - 3*b^3*B - 11*a^3*C + 3*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b
)*(a + b)^(3/2)*d) + (2*(3*b^3*B + a*b^2*(B - 3*C) + 9*a^3*C - 2*a^2*b*(3*B
 + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*Sqrt[a + b]*(b*
B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (2*b^2*(b*B - 2*a*C)*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(7*a^2*b*B - 3
*b^3*B - 11*a^3*C + 3*a*b^2*C)*Tan[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Rubi [A] time = 0.999887, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {24, 3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\left(7a^2bB-11a^3C+3ab^2C-3b^3B\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(bB-2aC)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(-2a^2b(3B+C)+9a^3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a + b*Sec[
c + d*x])^(7/2),x]
```


```
[Out] (2*(7*a^2*b*B - 3*b^3*B - 11*a^3*C + 3*a*b^2*C)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b
)*(a + b)^(3/2)*d) + (2*(3*b^3*B + a*b^2*(B - 3*C) + 9*a^3*C - 2*a^2*b*(3*B
 + C))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d) - (2*Sqrt[a + b]*(b*
B - a*C)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (2*b^2*(b*B - 2*a*C)*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(7*a^2*b*B - 3
*b^3*B - 11*a^3*C + 3*a*b^2*C)*Tan[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
+ b*Sec[c + d*x]])
```


Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{\int \frac{b^2(bB-aC)+b^3C\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx}{b^2}$$

$$= \frac{2b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}b^2\left(a^2-b^2\right)(bB-aC)+\frac{3}{2}}{}}{}$$

$$= \frac{2b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(7a^2bB-3b^3B-1\right.}{3a^2\left(a^2-b^2\right)^2}$$

$$= \frac{2b^2(bB-2aC)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(7a^2bB-3b^3B-1\right.}{3a^2\left(a^2-b^2\right)^2}$$

$$= \frac{2\left(7a^2bB-3b^3B-11a^3C+3ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{}\right.\right.}{3a^2(a-b)(a+b)}$$

$$= \frac{2\left(7a^2bB-3b^3B-11a^3C+3ab^2C\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{}\right.\right.}{3a^2(a-b)(a+b)}$$

Mathematica [A] time = 14.9712, size = 814, normalized size = 1.57

$$\frac{\sec^2(c+dx)(bB-aC+bC\sec(c+dx))\left(\frac{2b\left(11Ca^3-7bBa^2-3b^2Ca+3b^3B\right)\sin(c+dx)}{3a^2\left(b^2-a^2\right)^2}-\frac{2\left(b^4B\sin(c+dx)-2ab^3C\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)(b+a\cos(c+dx))^2}-\frac{2(4B\sin(c+dx)}{}\right.}{d(bC-a\cos(c+dx)C+bB\cos(c+dx))(a+b\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[c + d*x]^2)/(a +
b*Sec[c + d*x])^(7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(b*B - a*C + b*C*Sec[c + d*x])*((2*b
*(-7*a^2*b*B + 3*b^3*B + 11*a^3*C - 3*a*b^2*C)*Sin[c + d*x])/(3*a^2*(-a^2 +
 b^2)^2) - (2*(b^4*B*Sin[c + d*x] - 2*a*b^3*C*Sin[c + d*x]))/(3*a^2*(a^2 -
b^2)*(b + a*Cos[c + d*x])^2) - (2*(-8*a^2*b^3*B*Sin[c + d*x] + 4*b^5*B*Sin[
c + d*x] + 13*a^3*b^2*C*Sin[c + d*x] - 5*a*b^4*C*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(b*C + b*B*Cos[c + d*x] - a*C*Cos[c +
d*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*(b*B - a*C +
b*C*Sec[c + d*x])*(-(a*b*(a + b)*(-7*a^2*b*B + 3*b^3*B + 11*a^3*C - 3*a*b^2
*C)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]) - b*(a + b)*(3*b^
4*B - 2*a^2*b^2*(B - 3*C) - 12*a^4*C - 3*a*b^3*(2*B + C) + a^3*b*(9*B + C))
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 3*(a - b)^2*(a + b)
^2*(b*B - a*C)*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*b*(
-7*a^2*b*B + 3*b^3*B + 11*a^3*C - 3*a*b^2*C)*(b + a*Cos[c + d*x])*(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(3*a^3*(a^2
- b^2)^2*d*(b*C + b*B*Cos[c + d*x] - a*C*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c
```

```
+ d*x)/2]^2)^(3/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.466, size = 7862, normalized size = 15.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(7/2
),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Cb\sec(dx+c) - Ca + Bb)\sqrt{b\sec(dx+c) + a}}{b^3\sec(dx+c)^3 + 3\,ab^2\sec(dx+c)^2 + 3\,a^2b\sec(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c) - C*a + B*b)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d
*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*x+c)**2)/(a+b*sec(d*
x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cb^2\sec(dx+c)^2 + Bb^2\sec(dx+c) - Ca^2 + Bab}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2)/(a+b*sec(d*x+c)
)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)/(b*se
c(d*x + c) + a)^(7/2), x)
```

3.981 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(\right.$

Optimal. Leaf size=266

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(7aA+5aC+5bB)}{21d}+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(9aB+9Ab+7bC)}{45d}$$

```
[Out] (-2*(9*A*b + 9*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (2*(7*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(9*A*b + 9*a*B + 7*
b*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(7*a*A + 5*b*B + 5*a*C)*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(9*A*b + 9*a*B + 7*b*C)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*(b*B + a*C)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(7*d) + (2*b*C*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.300583, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(9aB+9Ab+7bC)}{45d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(7aA+5aC+5bB)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (-2*(9*A*b + 9*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(15*d) + (2*(7*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(9*A*b + 9*a*B + 7*
b*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(7*a*A + 5*b*B + 5*a*C)*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(9*A*b + 9*a*B + 7*b*C)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*(b*B + a*C)*Sec[c + d*x]^(7/2)*Sin[c
+ d*x])/(7*d) + (2*b*C*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2bC\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \mathrm{se}$$

$$=\frac{2bC\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}+\frac{2}{9}\int \mathrm{se}$$

$$=\frac{2(9Ab+9aB+7bC)\sec^{\frac{5}{2}}(c+dx)\sin(c}{45d}$$

$$=\frac{2(9Ab+9aB+7bC)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$=\frac{2(9Ab+9aB+7bC)\sqrt{\sec(c+dx)}\sin(c}{15d}$$

$$=-\frac{2(9Ab+9aB+7bC)\sqrt{\cos(c+dx)}E\left(\right.}{15d}$$

Mathematica [C] time = 7.30943, size = 1262, normalized size = 4.74

$$\frac{4aA\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^{\frac{7}{2}}(c+dx)}{3d(b+a\cos(c+dx))(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))}+\frac{20b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[2]*A*b*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] +
 E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/(5*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])) + (2*Sqrt[2]*a*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])) + (14*Sqrt[2]*b*C*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-
3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2))/(45*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*a*A*Cos[c + d*x]^(7/2)
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2))/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])) + (20*b*B*Cos[c + d*x]^(7/2)*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2))/(21*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])) + (20*a*C*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2))/(21*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])) + ((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((4*(9*A*b + 9*a*B + 7*b*C)*Cos[d*x]*Csc[c])/(15*d) + (4*b*C*Sec[c]*Sec[c
 + d*x]^4*Sin[d*x])/(9*d) + (4*Sec[c]*Sec[c + d*x]^3*(7*b*C*Sin[c] + 9*b*B*
Sin[d*x] + 9*a*C*Sin[d*x]))/(63*d) + (4*Sec[c]*Sec[c + d*x]*(63*A*b*Sin[c]
+ 63*a*B*Sin[c] + 49*b*C*Sin[c] + 105*a*A*Sin[d*x] + 75*b*B*Sin[d*x] + 75*a
*C*Sin[d*x]))/(315*d) + (4*Sec[c]*Sec[c + d*x]^2*(45*b*B*Sin[c] + 45*a*C*Si
n[c] + 63*A*b*Sin[d*x] + 63*a*B*Sin[d*x] + 49*b*C*Sin[d*x]))/(315*d) + (4*(
7*a*A + 5*b*B + 5*a*C)*Tan[c])/(21*d)))/((b + a*Cos[c + d*x])*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2))
```

Maple [B] time = 8.952, size = 1020, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*(A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*si
n(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*(B*b+C*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin
```

```
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+
5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+2*C*b*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*
c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec(dx+c)^5+(Ca+Bb)\sec(dx+c)^4+Aa\sec(dx+c)^2+(Ba+Ab)\sec(dx+c)^3\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^5 + (C*a + B*b)*sec(d*x + c)^4 + A*a*sec(d*x + c
)^2 + (B*a + A*b)*sec(d*x + c)^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(
d*x + c)^(5/2), x)
```

3.982 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=230

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(7aB+7Ab+5bC)}{21d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(7aB+7Ab+5b}{21d}$$

```
[Out] (-2*(5*a*A + 3*b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(7*A*b + 7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(5*a*A + 3*b*B + 3*a
*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(7*A*b + 7*a*B + 5*b*C)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*B + a*C)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d) + (2*b*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.276081, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(7aB+7Ab+5bC)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(5aA+3aC+3bB)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (-2*(5*a*A + 3*b*B + 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(7*A*b + 7*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(5*a*A + 3*b*B + 3*a
*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(7*A*b + 7*a*B + 5*b*C)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*B + a*C)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(5*d) + (2*b*C*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2bC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{3}{2}}($$

$$= \frac{2bC\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{3}{2}}($$

$$= \frac{2(7Ab+7aB+5bC)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{21d}$$

$$= \frac{2(5aA+3bB+3aC)\sqrt{\sec(c+dx)}\sin(c+}{5d}$$

$$= \frac{2(7Ab+7aB+5bC)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

$$= -\frac{2(5aA+3bB+3aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [C] time = 7.12165, size = 1202, normalized size = 5.23

$$\frac{4Ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^{\frac{7}{2}}(c+dx)}{3d(b+a\cos(c+dx))(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))}+\frac{4aB\text{El}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[2]*a*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] +
```

```
E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/(3*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])) + (2*Sqrt[2]*b*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])) + (2*Sqrt[2]*a*C*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3
*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*A*b*Cos[c + d*x]^(7/2)*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2))/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*a*B*Cos[c + d*x]^(7/2)*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2))/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])) + (20*b*C*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2))/(21*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])) + ((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((
4*(5*a*A + 3*b*B + 3*a*C)*Cos[d*x]*Csc[c])/(5*d) + (4*b*C*Sec[c]*Sec[c + d*
x]^3*Sin[d*x])/(7*d) + (4*Sec[c]*Sec[c + d*x]^2*(5*b*C*Sin[c] + 7*b*B*Sin[d
*x] + 7*a*C*Sin[d*x]))/(35*d) + (4*Sec[c]*Sec[c + d*x]*(21*b*B*Sin[c] + 21*
a*C*Sin[c] + 35*A*b*Sin[d*x] + 35*a*B*Sin[d*x] + 25*b*C*Sin[d*x]))/(105*d)
+ (4*(7*A*b + 7*a*B + 5*b*C)*Tan[c])/(21*d)))/((b + a*Cos[c + d*x])*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2))
```

Maple [B] time = 8.213, size = 851, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*(B*b+C*a)/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*C*b*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+
5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))+2*A*a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
```

```
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{4}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{3}+Aa\sec\left(dx+c\right)+\left(Ba+Ab\right)\sec\left(dx+c\right)^{2}\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^4 + (C*a + B*b)*sec(d*x + c)^3 + A*a*sec(d*x + c
) + (B*a + A*b)*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(
d*x + c)^(3/2), x)
```

3.983 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=192

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(a(3A+C)+bB)}{3d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(5aB+5Ab+3bC}{5d}$$

```
[Out] (-2*(5*A*b + 5*a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(b*B + a*(3*A + C))*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(5*A*b + 5*a*B + 3*b*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*B + a*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(3*d) + (2*b*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.234659, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(5aB+5Ab+3bC)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(3A+C)+bB)}{3d}-\frac{2\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (-2*(5*A*b + 5*a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(5*d) + (2*(b*B + a*(3*A + C))*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(5*A*b + 5*a*B + 3*b*C)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*B + a*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(3*d) + (2*b*C*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2bC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\sec\cdots}$$

$$= \frac{2bC\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\sec\cdots}$$

$$= \frac{2(5Ab+5aB+3bC)\sqrt{\sec(c+dx)}\sin(c+\cdots}{5d}$$

$$= \frac{2(5Ab+5aB+3bC)\sqrt{\sec(c+dx)}\sin(c+\cdots}{5d}$$

$$= -\frac{2(5Ab+5aB+3bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\cdots\right.}{5d}$$

Mathematica [C] time = 7.0227, size = 1140, normalized size = 5.94

$$\frac{4aA\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^{\frac{7}{2}}(c+dx)}{d(b+a\cos(c+dx))(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))}+\frac{4bB\text{El}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[2]*A*b*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] +
 E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/(3*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])) + (2*Sqrt[2]*a*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
```

```
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2))/(3*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])) + (2*Sqrt[2]*b*C*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3
*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*a*A*Cos[c + d*x]^(7/2)*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])) + (4*b*B*Cos[c + d*x]^(7/2)*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2))/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])) + (4*a*C*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
))/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])) + ((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(5
*A*b + 5*a*B + 3*b*C)*Cos[d*x]*Csc[c])/(5*d) + (4*b*C*Sec[c]*Sec[c + d*x]^2
*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]*(3*b*C*Sin[c] + 5*b*B*Sin[d*x] +
5*a*C*Sin[d*x]))/(15*d) + (4*(b*B + a*C)*Tan[c])/(3*d)))/((b + a*Cos[c + d*
x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2))
```

Maple [B] time = 6.992, size = 742, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(B*
b+C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C*b/(8*sin(1/2*d*x+1/2*
c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2
*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt
(sec(d*x + c)), x)
```

3.984 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(3aB+3Ab+bC)}{3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*(b*B - a*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (2*(3*A*b + 3*a*B + b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(b*B + a*C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/d + (2*b*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.223363, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)}{3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[Sec
[c + d*x]],x]
```

```
[Out] (-2*(b*B - a*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (2*(3*A*b + 3*a*B + b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(b*B + a*C)*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/d + (2*b*C*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{1}{2}(3Ab+3a$$

$$= \frac{2bC\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3aA}{2}+\frac{3}{2}(bB+aC)}{\sqrt{\sec(c+}}$$

$$= \frac{2(bB+aC)\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2bC\sec^{\frac{3}{2}}(c+dx}{3d}$$

$$= \frac{2(3Ab+3aB+bC)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3d}$$

$$= -\frac{2(bB-a(A-C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

Mathematica [C] time = 2.51004, size = 223, normalized size = 1.47

$$\frac{e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(i\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)(a(C-A)+bB)+2\,\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((3*I)*a*A - (3*I)*b*B - (3*I)*
a*C + (3*I)*a*A*Cos[2*(c + d*x)] - (3*I)*b*B*Cos[2*(c + d*x)] - (3*I)*a*C*C
os[2*(c + d*x)] + 2*(3*A*b + 3*a*B + b*C)*Cos[c + d*x]^(3/2)*EllipticF[(c +
 d*x)/2, 2] + I*(b*B + a*(-A + C))*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2*b*C*Sin[c + d*x] + 3*b*B
*Sin[2*(c + d*x)] + 3*a*C*Sin[2*(c + d*x)]))/(3*d*E^(I*d*x))
```

Maple [B] time = 5.243, size = 666, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b+C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec\left(dx+c\right)^3+\left(Ca+Bb\right)\sec\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right)\left(b \sec \left(dx+c\right)+a\right)}{\sqrt{\sec \left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt
(sec(d*x + c)), x)
```

3.985 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=146

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(a(A+3C)+3bB)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

```
[Out] (2*(A*b + a*B - b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*(3*b*B + a*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c
+ d*x]]) + (2*b*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.226589, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(3/2),x]
```

```
[Out] (2*(A*b + a*B - b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*(3*b*B + a*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c
+ d*x]]) + (2*b*C*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{1}{2}(3bB+a(A+$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{3}{2}bC\sec^2(c+d}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2bC\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - (-A$$

$$= \frac{2(3bB+a(A+3C))\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3d}$$

$$= \frac{2(Ab+aB-bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{d}$$

Mathematica [C] time = 2.76956, size = 197, normalized size = 1.35

$$\frac{e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-2ie^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)(aB+Ab-bC)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((6*I)*A*b*Cos[c + d*x] + (6*I)
*a*B*Cos[c + d*x] - (6*I)*b*C*Cos[c + d*x] + 2*(3*b*B + a*(A + 3*C))*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2*I)*(A*b + a*B - b*C)*E^(I*(c +
d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))] + 6*b*C*Sin[c + d*x] + a*A*Sin[2*(c + d*x)]))/(3*d*E^(I*d*x
))
```

Maple [B] time = 2.235, size = 388, normalized size = 2.7

$$-\frac{2}{3d}\left(4Aa\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^4 + Aa\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\text{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-2*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+3*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+3*a*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-6*C*b*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sec\left(c+dx\right)\right)\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/
2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sec(
c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(
d*x + c)^(3/2), x)
```

3.986 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(aB+Ab+3bC)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(}{5d}$$

```
[Out] (2*(3*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(A*b + a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec
[c + d*x]^(3/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.226559, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(5/2),x]
```

```
[Out] (2*(3*a*A + 5*b*B + 5*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(A*b + a*B + 3*b*C)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec
[c + d*x]^(3/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{1}{2}(3aA+5bB+}{\sec}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{5}{2}bC\sec^2(c+d}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-Ab-$$

$$= \frac{2(3aA+5bB+5aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5d}$$

$$= \frac{2(3aA+5bB+5aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5d}$$

Mathematica [C] time = 2.66076, size = 194, normalized size = 1.24

$$\frac{e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-4ie^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)(3aA+5aC+5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(A*b + a*B + 3*b*C)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (4*I)*(3*a*A + 5*b*B + 5*a*C)*E^(I*
(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))] + 2*Cos[c + d*x]*((6*I)*(3*a*A + 5*b*B + 5*a*C) + 10*(
A*b + a*B)*Sin[c + d*x] + 3*a*A*Sin[2*(c + d*x)])))/(30*d*E^(I*d*x))
```

Maple [B] time = 2.278, size = 465, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a+20*A*b+20*B*a)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5
*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+15*C*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sec\left(c+dx\right)\right)\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/
2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sec(
c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right)\left(b \sec \left(dx+c\right)+a\right)}{\sec \left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(
d*x + c)^(5/2), x)
```

3.987 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(5aA+7aC+7bB)}{21d}+\frac{2\sin(c+dx)(5aA+7aC+7bB)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos}}{}$$

```
[Out] (2*(3*A*b + 3*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(5*a*A + 7*b*B + 7*a*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(5/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2
)) + (2*(5*a*A + 7*b*B + 7*a*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.252596, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\sin(c+dx)(5aA+7aC+7bB)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+7aC+7bB)}{21d}+\frac{2\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(7/2),x]
```


```
[Out] (2*(3*A*b + 3*a*B + 5*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(5*d) + (2*(5*a*A + 7*b*B + 7*a*C)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*
d*Sec[c + d*x]^(5/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2
)) + (2*(5*a*A + 7*b*B + 7*a*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(5aA+7bB+$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{7}{2}bC\sec^2(c+d}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5aA+7}{21}$$

$$= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5aA+7}{21}$$

$$= \frac{2(3Ab+3aB+5bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5d}$$

Mathematica [C] time = 3.50242, size = 219, normalized size = 1.13

$$\frac{e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56ie^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)(3aB+3Ab+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(5*a*A + 7*b*B + 7*a*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(3*A*b + 3*a*B + 5*b*C)*
E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7
/4, -E^((2*I)*(c + d*x))] + 2*Cos[c + d*x]*((84*I)*(3*A*b + 3*a*B + 5*b*C)
```

```
+ 5*(23*a*A + 28*b*B + 28*a*C)*Sin[c + d*x] + 42*(A*b + a*B)*Sin[2*(c + d*x
)] + 15*a*A*Sin[3*(c + d*x)]))/(420*d*E^(I*d*x))
```

Maple [B] time = 2.316, size = 515, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a-168*A*b-168*B*a)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b+140*C*a)*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b-70*C*a)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*b+35*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a+35*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(7/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(
d*x + c)^(7/2), x)
```

3.988 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(5aB+5Ab+7bC)}{21d}+\frac{2\sin(c+dx)(7aA+9aC+9bB)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c$$

```
[Out] (2*(7*a*A + 9*b*B + 9*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (2*(5*A*b + 5*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(9
*d*Sec[c + d*x]^(7/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/
2)) + (2*(7*a*A + 9*b*B + 9*a*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) +
(2*(5*A*b + 5*a*B + 7*b*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.280518, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4074, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2\sin(c+dx)(7aA+9aC+9bB)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)(5aB+5Ab+7bC)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(9/2),x]
```


```
[Out] (2*(7*a*A + 9*b*B + 9*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (2*(5*A*b + 5*a*B + 7*b*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(9
*d*Sec[c + d*x]^(7/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/
2)) + (2*(7*a*A + 9*b*B + 9*a*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) +
(2*(5*A*b + 5*a*B + 7*b*C)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{9}{2}(Ab+aB)-\frac{1}{2}(7aA+9bB+$$
$$= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{9}{2}(Ab+aB)-\frac{9}{2}bC\sec^2(c+d}{\sec^{\frac{7}{2}}(c+dx)}$$
$$= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7aA+9}{45}$$
$$= \frac{2aA\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7aA+9}{45}$$
$$= \frac{2(7aA+9bB+9aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$
$$= \frac{2(7aA+9bB+9aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$

Mathematica [C] time = 4.68094, size = 249, normalized size = 1.08

$$\frac{e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-56ie^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)(7aA+9aC+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(120*(5*A*b + 5*a*B + 7*b*C)*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(7*a*A + 9*b*B + 9*a*C)
*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4,
7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((1176*I)*a*A + (1512*I)*b*B + (1
512*I)*a*C + 30*(23*A*b + 23*a*B + 28*b*C)*Sin[c + d*x] + 14*(19*a*A + 18*b
*B + 18*a*C)*Sin[2*(c + d*x)] + 90*A*b*Sin[3*(c + d*x)] + 90*a*B*Sin[3*(c +
 d*x)] + 35*a*A*Sin[4*(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [B] time = 2.036, size = 565, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*a*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a+720*A*b+720*B*a)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a-1080*A*b-1080*B*a-504*B*b-504*C*
a)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*A*a+840*A*b+840*B*a+504*B*b
+504*C*a+420*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a-240*A*b
-240*B*a-126*B*b-126*C*a-210*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-1
47*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a+75*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*b+75*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a+105*C*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec\left(dx+c\right)^{3} + \left(Ca+Bb\right)\sec\left(dx+c\right)^{2} + Aa + \left(Ba+Ab\right)\sec\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(9/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(
d*x + c)^(9/2), x)
```

3.989 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(9aA+11aC+11bB)}{231d}+\frac{2\sin(c+dx)(7aB+7Ab+9bC)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin}{}$$

```
[Out] (2*(7*A*b + 7*a*B + 9*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (10*(9*a*A + 11*b*B + 11*a*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a*A*Sin[c + d*x]
)/(11*d*Sec[c + d*x]^(9/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(9*d*Sec[c + d*x
]^(7/2)) + (2*(9*a*A + 11*b*B + 11*a*C)*Sin[c + d*x])/(77*d*Sec[c + d*x]^(5
/2)) + (2*(7*A*b + 7*a*B + 9*b*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) +
 (10*(9*a*A + 11*b*B + 11*a*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.308012, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\sin(c+dx)(7aB+7Ab+9bC)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)(9aA+11aC+11bB)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{10\sin(c+dx)(9aA+11aC+11bB)}{231d\sqrt{\sec(c+dx)}}+\frac{10\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c +
d*x]^(11/2),x]
```


```
[Out] (2*(7*A*b + 7*a*B + 9*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(15*d) + (10*(9*a*A + 11*b*B + 11*a*C)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a*A*Sin[c + d*x]
)/(11*d*Sec[c + d*x]^(9/2)) + (2*(A*b + a*B)*Sin[c + d*x])/(9*d*Sec[c + d*x
]^(7/2)) + (2*(9*a*A + 11*b*B + 11*a*C)*Sin[c + d*x])/(77*d*Sec[c + d*x]^(5
/2)) + (2*(7*A*b + 7*a*B + 9*b*C)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) +
 (10*(9*a*A + 11*b*B + 11*a*C)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```


Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{2}{11}\int \frac{-\frac{11}{2}(Ab+aB)-\frac{1}{2}(9aA+11b$$

$$= \frac{2aA\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{2}{11}\int \frac{-\frac{11}{2}(Ab+aB)-\frac{11}{2}bC\sec^2(c}{\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(9aA+}{}$$

$$= \frac{2aA\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(9aA+}{}$$

$$= \frac{2aA\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(9aA+}{}$$

$$= \frac{2(7Ab+7aB+9bC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{15d}$$

Mathematica [C] time = 6.91667, size = 1371, normalized size = 5.15

$$\frac{60aA\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\cos^{\frac{7}{2}}(c+dx)}{77d(b+a\cos(c+dx))(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))} + \frac{20bB}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(11/2),x]
```

```
[Out] (-14*Sqrt[2]*A*b*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/(45*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Co
s[2*c + 2*d*x])) - (14*Sqrt[2]*a*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c]*(-3*Sqrt[1 + E^
((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2))/(45*d*E^(I*d*x)*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])) - (2*Sqrt[2]*b*C*Sqrt[E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^3*Csc[c
]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(a + b*Sec[c + d*x])*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(5*d*E^(I*d*x)*(b + a*Cos[c + d*x])*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (60*a*A*Cos[c + d*x]^(
7/2)*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(77*d*(b + a*Cos[c + d*x])*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (20*b*B*Cos[c + d*x]^(7/2)*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2))/(21*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + (20*a*C*Cos[c + d*x]^(7/2)*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2))/(21*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])) + ((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*(-((149*A*b + 149*a*B + 198*b*C + 187*A*b*Cos[2*c] + 187*a*B*Cos[2*c
] + 234*b*C*Cos[2*c])*Cos[d*x]*Csc[c])/(180*d) + ((1041*a*A + 1144*b*B + 11
44*a*C)*Cos[2*d*x]*Sin[2*c])/(1848*d) + ((43*A*b + 43*a*B + 36*b*C)*Cos[3*d
*x]*Sin[3*c])/(180*d) + ((16*a*A + 11*b*B + 11*a*C)*Cos[4*d*x]*Sin[4*c])/(1
54*d) + ((A*b + a*B)*Cos[5*d*x]*Sin[5*c])/(36*d) + (a*A*Cos[6*d*x]*Sin[6*c]
)/(88*d) + ((187*A*b + 187*a*B + 234*b*C)*Cos[c]*Sin[d*x])/(90*d) + ((1041*
a*A + 1144*b*B + 1144*a*C)*Cos[2*c]*Sin[2*d*x])/(1848*d) + ((43*A*b + 43*a*
B + 36*b*C)*Cos[3*c]*Sin[3*d*x])/(180*d) + ((16*a*A + 11*b*B + 11*a*C)*Cos[
4*c]*Sin[4*d*x])/(154*d) + ((A*b + a*B)*Cos[5*c]*Sin[5*d*x])/(36*d) + (a*A*
Cos[6*c]*Sin[6*d*x])/(88*d)))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2))
```

Maple [B] time = 2.638, size = 611, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a-12320*A*b-12320*B*a)*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a+24640*A*b+24640*B*a+7920
*B*b+7920*C*a)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-34920*A*a-22792*A*
b-22792*B*a-11880*B*b-11880*C*a-5544*C*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(13860*A*a+10472*A*b+10472*B*a+9240*B*b+9240*C*a+5544*C*b)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2790*A*a-1848*A*b-1848*B*a-2640*B*b-2640*C
*a-1386*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-1617*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
```

```
,2^(1/2))*b+675*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a+825*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+825*
a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2
),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec(dx+c)^3 + (Ca+Bb)\sec(dx+c)^2 + Aa + (Ba+Ab)\sec(dx+c)}{\sec(dx+c)^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2
),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(11
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2
),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(
d*x + c)^(11/2), x)
```

3.990 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2\right.$

Optimal. Leaf size=343

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^2B+14aAb+10abC+5b^2B\right)}{21d}+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(4a^2C\right.}{45d}$$

```
[Out] (-2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(14*a*A*b + 7*a^2*B
+ 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B +
10*a*b*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(9*A*b^2 + 18*a*b*B
+ 4*a^2*C + 7*b^2*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*b*(9*b*B
+ 4*a*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*C*Sec[c + d*x]^(5/2)*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.587621, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4096, 4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(4a^2C+18abB+9Ab^2+7b^2C\right)}{45d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(7a^2B+14aAb+10abC+5b^2B\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(14*a*A*b + 7*a^2*B
+ 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(18*a*b*B + 3*a^2*(5*A + 3*C) + b^2*(9*A + 7*C))*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B +
10*a*b*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*(9*A*b^2 + 18*a*b*B
+ 4*a^2*C + 7*b^2*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*b*(9*b*B
+ 4*a*C)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*C*Sec[c + d*x]^(5/2)*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
```

```
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c}{9d}$$
$$= \frac{2b(9bB+4aC)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} +$$
$$= \frac{2b(9bB+4aC)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} +$$
$$= \frac{2\left(14aAb+7a^2B+5b^2B+10abC\right)\sec^{\frac{3}{2}}(c}{21d}$$
$$= \frac{2\left(18abB+3a^2(5A+3C)+b^2(9A+7C)\right.}{15d}$$
$$= \frac{2\left(14aAb+7a^2B+5b^2B+10abC\right)\sqrt{\cos(}}{21d}$$
$$= -\frac{2\left(18abB+3a^2(5A+3C)+b^2(9A+7C\right.}{}$$

Mathematica [A] time = 6.74075, size = 507, normalized size = 1.48

$$\frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4}{15}\sin(c+dx)\left(15a^2A+9a^2C+18abB+9Ab^2+7b^2C\right)+\frac{4}{45}\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^4*((2*(105*a^2*A + 63*A*b^2 + 126*a*b*B + 63*a^2*C + 49*b^
2*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2
*(-70*a*A*b - 35*a^2*B - 25*b^2*B - 50*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2))/(105*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])) + ((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((4*(15*a^2*A + 9*A*b^2 + 18*a*b*B + 9*a^2*C + 7*b^2*
C)*Sin[c + d*x])/15 + (4*Sec[c + d*x]^3*(b^2*B*Sin[c + d*x] + 2*a*b*C*Sin[c
 + d*x]))/7 + (4*Sec[c + d*x]*(14*a*A*b*Sin[c + d*x] + 7*a^2*B*Sin[c + d*x]
 + 5*b^2*B*Sin[c + d*x] + 10*a*b*C*Sin[c + d*x]))/21 + (4*Sec[c + d*x]^2*(9
*A*b^2*Sin[c + d*x] + 18*a*b*B*Sin[c + d*x] + 9*a^2*C*Sin[c + d*x] + 7*b^2*
C*Sin[c + d*x]))/45 + (4*b^2*C*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*C
os[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*
x]^(7/2))
```

Maple [B] time = 11.647, size = 1196, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(2*A*b+B*a)
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(B*b+2*C*a)*(-1/56*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*C*(-1/144*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)
^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))-2/5*(A
*b^2+2*B*a*b+C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*A*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec\left(dx+c\right)^5+\left(2\,Cab+Bb^2\right)\sec\left(dx+c\right)^4+Aa^2\sec\left(dx+c\right)+\left(Ca^2+2\,Bab+Ab^2\right)\sec\left(dx+c\right)^3+\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^5 + (2*C*a*b + B*b^2)*sec(d*x + c)^4 + A*a^2*s
ec(d*x + c) + (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^3 + (B*a^2 + 2*A*a*b)*
sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*se
c(d*x + c)^(3/2), x)
```

3.991 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2 \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=289

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)}{21d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.$$

```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*a^2*(3*A + C) +
 b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(5*d) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(7*b*B + 4*a*C)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(35*d) + (2*C*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(7*d)
```

Rubi [A] time = 0.529411, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4096, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(4a^2C+14abB+7Ab^2+5b^2C\right)}{21d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(5a^2B+10aAb+6abC+3b^2\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*a^2*(3*A + C) +
 b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(5*d) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(7*b*B + 4*a*C)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(35*d) + (2*C*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(7*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
```

```
!LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c}{7d}$$

$$= \frac{2b(7bB+4aC)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}$$

$$= \frac{2b(7bB+4aC)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B+6abC\right)\sqrt{\sec(c}}{5d}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B+6abC\right)\sqrt{\sec(c}}{5d}$$

$$= -\frac{2\left(10aAb+5a^2B+3b^2B+6abC\right)\sqrt{\cos}}{5d}$$

Mathematica [A] time = 2.41429, size = 333, normalized size = 1.15

$$\frac{4(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(5\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^2(3A+C)+14\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-21*(5*a
^2*B + 3*b^2*B + 2*a*b*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2] + 5*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + 210*a*A*b*Sin[c + d*x] + 105*a^2*B*Sin[c + d*x
] + 63*b^2*B*Sin[c + d*x] + 126*a*b*C*Sin[c + d*x] + 35*A*b^2*Tan[c + d*x]
+ 70*a*b*B*Tan[c + d*x] + 35*a^2*C*Tan[c + d*x] + 25*b^2*C*Tan[c + d*x] + 2
1*b^2*B*Sec[c + d*x]*Tan[c + d*x] + 42*a*b*C*Sec[c + d*x]*Tan[c + d*x] + 15
*b^2*C*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d*(b + a*Cos[c + d*x])^2*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 9.167, size = 947, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(
A*b^2+2*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*C*(-1/5
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b*
(B*b+2*C*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\text{se}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sq
rt(sec(d*x + c)), x)
```

3.992 $\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=241

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(4a^2C\right.}{5d}$$

```
[Out] (-2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*
(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*(5*A*b^2 + 10*a*b*B + 4*a^2*C + 3*b^2*C)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*d) + (2*b*(5*b*B + 4*a*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(1
5*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.51274, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4096, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(4a^2C+10abB+5Ab^2+3b^2C\right)}{5d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2a\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```

```
[Out] (-2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*
(3*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*(5*A*b^2 + 10*a*b*B + 4*a^2*C + 3*b^2*C)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*d) + (2*b*(5*b*B + 4*a*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(1
5*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d)
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2b(5bB+4aC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2C\sqrt{\sec(c}$$

$$= \frac{2b(5bB+4aC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2C\sqrt{\sec(c}$$

$$= \frac{2\left(5Ab^2+10abB+4a^2C+3b^2C\right)\sqrt{\sec(c+dx)}\sin(c}{5d}$$

$$= \frac{2\left(3a^2B+b^2B+2ab(3A+C)\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

$$= -\frac{2\left(10abB-5a^2(A-C)+b^2(5A+3C)\right)\sqrt{\cos(c+dx)}}{5d}$$

Mathematica [A] time = 2.08799, size = 271, normalized size = 1.12

$$\frac{4(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(5\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2B+2ab(3A+C)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(3*(-10*a
*b*B + 5*a^2*(A - C) - b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 5*(3*a^2*B + b^2*B + 2*a*b*(3*A + C))*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + 15*A*b^2*Sin[c + d*x] + 30*a*b*B*Sin[c + d*x] + 15*a^
2*C*Sin[c + d*x] + 9*b^2*C*Sin[c + d*x] + 5*b^2*B*Tan[c + d*x] + 10*a*b*C*T
an[c + d*x] + 3*b^2*C*Sec[c + d*x]*Tan[c + d*x]))/(15*d*(b + a*Cos[c + d*x]
)^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 7.509, size = 1000, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*A*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/
5*b^2*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(B*b+2*C*a)*(-1/6*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+2*(A*b^2+2*B*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\sec(dx+c)^4+\left(2\,Cab+Bb^2\right)\sec(dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\sec}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sq
rt(sec(d*x + c)), x)
```

3.993 $\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=224

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{}$$

```
[Out] (2*(a^2*B - b^2*B + 2*a*b*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*
b*(3*b*B - 2*a*(A - 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) - (2*b^2*(
A - C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + b*Sec[c + d*x])^2
*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.499758, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (2*(a^2*B - b^2*B + 2*a*b*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*
b*(3*b*B - 2*a*(A - 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) - (2*b^2*(
A - C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*(a + b*Sec[c + d*x])^2
*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int \frac{(a+b\sec(c}{}$$

$$= -\frac{2b^2(A-C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2A(a+b\sec(}{3d\sqrt{}}$$

$$= -\frac{2b^2(A-C)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2A(a+b\sec(}{3d\sqrt{}}$$

$$= -\frac{2b(2aA-3bB-6aC)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}-\frac{2b^2}{}$$

$$= \frac{2\left(6abB+b^2(3A+C)+a^2(A+3C)\right)\sqrt{\cos(c+dx)}F\left(\right.}{3d}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d}$$

Mathematica [A] time = 2.98645, size = 227, normalized size = 1.01

$$\frac{2(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(A+3C)+6abB+\right.\right.}{3d\sec^{\frac{7}{2}}(c+dx)(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*(a^2*B
 - b^2*B + 2*a*b*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*
(6*a*b*B + b^2*(3*A + C) + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2] + 6*b^2*B*Sin[c + d*x] + 12*a*b*C*Sin[c + d*x] + a^2*A*Sin[2*(c
 + d*x)] + 2*b^2*C*Tan[c + d*x]))/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 6.732, size = 1301, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-6*B*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-6*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b+6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*b^2*sin(1/2*d*x+1/2*c)^2+8*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4-2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a^2*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*b^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+24*C*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12*C*
a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+12*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b*sin(1/2*d*x+1/2*c)^2-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b*sin(1/2*d*x+1/2*c
)^2-12*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b*sin(1/2*d*x+1/2*c)^2-8*A*a^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+12*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-6*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2-6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin
(1/2*d*x+1/2*c)^2-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2+6*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2 \sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\text{se}}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\left(b\sec(dx+c) + a\right)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/se
c(d*x + c)^(3/2), x)
```

3.994 $\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=225

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{}$$

```
[Out] (2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*(
A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*a*(4*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) - (2*b
^2*(A - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A*(a + b*Sec[c + d
*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.517784, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(5/2),x]
```

```
[Out] (2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*(
A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*a*(4*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) - (2*b
^2*(A - 5*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A*(a + b*Sec[c + d
*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\sec(c\ldots}{\ldots}$$

$$= \frac{2a(4Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2A(a+b\sec(c+dx))^2\,\mathrm{s}\ldots}{5d\sec^{\frac{3}{2}}(c+dx\ldots}$$

$$= \frac{2a(4Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2A(a+b\sec(c+dx))^2\,\mathrm{s}\ldots}{5d\sec^{\frac{3}{2}}(c+dx\ldots}$$

$$= \frac{2a(4Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2(A-5C)\sqrt{\sec(c+dx\ldots}}{5d}$$

$$= \frac{2\left(a^2B+3b^2B+2ab(A+3C)\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\ldots\right.}{3d}$$

$$= \frac{2\left(10abB+5b^2(A-C)+a^2(3A+5C)\right)\sqrt{\cos(c+dx)}E\ldots}{5d}$$

Mathematica [A] time = 4.71443, size = 234, normalized size = 1.04

$$\frac{2(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2B+2ab(A+3C)\ldots\right.\right.}{15d\sec^{\frac{7}{2}}(c+dx)(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*(10*a*
b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*(a^2*B + 3*b^2*B + 2*a*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + (10*a*(2*A*b + a*B)*Cos[c + d*x] + 3*(a^2*A + 10*b^2*
C + a^2*A*Cos[2*(c + d*x)]))*Sin[c + d*x]))/(15*d*(b + a*Cos[c + d*x])^2*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 2.896, size = 932, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-24*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*a*(6*A*a+10*A*b+5*B*a)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*a^2+10*A*a*b
+5*B*a^2+15*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*a*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-9*A*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2-15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*b^2+5*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-30*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+30*a*b*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*C*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^2+15*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\sec(dx+c)^4+\left(2\,Cab+Bb^2\right)\sec(dx+c)^3+Aa^2+\left(Ca^2+2\,Bab+Ab^2\right)\sec(dx+c)^2+\left(Ba^2+2\,Aab\right)\text{se}}{\sec(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/s
ec(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/se
c(d*x + c)^(5/2), x)
```

3.995 $\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=242

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(5A+7C)+14abB+7b^2(A+3C)\right)}{21d}+\frac{2\sin(c+dx)\left(a^2(5A+\right.}{21d\sqrt{\sec}}$$

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*b^2*(A + 3*C) +
a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a*(4*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)
) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[
c + d*x]]) + (2*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5
/2))
```

Rubi [A] time = 0.521754, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2\sin(c+dx)\left(a^2(5A+7C)+14abB+4Ab^2\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)+14abB+\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(7/2),x]
```


```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*b*B + 7*b^2*(A + 3*C) +
a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a*(4*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)
) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*d*Sqrt[Sec[
c + d*x]]) + (2*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5
/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2}{7}\int \frac{(a+b\sec(c$$

$$= \frac{2a(4Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\sec(c+dx))^2 s}{7d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2a(4Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A(a+b\sec(c+dx))^2 s}{7d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{2a(4Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(4Ab^2+14abB+a^2(5\right.}{21d\sqrt{\sec(c}}$$

$$= \frac{2\left(6aAb+3a^2B+5b^2B+10abC\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

$$= \frac{2\left(6aAb+3a^2B+5b^2B+10abC\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.}{5d}$$

Mathematica [A] time = 6.57583, size = 251, normalized size = 1.04

$$\frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(20\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(5A+7C)+14\right.\right.}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(84*(3*a^2*
B + 5*b^2*B + 2*a*b*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + 20*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2] + (42*a*(2*A*b + a*B)*Cos[c + d*x] + 5*(14*A*b^2 +
 28*a*b*B + a^2*(13*A + 14*C) + 3*a^2*A*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]
))/(105*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c +
 d*x)])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 2.546, size = 706, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a
^2+280*B*a*b+140*C*a^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-
84*A*a*b-70*A*b^2-42*B*a^2-140*B*a*b-70*C*a^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-126*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+25*a^2*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1
05*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-210*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+105*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2 \sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\text{se}}{\sec(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/se
c(d*x + c)^(7/2), x)
```

3.996 $\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=290

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^2B+10aAb+14abC+7b^2B\right)}{21d}+\frac{2\sin(c+dx)\left(a^2(7A+9C)+\right.}{45d\sec^{\frac{3}{2}}(c+d}$$

```
[Out] (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(10*a*A*b + 5*a^2*B +
 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*a*(4*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(
5/2)) + (2*(4*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])
/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d
*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.552037, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4094, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\sin(c+dx)\left(a^2(7A+9C)+18abB+4Ab^2\right)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(5a^2B+10aAb+14abC+7b^2B\right)}{21d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(9/2),x]
```


```
[Out] (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(10*a*A*b + 5*a^2*B +
 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*a*(4*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(
5/2)) + (2*(4*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*Sin[c + d*x])
/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d
*Sec[c + d*x]^(7/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(4Ab^2+18abB+a\right.}{45d\,\mathrm{sec}}$$

$$= \frac{2a(4Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(4Ab^2+18abB+a\right.}{45d\,\mathrm{sec}}$$

$$= \frac{2\left(18abB+3b^2(3A+5C)+a^2(7A+9C)\right)\sqrt{\cos(c-}}{15d}$$

Mathematica [A] time = 3.65549, size = 286, normalized size = 0.99

$$\frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(120\sqrt{\cos(c+dx)}\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^2B+2ab(5A+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(9/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(168*(18*a*
b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2] + 120*(5*a^2*B + 7*b^2*B + 2*a*b*(5*A + 7*C))*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A*b^2 + 72*a*b*B + a^2*(43*A + 36*C))
*Cos[c + d*x] + 5*(156*a*A*b + 78*a^2*B + 84*b^2*B + 168*a*b*C + 18*a*(2*A*
b + a*B)*Cos[2*(c + d*x)] + 7*a^2*A*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(
630*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x
)])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 2.181, size = 784, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^2+1440*A*a*b+720*B*a^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^2-2160*A*a*b-504*A*b^2-
1080*B*a^2-1008*B*a*b-504*C*a^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(9
52*A*a^2+1680*A*a*b+504*A*b^2+840*B*a^2+1008*B*a*b+420*B*b^2+504*C*a^2+840*
C*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a^2-480*A*a*b-126*A*
```

```
b^2-240*B*a^2-252*B*a*b-210*B*b^2-126*C*a^2-420*C*a*b)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+150*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+75*B*a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+105*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b+210*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3
15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\text{se}}{\sec(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/se
c(d*x + c)^(9/2), x)
```

3.997 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=397

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^3(3A+C)+21a^2bB+3ab^2(7A+5C)+5b^3B\right)}{21d}+\frac{2b\sin(c+dx)s}{}$$

```
[Out] (-2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a
^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(15*a^3*B + 27*
a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(15*d) + (2*(54*a^2*b*B + 15*b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(63*d) + (2*b*(63*A*b^2 + 99*a*b*B + 24*a^
2*C + 49*b^2*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(3*b*B + 2*a*
C)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(21*d) + (2*C*Se
c[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.866576, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4096, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(24a^2C+99abB+63Ab^2+49b^2C\right)}{315d}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(54a^2bB+8a^3C+9ab^2(7A\right.}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a
^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3*a*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(15*a^3*B + 27*
a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(15*d) + (2*(54*a^2*b*B + 15*b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(63*d) + (2*b*(63*A*b^2 + 99*a*b*B + 24*a^
2*C + 49*b^2*C)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*(3*b*B + 2*a*
C)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(21*d) + (2*C*Se
c[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(9*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
```

```
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\sin(c}{9d}$$

$$= \frac{2(3bB+2aC)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+}{21d}$$

$$= \frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)\mathrm{se}}{315d}$$

$$= \frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)\mathrm{se}}{315d}$$

$$= \frac{2\left(15a^3B+27ab^2B+9a^2b(5A+3C)+b^3\right.}{15d}$$

$$= \frac{2\left(15a^3B+27ab^2B+9a^2b(5A+3C)+b^3\right.}{15d}$$

$$= -\frac{2\left(15a^3B+27ab^2B+9a^2b(5A+3C)+\right.}{}$$

Mathematica [A] time = 7.07986, size = 566, normalized size = 1.43

$$\frac{2\cos^5(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\mathrm{EllipticF}\left(\frac{1}{2}(c+dx)\right.\right.}{105d(a\cos(c+dx)+b)^3(A\cos(2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (2*Cos[c + d*x]^5*((2*(-315*a^2*A*b - 63*A*b^3 - 105*a^3*B - 189*a*b^2*B -
189*a^2*b*C - 49*b^3*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) + 2*(105*a^3*A + 105*a*A*b^2 + 105*a^2*b*B + 25*b^3*B + 35*
a^3*C + 75*a*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(
105*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])) + ((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*
(45*a^2*A*b + 9*A*b^3 + 15*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 7*b^3*C)*Sin[c
 + d*x])/15 + (4*Sec[c + d*x]^3*(b^3*B*Sin[c + d*x] + 3*a*b^2*C*Sin[c + d*x
]))/7 + (4*Sec[c + d*x]*(21*a*A*b^2*Sin[c + d*x] + 21*a^2*b*B*Sin[c + d*x]
+ 5*b^3*B*Sin[c + d*x] + 7*a^3*C*Sin[c + d*x] + 15*a*b^2*C*Sin[c + d*x]))/2
1 + (4*Sec[c + d*x]^2*(9*A*b^3*Sin[c + d*x] + 27*a*b^2*B*Sin[c + d*x] + 27*
a^2*b*C*Sin[c + d*x] + 7*b^3*C*Sin[c + d*x]))/45 + (4*b^3*C*Sec[c + d*x]^3*
Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 12.11, size = 1292, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C
*b^3*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/1
5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin
(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))))-2/5*b*(A*b^2+3*B*a*b+3*C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(B
*b+3*C*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2)))+2*a*(3*A*b^2+3*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))+2*a^2*(3*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\sec(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\sec(dx+c)^4 + Aa^3 + \left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + \left(Ca^3 + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))*sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sq
rt(sec(d*x + c)), x)
```

3.998 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=334

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)}{21d}+\frac{2b\sin(c+$$

```
[Out] (-2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*B
 + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(98*a^2*b*B + 21*b^
3*B + 24*a^3*C + 21*a*b^2*(5*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35
*d) + (2*b*(35*A*b^2 + 63*a*b*B + 24*a^2*C + 25*b^2*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(105*d) + (2*(7*b*B + 6*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
 d*x])^2*Sin[c + d*x])/(35*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^3*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.785919, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4096, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(24a^2C+63abB+35Ab^2+25b^2C\right)}{105d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(98a^2bB+24a^3C+21a\right.}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (-2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*B
 + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(98*a^2*b*B + 21*b^
3*B + 24*a^3*C + 21*a*b^2*(5*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35
*d) + (2*b*(35*A*b^2 + 63*a*b*B + 24*a^2*C + 25*b^2*C)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(105*d) + (2*(7*b*B + 6*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
 d*x])^2*Sin[c + d*x])/(35*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^3*Sin[c + d*x])/(7*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
```

```
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\sin(c+dx)}{7d}+\frac{2}{7}\int$$

$$= \frac{2(7bB+6aC)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+}{35d}$$

$$= \frac{2b\left(35Ab^2+63abB+24a^2C+25b^2C\right)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{105d}$$

$$= \frac{2b\left(35Ab^2+63abB+24a^2C+25b^2C\right)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{105d}$$

$$= \frac{2\left(98a^2bB+21b^3B+24a^3C+21ab^2(5A+3C)\right)\sqrt{\sec(}}{35d}$$

$$= \frac{2\left(21a^3B+21ab^2B+21a^2b(3A+C)+b^3(7A+5C)\right)\sqrt{}}{21d}$$

$$= -\frac{2\left(15a^2bB+3b^3B-5a^3(A-C)+3ab^2(5A+3C)\right)\sqrt{}}{5d}$$

Mathematica [A] time = 3.77841, size = 377, normalized size = 1.13

$$\frac{4(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(5\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(21a^2b(3A+C)+\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]

[Out] (4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(21*(-15*
a^2*b*B - 3*b^3*B + 5*a^3*(A - C) - 3*a*b^2*(5*A + 3*C))*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2] + 5*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*(3*A + C)
+ b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 315*a*A*b
^2*Sin[c + d*x] + 315*a^2*b*B*Sin[c + d*x] + 63*b^3*B*Sin[c + d*x] + 105*a^
3*C*Sin[c + d*x] + 189*a*b^2*C*Sin[c + d*x] + 35*A*b^3*Tan[c + d*x] + 105*a
*b^2*B*Tan[c + d*x] + 105*a^2*b*C*Tan[c + d*x] + 25*b^3*C*Tan[c + d*x] + 21
*b^3*B*Sec[c + d*x]*Tan[c + d*x] + 63*a*b^2*C*Sec[c + d*x]*Tan[c + d*x] + 1
5*b^3*C*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d*(b + a*Cos[c + d*x])^3*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(9/2))

Maple [B] time = 9.369, size = 1205, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a^2*b*(sin(1/2*d*x+1/2
c)^2)^(1/2)(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2/5*b^2*(B*b+3*C*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*b^3*(-1/56*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/
2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b^2+3*B*a*b+3
*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-

```
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(3*A*b^2+3*B*a*b+C*a^2
)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\sec(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\sec(dx+c)^4 + Aa^3 + \left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + \left(Ca^3 + 3\,Ba\right.}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sqrt(sec(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\left(b\sec(dx+c) + a\right)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sq
rt(sec(d*x + c)), x)
```

3.999 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)}{3d}+\frac{2b\sin(c+dx)\sqrt{\sec(}}{}$$

```
[Out] (2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*b*B
+ b^3*B + 3*a*b^2*(3*A + C) + a^3*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*(45*a*b*B - a^2*(10*A - 42*
C) + 3*b^2*(5*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(5
*a*A - 5*b*B - 9*a*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) - (2*b*(5*A -
 3*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(15*d) + (2*A
*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.829917, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4094, 4096, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2(-(10A-42C))+45abB+3b^2(5A+3C)\right)}{15d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*b*B
+ b^3*B + 3*a*b^2*(3*A + C) + a^3*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*(45*a*b*B - a^2*(10*A - 42*
C) + 3*b^2*(5*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(5
*a*A - 5*b*B - 9*a*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) - (2*b*(5*A -
 3*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(15*d) + (2*A
*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
```

```
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+b\sec(c}{}$$

$$= -\frac{2b(5A-3C)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+}{15d}$$

$$= -\frac{2b^2(5aA-5bB-9aC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} - \frac{2b}{}$$

$$= -\frac{2b^2(5aA-5bB-9aC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} - \frac{2b}{}$$

$$= \frac{2b\left(45abB-a^2(10A-42C)+3b^2(5A+3C)\right)\sqrt{\sec(c}}{15d}$$

$$= \frac{2\left(9a^2bB+b^3B+3ab^2(3A+C)+a^3(A+3C)\right)\sqrt{\cos(}}{3d}$$

$$= \frac{2\left(5a^3B-15ab^2B+15a^2b(A-C)-b^3(5A+3C)\right)\sqrt{\text{co}}}{5d}$$

Mathematica [A] time = 3.5338, size = 311, normalized size = 0.97

$$\frac{2(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^3(A+3C)+9a^2bB\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(6*(5*a^3
*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 10*(9*a^2*b*B + b^3*B + 3*a*b^2*(3*A + C) + a^3*(
A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 30*A*b^3*Sin[c + d
*x] + 90*a*b^2*B*Sin[c + d*x] + 90*a^2*b*C*Sin[c + d*x] + 18*b^3*C*Sin[c +
d*x] + 5*a^3*A*Sin[2*(c + d*x)] + 10*b^3*B*Tan[c + d*x] + 30*a*b^2*C*Tan[c
+ d*x] + 6*b^3*C*Sec[c + d*x]*Tan[c + d*x]))/(15*d*(b + a*Cos[c + d*x])^3*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 8.263, size = 1419, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*A*a^3*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-4*A*a^3+6*A*a^2*b+2*B*a^3)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*a^2*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a*b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2
*B*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+6*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+2*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*C*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*s
in(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1
/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*b^2*(B*b+3*C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b^2+3*B*a
*b+3*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\sec\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\sec\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\sec\left(dx+c\right)^3+\left(Ca^3+3\right.}{\sec\left(dx+c\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right) + a\right)^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/se
c(d*x + c)^(3/2), x)
```

3.1000 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=313

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2b(A+3C)+a^3B+9ab^2B+b^3(3A+C)\right)}{3d}-\frac{2b\sin(c+dx)\sqrt{s}}{}$$

```
[Out] (2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^3*B + 9*
a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b*(10*a^2*B - 15*b^2*B + 3*a*
b*(7*A - 15*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(9*A*b + 5
*a*B - 5*b*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(6*A*b + 5*a*B)*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a +
b*Sec[c + d*x])^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.833526, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4076, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}\left(10a^2B+3ab(7A-15C)-15b^2B\right)}{15d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(5/2),x]
```


```
[Out] (2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(a^3*B + 9*
a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b*(10*a^2*B - 15*b^2*B + 3*a*
b*(7*A - 15*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(9*A*b + 5
*a*B - 5*b*C)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(6*A*b + 5*a*B)*
(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a +
b*Sec[c + d*x])^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
```

```
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\text{se}}{}$$

$$= \frac{2(6Ab+5aB)(a+b\sec(c+dx))^2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2A}{}$$

$$= -\frac{2b^2(9Ab+5aB-5bC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} +$$

$$= -\frac{2b^2(9Ab+5aB-5bC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} +$$

$$= -\frac{2b\left(10a^2B-15b^2B+3ab(7A-15C)\right)\sqrt{\sec(c+dx}}{15d}$$

$$= \frac{2\left(a^3B+9ab^2B+b^3(3A+C)+3a^2b(A+3C)\right)\sqrt{\text{c}}}{3d}$$

$$= \frac{2\left(15a^2bB-5b^3B+15ab^2(A-C)+a^3(3A+5C)\right)}{5d}$$

Mathematica [A] time = 2.69708, size = 295, normalized size = 0.94

$$\frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(20\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2b(A+3C)+a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(12*(15*a^2
*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2] + 20*(a^3*B + 9*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A
 + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*a^3*A*Sin[c + d*x
] + 60*b^3*B*Sin[c + d*x] + 180*a*b^2*C*Sin[c + d*x] + 30*a^2*A*b*Sin[2*(c
+ d*x)] + 10*a^3*B*Sin[2*(c + d*x)] + 3*a^3*A*Sin[3*(c + d*x)] + 20*b^3*C*T
an[c + d*x]))/(15*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*(c + d*x)])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 7.42, size = 1837, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-45*C*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*a^2*b+30*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+90
*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+120*A*a^2*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^6-120*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4-180*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*a^3-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-15*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3+15*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-36*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4-40*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*B*b^3*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+6*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+10*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+30*B*b^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+10*C*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48
*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*A*a^3*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^6+40*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+30*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-90*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1
/2*d*x+1/2*c)^2-90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+90
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+90*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-45*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2+30
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-18*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*
d*x+1/2*c)^2+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+10*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-30*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*a^3*sin(1/2*d*x+1/2*c)^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+45*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*a*b^2-45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2+45*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*a^2*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
```

```
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\sec(dx+c)^5+\left(3\,Cab^2+Bb^3\right)\sec(dx+c)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\sec(dx+c)^3+\left(Ca^3+3\right.}{\sec(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(5
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/se
c(d*x + c)^(5/2), x)
```

3.1001 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=317

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)}{21d}+\frac{2a\sin(c+dx)}{}$$

```
[Out] (2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^2*b*
B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(24*A*b^2 + 63*a*b
*B + 5*a^2*(5*A + 7*C))*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(
11*A*b + 7*a*B - 35*b*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35*d) + (2*(6*A*
b + 7*a*B)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) +
 (2*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.831875, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\sin(c+dx)\left(5a^2(5A+7C)+63abB+24Ab^2\right)}{105d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(5A+7C)+21a^2bB\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(7/2),x]
```


```
[Out] (2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^2*b*
B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(24*A*b^2 + 63*a*b
*B + 5*a^2*(5*A + 7*C))*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(
11*A*b + 7*a*B - 35*b*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35*d) + (2*(6*A*
b + 7*a*B)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) +
 (2*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2}{7}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2(6Ab+7aB)(a+b\sec(c+dx))^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A}{}$$

$$= \frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+$$

$$= \frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+$$

$$= \frac{2a\left(24Ab^2+63abB+5a^2(5A+7C)\right)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}-$$

$$= \frac{2\left(21a^2bB+21b^3B+21ab^2(A+3C)+a^3(5A+7C\right.}{21d}$$

$$= \frac{2\left(3a^3B+15ab^2B+5b^3(A-C)+3a^2b(3A+5C)\right)}{5d}$$

Mathematica [A] time = 5.06322, size = 234, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)+2\sin(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3
*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 40*(21*a^2*b*B +
21*b^3*B + 21*a*b^2*(A + 3*C) + a^3*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2] + 2*(42*(3*a^2*A*b + a^3*B + 10*b^3*C) + 5*a*(84*A*b^2 +
 84*a*b*B + a^2*(29*A + 28*C))*Cos[c + d*x] + 42*a^2*(3*A*b + a*B)*Cos[2*(c
 + d*x)] + 15*a^3*A*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] time = 2.813, size = 1278, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*(240*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^2*(15*A*a+21*A*b+7*B*a)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+28*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(10*A*a^2
+18*A*a*b+15*A*b^2+6*B*a^2+15*B*a*b+5*C*a^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(40*A*a^3+
63*A*a^2*b+105*A*a*b^2+21*B*a^3+105*B*a^2*b+35*C*a^3+105*C*b^3)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+105*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-189*A*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-105*
A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*b^3+105*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+105*B*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-63*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-315*B*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+35*a^3*C*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
+315*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)-315*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
```

```
(1/2*d*x+1/2*c),2^(1/2))*a^2*b+105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\sec(dx+c)^5+\left(3\,Cab^2+Bb^3\right)\sec(dx+c)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\sec(dx+c)^3+\left(Ca^3+3\right.}{\sec(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(7
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/se
c(d*x + c)^(7/2), x)
```

3.1002 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=336

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2b(5A+7C)+5a^3B+21ab^2B+7b^3(A+3C)\right)}{21d}+\frac{2a\sin(c+d}{}$$

```
[Out] (2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(5*a^3
*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(24*A*b^2 + 99
*a*b*B + 7*a^2*(7*A + 9*C))*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(
8*A*b^3 + 15*a^3*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*Sin[c + d*x])/(63*d*
Sqrt[Sec[c + d*x]]) + (2*(2*A*b + 3*a*B)*(a + b*Sec[c + d*x])^2*Sin[c + d*x
])/(21*d*Sec[c + d*x]^(5/2)) + (2*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(9
*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.856031, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a\sin(c+dx)\left(7a^2(7A+9C)+99abB+24Ab^2\right)}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(9a^2b(5A+7C)+15a^3B+54ab^2B+8Ab^3\right)}{63d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(9/2),x]
```


```
[Out] (2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(5*a^3
*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(24*A*b^2 + 99
*a*b*B + 7*a^2*(7*A + 9*C))*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(
8*A*b^3 + 15*a^3*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*Sin[c + d*x])/(63*d*
Sqrt[Sec[c + d*x]]) + (2*(2*A*b + 3*a*B)*(a + b*Sec[c + d*x])^2*Sin[c + d*x
])/(21*d*Sec[c + d*x]^(5/2)) + (2*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(9
*d*Sec[c + d*x]^(7/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2(2Ab+3aB)(a+b\sec(c+dx))^2\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2\left(27a^2bB+15b^3B+9ab^2(3A+5C)+a^3(7A+9C\right.}{15d}$$

$$= \frac{2\left(27a^2bB+15b^3B+9ab^2(3A+5C)+a^3(7A+9C\right.}{15d}$$

Mathematica [A] time = 6.13451, size = 323, normalized size = 0.96

$$\frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(240\sqrt{\cos(c+dx)}\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2b(5A+7C)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(9/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(336*(27*a^
2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2] + 240*(5*a^3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) +
 3*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(7*a
*(108*A*b^2 + 108*a*b*B + a^2*(43*A + 36*C))*Cos[c + d*x] + 5*(84*A*b^3 + 7
8*a^3*B + 252*a*b^2*B + 6*a^2*(39*A*b + 42*b*C) + 18*a^2*(3*A*b + a*B)*Cos[
2*(c + d*x)] + 7*a^3*A*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d*(b + a
*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c +
d*x]^(9/2))
```

Maple [B] time = 2.434, size = 975, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*a^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^3+2160*A*a^2*b+720*B*a^
```

```
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^3-3240*A*a^2*b-1512*A
*a*b^2-1080*B*a^3-1512*B*a^2*b-504*C*a^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(952*A*a^3+2520*A*a^2*b+1512*A*a*b^2+420*A*b^3+840*B*a^3+1512*B*a^2*
b+1260*B*a*b^2+504*C*a^3+1260*C*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-168*A*a^3-720*A*a^2*b-378*A*a*b^2-210*A*b^3-240*B*a^3-378*B*a^2*b-630
*B*a*b^2-126*C*a^3-630*C*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-147*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*a^3-567*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2+75*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*a^3+315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2-567*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*a^2*b-315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3+315*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*a^2*b+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-189*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*a^3-945*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Cb^3\sec(dx+c)^5+\left(3\,Cab^2+Bb^3\right)\sec(dx+c)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\sec(dx+c)^3+\left(Ca^3+3\,Ba\cdots\right.}{\sec(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(9
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/se
c(d*x + c)^(9/2), x)
```

3.1003 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=401

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^3(9A+11C)+165a^2bB+33ab^2(5A+7C)+77b^3B\right)}{231d}+\frac{2\sin(c+\ldots}{}$$

```
[Out] (2*(7*a^3*B + 27*a*b^2*B + 3*b^3*(3*A + 5*C) + 3*a^2*b*(7*A + 9*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(165*
a^2*b*B + 77*b^3*B + 33*a*b^2*(5*A + 7*C) + 5*a^3*(9*A + 11*C))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a*(24*A*
b^2 + 143*a*b*B + 9*a^2*(9*A + 11*C))*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/
2)) + (2*(24*A*b^3 + 77*a^3*B + 242*a*b^2*B + 33*a^2*b*(7*A + 9*C))*Sin[c +
 d*x])/(495*d*Sec[c + d*x]^(3/2)) + (2*(165*a^2*b*B + 77*b^3*B + 33*a*b^2*(
5*A + 7*C) + 5*a^3*(9*A + 11*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) +
 (2*(6*A*b + 11*a*B)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(99*d*Sec[c + d*x
]^(7/2)) + (2*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/
2))
```

Rubi [A] time = 0.914722, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4094, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\sin(c+dx)\left(33a^2b(7A+9C)+77a^3B+242ab^2B+24Ab^3\right)}{495d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\left(9a^2(9A+11C)+143abB+24Ab^2\right)}{693d\sec^{\frac{5}{2}}(c+dx)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(11/2),x]
```


```
[Out] (2*(7*a^3*B + 27*a*b^2*B + 3*b^3*(3*A + 5*C) + 3*a^2*b*(7*A + 9*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(165*
a^2*b*B + 77*b^3*B + 33*a*b^2*(5*A + 7*C) + 5*a^3*(9*A + 11*C))*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a*(24*A*
b^2 + 143*a*b*B + 9*a^2*(9*A + 11*C))*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/
2)) + (2*(24*A*b^3 + 77*a^3*B + 242*a*b^2*B + 33*a^2*b*(7*A + 9*C))*Sin[c +
 d*x])/(495*d*Sec[c + d*x]^(3/2)) + (2*(165*a^2*b*B + 77*b^3*B + 33*a*b^2*(
5*A + 7*C) + 5*a^3*(9*A + 11*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) +
 (2*(6*A*b + 11*a*B)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(99*d*Sec[c + d*x
]^(7/2)) + (2*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/
2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^3\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\sec(}{}$$

$$= \frac{2(6Ab+11aB)(a+b\sec(c+dx))^2\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2a\left(24Ab^2+143abB+9a^2(9A+11C)\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2a\left(24Ab^2+143abB+9a^2(9A+11C)\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2a\left(24Ab^2+143abB+9a^2(9A+11C)\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2a\left(24Ab^2+143abB+9a^2(9A+11C)\right)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2\left(7a^3B+27ab^2B+3b^3(3A+5C)+3a^2b(7A+9C)\right)}{15d}$$

Mathematica [A] time = 6.83624, size = 538, normalized size = 1.34

$$\frac{2\cos^5(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx\right.\right.}{1155d(a\cos(c+dx)+b)^3(A}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*Cos[c + d*x]^5*((2*(1617*a^2*A*b + 693*A*b^3 + 539*a^3*B + 2079*a*b^2*B
+ 2079*a^2*b*C + 1155*b^3*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]) + 2*(225*a^3*A + 825*a*A*b^2 + 825*a^2*b*B + 385*b^3*B
 + 275*a^3*C + 1155*a*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2))/(1155*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) + ((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*(((57*a^2*A*b + 18*A*b^3 + 19*a^3*B + 54*a*b^2*B + 54*a^2*b*C)*Sin[c
+ d*x])/90 + ((1041*a^3*A + 3432*a*A*b^2 + 3432*a^2*b*B + 1232*b^3*B + 1144
*a^3*C + 3696*a*b^2*C)*Sin[2*(c + d*x)])/1848 + ((129*a^2*A*b + 36*A*b^3 +
43*a^3*B + 108*a*b^2*B + 108*a^2*b*C)*Sin[3*(c + d*x)])/180 + (a*(16*a^2*A
+ 33*A*b^2 + 33*a*b*B + 11*a^2*C)*Sin[4*(c + d*x)])/154 + (a^2*(3*A*b + a*B
)*Sin[5*(c + d*x)])/36 + (a^3*A*Sin[6*(c + d*x)])/88))/(d*(b + a*Cos[c + d*
x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 2.287, size = 1082, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a^3-36960*A*a^2*b-1232
0*B*a^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^3+73920*A*a^2*
b+23760*A*a*b^2+24640*B*a^3+23760*B*a^2*b+7920*C*a^3)*sin(1/2*d*x+1/2*c)^8*
cos(1/2*d*x+1/2*c)+(-34920*A*a^3-68376*A*a^2*b-35640*A*a*b^2-5544*A*b^3-227
92*B*a^3-35640*B*a^2*b-16632*B*a*b^2-11880*C*a^3-16632*C*a^2*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A*a^3+31416*A*a^2*b+27720*A*a*b^2+5544*
A*b^3+10472*B*a^3+27720*B*a^2*b+16632*B*a*b^2+4620*B*b^3+9240*C*a^3+16632*C
*a^2*b+13860*C*a*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2790*A*a^3-
5544*A*a^2*b-7920*A*a*b^2-1386*A*b^3-1848*B*a^3-7920*B*a^2*b-4158*B*a*b^2-2
310*B*b^3-2640*C*a^3-4158*C*a^2*b-6930*C*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-4851*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-2079*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b^3+675*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2475*A*a*b^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6237*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^2+2475*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1155*B*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-6237*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3465*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*b^3+825*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3465*C*a*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3 \sec(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\sec(dx+c)^4 + Aa^3 + \left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + \left(Ca^3 + 3\right.}{\sec(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sec(d*x + c)^(1
1/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{3}}{\sec\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/se
c(d*x + c)^(11/2), x)
```

3.1004 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=515

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(66a^2b^2(7A+5C)+77a^4(3A+C)+308a^3bB+220ab^3B+5b^4(\right.}{231d}$$

```
[Out] (-2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*
A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (2*(308*a^3*b*B + 220*a*b^3*B + 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A
 + 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(231*d) + (2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^
3*b*(5*A + 3*C) + 4*a*b^3*(9*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*d) + (2*(682*a^3*b*B + 660*a*b^3*B + 64*a^4*C + 15*b^4*(11*A + 9*C) + 9*a^
2*b^2*(143*A + 101*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(693*d) + (2*b*(135
3*a^2*b*B + 539*b^3*B + 192*a^3*C + 2*a*b^2*(891*A + 673*C))*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(3465*d) + (2*(33*A*b^2 + 55*a*b*B + 16*a^2*C + 27*b^2*C
)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(231*d) + (2*(11*
b*B + 8*a*C)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(99*d)
 + (2*C*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 1.3055, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4096, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(16a^2C+55abB+33Ab^2+27b^2C\right)(a+b\sec(c+dx))^2}{231d}+\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(135\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^3*b*(5*A + 3*C) + 4*a*b^3*(9*
A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (2*(308*a^3*b*B + 220*a*b^3*B + 77*a^4*(3*A + C) + 66*a^2*b^2*(7*A
 + 5*C) + 5*b^4*(11*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(231*d) + (2*(15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 12*a^
3*b*(5*A + 3*C) + 4*a*b^3*(9*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*d) + (2*(682*a^3*b*B + 660*a*b^3*B + 64*a^4*C + 15*b^4*(11*A + 9*C) + 9*a^
2*b^2*(143*A + 101*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(693*d) + (2*b*(135
3*a^2*b*B + 539*b^3*B + 192*a^3*C + 2*a*b^2*(891*A + 673*C))*Sec[c + d*x]^(
5/2)*Sin[c + d*x])/(3465*d) + (2*(33*A*b^2 + 55*a*b*B + 16*a^2*C + 27*b^2*C
)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(231*d) + (2*(11*
b*B + 8*a*C)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(99*d)
 + (2*C*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(11*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4 \text{si}}{11d}$$

$$= \frac{2(11bB+8aC)\sec^{\frac{3}{2}}(c+dx)(a+b\sec}{99d}$$

$$= \frac{2\left(33Ab^2+55abB+16a^2C+27b^2C\right)}{}$$

$$= \frac{2b\left(1353a^2bB+539b^3B+192a^3C+2\right.}{}$$

$$= \frac{2b\left(1353a^2bB+539b^3B+192a^3C+2\right.}{}$$

$$= \frac{2\left(15a^4B+54a^2b^2B+7b^4B+12a^3b(5\right.}{}$$

$$= \frac{2\left(15a^4B+54a^2b^2B+7b^4B+12a^3b(5\right.}{}$$

$$= -\frac{2\left(15a^4B+54a^2b^2B+7b^4B+12a^3b\right.}{}$$

Mathematica [A] time = 7.50476, size = 713, normalized size = 1.38

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{1155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(-4620*a^3*A*b - 2772*a*A*b^3 - 1155*a^4*B - 4158*a^2
*b^2*B - 539*b^4*B - 2772*a^3*b*C - 2156*a*b^3*C)*EllipticE[(c + d*x)/2, 2]
)/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(1155*a^4*A + 2310*a^2*A*b^2
+ 275*A*b^4 + 1540*a^3*b*B + 1100*a*b^3*B + 385*a^4*C + 1650*a^2*b^2*C + 22
5*b^4*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(
a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(1155*d*(b +
 a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((a
 + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(60*a^3*A*
b + 36*a*A*b^3 + 15*a^4*B + 54*a^2*b^2*B + 7*b^4*B + 36*a^3*b*C + 28*a*b^3*
C)*Sin[c + d*x])/15 + (4*Sec[c + d*x]^4*(b^4*B*Sin[c + d*x] + 4*a*b^3*C*Sin
[c + d*x]))/9 + (4*Sec[c + d*x]^2*(36*a*A*b^3*Sin[c + d*x] + 54*a^2*b^2*B*S
in[c + d*x] + 7*b^4*B*Sin[c + d*x] + 36*a^3*b*C*Sin[c + d*x] + 28*a*b^3*C*S
in[c + d*x]))/45 + (4*Sec[c + d*x]^3*(11*A*b^4*Sin[c + d*x] + 44*a*b^3*B*Si
n[c + d*x] + 66*a^2*b^2*C*Sin[c + d*x] + 9*b^4*C*Sin[c + d*x]))/77 + (4*Sec
[c + d*x]*(462*a^2*A*b^2*Sin[c + d*x] + 55*A*b^4*Sin[c + d*x] + 308*a^3*b*B
*Sin[c + d*x] + 220*a*b^3*B*Sin[c + d*x] + 77*a^4*C*Sin[c + d*x] + 330*a^2*
b^2*C*Sin[c + d*x] + 45*b^4*C*Sin[c + d*x]))/231 + (4*b^4*C*Sec[c + d*x]^4*
Tan[c + d*x])/11))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(11/2))
```

Maple [B] time = 14.859, size = 1550, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5
*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b^2+4*B
*a*b+6*C*a^2)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+2*C*b^4*(-1/352*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^6-9/616*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-15/154*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*(B*b+4*C*a)*(
-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
))+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2
*a^3*(4*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4 \sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\sec(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\sec(dx+c)^4 + 2\left(2\,Ca\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)^4 \sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*sq
rt(sec(d*x + c)), x)
```

3.1005 $\int \frac{(a+b\sec(c+dx))^4(A+B\sec(c+dx)+C\sec^2(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=441

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(28a^3b(3A+C)+42a^2b^2B+21a^4B+4ab^3(7A+5C)+5b^4B\right)}{21d}+\frac{2b}{}$$

```
[Out] (-2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^
4*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(15*d) + (2*(21*a^4*B + 42*a^2*b^2*B + 5*b^4*B + 28*a^3*b*(3*A + C) +
4*a*b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*(1098*a^3*b*B + 756*a*b^3*B + 192*a^4*C + 21*b^4*(9*
A + 7*C) + 7*a^2*b^2*(261*A + 155*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315
*d) + (2*b*(261*a^2*b*B + 75*b^3*B + 64*a^3*C + 2*a*b^2*(147*A + 101*C))*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) + (2*(63*A*b^2 + 117*a*b*B + 48*a^2*
C + 49*b^2*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(315*
d) + (2*(9*b*B + 8*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(63*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/
(9*d)
```

Rubi [A] time = 1.23921, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4096, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(261a^2bB+64a^3C+2ab^2(147A+101C)+75b^3B\right)}{315d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(48a^2C+11\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[S
ec[c + d*x]],x]
```


```
[Out] (-2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^
4*(9*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(15*d) + (2*(21*a^4*B + 42*a^2*b^2*B + 5*b^4*B + 28*a^3*b*(3*A + C) +
4*a*b^3*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*(1098*a^3*b*B + 756*a*b^3*B + 192*a^4*C + 21*b^4*(9*
A + 7*C) + 7*a^2*b^2*(261*A + 155*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315
*d) + (2*b*(261*a^2*b*B + 75*b^3*B + 64*a^3*C + 2*a*b^2*(147*A + 101*C))*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(315*d) + (2*(63*A*b^2 + 117*a*b*B + 48*a^2*
C + 49*b^2*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(315*
d) + (2*(9*b*B + 8*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(63*d) + (2*C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/
(9*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2C\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\sin(c+dx)}{9d}+\frac{2}{9}\int$$

$$=\frac{2(9bB+8aC)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\sin(c+$$

$$=\frac{2\left(63Ab^2+117abB+48a^2C+49b^2C\right)\sqrt{\sec(c+dx)}(a}{315d}$$

$$=\frac{2b\left(261a^2bB+75b^3B+64a^3C+2ab^2(147A+101C)\right)}{315d}$$

$$=\frac{2b\left(261a^2bB+75b^3B+64a^3C+2ab^2(147A+101C)\right)}{315d}$$

$$=\frac{2\left(1098a^3bB+756ab^3B+192a^4C+21b^4(9A+7C)+\right.}{315d}$$

$$=\frac{2\left(21a^4B+42a^2b^2B+5b^4B+28a^3b(3A+C)+4ab^3(\right.}{21}$$

$$=-\frac{2\left(60a^3bB+36ab^3B-15a^4(A-C)+18a^2b^2(5A+3\right.}{}$$

Mathematica [A] time = 7.38383, size = 609, normalized size = 1.38

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.\right.}{105d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(105*a^4*A - 630*a^2*A*b^2 - 63*A*b^4 - 420*a^3*b*B -
 252*a*b^3*B - 105*a^4*C - 378*a^2*b^2*C - 49*b^4*C)*EllipticE[(c + d*x)/2,
 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(420*a^3*A*b + 140*a*A*b^3
 + 105*a^4*B + 210*a^2*b^2*B + 25*b^4*B + 140*a^3*b*C + 100*a*b^3*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d
*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(105*d*(b + a*Cos[c + d*x])
^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((a + b*Sec[c + d*x
])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(90*a^2*A*b^2 + 9*A*b^4 +
60*a^3*b*B + 36*a*b^3*B + 15*a^4*C + 54*a^2*b^2*C + 7*b^4*C)*Sin[c + d*x])/
15 + (4*Sec[c + d*x]^3*(b^4*B*Sin[c + d*x] + 4*a*b^3*C*Sin[c + d*x]))/7 + (
4*Sec[c + d*x]*(28*a*A*b^3*Sin[c + d*x] + 42*a^2*b^2*B*Sin[c + d*x] + 5*b^4
*B*Sin[c + d*x] + 28*a^3*b*C*Sin[c + d*x] + 20*a*b^3*C*Sin[c + d*x]))/21 +
(4*Sec[c + d*x]^2*(9*A*b^4*Sin[c + d*x] + 36*a*b^3*B*Sin[c + d*x] + 54*a^2*
b^2*C*Sin[c + d*x] + 7*b^4*C*Sin[c + d*x]))/45 + (4*b^4*C*Sec[c + d*x]^3*Ta
n[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*Sec[c + d*x]^(11/2))
```

Maple [B] time = 12.626, size = 1550, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*A*a^3*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^4*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2/5*b^2*(A*b^2+4*B*a*b+6*C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*
c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^3*(B*b+4*C*
a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)+2*C*b^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3
-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))+4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4 \sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\sec(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\sec(dx+c)^4 + 2\left(2\,Ca^3b + \right.}{\sqrt{\sec(dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/sq
rt(sec(d*x + c)), x)
```

3.1006 $\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5\right.}{21d}$$

```
[Out] (2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3*
C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
+ (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^
4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*b*(609*a^2*b*B + 63*b^3*B - a^3*(70*A - 366*C) + 84*a*b^2*
(5*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*b^2*(98*a*b*B -
a^2*(35*A - 87*C) + 5*b^2*(7*A + 5*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d) - (2*b*(35*a*A - 21*b*B - 39*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*
x])^2*Sin[c + d*x])/(105*d) - (2*b*(7*A - 3*C)*Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x])^3*Sin[c + d*x])/(21*d) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*
x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.22565, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4094, 4096, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2(-(35A-87C))+98abB+5b^2(7A+5C)\right)}{105d}+\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^3(-(70A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(3/2),x]
```


```
[Out] (2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3*
C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
+ (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^2*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^
4*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*b*(609*a^2*b*B + 63*b^3*B - a^3*(70*A - 366*C) + 84*a*b^2*
(5*A + 3*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*b^2*(98*a*b*B -
a^2*(35*A - 87*C) + 5*b^2*(7*A + 5*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d) - (2*b*(35*a*A - 21*b*B - 39*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*
x])^2*Sin[c + d*x])/(105*d) - (2*b*(7*A - 3*C)*Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x])^3*Sin[c + d*x])/(21*d) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*
x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= -\frac{2b(7A-3C)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\sin}{21d}$$

$$= -\frac{2b(35aA-21bB-39aC)\sqrt{\sec(c+dx)}(a+b\sec(}{105d}$$

$$= \frac{2b^2\left(98abB-a^2(35A-87C)+5b^2(7A+5C)\right)\sec^{\frac{3}{2}}}{105d}$$

$$= \frac{2b^2\left(98abB-a^2(35A-87C)+5b^2(7A+5C)\right)\sec^{\frac{3}{2}}}{105d}$$

$$= \frac{2b\left(609a^2bB+63b^3B-a^3(70A-366C)+84ab^2(5\right.}{105d}$$

$$= \frac{2\left(84a^3bB+28ab^3B+42a^2b^2(3A+C)+7a^4(A+\right.}{}$$

$$= \frac{2\left(5a^4B-30a^2b^2B-3b^4B+20a^3b(A-C)-4ab^3(\right.}{}$$

Mathematica [A] time = 7.36756, size = 530, normalized size = 1.26

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{105d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(420*a^3*A*b - 420*a*A*b^3 + 105*a^4*B - 630*a^2*b^2*
B - 63*b^4*B - 420*a^3*b*C - 252*a*b^3*C)*EllipticE[(c + d*x)/2, 2])/(Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(35*a^4*A + 630*a^2*A*b^2 + 35*A*b^4
+ 420*a^3*b*B + 140*a*b^3*B + 105*a^4*C + 210*a^2*b^2*C + 25*b^4*C)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d*
x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(105*d*(b + a*Cos[c + d*x])^
4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((a + b*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*b*(20*a*A*b^2 + 30*a^2*b*B
+ 3*b^3*B + 20*a^3*C + 12*a*b^2*C)*Sin[c + d*x])/5 + (4*Sec[c + d*x]^2*(b^4
*B*Sin[c + d*x] + 4*a*b^3*C*Sin[c + d*x]))/5 + (4*Sec[c + d*x]*(7*A*b^4*Sin
[c + d*x] + 28*a*b^3*B*Sin[c + d*x] + 42*a^2*b^2*C*Sin[c + d*x] + 5*b^4*C*S
in[c + d*x]))/21 + (2*a^4*A*Sin[2*(c + d*x)])/3 + (4*b^4*C*Sec[c + d*x]^2*T
an[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sec[c + d*x]^(11/2))
```

Maple [B] time = 10.528, size = 1624, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*A*a^4*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-4*A*a^4+8*A*a^3*b+2*B*a^4)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8*A*a^3*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*A*a^2*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-2*B*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+8*B*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+2*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b^4*(-1/56*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3*(B*b+4*C*a)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

<u>Maxima</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
```

```
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\sec(dx+c)^6+\left(4\,Cab^3+Bb^4\right)\sec(dx+c)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\sec(dx+c)^4+2\left(2\,Ca^3\right.}{\sec(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/se
c(d*x + c)^(3/2), x)
```

3.1007 $\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=409

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(4a^3b(A+3C)+18a^2b^2B+a^4B+4ab^3(3A+C)+b^4B\right)}{3d}-\frac{2b^2\sin(c$$

```
[Out] (2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3
*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) - (2*b*(10*a^3*B - 60*a*b^2*B + a^2*b*(31*A - 87*C) - 3*b^3*(5*A + 3*C)
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(5*a^2*B - 5*b^2*B + 14*
a*b*(A - C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) - (2*b*(11*A*b + 5*a*B
 - 3*b*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(15*d) +
(2*(8*A*b + 5*a*B)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(15*d*Sqrt[Sec[c +
d*x]]) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 1.24684, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4094, 4096, 4076, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2B+14ab(A-C)-5b^2B\right)}{15d}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2b(31A-87C)+10a^3B-6\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(5/2),x]
```


```
[Out] (2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3
*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b^4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) - (2*b*(10*a^3*B - 60*a*b^2*B + a^2*b*(31*A - 87*C) - 3*b^3*(5*A + 3*C)
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) - (2*b^2*(5*a^2*B - 5*b^2*B + 14*
a*b*(A - C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) - (2*b*(11*A*b + 5*a*B
 - 3*b*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(15*d) +
(2*(8*A*b + 5*a*B)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(15*d*Sqrt[Sec[c +
d*x]]) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\sec(c}{}$$

$$= \frac{2(8Ab+5aB)(a+b\sec(c+dx))^3\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2A(a}{}$$

$$= -\frac{2b(11Ab+5aB-3bC)\sqrt{\sec(c+dx)}(a+b\sec(c+dx}{15d}$$

$$= -\frac{2b^2\left(5a^2B-5b^2B+14ab(A-C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15d}$$

$$= -\frac{2b^2\left(5a^2B-5b^2B+14ab(A-C)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15d}$$

$$= -\frac{2b\left(10a^3B-60ab^2B+a^2b(31A-87C)-3b^3(5A+3\right.}{15d}$$

$$= \frac{2\left(a^4B+18a^2b^2B+b^4B+4ab^3(3A+C)+4a^3b(A+3\right.}{3d}$$

$$= \frac{2\left(20a^3bB-20ab^3B+30a^2b^2(A-C)-b^4(5A+3C)+\right.}{}$$

Mathematica [A] time = 7.36504, size = 485, normalized size = 1.19

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.\right.}{15d(a\cos(c+dx)+b)^4(A}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(9*a^4*A + 90*a^2*A*b^2 - 15*A*b^4 + 60*a^3*b*B - 60*
a*b^3*B + 15*a^4*C - 90*a^2*b^2*C - 9*b^4*C)*EllipticE[(c + d*x)/2, 2])/(Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(20*a^3*A*b + 60*a*A*b^3 + 5*a^4*B
 + 90*a^2*b^2*B + 5*b^4*B + 60*a^3*b*C + 20*a*b^3*C)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d*x])^4*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2))/(15*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((a + b*Sec[c + d*x])^4*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*(((a^4*A + 20*A*b^4 + 80*a*b^3*B + 120*a^2*b^2*
C + 12*b^4*C)*Sin[c + d*x])/5 + (4*Sec[c + d*x]*(b^4*B*Sin[c + d*x] + 4*a*b
^3*C*Sin[c + d*x]))/3 + (2*a^3*(4*A*b + a*B)*Sin[2*(c + d*x)])/3 + (a^4*A*S
in[3*(c + d*x)])/5 + (4*b^4*C*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[
c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^
(11/2))
```

Maple [B] time = 10.602, size = 1884, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*A*a^4*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+1/3*(-12*A*a^4+16*A*a^3*b+4*B*a^4)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+(6*A*a^4-16*A*a^3*b+12*A*a^2*b^2-4*B*a^4+8*B*a^3*b+2*C*a^4)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*A*a^3*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*a^2*b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+8*A*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+2*B*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-8*B*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a^2*b^2*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*a^4*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8
*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-2/5*C*b^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*si
n(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin
(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^3*(B*b+4*C*a)*(-1/6*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\sec(dx+c)^6+\left(4\,Cab^3+Bb^4\right)\sec(dx+c)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\sec(dx+c)^4+2\left(2\,Ca^3b+\right.}{\sec(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/se
c(d*x + c)^(5/2), x)
```

3.1008 $\int \frac{(a+b\sec(c+dx))^4(A+B\sec(c+dx)+C\sec^2(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=429

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(42a^2b^2(A+3C)+a^4(5A+7C)+28a^3bB+84ab^3B+7b^4(3A+\right.}{21d}$$

```
[Out] (2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5*
C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
+ (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^
4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) - (2*b*(217*a^2*b*B - 105*b^3*B + 12*a*b^2*(19*A - 35*C) + 10*a
^3*(5*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) - (2*b^2*(98*a*b*B
 + b^2*(87*A - 35*C) + 5*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(105*d) + (2*(48*A*b^2 + 77*a*b*B + 5*a^2*(5*A + 7*C))*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*(8*A*b + 7*a*B)*(a + b*Sec
[c + d*x])^3*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + b*Sec[c +
d*x])^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 1.30936, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4076, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2(5A+7C)+98abB+b^2(87A-35C)\right)}{105d}+\frac{2\sin(c+dx)\left(5a^2(5A+7C)+77abB+4\right.}{105d\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(7/2),x]
```


```
[Out] (2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5*
C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
+ (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^
4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) - (2*b*(217*a^2*b*B - 105*b^3*B + 12*a*b^2*(19*A - 35*C) + 10*a
^3*(5*A + 7*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d) - (2*b^2*(98*a*b*B
 + b^2*(87*A - 35*C) + 5*a^2*(5*A + 7*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(105*d) + (2*(48*A*b^2 + 77*a*b*B + 5*a^2*(5*A + 7*C))*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*(8*A*b + 7*a*B)*(a + b*Sec
[c + d*x])^3*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + b*Sec[c +
d*x])^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2(8Ab+7aB)(a+b\sec(c+dx))^3\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2\left(48Ab^2+77abB+5a^2(5A+7C)\right)(a+b\sec(c+}{105d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2\left(98abB+b^2(87A-35C)+5a^2(5A+7C)\right)\mathrm{se}}{105d}$$

$$= -\frac{2b^2\left(98abB+b^2(87A-35C)+5a^2(5A+7C)\right)\mathrm{se}}{105d}$$

$$= -\frac{2b\left(217a^2bB-105b^3B+12ab^2(19A-35C)+10a\right.}{105d}$$

$$= \frac{2\left(28a^3bB+84ab^3B+7b^4(3A+C)+42a^2b^2(A+\right.}{}$$

$$= \frac{2\left(3a^4B+30a^2b^2B-5b^4B+20ab^3(A-C)+4a^3b(\right.}{}$$

Mathematica [A] time = 6.66818, size = 394, normalized size = 0.92

$$\frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(40\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(42a^2b^2(A+3C)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(168*(3*a^4
*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5*C))*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 40*(28*a^3*b*B + 84*a*b^3*B + 7*b
^4*(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2] + 168*a^3*A*b*Sin[c + d*x] + 42*a^4*B*Sin[c + d*x]
 + 840*b^4*B*Sin[c + d*x] + 3360*a*b^3*C*Sin[c + d*x] + 130*a^4*A*Sin[2*(c
+ d*x)] + 840*a^2*A*b^2*Sin[2*(c + d*x)] + 560*a^3*b*B*Sin[2*(c + d*x)] + 1
40*a^4*C*Sin[2*(c + d*x)] + 168*a^3*A*b*Sin[3*(c + d*x)] + 42*a^4*B*Sin[3*(
c + d*x)] + 15*a^4*A*Sin[4*(c + d*x)] + 280*b^4*C*Tan[c + d*x]))/(210*d*(b
+ a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c
 + d*x]^(11/2))
```

Maple [B] time = 9.3, size = 2507, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)

[Out] -2/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-630*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-50*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^2-210*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^2+126*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^2-210*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^2-70*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^2-70*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^2-252*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-420*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+210*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-504*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-280*C*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+440*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+252*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+420*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+280*C*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-80*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-42*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-210*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-70*C*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-70*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-480*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+960*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+336*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-920*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+504*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b*sin(1/2*d*x+1/2*c)^2+840*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3*sin(1/2*d*x+1/2*c)^2-420*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^2+1260*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^2-280*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b*sin(1/2*d*x+1/2*c)^2-840*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3*sin(1/2*d*x+1/2*c)^2+840*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b*sin(1/2*d*x+1/2*c)^2-840*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3*sin(1/2*d*x+1/2*c)^2-1260*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^2-2016*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1680*A*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1120*B*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+1008*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1680*A*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1120*B*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1680*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-168*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-420*A*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-280*B*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-840*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+105*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-63*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+105*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2

```
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+35
*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*a^4+35*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1344*A*a^
3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+140*B*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a^3*b+420*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-420*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^3*b+420*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+630*C*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\sec(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\sec(dx+c)^4 + 2\left(2\,Ca^3\right.}{\sec(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{4}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/se
c(d*x + c)^(7/2), x)
```

3.1009 $\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=426

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(4a^3b(5A+7C)+42a^2b^2B+5a^4B+28ab^3(A+3C)+21b^4B\right)}{21d}+$$

```
[Out] (2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4
*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2*B + 21*b^4*B + 28*a*b^3*(A + 3*C) + 4
*a^3*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(21*d) + (2*a*(64*A*b^3 + 75*a^3*B + 261*a*b^2*B + a^2*(202*A*b +
 294*b*C))*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(162*a*b*B + 3
*b^2*(41*A - 105*C) + 7*a^2*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
315*d) + (2*(48*A*b^2 + 117*a*b*B + 7*a^2*(7*A + 9*C))*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(8*A*b + 9*a*B)*(a + b*Sec
[c + d*x])^3*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*A*(a + b*Sec[c +
d*x])^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 1.31303, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\sin(c+dx)\left(7a^2(7A+9C)+117abB+48Ab^2\right)(a+b\sec(c+dx))^2}{315d\sec^{\frac{3}{2}}(c+dx)}-\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}\left(7a^2(7A+9C)+\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
+ d*x]^(9/2),x]
```


```
[Out] (2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4
*(7*A + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2*B + 21*b^4*B + 28*a*b^3*(A + 3*C) + 4
*a^3*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(21*d) + (2*a*(64*A*b^3 + 75*a^3*B + 261*a*b^2*B + a^2*(202*A*b +
 294*b*C))*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(162*a*b*B + 3
*b^2*(41*A - 105*C) + 7*a^2*(7*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
315*d) + (2*(48*A*b^2 + 117*a*b*B + 7*a^2*(7*A + 9*C))*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(8*A*b + 9*a*B)*(a + b*Sec
[c + d*x])^3*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*A*(a + b*Sec[c +
d*x])^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{(a+b\text{se}}{}$$

$$= \frac{2(8Ab+9aB)(a+b\sec(c+dx))^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2A}{}$$

$$= \frac{2\left(48Ab^2+117abB+7a^2(7A+9C)\right)(a+b\sec(c+}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(202Ab+294b\right.}{315d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(202Ab+294b\right.}{315d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\left(64Ab^3+75a^3B+261ab^2B+a^2(202Ab+294b\right.}{315d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(5a^4B+42a^2b^2B+21b^4B+28ab^3(A+3C)+4a\right.}{}$$

$$= \frac{2\left(36a^3bB+60ab^3B+15b^4(A-C)+18a^2b^2(3A+\right.}{}$$

Mathematica [A] time = 7.33785, size = 517, normalized size = 1.21

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{105d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(49*a^4*A + 378*a^2*A*b^2 + 105*A*b^4 + 252*a^3*b*B +
 420*a*b^3*B + 63*a^4*C + 630*a^2*b^2*C - 105*b^4*C)*EllipticE[(c + d*x)/2,
 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(100*a^3*A*b + 140*a*A*b^3
 + 25*a^4*B + 210*a^2*b^2*B + 105*b^4*B + 140*a^3*b*C + 420*a*b^3*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d
*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(105*d*(b + a*Cos[c + d*x])
^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((a + b*Sec[c + d*x
])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((19*a^4*A + 108*a^2*A*b^2 +
72*a^3*b*B + 18*a^4*C + 360*b^4*C)*Sin[c + d*x])/90 + (a*(52*a^2*A*b + 56*A
*b^3 + 13*a^3*B + 84*a*b^2*B + 56*a^2*b*C)*Sin[2*(c + d*x)])/21 + (a^2*(43*
a^2*A + 216*A*b^2 + 144*a*b*B + 36*a^2*C)*Sin[3*(c + d*x)])/180 + (a^3*(4*A
*b + a*B)*Sin[4*(c + d*x)])/14 + (a^4*A*Sin[5*(c + d*x)])/36))/(d*(b + a*Co
s[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x
]^(11/2))
```

Maple [B] time = 3.615, size = 1652, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*(-1120*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+80*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^3*(28*A*a+36*A*b+9*B*a)*sin(1/2*d*x+1/2*c)^8*cos(1/2*
d*x+1/2*c)-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(259*
A*a^2+540*A*a*b+378*A*b^2+135*B*a^2+252*B*a*b+63*C*a^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+56*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*a*(17*A*a^3+60*A*a^2*b+54*A*a*b^2+30*A*b^3+15*B*a^3+36*B*a^2*b+45*B*a*b^2
+9*C*a^3+30*C*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*A*a^4+160*A*a^3*b+126*A*a^2*b^
2+140*A*a*b^3+40*B*a^4+84*B*a^3*b+210*B*a^2*b^2+21*C*a^4+140*C*a^3*b+105*C*
b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*A*a^3*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+420*A*a*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-
147*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a^4-1134*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2*b^2-315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+75*B*a^4*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+630*a^2*b^2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+31
5*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)-756*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a^3*b-1260*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+420*a^3*b*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1260*C*a*b^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-18
9*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^4-1890*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^2+315*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
```

```
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\sec(dx+c)^6+\left(4\,Cab^3+Bb^4\right)\sec(dx+c)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\sec(dx+c)^4+2\left(2\,Ca^3\right.}{\sec(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/se
c(d*x + c)^(9/2), x)
```

3.1010 $\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=444

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A\right.}{231d}$$

```
[Out] (2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*A
 + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (2*(220*a^3*b*B + 308*a*b^3*B + 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A
+ 7*C) + 5*a^4*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(231*d) + (2*a*(192*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2
*a^2*b*(673*A + 891*C))*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)) + (2*(64*
A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a^4*(9*A + 11*C) + 9*a^2*b^2*(101*A
+ 143*C))*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]) + (2*(16*A*b^2 + 55*a*b*
B + 3*a^2*(9*A + 11*C))*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(231*d*Sec[c +
 d*x]^(5/2)) + (2*(8*A*b + 11*a*B)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(99
*d*Sec[c + d*x]^(7/2)) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(11*d*Se
c[c + d*x]^(9/2))
```

Rubi [A] time = 1.31581, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4094, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2\sin(c+dx)\left(3a^2(9A+11C)+55abB+16Ab^2\right)(a+b\sec(c+dx))^2}{231d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)\left(2a^2b(673A+891C)+539a^3B+1\right.}{3465d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(11/2),x]
```


```
[Out] (2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*A
 + 9*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (2*(220*a^3*b*B + 308*a*b^3*B + 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A
+ 7*C) + 5*a^4*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(231*d) + (2*a*(192*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2
*a^2*b*(673*A + 891*C))*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(3/2)) + (2*(64*
A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a^4*(9*A + 11*C) + 9*a^2*b^2*(101*A
+ 143*C))*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]) + (2*(16*A*b^2 + 55*a*b*
B + 3*a^2*(9*A + 11*C))*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(231*d*Sec[c +
 d*x]^(5/2)) + (2*(8*A*b + 11*a*B)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(99
*d*Sec[c + d*x]^(7/2)) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(11*d*Se
c[c + d*x]^(9/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\sec(}{}$$

$$= \frac{2(8Ab+11aB)(a+b\sec(c+dx))^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2A(a}{}$$

$$= \frac{2\left(16Ab^2+55abB+3a^2(9A+11C)\right)(a+b\sec(c+dx}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b(673A+891\right.}{3465d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b(673A+891\right.}{3465d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2b(673A+891\right.}{3465d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(7a^4B+54a^2b^2B+15b^4B+12ab^3(3A+5C)+4a^3b\right.}{15}$$

$$= \frac{2\left(7a^4B+54a^2b^2B+15b^4B+12ab^3(3A+5C)+4a^3b\right.}{15}$$

Mathematica [A] time = 7.06534, size = 580, normalized size = 1.31

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.\right.}{1155d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(2156*a^3*A*b + 2772*a*A*b^3 + 539*a^4*B + 4158*a^2*b
^2*B + 1155*b^4*B + 2772*a^3*b*C + 4620*a*b^3*C)*EllipticE[(c + d*x)/2, 2])
/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(225*a^4*A + 1650*a^2*A*b^2 +
385*A*b^4 + 1100*a^3*b*B + 1540*a*b^3*B + 275*a^4*C + 2310*a^2*b^2*C + 1155
*b^4*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a
 + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(1155*d*(b +
a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((a
+ b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((a*(76*a^2*A*b
 + 72*A*b^3 + 19*a^3*B + 108*a*b^2*B + 72*a^2*b*C)*Sin[c + d*x])/90 + ((104
1*a^4*A + 6864*a^2*A*b^2 + 1232*A*b^4 + 4576*a^3*b*B + 4928*a*b^3*B + 1144*
a^4*C + 7392*a^2*b^2*C)*Sin[2*(c + d*x)])/1848 + (a*(172*a^2*A*b + 144*A*b^
3 + 43*a^3*B + 216*a*b^2*B + 144*a^2*b*C)*Sin[3*(c + d*x)])/180 + (a^2*(16*
a^2*A + 66*A*b^2 + 44*a*b*B + 11*a^2*C)*Sin[4*(c + d*x)])/154 + (a^3*(4*A*b
 + a*B)*Sin[5*(c + d*x)])/36 + (a^4*A*Sin[6*(c + d*x)])/88))/(d*(b + a*Cos[
c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^
(11/2))
```

Maple [B] time = 2.626, size = 1273, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a^4-49280*A*a^3*b-1232
0*B*a^4)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^4+98560*A*a^3*
b+47520*A*a^2*b^2+24640*B*a^4+31680*B*a^3*b+7920*C*a^4)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-34920*A*a^4-91168*A*a^3*b-71280*A*a^2*b^2-22176*A*a*
b^3-22792*B*a^4-47520*B*a^3*b-33264*B*a^2*b^2-11880*C*a^4-22176*C*a^3*b)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A*a^4+41888*A*a^3*b+55440*A*a^
2*b^2+22176*A*a*b^3+4620*A*b^4+10472*B*a^4+36960*B*a^3*b+33264*B*a^2*b^2+18
480*B*a*b^3+9240*C*a^4+22176*C*a^3*b+27720*C*a^2*b^2)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-2790*A*a^4-7392*A*a^3*b-15840*A*a^2*b^2-5544*A*a*b^3-2
310*A*b^4-1848*B*a^4-10560*B*a^3*b-8316*B*a^2*b^2-9240*B*a*b^3-2640*C*a^4-5
544*C*a^3*b-13860*C*a^2*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+675*A*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a^4+4950*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+1155*A
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b^4-6468*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*A*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b^3+3300*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+4620*B
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b^3-1617*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12474*B
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-3465*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+825*C
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^4+6930*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+3465*
C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*b^4-8316*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-13860*
C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4 \sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\sec(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\sec(dx+c)^4 + 2\left(2\,Ca^3b + \right.}{\sec(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\left(b\sec(dx+c) + a\right)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11
/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/se
c(d*x + c)^(11/2), x)
```

3.1011 $\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=516

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(20a^3b(9A+11C)+330a^2b^2B+45a^4B+44ab^3(5A+7C)+77b\right.}{231d}$$

```
[Out] (2*(364*a^3*b*B + 468*a*b^3*B + 39*b^4*(3*A + 5*C) + 78*a^2*b^2*(7*A + 9*C)
 + a^4*(77*A + 91*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(195*d) + (2*(45*a^4*B + 330*a^2*b^2*B + 77*b^4*B + 44*a*b^3*(5
*A + 7*C) + 20*a^3*b*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a*(192*A*b^3 + 1053*a^3*B + 2171*a*b^
2*B + a^2*(2518*A*b + 3146*b*C))*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2))
+ (2*(192*A*b^4 + 4004*a^3*b*B + 3458*a*b^3*B + 77*a^4*(11*A + 13*C) + 11*a
^2*b^2*(491*A + 637*C))*Sin[c + d*x])/(6435*d*Sec[c + d*x]^(3/2)) + (2*(45*
a^4*B + 330*a^2*b^2*B + 77*b^4*B + 44*a*b^3*(5*A + 7*C) + 20*a^3*b*(9*A + 1
1*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*(48*A*b^2 + 221*a*b*B +
 11*a^2*(11*A + 13*C))*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(1287*d*Sec[c +
 d*x]^(7/2)) + (2*(8*A*b + 13*a*B)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(14
3*d*Sec[c + d*x]^(9/2)) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(13*d*S
ec[c + d*x]^(11/2))
```

Rubi [A] time = 1.40058, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4094, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\sin(c+dx)\left(11a^2(11A+13C)+221abB+48Ab^2\right)(a+b\sec(c+dx))^2}{1287d\sec^{\frac{7}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(11a^2b^2(491A+637C)+77a\right.}{6435d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c
 + d*x]^(13/2),x]
```


```
[Out] (2*(364*a^3*b*B + 468*a*b^3*B + 39*b^4*(3*A + 5*C) + 78*a^2*b^2*(7*A + 9*C)
 + a^4*(77*A + 91*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(195*d) + (2*(45*a^4*B + 330*a^2*b^2*B + 77*b^4*B + 44*a*b^3*(5
*A + 7*C) + 20*a^3*b*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (2*a*(192*A*b^3 + 1053*a^3*B + 2171*a*b^
2*B + a^2*(2518*A*b + 3146*b*C))*Sin[c + d*x])/(9009*d*Sec[c + d*x]^(5/2))
+ (2*(192*A*b^4 + 4004*a^3*b*B + 3458*a*b^3*B + 77*a^4*(11*A + 13*C) + 11*a
^2*b^2*(491*A + 637*C))*Sin[c + d*x])/(6435*d*Sec[c + d*x]^(3/2)) + (2*(45*
a^4*B + 330*a^2*b^2*B + 77*b^4*B + 44*a*b^3*(5*A + 7*C) + 20*a^3*b*(9*A + 1
1*C))*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*(48*A*b^2 + 221*a*b*B +
 11*a^2*(11*A + 13*C))*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(1287*d*Sec[c +
 d*x]^(7/2)) + (2*(8*A*b + 13*a*B)*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(14
3*d*Sec[c + d*x]^(9/2)) + (2*A*(a + b*Sec[c + d*x])^4*Sin[c + d*x])/(13*d*S
ec[c + d*x]^(11/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{13}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^4\sin(c+dx)}{13d\sec^{\frac{11}{2}}(c+dx)} + \frac{2}{13}\int \frac{(a+b\,\mathrm{s}}{}$$

$$= \frac{2(8Ab+13aB)(a+b\sec(c+dx))^3\sin(c+dx)}{143d\sec^{\frac{9}{2}}(c+dx)} + \frac{2.}{}$$

$$= \frac{2\left(48Ab^2+221abB+11a^2(11A+13C)\right)(a+b\sec(}{1287d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+1053a^3B+2171ab^2B+a^2(2518Ab+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+1053a^3B+2171ab^2B+a^2(2518Ab+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+1053a^3B+2171ab^2B+a^2(2518Ab+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(192Ab^3+1053a^3B+2171ab^2B+a^2(2518Ab+\right.}{9009d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(364a^3bB+468ab^3B+39b^4(3A+5C)+78a^2b^2(\right.}{}$$

Mathematica [A] time = 7.14017, size = 658, normalized size = 1.28

$$\frac{2\cos^6(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sec[c + d*x]^(13/2),x]
```

```
[Out] (2*Cos[c + d*x]^6*((2*(5929*a^4*A + 42042*a^2*A*b^2 + 9009*A*b^4 + 28028*a^
3*b*B + 36036*a*b^3*B + 7007*a^4*C + 54054*a^2*b^2*C + 15015*b^4*C)*Ellipti
cE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(11700*a^3*
A*b + 14300*a*A*b^3 + 2925*a^4*B + 21450*a^2*b^2*B + 5005*b^4*B + 14300*a^3
*b*C + 20020*a*b^3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))
/(15015*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])) + ((a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
(((1897*a^4*A + 11856*a^2*A*b^2 + 1872*A*b^4 + 7904*a^3*b*B + 7488*a*b^3*B
+ 1976*a^4*C + 11232*a^2*b^2*C)*Sin[c + d*x])/9360 + ((4164*a^3*A*b + 4576*
a*A*b^3 + 1041*a^4*B + 6864*a^2*b^2*B + 1232*b^4*B + 4576*a^3*b*C + 4928*a*
b^3*C)*Sin[2*(c + d*x)])/1848 + ((2297*a^4*A + 13416*a^2*A*b^2 + 1872*A*b^4
 + 8944*a^3*b*B + 7488*a*b^3*B + 2236*a^4*C + 11232*a^2*b^2*C)*Sin[3*(c + d
*x)])/9360 + (a*(32*a^2*A*b + 22*A*b^3 + 8*a^3*B + 33*a*b^2*B + 22*a^2*b*C)
*Sin[4*(c + d*x)])/77 + (a^2*(89*a^2*A + 312*A*b^2 + 208*a*b*B + 52*a^2*C)*
Sin[5*(c + d*x)])/1872 + (a^3*(4*A*b + a*B)*Sin[6*(c + d*x)])/88 + (a^4*A*S
```

```
in[7*(c + d*x)])/208))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(11/2))
```

Maple [B] time = 2.689, size = 1407, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13/2),x)
```

```
[Out] -2/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-443520*A
*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(1330560*A*a^4+1048320*A*a^3*
b+262080*B*a^4)*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-1798720*A*a^4-26
20800*A*a^3*b-960960*A*a^2*b^2-655200*B*a^4-640640*B*a^3*b-160160*C*a^4)*si
n(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(1379840*A*a^4+2957760*A*a^3*b+19219
20*A*a^2*b^2+411840*A*a*b^3+739440*B*a^4+1281280*B*a^3*b+617760*B*a^2*b^2+3
20320*C*a^4+411840*C*a^3*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-66651
2*A*a^4-1815840*A*a^3*b-1777776*A*a^2*b^2-617760*A*a*b^3-72072*A*b^4-453960
*B*a^4-1185184*B*a^3*b-926640*B*a^2*b^2-288288*B*a*b^3-296296*C*a^4-617760*
C*a^3*b-432432*C*a^2*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(198352*A
*a^4+720720*A*a^3*b+816816*A*a^2*b^2+480480*A*a*b^3+72072*A*b^4+180180*B*a^
4+544544*B*a^3*b+720720*B*a^2*b^2+288288*B*a*b^3+60060*B*b^4+136136*C*a^4+4
80480*C*a^3*b+432432*C*a^2*b^2+240240*C*a*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-27258*A*a^4-145080*A*a^3*b-144144*A*a^2*b^2-137280*A*a*b^3-18
018*A*b^4-36270*B*a^4-96096*B*a^3*b-205920*B*a^2*b^2-72072*B*a*b^3-30030*B*
b^4-24024*C*a^4-137280*C*a^3*b-108108*C*a^2*b^2-120120*C*a*b^3)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+35100*A*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+42900*A
*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-17787*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-126126
*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-27027*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+87
75*B*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+64350*a^2*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
15015*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-84084*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b
-108108*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+42900*a^3*b*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+60060*C*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21021*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^4-162162*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-45045*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13
/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^4\sec(dx+c)^6+\left(4\,Cab^3+Bb^4\right)\sec(dx+c)^5+Aa^4+\left(6\,Ca^2b^2+4\,Bab^3+Ab^4\right)\sec(dx+c)^4+2\left(2\,Ca^{}\right.}{\sec(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13
/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))/sec(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**
(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\left(b\sec(dx+c)+a\right)^4}{\sec(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(13
/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4/se
c(d*x + c)^(13/2), x)
```

3.1012 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=296

$$\frac{2(bB-aC)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d}$$

```
[Out] (-2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(b*B - a*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*d) - (2*a*(A*b^2
- a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(b^3*(a + b)*d) + (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C +
3*b^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*b^3*d) + (2*(b*B - a*C)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*b^2*d) + (2*C*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(5*b*d)
```

Rubi [A] time = 1.11021, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2C-5abB+\right.}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```

```
[Out] (-2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(b*B - a*C)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*d) - (2*a*(A*b^2
- a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(b^3*(a + b)*d) + (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C +
3*b^2*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*b^3*d) + (2*(b*B - a*C)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*b^2*d) + (2*C*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(5*b*d)
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
```

```
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx = \frac{2C\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aC}{2}+\frac{1}{2}b(5A+3C)\sec\right.}{a+b\sec(}}{5b}$$

$$= \frac{2(bB-aC)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2d} + \frac{2C\sec^{\frac{5}{2}}(c+dx)\sin(c}{5bd}$$

$$= \frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5b^3d} +$$

$$= \frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5b^3d} +$$

$$= \frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5b^3d} +$$

$$= -\frac{2a\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{b^3(a+b)d}$$

$$= -\frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{5b^3d}$$

Mathematica [F] time = 80.3292, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]),x]
```

```
[Out] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]), x]
```

Maple [B] time = 9.906, size = 800, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*C/b/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*(A*b^2-B*a*b+C*a^2)*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+2*(B*b-C*a)/b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2-B*a*b+C
*a^2)/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2
*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec
(d*x + c) + a), x)
```

3.1013 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=218

$$\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\right.}{b^2d(a+b)}$$

```
[Out] (-2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(b^2*d) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) + (2*
(b*B - a*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d) + (2*C*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.773251, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(bB-aC)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}-\frac{2(bB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```


```
[Out] (-2*(b*B - a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(b^2*d) + (2*C*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) + (2*
(b*B - a*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d) + (2*C*Sec[c + d*x]^(3
/2)*Sin[c + d*x])/(3*b*d)
```


Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{2C\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aC}{2}+\frac{1}{2}b(3A+C)\sec(c\cdots\right.}{a+b\sec(c\cdots}}{3b}$$

$$= \frac{2(bB-aC)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2C\sec^{\frac{3}{2}}(c+dx)\sin(c\cdots}{3bd}$$

$$= \frac{2(bB-aC)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2C\sec^{\frac{3}{2}}(c+dx)\sin(c\cdots}{3bd}$$

$$= \frac{2(bB-aC)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2C\sec^{\frac{3}{2}}(c+dx)\sin(c\cdots}{3bd}$$

$$= \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+\cdots}}{(a+b)d}$$

$$= -\frac{2(bB-aC)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2C\cdots}{}$$

Mathematica [F] time = 58.2649, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]),x]
```

```
[Out] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]), x]
```

Maple [A] time = 7.304, size = 472, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b^2-B*a*b
+C*a^2)/b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*C/b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+2*(B*b-C*a)/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec
(d*x + c) + a), x)
```

3.1014 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\ \cdots\right.}{abd(a+b)}$$

```
[Out] (-2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a*d) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a + b)*d) + (2*C*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.477231, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|\ \cdots}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```

```
[Out] (-2*C*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a*d) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a + b)*d) + (2*C*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(b*d)
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
```

```
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \frac{2C\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{2\int \frac{-\frac{aC}{2}+\frac{1}{2}b(A-C)\sec(c+dx)+\frac{1}{2}(bB-}{\sqrt{\sec(c+dx)}(a+b\sec(c+}}{b}$$

$$= \frac{2C\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{2\int \frac{-\frac{a^2C}{2}-\left(-\frac{1}{2}ab(A-C)-\frac{abC}{2}\right)\sec(c+}{\sqrt{\sec(c+dx)}}}{a^2b}$$

$$= \frac{2C\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} + \frac{A\int\sqrt{\sec(c+dx)}\,dx}{a} - \frac{C\int -}{}$$

$$= -\frac{2\left(\frac{Ab}{a}-B+\frac{aC}{b}\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{(a+b)d}$$

$$= -\frac{2C\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2A\sqrt{\cos(}}{}$$

Mathematica [F] time = 44.5846, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]),x]
```

```
[Out] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]), x]
```

Maple [A] time = 4.575, size = 409, normalized size = 2.3

$$-\frac{1}{d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\text{Ellipti}}{a\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*(-A
*b^2+B*a*b-C*a^2)/b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*C/b*(-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec
(d*x + c) + a), x)
```

3.1015 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=157

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi}{a^2d(a+b)}$$

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rubi [A] time = 0.288498, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[
c + d*x])),x]
```

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a^2*d) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{\int \frac{aA-(Ab-aB)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{\left(Ab^2-abB+a^2C\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2}$$

$$= \frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \sqrt{\sec(c+dx)}\,dx}{a^2} + \frac{\left(\left(Ab^2-abB+a^2C\right)\sqrt{\mathrm{co}}\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} + \frac{(A}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)\sqrt{\cos(c+dx)}F\left(\right.}{a^2}$$

Mathematica [F] time = 13.2909, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
b*Sec[c + d*x])),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
b*Sec[c + d*x])), x]
```

Maple [A] time = 2.664, size = 323, normalized size = 2.1

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\,a^2\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\,\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(A\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b+C*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))*a^2)/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{(a+b\sec(c+dx))\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/((a + b*sec(c + d*x))*sqr
t(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{(b \sec (dx + c) + a)\sqrt{\sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqr
t(sec(d*x + c))), x)
```

3.1016 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=207

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(A+3C)-3abB+3Ab^2\right)}{3a^3d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab\right.}{a^3d}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a^2*d) + (2*(3*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^3*d) - (2*b*(A*b^2 - a*(b
*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d
*x]])
```

Rubi [A] time = 0.530612, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)-3abB+3Ab^2\right)}{3a^3d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(Ab^2-a(bB\right.}{a^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])),x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a^2*d) + (2*(3*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^3*d) - (2*b*(A*b^2 - a*(b
*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d
*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{2}(Ab-aB)-\frac{1}{2}a(A+3C)\sec(c+dx)-\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{2}a(Ab-aB)-\left(\frac{3}{2}b(Ab-aB)+\frac{1}{2}a^2(A+3C)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^3} - \frac{\left(b\left(Ab^2\right.\right.}{}$$

$$= \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{\left(3Ab^2-3abB+a^2(A+3C\right.}{3a^3}$$

$$= -\frac{2b\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d} + $$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\left(3Ab^2-3abB+\right.}{}$$

Mathematica [F] time = 59.8327, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])), x]
```

Maple [B] time = 2.712, size = 945, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*A*a^3-4*A*
a^2*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^3+2*A*a^2*b)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*a^2*b+3*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*b^3+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^3-3*B*a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))*a*b^2+3*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a^2*b+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a^2*b)/a^3/(a-b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx + c)^2 + B \sec(dx + c) + A}{(b \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A}{\left(b\sec\left(dx+c\right) + a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec
(d*x + c)^(3/2)), x)
```

3.1017 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=269

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2b(A+3C)+a^3(-B)-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

```
[Out] (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(3*A*b^3 - a^3*B - 3*a*b^2*B
+ a^2*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*a^4*d) + (2*b^2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a + b)*d) +
 (2*A*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(A*b - a*B)*Sin[c + d*x
])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.855461, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b(A+3C)+a^3(-B)-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])),x]
```


```
[Out] (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(3*A*b^3 - a^3*B - 3*a*b^2*B
+ a^2*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*a^4*d) + (2*b^2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a + b)*d) +
 (2*A*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(A*b - a*B)*Sin[c + d*x
])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx = \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{5}{2}(Ab-aB)-\frac{1}{2}a(3A+5C)\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{4\int \frac{\frac{3}{4}(5Ab^2-5abB+a^2(3A+5C))+}{\sqrt{\sec}}}{}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{4\int \frac{3}{4}a(5Ab^2-5abB+a^2(3A+5C))-}{}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3Ab^3-a^3B-3ab^2B+a^2b}{3}$$

$$= \frac{2b^2(Ab^2-a(bB-aC))\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^4(a+b)d}$$

$$= \frac{2(5Ab^2-5abB+a^2(3A+5C))\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5a^3d}$$

Mathematica [F] time = 55.2983, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])), x]
```

Maple [B] time = 5.605, size = 801, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*A/a*(-4*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)-4/3/a^2*(3*A*a+A*b-B*a)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos
```

```
(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/a^3
*(3*A*a^2+2*A*a*b+A*b^2-2*B*a^2-B*a*b+C*a^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*(A*a^3+A*a^2*b+A*a*b^2+A*b^3-B*a^3-B*a^2*b-B*a*b^2+C*a^3+C*
a^2*b)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-2*b^2*(A*b^2-B*a*b+C*a^2)/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec
(d*x + c)^(5/2)), x)
```

3.1018 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=342

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^2b^2(A+3C)+a^4(5A+7C)-7a^3bB-21ab^3B+21Ab^4\right)}{21a^5d}+\frac{2\sin}{}$$

```
[Out] (-2*(5*A*b^3 - 3*a^3*B - 5*a*b^2*B + a^2*b*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^4*d) + (2*(21*A*b^4 - 7*
a^3*b*B - 21*a*b^3*B + 7*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*a^5*d) - (2*b^3*(A*
b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(a^5*(a + b)*d) + (2*A*Sin[c + d*x])/(7*a*d*Sec[
c + d*x]^(5/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(5*a^2*d*Sec[c + d*x]^(3/2))
 + (2*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*a^3*d*Sqrt[Se
c[c + d*x]])
```

Rubi [A] time = 1.22208, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\sin(c+dx)\left(a^2(5A+7C)-7abB+7Ab^2\right)}{21a^3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2b^2(A+3C)+a^4(5A+7C\right.}{21a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*(a + b*Sec[
c + d*x])),x]
```


```
[Out] (-2*(5*A*b^3 - 3*a^3*B - 5*a*b^2*B + a^2*b*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^4*d) + (2*(21*A*b^4 - 7*
a^3*b*B - 21*a*b^3*B + 7*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*a^5*d) - (2*b^3*(A*
b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(a^5*(a + b)*d) + (2*A*Sin[c + d*x])/(7*a*d*Sec[
c + d*x]^(5/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(5*a^2*d*Sec[c + d*x]^(3/2))
 + (2*(7*A*b^2 - 7*a*b*B + a^2*(5*A + 7*C))*Sin[c + d*x])/(21*a^3*d*Sqrt[Se
c[c + d*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2A\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2\int \frac{\frac{7}{2}(Ab-aB)-\frac{1}{2}a(5A+7C)\sec(c+dx)-\frac{5}{2}Ab\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx}{7a}$$

$$= \frac{2A\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{5}{4}(7Ab^2-7abB+a^2(5A+7C))+\frac{1}{4}a(4}{\sec^{\frac{3}{2}}(c+}}{}$$

$$= \frac{2A\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+a^2(5A+7C\right.}{21a^3d\sqrt{\sec(c+dx}}$$

$$= \frac{2A\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+a^2(5A+7C\right.}{21a^3d\sqrt{\sec(c+dx}}$$

$$= \frac{2A\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{5a^2d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7Ab^2-7abB+a^2(5A+7C\right.}{21a^3d\sqrt{\sec(c+dx}}$$

$$= -\frac{2b^3\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^5(a+b)d} +$$

$$= -\frac{2\left(5Ab^3-3a^3B-5ab^2B+a^2b(3A+5C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{5a^4d}$$

Mathematica [F] time = 68.7677, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*(a +
b*Sec[c + d*x])),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*(a +
b*Sec[c + d*x])), x]
```

Maple [B] time = 7.123, size = 1095, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/105*A/a*(60*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-168*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
```

```
c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/5/a^2*(4*A*a+A*b
-B*a)*(-4*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+4/3/a^3*(6*A*a^2+3*A*a*b+A*b^2-3*B*a^2-B*a*b+C*a^2)*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(4*A*a^3+3*A*a^2*b+2*A*a*b^2+A*b^
3-3*B*a^3-2*B*a^2*b-B*a*b^2+2*C*a^3+C*a^2*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*(A*a^4+A*a^3*b+A*a^2*b^2+A*a*b^3+A*b^4-B*a^4-B*a^3*b-B*a^2*
b^2-B*a*b^3+C*a^4+C*a^3*b+C*a^2*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b^3*(A*b^2-B*a*b+C*a^2)/a^4/
(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec
(d*x + c)^(7/2)), x)
```

3.1019 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=447

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^2C-3abB+3Ab^2-2b^2C\right)}{3b^2d\left(a^2-b^2\right)}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab^2-\right.}{bd\left(a^2-b^2\right)(a+b\sec(c+\right.}$$

```
[Out] -(((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) + ((3*A*b^2
 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - 5*a*b
^3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) + ((3*a
^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(b^3*(a^2 - b^2)*d) + ((3*A*b^2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.37041, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2C-3abB+3Ab^2-2b^2C\right)}{3b^2d\left(a^2-b^2\right)}+\frac{\sin(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```


```
[Out] -(((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) + ((3*A*b^2
 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - 5*a*b
^3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) + ((3*a
^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(b^3*(a^2 - b^2)*d) + ((3*A*b^2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C))*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\sec^{\frac{3}{2}}(c+\ldots}{\ldots}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C-2b^2C\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \ldots$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)\sqrt{\sec(c+dx)}\sin(c\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)\sqrt{\sec(c+dx)}\sin(c\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)\sqrt{\sec(c+dx)}\sin(c\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(3Ab^4+3a^3bB-5ab^3B-a^2b^2(A-7C)-5a^4C\right)\sqrt{\cos(c\ldots}}{(a-b)b^3(a+b)^2\ldots}$$

$$= -\frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)\sqrt{\cos(c+dx)}E\left(\ldots\right.}{b^3\left(a^2-b^2\right)d}$$

Mathematica [B] time = 7.42214, size = 931, normalized size = 2.08

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(12Bb^4+12aAb^3-28aCb^3-24a^2Bb^2+40a^3Cb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec\ldots}}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(12*a*
A*b^3 - 24*a^2*b^2*B + 12*b^4*B + 40*a^3*b*C - 28*a*b^3*C)*Cos[c + d*x]^2*E
llipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqr
t[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)) + (2*(9*a^2*A*b^2 - 12*A*b^4 - 27*a^3*b*B + 30*a*b^3*B + 45*a^4*C -
44*a^2*b^2*C - 4*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]
], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c
+ d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 -
 Cos[c + d*x]^2)) - (2*(3*a^2*A*b^2 - 9*a^3*b*B + 6*a*b^3*B + 15*a^4*C - 12
*a^2*b^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*
x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(6*(a - b)*b^
3*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c
+ d*x])^2) + ((b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((2*(a*A*b^2 - 3*a^2*b*B + 2*b^3*B + 5*a^3*C - 4*a*b^2
```

```
*C)*Sin[c + d*x])/(b^3*(-a^2 + b^2)) - (2*(a*A*b^2*Sin[c + d*x] - a^2*b*B*S
in[c + d*x] + a^3*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x]))
+ (4*C*Tan[c + d*x])/(3*b^2)))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 11.463, size = 1031, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b^2-B*a*b+
C*a^2)/b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2
*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*(B*b-2*C*a)/b^
3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+2*C/b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
+2*(B*b-2*C*a)/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec
(d*x + c) + a)^2, x)
```

3.1020 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

```
[Out] -(((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d)) - ((A*b^2 - a*(b*B - a*C)
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^
2 - b^2)*d) + ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 5*C))*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C
))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.959012, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C-abB+Ab^2-2b^2C\right)}{b^2d\left(a^2-b^2\right)}-\frac{\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```


```
[Out] -(((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d)) - ((A*b^2 - a*(b*B - a*C)
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^
2 - b^2)*d) + ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 5*C))*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a*(a - b)*b^2*(a + b)^2*d) + ((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) - ((A*b^2 - a*(b*B - a*C
))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\sqrt{\sec(c+dx)}\Big(}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{\Big(Ab^2}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{\Big(Ab^2}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{\Big(Ab^2}{}$$

$$= \frac{\left(Ab^4+a^3bB-3ab^3B-3a^4C+a^2b^2(A+5C)\right)\sqrt{\cos(c+dx)}}{a(a-b)b^2(a+b)^2d}$$

$$= -\frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d}$$

Mathematica [B] time = 7.18645, size = 865, normalized size = 2.38

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(4Ab^3-4Cb^3-4aBb^2+8a^2Cb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(4*A*b
^3 - 4*a*b^2*B + 8*a^2*b*C - 4*b^3*C)*Cos[c + d*x]^2*EllipticPi[-(b/a), -Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]
*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(a*A*b^
2) - 3*a^2*b*B + 4*b^3*B + 9*a^3*C - 10*a*b^2*C)*Cos[c + d*x]^2*(EllipticF[
ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d
*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*
(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(a*A*b^2 - a^2*b*B + 3*a^3*
C - 2*a*b^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c +
 d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a)
, -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(2*b^2*(-a
 + b)*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Se
c[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*((-2*(A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sin[c + d
*x])/(b^2*(-a^2 + b^2)) + (2*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2
*C*Sin[c + d*x]))/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 7.709, size = 897, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b^2+B*a*b
-C*a^2)/a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(A*b^2-C*a^2)/b^2
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+2*C/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2
*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^2, x)
```

3.1021 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(-(2A+C))+abB+Ab^2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-\right.}{bd\left(a^2-b^2\right)(a+b\sec(c\right.}$$

```
[Out] ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(a*b*(a^2 - b^2)*d) - ((A*b^2 + a*b*B - a^2*(2*A + C))*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2
)*d) + ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
(a - b)*b*(a + b)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.672626, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A+C))+abB+\right.}{a^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```


```
[Out] ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(a*b*(a^2 - b^2)*d) - ((A*b^2 + a*b*B - a^2*(2*A + C))*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2
)*d) + ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
(a - b)*b*(a + b)^2*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{1}{2}\left(-Ab^2+a\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{1}{2}a\left(-Ab^2+\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(Ab^2+abB\right.}{}$$

$$= \frac{\left(Ab^4+a^3bB+ab^3B+a^4C-3a^2b^2(A+C)\right)\sqrt{\cos(c+dx)}\Pi}{a^2(a-b)b(a+b)^2d}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{ab\left(a^2-b^2\right)d}$$

Mathematica [B] time = 7.04354, size = 829, normalized size = 2.77

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(-4Bb^2+4aAb+4aCb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(4*a*A
*b - 4*b^2*B + 4*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[
c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-(A*b^2) + a*b*B + 3*a
^2*C - 4*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] +
 EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*
Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c +
 d*x]^2)) - (2*(A*b^2 - a*b*B + a^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x]
)*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^
2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c
 + d*x]^2))))/(2*(a - b)*b*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sqrt[Sec[c +
d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(A*b^2 - a*b*B + a^2*C)*S
in[c + d*x])/(a*b*(-a^2 + b^2)) + (2*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*
x] + a^2*C*Sin[c + d*x]))/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 5.817, size = 809, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a
^2*(A*b^2-B*a*b+C*a^2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+
b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a
*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-2*A*
b+B*a)/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec
(d*x + c) + a)^2, x)
```

3.1022 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=317

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b(4A+C)+2a^3B-ab^2B+3Ab^3\right)}{a^3d\left(a^2-b^2\right)}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(A\right.}{ad\left(a^2-b^2\right)(a+b\sec\right.}$$

```
[Out] -(((3*A*b^2 - a*b*B - a^2*(2*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d)) + ((3*A*b^3 + 2*a^3*B - a*b
^2*B - a^2*b*(4*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(a^3*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C
 - a^2*b^2*(5*A + C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + ((A*b^2 - a*(b*B -
a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
))
```

Rubi [A] time = 0.678562, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b(4A+C)+2a^3B-ab^2B\right.}{a^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[
c + d*x])^2),x]
```


```
[Out] -(((3*A*b^2 - a*b*B - a^2*(2*A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d)) + ((3*A*b^3 + 2*a^3*B - a*b
^2*B - a^2*b*(4*A + C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(a^3*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C
 - a^2*b^2*(5*A + C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + ((A*b^2 - a*(b*B -
a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
```

```
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx = \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\frac{1}{2}\left(3Ab^2-abB-a^2(2A-C)\right)+a(Ab-\cdots}{\sqrt{\sec(\cdots}}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\frac{1}{2}a\left(3Ab^2-abB-a^2(2A-C)\right)-\left(\frac{1}{2}b(3\cdots\right.}{\cdots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(3Ab^2-abB-a^2(2A-C)\right)\cdots}{2a^2\left(a^2-b^2\right)\cdots}$$

$$= -\frac{\left(3Ab^4+3a^3bB-ab^3B-a^4C-a^2b^2(5A+C)\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\cdots\right.}{a^3(a-b)(a+b)^2d}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} + \cdots$$

Mathematica [B] time = 7.13084, size = 835, normalized size = 2.63

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(-4Ba^2+4Aba+4bCa\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)\cos^2(c+\cdots}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
b*Sec[c + d*x])^2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(4*a*A
*b - 4*a^2*B + 4*a*b*C)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[
c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-2*a^2*A + A*b^2 + a*b
*B - a^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ell
ipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt
[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)) - (2*(-2*a^2*A + 3*A*b^2 - a*b*B + a^2*C)*Cos[2*(c + d*x)]*(a + b*Sec
[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*E
llipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(
2 - Sec[c + d*x]^2))))/(2*a*(-a + b)*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sqr
t[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(A*b^2 - a*b*B
+ a^2*C)*Sin[c + d*x])/(a^2*(a^2 - b^2)) - (2*(A*b^3*Sin[c + d*x] - a*b^2*B
*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]
))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*
x])^2)
```

Maple [B] time = 7.458, size = 856, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2*b*(A*b^2-B*a*b+C*a^2)/a^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b
/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2)))-2/a^2*(3*A*b^2-2*B*a*b+C*a^2)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*s
qrt(sec(d*x + c))), x)
```

3.1023 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=406

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^2(16A-3C)-2a^4(A+3C)+12a^3bB-9ab^3B+15Ab^4\right)}{3a^4d\left(a^2-b^2\right)}-$$

```
[Out] ((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) - ((15*A*b^4 +
 12*a^3*b*B - 9*a*b^3*B - a^2*b^2*(16*A - 3*C) - 2*a^4*(A + 3*C))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*
d) + (b*(5*A*b^4 + 5*a^3*b*B - 3*a*b^3*B - a^2*b^2*(7*A - C) - 3*a^4*C)*Sqr
t[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^4*(a - b)*(a + b)^2*d) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + ((A*b^2 - a*(b*B - a*C)
)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.03009, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\left(a^2(-(2A-3C))-3abB+5Ab^2\right)}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^2),x]
```


```
[Out] ((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) - ((15*A*b^4 +
 12*a^3*b*B - 9*a*b^3*B - a^2*b^2*(16*A - 3*C) - 2*a^4*(A + 3*C))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*
d) + (b*(5*A*b^4 + 5*a^3*b*B - 3*a*b^3*B - a^2*b^2*(7*A - C) - 3*a^4*C)*Sqr
t[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^4*(a - b)*(a + b)^2*d) - ((5*A*b^2 - 3*a*b*B - a^2*(2*A - 3*C))*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + ((A*b^2 - a*(b*B - a*C)
)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(5Ab^2-3abB-a^2(2A-3C)\right)+a}{\sec}}{}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{\left(Ab^2-a(bB-aC)\right)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{\left(Ab^2-a(bB-aC)\right)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{\left(Ab^2-a(bB-aC)\right)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(}$$

$$= \frac{b\left(5Ab^4+5a^3bB-3ab^3B-a^2b^2(7A-C)-3a^4C\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\right.}{a^4(a-b)(a+b)^2d}$$

$$= \frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{a^3\left(a^2-b^2\right)d}$$

Mathematica [B] time = 7.273, size = 887, normalized size = 2.18

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(4Aa^3+12Ca^3-12bBa^2+8Ab^2a\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])^2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(4*a^3
*A + 8*a*A*b^2 - 12*a^2*b*B + 12*a^3*C)*Cos[c + d*x]^2*EllipticPi[-(b/a), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^
2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-8*a^2
*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B - 3*a^2*b*C)*Cos[c + d*x]^2*(EllipticF
[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c +
d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b
*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(-12*a^2*A*b + 15*A*b^3 +
6*a^3*B - 9*a*b^2*B + 3*a^2*b*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a
*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2
*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b
 + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x
]^2))))/(6*a^2*(a - b)*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]
]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*b*(A*b^2 - a*b*B + a^2*C)*Sin
[c + d*x])/(a^3*(-a^2 + b^2)) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*
x] + a^2*b^2*C*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + (2*A
*Sin[2*(c + d*x)])/(3*a^2)))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 9.103, size = 1123, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*
A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b^2-B*a*b+C*a^2)/a^4*(a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b
/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^3*b*(4*A*b^2-3*B*a*b+2*C*a^2)/(a
^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*s
ec(d*x + c)^(3/2)), x)
```

3.1024 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=507

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^3(20A-9C)-4a^4b(A+3C)+16a^3b^2B+2a^5B-15ab^4B+21A\right.}{3a^5d\left(a^2-b^2\right)}$$

```
[Out] -((35*A*b^4 + 20*a^3*b*B - 25*a*b^3*B - 3*a^2*b^2*(8*A - 5*C) - 2*a^4*(3*A
+ 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*a^4*(a^2 - b^2)*d) + ((21*A*b^5 + 2*a^5*B + 16*a^3*b^2*B - 15*a*b^4*B - a^
2*b^3*(20*A - 9*C) - 4*a^4*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^5*(a^2 - b^2)*d) - (b^2*(7*A*b^4 + 7*a^3
*b*B - 5*a*b^3*B - 3*a^2*b^2*(3*A - C) - 5*a^4*C)*Sqrt[Cos[c + d*x]]*Ellipt
icPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^5*(a - b)*(a + b
)^2*d) - ((7*A*b^2 - 5*a*b*B - a^2*(2*A - 5*C))*Sin[c + d*x])/(5*a^2*(a^2 -
 b^2)*d*Sec[c + d*x]^(3/2)) + ((7*A*b^3 + 2*a^3*B - 5*a*b^2*B - a^2*(4*A*b
- 3*b*C))*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + ((A*b^2
- a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*S
ec[c + d*x]))
```

Rubi [A] time = 1.51598, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}-\frac{\sin(c+dx)\left(a^2(-(2A-5C))-5abB+7Ab^2\right)}{5a^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(-a^2(4Ab-3b\right.}{3a^3d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])^2),x]
```


```
[Out] -((35*A*b^4 + 20*a^3*b*B - 25*a*b^3*B - 3*a^2*b^2*(8*A - 5*C) - 2*a^4*(3*A
+ 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*a^4*(a^2 - b^2)*d) + ((21*A*b^5 + 2*a^5*B + 16*a^3*b^2*B - 15*a*b^4*B - a^
2*b^3*(20*A - 9*C) - 4*a^4*b*(A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^5*(a^2 - b^2)*d) - (b^2*(7*A*b^4 + 7*a^3
*b*B - 5*a*b^3*B - 3*a^2*b^2*(3*A - C) - 5*a^4*C)*Sqrt[Cos[c + d*x]]*Ellipt
icPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^5*(a - b)*(a + b
)^2*d) - ((7*A*b^2 - 5*a*b*B - a^2*(2*A - 5*C))*Sin[c + d*x])/(5*a^2*(a^2 -
 b^2)*d*Sec[c + d*x]^(3/2)) + ((7*A*b^3 + 2*a^3*B - 5*a*b^2*B - a^2*(4*A*b
- 3*b*C))*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + ((A*b^2
- a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*S
ec[c + d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
```

```
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(7Ab^2-5abB-a^2(2A-5C)\right)+a(Ab-}{\sec^{\frac{5}{2}}(}}{}$$

$$= -\frac{\left(7Ab^2-5abB-a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(Ab^2-a(bB-aC)\right)\sin}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{\left(7Ab^2-5abB-a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(7Ab^3+2a^3B-5ab^2B-a^2(4\right.}{3a^3\left(a^2-b^2\right)d\sqrt{s}}$$

$$= -\frac{\left(7Ab^2-5abB-a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(7Ab^3+2a^3B-5ab^2B-a^2(4\right.}{3a^3\left(a^2-b^2\right)d\sqrt{s}}$$

$$= -\frac{\left(7Ab^2-5abB-a^2(2A-5C)\right)\sin(c+dx)}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(7Ab^3+2a^3B-5ab^2B-a^2(4\right.}{3a^3\left(a^2-b^2\right)d\sqrt{s}}$$

$$= -\frac{b^2\left(7Ab^4+7a^3bB-5ab^3B-3a^2b^2(3A-C)-5a^4C\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\right.}{a^5(a-b)(a+b)^2d}$$

$$= -\frac{\left(35Ab^4+20a^3bB-25ab^3B-3a^2b^2(8A-5C)-2a^4(3A+5C)\right)\sqrt{\cos(c+}}{5a^4\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.59615, size = 976, normalized size = 1.93

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(-20Ba^4+4Aba^3+60bCa^3-40b^2Ba^2+56Ab^3a\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c}}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])^2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(4*a^3
*A*b + 56*a*A*b^3 - 20*a^4*B - 40*a^2*b^2*B + 60*a^3*b*C)*Cos[c + d*x]^2*El
lipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt
[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)) + (2*(-18*a^4*A - 32*a^2*A*b^2 + 35*A*b^4 + 40*a^3*b*B - 25*a*b^3*B -
 30*a^4*C + 15*a^2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x
]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[
c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) - (2*(-18*a^4*A - 72*a^2*A*b^2 + 105*A*b^4 + 60*a^3*b*B
 - 75*a*b^3*B - 30*a^4*C + 45*a^2*b^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*
x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c
```

```
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(
a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec
[c + d*x]^2))))/(30*a^3*(-a + b)*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sqrt[Se
c[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((a^4*A - a^2*A*b^2 +
10*A*b^4 - 10*a*b^3*B + 10*a^2*b^2*C)*Sin[c + d*x])/(5*a^4*(a^2 - b^2)) - (
2*(A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x]))/(a^
4*(a^2 - b^2)*(b + a*Cos[c + d*x])) + (2*(-2*A*b + a*B)*Sin[2*(c + d*x)])/(
3*a^3) + (A*Sin[3*(c + d*x)])/(5*a^2)))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 8.982, size = 1377, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*A/a^2*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)-4/3/a^3*(3*A*a+2*A*b-B*a)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
/a^4*(3*A*a^2+4*A*a*b+3*A*b^2-2*B*a^2-2*B*a*b+C*a^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-2*(A*a^3+2*A*a^2*b+3*A*a*b^2+4*A*b^3-B*a^3-2*B*a^2*b-
3*B*a*b^2+C*a^3+2*C*a^2*b)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*(A*b^2-B*a*b+C*a^2)/a^5*(a^2/b/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2
-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2/a^4*b^2*(5*A*b^2-4*B*a*b+3*C*a^2)/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*s
ec(d*x + c)^(5/2)), x)
```

3.1025 $\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=667

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^2(3A-61C)+15a^3bB-35a^4C-33ab^3B+b^4(21A-8C)\right)}{12b^3d\left(a^2-b^2\right)^2}$$

```
[Out] -((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*
A - 8*C) - 35*a^5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*b*B - 33*a*b^3*B - a^2*b^2*(3
*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^3*(a^2 - b^2)^2*d) + ((15*A*b^6 - 15*
a^5*b*B + 38*a^3*b^3*B - 35*a*b^5*B + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A
 - 21*C) + 35*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + ((15*a^4*b*B - 2
9*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*A - 8*C) - 35*a^5
*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*b*B
 - 33*a*b^3*B - a^2*b^2*(3*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C
))*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) + ((5*A*b^4 + 3*a^3*b*B - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A + 13*C))*Sec
[c + d*x]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 2.24793, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^2b^2(A+13C)+3a^3bB-7a^4C-9ab^3B+\right.}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```


```
[Out] -((15*a^4*b*B - 29*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*
A - 8*C) - 35*a^5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*b*B - 33*a*b^3*B - a^2*b^2*(3
*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^3*(a^2 - b^2)^2*d) + ((15*A*b^6 - 15*
a^5*b*B + 38*a^3*b^3*B - 35*a*b^5*B + a^4*b^2*(3*A - 86*C) - 3*a^2*b^4*(2*A
 - 21*C) + 35*a^6*C)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + ((15*a^4*b*B - 2
9*a^2*b^3*B + 8*b^5*B - a^3*b^2*(3*A - 65*C) + 3*a*b^4*(3*A - 8*C) - 35*a^5
*C)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*b*B
 - 33*a*b^3*B - a^2*b^2*(3*A - 61*C) + b^4*(21*A - 8*C) - 35*a^4*C)*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - ((A*b^2 - a*(b*B - a*C
))*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) + ((5*A*b^4 + 3*a^3*b*B - 9*a*b^3*B - 7*a^4*C + a^2*b^2*(A + 13*C))*Sec
[c + d*x]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\sec^{\frac{5}{2}}(c+ \ldots}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(5Ab^4+\ldots\right.}{\ldots}$$

$$= -\frac{\left(15a^3bB-33ab^3B-a^2b^2(3A-61C)+b^4(21A-8C)-\ldots\right.}{12b^3\left(a^2-b^2\right)^2 d \ldots}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A\ldots\right.}{4b^4\left(a^2-b^2\right)^2 d \ldots}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A\ldots\right.}{4b^4\left(a^2-b^2\right)^2 d \ldots}$$

$$= \frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A\ldots\right.}{4b^4\left(a^2-b^2\right)^2 d \ldots}$$

$$= \frac{\left(15Ab^6-15a^5bB+38a^3b^3B-35ab^5B+a^4b^2(3A-86C)\ldots\right.}{\ldots}$$

$$= -\frac{\left(15a^4bB-29a^2b^3B+8b^5B-a^3b^2(3A-65C)+3ab^4(3A\ldots\right.}{4b^4\left(a\ldots\right.}$$

Mathematica [A] time = 7.78109, size = 1161, normalized size = 1.74

$$\frac{\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(-48Bb^6-96aAb^5+160aCb^5+240a^2Bb^4+24a^3Ab^3-512a^3Cb^3-120a^4Bb^2+280a^5Cb\right)\Pi\left(\ldots\right.}{a(b+a\cos(c+dx))\left(\ldots\right.}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-2*(24*a^3*A*b^3 - 96*a*A*b^5 - 120*a^4*b^2*B + 240*a^2*b^4*B - 48*b^6*
B + 280*a^5*b*C - 512*a^3*b^3*C + 160*a*b^5*C)*Cos[c + d*x]^2*EllipticPi[-(
b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c
```

```
+ d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*
(27*a^4*A*b^2 - 57*a^2*A*b^4 + 48*A*b^6 - 135*a^5*b*B + 285*a^3*b^3*B - 168
*a*b^5*B + 315*a^6*C - 641*a^4*b^2*C + 328*a^2*b^4*C + 16*b^6*C)*Cos[c + d*
x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*S
in[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*(9*a^4*A*b^
2 - 27*a^2*A*b^4 - 45*a^5*b*B + 87*a^3*b^3*B - 24*a*b^5*B + 105*a^6*C - 195
*a^4*b^2*C + 72*a^2*b^4*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2
*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*C
os[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))
)/(24*(a - b)^2*b^4*(a + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-((3*a^3*A*b^2 - 9*a*A*b^4 - 15*a
^4*b*B + 29*a^2*b^3*B - 8*b^5*B + 35*a^5*C - 65*a^3*b^2*C + 24*a*b^4*C)*Sin
[c + d*x])/(2*b^4*(a^2 - b^2)^2) + (-(a*A*b^2*Sin[c + d*x]) + a^2*b*B*Sin[c
 + d*x] - a^3*C*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (
a^3*A*b^2*Sin[c + d*x] - 7*a*A*b^4*Sin[c + d*x] - 5*a^4*b*B*Sin[c + d*x] +
11*a^2*b^3*B*Sin[c + d*x] + 9*a^5*C*Sin[c + d*x] - 15*a^3*b^2*C*Sin[c + d*x
])/(2*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (4*C*Tan[c + d*x])/(3*b^3)
))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x]
)^3)
```

Maple [B] time = 20.032, size = 2185, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(B*b-2*C*a
)/b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*(B*b-3*C*a)/b^4/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+2*(A*b^2-B*a*b+C*a^2)/b^2*(1/2*a^2/b/(a^2-b^2)*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
```

```
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/
8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8
*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^
2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*C/b
^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b-3*C*a)/b^4*(-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec
(d*x + c) + a)^3, x)
```

3.1026 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=556

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+3Ab^4\right)}{4ab^2d\left(a^2-b^2\right)^2}-\frac{\sin(c+dx)\,\mathrm{se}}{2bd\left(a^2\right.}$$

```
[Out] ((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^
2 - b^2)^2*d) + ((3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A +
11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*
a*b^2*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B + 6*a^3*b^3*B - 15*a*b^5*B +
 15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*Sqrt[Cos[c + d*x]]*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^
2*b^3*(a + b)^3*d) - ((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C +
 a^2*b^2*(A + 29*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*
d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^2) + ((3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C
+ a^2*b^2*(3*A + 11*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)
^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.69225, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+\right.}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```


```
[Out] ((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^
2 - b^2)^2*d) + ((3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A +
11*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*
a*b^2*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B + 6*a^3*b^3*B - 15*a*b^5*B +
 15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*Sqrt[Cos[c + d*x]]*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^
2*b^3*(a + b)^3*d) - ((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C +
 a^2*b^2*(A + 29*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*
d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^2) + ((3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C
+ a^2*b^2*(3*A + 11*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)
^2*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
```

```
x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\sec^{\frac{3}{2}}(c+}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(3Ab^4+\right.}{}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C\right.}{4b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C\right.}{4b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C\right.}{4b^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(3Ab^6-3a^5bB+6a^3b^3B-15ab^5B+15a^6C+5a^2b^4(2A\right.}{4a}$$

$$= \frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right.}{4b^3\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 7.48164, size = 1092, normalized size = 1.96

$$\frac{(b+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{\left(15Ca^4-3bBa^3-Ab^2a^2-29b^2Ca^2+9b^3Ba-5Ab^4+8b^4C\right)\sin(c}{2b^3\left(b^2-a^2\right)^2}\right.}{d(\cos(2c+2dx)A+A+2C}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^3,x]
```

```
[Out] -((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*((-2*(-8*a^2*A*b^3 - 16*A*b^5 - 8*a^3*b^2*B + 32*a*b^4*B + 40*a^4*b*C -
80*a^2*b^3*C + 16*b^5*C)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-3*a^3*A*b^2 + 9*a*A*
b^4 - 9*a^4*b*B + 19*a^2*b^3*B - 16*b^5*B + 45*a^5*C - 95*a^3*b^2*C + 56*a*
b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 - (2*(-(a^3*A*b^2) - 5*a*A*b^4 - 3*a^4*b*B + 9*a^2*b^3*B + 15*a^5*C - 29*a
^3*b^2*C + 8*a*b^4*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*
Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
 + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi
```

```
[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c
+ d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(8*
(a - b)^2*b^3*(a + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((-(a^2*A*b^2) - 5*A*b^4 - 3*a^3*b*B +
9*a*b^3*B + 15*a^4*C - 29*a^2*b^2*C + 8*b^4*C)*Sin[c + d*x])/(2*b^3*(-a^2 +
 b^2)^2) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x])/(
b*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (3*a^2*A*b^2*Sin[c + d*x] + 3*A*b^
4*Sin[c + d*x] + a^3*b*B*Sin[c + d*x] - 7*a*b^3*B*Sin[c + d*x] - 5*a^4*C*Si
n[c + d*x] + 11*a^2*b^2*C*Sin[c + d*x])/(2*b^2*(-a^2 + b^2)^2*(b + a*Cos[c
+ d*x]))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[
c + d*x])^3)
```

Maple [B] time = 12.217, size = 2049, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b^2-C*a^2)
/b^2/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*C/b^3/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+2*(-A*b^2+B*a*b-C*a^2)/a/b*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-
a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
```

```
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a
*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*C/b^3*(-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+
2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec
(d*x + c) + a)^3, x)
```

3.1027 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=469

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{\sin(c+dx)\sec}{2bd\left(a^2-\right.}$$

```
[Out] -((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^2)^
2*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B + a^4*C - 7*a^2*b^2*(A + C))*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b
^2)^2*d) - ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^2*(A - 2*
C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b^2*(a + b)^3*
d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^2) + ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C +
a^2*b^2*(5*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*
d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.17975, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2b^2(5A+9C)-a^3bB-3a^4C-5ab^3B+\right.}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```


```
[Out] -((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b^2*(a^2 - b^2)^
2*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B + a^4*C - 7*a^2*b^2*(A + C))*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b
^2)^2*d) - ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^2*(A - 2*
C) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b^2*(a + b)^3*
d) - ((A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 -
b^2)*d*(a + b*Sec[c + d*x])^2) + ((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C +
a^2*b^2*(5*A + 9*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*
d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
```

```
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \sqrt{\sec(c+}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(Ab^4-a\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(Ab^4-a\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(Ab^4-a\right.}{}$$

$$= -\frac{\left(Ab^6-a^5bB+10a^3b^3B+3ab^5B-3a^4b^2(A-2C)-3a^6\right.}{4a^2(}$$

$$= -\frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)\sqrt{\cos(c}}{4ab^2\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 7.23851, size = 1051, normalized size = 2.24

$$\frac{\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(16Bb^4-24aAb^3-32aCb^3+8a^2Bb^2+8a^3Cb\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-2*(-24*a*A*b^3 + 8*a^2*b^2*B + 16*b^4*B + 8*a^3*b*C - 32*a*b^3*C)*Cos[
c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + (2*(a^2*A*b^2 + 5*A*b^4 + 3*a^3*b*B - 9*a*b^3*B + 9*a^
4*C - 19*a^2*b^2*C + 16*b^4*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) - (2*(-5*a^2*A*b^2 - A*b^4 + a^3*b*B + 5*a*b^3*B +
 3*a^4*C - 9*a^2*b^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*
b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*Elliptic
Pi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[
c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(
8*(a - b)^2*b^2*(a + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(((5*a^2*A*b^2 + A*b^4 - a^3*b*B - 5*a
*b^3*B - 3*a^4*C + 9*a^2*b^2*C)*Sin[c + d*x])/(2*a*b^2*(-a^2 + b^2)^2) + (A
*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x])/(a*(a^2 - b^2)
*(b + a*Cos[c + d*x])^2) + (-7*a^2*A*b^2*Sin[c + d*x] + A*b^4*Sin[c + d*x]
```

```
+ 3*a^3*b*B*Sin[c + d*x] + 3*a*b^3*B*Sin[c + d*x] + a^4*C*Sin[c + d*x] - 7*
a^2*b^2*C*Sin[c + d*x])/(2*a*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 10.419, size = 1879, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-2*A*b+B*a)/
a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*A/a/(a^2-a*b)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+2*(A*b^2-B*a*b+C*a^2)/a^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a
+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(
a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a
-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^3, x)
```

3.1028 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=478

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^2(5A-3C)+a^4(8A+3C)-7a^3bB+ab^3B+3Ab^4\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{\sin(c+\ldots}{\ldots}$$

```
[Out] -((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^
2*d) + ((3*A*b^4 - 7*a^3*b*B + a*b^3*B - a^2*b^2*(5*A - 3*C) + a^4*(8*A + 3
*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^
3*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2
*b^4*(2*A - C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*b*(a
 + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B +
a^4*C - 7*a^2*b^2*(A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*b*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.20769, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4098, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)}{4abd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```


```
[Out] -((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*b*(a^2 - b^2)^
2*d) + ((3*A*b^4 - 7*a^3*b*B + a*b^3*B - a^2*b^2*(5*A - 3*C) + a^4*(8*A + 3
*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^
3*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2
*b^4*(2*A - C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*b*(a
 + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B +
a^4*C - 7*a^2*b^2*(A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*b*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
```

```
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\frac{1}{2}\left(-Ab^2+a\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(Ab^4+3a^3\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(Ab^4+3a^3\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(Ab^4+3a^3\right.}{}$$

$$= -\frac{\left(3Ab^6-3a^5bB-10a^3b^3B+ab^5B-3a^2b^4(2A-C)-a^6C+\right.}{4a^3(a-}$$

$$= -\frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)\sqrt{\cos(c+}}{4a^2b\left(a^2-b^2\right)^2d}$$

Mathematica [B] time = 7.19892, size = 1051, normalized size = 2.2

$$\frac{\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(16Aba^3+8bCa^3-24b^2Ba^2+8Ab^3a+16b^3Ca\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+d}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-2*(16*a^3*A*b + 8*a*A*b^3 - 24*a^2*b^2*B + 8*a^3*b*C + 16*a*b^3*C)*Cos
[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[
c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) + (2*(-5*a^2*A*b^2 - A*b^4 + a^3*b*B + 5*a*b^3*B + 3*a^
4*C - 9*a^2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x
])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)) - (2*(9*a^2*A*b^2 - 3*A*b^4 - 5*a^3*b*B - a*b^3*B + a^4*C + 5*
a^2*b^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x
]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
- 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1
- Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(8*a*(a - b)^2
*b*(a + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec
[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*(((-9*a^2*A*b^2 + 3*A*b^4 + 5*a^3*b*B + a*b^3*B - a
```

```
^4*C - 5*a^2*b^2*C)*Sin[c + d*x])/(2*a^2*b*(-a^2 + b^2)^2) - (A*b^3*Sin[c +
 d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x])/(a^2*(a^2 - b^2)*(b +
a*Cos[c + d*x])^2) + (11*a^2*A*b^2*Sin[c + d*x] - 5*A*b^4*Sin[c + d*x] - 7*
a^3*b*B*Sin[c + d*x] + a*b^3*B*Sin[c + d*x] + 3*a^4*C*Sin[c + d*x] + 3*a^2*
b^2*C*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])))))/(d*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 10.687, size = 1972, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a
^3*(3*A*b^2-2*B*a*b+C*a^2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2
/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a
^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-
3*A*b+B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b*(A*b^2-B*a*b+C*a^2)/a^3*(1/2*a^
2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/
(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b
)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec
(d*x + c) + a)^3, x)
```

3.1029 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=486

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^3(33A+C)+a^4b(24A+7C)+5a^3b^2B-8a^5B-3ab^4B+15A\right.}{4a^4d\left(a^2-b^2\right)^2}$$

```
[Out] ((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^
2 - b^2)^2*d) - ((15*A*b^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*B - a^2*b^3*(3
3*A + C) + a^4*b*(24*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((15*A*b^6 - 15*a^5*b*B + 6*
a^3*b^3*B - 3*a*b^5*B + 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C
))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((5*A*
b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.22286, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-a^2b^2(11A+3C)+7a^3bB-3a^4C-ab^3B+5Ab^4\right)}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-\right.}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[
c + d*x])^3),x]
```


```
[Out] ((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^
2 - b^2)^2*d) - ((15*A*b^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*B - a^2*b^3*(3
3*A + C) + a^4*b*(24*A + 7*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((15*A*b^6 - 15*a^5*b*B + 6*
a^3*b^3*B - 3*a*b^5*B + 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C
))*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((5*A*
b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a^2*b^2*(11*A + 3*C))*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
```

```
&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \int \frac{\frac{1}{2}\left(5Ab^2-abB-a^2(4A-C)\right)+2a(Ab-}{\sqrt{\sec(}}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(5Ab^4+7a^3bB-ab^3B-3a\right.}{4a^2\left(a^2\right.}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(5Ab^4+7a^3bB-ab^3B-3a\right.}{4a^2\left(a^2\right.}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(5Ab^4+7a^3bB-ab^3B-3a\right.}{4a^2\left(a^2\right.}$$

$$= \frac{\left(15Ab^6-15a^5bB+6a^3b^3B-3ab^5B+3a^6C-a^2b^4(38A+C)+5a^4b^2(7A+\right.}{4a^4(a-b)^2(a+b)^3d}$$

$$= \frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)\sqrt{\cos(c+dx)}E\left(\right.}{4a^3\left(a^2-b^2\right)^2d}$$

Mathematica [B] time = 7.37044, size = 1064, normalized size = 2.19

$$\frac{\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(16Ba^4-32Aba^3-24bCa^3+8b^2Ba^2+8Ab^3a\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
b*Sec[c + d*x])^3),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-2*(-32*a^3*A*b + 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B - 24*a^3*b*C)*Cos[
c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + (2*(8*a^4*A - 7*a^2*A*b^2 + 5*A*b^4 - 5*a^3*b*B - a*b^
3*B + a^4*C + 5*a^2*b^2*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*
x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) - (2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B -
3*a*b^3*B - 5*a^4*C - a^2*b^2*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a
*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2
*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b
 + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x
]^2))))/(8*a^2*(a - b)^2*(a + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^
(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-((-13*a^2*A*b^2 + 7*A*b^4 +
 9*a^3*b*B - 3*a*b^3*B - 5*a^4*C - a^2*b^2*C)*Sin[c + d*x])/(2*a^3*(-a^2 +
b^2)^2) - (-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x] - a^2*b^2*C*Sin[c +
 d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (-15*a^2*A*b^3*Sin[c + d*
```

```
x] + 9*A*b^5*Sin[c + d*x] + 11*a^3*b^2*B*Sin[c + d*x] - 5*a*b^4*B*Sin[c + d
*x] - 7*a^4*b*C*Sin[c + d*x] + a^2*b^3*C*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2
*(b + a*Cos[c + d*x]))))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 12.197, size = 2022, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2/a^4*b*(4*A*b^2-3*B*a*b+2*C*a^2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2)))-2/a^3*(6*A*b^2-3*B*a*b+C*a^2)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^2
*(A*b^2-B*a*b+C*a^2)/a^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2
+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
```

```
^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(
a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(b \sec(dx+c) + a)^3 \sqrt{\sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*s
qrt(sec(d*x + c))), x)
```

3.1030 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=598

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^4b^2(128A-15C)-a^2b^4(223A-9C)+8a^6(A+3C)+99a^3b^3B-7\right.}{12a^5d\left(a^2-b^2\right)^2}$$

```
[Out] -((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a
^2*b^3*(65*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*
B - 45*a*b^5*B + a^4*b^2*(128*A - 15*C) - a^2*b^4*(223*A - 9*C) + 8*a^6*(A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
2*a^5*(a^2 - b^2)^2*d) - (b*(35*A*b^6 - 35*a^5*b*B + 38*a^3*b^3*B - 15*a*b^
5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^
5*(a - b)^2*(a + b)^3*d) + ((35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A
- 21*C) - a^2*b^2*(61*A - 3*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[
Sec[c + d*x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*
Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^
3*B - 5*a^4*C - a^2*b^2*(13*A + C))*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sq
rt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.8052, antiderivative size = 598, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\left(-a^2b^2(61A-3C)+a^4(8A-21C)+33a^3bB-15ab^3B+35Ab^4\right)}{12a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)\left(-a^2b^2(13A+C)+9a^3bB-5\right.}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^3),x]
```

```
[Out] -((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B + 3*a^4*b*(8*A - 3*C) - a
^2*b^3*(65*A - 3*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*
B - 45*a*b^5*B + a^4*b^2*(128*A - 15*C) - a^2*b^4*(223*A - 9*C) + 8*a^6*(A
+ 3*C))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
2*a^5*(a^2 - b^2)^2*d) - (b*(35*A*b^6 - 35*a^5*b*B + 38*a^3*b^3*B - 15*a*b^
5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^
5*(a - b)^2*(a + b)^3*d) + ((35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A
- 21*C) - a^2*b^2*(61*A - 3*C))*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[
Sec[c + d*x]]) + ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*
Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^
3*B - 5*a^4*C - a^2*b^2*(13*A + C))*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sq
rt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx = \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} - \int \frac{\frac{1}{2}\left(7Ab^2-3abB-a^2(4A-3C)\right)+2a}{\sec}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} - \frac{\left(7Ab^4+9a^3bB-3ab^3B-\right.}{4a^2\left(a^2-b^2\right)^2d\sqrt{\text{se}}}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C)\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{b\left(35Ab^6-35a^5bB+38a^3b^3B-15ab^5B-a^2b^4(86A-3C)+3a^4b^2(21A-\right.}{4a^5(a-b)^2(a+}$$

$$= -\frac{\left(35Ab^5-8a^5B+29a^3b^2B-15ab^4B+3a^4b(8A-3C)-a^2b^3(65A-3C)\right)}{4a^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.59956, size = 1121, normalized size = 1.87

$$\frac{\sec(c+dx)\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{2\left(16Aa^5+48Ca^5-96bBa^4+112Ab^2a^3+24b^2Ca^3+24b^3Ba^2-56Ab^4a\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+}\right.\right.}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])^3),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((-2*(16*a^5*A + 112*a^3*A*b^2 - 56*a*A*b^4 - 96*a^4*b*B + 24*a^2*b^3*B +
 48*a^5*C + 24*a^3*b^2*C)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x
])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-56*a^4*A*b + 73*a^2
*A*b^3 - 35*A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a*b^4*B - 15*a^4*b*C - 3*a
^2*b^3*C)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)) - (2*(-72*a^4*A*b + 195*a^2*A*b^3 - 105*A*b^5 + 24*a^5*B - 87*a^3*b^2*B
 + 45*a*b^4*B + 27*a^4*b*C - 9*a^2*b^3*C)*Cos[2*(c + d*x)]*(a + b*Sec[c + d
*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*
```

```
x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/
(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Se
c[c + d*x]^2))))/(24*a^3*(a - b)^2*(a + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec
[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((b*(-17*a^2*A*b^2
+ 11*A*b^4 + 13*a^3*b*B - 7*a*b^3*B - 9*a^4*C + 3*a^2*b^2*C)*Sin[c + d*x])/
(2*a^4*(-a^2 + b^2)^2) - (A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x] + a^2*b
^3*C*Sin[c + d*x])/(a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (19*a^2*A*b^4
*Sin[c + d*x] - 13*A*b^6*Sin[c + d*x] - 15*a^3*b^3*B*Sin[c + d*x] + 9*a*b^5
*B*Sin[c + d*x] + 11*a^4*b^2*C*Sin[c + d*x] - 5*a^2*b^4*C*Sin[c + d*x])/(2*
a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + (2*A*Sin[2*(c + d*x)])/(3*a^3)))/
(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^3
)
```

Maple [B] time = 13.84, size = 2289, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^5*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18
*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/a^5*b^2*(5*A*b^2-4*B*a*b+3*C*a^2
)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^4*b*(10*A*b^2-6*B*a*b+3*C
*a^2)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^3*(A*b^2-B*a*b+C*a^2)/a^5*(1/2*a^2/b/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*co
```

```
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)
/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-
a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(b \sec(dx+c) + a)^3 \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*s
ec(d*x + c)^(3/2)), x)
```

3.1031 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=447

$$\frac{\sqrt{\sec(c+dx)}\left(a^2(-C)+18abB+24Ab^2+16b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24bd\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-\right.}{}$$

```
[Out] ((24*A*b^2 + 18*a*b*B - a^2*C + 16*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*b*d*Sqrt[a
+ b*Sec[c + d*x]]) - ((2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(8*b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 + 6*a*b*
B - 3*a^2*C + 16*b^2*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(24*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + ((24*A*b^2 + 6*a*b*B - 3*a^2*C + 16*b^2*C)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(24*b^2*d) + ((6*b*B + a*C)*Sec[c + d*x]^(3/
2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*b*d) + (C*Sec[c + d*x]^(5/2)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.62475, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(-3a^2C+6abB+24Ab^2+16b^2C\right)\sqrt{a+b\sec(c+dx)}}{24b^2d}+\frac{\sqrt{\sec(c+dx)}\left(a^2(-C)+18abB+24Ab^2\right.}{24bd\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] ((24*A*b^2 + 18*a*b*B - a^2*C + 16*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*b*d*Sqrt[a
+ b*Sec[c + d*x]]) - ((2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(8*b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 + 6*a*b*
B - 3*a^2*C + 16*b^2*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(24*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + ((24*A*b^2 + 6*a*b*B - 3*a^2*C + 16*b^2*C)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(24*b^2*d) + ((6*b*B + a*C)*Sec[c + d*x]^(3/
2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*b*d) + (C*Sec[c + d*x]^(5/2)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c}{3d}$$

$$= \frac{(6bB+aC)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}{12bd}$$

$$= \frac{\left(24Ab^2+6abB-3a^2C+16b^2C\right)\sqrt{\sec(}{24b}$$

$$= \frac{\left(24Ab^2+6abB-3a^2C+16b^2C\right)\sqrt{\sec(}{24b}$$

$$= \frac{\left(24Ab^2+6abB-3a^2C+16b^2C\right)\sqrt{\sec(}{24b}$$

$$= \frac{\left(24Ab^2+6abB-3a^2C+16b^2C\right)\sqrt{\sec(}{24b}$$

$$= -\frac{\left(2a^2bB-8b^3B-a^3C-4ab^2(2A+C)\right.}{8b^2d\sqrt{}}$$

$$= \frac{\left(24Ab^2+18abB-a^2C+16b^2C\right)\sqrt{\frac{b+a}{}}}{24bd\sqrt{a+b}}$$

Mathematica [C] time = 6.88709, size = 782, normalized size = 1.75

$$\frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{\sec(c+dx)\left(-3a^2C\sin(c+dx)+6abB\sin(c+dx)+24Ab^2\sin(c+dx)+16b^2C\sin(c+d}{12b^2}\right.}{d\sec^{\frac{5}{2}}(c+dx)(A\cos(2c+2dx)+A+2B\cos(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(24*a
*b^2*B + 4*a^2*b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(24*a*A*b^2 - 18*a^2*b*B +
 48*b^3*B + 9*a^3*C + 8*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-24*
a*A*b^2 - 6*a^2*b*B + 3*a^3*C - 16*a*b^2*C)*Sqrt[(a - a*Cos[c + d*x])/(a +
b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*Ellip
ticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]
]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*
Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^
(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2
 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(48*b^2*
d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*Sec[c + d*x]^(5/2)) + (Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((Sec[c + d*x]^2*(6*b*B*Sin[c + d*x] + a*C*Sin[c + d*x]))/(6
```

```
*b) + (Sec[c + d*x]*(24*A*b^2*Sin[c + d*x] + 6*a*b*B*Sin[c + d*x] - 3*a^2*C
*Sin[c + d*x] + 16*b^2*C*Sin[c + d*x]))/(12*b^2) + (2*C*Sec[c + d*x]^2*Tan[
c + d*x])/3))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c +
d*x]^(5/2))
```

Maple [C] time = 0.628, size = 4821, normalized size = 10.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/24/d/((a-b)/(a+b))^(1/2)/b^2*(24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2+3*C*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3-24*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*b^
3-3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-16*C*((a-b)/(a+b))^(1/2)*cos(d*x
+c)^3*b^3+24*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^3+8*C*((a-b)/(a+b))^(1/2)
*cos(d*x+c)^2*b^3-12*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*b^3+12*B*((a-b)/(a+
b))^(1/2)*b^3*cos(d*x+c)+12*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b
+3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+6*C*((a-b)/(a+b))^(1/2)*cos(d*x
+c)^3*a*b^2+18*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2-C*((a-b)/(a+b))^(1/
2)*cos(d*x+c)^2*a^2*b+10*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+12*B*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b
+16*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-4
*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^4*a^3-24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+24*B*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b
^3-48*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3-3*C*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^3*a^3-16*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+6*C*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3
```

```
*a^3-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-24*A*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^4*b^3+24*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^3-48*B*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^4*b^3-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3-16*C*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^4*b^3+6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3-24*A*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^4*a*b^2-6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^2*b-12*B*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^4*a*b^2-2*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^2*b-1
6*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a*b^2+24*A*((a-b)/(a+b))^(1/2)*cos(d*x
+c)^3*a*b^2+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b-6*B*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^3*a*b^2+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b+16*C*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^4*a*b^2-2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b-4*C*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^4*a*b^2-24*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2+24*A*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*sin(d*x+c)*cos(d*x+c)^3*a*b^2-48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3
*a*b^2+6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-6*B*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*
b^2-12*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b-12*B*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b
^2+8*C*((a-b)/(a+b))^(1/2)*b^3-24*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3
*a*b^2-48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2+6*B*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^4*a^2*b-6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
```

```
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2-12*B*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^4*a^2*b-12*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2)*(1/cos(d*x+c))^(3/2)
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sec(d*x + c)^(3/2), x)
```

3.1032 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=346

$$\frac{\sqrt{\sec(c+dx)}(8aA+3aC+4bB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\left(a^2(-C)+4abB+8Ab^2+4b^2C\right.}{4bd\sqrt{a+b\sec(c}}$$

```
[Out] ((8*a*A + 4*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) +
 ((8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a
 + b*Sec[c + d*x]]) - ((4*b*B + a*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + ((4*b*B + a*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(4*b*d) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(2*d)
```

Rubi [A] time = 1.18589, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sqrt{\sec(c+dx)}\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}(8aA+3aC+4bB)\sqrt{\frac{a}{}}}{4d\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] ((8*a*A + 4*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) +
 ((8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a
 + b*Sec[c + d*x]]) - ((4*b*B + a*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + ((4*b*B + a*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(4*b*d) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(2*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{2d}$$

$$= \frac{(4bB+aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{4bd}$$

$$= \frac{(4bB+aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{4bd}$$

$$= \frac{(4bB+aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{4bd}$$

$$= \frac{(4bB+aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}{4bd}$$

$$= \frac{\left(8Ab^2+4abB-a^2C+4b^2C\right)\sqrt{\frac{b+a\cos(c+d}{a+b}}}{4bd\sqrt{a+b\sec}}$$

$$= \frac{(8aA+4bB+3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{4d\sqrt{a+b\sec(c+dx}}$$

Mathematica [C] time = 6.2557, size = 478, normalized size = 1.38

$$\frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{8a(4A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2i(aC+4bB)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*a*(4*
A + C)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((a + b)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*(16*A*b^2 + 4*a*b*B - 3*a^2*C + 8*b^2*C)*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/(b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
) - ((2*I)*(4*b*B + a*C)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(
1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[
Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*El
lipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a
 + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[
c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b^2*Sqrt[b + a*Cos[c
 + d*x]]) + (4*(4*b*B + a*C)*Tan[c + d*x])/b + 8*C*Sec[c + d*x]*Tan[c + d*x
]))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/
2))
```

Maple [C] time = 0.461, size = 3182, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/4/d/b/((a-b)/(a+b))^(1/2)*(8*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-4*B*cos(d*x
+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a*b+8*B*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+C*co
s(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*a*b+2*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+8*A*cos(d*x+c
)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*a*b-4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+8*B*cos(d*x+c)^2*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a*b+C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
```

```
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+2*C*cos(d*x+c)^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*a*b+C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+4*B*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^2*b^2-C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2-4*B*((a-b)/(
a+b))^(1/2)*cos(d*x+c)*b^2+2*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-8*A*cos
(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*sin(d*x+c)*b^2+16*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2
+4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+2*C*cos(
d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*a^2-4*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-2*C*cos(d*x+c)^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+8*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c
)*b^2-2*C*((a-b)/(a+b))^(1/2)*b^2-8*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2+16*A*co
s(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+4*B*cos(d*x+c)^3*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2
-C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+2*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-4*C*cos(
d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*b^2-2*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+8
*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+
b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+4*B*((a-b)/(a+b))^(1/2)*cos(
d*x+c)^3*a*b+2*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a*b-4*B*((a-b)/(a+b))^(1/
2)*cos(d*x+c)^2*a*b+C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-3*C*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)*a*b)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sqrt(sec(d*x + c)), x)
```

3.1033 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=258

$$\frac{(2aB+bC)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2A-C)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(aC$$

```
[Out] ((2*a*B + b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B + a
*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A - C)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[Sec[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.832207, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2A-C)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2aB+bC)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(aC+2bB)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[Sec[c + d*x]],x]
```


```
[Out] ((2*a*B + b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B + a
*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A - C)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[Sec[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{\frac{1}{2}a(}{}$$

$$= \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(2bB$$

$$= \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(2A$$

$$= \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{((-2aB}{}$$

$$= \frac{(2bB+aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2aB+bC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 3.72265, size = 438, normalized size = 1.7

$$\frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{8(aB+Ab)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2i(2A-C)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos}{}}}{}\right.}{2d\,\text{se}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*(A*b
+ a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((a + b)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*(4*b*B + a*(2*A + C))*EllipticPi[2, (c + d*x)/2, (2*a)
/(a + b)])/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + ((2*I)*(2*A - C)*
Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b
)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b
```

```
 + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 -
 a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)])))/(a*Sqrt[(a - b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*C*Tan[c + d*x
]))/(2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/
2))
```

Maple [C] time = 0.456, size = 2345, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(-C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+C*cos(d*x+c)^2*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*b+2*C*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-2*A*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a+2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*A*cos(d*x+c)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-2*
A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*b+2*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*B*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*b+4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-C*cos(d*x+c)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a+C*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+2*C*c
os(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*A*
cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*a-2*A*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
```

```
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-2*A*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a+2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b+C*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-C*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a+C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b+2*A*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a+2*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*B*cos(d*x+c)^2*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*b+4*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-C*((a-b)/(a+b))^(1/
2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(
b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sqrt(sec(d*x + c)), x)
```

3.1034 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=277

$$-\frac{2\left(Ab^2-a^2(A+3C)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*(A*b^2 - a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x
]]) + (2*b*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b +
 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*
a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.90365, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(Ab^2-a^2(A+3C)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(3/2),x]
```

```
[Out] (-2*(A*b^2 - a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x
]]) + (2*b*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b +
 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*
a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)}{}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)}{\sqrt{\sec}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{(Ab+3aB)\int \frac{\sqrt{a+b}}{\sqrt{s}}}{3a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\left(-\frac{Ab^2}{a}+a(A+3\right.\right.}{}$$

$$= \frac{2bC\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2\left(\frac{Ab^2}{a}-a(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{}}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [F] time = 31.6863, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(3/2), x]
```

Maple [C] time = 0.431, size = 2548, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)/a*(-A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b+A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-3*B*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*C*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b*sin(d*x+c)+A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*B*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b*sin(d*x+c)-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+6*C*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A
*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+
3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*B*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*C*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b+6*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-3*B*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+3*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+A*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-A*b^2*(
```

```
(a-b)/(a+b))^(1/2)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2-A*((a-b)/(a+b))^(
1/2)*cos(d*x+c)*a^2-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+3*B*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^2*a^2+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-A*a*b*((a-b)/
(a+b))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec\left(c+dx\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/
sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(3/2), x)
```

3.1035 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=273

$$-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{a}\cdots}{15a^2d\sqrt{\sec(c+dx)}\cdots}$$

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[
c + d*x]]) - (2*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.806691, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c\cdots}{a\cdots}}}{15a^2d\sqrt{a+b\sec(c+d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(5/2),x]
```


```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[
c + d*x]]) - (2*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
```

```
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5aB)}{}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+}}{15ad}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+}}{15ad}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+}}{15ad}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+}}{15ad}$$

$$= -\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{15a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.60833, size = 3426, normalized size = 12.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(9*a
^2*A - 2*A*b^2 + 5*a*b*B + 15*a^2*C)*Cot[c])/(15*a^2*d) + (4*(A*b + 5*a*B)*
Cos[d*x]*Sin[c])/(15*a*d) + (2*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*(A*b + 5*a
*B)*Cos[c]*Sin[d*x])/(15*a*d) + (2*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)) - (28*A*b*App
ellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c
]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*
Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x -
 ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c
]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2
] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]]])/(15*a*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(5/2)) - (4*B*AppellF1[1/2, 1/
2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc
[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1
+ Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*Sqrt[a + b*Se
c[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c
]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt
[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])
]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
```

t[1 + Cot[c]^2]*Sec[c + d*x]^(5/2)) - (4*b*C*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(a*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(5/2)) - (6*a*A*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)) + (4*A*b^2*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*a*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)) - (2*b*B*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(3*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)) - (2*a*C*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))

```
))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 +
Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*S
qrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*Sqrt[b + a*Cos[c +
 d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)
)
```

Maple [B] time = 0.505, size = 3639, normalized size = 13.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] 2/15/d/((a-b)/(a+b))^(1/2)/a^2*(-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^2-9*A*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3-9*A*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*a^3-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-5*B*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
cos(d*x+c)*a^3+15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3-15*C*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a^3-7*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*A*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3+5*B*a^3*((a-b)/(a+b))^(1/2)*cos(d*
x+c)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3-6*A*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^2*a^3-15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3+9*A*((a-b)/(a+b))^(1/
2)*cos(d*x+c)*a^3+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3+15*C*((a-b)/(a+b))
^(1/2)*cos(d*x+c)*a^3-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+9*A*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
```

```
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*
b+2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b-5*B*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*sin(d*x+c)*cos(d*x+c)*a^2*b+5*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-15*C*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*cos(d*x+c)*a^2*b+15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-7*A*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
cos(d*x+c)*a^2*b+9*A*a^2*b*((a-b)/(a+b))^(1/2)+A*a*b^2*((a-b)/(a+b))^(1/2)+
5*B*a^2*b*((a-b)/(a+b))^(1/2)+5*B*a*b^2*((a-b)/(a+b))^(1/2)+15*C*((a-b)/(a+
b))^(1/2)*a^2*b-5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-15*C*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d
*x+c)+9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)-2*A*b^3*((a-b)/(a+b))^(1/2)-4*A*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^3*a^2*b+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2-10*B*((a-b)
/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-5*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-2
*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*
a^2*b-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2-15*C*((a-b)/(a+b))^(1/2)*cos
(d*x+c)*a^2*b+2*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*B*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+5*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c))*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+
c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
```

```
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/s
ec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(5/2), x)
```

3.1036 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=360

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(25a^2A+35a^2C-14abB+8Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^3d\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(\cdot\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])
/(105*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B
 + a^2*b*(19*A + 35*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x
]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
+ (2*(A*b + 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d
*x]^(3/2)) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A + 7*C))*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(105*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.17314, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(-5a^2(5A+7C)-7abB+4Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(25a^2A+35a^2C-14a\right.}{105a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(7/2),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])
/(105*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B
 + a^2*b*(19*A + 35*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x
]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
+ (2*(A*b + 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d
*x]^(3/2)) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A + 7*C))*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(105*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7a}{}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a}}{35}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A+8Ab^2-14abB+35a^2C\right)\sqrt{\frac{b+a}{}}}{105a^3d\sqrt{a+b\sec(c}}$$

Mathematica [C] time = 6.76679, size = 4441, normalized size = 12.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(19*
a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B + 35*a^2*b*C)*Cot[c])/(105*a^3*d)
 + ((115*a^2*A - 16*A*b^2 + 28*a*b*B + 140*a^2*C)*Cos[d*x]*Sin[c])/(105*a^2
*d) + (2*(A*b + 7*a*B)*Cos[2*d*x]*Sin[2*c])/(35*a*d) + (A*Cos[3*d*x]*Sin[3*
c])/(7*d) + ((115*a^2*A - 16*A*b^2 + 28*a*b*B + 140*a^2*C)*Cos[c]*Sin[d*x])
/(105*a^2*d) + (2*(A*b + 7*a*B)*Cos[2*c]*Sin[2*d*x])/(35*a*d) + (A*Cos[3*c]
*Sin[3*d*x])/(7*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec
[c + d*x]^(5/2)) - (20*A*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 +
(b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sq
rt[1 + Cot[c]^2])))]*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*
Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc
[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Co
t[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(5/2
)) - (8*A*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/
(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[
```

c]^2])))]*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Co
t[c]^2]*Sin[d*x - ArcTan[Cot[c]]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[
(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]]])/(a*
Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]]]])/(105*a^2*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(5/2)) - (
28*b*B*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt
[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))
)]*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]
*Sin[d*x - ArcTan[Cot[c]]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt
[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]]])/(a*Sqrt[1
+ Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]]]])/(15*a*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(5/2)) - (4*C*Appell
F1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2
]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]]))/
(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*Sqr
t[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - Ar
cTan[Cot[c]]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2
] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]]])/(a*Sqrt[1 + Cot[c]^2] +
 b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]]]
)/(3*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(5/2)) - (38*A*b*Csc[c]*Sqrt[a + b*S
ec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2,
 -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -(
(Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqr
t[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]
*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(105*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)) - (16*A*b^3*C
sc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Ap
pellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 +
 Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))
)]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + T
an[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sq
rt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(105*a^2*d*Sqrt[b + a*
Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x
]^(5/2)) - (6*a*B*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Co

```
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*
Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sq
rt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]
*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2]
)]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c
]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*
d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*Sec[c + d*x]^(5/2)) + (4*b^2*B*Csc[c]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(
b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan
[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (
b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c
]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqr
t[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos
[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c
]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]))/(15*a*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2)) - (2*b*C*Csc[c]*Sqrt[a + b*Sec[c +
d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2,
1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))
/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]
*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + T
an[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(
a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b
*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(3*d*Sqrt[b + a*Cos[c + d*x]]*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.686, size = 4764, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2)
,x)
```

```
[Out] -2/105/d/((a-b)/(a+b))^(1/2)/a^3*(-8*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-19*A*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*c
os(d*x+c)*a^3*b-25*A*a^3*b*((a-b)/(a+b))^(1/2)-19*A*a^2*b^2*((a-b)/(a+b))^(
```

1/2)+4*A*a*b^3*((a-b)/(a+b))^(1/2)-63*B*a^3*b*((a-b)/(a+b))^(1/2)-7*B*a^2*b ^2*((a-b)/(a+b))^(1/2)+14*B*a*b^3*((a-b)/(a+b))^(1/2)-35*C*a^3*b*((a-b)/(a+ b))^(1/2)-35*C*a^2*b^2*((a-b)/(a+b))^(1/2)+63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d *x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+35*C*Ellip ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a ^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2) *sin(d*x+c)+25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1 /(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+18*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3 *b-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+28*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+26*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+4*A*cos(d*x+c)^2 *((a-b)/(a+b))^(1/2)*a*b^3-7*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2+70* C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-19*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+20*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+14*B*cos(d*x+c) *((a-b)/(a+b))^(1/2)*a^2*b^2-14*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-35*C *cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a ^2*b^2-8*A*b^4*((a-b)/(a+b))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4- 35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/ 2)*a^4+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4+35*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4+21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2 *((a-b)/(a+b))^(1/2)*a^4+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-63*B*cos(d* x+c)*((a-b)/(a+b))^(1/2)*a^4-63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d* x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+25*A*EllipticF((-1+cos(d *x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d* x+c)*a^4-8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c) +1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b) /(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4-63*B*EllipticF((-1+cos(d*x+c))*((a -b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)) /(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4+6 3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+35*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x +c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4-19*A*Ellip ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a ^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/ 2)*sin(d*x+c)+2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c), (-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/ 2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticF((-1+cos(d*x+c))*((a-b) /(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+ c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+19*A*Elliptic E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3* b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)* sin(d*x+c)-19*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(- (a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2) *(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+49*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b* (1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si n(d*x+c)+14*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a +b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a +b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c)) /(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-14*B*EllipticE((

```
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+14*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-35*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+35*C*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-35*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^2-8*A*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*cos(d*x+c)*a*b^3+19*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-19*A*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*a^2*b^2+8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+49*B*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*cos(d*x+c)*a^3*b+14*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^2-63*B*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a^3*b-14*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+14*B*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b^3-35*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b+35*C*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a^3*b-35*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
```

```
sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/s
ec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(7/2), x)
```

3.1037 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=457

$$-\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(6a^2b(6A+7C)-75a^3B-24ab^2B+16Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^4d\sqrt{a+b\sec(c+dx)}}-\frac{2\text{s}}{}$$

```
[Out] (-2*(a^2 - b^2)*(16*A*b^3 - 75*a^3*B - 24*a*b^2*B + 6*a^2*b*(6*A + 7*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(315*a^4*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(16*A*b^4 - 57*a^
3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^4*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(A*b + 9*a*B)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(63*a*d*Sec[c + d*x]^(5/2)) - (2*(6*A*b^2 - 9*a*b*B
 - 7*a^2*(7*A + 9*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Sec
[c + d*x]^(3/2)) + (2*(8*A*b^3 + 75*a^3*B - 12*a*b^2*B + a^2*b*(13*A + 21*C
))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.64362, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(-7a^2(7A+9C)-9abB+6Ab^2\right)\sqrt{a+b\sec(c+dx)}}{315a^2d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(a^2b(13A+21C)+75a^3B-12ab^2\right.}{315a^3d\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[
c + d*x]^(9/2),x]
```


```
[Out] (-2*(a^2 - b^2)*(16*A*b^3 - 75*a^3*B - 24*a*b^2*B + 6*a^2*b*(6*A + 7*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(315*a^4*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(16*A*b^4 - 57*a^
3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^4*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*(A*b + 9*a*B)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(63*a*d*Sec[c + d*x]^(5/2)) - (2*(6*A*b^2 - 9*a*b*B
 - 7*a^2*(7*A + 9*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Sec
[c + d*x]^(3/2)) + (2*(8*A*b^3 + 75*a^3*B - 12*a*b^2*B + a^2*b*(13*A + 21*C
))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^3*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{1}{2}(Ab+9a}{}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+9aB)\sqrt{a}}{63}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+9aB)\sqrt{a}}{63}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+9aB)\sqrt{a}}{63}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+9aB)\sqrt{a}}{63}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+9aB)\sqrt{a}}{63}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+9aB)\sqrt{a}}{63}$$

$$= -\frac{2\left(a^2-b^2\right)\left(16Ab^3-75a^3B-24ab^2B+6a^2b(6A+\right.}{315a^4d\sqrt{a+}}$$

Mathematica [C] time = 6.97829, size = 5993, normalized size = 13.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sec[c + d*x]^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.9, size = 6551, normalized size = 14.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/s
ec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sec(d*x + c)^(9/2), x)
```

3.1038 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=551

$$\frac{\sqrt{\sec(c+dx)}\left(136a^2bB-3a^3C+12ab^2(28A+19C)+128b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{192bd\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)}{}$$

```
[Out] ((136*a^2*b*B + 128*b^3*B - 3*a^3*C + 12*a*b^2*(28*A + 19*C))*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(192*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((8*a^3*b*B - 96*a*b^3*B - 3*a^4*
C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(64*b^
2*d*Sqrt[a + b*Sec[c + d*x]]) - ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b
^2*(20*A + 13*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(192*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) +
((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[Sec[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d) + ((48*A*b^2 + 56*a
*b*B + 3*a^2*C + 36*b^2*C)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(96*b*d) + ((8*b*B + 3*a*C)*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(24*d) + (C*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 2.16215, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(3a^2C+56abB+48Ab^2+36b^2C\right)\sqrt{a+b\sec(c+dx)}}{96bd}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(24a^2bB-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] ((136*a^2*b*B + 128*b^3*B - 3*a^3*C + 12*a*b^2*(28*A + 19*C))*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(192*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((8*a^3*b*B - 96*a*b^3*B - 3*a^4*
C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(64*b^
2*d*Sqrt[a + b*Sec[c + d*x]]) - ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b
^2*(20*A + 13*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(192*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) +
((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[Sec[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d) + ((48*A*b^2 + 56*a
*b*B + 3*a^2*C + 36*b^2*C)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(96*b*d) + ((8*b*B + 3*a*C)*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(24*d) + (C*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(4*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(}{4d}$$

$$= \frac{(8bB+3aC)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}}{24d}$$

$$= \frac{\left(48Ab^2+56abB+3a^2C+36b^2C\right)\sec^{\frac{3}{2}}(}{96b}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+12ab^2(20A\right.}{}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+12ab^2(20A\right.}{}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+12ab^2(20A\right.}{}$$

$$= \frac{\left(24a^2bB+128b^3B-9a^3C+12ab^2(20A\right.}{}$$

$$= -\frac{\left(8a^3bB-96ab^3B-3a^4C-24a^2b^2(2A\right.}{}$$

$$= \frac{\left(136a^2bB+128b^3B-3a^3C+12ab^2(28A\right.}{192ba}$$

Mathematica [C] time = 7.05807, size = 916, normalized size = 1.66

$$\frac{\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{2\left(12bCa^3+224b^2Ba^2+192Ab^3a+144b^3Ca\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(27Ca^4-72bBa^3+48A\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(19
2*a*A*b^3 + 224*a^2*b^2*B + 12*a^3*b*C + 144*a*b^3*C)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x
]] + (2*(48*a^2*A*b^2 + 384*A*b^4 - 72*a^3*b*B + 448*a*b^3*B + 27*a^4*C - 1
2*a^2*b^2*C + 288*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-240*a^2*A*b
^2 - 24*a^3*b*B - 128*a*b^3*B + 9*a^4*C - 156*a^2*b^2*C)*Sqrt[(a - a*Cos[c
+ d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*
(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (
-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a
*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(
a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(
Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^
```

```
2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2
))))/(384*b^2*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)) + ((a + b*Sec[c + d*x])^(3/2)*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((Sec[c + d*x]^3*(8*b*B*Sin[c + d*x] + 9*a
*C*Sin[c + d*x]))/12 + (Sec[c + d*x]^2*(48*A*b^2*Sin[c + d*x] + 56*a*b*B*Si
n[c + d*x] + 3*a^2*C*Sin[c + d*x] + 36*b^2*C*Sin[c + d*x]))/(48*b) + (Sec[c
 + d*x]*(240*a*A*b^2*Sin[c + d*x] + 24*a^2*b*B*Sin[c + d*x] + 128*b^3*B*Sin
[c + d*x] - 9*a^3*C*Sin[c + d*x] + 156*a*b^2*C*Sin[c + d*x]))/(96*b^2) + (b
*C*Sec[c + d*x]^3*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2))
```

Maple [C] time = 0.855, size = 7134, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")

[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sec(d*x + c)^(3/2), x)
```

3.1039 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+\right.$

Optimal. Leaf size=446

$$\frac{\sqrt{\sec(c+dx)}\left(a^2(48A+17C)+42abB+8b^2(3A+2C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(}}{}$$

```
[Out] ((42*a*b*B + 8*b^2*(3*A + 2*C) + a^2*(48*A + 17*C))*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d
*Sqrt[a + b*Sec[c + d*x]]) + ((6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A
+ C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(
a + b)]*Sqrt[Sec[c + d*x]])/(8*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 +
 30*a*b*B + 3*a^2*C + 16*b^2*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + ((24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[Sec[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d) + ((2*b*B + a*C)*Sec[c + d*x
]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sec[c + d*x]^(3/2
)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.65448, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C+30abB+24Ab^2+16b^2C\right)\sqrt{a+b\sec(c+dx)}}{24bd}+\frac{\sqrt{\sec(c+dx)}\left(a^2(48A+17C)+42a\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```



```
[Out] ((42*a*b*B + 8*b^2*(3*A + 2*C) + a^2*(48*A + 17*C))*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d
*Sqrt[a + b*Sec[c + d*x]]) + ((6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A
+ C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(
a + b)]*Sqrt[Sec[c + d*x]])/(8*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 +
 30*a*b*B + 3*a^2*C + 16*b^2*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + ((24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[Sec[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d) + ((2*b*B + a*C)*Sec[c + d*x
]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sec[c + d*x]^(3/2
)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin}{3d}$$

$$= \frac{(2bB+aC)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{4d}$$

$$= \frac{\left(24Ab^2+30abB+3a^2C+16b^2C\right)\sqrt{\sec}}{24}$$

$$= \frac{\left(24Ab^2+30abB+3a^2C+16b^2C\right)\sqrt{\sec}}{24}$$

$$= \frac{\left(24Ab^2+30abB+3a^2C+16b^2C\right)\sqrt{\sec}}{24}$$

$$= \frac{\left(24Ab^2+30abB+3a^2C+16b^2C\right)\sqrt{\sec}}{24}$$

$$= \frac{\left(6a^2bB+8b^3B-a^3C+12ab^2(2A+C)\right)}{8bd\sqrt{a}}$$

$$= \frac{\left(42abB+8b^2(3A+2C)+a^2(48A+17\right.}{24d\sqrt{a}}$$

Mathematica [C] time = 6.95525, size = 800, normalized size = 1.79

$$\frac{(a+b\sec(c+dx))^{3/2}\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{1}{6}(6bB\sin(c+dx)+7aC\sin(c+dx))\sec^2(c+dx)+\frac{2}{3}bC\tan\right.}{d(b+a\cos(c+dx))(\cos(2c+2dx)A+A+2C+2B\cos}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] -((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(-
96*a^2*A*b - 24*a*b^2*B - 28*a^2*b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(-120*a*
A*b^2 - 6*a^2*b*B - 48*b^3*B + 9*a^3*C - 56*a*b^2*C)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
d*x]] + ((2*I)*(24*a*A*b^2 + 30*a^2*b*B + 3*a^3*C + 16*a*b^2*C)*Sqrt[(a - a
*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]
*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqr
t[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c +
d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(48*b*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)) + ((a + b*Sec[c + d*x])^(3/2)*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Sec[c + d*x]^2*(6*b*B*Sin[c + d*x] +
 7*a*C*Sin[c + d*x]))/6 + (Sec[c + d*x]*(24*A*b^2*Sin[c + d*x] + 30*a*b*B*S
```

```
in[c + d*x] + 3*a^2*C*Sin[c + d*x] + 16*b^2*C*Sin[c + d*x]))/(12*b) + (2*b*
C*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2))
```

Maple [C] time = 0.583, size = 5245, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sqrt(sec(d*x + c)), x)
```

3.1040 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=353

$$\frac{\sqrt{\sec(c+dx)}\left(8a^2B+ab(8A+7C)+4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\left(3a^2C+12abB\right.}{4}$$

```
[Out] ((8*a^2*B + 4*b^2*B + a*b*(8*A + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Se
c[c + d*x]]) + ((8*A*b^2 + 12*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*a*A - 4*b*B - 5*a*C)*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((4*b*B + 3*a*C)*Sqrt[Sec[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Sec[c + d*x]]*(a + b*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.1819, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sqrt{\sec(c+dx)}\left(8a^2B+ab(8A+7C)+4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\left(3a^2C+12abB+8Ab^2+\right.}{4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```


```
[Out] ((8*a^2*B + 4*b^2*B + a*b*(8*A + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Se
c[c + d*x]]) + ((8*A*b^2 + 12*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*a*A - 4*b*B - 5*a*C)*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((4*b*B + 3*a*C)*Sqrt[Sec[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Sqrt[Sec[c + d*x]]*(a + b*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d} +$$

$$= \frac{(4bB+3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c}{4d}$$

$$= \frac{(4bB+3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c}{4d}$$

$$= \frac{(4bB+3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c}{4d}$$

$$= \frac{(4bB+3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c}{4d}$$

$$= \frac{\left(8Ab^2+12abB+3a^2C+4b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(\right.}{4d\sqrt{a+b\sec(c+dx}}$$

$$= \frac{\left(8a^2B+4b^2B+ab(8A+7C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.93808, size = 709, normalized size = 2.01

$$\frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{1}{2}\sec(c+dx)(5aC\sin(c+dx)+4bB\sin(c+dx))+bC\tan\right.}{d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(32
*a*A*b + 16*a^2*B + 4*a*b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(8*a^2*A + 16*A*b
^2 + 20*a*b*B + a^2*C + 8*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(8*a^
2*A - 4*a*b*B - 5*a^2*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos
[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt
[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ellipt
icF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b
)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Co
s[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b +
 a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(8*d*(b + a*Cos[c + d*x])^(
3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2))
+ ((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Sec
[c + d*x]*(4*b*B*Sin[c + d*x] + 5*a*C*Sin[c + d*x]))/2 + b*C*Sec[c + d*x]*T
an[c + d*x]))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*Sec[c + d*x]^(7/2))
```

Maple [C] time = 0.53, size = 4335, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(16*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-8*A*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+24*B*cos(d*x
+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+5*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+2*C*
cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*sin(d*x+c)*a*b+16*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-4*B*cos(d*
x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*sin(d*x+c)*a*b+24*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+5*
C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*sin(d*x+c)*a*b+2*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+8*A*cos(d
*x+c)^4*((a-b)/(a+b))^(1/2)*a^2+5*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+4*
B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-5*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2
*a^2-4*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2+2*C*((a-b)/(a+b))^(1/2)*cos(d*x
```

```
+c)^2*b^2-8*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+c)*cos(d*x+c)^3*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b-8*B*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+8*A*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-8*A*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+8*B*
sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^2+8*A*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-8*A*sin(d*x
+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+8*A*sin(d*x+c)*cos
(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2-8*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2+16*A*cos(d*x+c)^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*b^2+4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-5*C*cos(d*x+c)^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*a^2+2*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-4*C*cos(d*x+c)^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d
*x+c)*b^2+6*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+8*C*cos(d*x+c)^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^2-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2-2*C
*((a-b)/(a+b))^(1/2)*b^2-8*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2+16*A*cos(d*x+c)^
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+4*B*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-5*C*cos(
d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*a^2+2*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-4*C*cos(d*x+c)^
```

```
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*b^2+6*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+8*C*cos(
d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+4*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^
3*a*b+2*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a*b-4*B*((a-b)/(a+b))^(1/2)*cos(
d*x+c)^2*a*b+5*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-7*C*((a-b)/(a+b))^(1/
2)*cos(d*x+c)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)
/sin(d*x+c)/(b+a*cos(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sqrt(sec(d*x + c)), x)
```

3.1041 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=340

$$\frac{\sqrt{\sec(c+dx)}\left(2a^2(A+3C)+6abB-b^2(2A-3C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{(6aB+8Ab-3bC)\sqrt{a+b}}{3d\sqrt{\sec(c+}}$$

```
[Out] ((6*a*b*B - b^2*(2*A - 3*C) + 2*a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[
a + b*Sec[c + d*x]]) + (b*(2*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) + ((8*A*b + 6*a*B - 3*b*C)*EllipticE[(c + d*x)/2, (2*a)/(
a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) - (b*(2*A - 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.23136, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4094, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sqrt{\sec(c+dx)}\left(2a^2(A+3C)+6abB-b^2(2A-3C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{(6aB+8Ab-3bC)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```


```
[Out] ((6*a*b*B - b^2*(2*A - 3*C) + 2*a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[
a + b*Sec[c + d*x]]) + (b*(2*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) + ((8*A*b + 6*a*B - 3*b*C)*EllipticE[(c + d*x)/2, (2*a)/(
a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) - (b*(2*A - 3*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(3*d) + (2*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
```

```
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\sqrt{a+b\,\mathrm{se}\ldots}}{\ldots}$$

$$= -\frac{b(2A-3C)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+\ldots}{3d}$$

$$= -\frac{b(2A-3C)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+\ldots}{3d}$$

$$= -\frac{b(2A-3C)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+\ldots}{3d}$$

$$= -\frac{b(2A-3C)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+\ldots}{3d}$$

$$= \frac{b(2bB+3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec\ldots}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(6abB-b^2(2A-3C)+2a^2(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\ldots}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.89698, size = 685, normalized size = 2.01

$$\frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4}{3}aA\sin(c+dx)+2bC\tan(c+dx)\right)}{d\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)}+\frac{(a+b\sec(c+dx)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(4*
a^2*A + 12*A*b^2 + 24*a*b*B + 12*a^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(8*a*A
*b + 6*a^2*B + 12*b^2*B + 15*a*b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(8
*a*A*b + 6*a^2*B - 3*a*b*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*
Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[S
qrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ell
ipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a
+ b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
 + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 -
 Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(
b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(6*d*(b + a*Cos[c + d*x]
)^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2
)) + ((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((
4*a*A*Sin[c + d*x])/3 + 2*b*C*Tan[c + d*x]))/(d*(b + a*Cos[c + d*x])*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2))
```

Maple [C] time = 0.465, size = 3823, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] -1/3/d/((a-b)/(a+b))^(1/2)*(-8*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*A*cos(d*x+c)
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2-8*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-6*B*cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2+6*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-8*A*
((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b
-6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-8*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(
```

d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-6
*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-3*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-6*C*cos(
d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*a*b+2*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2+8*A*cos(
d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b+12*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*B*cos(d*x+c)*sin
(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b-6*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+18*C*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/
(a+b))^(1/2))*a*b-3*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+18*C*cos(d*x+c)^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b+6*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+8*A*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*b^2+3*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-2*A*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+12*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+8*A*
sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+2*A
*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2-6*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*A*cos(d*
x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*sin(d*x+c)*b^2+6*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-6*B*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^2*a^2-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-3*C*((a-b)/(
a+b))^(1/2)*b^2+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+3*C*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^2*a*b-3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-8*A*cos(d*x+c
)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*b^2-6*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2+6*B*cos(d*x+c)^2*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*a^2+12*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+3*C*cos(d*x+
c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/

```
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*b^2+6*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-6*B*cos(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(
d*x+c)*b^2+12*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+3*C*cos(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
sin(d*x+c)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(1/cos(d*x+c
))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\sec(dx+c)^3 + (Ca+Bb)\sec(dx+c)^2 + Aa + (Ba+Ab)\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(3/2), x)
```

3.1042 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=356

$$-\frac{2\sqrt{\sec(c+dx)}\left(-3a^2b(A+5C)-5a^3B+5ab^2B+3Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2(3A+5C\right.}{}$$

```
[Out] (-2*(3*A*b^3 - 5*a^3*B + 5*a*b^2*B - 3*a^2*b*(A + 5*C))*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(
15*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*C*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
 + b*Sec[c + d*x]]) + (2*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(3*A*b + 5*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c +
 d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 1.25456, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sqrt{\sec(c+dx)}\left(-3a^2b(A+5C)-5a^3B+5ab^2B+3Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2(3A+5C)+20ab\right.}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(5/2),x]
```

```
[Out] (-2*(3*A*b^3 - 5*a^3*B + 5*a*b^2*B - 3*a^2*b*(A + 5*C))*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(
15*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*C*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
 + b*Sec[c + d*x]]) + (2*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(3*A*b + 5*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c +
 d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{\sqrt{a+}}{}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A}{}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A}{}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A}{}$$

$$= \frac{2(3Ab+5aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A}{}$$

$$= \frac{2b^2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2\left(3Ab^3-5a^3B+5ab^2B-3a^2b(A+5C)\right)\sqrt{\frac{b+a}{}}}{15ad\sqrt{a+b\sec(}}$$

Mathematica [F] time = 39.345, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(5/2), x]
```

Maple [C] time = 0.501, size = 4247, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] -2/15/d/a/((a-b)/(a+b))^(1/2)*(-3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^2+9*A*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3+9*A*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a^3-3*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+5*B*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*c
os(d*x+c)*a^3-15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3+15*C*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*a^3+12*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*A*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*cos(d*
x+c)+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^2*a^3+15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3-9*A*((a-b)/(a+b))^(1/
2)*cos(d*x+c)*a^3+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3-15*C*((a-b)/(a+b))
^(1/2)*cos(d*x+c)*a^3-3*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-9*A*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*
b+3*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-20*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b+20*B
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
```

```
))*sin(d*x+c)*cos(d*x+c)*a^2*b-20*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+30*C*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*cos(d*x+c)*a^2*b-15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+12*A*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*cos(d*x+c)*a^2*b-9*A*a^2*b*((a-b)/(a+b))^(1/2)-6*A*a*b^2*((a-b)/(a+b))^
(1/2)-5*B*a^2*b*((a-b)/(a+b))^(1/2)-20*B*a*b^2*((a-b)/(a+b))^(1/2)-15*C*((a
-b)/(a+b))^(1/2)*a^2*b+15*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-15*C*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+30*C
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-15*C*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c
)+15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^3*sin(d*x+c)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*A*b^3*((a-b)/(a+b))^(1/2)+9*
A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)
^2*a*b^2+25*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-3*A*((a-b)/(a+b))^(1/2
)*cos(d*x+c)*a*b^2-20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b+20*B*((a-b)/(a
+b))^(1/2)*cos(d*x+c)*a*b^2+15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b+15*B*
cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*a*b^2-15*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+30*C*cos(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+3*A*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-20*B*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+20*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-20*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+30*C*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin
(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5
/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\sec\left(dx+c\right)^3+\left(Ca+Bb\right)\sec\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(5/2), x)
```

3.1043 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=359

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(25a^2A+35a^2C+21abB-6Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\,(5}{}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])
/(105*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B
 - 2*a^2*b*(41*A + 70*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]) + (2*(3*A*b + 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec
[c + d*x]^(3/2)) + (2*(3*A*b^2 + 42*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(105*a*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 1.23594, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)+42abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(25a^2A+35a^2C+21ab\right.}{105a^2d\sqrt{a+b\,\text{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(7/2),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])
/(105*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B
 - 2*a^2*b*(41*A + 70*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]) + (2*(3*A*b + 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec
[c + d*x]^(3/2)) + (2*(3*A*b^2 + 42*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(105*a*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+}}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{2(3Ab+7aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A-6Ab^2+21abB+35a^2C\right)\sqrt{\frac{b}{}}}{105a^2d\sqrt{a+b\sec}}$$

Mathematica [C] time = 6.82411, size = 4862, normalized size = 13.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(8
2*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 140*a^2*b*C)*Cot[c])/(105*a^2
*d) + ((115*a^2*A + 12*A*b^2 + 168*a*b*B + 140*a^2*C)*Cos[d*x]*Sin[c])/(105
*a*d) + (2*(8*A*b + 7*a*B)*Cos[2*d*x]*Sin[2*c])/(35*d) + (a*A*Cos[3*d*x]*Si
n[3*c])/(7*d) + ((115*a^2*A + 12*A*b^2 + 168*a*b*B + 140*a^2*C)*Cos[c]*Sin[
d*x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Cos[2*c]*Sin[2*d*x])/(35*d) + (a*A*Cos
[3*c]*Sin[3*d*x])/(7*d)))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)) - (20*a*A*AppellF1[1/2, 1/2, 1/2
, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))
/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b
 - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[
c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*(a + b*Sec[c + d*x]
)^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])
/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 +
 Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sq
rt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(b + a
*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2]*Sec[c + d*x]^(7/2)) - (68*A*b^2*AppellF1[1/2, 1/2, 1/2, 3/2,
 (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sq
rt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*
```

(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*(a + b*Sec[c + d*x])^(3/2
)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a
*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sq
rt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c
]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b -
 a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(35*a*d*(b + a*Cos
[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
 Cot[c]^2]*Sec[c + d*x]^(7/2)) - (16*b*B*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[
c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 +
 Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 +
(b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[
1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 +
 Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*S
in[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(5*d*(b + a*Cos[c + d*x]
)^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2
]*Sec[c + d*x]^(7/2)) - (4*a*C*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]
*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])
/(a*Sqrt[1 + Cot[c]^2])))]*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]
^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
 - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - A
rcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^(3/2)*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c +
d*x]^(7/2)) - (4*b^2*C*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b
*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt
[1 + Cot[c]^2])))]*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*
Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc
[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Co
t[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]]])/(a*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(7/
2)) - (164*a*A*b*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b
*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*S
qrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2
]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2
])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[
c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(1
05*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*Sec[c + d*x]^(7/2)) + (4*A*b^3*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((S
ec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[
1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*

```
(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1
+ Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c])
+ a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^
2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]]))/(35*a*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)) - (6*a^2*B*Csc[c]*(a + b*S
ec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2,
 -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))
)), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/
(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c
]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^(3/2)*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)) - (2*b^
2*B*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*S
qrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c
]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqr
t[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c]
 + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b +
a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b
 + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos
[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c +
d*x]^(7/2)) - (8*a*b*C*Csc[c]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(
1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c]
)/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Ta
n[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Ta
n[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b +
 a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^
2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]))/(3*d*(b + a*Cos[c + d*x])^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])*Sec[c + d*x]^(7/2))
```

Maple [B] time = 0.622, size = 4944, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2),x)

[Out] -2/105/d/a^2/((a-b)/(a+b))^(1/2)*(6*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-82*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b-25*A*a^3*b*((a-b)/(a+b))^(1/2)-82*A*a^2*b^2*((a-b)/(a+b))^(1/2)-3*A*a*b^3*((a-b)/(a+b))^(1/2)-63*B*a^3*b*((a-b)/(a+b))^(1/2)-42*B*a^2*b^2*((a-b)/(a+b))^(1/2)-21*B*a*b^3*((a-b)/(a+b))^(1/2)-35*C*a^3*b*((a-b)/(a+b))^(1/2)-140*C*a^2*b^2*((a-b)/(a+b))^(1/2)+63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+35*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+39*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+27*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+63*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+68*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+63*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2+175*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-82*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+55*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-21*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+21*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-140*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+140*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+105*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+6*A*b^4*((a-b)/(a+b))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4+35*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4+21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4+6*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4-63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4+63*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+35*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4-82*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+51*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+82*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(

```
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-82*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-6*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+84*B*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-21*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-63*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21*B*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)-21*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-140*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+140*C*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-140*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+105*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+51*A*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*cos(d*x+c)*a^2*b^2+6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^3+82*A*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a^3*b-82*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-6*A*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a*b^3+84*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b-21*B*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*co
s(d*x+c)*a^2*b^2-63*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+21*B*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*a^2*b^2-21*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-140*C*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x
+c)*a^3*b+140*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-140*C*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)
```

```
*a^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(
7/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\sec\left(dx+c\right)^3+\left(Ca+Bb\right)\sec\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(7/2), x)
```

3.1044 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=455

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(a^2(39Ab+63bC)+75a^3B-18ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(}{}$$

```
[Out] (2*(a^2 - b^2)*(8*A*b^3 + 75*a^3*B - 18*a*b^2*B + a^2*(39*A*b + 63*b*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(315*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 246*a^
3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Elliptic
E[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b + 3*a*B)*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*(3*A*b^2 + 72*
a*b*B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*
Sec[c + d*x]^(3/2)) - (2*(4*A*b^3 - 75*a^3*B - 9*a*b^2*B - 2*a^2*b*(44*A +
63*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Sqrt[Sec[c + d*x]]
) + (2*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 1.70909, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(7a^2(7A+9C)+72abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}}{315ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\sin(c+dx)\left(-2a^2b(44A+63C)-75a^3B-9ab^2B\right.}{315a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(9/2),x]
```


```
[Out] (2*(a^2 - b^2)*(8*A*b^3 + 75*a^3*B - 18*a*b^2*B + a^2*(39*A*b + 63*b*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(315*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 246*a^
3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Elliptic
E[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b + 3*a*B)*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*(3*A*b^2 + 72*
a*b*B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*
Sec[c + d*x]^(3/2)) - (2*(4*A*b^3 - 75*a^3*B - 9*a*b^2*B - 2*a^2*b*(44*A +
63*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Sqrt[Sec[c + d*x]]
) + (2*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\,\mathrm{se}}}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a-}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab\right.}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab\right.}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab\right.}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab\right.}{}$$

$$= \frac{2(Ab+3aB)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(3Ab\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(8Ab^3+75a^3B-18ab^2B+a^2(39Ab+63\right.}{315a^3d\sqrt{a+b}}$$

Mathematica [C] time = 7.03093, size = 5997, normalized size = 13.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.868, size = 6526, normalized size = 14.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb\sec\left(dx+c\right)^3+\left(Ca+Bb\right)\sec\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sec(d*x + c)^(9/2), x)
```

3.1045 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=550

$$\frac{\sqrt{\sec(c+dx)}\left(a^3(384A+133C)+472a^2bB+4ab^2(132A+89C)+128b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{192d\sqrt{a+b\sec(c+dx)}}+$$

```
[Out] ((472*a^2*b*B + 128*b^3*B + 4*a*b^2*(132*A + 89*C) + a^3*(384*A + 133*C))*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[Sec[c + d*x]])/(192*d*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*b*B + 160*a*b^
3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
d*x]])/(64*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((264*a^2*b*B + 128*b^3*B + 15*a
^3*C + 4*a*b^2*(108*A + 71*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A + 71*C))*S
qrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*b*d) + ((16*A
*b^2 + 24*a*b*B + 5*a^2*C + 12*b^2*C)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(32*d) + ((8*b*B + 5*a*C)*Sec[c + d*x]^(3/2)*(a + b*Se
c[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sec[c + d*x]^(3/2)*(a + b*Sec[c
 + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 2.1895, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(5a^2C+24abB+16Ab^2+12b^2C\right)\sqrt{a+b\sec(c+dx)}}{32d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(264a^2bB+15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] ((472*a^2*b*B + 128*b^3*B + 4*a*b^2*(132*A + 89*C) + a^3*(384*A + 133*C))*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[Sec[c + d*x]])/(192*d*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*b*B + 160*a*b^
3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
d*x]])/(64*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((264*a^2*b*B + 128*b^3*B + 15*a
^3*C + 4*a*b^2*(108*A + 71*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + ((264*a^2*b*B + 128*b^3*B + 15*a^3*C + 4*a*b^2*(108*A + 71*C))*S
qrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*b*d) + ((16*A
*b^2 + 24*a*b*B + 5*a^2*C + 12*b^2*C)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(32*d) + ((8*b*B + 5*a*C)*Sec[c + d*x]^(3/2)*(a + b*Se
c[c + d*x])^(3/2)*Sin[c + d*x])/(24*d) + (C*Sec[c + d*x]^(3/2)*(a + b*Sec[c
 + d*x])^(5/2)*Sin[c + d*x])/(4*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{C\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}{4d}$$

$$= \frac{(8bB+5aC)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(}{24d}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\mathrm{se}}{}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+4ab^2\right.}{}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+4ab^2\right.}{}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+4ab^2\right.}{}$$

$$= \frac{\left(264a^2bB+128b^3B+15a^3C+4ab^2\right.}{}$$

$$= \frac{\left(40a^3bB+160ab^3B-5a^4C+120a^2\right.}{}$$

$$= \frac{\left(472a^2bB+128b^3B+4ab^2(132A+\right.}{}$$

Mathematica [C] time = 7.21986, size = 925, normalized size = 1.68

$$\frac{(a+b\sec(c+dx))^{5/2}\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{1}{12}\left(8B\sin(c+dx)b^2+17aC\sin(c+dx)b\right)\sec^3(c+dx)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] -((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(-
768*a^3*A*b - 192*a*A*b^3 - 416*a^2*b^2*B - 236*a^3*b*C - 144*a*b^3*C)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[
b + a*Cos[c + d*x]] + (2*(-1008*a^2*A*b^2 - 384*A*b^4 + 24*a^3*b*B - 832*a*
b^3*B + 45*a^4*C - 436*a^2*b^2*C - 288*b^4*C)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] +
 ((2*I)*(432*a^2*A*b^2 + 264*a^3*b*B + 128*a*b^3*B + 15*a^4*C + 284*a^2*b^2
*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b
 + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 -
 a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a
^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*
```

```
(b + a*Cos[c + d*x])^2))))/(384*b*d*(b + a*Cos[c + d*x])^(5/2)*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2)) + ((a + b*Sec[c +
 d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Sec[c + d*x]^3*(8*b^
2*B*Sin[c + d*x] + 17*a*b*C*Sin[c + d*x]))/12 + (Sec[c + d*x]^2*(48*A*b^2*S
in[c + d*x] + 104*a*b*B*Sin[c + d*x] + 59*a^2*C*Sin[c + d*x] + 36*b^2*C*Sin
[c + d*x]))/48 + (Sec[c + d*x]*(432*a*A*b^2*Sin[c + d*x] + 264*a^2*b*B*Sin[
c + d*x] + 128*b^3*B*Sin[c + d*x] + 15*a^3*C*Sin[c + d*x] + 284*a*b^2*C*Sin
[c + d*x]))/(96*b) + (b^2*C*Sec[c + d*x]^3*Tan[c + d*x])/2))/(d*(b + a*Cos[
c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^
(9/2))
```

Maple [C] time = 0.816, size = 7346, normalized size = 13.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)
,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec{(dx+c)}^2 + B\sec{(dx+c)} + A\right)(b\sec{(dx+c)} + a)^{\frac{5}{2}}\sqrt{\sec{(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*sqrt(sec(d*x + c)), x)
```

3.1046 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=453

$$\frac{\sqrt{\sec(c+dx)}\left(a^2b(96A+59C)+48a^3B+66ab^2B+8b^3(3A+2C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+d\ldots}{\ldots}$$

```
[Out] ((48*a^3*B + 66*a*b^2*B + 8*b^3*(3*A + 2*C) + a^2*b*(96*A + 59*C))*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + ((30*a^2*b*B + 8*b^3*B + 5*a^3*
C + 20*a*b^2*(2*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]
) - ((54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*Sqrt[Sec[c + d*x]]) + ((24*A*b^2 + 42*a*b*B + 15*a^2*C + 16*b^2*
C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + ((6*b
*B + 5*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12
*d) + (C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.68136, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(15a^2C+42abB+24Ab^2+16b^2C\right)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{\sec(c+dx)}\left(a^2b(96A+59C)+48a\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Sec[c + d*x]],x]
```


```
[Out] ((48*a^3*B + 66*a*b^2*B + 8*b^3*(3*A + 2*C) + a^2*b*(96*A + 59*C))*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + ((30*a^2*b*B + 8*b^3*B + 5*a^3*
C + 20*a*b^2*(2*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]
) - ((54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*Sqrt[Sec[c + d*x]]) + ((24*A*b^2 + 42*a*b*B + 15*a^2*C + 16*b^2*
C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + ((6*b
*B + 5*a*C)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12
*d) + (C*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
```

```
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{3d}+\frac{1}{3}$$

$$= \frac{(6bB+5aC)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\sin(c}{12d}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{\sec(c+dx)}\sqrt{a}}{24d}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{\sec(c+dx)}\sqrt{a}}{24d}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{\sec(c+dx)}\sqrt{a}}{24d}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{\sec(c+dx)}\sqrt{a}}{24d}$$

$$= \frac{\left(30a^2bB+8b^3B+5a^3C+20ab^2(2A+C)\right)\sqrt{\frac{b+a\cos(c}{a+b}}}{8d\sqrt{a+b\sec(c+}}$$

$$= \frac{\left(48a^3B+66ab^2B+8b^3(3A+2C)+a^2b(96A+59C\right.}{24d\sqrt{a+b\sec}}$$

Mathematica [C] time = 7.30106, size = 817, normalized size = 1.8

$$\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(\frac{2\left(96Ba^3+288Aba^2+52bCa^2+24b^2Ba\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(48Aa^3-3Ca^3+126bBa^2+\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(28
8*a^2*A*b + 96*a^3*B + 24*a*b^2*B + 52*a^2*b*C)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (
2*(48*a^3*A + 216*a*A*b^2 + 126*a^2*b*B + 48*b^3*B - 3*a^3*C + 104*a*b^2*C)
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b
)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(48*a^3*A - 24*a*A*b^2 - 54*a^2*b*B -
 33*a^3*C - 16*a*b^2*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[
c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ellipti
cF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)
] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b +
a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(48*d*(b + a*Cos[c + d*x])^(
5/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
+ ((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Sec
[c + d*x]^2*(6*b^2*B*Sin[c + d*x] + 13*a*b*C*Sin[c + d*x]))/6 + (Sec[c + d*
x]*(24*A*b^2*Sin[c + d*x] + 54*a*b*B*Sin[c + d*x] + 33*a^2*C*Sin[c + d*x] +
 16*b^2*C*Sin[c + d*x]))/12 + (2*b^2*C*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*
(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Se
c[c + d*x]^(9/2))
```

Maple [C] time = 0.672, size = 6194, normalized size = 13.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sqrt(sec(d*x + c)), x)
```

3.1047 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=427

$$\frac{\sqrt{\sec(c+dx)}\left(8a^3(A+3C)+48a^2bB+ab^2(16A+33C)+12b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{12d\sqrt{a+b\sec(c+dx)}}+\frac{\left(24a^2B\right.}{}$$

```
[Out] ((48*a^2*b*B + 12*b^3*B + 8*a^3*(A + 3*C) + a*b^2*(16*A + 33*C))*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(12*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(8*A*b^2 + 20*a*b*B + 15*a^2*C
 + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((24*a^2
*B - 12*b^2*B + a*b*(56*A - 27*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(12*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]) - (b*(8*a*A - 12*b*B - 21*a*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(12*d) - (b*(4*A - 3*C)*Sqrt[Sec[c + d*x]]*(a + b*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.66229, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4094, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sqrt{\sec(c+dx)}\left(8a^3(A+3C)+48a^2bB+ab^2(16A+33C)+12b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{12d\sqrt{a+b\sec(c+dx)}}+\frac{\left(24a^2B+ab(56A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(3/2),x]
```


```
[Out] ((48*a^2*b*B + 12*b^3*B + 8*a^3*(A + 3*C) + a*b^2*(16*A + 33*C))*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(12*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(8*A*b^2 + 20*a*b*B + 15*a^2*C
 + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((24*a^2
*B - 12*b^2*B + a*b*(56*A - 27*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(12*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]) - (b*(8*a*A - 12*b*B - 21*a*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(12*d) - (b*(4*A - 3*C)*Sqrt[Sec[c + d*x]]*(a + b*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= -\frac{b(4A-3C)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\sin(c}{6d}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{12d}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{12d}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{12d}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{12d}$$

$$= \frac{b\left(8Ab^2+20abB+15a^2C+4b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(48a^2bB+12b^3B+8a^3(A+3C)+ab^2(16A+33C)\right)}{12d\sqrt{a+b\sec(}}$$

Mathematica [C] time = 7.05312, size = 766, normalized size = 1.79

$$\frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{4}{3}a^2A\sin(c+dx)+\frac{1}{2}\sec(c+dx)\left(9abC\sin(c+dx)+4b^2B\,\mathrm{s}\right.\right.}{d\sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+b)^2(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(16
*a^3*A + 144*a*A*b^2 + 144*a^2*b*B + 48*a^3*C + 12*a*b^2*C)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c
 + d*x]] + (2*(56*a^2*A*b + 48*A*b^3 + 24*a^3*B + 108*a*b^2*B + 63*a^2*b*C
+ 24*b^3*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(56*a^2*A*b + 24*a^3*B - 1
2*a*b^2*B - 27*a^2*b*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[
c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ellipti
cF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)
] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b +
a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(24*d*(b + a*Cos[c + d*x])^(
5/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
+ ((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*a
^2*A*Sin[c + d*x])/3 + (Sec[c + d*x]*(4*b^2*B*Sin[c + d*x] + 9*a*b*C*Sin[c
```

```
+ d*x]))/2 + b^2*C*Sec[c + d*x]*Tan[c + d*x]))/(d*(b + a*Cos[c + d*x])^2*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [C] time = 0.642, size = 5629, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2 \sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aa\right.\right.}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(3/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(3/2), x)
```

3.1048 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{\sqrt{\sec(c+dx)}\left(4a^2b(4A+15C)+10a^3B+20ab^2B-b^3(16A-15C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{(6a^2(3\cdots}{\cdots}$$

```
[Out] ((10*a^3*B + 20*a*b^2*B - b^3*(16*A - 15*C) + 4*a^2*b*(4*A + 15*C))*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[
c + d*x]])/(15*d*Sqrt[a + b*Sec[c + d*x]]) + (b^2*(2*b*B + 5*a*C)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Se
c[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((70*a*b*B + b^2*(46*A - 15*C)
+ 6*a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (b
*(16*A*b + 10*a*B - 15*b*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*(A*b + a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(
5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 1.64518, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4094, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sqrt{\sec(c+dx)}\left(4a^2b(4A+15C)+10a^3B+20ab^2B-b^3(16A-15C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{(6a^2(3A+5C)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(5/2),x]
```


```
[Out] ((10*a^3*B + 20*a*b^2*B - b^3*(16*A - 15*C) + 4*a^2*b*(4*A + 15*C))*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[
c + d*x]])/(15*d*Sqrt[a + b*Sec[c + d*x]]) + (b^2*(2*b*B + 5*a*C)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Se
c[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((70*a*b*B + b^2*(46*A - 15*C)
+ 6*a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (b
*(16*A*b + 10*a*B - 15*b*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*(A*b + a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(
5*d*Sec[c + d*x]^(3/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2(Ab+aB)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2A(a}{}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= \frac{b^2(2bB+5aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(10a^3B+20ab^2B-b^3(16A-15C)+4a^2b(4A+15C\right.}{15d\sqrt{a+b\,\mathrm{sec}}}$$

Mathematica [C] time = 7.05177, size = 755, normalized size = 1.8

$$\frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2}{5}a^2A\sin(2(c+dx))+\frac{4}{15}a(5aB+11Ab)\sin(c+dx)+2b^2C\,\mathrm{t}\right.}{d\sec^{\frac{9}{2}}(c+dx)(a\cos(c+dx)+b)^2(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(68
*a^2*A*b + 60*A*b^3 + 20*a^3*B + 180*a*b^2*B + 180*a^2*b*C)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c
 + d*x]] + (2*(18*a^3*A + 46*a*A*b^2 + 70*a^2*b*B + 60*b^3*B + 30*a^3*C + 1
35*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(18*a^3*A + 46*a*A*b^2 + 7
0*a^2*b*B + 30*a^3*C - 15*a*b^2*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[
(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*Ar
cSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(
2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*S
qrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2
- 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(30*d*(b + a*Cos[
c + d*x])^(5/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d
```

```
*x]^(9/2)) + ((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((4*a*(11*A*b + 5*a*B)*Sin[c + d*x])/15 + (2*a^2*A*Sin[2*(c + d*x)])/
5 + 2*b^2*C*Tan[c + d*x]))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [C] time = 0.646, size = 5634, normalized size = 13.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aa\right.}{\sec(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(5/2), x)
```

3.1049 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=441

$$-\frac{2\sqrt{\sec(c+dx)}\left(10a^2b^2(A-7C)-5a^4(5A+7C)-56a^3bB+56ab^3B+15Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2}{a+}\right.}{105ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*(15*A*b^4 - 56*a^3*b*B + 56*a*b^3*B + 10*a^2*b^2*(A - 7*C) - 5*a^4*(5*A
 + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[Sec[c + d*x]])/(105*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^3*C*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(15*A*b^3 + 63*a^3*B
+ 161*a*b^2*B + 5*a^2*b*(29*A + 49*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[a + b*Sec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b + 7*a*B)
*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*
(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 1.67083, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)+56abB+15Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{\sec(c+dx)}\left(10a^2b^2(A-7C)-5a^4(5A+7C\right.}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(7/2),x]
```

```
[Out] (-2*(15*A*b^4 - 56*a^3*b*B + 56*a*b^3*B + 10*a^2*b^2*(A - 7*C) - 5*a^4*(5*A
 + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[Sec[c + d*x]])/(105*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^3*C*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(15*A*b^3 + 63*a^3*B
+ 161*a*b^2*B + 5*a^2*b*(29*A + 49*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[a + b*Sec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]) + (2*(5*A*b + 7*a*B)
*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*
(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2}{7}\int \frac{(a+b}{}$$

$$= \frac{2(5Ab+7aB)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{a+b\sec(c}}{105d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{a+b\sec(c}}{105d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{a+b\sec(c}}{105d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)\sqrt{a+b\sec(c}}{105d\sqrt{\sec(c+dx)}}$$

$$= \frac{2b^3C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2\left(15Ab^4-56a^3bB+56ab^3B+10a^2b^2(A-7C)\right.}{105a}$$

Mathematica [F] time = 51.1526, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(7/2),x]
```

```
[Out] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(7/2), x]
```

Maple [C] time = 0.731, size = 5602, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(7/2), x)
```

3.1050 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=452

$$-\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(-6a^2b(19A+28C)-75a^3B-45ab^2B+10Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2}{}$$

```
[Out] (-2*(a^2 - b^2)*(10*A*b^3 - 75*a^3*B - 45*a*b^2*B - 6*a^2*b*(19*A + 28*C))*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(315*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(10*A*b^4 - 435
*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(15*A*b^2 + 90*a*b*
B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sec[c
+ d*x]^(3/2)) + (2*(5*A*b^3 + 75*a^3*B + 135*a*b^2*B + a^2*b*(163*A + 231*C
))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sqrt[Sec[c + d*x]]) + (2
*(5*A*b + 9*a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(63*d*Sec[c + d*x
]^(5/2)) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^
(7/2))
```

Rubi [A] time = 1.74587, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(7a^2(7A+9C)+90abB+15Ab^2\right)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(a^2b(163A+231C)+75a^3B+135ab\right.}{315ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(9/2),x]
```


```
[Out] (-2*(a^2 - b^2)*(10*A*b^3 - 75*a^3*B - 45*a*b^2*B - 6*a^2*b*(19*A + 28*C))*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(315*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(10*A*b^4 - 435
*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(15*A*b^2 + 90*a*b*
B + 7*a^2*(7*A + 9*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sec[c
+ d*x]^(3/2)) + (2*(5*A*b^3 + 75*a^3*B + 135*a*b^2*B + a^2*b*(163*A + 231*C
))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sqrt[Sec[c + d*x]]) + (2
*(5*A*b + 9*a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(63*d*Sec[c + d*x
]^(5/2)) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^
(7/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
```

```
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}dx = \frac{2A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2(5Ab+9aB)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A}{}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\sec(c+d}}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\sec(c+d}}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\sec(c+d}}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\sec(c+d}}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\sqrt{a+b\sec(c+d}}{315d\sec^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\left(a^2-b^2\right)\left(10Ab^3-75a^3B-45ab^2B-6a^2b(19A+\right.}{315a^2d\sqrt{a+}}$$

Mathematica [C] time = 7.03576, size = 6410, normalized size = 14.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.892, size = 6758, normalized size = 15.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(9/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Cb^2 \sec\left(dx+c\right)^4 + \left(2\,Cab + Bb^2\right)\sec\left(dx+c\right)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec\left(dx+c\right)^2 + \left(Ba^2 + 2\,Aa\right.}{\sec\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right) + a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(9/2), x)
```

3.1051 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=565

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\left(15a^2b^2(19A+33C)+75a^4(9A+11C)+1254a^3bB-110ab^3B+40Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Elliptic}}{3465a^3d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*(a^2 - b^2)*(40*A*b^4 + 1254*a^3*b*B - 110*a*b^3*B + 75*a^4*(9*A + 11*C)
 + 15*a^2*b^2*(19*A + 33*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3465*a^3*d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a
^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*a^3*d*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^2 + 44*a*b*B + 3*a^2*(9*A + 11*C)
)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(231*d*Sec[c + d*x]^(5/2)) + (2*(1
5*A*b^3 + 539*a^3*B + 825*a*b^2*B + 5*a^2*b*(229*A + 297*C))*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(3465*a*d*Sec[c + d*x]^(3/2)) - (2*(20*A*b^4 - 179
3*a^3*b*B - 55*a*b^3*B - 75*a^4*(9*A + 11*C) - 5*a^2*b^2*(205*A + 297*C))*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Sqrt[Sec[c + d*x]]) + (2*
(5*A*b + 11*a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d*Sec[c + d*x
]^(7/2)) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]
^(9/2))
```

Rubi [A] time = 2.29866, antiderivative size = 565, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(5a^2b(229A+297C)+539a^3B+825ab^2B+15Ab^3\right)\sqrt{a+b\sec(c+dx)}}{3465ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(3a^2(9A+11C)+\right.}{231d\,\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Se
c[c + d*x]^(11/2),x]
```


```
[Out] (2*(a^2 - b^2)*(40*A*b^4 + 1254*a^3*b*B - 110*a*b^3*B + 75*a^4*(9*A + 11*C)
 + 15*a^2*b^2*(19*A + 33*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3465*a^3*d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a
^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*a^3*d*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(5*A*b^2 + 44*a*b*B + 3*a^2*(9*A + 11*C)
)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(231*d*Sec[c + d*x]^(5/2)) + (2*(1
5*A*b^3 + 539*a^3*B + 825*a*b^2*B + 5*a^2*b*(229*A + 297*C))*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(3465*a*d*Sec[c + d*x]^(3/2)) - (2*(20*A*b^4 - 179
3*a^3*b*B - 55*a*b^3*B - 75*a^4*(9*A + 11*C) - 5*a^2*b^2*(205*A + 297*C))*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Sqrt[Sec[c + d*x]]) + (2*
(5*A*b + 11*a*B)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d*Sec[c + d*x
]^(7/2)) + (2*A*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]
^(9/2))
```


Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2A(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{2}{11}\int \frac{(a+b\,\mathrm{s}}{}$$

$$= \frac{2(5Ab+11aB)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2A}{}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a+b\sec(c+d}}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a+b\sec(c+d}}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a+b\sec(c+d}}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a+b\sec(c+d}}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a+b\sec(c+d}}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\sqrt{a+b\sec(c+d}}{231d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(a^2-b^2\right)\left(40Ab^4+1254a^3bB-110ab^3B+75a^4(9A\right.}{3}$$

Mathematica [C] time = 7.3617, size = 7479, normalized size = 13.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sec[c + d*x]^(11/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.17, size = 7971, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(11/2
),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cb^2 \sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aa\right.}{\sec(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+
c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)
^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sec(d*x + c)^(11/2), x)
```

3.1052 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=350

$$\frac{(4bB-aC)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos}{}}}{4b^2d\sqrt{a+b\sec(c+dx}}$$

```
[Out] ((4*b*B - a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b
^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b
*Sec[c + d*x]]) - ((4*b*B - 3*a*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + ((4*b*B - 3*a*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(4*b^2*d) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(2*b*d)
```

Rubi [A] time = 1.14485, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sqrt{\sec(c+dx)}\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}+\frac{(4bB-3aC)\sin(c+dx)\sqrt{\sec(c+d}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```

```
[Out] ((4*b*B - a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b
^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b
*Sec[c + d*x]]) - ((4*b*B - 3*a*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + ((4*b*B - 3*a*C)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(4*b^2*d) + (C*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(2*b*d)
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \frac{C\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd} + \frac{\int \frac{\sqrt{\sec(c+\ldots}}{\ldots}}{\ldots}$$

$$= \frac{(4bB-3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \ldots$$

$$= \frac{(4bB-3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \ldots$$

$$= \frac{(4bB-3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \ldots$$

$$= \frac{(4bB-3aC)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \ldots$$

$$= \frac{\left(8Ab^2-4abB+3a^2C+4b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\ldots\right.}{4b^2d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(4bB-aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} + \ldots$$

Mathematica [C] time = 4.966, size = 503, normalized size = 1.44

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2i(3aC-4bB)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)+b}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\ldots\right.\right.\right.\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + b*Sec[c + d*x]],x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*a*C*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/b + (2*(16*A*b^2 - 12*a*b*B
 + 9*a^2*C + 8*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/b^2 + ((2*I)*(-4*b*B + 3*a*C)*Sqrt[-((a*(-1 + Cos[
c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c
+ d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sq
rt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticP
i[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)
/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b^3) - (4*a*(-4*b*B + 3*a*C)*Sin[c + d*x
])/b^2 + (8*a*C*Tan[c + d*x])/b + (4*(4*b*B - 3*a*C)*Tan[c + d*x])/b + 8*C*
Sec[c + d*x]*Tan[c + d*x]))/(8*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(c +
 d*x)])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.462, size = 3178, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/4/d/b^2/((a-b)/(a+b))^(1/2)*(4*B*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+8*B*cos(d*
x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-2*C
*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+8*B*cos(d*
x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-2*C
*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*sin(d*x+c)*a*b+3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2-4*B
*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*
a^2+4*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-2*C*((a-b)/(a+b))^(1/2)*cos(d*x+
c)^2*b^2-8*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*B*sin(d*x+c)*cos(d*x+c)^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b+8*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-16*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d
*x+c)*b^2-4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
```

```
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-3*C*cos(d*x+c)^2*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a
^2+6*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+4*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-6*C*
cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-8*C*cos(d*x+c)^2*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*b^2+2*C*((a-b)/(a+b))^(1/2)*b^2+8*A*cos(d*x+c)^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*
b^2-16*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-4*B*cos(d*x+c)^3*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin
(d*x+c)*b^2-3*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+6*C*cos(d*x+c)^3*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)
*a^2+4*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-6*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^2-8*C*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-4*B*((a-b)/(a+b
))^(1/2)*cos(d*x+c)^3*a*b-2*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a*b+4*B*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-C
*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1
/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b
*sec(d*x + c) + a), x)
```

3.1053 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=260

$$\frac{(2A+C)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2bB-aC)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx\right.}{bd\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((2*A + C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - a*C)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (C*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.839786, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2A+C)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2bB-aC)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```


```
[Out] ((2*A + C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - a*C)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (C*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (C*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(b*d)
```


Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \int \frac{-\frac{aC}{2}+Ab \cdots}{\cdots}$$

$$= \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \int \frac{-\frac{aC}{2} \cdots}{\sqrt{\sec(c+ \cdots}}$$

$$= \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} - \frac{C\int \frac{\sqrt{a+b \cdots}}{\sqrt{\mathrm{se} \cdots}}}{\cdots}$$

$$= \frac{C\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{((2A+C \cdots}{\cdots}$$

$$= \frac{(2bB-aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx \cdots}}{bd\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2A+C)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \cdots$$

Mathematica [C] time = 6.48538, size = 427, normalized size = 1.64

$$\frac{\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(8Ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-\frac{2iC\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+d \cdots}{a-b}}\cdots}{\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + b*Sec[c + d*x]],x]
```

```
[Out] ((A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(8*A*b*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*(4*b*B - 3*a*C)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] - ((2*I
)*C*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a
 - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcS
```

```
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*
b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b
)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b) + 4*C*(b + a*
Cos[c + d*x])*Tan[c + d*x]))/(2*b*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*(
c + d*x)])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.389, size = 1638, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d/b/((a-b)/(a+b))^(1/2)*(2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*B*cos(d*x+c)^
2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b+4*B*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b+2*C*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)^2*a-2*C*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-C*cos(d*x
+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*a+C*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+2*A*cos(d*x+c)*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b-2*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*b+2*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a-2*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*a-C*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+C*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+C*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a-C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+C*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*b-C*((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b
*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b
*sec(d*x + c) + a), x)
```

3.1054 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=219

$$-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2C\sqrt{\sec(}{}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.618127, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2C\sqrt{\sec(c+dx)}\sqrt{}}{d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = C\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{A\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} + \frac{\left(C\sqrt{b+a\cos(c+dx)}\right.}{\sqrt{a}}$$
$$= -\frac{\left((Ab-aB)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\left(C\sqrt{\frac{b+a\,\cdots}{}}\right.}{}$$
$$= \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{\left((Ab-aB)\sqrt{\frac{b+a\cos(c+\cdots}{a+b}}\right.}{a\sqrt{\cdots}}$$
$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{\frac{b+a\cos(c+dx\cdots}{a+b}}}{\cdots}$$

Mathematica [F] time = 16.5248, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]), x]
```

Maple [C] time = 0.431, size = 1358, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)/a*(-A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+A*cos(d*x+c)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a-A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
```

```
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-C*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)*cos(d*x+c)*a+2*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-A*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-C*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*
C*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-A*((a-b
)/(a+b))^(1/2)*cos(d*x+c)*a+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-A*b*((a-b)/(
a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d
*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^2 + a\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A}{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sqrt(sec(d*x + c))), x)
```

3.1055 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=216

$$\frac{2\sqrt{\sec(c+dx)}\left(a^2(A+3C)-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{a+b\sec(c+a}}{3a^2d\sqrt{\sec(c+dx)}\sqrt{}}$$

```
[Out] (2*(2*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a +
b*Sec[c + d*x]]) - (2*(2*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.52101, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sqrt{\sec(c+dx)}\left(a^2(A+3C)-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]),x]
```

```
[Out] (2*(2*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a +
b*Sec[c + d*x]]) - (2*(2*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[S
ec[c + d*x]])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(2Ab-3aB)-\frac{1}{2}a(A+3C)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(2Ab-3aB)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a^2} + \frac{1}{3}\Big(A$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(A+\frac{b(2Ab-3aB)}{a^2}+3C\right)\sqrt{b+a\cos(c+}\right.}{3\sqrt{a+b\sec}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(A+\frac{b(2Ab-3aB)}{a^2}+3C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right.}{3\sqrt{a+b\sec(c}}$$

$$= \frac{2\left(A+\frac{b(2Ab-3aB)}{a^2}+3C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} - \frac{2(2}{}$$

Mathematica [C] time = 6.5376, size = 1959, normalized size = 9.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*Sqrt[
a + b*Sec[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(-2*A*b
+ 3*a*B)*Cot[c])/(3*a^2*d) + (4*A*Cos[d*x]*Sin[c])/(3*a*d) + (4*A*Cos[c]*Si
n[d*x])/(3*a*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c
+ d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (4*A*AppellF1[1/2, 1/2, 1/2, 3/2,
(Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqr
t[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(
-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sq
rt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[
1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2
]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(3*a*d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(3/2)*S
qrt[a + b*Sec[c + d*x]]) - (4*C*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a
*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2
]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c]
)/(a*Sqrt[1 + Cot[c]^2])))]*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
- b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - Ar
cTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(a*d*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
+ d*x]]) + (4*A*b*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*
Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sq
rt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]
*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2]
)]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c
]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(3*
a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sq
rt[a + b*Sec[c + d*x]]) - (2*B*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b
+ a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c
]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*
Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^
2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[
1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c
```

```
]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^
2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]
^(3/2)*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.406, size = 1931, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d/a^2/((a-b)/(a+b))^(1/2)*(2*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b+A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-3*B
*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2+3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*C*cos(d*x+c)
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2+2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-2*A*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*B*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d
*x+c)-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+2*A*((a-b)/(a+b))^(1/2)*cos(d*
x+c)*a*b+3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-2*A*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b-3*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*B*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*sin(d*x+c)+A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*A*b^2*((a-b)/(a+b))^(1/2)+A*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2-3*
B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a
^2-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-A*a*b*((a-b)/(a+b))^(1/2)-3*B*((a
```

```
-b)/(a+b))^(1/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/c
os(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(sqrt(a + b*sec(c + d*x))
*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sec(d*x + c)^(3/2)), x)
```

3.1056 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=291

$$-\frac{2\sqrt{\sec(c+dx)}\left(a^2b(7A+15C)-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2(3A+5C\right.}{}$$

```
[Out] (-2*(8*A*b^3 - 5*a^3*B - 10*a*b^2*B + a^2*b*(7*A + 15*C))*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])
/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A
+ 5*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15
*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(4*A*b - 5
*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.817101, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sqrt{\sec(c+dx)}\left(a^2b(7A+15C)-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2(3A+5C)-10ab\right.}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*Sqrt[a + b*
Sec[c + d*x]]),x]
```

```
[Out] (-2*(8*A*b^3 - 5*a^3*B - 10*a*b^2*B + a^2*b*(7*A + 15*C))*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])
/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A
+ 5*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15
*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(4*A*b - 5
*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{1}{2}(4Ab-5aB)-\frac{1}{2}a(3A+5C)\sec(c+dx)-Ab\,\mathrm{s}}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{5a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c+dx)}\sin}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\left(8Ab^3-5a^3B-10ab^2B+a^2b(7A+15C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{15a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.65816, size = 3039, normalized size = 10.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*Sqrt[
a + b*Sec[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(9*a^2*A
 + 8*A*b^2 - 10*a*b*B + 15*a^2*C)*Cot[c])/(15*a^3*d) + (4*(-4*A*b + 5*a*B)*
Cos[d*x]*Sin[c])/(15*a^2*d) + (2*A*Cos[2*d*x]*Sin[2*c])/(5*a*d) + (4*(-4*A*
b + 5*a*B)*Cos[c]*Sin[d*x])/(15*a^2*d) + (2*A*Cos[2*c]*Sin[2*d*x])/(5*a*d))
)/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqr
t[a + b*Sec[c + d*x]]) - (8*A*b*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a
*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2
]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c]
)/(a*Sqrt[1 + Cot[c]^2])))]*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
- b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - Ar
cTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(15*a^2*d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*S
ec[c + d*x]]) - (4*B*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*C
sc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1
 + Cot[c]^2])))]*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt
[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])
]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]
]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]
```

*Sin[d*x - ArcTan[Cot[c]]]])/(3*a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (6*A*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (16*A*b^2*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*a^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (4*b*B*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(3*a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (2*C*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]])

Maple [B] time = 0.532, size = 3439, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/15/d/a^3/((a-b)/(a+b))^(1/2)*(-8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^2+9*A*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3+9*A*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*sin(d*x+c)*cos(d*x+c)*a^3-8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+5*B*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*cos(d*x+c)*a^3-15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3+15*C*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*a^3+2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*A*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*a^3*((a-b)/(a+b))^(1/2)*cos(d
*x+c)+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(
d*x+c)^2*a^3+15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3-9*A*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*a^3+8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3-15*C*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)*a^3-8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-9*A*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2
*b+8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+10*B*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b-10*
B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*cos(d*x+c)*a^2*b+10*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
```

```
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-15*C*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a^2*b+2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b-9*A*a^2*b*((a-
b)/(a+b))^(1/2)+4*A*a*b^2*((a-b)/(a+b))^(1/2)-5*B*a^2*b*((a-b)/(a+b))^(1/2)
+10*B*a*b^2*((a-b)/(a+b))^(1/2)-15*C*((a-b)/(a+b))^(1/2)*a^2*b+5*B*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)-15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3*sin(d*x+c)+15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-9*A*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8
*A*b^3*((a-b)/(a+b))^(1/2)-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+4*A*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^
2*b+10*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-8*A*((a-b)/(a+b))^(1/2)*cos(d
*x+c)*a*b^2+10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-10*B*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*a*b^2+15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b+8*A*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+10*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-10*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+10*B*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5
/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b\sec\left(dx+c\right)^{4}+a\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b*sec(d*x + c)^4 + a*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sec(d*x + c)^(5/2)), x)
```

3.1057 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=380

$$\frac{2\sqrt{\sec(c+dx)}\left(2a^2b^2(16A+35C)+5a^4(5A+7C)-49a^3bB-56ab^3B+48Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^4d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*(48*A*b^4 - 49*a^3*b*B - 56*a*b^3*B + 5*a^4*(5*A + 7*C) + 2*a^2*b^2*(16*
A + 35*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^4*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*
A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^4*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(7*a*d*Sec[c + d*x]^(5/2)) - (2*(6*A*b - 7*a*B)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(35*a^2*d*Sec[c + d*x]^(3/2)) + (2*(24*A*b^2 - 28*a*b*B +
 5*a^2*(5*A + 7*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^3*d*Sqrt[
Sec[c + d*x]])
```

Rubi [A] time = 1.17775, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(5a^2(5A+7C)-28abB+24Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105a^3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\sec(c+dx)}\left(2a^2b^2(16A+35C)+5a^4(5A+7C)\right.}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*Sqrt[a + b*
Sec[c + d*x]]),x]
```


```
[Out] (2*(48*A*b^4 - 49*a^3*b*B - 56*a*b^3*B + 5*a^4*(5*A + 7*C) + 2*a^2*b^2*(16*
A + 35*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^4*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*
A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^4*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(7*a*d*Sec[c + d*x]^(5/2)) - (2*(6*A*b - 7*a*B)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(35*a^2*d*Sec[c + d*x]^(3/2)) + (2*(24*A*b^2 - 28*a*b*B +
 5*a^2*(5*A + 7*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^3*d*Sqrt[
Sec[c + d*x]])
```


Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2\int \frac{\frac{1}{2}(6Ab-7aB)-\frac{1}{2}a(5A+7C)\sec(c+dx)-2Ab\sec^2}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{7a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{35a^2d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{35a^2d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{35a^2d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{35a^2d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{2(6Ab-7aB)\sqrt{a+b\sec(c+dx)}\sin(c+}{35a^2d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(48Ab^4-49a^3bB-56ab^3B+5a^4(5A+7C)+2a^2b^2(16A+35C)\right)\sqrt{\frac{b+a\cos}{a}}}{105a^4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 6.92929, size = 4470, normalized size = 11.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(7/2)*Sqrt[
a + b*Sec[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(-44*a^2
*A*b - 48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 70*a^2*b*C)*Cot[c])/(105*a^4*d) +
 ((115*a^2*A + 96*A*b^2 - 112*a*b*B + 140*a^2*C)*Cos[d*x]*Sin[c])/(105*a^3*
d) + (2*(-6*A*b + 7*a*B)*Cos[2*d*x]*Sin[2*c])/(35*a^2*d) + (A*Cos[3*d*x]*Si
n[3*c])/(7*a*d) + ((115*a^2*A + 96*A*b^2 - 112*a*b*B + 140*a^2*C)*Cos[c]*Si
n[d*x])/(105*a^3*d) + (2*(-6*A*b + 7*a*B)*Cos[2*c]*Sin[2*d*x])/(35*a^2*d) +
 (A*Cos[3*c]*Sin[3*d*x])/(7*a*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (20*A*AppellF1[1/
2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))),
 (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sq
rt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Sqrt[b + a*Cos[
c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[
Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a
*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Cs
c[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(21
*a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*S
ec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (16*A*b^2*AppellF1[1/2, 1/2,
1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]
```

```
*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + C
ot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))))]*Sqrt[b + a*Cos[c + d*x]
]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]]
)/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1
+ Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*S
qrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(35*a^3*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c +
 d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (8*b*B*AppellF1[1/2, 1/2, 1/2, 3/2,
 (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sq
rt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*
(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*S
qrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt
[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^
2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a
*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(15*a^2*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(3/
2)*Sqrt[a + b*Sec[c + d*x]]) - (4*C*AppellF1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b
 - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[
c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[c]*(b - a*Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Cs
c[c])/(a*Sqrt[1 + Cot[c]^2])))]*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot
[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]
^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x
- ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c])]*Sqrt[b - a*Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(3*a*d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*Sec[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]) + (88*A*b*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(
1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c]
)/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Ta
n[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Ta
n[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b +
 a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^
2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]))/(105*a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]
^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (32*A*b^3*Sqrt[b + a*Cos[c + d*x]]*Csc[c
]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2,
 -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*
Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b
+ a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c
]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sq
rt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec
[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
)/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]))/(35*a^3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (6*B*Sqrt[b + a*Cos
```

```
[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2,
-1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))
), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(
a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]
^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (16*b^2*
B*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(
(AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[
1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]
))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1
+ Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a
*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a
*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*a^2*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]) + (4*b*C*Sqrt[b + a*Cos[c + d*x]]*Csc[c]*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (
b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*
Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^
2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^
2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin
[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
3*a*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(3/2)*
Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.685, size = 4764, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 2/105/d/a^4/((a-b)/(a+b))^(1/2)*(-48*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-44*A*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
```

b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*c
os(d*x+c)*a^3*b+25*A*a^3*b*((a-b)/(a+b))^(1/2)-44*A*a^2*b^2*((a-b)/(a+b))^(
1/2)+24*A*a*b^3*((a-b)/(a+b))^(1/2)+63*B*a^3*b*((a-b)/(a+b))^(1/2)-28*B*a^2
b^2((a-b)/(a+b))^(1/2)+56*B*a*b^3*((a-b)/(a+b))^(1/2)+35*C*a^3*b*((a-b)/(
a+b))^(1/2)-70*C*a^2*b^2*((a-b)/(a+b))^(1/2)-63*B*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-35*C*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
a^4(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^
3*b-6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+7*B*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^3*b+16*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+24*A*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a*b^3-28*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2
+35*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-44*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a^3*b+50*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-48*A*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b^3-70*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+56*B*cos(d
x+c)((a-b)/(a+b))^(1/2)*a^2*b^2-56*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3
-70*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+70*C*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a^2*b^2-48*A*b^4*((a-b)/(a+b))^(1/2)+25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a^4+35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-15*A*cos(d*x+c)^5*((a-b)/(a+b
))^(1/2)*a^4-10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-35*C*cos(d*x+c)^3*((
a-b)/(a+b))^(1/2)*a^4-21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4-42*B*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^4+48*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+63*B
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-25*A*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*cos(d*x+c)*a^4-48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4+63*B*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+
c)*a^4-63*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-35*C*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4-44
*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+12*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-48*A*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+44*
A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-44*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+48*A*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-14*B
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+56*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+63*B*EllipticE((-1+cos(d*x+c)

```
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-56*B*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)+56*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-70*C*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+70*C*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-70*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+12*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^
2*b^2-48*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^3+44*A*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b
-44*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-14
*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b+56*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^2+63*B*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)*a^3*b-56*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+56*B*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a*b^3-70*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b+70*C*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a^3*b-70*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2)*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(
d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b\sec\left(dx+c\right)^{5}+a\sec\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b*sec(d*x + c)^5 + a*sec(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(7/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sec(d*x + c)^(7/2)), x)
```

3.1058 $\int \frac{\sqrt{\sec(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{(2aA+bB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.07895, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {4072, 4031, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2aA+bB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c + d*x]^
2))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d
```


Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4031

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n -
1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B*m
 + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B},
```

```
x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(-abB+b(Ab\right.}{b^2}$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + $$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + $$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}$$

$$= \frac{B\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + $$

$$= \frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 4.35286, size = 377, normalized size = 1.49

$$\frac{\sqrt{\sec(c+dx)}\left(8aA\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-\frac{2iB\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)+b}\left(a\left(2b\text{EllipticF}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c +
 d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(8*a*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*a)/(a + b)] + 2*(4*A*b + a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] - ((2*I)*B*Sqrt[-((a*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c
 + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*
Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[S
qrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Elliptic
Pi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b
)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b) + 4*B*(b + a*Cos[c + d*x])*Tan[c + d
*x]))/(4*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.468, size = 1431, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*a+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d
*x+c))^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)^2*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b+4*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b+2*B*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^2*a-B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a+B*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*b+2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b+4*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+2*B*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*a-B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a+B*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+B*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a-B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a+B*((a-b
)/(a+b))^(1/2)*cos(d*x+c)*b-B*((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(
d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(sec(d*
x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(sec(d*
x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.1059 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=393

$$\frac{C\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx}{}$$

```
[Out] (C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 3*a*C)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((2*A*b^2 - 2*a*b*B +
3*a^2*C - b^2*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*
x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2
- b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)
*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)
*d)
```

Rubi [A] time = 1.3543, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(3a^2C-2abB+2Ab^2-b^2C\right)\sqrt{a+b\sec(}}{b^2d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```


```
[Out] (C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 3*a*C)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((2*A*b^2 - 2*a*b*B +
3*a^2*C - b^2*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*
x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2
- b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)
*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)
*d)
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
```

```
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sqrt{\sec(c+}}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2\right.}{}$$

$$= \frac{(2bB-3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{(2bB-3aC}{}$$

Mathematica [C] time = 7.03888, size = 774, normalized size = 1.97

$$\frac{(a\cos(c+dx)+b)^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{2C\tan(c+dx)}{b^2}-\frac{4\left(-a^2bB\sin(c+dx)+a^3C\sin(c+dx)+aAb^2\sin(c+dx)\right)}{b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)}\right)}{d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)} + \frac{(a\,c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*(4*
A*b^3 - 4*a*b^2*B + 4*a^2*b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(2*a*A*b^2 - 6*
a^2*b*B + 4*b^3*B + 9*a^3*C - 7*a*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*
I)*(2*a*A*b^2 - 2*a^2*b*B + 3*a^3*C - a*b^2*C)*Sqrt[(a - a*Cos[c + d*x])/(a
 + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*El
lipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a
 + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1
)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a -
b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-
a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(2*b^
2*(-a + b)*(a + b)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt
[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(a*A*b^2*Sin[c + d*x] - a^2*b*B*Sin[
c + d*x] + a^3*C*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (
2*C*Tan[c + d*x])/b^2))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
```

```
)*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.41, size = 3121, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/(a+b)/b^2*(-2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*C*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2+2*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+2*B*cos(d*
x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*C
*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*sin(d*x+c)*a*b-4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*cos(d*x+
c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b+4*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*C*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*a*b+3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2-6*C*cos(d*x
+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*a*b+2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+C*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*b^2-4*B*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-C*((a-b)/(a+b))^(1/2)*a*b+2*A*
cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*sin(d*x+c)*b^2-3*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+6*C*cos(d*x
+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a^2-6*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-2*A*
((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2-C
*((a-b)/(a+b))^(1/2)*b^2-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
```

```
n(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-6*C*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a^2-2*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+C*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-2*B*cos(
d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*sin(d*x+c)*b^2+4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+C
*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*sin(d*x+c)*b^2+2*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2-2*B*cos(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*b^2+4*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+C*cos(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^(3/2), x)
```

3.1060 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=311

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(Ab^2-a(b\right.}{aba}$$

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*b*(a^2 - b^2)*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*
B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[
c + d*x]])
```

Rubi [A] time = 0.977017, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{abd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```


```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*b*(a^2 - b^2)*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*
B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[
c + d*x]])
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-Ab}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-Ab}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{A\int \frac{\sqrt{se}}{\sqrt{a+b}}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{b+a}\right.}{}$$

$$= \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2\left(Ab^2\right.}{}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{\frac{b+a\,c}{}}}{}$$

Mathematica [F] time = 33.5075, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(3/2), x]
```

Maple [C] time = 0.467, size = 2053, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 2/d/a/b/(a+b)/((a-b)/(a+b))^(1/2)*(-A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-A*cos(d*x
+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^2-B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+B*cos(d*x+c)*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b+2*C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+C*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-C*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a^2-2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-2*C*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-A*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+C*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d
*x+c)-C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*sin(d*x+c)-2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-2*C*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-B*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*a*b+C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2-A*b^2*((a-b)/(a+b))
^(1/2)+B*((a-b)/(a+b))^(1/2)*a*b-C*((a-b)/(a+b))^(1/2)*a^2)*cos(d*x+c)*(1/c
os(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec
(d*x + c) + a)^(3/2), x)
```

3.1061 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{2(2Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(
2*A*b^2 - a*b*B - a^2*(A - C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.605477, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(a^2(-(A-C))-abB+2Ab^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(
2*A*b^2 - a*b*B - a^2*(A - C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2 - a(bB - aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(2Ab^2-abB-a^2(A-C)\right)+}{\sqrt{\sec(c+dx)}\sqrt{}}}{a\left(a^2\right.}$$

$$= \frac{2\left(Ab^2 - a(bB - aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2Ab - aB)\int \frac{\sqrt{\sec(c+d}}{\sqrt{a+b\sec(c-}}}{a^2}$$

$$= \frac{2\left(Ab^2 - a(bB - aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab - aB)\sqrt{b + a\cos}\right.}{a^2}$$

$$= \frac{2\left(Ab^2 - a(bB - aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab - aB)\sqrt{\frac{b+a\cos(c+}{a+b}}\right.}{a^2\sqrt{}}$$

$$= -\frac{2(2Ab - aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2 d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(2Ab^2 - a\right.}{}$$

Mathematica [C] time = 7.04799, size = 3541, normalized size = 14.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a +
b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(a^2*A
 - 3*A*b^2 + 2*a*b*B - 2*a^2*C + a^2*A*Cos[2*c] - A*b^2*Cos[2*c])*Csc[c]*Se
c[c])/(a^2*(a^2 - b^2)*d) - (4*Sec[c]*(A*b^3*Sin[c] - a*b^2*B*Sin[c] + a^2*
b*C*Sin[c] - a*A*b^2*Sin[d*x] + a^2*b*B*Sin[d*x] - a^3*C*Sin[d*x]))/(a^2*(a
^2 - b^2)*d*(b + a*Cos[c + d*x]))))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*A*b*AppellF
1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]
))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(
a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[
c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - A
rcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
+ b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
])/(a*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (4*B*AppellF
1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]
))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(
a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[
c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - A
rcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
+ b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
])/((a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*b*C*AppellF
1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
```

cTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]
))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(
a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[
c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - A
rcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
+ b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
])/(a*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*a*A*(b +
a*Cos[c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Appe
llF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + T
an[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan
[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt
[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/((a^2 - b^2)*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + (4*A*b^2*(b + a*Cos[c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*
(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c
])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + T
an[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] +
a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + T
an[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
 - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b
+ a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a
^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(a*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*b*B*(b + a*Cos[c + d*x])^(
3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2,
-1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((
Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt
[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*
Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
)/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/((a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2
*a*C*(b + a*Cos[c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*
Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[
c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sq

```
rt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c
] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b +
 a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[
b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/((a^2 - b^2)*d
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a +
b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.457, size = 1889, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
,x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)/(a+b)/a^2*(-C*((a-b)/(a+b))^(1/2)*a^2-2*A*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)-2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*A*
cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^2+B*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+C*cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+A*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^2*a*b-B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+B*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b+B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+A*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a^2+C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*sin(d*x+c)-A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*A*b^2*((a-b)/(a+b))^(1/2
)+A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*sin(d*x+c)*a^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+2*A*((
a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2+C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+A*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
```

```
*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-A*a*b*((a-b)/(a+b))^(1/2)+B*((a
-b)/(a+b))^(1/2)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1
/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{2}\sec\left(dx+c\right)^{3}+2\,ab\sec\left(dx+c\right)^{2}+a^{2}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/
2)*sqrt(sec(d*x + c))), x)
```

3.1062 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=350

$$\frac{2\sqrt{\sec(c+dx)}\left(a^2(A+3C)-6abB+8Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(a^2(-(A-3C))-3a\right.}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] (2*(8*A*b^2 - 6*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a +
b*Sec[c + d*x]]) + (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^
2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.943284, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(a^2(-(A-3C))-3abB+4Ab^2\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] (2*(8*A*b^2 - 6*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a +
b*Sec[c + d*x]]) + (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^
2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(4Ab^2-3abB-a^2(A-3C)\right)+\frac{1}{2}a(}{\sec^{\frac{3}{2}}}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3abB-a^2(A-3C\right.}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3abB-a^2(A-3C\right.}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3abB-a^2(A-3C\right.}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3abB-a^2(A-3C\right.}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2\left(8Ab^2-6abB+a^2(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 7.56605, size = 4557, normalized size = 13.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(-5*a^
2*A*b + 11*A*b^3 + 3*a^3*B - 9*a*b^2*B + 6*a^2*b*C - 5*a^2*A*b*Cos[2*c] + 5
*A*b^3*Cos[2*c] + 3*a^3*B*Cos[2*c] - 3*a*b^2*B*Cos[2*c])*Csc[c]*Sec[c])/(3*
a^3*(a^2 - b^2)*d) + (4*A*Cos[d*x]*Sin[c])/(3*a^2*d) + (4*A*Cos[c]*Sin[d*x]
)/(3*a^2*d) + (4*Sec[c]*(A*b^4*Sin[c] - a*b^3*B*Sin[c] + a^2*b^2*C*Sin[c] -
 a*A*b^3*Sin[d*x] + a^2*b^2*B*Sin[d*x] - a^3*b*C*Sin[d*x]))/(a^3*(a^2 - b^2
)*d*(b + a*Cos[c + d*x]))))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (4*A*AppellF1[1/2, 1/2
, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (Csc[
c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt[1 +
 Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[c + d*x])^
(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[
c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a*Sqr
t[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Csc[c]
)]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(3*(a^2
 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]
^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (8*A*b^2*AppellF1[1/2,
 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]))), (
Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(a*Sqrt
[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[c + d*
x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[
```

Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^2] + a
*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] + b*Cs
c[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]])/(3*
a^2*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*b*B*AppellF
1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]
))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(
a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[
c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - A
rcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
+ b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
])/(a*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (4*C*AppellF
1[1/2, 1/2, 1/2, 3/2, (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]]))/(a*Sqrt[1 + Cot[c]^2]*(1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2]
))), (Csc[c]*(b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]))/(
a*Sqrt[1 + Cot[c]^2]*(-1 + (b*Csc[c])/(a*Sqrt[1 + Cot[c]^2])))]*(b + a*Cos[
c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[(a*Sqrt[1 + Cot[c]^2] - a*Sqrt[1 + Cot[c]^2]*Sin[d*x - A
rcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2] - b*Csc[c])]*Sqrt[(a*Sqrt[1 + Cot[c]^
2] + a*Sqrt[1 + Cot[c]^2]*Sin[d*x - ArcTan[Cot[c]]])/(a*Sqrt[1 + Cot[c]^2]
+ b*Csc[c])]*Sqrt[b - a*Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
])/((a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2]*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (10*A*b*(b + a
*Cos[c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Appel
lF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Ta
n[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[
c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[
1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(3*(a^2 - b^2)*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
 d*x])^(3/2)) - (16*A*b^3*(b + a*Cos[c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a
*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2
]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec
[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 +
 Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2]
+ a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 +
 Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(
b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 +
 a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(3*a^2*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*a*B*(b + a*Cos[c + d
*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2,
-1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))

```
), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(
a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]
^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/((a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)
) + (4*b^2*B*(b + a*Cos[c + d*x])^(3/2)*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[c]*(b + a*Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(1 - (b*Sec
[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]^2]*(-1 - (b*Sec[c])/(a*Sqrt[
1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 + Tan[c]^2]*Sq
rt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqrt[1 + Tan[c]^2] + a*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[c]) + a*Sqrt[1 + Tan[c]^2])]*
Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*a*Cos[c]*(b + a*Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a^2*Cos[c]^2 + a^2*Sin[c]^2
))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(a*(a^
2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*b*C*(b + a*Cos[c + d*x])^(3/2)*Csc[c]
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((AppellF1[-1/2, -1/2, -1/2, 1/2,
-((Sec[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*S
qrt[1 + Tan[c]^2]*(1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2])))), -((Sec[c]*(b +
 a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(a*Sqrt[1 + Tan[c]
^2]*(-1 - (b*Sec[c])/(a*Sqrt[1 + Tan[c]^2]))))]*Sin[d*x + ArcTan[Tan[c]]]*T
an[c])/(Sqrt[1 + Tan[c]^2]*Sqrt[(a*Sqrt[1 + Tan[c]^2] - a*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2])/(b*Sec[c] + a*Sqrt[1 + Tan[c]^2])]*Sqrt[(a*Sqr
t[1 + Tan[c]^2] + a*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(-(b*Sec[
c]) + a*Sqrt[1 + Tan[c]^2])]*Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
 + (2*a*Cos[c]*(b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))
/(a^2*Cos[c]^2 + a^2*Sin[c]^2))/Sqrt[b + a*Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]))/((a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.404, size = 2733, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -2/3/d/a^3/(a+b)/((a-b)/(a+b))^(1/2)*(8*A*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^2+3*B
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*sin(d*x+c)+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
```

```
),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+A*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3+8*A*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)*b^3-3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3+3*C*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*cos(d*x+c)*a^3+6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*A*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*A*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-3*B*a^3*((a-b)/(a+b))^(1/2)*cos(d*x+c)-A*((a-b)/(a+b))^(1/2)*cos
(d*x+c)*a^3-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3+8*A*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-5*A
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*a^2*b-6*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b-6*B*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a*b^2+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+6*A*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
cos(d*x+c)*a^2*b+A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+3*B*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a^3+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-A*a^2*b*((a-b)/(a+b))
^(1/2)+4*A*a*b^2*((a-b)/(a+b))^(1/2)-3*B*a^2*b*((a-b)/(a+b))^(1/2)-6*B*a*b^
2*((a-b)/(a+b))^(1/2)+3*C*((a-b)/(a+b))^(1/2)*a^2*b-4*A*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a^2*b-3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+A*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+8*A*b^3*((a-b)/(a+b))^(1/2)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b-
4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2+3*B*((a-b)/(a+b))^(1/2)*cos(d*x+
c)^2*a^2*b+4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b+6*B*((a-b)/(a+b))^(1/2)
*cos(d*x+c)*a*b^2-3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b-6*B*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-6*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+
c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{2}\sec\left(dx+c\right)^{4}+2\,ab\sec\left(dx+c\right)^{3}+a^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x
+ c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/
2)*sec(d*x + c)^(3/2)), x)
```

3.1063 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=461

$$-\frac{2\sqrt{\sec(c+dx)}\left(6a^2b(2A+5C)-5a^3B-40ab^2B+48Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^4d\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(\right.}{}$$

```
[Out] (-2*(48*A*b^3 - 5*a^3*B - 40*a*b^2*B + 6*a^2*b*(2*A + 5*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]
])/(15*a^4*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*A*b^4 + 25*a^3*b*B - 40*a*b
^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c
 + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (2
*(6*A*b^2 - 5*a*b*B - a^2*(A - 5*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) + (2*(24*A*b^3 + 5*a^3*B - 20*a*b
^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3
*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.37268, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(a^2(-(A-5C))-5abB+6Ab^2\right)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] (-2*(48*A*b^3 - 5*a^3*B - 40*a*b^2*B + 6*a^2*b*(2*A + 5*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]
])/(15*a^4*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*A*b^4 + 25*a^3*b*B - 40*a*b
^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c
 + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - (2
*(6*A*b^2 - 5*a*b*B - a^2*(A - 5*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) + (2*(24*A*b^3 + 5*a^3*B - 20*a*b
^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3
*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(6Ab^2-5abB-a^2(A-5C)\right)+}{}}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-\right.}{5a^2\left(a^2-\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-\right.}{5a^2\left(a^2-\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-\right.}{5a^2\left(a^2-\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-\right.}{5a^2\left(a^2-\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\left(6Ab^2-5abB-a^2(A-\right.}{5a^2\left(a^2-\right.}$$

$$= -\frac{2\left(48Ab^3-5a^3B-40ab^2B+6a^2b(2A+5C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx\right.}{15a^4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 8.06046, size = 6134, normalized size = 13.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + 
b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.52, size = 4114, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -2/15/d/a^4/(a+b)/((a-b)/(a+b))^(1/2)*(-48*A*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+
```

c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-12*A*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)*a^3*b-9*A*a^3*b*((a-b)/(a+b))^(1/2)-24*A*a*b^3*((a-b)/(a+b)
)^(1/2)-5*B*a^3*b*((a-b)/(a+b))^(1/2)+20*B*a^2*b^2*((a-b)/(a+b))^(1/2)+40*B
*a*b^3*((a-b)/(a+b))^(1/2)-15*C*a^3*b*((a-b)/(a+b))^(1/2)-30*C*a^2*b^2*((a-
b)/(a+b))^(1/2)-15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*A*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*A*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*cos(d*x+c)*a^4+15*C*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^4+9*A*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+15*C*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^4*sin(d*x+c)+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-6*A*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*a^2*b^2+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b+6*
A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a*b^3-20*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2+15*C*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a^3*b+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-18*A*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+20*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*
b-40*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+30*C*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b^2+24*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^2-20*B*((a-b)/(a+b
))^(1/2)*cos(d*x+c)^2*a^3*b-6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b+3*A*
((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a
^4+15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4+5*B*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^3*a^4-48*A*b^4*((a-b)/(a+b))^(1/2)-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^4-15*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+48*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b^4-5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+5*B*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*A*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)*cos(d*x+c)*a^4-48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4+5*B*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*cos(d*x+c)*a^4-15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4-12*A*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-36
*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-48*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+24*A*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3
0*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+40*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+

```
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-25*B*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+40
*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-30*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-30*C*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-36*
A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^2-48*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^3+24*A*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*sin(d*x+c)*cos(d*x+c)*a^2*b^2+30*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b+40*B*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)*cos(d*x+c)*a^2*b^2-25*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+40*B*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*a*b^3-30*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b-30*C*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*a^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/co
s(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{2}\sec\left(dx+c\right)^{5}+2\,ab\sec\left(dx+c\right)^{4}+a^{2}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x
+ c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec{(dx+c)}^2 + B \sec{(dx+c)} + A}{(b \sec{(dx+c)} + a)^{\frac{3}{2}} \sec{(dx+c)}^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/
2)*sec(d*x + c)^(5/2)), x)
```

3.1064 $\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=563

$$\frac{\sqrt{\sec(c+dx)}\left(5a^2C-2abB+2Ab^2-3b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab\right.}{3bd\left(a^2-b^2\right)(a+b\sec(\right.}$$

```
[Out] ((2*A*b^2 - 2*a*b*B + 5*a^2*C - 3*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 5*a*C)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^
3*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b^4 + 6*a^3*b*B - 14*a*b^3*B - 15*a^4
*C + 26*a^2*b^2*C - 3*b^4*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
 b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^
3*b*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((8*A*b^4 + 6*a^3
*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[Sec[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.94704, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B\right.}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```


```
[Out] ((2*A*b^2 - 2*a*b*B + 5*a^2*C - 3*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 5*a*C)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^
3*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b^4 + 6*a^3*b*B - 14*a*b^3*B - 15*a^4
*C + 26*a^2*b^2*C - 3*b^4*C)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
 b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*B - a*C))*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^
3*b*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((8*A*b^4 + 6*a^3
*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[Sec[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d)
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^{\frac{3}{2}}(c+}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4+\right.}{}$$

$$= \frac{(2bB-5aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^3d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(2Ab^2-2abB+5a^2C-3b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{3b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 7.34284, size = 938, normalized size = 1.67

$$\frac{(b+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\left(C\sec^2(c+dx)+B\sec(c+dx)+A\right)\left(-\frac{4\left(C\sin(c+dx)a^3-bB\sin(c+dx)a^2+Ab^2\sin(c+dx)a\right)}{3b^2\left(b^2-a^2\right)(b+a\cos(c+dx))^2}-\frac{4(-6C}{}\right.}{d(\cos(2c+2dx)A+A+2C+2B\cos(c+dx))(a+b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((b + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2)*((2*(-4*a^2*A*b^3 - 12*A*b^5 - 8*a^3*b^2*B + 24*a*b^4*B + 20*a
^4*b*C - 36*a^2*b^3*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(-8*a*A*b^4 - 18*a^4*b*
B + 38*a^2*b^3*B - 12*b^5*B + 45*a^5*C - 86*a^3*b^2*C + 33*a*b^4*C)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt
[b + a*Cos[c + d*x]] + ((2*I)*(-8*a*A*b^4 - 6*a^4*b*B + 14*a^2*b^3*B + 15*a
^5*C - 26*a^3*b^2*C + 3*a*b^4*C)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a
 + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*
b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b
)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqr
```

```
t[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 -
4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(6*(a - b)^2*b^3*(a
 + b)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c +
d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((-4*(a*A*b^2*Sin[c + d*x] - a^2*b*B*Sin[c + d*x] +
a^3*C*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (4*(4*a*
A*b^4*Sin[c + d*x] + 3*a^4*b*B*Sin[c + d*x] - 7*a^2*b^3*B*Sin[c + d*x] - 6*
a^5*C*Sin[c + d*x] + 10*a^3*b^2*C*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b +
 a*Cos[c + d*x])) + (2*C*Tan[c + d*x])/b^3))/(d*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2))
```

Maple [C] time = 0.615, size = 9944, normalized size = 17.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right) \sec \left(dx+c\right)^{\frac{5}{2}}}{\left(b \sec \left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec
(d*x + c) + a)^(5/2), x)
```

3.1065 $\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=447

$$-\frac{2\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3abd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB\right.}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*b*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*
x]]) - (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*b^2*(a^2 - b^2)^2*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*
B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c
+ d*x])^(3/2)) + (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x
]])
```

Rubi [A] time = 1.44901, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(a^2b^2(3A+7C)-3a^4C-4ab^3B+Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```


```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*b*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*
x]]) - (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*b^2*(a^2 - b^2)^2*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b^2 - a*(b*
B - a*C))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c
+ d*x])^(3/2)) + (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x
]])
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
```

```
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - 2\int \frac{\sqrt{\mathrm{se}}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4\right.}{}$$

$$= \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(A\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}}}{3ab\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [F] time = 52.9623, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2), x]
```

Maple [C] time = 0.478, size = 7030, normalized size = 15.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec \left(dx+c\right)^2+B\sec \left(dx+c\right)+A\right)\sec \left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec \left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^(5/2), x)
```

3.1066 $\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=378

$$-\frac{2\sqrt{\sec(c+dx)}\left(a^2(-(3A+C))+abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-5\right.}{3abd\left(a^2\right.}$$

```
[Out] (-2*(2*A*b^2 + a*b*B - a^2*(3*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]]) - (2*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*
A + 2*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]
) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(A*b^4 + 2*a^3*b*B + 2*a*b^3*B + a
^4*C - 5*a^2*b^2*(A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*b*(a^2 - b^
2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.03745, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4098, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-5a^2b^2(A+C)+2a^3bB+a^4C+2ab^3B+Ab^4\right)}{3abd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB-a\right.}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```


```
[Out] (-2*(2*A*b^2 + a*b*B - a^2*(3*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]]) - (2*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*
A + 2*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]
) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(A*b^4 + 2*a^3*b*B + 2*a*b^3*B + a
^4*C - 5*a^2*b^2*(A + C))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*b*(a^2 - b^
2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-Ab}{}}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4+\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4+\right.}{}$$

$$= -\frac{2\left(2Ab^2+abB-a^2(3A+C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{3a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 7.61037, size = 5040, normalized size = 13.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.463, size = 5169, normalized size = 13.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec
(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*se
c(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec
(d*x + c) + a)^(5/2), x)
```

3.1067 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=401

$$\frac{2\sqrt{\sec(c+dx)}\left(-a^2b(9A+C)+3a^3B-2ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\sec(c+}{3}$$

```
[Out] (2*(8*A*b^3 + 3*a^3*B - 2*a*b^2*B - a^2*b*(9*A + C))*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a
^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 6*a^3*b*B - 2*a*
b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*
(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2*a^2*b^2*(4*A + C))*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.03434, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(-2a^2b^2(4A+C)+5a^3bB-2a^4C-ab^3B+4Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\left(Ab^2-a(bB\right.}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] (2*(8*A*b^3 + 3*a^3*B - 2*a*b^2*B - a^2*b*(9*A + C))*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a
^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 6*a^3*b*B - 2*a*
b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*
(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2*a^2*b^2*(4*A + C))*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - 2\int \frac{\frac{1}{2}\left(4Ab^2-abB-a^2(3A-C)\right)+\frac{3}{2}\ldots}{\ldots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab^4+5a^3bB-ab^3B-\ldots\right.}{3a^2\left(\ldots\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab^4+5a^3bB-ab^3B-\ldots\right.}{3a^2\left(\ldots\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab^4+5a^3bB-ab^3B-\ldots\right.}{3a^2\left(\ldots\right.}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab^4+5a^3bB-ab^3B-\ldots\right.}{3a^2\left(\ldots\right.}$$

$$= \frac{2\left(8Ab^3+3a^3B-2ab^2B-a^2b(9A+C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\ldots}}{3a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 8.30899, size = 6142, normalized size = 15.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.51, size = 6945, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{4}+3\,ab^{2}\sec\left(dx+c\right)^{3}+3\,a^{2}b\sec\left(dx+c\right)^{2}+a^{3}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^3*sec(d*x + c)^4 + 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*se
c(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2)/sec(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*sqrt(sec(d*x + c))), x)
```

3.1068 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=521

$$-\frac{2\sqrt{\sec(c+dx)}\left(-2a^2b^2(8A-C)+a^4(-(A+3C))+9a^3bB-8ab^3B+16Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+$$

```
[Out] (-2*(16*A*b^4 + 9*a^3*b*B - 8*a*b^3*B - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*C)
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(16
*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) + a^4*(8
*A*b - 6*b*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[S
ec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a
^3*b*B + 3*a*b^3*B + 4*a^4*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B
+ a^4*(A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.60248, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(-a^2b^2(13A-C)+a^4(A-5C)+8a^3bB-4ab^3B+8Ab^4\right)\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{2\sin(c+dx)\left(10a^2Ab^2-7\ldots\right.}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] (-2*(16*A*b^4 + 9*a^3*b*B - 8*a*b^3*B - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*C)
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(16
*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) + a^4*(8
*A*b - 6*b*C))*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[S
ec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a
^3*b*B + 3*a*b^3*B + 4*a^4*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B
+ a^4*(A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```


Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
```

```
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}\left(2Ab^2-abB-a^2(A-C)\right)+\frac{3}{2}\cdots}{\cdots}}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\sec\cdots}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\sec\cdots}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\sec\cdots}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\sec\cdots}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2Ab^2-6Ab^4-7a^3\cdots\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{\sec\cdots}}$$

$$= -\frac{2\left(16Ab^4+9a^3bB-8ab^3B-2a^2b^2(8A-C)-a^4(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\cdots}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}\cdots}$$

Mathematica [C] time = 9.23639, size = 7608, normalized size = 14.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.599, size = 8777, normalized size = 16.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{5}+3\,ab^{2}\sec\left(dx+c\right)^{4}+3\,a^{2}b\sec\left(dx+c\right)^{3}+a^{3}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 + 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*se
c(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*sec(d*x + c)^(3/2)), x)
```

3.1069 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=663

$$\frac{2\sqrt{\sec(c+dx)}\left(-4a^2b^3(29A-10C)-a^4b(17A+45C)+80a^3b^2B+5a^5B-80ab^4B+128Ab^5\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\right.}{15a^5d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*(128*A*b^5 + 5*a^5*B + 80*a^3*b^2*B - 80*a*b^4*B - 4*a^2*b^3*(29*A - 10*
C) - a^4*b*(17*A + 45*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]]) + (2*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B
+ 5*a^4*b^2*(11*A - 15*C) - 4*a^2*b^4*(53*A - 10*C) + 3*a^6*(3*A + 5*C))*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^5*(a^2
- b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a
+ b*Sec[c + d*x])^(3/2)) - (2*(8*A*b^4 + 9*a^3*b*B - 5*a*b^3*B - 2*a^2*b^2*
(6*A - C) - 6*a^4*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2
)*Sqrt[a + b*Sec[c + d*x]]) + (2*(48*A*b^4 + 50*a^3*b*B - 30*a*b^3*B + a^4*
(3*A - 35*C) - a^2*b^2*(71*A - 15*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(15*a^3*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)) - (2*(64*A*b^5 - 5*a^5*B + 65
*a^3*b^2*B - 40*a*b^4*B + 2*a^4*b*(7*A - 20*C) - 2*a^2*b^3*(49*A - 10*C))*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] time = 2.22685, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(-a^2b^2(71A-15C)+a^4(3A-35C)+50a^3bB-30ab^3B+48Ab^4\right)\sqrt{a+b\sec(c+dx)}}{15a^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}-\frac{2\sin(c+dx)\left(-2a^2\right.}{3a^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] (2*(128*A*b^5 + 5*a^5*B + 80*a^3*b^2*B - 80*a*b^4*B - 4*a^2*b^3*(29*A - 10*
C) - a^4*b*(17*A + 45*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]]) + (2*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B
+ 5*a^4*b^2*(11*A - 15*C) - 4*a^2*b^4*(53*A - 10*C) + 3*a^6*(3*A + 5*C))*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^5*(a^2
- b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*(A*b
^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a
+ b*Sec[c + d*x])^(3/2)) - (2*(8*A*b^4 + 9*a^3*b*B - 5*a*b^3*B - 2*a^2*b^2*
(6*A - C) - 6*a^4*C)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2
)*Sqrt[a + b*Sec[c + d*x]]) + (2*(48*A*b^4 + 50*a^3*b*B - 30*a*b^3*B + a^4*
(3*A - 35*C) - a^2*b^2*(71*A - 15*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(15*a^3*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)) - (2*(64*A*b^5 - 5*a^5*B + 65
*a^3*b^2*B - 40*a*b^4*B + 2*a^4*b*(7*A - 20*C) - 2*a^2*b^3*(49*A - 10*C))*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c +
d*x]])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx = \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(8Ab^2-5abB-a^2(3A-5C)\right)\cdots}{\cdots}}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9a^3bB-5ab^3B\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{se}\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9a^3bB-5ab^3B\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{se}\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9a^3bB-5ab^3B\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{se}\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9a^3bB-5ab^3B\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{se}\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9a^3bB-5ab^3B\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{se}\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9a^3bB-5ab^3B\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{se}\cdots}$$

$$= \frac{2\left(128Ab^5+5a^5B+80a^3b^2B-80ab^4B-4a^2b^3(29A-10C)-a^4b(17A+45\cdots\right.}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+\cdots}}$$

Mathematica [C] time = 10.3919, size = 9192, normalized size = 13.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sec[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.879, size = 11337, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{6}+3\,ab^{2}\sec\left(dx+c\right)^{5}+3\,a^{2}b\sec\left(dx+c\right)^{4}+a^{3}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(sec(d*x + c))/(b^3*sec(d*x + c)^6 + 3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*se
c(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*sec(d*x + c)^(5/2)), x)
```

3.1070 $\int (a+b\sec(c+dx))^{2/3} \left(A + B\sec(c+dx) + C\sec^2(c+dx\right.$

Optimal. Leaf size=247

$$A\text{Unintegrable}\left((a+b\sec(c+dx))^{2/3},x\right) + \frac{\sqrt{2}(bB-aC)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c\right.}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Unintegrable[(a + b*Se
c[c + d*x])^(2/3), x]
```

Rubi [A] time = 0.322853, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3} \left(A + B\sec(c+dx) + C\sec^2(c+dx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Defer[Int][(a + b*Sec[
c + d*x])^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \frac{\int (a+b\sec(c+dx))^{2/3}(Ab+(bB-aC)\sec(c+dx))\,d}{b}$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx + \frac{(bB-aC)\int \sec(c+d}{}$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx - \frac{((bB-aC)\tan(c+dx}{bd\sqrt{1-s}}$$

$$= \frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(}{a+b}\right.}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b}{}\right.}$$

$$= \frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(}{a+b}\right.}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b}{}\right.}$$

Mathematica [A] time = 52.5458, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{2/3}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
), x]
```

Maple [A] time = 0.168, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{2}{3}}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec (c+dx))^{\frac{2}{3}}\left(A+B\sec (c+dx)+C\sec ^{2}(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x) + C*sec(c + d*x)**
2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec (dx+c)^{2}+B\sec (dx+c)+A\right)(b\sec (dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3
), x)
```

3.1071 $\int \sqrt[3]{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)$

Optimal. Leaf size=247

$$A\text{Unintegrable}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)+\frac{\sqrt{2}(bB-aC)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\right.}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Unintegrable[(a + b*Se
c[c + d*x])^(1/3), x]
```

Rubi [A] time = 0.304566, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B
 - a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Defer[Int][(a + b*Sec[
c + d*x])^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\frac{\int \sqrt[3]{a+b\sec(c+dx)}(Ab+(bB-aC)\sec(c+dx))\,dx}{b}+$$

$$=A\int \sqrt[3]{a+b\sec(c+dx)}\,dx+\frac{(bB-aC)\int \sec(c+dx)\sqrt[3]{}}{b}$$

$$=A\int \sqrt[3]{a+b\sec(c+dx)}\,dx-\frac{((bB-aC)\tan(c+dx))\,\mathrm{S}}{bd\sqrt{1-\sec(}}$$

$$=\frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+}{a+b}\right.}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\mathrm{s}}{}}}$$

$$=\frac{\sqrt{2}(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+}{a+b}\right.}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\mathrm{s}}{}}}$$

Mathematica [A] time = 27.2856, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
), x]
```

Maple [A] time = 0.17, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(dx+c)}\left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(1/3)*(A + B*sec(c + d*x) + C*sec(c + d*x)**
2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3
), x)
```

3.1072 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$A\text{Unintegrable}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)+\frac{\sqrt{2}(bB-aC)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1}{}\right.}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Unintegrable[(a + b*Sec[c + d*x
])^(-1/3), x]
```

Rubi [A] time = 0.303332, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Defer[Int][(a + b*Sec[c + d*x])
^(-1/3), x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\int \frac{Ab+(bB-aC)\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b} + \frac{C\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b}$$

$$= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx + \frac{(bB-aC)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b} - \frac{(C\tan(c+dx)}{bd\sqrt{1}}$$

$$= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx - \frac{((bB-aC)\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a}}\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\,\mathrm{ta}}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\,\mathrm{ta}}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] time = 49.6443, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3
),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(1/3
), x]
```

Maple [A] time = 0.164, size = 0, normalized size = 0.

$$\int (A+B\sec(dx+c)+C(\sec(dx+c))^2)\frac{1}{\sqrt[3]{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B \sec{\left(c + dx \right)} + C \sec^{2}{\left(c + dx \right)}}{\sqrt[3]{a + b \sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(1/
3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec\left(dx + c\right)^{2} + B \sec\left(dx + c\right) + A}{\left(b \sec\left(dx + c\right) + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3
), x)
```

3.1073 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=244

$$A\text{Unintegrable}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)+\frac{\sqrt{2}(bB-aC)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\right.}{bd\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Unintegrable[(a + b*Sec[c + d*x
])^(-2/3), x]
```

Rubi [A] time = 0.311674, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(b*B - a*C)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(
a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Defer[Int][(a + b*Sec[c + d*x])
^(-2/3), x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{\int \frac{Ab+(bB-aC)\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{b} + \frac{C\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{b}$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx + \frac{(bB-aC)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{b} - \frac{(C\tan(}{}$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx - \frac{((bB-aC)\tan(c+dx))\,\text{Subst}\left(\int \frac{}{\sqrt{1-x}\sqrt{1}}\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\text{t}}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

$$= \frac{\sqrt{2}CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\text{t}}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [A] time = 26.5233, size = 0, normalized size = 0.

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3
),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(a + b*Sec[c + d*x])^(2/3
), x]
```

Maple [A] time = 0.176, size = 0, normalized size = 0.

$$\int \left(A+B\sec(dx+c)+C(\sec(dx+c))^2\right)(a+b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+B\sec(dx+c)+A}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec{\left(c + dx \right)} + C\sec^{2}{\left(c + dx \right)}}{\left(a + b\sec{\left(c + dx \right)}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(a + b*sec(c + d*x))**(2/
3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A}{\left(b\sec\left(dx + c\right) + a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(2/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3
), x)
```

3.1074 $\int (a+b\sec(c+dx))^m \left(abB-a^2C+b^2B\sec(c+dx)+b^2C\right.$

Optimal. Leaf size=136

$$(bB-aC)\text{Unintegrable}\left((a+b\sec(c+dx))^{m+1},x\right)+\frac{\sqrt{2}bC(a+b)\tan(c+dx)(a+b\sec(c+dx))^m\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-m}F}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*b*(a + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[c + d*x])/2,
(b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^m*Tan[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^m) + (b*B - a*C)*Uninteg
rable[(a + b*Sec[c + d*x])^(1 + m), x]
```

Rubi [A] time = 0.24001, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a+b\sec(c+dx))^m \left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^m*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*C*Sec[
c + d*x]^2),x]
```

```
[Out] (Sqrt[2]*b*(a + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[c + d*x])/2,
(b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^m*Tan[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^m) + (b*B - a*C)*Defer[I
nt][(a + b*Sec[c + d*x])^(1 + m), x]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))^m \left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right) dx &= \frac{\int (a+b\sec(c+dx))^{1+m}\left(b^2(bB-aC)\right.}{b^2} \\
&= (bC)\int \sec(c+dx)(a+b\sec(c+dx))^1 \\
&= (bB-aC)\int (a+b\sec(c+dx))^{1+m}\, dx \\
&= (bB-aC)\int (a+b\sec(c+dx))^{1+m}\, dx \\
&= \frac{\sqrt{2}b(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\right.}{}
\end{aligned}$$

Mathematica [A] time = 8.98946, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^m \left(abB-a^2C+b^2B\sec(c+dx)+b^2C\sec^2(c+dx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^m*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*
C*Sec[c + d*x]^2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^m*(a*b*B - a^2*C + b^2*B*Sec[c + d*x] + b^2*
C*Sec[c + d*x]^2), x]
```

Maple [A] time = 0.422, size = 0, normalized size = 0.

$$\int (a+b\sec (dx+c))^{m} \left(Bab-a^{2}C+b^{2}B\sec (dx+c)+b^{2}C(\sec (dx+c))^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^m*(B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2),x)
```

```
[Out] int((a+b*sec(d*x+c))^m*(B*a*b-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^{2}\sec (dx+c)^{2}+Bb^{2}\sec (dx+c)-Ca^{2}+Bab\right)(b\sec (dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^m*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*(b*se
c(d*x + c) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^{2}\sec (dx+c)^{2}+Bb^{2}\sec (dx+c)-Ca^{2}+Bab\right)(b\sec (dx+c)+a)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^m*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*(b*sec
(d*x + c) + a)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int Ca^{2}(a+b\sec (c+dx))^{m}\,dx-\int -Bab(a+b\sec (c+dx))^{m}\,dx-\int -Bb^{2}(a+b\sec (c+dx))^{m}\sec (c+dx)\,dx-\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**m*(a*b*B-a**2*C+b**2*B*sec(d*x+c)+b**2*C*sec(d*
x+c)**2),x)
```

```
[Out] -Integral(C*a**2*(a + b*sec(c + d*x))**m, x) - Integral(-B*a*b*(a + b*sec(c
 + d*x))**m, x) - Integral(-B*b**2*(a + b*sec(c + d*x))**m*sec(c + d*x), x)
 - Integral(-C*b**2*(a + b*sec(c + d*x))**m*sec(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(Cb^2 \sec(dx+c)^2 + Bb^2 \sec(dx+c) - Ca^2 + Bab\right)(b\sec(dx+c)+a)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^m*(a*b*B-a^2*C+b^2*B*sec(d*x+c)+b^2*C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*b^2*sec(d*x + c)^2 + B*b^2*sec(d*x + c) - C*a^2 + B*a*b)*(b*se
c(d*x + c) + a)^m, x)
```

3.1075 $\int \cos^{\frac{9}{2}}(c+dx) \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=80

$$\frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(7A+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2A\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

```
[Out] (2*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*A + 9*C)*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*A*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d
)
```

Rubi [A] time = 0.0790408, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4066, 3014, 2635, 2639}

$$\frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(7A+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2A\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*A + 9*C)*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*A*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d
)
```

Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{5}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)dx$$
$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}(7A+9C)\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2(7A+9C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2A\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{15}$$
$$= \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2(7A+9C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2A}{}$$

Mathematica [A] time = 0.329501, size = 65, normalized size = 0.81

$$\frac{12(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\sqrt{\cos(c+dx)}(5A\cos(2(c+dx))+19A+18C)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (12*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(19*A + 18*C
 + 5*A*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(90*d)
```

Maple [B] time = 1.864, size = 313, normalized size = 3.9

$$-\frac{2}{45d}\sqrt{\left(2(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-160\,A\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^{10}+320\,A(\sin(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*A*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+
1/2*c)+(-296*A-72*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(136*A+72*C)*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-24*A-18*C)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-27*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4*sec(d*x + c)^2 + A*cos(d*x + c)^4)*sqrt(cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(9/2), x)
```

3.1076 $\int \cos^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=80

$$\frac{2(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(5A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

```
[Out] (2*(5*A + 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*A + 7*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rubi [A] time = 0.0743782, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4066, 3014, 2635, 2641}

$$\frac{2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*A + 7*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)dx$$
$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}(5A+7C)\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{21}(5A$$
$$= \frac{2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A\cos}{}$$

Mathematica [A] time = 0.303661, size = 63, normalized size = 0.79

$$\frac{2(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3A\cos(2(c+dx))+13A+14C)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 7*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(13*A + 14*C
+ 3*A*Cos[2*(c + d*x)])*Sin[c + d*x])/(21*d)
```

Maple [B] time = 2.288, size = 285, normalized size = 3.6

$$-\frac{2}{21\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48\,A\,(\sin(1/2\,dx+c/2))^8\cos(1/2\,dx+c/2)-72\,A\,(\sin(1/2\,dx+c/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*A*sin(1/2
*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(56*A+28*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-16*A-14*C)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+7*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)^3\sec(dx+c)^2 + A\cos(dx+c)^3\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + A*cos(d*x + c)^3)*sqrt(cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(7/2), x)
```

3.1077 $\int \cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*A*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*d)
```

Rubi [A] time = 0.0592337, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4066, 3014, 2639}

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*A*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*d)
```

Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx &= \int \sqrt{\cos(c+dx)}\left(C+A\cos^2(c+dx)\right)\,dx\\
&= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}(3A+5C)\int \sqrt{\cos(c+dx)}\,dx\\
&= \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.108248, size = 48, normalized size = 0.96

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(2(c+dx))\sqrt{\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]*Sin[2*(c +
d*x)])/(5*d)
```

Maple [B] time = 2.193, size = 252, normalized size = 5.

$$-\frac{2}{5d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8A\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+8A\cos(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*A*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-5
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec(dx+c)^2+A\right)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos(dx+c)^2\sec(dx+c)^2+A\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2), x)
```

3.1078 $\int \cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{2(A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*(A + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*A*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(3*d)
```

Rubi [A] time = 0.0600691, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4066, 3014, 2641}

$$\frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(A + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*A*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(3*d)
```

Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)\,dx &= \int \frac{C+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}(A+3C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [C] time = 0.940077, size = 124, normalized size = 2.58

$$-\frac{4\sin(c)\sqrt{\cos(c+dx)}\left(A\cos^2(c+dx)+C\right)\left((A+3C)\sqrt{\csc^2(c)}\sqrt{\cos^2\left(dx-\tan^{-1}(\cot(c))\right)}\sec\left(dx-\tan^{-1}(\cot(c))\right)\text{Hy}\right.}{3d(A\cos(2(c+dx))+A+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-4*Sqrt[Cos[c + d*x]]*(C + A*Cos[c + d*x]^2)*Sin[c]*((A + 3*C)*Sqrt[Cos[d*
x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Csc[c]*Sin[c + d
*x]))/(3*d*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 1.782, size = 228, normalized size = 4.8

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)*A+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec(dx+c)^2+A\right)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)\sec(dx+c)^2+A\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2), x)
```

3.1079 $\int \sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=44

$$\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*C*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]])
```

Rubi [A] time = 0.0618465, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4066, 3012, 2639}

$$\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*C*Sin[c + d*x])/(d*Sqrt[Cos[c
+ d*x]])
```

Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)\,dx &= \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-(-A+C)\int \sqrt{\cos(c+dx)}\,dx\\
&= \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 1.61253, size = 289, normalized size = 6.57

$$\frac{\cos^2(c+dx)\left(A+C\sec^2(c+dx)\right)\left(-\frac{4\csc(c)(A\cos(2c+dx)+(A-2C)\cos(dx))}{d\sqrt{\cos(c+dx)}}+\frac{2(A-C)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{e^{-idx}\left(2i\sin(c)\left(-1+e^{2idx}\right)+2\cos(c)\left(1+\right.\right.}}{}\right)}{2(A\cos(2(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^2*(A + C*Sec[c + d*x]^2)*((-4*((A - 2*C)*Cos[d*x] + A*Cos[2*c
 + d*x])*Csc[c])/(d*Sqrt[Cos[c + d*x]]) + (2*(A - C)*Csc[c/2]*(3*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)] + E^((2*I)*
d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)])*Sec[c/2]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/(3*d*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]))))
/(2*(A + 2*C + A*Cos[2*(c + d*x)]))
```

Maple [B] time = 2.312, size = 149, normalized size = 3.4

$$2\,\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-C\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)*C)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec(dx+c)^2+A\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+C*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c)), x)
```

3.1080 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=48

$$\frac{2(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*(3*A + C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*C*Sin[c + d*x])/(3*d*Cos
[c + d*x]^(3/2))
```

Rubi [A] time = 0.0603084, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4066, 3012, 2641}

$$\frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*C*Sin[c + d*x])/(3*d*Cos
[c + d*x]^(3/2))
```

Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{1}{3}(-3A-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.183953, size = 43, normalized size = 0.9

$$\frac{2\left((3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{C\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*((3*A + C)*EllipticF[(c + d*x)/2, 2] + (C*Sin[c + d*x])/Cos[c + d*x]^(3/
2)))/(3*d)
```

Maple [B] time = 2.162, size = 266, normalized size = 5.5

$$-\frac{2}{3d}\left(-2\,(\sin(1/2\,dx+c/2))^2\cos(1/2\,dx+c/2)\,C-2\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticF}\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*(-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C-2*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(3*A+C)*sin(1/2*d*x+1/2*c)^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^2+A}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2(c + dx)}{\sqrt{\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{\sqrt{\cos(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/sqrt(cos(d*x + c)), x)
```

3.1081 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*C*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2)) + (2*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0747611, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4066, 3012, 2636, 2639}

$$-\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*C*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2)) + (2*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{5}(-5A-3C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(5A+3C)\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.363833, size = 73, normalized size = 0.91

$$\frac{(5A+3C)\sin(2(c+dx)) - 2(5A+3C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2C\tan(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*A + 3*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + (5*A + 3*C)*
Sin[2*(c + d*x)] + 2*C*Tan[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 5.565, size = 593, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1/2*d*
x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/
2*c)^3*(20*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*A*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+12*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-20*A*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2+40*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12*C
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*C*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-10*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)*A+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\sec(dx+c)^2 + A}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/cos(d*x + c)^(3/2), x)
```

3.1082 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2(7A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] (2*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*C*Sin[c + d*x])/(7*d*
Cos[c + d*x]^(7/2)) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)
)
```

Rubi [A] time = 0.0755043, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4066, 3012, 2636, 2641}

$$\frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (2*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*C*Sin[c + d*x])/(7*d*
Cos[c + d*x]^(7/2)) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)
)
```


Rule 4066

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[e + f*x])^(m - 2)*(C + A*Cos[e +
 f*x]^2), x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !IntegerQ[m]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{1}{7}(-7A-5C)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{21}(-7A-5C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.573954, size = 73, normalized size = 0.91

$$\frac{2(7A+5C)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(7A+5C)\sin(2(c+dx))+6C\tan(c+dx)}{21d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(7*A + 5*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + (7*A + 5*C)*S
in[2*(c + d*x)] + 6*C*Tan[c + d*x])/(21*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 4.45, size = 376, normalized size = 4.7

$$-\frac{1}{d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,C\left(-\frac{\cos(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+}{56\,\left((\cos(1/2\,dx+c/2))^2-1/2\right)^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/56*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2+A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec\left(dx+c\right)^{2}+A}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/cos(d*x + c)^(5/2), x)
```

3.1083 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=165

$$\frac{2a(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(7A+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a(5A$$

```
[Out] (2*a*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(5*A + 7*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*a*(7*A + 9*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (
2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*A*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```

Rubi [A] time = 0.241436, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4114, 3034, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(7A+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a(5A+7C)\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(5*A + 7*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*a*(7*A + 9*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (
2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*A*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right)$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{9aC}{2}\right.$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c}{9d}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c}{9d}$$

$$= \frac{2a(5A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a(7A+9C)\,c}{}$$

$$= \frac{2a(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2a(5A+7C)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] time = 6.29184, size = 918, normalized size = 5.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((7*A + 9*C
)*Cot[c])/(15*d) + ((23*A + 28*C)*Cos[d*x]*Sin[c])/(84*d) + ((19*A + 18*C)*
Cos[2*d*x]*Sin[2*c])/(180*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) + (A*Cos[4*d*
x]*Sin[4*c])/(72*d) + ((23*A + 28*C)*Cos[c]*Sin[d*x])/(84*d) + ((19*A + 18*
C)*Cos[2*c]*Sin[2*d*x])/(180*d) + (A*Cos[3*c]*Sin[3*d*x])/(28*d) + (A*Cos[4
```

```
*c]*Sin[4*d*x])/(72*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(21*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (7*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 +
 (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(30*d) - (3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]
^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(10*d))
```

Maple [B] time = 3.306, size = 406, normalized size = 2.5

$$-\frac{2a}{315d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-1120\,A\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^{10}+2960\,A\,(\sin(1/$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2960*A*sin(1/2*d*x+1/2*c)^8*cos(1/2
*d*x+1/2*c)+(-3152*A-504*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1792*A
+924*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-408*A-336*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+Ca\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+Aa\cos\left(dx+c\right)^4\sec\left(dx+c\right)+Aa\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^4*sec(d*x + c)^3 + C*a*cos(d*x + c)^4*sec(d*x +
c)^2 + A*a*cos(d*x + c)^4*sec(d*x + c) + A*a*cos(d*x + c)^4)*sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x
)
```

3.1084 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=134

$$\frac{2a(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aA\sin}{}$$

```
[Out] (2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(21*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.218576, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4114, 3034, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aA\sin(c+dx}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*C)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(21*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right.$$
$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int\sqrt{\cos(c+dx)}\left(\frac{7a}{2}\right.$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c}{7d}$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c}{7d}$$
$$= \frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7C)\sqrt{\cos(c+}}{21d}$$
$$= \frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7C)F\left(\frac{1}{2}(c+d\right.}{21d}$$

Mathematica [C] time = 6.23881, size = 872, normalized size = 6.51

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(3A+5C)\cot(c)}{5d}+\frac{(23A+28C)\cos(dx)\sin(c)}{84d}+\frac{A\cos(2dx)\sin(2c)}{10d}+\frac{A\cos(3}{2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((3*A + 5*C
)*Cot[c])/(5*d) + ((23*A + 28*C)*Cos[d*x]*Sin[c])/(84*d) + (A*Cos[2*d*x]*Si
n[2*c])/(10*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) + ((23*A + 28*C)*Cos[c]*Sin
[d*x])/(84*d) + (A*Cos[2*c]*Sin[2*d*x])/(10*d) + (A*Cos[3*c]*Sin[3*d*x])/(2
8*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(21*d*Sqrt[1 +
 Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(3*d*Sqrt[
1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
10*d) - (C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricP
FQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
+ Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*
Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2
))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.119, size = 378, normalized size = 2.8

$$-\frac{2\,a}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,A\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}\cos\left(1/2\,dx+c/2\right)-528\,A\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-528*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(448*A+140*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-122*A-70*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+Aa\cos\left(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3*sec(d*x + c)^3 + C*a*cos(d*x + c)^3*sec(d*x +
c)^2 + A*a*cos(d*x + c)^3*sec(d*x + c) + A*a*cos(d*x + c)^3)*sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x
)
```

3.1085 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=101

$$\frac{2a(A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)\sqrt{}}{3d}$$

```
[Out] (2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + 3*C)*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.203163, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4114, 3034, 3023, 2748, 2641, 2639}

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + 3*C)*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3034

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 3))
, x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m
 + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m + 3)
)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{5aC}{2}+\frac{1}{2}a(3A+5C}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{2a(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+3C)F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [C] time = 6.29581, size = 824, normalized size = 8.16

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(3A+5C)\cot(c)}{5d}+\frac{A\cos(dx)\sin(c)}{3d}+\frac{A\cos(2dx)\sin(2c)}{10d}+\frac{A\cos(c)\sin(dx)}{3d}+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((3*A + 5*C
)*Cot[c])/(5*d) + (A*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[2*d*x]*Sin[2*c])/(10*d
) + (A*Cos[c]*Sin[d*x])/(3*d) + (A*Cos[2*c]*Sin[2*d*x])/(10*d)) - (A*(1 + C
os[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1
+ Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
```

```
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (3*A*
(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (C*(1 + Cos[c
+ d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.169, size = 345, normalized size = 3.4

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,A\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+44\,A\cos(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+44*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca\cos(dx+c)^2\sec(dx+c)^3+Ca\cos(dx+c)^2\sec(dx+c)^2+Aa\cos(dx+c)^2\sec(dx+c)+Aa\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^2*sec(d*x + c)^3 + C*a*cos(d*x + c)^2*sec(d*x +
c)^2 + A*a*cos(d*x + c)^2*sec(d*x + c) + A*a*cos(d*x + c)^2)*sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^{2}+A\right)\left(a \sec \left(dx+c\right)+a\right) \cos \left(dx+c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x
)
```

3.1086 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{2a(A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*C)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.203875, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4114, 3032, 3023, 2748, 2641, 2639}

$$\frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*C)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$= \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{aC}{2}+\frac{1}{2}a(A-C)\cos(c+dx)+\frac{1}{2}}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{4}{3}\int$$

$$= \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + (a(A$$

$$= \frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+3C)F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

Mathematica [C] time = 6.35124, size = 813, normalized size = 8.56

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(-\frac{(\cos(2c)A+A-2C)\csc(c)\sec(c)}{2d}+\frac{C\sec(c+dx)\sin(dx)\sec(c)}{d}+\frac{A\cos(dx)\sin}{3d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((A - 2*C +
 A*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (A*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[c]*S
in[d*x])/(3*d) + (C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d) - (A*(1 + Cos[c + d*x]
)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d
*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
```

```
2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (A*(1 + Cos[c +
d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (C*(1 + Cos[c + d*x])*Csc[c]
*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.524, size = 458, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*a*(4*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+3*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + A\right)(a \sec (dx+c) + a) \cos (dx+c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+Ca\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)\sec\left(dx+c\right)+Aa\cos\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)*sec(d*x + c)^3 + C*a*cos(d*x + c)*sec(d*x + c)^2
 + A*a*cos(d*x + c)*sec(d*x + c) + A*a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x
)
```

3.1087 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=95

$$\frac{2a(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + C)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*C*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.207393, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4114, 3032, 3021, 2748, 2641, 2639}

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + C)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*C*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$= \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3aC}{2}+\frac{1}{2}a(3A+C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{4}{3}\int \frac{\frac{1}{4}a(3A+C)}{}$$

$$= \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (a(A-C))\int \sqrt{c}$$

$$= \frac{2a(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [C] time = 6.37, size = 817, normalized size = 8.6

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{C\sec(c)\sin(dx)\sec^2(c+dx)}{3d} + \frac{\sec(c)(C\sin(c)+3C\sin(dx))\sec(c+dx)}{3d} - \frac{(\cos(2c}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((A - 2*C +
 A*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d
) + (Sec[c]*Sec[c + d*x]*(C*Sin[c] + 3*C*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c
+ d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c
```

```
 + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (A*(1 + Co
s[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (C*(1 + Cos[c + d*x])*
Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*
x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 4.892, size = 437, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+1/2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\sec\left(dx+c\right)^3+Ca\sec\left(dx+c\right)^2+Aa\sec\left(dx+c\right)+Aa\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x
)
```

3.1088 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=132

$$\frac{2a(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\cdots}{5d\cdots}$$

```
[Out] (-2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + C)*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (2*a*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*C)*Sin[c + d
*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.222512, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4114, 3032, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2aC\sin(c\cdots}{5d\cos^{\frac{5}{2}}(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + C)*Ellipti
cF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) +
 (2*a*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*C)*Sin[c + d
*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
```

```
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5aC}{2}+\frac{1}{2}a(5A+3C)\cos(c+dx)+\frac{5}{2}aA\cos^2}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4}{15}\int \frac{\frac{3}{4}a(5A+3C)+\frac{5}{4}a(3A}{\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a(3A+C))\int \frac{1}{\sqrt{\cos(c+dx}}$$

$$= \frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a}{5d}$$

Mathematica [C] time = 6.4438, size = 851, normalized size = 6.45

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{C\sec(c)\sin(dx)\sec^3(c+dx)}{5d} + \frac{\sec(c)(3C\sin(c)+5C\sin(dx))\sec^2(c+dx)}{15d} + \frac{\sec(}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 3*C)
*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*C*Sin[c] + 5*C*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(
5*C*Sin[c] + 15*A*Sin[d*x] + 9*C*Sin[d*x]))/(15*d)) - (A*(1 + Cos[c + d*x])
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x]
)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c + d
*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (3*C*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 6.617, size = 729, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/10*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4
-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin
(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2)))+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
```

```
+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+Ca\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)+Aa}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x
)
```

3.1089 $\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{2a(7A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)
) + (2*a*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(7*A + 5*C)*Sin[c
+ d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqr
t[Cos[c + d*x]])
```

Rubi [A] time = 0.244699, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4114, 3032, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)
) + (2*a*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(7*A + 5*C)*Sin[c
+ d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqr
t[Cos[c + d*x]])
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3032

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp
[((b*c - a*d)*(A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b
^2*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a +
b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d))
- ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*C*
d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\cos(c+dx))\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{7aC}{2}+\frac{1}{2}a(7A+5C)\cos(c+dx)+\frac{7}{2}aA\cos^2(c+}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{35}\int \frac{\frac{5}{4}a(7A+5C)+\frac{7}{4}a(5A+3}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(a(5A+3C))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,d$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(5}{5}$$

$$= -\frac{2a(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2aC}{7d\,co}$$

Mathematica [C] time = 6.51491, size = 895, normalized size = 5.42

$$a\left(\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\left(\frac{C\sec(c)\sin(dx)\sec^4(c+dx)}{7d}+\frac{\sec(c)(5C\sin(c)+7C\sin(dx))\sec^3(c+dx)}{35d}+\sec(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 3*C)
*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(7*d) + (Sec[c]*
Sec[c + d*x]^3*(5*C*Sin[c] + 7*C*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]^2
*(21*C*Sin[c] + 35*A*Sin[d*x] + 25*C*Sin[d*x]))/(105*d) + (Sec[c]*Sec[c + d
*x]*(35*A*Sin[c] + 25*C*Sin[c] + 105*A*Sin[d*x] + 63*C*Sin[d*x]))/(105*d))
- (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2
]) - (5*C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Co
t[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (
3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 7.763, size = 838, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*C*(-1/5
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*C
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+Ca\sec\left(dx+c\right)^{2}+Aa\sec\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + C*a*sec(d*x + c)^2 + A*a*sec(d*x + c) + A*a)
/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x
)
```

3.1090 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^2 \left(A+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=230

$$\frac{8a^2(25A+33C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(89A+99C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+$$

```
[Out] (4*a^2*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^2*(25*A + 33*C)
*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^2*(25*A + 33*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(231*d) + (4*a^2*(7*A + 9*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(45*d) + (2*a^2*(89*A + 99*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d)
+ (2*A*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d) + (8*
A*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```

Rubi [A] time = 0.518157, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3046, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{8a^2(25A+33C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(89A+99C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d}+\frac{4a^2(7A$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^2*(7*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^2*(25*A + 33*C)
*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^2*(25*A + 33*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(231*d) + (4*a^2*(7*A + 9*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(45*d) + (2*a^2*(89*A + 99*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d)
+ (2*A*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(11*d) + (8*
A*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{2\int \text{cc}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{8A\,\text{cc}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d} + \frac{8A\,\text{cc}}{}$$

$$= \frac{2a^2(89A+99C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2A\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{2a^2(89A+99C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2A\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{8a^2(25A+33C)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} + \frac{4a^2(7A+9\text{C}}{}$$

$$= \frac{4a^2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{8a^2(25A+33C)F\left(\frac{1}{2}(c\right.}{231d}$$

Mathematica [C] time = 6.31492, size = 976, normalized size = 4.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^2*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((7*A +
 9*C)*Cot[c])/(15*d) + ((941*A + 1122*C)*Cos[d*x]*Sin[c])/(3696*d) + ((19*A
 + 18*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + ((101*A + 44*C)*Cos[3*d*x]*Sin[3*c]
)/(2464*d) + (A*Cos[4*d*x]*Sin[4*c])/(72*d) + (A*Cos[5*d*x]*Sin[5*c])/(352*
d) + ((941*A + 1122*C)*Cos[c]*Sin[d*x])/(3696*d) + ((19*A + 18*C)*Cos[2*c]*
Sin[2*d*x])/(180*d) + ((101*A + 44*C)*Cos[3*c]*Sin[3*d*x])/(2464*d) + (A*Co
s[4*c]*Sin[4*d*x])/(72*d) + (A*Cos[5*c]*Sin[5*d*x])/(352*d)) - (50*A*(1 + C
os[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*d*Sqrt[1 + Cot[c]^2]) - (2
*C*(1 + Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2
]) - (7*A*(1 + Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d) - (3
*C*(1 + Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
```

```
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [A] time = 2.239, size = 436, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-37520*A*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^10+(57040*A+3960*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-46192*A-11484*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(22022*A+12474*C)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4563*A-3861*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+750*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+990*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^5\sec(dx+c)^4+2\,Ca^2\cos(dx+c)^5\sec(dx+c)^3+(A+C)a^2\cos(dx+c)^5\sec(dx+c)^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^5*sec(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^5*sec(
d*x + c)^3 + (A + C)*a^2*cos(d*x + c)^5*sec(d*x + c)^2 + 2*A*a^2*cos(d*x +
c)^5*sec(d*x + c) + A*a^2*cos(d*x + c)^5)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algori
thm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(11/2)
, x)
```

3.1091 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=197

$$\frac{4a^2(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(19A+21C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}$$

```
[Out] (16*a^2*(2*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 7*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(21*d) + (2*a^2*(19*A + 21*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(105*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d) + (8*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d)
```

Rubi [A] time = 0.485966, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3046, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(19A+21C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+\frac{4a^2(5$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (16*a^2*(2*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 7*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(21*d) + (2*a^2*(19*A + 21*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(105*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d) + (8*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx=\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\left(C+A\cos^2(c+d\right.$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}+\frac{2\int}{}$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}+\frac{8A}{}$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}+\frac{8A}{}$$

$$=\frac{2a^2(19A+21C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2A\cos^{\frac{3}{2}}(c}{}$$

$$=\frac{2a^2(19A+21C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d}+\frac{2A\cos^{\frac{3}{2}}(c}{}$$

$$=\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+7C)\sqrt{\cos(}}{21}$$

$$=\frac{16a^2(2A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+7C)F\left(\frac{1}{2}(c\right.}{21d}$$

Mathematica [C] time = 6.30396, size = 1118, normalized size = 5.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*((-8*(2*A + 3*C)*Cot[c])/(15*d) + ((23*A + 28*C)*Cos[d*x]*Sin[c
])/(42*d) + ((37*A + 18*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + (A*Cos[3*d*x]*Sin
[3*c])/(14*d) + (A*Cos[4*d*x]*Sin[4*c])/(72*d) + ((23*A + 28*C)*Cos[c]*Sin[
d*x])/(42*d) + ((37*A + 18*C)*Cos[2*c]*Sin[2*d*x])/(180*d) + (A*Cos[3*c]*Si
n[3*d*x])/(14*d) + (A*Cos[4*c]*Sin[4*d*x])/(72*d)))/(A + 2*C + A*Cos[2*c +
2*d*x]) - (10*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2]) - (2*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^
2*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTa
n[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2]) - (8*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Se
c[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d*(A + 2*C + A*Cos[2*c + 2*d*
x])) - (4*C*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])
```

```
^2*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 2.286, size = 408, normalized size = 2.1

$$-\frac{4\,a^2}{315\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-560\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}+1840\,A\left(\sin\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+1840*A*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-2368*A-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(1568*
A+672*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-273*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+(A+C)a^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4*sec(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^4*sec(
d*x + c)^3 + (A + C)*a^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*A*a^2*cos(d*x +
c)^4*sec(d*x + c) + A*a^2*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2),
 x)
```

3.1092 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=164

$$\frac{8a^2(3A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{8...}{}$$

```
[Out] (4*a^2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(3*A + 7*C)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 35*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(7*d) + (8*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(35*d)
```

Rubi [A] time = 0.466033, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3046, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^2(3A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{8A\sin(c ...}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(3*A + 7*C)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 35*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(7*d) + (8*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(35*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d}+\frac{2\int \frac{(a+}{}}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d}+\frac{8A\sqrt{c}}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c+dx)}{7d}+\frac{8A\sqrt{c}}{}$$

$$=\frac{2a^2(33A+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d}+\frac{2A\sqrt{\cos(c+}}{}$$

$$=\frac{2a^2(33A+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d}+\frac{2A\sqrt{\cos(c+}}{}$$

$$=\frac{4a^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2(3A+7C)F\left(\frac{1}{2}(c+d\right.}{21d}$$

Mathematica [C] time = 6.37914, size = 1070, normalized size = 6.52

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*((-2*(3*A + 5*C)*Cot[c])/(5*d) + ((51*A + 28*C)*Cos[d*x]*Sin[c]
)/(84*d) + (A*Cos[2*d*x]*Sin[2*c])/(5*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) +
 ((51*A + 28*C)*Cos[c]*Sin[d*x])/(84*d) + (A*Cos[2*c]*Sin[2*d*x])/(5*d) + (
A*Cos[3*c]*Sin[3*d*x])/(28*d)))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c
 + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
ot[c]]]])/(7*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Co
s[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^
2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (3*A
*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*S
ec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Ta
n[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + A*Cos[2*c + 2*d*x])) - (C*Cos[c + d*x]^
4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
```

```
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 2.148, size = 380, normalized size = 2.3

$$-\frac{4a^2}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\,A\left(\sin\left(1/2\,dx+c/2\right)\right)^8\cos\left(1/2\,dx+c/2\right)-348\,A\left(\sin\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-348*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(378*A+70*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-117*A-35*C
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+70*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+\left(A+C\right)a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^3*sec(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^3*sec(
d*x + c)^3 + (A + C)*a^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*A*a^2*cos(d*x +
c)^3*sec(d*x + c) + A*a^2*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2),
 x)
```

3.1093 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=158

$$\frac{4a^2(A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(7A-15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(A-5C)\sin(c+dx)\sqrt{\cos(c+dx}}{5d}$$

```
[Out] (16*a^2*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 3*C)*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*a^2*(7*A - 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(15*d) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) +
 (2*(A - 5*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*
d)
```

Rubi [A] time = 0.464138, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3044, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(7A-15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (16*a^2*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 3*C)*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*a^2*(7*A - 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(15*d) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) +
 (2*(A - 5*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*
d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
```

```
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))^2}{}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2(A-5C)\sqrt{\cos(}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2(A-5C)\sqrt{\cos(}}{}$$

$$= \frac{2a^2(7A-15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2C(a+a\,\mathrm{co}}{d}$$

$$= \frac{2a^2(7A-15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2C(a+a\,\mathrm{co}}{d}$$

$$= \frac{16a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [C] time = 6.4492, size = 799, normalized size = 5.06

$$\frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(\sec(c+dx)a+a)^2\left(C\sec^2(c+dx)+A\right)\left(-\frac{(8\cos(2c)A+8A-5C+5C\cos(2c))\csc(c)\sec(c)}{10d}+\frac{C\sec(c+dx)\sin(dx)\sec(c)}{d}\right.}{\cos(2c+2dx)A+A+2C}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*(-((8*A - 5*C + 8*A*Cos[2*c] + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(10
*d) + (2*A*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[2*d*x]*Sin[2*c])/(10*d) + (2*A*C
os[c]*Sin[d*x])/(3*d) + (C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (A*Cos[2*c]*Si
n[2*d*x])/(10*d)))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (2*A*Cos[c + d*x]^4*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c + d*x]^4*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*Cos[c + d*x]^4
*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
```

```
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(5*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.156, size = 440, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/15*a^2*(-12*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+15*C)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3+(A+C)a^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+2\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^2*sec(d*x + c)^4 + 2*C*a^2*cos(d*x + c)^2*sec(
d*x + c)^3 + (A + C)*a^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*A*a^2*cos(d*x +
c)^2*sec(d*x + c) + A*a^2*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec \left(dx+c\right)^2+A\right)(a\sec \left(dx+c\right)+a)^2\cos \left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2),
 x)
```

3.1094 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=154

$$\frac{8a^2(A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8C\sin(c+$$

```
[Out] (4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*(A + C)*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (2*a^2*(A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (8
*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.467617, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3044, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(A-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8C\sin(c+dx)\left(a^2\right.}{3d\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*(A + C)*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (2*a^2*(A - 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (8
*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^2(2a\cdots}{\cos^{\frac{3}{2}}\cdots}}{3\cdots}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(c+\cdots\right.}{3d\sqrt{\cos(c\cdots}}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(c+\cdots\right.}{3d\sqrt{\cos(c\cdots}}$$

$$= \frac{2a^2(A-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(a+a\cos(c+\cdots}{3d\cos^{\frac{3}{2}}\cdots}$$

$$= \frac{2a^2(A-5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(a+a\cos(c+\cdots}{3d\cos^{\frac{3}{2}}\cdots}$$

$$= \frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}\cdots$$

Mathematica [C] time = 6.46694, size = 1040, normalized size = 6.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*(-(((A - 2*C + A*Cos[2*c])*Csc[c]*Sec[c])/d) + (A*Cos[d*x]*Sin[
c])/(3*d) + (A*Cos[c]*Sin[d*x])/(3*d) + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/
(3*d) + (Sec[c]*Sec[c + d*x]*(C*Sin[c] + 6*C*Sin[d*x]))/(3*d)))/(A + 2*C +
A*Cos[2*c + 2*d*x]) - (4*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
 d*x])^2*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x
])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[
c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*
d*x])*Sqrt[1 + Cot[c]^2]) - (A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(
a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(A + 2*C + A*Cos[2*c +
```

```
2*d*x])) + (C*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*
x])^2*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 5.49, size = 651, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*A*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-4*A*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4+4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2
+6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*C*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^4-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-2*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos\left(dx+c\right)\sec\left(dx+c\right)^4+2\,Ca^2\cos\left(dx+c\right)\sec\left(dx+c\right)^3+\left(A+C\right)a^2\cos\left(dx+c\right)\sec\left(dx+c\right)^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)*sec(d*x + c)^4 + 2*C*a^2*cos(d*x + c)*sec(d*x
+ c)^3 + (A + C)*a^2*cos(d*x + c)*sec(d*x + c)^2 + 2*A*a^2*cos(d*x + c)*sec
(d*x + c) + A*a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2),
 x)
```

3.1095 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)$

Optimal. Leaf size=156

$$\frac{4a^2(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(15A+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{16a^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8C\sin(c+dx)\left(a^2\cos\right.}{15d\cos^{\frac{3}{2}}(c}$$

```
[Out] (-16*a^2*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + C)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a^2*(15*A + 17*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c
+ d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.482327, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3044, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(15A+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{16a^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8C\sin(c+dx)\left(a^2\cos(c+dx\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (-16*a^2*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + C)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a^2*(15*A + 17*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c
+ d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))}{}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(\right.}{15d\,\mathrm{co}}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8C\left(a^2+a^2\cos(\right.}{15d\,\mathrm{co}}$$

$$= \frac{2a^2(15A+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2C(a+a\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+d}$$

$$= \frac{2a^2(15A+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2C(a+a\cos(c+dx))^2}{5d\cos^{\frac{5}{2}}(c+d}$$

$$= -\frac{16a^2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

Mathematica [C] time = 6.54155, size = 800, normalized size = 5.13

$$\frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(\sec(c+dx)a+a)^2\left(C\sec^2(c+dx)+A\right)\left(\frac{C\sec(c)\sin(dx)\sec^3(c+dx)}{5d}+\frac{\sec(c)(3C\sin(c)+10C\sin(dx))\sec^2(c+dx)}{15d}+\frac{\sec}{}\right.}{\cos(2c+2dx)A+A+2C}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*(-((-5*A - 16*C + 5*A*Cos[2*c])*Csc[c]*Sec[c])/(10*d) + (C*Sec[
c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]^2*(3*C*Sin[c] + 10
*C*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(10*C*Sin[c] + 15*A*Sin[d*x] +
24*C*Sin[d*x]))/(15*d)))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (2*A*Cos[c + d*x]
^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c + d*x
]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (4*C*Cos[c +
d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*
```

```
x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(5*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 6.708, size = 756, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*A*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+20*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^4+48*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-96*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+60*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*sin(1/2*d*x+1/2*c)^2-48*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+116*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A+5*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+12*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-37*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)*C)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\sec(dx+c)^4 + 2\,Ca^2\sec(dx+c)^3 + (A+C)a^2\sec(dx+c)^2 + 2\,Aa^2\sec(dx+c) + Aa^2\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)),
 x)
```

3.1096 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=197

$$\frac{8a^2(7A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+3C)\text{s}}{5d\sqrt{\cos(c}}$$

```
[Out] (-4*a^2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(7*A + 3*C)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 33*C)*Sin[c + d*x])/(105*
d*Cos[c + d*x]^(3/2)) + (4*a^2*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
 + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.512475, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3044, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{8a^2(7A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+3C)\sin(c+d}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-4*a^2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*(7*A + 3*C)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 33*C)*Sin[c + d*x])/(105*
d*Cos[c + d*x]^(3/2)) + (4*a^2*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
 + (8*C*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx=\int \frac{(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\int\frac{(a+a\cos(c+dx))^2\left(2aC+\frac{1}{2}a(7A+C)\cos\right.}{\cos^{\frac{7}{2}}(c+dx)}}{7a}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c+}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2a^2(35A+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{8}{}$$

$$=\frac{2a^2(35A+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{8}{}$$

$$=\frac{8a^2(7A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2(35A+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5}{5}$$

$$=-\frac{4a^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2(7A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a}{}$$

Mathematica [C] time = 6.61354, size = 1092, normalized size = 5.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*((2*(5*A + 3*C)*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^4
*Sin[d*x])/(7*d) + (Sec[c]*Sec[c + d*x]^3*(5*C*Sin[c] + 14*C*Sin[d*x]))/(35
*d) + (Sec[c]*Sec[c + d*x]^2*(42*C*Sin[c] + 35*A*Sin[d*x] + 60*C*Sin[d*x]))
/(105*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 60*C*Sin[c] + 210*A*Sin[d*x]
 + 126*C*Sin[d*x]))/(105*d)))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c +
 d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[
c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(7*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (A*Cos
[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c
 + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]
```

```
]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (3*C*Cos[c + d*x]^4*Cs
c[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((H
ypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + T
an[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c
]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
)/(5*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 8.003, size = 918, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/10*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/4*C*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-
5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/4*A+1/4*C)*(-1/6*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec\left(dx+c\right)^4+2\,Ca^2\sec\left(dx+c\right)^3+(A+C)a^2\sec\left(dx+c\right)^2+2\,Aa^2\sec\left(dx+c\right)+Aa^2}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^2}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)),
 x)
```

3.1097 $\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{4a^2(7A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(21A+19}{105d\cos}$$

```
[Out] (-16*a^2*(3*A + 2*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A + 5*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(21*A + 19*C)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(5/2)) + (4*a^2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Cos[c + d*
x]^(3/2)) + (16*a^2*(3*A + 2*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (
2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (8*C*(a
^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.541732, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3044, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(21A+19C)\sin(}{105d\cos^{\frac{5}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-16*a^2*(3*A + 2*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A + 5*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(21*A + 19*C)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(5/2)) + (4*a^2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Cos[c + d*
x]^(3/2)) + (16*a^2*(3*A + 2*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (
2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (8*C*(a
^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx=\int \frac{(a+a\cos(c+dx))^2\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2\int \frac{(a+a\cos(c+dx))^2\left(2aC+\frac{3}{2}a(3A+C\right.}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{8C\left(a^2+a^2\cos(c+dx)\right)\sin(c}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2a^2(21A+19C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$=\frac{2a^2(21A+19C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$=\frac{2a^2(21A+19C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2(3A}{15}$$

$$=-\frac{16a^2(3A+2C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [C] time = 6.67063, size = 1137, normalized size = 4.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + C*Sec[
c + d*x]^2)*((8*(3*A + 2*C)*Csc[c]*Sec[c])/(15*d) + (C*Sec[c]*Sec[c + d*x]^
5*Sin[d*x])/(9*d) + (Sec[c]*Sec[c + d*x]^4*(7*C*Sin[c] + 18*C*Sin[d*x]))/(6
3*d) + (2*Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 25*C*Sin[c] + 84*A*Sin[d*x] +
56*C*Sin[d*x]))/(105*d) + (Sec[c]*Sec[c + d*x]^3*(90*C*Sin[c] + 63*A*Sin[d*
x] + 112*C*Sin[d*x]))/(315*d) + (Sec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 112*C
*Sin[c] + 210*A*Sin[d*x] + 150*C*Sin[d*x]))/(315*d)))/(A + 2*C + A*Cos[2*c
+ 2*d*x]) - (2*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(
A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2]) - (10*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])
^2*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sq
rt[1 + Cot[c]^2]) + (4*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*
```

```
Sec[c + d*x])^2*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + A*Cos[2*c + 2*d
*x])) + (8*C*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
)^2*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 9.685, size = 1168, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/5*(1/4
*A+1/4*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/4*C*(-1/144*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)
^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+1/2*C*(-1/56*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*A*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec\left(dx+c\right)^4+2\,Ca^2\sec\left(dx+c\right)^3+\left(A+C\right)a^2\sec\left(dx+c\right)^2+2\,Aa^2\sec\left(dx+c\right)+Aa^2}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + 2*C*a^2*sec(d*x + c)^3 + (A + C)*a^2*sec(d
*x + c)^2 + 2*A*a^2*sec(d*x + c) + A*a^2)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2),
 x)
```

3.1098 $\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=279

$$\frac{4a^3(95A+121C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(175A+221C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{40a^3(118A+143C)\sin(c+dx)\cos^{\frac{5}{2}}(c}{9009d}$$

```
[Out] (4*a^3*(175*A + 221*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (4*a^3*(95*A +
121*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(95*A + 121*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(175*A + 221*C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(585*d) + (40*a^3*(118*A + 143*C)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(9009*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*
x])/(13*d) + (12*A*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(143*a*d) + (2*(145*A + 143*C)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*
x])*Sin[c + d*x])/(1287*d)
```

Rubi [A] time = 0.685408, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3046, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(95A+121C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(175A+221C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d}+\frac{40a^3(118A+143C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{9009d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(175*A + 221*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (4*a^3*(95*A +
121*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(95*A + 121*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(175*A + 221*C)*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(585*d) + (40*a^3*(118*A + 143*C)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(9009*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*
x])/(13*d) + (12*A*Cos[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(143*a*d) + (2*(145*A + 143*C)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*
x])*Sin[c + d*x])/(1287*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d} + \frac{2\int \mathrm{co}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d} + \frac{12A\,\mathrm{c}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d} + \frac{12A\,\mathrm{c}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{13d} + \frac{12A\,\mathrm{c}}{}$$

$$= \frac{40a^3(118A+143C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9009d} + \frac{2A\cos^{\frac{5}{2}}(}{}$$

$$= \frac{40a^3(118A+143C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9009d} + \frac{2A\cos^{\frac{5}{2}}(}{}$$

$$= \frac{4a^3(95A+121C)\sqrt{\cos(c+dx)}\sin(c+dx)}{231d} + \frac{4a^3(175A-}{}$$

$$= \frac{4a^3(175A+221C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{195d} + \frac{4a^3(95A+121C)F}{231d}$$

Mathematica [C] time = 6.37319, size = 1022, normalized size = 3.66

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^3*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((175*A
 + 221*C)*Cot[c])/(390*d) + ((1811*A + 2134*C)*Cos[d*x]*Sin[c])/(7392*d) +
((7825*A + 7592*C)*Cos[2*d*x]*Sin[2*c])/(74880*d) + ((215*A + 132*C)*Cos[3*
d*x]*Sin[3*c])/(4928*d) + ((59*A + 13*C)*Cos[4*d*x]*Sin[4*c])/(3744*d) + (3
*A*Cos[5*d*x]*Sin[5*c])/(704*d) + (A*Cos[6*d*x]*Sin[6*c])/(1664*d) + ((1811
*A + 2134*C)*Cos[c]*Sin[d*x])/(7392*d) + ((7825*A + 7592*C)*Cos[2*c]*Sin[2*
d*x])/(74880*d) + ((215*A + 132*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + ((59*A +
 13*C)*Cos[4*c]*Sin[4*d*x])/(3744*d) + (3*A*Cos[5*c]*Sin[5*d*x])/(704*d) +
(A*Cos[6*c]*Sin[6*d*x])/(1664*d)) - (95*A*(1 + Cos[c + d*x])^3*Csc[c]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x
)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-
(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - A
rcTan[Cot[c]]]])/(462*d*Sqrt[1 + Cot[c]^2]) - (11*C*(1 + Cos[c + d*x])^3*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (35*A*(1 + Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
```

```
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(156*d) - (17*C*(1 + Cos[c + d*x])^
3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d))
```

Maple [A] time = 2.321, size = 464, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-2217
60*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+1058400*A*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^12+(-2122400*A-80080*C)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*
x+1/2*c)+(2331040*A+314600*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-153
5860*A-487916*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(633710*A+386386*C
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-121230*A-105534*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)+18525*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-40425*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+23595*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-51051*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^6\sec(dx+c)^5 + 3\,Ca^3\cos(dx+c)^6\sec(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^6\sec(dx+c)^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^6*sec(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^6*sec(
d*x + c)^4 + (A + 3*C)*a^3*cos(d*x + c)^6*sec(d*x + c)^3 + (3*A + C)*a^3*co
s(d*x + c)^6*sec(d*x + c)^2 + 3*A*a^3*cos(d*x + c)^6*sec(d*x + c) + A*a^3*c
os(d*x + c)^6)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(13/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algori
thm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*cos(d*x + c)^(13/2)
, x)
```

3.1099 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=246

$$\frac{4a^3(105A+143C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^3(35A+44C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{385d}$$

```
[Out] (4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(105*A + 143*C
)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(105*A + 143*C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(231*d) + (8*a^3*(35*A + 44*C)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(385*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(11*d) + (4*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(33*a*d) + (2*(35*A + 33*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])
*Sin[c + d*x])/(231*d)
```

Rubi [A] time = 0.645205, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3046, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(105A+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^3(35A+44C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{385d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(105*A + 143*C
)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(105*A + 143*C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(231*d) + (8*a^3*(35*A + 44*C)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(385*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(11*d) + (4*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*
x])/(33*a*d) + (2*(35*A + 33*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])
*Sin[c + d*x])/(231*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(C+A\cos^2(c+\right.$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d} + \frac{2}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d} + \frac{4A}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d} + \frac{4A}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{11d} + \frac{4A}{}$$

$$= \frac{8a^3(35A+44C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{385d} + \frac{2A\cos^{\frac{3}{2}}(}{}$$

$$= \frac{8a^3(35A+44C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{385d} + \frac{2A\cos^{\frac{3}{2}}(}{}$$

$$= \frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(105A+143C)\sqrt{}}{2}$$

$$= \frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(105A+143C)F}{231d}$$

Mathematica [C] time = 6.32568, size = 976, normalized size = 3.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)
,x]
```

```
[Out] a^3*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((5*A +
 7*C)*Cot[c])/(10*d) + ((1953*A + 2354*C)*Cos[d*x]*Sin[c])/(7392*d) + ((25*
A + 18*C)*Cos[2*d*x]*Sin[2*c])/(240*d) + ((189*A + 44*C)*Cos[3*d*x]*Sin[3*c
])/(4928*d) + (A*Cos[4*d*x]*Sin[4*c])/(96*d) + (A*Cos[5*d*x]*Sin[5*c])/(704
*d) + ((1953*A + 2354*C)*Cos[c]*Sin[d*x])/(7392*d) + ((25*A + 18*C)*Cos[2*c
]*Sin[2*d*x])/(240*d) + ((189*A + 44*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + (A*
Cos[4*c]*Sin[4*d*x])/(96*d) + (A*Cos[5*c]*Sin[5*d*x])/(704*d)) - (5*A*(1 +
Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c]^2]) - (1
3*C*(1 + Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]
^2]) - (A*(1 + Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d) - (7*
C*(1 + Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/
```

```
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d))
```

Maple [A] time = 2.155, size = 436, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/1155*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(3360*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-14560*A*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^10+(25760*A+1320*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-
24080*A-4752*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13090*A+6622*C)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2940*A-2288*C)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+525*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1155*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+715*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algori
thm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\Big(\big(Ca^3\cos(dx+c)^5\sec(dx+c)^5 + 3\,Ca^3\cos(dx+c)^5\sec(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)^5\sec(dx+c)^3 + ($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algori
thm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5*sec(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^5*sec(
d*x + c)^4 + (A + 3*C)*a^3*cos(d*x + c)^5*sec(d*x + c)^3 + (3*A + C)*a^3*co
s(d*x + c)^5*sec(d*x + c)^2 + 3*A*a^3*cos(d*x + c)^5*sec(d*x + c) + A*a^3*c
```

```
os(d*x + c)^5)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + A\right)(a \sec (dx+c) + a)^3 \cos (dx+c)^{\frac{11}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algori
thm="giac")

[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*cos(d*x + c)^(11/2)
, x)
```

3.1100 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=213

$$\frac{4a^3(11A+21C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(17A+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(16A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}$$

```
[Out] (4*a^3*(17*A + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 21*
C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(16*A + 21*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*
Sin[c + d*x])/(9*d) + (4*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Si
n[c + d*x])/(21*a*d) + (2*(73*A + 63*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c
 + d*x])*Sin[c + d*x])/(315*d)
```

Rubi [A] time = 0.622529, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3046, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(11A+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(17A+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(16A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(73A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(17*A + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 21*
C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(16*A + 21*C)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*
Sin[c + d*x])/(9*d) + (4*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Si
n[c + d*x])/(21*a*d) + (2*(73*A + 63*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c
 + d*x])*Sin[c + d*x])/(315*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d}+\frac{2\int \frac{(a+}{}}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d}+\frac{4A\sqrt{c}}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d}+\frac{4A\sqrt{c}}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\sin(c+dx)}{9d}+\frac{4A\sqrt{c}}{}$$

$$=\frac{8a^3(16A+21C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d}+\frac{2A\sqrt{\cos(c+}}{}$$

$$=\frac{8a^3(16A+21C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d}+\frac{2A\sqrt{\cos(c+}}{}$$

$$=\frac{4a^3(17A+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+21C)F\left(\frac{1}{2}(\right.}{21d}$$

Mathematica [C] time = 6.43928, size = 1116, normalized size = 5.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(-((17*A + 27*C)*Cot[c])/(15*d) + ((97*A + 84*C)*Cos[d*x]*Sin[
c])/(168*d) + ((73*A + 18*C)*Cos[2*d*x]*Sin[2*c])/(360*d) + (3*A*Cos[3*d*x]
*Sin[3*c])/(56*d) + (A*Cos[4*d*x]*Sin[4*c])/(144*d) + ((97*A + 84*C)*Cos[c]
*Sin[d*x])/(168*d) + ((73*A + 18*C)*Cos[2*c]*Sin[2*d*x])/(360*d) + (3*A*Cos
[3*c]*Sin[3*d*x])/(56*d) + (A*Cos[4*c]*Sin[4*d*x])/(144*d)))/(A + 2*C + A*C
os[2*c + 2*d*x]) - (11*A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d
*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x]
)*Sqrt[1 + Cot[c]^2]) - (C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])
*Sqrt[1 + Cot[c]^2]) - (17*A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sq
rt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*
```

```
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d*(A + 2*C + A*Cos[2*c
+ 2*d*x])) - (9*C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Co
s[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 2.142, size = 408, normalized size = 1.9

$$-\frac{4a^3}{315d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-560\,A\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}+2200\,A\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2200*A*sin(1/2*d*x+1/2*c)^8*cos(1/
2*d*x+1/2*c)+(-3412*A-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(2702*
A+882*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-738*A-378*C)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^4+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^4*sec(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^4*sec(
d*x + c)^4 + (A + 3*C)*a^3*cos(d*x + c)^4*sec(d*x + c)^3 + (3*A + C)*a^3*co
s(d*x + c)^4*sec(d*x + c)^2 + 3*A*a^3*cos(d*x + c)^4*sec(d*x + c) + A*a^3*c
os(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2),
 x)
```

3.1101 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=215

$$\frac{4a^3(13A+35C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A-35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}$$

```
[Out] (4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 35*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A - 35*C)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]]) + (2*(A - 7*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(7*a*d) + (2*(11*A - 35*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos
[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.62801, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3044, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(13A+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A-35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(11}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 35*C)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A - 35*C)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]]) + (2*(A - 7*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2
*Sin[c + d*x])/(7*a*d) + (2*(11*A - 35*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos
[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2\int \frac{(a+a\cos(c+dx))^3}{\cdots}}{\cdots}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2(A-7C)\sqrt{\cos(\cdots}}{\cdots}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2(A-7C)\sqrt{\cos(\cdots}}{\cdots}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2(A-7C)\sqrt{\cos(\cdots}}{\cdots}$$

$$= \frac{4a^3(41A-35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C(a+a\,\cdots}{\cdots}$$

$$= \frac{4a^3(41A-35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d} + \frac{2C(a+a\,\cdots}{\cdots}$$

$$= \frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(13A+35C)F\left(\frac{1}{2}\cdots\right.}{21d}$$

Mathematica [C] time = 6.56152, size = 1108, normalized size = 5.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(-((14*A + 5*C + 14*A*Cos[2*c] + 15*C*Cos[2*c])*Csc[c]*Sec[c])
/(20*d) + ((107*A + 28*C)*Cos[d*x]*Sin[c])/(168*d) + (3*A*Cos[2*d*x]*Sin[2*
c])/(20*d) + (A*Cos[3*d*x]*Sin[3*c])/(56*d) + ((107*A + 28*C)*Cos[c]*Sin[d*
x])/(168*d) + (C*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d) + (3*A*Cos[2*c]*Sin[2*
d*x])/(20*d) + (A*Cos[3*c]*Sin[3*d*x])/(56*d)))/(A + 2*C + A*Cos[2*c + 2*d*
x]) - (13*A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C
*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]) - (5*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 +
 Cot[c]^2]) - (7*A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c
+ d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
```

```
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
 - (C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A
+ C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.736, size = 569, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/105*a^3*(120*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A+5*C)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)-4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(52
*A+35*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+175*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-
105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^4+\left(A+3\,C\right)a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^3*sec(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^3*sec(
d*x + c)^4 + (A + 3*C)*a^3*cos(d*x + c)^3*sec(d*x + c)^3 + (3*A + C)*a^3*co
s(d*x + c)^3*sec(d*x + c)^2 + 3*A*a^3*cos(d*x + c)^3*sec(d*x + c) + A*a^3*c
os(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2),
 x)
```

3.1102 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=211

$$\frac{4a^3(3A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(9A-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^3(3A-10C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(3}{}$$

```
[Out] (4*a^3*(9*A - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*C)*El
lipticF[(c + d*x)/2, 2])/(3*d) + (8*a^3*(3*A - 10*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(3*d*Cos[c +
d*x]^(3/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(a*d*Sqrt[Cos[c
 + d*x]]) + (2*(3*A - 35*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin
[c + d*x])/(15*d)
```

Rubi [A] time = 0.62879, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3044, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(9A-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^3(3A-10C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(3A-35C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(9*A - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*C)*El
lipticF[(c + d*x)/2, 2])/(3*d) + (8*a^3*(3*A - 10*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(3*d*Cos[c +
d*x]^(3/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(a*d*Sqrt[Cos[c
 + d*x]]) + (2*(3*A - 35*C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin
[c + d*x])/(15*d)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^3(3a}{\cos}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+\right.}{ad\sqrt{\cos(c}}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+\right.}{ad\sqrt{\cos(c}}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+\right.}{ad\sqrt{\cos(c}}$$

$$= \frac{8a^3(3A-10C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2C(a+a\cos(c}{3d\cos}$$

$$= \frac{8a^3(3A-10C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2C(a+a\cos(c}{3d\cos}$$

$$= \frac{4a^3(9A-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(3A+5C)F\left(\frac{1}{2}(c+d\right.}{3d}$$

Mathematica [C] time = 6.58577, size = 1089, normalized size = 5.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(-((18*A - 25*C + 18*A*Cos[2*c] + 5*C*Cos[2*c])*Csc[c]*Sec[c])
/(20*d) + (A*Cos[d*x]*Sin[c])/(2*d) + (A*Cos[2*d*x]*Sin[2*c])/(20*d) + (A*C
os[c]*Sin[d*x])/(2*d) + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*
Sec[c + d*x]*(C*Sin[c] + 9*C*Sin[d*x]))/(6*d) + (A*Cos[2*c]*Sin[2*d*x])/(20
*d)))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (A*Cos[c + d*x]^5*Csc[c]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6
*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + A*Co
s[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (5*C*Cos[c + d*x]^5*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A
*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (9*A*Cos[c + d*x]^5*Csc[c]*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((Hypergeometri
cPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan
```

```
[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]
^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A +
 2*C + A*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6
*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2*C + A*Cos[2
*c + 2*d*x]))
```

Maple [B] time = 2.693, size = 704, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -4/15*(24*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+15*C)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A+25*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+15*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+15*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos\left(dx+c\right)^2\sec\left(dx+c\right)^5+3\,Ca^3\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4+(A+3\,C)a^3\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3+\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^2*sec(d*x + c)^5 + 3*C*a^3*cos(d*x + c)^2*sec(
d*x + c)^4 + (A + 3*C)*a^3*cos(d*x + c)^2*sec(d*x + c)^3 + (3*A + C)*a^3*co
s(d*x + c)^2*sec(d*x + c)^2 + 3*A*a^3*cos(d*x + c)^2*sec(d*x + c) + A*a^3*c
os(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2),
 x)
```

3.1103 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=213

$$\frac{4a^3(5A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(5A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+$$

```
[Out] (4*a^3*(5*A - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*C)*El
lipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A + 21*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c +
d*x]^(5/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(5*a*d*Cos[c +
d*x]^(3/2)) + (2*(5*A + 11*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rubi [A] time = 0.63446, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3044, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(5A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(5*A - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*C)*El
lipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A + 21*C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c +
d*x]^(5/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(5*a*d*Cos[c +
d*x]^(3/2)) + (2*(5*A + 11*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\int \frac{(a+a\cos(c+dx))^3}{\cdots}}{\cdots}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4C\left(a^2+a^2\cos(\cdots\right.}{5ad\cos\cdots}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4C\left(a^2+a^2\cos(\cdots\right.}{5ad\cos\cdots}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4C\left(a^2+a^2\cos(\cdots\right.}{5ad\cos\cdots}$$

$$=-\frac{4a^3(5A+21C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2C(a+a\cdots}{\cdots}$$

$$=-\frac{4a^3(5A+21C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2C(a+a\cdots}{\cdots}$$

$$=\frac{4a^3(5A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(5A+3C)F\left(\frac{1}{2}(c\cdots\right.}{3d}$$

Mathematica [C] time = 6.646, size = 1085, normalized size = 5.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(-((5*A - 36*C + 15*A*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + (A*Cos
[d*x]*Sin[c])/(6*d) + (A*Cos[c]*Sin[d*x])/(6*d) + (C*Sec[c]*Sec[c + d*x]^3*
Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*x]^2*(C*Sin[c] + 5*C*Sin[d*x]))/(10*d)
 + (Sec[c]*Sec[c + d*x]*(5*C*Sin[c] + 5*A*Sin[d*x] + 18*C*Sin[d*x]))/(10*d)
))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (5*A*Cos[c + d*x]^5*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*
(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*C
os[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (C*Cos[c + d*x]^5*Csc[c]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6
*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + A*Co
s[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d
*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
```

```
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2*C +
 A*Cos[2*c + 2*d*x])) + (9*C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sq
rt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + A*Cos[2*c
+ 2*d*x]))
```

Maple [B] time = 7.404, size = 939, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(40*A*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+100*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^4-120*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+60*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+108*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-216*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2+90*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^2-108*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+246*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+25*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)*A+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^3\cos(dx+c)\sec(dx+c)^5 + 3\,Ca^3\cos(dx+c)\sec(dx+c)^4 + (A+3\,C)a^3\cos(dx+c)\sec(dx+c)^3 + (\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)*sec(d*x + c)^5 + 3*C*a^3*cos(d*x + c)*sec(d*x
+ c)^4 + (A + 3*C)*a^3*cos(d*x + c)*sec(d*x + c)^3 + (3*A + C)*a^3*cos(d*x
+ c)*sec(d*x + c)^2 + 3*A*a^3*cos(d*x + c)*sec(d*x + c) + A*a^3*cos(d*x + c
))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2),
 x)
```

3.1104 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=213

$$\frac{4a^3(35A+13C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5A+7C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A + 13*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(70*A + 53*C)*Sin[c + d*x])/(10
5*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d*Co
s[c + d*x]^(7/2)) + (12*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(35*a*d*
Cos[c + d*x]^(5/2)) + (2*(5*A + 7*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])
/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.651524, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {4114, 3044, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(35A+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5A+7C)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (-4*a^3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A + 13*C)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(70*A + 53*C)*Sin[c + d*x])/(10
5*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(7*d*Co
s[c + d*x]^(7/2)) + (12*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(35*a*d*
Cos[c + d*x]^(5/2)) + (2*(5*A + 7*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])
/(15*d*Cos[c + d*x]^(3/2))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\int \frac{(a+a\cos(c+dx))^3(3a}{\cos}}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12C\left(a^2+a^2\cos(c\right.}{35ad\cos}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12C\left(a^2+a^2\cos(c\right.}{35ad\cos}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12C\left(a^2+a^2\cos(c\right.}{35ad\cos}$$

$$=\frac{8a^3(70A+53C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}}+\frac{2C(a+a\cos(c+dx))^3\sin}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{8a^3(70A+53C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}}+\frac{2C(a+a\cos(c+dx))^3\sin}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$=-\frac{4a^3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(35A+13C)F\left(\frac{1}{2}(\right.}{21d}$$

Mathematica [C] time = 6.71717, size = 1102, normalized size = 5.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2),
x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(-((-25*A - 28*C + 5*A*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + (C*Se
c[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c]*Sec[c + d*x]^3*(5*C*Sin[c] +
 21*C*Sin[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]^2*(63*C*Sin[c] + 35*A*Sin[d*
x] + 130*C*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 130*C*S
in[c] + 315*A*Sin[d*x] + 294*C*Sin[d*x]))/(210*d)))/(A + 2*C + A*Cos[2*c +
2*d*x]) - (5*A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A
+ C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 +
Cot[c]^2]) - (13*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3
*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2]) + (A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[
```

```
c + d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2*C + A*Cos[2*c + 2*d*x])
) + (7*C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*
(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 8.409, size = 1012, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/40*C/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+1/8*C*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+(1/8*A+3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/
8*A+1/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\sec(dx+c)^5 + 3\,Ca^3\sec(dx+c)^4 + (A+3\,C)a^3\sec(dx+c)^3 + (3\,A+C)a^3\sec(dx+c)^2 + 3\,Aa^3\sec(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)),
 x)
```

3.1105 $\int \frac{(a+a\sec(c+dx))^3(A+C\sec^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{4a^3(21A+11C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^3(27A+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(21A+16C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(63A\ \cdots}{}$$

```
[Out] (-4*a^3*(27*A + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21*A + 11
*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(21*A + 16*C)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(27*A + 17*C)*Sin[c + d*x])/(15*d*Sqrt[
Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*
x]^(7/2)) + (2*(63*A + 73*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*
Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.678175, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3044, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(21A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(27A+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{8a^3(21A+16C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(63A+73C)\,\text{s}\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-4*a^3*(27*A + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21*A + 11
*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (8*a^3*(21*A + 16*C)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(27*A + 17*C)*Sin[c + d*x])/(15*d*Sqrt[
Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x
]^(9/2)) + (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*
x]^(7/2)) + (2*(63*A + 73*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*
Cos[c + d*x]^(5/2))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{2\int\frac{(a+a\cos(c+dx))^3\left(3aC+\frac{1}{2}a(9A+C\right.}{\cos^{\frac{9}{2}}(c+dx)}}{9a}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin}{21ad\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin}{21ad\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+\frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin}{21ad\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{8a^3(21A+16C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$=\frac{8a^3(21A+16C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$=\frac{4a^3(21A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{8a^3(21A+16C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+$$

$$=-\frac{4a^3(27A+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(21A+11C)F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

Mathematica [C] time = 6.76038, size = 1135, normalized size = 4.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(((27*A + 17*C)*Csc[c]*Sec[c])/(15*d) + (C*Sec[c]*Sec[c + d*x]
^5*Sin[d*x])/(18*d) + (Sec[c]*Sec[c + d*x]^4*(7*C*Sin[c] + 27*C*Sin[d*x]))/
(126*d) + (Sec[c]*Sec[c + d*x]^3*(135*C*Sin[c] + 63*A*Sin[d*x] + 238*C*Sin[
d*x]))/(630*d) + (Sec[c]*Sec[c + d*x]*(105*A*Sin[c] + 110*C*Sin[c] + 378*A*
Sin[d*x] + 238*C*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]^2*(63*A*Sin[c] +
 238*C*Sin[c] + 315*A*Sin[d*x] + 330*C*Sin[d*x]))/(630*d)))/(A + 2*C + A*Co
s[2*c + 2*d*x]) - (A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]) - (11*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
```

```
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + A*Cos[2*c + 2*d*x])*
Sqrt[1 + Cot[c]^2]) + (9*A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a +
a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt
[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + A*Cos[2*c +
2*d*x])) + (17*C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
d*x])^3*(A + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d*(A + 2*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 9.904, size = 1246, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/5*(1/8*A+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*C*(-1/144*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7
/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+3/8*C*(-1/56*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A+1/8*C
)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
```

```
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+3/8*A*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3 \sec(dx+c)^5 + 3\,Ca^3 \sec(dx+c)^4 + (A+3\,C)a^3 \sec(dx+c)^3 + (3\,A+C)a^3 \sec(dx+c)^2 + 3\,Aa^3 \sec(\right.$$
$$\overline{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)(a\sec(dx+c) + a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)),
 x)
```

3.1106 $\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{4a^3(143A+105C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}-\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(143A+105C)\sin(c+dx)}{231d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(4$$

```
[Out] (-4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(143*A + 105*
C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^3*(44*A + 35*C)*Sin[c + d*x])/
(385*d*Cos[c + d*x]^(5/2)) + (4*a^3*(143*A + 105*C)*Sin[c + d*x])/(231*d*Co
s[c + d*x]^(3/2)) + (4*a^3*(7*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) +
 (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(33*a*d*Cos[c + d*x]^(9/2))
+ (2*(33*A + 35*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(231*d*Cos[c + d*
x]^(7/2))
```

Rubi [A] time = 0.713054, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 3044, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^3(143A+105C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(143A+105C)\sin(c+dx)}{231d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^3(44A+3}{385d\,\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-4*a^3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(143*A + 105*
C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^3*(44*A + 35*C)*Sin[c + d*x])/
(385*d*Cos[c + d*x]^(5/2)) + (4*a^3*(143*A + 105*C)*Sin[c + d*x])/(231*d*Co
s[c + d*x]^(3/2)) + (4*a^3*(7*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) +
 (4*C*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(33*a*d*Cos[c + d*x]^(9/2))
+ (2*(33*A + 35*C)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(231*d*Cos[c + d*
x]^(7/2))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\cos(c+dx))^3\left(C+A\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^3\left(3aC+\frac{1}{2}a(11A+}{\cos^{\frac{11}{2}}(c+dx)}}{11a}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin(}{33ad\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin(}{33ad\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{4C\left(a^2+a^2\cos(c+dx)\right)^2\sin(}{33ad\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{8a^3(44A+35C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{8a^3(44A+35C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{8a^3(44A+35C)\sin(c+dx)}{385d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^3(143A+105C)\sin(c+dx)}{231d\cos^{\frac{3}{2}}(c+dx)} + 4a$$

$$= -\frac{4a^3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(143A+105C)F\left(\frac{1}{2}(c+dx)\right|}{231d}$$

Mathematica [C] time = 6.87191, size = 1179, normalized size = 4.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec
[c + d*x]^2)*(((7*A + 5*C)*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^6*
Sin[d*x])/(22*d) + (Sec[c]*Sec[c + d*x]^5*(3*C*Sin[c] + 11*C*Sin[d*x]))/(66
*d) + (Sec[c]*Sec[c + d*x]^4*(77*C*Sin[c] + 33*A*Sin[d*x] + 126*C*Sin[d*x])
)/(462*d) + (Sec[c]*Sec[c + d*x]^3*(165*A*Sin[c] + 630*C*Sin[c] + 693*A*Sin
[d*x] + 770*C*Sin[d*x]))/(2310*d) + (Sec[c]*Sec[c + d*x]^2*(693*A*Sin[c] +
770*C*Sin[c] + 1430*A*Sin[d*x] + 1050*C*Sin[d*x]))/(2310*d) + (Sec[c]*Sec[c
 + d*x]*(715*A*Sin[c] + 525*C*Sin[c] + 1617*A*Sin[d*x] + 1155*C*Sin[d*x]))/
(1155*d)))/(A + 2*C + A*Cos[2*c + 2*d*x]) - (13*A*Cos[c + d*x]^5*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A +
```

```
 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (5*C*Cos[c + d*x]^5*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(11*d*
(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (7*A*Cos[c + d*x]^5*Cs
c[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((H
ypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + T
an[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c
]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
)/(10*d*(A + 2*C + A*Cos[2*c + 2*d*x])) + (C*Cos[c + d*x]^5*Csc[c]*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + C*Sec[c + d*x]^2)*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2
*C + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 10.798, size = 1408, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/5*(3/
8*A+1/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3/8*C*(-1/144*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7
/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+(1/8*A+3/8*C)*(-1/56*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*C*(-1/352
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
```

```
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-15/154*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x
+1/2*c)^2-1/2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))+3/8*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+
1/8*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^
2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + 3\,Ca^3\sec(dx+c)^4 + (A+3\,C)a^3\sec(dx+c)^3 + (3\,A+C)a^3\sec(dx+c)^2 + 3\,Aa^3\sec(\right.}{\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + 3*C*a^3*sec(d*x + c)^4 + (A + 3*C)*a^3*sec
(d*x + c)^3 + (3*A + C)*a^3*sec(d*x + c)^2 + 3*A*a^3*sec(d*x + c) + A*a^3)/
cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2),
 x)
```

3.1107 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{5(9A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21ad}-\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(9A+7C)\text{s}}{}$$

```
[Out] (-3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(9*A + 7*C)*Ellipti
cF[(c + d*x)/2, 2])/(21*a*d) + (5*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(21*a*d) - ((7*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((9*
A + 7*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*a*d) - ((A + C)*Cos[c + d*x]^(
7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.287431, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2748, 2635, 2639, 2641}

$$\frac{5(9A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(9A+7C)\sin(c+}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```


```
[Out] (-3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(9*A + 7*C)*Ellipti
cF[(c + d*x)/2, 2])/(21*a*d) + (5*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(21*a*d) - ((7*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((9*
A + 7*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*a*d) - ((A + C)*Cos[c + d*x]^(
7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(7A+5C)+\frac{1}{2}a(9A\right.}{a^2}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(7A+5C)\int\cos^{\frac{5}{2}}(c+dx)\,dx}{2a} + \frac{(9A+7}{}$$

$$= -\frac{(7A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{(9A+7C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7ad} -$$

$$= -\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(9A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21ad} - \frac{(7A}{}$$

$$= -\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(9A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad} + \frac{5(9A+7C)\sqrt{}}{}$$

Mathematica [C] time = 6.72502, size = 1393, normalized size = 7.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] (((-21*I)/10)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*
Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E
^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d
```

```
*x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (((3
*I)/2)*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 +
(d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((4*(5*A + 5*C + 16*A*
Cos[c] + 10*C*Cos[c])*Csc[c])/(5*d) + (2*(51*A + 28*C)*Cos[d*x]*Sin[c])/(21
*d) - (4*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (2*A*Cos[3*d*x]*Sin[3*c])/(7*d) + (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (2*(51
*A + 28*C)*Cos[c]*Sin[d*x])/(21*d) - (4*A*Cos[2*c]*Sin[2*d*x])/(5*d) + (2*A
*Cos[3*c]*Sin[3*d*x])/(7*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
+ d*x])) - (30*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(7*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a
 + a*Sec[c + d*x])) - (10*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A +
 C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + C
ot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.47, size = 295, normalized size = 1.5

$$-\frac{1}{105\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(225*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+175*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*C*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))-480*A*sin(1/2*d*x+1/2*c)^10+864*A*sin(1/2*d*x+1/2*c)
^8+(-888*A-280*C)*sin(1/2*d*x+1/2*c)^6+(930*A+630*C)*sin(1/2*d*x+1/2*c)^4+(
-321*A-245*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + A*cos(d*x + c)^3)*sqrt(cos(d*x
+ c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x
)
```

3.1108 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{(5A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A+5C)\sin(c+dx)\ldots}{\ldots}$$

```
[Out] (3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A + 3*C)*EllipticF[
(c + d*x)/2, 2])/(3*a*d) - ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a*d) + ((7*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.26467, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2748, 2635, 2641, 2639}

$$-\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A+5C)\sin(c+\ldots}{5a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*(7*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A + 3*C)*EllipticF[
(c + d*x)/2, 2])/(3*a*d) - ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a*d) + ((7*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A + C)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(5A+3C)+\frac{1}{2}a(7A\right.}{a^2}$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A+3C)\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{(7A+5}{}$$

$$= -\frac{(5A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{(7A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}$$

$$= \frac{3(7A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{(5A+3C)\sqrt{\cos(}}{3}$$

Mathematica [C] time = 6.64194, size = 1345, normalized size = 8.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] (((21*I)/10)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*S
ec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (((3*
I)/2)*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c +
d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
```

```
)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*
d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin
[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (
d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((-4*(5*A + 5*C + 16*A*
Cos[c] + 10*C*Cos[c])*Csc[c])/(5*d) - (8*A*Cos[d*x]*Sin[c])/(3*d) + (4*A*Co
s[2*d*x]*Sin[2*c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] +
 C*Sin[(d*x)/2]))/d - (8*A*Cos[c]*Sin[d*x])/(3*d) + (4*A*Cos[2*c]*Sin[2*d*x
])/(5*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (10*A*Co
s[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
 + (2*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c
+ d*x]))
```

Maple [A] time = 2.252, size = 277, normalized size = 1.7

$$-\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+15*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*C*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8-56*A*sin(1/2*d*x+1/2*c)^6+(-30
*A-30*C)*sin(1/2*d*x+1/2*c)^4+(23*A+15*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(cos(d*x
+ c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x
)
```

3.1109 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{(5A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A+3C)\sin(\ldots}{}$$

```
[Out] -(((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((5*A + 3*C)*EllipticF[(c
+ d*x)/2, 2])/(3*a*d) + ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.240637, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2748, 2639, 2635, 2641}

$$\frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A+3C)\sin(c+dx)\sqrt{\ldots}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((5*A + 3*C)*EllipticF[(c
+ d*x)/2, 2])/(3*a*d) + ((5*A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+A\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$
$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\left(-\frac{1}{2}a(3A+C)+\frac{1}{2}a(5A\right.}{a^2}$$
$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3A+C)\int\sqrt{\cos(c+dx)}\,dx}{2a} + \frac{(5A+3}{}$$
$$= -\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{(A+C)}{d(}$$
$$= -\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A+3C)\sqrt{\cos(}}{3}$$

Mathematica [C] time = 6.56992, size = 1300, normalized size = 10.66

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] (((-3*I)/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - ((I/2)
*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^
2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))
```

```
*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))
)/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d*x)/
2]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((4*(A + C + 2*A*Cos[c])*Csc
[c])/d + (8*A*Cos[d*x]*Sin[c])/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Si
n[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*A*Cos[c]*Sin[d*x])/(3*d)))/((A + 2*C +
 A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (10*A*Cos[c/2 + (d*x)/2]^2*Cos
[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2
*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (2*C*Cos[c/2 + (d*x
)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C +
 A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.121, size = 262, normalized size = 2.2

$$-\frac{1}{3\,ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+3*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A+6*C)*sin(1/2*d*x+1/2*c)^4+(-7*A
-3*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\cos(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c)
)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x
)
```

3.1110 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{(A-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] ((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - C)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rubi [A] time = 0.230836, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4114, 3042, 2748, 2641, 2639}

$$-\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((3*A + C)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - C)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx=\int \frac{C+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$$

$$=-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-C)+\frac{1}{2}a(3A+C)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$=-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{(3A+C)\int \cdot}{}$$

$$=\frac{(3A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sqrt{\cos(c+dx)}}{d(a+a\cos(c+}$$

Mathematica [C] time = 6.52412, size = 1270, normalized size = 15.12

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x]),
x]
```

```
[Out] (((3*I)/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec
[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((
2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x)
)*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + ((I/2)*
C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2
)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*
Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))
/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d*x)/2
]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((-4*(A + C + 2*A*Cos[c])*Csc
[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/
d))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Cos[c/2 +
(d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Co
```

```
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2
*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (2*C*Co
s[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.263, size = 245, normalized size = 2.9

$$\frac{1}{ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\left(AE\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)+(2*A+2*C)*sin(1/2*d*x+1/2*c)^4+(-A-C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{\cos(c+dx)}}{\sec(c+dx)+1}\,dx + \int \frac{C\sqrt{\cos(c+dx)}\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x) + Integral(C*sqrt(cos
(c + d*x))*sec(c + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x
)
```

3.1111 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=112

$$\frac{(A-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+$$

```
[Out] -(((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A - C)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((A + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A +
 C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.243782, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2748, 2636, 2639, 2641}

$$\frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```


```
[Out] -(((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A - C)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((A + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A +
 C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A+3C)+\frac{1}{2}a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{(A-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{(A+3C)\int \frac{}{\cos}}{2a}$$

$$= \frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+$$

$$= -\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.62202, size = 1304, normalized size = 11.64

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),
x]
```

```
[Out] ((-I/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c
+ d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (((3*I)/2)
*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^
2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
```

```
)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))
*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))
)/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d*x)/
2]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((2*(2*C + A*Cos[c] + C*Cos[
c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*
x)/2] + C*Sin[(d*x)/2]))/d + (8*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/((A + 2
*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*A*Cos[c/2 + (d*x)/2]^2*
Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (2*C*Cos[c/2 + (d*
x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C
+ A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 4.679, size = 316, normalized size = 2.8

$$-\frac{1}{ad}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^4+(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+5*C)*sin(1/2*d*x+1/2*c)^
2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)\sec\left(dx+c\right)+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)*sec(d*x
+ c) + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{C\sec^2\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) + In
tegral(C*sec(c + d*x)**2/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*
x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))),
 x)
```

3.1112 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{(3A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A+5C)\sin(}{3ad\cos^{\frac{3}{2}}(c+}$$

```
[Out] ((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((3*A + 5*C)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- ((A + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.262772, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2748, 2636, 2641, 2639}

$$\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A+5C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((A + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((3*A + 5*C)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- ((A + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A+5C)-\frac{1}{2}a(A+3C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(A+3C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(3A+5C)\int -}{2a}$$

$$= \frac{(3A+5C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(A+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+d}$$

$$= \frac{(A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(3A+5C)\sin(c+dx}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 6.95902, size = 1337, normalized size = 8.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),
x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (((3*I)/2)*
C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2
```

```
)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*
Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))
/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 + (d*x)/2
]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((-2*(2*C + A*Cos[c] + C*Cos[
c])*Csc[c/2]*Sec[c/2]*Sec[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*
x)/2] + C*Sin[(d*x)/2]))/d + (8*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (
8*Sec[c]*Sec[c + d*x]*(C*Sin[c] - 3*C*Sin[d*x]))/(3*d)))/((A + 2*C + A*Cos[
2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x
]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x
])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (10*C*Cos[c/2 + (d*x)/2]^2*Co
s[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 6.177, size = 486, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A+C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)-2*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^2\sec\left(dx+c\right)+a\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^2*sec(d*
x + c) + a*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)),
 x)
```

3.1113 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=192

$$-\frac{(3A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A+5C)}{3ad\cos^{\frac{3}{2}}}$$

```
[Out] (-3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A + 5*C)*EllipticF
[(c + d*x)/2, 2])/(3*a*d) + ((5*A + 7*C)*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^
(5/2)) - ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) + (3*(5*A +
7*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x])/(d*C
os[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.273037, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2748, 2636, 2639, 2641}

$$-\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A+5C)\sin(c+}{3ad\cos^{\frac{3}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-3*(5*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A + 5*C)*EllipticF
[(c + d*x)/2, 2])/(3*a*d) + ((5*A + 7*C)*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^
(5/2)) - ((3*A + 5*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) + (3*(5*A +
7*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]) - ((A + C)*Sin[c + d*x])/(d*C
os[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx=\int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$=-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}a(5A+7C)-\frac{1}{2}a(3A+5C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx}{a^2}$$

$$=-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}-\frac{(3A+5C)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a}+\frac{(5A+7C)\int}{2a}$$

$$=\frac{(5A+7C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{(3A+5C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{(A+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+$$

$$=-\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(5A+7C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)}-\frac{(3A+5C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-$$

$$=-\frac{3(5A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(3A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(5A+7C)\sin(c+}{5ad\cos^{\frac{5}{2}}(c+d}$$

Mathematica [C] time = 7.24337, size = 1382, normalized size = 7.2

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),
x]
```

```
[Out] (((-3*I)/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
```

```
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (((21*
I)/10)*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (Cos[c/2 +
(d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2)*((2*(10*A + 16*C + 5*A
*Cos[c] + 5*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (4*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*C*Sec[c]*Sec[c + d*x]
^3*Sin[d*x])/(5*d) - (8*Sec[c]*Sec[c + d*x]*(5*C*Sin[c] - 15*A*Sin[d*x] - 2
4*C*Sin[d*x]))/(15*d) + (8*Sec[c]*Sec[c + d*x]^2*(3*C*Sin[c] - 5*C*Sin[d*x]
))/(15*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Co
s[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) +
 (10*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c
 + d*x]))
```

Maple [B] time = 8.174, size = 803, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-2/5*C/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+(-A-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)-2*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(2*A+2*C)*(-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+a\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^3*sec(d*
x + c) + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)),
 x)
```

3.1114 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=196

$$-\frac{5(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{4(14A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{4(14A+5C}{}$$

```
[Out] (4*(14*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A + C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*a^2*d) + (4*(14*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
- ((3*A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) -
((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.404656, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4(14A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{4(14A+5C)\sin(c+}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (4*(14*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A + C)*Ellipti
cF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*a^2*d) + (4*(14*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
- ((3*A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) -
((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(7A+C)+\frac{1}{2}a(11A+5C)\cos(\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(3A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} +$$

$$= -\frac{(3A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} -$$

$$= -\frac{5(3A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{4(14A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{15a^2d}$$

$$= \frac{4(14A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(3A+C)\sqrt{}}{}$$

Mathematica [C] time = 6.80351, size = 1398, normalized size = 7.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
2,x]
```

```
[Out] (((56*I)/5)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)
*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*S
in[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/
((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + ((4*I)*C*Cos[c/2
+ (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + A*Cos[2*c
+ 2*d*x])*(a + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A
+ C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) + (20*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*
(A + C*Sec[c + d*x]^2)*((-16*(10*A + 5*C + 18*A*Cos[c] + 5*C*Cos[c])*Csc[c]
)/(5*d) - (32*A*Cos[d*x]*Sin[c])/(3*d) + (8*A*Cos[2*d*x]*Sin[2*c])/(5*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) -
 (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d - (
32*A*Cos[c]*Sin[d*x])/(3*d) + (8*A*Cos[2*c]*Sin[2*d*x])/(5*d) + (4*(A + C)*
Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a +
 a*Sec[c + d*x])^2)
```

Maple [A] time = 2.513, size = 451, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*A*cos(1/2
*d*x+1/2*c)^10-352*A*cos(1/2*d*x+1/2*c)^8+120*A*cos(1/2*d*x+1/2*c)^6-150*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*A*cos(1/2*d*x+1/2*c)^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-120*C*cos(1/2*d*x+1/2*c)^6-50*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*cos(1/2*d*x+1/2*c)^3-120*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+266*A*cos(1/2*d*x+1/2*c)^4+190*C*cos(1/2*d*x+1/2*c)^4-135*A*cos(1/2*
d*x+1/2*c)^2-75*C*cos(1/2*d*x+1/2*c)^2+5*A+5*C)/a^2/cos(1/2*d*x+1/2*c)^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(cos(d*x
+ c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2,
 x)
```

3.1115 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{2(5A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{2(5A+C)\sin}{}$$

```
[Out] -(((7*A + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (2*(5*A + C)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + (2*(5*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d) - ((7*A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
+ d*x])) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```

Rubi [A] time = 0.381212, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2977, 2748, 2639, 2635, 2641}

$$\frac{2(5A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{2(5A+C)\sin(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((7*A + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (2*(5*A + C)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + (2*(5*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d) - ((7*A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
+ d*x])) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(5A-C)+\frac{3}{2}a(3A+C)\cos(c+dx)\right)}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(7A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{(7A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + (5$$

$$= -\frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(5A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{(7A+C}{3a}$$

$$= -\frac{(7A+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(5A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{2(5A+C)\sqrt{\cos(}}{3a}$$

Mathematica [C] time = 6.73211, size = 1355, normalized size = 8.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
2,x]
```

```
[Out] ((-7*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2
*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A
+ 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (I*C*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^2) - (40*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[
c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2
]*(a + a*Sec[c + d*x])^2) - (8*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c
 + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]
*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + C*
Sec[c + d*x]^2)*((8*(3*A + C + 4*A*Cos[c])*Csc[c])/d + (16*A*Cos[d*x]*Sin[c
])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2
]))/(3*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + C*Sin[(d*x)/
2]))/d + (16*A*Cos[c]*Sin[d*x])/(3*d) - (4*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan
[c/2])/(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.59, size = 437, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*cos(1/2*
d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+42*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*co
s(1/2*d*x+1/2*c)^6+4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*C*c
os(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*A*cos(1/2*d*x+1/2*c)^4-20*
C*cos(1/2*d*x+1/2*c)^4+21*A*cos(1/2*d*x+1/2*c)^2+9*C*cos(1/2*d*x+1/2*c)^2-A
```

```
-C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c)
)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2,
 x)
```

3.1116 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$-\frac{(5A-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{4AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (4*A*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - C)*EllipticF[(c + d*x)/2,
 2])/(3*a^2*d) - ((5*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[
c + d*x])^2)
```

Rubi [A] time = 0.348991, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2977, 2748, 2641, 2639}

$$-\frac{(5A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{4AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*A*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - C)*EllipticF[(c + d*x)/2,
 2])/(3*a^2*d) - ((5*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[
c + d*x])^2)
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(A-C)+\frac{1}{2}a(7A+C)\cos(c+\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(5A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} -$$

$$= -\frac{(5A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} -$$

$$= \frac{4AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(5A-C)\sqrt{\cos(c+dx)}}{3a^2d(1+\cos(c+}$$

Mathematica [C] time = 6.58043, size = 934, normalized size = 7.18

$$\frac{4iA\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(C\sec^2(c+dx)+A\right)\left(\frac{2e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\right.}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}\right.}{(\cos(2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
2,x]
```

```
[Out] ((4*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
```

```
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A +
 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c +
2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (4*C*Cos[c/2 + (d*x)/2
]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2
*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*S
qrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2)*((-16*A*Cot[c/2])/d - (16*A*Sec[c/
2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A
*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (4*(A + C)*Sec[c/2 + (d*x)/2]^2*Ta
n[c/2])/(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.494, size = 352, normalized size = 2.7

$$\frac{1}{6\,da^2}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+10\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d
*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*cos(1/
2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(
1/2*d*x+1/2*c)^3-38*A*cos(1/2*d*x+1/2*c)^4-2*C*cos(1/2*d*x+1/2*c)^4+15*A*co
s(1/2*d*x+1/2*c)^2+3*C*cos(1/2*d*x+1/2*c)^2-A-C)/a^2/cos(1/2*d*x+1/2*c)^3/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2,
 x)
```

3.1117 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=125

$$\frac{2(A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A+C)\sin(c+dx)\sqrt{\cdots}}{3d(a\cos(c+dx\cdots}$$

```
[Out] -(((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (2*(A + C)*EllipticF[(c +
d*x)/2, 2])/(3*a^2*d) + ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1
 + Cos[c + d*x])) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*C
os[c + d*x])^2)
```

Rubi [A] time = 0.358232, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2978, 2748, 2641, 2639}

$$\frac{2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+\cdots}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] -(((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (2*(A + C)*EllipticF[(c +
d*x)/2, 2])/(3*a^2*d) + ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1
 + Cos[c + d*x])) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*C
os[c + d*x])^2)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{C+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-5C)+\frac{1}{2}a(5A-C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \cdots}{\cdots}$$

$$= \frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{(A \cdots}{\cdots}$$

$$= -\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(A-C)\sqrt{\cos(c+\cdots}}{a^2d(1+\cos\cdots}$$

Mathematica [C] time = 6.6479, size = 1322, normalized size = 10.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2
),x]
```

```
[Out] ((-I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))/((A +
```

```
2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (I*C*Cos[c/2 + (d*x)/2]
^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometri
c2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I
)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1
/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)
*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((
2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))
*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*
(a + a*Sec[c + d*x])^2) - (8*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c +
 d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(
a + a*Sec[c + d*x])^2) - (8*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a
 + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + C*Sec
[c + d*x]^2)*((8*(A - C)*Csc[c])/d + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[
(d*x)/2] - C*Sin[(d*x)/2]))/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*
x)/2] + C*Sin[(d*x)/2]))/(3*d) - (4*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/
(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.459, size = 423, normalized size = 3.4

$$-\frac{1}{6\,da^2}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,A\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\,A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^6+4*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+16*C*cos(1/2*d*x+1/2*c)^4+9*A*c
os(1/2*d*x+1/2*c)^2-3*C*cos(1/2*d*x+1/2*c)^2-A-C)/a^2/cos(1/2*d*x+1/2*c)^3/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)*sec(d*
x + c)^2 + 2*a^2*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))
), x)
```

3.1118 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=151

$$\frac{(A-5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{4CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4C\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{}{3d\sqrt{}}$$

```
[Out] (-4*C*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A - 5*C)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (4*C*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) + ((A - 5*C
)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - ((A + C)*
Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.376276, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2978, 2748, 2636, 2639, 2641}

$$\frac{(A-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{4CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4C\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(A}{3d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (-4*C*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A - 5*C)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (4*C*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) + ((A - 5*C
)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - ((A + C)*
Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx=\int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(A+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{1}{2}a(A+7C)+\frac{3}{2}a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$=\frac{(A-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}+$$

$$=\frac{(A-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}+$$

$$=\frac{(A-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4C\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(A-5C)\sin(c+dx}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(}$$

$$=-\frac{4CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4C\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{}{3a^2}$$

Mathematica [C] time = 6.68475, size = 954, normalized size = 6.32

$$-\frac{4iC\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(C\sec^2(c+dx)+A\right)\left(\frac{2e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right.\right.}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}}{(\cos(2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2
),x]
```

```
[Out] ((-4*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2
*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A
+ 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (4*A*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c +
2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (20*C*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c +
2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*
Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2)*((16*C*Cot[c/2]*Sec[c])/d + (16*C
*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/
2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (16*C*Sec[c]*Sec[c + d*x]*S
in[d*x])/d + (4*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C +
A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.887, size = 450, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))*cos(1/2*d*x+1/2*c)-48*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(A+43*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A+37*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^
3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2*sec(
d*x + c)^2 + 2*a^2*cos(d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)
), x)
```

3.1119 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{2(A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{2(A+5C)\sin(c}{3a^2d\cos^{\frac{3}{2}}(c+}$$

```
[Out] ((A + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(A + 5*C)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) + (2*(A + 5*C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(
3/2)) - ((A + 7*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((A + 7*C)*Si
n[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A + C)*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.414405, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2978, 2748, 2636, 2641, 2639}

$$\frac{2(A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{2(A+5C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((A + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(A + 5*C)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) + (2*(A + 5*C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(
3/2)) - ((A + 7*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((A + 7*C)*Si
n[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A + C)*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx=\int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(A+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{3}{2}a(A+3C)+\frac{1}{2}a(A-5C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$=-\frac{(A+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$=-\frac{(A+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}-\frac{(A+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$=\frac{2(A+5C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{(A+7C)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(A+7C)\sin(c+dx}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos}$$

$$=\frac{(A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{2(A+5C)\sin(c}{3a^2d\cos^{\frac{3}{2}}(c+}$$

Mathematica [C] time = 7.25498, size = 1391, normalized size = 7.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2
),x]
```

```
[Out] (I*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*E^((
2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
(2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I
)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C
 + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + ((7*I)*C*Cos[c/2 + (d*x)/2
]^4*Csc[c/2]*Sec[c/2]*(A + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*
I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-
1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])^2) - (8*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*
(a + a*Sec[c + d*x])^2) - (40*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*
(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + C*S
ec[c + d*x]^2)*((-4*(4*C + A*Cos[c] + 3*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])
/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3
*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/d
 + (16*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (16*Sec[c]*Sec[c + d*x]*(C
*Sin[c] - 6*C*Sin[d*x]))/(3*d) - (4*(A + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/
(3*d)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 7.296, size = 738, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A+
C)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c
)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
```

```
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)+4
*C*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*C*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))-8*C*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3*sec(
d*x + c)^2 + 2*a^2*cos(d*x + c)^3*sec(d*x + c) + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)
), x)
```

3.1120 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=250

$$-\frac{(63A+13C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{7(33A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(63A+13C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{7(3}{}$$

```
[Out] (7*(33*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((63*A + 13*C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((63*A + 13*C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + (7*(33*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*a^
3*d) - ((A + C)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^
3) - (2*(6*A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d
*x])^2) - ((63*A + 13*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```

Rubi [A] time = 0.588112, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2977, 2748, 2635, 2641, 2639}

$$-\frac{(63A+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(33A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(63A+13C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{7(33A+7}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (7*(33*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((63*A + 13*C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((63*A + 13*C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + (7*(33*A + 7*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*a^
3*d) - ((A + C)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^
3) - (2*(6*A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d
*x])^2) - ((63*A + 13*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}dx$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(-\frac{1}{2}a(9A-C)+\frac{5}{2}a(3A+C)\cos(c+\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(6A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(6A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(6A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(63A+13C)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} + \frac{7(33A+7C)\cos^{\frac{3}{2}}(c+dx)\sin(}{30a^3d}$$

$$= \frac{7(33A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(63A+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(63A+1}{}$$

Mathematica [C] time = 7.11061, size = 1507, normalized size = 6.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (((231*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*S
ec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (((
49*I)/5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c
 + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*
x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
 E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((
2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*
I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*
Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (84*A*C
os[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
```

```
(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3
) + (52*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Se
c[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((-8*(99*A +
29*C + 132*A*Cos[c] + 20*C*Cos[c])*Csc[c])/(5*d) - (32*A*Cos[d*x]*Sin[c])/d
 + (16*A*Cos[2*d*x]*Sin[2*c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*S
in[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(1
2*A*Sin[(d*x)/2] + 7*C*Sin[(d*x)/2]))/(15*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/
2]*(99*A*Sin[(d*x)/2] + 29*C*Sin[(d*x)/2]))/(5*d) - (32*A*Cos[c]*Sin[d*x])/
d + (16*A*Cos[2*c]*Sin[2*d*x])/(5*d) + (16*(12*A + 7*C)*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(15*d) - (4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqr
t[Cos[c + d*x]]*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.33, size = 479, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*cos(1/
2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10-228*A*cos(1/2*d*x+1/2*c)^8-630*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-1386*A*cos(1/2*d*x+1/2*c)^
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6+578*C*cos(1/2*d*x+1/2*c)^6-744*A*cos
(1/2*d*x+1/2*c)^4-264*C*cos(1/2*d*x+1/2*c)^4+57*A*cos(1/2*d*x+1/2*c)^2+37*C
*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + A*cos(d*x + c)^2)*sqrt(cos(d*x
 + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3,
 x)
```

3.1121 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=209

$$\frac{(11A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(11A+C)}{}$$

```
[Out] -((119*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A + C)*Ellipti
cF[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(2*a^3*d) - ((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c +
 d*x])^3) - (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x
])^2) - ((119*A + 9*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Co
s[c + d*x]))
```

Rubi [A] time = 0.542244, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2977, 2748, 2639, 2635, 2641}

$$\frac{(11A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(11A+C)\sin(c+}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -((119*A + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A + C)*Ellipti
cF[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(2*a^3*d) - ((A + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c +
 d*x])^3) - (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x
])^2) - ((119*A + 9*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Co
s[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(7A-3C)+\frac{1}{2}a(13A+3C)\cos\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos}{}}{}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{(119}{}$$

$$= -\frac{(A+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{(119}{}$$

$$= -\frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(11A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{(}{}$$

$$= -\frac{(119A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(11A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{(11A+C)}{}$$

Mathematica [C] time = 6.94749, size = 1470, normalized size = 7.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (((-119*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*
Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E
^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d
*x))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - ((
(9*I)/5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c
 + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*
x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
 E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((
2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*
I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*
Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (44*A*C
os[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3
) - (4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
ot[c]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c
 + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((8*(59*A + 9*C
+ 60*A*Cos[c])*Csc[c])/(5*d) + (32*A*Cos[d*x]*Sin[c])/(3*d) + (4*Sec[c/2]*S
ec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (8*Sec[c/2]*
Sec[c/2 + (d*x)/2]^3*(19*A*Sin[(d*x)/2] + 9*C*Sin[(d*x)/2]))/(15*d) + (8*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] + 9*C*Sin[(d*x)/2]))/(5*d) + (
32*A*Cos[c]*Sin[d*x])/(3*d) - (8*(19*A + 9*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2]
)/(15*d) + (4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c +
d*x]]*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A] time = 2.751, size = 465, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*cos(1/
2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1
08*C*cos(1/2*d*x+1/2*c)^8+30*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)
^5+54*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1
/2*c)^6-198*C*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4+114*C*cos(1/2
*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2-27*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/
a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c)
)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3,
 x)
```

3.1122 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=186

$$-\frac{(13A-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(49A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\text{si}}{5d(a}$$

```
[Out] ((49*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - C)*EllipticF[(
c + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*
(a + a*Cos[c + d*x])^3) - (2*(4*A - C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15
*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.540663, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2977, 2748, 2641, 2639}

$$-\frac{(13A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+dx}{5d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((49*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - C)*EllipticF[(
c + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*
(a + a*Cos[c + d*x])^3) - (2*(4*A - C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15
*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-C)+\frac{1}{2}a(11A+C)\cos(c+dx)\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(4A-C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} +$$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(4A-C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - ($$

$$= -\frac{(A+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2(4A-C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - ($$

$$= \frac{(49A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+C)\cos^{\frac{5}{2}}(c+}{5d(a+a\cos}$$

Mathematica [C] time = 6.83538, size = 1446, normalized size = 7.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
3,x]
```

```
[Out] (((49*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
```

```
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - ((I/
5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x
]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x
))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]
)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (52*A*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(
A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) -
(4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c +
 d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((-8*(29*A - C + 2
0*A*Cos[c])*Csc[c])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/
2] - C*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)
/2] + C*Sin[(d*x)/2]))/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(
d*x)/2] + 2*C*Sin[(d*x)/2]))/(15*d) + (16*(7*A + 2*C)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) - (4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[
Cos[c + d*x]]*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.496, size = 451, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos(1/2
*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+1/2*c)^6
+264*A*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*
c)^2-17*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*
a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3,
 x)
```

3.1123 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=184

$$\frac{(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(9A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+}{5d(a\cos(c}$$

```
[Out] -((9*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + C)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) - (2*(3*A - 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1
5*a*d*(a + a*Cos[c + d*x])^2) + ((9*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.531016, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2977, 2978, 2748, 2641, 2639}

$$\frac{(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A+C)\sin(c+dx)\cos}{5d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -((9*A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + C)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) - (2*(3*A - 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1
5*a*d*(a + a*Cos[c + d*x])^2) + ((9*A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx=\int \frac{\sqrt{\cos(c+dx)}\left(C+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$=-\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{1}{2}a(3A-7C)+\frac{1}{2}a(9A-C)\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$=-\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2(3A-2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+$$

$$=-\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2(3A-2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+$$

$$=-\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2(3A-2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+$$

$$=-\frac{(9A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)}{5d(a+a\cos(c}$$

Mathematica [C] time = 6.77459, size = 1439, normalized size = 7.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] (((-9*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + ((I/
5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x
]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x
))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]
)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (4*A*Cos[c/2
+ (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A +
 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*
C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((8*(9*A - C)*Csc[c])
/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] - C*Sin[(d*x)/2])
)/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2] - C*Sin[(d*x)/
2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x
)/2]))/(5*d) - (8*(9*A - C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A +
 C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A + 2*C + A
*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.484, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
```

```
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8+10*C*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+22*C*cos(1/2*d*x+1/2*c)^6
+114*A*cos(1/2*d*x+1/2*c)^4-6*C*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c
)^2-7*C*cos(1/2*d*x+1/2*c)^2+3*A+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)\sec\left(dx+c\right)+a^3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)*sec(d*
x + c)^3 + 3*a^3*cos(d*x + c)*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)*sec(d*x +
 c) + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)^3\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))
), x)
```

3.1124 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{(A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2(2A-3C)\sin(c+}{15ad(a\cos(c}$$

```
[Out] -((A - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*C)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) + (2*(2*A - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1
5*a*d*(a + a*Cos[c + d*x])^2) + ((A - 9*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.523194, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4114, 3042, 2978, 2748, 2641, 2639}

$$\frac{(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2(2A-3C)\sin(c+dx)\sqrt{c}}{15ad(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -((A - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*C)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) + (2*(2*A - 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1
5*a*d*(a + a*Cos[c + d*x])^2) + ((A - 9*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{C+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-9C)+\frac{1}{2}a(7A-3C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(2A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(2A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2(2A-3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+C)\sqrt{\cos(c+}}{5d(a+a\cos}$$

Mathematica [C] time = 6.763, size = 1436, normalized size = 7.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] ((-I/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c
+ d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
```

```
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (((9*I)/
5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x
]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x
))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]
)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (4*A*Cos[c/2
+ (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A
 + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (
4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((8*(A - 9*C)*Csc[c])
/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 9*C*Sin[(d*x)/2])
)/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] - 3*C*Sin[(d*
x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(
d*x)/2]))/(5*d) + (16*(2*A - 3*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (
4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A + 2
*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.521, size = 451, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8+30*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*C*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+138*C*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4-24*C*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)
^2-3*C*cos(1/2*d*x+1/2*c)^2-3*A-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a^{3}\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^2*sec(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^2*sec
(d*x + c) + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)
), x)
```

3.1125 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=209

$$\frac{(A-13C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-49C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(A-13C)\sin(c+dx}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+\right.}$$

```
[Out] ((A - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A - 13*C)*EllipticF[(
c + d*x)/2, 2])/(6*a^3*d) - ((A - 49*C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c
+ d*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*
x])^3) + (2*(A - 4*C)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c
 + d*x])^2) + ((A - 13*C)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*
Cos[c + d*x]))
```

Rubi [A] time = 0.565463, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2978, 2748, 2636, 2639, 2641}

$$\frac{(A-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-49C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(A-13C)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] ((A - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A - 13*C)*EllipticF[(
c + d*x)/2, 2])/(6*a^3*d) - ((A - 49*C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c
+ d*x]]) - ((A + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*
x])^3) + (2*(A - 4*C)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c
 + d*x])^2) + ((A - 13*C)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*
Cos[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] &&  !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A+11C)+\frac{5}{2}a(A-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{2(A-4C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{2(A-4C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{2(A-4C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= \frac{(A-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-49C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+}{5d\sqrt{\cos(c+dx)}(a+a\,\mathrm{co}}$$

$$= \frac{(A-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-49C)\sin(c+d}{10a^3d\sqrt{\cos(c+dx}}$$

Mathematica [C] time = 6.97649, size = 1473, normalized size = 7.05

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] ((I/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (((49*I)/
5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x
]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x
))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]
)))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (4*A*Cos[c/2
+ (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
```

```
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A
 + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (
52*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(3*d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c +
 d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((-4*(-20*C + A*Co
s[c] - 29*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) - (8*Sec[c/2]*Sec[c/2 +
 (d*x)/2]*(A*Sin[(d*x)/2] - 29*C*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2
 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/
2 + (d*x)/2]^3*(A*Sin[(d*x)/2] + 11*C*Sin[(d*x)/2]))/(15*d) + (32*C*Sec[c]*
Sec[c + d*x]*Sin[d*x])/d + (8*(A + 11*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15
*d) + (4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]
*(A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 3.061, size = 679, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-65*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)
+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-49*C)*sin(1/2*d
*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A-81
7*C)*sin(1/2*d*x+1/2*c)^6+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(A-124*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(A-439*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^3\cos\left(dx+c\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3*sec(
d*x + c)^3 + 3*a^3*cos(d*x + c)^3*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^3*sec
(d*x + c) + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)
), x)
```

3.1126 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=242

$$\frac{(A+11C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+119C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+1}{2a^3d}$$

```
[Out] ((9*A + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 11*C)*Elliptic
F[(c + d*x)/2, 2])/(2*a^3*d) + ((A + 11*C)*Sin[c + d*x])/(2*a^3*d*Cos[c + d
*x]^(3/2)) - ((9*A + 119*C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - (
(A + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3) - (2*
C*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((9*A +
 119*C)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.566726, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4114, 3042, 2978, 2748, 2636, 2641, 2639}

$$\frac{(A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+119C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+11C)\sin(}{2a^3d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] ((9*A + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 11*C)*Elliptic
F[(c + d*x)/2, 2])/(2*a^3*d) + ((A + 11*C)*Sin[c + d*x])/(2*a^3*d*Cos[c + d
*x]^(3/2)) - ((9*A + 119*C)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - (
(A + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3) - (2*
C*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((9*A +
 119*C)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4114

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[d^(
m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e + f*x])^(n - m - 2)*(C + A*Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && !IntegerQ[n]
&& IntegerQ[m]
```

Rule 3042

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(a*(A + C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
 + 1))/(f*(b*c - a*d)*(2*m + 1)), x] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) -
 b*d*(2*m + n + 2)) - C*(a*c*m + b*d*(n + 1)) + (a*A*d*(m + n + 2) + C*(b*c
*(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{C+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(3A+13C)+\frac{1}{2}a(3A-7C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2C\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2C\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{(A+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2C\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= \frac{(A+11C)\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{(9A+119C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A+C)\sin(c+}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\,c}$$

$$= \frac{(9A+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{(A+11C)\sin}{2a^3d\cos^{\frac{3}{2}}(c}$$

Mathematica [C] time = 7.5439, size = 1505, normalized size = 6.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3
),x]
```

```
[Out] (((9*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec
[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((
2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x)
)*Sin[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (((11
9*I)/5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c
+ d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (4*A*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
```

```
rcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d
*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3)
- (44*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + C*Sec[c + d*x]^2)*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(d*(A + 2*C + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c
+ d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + C*Sec[c + d*x]^2)*((-4*(60*C + 9*A*
Cos[c] + 59*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) - (4*Sec[c/2]*Sec[c/2
 + (d*x)/2]^5*(A*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (16*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*(3*A*Sin[(d*x)/2] + 8*C*Sin[(d*x)/2]))/(15*d) - (8*Sec[c/2]
*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] + 59*C*Sin[(d*x)/2]))/(5*d) + (32*C*S
ec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (32*Sec[c]*Sec[c + d*x]*(C*Sin[c] -
9*C*Sin[d*x]))/(3*d) - (16*(3*A + 8*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d
) - (4*(A + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(
A + 2*C + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 3.187, size = 876, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+55*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-119*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-30*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+55*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+24*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+55*C*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-119*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+55*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-119*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(9*A+119*C)*sin(1/2*d*x+1/2*c)^10+24*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(29*A+389*C)*sin(1/2*d*x+1/2*c)^8-10*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(81*A+1111*C)*sin(1/2*d*x+1/2*c
)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(99*A+1414*C)*si
n(1/2*d*x+1/2*c)^4-3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
23*A+343*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1
/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+a^{3}\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4*sec(
d*x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^4*sec
(d*x + c) + a^3*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorit
hm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)
), x)
```

3.1127 $\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=213

$$\frac{2a(16A+21C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)}}$$

```
[Out] (16*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (8*a*(16*A + 21*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqr
t[a + a*Sec[c + d*x]]) + (2*a*(16*A + 21*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x]
)/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[
c + d*x]]*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.570434, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4087, 4015, 3805, 3804}

$$\frac{2a(16A+21C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (16*a*(16*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (8*a*(16*A + 21*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqr
t[a + a*Sec[c + d*x]]) + (2*a*(16*A + 21*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x]
)/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[
c + d*x]]*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}\left(A\right.}{\sec^{\frac{9}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d} + \frac{\left(2\sqrt{c\right.}}{}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{7}{2}}(c+dx)\sqrt{a+}}{9}$$

$$= \frac{2a(16A+21C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{5}{2}}(c}{63d\sqrt{a+}}$$

$$= \frac{8a(16A+21C)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a(16A+2}{105}$$

$$= \frac{16a(16A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a(16A+21C)}{315d\sqrt{}}$$

Mathematica [A] time = 0.306625, size = 109, normalized size = 0.51

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left((48A+63C)\cos^2(c+dx)+(64A+84C)\cos(c+dx)+35A\cos^4(c+\right.}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(8*(16*A + 21*C) + (64*A + 84*C)*Cos[c + d*x] + (48*A
 + 63*C)*Cos[c + d*x]^2 + 40*A*Cos[c + d*x]^3 + 35*A*Cos[c + d*x]^4)*Sqrt[a
*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.358, size = 119, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(35A(\cos(dx+c))^4+40A(\cos(dx+c))^3+48A(\cos(dx+c))^2+63C(\cos(dx+c))^2+64\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+40*A*cos(d*x+c)^3+48*A*cos(d*x+
c)^2+63*C*cos(d*x+c)^2+64*A*cos(d*x+c)+84*C*cos(d*x+c)+128*A+168*C)*(a*(cos
(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.17758, size = 684, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(1890*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c)))*sin(9/2*d*x + 9/2*c) + 420*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 252*cos(4/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 45*cos(2/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 1890*c
os(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2
*c))) - 420*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) - 252*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) - 45*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*sin(9/2*d*x + 9/2*c) + 45*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) - 84*sqrt(2)*(5*(6*sin(2*d
*x + 2*c) + sin(d*x + c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - (3
0*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 6)*sin(5/2*arctan2(sin(d*x + c), cos(
d*x + c))) - 5*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 30*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.51126, size = 302, normalized size = 1.42

$$\frac{2\left(35\,A\cos\left(dx+c\right)^{4}+40\,A\cos\left(dx+c\right)^{3}+3\,(16\,A+21\,C)\cos\left(dx+c\right)^{2}+4\,(16\,A+21\,C)\cos\left(dx+c\right)+128\,A+168\,C\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/315*(35*A*cos(d*x + c)^4 + 40*A*cos(d*x + c)^3 + 3*(16*A + 21*C)*cos(d*x
+ c)^2 + 4*(16*A + 21*C)*cos(d*x + c) + 128*A + 168*C)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1128 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=168

$$\frac{2a(24A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a(24A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c}}{7d}$$

```
[Out] (4*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
 + d*x]]) + (2*a*(24*A + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt
[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(7*d)
```

Rubi [A] time = 0.492803, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4087, 4015, 3805, 3804}

$$\frac{2a(24A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a(24A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (4*a*(24*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
 + d*x]]) + (2*a*(24*A + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt
[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(7*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}\,(A}{\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{7d} + \frac{(2\sqrt{c}}{}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}{7}$$

$$= \frac{2a(24A+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{3}{2}}(c}{35d\sqrt{a-}}$$

$$= \frac{4a(24A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a(24A+35C}{105d\sqrt{}}$$

Mathematica [A] time = 0.288903, size = 90, normalized size = 0.54

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}((141A+140C)\cos(c+dx)+36A\cos(2(c+dx))+15A\cos(3(c+dx))+}{210d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(228*A + 280*C + (141*A + 140*C)*Cos[c + d*x] + 36*A*Co
s[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c +
d*x])/(210*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.338, size = 97, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^3+18A(\cos(dx+c))^2+24A\cos(dx+c)+35C\cos(dx+c)+48A+7}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+24*A*cos(d*x+
c)+35*C*cos(d*x+c)+48*A+70*C)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c
)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.11271, size = 522, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) - 140*(3*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d
*x + c)))*sin(d*x + c) - (3*sqrt(2)*cos(d*x + c) + 2*sqrt(2))*sin(3/2*arcta
n2(sin(d*x + c), cos(d*x + c))) - 3*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.487439, size = 252, normalized size = 1.5

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{3}+18\,A\cos\left(dx+c\right)^{2}+\left(24\,A+35\,C\right)\cos\left(dx+c\right)+48\,A+70\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^3 + 18*A*cos(d*x + c)^2 + (24*A + 35*C)*cos(d*x +
c) + 48*A + 70*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2
), x)
```

3.1129 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=122

$$\frac{2a(7A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a}}{15d}$$

```
[Out] (2*a*(7*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1
5*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.422875, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {4265, 4087, 4013, 3804}

$$\frac{2a(7A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a*(7*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1
5*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
```

```
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}\,(A\ldots}{\sec^{\frac{5}{2}}(c+\ldots}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d} + \frac{\left(2\sqrt{c\ldots\right.}{\ldots}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2A\,c\ldots}{\ldots}$$

$$= \frac{2a(7A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx\ldots}}{\ldots}$$

Mathematica [A] time = 0.243768, size = 68, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(8A\cos(c+dx)+3A\cos(2(c+dx))+19A+30C)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(19*A + 30*C + 8*A*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])
*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.327, size = 77, normalized size = 0.6

$$-\frac{(-2+2\cos(dx+c))\left(3A(\cos(dx+c))^2+4A\cos(dx+c)+8A+15C\right)}{15d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+8*A+15*C)*cos(d*x+
c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.02017, size = 312, normalized size = 2.56

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\ldots\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 + 120*sqrt(2)*C*sqrt(a)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/d
```

Fricas [A] time = 0.500842, size = 205, normalized size = 1.68

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{2}+4\,A\cos\left(dx+c\right)+8\,A+15\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + 8*A + 15*C)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2
), x)
```

3.1130 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)$ a

Optimal. Leaf size=136

$$\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.399067, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4087, 4015, 3801, 215}

$$\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{\sqrt{a+a\sec(c+dx)}\left(A+C\right.}{\sec^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}+\frac{\left(2\sqrt{\cos(\right.}{}$$

$$=\frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

$$=\frac{2aA\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{a}}{}$$

$$=\frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+$$

Mathematica [A] time = 0.567713, size = 92, normalized size = 0.68

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(A\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)+3\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c\right.\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*
C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + A*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d
*x))/2])))/(3*d)
```

Maple [A] time = 0.352, size = 199, normalized size = 1.5

$$-\frac{-1+\cos(dx+c)}{3d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}+4A\sin(dx+c)\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)-3*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))+3*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c))))*cos(d*x+c)^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/
2)
```

Maxima [B] time = 2.08302, size = 479, normalized size = 3.52

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 3*C*sqrt(a)*(
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [A] time = 0.567729, size = 867, normalized size = 6.38

$$\left[\frac{4\left(A\cos\left(dx+c\right)+2A\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(C\cos\left(dx+c\right)+C\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos}{\cos}}}{}\right.}{6\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*(4*(A*cos(d*x + c) + 2*A)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + 3*(C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x +
c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(A*cos(d*x + c) + 2*A
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
 3*(C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d
*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2
), x)
```

3.1131 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)$

Optimal. Leaf size=135

$$\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\,\mathrm{ta}}{\sqrt{a\sec}}\right.}{d}$$

```
[Out] (Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*(2*A - C)*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d
*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.409491, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4089, 4015, 3801, 215}

$$\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\,\mathrm{ta}}{\sqrt{a\sec}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*(2*A - C)*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d
*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+C}{\sqrt{\sec(c+dx}}$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.}{}$$

$$= \frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+dx)}}{d\sqrt{\cos(c+d}}$$

$$= \frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+dx)}}{d\sqrt{\cos(c+d}}$$

$$= \frac{\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [A] time = 0.631644, size = 90, normalized size = 0.67

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2A+C\sec(c+dx))+\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*C*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(2*A + C*Sec[c + d*x])*Sin[(c + d*x)/
2]))/(2*d)
```

Maple [A] time = 0.358, size = 210, normalized size = 1.6

$$-\frac{-1+\cos(dx+c)}{2d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(4A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}+C\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4*A*cos(d*x+c)*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-C*2^(1/2)*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+2*
C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1
/2)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.22962, size = 987, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*(8*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) - (4*sqrt(2)*cos(5/2*d*x + 5/
2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4
*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
qrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c
) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
4*sqrt(2)*sin(3/2*d*x + 3/2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.57613, size = 942, normalized size = 6.98

$$\left[\frac{4\,(2\,A\cos\left(dx+c\right)+C)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(C\cos\left(dx+c\right)^{2}+C\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c}{\quad}\right.}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(4*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(a)*l
og((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(c
os(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a
)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)),
1/2*(2*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + (C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(-a)*a
rctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)
), x)
```

3.1132 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{a}(8A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{aC\sin}{4d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*C*Sin[c + d*x])/(4*d*C
os[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.406422, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4089, 4016, 3801, 215}

$$\frac{\sqrt{a}(8A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{C\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{aC\sin}{4d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(8*A + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*C*Sin[c + d*x])/(4*d*C
os[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,(A+C$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\sqrt{\sec}}{}$$

$$= \frac{aC\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\sqrt{a}(8A+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{}{4d\,\mathrm{c}}$$

Mathematica [A] time = 0.571066, size = 105, normalized size = 0.73

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(8A+3C)\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\left(\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\right.\right.}{8d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(8*A + 3*C)*ArcTanh[S
qrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + C*(Sin[(c + d*x)/2] + 3*Sin[(3*(c
 + d*x))/2])))/(8*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 0.349, size = 313, normalized size = 2.2

$$-\frac{-1+\cos(dx+c)}{8d(\sin(dx+c))^2}\left(8A\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(8*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-8*A*2^(1/2)*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+3*C*2^(
1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)*cos(d*x+c)^2-3*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+6*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)*sin(d*x+c)+4*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.31847, size = 2034, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*(8*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2)) - (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(
7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*
sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*
(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt
(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + s
in(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c
)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
```

```
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.688608, size = 1014, normalized size = 7.04

$$\left[\frac{4\,(3\,C\cos\left(dx+c\right)+2\,C)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left((8\,A+3\,C)\cos\left(dx+c\right)^{3}+(8\,A+3\,C)\cos\left(dx+c\right)\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*(4*(3*C*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 3*C)*cos(d*x + c)^3 + (8*A + 3*C)*
cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) -
 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2), 1/8*(2*(3*C*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 3*C)*cos(
d*x + c)^3 + (8*A + 3*C)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
 + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+C\sec^{2}\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + C*sec(c + d*x)**2)/sqrt(cos(c + d*
x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)
), x)
```

3.1133 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=189

$$\frac{a(8A+5C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{C\sin(c+dx)\sqrt{a}}{3d\cos^{\frac{5}{2}}(}$$

```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*C*Sin[c + d*x])/(12*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*A + 5*C)*Sin[c + d*x])
/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.484695, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4089, 4016, 3803, 3801, 215}

$$\frac{a(8A+5C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{C\sin(c+dx)\sqrt{a}}{3d\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```


```
[Out] (Sqrt[a]*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*C*Sin[c + d*x])/(12*d*
Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*A + 5*C)*Sin[c + d*x])
/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
```

```
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int s}{}$$

$$= \frac{aC\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+}{3d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(8A+5C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}}$$

$$= \frac{aC\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(8A+5C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}}$$

$$= \frac{\sqrt{a}(8A+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{}{1}$$

Mathematica [A] time = 0.98151, size = 125, normalized size = 0.66

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+5C)\cos(2(c+dx))+24A+20C\cos(c+dx)+31C)+3\right.}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(8*A + 5*C)*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*A + 31*C + 20*C*Cos[c + d*x
] + 3*(8*A + 5*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c + d*x]^(
5/2))
```

Maple [B] time = 0.337, size = 375, normalized size = 2.

$$\frac{-1+\cos(dx+c)}{48\,d\,(\sin(dx+c))^2}\left(24\,A\,(\cos(dx+c))^3\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\sqrt{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/48/d*(-1+cos(d*x+c))*(24*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-24*A*cos(d*x+c)^3*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+15*C*
cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin
(d*x+c)))*2^(1/2)-15*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2)-30*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-20*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-16*C*(-2/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1
))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.69015, size = 3699, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
```

```
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*si
n(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)
*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c)
)*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*sin(6*d*x + 6
*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arct
an2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2
)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4
*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 15*(2*(3*
cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6
*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2
+ 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 15*(2*(3*co
s(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c
)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 +
9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(2*(3*cos(
4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^
2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*
cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 15*(2*(3*cos(4*
d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
+ 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*co
s(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
 + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sq
rt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 60*(sqrt(2)*cos(6
*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(6*d*x
 + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*cos(6*d*x + 6
*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*si
n(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*cos(6*d*x + 6*c)
+ 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(6*d*x + 6*c) + 3*s
qrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2
)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(s
```

```
in(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 +
 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x
+ 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.699942, size = 1107, normalized size = 5.86

$$\left[\frac{4\left(3\left(8A+5C\right)\cos\left(dx+c\right)^2+10C\cos\left(dx+c\right)+8C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(8A+5C\right)\cos\left(dx\right.\right.}{96\left(d\cos\left(dx+c\right)^4+d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(4*(3*(8*A + 5*C)*cos(d*x + c)^2 + 10*C*cos(d*x + c) + 8*C)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((8*A +
5*C)*cos(d*x + c)^4 + (8*A + 5*C)*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x +
c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)
*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^
3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(3*(8*
A + 5*C)*cos(d*x + c)^2 + 10*C*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((8*A + 5*C)*cos(d*x +
c)^4 + (8*A + 5*C)*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

3.1134 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{a(48A+35C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{64d}$$

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*C*Sin[c + d*x])/(24
*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A + 35*C)*Sin[c +
d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A + 35*C)
*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.572557, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4089, 4016, 3803, 3801, 215}

$$\frac{a(48A+35C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+35C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a]*(48*A + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*C*Sin[c + d*x])/(24
*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A + 35*C)*Sin[c +
d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A + 35*C)
*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
```

```
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+C\right.$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}}{}$$

$$= \frac{aC\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{aC\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{aC\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{aC\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{\sqrt{a}(48A+35C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{}{24}$$

Mathematica [A] time = 1.63234, size = 152, normalized size = 0.65

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((432A+539C)\cos(c+dx)+4(48A+35C)\cos(2(c+dx))+144A\right.}{768d\cos^{\frac{7}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(48*A + 35*C)*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (192*A + 332*C + (432*A + 539
*C)*Cos[c + d*x] + 4*(48*A + 35*C)*Cos[2*(c + d*x)] + 144*A*Cos[3*(c + d*x)
] + 105*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 0.352, size = 437, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/384/d*(-1+cos(d*x+c))*(144*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-144*A*cos(d*x+c)^4*2^(1/2
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+1
05*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*2^(1/2)-105*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+288*A*sin(d*x+c)*cos(d*x+c)^
3*(-2/(cos(d*x+c)+1))^(1/2)+210*C*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+140*C*cos(
d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+112*C*(-2/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)*sin(d*x+c)+96*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos
(d*x+c)+1)/cos(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x
+c)^(7/2)
```

Maxima [B] time = 3.50367, size = 5963, normalized size = 25.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] -1/768*(48*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(
7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*
sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*
(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
```

```
), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt
(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + s
in(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c
)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1) + (420*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d
*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(15
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 140*(sqrt(2)*sin(8*d*x + 8*c) + 4
*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*
x + 2*c))*cos(13/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1596*(sqrt(2)*sin
(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4
*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c))) +
500*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(
4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 500*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) +
 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 1596*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*si
n(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*c
os(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 140*(sqrt(2)*sin(8*d*x + 8*c)
 + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(
2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 420*(sqrt(2)*s
in(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) +
 4*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*
cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
 + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) +
 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2
*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)
^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2
*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*a
```

```
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 2) - 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c))) + 2) + 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) + 2) - 420*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c)
 + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1
5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 140*(sqrt(2)*cos(8*d*x + 8*c) +
4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*sin(13/2*arctan2(sin(d*x + c), cos(d*x + c))) - 1596*(
sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x
 + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 500*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x +
6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*s
in(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 500*(sqrt(2)*cos(8*d*x + 8*c)
 + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1596
*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d
*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 140*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x +
 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 420*(sqrt(2)*cos(8*d*x + 8*c
) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sq
rt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)
*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
+ 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*
x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c)
 + 1))/d
```

Fricas [A] time = 0.837242, size = 1218, normalized size = 5.21

$$\left[\frac{4\left(3\,(48\,A+35\,C)\cos\left(dx+c\right)^{3}+2\,(48\,A+35\,C)\cos\left(dx+c\right)^{2}+56\,C\cos\left(dx+c\right)+48\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\,\mathrm{s}}{76}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/768*(4*(3*(48*A + 35*C)*cos(d*x + c)^3 + 2*(48*A + 35*C)*cos(d*x + c)^2
+ 56*C*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + 3*((48*A + 35*C)*cos(d*x + c)^5 + (48*A + 35*C)*
cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) -
 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x +
 c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(48*A + 35*C)*cos(d*x + c)^3 + 2*(48
*A + 35*C)*cos(d*x + c)^2 + 56*C*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((48*A + 35*C)*cos(d
*x + c)^5 + (48*A + 35*C)*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

3.1135 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)$

Optimal. Leaf size=266

$$\frac{2a^2(28A+33C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{231d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(112A+143C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{385d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(112A+143C)\sin(c+}{1155d\sqrt{a\sec(c+}$$

```
[Out] (16*a^2*(112*A + 143*C)*Sin[c + d*x])/(1155*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(112*A + 143*C)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(385*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(28*A + 33*C)*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(231*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos
[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(33*d) + (2*A*Cos[c
+ d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.806646, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4017, 4015, 3805, 3804}

$$\frac{2a^2(28A+33C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{231d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(112A+143C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{385d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(112A+143C)\sin(c+}{1155d\sqrt{a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x
]
```

```
[Out] (16*a^2*(112*A + 143*C)*Sin[c + d*x])/(1155*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (8*a^2*(112*A + 143*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(112*A + 143*C)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(385*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(28*A + 33*C)*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(231*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos
[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(33*d) + (2*A*Cos[c
+ d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}\left(A\right.}{\sec^{\frac{11}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d} + \frac{(2}{}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{33d} + \frac{2A}{}$$

$$= \frac{2a^2(28A+33C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{231d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{7}{2}}(}{}$$

$$= \frac{2a^2(112A+143C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{385d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(28A}{}$$

$$= \frac{8a^2(112A+143C)\sqrt{\cos(c+dx)}\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(112}{}$$

$$= \frac{16a^2(112A+143C)\sin(c+dx)}{1155d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a^2(112A+1}{1155}$$

Mathematica [A] time = 1.99648, size = 125, normalized size = 0.47

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(5789A+5566C)\cos(c+dx)+8(581A+429C)\cos(2(c+dx))+1}{9240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(18494*A + 21736*C + 2*(5789*A + 5566*C)*Cos[c + d*x]
 + 8*(581*A + 429*C)*Cos[2*(c + d*x)] + 1645*A*Cos[3*(c + d*x)] + 660*C*Cos
[3*(c + d*x)] + 490*A*Cos[4*(c + d*x)] + 105*A*Cos[5*(c + d*x)])*Sqrt[a*(1
+ Sec[c + d*x])]*Tan[(c + d*x)/2])/(9240*d)
```

Maple [A] time = 0.351, size = 142, normalized size = 0.5

$$-\frac{2\,a\left(-1+\cos\left(dx+c\right)\right)\left(105\,A\left(\cos\left(dx+c\right)\right)^{5}+245\,A\left(\cos\left(dx+c\right)\right)^{4}+280\,A\left(\cos\left(dx+c\right)\right)^{3}+165\,C\left(\cos\left(dx+c\right)\right)^{3}\right.}{1155\,d\sin\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/1155/d*a*(-1+cos(d*x+c))*(105*A*cos(d*x+c)^5+245*A*cos(d*x+c)^4+280*A*co
s(d*x+c)^3+165*C*cos(d*x+c)^3+336*A*cos(d*x+c)^2+429*C*cos(d*x+c)^2+448*A*c
os(d*x+c)+572*C*cos(d*x+c)+896*A+1144*C)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.24813, size = 880, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/36960*(7*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11/2
*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*cos
(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x
 + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 11/
2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d*x
 + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)
)) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c),
cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/2*
c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55*a
*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d
*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11/2
*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/2*
d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/2*
d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) - 44*sqrt(2)*(175*a*cos(
7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 5*(35*a
*cos(2*d*x + 2*c) + 6*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
)) - 126*a*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 175*a*sin
(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1470*a*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.499507, size = 378, normalized size = 1.42

$$\frac{2\left(105\,Aa\cos\left(dx+c\right)^{5}+245\,Aa\cos\left(dx+c\right)^{4}+5\,(56\,A+33\,C)a\cos\left(dx+c\right)^{3}+3\,(112\,A+143\,C)a\cos\left(dx+c\right)^{2}+4\,(1}{1155\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 2/1155*(105*A*a*cos(d*x + c)^5 + 245*A*a*cos(d*x + c)^4 + 5*(56*A + 33*C)*a
*cos(d*x + c)^3 + 3*(112*A + 143*C)*a*cos(d*x + c)^2 + 4*(112*A + 143*C)*a*
cos(d*x + c) + 8*(112*A + 143*C)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(1
1/2), x)
```

3.1136 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)d$

Optimal. Leaf size=219

$$\frac{2a^2(52A+63C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+189C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+189C)\sin}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(}}$$

```
[Out] (4*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(52*A + 63*C)*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(7/2)*(a + a
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.722911, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4017, 4015, 3805, 3804}

$$\frac{2a^2(52A+63C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+189C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+189C)\sin}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (4*a^2*(136*A + 189*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^2*(136*A + 189*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(52*A + 63*C)*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(7/2)*(a + a
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}\left(A\right.}{\sec^{\frac{9}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} + \frac{2A\,c}{}$$

$$= \frac{2a^2(52A+63C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cos^{\frac{5}{2}}(c}{}$$

$$= \frac{2a^2(136A+189C)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A}{}$$

$$= \frac{4a^2(136A+189C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(136A+189}{315d\surd}$$

Mathematica [A] time = 1.30907, size = 103, normalized size = 0.47

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(799A+756C)\cos(c+dx)+4(137A+63C)\cos(2(c+dx))+170A}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(2689*A + 3276*C + 2*(799*A + 756*C)*Cos[c + d*x] + 4
*(137*A + 63*C)*Cos[2*(c + d*x)] + 170*A*Cos[3*(c + d*x)] + 35*A*Cos[4*(c +
 d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.332, size = 120, normalized size = 0.6

$$-\frac{2\,a\left(-1+\cos\left(dx+c\right)\right)\left(35\,A\left(\cos\left(dx+c\right)\right)^{4}+85\,A\left(\cos\left(dx+c\right)\right)^{3}+102\,A\left(\cos\left(dx+c\right)\right)^{2}+63\,C\left(\cos\left(dx+c\right)\right)^{2}+1\right.}{315\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+102*A*cos(d
*x+c)^2+63*C*cos(d*x+c)^2+136*A*cos(d*x+c)+189*C*cos(d*x+c)+272*A+378*C)*co
s(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.19762, size = 734, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) - 504*(10*sqrt(2)*a*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))*sin(2*d*x + 2*c) - 5*sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 10*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - (10*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.499324, size = 321, normalized size = 1.47

$$\frac{2\left(35\,Aa\cos\left(dx+c\right)^{4}+85\,Aa\cos\left(dx+c\right)^{3}+3\left(34\,A+21\,C\right)a\cos\left(dx+c\right)^{2}+\left(136\,A+189\,C\right)a\cos\left(dx+c\right)+2\left(136\right.\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^4 + 85*A*a*cos(d*x + c)^3 + 3*(34*A + 21*C)*a*co
s(d*x + c)^2 + (136*A + 189*C)*a*cos(d*x + c) + 2*(136*A + 189*C)*a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*
x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1137 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)d$

Optimal. Leaf size=169

$$\frac{8a^2(19A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2A\sin(c+dx)}{}$$

```
[Out] (8*a^2*(19*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*a*(19*A + 35*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (6*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2
)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.544518, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4087, 4013, 3809, 3804}

$$\frac{8a^2(19A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2A\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (8*a^2*(19*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*a*(19*A + 35*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (6*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2
)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}\left(A\right.}{\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{6A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2A}{}$$

$$= \frac{2a(19A+35C)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+}{105d}$$

$$= \frac{8a^2(19A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a(19A+35C)}{}$$

Mathematica [A] time = 0.887585, size = 85, normalized size = 0.5

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((253A+140C)\cos(c+dx)+78A\cos(2(c+dx))+15A\cos(3(c+d}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(494*A + 700*C + (253*A + 140*C)*Cos[c + d*x] + 78*A*
Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
 + d*x)/2])/(210*d)
```

Maple [A] time = 0.293, size = 98, normalized size = 0.6

$$-\frac{2a(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+39A(\cos(dx+c))^2+52A\cos(dx+c)+35C\cos(dx+c)+104A+17\right.}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+52*A*cos(d*
x+c)+35*C*cos(d*x+c)+104*A+175*C)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*
x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.12708, size = 497, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 280*(sqrt(2)*a*sin(3/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 9*sqrt(2)*a*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.491414, size = 269, normalized size = 1.59

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{3}+39\,Aa\cos\left(dx+c\right)^{2}+\left(52\,A+35\,C\right)a\cos\left(dx+c\right)+\left(104\,A+175\,C\right)a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(d\right.}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^3 + 39*A*a*cos(d*x + c)^2 + (52*A + 35*C)*a*cos(
d*x + c) + (104*A + 175*C)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7
/2), x)
```

3.1138 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)d$

Optimal. Leaf size=183

$$\frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(}{}$$

```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 5*C)*Sin[c + d*x])/(5*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A*Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.596167, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4017, 4015, 3801, 215}

$$\frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 5*C)*Sin[c + d*x])/(5*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A*Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}\left(A\right.}{\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{(2\sqrt{}}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2A\,\mathrm{c}}{}$$

$$= \frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}}{}$$

$$= \frac{2a^2(4A+5C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}}{}$$

$$= \frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.867234, size = 105, normalized size = 0.57

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(6A\cos(c+dx)+A\cos(2(c+dx))+13A+10C)+\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(5*Sqrt[2
]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (13*A + 10*C + 6*A*Cos[c + d*x] + A
*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(5*d)
```

Maple [A] time = 0.336, size = 212, normalized size = 1.2

$$-\frac{a}{10\,d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(5\,C\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/10/d*a*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*C*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-5*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+4*A*cos(d*x+c)^3+8*A*cos(d*x+c)^2+12*A*cos(d*x+c)+20*C*cos(d*x+c)-24*
A-20*C)/sin(d*x+c)
```

Maxima [B] time = 2.20555, size = 937, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/20*(sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5
/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*si
n(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))))*A*sqrt(a) + 10*(4*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - a*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - a*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*C*
sqrt(a))/d
```

Fricas [A] time = 0.577488, size = 973, normalized size = 5.32

$$\left[\frac{4\left(Aa\cos\left(dx+c\right)^{2}+3\,Aa\cos\left(dx+c\right)+\left(6\,A+5\,C\right)a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(Ca\cos\left(dx+c\right)+C\right.}{10\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/10*(4*(A*a*cos(d*x + c)^2 + 3*A*a*cos(d*x + c) + (6*A + 5*C)*a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(C*a*co
s(d*x + c) + C*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c)
 - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x
 + c) + d), 1/5*(2*(A*a*cos(d*x + c)^2 + 3*A*a*cos(d*x + c) + (6*A + 5*C)*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
 5*(C*a*cos(d*x + c) + C*a)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5
/2), x)
```

3.1139 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=189

$$\frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c+a}{3d\sqrt{\cos}}$$

```
[Out] (3*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(8*A - 3*C)*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a*(2*A - 3*C)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.600252, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4018, 4015, 3801, 215}

$$\frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c+a}{3d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (3*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(8*A - 3*C)*Sin[c + d*x])/(3*d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a*(2*A - 3*C)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}\left(A\right.}{\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{(2\surd}{}$$

$$= -\frac{a(2A-3C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(}}{}$$

$$= \frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a(2A-3C)\sqrt{a+a}}{3d\sqrt{c}}$$

$$= \frac{a^2(8A-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a(2A-3C)\sqrt{a+a}}{3d\sqrt{c}}$$

$$= \frac{3a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.923981, size = 110, normalized size = 0.58

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(10A\cos(c+dx)+A\cos(2(c+dx))+A+3C)+9\sqrt{2}C\cos(c+\right.}{6d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(9*Sqrt[2]*C*ArcTanh[Sqrt[2]
*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 3*C + 10*A*Cos[c + d*x] + A*Cos[2*
(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.346, size = 243, normalized size = 1.3

$$-\frac{a\left(-1+\cos\left(dx+c\right)\right)}{6d\left(\sin\left(dx+c\right)\right)^{2}}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(4\,A\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}+20\,A\cos\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/6/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4*A*cos(d*x+c
)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+20*A*cos(d*x+c)*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)+9*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-9*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+6*C*(-2/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)
^(1/2)
```

Maxima [B] time = 2.12623, size = 1828, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))
*A*sqrt(a) - 3*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(
2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2
*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x
+ 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/
2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*
```

```
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt
(2)*a)*sin(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/2*d*x + 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/
2*d*x + 1/2*c))*sin(2*d*x + 2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.585269, size = 1041, normalized size = 5.51

$$\left[\frac{4\left(2\,Aa\cos\left(dx+c\right)^2+10\,Aa\cos\left(dx+c\right)+3\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+9\left(Ca\cos\left(dx+c\right)^2+Ca\cos\right.}{12\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/12*(4*(2*A*a*cos(d*x + c)^2 + 10*A*a*cos(d*x + c) + 3*C*a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 9*(C*a*cos(d*x
 + c)^2 + C*a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*si
n(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/
(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/6*(2*(2*A*a*cos(d*x + c)^2 + 10*A*a*
cos(d*x + c) + 3*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) + 9*(C*a*cos(d*x + c)^2 + C*a*cos(d*x + c))*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 +
 d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3
/2), x)
```

3.1140 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=191

$$\frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{3aC\sin(c+dx)}{4d\sqrt{co}}$$

```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(8*A - 5*C)*Sin[c +
d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (3*a*C*Sqrt[a + a
*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (C*(a + a*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.613235, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4089, 4018, 4015, 3801, 215}

$$\frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{3aC\sin(c+dx)}{4d\sqrt{co}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(8*A - 5*C)*Sin[c +
d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (3*a*C*Sqrt[a + a
*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (C*(a + a*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(}}$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{3aC\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{C(a+a\sec(c}{2d\sqrt{c}}$$

$$= \frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{3aC\sqrt{a+a\sec}}{4d\sqrt{c}}$$

$$= \frac{a^2(8A-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{3aC\sqrt{a+a\sec}}{4d\sqrt{c}}$$

$$= \frac{a^{3/2}(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{}}{4d}$$

Mathematica [A] time = 1.42929, size = 120, normalized size = 0.63

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(4A\cos(2(c+dx))+4A+7C\cos(c+dx)+2C)+\sqrt{2}(8A+\right.}{8d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(8*A + 7*C)*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(4*A + 2*C + 7*C*Cos[c + d*x]
 + 4*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 0.365, size = 345, normalized size = 1.8

$$\frac{a(-1+\cos(dx+c))}{8d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(8A\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 1/8/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(8*A*2^(1/2)*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d
*x+c)^2-8*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))*cos(d*x+c)^2-16*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+
1))^(1/2)+7*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)^2-7*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-14*C*(-2/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/c
os(d*x+c)^(3/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.45456, size = 3402, normalized size = 17.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/16*(4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A
*sqrt(a) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
4*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*s
in(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*si
n(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*si
n(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos
```

(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(
2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(
2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sqrt(a)/(2*(2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2

```
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d
```

Fricas [A] time = 0.704489, size = 1098, normalized size = 5.75

$$\left[\frac{4\left(8\,Aa\cos\left(dx+c\right)^{2}+7\,Ca\cos\left(dx+c\right)+2\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\left(8\,A+7\,C\right)a\cos\left(dx+c\right)^{3}+\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\,\mathrm{cc}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*(4*(8*A*a*cos(d*x + c)^2 + 7*C*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 7*C)*a*
cos(d*x + c)^3 + (8*A + 7*C)*a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^
3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sq
rt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*(8*A*a*cos
(d*x + c)^2 + 7*C*a*cos(d*x + c) + 2*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 7*C)*a*cos(d*x + c)^3 + (8
*A + 7*C)*a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
c)), x)
```

3.1141 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=191

$$\frac{a^2(24A+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aC\sin(c+d}{4d\,c}$$

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 19*C)*Sin[
c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*C*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (C*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.61979, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4089, 4018, 4016, 3801, 215}

$$\frac{a^2(24A+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aC\sin(c+d}{4d\,c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (a^(3/2)*(24*A + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 19*C)*Sin[
c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*C*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (C*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{aC\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{C(a+a\sec(c+dx))^{3/2}\sin(c}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(24A+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aC\sqrt{a+a\sec(c+dx)}\sin(}{4d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(24A+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aC\sqrt{a+a\sec(c+dx)}\sin(}{4d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^{3/2}(24A+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d}$$

Mathematica [A] time = 1.87281, size = 126, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+11C)\cos(2(c+dx))+24A+44C\cos(c+dx)+49C)\right.}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(24*A + 11*C)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*A + 49*C + 44*C*Cos[c +
 d*x] + 3*(8*A + 11*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c + d
*x]^(5/2))
```

Maple [B] time = 0.319, size = 376, normalized size = 2.

$$\frac{a\left(-1+\cos\left(dx+c\right)\right)}{48\,d\left(\sin\left(dx+c\right)\right)^{2}}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(72\,A\left(\cos\left(dx+c\right)\right)^{3}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\left(\cos\left(dx+c\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/48/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(72*A*cos(d*x+
c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)*2^(1/2)-72*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*2^(1/2)+33*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-33*C*cos(d*x+c)^3*a
rctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1
/2)-48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-66*C*cos(d*x+c)^
2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-44*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)*sin(d*x+c)-16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/(-2/(cos(d*x+c)
+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.67405, size = 4733, normalized size = 24.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/96*(24*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d
*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
```

+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin
(2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - (132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x
+ 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*
cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*
x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x +
2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) +
a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2
+ a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(
2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x
+ 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6
*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c
)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a
*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*c
os(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2
*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d
*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x
+ 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2
+ 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6

```
*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*
sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a
*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*s
in(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*
d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(
2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*
x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2
)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*C*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos
(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*
c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(
2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.700879, size = 1148, normalized size = 6.01

$$\left[\frac{4\left(3\,(8\,A+11\,C)a\cos\left(dx+c\right)^2+22\,Ca\cos\left(dx+c\right)+8\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left((24\,A+11\,C)a\right.}{96\left(d\cos\left(dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(4*(3*(8*A + 11*C)*a*cos(d*x + c)^2 + 22*C*a*cos(d*x + c) + 8*C*a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(
(24*A + 11*C)*a*cos(d*x + c)^4 + (24*A + 11*C)*a*cos(d*x + c)^3)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(co
s(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3),
 1/48*(2*(3*(8*A + 11*C)*a*cos(d*x + c)^2 + 22*C*a*cos(d*x + c) + 8*C*a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(
(24*A + 11*C)*a*cos(d*x + c)^4 + (24*A + 11*C)*a*cos(d*x + c)^3)*sqrt(-a)*a
rctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x +
c)), x)
```

3.1142 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=238

$$\frac{a^2(112A+75C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+13C)\sin(c+dx)}{32d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{64d}$$

```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(16*A + 13*C)*Si
n[c + d*x])/(32*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(112*
A + 75*C)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (a*C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (C
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.731376, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(112A+75C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+13C)\sin(c+dx)}{32d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+75C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (a^(3/2)*(112*A + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(16*A + 13*C)*Si
n[c + d*x])/(32*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(112*
A + 75*C)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (a*C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (C
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a + a\sec(c + dx))^{3/2} \left(A + C\sec^2(c + dx)\right)}{\cos^{\frac{3}{2}}(c + dx)} dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \sec^{\frac{3}{2}}(c + dx)(a + a\sec(c + dx))^{3/2} \left(A\right.$$

$$= \frac{C(a + a\sec(c + dx))^{3/2}\sin(c + dx)}{4d\cos^{\frac{5}{2}}(c + dx)} + \frac{\left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int}{}$$

$$= \frac{aC\sqrt{a + a\sec(c + dx)}\sin(c + dx)}{8d\cos^{\frac{5}{2}}(c + dx)} + \frac{C(a + a\sec(c + dx))^{3/2}\sin(c + d}{4d\cos^{\frac{5}{2}}(c + dx)}$$

$$= \frac{a^2(16A + 13C)\sin(c + dx)}{32d\cos^{\frac{5}{2}}(c + dx)\sqrt{a + a\sec(c + dx)}} + \frac{aC\sqrt{a + a\sec(c + dx)}\sin(c +}{8d\cos^{\frac{5}{2}}(c + dx)}$$

$$= \frac{a^2(16A + 13C)\sin(c + dx)}{32d\cos^{\frac{5}{2}}(c + dx)\sqrt{a + a\sec(c + dx)}} + \frac{a^2(112A + 75C)\sin(c + d}{64d\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\sec(c}$$

$$= \frac{a^2(16A + 13C)\sin(c + dx)}{32d\cos^{\frac{5}{2}}(c + dx)\sqrt{a + a\sec(c + dx)}} + \frac{a^2(112A + 75C)\sin(c + d}{64d\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\sec(c}$$

$$= \frac{a^{3/2}(112A + 75C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}}{64d} +$$

Mathematica [A] time = 3.07343, size = 154, normalized size = 0.65

$$\frac{a\sec\left(\frac{1}{2}(c + dx)\right)\sqrt{a(\sec(c + dx) + 1)}\left(\sin\left(\frac{1}{2}(c + dx)\right)(7(48A + 55C)\cos(c + dx) + 4(16A + 25C)\cos(2(c + dx)) + 112A\right.}{256d\cos^{\frac{7}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(2*Sqrt[2]*(112*A + 75*C)*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (64*A + 164*C + 7*(48*A +
55*C)*Cos[c + d*x] + 4*(16*A + 25*C)*Cos[2*(c + d*x)] + 112*A*Cos[3*(c + d*
x)] + 75*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(256*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 0.322, size = 438, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/128/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(112*A*cos(d*
x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))-112*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+75*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-75*C*cos(d*x+c)^
```

```
4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2
^(1/2)-224*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-150*C*cos(d*
x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-64*A*cos(d*x+c)^2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)-100*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-80*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-32*C*(-2/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c))/cos(d*x+c)^(7/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))
^(1/2)
```

Maxima [B] time = 3.72067, size = 7777, normalized size = 32.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] -1/256*(16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
4*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*s
in(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*si
n(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*si
n(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(
2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
```

), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(
2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(2*(2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (300*(sqrt(2)*a*sin(8*d*x
+ 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c))) + 100(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*
sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(13/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*sin(8*d*x + 8*c) +
4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*s
in(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 22
8*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*
sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*
sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a
*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x +
4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 100*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x +
6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(sqrt(2)*a*sin(8*d*x +
8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt
(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4

```
*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x + 6
*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) +
4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*a*
cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*cos(
4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4*d
*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x + 4*
c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x +
 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x +
 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c)
+ 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) + 4*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*co
s(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin(4
*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x +
4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x
 + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*x
 + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c
) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) +
4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)*
cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*sin
(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x
+ 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d
*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d
*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4
*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c)
+ 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a
)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*s
in(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*
x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
+ 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*a*cos(
8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c)
+ 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(13/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos
(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c)
 + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 228*(
sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos
(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*s
qrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(
```

```
2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*s
qrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*a*cos(8*d*x
+ 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sq
rt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x +
 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(
2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(4
*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x
+ 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(
6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.838952, size = 1247, normalized size = 5.24

$$\left[\frac{4\left((112\,A + 75\,C)a\cos\left(dx + c\right)^3 + 2\,(16\,A + 25\,C)a\cos\left(dx + c\right)^2 + 40\,Ca\cos\left(dx + c\right) + 16\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx + \right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/256*(4*((112*A + 75*C)*a*cos(d*x + c)^3 + 2*(16*A + 25*C)*a*cos(d*x + c)
^2 + 40*C*a*cos(d*x + c) + 16*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + ((112*A + 75*C)*a*cos(d*x + c)^5 + (112*A
 + 75*C)*a*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin
(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(
d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/128*(2*((112*A + 75*C)*a*cos(d*x +
c)^3 + 2*(16*A + 25*C)*a*cos(d*x + c)^2 + 40*C*a*cos(d*x + c) + 16*C*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((11
2*A + 75*C)*a*cos(d*x + c)^5 + (112*A + 75*C)*a*cos(d*x + c)^4)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5
+ d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx+c\right)^2+A\right)\left(a \sec \left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos \left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3
/2), x)
```

3.1143 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{a^2(176A+133C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A + 67*C)*
Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(1
76*A + 133*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*
x]]) + (a^2*(176*A + 133*C)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ a*Sec[c + d*x]]) + (3*a*C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d*Co
s[c + d*x]^(7/2)) + (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(7/2))
```

Rubi [A] time = 0.807993, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(176A+133C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+133C)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```


```
[Out] (a^(3/2)*(176*A + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A + 67*C)*
Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(1
76*A + 133*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*
x]]) + (a^2*(176*A + 133*C)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ a*Sec[c + d*x]]) + (3*a*C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d*Co
s[c + d*x]^(7/2)) + (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(7/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A\right.$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{3aC\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{40d\cos^{\frac{7}{2}}(c+dx)} + \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+}{5d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3aC\sqrt{a+a\sec(c+dx)}\sin(c}{40d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A+133C)\sin(c+}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A+133C)\sin(c+}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{a^2(80A+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A+133C)\sin(c+}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{a^{3/2}(176A+133C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{128d}$$

Mathematica [A] time = 4.62212, size = 176, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(880A+1273C)\cos(c+dx)+4(3280A+3059C)\cos(2(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(176*A + 133*C)*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (10480*A + 13313*C + 12*
(880*A + 1273*C)*Cos[c + d*x] + 4*(3280*A + 3059*C)*Cos[2*(c + d*x)] + 3520
*A*Cos[3*(c + d*x)] + 2660*C*Cos[3*(c + d*x)] + 2640*A*Cos[4*(c + d*x)] + 1
995*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d*Cos[c + d*x]^(9/2))
```

Maple [B] time = 0.317, size = 500, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3840/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2640*A*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*
```

```
x+c)^5*2^(1/2)-2640*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)^5*2^(1/2)+1995*C*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^5*2^(1/2)-1995*C*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*
x+c)^5*2^(1/2)+5280*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+399
0*C*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+3520*A*sin(d*x+c)*cos
(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+2660*C*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+1280*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+
2128*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+1824*C*(-2/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+768*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c))/sin(d*x+c)^2/cos(d*x+c)^(9/2)/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 5.97014, size = 9767, normalized size = 34.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] -1/7680*(80*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
 + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d
*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c
) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) +
a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*
sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*c
os(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin
(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x +
 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c)
 + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 +
 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9
*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*
d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*
x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x +
2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*lo
g(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arct
```

an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*
c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)
^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3
*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*c
os(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d
*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) +
 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*
d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt
(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6
*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)
*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*co
s(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c
) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(
sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos
(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*A*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x
 + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 +
18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
 2*c) + 1) + (7980*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x +
8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2660*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) +
10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a
*sin(2*d*x + 2*c))*cos(17/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
38304*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqr
t(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2
*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12160*
(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a
*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x +
 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 45400*(sqrt(
2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6
*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))
*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 45400*(sqrt(2)*a*s
in(10*d*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x +
 6*c) + 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12160*(sqrt(2)*a*sin(10*d
*x + 10*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) +
 10*sqrt(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 38304*(sqrt(2)*a*sin(10*d*x + 10
*c) + 5*sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqr
t(2)*a*sin(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2660*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*
sqrt(2)*a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*s
in(4*d*x + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 7980*(sqrt(2)*a*sin(10*d*x + 10*c) + 5*sqrt(2)*
a*sin(8*d*x + 8*c) + 10*sqrt(2)*a*sin(6*d*x + 6*c) + 10*sqrt(2)*a*sin(4*d*x
 + 4*c) + 5*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8*c)^2

```
+ 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*x + 2*
c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6*d*x +
 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*x + 6*
c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 10*c) +
 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) +
 a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a
)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 10*a*
cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*a*sin(
4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6*d*x +
 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(
2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) + 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8*c)^
2 + 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*x +
2*c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6*d*x
 + 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*x +
6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 10*c)
 + 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c)
 + a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) +
 a)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 10*
a*cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*a*si
n(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6*d*x
 + 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100
*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) - 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8*c
)^2 + 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*x
+ 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6*d
*x + 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*x
+ 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 10*
c) + 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*
c) + a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c)
 + a)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 1
0*a*cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*a*
sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6*d
*x + 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 1
00*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) + 1995*(a*cos(10*d*x + 10*c)^2 + 25*a*cos(8*d*x + 8
*c)^2 + 100*a*cos(6*d*x + 6*c)^2 + 100*a*cos(4*d*x + 4*c)^2 + 25*a*cos(2*d*
x + 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 25*a*sin(8*d*x + 8*c)^2 + 100*a*sin(6
*d*x + 6*c)^2 + 100*a*sin(4*d*x + 4*c)^2 + 100*a*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 25*a*sin(2*d*x + 2*c)^2 + 2*(5*a*cos(8*d*x + 8*c) + 10*a*cos(6*d*
x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*c) + a)*cos(10*d*x + 1
0*c) + 10*(10*a*cos(6*d*x + 6*c) + 10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x +
2*c) + a)*cos(8*d*x + 8*c) + 20*(10*a*cos(4*d*x + 4*c) + 5*a*cos(2*d*x + 2*
c) + a)*cos(6*d*x + 6*c) + 20*(5*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 10*a*cos(2*d*x + 2*c) + 10*(a*sin(8*d*x + 8*c) + 2*a*sin(6*d*x + 6*c) + 2*
a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*a*sin(6
*d*x + 6*c) + 2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
```

```
 100*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) - 7980*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*
a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x
 + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(19/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2660*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqr
t(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(
4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(17/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 38304*(sqrt(2)*a*cos(10*d*x + 10*c) +
 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*
a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(15/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12160*(sqrt(2)*a*cos(10*d*x + 1
0*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) + 10*sq
rt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(13
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 45400*(sqrt(2)*a*cos(10*d
*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x + 6*c) +
 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 45400*(sqrt(2)*a*co
s(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d*x +
6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(
2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12160*(sqrt(2)
*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*cos(6*d
*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 38304*(sq
rt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*a*co
s(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2660
*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt(2)*
a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
7980*(sqrt(2)*a*cos(10*d*x + 10*c) + 5*sqrt(2)*a*cos(8*d*x + 8*c) + 10*sqrt
(2)*a*cos(6*d*x + 6*c) + 10*sqrt(2)*a*cos(4*d*x + 4*c) + 5*sqrt(2)*a*cos(2*
d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*C*sqrt(a)/(2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*d*x +
4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 +
10*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)
^2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2
*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x +
6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d
*x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.863369, size = 1385, normalized size = 4.86

$$\left[\frac{4\left(15\,(176\,A+133\,C)a\cos{(dx+c)}^4+10\,(176\,A+133\,C)a\cos{(dx+c)}^3+8\,(80\,A+133\,C)a\cos{(dx+c)}^2+912\,Ca\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/7680*(4*(15*(176*A + 133*C)*a*cos(d*x + c)^4 + 10*(176*A + 133*C)*a*cos(
d*x + c)^3 + 8*(80*A + 133*C)*a*cos(d*x + c)^2 + 912*C*a*cos(d*x + c) + 384
*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c) + 15*((176*A + 133*C)*a*cos(d*x + c)^6 + (176*A + 133*C)*a*cos(d*x + c)^
5)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x
+ c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^6 + d*cos
(d*x + c)^5), 1/3840*(2*(15*(176*A + 133*C)*a*cos(d*x + c)^4 + 10*(176*A +
133*C)*a*cos(d*x + c)^3 + 8*(80*A + 133*C)*a*cos(d*x + c)^2 + 912*C*a*cos(d
*x + c) + 384*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 15*((176*A + 133*C)*a*cos(d*x + c)^6 + (176*A + 133*C)*a*
cos(d*x + c)^5)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
- 2*a)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5
/2), x)
```

3.1144 $\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=313

$$\frac{2a^2(136A+143C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2717C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{9009d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(8$$

```
[Out] (16*a^3*(8368*A + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Sec[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(45045*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(222
4*A + 2717*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Sec[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(1287*d) + (10*a*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(143*d) + (2*A*Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(
5/2)*Sin[c + d*x])/(13*d)
```

Rubi [A] time = 1.01576, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4017, 4015, 3805, 3804}

$$\frac{2a^2(136A+143C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2717C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{9009d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(8$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x
]
```


```
[Out] (16*a^3*(8368*A + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Sec[c + d*x]]) + (8*a^3*(8368*A + 10439*C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(45045*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(8368*A + 10439*C)*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(222
4*A + 2717*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Sec[c + d
*x]]) + (2*a^2*(136*A + 143*C)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(1287*d) + (10*a*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(143*d) + (2*A*Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(
5/2)*Sin[c + d*x])/(13*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}\left(A\right.}{\sec^{\frac{13}{2}}(c+\right.}$$

$$= \frac{2A\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{13d} + \frac{(2}{}$$

$$= \frac{10aA\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{143d} + \frac{2}{}$$

$$= \frac{2a^2(136A+143C)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(}{1287d}$$

$$= \frac{2a^3(2224A+2717C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(13}{}$$

$$= \frac{2a^3(8368A+10439C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15015d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(}{}$$

$$= \frac{8a^3(8368A+10439C)\sqrt{\cos(c+dx)}\sin(c+dx)}{45045d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(}{}$$

$$= \frac{16a^3(8368A+10439C)\sin(c+dx)}{45045d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a^3(8368A}{45}$$

Mathematica [A] time = 3.51184, size = 148, normalized size = 0.47

$$\frac{a^2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(8(226573A+222794C)\cos(c+dx)+(746519A+581152C)\cos(2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(2798182*A + 3233516*C + 8*(226573*A + 222794*C)*Co
s[c + d*x] + (746519*A + 581152*C)*Cos[2*(c + d*x)] + 287060*A*Cos[3*(c + d
*x)] + 148720*C*Cos[3*(c + d*x)] + 94010*A*Cos[4*(c + d*x)] + 20020*C*Cos[4
*(c + d*x)] + 23940*A*Cos[5*(c + d*x)] + 3465*A*Cos[6*(c + d*x)])*Sqrt[a*(1
 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(720720*d)
```

Maple [A] time = 0.368, size = 166, normalized size = 0.5

$$-\frac{2a^2(-1+\cos(dx+c))\left(3465A(\cos(dx+c))^6+11970A(\cos(dx+c))^5+18305A(\cos(dx+c))^4+5005C(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(3465*A*cos(d*x+c)^6+11970*A*cos(d*x+c)^5+18
305*A*cos(d*x+c)^4+5005*C*cos(d*x+c)^4+20920*A*cos(d*x+c)^3+18590*C*cos(d*x
+c)^3+25104*A*cos(d*x+c)^2+31317*C*cos(d*x+c)^2+33472*A*cos(d*x+c)+41756*C*
cos(d*x+c)+66944*A+83512*C)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.2674, size = 1150, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2882880*(sqrt(2)*(3783780*a^2*cos(12/13*arctan2(sin(13/2*d*x + 13/2*c), c
os(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 1066065*a^2*cos(10/13*arct
an2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c)
 + 459459*a^2*cos(8/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*
c)))*sin(13/2*d*x + 13/2*c) + 193050*a^2*cos(6/13*arctan2(sin(13/2*d*x + 13
/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 70070*a^2*cos(4/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13
/2*c) + 20475*a^2*cos(2/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c)))*sin(13/2*d*x + 13/2*c) - 3783780*a^2*cos(13/2*d*x + 13/2*c)*sin(12
/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 1066065*a^2*
cos(13/2*d*x + 13/2*c)*sin(10/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d
*x + 13/2*c))) - 459459*a^2*cos(13/2*d*x + 13/2*c)*sin(8/13*arctan2(sin(13/
2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 193050*a^2*cos(13/2*d*x + 13/2*
c)*sin(6/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 7007
0*a^2*cos(13/2*d*x + 13/2*c)*sin(4/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) - 20475*a^2*cos(13/2*d*x + 13/2*c)*sin(2/13*arctan2(sin
(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 6930*a^2*sin(13/2*d*x + 13/
2*c) + 20475*a^2*sin(11/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c))) + 70070*a^2*sin(9/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x
+ 13/2*c))) + 193050*a^2*sin(7/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*
d*x + 13/2*c))) + 459459*a^2*sin(5/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) + 1066065*a^2*sin(3/13*arctan2(sin(13/2*d*x + 13/2*c),
cos(13/2*d*x + 13/2*c))) + 3783780*a^2*sin(1/13*arctan2(sin(13/2*d*x + 13/2
*c), cos(13/2*d*x + 13/2*c))))*A*sqrt(a) + 1144*sqrt(2)*(225*a^2*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 378*a^2*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2100*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4095*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 63*(65*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*cos(9/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(585*a^2*cos(4*d*x + 4*c) +
54*a^2*cos(2*d*x + 2*c) + 5*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.510444, size = 473, normalized size = 1.51

$$2\left(3465\,Aa^2\cos\left(dx+c\right)^6+11970\,Aa^2\cos\left(dx+c\right)^5+35\,(523\,A+143\,C)a^2\cos\left(dx+c\right)^4+10\,(2092\,A+1859\,C)a^2\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 2/45045*(3465*A*a^2*cos(d*x + c)^6 + 11970*A*a^2*cos(d*x + c)^5 + 35*(523*A
 + 143*C)*a^2*cos(d*x + c)^4 + 10*(2092*A + 1859*C)*a^2*cos(d*x + c)^3 + 3*
(8368*A + 10439*C)*a^2*cos(d*x + c)^2 + 4*(8368*A + 10439*C)*a^2*cos(d*x +
c) + 8*(8368*A + 10439*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(13/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(1
3/2), x)
```

3.1145 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=266

$$\frac{2a^2(32A+33C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{231d}+\frac{2a^3(232A+297C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(56$$

```
[Out] (4*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(69
3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(232*A + 297*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(32*A + 33*C)*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(231*d) + (10*a*A*Cos[c
 + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d) + (2*A*Cos[c
+ d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.937103, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4017, 4015, 3805, 3804}

$$\frac{2a^2(32A+33C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{231d}+\frac{2a^3(232A+297C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(56$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x
]
```

```
[Out] (4*a^3*(568*A + 759*C)*Sin[c + d*x])/(693*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(568*A + 759*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(69
3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(232*A + 297*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(32*A + 33*C)*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(231*d) + (10*a*A*Cos[c
 + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d) + (2*A*Cos[c
+ d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(11*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}\left(A\right.}{\sec^{\frac{11}{2}}(c+}$$

$$=\frac{2A\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{11d}+\frac{(2}{}$$

$$=\frac{10aA\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{99d}+\frac{2}{}$$

$$=\frac{2a^2(32A+33C)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c}{231d}$$

$$=\frac{2a^3(232A+297C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(32A}{}$$

$$=\frac{2a^3(568A+759C)\sqrt{\cos(c+dx)}\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(232}{}$$

$$=\frac{4a^3(568A+759C)\sin(c+dx)}{693d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(568A+75}{693d}$$

Mathematica [A] time = 2.42434, size = 127, normalized size = 0.48

$$\frac{a^2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(6989A+6666C)\cos(c+dx)+16(325A+198C)\cos(2(c+dx}{554}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x
]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(22928*A + 27456*C + 2*(6989*A + 6666*C)*Cos[c + d*
x] + 16*(325*A + 198*C)*Cos[2*(c + d*x)] + 1735*A*Cos[3*(c + d*x)] + 396*C*
Cos[3*(c + d*x)] + 448*A*Cos[4*(c + d*x)] + 63*A*Cos[5*(c + d*x)])*Sqrt[a*(
1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(5544*d)
```

Maple [A] time = 0.345, size = 144, normalized size = 0.5

$$-\frac{2a^2(-1+\cos(dx+c))\left(63A(\cos(dx+c))^5+224A(\cos(dx+c))^4+355A(\cos(dx+c))^3+99C(\cos(dx+c))^3+}{693d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/693/d*a^2*(-1+cos(d*x+c))*(63*A*cos(d*x+c)^5+224*A*cos(d*x+c)^4+355*A*co
s(d*x+c)^3+99*C*cos(d*x+c)^3+426*A*cos(d*x+c)^2+396*C*cos(d*x+c)^2+568*A*co
s(d*x+c)+759*C*cos(d*x+c)+1136*A+1518*C)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.19197, size = 938, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/22176*(sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(1
1/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(
11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*
a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(1
1/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(1
1/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*
a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*
c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287
*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c)
 + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))
```

```
) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c
))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2
*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11
/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
 11/2*c))))*A*sqrt(a) - 132*sqrt(2)*(77*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 42*a^2*sin(5/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 77*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 630*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - (77*a^2*cos(2*d*x + 2*c) + 6*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.517164, size = 385, normalized size = 1.45

$$\frac{2\left(63\,Aa^2\cos\left(dx+c\right)^5+224\,Aa^2\cos\left(dx+c\right)^4+\left(355\,A+99\,C\right)a^2\cos\left(dx+c\right)^3+6\left(71\,A+66\,C\right)a^2\cos\left(dx+c\right)^2+(56\cdots}{693\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 2/693*(63*A*a^2*cos(d*x + c)^5 + 224*A*a^2*cos(d*x + c)^4 + (355*A + 99*C)*
a^2*cos(d*x + c)^3 + 6*(71*A + 66*C)*a^2*cos(d*x + c)^2 + (568*A + 759*C)*a
^2*cos(d*x + c) + 2*(568*A + 759*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(1
1/2), x)
```

3.1146 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=216

$$\frac{16a^2(13A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(13A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(13A+21$$

```
[Out] (64*a^3*(13*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Se
c[c + d*x]]) + (16*a^2*(13*A + 21*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(315*d) + (2*a*(13*A + 21*C)*Cos[c + d*x]^(3/2)*(a + a*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (10*A*Cos[c + d*x]^(5/2)*(a + a
*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*A*Cos[c + d*x]^(7/2)*(a + a*
Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.631934, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4087, 4013, 3809, 3804}

$$\frac{16a^2(13A+21C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(13A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(13A+21$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (64*a^3*(13*A + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Se
c[c + d*x]]) + (16*a^2*(13*A + 21*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(315*d) + (2*a*(13*A + 21*C)*Cos[c + d*x]^(3/2)*(a + a*
Sec[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (10*A*Cos[c + d*x]^(5/2)*(a + a
*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*A*Cos[c + d*x]^(7/2)*(a + a*
Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
```

```
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}\left(A\right.}{\sec^{\frac{9}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{9d} + \frac{\left(2\surd\right.}{}$$

$$= \frac{10A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2A}{}$$

$$= \frac{2a(13A+21C)\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c-}{105d}$$

$$= \frac{16a^2(13A+21C)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c-}{315d}$$

$$= \frac{64a^3(13A+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(13A+21C}{}$$

Mathematica [A] time = 1.58165, size = 105, normalized size = 0.49

$$\frac{a^2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(4(779A+588C)\cos(c+dx)+4(254A+63C)\cos(2(c+dx))+260}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(5653*A + 7476*C + 4*(779*A + 588*C)*Cos[c + d*x] +
 4*(254*A + 63*C)*Cos[2*(c + d*x)] + 260*A*Cos[3*(c + d*x)] + 35*A*Cos[4*(c
 + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.305, size = 122, normalized size = 0.6

$$-\frac{2 a^2 (-1+\cos(dx+c)) \left(35 A (\cos(dx+c))^4+130 A (\cos(dx+c))^3+219 A (\cos(dx+c))^2+63 C (\cos(dx+c))^2+\right.}{315 d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+219*A*co
s(d*x+c)^2+63*C*cos(d*x+c)^2+292*A*cos(d*x+c)+294*C*cos(d*x+c)+584*A+903*C)
*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.17587, size = 783, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) - 168*(75*sqrt(2)*a^2*cos(5/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 25*sqrt(2)*a^2*sin(3/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*sqrt(2)*a^2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqr
t(a))/d
```

Fricas [A] time = 0.495154, size = 336, normalized size = 1.56

$$\frac{2 \left(35 A a^2 \cos(dx+c)^4+130 A a^2 \cos(dx+c)^3+3 (73 A+21 C) a^2 \cos(dx+c)^2+2 (146 A+147 C) a^2 \cos(dx+c)+\right.}{315 (d \cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^4 + 130*A*a^2*cos(d*x + c)^3 + 3*(73*A + 21*C)
*a^2*cos(d*x + c)^2 + 2*(146*A + 147*C)*a^2*cos(d*x + c) + (584*A + 903*C)*
a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.1147 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=230

$$\frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a^{5/2}C\sqrt{\cos(c+d}}{}$$

```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 49*C)*Sin[c + d*x])/(2
1*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c
+ d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*A*Cos[c +
d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.781066, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4017, 4015, 3801, 215}

$$\frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a^{5/2}C\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 49*C)*Sin[c + d*x])/(2
1*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 7*C)*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c
+ d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*A*Cos[c +
d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}\left(A\right.}{\sec^{\frac{7}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{\left(2\sqrt{\right.}{}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2A}{}$$

$$= \frac{2a^2(8A+7C)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+d}{21d}$$

$$= \frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+7C)\sqrt{c}}{}$$

$$= \frac{2a^3(32A+49C)\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+7C)\sqrt{c}}{}$$

$$= \frac{2a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.50132, size = 125, normalized size = 0.54

$$\frac{a^2\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((101A+28C)\cos(c+dx)+24A\cos(2(c+dx))+\right.}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(84*Sqr
t[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(208*A + 224*C + (101*A + 28*C
)*Cos[c + d*x] + 24*A*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sin[(c + d*x
)/2]))/(84*d)
```

Maple [A] time = 0.272, size = 236, normalized size = 1.

$$-\frac{a^2}{42\,d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(12\,A(\cos(dx+c))^4-21\,C\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/42/d*a^2*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(12*A*cos(
d*x+c)^4-21*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21*C*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))*sin(d*x+c)+36*A*cos(d*x+c)^3+44*A*cos(d*x+c)^2+28*C*cos(d*
x+c)^2+92*A*cos(d*x+c)+196*C*cos(d*x+c)-184*A-224*C)/sin(d*x+c)
```

Maxima [B] time = 2.22792, size = 1146, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/168*(sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*co
s(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), co
s(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*
sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3
/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 28*(2*sqrt
(2)*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 30*sqrt(2)*a
^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*log(2*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2) - 3*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*a^2*log(2*cos(
```

```
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) - 3*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 -
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*C*sqrt(a))/d
```

Fricas [A] time = 0.587811, size = 1115, normalized size = 4.85

$$\left[\frac{4\left(3\,Aa^2\cos(dx+c)^3+12\,Aa^2\cos(dx+c)^2+(23\,A+7\,C)a^2\cos(dx+c)+2\,(23\,A+28\,C)a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx}{42\,(d\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/42*(4*(3*A*a^2*cos(d*x + c)^3 + 12*A*a^2*cos(d*x + c)^2 + (23*A + 7*C)*a
^2*cos(d*x + c) + 2*(23*A + 28*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + 21*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(
a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/21*(2*(3*
A*a^2*cos(d*x + c)^3 + 12*A*a^2*cos(d*x + c)^2 + (23*A + 7*C)*a^2*cos(d*x +
 c) + 2*(23*A + 28*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c) + 21*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(-a)*arctan(2
*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+A\right)(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7
/2), x)
```

3.1148 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=230

$$\frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A-15C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (5*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(64*A + 15*C)*Sin[c + d*x])/(15*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(16*A - 15*C)*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[Cos
[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*A*Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.798274, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4087, 4017, 4018, 4015, 3801, 215}

$$\frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A-15C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```

```
[Out] (5*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(64*A + 15*C)*Sin[c + d*x])/(15*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(16*A - 15*C)*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[Cos
[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*A*Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}\left(A\right.}{\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{(2\sqrt{}}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2A}{}$$

$$= -\frac{a^2(16A-15C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{}}{}$$

$$= \frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(16A-15C)\sqrt{a}}{15d}$$

$$= \frac{a^3(64A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(16A-15C)\sqrt{a}}{15d}$$

$$= \frac{5a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.57593, size = 131, normalized size = 0.57

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((181A+60C)\cos(c+dx)+28A\cos(2(c+dx))+3A\cos(3(c\right.}{60d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(150*Sqrt[2]*C*ArcTanh[Sqr
t[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(28*A + 30*C + (181*A + 60*C)*Cos[c
 + d*x] + 28*A*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/
(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.271, size = 245, normalized size = 1.1

$$-\frac{a^2}{60\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(75\,C\sin(dx+c)\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/60/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(75*C*sin(d*x+c)*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*cos(d*x+c)-75*C*sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
```

```
)))*cos(d*x+c)+24*A*cos(d*x+c)^4+88*A*cos(d*x+c)^3+232*A*cos(d*x+c)^2+120*C
*cos(d*x+c)^2-344*A*cos(d*x+c)-60*C*cos(d*x+c)-60*C)/sin(d*x+c)/cos(d*x+c)^
(1/2)
```

Maxima [B] time = 3.30519, size = 11036, normalized size = 47.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/1260*(42*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) - 5*(1449*sqrt(
2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d
*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
+ 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 60*sqrt(2)*a^2*s
in(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^
2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*cos(5/2*d*x
+ 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*
c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 69*sqrt
(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 198*sqrt(2)*a^2*sin(1/2*d*x
 + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 198*sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 12
*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c)^2
 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c))*sin(2*
d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*
sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sq
rt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(5/2*d
*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1/2*d*
x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(13
/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c)
 + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)
 + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(
5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*c
os(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x +
2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x +
2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*
sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c
))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin(2*d*x
 + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin(2*d*x
 + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a
^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x +
2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*
d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin
(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)
*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x
+ 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(2*
```

d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 18
9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) - 21*(60*sqrt(2)*a^2*sin(1/2*d*x
+ 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) - 315*(a^2*cos(
1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 +
a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c
)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*
sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(
1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x +
5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2
*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*
x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*
c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/2*d*x + 1/2*c)^2 +
a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d
*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2
*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(
5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)
^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a
^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2
*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos
(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2
*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d
*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x
+ 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1
/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*
sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*
sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 +
(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)
^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2
*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2
+ a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*
sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*
cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d
*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2
*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(
5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)
^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x +
1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*

x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)
^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)
)*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x +
1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)
+ a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin
(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 35*(sqrt(2)*a^2*co
s(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*co
s(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d
*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x
+ 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x
+ 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x +
1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x +
1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(13/2*
d*x + 13/2*c) + 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*
x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5
/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)
*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c))*sin(5/2*d*x + 5/2*c))*sin(11/2*d*x + 11/2*c) + 7*(9*sqrt(2)*a^2*cos(1/2
*d*x + 1/2*c)^2 + 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 - (5*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*
x + 2*c) - 9*sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d
*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 - (5
*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 - 5*(sqrt(
2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d
*x + 2*c)^2 - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + 5*
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d
*x + 2*c)*cos(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/
2*d*x + 5/2*c) + 4*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*si
n(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)
- 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(9/2*d*x + 9/2*c) - 390*(sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*
a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*c
os(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(
2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5
/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2
*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin
(7/2*d*x + 7/2*c) - 21*(69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 189*sqrt(2)
*a^2*sin(1/2*d*x + 1/2*c)^2 + 69*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^
2)*cos(5/2*d*x + 5/2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*
d*x + 1/2*c) - 6*sqrt(2)*a^2)*cos(2*d*x + 2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/
2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*sin(2*d*x + 2*c)^2 + 1
2*sqrt(2)*a^2 + 138*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - sq
rt(2)*a^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x +
1/2*c))*cos(5/2*d*x + 5/2*c) + (69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 - 50
*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 189*sqrt(2)*a^2*si
n(1/2*d*x + 1/2*c)^2 + 24*sqrt(2)*a^2)*cos(2*d*x + 2*c) - 10*(5*sqrt(2)*a^2
*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x + 5/2*c) + 105*(1

2*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*
sin(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^
2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) - 252*(5*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c) - 135
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 +
(sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(
2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^
2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 +
2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sq
rt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(
2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*
x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d
*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqr
t(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x +
1/2*c))*sin(5/2*d*x + 5/2*c))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 63*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*
x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/
2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/
2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*
sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*
d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(
1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*
sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x
+ 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + s
qrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*
d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*
d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(5/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1260*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c
)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2
+ sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)
*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x +
2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) +
sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2
+ sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*c
os(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*s
in(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) +
2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2
*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sqrt(a)/((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(5/2*d*x +
5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2
*c)^2 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*
sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + cos(1/2*d
*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) +
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + cos(1/2*d*x + 1/2*c)^2 + 2*(cos(2*
d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c)

```
 + 2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(5/2*
d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2))/d
```

Fricas [A] time = 0.594323, size = 1175, normalized size = 5.11

$$\left[\frac{4\left(6\,Aa^{2}\cos\left(dx+c\right)^{3}+28\,Aa^{2}\cos\left(dx+c\right)^{2}+2\,(43\,A+15\,C)a^{2}\cos\left(dx+c\right)+15\,Ca^{2}\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\,\mathrm{s}}{60\left(d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/60*(4*(6*A*a^2*cos(d*x + c)^3 + 28*A*a^2*cos(d*x + c)^2 + 2*(43*A + 15*C
)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + 75*(C*a^2*cos(d*x + c)^2 + C*a^2*cos(d*x + c)
)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
 c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(
d*x + c)), 1/30*(2*(6*A*a^2*cos(d*x + c)^3 + 28*A*a^2*cos(d*x + c)^2 + 2*(4
3*A + 15*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(C*a^2*cos(d*x + c)^2 + C*a^2*co
s(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*
a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5
/2), x)
```

3.1149 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=244

$$\frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{12d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{4d}$$

```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(56*A - 27*C)*Sin[c
 + d*x])/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(8*A - 2
1*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x]]) - (a*
(4*A - 3*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.812885, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4018, 4015, 3801, 215}

$$\frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{12d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```



```
[Out] (a^(5/2)*(8*A + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(56*A - 27*C)*Sin[c
 + d*x])/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(8*A - 2
1*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x]]) - (a*
(4*A - 3*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \cdots$$

$$= -\frac{a(4A-3C)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{6d\sqrt{\cos(c+dx)}} + \frac{2A\cdots}{}$$

$$= -\frac{a^2(8A-21C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{12d\sqrt{\cos(c+dx)}} - \frac{a(4\cdots}{}$$

$$= \frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(8A-21C)\cdots}{}$$

$$= \frac{a^3(56A-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{a^2(8A-21C)\cdots}{}$$

$$= \frac{a^{5/2}(8A+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\cdots}}{4d}$$

Mathematica [A] time = 1.71225, size = 139, normalized size = 0.57

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)((6A+33C)\cos(c+dx)+32A\cos(2(c+dx))+2A\cos(3(c\cdots\right.}{48d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(8*A + 19*C)*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 4*(32*A + 6*C + (6*A + 33*
C)*Cos[c + d*x] + 32*A*Cos[2*(c + d*x)] + 2*A*Cos[3*(c + d*x)])*Sin[(c + d*
x)/2]))/(48*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.316, size = 378, normalized size = 1.6

$$-\frac{a^2(-1+\cos(dx+c))}{24\,d\,(\sin(dx+c))^2}\left(16\,A\sin(dx+c)\,(\cos(dx+c))^3\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+128\,A\,(\cos(dx+c))^2\sin(dx+c)\,\sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(16*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1
))^(1/2)+128*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-24*A*2^(1/
2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*
cos(d*x+c)^2+24*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-57*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+57*C*2^(1/2)*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*
x+c)^2+66*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+12*C*(-2/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^
(3/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 21.4554, size = 4618, normalized size = 18.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="maxima")
```

```
[Out] 1/48*(4*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
```

```
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*
d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*A*sqrt(a) - 3*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c
) - 56*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*s
in(3/2*d*x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4
*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))
*sin(4*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(
7/2*d*x + 7/2*c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin
(3/2*d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ 38*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
```

```
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*
d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(
2*d*x + 2*c) + 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5
/2*d*x + 5/2*c) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2
)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a
^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c))*C*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.710428, size = 1210, normalized size = 4.96

$$\left[\frac{4\left(8\,Aa^2\cos\left(dx+c\right)^3+64\,Aa^2\cos\left(dx+c\right)^2+33\,Ca^2\cos\left(dx+c\right)+6\,Ca^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3}{48\left(d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="fricas")
```

```
[Out] [1/48*(4*(8*A*a^2*cos(d*x + c)^3 + 64*A*a^2*cos(d*x + c)^2 + 33*C*a^2*cos(d
*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 3*((8*A + 19*C)*a^2*cos(d*x + c)^3 + (8*A + 19*C)*a^2*cos
(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*
a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)
^3 + d*cos(d*x + c)^2), 1/24*(2*(8*A*a^2*cos(d*x + c)^3 + 64*A*a^2*cos(d*x
+ c)^2 + 33*C*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((8*A + 19*C)*a^2*cos(d*x + c)^3
 + (8*A + 19*C)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3
/2), x)
```

3.1150 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right) d$

Optimal. Leaf size=238

$$\frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+31C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{24d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{se}}{8a}$$

```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(24*A - 49*C)*Sin[
c + d*x])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*A +
 31*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (
5*a*C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x]]) +
(C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.792775, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4089, 4018, 4015, 3801, 215}

$$\frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+31C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{24d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}(8A+5C)\sqrt{\cos(c+dx)}\sqrt{se}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2),x]
```


```
[Out] (5*a^(5/2)*(8*A + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(24*A - 49*C)*Sin[
c + d*x])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*A +
 31*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (
5*a*C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x]]) +
(C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(}}$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{5aC(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{12d\sqrt{\cos(c+dx)}} + \frac{C(a+a\sec(}{3d}$$

$$= \frac{a^2(24A+31C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{5aC}{}$$

$$= \frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2(24A+31C}{}$$

$$= \frac{a^3(24A-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2(24A+31C}{}$$

$$= \frac{5a^{5/2}(8A+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{8d}$$

Mathematica [A] time = 2.38492, size = 144, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((72A+68C)\cos(c+dx)+3(8A+25C)\cos(2(c+dx))+24A}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]
^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[2]*(8*A + 5*C)*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*A + 91*C + (72*A + 68*
C)*Cos[c + d*x] + 3*(8*A + 25*C)*Cos[2*(c + d*x)] + 24*A*Cos[3*(c + d*x)])*
Sin[(c + d*x)/2]))/(48*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.309, size = 409, normalized size = 1.7

$$-\frac{a^2(-1+\cos(dx+c))}{48\,d\,(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(120\,A\,(\cos(dx+c))^3\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(120*A*cos(
d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c)))*2^(1/2)-120*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+96*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos
(d*x+c)+1))^(1/2)+75*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-75*C*cos(d*x+c)^3*arctan(1/4*2^(1/
2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+48*A*cos(d*
x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+150*C*cos(d*x+c)^2*(-2/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+68*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+
c)+16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/(-2/(cos(d*x+c)+1))^(1/2)/cos
(d*x+c)^(5/2)/sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.716152, size = 1237, normalized size = 5.2

$$\left[\frac{4\left(48\,Aa^2\cos(dx+c)^3+3\,(8\,A+25\,C)a^2\cos(dx+c)^2+34\,Ca^2\cos(dx+c)+8\,Ca^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(\ldots}{96\,(\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(4*(48*A*a^2*cos(d*x + c)^3 + 3*(8*A + 25*C)*a^2*cos(d*x + c)^2 + 34*
C*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 15*((8*A + 5*C)*a^2*cos(d*x + c)^4 + (8*A + 5*
C)*a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*co
s(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(48*A*a^2*cos(d*x + c)^3 + 3*(8*A
 + 25*C)*a^2*cos(d*x + c)^2 + 34*C*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((8*A + 5*
C)*a^2*cos(d*x + c)^4 + (8*A + 5*C)*a^2*cos(d*x + c)^3)*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(
d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + A\right)(a \sec (dx+c) + a)^{\frac{5}{2}} \sqrt{\cos (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
c)), x)
```

3.1151 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=238

$$\frac{a^3(432A+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+17C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{32d\cos^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(304A+163C)\sqrt{\cos(c+dx)}}{}$$

```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A + 299*C)
*Sin[c + d*x])/(192*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(
16*A + 17*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(32*d*Cos[c + d*x]^(3/2
)) + (5*a*C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Cos[c + d*x]^(3/
2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.834265, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4089, 4018, 4016, 3801, 215}

$$\frac{a^3(432A+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+17C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{32d\cos^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(304A+163C)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```


```
[Out] (a^(5/2)*(304*A + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A + 299*C)
*Sin[c + d*x])/(192*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(
16*A + 17*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(32*d*Cos[c + d*x]^(3/2
)) + (5*a*C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(24*d*Cos[c + d*x]^(3/
2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
```

```
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{5aC(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)} + \frac{C(a+a\sec(c+dx))^{5/2}\sin}{4d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2(16A+17C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{32d\cos^{\frac{3}{2}}(c+dx)} + \frac{5aC(a+a\sec(c}{24d\cos}$$

$$= \frac{a^3(432A+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A+17C)\sqrt{a+a\sec}}{32d\cos^{\frac{3}{2}}(c}$$

$$= \frac{a^3(432A+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A+17C)\sqrt{a+a\sec}}{32d\cos^{\frac{3}{2}}(c}$$

$$= \frac{a^{5/2}(304A+163C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{64d}$$

Mathematica [A] time = 3.62775, size = 155, normalized size = 0.65

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((1584A+2203C)\cos(c+dx)+4(48A+163C)\cos(2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(304*A + 163*C)
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (192*A + 844*C + (1584*
A + 2203*C)*Cos[c + d*x] + 4*(48*A + 163*C)*Cos[2*(c + d*x)] + 528*A*Cos[3*
(c + d*x)] + 489*C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]
^(7/2))
```

Maple [B] time = 0.328, size = 440, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/384/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(912*A*cos
(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))-912*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+489*C*cos(d*x+c)^4*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-489*C*cos(d*
x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))*2^(1/2)+1056*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+978*C*
cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+192*A*cos(d*x+c)^2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2)+652*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+368*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*C*(-2/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/co
s(d*x+c)^(7/2)
```

Maxima [B] time = 22.4141, size = 9027, normalized size = 37.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/768*(48*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(
2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x +
 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^
2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
```

+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x +
4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/
2*c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
- 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)
+ 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2
*c) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2
c) - 19(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq

```
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(
2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*
d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d
*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*A*sqrt
(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + (1956*(sqrt(2)*a^2*sin
(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6
*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^
2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d
*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqr
t(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*
sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(
2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*si
n(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*
(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*
a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*s
qrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^
2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x
+ 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(
6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(
4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) +
a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c)
+ a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c
) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8
*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(
2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^
2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*si
n(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c)
 + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c)
+ 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c
) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x
 + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c)
 + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*l
og(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d
*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*s
in(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 +
 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a
^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4
```

```
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x +
 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*
x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*
d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x
 + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*c
os(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16
*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) +
a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*
x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*
d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin
(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
- 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1956*(sqrt(2)
*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(
4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*cos(8*d*x + 8*c)
 + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos
(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x +
2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*co
s(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x +
 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sq
rt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2
*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x +
6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sq
rt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqr
t(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*
cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(4*cos(6*d*x + 6*c
) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)
^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin
(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.841422, size = 1304, normalized size = 5.48

$$\left[\frac{4\left(3\,(176\,A+163\,C)a^2\cos\left(dx+c\right)^3+2\,(48\,A+163\,C)a^2\cos\left(dx+c\right)^2+184\,Ca^2\cos\left(dx+c\right)+48\,Ca^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/768*(4*(3*(176*A + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 163*C)*a^2*cos(
d*x + c)^2 + 184*C*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((304*A + 163*C)*a^2*cos(d
*x + c)^5 + (304*A + 163*C)*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)
^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*s
qrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(176
*A + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 163*C)*a^2*cos(d*x + c)^2 + 184*
C*a^2*cos(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + 3*((304*A + 163*C)*a^2*cos(d*x + c)^5 + (304*
A + 163*C)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x +
c)), x)
```

3.1152 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=285

$$\frac{a^3(400A+283C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1040A+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+dx)\sqrt{a\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*A + 787*
C)*Sin[c + d*x])/(960*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3
*(400*A + 283*C)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
 d*x]]) + (a^2*(80*A + 79*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(240*d*
Cos[c + d*x]^(5/2)) + (a*C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d*Co
s[c + d*x]^(5/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2))
```

Rubi [A] time = 0.932856, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(400A+283C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1040A+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+79C)\sin(c+dx)\sqrt{a\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
```


```
[Out] (a^(5/2)*(400*A + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*A + 787*
C)*Sin[c + d*x])/(960*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3
*(400*A + 283*C)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
 d*x]]) + (a^2*(80*A + 79*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(240*d*
Cos[c + d*x]^(5/2)) + (a*C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d*Co
s[c + d*x]^(5/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{aC(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{C(a+a\sec(c+dx))^{5/2}\sin(}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^2(80A+79C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{240d\cos^{\frac{5}{2}}(c+dx)} + \frac{aC(a+a\sec(c+}{8d\cos}$$

$$= \frac{a^3(1040A+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(80A+79C)\sqrt{a+a\sec}}{240d\cos^{\frac{5}{2}}(}$$

$$= \frac{a^3(1040A+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(400A+283C)\sin}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}}$$

$$= \frac{a^3(1040A+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(400A+283C)\sin}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}}$$

$$= \frac{a^{5/2}(400A+283C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{128d}$$

Mathematica [A] time = 3.57572, size = 178, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(1360A+2343C)\cos(c+dx)+4(6640A+6509C)\cos(2(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(400*A + 283*C
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (20560*A + 24863*C + 1
2*(1360*A + 2343*C)*Cos[c + d*x] + 4*(6640*A + 6509*C)*Cos[2*(c + d*x)] + 5
440*A*Cos[3*(c + d*x)] + 5660*C*Cos[3*(c + d*x)] + 6000*A*Cos[4*(c + d*x)]
+ 4245*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d*Cos[c + d*x]^(9/2))
```

Maple [B] time = 0.346, size = 502, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/3840/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-6000*A*
arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos
```

```
(d*x+c)^5*2^(1/2)+6000*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^5*2^(1/2)-4245*C*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^5*2^(1/2)+4245*C*
arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos
(d*x+c)^5*2^(1/2)+12000*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4
+8490*C*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+5440*A*sin(d*x+c)
*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+5660*C*cos(d*x+c)^3*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+1280*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2)+4528*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2784*C*(-2/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+768*C*(-2/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c))/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 6.46128, size = 11950, normalized size = 41.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/7680*(80*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*s
qrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sq
rt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d
*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)
*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*si
n(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2
+ 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*si
n(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c
) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^
2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*co
s(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
```

), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c
) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6
*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6
*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6
*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^
2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6
*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^
2*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c
) + 3*sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*
x + 3/2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*
x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x

+ 3/2*c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) - 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(cos(6*d*x + 6*c)^2 +
6*(cos(6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 9*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(co
s(6*d*x + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
sin(6*d*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*cos(6*d*x + 6*c) + 1) - (16980*(sqrt(2)*a^2*sin(10*d*x + 10*c)
 + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(19/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*sin(10*d*x + 10
*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c) + 10
*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(17/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 81504*(sqrt(2)*a^2*sin(10*d*x
+ 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*c)
+ 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8320*(sqrt(2)*a^2*sin(10*d
*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x + 6*
c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(
13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 86440*(sqrt(2)*a^2*sin(
10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c))*
cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*a^2*
sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(6*
d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^
2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x +
2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 81504*(sqrt(2)
*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2*s
in(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 5660*(sqrt(
2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*a^2
*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(2*d
*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16980*(sq
rt(2)*a^2*sin(10*d*x + 10*c) + 5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 10*sqrt(2)*
a^2*sin(6*d*x + 6*c) + 10*sqrt(2)*a^2*sin(4*d*x + 4*c) + 5*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4245*(
a^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x +
6*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(1
0*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 +
100*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25
*a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*
x + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*
d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2
*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10
*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20
*(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c)
 + 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*
sin(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) +

a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*
sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a
^2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6
*c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10
*d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 1
00*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*
a^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x
 + 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d
*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*
cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*
a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*
(5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c)
+ 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*s
in(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a
^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*s
in(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 4245*(a^
2*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*
c)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*
d*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 10
0*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a
^2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x
+ 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*
x + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*c
os(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a
^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(
5*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) +
 2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*si
n(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^
2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 4245*(a^2
*cos(10*d*x + 10*c)^2 + 25*a^2*cos(8*d*x + 8*c)^2 + 100*a^2*cos(6*d*x + 6*c
)^2 + 100*a^2*cos(4*d*x + 4*c)^2 + 25*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d
*x + 10*c)^2 + 25*a^2*sin(8*d*x + 8*c)^2 + 100*a^2*sin(6*d*x + 6*c)^2 + 100
*a^2*sin(4*d*x + 4*c)^2 + 100*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^
2*sin(2*d*x + 2*c)^2 + 10*a^2*cos(2*d*x + 2*c) + a^2 + 2*(5*a^2*cos(8*d*x +
 8*c) + 10*a^2*cos(6*d*x + 6*c) + 10*a^2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x
 + 2*c) + a^2)*cos(10*d*x + 10*c) + 10*(10*a^2*cos(6*d*x + 6*c) + 10*a^2*co
s(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 20*(10*a^
2*cos(4*d*x + 4*c) + 5*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 20*(5
*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 10*(a^2*sin(8*d*x + 8*c) +
2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin
(10*d*x + 10*c) + 50*(2*a^2*sin(6*d*x + 6*c) + 2*a^2*sin(4*d*x + 4*c) + a^2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*a^2*sin(4*d*x + 4*c) + a^2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16980*(sqr
t(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a
^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2
*d*x + 2*c) + sqrt(2)*a^2)*sin(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2

```
*c))) - 5660*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*
c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(17/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*
a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8320*(sqrt(2)*a^2*cos(10*d*x + 1
0*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 1
0*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a
^2)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 86440*(sqrt(2)*
a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*co
s(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 86440*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) +
 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 8320*(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*co
s(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x
 + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 81504*(sqrt(2)*a^2*cos(10*d*x + 10*c) +
 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x + 6*c) + 10*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 5660*(sqrt(2)*a^2*cos(
10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(2)*a^2*cos(6*d*x
+ 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*cos(2*d*x + 2*c) +
 sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16980*
(sqrt(2)*a^2*cos(10*d*x + 10*c) + 5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 10*sqrt(
2)*a^2*cos(6*d*x + 6*c) + 10*sqrt(2)*a^2*cos(4*d*x + 4*c) + 5*sqrt(2)*a^2*c
os(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))))*C*sqrt(a)/(2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4
*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*
c)^2 + 10*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) +
 1)*cos(8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*
cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x
 + 2*c)^2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c)
+ sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6
*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*
sin(8*d*x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.856404, size = 1426, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/7680*(4*(15*(400*A + 283*C)*a^2*cos(d*x + c)^4 + 10*(272*A + 283*C)*a^2*
cos(d*x + c)^3 + 8*(80*A + 283*C)*a^2*cos(d*x + c)^2 + 1392*C*a^2*cos(d*x +
 c) + 384*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) + 15*((400*A + 283*C)*a^2*cos(d*x + c)^6 + (400*A + 283*C)*a^
2*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c)
```

```
 - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x
 + c)^6 + d*cos(d*x + c)^5), 1/3840*(2*(15*(400*A + 283*C)*a^2*cos(d*x + c)
^4 + 10*(272*A + 283*C)*a^2*cos(d*x + c)^3 + 8*(80*A + 283*C)*a^2*cos(d*x +
 c)^2 + 1392*C*a^2*cos(d*x + c) + 384*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((400*A + 283*C)*a^2*cos(d*x
 + c)^6 + (400*A + 283*C)*a^2*cos(d*x + c)^5)*sqrt(-a)*arctan(2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3
/2), x)
```

3.1153 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=332

$$\frac{a^3(1304A+1015C)\sin(c+dx)}{512d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1304A+1015C)\sin(c+dx)}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(136A+109C)\sin(c+dx)}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(136*A + 109
*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^
3*(1304*A + 1015*C)*Sin[c + d*x])/(768*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[
c + d*x]]) + (a^3*(1304*A + 1015*C)*Sin[c + d*x])/(512*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*A + 23*C)*Sqrt[a + a*Sec[c + d*x]]*Si
n[c + d*x])/(96*d*Cos[c + d*x]^(7/2)) + (a*C*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(12*d*Cos[c + d*x]^(7/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c
+ d*x])/(6*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.03235, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4089, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(1304A+1015C)\sin(c+dx)}{512d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1304A+1015C)\sin(c+dx)}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(136A+109C)\sin(c+dx)}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (a^(5/2)*(1304*A + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(136*A + 109
*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^
3*(1304*A + 1015*C)*Sin[c + d*x])/(768*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[
c + d*x]]) + (a^3*(1304*A + 1015*C)*Sin[c + d*x])/(512*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(24*A + 23*C)*Sqrt[a + a*Sec[c + d*x]]*Si
n[c + d*x])/(96*d*Cos[c + d*x]^(7/2)) + (a*C*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(12*d*Cos[c + d*x]^(7/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c
+ d*x])/(6*d*Cos[c + d*x]^(7/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !Lt
```

```
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A\right.$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{6d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= \frac{aC(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{12d\cos^{\frac{7}{2}}(c+dx)} + \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+}{6d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{a^2(24A+23C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{96d\cos^{\frac{7}{2}}(c+dx)} + \frac{aC(a+a\sec(c+dx}{12d\cos^{\frac{7}{2}}(c}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(24A+23C)\sqrt{a+a\sec(c}}{96d\cos^{\frac{7}{2}}(c+c}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1304A+1015C)\sin(c}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1304A+1015C)\sin(c}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{a^3(136A+109C)\sin(c+dx)}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1304A+1015C)\sin(c}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{a^{5/2}(1304A+1015C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{512d}$$

Mathematica [A] time = 4.76885, size = 200, normalized size = 0.6

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(14(4056A+4591C)\cos(c+dx)+16(1496A+1711C)\cos(2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + C*Sec[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(24*Sqrt[2]*(1304*A + 1015
*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^6 + (18720*A + 27412*C +
 14*(4056*A + 4591*C)*Cos[c + d*x] + 16*(1496*A + 1711*C)*Cos[2*(c + d*x)]
+ 25448*A*Cos[3*(c + d*x)] + 21721*C*Cos[3*(c + d*x)] + 5216*A*Cos[4*(c + d
*x)] + 4060*C*Cos[4*(c + d*x)] + 3912*A*Cos[5*(c + d*x)] + 3045*C*Cos[5*(c
+ d*x)])*Sin[(c + d*x)/2]))/(24576*d*Cos[c + d*x]^(11/2))
```

Maple [A] time = 0.373, size = 564, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/3072/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3912*A*2^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)
))*cos(d*x+c)^6-3912*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^6+3045*C*2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^6-3045*C*2^
(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*cos(d*x+c)^6-7824*A*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^5-60
90*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^5-5216*A*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4-4060*C*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)^4-2944*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)
-3248*C*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-768*A*cos(d*x+c)^
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-2784*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-1792*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)
-512*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/(-2/(cos(d*x+c)+1))^(1/2)/sin(
d*x+c)^2/cos(d*x+c)^(11/2)
```

Maxima [B] time = 11.3096, size = 14959, normalized size = 45.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="maxima")
```

```
[Out] -1/6144*(8*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos
(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8
*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*
d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c
))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^
2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d
*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*
a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2
*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(s
qrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^
2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6
*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2
*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2
 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8
*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x +
 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x
 + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(
4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d
*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2
```

(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2
*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 +
16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c)
+ a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*
x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(
4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*co
s(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 +
16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c
)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6
*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*
d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(
6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^
2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*
a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c
)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x
 + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2
*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*co
s(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*
cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a
^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3
*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^
2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(
6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt
(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*si
n(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos
(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x +
4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sq
rt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2
*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x +
 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + s
qrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sq
rt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2
*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x +
8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*

sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*
cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*A*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c
) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*co
s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2
c)^2 + 4(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
+ 2*c) + 1) + (12180*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10
*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(23/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4060*(sqrt(2)*a^2*si
n(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*
d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(21/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 70644*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*si
n(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d
*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c
))*cos(19/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 22620*(sqrt(2)*a
^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*s
in(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*
x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(17/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 147592*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*
a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*s
in(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 37800*(sqr
t(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)
*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 37800*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqr
t(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*
a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 14759
2*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*s
qrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*
a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 22620*(sqrt(2)*a^2*sin(12*d*x + 12*c) +
6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqr
t(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2
*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 7
0644*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 1
5*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(
2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4060*(sqrt(2)*a^2*sin(12*d*x + 12*c)
+ 6*sqrt(2)*a^2*sin(10*d*x + 10*c) + 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*s
qrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a
^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 12180*(sqrt(2)*a^2*sin(12*d*x + 12*c) + 6*sqrt(2)*a^2*sin(10*d*x + 10*c) +
 15*sqrt(2)*a^2*sin(8*d*x + 8*c) + 20*sqrt(2)*a^2*sin(6*d*x + 6*c) + 15*sqr
t(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3045*(a^2*cos(12*d*x + 12*c)^2 + 36
*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*cos(8*d*x + 8*c)^2 + 400*a^2*cos(6*d*x
+ 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^2 + 36*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*x + 10*c)^2 + 225*a^2*sin(8*d*x + 8*c)^
2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225*a^2*sin(4*d*x + 4*c)^2 + 180*a^2*sin(4

*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^2*sin(2*d*x + 2*c)^2 + 12*a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x + 10*c) + 15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 40*(15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 30*(6*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*d*x + 10*c) + 15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3045*(a^2*cos(12*d*x + 12*c)^2 + 36*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*cos(8*d*x + 8*c)^2 + 400*a^2*cos(6*d*x + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^2 + 36*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*x + 10*c)^2 + 225*a^2*sin(8*d*x + 8*c)^2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225*a^2*sin(4*d*x + 4*c)^2 + 180*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^2*sin(2*d*x + 2*c)^2 + 12*a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x + 10*c) + 15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 40*(15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 30*(6*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*d*x + 10*c) + 15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3045*(a^2*cos(12*d*x + 12*c)^2 + 36*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*cos(8*d*x + 8*c)^2 + 400*a^2*cos(6*d*x + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^2 + 36*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*x + 10*c)^2 + 225*a^2*sin(8*d*x + 8*c)^2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225*a^2*sin(4*d*x + 4*c)^2 + 180*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^2*sin(2*d*x + 2*c)^2 + 12*a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x + 10*c) + 15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 40*(15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 30*(6*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*d*x + 10*c) + 15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*

d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
) + 3045*(a^2*cos(12*d*x + 12*c)^2 + 36*a^2*cos(10*d*x + 10*c)^2 + 225*a^2*
cos(8*d*x + 8*c)^2 + 400*a^2*cos(6*d*x + 6*c)^2 + 225*a^2*cos(4*d*x + 4*c)^
2 + 36*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(12*d*x + 12*c)^2 + 36*a^2*sin(10*d*
x + 10*c)^2 + 225*a^2*sin(8*d*x + 8*c)^2 + 400*a^2*sin(6*d*x + 6*c)^2 + 225
*a^2*sin(4*d*x + 4*c)^2 + 180*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*a^
2*sin(2*d*x + 2*c)^2 + 12*a^2*cos(2*d*x + 2*c) + a^2 + 2*(6*a^2*cos(10*d*x
+ 10*c) + 15*a^2*cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*
d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(12*d*x + 12*c) + 12*(15*a^2*
cos(8*d*x + 8*c) + 20*a^2*cos(6*d*x + 6*c) + 15*a^2*cos(4*d*x + 4*c) + 6*a^
2*cos(2*d*x + 2*c) + a^2)*cos(10*d*x + 10*c) + 30*(20*a^2*cos(6*d*x + 6*c)
+ 15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c)
+ 40*(15*a^2*cos(4*d*x + 4*c) + 6*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6
c) + 30(6*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 2*(6*a^2*sin(10*
d*x + 10*c) + 15*a^2*sin(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*si
n(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 12*(15*a^2*si
n(8*d*x + 8*c) + 20*a^2*sin(6*d*x + 6*c) + 15*a^2*sin(4*d*x + 4*c) + 6*a^2*
sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 30*(20*a^2*sin(6*d*x + 6*c) + 15*a^2
*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 120*(5*a^2*s
in(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2) - 12180*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(
10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x
 + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(23/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4060
*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sq
rt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a
^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(21/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 70644*(sqrt(2)*a^2*cos(12*
d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x +
 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) +
 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(19/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 22620*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(
2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^
2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*
d*x + 2*c) + sqrt(2)*a^2)*sin(17/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 147592*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x +
10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) +
 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)
*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 37800*(sqrt(2
)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^
2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4
*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 37800*(sqrt(2)*a^2*cos(12*d*x + 12
*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) +
20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(
2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 147592*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*
cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6
*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
2620*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 1
5*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(
2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin
(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 70644*(sqrt(2)*a^2*cos(

```
12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*
x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c
) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4060*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt
(2)*a^2*cos(10*d*x + 10*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a
^2*cos(6*d*x + 6*c) + 15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2
*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 12180*(sqrt(2)*a^2*cos(12*d*x + 12*c) + 6*sqrt(2)*a^2*cos(10*d*x + 1
0*c) + 15*sqrt(2)*a^2*cos(8*d*x + 8*c) + 20*sqrt(2)*a^2*cos(6*d*x + 6*c) +
15*sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(6*
cos(10*d*x + 10*c) + 15*cos(8*d*x + 8*c) + 20*cos(6*d*x + 6*c) + 15*cos(4*d
*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(12*d*x + 12*c) + cos(12*d*x + 12*c)
^2 + 12*(15*cos(8*d*x + 8*c) + 20*cos(6*d*x + 6*c) + 15*cos(4*d*x + 4*c) +
6*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + 36*cos(10*d*x + 10*c)^2 + 30*(
20*cos(6*d*x + 6*c) + 15*cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(8*d
*x + 8*c) + 225*cos(8*d*x + 8*c)^2 + 40*(15*cos(4*d*x + 4*c) + 6*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 400*cos(6*d*x + 6*c)^2 + 30*(6*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + 225*cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2
+ 2*(6*sin(10*d*x + 10*c) + 15*sin(8*d*x + 8*c) + 20*sin(6*d*x + 6*c) + 15*
sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + sin(12*d*x + 12
*c)^2 + 12*(15*sin(8*d*x + 8*c) + 20*sin(6*d*x + 6*c) + 15*sin(4*d*x + 4*c)
 + 6*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 36*sin(10*d*x + 10*c)^2 + 30*(2
0*sin(6*d*x + 6*c) + 15*sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + 225*sin(8*d*x + 8*c)^2 + 120*(5*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 400*sin(6*d*x + 6*c)^2 + 225*sin(4*d*x + 4*c)^2 + 180
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 12*cos(2*d*x +
 2*c) + 1))/d
```

Fricas [A] time = 0.864351, size = 1539, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/6144*(4*(3*(1304*A + 1015*C)*a^2*cos(d*x + c)^5 + 2*(1304*A + 1015*C)*a^
2*cos(d*x + c)^4 + 8*(184*A + 203*C)*a^2*cos(d*x + c)^3 + 48*(8*A + 29*C)*a
^2*cos(d*x + c)^2 + 896*C*a^2*cos(d*x + c) + 256*C*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((1304*A + 1015*C)
*a^2*cos(d*x + c)^7 + (1304*A + 1015*C)*a^2*cos(d*x + c)^6)*sqrt(a)*log((a*
cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
 + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos
(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6), 1/30
72*(2*(3*(1304*A + 1015*C)*a^2*cos(d*x + c)^5 + 2*(1304*A + 1015*C)*a^2*cos
(d*x + c)^4 + 8*(184*A + 203*C)*a^2*cos(d*x + c)^3 + 48*(8*A + 29*C)*a^2*co
s(d*x + c)^2 + 896*C*a^2*cos(d*x + c) + 256*C*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((1304*A + 1015*C)*a^2*
cos(d*x + c)^7 + (1304*A + 1015*C)*a^2*cos(d*x + c)^6)*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^7 + d*cos(d
*x + c)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5
/2), x)
```

3.1154 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{2(31A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(43*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(31*A + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sq
rt[a + a*Sec[c + d*x]]) - (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a +
a*Sec[c + d*x]])
```

Rubi [A] time = 0.819936, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4022, 4013, 3808, 206}

$$\frac{2(31A+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(43*A + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(31*A + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sq
rt[a + a*Sec[c + d*x]]) - (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a +
a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(6A}{\sec^{\frac{5}{2}}(c+dx)}}{7a}$$

$$= -\frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(4\sqrt{\cos(c\right.}}{}$$

$$= \frac{2(31A+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{2(43A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A+35C)\sqrt{\cos(c+dx)}\sin(}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{2(43A+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A+35C)\sqrt{\cos(c+dx)}\sin(}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{}{10}$$

Mathematica [A] time = 1.7355, size = 166, normalized size = 0.68

$$-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(105\sqrt{2}(A+C)\sec^{\frac{7}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2\sin^2\left(\frac{1}{2}(c+dx)\right)\sqrt{1-\sec(c+dx)}\sec}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] -(Cos[c + d*x]^(5/2)*(105*Sqrt[2]*(A + C)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(7/2) + 2*(101*A + 70*C + 24*A*Cos[
c + d*x] + 15*A*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^3*Sin
[(c + d*x)/2]^2)*Sin[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Se
c[c + d*x])])
```

Maple [A] time = 0.296, size = 206, normalized size = 0.8

$$-\frac{1}{105\,ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(30\,A(\cos(dx+c))^4+105\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/105/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(30*A*cos(d*x
+c)^4+105*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+
1))^(1/2)*A*sin(d*x+c)-36*A*cos(d*x+c)^3+105*C*(-2/(cos(d*x+c)+1))^(1/2)*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+68*A*cos(d*x+c)^2
+70*C*cos(d*x+c)^2-148*A*cos(d*x+c)-140*C*cos(d*x+c)+86*A+70*C)/a/sin(d*x+c
)
```

Maxima [B] time = 2.25003, size = 906, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) - 140*
(3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin
```

```
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*sqrt(2)*log(cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + 2*sqrt(2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/sqrt(a))/d
```

Fricas [A] time = 0.544241, size = 1062, normalized size = 4.35

$$\left[\frac{4\left(15\,A\cos\left(dx+c\right)^{3}-3\,A\cos\left(dx+c\right)^{2}+\left(31\,A+35\,C\right)\cos\left(dx+c\right)-43\,A-35\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin}{210\left(ad\cos\left(dx+c\right)+ad\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/210*(4*(15*A*cos(d*x + c)^3 - 3*A*cos(d*x + c)^2 + (31*A + 35*C)*cos(d*x
 + c) - 43*A - 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) + 105*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(
cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/105*(105*sqrt(2)*((
A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(
15*A*cos(d*x + c)^3 - 3*A*cos(d*x + c)^2 + (31*A + 35*C)*cos(d*x + c) - 43*
A - 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/sqrt(a*sec(d*x + c) + a
), x)
```

3.1155 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=201

$$\frac{2(13A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(}{5d\sqrt{}}$$

```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*(13*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) - (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Se
c[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c
+ d*x]])
```

Rubi [A] time = 0.625252, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4087, 4022, 4013, 3808, 206}

$$\frac{2(13A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(}{5d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*(13*A + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) - (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Se
c[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c
+ d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
```

```
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(4A+5C}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+}}}{5a}$$

$$= -\frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(4\sqrt{\cos(c+\right.}}{}$$

$$= \frac{2(13A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos}{5d}$$

$$= \frac{2(13A+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos}{5d}$$

$$= -\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{}{15d}$$

Mathematica [A] time = 0.429578, size = 153, normalized size = 0.76

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(\sqrt{1-\sec(c+dx)}\sec^2(c+dx)(-2A\cos(c+dx)+3A\cos(2(c+dx))+29A+30C)+15\sqrt{2}(A\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*((29*A + 30*C - 2*A*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)]
)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^2 + 15*Sqrt[2]*(A + C)*ArcTan[(Sqrt[2
]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(5/2))*Sin[c + d
*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.393, size = 184, normalized size = 0.9

$$\frac{1}{15\,ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\arctan\left(1/2\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c
)-6*A*cos(d*x+c)^3+15*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+8*A*cos(d*x+c)^2-28*A*cos(d*x+c)-30*C*co
s(d*x+c)+26*A+30*C)/a/sin(d*x+c)
```

Maxima [B] time = 2.22375, size = 747, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) - 30*(sqrt(2)*log(cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1) - sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/sqrt(a))/d
```

Fricas [A] time = 0.536387, size = 964, normalized size = 4.8

$$\left[\frac{4\left(3A\cos(dx+c)^2 - A\cos(dx+c) + 13A + 15C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) + \frac{15\sqrt{2}((A+C)a\cos(dx+c)+(A+C)}{}}{30\,(ad\cos(dx+c) + ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/30*(4*(3*A*cos(d*x + c)^2 - A*cos(d*x + c) + 13*A + 15*C)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*((A
+ C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*
cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(
d*x + c) + a*d), 1/15*(15*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt
(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sq
rt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x + c)^2 - A*cos(d*x + c) + 1
3*A + 15*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + A\right)\cos(dx+c)^{\frac{5}{2}}}{\sqrt{a\sec(dx+c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a
), x)
```

3.1156 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2A\,\mathrm{si}}{3d\sqrt{\cos(c+dx}}$$

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.452282, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4087, 4013, 3808, 206}

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2A\,\mathrm{si}}{3d\sqrt{\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*A*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$=\frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aA}{2}+\frac{1}{2}a(2A+3C}{\sqrt{\sec(c+dx)}\sqrt{a+a}}}{3a}$$

$$=-\frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\left((A+\right.$$

$$=-\frac{2A\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}-\frac{\left(2(A\right.}{}$$

$$=\frac{\sqrt{2}(A+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{ad}}-\frac{}{3d\sqrt{c}}$$

Mathematica [A] time = 0.599132, size = 73, normalized size = 0.47

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(3(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4A\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(3*(A + C)*ArcTanh[Sin[(c + d*x)/2]] - 4*A*Sin[(c + d*x
)/2]^3))/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.335, size = 171, normalized size = 1.1

$$-\frac{-2+2\cos(dx+c)}{3ad(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}-A\sin(dx+c)\sqrt{-2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(A*cos(d*x+c)*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3
*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+3*C*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)^(1/2)/a/(-2/(cos(d*x+c)+1))^(1/2)
/sin(d*x+c)^2
```

Maxima [B] time = 2.14796, size = 504, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) - 3*(sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*C/sqrt(a))/d
```

Fricas [A] time = 0.531383, size = 876, normalized size = 5.62

$$\left[\frac{4\left(A\cos\left(dx+c\right)-A\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\frac{3\sqrt{2}((A+C)a\cos(dx+c)+(A+C)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)\cdots}{\cos(dx+c)}}\cdots}{\cdots}}{\cos(dx+c\cdots}\right)}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*(4*(A*cos(d*x + c) - A)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + 3*sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*
log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*
((A + C)*a*cos(d*x + c) + (A + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2
*(A*cos(d*x + c) - A)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
```

```
+ c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a
), x)
```

3.1157 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+2C\sqrt{\cos}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.485774, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4087, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+2C\sqrt{\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4087

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(
b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*
(A*(m + n + 1) + C*n)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A,
C, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] ||
EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+\cdots}}{\cdots}}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c\cdots}}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\cdots\right)}{ad}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.523765, size = 93, normalized size = 0.53

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-(A+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/Sqrt[a + a*Sec[c + d*
x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-((A + C)*ArcTanh[Sin[(c + d*x)/2]]) + Sqrt[2]*C*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x]]
```

```
*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.366, size = 224, normalized size = 1.3

$$-\frac{-1+\cos(dx+c)}{ad(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2A\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}-C\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)-C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))+C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-2*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2))-2*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos(d
*x+c)^(1/2)/a/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.21642, size = 961, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) + (sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - sq
rt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + log(2*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 2) + log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 2))*C/sqrt(a))/d
```

Fricas [A] time = 0.616374, size = 1331, normalized size = 7.61

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(C\cos\left(dx+c\right)+C\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2\left(ad\cos\left(dx+c\right)\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 + sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*a)*log(-(cos(d*x + c)^2 + 2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqr
t(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A + C)*a*cos(d*x + c) + (A + C)*
a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x + c) + C)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)
+ a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + C\sec^{2}\left(c + dx\right)\right)\sqrt{\cos\left(c + dx\right)}}{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)*sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) +
1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx + c\right)^{2} + A\right)\sqrt{\cos\left(dx + c\right)}}{\sqrt{a\sec\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a
), x)
```

3.1158 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-C\sqrt{\cos(c+d\cdots}$$

```
[Out] -((C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.484263, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4089, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-C\sqrt{\cos(c+d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```


```
[Out] -((C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*(A + C)*ArcTanh[(Sqrt[a]*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{}}{a}$$

$$= \frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(}}{2a}$$

$$= \frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\Big(}{ad}$$

$$= -\frac{C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tanh^{-1}\Big(}{}$$

Mathematica [A] time = 0.44548, size = 105, normalized size = 0.61

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A+C)\cos(c+dx)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2C\sin\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}C\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\right.\right.}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A + C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x] - Sqrt[
2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*C*Sin[(c + d*x)/2])
)/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.355, size = 248, normalized size = 1.4

$$-\frac{-1+\cos(dx+c)}{2\,ad\,(\sin(dx+c))^2}\left(C\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\cos(dx+c)-C\sqrt{2}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+4*A*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+4*C*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*
x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.624187, size = 1465, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(4*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
+ 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt
(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)) + 2*sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos(d*x +
 c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c
)), -1/2*(2*sqrt(2)*((A + C)*a*cos(d*x + c)^2 + (A + C)*a*cos(d*x + c))*sqr
t(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*s
```

```
qrt(cos(d*x + c))/sin(d*x + c)) - 2*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x + c)^2 + C*cos(d*x + c))*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(
d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C\sec^2{(c + dx)}}{\sqrt{a(\sec{(c + dx)} + 1)}\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*sec(c + d*x)**2)/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d
*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + A}{\sqrt{a\sec(dx + c) + a}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c
))), x)
```

3.1159 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{4\sqrt{a}d}$$

```
[Out] ((8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(2
*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (C*Sin[c + d*x])/(4*d*Cos
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.678429, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4265, 4089, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(8A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{4\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```


```
[Out] ((8*A + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A + C)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(2
*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (C*Sin[c + d*x])/(4*d*Cos
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
```

```
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx$$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}\right.}{}}{2a}$$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(\sqrt{c}}{}$$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - ((A$$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(2(A}{}$$

$$= \frac{(8A+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} - \frac{\sqrt{2}(A+C)\tanh}{}$$

Mathematica [A] time = 0.93857, size = 130, normalized size = 0.58

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(8(A+C)\cos^2(c+dx)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{2}(8A+7C)\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{4d\cos^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] -(Cos[(c + d*x)/2]*(8*(A + C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - Sq
rt[2]*(8*A + 7*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + C*(-5*
Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(4*d*Cos[c + d*x]^(5/2)*Sqrt[a*(
1 + Sec[c + d*x])])
```

Maple [B] time = 0.326, size = 384, normalized size = 1.7

$$\frac{-1+\cos(dx+c)}{8\,ad\,(\sin(dx+c))^2}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-8\,A\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-8*A*2^(1/2)*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*
x+c)^2+8*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1-sin(d*x+c)))*cos(d*x+c)^2-7*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)
```

```
+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+7*C*2^(1/2)*arctan(1/4*2
^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+16
*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+16*C*arcta
n(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+2*C*(-2/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))
/a/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.54031, size = 3110, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/16*(8*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) - (4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos
(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(2)*sin(4*d*x +
 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 7*(2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 7*(2*(2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2
*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 7*(
```

```
2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*
d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)
 - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*s
in(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)
*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(
4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*si
n(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c
) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt
(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((2*(2*cos(2*d*x + 2*c) + 1)*cos(
4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d
*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.783716, size = 1593, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/16*(4*(C*cos(d*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - ((8*A + 7*C)*cos(d*x + c)^3 + (8*A + 7*C)*c
os(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) -
7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*((
A + C)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*log(-(cos(d*x + c)^2 +
2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), 1/8*(8*sqrt(2)*((A + C
)*a*cos(d*x + c)^3 + (A + C)*a*cos(d*x + c)^2)*sqrt(-1/a)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x
 + c)) - 2*(C*cos(d*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 7*C)*cos(d*x + c)^3 + (8*A + 7*C)*
cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
- 2*a)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/
2)), x)
```

3.1160 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=266

$$\frac{(8A+7C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A+9C)\sqrt{}}{}$$

```
[Out] -((8*A + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTanh
[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) - (C*Sin[c + d*x])/(12*d*C
os[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((8*A + 7*C)*Sin[c + d*x])/(8
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.848073, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4265, 4089, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(8A+7C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A+9C)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```


```
[Out] -((8*A + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A + C)*ArcTanh
[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) - (C*Sin[c + d*x])/(12*d*C
os[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((8*A + 7*C)*Sin[c + d*x])/(8
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4089

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(C*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(m + n + 1)), x]
 + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n*Si
mp[A*b*(m + n + 1) + b*C*n + a*C*m*Csc[e + f*x], x], x], x] /; FreeQ[{a, b,
 d, e, f, A, C, m, n}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !Lt
Q[n, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}\right.}{}}{3a}$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{8d}$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{8d}$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{C\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{8d}$$

$$= -\frac{(8A+9C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8\sqrt{a}d} + \frac{\sqrt{2}(A+C)\tan}{}$$

Mathematica [A] time = 1.49837, size = 149, normalized size = 0.56

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+7C)\cos(2(c+dx))+24A-4C\cos(c+dx)+37C)+48(A+C)\cos^3(c+dx)\tan\right.}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(48*(A + C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x]^3 - 3*
Sqrt[2]*(8*A + 9*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*
A + 37*C - 4*C*Cos[c + d*x] + 3*(8*A + 7*C)*Cos[2*(c + d*x)])*Sin[(c + d*x)
/2]))/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.362, size = 446, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/48/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(24*A*cos(d*x+c)
^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*
```

```
2^(1/2)-24*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1-sin(d*x+c)))*2^(1/2)+27*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-27*C*cos(d*x+c)^3*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2
)-48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-96*A*arctan(1/2*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-42*C*cos(d*x+c)^2*(-2/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-96*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)^3+4*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-16
*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/a/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))
^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.87779, size = 5084, normalized size = 19.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x +
2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) + (84*(sqrt(2
)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c
))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*sin
(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*co
s(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 312*(sqrt(2)*sin(6*d*x
 + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(7/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 312*(sqrt(2)*sin(6*d*x + 6*c
) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*sin(6*d*x + 6*c) + 3*
sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(
```

```
2*d*x + 2*c), cos(2*d*x + 2*c))) - 84*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)
*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c)
+ 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x
+ 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*
d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2
+ 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 27*(2*(3*cos(4*d*
x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 +
6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(
2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x
+ 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 +
18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c) + 3*co
s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2
+ 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*
c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(
2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 48
*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos
(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2
 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c
)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*
c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 48
*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos
(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2
 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c
)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*
c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 84
*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 100*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 312*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2
```

```
)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 312*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 100*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) +
3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 84*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c)
 + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*C/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos
(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*
c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(
2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 0.783738, size = 1692, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/96*(4*(3*(8*A + 7*C)*cos(d*x + c)^2 - 2*C*cos(d*x + c) + 8*C)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((8*A + 9
*C)*cos(d*x + c)^4 + (8*A + 9*C)*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c
)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*
sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 48*sqrt(2)*((A + C)*a*cos(d*x + c)^4 + (A + C)*a*cos(
d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(
d*x + c)^3), -1/48*(48*sqrt(2)*((A + C)*a*cos(d*x + c)^4 + (A + C)*a*cos(d*
x + c)^3)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(3*(8*A + 7*C)*cos(d*x + c
)^2 - 2*C*cos(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*((8*A + 9*C)*cos(d*x + c)^4 + (8*A + 9*C)*co
s(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
2*a)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + A}{\sqrt{a \sec (dx + c) + a} \cos (dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/
2)), x)
```

3.1161 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{(15A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A+5C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C}{2}$$

```
[Out] -((15*A + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]
*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c +
 d*x])^(3/2)) + ((49*A + 25*C)*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Sec[c + d*x]]) - ((13*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.865277, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4085, 4022, 4013, 3808, 206}

$$-\frac{(15A+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A+5C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{(A+C}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] -((15*A + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]
*a^(3/2)*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c +
 d*x])^(3/2)) + ((49*A + 25*C)*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Sec[c + d*x]]) - ((13*A + 5*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(9A+5}{\sec^{\frac{5}{2}}(c}}{2a^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - $$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(13A+5C)\sqrt{\cos(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + $$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(49A+25C)\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - $$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(49A+25C)\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - $$

$$= -\frac{(15A+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{(A}{}$$

Mathematica [A] time = 2.1879, size = 118, normalized size = 0.44

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((39A+20C)\cos(c+dx)-2A\cos(2(c+dx))+A\cos(3(c+dx))+47A+25C)-5(15A+7C)\cos\left(\right.}{10ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(3/2),x]
```

```
[Out] (-5*(15*A + 7*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (47*A + 25*C
+ (39*A + 20*C)*Cos[c + d*x] - 2*A*Cos[2*(c + d*x)] + A*Cos[3*(c + d*x)])*T
an[(c + d*x)/2])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.375, size = 318, normalized size = 1.2

$$\frac{-1+\cos(dx+c)}{20\,da^2(\sin(dx+c))^3}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(8\,A(\cos(dx+c))^4-75\,A\sin(dx+c)\cos(dx+c)\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/20/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
*(8*A*cos(d*x+c)^4-75*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-35*C*cos(d*x+c)*sin(d*x+c)*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-16
*A*cos(d*x+c)^3-75*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(co
s(d*x+c)+1))^(1/2)*A*sin(d*x+c)-35*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+80*A*cos(d*x+c)^2+40*C*cos(
d*x+c)^2+26*A*cos(d*x+c)+10*C*cos(d*x+c)-98*A-50*C)/a^2/sin(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.557679, size = 1234, normalized size = 4.6

$$\left[\frac{5\sqrt{2}\left((15\,A+7\,C)\cos(dx+c)^2+2\,(15\,A+7\,C)\cos(dx+c)+15\,A+7\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\mathrm{co}}}{\cos(dx+c)^2+2\,\mathrm{c}}\right.}{40\,(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/40*(5*sqrt(2)*((15*A + 7*C)*cos(d*x + c)^2 + 2*(15*A + 7*C)*cos(d*x + c)
 + 15*A + 7*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*A*cos(d*x + c)^
3 - 4*A*cos(d*x + c)^2 + 4*(9*A + 5*C)*cos(d*x + c) + 49*A + 25*C)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(5*sqrt(2)*((15*A + 7*C)*
cos(d*x + c)^2 + 2*(15*A + 7*C)*cos(d*x + c) + 15*A + 7*C)*sqrt(-a)*arctan(
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c)^3 - 4*A*cos(d*x + c)^2 + 4*(9*A +
5*C)*cos(d*x + c) + 49*A + 25*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3
/2), x)
```

3.1162 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(11A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{(1}{6ad\sqrt{co}}$$

```
[Out] ((11*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c +
d*x])^(3/2)) - ((19*A + 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Sec[c + d*x]]) + ((7*A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.66801, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4085, 4022, 4013, 3808, 206}

$$\frac{(11A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{(1}{6ad\sqrt{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```


```
[Out] ((11*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c +
d*x])^(3/2)) - ((19*A + 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Sec[c + d*x]]) + ((7*A + 3*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d*
Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$=-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(7A+3}{\sec^{\frac{3}{2}}(c+d}}{2a^2}$$

$$=-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(7A+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}}-$$

$$=-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(19A+3C)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+$$

$$=-\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(19A+3C)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+$$

$$=\frac{(11A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}-\frac{(A+}{2}$$

Mathematica [A] time = 1.64511, size = 104, normalized size = 0.47

$$\frac{3(11A+3C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\tan\left(\frac{1}{2}(c+dx)\right)(12A\cos(c+dx)-2A\cos(2(c+dx))+17A+3}{6ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(3/2),x]
```

```
[Out] (3*(11*A + 3*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - (17*A + 3*C +
12*A*Cos[c + d*x] - 2*A*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(6*a*d*Sqrt[Cos
[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.355, size = 262, normalized size = 1.2

$$-\frac{-1+\cos(dx+c)}{6\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-4\,A\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^3+16\,A(\cos(dx+c))^2\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-4*A*(-2/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)^3+16*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+33
*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+7*A*cos(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)+9*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))+3*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-19*A*(-2/(cos(d*
x+c)+1))^(1/2)-3*C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)/a^2/sin(d*x+
c)^3/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.545096, size = 1152, normalized size = 5.21

$$\left[\frac{3\sqrt{2}\left((11\,A+3\,C)\cos(dx+c)^2+2\,(11\,A+3\,C)\cos(dx+c)+11\,A+3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos}}{\cos(dx+c)^2+2\cos}\right.}{24\left(a^2d\cos(dx+c)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((11*A + 3*C)*cos(d*x + c)^2 + 2*(11*A + 3*C)*cos(d*x + c)
 + 11*A + 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*c
```

```
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*A*cos(d*x + c)^
2 - 12*A*cos(d*x + c) - 19*A - 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
 c) + a^2*d), -1/12*(3*sqrt(2)*((11*A + 3*C)*cos(d*x + c)^2 + 2*(11*A + 3*C
)*cos(d*x + c) + 11*A + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*A*co
s(d*x + c)^2 - 12*A*cos(d*x + c) - 19*A - 3*C)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3
/2), x)
```

3.1163 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{(7A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{}{2d\sqrt{\cos}}$$

```
[Out] -((7*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^
(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*
x])^(3/2)) + ((5*A + C)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]])
```

Rubi [A] time = 0.490961, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4085, 4013, 3808, 206}

$$-\frac{(7A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{}{2d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] -((7*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^
(3/2)*d) - ((A + C)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*
x])^(3/2)) + ((5*A + C)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
```

```
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a}{\sqrt{s}}}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(7A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{}{2d\sqrt{c}}$$

Mathematica [A] time = 1.85802, size = 114, normalized size = 0.66

$$-\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)(4A\cos(c+dx)+5A+C)-(7A-C)(\cos(c+dx)+1)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(3/2),x]
```

```
[Out] -(Cos[(c + d*x)/2]^3*(-((7*A - C)*ArcTanh[Sin[(c + d*x)/2]]*(1 + Cos[c + d*
x])) + 2*(5*A + C + 4*A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(2*d*Cos[c + d*x]^
(3/2)*(a*(1 + Sec[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

Maple [A] time = 0.332, size = 235, normalized size = 1.4

$$\frac{-1+\cos(dx+c)}{2da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(4A(\cos(dx+c))^2\sqrt{-2(\cos(dx+c)+1)^{-1}}+A\cos(dx+c)\sqrt{-2(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4*A*cos(d*x+c)^2
*(-2/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+7*A*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+C*cos(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)-C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(
1/2))-5*A*(-2/(cos(d*x+c)+1))^(1/2)-C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
^(1/2)/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.536295, size = 1052, normalized size = 6.12

$$\left[-\frac{\sqrt{2}\left((7A-C)\cos(dx+c)^2+2(7A-C)\cos(dx+c)+7A-C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - C)*cos(d*x + c)^2 + 2*(7*A - C)*cos(d*x + c) + 7*A -
 C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c) + 5*A + C)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((7*A - C
)*cos(d*x + c)^2 + 2*(7*A - C)*cos(d*x + c) + 7*A - C)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*
sin(d*x + c))) + 2*(4*A*cos(d*x + c) + 5*A + C)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+A\right)\sqrt{\cos(dx+c)}}{\left(a\sec(dx+c)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3
/2), x)
```

3.1164 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{(3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+}{\sqrt{a\sec(c+d}}\right.}{a^{3/2}d}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/
(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.528113, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4085, 4023, 3808, 206, 3801, 215}

$$\frac{(3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+}{\sqrt{a\sec(c+d}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),
x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/
(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
```

```
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec}\cdots}{\cdots}}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left((3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\cdots}\right.}{4a}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left((3A-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\cdots}\right.}{\cdots}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(3A-5C)\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.60871, size = 114, normalized size = 0.62

$$\frac{-(A+C)\tan\left(\frac{1}{2}(c+dx)\right)+(3A-5C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\sqrt{2}C\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\cdots\right.}{2ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(
3/2)),x]
```

```
[Out] ((3*A - 5*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + 4*Sqrt[2]*C*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - (A + C)*Tan[(c + d*x)/2])/(
2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.321, size = 304, normalized size = 1.6

$$-\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\left(-2\,C\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\sqrt{2}\sin(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(-2*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*sin(d*x+c)+2*C*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*sin(d*x+c)+A*cos(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)+3*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))+C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*C*sin(d*x+c)*arc
tan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*(-2/(cos(d*x+c)+1))^(1/2)-C
*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [B] time = 2.43745, size = 4257, normalized size = 23.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
 + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin
(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2
*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
```

2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c
)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
+ 4*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*co
s(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*
sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2
*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a))
 + (4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*co
s(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x +
 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 +
 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*
x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2
*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*

```
x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*C/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*
sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)
*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)))/d
```

Fricas [A] time = 0.666074, size = 1593, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((3*A - 5*C)*cos(d*x + c)^2 + 2*(3*A - 5*C)*cos(d*x + c) + 3
*A - 5*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(A + C)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(C*cos(d*x + c
)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*si
n(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A
- 5*C)*cos(d*x + c)^2 + 2*(3*A - 5*C)*cos(d*x + c) + 3*A - 5*C)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) + 2*(A + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) +
 C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + A}{(a \sec (dx+c) + a)^{\frac{3}{2}} \sqrt{\cos (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x +
 c))), x)
```

3.1165 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=228

$$\frac{(A+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] (-3*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A + 9*C)*ArcTanh[(Sqrt[a]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(
2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) + ((A + 3*C)*Sin[c + d*x
])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.7036, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4265, 4085, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(A+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),
x]
```

```
[Out] (-3*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A + 9*C)*ArcTanh[(Sqrt[a]*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Sin[c + d*x])/(
2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) + ((A + 3*C)*Sin[c + d*x
])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+3C)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+3C)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+3C)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= -\frac{3C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A+9C)\tanh^{-1}}{}$$

Mathematica [A] time = 2.23857, size = 169, normalized size = 0.74

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)\left(2(A+9C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{12\sqrt{2}C\cos^2\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}(\sqrt{}}{}\right.}{d(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+dx))+A+2C)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(
3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2)*(2*(A + 9*C)*
ArcTanh[Sin[(c + d*x)/2]] + (12*Sqrt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]
*Cos[(c + d*x)/2]^2 - 2*(A + 3*C + 2*C*Sec[c + d*x])*Sin[(c + d*x)/2])/(-1
+ Sin[(c + d*x)/2]^2)))/(d*(A + 2*C + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d
*x]))^(3/2))
```

Maple [A] time = 0.341, size = 362, normalized size = 1.6

$$\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\,C\sin(dx+c)\sqrt{2}\cos(dx+c)\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*C*sin(d*x+c)*2
^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))-3*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(co
```

```
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+A*cos(d*x+c)^2*(-2/(cos(d*x+c
)+1))^(1/2)-A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))+3*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-9*C*sin(d*x+c)*cos(d*x
+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*cos(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2)-C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-2*C*(-2/(cos(d*x+c)
+1))^(1/2))/a^2/sin(d*x+c)^3/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.680746, size = 1764, normalized size = 7.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((A + 9*C)*cos(d*x + c)^3 + 2*(A + 9*C)*cos(d*x + c)^2 + (A +
 9*C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*c
os(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A + 3*C)*co
s(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c) + 6*(C*cos(d*x + c)^3 + 2*C*cos(d*x + c)^2 + C*cos(d*x + c))
*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a
^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/4*(sqrt(2)*((A + 9*C)*cos(d*x
 + c)^3 + 2*(A + 9*C)*cos(d*x + c)^2 + (A + 9*C)*cos(d*x + c))*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))/(a*sin(d*x + c))) - 2*((A + 3*C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(C*cos(d*x + c)^
3 + 2*C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
 + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x +
 c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(
3/2)), x)
```

3.1166 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{(5A+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\cdots\right.}{4a^{3/2}d}$$

```
[Out] ((8*A + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A + 13*C)*ArcTanh[(S
qrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Si
n[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)) + ((A + 2*C
)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2*A
 + 7*C)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.910585, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4265, 4085, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(5A+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(8A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\cdots\right.}{4a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),
x]
```


```
[Out] ((8*A + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A + 13*C)*ArcTanh[(S
qrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A + C)*Si
n[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)) + ((A + 2*C
)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2*A
 + 7*C)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(}{}}{2a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(A+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(8A+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{3/2}d} - \frac{(5A+13C)\,\mathrm{t}}{}$$

Mathematica [A] time = 3.76652, size = 213, normalized size = 0.75

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(A\cos^2(c+dx)+C\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)((2A+7C)\cos(2(c+dx))+2A+6C\cos(c+dx)+3C)+(5A+\right.}{4ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(
3/2)),x]
```

```
[Out] -((C + A*Cos[c + d*x]^2)*Sec[(c + d*x)/2]*((5*A + 13*C)*ArcTanh[Sin[(c + d*
x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2 - ((8*A + 19*C)*ArcTanh[
Sqrt[2]*Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2)/Sqrt
[2] + (2*A + 3*C + 6*C*Cos[c + d*x] + (2*A + 7*C)*Cos[2*(c + d*x)])*Sin[(c
+ d*x)/2]))/(4*a*d*Cos[c + d*x]^(5/2)*(A + 2*C + A*Cos[2*(c + d*x)])*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.3, size = 508, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*A*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2
^(1/2)*sin(d*x+c)-8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+19*C*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*s
in(d*x+c)-19*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+4*A*(-2/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^3-20*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c
)^2*sin(d*x+c)+14*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-52*C*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-4*A*cos(d*x+c
)^2*(-2/(cos(d*x+c)+1))^(1/2)-8*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-10
*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*C*(-2/(cos(d*x+c)+1))^(1/2))/a^2/
sin(d*x+c)^3/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.869465, size = 1956, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*(2*sqrt(2)*((5*A + 13*C)*cos(d*x + c)^4 + 2*(5*A + 13*C)*cos(d*x + c)
^3 + (5*A + 13*C)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4
*((2*A + 7*C)*cos(d*x + c)^2 + 3*C*cos(d*x + c) - 2*C)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 19*C)*cos(d*x
 + c)^4 + 2*(8*A + 19*C)*cos(d*x + c)^3 + (8*A + 19*C)*cos(d*x + c)^2)*sqrt
(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2
+ 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*
cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), 1/8*(2*sqrt(2)*((5*A + 13*C)*cos(d*
x + c)^4 + 2*(5*A + 13*C)*cos(d*x + c)^3 + (5*A + 13*C)*cos(d*x + c)^2)*sqr
t(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))/(a*sin(d*x + c))) - 2*((2*A + 7*C)*cos(d*x + c)^2 + 3*C*cos(d
*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + ((8*A + 19*C)*cos(d*x + c)^4 + 2*(8*A + 19*C)*cos(d*x + c)^3
+ (8*A + 19*C)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^
2*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(
5/2)), x)
```

3.1167 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=315

$$\frac{(157A+45C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A+195C)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A+735C)\sin(c+d}{240a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+}$$

```
[Out] -((283*A + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt
[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Sec[
c + d*x])^(5/2)) - ((21*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(
a + a*Sec[c + d*x])^(3/2)) + ((2671*A + 735*C)*Sin[c + d*x])/(240*a^2*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((787*A + 195*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((157*A + 45*C)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 1.07039, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4085, 4020, 4022, 4013, 3808, 206}

$$\frac{(157A+45C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A+195C)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A+735C)\sin(c+d}{240a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```


```
[Out] -((283*A + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt
[2]*a^(5/2)*d) - ((A + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Sec[
c + d*x])^(5/2)) - ((21*A + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(
a + a*Sec[c + d*x])^(3/2)) + ((2671*A + 735*C)*Sin[c + d*x])/(240*a^2*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((787*A + 195*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((157*A + 45*C)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(1}{\sec^{\frac{5}{2}}(}}{4a^2}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(283A+75C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 3.97109, size = 150, normalized size = 0.48

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)(5(887A+255C)\cos(c+dx)+16(52A+15C)\cos(2(c+dx))-40A\cos(3(c+dx)\right.}{960ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*(-120*(283*A + 75*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d
*x)/2]^4 + 4*(3491*A + 975*C + 5*(887*A + 255*C)*Cos[c + d*x] + 16*(52*A +
15*C)*Cos[2*(c + d*x)] - 40*A*Cos[3*(c + d*x)] + 12*A*Cos[4*(c + d*x)])*Sin
[(c + d*x)/2]))/(960*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.39, size = 450, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/480/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(192*A*cos(d*x+c)^5-4245*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+
```

```
c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-1125*C*cos(d*x+c)^2
*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c
)+1))^(1/2)-512*A*cos(d*x+c)^4-8490*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-2250*C*cos(d*x+
c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x
+c)+1))^(1/2)+3456*A*cos(d*x+c)^3-4245*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c
)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+960*C*cos(d*x+c)^3-1125
*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*sin(d*x+c)+5974*A*cos(d*x+c)^2+1590*C*cos(d*x+c)^2-3768*A*cos(d*x+c)-108
0*C*cos(d*x+c)-5342*A-1470*C)/a^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.580757, size = 1534, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/960*(15*sqrt(2)*((283*A + 75*C)*cos(d*x + c)^3 + 3*(283*A + 75*C)*cos(d*
x + c)^2 + 3*(283*A + 75*C)*cos(d*x + c) + 283*A + 75*C)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(96*A*cos(d*x + c)^4 - 160*A*cos(d*x + c)^3 + 32*(4
9*A + 15*C)*cos(d*x + c)^2 + 5*(911*A + 255*C)*cos(d*x + c) + 2671*A + 735*
C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^
3*d), 1/480*(15*sqrt(2)*((283*A + 75*C)*cos(d*x + c)^3 + 3*(283*A + 75*C)*c
os(d*x + c)^2 + 3*(283*A + 75*C)*cos(d*x + c) + 283*A + 75*C)*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^4 - 160*A*cos(d*x + c)^3 + 32*
(49*A + 15*C)*cos(d*x + c)^2 + 5*(911*A + 255*C)*cos(d*x + c) + 2671*A + 73
5*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5
/2), x)
```

3.1168 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=266

$$\frac{5(19A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{16\sqrt{2}a^{5/}}$$

```
[Out] ((163*A + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c
 + d*x])^(5/2)) - ((17*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a +
 a*Sec[c + d*x])^(3/2)) - ((299*A + 27*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (5*(19*A + 3*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.859293, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {4265, 4085, 4020, 4022, 4013, 3808, 206}

$$\frac{5(19A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{16\sqrt{2}a^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((163*A + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c
 + d*x])^(5/2)) - ((17*A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a +
 a*Sec[c + d*x])^(3/2)) - ((299*A + 27*C)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (5*(19*A + 3*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(11A+}{\sec^{\frac{3}{2}}(c+}}{4a^2}$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} +$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} -$$

$$= -\frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} -$$

$$= \frac{(163A+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{(A}{}$$

Mathematica [A] time = 3.18382, size = 132, normalized size = 0.5

$$\frac{6(163A+19C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \tan\left(\frac{1}{2}(c+dx)\right)((479A+39C)\cos(c+dx)+80A\cos(2(c+}{48a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(5/2),x]
```

```
[Out] (6*(163*A + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 - (379*A + 2
7*C + (479*A + 39*C)*Cos[c + d*x] + 80*A*Cos[2*(c + d*x)] - 8*A*Cos[3*(c +
d*x)])*Tan[(c + d*x)/2])/(48*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.387, size = 390, normalized size = 1.5

$$-\frac{(-1+\cos(dx+c))^2}{48\,da^3(\sin(dx+c))^5}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(32\,A(\cos(dx+c))^4\sqrt{-2\,(\cos(dx+c)+1)^{-1}} - 192\,A\sqrt{-2\,(\cos(dx+c)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/48/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(32*A*cos(d*x
+c)^4*(-2/(cos(d*x+c)+1))^(1/2)-192*A*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^
3-343*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-489*A*sin(d*x+c)*cos(d*x+c)*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-39*C*cos(d*x+c)^2*(-2/(cos
```

```
(d*x+c)+1))^(1/2)-57*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2))+204*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-489*A*sin(d*x
+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+12*C*cos(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2)-57*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))+299*A*(-2/(cos(d*x+c)+1))^(1/2)+27*C*(-2/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)^(1/2)/a^3/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")

[Out] Timed out
```

Fricas [A] time = 0.562538, size = 1426, normalized size = 5.36

$$\left[\frac{3\sqrt{2}\left((163\,A+19\,C)\cos(dx+c)^3+3\,(163\,A+19\,C)\cos(dx+c)^2+3\,(163\,A+19\,C)\cos(dx+c)+163\,A+19\,C\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")

[Out] [1/192*(3*sqrt(2)*((163*A + 19*C)*cos(d*x + c)^3 + 3*(163*A + 19*C)*cos(d*x
 + c)^2 + 3*(163*A + 19*C)*cos(d*x + c) + 163*A + 19*C)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(32*A*cos(d*x + c)^3 - 160*A*cos(d*x + c)^2 - (503*A
 + 39*C)*cos(d*x + c) - 299*A - 27*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
 + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163*A + 19*C)*c
os(d*x + c)^3 + 3*(163*A + 19*C)*cos(d*x + c)^2 + 3*(163*A + 19*C)*cos(d*x
+ c) + 163*A + 19*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(32*A*cos(d*x +
 c)^3 - 160*A*cos(d*x + c)^2 - (503*A + 39*C)*cos(d*x + c) - 299*A - 27*C)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5
/2), x)
```

3.1169 $\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{(49A+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{5(15A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16ad}$$

```
[Out] (-5*(15*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c
+ d*x])^(5/2)) - ((13*A - 3*C)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a
+ a*Sec[c + d*x])^(3/2)) + ((49*A + C)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.687886, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {4265, 4085, 4020, 4013, 3808, 206}

$$\frac{(49A+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{5(15A-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] (-5*(15*A - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*S
qrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c
+ d*x])^(5/2)) - ((13*A - 3*C)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a
+ a*Sec[c + d*x])^(3/2)) + ((49*A + C)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
```

```
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a\cdots}{\sqrt{\mathrm{se}\cdots}}}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx\cdots}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx\cdots}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-3C)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx\cdots}$$

$$= -\frac{5(15A-C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\cdots}{4d\cdots}$$

Mathematica [A] time = 2.67953, size = 118, normalized size = 0.54

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)(5(17A+C)\cos(c+dx)+16A\cos(2(c+dx))+65A+C)-40(15A-C)\cos^4\left(\frac{1}{2}(c+\cdots\right.\right.}{64ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Sec[c + d*x]^2))/(a + a*Sec[c + d*x])^
(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*(-40*(15*A - C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2
]^4 + 4*(65*A + C + 5*(17*A + C)*Cos[c + d*x] + 16*A*Cos[2*(c + d*x)])*Sin[
(c + d*x)/2]))/(64*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.354, size = 365, normalized size = 1.7

$$-\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(32\,A\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^3+75\,A\sin(dx+c)\cos(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(32*A*(-2/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)^3+75*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+53*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2
)-5*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)+5*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+75*A*sin(d*x+c)*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-36*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)-5*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*C*cos(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-49*A*(-2/(cos(d*x+c)+1))^(1/2)-C*(-2/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)/a^3/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^
5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.553944, size = 1303, normalized size = 5.95

$$\left[-\frac{5\sqrt{2}\left((15\,A-C)\cos(dx+c)^3+3\,(15\,A-C)\cos(dx+c)^2+3\,(15\,A-C)\cos(dx+c)+15\,A-C\right)\sqrt{a}\log\left(-\frac{a\cos(dx\cdots}{\cdots}\right.}{64\left(a^3d\cos(\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [-1/64*(5*sqrt(2)*((15*A - C)*cos(d*x + c)^3 + 3*(15*A - C)*cos(d*x + c)^2
+ 3*(15*A - C)*cos(d*x + c) + 15*A - C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*
sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 4*(32*A*cos(d*x + c)^2 + 5*(17*A + C)*cos(d*x + c) + 49*A + C)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32
*(5*sqrt(2)*((15*A - C)*cos(d*x + c)^3 + 3*(15*A - C)*cos(d*x + c)^2 + 3*(1
5*A - C)*cos(d*x + c) + 15*A - C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(3
2*A*cos(d*x + c)^2 + 5*(17*A + C)*cos(d*x + c) + 49*A + C)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5
/2), x)
```

3.1170 $\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{(19A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d\,c}$$

```
[Out] ((19*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c +
 d*x])^(5/2)) - ((9*A - 7*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.507534, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4265, 4085, 4012, 3808, 206}

$$\frac{(19A+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),
x]
```

```
[Out] ((19*A + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) - ((A + C)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c +
 d*x])^(5/2)) - ((9*A - 7*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
```

```
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec}}{}}{4a}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(9A-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(9A-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= \frac{(19A+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{}{4d\,\mathrm{co}}$$

Mathematica [A] time = 1.95138, size = 110, normalized size = 0.63

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((3C-13A)\cos(c+dx)-9A+7C)+8(19A+3C)\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c\right.\right.}{64ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(
5/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*(8*(19*A + 3*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2
]^4 + 4*(-9*A + 7*C + (-13*A + 3*C)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(64*a*
d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.323, size = 340, normalized size = 2.

$$\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(13\,A(\cos(dx+c))^2\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+19\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(13*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+19*A*
sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*C*
cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+3*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(
1/2)+19*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*C*c
os(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))-9*A*(-2/(cos(d*x+c)+1))^(1/2)+7*C*(-2/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a^3/sin(d*x
+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 4.87552, size = 7466, normalized size = 42.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="maxima")
```

```
[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2
+ 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x
+ 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
 + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x +
4*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7
/2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
```

^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) + 57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x + 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) + 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d*x + c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(3*d*x + 3*c))*sqrt(a)) - (12*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(11*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*

```
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) +
4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*
c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(6*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2
 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(c
os(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2
*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(2*(6*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x
+ 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
12*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(11*co
s(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*cos(3/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*
(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 48*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 6
*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 48*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 +
36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*a
^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 +
16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*
sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*sqr
t(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x + 2*c)
```

```
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt
(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8
*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)
*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*
sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sqrt(a)))/d
```

Fricas [A] time = 0.547532, size = 1262, normalized size = 7.25

$$\left[\frac{\sqrt{2}\left((19\,A+3\,C)\cos\left(dx+c\right)^{3}+3\,(19\,A+3\,C)\cos\left(dx+c\right)^{2}+3\,(19\,A+3\,C)\cos\left(dx+c\right)+19\,A+3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(d\cdots}{\cdots}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 3*C)*cos(d*x + c)^3 + 3*(19*A + 3*C)*cos(d*x + c)^2
 + 3*(19*A + 3*C)*cos(d*x + c) + 19*A + 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((13*A - 3*C)*cos(d*x + c) + 9*A - 7*C)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*
A + 3*C)*cos(d*x + c)^3 + 3*(19*A + 3*C)*cos(d*x + c)^2 + 3*(19*A + 3*C)*co
s(d*x + c) + 19*A + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 3*
C)*cos(d*x + c) + 9*A - 7*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+A}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x +
 c))), x)
```

3.1171 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=232

$$\frac{(5A-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a\sec(c+dx)+}}\right.}{a^{5/2}d}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A - 43*C)*ArcTanh[(Sqrt[a]*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x]
)/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((5*A - 11*C)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.711181, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {4265, 4085, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(5A-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a\sec(c+dx)+}}\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),
x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A - 43*C)*ArcTanh[(Sqrt[a]*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*x]
)/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((5*A - 11*C)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(}{}}{4a^2}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A-11C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A-11C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A-11C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(5A-43C)\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 3.53683, size = 144, normalized size = 0.62

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((5A-11C)\cos(c+dx)+A-15C)+2(5A-43C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64\sqrt{2}C}{16a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(
5/2)),x]
```

```
[Out] (2*(5*A - 43*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqrt[2]*C
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A - 15*C + (5*A -
11*C)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos
[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.29, size = 539, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(16*C*sin(d*x+
c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1+sin(d*x+c)))-16*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+5*A*sin(d*x+c)*cos(d*x+c
)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*A*cos(d*x+c)^2*(-2/(co
s(d*x+c)+1))^(1/2)-43*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
```

```
s(d*x+c)+1))^(1/2))+16*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c)))*2^(1/2)*sin(d*x+c)-16*C*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*sin(d*x+c)+11*C*cos(d*x+
c)^2*(-2/(cos(d*x+c)+1))^(1/2)+5*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))+4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-43*C*sin(d*x+c
)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+4*C*cos(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2)+A*(-2/(cos(d*x+c)+1))^(1/2)-15*C*(-2/(cos(d*x+c)+1))^(1/2)
)*cos(d*x+c)^(1/2)/a^3/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [B] time = 4.66789, size = 10615, normalized size = 45.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/32*((4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*si
n(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x
+ 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*
c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x
+ 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*
cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*
d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8
*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
```

d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3
*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c)
 + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*
cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))*
A/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*
x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c)
 + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2
*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^
2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)) + (44*(sin(4*d*x + 4*c) + 6*sin(2
*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*si

```
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*sin(5/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 19*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 11*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x +
 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*s
in(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*
(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2
+ 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c
)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 1
6*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x +
4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*s
in(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4
*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2
*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt
(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sq
rt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqr
t(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos
(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) +
 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c)
```

+ 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c)
+ 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*s
qrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x
+ 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*
d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos
(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x
+ 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2
*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 43*(2*(6*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^
2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(
cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(
2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 43*(2*(
6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*
x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
- 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(19*

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cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*cos(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 176*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(cos(4*d*x + 4*c)
 + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 176*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((sqrt(2)*a^2*cos(4*d*x + 4*c)
^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt
(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)
^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 1
2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6
*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*
x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sq
rt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c)
 + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sqrt(a)))/d
```

Fricas [A] time = 0.698678, size = 1901, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*A - 43*C)*cos(d*x + c)^3 + 3*(5*A - 43*C)*cos(d*x + c)^
2 + 3*(5*A - 43*C)*cos(d*x + c) + 5*A - 43*C)*sqrt(a)*log(-(a*cos(d*x + c)^
2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 4*((5*A - 11*C)*cos(d*x + c) + A - 15*C)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(C*cos(d*x + c)^3 + 3
*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 4
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(co
s(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A - 43*C)*cos(d*x + c)^3 + 3*(5*A - 4
3*C)*cos(d*x + c)^2 + 3*(5*A - 43*C)*cos(d*x + c) + 5*A - 43*C)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*((5*A - 11*C)*cos(d*x + c) + A - 15*C)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(C*cos
(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^3 + 3
```

```
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+A}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(
3/2)), x)
```

3.1172 $\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{(3A+35C)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(3A+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sqrt{}}{}$$

```
[Out] (-5*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A + 115*C)*ArcTanh[(Sqrt[a]*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*
x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) + ((A - 15*C)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) + ((3*A + 3
5*C)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.919193, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {4265, 4085, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(3A+35C)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(3A+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5C\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),
x]
```


```
[Out] (-5*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A + 115*C)*ArcTanh[(Sqrt[a]*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A + C)*Sin[c + d*
x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) + ((A - 15*C)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) + ((3*A + 3
5*C)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4085

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(d_.
))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(a*
(A + C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m +
 1)), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[
e + f*x])^n*Simp[b*C*n + A*b*(2*m + n + 1) - (a*(A*(m + n + 1) - C*(m - n))
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, C, n}, x] && EqQ[a^2
 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(A+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{5C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A+115C)\tan}{}$$

Mathematica [A] time = 3.57722, size = 187, normalized size = 0.68

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(A+C\sec^2(c+dx)\right)\left((6A+230C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^3\left(\frac{1}{2}\right.\right.}{4d\sqrt{\cos(c+dx)}(a(\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(
5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(A + C*Sec[c + d*x]^2)*((6*A + 230*C)*ArcTanh[Sin[(c +
d*x)/2]] - 160*Sqrt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + ((3*A + 67*C +
 2*(7*A + 55*C)*Cos[c + d*x] + (3*A + 35*C)*Cos[2*(c + d*x)])*Sec[(c + d*x)
/2]^3*Sec[c + d*x]*Tan[(c + d*x)/2])/2))/(4*d*Sqrt[Cos[c + d*x]]*(A + 2*C +
 A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.305, size = 605, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(40*C*arctan(1
/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^
```

```
2*2^(1/2)*sin(d*x+c)-40*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+3*A*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-3*A*(-2/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^3+40*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-40*C*sin(d*x+c)*2^
(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
+sin(d*x+c)))+115*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)-35*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+3*A*sin(d*x+c
)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A*cos(d*x+c
)^2*(-2/(cos(d*x+c)+1))^(1/2)+115*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))-20*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)
+7*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+39*C*cos(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2)+16*C*(-2/(cos(d*x+c)+1))^(1/2))/a^3/(-2/(cos(d*x+c)+1))^(1/2)/sin(
d*x+c)^5/cos(d*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.717105, size = 2107, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="fricas")

[Out] [1/64*(sqrt(2)*((3*A + 115*C)*cos(d*x + c)^4 + 3*(3*A + 115*C)*cos(d*x + c)
^3 + 3*(3*A + 115*C)*cos(d*x + c)^2 + (3*A + 115*C)*cos(d*x + c))*sqrt(a)*l
og(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A + 35*C)*cos(d*x + c)^2 + (7*A + 55*C
)*cos(d*x + c) + 16*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) + 80*(C*cos(d*x + c)^4 + 3*C*cos(d*x + c)^3 + 3*C*cos(d
*x + c)^2 + C*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*si
n(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/
(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a
^3*d*cos(d*x + c)), -1/32*(sqrt(2)*((3*A + 115*C)*cos(d*x + c)^4 + 3*(3*A +
 115*C)*cos(d*x + c)^3 + 3*(3*A + 115*C)*cos(d*x + c)^2 + (3*A + 115*C)*cos
(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((3*A + 35*C)*cos(d*x + c
)^2 + (7*A + 55*C)*cos(d*x + c) + 16*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) + 80*(C*cos(d*x + c)^4 + 3*C*cos(d*x +
 c)^3 + 3*C*cos(d*x + c)^2 + C*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos
(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
```

```
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + A}{(a \sec(dx+c) + a)^{\frac{5}{2}} \cos(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(
5/2)), x)
```

3.1173 $\int \cos^{\frac{9}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=111

$$\frac{10B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10B\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\text{s}}{}$$

```
[Out] (6*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*B*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*C*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.094935, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4064, 2748, 2635, 2639, 2641}

$$\frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10B\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```


```
[Out] (6*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*B*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*C*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{5}{2}}(c+dx)(C+B\cos(c+dx))\,dx$$
$$= B\int \cos^{\frac{7}{2}}(c+dx)\,dx + C\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} +$$
$$= \frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C\cos^{\frac{3}{2}}}{}$$
$$= \frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10B\sqrt{\cos(c+dx)}}{21d}$$

Mathematica [A] time = 0.495843, size = 77, normalized size = 0.69

$$\frac{50B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(15B\cos(2(c+dx))+65B+42C\cos(c+dx))+126CE\left(\frac{1}{2}(c+\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (126*C*EllipticE[(c + d*x)/2, 2] + 50*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*B + 42*C*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

Maple [A] time = 2.125, size = 290, normalized size = 2.6

$$-\frac{2}{105\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^8+(-360\,B-168\,C\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*B-168*C)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*B+168*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*B
-42*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-63*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)\right)\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^4*sec(d*x + c)^2 + B*cos(d*x + c)^4*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(9/2), x)
```

3.1174 $\int \cos^{\frac{7}{2}}(c+dx) \left(B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=87

$$\frac{2C\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0833361, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4064, 2748, 2635, 2641, 2639}

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*C*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(C+B\cos(c+dx))\,dx$$
$$= B\int \cos^{\frac{5}{2}}(c+dx)\,dx + C\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2C\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sqrt{\cos(c+dx)}\sin(c}{3d}$$

Mathematica [A] time = 0.229573, size = 66, normalized size = 0.76

$$\frac{2\left(5C\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3B\cos(c+dx)+5C)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(9*B*EllipticE[(c + d*x)/2, 2] + 5*C*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*C + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 2.036, size = 262, normalized size = 3.

$$-\frac{2}{15\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6+(24\,B+20\,C)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*B+20*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*B-10*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)\right)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+B\cos\left(dx+c\right)^3\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(7/2), x)
```

3.1175 $\int \cos^{\frac{5}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=61

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0714616, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4064, 2748, 2639, 2635, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sqrt{\cos(c+dx)}(C+B\cos(c+dx))\,dx$$
$$= B\int \cos^{\frac{3}{2}}(c+dx)\,dx + C\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}B\int \frac{}{\sqrt{c}}$$
$$= \frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\,\text{si}}{3d}$$

Mathematica [A] time = 0.108685, size = 53, normalized size = 0.87

$$\frac{2\left(B\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)+3CE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*C*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

Maple [B] time = 2.279, size = 228, normalized size = 3.7

$$-\frac{2}{3\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4+B\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(5/2), x)
```

3.1176 $\int \cos^{\frac{3}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=35

$$\frac{2C\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0613801, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4064, 2748, 2641, 2639}

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/d
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx &= \int \frac{C+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= B\int \sqrt{\cos(c+dx)}\,dx+C\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0637345, size = 35, normalized size = 1.

$$\frac{2\text{CEllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/d
```

Maple [A] time = 1.767, size = 152, normalized size = 4.3

$$2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\left(B\text{EllipticE}\left(\cos\right.\right.}{\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c))*sqrt
(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*cos(d*x + c)^(3/2), x)
```

3.1177 $\int \sqrt{\cos(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx$

Optimal. Leaf size=57

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d + (2
*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0695075, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4064, 2748, 2636, 2639, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d + (2
*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\left(B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{C+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$
$$= B\int \frac{1}{\sqrt{\cos(c+dx)}}dx + C\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - C\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.139659, size = 51, normalized size = 0.89

$$\frac{2\left(B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-CE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{C\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(-(C*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (C*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 2.441, size = 148, normalized size = 2.6

$$-2\,\frac{B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+C\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2*(B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)*C)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)\right)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B+C\sec\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sqrt(cos(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

3.1178 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=83

$$\frac{2C\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 0.0789359, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4064, 2748, 2636, 2641, 2639}

$$-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*C*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{C+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + C\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - B\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}C\int \frac{1}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.401656, size = 65, normalized size = 0.78

$$\frac{2C\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{2\sin(c+dx)(3B\cos(c+dx)+C)}{\cos^{\frac{3}{2}}(c+dx)}-6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2] + 2*C*EllipticF[(c + d*x)/2, 2] + (2*(C + 3
*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] time = 4.829, size = 397, normalized size = 4.8

$$\frac{2}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(6\,B\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\text{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(6*B*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4+2*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/sqrt(cos(d*x + c)), x)
```

3.1179 $\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6C\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*C*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0936062, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4064, 2748, 2636, 2639, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6C\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*C*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*C*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] && !IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{C+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + C\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3C)\int \frac{}{\cos}$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6C\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} -$$
$$= -\frac{6CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+d}{3d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A] time = 0.301493, size = 95, normalized size = 0.86

$$\frac{10B\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10B\sin(c+dx)+9C\sin(2(c+dx))+6C\tan(c+dx)-18C\cos^{\frac{3}{2}}(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-18*C*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 9*C*Sin[2*(c + d*x)] + 6
*C*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 5.675, size = 502, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1
2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
```

```
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C\sec(c+dx))\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((B + C*sec(c + d*x))*sec(c + d*x)/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c))/cos(d*x + c)^(3/2), x)
```

3.1180 $\int \cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=123

$$\frac{2(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(5A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6BE\left(\frac{1}{2}\right.}{}$$

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*A + 7*C)*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2
*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(7*d)
```

Rubi [A] time = 0.129653, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4064, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5A+7C)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6BE\left(\frac{1}{2}(c+dx)\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*A + 7*C)*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2
*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*A*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(7*d)
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)dx$$
$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}(5A+7C\right.$$
$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + B\int \cos^{\frac{5}{2}}(c+dx)\,dx + \frac{1}{7}(5A$$
$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c}{5d}$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5A+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2(5A+}{}$$

Mathematica [A] time = 0.593183, size = 86, normalized size = 0.7

$$\frac{10(5A+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(15A\cos(2(c+dx))+65A+42B\cos(c+dx)+70C)+}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (126*B*EllipticE[(c + d*x)/2, 2] + 10*(5*A + 7*C)*EllipticF[(c + d*x)/2, 2]
 + Sqrt[Cos[c + d*x]]*(65*A + 70*C + 42*B*Cos[c + d*x] + 15*A*Cos[2*(c + d*
x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 2.33, size = 342, normalized size = 2.8

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,A\left(\sin\left(1/2\,dx+c/2\right)\right)^8\cos\left(1/2\,dx+c/2\right)+\left(-360\,A-168\,B\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-360*A-168*B)*sin(1/2*d*x+1/2*c)^6*cos(
```

```
1/2*d*x+1/2*c)+(280*A+168*B+140*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-80*A-42*B-70*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(7/2), x)
```

3.1181 $\int \cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=93

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.116145, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4064, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*A*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] && !IntegerQ[m]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)dx$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}(3A+5$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+B\int \cos^{\frac{3}{2}}(c+dx)\,dx+\frac{1}{5}(3A$$

$$=\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+$$

$$=\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{\cos}}{}$$

Mathematica [A] time = 0.287705, size = 72, normalized size = 0.77

$$\frac{2\left(5B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3A\cos(c+dx)+5B)+3(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(3*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2]
 + Sqrt[Cos[c + d*x]]*(5*B + 3*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 2.305, size = 308, normalized size = 3.3

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,A\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+\left(24\,A+20\,B\right)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(24*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
```

```
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

3.1182 $\int \cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=65

$$\frac{2(A+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(A + 3*C)*EllipticF[(c + d*x)/2, 2])
/(3*d) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.103866, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4064, 3023, 2748, 2641, 2639}

$$\frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(A + 3*C)*EllipticF[(c + d*x)/2, 2])
/(3*d) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] && !IntegerQ[m]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(A+3C)+\frac{3}{2}B\cos(}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + B\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}($$

$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(A+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sqrt{c}}{}$$

Mathematica [C] time = 6.20482, size = 682, normalized size = 10.49

$$-\frac{2B\csc(c)\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{}}\right.}{d(A\cos(2c+2dx)+A+2B\cos(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*B*Cot[c])/
d + (4*A*Cos[d*x]*Sin[c])/(3*d) + (4*A*Cos[c]*Sin[d*x])/(3*d)))/(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c + d*x]^2*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^2*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^2*Csc[c]*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Arc
Tan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x]))
```

Maple [B] time = 2.191, size = 274, normalized size = 4.2

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4A\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)*A-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

3.1183 $\int \sqrt{\cos(c+dx)} \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx$

Optimal. Leaf size=61

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d
 + (2*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.106751, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4064, 3021, 2748, 2641, 2639}

$$\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d
 + (2*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$
$$= \frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{B}{2}+\frac{1}{2}(A-C)\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx$$
$$= \frac{2C\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + B\int \frac{1}{\sqrt{\cos(c+dx)}}dx + (A-C)\int \sqrt{\text{co}$$
$$= \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2C\sin}{d\sqrt{\cos}}$$

Mathematica [C] time = 6.25351, size = 759, normalized size = 12.44

$$-\frac{2A\csc(c)\cos^2(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{}}\right.}{d(A\cos(2c+2dx)+A+2B\cos(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(A - 2*C +
 A*Cos[2*c])*Csc[c]*Sec[c])/d + (4*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (4*B*Cos[c + d*x]^2*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*A*Cos[c + d*x]^2*Csc[c]*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Co
s[2*c + 2*d*x])) + (2*C*Cos[c + d*x]^2*Csc[c]*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]
]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 2.326, size = 195, normalized size = 3.2

$$2\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-B\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

3.1184 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A + C)*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (2*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.120972, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4064, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A + C)*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (2*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] && !IntegerQ[m]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3B}{2}+\frac{1}{2}(3A+C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(3A+C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d$$
$$= \frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - B\int \sqrt{}$$
$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2C\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} +$$

Mathematica [A] time = 0.585318, size = 69, normalized size = 0.79

$$\frac{2(3A+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{2\sin(c+dx)(3B\cos(c+dx)+C)}{\cos^{\frac{3}{2}}(c+dx)}-6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*EllipticF[(c + d*x)/2, 2] + (
2*(C + 3*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] time = 5.282, size = 500, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(6*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

3.1185 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{2B\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos
[c + d*x]])
```

Rubi [A] time = 0.134258, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4064, 3021, 2748, 2636, 2641, 2639}

$$-\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*C*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*A + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos
[c + d*x]])
```


Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\frac{5B}{2}+\frac{1}{2}(5A+3C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{5}(5A+3C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,d$$
$$=\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5A+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{1}{3}B\int -$$
$$=-\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2C\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+$$

Mathematica [A] time = 0.516547, size = 112, normalized size = 0.91

$$\frac{10B\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-6(5A+3C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+15A\sin(2(c+dx))+10B\sin(}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*(5*A + 3*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c +
 d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 15*A*Sin[2*(c +
 d*x)] + 9*C*Sin[2*(c + d*x)] + 6*C*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 6.925, size = 799, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^3*(60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*A*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^4+36*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*C*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+120*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-36*C
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*C*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)*A+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

3.1186 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{2(7A+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6B\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A + 5*C)*EllipticF[(c + d*x)
/2, 2])/(21*d) + (2*C*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*B*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Cos[c
+ d*x]^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.152271, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4064, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6B\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2C}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A + 5*C)*EllipticF[(c + d*x)
/2, 2])/(21*d) + (2*C*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*B*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*d*Cos[c
+ d*x]^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4064

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(m_)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(
x_)] + (C_.)*sec[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[b^2, Int[(b*Cos[
e + f*x])^(m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; FreeQ[
{b, e, f, A, B, C, m}, x] &&  !IntegerQ[m]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{7B}{2}+\frac{1}{2}(7A+5C)\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + B\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + \frac{1}{7}(7A+5C)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,d$$
$$= \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(7A+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3B)\int$$
$$= \frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(7}{2}$$
$$= -\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2C\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} +$$

Mathematica [A] time = 0.560619, size = 129, normalized size = 0.88

$$\frac{10(7A+5C)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+35A\sin(2(c+dx))+42B\sin(c+dx)+126B\sin(c+dx)\cos^2(c}{105d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-126*B*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Cos[c
 + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 42*B*Sin[c + d*x] + 126*B*Cos[c +
 d*x]^2*Sin[c + d*x] + 35*A*Sin[2*(c + d*x)] + 25*C*Sin[2*(c + d*x)] + 30*C
*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 7.17, size = 684, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

3.1187 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=175

$$\frac{2a(5(A+B)+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(7A+9(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(7A+9(B+C))\sin(c+dx)\cos^{\frac{3}{2}}(}{45d}$$

```
[Out] (2*a*(7*A + 9*(B + C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(5*(A + B)
+ 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*(A + B) + 7*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(7*A + 9*(B + C))*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(45*d) + (2*a*(A + B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
+ (2*a*A*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.307665, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(5(A+B)+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(7A+9(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(7A+9(B+C))\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*a*(7*A + 9*(B + C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(5*(A + B)
+ 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*(A + B) + 7*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(7*A + 9*(B + C))*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(45*d) + (2*a*(A + B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
+ (2*a*A*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\left(C+\right.$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \text{co}$$

$$= \frac{2a(A+B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{}$$

$$= \frac{2a(A+B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{}$$

$$= \frac{2a(5(A+B)+7C)\sqrt{\cos(c+dx)}\sin(c+}{21d}$$

$$= \frac{2a(7A+9(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2}{}$$

Mathematica [C] time = 6.36243, size = 1292, normalized size = 7.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((7*A + 9*B
 + 9*C)*Cot[c])/(15*d) + ((23*A + 23*B + 28*C)*Cos[d*x]*Sin[c])/(84*d) + ((
```

```
19*A + 18*B + 18*C)*Cos[2*d*x]*Sin[2*c])/(180*d) + ((A + B)*Cos[3*d*x]*Sin[
3*c])/(28*d) + (A*Cos[4*d*x]*Sin[4*c])/(72*d) + ((23*A + 23*B + 28*C)*Cos[c
]*Sin[d*x])/(84*d) + ((19*A + 18*B + 18*C)*Cos[2*c]*Sin[2*d*x])/(180*d) + (
(A + B)*Cos[3*c]*Sin[3*d*x])/(28*d) + (A*Cos[4*c]*Sin[4*d*x])/(72*d)) - (5*
A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2])
 - (5*B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[
c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + C
ot[c]^2]) - (7*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*S
qrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
 + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Si
n[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d)
 - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (3*C*(
1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1
/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sq
rt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
 - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 2.536, size = 512, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2960*A+720*B)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-3152*A-1584*B-504*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(1792*A+1344*B+924*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-408
*A-366*B-336*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+\left(B+C\right)a\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+\left(A+B\right)a\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^4*sec(d*x + c)^3 + (B + C)*a*cos(d*x + c)^4*sec(
d*x + c)^2 + (A + B)*a*cos(d*x + c)^4*sec(d*x + c) + A*a*cos(d*x + c)^4)*sq
rt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(
d*x + c)^(9/2), x)
```

3.1188 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=142

$$\frac{2a(5A+7(B+C))\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a(3(A+B)+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7(B+C))\sin(c+dx)\sqrt{\cos(}}{21d}$$

```
[Out] (2*a*(3*(A + B) + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*(B
+ C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*(B + C))*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.27949, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(5A+7(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(3(A+B)+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7(B+C))\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*a*(3*(A + B) + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*(B
+ C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*(B + C))*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\left(C+\right.$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{}$$

$$= \frac{2a(A+B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{}$$

$$= \frac{2a(A+B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{}$$

$$= \frac{2a(3(A+B)+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2}{}$$

$$= \frac{2a(3(A+B)+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2}{}$$

Mathematica [C] time = 6.31574, size = 1240, normalized size = 8.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((3*A + 3*B
 + 5*C)*Cot[c])/(5*d) + ((23*A + 28*B + 28*C)*Cos[d*x]*Sin[c])/(84*d) + ((A
 + B)*Cos[2*d*x]*Sin[2*c])/(10*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) + ((23*A
 + 28*B + 28*C)*Cos[c]*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10
```

```
*d) + (A*Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[
c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]
]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin
[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])
*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*S
ec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d
*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.479, size = 481, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-528*A-168*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(448*A+308*B+140*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-122*A-112*B-70*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+35*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos(dx+c)^3\sec(dx+c)^3 + (B+C)a\cos(dx+c)^3\sec(dx+c)^2 + (A+B)a\cos(dx+c)^3\sec(dx+c) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^3*sec(d*x + c)^3 + (B + C)*a*cos(d*x + c)^3*sec(
d*x + c)^2 + (A + B)*a*cos(d*x + c)^3*sec(d*x + c) + A*a*cos(d*x + c)^3)*sq
rt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c) + a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(
d*x + c)^(7/2), x)
```

3.1189 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=106

$$\frac{2a(A+B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(3A+5(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+$$

```
[Out] (2*a*(3*A + 5*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B + 3*C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.244299, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4112, 3033, 3023, 2748, 2641, 2639}

$$\frac{2a(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*a*(3*A + 5*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B + 3*C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))\left(C+B\cos(c+d\right.}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{5a}{-}$$
$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{-}$$
$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{-}$$
$$= \frac{2a(3A+5(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2}{-}$$

Mathematica [C] time = 6.37835, size = 1186, normalized size = 11.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((3*A + 5*B
 + 5*C)*Cot[c])/(5*d) + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[2*d*x]*Sin
[2*c])/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (A*Cos[2*c]*Sin[2*d*x])/(
10*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + C
ot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1
+ Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1
```

```
 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(1
0*d) - (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (C*(1
 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.425, size = 447, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(44*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-16*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3+(B+C)a\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+(A+B)a\cos\left(dx+c\right)^2\sec\left(dx+c\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)^2*sec(d*x + c)^3 + (B + C)*a*cos(d*x + c)^2*sec(
d*x + c)^2 + (A + B)*a*cos(d*x + c)^2*sec(d*x + c) + A*a*cos(d*x + c)^2)*sq
rt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(
d*x + c)^(5/2), x)
```

3.1190 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=98

$$\frac{2a(A+3(B+C))\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aC\sin(c}{d\sqrt{\cos(c}}$$

```
[Out] (2*a*(A + B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*(B + C))*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) +
(2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.239549, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4112, 3031, 3023, 2748, 2641, 2639}

$$\frac{2a(A+3(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```

```
[Out] (2*a*(A + B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*(B + C))*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) +
(2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))\left(C+B\cos(c+d\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{-\frac{1}{2}a(B+C)-\frac{1}{2}}{}$$

$$= \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sin}{3d}$$

$$= \frac{2aC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sin}{3d}$$

$$= \frac{2a(A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A}{}$$

Mathematica [C] time = 6.44697, size = 1173, normalized size = 11.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((A + B - 2
*C + A*Cos[2*c] + B*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (A*Cos[d*x]*Sin[c])/(3
*d) + (A*Cos[c]*Sin[d*x])/(3*d) + (C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d) - (A*
(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) -
(B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) -
 (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
```

```
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(d*Sqrt[1 + Cot[c]^2])
- (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (B*(1 + Co
s[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (C*(1 + Cos[c + d*x])*
Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*
x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 2.57, size = 380, normalized size = 3.9

$$-\frac{2a}{3d}\left(4A\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+A\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*a*(4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*
A+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)*C)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+\left(B+C\right)a\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+\left(A+B\right)a\cos\left(dx+c\right)\sec\left(dx+c\right)+A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*cos(d*x + c)*sec(d*x + c)^3 + (B + C)*a*cos(d*x + c)*sec(d*x
+ c)^2 + (A + B)*a*cos(d*x + c)*sec(d*x + c) + A*a*cos(d*x + c))*sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

3.1191 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=103

$$\frac{2a(3A+3B+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a(A-B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*a*(A - B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + 3*B + C)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
+ (2*a*(B + C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.248645, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4112, 3031, 3021, 2748, 2641, 2639}

$$\frac{2a(3A+3B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A-B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*a*(A - B - C)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + 3*B + C)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
+ (2*a*(B + C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))\left(C+B\cos(c+d\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}a(B+C)-}{}$$

$$= \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+d}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2aC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+d}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a(A-B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(3.}{}$$

Mathematica [C] time = 6.50473, size = 1180, normalized size = 11.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-((A - 2*B -
 2*C + A*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x]
)/(3*d) + (Sec[c]*Sec[c + d*x]*(C*Sin[c] + 3*B*Sin[d*x] + 3*C*Sin[d*x]))/(3
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c
]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
```

```
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[
c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + C
ot[c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqr
t[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
 (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[
c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) +
(B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (C*(1 + Cos[
c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
- Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] time = 5.812, size = 515, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*C*(-1/6*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))+(1/2*B+1/2*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right)(a \sec(dx+c) + a)\sqrt{\cos(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca\sec\left(dx+c\right)^{3}+(B+C)a\sec\left(dx+c\right)^{2}+(A+B)a\sec\left(dx+c\right)+Aa\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

3.1192 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2a(3A+B+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2a(5A+5B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+5B+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a(B+}{3d\,}$$

```
[Out] (-2*a*(5*A + 5*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + B +
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)) + (2*a*(B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A
+ 5*B + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.266809, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+5B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+5B+3C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a(B+C)\sin(}{3d\cos^{\frac{3}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```


```
[Out] (-2*a*(5*A + 5*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(3*A + B +
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*C*Sin[c + d*x])/(5*d*Cos[c + d*x
]^(5/2)) + (2*a*(B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A
+ 5*B + 3*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+a\cos(c+dx))\left(C+B\cos(c+dx)+A\cos^2(c\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(B+C)-\frac{1}{2}a(5A+5B+}{\text{co}}$$

$$= \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int$$

$$= \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a(3A$$

$$= \frac{2a(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2a(5A+5B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(3A+B+}{}$$

Mathematica [C] time = 6.57695, size = 1228, normalized size = 8.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 5*B
+ 3*C)*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (S
ec[c]*Sec[c + d*x]^2*(3*C*Sin[c] + 5*B*Sin[d*x] + 5*C*Sin[d*x]))/(15*d) + (
Sec[c]*Sec[c + d*x]*(5*B*Sin[c] + 5*C*Sin[c] + 15*A*Sin[d*x] + 15*B*Sin[d*x
] + 9*C*Sin[d*x]))/(15*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (C*(1 + Cos[c + d*x])*Csc[c]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 +
(d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]
]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(2*d) + (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*(
(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
 Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]))/(2*d) + (3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*S
qrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
 + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Si
n[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
)
```

Maple [B] time = 7.517, size = 739, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*B+1/2*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*C/(8*sin(1/2*d*x
```

```
+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2
*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*
c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)+(1/2*A+1/2*B)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec(dx+c)^3+(B+C)a\sec(dx+c)^2+(A+B)a\sec(dx+c)+Aa}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sqrt{\cos(c+dx)}}\,dx+\int\frac{A\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int\frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int\frac{B\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int\frac{C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/
2),x)
```

```
[Out] a*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(A*sec(c + d*x)/sqrt(cos(c +
 d*x)), x) + Integral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(B*se
c(c + d*x)**2/sqrt(cos(c + d*x)), x) + Integral(C*sec(c + d*x)**2/sqrt(cos(
```

```
c + d*x)), x) + Integral(C*sec(c + d*x)**3/sqrt(cos(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec{(dx+c)}^2 + B\sec{(dx+c)} + A\right)(a\sec{(dx+c)} + a)}{\sqrt{\cos{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

3.1193 $\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{2a(7A+7B+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+7B+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (-2*a*(5*A + 3*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 7*B
+ 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Cos[c
+ d*x]^(7/2)) + (2*a*(B + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*
(7*A + 7*B + 5*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*(
B + C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.30244, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(7A+7B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(7A+7B+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```


```
[Out] (-2*a*(5*A + 3*(B + C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 7*B
+ 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*C*Sin[c + d*x])/(7*d*Cos[c
+ d*x]^(7/2)) + (2*a*(B + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*
(7*A + 7*B + 5*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*(
B + C))*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\cos(c+dx))\left(C+B\cos(c+dx)+A\cos^2(c+\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(B+C)-\frac{1}{2}a(7A+7B+5C}{\cos^{\frac{7}{2}}}$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a(}{}$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(a(7A+7$$

$$= \frac{2aC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(B+C)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(7A+7B}{21d\,c}$$

$$= -\frac{2a(5A+3(B+C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(7A+7B+5}{}$$

Mathematica [C] time = 6.65058, size = 1284, normalized size = 7.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 3*B
+ 3*C)*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(7*d) + (S
ec[c]*Sec[c + d*x]^3*(5*C*Sin[c] + 7*B*Sin[d*x] + 7*C*Sin[d*x]))/(35*d) + (
Sec[c]*Sec[c + d*x]^2*(21*B*Sin[c] + 21*C*Sin[c] + 35*A*Sin[d*x] + 35*B*Sin
[d*x] + 25*C*Sin[d*x]))/(105*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 35*B*
Sin[c] + 25*C*Sin[c] + 105*A*Sin[d*x] + 63*B*Sin[d*x] + 63*C*Sin[d*x]))/(10
5*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Co
t[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
 Cot[c]^2]) - (5*C*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sq
rt[1 + Cot[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(2*d) + (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometri
cPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]
^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) + (
3*C*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [B] time = 8.987, size = 849, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*((1/2*A+1/2*
B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*C*(-1/56*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
```

```
-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/2*B+1/2*C)/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+1/2*A*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca\sec\left(dx+c\right)^{3}+\left(B+C\right)a\sec\left(dx+c\right)^{2}+\left(A+B\right)a\sec\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*a*sec(d*x + c)^3 + (B + C)*a*sec(d*x + c)^2 + (A + B)*a*sec(d*x
 + c) + A*a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

3.1194 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=251

$$\frac{4a^2(50A+55B+66C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^2(7A+8B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(89A+121B+99C)\sin(c+}{693d}$$

```
[Out] (4*a^2*(7*A + 8*B + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(50*A +
 55*B + 66*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^2*(50*A + 55*B + 66
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^2*(7*A + 8*B + 9*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a^2*(89*A + 121*B + 99*C)*Cos[c +
 d*x]^(5/2)*Sin[c + d*x])/(693*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(11*d) + (2*(4*A + 11*B)*Cos[c + d*x]^(5/2)*(a^2 + a^
2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```

Rubi [A] time = 0.596742, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(50A+55B+66C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^2(7A+8B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(89A+121B+99C)\sin(c+dx)\cos}{693d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2),x]
```

```
[Out] (4*a^2*(7*A + 8*B + 9*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(50*A +
 55*B + 66*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^2*(50*A + 55*B + 66
*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^2*(7*A + 8*B + 9*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a^2*(89*A + 121*B + 99*C)*Cos[c +
 d*x]^(5/2)*Sin[c + d*x])/(693*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(11*d) + (2*(4*A + 11*B)*Cos[c + d*x]^(5/2)*(a^2 + a^
2*Cos[c + d*x])*Sin[c + d*x])/(99*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\left(C+\right.$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(}{11d}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(}{11d}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(}{11d}$$

$$= \frac{2a^2(89A+121B+99C)\cos^{\frac{5}{2}}(c+dx)\sin}{693d}$$

$$= \frac{2a^2(89A+121B+99C)\cos^{\frac{5}{2}}(c+dx)\sin}{693d}$$

$$= \frac{4a^2(50A+55B+66C)\sqrt{\cos(c+dx)}\sin(}{231d}$$

$$= \frac{4a^2(7A+8B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a}{}$$

Mathematica [C] time = 6.45479, size = 1364, normalized size = 5.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2),x]
```

```
[Out] a^2*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-((7*A +
 8*B + 9*C)*Cot[c])/(15*d) + ((941*A + 1012*B + 1122*C)*Cos[d*x]*Sin[c])/(3
696*d) + ((38*A + 37*B + 36*C)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((101*A + 88*
B + 44*C)*Cos[3*d*x]*Sin[3*c])/(2464*d) + ((2*A + B)*Cos[4*d*x]*Sin[4*c])/(
144*d) + (A*Cos[5*d*x]*Sin[5*c])/(352*d) + ((941*A + 1012*B + 1122*C)*Cos[c
]*Sin[d*x])/(3696*d) + ((38*A + 37*B + 36*C)*Cos[2*c]*Sin[2*d*x])/(360*d) +
 ((101*A + 88*B + 44*C)*Cos[3*c]*Sin[3*d*x])/(2464*d) + ((2*A + B)*Cos[4*c]
*Sin[4*d*x])/(144*d) + (A*Cos[5*c]*Sin[5*d*x])/(352*d)) - (50*A*(1 + Cos[c
+ d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1
 + Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) -
 (2*C*(1 + Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c
]^2]) - (7*A*(1 + Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqr
t[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
```

```
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[
c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d) -
 (4*B*(1 + Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d) - (3*C*(
1 + Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [A] time = 2.334, size = 545, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-37520*A-6160*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(57040*A+20240*B+3960*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-46192*A-26048*B-11484*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(22022*A+17248*B+12474*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-4563*A-4257*B-3861*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+750*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+825*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-1848*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+990*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^5\sec(dx+c)^4+(B+2C)a^2\cos(dx+c)^5\sec(dx+c)^3+(A+2B+C)a^2\cos(dx+c)^5\sec(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^5*sec(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^
5*sec(d*x + c)^3 + (A + 2*B + C)*a^2*cos(d*x + c)^5*sec(d*x + c)^2 + (2*A +
 B)*a^2*cos(d*x + c)^5*sec(d*x + c) + A*a^2*cos(d*x + c)^5)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*co
s(d*x + c)^(11/2), x)
```

3.1195 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2 \left(A+B\sec(c+dx)+C s\right.$

Optimal. Leaf size=215

$$\frac{4a^2(5A+6B+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(19A+27B+21C)\sin(c+}{105d}$$

```
[Out] (4*a^2*(8*A + 9*B + 12*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A +
 6*B + 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 6*B + 7*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a^2*(19*A + 27*B + 21*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(9*d) + (2*(4*A + 9*B)*Cos[c + d*x]^(3/2)*(a^2 + a^2*
Cos[c + d*x])*Sin[c + d*x])/(63*d)
```

Rubi [A] time = 0.553975, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(5A+6B+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(19A+27B+21C)\sin(c+dx)\cos}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (4*a^2*(8*A + 9*B + 12*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A +
 6*B + 7*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 6*B + 7*C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a^2*(19*A + 27*B + 21*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c +
d*x])^2*Sin[c + d*x])/(9*d) + (2*(4*A + 9*B)*Cos[c + d*x]^(3/2)*(a^2 + a^2*
Cos[c + d*x])*Sin[c + d*x])/(63*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2 \left(C\right.$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2 \mathrm{si}}{9d}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2 \mathrm{si}}{9d}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2 \mathrm{si}}{9d}$$

$$= \frac{2a^2(19A+27B+21C)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{105d}$$

$$= \frac{2a^2(19A+27B+21C)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{105d}$$

$$= \frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

$$= \frac{4a^2(8A+9B+12C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

Mathematica [C] time = 6.41589, size = 1699, normalized size = 7.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((-2*(8*A + 9*B + 12*C)*Cot[c])/(15*d) + ((46*
A + 51*B + 56*C)*Cos[d*x]*Sin[c])/(84*d) + ((37*A + 36*B + 18*C)*Cos[2*d*x]
*Sin[2*c])/(180*d) + ((2*A + B)*Cos[3*d*x]*Sin[3*c])/(28*d) + (A*Cos[4*d*x]
*Sin[4*c])/(72*d) + ((46*A + 51*B + 56*C)*Cos[c]*Sin[d*x])/(84*d) + ((37*A
+ 36*B + 18*C)*Cos[2*c]*Sin[2*d*x])/(180*d) + ((2*A + B)*Cos[3*c]*Sin[3*d*x
])/(28*d) + (A*Cos[4*c]*Sin[4*d*x])/(72*d)))/(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x]) - (10*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
+ d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*B*Cos[c +
 d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]
]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin
[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Se
c[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (8*A*Cos[
c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c
```

```
+ d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])) - (3*B*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[
c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*C*Cos[c + d*x]^4*Csc[c]*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.522, size = 514, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(1568*A+1134*B+672*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-38
7*A-351*B-273*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-168*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18
9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^4\sec(dx+c)^4 + (B+2C)a^2\cos(dx+c)^4\sec(dx+c)^3 + (A+2B+C)a^2\cos(dx+c)^4\sec\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^4*sec(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^
4*sec(d*x + c)^3 + (A + 2*B + C)*a^2*cos(d*x + c)^4*sec(d*x + c)^2 + (2*A +
 B)*a^2*cos(d*x + c)^4*sec(d*x + c) + A*a^2*cos(d*x + c)^4)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*co
s(d*x + c)^(9/2), x)
```

3.1196 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=179

$$\frac{4a^2(6A+7B+14C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^2(3A+4B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+49B+35C)\sin(c+dx}{105d}$$

```
[Out] (4*a^2*(3*A + 4*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7
*B + 14*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 49*B + 35*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^2*Sin[c + d*x])/(7*d) + (2*(4*A + 7*B)*Sqrt[Cos[c + d*x]]*(a^2
 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.546143, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(6A+7B+14C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+4B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(33A+49B+35C)\sin(c+dx)\sqrt{\cos(c}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (4*a^2*(3*A + 4*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7
*B + 14*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(33*A + 49*B + 35*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*C
os[c + d*x])^2*Sin[c + d*x])/(7*d) + (2*(4*A + 7*B)*Sqrt[Cos[c + d*x]]*(a^2
 + a^2*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^2\left(C+B\cos(c+dx\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c}{7d}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c}{7d}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\sin(c}{7d}$$

$$= \frac{2a^2(33A+49B+35C)\sqrt{\cos(c+dx)}\sin(c}{105d}$$

$$= \frac{2a^2(33A+49B+35C)\sqrt{\cos(c+dx)}\sin(c}{105d}$$

$$= \frac{4a^2(3A+4B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a}{}$$

Mathematica [C] time = 6.72706, size = 2001, normalized size = 11.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (((3*I)/10)*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x
])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*
I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-
1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x]) + (((2*I)/5)*B*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^
4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*
I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2
*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*
Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(
I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*
d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) + ((I/2)*C*Cos[c + d*x]^4*Csc[c]*Sec
[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*
```

```
x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
+ I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d
*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*S
in[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c
])))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) + (Cos[c + d*x]^(9/2
)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*((-2*(3*A + 4*B + 5*C)*Cot[c])/(5*d) + ((51*A + 56*B + 28*C)*Cos
[d*x]*Sin[c])/(84*d) + ((2*A + B)*Cos[2*d*x]*Sin[2*c])/(10*d) + (A*Cos[3*d*
x]*Sin[3*c])/(28*d) + ((51*A + 56*B + 28*C)*Cos[c]*Sin[d*x])/(84*d) + ((2*A
 + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (A*Cos[3*c]*Sin[3*d*x])/(28*d)))/(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c + d*x]^4*Csc[c]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot
[c]^2]) - (2*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^4*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
Cot[c]^2])
```

Maple [B] time = 2.266, size = 483, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-348*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(378*A+224*B+70*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-117*A-91*B-35*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-84*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+70*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\cos(dx+c)^3\sec(dx+c)^4+(B+2C)a^2\cos(dx+c)^3\sec(dx+c)^3+(A+2B+C)a^2\cos(dx+c)^3\sec(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^3*sec(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^
3*sec(d*x + c)^3 + (A + 2*B + C)*a^2*cos(d*x + c)^3*sec(d*x + c)^2 + (2*A +
 B)*a^2*cos(d*x + c)^3*sec(d*x + c) + A*a^2*cos(d*x + c)^3)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*co
s(d*x + c)^(7/2), x)
```

3.1197 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2 \left(A + B\sec(c+dx) + C s\right.$

Optimal. Leaf size=170

$$\frac{4a^2(A+2B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(7A+5B-15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+\right.}{5d}$$

```
[Out] (4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B + 3*C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B - 15*C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) + (2*(A - 5*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*
x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.529353, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(A+2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(7A+5B-15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```



```
[Out] (4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B + 3*C
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B - 15*C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) + (2*(A - 5*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*
x])*Sin[c + d*x])/(5*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^2\left(C+B\cos(c+\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + $$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + $$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + $$

$$= \frac{2a^2(7A+5B-15C)\sqrt{\cos(c+dx)}\sin(}{15d}$$

$$= \frac{2a^2(7A+5B-15C)\sqrt{\cos(c+dx)}\sin(}{15d}$$

$$= \frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(}{}$$

Mathematica [C] time = 6.63849, size = 1356, normalized size = 7.98

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*(-((8*A + 10*B - 5*C + 8*A*Cos[2*c] + 10*B*Cos
[2*c] + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(10*d) + ((2*A + B)*Cos[d*x]*Sin[c])/(
3*d) + (A*Cos[2*d*x]*Sin[2*c])/(10*d) + ((2*A + B)*Cos[c]*Sin[d*x])/(3*d) +
 (C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/(10*d)))/(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (2*A*Cos[c + d*x]^4*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
ot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + C
ot[c]^2]) - (4*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c + d*x]^4*Csc[
c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
Cot[c]^2]) - (4*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
 d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2,
```

```
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])) - (B*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.99, size = 595, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/15*a^2*(-12*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(16*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+5*B+15*C)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^2\cos(dx+c)^2\sec(dx+c)^4+(B+2C)a^2\cos(dx+c)^2\sec(dx+c)^3+(A+2B+C)a^2\cos(dx+c)^2\sec\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)^2*sec(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)^
2*sec(d*x + c)^3 + (A + 2*B + C)*a^2*cos(d*x + c)^2*sec(d*x + c)^2 + (2*A +
 B)*a^2*cos(d*x + c)^2*sec(d*x + c) + A*a^2*cos(d*x + c)^2)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*co
s(d*x + c)^(5/2), x)
```

3.1198 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=170

$$\frac{4a^2(2A+3B+2C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(A-3B-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B + 2*C)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(A - 3*B - 5*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(3*d) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c +
d*x]^(3/2)) + (2*(3*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sq
rt[Cos[c + d*x]])
```

Rubi [A] time = 0.537506, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(2A+3B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(A-3B-5C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (4*a^2*(A - C)*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B + 2*C)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(A - 3*B - 5*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(3*d) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c +
d*x]^(3/2)) + (2*(3*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sq
rt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^2\left(C+B\cos(c+dx\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\int}{}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3I}{}$$

$$=\frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3I}{}$$

$$=\frac{2a^2(A-3B-5C)\sqrt{\cos(c+dx)}\sin(c+dx}{3d}$$

$$=\frac{2a^2(A-3B-5C)\sqrt{\cos(c+dx)}\sin(c+dx}{3d}$$

$$=\frac{4a^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(2A+3}{}$$

Mathematica [C] time = 6.84113, size = 1583, normalized size = 9.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((I/2)*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1
[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*
x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4,
1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)
*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos
[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x]) - ((I/2)*C*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*
Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*H
ypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*
(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^
((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x]) + (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-((2*A - B -
```

```
4*C + 2*A*Cos[2*c] + B*Cos[2*c])*Csc[c]*Sec[c])/(2*d) + (A*Cos[d*x]*Sin[c])
/(3*d) + (A*Cos[c]*Sin[d*x])/(3*d) + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*
d) + (Sec[c]*Sec[c + d*x]*(C*Sin[c] + 3*B*Sin[d*x] + 6*C*Sin[d*x]))/(3*d)))
/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c + d*x]^4*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
)^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^4*C
sc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec
[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2])
```

Maple [B] time = 6.76, size = 800, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*A*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-4*A*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2
-6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*C*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-3*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+3*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^2\cos\left(dx+c\right)\sec\left(dx+c\right)^4+(B+2\,C)a^2\cos\left(dx+c\right)\sec\left(dx+c\right)^3+(A+2\,B+C)a^2\cos\left(dx+c\right)\sec\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*cos(d*x + c)*sec(d*x + c)^4 + (B + 2*C)*a^2*cos(d*x + c)*se
c(d*x + c)^3 + (A + 2*B + C)*a^2*cos(d*x + c)*sec(d*x + c)^2 + (2*A + B)*a^
2*cos(d*x + c)*sec(d*x + c) + A*a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*co
s(d*x + c)^(3/2), x)
```

3.1199 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2 \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=174

$$\frac{4a^2(3A+2B+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2(15A+25B+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{4a^2(5B+4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2}{}$$

```
[Out] (-4*a^2*(5*B + 4*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + 2*B +
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(15*A + 25*B + 17*C)*Sin[c + d
*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/
(5*d*Cos[c + d*x]^(5/2)) + (2*(5*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c +
d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.541798, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(3A+2B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(15A+25B+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}}-\frac{4a^2(5B+4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5B+4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (-4*a^2*(5*B + 4*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(3*A + 2*B +
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(15*A + 25*B + 17*C)*Sin[c + d
*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/
(5*d*Cos[c + d*x]^(5/2)) + (2*(5*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c +
d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \int \frac{(a+a\cos(c+dx))^2\left(C+B\cos(c+\right.}{\cos^{\frac{7}{2}}(c+dx}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2a^2(15A+25B+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} +$$

$$= \frac{2a^2(15A+25B+17C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} +$$

$$= -\frac{4a^2(5B+4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2}{}$$

Mathematica [C] time = 6.94642, size = 1599, normalized size = 9.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((-I/2)*B*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F
1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d
*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4,
 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*
x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I
)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Co
s[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x]) - (((2*I)/5)*C*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x]) + (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]
```

```
^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(-((-5*A
- 20*B - 16*C + 5*A*Cos[2*c])*Csc[c]*Sec[c])/(10*d) + (C*Sec[c]*Sec[c + d*x
]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]^2*(3*C*Sin[c] + 5*B*Sin[d*x] + 1
0*C*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*B*Sin[c] + 10*C*Sin[c] + 15
*A*Sin[d*x] + 30*B*Sin[d*x] + 24*C*Sin[d*x]))/(15*d)))/(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x]) - (2*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*B*C
os[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*C*Cos[c + d*x]^4*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 7.461, size = 906, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(1/4*B+1/2*C)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/20*C/(8*sin(1/2*d*x+1/2*c)^6-12*
sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*
d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
+(1/4*A+1/2*B+1/4*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^2\sec\left(dx+c\right)^4+(B+2C)a^2\sec\left(dx+c\right)^3+(A+2B+C)a^2\sec\left(dx+c\right)^2+(2A+B)a^2\sec\left(dx+c\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sq
rt(cos(d*x + c)), x)
```

3.1200 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=215

$$\frac{4a^2(14A+7B+6C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^2(5A+4B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+49B+33C)\sin(c+dx}{105d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4*a^2*(5*A + 4*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(14*A +
 7*B + 6*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 49*B + 33*C)
*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^2*(5*A + 4*B + 3*C)*Sin[c
+ d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d*Cos[c + d*x]^(7/2)) + (2*(7*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c
+ d*x])/(35*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.573418, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^2(14A+7B+6C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(5A+4B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] (-4*a^2*(5*A + 4*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(14*A +
 7*B + 6*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(35*A + 49*B + 33*C)
*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^2*(5*A + 4*B + 3*C)*Sin[c
+ d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d*Cos[c + d*x]^(7/2)) + (2*(7*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c
+ d*x])/(35*d*Cos[c + d*x]^(5/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \frac{(a+a\cos(c+dx))^2\left(C+B\cos(c+dx)+A\cos^2(c\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^2}{}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(7B+4C)\left(a^2+\right.}{35}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(7B+4C)\left(a^2+\right.}{35}$$

$$= \frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{7d\cos}$$

$$= \frac{2a^2(35A+49B+33C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos(c}{7d\cos}$$

$$= \frac{4a^2(14A+7B+6C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^2(35A+49}{105d}$$

$$= -\frac{4a^2(5A+4B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(14A+7B}{}$$

Mathematica [C] time = 7.04979, size = 2041, normalized size = 9.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((-I/2)*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F
1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d
*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4,
 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*
x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I
)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Co
s[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x]) - (((2*I)/5)*B*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
```

```
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (((3*I)/10)*C*Cos[c + d*x]^4*Csc[c]*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) + (Cos[c + d*x]^(9/
2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((2*(5*A + 4*B + 3*C)*Csc[c]*Sec[c])/(5*d) + (C*Sec[c]*Sec[c +
d*x]^4*Sin[d*x])/(7*d) + (Sec[c]*Sec[c + d*x]^3*(5*C*Sin[c] + 7*B*Sin[d*x]
+ 14*C*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]^2*(21*B*Sin[c] + 42*C*Sin[c
] + 35*A*Sin[d*x] + 70*B*Sin[d*x] + 60*C*Sin[d*x]))/(105*d) + (Sec[c]*Sec[c
 + d*x]*(35*A*Sin[c] + 70*B*Sin[c] + 60*C*Sin[c] + 210*A*Sin[d*x] + 168*B*S
in[d*x] + 126*C*Sin[d*x]))/(105*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x]) - (4*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^4*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
)^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 8.954, size = 932, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+(1/4*A+1/2*B+1/4*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/4*B+1/
2*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
```

```
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/4*C*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/4
2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/4*B)*(-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4+(B+2C)a^2\sec(dx+c)^3+(A+2B+C)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+Aa^2}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^2}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sq
rt(cos(d*x + c)), x)
```

3.1201 $\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=251

$$\frac{4a^2(7A+6B+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(7A+6B+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (-4*a^2*(12*A + 9*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A
+ 6*B + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(21*A + 27*B + 19*C
)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)) + (4*a^2*(7*A + 6*B + 5*C)*Sin[c
 + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (4*a^2*(12*A + 9*B + 8*C)*Sin[c + d*x]
)/(15*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d*Cos[c + d*x]^(9/2)) + (2*(9*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(63*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.599084, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(7A+6B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(7A+6B+5C)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(21A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```


```
[Out] (-4*a^2*(12*A + 9*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(7*A
+ 6*B + 5*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(21*A + 27*B + 19*C
)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)) + (4*a^2*(7*A + 6*B + 5*C)*Sin[c
 + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (4*a^2*(12*A + 9*B + 8*C)*Sin[c + d*x]
)/(15*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*
d*Cos[c + d*x]^(9/2)) + (2*(9*B + 4*C)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(63*d*Cos[c + d*x]^(7/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \int \frac{(a+a\cos(c+dx))^2\left(C+B\cos(c+dx)+A\cos^2(c\right.}{\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+dx))^2}{}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(9B+4C)\left(a^2+\right.}{63}$$

$$= \frac{2C(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(9B+4C)\left(a^2+\right.}{63}$$

$$= \frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+}{9d\cos}$$

$$= \frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C(a+a\cos(c+}{9d\cos}$$

$$= \frac{2a^2(21A+27B+19C)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+6B+}{21d\cos^{\frac{3}{2}}}$$

$$= -\frac{4a^2(12A+9B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(7A+6B}{}$$

Mathematica [C] time = 6.98737, size = 1741, normalized size = 6.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((2*(12*A + 9*B + 8*C)*Csc[c]*Sec[c])/(15*d) +
 (C*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(9*d) + (Sec[c]*Sec[c + d*x]^4*(7*C*Sin
[c] + 9*B*Sin[d*x] + 18*C*Sin[d*x]))/(63*d) + (2*Sec[c]*Sec[c + d*x]*(35*A*
Sin[c] + 30*B*Sin[c] + 25*C*Sin[c] + 84*A*Sin[d*x] + 63*B*Sin[d*x] + 56*C*S
in[d*x]))/(105*d) + (Sec[c]*Sec[c + d*x]^3*(45*B*Sin[c] + 90*C*Sin[c] + 63*
A*Sin[d*x] + 126*B*Sin[d*x] + 112*C*Sin[d*x]))/(315*d) + (Sec[c]*Sec[c + d*
x]^2*(63*A*Sin[c] + 126*B*Sin[c] + 112*C*Sin[c] + 210*A*Sin[d*x] + 180*B*Si
n[d*x] + 150*C*Sin[d*x]))/(315*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x]) - (2*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*B*Cos[c + d*x]^4*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
```

rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (10*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (4*A*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (3*B*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (8*C*Cos[c + d*x]^4*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))

Maple [B] time = 10.903, size = 1181, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*((1/2*A+1/
4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/4*A+1/2*B+1/4*C)/(8*si
n(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)+1/4*C*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/
```

```
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))))+(1/4*B+1/2*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/
2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^2\sec(dx+c)^4+(B+2C)a^2\sec(dx+c)^3+(A+2B+C)a^2\sec(dx+c)^2+(2A+B)a^2\sec(dx+c)+Aa^2}{\cos(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^2*sec(d*x + c)^4 + (B + 2*C)*a^2*sec(d*x + c)^3 + (A + 2*B +
C)*a^2*sec(d*x + c)^2 + (2*A + B)*a^2*sec(d*x + c) + A*a^2)/cos(d*x + c)^(3
/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**
(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{2}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/co
s(d*x + c)^(3/2), x)
```

3.1202 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=267

$$\frac{4a^3(105A+121B+143C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(210A+253B+264C}{11}$$

```
[Out] (4*a^3*(15*A + 17*B + 21*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(105
*A + 121*B + 143*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(105*A + 12
1*B + 143*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(210*A + 253
*B + 264*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1155*d) + (2*A*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d) + (2*(6*A + 11*B)*Cos[c +
d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d) + (2*(105*A +
143*B + 99*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(69
3*d)
```

Rubi [A] time = 0.731935, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(105A+121B+143C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(210A+253B+264C)\sin(c}{1155d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2),x]
```


```
[Out] (4*a^3*(15*A + 17*B + 21*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(105
*A + 121*B + 143*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(105*A + 12
1*B + 143*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(210*A + 253
*B + 264*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1155*d) + (2*A*Cos[c + d*x]^(
3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*d) + (2*(6*A + 11*B)*Cos[c +
d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(99*a*d) + (2*(105*A +
143*B + 99*C)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(69
3*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\left(C+\right.$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$=\frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\sin(}{11d}$$

$$=\frac{4a^3(210A+253B+264C)\cos^{\frac{3}{2}}(c+dx)}{1155d}$$

$$=\frac{4a^3(210A+253B+264C)\cos^{\frac{3}{2}}(c+dx)}{1155d}$$

$$=\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

$$=\frac{4a^3(15A+17B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

Mathematica [C] time = 6.46387, size = 1364, normalized size = 5.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2),x]
```

```
[Out] a^3*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((15*A
+ 17*B + 21*C)*Cot[c])/(30*d) + ((1953*A + 2134*B + 2354*C)*Cos[d*x]*Sin[c]
)/(7392*d) + ((75*A + 73*B + 54*C)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((189*A +
 132*B + 44*C)*Cos[3*d*x]*Sin[3*c])/(4928*d) + ((3*A + B)*Cos[4*d*x]*Sin[4*
c])/(288*d) + (A*Cos[5*d*x]*Sin[5*c])/(704*d) + ((1953*A + 2134*B + 2354*C)
*Cos[c]*Sin[d*x])/(7392*d) + ((75*A + 73*B + 54*C)*Cos[2*c]*Sin[2*d*x])/(72
0*d) + ((189*A + 132*B + 44*C)*Cos[3*c]*Sin[3*d*x])/(4928*d) + ((3*A + B)*C
os[4*c]*Sin[4*d*x])/(288*d) + (A*Cos[5*c]*Sin[5*d*x])/(704*d)) - (5*A*(1 +
Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(22*d*Sqrt[1 + Cot[c]^2]) - (1
1*B*(1 + Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]
^2]) - (13*C*(1 + Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1
 + Cot[c]^2]) - (A*(1 + Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
```

```
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4
*d) - (17*B*(1 + Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d) -
(7*C*(1 + Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{
-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Ta
n[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[
c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d))
```

Maple [A] time = 2.18, size = 545, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-43680*A-6160*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(77280*A+24200*B+3960*C)*sin(1/2*d*x+1/2*c)^8*co
s(1/2*d*x+1/2*c)+(-72240*A-37532*B-14256*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(39270*A+29722*B+19866*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-8820*A-8118*B-6864*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1575*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-3465*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1815*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3927*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2145*C*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4
851*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^5\sec(dx+c)^5+(B+3C)a^3\cos(dx+c)^5\sec(dx+c)^4+(A+3B+3C)a^3\cos(dx+c)^5\sec(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^5*sec(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^
5*sec(d*x + c)^4 + (A + 3*B + 3*C)*a^3*cos(d*x + c)^5*sec(d*x + c)^3 + (3*A
 + 3*B + C)*a^3*cos(d*x + c)^5*sec(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c)^
5*sec(d*x + c) + A*a^3*cos(d*x + c)^5)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*co
s(d*x + c)^(11/2), x)
```

3.1203 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=231

$$\frac{4a^3(11A+13B+21C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(17A+21B+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(32A+41B+42C)\text{si}}{105}$$

```
[Out] (4*a^3*(17*A + 21*B + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*
A + 13*B + 21*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(32*A + 41*B +
42*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a
 + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*(2*A + 3*B)*Sqrt[Cos[c + d*x]
]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d) + (2*(73*A + 99*B + 63*
C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d)
```

Rubi [A] time = 0.71653, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(11A+13B+21C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(17A+21B+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(32A+41B+42C)\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (4*a^3*(17*A + 21*B + 27*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*
A + 13*B + 21*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(32*A + 41*B +
42*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*A*Sqrt[Cos[c + d*x]]*(a
 + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*(2*A + 3*B)*Sqrt[Cos[c + d*x]
]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*a*d) + (2*(73*A + 99*B + 63*
C)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\sqrt{\cos(c+dx)}}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,\mathrm{si}}{9d}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,\mathrm{si}}{9d}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,\mathrm{si}}{9d}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,\mathrm{si}}{9d}$$

$$=\frac{4a^3(32A+41B+42C)\sqrt{\cos(c+dx)}\,\mathrm{si}}{105d}$$

$$=\frac{4a^3(32A+41B+42C)\sqrt{\cos(c+dx)}\,\mathrm{si}}{105d}$$

$$=\frac{4a^3(17A+21B+27C)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

Mathematica [C] time = 6.58259, size = 1697, normalized size = 7.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((17*A + 21*B + 27*C)*Cot[c])/(15*d) + ((97
*A + 107*B + 84*C)*Cos[d*x]*Sin[c])/(168*d) + ((73*A + 54*B + 18*C)*Cos[2*d
*x]*Sin[2*c])/(360*d) + ((3*A + B)*Cos[3*d*x]*Sin[3*c])/(56*d) + (A*Cos[4*d
*x]*Sin[4*c])/(144*d) + ((97*A + 107*B + 84*C)*Cos[c]*Sin[d*x])/(168*d) + (
(73*A + 54*B + 18*C)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((3*A + B)*Cos[3*c]*Sin
[3*d*x])/(56*d) + (A*Cos[4*c]*Sin[4*d*x])/(144*d)))/(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x]) - (11*A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a
*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (13*B
*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (C*Cos[c + d*x]^5*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a
 + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(
```

```
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (17*
A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])) - (7*B*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a +
a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricP
FQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
+ Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*
Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2
))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (9*C*Cos[c + d*x]^5*Csc[c]*S
ec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 2.302, size = 514, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2200*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-3412*A-1296*B-252*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)+(2702*A+1806*B+882*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-73
8*A-624*B-378*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-357*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+195*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
441*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+315*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^4\sec(dx+c)^5 + (B+3C)a^3\cos(dx+c)^4\sec(dx+c)^4 + (A+3B+3C)a^3\cos(dx+c)^4\text{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^4*sec(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^
4*sec(d*x + c)^4 + (A + 3*B + 3*C)*a^3*cos(d*x + c)^4*sec(d*x + c)^3 + (3*A
 + 3*B + C)*a^3*cos(d*x + c)^4*sec(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c)^
4*sec(d*x + c) + A*a^3*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*co
s(d*x + c)^(9/2), x)
```

3.1204 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=227

$$\frac{4a^3(13A+21B+35C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^3(7A+9B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B-35C)\sin(c+}{105d}$$

```
[Out] (4*a^3*(7*A + 9*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A +
21*B + 35*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A + 42*B - 35*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*(A - 7*C)*Sqrt[Cos[c + d*x]]*(a^2 +
 a^2*Cos[c + d*x])^2*Sin[c + d*x])/(7*a*d) + (2*(11*A + 7*B - 35*C)*Sqrt[Co
s[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```

Rubi [A] time = 0.704556, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(13A+21B+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(7A+9B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B-35C)\sin(c+dx)\sqrt{\cos}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (4*a^3*(7*A + 9*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A +
21*B + 35*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A + 42*B - 35*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*C*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*(A - 7*C)*Sqrt[Cos[c + d*x]]*(a^2 +
 a^2*Cos[c + d*x])^2*Sin[c + d*x])/(7*a*d) + (2*(11*A + 7*B - 35*C)*Sqrt[Co
s[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^3\left(C+B\cos(c+dx\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2\int}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2(A}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2(A}{}$$

$$=\frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2(A}{}$$

$$=\frac{4a^3(41A+42B-35C)\sqrt{\cos(c+dx)}\sin(c}{105d}$$

$$=\frac{4a^3(41A+42B-35C)\sqrt{\cos(c+dx)}\sin(c}{105d}$$

$$=\frac{4a^3(7A+9B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a}{}$$

Mathematica [C] time = 6.73962, size = 1688, normalized size = 7.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((14*A + 18*B + 5*C + 14*A*Cos[2*c] + 18*B*
Cos[2*c] + 15*C*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + ((107*A + 84*B + 28*C)*Co
s[d*x]*Sin[c])/(168*d) + ((3*A + B)*Cos[2*d*x]*Sin[2*c])/(20*d) + (A*Cos[3*
d*x]*Sin[3*c])/(56*d) + ((107*A + 84*B + 28*C)*Cos[c]*Sin[d*x])/(168*d) + (
C*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d) + ((3*A + B)*Cos[2*c]*Sin[2*d*x])/(20
*d) + (A*Cos[3*c]*Sin[3*d*x])/(56*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x]) - (13*A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x]
)^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^5*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-
(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - A
rcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]) - (5*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x]
)^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
```

```
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (7*A*Cos[c + d*x]^5
*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[
Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])) - (9*B*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^
3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) - (C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^
6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.993, size = 727, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/105*a^3*(120*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(36*A+7*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A+21*B+5*C)*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(104*A+63*B+70*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+105*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-189*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+175*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
```

```
)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\Big(\big(Ca^3\cos{(dx+c)}^3\sec{(dx+c)}^5 + (B+3\,C)a^3\cos{(dx+c)}^3\sec{(dx+c)}^4 + (A+3\,B+3\,C)a^3\cos{(dx+c)}^3\sec($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^3*sec(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^
3*sec(d*x + c)^4 + (A + 3*B + 3*C)*a^3*cos(d*x + c)^3*sec(d*x + c)^3 + (3*A
 + 3*B + C)*a^3*cos(d*x + c)^3*sec(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c)^
3*sec(d*x + c) + A*a^3*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec{(dx+c)}^2 + B\sec{(dx+c)} + A\right)(a\sec{(dx+c)}+a)^3\cos{(dx+c)}^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*co
s(d*x + c)^(7/2), x)
```

3.1205 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\,\text{s}\right.$

Optimal. Leaf size=226

$$\frac{4a^3(3A+5(B+C))\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(9A+5B-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(6A-5B-20C)\sin(c+dx)}{15d}$$

```
[Out] (4*a^3*(9*A + 5*B - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5
*(B + C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (4*a^3*(6*A - 5*B - 20*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(B + 2*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin
[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + (2*(3*A - 15*B - 35*C)*Sqrt[Cos[c + d
*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.733771, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(3A+5(B+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(9A+5B-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(6A-5B-20C)\sin(c+dx)\sqrt{\cos(c}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (4*a^3*(9*A + 5*B - 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5
*(B + C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (4*a^3*(6*A - 5*B - 20*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(B + 2*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin
[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + (2*(3*A - 15*B - 35*C)*Sqrt[Cos[c + d
*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \cdots$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \cdots$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \cdots$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \cdots$$

$$= \frac{4a^3(6A-5B-20C)\sqrt{\cos(c+dx)}\sin(\cdots}{15d}$$

$$= \frac{4a^3(6A-5B-20C)\sqrt{\cos(c+dx)}\sin(\cdots}{15d}$$

$$= \frac{4a^3(9A+5B-5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \cdots$$

Mathematica [C] time = 6.81363, size = 1672, normalized size = 7.4

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((18*A + 5*B - 25*C + 18*A*Cos[2*c] + 15*B*
Cos[2*c] + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(20*d) + ((3*A + B)*Cos[d*x]*Sin[c]
)/(6*d) + (A*Cos[2*d*x]*Sin[2*c])/(20*d) + ((3*A + B)*Cos[c]*Sin[d*x])/(6*d
) + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(C*Sin[
c] + 3*B*Sin[d*x] + 9*C*Sin[d*x]))/(6*d) + (A*Cos[2*c]*Sin[2*d*x])/(20*d)))
/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (A*Cos[c + d*x]^5*Csc[
c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
Cot[c]^2]) - (5*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (5*C*Cos[c + d*x]^5*Csc
```

```
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
an[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
 + Cot[c]^2]) - (9*A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[
c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (B*Cos[c + d*x]^5*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (C*Cos[c + d
*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x]))
```

Maple [B] time = 7.306, size = 950, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-24*A*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+96*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+30*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2-54*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
2-78*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+50*B*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*sin(1/2*d*x+1/2*c)^2-30*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-5
0*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+50*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+30*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-90*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*A
```

```
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+18*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-25*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+20*B*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)-25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))+50*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^3\cos(dx+c)^2\sec(dx+c)^5 + (B+3C)a^3\cos(dx+c)^2\sec(dx+c)^4 + (A+3B+3C)a^3\cos(dx+c)^2\text{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)^2*sec(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)^
2*sec(d*x + c)^4 + (A + 3*B + 3*C)*a^3*cos(d*x + c)^2*sec(d*x + c)^3 + (3*A
 + 3*B + C)*a^3*cos(d*x + c)^2*sec(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c)^
2*sec(d*x + c) + A*a^3*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*co
s(d*x + c)^(5/2), x)
```

3.1206 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=231

$$\frac{4a^3(5A+5B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3(5A-5B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A+20B+21C)\sin(c+dx}{15d}$$

```
[Out] (4*a^3*(5*A - 5*B - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 5
*B + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A + 20*B + 21*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c +
d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(5*B + 6*C)*(a^2 + a^2*Cos[c + d*x])^2*
Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2)) + (2*(15*A + 35*B + 33*C)*(a^3 +
a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.709355, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(5A+5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A-5B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A+20B+21C)\sin(c+dx)\sqrt{\cos(c}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (4*a^3*(5*A - 5*B - 9*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 5
*B + 3*C)*EllipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A + 20*B + 21*C)*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*C*(a + a*Cos[c + d*x])^3*Sin[c +
d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(5*B + 6*C)*(a^2 + a^2*Cos[c + d*x])^2*
Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2)) + (2*(15*A + 35*B + 33*C)*(a^3 +
a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + $$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + $$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + $$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + $$

$$= -\frac{4a^3(5A+20B+21C)\sqrt{\cos(c+dx)}\,\mathrm{s}}{15d}$$

$$= -\frac{4a^3(5A+20B+21C)\sqrt{\cos(c+dx)}\,\mathrm{s}}{15d}$$

$$= \frac{4a^3(5A-5B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + $$

Mathematica [C] time = 6.92319, size = 1673, normalized size = 7.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((5*A - 25*B - 36*C + 15*A*Cos[2*c] + 5*B*C
os[2*c])*Csc[c]*Sec[c])/(20*d) + (A*Cos[d*x]*Sin[c])/(6*d) + (A*Cos[c]*Sin[
d*x])/(6*d) + (C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d
*x]^2*(3*C*Sin[c] + 5*B*Sin[d*x] + 15*C*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c +
 d*x]*(5*B*Sin[c] + 15*C*Sin[c] + 15*A*Sin[d*x] + 45*B*Sin[d*x] + 54*C*Sin[
d*x]))/(30*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (5*A*Co
s[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*Sqrt[1 + Cot[c]^2]) - (5*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a +
 a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (C*C
```

```
os[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*Sqrt[1 + Cot[c]^2]) - (A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^
6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (B*Cos[c + d*x]^5*Csc[
c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
(9*C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 8.903, size = 1328, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(40*A*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+190*B*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-
120*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+25*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-50*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-100*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*sin(1/2*d*x+1/2*c)^2-72*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C-60*A*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+100*B*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^4+108*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+60
*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-100*B*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-108*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2-60*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*C*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2+60*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2
*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+100*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+90*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-216*C*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^6+246*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^3 \cos(dx+c) \sec(dx+c)^5 + (B+3C)a^3 \cos(dx+c) \sec(dx+c)^4 + (A+3B+3C)a^3 \cos(dx+c) \sec\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*cos(d*x + c)*sec(d*x + c)^5 + (B + 3*C)*a^3*cos(d*x + c)*se
c(d*x + c)^4 + (A + 3*B + 3*C)*a^3*cos(d*x + c)*sec(d*x + c)^3 + (3*A + 3*B
 + C)*a^3*cos(d*x + c)*sec(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c)*sec(d*x
+ c) + A*a^3*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*co
s(d*x + c)^(3/2), x)
```

3.1207 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3 \left(A + B\sec(c+dx) + C\right.$

Optimal. Leaf size=231

$$\frac{4a^3(35A+21B+13C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} - \frac{4a^3(5A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5A+9B+7C)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c}$$

```
[Out] (-4*a^3*(5*A + 9*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A +
 21*B + 13*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(140*A + 147*B + 1
06*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^
3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(7*B + 6*C)*(a^2 + a^2*Cos[c
+ d*x])^2*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(5*A + 9*B + 7*C)*
(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.727096, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(35A+21B+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} - \frac{4a^3(5A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5A+9B+7C)\sin(c+dx)\left(a^3\cos\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-4*a^3*(5*A + 9*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(35*A +
 21*B + 13*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(140*A + 147*B + 1
06*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^
3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(7*B + 6*C)*(a^2 + a^2*Cos[c
+ d*x])^2*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(5*A + 9*B + 7*C)*
(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\cos^{\frac{9}{2}}(c+dx}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + $$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + $$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + $$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + $$

$$= \frac{4a^3(140A+147B+106C)\sin(c+dx}{105d\sqrt{\cos(c+dx)}}$$

$$= \frac{4a^3(140A+147B+106C)\sin(c+dx}{105d\sqrt{\cos(c+dx)}}$$

$$= -\frac{4a^3(5A+9B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Mathematica [C] time = 6.96214, size = 1692, normalized size = 7.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((-25*A - 36*B - 28*C + 5*A*Cos[2*c])*Csc[c
]*Sec[c])/(20*d) + (C*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c]*Sec[
c + d*x]^3*(5*C*Sin[c] + 7*B*Sin[d*x] + 21*C*Sin[d*x]))/(70*d) + (Sec[c]*Se
c[c + d*x]^2*(21*B*Sin[c] + 63*C*Sin[c] + 35*A*Sin[d*x] + 105*B*Sin[d*x] +
130*C*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 105*B*Sin[c]
 + 130*C*Sin[c] + 315*A*Sin[d*x] + 378*B*Sin[d*x] + 294*C*Sin[d*x]))/(210*d
)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (5*A*Cos[c + d*x]^5
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sq
rt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
```

```
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (13*C*Cos[c + d*x]^5
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*S
qrt[1 + Cot[c]^2]) + (A*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*S
ec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (9*B*Cos[c + d*x]^5*Csc[c]*Sec[c
/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (7*C*Co
s[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x]))
```

Maple [B] time = 9.569, size = 1097, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+1/4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/8*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(1/8*A+3/8*B+
3/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*C*(-1/56*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/8*B+3/8*C)/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1
/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+(3/8*A+3/8*B+1/8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^3 \sec(dx+c)^5 + (B+3C)a^3 \sec(dx+c)^4 + (A+3B+3C)a^3 \sec(dx+c)^3 + (3A+3B+C)a^3 \sec(dx\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sq
rt(cos(d*x + c)), x)
```

3.1208 $\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=267

$$\frac{4a^3(21A+13B+11C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{4a^3(27A+21B+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(42A+41B+32C)\,\text{si}}{105d\cos^{\frac{3}{2}}(c+dx}$$

```
[Out] (-4*a^3*(27*A + 21*B + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21
*A + 13*B + 11*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B +
 32*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(27*A + 21*B + 17*
C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(3*B + 2*C)*(a^2 + a^2*Cos[c + d*
x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*x]^(7/2)) + (2*(63*A + 99*B + 73*C)*(
a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.750642, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(21A+13B+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(27A+21B+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(42A+41B+32C)\sin(c+dx}{105d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] (-4*a^3*(27*A + 21*B + 17*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(21
*A + 13*B + 11*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B +
 32*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*a^3*(27*A + 21*B + 17*
C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^3*Si
n[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(3*B + 2*C)*(a^2 + a^2*Cos[c + d*
x])^2*Sin[c + d*x])/(21*a*d*Cos[c + d*x]^(7/2)) + (2*(63*A + 99*B + 73*C)*(
a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \frac{(a+a\cos(c+dx))^3\left(C+B\cos(c+dx)+A\cos^2\right.}{\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\int \frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(3B+2C)\left(a\right.}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(3B+2C)\left(a\right.}{}$$

$$= \frac{2C(a+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(3B+2C)\left(a\right.}{}$$

$$= \frac{4a^3(42A+41B+32C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos}{9d\,c}$$

$$= \frac{4a^3(42A+41B+32C)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C(a+a\cos}{9d\,c}$$

$$= \frac{4a^3(21A+13B+11C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^3(42A}{1}$$

$$= -\frac{4a^3(27A+21B+17C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(21}{}$$

Mathematica [C] time = 7.06482, size = 1739, normalized size = 6.51

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(11/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(((27*A + 21*B + 17*C)*Csc[c]*Sec[c])/(15*d)
+ (C*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(18*d) + (Sec[c]*Sec[c + d*x]^4*(7*C*S
in[c] + 9*B*Sin[d*x] + 27*C*Sin[d*x]))/(126*d) + (Sec[c]*Sec[c + d*x]^3*(45
*B*Sin[c] + 135*C*Sin[c] + 63*A*Sin[d*x] + 189*B*Sin[d*x] + 238*C*Sin[d*x])
)/(630*d) + (Sec[c]*Sec[c + d*x]*(105*A*Sin[c] + 130*B*Sin[c] + 110*C*Sin[c
] + 378*A*Sin[d*x] + 294*B*Sin[d*x] + 238*C*Sin[d*x]))/(210*d) + (Sec[c]*Se
c[c + d*x]^2*(63*A*Sin[c] + 189*B*Sin[c] + 238*C*Sin[c] + 315*A*Sin[d*x] +
390*B*Sin[d*x] + 330*C*Sin[d*x]))/(630*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x]) - (A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*
x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
```

```
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (13*B*Cos[c + d*x]^
5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
Sqrt[1 + Cot[c]^2]) - (11*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c
+ d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (9*A*Cos[c +
 d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])) + (7*B*Cos[c + d*x]^5*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (17*C*Cos[c + d*x]^5*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(30*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 11.067, size = 1262, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+(3/8*A+3/8*B+1/8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/8*B+3/8*C
)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
-1/5*(1/8*A+3/8*B+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*
sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*C*(-1/144*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*
x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1
/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+(3/8*A+1/8*B
)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^3\sec(dx+c)^5 + (B+3C)a^3\sec(dx+c)^4 + (A+3B+3C)a^3\sec(dx+c)^3 + (3A+3B+C)a^3\sec(dx+c)^3 + (3A+3B+C)a^3\sec(dx+c)^3}{\sqrt{\cos(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^3*sec(d*x + c)^5 + (B + 3*C)*a^3*sec(d*x + c)^4 + (A + 3*B +
3*C)*a^3*sec(d*x + c)^3 + (3*A + 3*B + C)*a^3*sec(d*x + c)^2 + (3*A + B)*a^
3*sec(d*x + c) + A*a^3)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sq
rt(cos(d*x + c)), x)
```

3.1209 $\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^4 \left(A + B\sec(c + dx) + C\right.$

Optimal. Leaf size=310

$$\frac{8a^4(100A + 113B + 132C)\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{231d} + \frac{8a^4(185A + 208B + 247C)E\left(\frac{1}{2}(c + dx)\middle|2\right)}{195d} + \frac{4a^4(5255A + 601}{}$$

```
[Out] (8*a^4*(185*A + 208*B + 247*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (8*a^4*
(100*A + 113*B + 132*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^4*(100*A
+ 113*B + 132*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^4*(5255*A
+ 6019*B + 6721*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15015*d) + (2*a*(8*A +
 13*B)*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(143*d) + (2
*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(13*d) + (2*(13*
A + 17*B + 11*C)*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x]
)/(99*d) + (4*(1355*A + 1612*B + 1573*C)*Cos[c + d*x]^(3/2)*(a^4 + a^4*Cos[
c + d*x])*Sin[c + d*x])/(9009*d)
```

Rubi [A] time = 0.909167, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{8a^4(100A + 113B + 132C)F\left(\frac{1}{2}(c + dx)\middle|2\right)}{231d} + \frac{8a^4(185A + 208B + 247C)E\left(\frac{1}{2}(c + dx)\middle|2\right)}{195d} + \frac{4a^4(5255A + 6019B + 67}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2),x]
```


```
[Out] (8*a^4*(185*A + 208*B + 247*C)*EllipticE[(c + d*x)/2, 2])/(195*d) + (8*a^4*
(100*A + 113*B + 132*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (8*a^4*(100*A
+ 113*B + 132*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (4*a^4*(5255*A
+ 6019*B + 6721*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15015*d) + (2*a*(8*A +
 13*B)*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(143*d) + (2
*A*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(13*d) + (2*(13*
A + 17*B + 11*C)*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x]
)/(99*d) + (4*(1355*A + 1612*B + 1573*C)*Cos[c + d*x]^(3/2)*(a^4 + a^4*Cos[
c + d*x])*Sin[c + d*x])/(9009*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
```

```
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4 \big($$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4 \mathrm{s}}{13d}$$

$$= \frac{2a(8A+13B)\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}{143d}$$

$$= \frac{2a(8A+13B)\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}{143d}$$

$$= \frac{2a(8A+13B)\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}{143d}$$

$$= \frac{2a(8A+13B)\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}{143d}$$

$$= \frac{4a^4(5255A+6019B+6721C)\cos^{\frac{3}{2}}(c}{15015d}$$

$$= \frac{4a^4(5255A+6019B+6721C)\cos^{\frac{3}{2}}(c}{15015d}$$

$$= \frac{8a^4(185A+208B+247C)E\left(\frac{1}{2}(c+d\right.}{195d}$$

$$= \frac{8a^4(185A+208B+247C)E\left(\frac{1}{2}(c+d\right.}{195d}$$

Mathematica [C] time = 6.51246, size = 1416, normalized size = 4.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2),x]
```

```
[Out] a^4*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(-((185*A
 + 208*B + 247*C)*Cot[c])/(390*d) + ((3764*A + 4087*B + 4488*C)*Cos[d*x]*Si
n[c])/(14784*d) + ((15625*A + 15392*B + 13208*C)*Cos[2*d*x]*Sin[2*c])/(1497
60*d) + ((404*A + 321*B + 176*C)*Cos[3*d*x]*Sin[3*c])/(9856*d) + ((98*A + 5
2*B + 13*C)*Cos[4*d*x]*Sin[4*c])/(7488*d) + ((4*A + B)*Cos[5*d*x]*Sin[5*c])
/(1408*d) + (A*Cos[6*d*x]*Sin[6*c])/(3328*d) + ((3764*A + 4087*B + 4488*C)*
Cos[c]*Sin[d*x])/(14784*d) + ((15625*A + 15392*B + 13208*C)*Cos[2*c]*Sin[2*
d*x])/(149760*d) + ((404*A + 321*B + 176*C)*Cos[3*c]*Sin[3*d*x])/(9856*d) +
 ((98*A + 52*B + 13*C)*Cos[4*c]*Sin[4*d*x])/(7488*d) + ((4*A + B)*Cos[5*c]*
Sin[5*d*x])/(1408*d) + (A*Cos[6*c]*Sin[6*d*x])/(3328*d)) - (50*A*(1 + Cos[c
 + d*x])^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*d*Sqrt[1 + Cot[c]^2]) - (113*B
*(1 + Cos[c + d*x])^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(462*d*Sqrt[1 + Cot[c]^2
]) - (2*C*(1 + Cos[c + d*x])^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]
```

```
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + C
ot[c]^2]) - (37*A*(1 + Cos[c + d*x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15
6*d) - (4*B*(1 + Cos[c + d*x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d) -
(19*C*(1 + Cos[c + d*x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((HypergeometricPFQ[
{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + T
an[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos
[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d))
```

Maple [A] time = 2.398, size = 576, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -8/45045*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(-1108
80*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14+(594720*A+65520*B)*sin(1/2*d*
x+1/2*c)^12*cos(1/2*d*x+1/2*c)+(-1345120*A-323960*B-40040*C)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(1667840*A+659620*B+183040*C)*sin(1/2*d*x+1/2*c)
^8*cos(1/2*d*x+1/2*c)+(-1237490*A-713518*B-336622*C)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(572110*A+448448*B+322322*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-117945*A-110097*B-97383*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+19500*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-42735*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+220
35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-48048*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25740*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-57057*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ca^4\cos(dx+c)^6\sec(dx+c)^6+(B+4C)a^4\cos(dx+c)^6\sec(dx+c)^5+(A+4B+6C)a^4\cos(dx+c)^6\,\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*cos(d*x + c)^6*sec(d*x + c)^6 + (B + 4*C)*a^4*cos(d*x + c)^
6*sec(d*x + c)^5 + (A + 4*B + 6*C)*a^4*cos(d*x + c)^6*sec(d*x + c)^4 + 2*(2
*A + 3*B + 2*C)*a^4*cos(d*x + c)^6*sec(d*x + c)^3 + (6*A + 4*B + C)*a^4*cos
(d*x + c)^6*sec(d*x + c)^2 + (4*A + B)*a^4*cos(d*x + c)^6*sec(d*x + c) + A*
a^4*cos(d*x + c)^6)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(13/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^4\cos(dx+c)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*co
s(d*x + c)^(13/2), x)
```

3.1210 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=274

$$\frac{8a^4(113A+132B+187C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{8a^4(16A+19B+24C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^4(667A+803B+913C)}{11}$$

```
[Out] (8*a^4*(16*A + 19*B + 24*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(113
*A + 132*B + 187*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^4*(667*A + 80
3*B + 913*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1155*d) + (2*a*(8*A + 11*B)*
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(99*d) + (2*A*Sqrt[
Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(11*d) + (2*(43*A + 55*B
 + 33*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(231*d
) + (4*(769*A + 946*B + 891*C)*Sqrt[Cos[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*
Sin[c + d*x])/(3465*d)
```

Rubi [A] time = 0.882976, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3045, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^4(113A+132B+187C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{8a^4(16A+19B+24C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^4(667A+803B+913C)\sin(c}{1155d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2),x]
```


```
[Out] (8*a^4*(16*A + 19*B + 24*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(113
*A + 132*B + 187*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^4*(667*A + 80
3*B + 913*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1155*d) + (2*a*(8*A + 11*B)*
Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(99*d) + (2*A*Sqrt[
Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(11*d) + (2*(43*A + 55*B
 + 33*C)*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(231*d
) + (4*(769*A + 946*B + 891*C)*Sqrt[Cos[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*
Sin[c + d*x])/(3465*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
```

```
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+dx\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\sin(}{11d}$$

$$= \frac{2a(8A+11B)\sqrt{\cos(c+dx)}(a+a\cos(c+}{99d}$$

$$= \frac{2a(8A+11B)\sqrt{\cos(c+dx)}(a+a\cos(c+}{99d}$$

$$= \frac{2a(8A+11B)\sqrt{\cos(c+dx)}(a+a\cos(c+}{99d}$$

$$= \frac{2a(8A+11B)\sqrt{\cos(c+dx)}(a+a\cos(c+}{99d}$$

$$= \frac{4a^4(667A+803B+913C)\sqrt{\cos(c+dx)}}{1155d}$$

$$= \frac{4a^4(667A+803B+913C)\sqrt{\cos(c+dx)}}{1155d}$$

$$= \frac{8a^4(16A+19B+24C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} +$$

Mathematica [C] time = 6.71311, size = 1751, normalized size = 6.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((16*A + 19*B + 24*C)*Cot[c])/(15*d) + ((40
87*A + 4488*B + 4202*C)*Cos[d*x]*Sin[c])/(7392*d) + ((148*A + 127*B + 72*C)
*Cos[2*d*x]*Sin[2*c])/(720*d) + ((321*A + 176*B + 44*C)*Cos[3*d*x]*Sin[3*c]
)/(4928*d) + ((4*A + B)*Cos[4*d*x]*Sin[4*c])/(288*d) + (A*Cos[5*d*x]*Sin[5*
c])/(704*d) + ((4087*A + 4488*B + 4202*C)*Cos[c]*Sin[d*x])/(7392*d) + ((148
*A + 127*B + 72*C)*Cos[2*c]*Sin[2*d*x])/(720*d) + ((321*A + 176*B + 44*C)*C
os[3*c]*Sin[3*d*x])/(4928*d) + ((4*A + B)*Cos[4*c]*Sin[4*d*x])/(288*d) + (A
*Cos[5*c]*Sin[5*d*x])/(704*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x]) - (113*A*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*B*Cos[c + d*x]^6*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
```

```
an[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
 + Cot[c]^2]) - (17*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (8*A*Cos[c + d*x]^
6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]))/(15*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])) - (19*B*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(S
qrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
 ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])) - (4*C*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)
/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [A] time = 2.405, size = 545, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-24920*A-3080*B)*sin(1/2*d*x+1/2
*c)^10*cos(1/2*d*x+1/2*c)+(50740*A+14080*B+1980*C)*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-54886*A-25894*B-8514*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(34496*A+24794*B+14784*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-
8469*A-7491*B-5511*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1695*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-3696*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1980*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-4389*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2805*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-554
4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^4\cos(dx+c)^5\sec(dx+c)^6+(B+4C)a^4\cos(dx+c)^5\sec(dx+c)^5+(A+4B+6C)a^4\cos(dx+c)^5\sec(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*cos(d*x + c)^5*sec(d*x + c)^6 + (B + 4*C)*a^4*cos(d*x + c)^
5*sec(d*x + c)^5 + (A + 4*B + 6*C)*a^4*cos(d*x + c)^5*sec(d*x + c)^4 + 2*(2
*A + 3*B + 2*C)*a^4*cos(d*x + c)^5*sec(d*x + c)^3 + (6*A + 4*B + C)*a^4*cos
(d*x + c)^5*sec(d*x + c)^2 + (4*A + B)*a^4*cos(d*x + c)^5*sec(d*x + c) + A*
a^4*cos(d*x + c)^5)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^4\cos(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*co
s(d*x + c)^(11/2), x)
```

3.1211 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=270

$$\frac{8a^4(12A+17B+28C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{8a^4(19A+24B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^4(73A+83B+7C)\sin}{105}$$

```
[Out] (8*a^4*(19*A + 24*B + 21*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(12*
A + 17*B + 28*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^4*(73*A + 83*B +
7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(A - 9*C)*Sqrt[Cos[c +
 d*x]]*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*C*(a + a*Cos[c + d*x
])^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 3*B - 21*C)*Sqrt[Cos[
c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*d) + (4*(86*A + 81*B
 - 126*C)*Sqrt[Cos[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(315*d)
```

Rubi [A] time = 0.887795, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^4(12A+17B+28C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{8a^4(19A+24B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^4(73A+83B+7C)\sin(c+dx}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (8*a^4*(19*A + 24*B + 21*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(12*
A + 17*B + 28*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^4*(73*A + 83*B +
7*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(A - 9*C)*Sqrt[Cos[c +
 d*x]]*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*C*(a + a*Cos[c + d*x
])^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 3*B - 21*C)*Sqrt[Cos[
c + d*x]]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(21*d) + (4*(86*A + 81*B
 - 126*C)*Sqrt[Cos[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(315*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+$$

$$=\frac{2a(A-9C)\sqrt{\cos(c+dx)}(a+a\cos(c+}{9d}$$

$$=\frac{2a(A-9C)\sqrt{\cos(c+dx)}(a+a\cos(c+}{9d}$$

$$=\frac{2a(A-9C)\sqrt{\cos(c+dx)}(a+a\cos(c+}{9d}$$

$$=\frac{2a(A-9C)\sqrt{\cos(c+dx)}(a+a\cos(c+}{9d}$$

$$=\frac{4a^4(73A+83B+7C)\sqrt{\cos(c+dx)}\sin}{105d}$$

$$=\frac{4a^4(73A+83B+7C)\sqrt{\cos(c+dx)}\sin}{105d}$$

$$=\frac{8a^4(19A+24B+21C)E\left(\frac{1}{2}(c+dx)\middle|2\right.}{15d}$$

Mathematica [C] time = 6.85404, size = 1742, normalized size = 6.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((76*A + 96*B + 69*C + 76*A*Cos[2*c] + 96*B
*Cos[2*c] + 99*C*Cos[2*c])*Csc[c]*Sec[c])/(120*d) + ((204*A + 191*B + 112*C
)*Cos[d*x]*Sin[c])/(336*d) + ((127*A + 72*B + 18*C)*Cos[2*d*x]*Sin[2*c])/(7
20*d) + ((4*A + B)*Cos[3*d*x]*Sin[3*c])/(112*d) + (A*Cos[4*d*x]*Sin[4*c])/(
288*d) + ((204*A + 191*B + 112*C)*Cos[c]*Sin[d*x])/(336*d) + (C*Sec[c]*Sec[
c + d*x]*Sin[d*x])/(4*d) + ((127*A + 72*B + 18*C)*Cos[2*c]*Sin[2*d*x])/(720
*d) + ((4*A + B)*Cos[3*c]*Sin[3*d*x])/(112*d) + (A*Cos[4*c]*Sin[4*d*x])/(28
8*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c + d*x
]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sqrt[1 + Cot[c]^2]) - (17*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
 + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]
```

```
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c
+ d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*Sqrt[1 + Cot[c]^2]) - (19*A*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^
8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*B*Cos[c + d*x]^6*C
sc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Ta
n[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
 - (7*C*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 3.204, size = 786, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/315*a^4*(-560*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+40*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(64*A+9*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-4*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1177*A+387*B+63*C)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+28*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(161*A+96*B+39*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(227*A+167*B+133*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+360*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-798*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+510*B*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1008*B*(-
```

```
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+840*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)-882*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^4\cos(dx+c)^4\sec(dx+c)^6+(B+4C)a^4\cos(dx+c)^4\sec(dx+c)^5+(A+4B+6C)a^4\cos(dx+c)^4\,\mathrm{s}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*cos(d*x + c)^4*sec(d*x + c)^6 + (B + 4*C)*a^4*cos(d*x + c)^
4*sec(d*x + c)^5 + (A + 4*B + 6*C)*a^4*cos(d*x + c)^4*sec(d*x + c)^4 + 2*(2
*A + 3*B + 2*C)*a^4*cos(d*x + c)^4*sec(d*x + c)^3 + (6*A + 4*B + C)*a^4*cos
(d*x + c)^4*sec(d*x + c)^2 + (4*A + B)*a^4*cos(d*x + c)^4*sec(d*x + c) + A*
a^4*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^4\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*co
s(d*x + c)^(9/2), x)
```

3.1212 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=269

$$\frac{8a^4(17A+28B+35C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{4a^4(83A+7B-175C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(A-7B-21C}{}$$

```
[Out] (8*a^4*(8*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^4*(17*A + 28*B +
 35*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^4*(83*A + 7*B - 175*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(3*B + 8*C)*(a + a*Cos[c + d*x]
)^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^4*Si
n[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(A - 7*B - 21*C)*Sqrt[Cos[c + d*x
]]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + (4*(27*A - 42*B - 175*C
)*Sqrt[Cos[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(105*d)
```

Rubi [A] time = 0.87912, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^4(17A+28B+35C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^4(83A+7B-175C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2(A-7B-21C)\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (8*a^4*(8*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^4*(17*A + 28*B +
 35*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^4*(83*A + 7*B - 175*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(3*B + 8*C)*(a + a*Cos[c + d*x]
)^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*C*(a + a*Cos[c + d*x])^4*Si
n[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(A - 7*B - 21*C)*Sqrt[Cos[c + d*x
]]*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) + (4*(27*A - 42*B - 175*C
)*Sqrt[Cos[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(105*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + $$

$$= \frac{2a(3B+8C)(a+a\cos(c+dx))^3\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a(3B+8C)(a+a\cos(c+dx))^3\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a(3B+8C)(a+a\cos(c+dx))^3\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a(3B+8C)(a+a\cos(c+dx))^3\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{4a^4(83A+7B-175C)\sqrt{\cos(c+dx)}\,\mathrm{si}}{105d}$$

$$= \frac{4a^4(83A+7B-175C)\sqrt{\cos(c+dx)}\,\mathrm{si}}{105d}$$

$$= \frac{8a^4(8A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^4(1}{}$$

Mathematica [C] time = 6.95355, size = 1451, normalized size = 5.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((32*A + 23*B - 20*C + 32*A*Cos[2*c] + 33*B
*Cos[2*c] + 20*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + ((191*A + 112*B + 28*C)*
Cos[d*x]*Sin[c])/(336*d) + ((4*A + B)*Cos[2*d*x]*Sin[2*c])/(40*d) + (A*Cos[
3*d*x]*Sin[3*c])/(112*d) + ((191*A + 112*B + 28*C)*Cos[c]*Sin[d*x])/(336*d)
 + (C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Sec[c]*Sec[c + d*x]*(C*Sin[
c] + 3*B*Sin[d*x] + 12*C*Sin[d*x]))/(12*d) + ((4*A + B)*Cos[2*c]*Sin[2*d*x]
)/(40*d) + (A*Cos[3*c]*Sin[3*d*x])/(112*d)))/(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x]) - (17*A*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*B*Cos[c +
```

```
d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]
]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin
[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*Sqrt[1 + Cot[c]^2]) - (5*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Se
c[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*Cos[
c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])) - (7*B*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
 + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 3.245, size = 864, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -4/105*(-240*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(48*A+7*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-4*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(577*A+203*B+35*C)*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(391*A+224*B+245*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(167*A+133*B+245*C)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(168*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-85*A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-140*B*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-175*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*si
n(1/2*d*x+1/2*c)^2-336*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+170*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-294*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+280*B*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
```

```
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+350*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^4\cos(dx+c)^3\sec(dx+c)^6 + (B+4C)a^4\cos(dx+c)^3\sec(dx+c)^5 + (A+4B+6C)a^4\cos(dx+c)^3 \mathrm{s}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*cos(d*x + c)^3*sec(d*x + c)^6 + (B + 4*C)*a^4*cos(d*x + c)^
3*sec(d*x + c)^5 + (A + 4*B + 6*C)*a^4*cos(d*x + c)^3*sec(d*x + c)^4 + 2*(2
*A + 3*B + 2*C)*a^4*cos(d*x + c)^3*sec(d*x + c)^3 + (6*A + 4*B + C)*a^4*cos
(d*x + c)^3*sec(d*x + c)^2 + (4*A + B)*a^4*cos(d*x + c)^3*sec(d*x + c) + A*
a^4*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(a\sec(dx+c)+a)^4\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*co
s(d*x + c)^(7/2), x)
```

3.1213 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\,s\right.$

Optimal. Leaf size=267

$$\frac{8a^4(4A+5B+4C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^4(A-25B-41C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(5A+15B+19C)\sin}{5d}$$

```
[Out] (56*a^4*(A - C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^4*(4*A + 5*B + 4*C)
*EllipticF[(c + d*x)/2, 2])/(3*d) + (4*a^4*(A - 25*B - 41*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(15*d) + (2*a*(5*B + 8*C)*(a + a*Cos[c + d*x])^3*Sin[c +
 d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^4*Sin[c + d*x]
)/(5*d*Cos[c + d*x]^(5/2)) + (2*(5*A + 15*B + 19*C)*(a^2 + a^2*Cos[c + d*x]
)^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) - (4*(6*A + 25*B + 34*C)*Sqrt[Co
s[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(15*d)
```

Rubi [A] time = 0.874518, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^4(4A+5B+4C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^4(A-25B-41C)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2(5A+15B+19C)\sin(c+dx}{5d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (56*a^4*(A - C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^4*(4*A + 5*B + 4*C)
*EllipticF[(c + d*x)/2, 2])/(3*d) + (4*a^4*(A - 25*B - 41*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(15*d) + (2*a*(5*B + 8*C)*(a + a*Cos[c + d*x])^3*Sin[c +
 d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*C*(a + a*Cos[c + d*x])^4*Sin[c + d*x]
)/(5*d*Cos[c + d*x]^(5/2)) + (2*(5*A + 15*B + 19*C)*(a^2 + a^2*Cos[c + d*x]
)^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) - (4*(6*A + 25*B + 34*C)*Sqrt[Co
s[c + d*x]]*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(15*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
```

```
-2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + $$

$$= \frac{2a(5B+8C)(a+a\cos(c+dx))^3\sin(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a(5B+8C)(a+a\cos(c+dx))^3\sin(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a(5B+8C)(a+a\cos(c+dx))^3\sin(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a(5B+8C)(a+a\cos(c+dx))^3\sin(c}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{4a^4(A-25B-41C)\sqrt{\cos(c+dx)}\sin(}{15d}$$

$$= \frac{4a^4(A-25B-41C)\sqrt{\cos(c+dx)}\sin(}{15d}$$

$$= \frac{56a^4(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^4(4A}{}$$

Mathematica [C] time = 7.07729, size = 1449, normalized size = 5.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((23*A - 20*B - 61*C + 33*A*Cos[2*c] + 20*B
*Cos[2*c] + 5*C*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + ((4*A + B)*Cos[d*x]*Sin[c
])/(12*d) + (A*Cos[2*d*x]*Sin[2*c])/(40*d) + ((4*A + B)*Cos[c]*Sin[d*x])/(1
2*d) + (C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d*x]^2*(
3*C*Sin[c] + 5*B*Sin[d*x] + 20*C*Sin[d*x]))/(60*d) + (Sec[c]*Sec[c + d*x]*(
5*B*Sin[c] + 20*C*Sin[c] + 15*A*Sin[d*x] + 60*B*Sin[d*x] + 99*C*Sin[d*x]))/
(60*d) + (A*Cos[2*c]*Sin[2*d*x])/(40*d)))/(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x]) - (4*A*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*
x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (5*B*Cos[c + d*x]
```

```
^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c +
 d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (7*A*Cos[c + d
*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])) + (7*C*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d
*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 9.332, size = 1214, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 8/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+128*A*sin(1/2*d
*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8
+140*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-186*A*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+20*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-35*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-80*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-61*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)*C+84*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-100*B*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-84*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
```

```
d*x+1/2*c)^2-80*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+80*C*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+80*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^4-84*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+100*B*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+84*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4+102*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-19*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A+218*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4-150*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-198*C*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^6)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ca^4\cos{(dx+c)}^2\sec{(dx+c)}^6 + (B+4\,C)a^4\cos{(dx+c)}^2\sec{(dx+c)}^5 + (A+4\,B+6\,C)a^4\cos{(dx+c)}^2\,\mathrm{s}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*cos(d*x + c)^2*sec(d*x + c)^6 + (B + 4*C)*a^4*cos(d*x + c)^
2*sec(d*x + c)^5 + (A + 4*B + 6*C)*a^4*cos(d*x + c)^2*sec(d*x + c)^4 + 2*(2
*A + 3*B + 2*C)*a^4*cos(d*x + c)^2*sec(d*x + c)^3 + (6*A + 4*B + C)*a^4*cos
(d*x + c)^2*sec(d*x + c)^2 + (4*A + B)*a^4*cos(d*x + c)^2*sec(d*x + c) + A*
a^4*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + B \sec (dx+c) + A\right)(a \sec (dx+c) + a)^4 \cos (dx+c)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*co
s(d*x + c)^(5/2), x)
```

3.1214 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\,s\right.$

Optimal. Leaf size=271

$$\frac{8a^4(35A+28B+17C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(35A+77B+73C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{105d\cos^{\frac{3}{2}}(c+dx)}-\frac{4a^4(175A}{}$$

```
[Out] (-8*a^4*(7*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^4*(35*A + 28*B
+ 17*C)*EllipticF[(c + d*x)/2, 2])/(21*d) - (4*a^4*(175*A + 287*B + 253*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(7*B + 8*C)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])
^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(35*A + 77*B + 73*C)*(a^2 +
a^2*Cos[c + d*x])^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*(175*A +
238*B + 197*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d
*x]])
```

Rubi [A] time = 0.870524, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{8a^4(35A+28B+17C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(35A+77B+73C)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{105d\cos^{\frac{3}{2}}(c+dx)}-\frac{4a^4(175A+287B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-8*a^4*(7*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^4*(35*A + 28*B
+ 17*C)*EllipticF[(c + d*x)/2, 2])/(21*d) - (4*a^4*(175*A + 287*B + 253*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(7*B + 8*C)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*C*(a + a*Cos[c + d*x])
^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(35*A + 77*B + 73*C)*(a^2 +
a^2*Cos[c + d*x])^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (4*(175*A +
238*B + 197*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d
*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int\frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$=\frac{2C(a+a\cos(c+dx))^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+$$

$$=\frac{2a(7B+8C)(a+a\cos(c+dx))^3\sin(c}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2a(7B+8C)(a+a\cos(c+dx))^3\sin(c}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2a(7B+8C)(a+a\cos(c+dx))^3\sin(c}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2a(7B+8C)(a+a\cos(c+dx))^3\sin(c}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$=-\frac{4a^4(175A+287B+253C)\sqrt{\cos(c+}}{105d}$$

$$=-\frac{4a^4(175A+287B+253C)\sqrt{\cos(c+}}{105d}$$

$$=-\frac{8a^4(7B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^4}{}$$

Mathematica [C] time = 7.22779, size = 1454, normalized size = 5.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((-20*A - 61*B - 64*C + 20*A*Cos[2*c] + 5*B
*Cos[2*c])*Csc[c]*Sec[c])/(40*d) + (A*Cos[d*x]*Sin[c])/(12*d) + (A*Cos[c]*S
in[d*x])/(12*d) + (C*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c
 + d*x]^3*(5*C*Sin[c] + 7*B*Sin[d*x] + 28*C*Sin[d*x]))/(140*d) + (Sec[c]*Se
c[c + d*x]^2*(21*B*Sin[c] + 84*C*Sin[c] + 35*A*Sin[d*x] + 140*B*Sin[d*x] +
235*C*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 140*B*Sin[c]
 + 235*C*Sin[c] + 420*A*Sin[d*x] + 693*B*Sin[d*x] + 672*C*Sin[d*x]))/(420*d
)))/(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (5*A*Cos[c + d*x]^6
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sq
```

```
rt[1 + Cot[c]^2]) - (4*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d
*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (17*C*Cos[c + d*
x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*Sqrt[1 + Cot[c]^2]) + (7*B*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a
 + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*C*Cos[c + d*x]^6*Csc[c
]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]
]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 10.01, size = 1535, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 8/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(16*sin
(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)^4-8*sin(1/2
*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(280*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^10-2240*A*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-2772*B*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2380*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
4+3010*A*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-140*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-85*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-175*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+427*B*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+1050*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^
2+503*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*C+1120*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^6-1764*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^4+1400*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+1176*B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+680*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^6+1344*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+840*B*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+1008*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+882*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+510
*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-2688*C*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^8-1020*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-2
100*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-1680*B*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^4-2016*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^4-1470*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+245*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)*A-2570*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+44
38*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4502*C*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^6)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Ca^4\cos(dx+c)\sec(dx+c)^6 + (B+4C)a^4\cos(dx+c)\sec(dx+c)^5 + (A+4B+6C)a^4\cos(dx+c)\sec\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*cos(d*x + c)*sec(d*x + c)^6 + (B + 4*C)*a^4*cos(d*x + c)*se
c(d*x + c)^5 + (A + 4*B + 6*C)*a^4*cos(d*x + c)*sec(d*x + c)^4 + 2*(2*A + 3
*B + 2*C)*a^4*cos(d*x + c)*sec(d*x + c)^3 + (6*A + 4*B + C)*a^4*cos(d*x + c
)*sec(d*x + c)^2 + (4*A + B)*a^4*cos(d*x + c)*sec(d*x + c) + A*a^4*cos(d*x
+ c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^4\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*co
s(d*x + c)^(3/2), x)
```

3.1215 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=274

$$\frac{8a^4(28A+17B+12C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{8a^4(21A+24B+19C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(63A+117B+97C)\sin}{315d\,c}$$

```
[Out] (-8*a^4*(21*A + 24*B + 19*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(28
*A + 17*B + 12*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^4*(287*A + 253*B
 + 193*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) + (2*a*(9*B + 8*C)*(a +
a*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*C*(a + a*Cos
[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(63*A + 117*B + 97
*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (
4*(21*A + 24*B + 19*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(45*d*Cos[c +
 d*x]^(3/2))
```

Rubi [A] time = 0.89431, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{8a^4(28A+17B+12C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{8a^4(21A+24B+19C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(63A+117B+97C)\sin(c+dx}{315d\cos^{\frac{5}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (-8*a^4*(21*A + 24*B + 19*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(28
*A + 17*B + 12*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^4*(287*A + 253*B
 + 193*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) + (2*a*(9*B + 8*C)*(a +
a*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*C*(a + a*Cos
[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(63*A + 117*B + 97
*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (
4*(21*A + 24*B + 19*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(45*d*Cos[c +
 d*x]^(3/2))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
```

```
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\int \frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{11}{2}}(c+dx}$$

$$=\frac{2C(a+a\cos(c+dx))^4\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)}+$$

$$=\frac{2a(9B+8C)(a+a\cos(c+dx))^3\sin(c}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2a(9B+8C)(a+a\cos(c+dx))^3\sin(c}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2a(9B+8C)(a+a\cos(c+dx))^3\sin(c}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2a(9B+8C)(a+a\cos(c+dx))^3\sin(c}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$=\frac{4a^4(287A+253B+193C)\sin(c+dx}{105d\sqrt{\cos(c+dx)}}$$

$$=\frac{4a^4(287A+253B+193C)\sin(c+dx}{105d\sqrt{\cos(c+dx)}}$$

$$=-\frac{8a^4(21A+24B+19C)E\left(\frac{1}{2}(c+dx)\right|}{15d}$$

Mathematica [C] time = 7.35317, size = 1748, normalized size = 6.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(-((-183*A - 192*B - 152*C + 15*A*Cos[2*c])*C
sc[c]*Sec[c])/(120*d) + (C*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d) + (Sec[c]
*Sec[c + d*x]^4*(7*C*Sin[c] + 9*B*Sin[d*x] + 36*C*Sin[d*x]))/(252*d) + (Sec
[c]*Sec[c + d*x]^3*(45*B*Sin[c] + 180*C*Sin[c] + 63*A*Sin[d*x] + 252*B*Sin[
d*x] + 427*C*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]*(140*A*Sin[c] + 235
*B*Sin[c] + 240*C*Sin[c] + 693*A*Sin[d*x] + 672*B*Sin[d*x] + 532*C*Sin[d*x]
))/(420*d) + (Sec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 252*B*Sin[c] + 427*C*Sin
[c] + 420*A*Sin[d*x] + 705*B*Sin[d*x] + 720*C*Sin[d*x]))/(1260*d)))/(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]) - (4*A*Cos[c + d*x]^6*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
```

```
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[
c]^2]) - (17*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^6*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
 Cot[c]^2]) + (7*A*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (4*B*Cos[c + d*x]^6*Csc[c]*Sec[c/2 +
 (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (19*C*Cos[c
+ d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
 ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x]))
```

Maple [B] time = 11.766, size = 1427, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(1/16*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+3/16*A*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/16*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(1/4*A+3/8
*B+1/4*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/16*B+1/4*C)*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*(1/16*
A+1/4*B+3/8*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/16*C*(-1/144*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))+(3/8*A+1/4*B+1/16*C)*
(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\Big(\big(Ca^4\sec(dx+c)^6 + (B+4C)a^4\sec(dx+c)^5 + (A+4B+6C)a^4\sec(dx+c)^4 + 2(2A+3B+2C)a^4\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*sec(d*x + c)^6 + (B + 4*C)*a^4*sec(d*x + c)^5 + (A + 4*B +
6*C)*a^4*sec(d*x + c)^4 + 2*(2*A + 3*B + 2*C)*a^4*sec(d*x + c)^3 + (6*A + 4
*B + C)*a^4*sec(d*x + c)^2 + (4*A + B)*a^4*sec(d*x + c) + A*a^4)*sqrt(cos(d
```

```
*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^4\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4*sq
rt(cos(d*x + c)), x)
```

3.1216 $\int \frac{(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=310

$$\frac{8a^4(187A+132B+113C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}-\frac{8a^4(24A+19B+16C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^4(913A+803B+6}{1155d\cos^{\frac{3}{2}}(}$$

```
[Out] (-8*a^4*(24*A + 19*B + 16*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(18
7*A + 132*B + 113*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^4*(913*A + 8
03*B + 667*C)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(3/2)) + (8*a^4*(24*A + 19
*B + 16*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a*(11*B + 8*C)*(a +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*C*(a + a*Co
s[c + d*x])^4*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + (2*(33*A + 55*B +
43*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) +
 (4*(891*A + 946*B + 769*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(3465*d*
Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.924663, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {4112, 3043, 2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{8a^4(187A+132B+113C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}-\frac{8a^4(24A+19B+16C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^4(913A+803B+667C)\sin}{1155d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] (-8*a^4*(24*A + 19*B + 16*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (8*a^4*(18
7*A + 132*B + 113*C)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^4*(913*A + 8
03*B + 667*C)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(3/2)) + (8*a^4*(24*A + 19
*B + 16*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a*(11*B + 8*C)*(a +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*C*(a + a*Co
s[c + d*x])^4*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + (2*(33*A + 55*B +
43*C)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(231*d*Cos[c + d*x]^(7/2)) +
 (4*(891*A + 946*B + 769*C)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(3465*d*
Cos[c + d*x]^(5/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3043

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c
```

```
+ d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*m + b*c*(n + 1)) + (c*C - B*d)*(a*c
*m + b*d*(n + 1)) + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n
+ 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m,
 -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \frac{(a+a\cos(c+dx))^4\left(C+B\cos(c+dx)+A\cos^2\right.}{\cos^{\frac{13}{2}}(c+dx)}$$

$$= \frac{2C(a+a\cos(c+dx))^4\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2\int\frac{(a+a\cos(c+}{}}{}$$

$$= \frac{2a(11B+8C)(a+a\cos(c+dx))^3\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{2a(11B+8C)(a+a\cos(c+dx))^3\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{2a(11B+8C)(a+a\cos(c+dx))^3\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{2a(11B+8C)(a+a\cos(c+dx))^3\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{4a^4(913A+803B+667C)\sin(c+dx)}{1155d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(11B+}{}$$

$$= \frac{4a^4(913A+803B+667C)\sin(c+dx)}{1155d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(11B+}{}$$

$$= \frac{8a^4(187A+132B+113C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d} + \frac{4a^4(9}{}$$

$$= -\frac{8a^4(24A+19B+16C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{8a^4(18}{}$$

Mathematica [C] time = 7.41277, size = 1795, normalized size = 5.79

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(13/2)*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*(((24*A + 19*B + 16*C)*Csc[c]*Sec[c])/(15*d)
+ (C*Sec[c]*Sec[c + d*x]^6*Sin[d*x])/(44*d) + (Sec[c]*Sec[c + d*x]^5*(9*C*S
in[c] + 11*B*Sin[d*x] + 44*C*Sin[d*x]))/(396*d) + (Sec[c]*Sec[c + d*x]^4*(7
7*B*Sin[c] + 308*C*Sin[c] + 99*A*Sin[d*x] + 396*B*Sin[d*x] + 675*C*Sin[d*x]
))/(2772*d) + (Sec[c]*Sec[c + d*x]^3*(495*A*Sin[c] + 1980*B*Sin[c] + 3375*C
*Sin[c] + 2772*A*Sin[d*x] + 4697*B*Sin[d*x] + 4928*C*Sin[d*x]))/(13860*d) +
 (Sec[c]*Sec[c + d*x]*(2585*A*Sin[c] + 2640*B*Sin[c] + 2260*C*Sin[c] + 7392
*A*Sin[d*x] + 5852*B*Sin[d*x] + 4928*C*Sin[d*x]))/(4620*d) + (Sec[c]*Sec[c
```

```
+ d*x]^2*(2772*A*Sin[c] + 4697*B*Sin[c] + 4928*C*Sin[c] + 7755*A*Sin[d*x] +
 7920*B*Sin[d*x] + 6780*C*Sin[d*x]))/(13860*d)))/(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x]) - (17*A*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Se
c[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[
c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*B*Cos
[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*Sqrt[1 + Cot[c]^2]) - (113*C*Cos[c + d*x]^6*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^8*(a
+ a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(231*
d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (
4*A*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])) + (19*B*Cos[c + d*x]^6*Csc[c]*Sec[c/2 + (d*x)/2]^8*(a
+ a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeometri
cPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]
^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(30*d*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (8*C*Cos[c + d*x]^6*Csc[c]
*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]
]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]))/(15*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 12.971, size = 1505, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(1/16*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+(3/8*A+1/4*B+1/16*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/16*A+1/
4*B+3/8*C)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+
5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))-1/5*(1/4*A+3/8*B+1/4*C)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/
2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/16*B+1/4*C
)*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))))+1/16*C*(-1/352*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^6-9/616*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^4-15/154*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+15/77*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/4*A+1/16*B)*(-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Ca^4\sec(dx+c)^6 + (B+4C)a^4\sec(dx+c)^5 + (A+4B+6C)a^4\sec(dx+c)^4 + 2(2A+3B+2C)a^4\sec(}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*a^4*sec(d*x + c)^6 + (B + 4*C)*a^4*sec(d*x + c)^5 + (A + 4*B +
6*C)*a^4*sec(d*x + c)^4 + 2*(2*A + 3*B + 2*C)*a^4*sec(d*x + c)^3 + (6*A + 4
*B + C)*a^4*sec(d*x + c)^2 + (4*A + B)*a^4*sec(d*x + c) + A*a^4)/sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^2 + B \sec\left(dx+c\right) + A\right)\left(a \sec\left(dx+c\right) + a\right)^4}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^4/sq
rt(cos(d*x + c)), x)
```

3.1217 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{5(9A-7B+7C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21ad}-\frac{3(7A-7B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] (-3*(7*A - 7*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(9*A - 7*B +
7*C)*EllipticF[(c + d*x)/2, 2])/(21*a*d) + (5*(9*A - 7*B + 7*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*a*d) - ((7*A - 7*B + 5*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(5*a*d) + ((9*A - 7*B + 7*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*
a*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
))
```

Rubi [A] time = 0.329961, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2748, 2635, 2639, 2641}

$$\frac{5(9A-7B+7C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{3(7A-7B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(9A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```


```
[Out] (-3*(7*A - 7*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(9*A - 7*B +
7*C)*EllipticF[(c + d*x)/2, 2])/(21*a*d) + (5*(9*A - 7*B + 7*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*a*d) - ((7*A - 7*B + 5*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(5*a*d) + ((9*A - 7*B + 7*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*
a*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]
))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$=-\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}\right.}{}$$

$$=-\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(7A-7B+5C)\int c}{2a}$$

$$=-\frac{(7A-7B+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}+\frac{(9A-7B+7C}{}$$

$$=-\frac{3(7A-7B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{5(9A-7B+7C)\sqrt{\cos}}{21ad}$$

$$=-\frac{3(7A-7B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{5(9A-7B+7C)F\left(\frac{1}{2}(\right.}{21ad}$$

Mathematica [C] time = 6.89322, size = 2117, normalized size = 10.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] (((-21*I)/10)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3
/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x)
)*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*
d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos
```

```
[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d
*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(a + a*Sec[c + d*x])) + (((21*I)/10)*B*Cos[c/2 + (d*x)/2]^2*Cos[c +
 d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I
)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*
Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d
*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (((3*I)/2)*C
*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((
2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I
)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
 I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 +
 E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)
*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*
Sec[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((4*(5*A - 5*B + 5*C + 16*A*Cos[c] - 16*B*Cos[c] + 1
0*C*Cos[c])*Csc[c])/(5*d) + (2*(51*A - 28*B + 28*C)*Cos[d*x]*Sin[c])/(21*d)
 - (4*(A - B)*Cos[2*d*x]*Sin[2*c])/(5*d) + (2*A*Cos[3*d*x]*Sin[3*c])/(7*d)
+ (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(
d*x)/2]))/d + (2*(51*A - 28*B + 28*C)*Cos[c]*Sin[d*x])/(21*d) - (4*(A - B)*
Cos[2*c]*Sin[2*d*x])/(5*d) + (2*A*Cos[3*c]*Sin[3*d*x])/(7*d)))/((A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (30*A*Cos[c/
2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
 Cot[c]^2]*(a + a*Sec[c + d*x])) + (10*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])) - (10*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.439, size = 341, normalized size = 1.6

$$-\frac{1}{105\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(225*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-175*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))+175*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*C*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-480*A*sin(1/2*d*x+1/2*c)^10+(864*A+336*
B)*sin(1/2*d*x+1/2*c)^8+(-888*A-392*B-280*C)*sin(1/2*d*x+1/2*c)^6+(930*A-21
0*B+630*C)*sin(1/2*d*x+1/2*c)^4+(-321*A+161*B-245*C)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*sec
(d*x + c) + a), x)
```

3.1218 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$-\frac{(5A-5B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{3(7A-5B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+$$

```
[Out] (3*(7*A - 5*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A - 5*B + 3*C
)*EllipticF[(c + d*x)/2, 2])/(3*a*d) - ((5*A - 5*B + 3*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d) + ((7*A - 5*B + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*a*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c
+ d*x]))
```

Rubi [A] time = 0.309413, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2748, 2635, 2641, 2639}

$$-\frac{(5A-5B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(7A-5B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A-5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```

```
[Out] (3*(7*A - 5*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((5*A - 5*B + 3*C
)*EllipticF[(c + d*x)/2, 2])/(3*a*d) - ((5*A - 5*B + 3*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d) + ((7*A - 5*B + 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*a*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c
+ d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\,(\ldots}{\ldots}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A-5B+3C)\ldots}{\ldots}$$

$$= -\frac{(5A-5B+3C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{(7A-5B+\ldots}{\ldots}$$

$$= \frac{3(7A-5B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(5A-5B+3C)F\left(\frac{1}{2}(\ldots\right.}{3ad}$$

Mathematica [C] time = 6.8413, size = 2063, normalized size = 11.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] (((21*I)/10)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/
4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[
c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*
```

```
(-1 + E^((2*I)*d*x))*Sin[c]))))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])) - (((3*I)/2)*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*
x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (((3*I)/2)*C*Co
s[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*((-4*(5*A - 5*B + 5*C + 16*A*Cos[c] - 10*B*Cos[c] + 10*
C*Cos[c])*Csc[c])/(5*d) - (8*(A - B)*Cos[d*x]*Sin[c])/(3*d) + (4*A*Cos[2*d*
x]*Sin[2*c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2] + C*Sin[(d*x)/2]))/d - (8*(A - B)*Cos[c]*Sin[d*x])/(3*d) + (4*A*C
os[2*c]*Sin[2*d*x])/(5*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*(a + a*Sec[c + d*x])) + (10*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) -
(10*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (2*C*Cos[c/2 + (d*x)/2]^2*Co
s[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*S
ec[c + d*x]))
```

Maple [A] time = 2.26, size = 320, normalized size = 1.8

$$-\frac{1}{15\,ad}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^{2}-1}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+15*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*C*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/
2*d*x+1/2*c)^6+(-30*A+90*B-30*C)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B+15*C)*sin(
1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec
(d*x + c) + a), x)
```

3.1219 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{(5A-3B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+$$

```
[Out] -(((3*A - 3*B + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((5*A - 3*B + 3*C)*E
llipticF[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*a*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rubi [A] time = 0.28913, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2748, 2639, 2635, 2641}

$$\frac{(5A-3B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A-3B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```


```
[Out] -(((3*A - 3*B + C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((5*A - 3*B + 3*C)*E
llipticF[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(3*a*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx=\int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{a+a\cos(c+dx)}\,dx$$

$$=-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int\sqrt{\cos(c+dx)}\left(-\right.}{}$$

$$=-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(3A-3B+C)\int\sqrt{}}{2a}$$

$$=-\frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(5A-3B+3C)\sqrt{\cos(c+}}{3ad}$$

$$=-\frac{(3A-3B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(5A-3B+3C)F\left(\frac{1}{2}(c+\right.}{3ad}$$

Mathematica [C] time = 6.70323, size = 2008, normalized size = 14.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] (((-3*I)/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c
] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(
-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*(a + a*Sec[c + d*x])) + (((3*I)/2)*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x
```

```
]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I
)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
+ E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - ((I/2)*C*Cos[c/2
 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
d*x])) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((4*(A - B + C + 2*A*Cos[c] - 2*B*Cos[c])*Csc[c])/d + (8*A*C
os[d*x]*Sin[c])/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*
Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*A*Cos[c]*Sin[d*x])/(3*d)))/((A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (10*A*Cos
[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (2*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x
]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])) - (2*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.276, size = 300, normalized size = 2.2

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
```

```
c),2^(1/2))+3*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B+6*C)*sin(1/2*d*x+1
/2*c)^4+(-7*A+3*B-3*C)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a), x)
```

3.1220 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=93

$$-\frac{(A-B-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{(3A-B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] ((3*A - B + C)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - B - C)*EllipticF[(c
 + d*x)/2, 2])/(a*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rubi [A] time = 0.265925, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {4112, 3041, 2748, 2641, 2639}

$$-\frac{(A-B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x]),x]
```

```
[Out] ((3*A - B + C)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - B - C)*EllipticF[(c
 + d*x)/2, 2])/(a*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a
+ a*Cos[c + d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+a\sec(c+dx)}\,dx=\int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$$

$$=-\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-B-C)+\frac{1}{2}}{\sqrt{c}}}{}$$

$$=-\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-B-C)\int}{2a}$$

$$=\frac{(3A-B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B-C)F\left(\frac{1}{2}(c+dx)\right.}{ad}$$

Mathematica [C] time = 6.65543, size = 1973, normalized size = 21.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x]),x]
```

```
[Out] (((3*I)/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
-(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
+ (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[
2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-
1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])) - ((I/2)*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc
[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + ((I/2)*C*Cos[c/2 + (d
*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
```

```
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x
))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]
)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])
) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*((-4*(A - B + C + 2*A*Cos[c])*Csc[c])/d - (4*Sec[c/2]*Sec[c/2 + (
d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d))/((A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Cos[c/
2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + C
ot[c]^2]*(a + a*Sec[c + d*x])) - (2*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc
[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec
[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
- (2*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]
]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin
[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A] time = 2.697, size = 281, normalized size = 3.

$$\frac{1}{ad}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(A\mathrm{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+(2*A-2*B+2*C)*sin(1/2*d*x+1/2*c)^4+(-A+B-C)*sin(1/2*d*x+1/2*c)^2)/a/
cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(
d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A\sqrt{\cos\left(c+dx\right)}}{\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int\frac{C\sqrt{\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sqrt(cos
(c + d*x))*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(C*sqrt(cos(c + d*
x))*sec(c + d*x)**2/(sec(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a), x)
```

3.1221 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=122

$$\frac{(A+B-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{(A-B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B+C)\sin}{d\sqrt{\cos(c+dx)}(a\cos}$$

```
[Out] -(((A - B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A + B - C)*EllipticF
[(c + d*x)/2, 2])/(a*d) + ((A - B + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*
x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
))
```

Rubi [A] time = 0.284946, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2748, 2636, 2639, 2641}

$$\frac{(A+B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[
c + d*x])),x]
```

```
[Out] -(((A - B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A + B - C)*EllipticF
[(c + d*x)/2, 2])/(a*d) + ((A - B + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*
x]]) - ((A - B + C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B+3C)+\frac{1}{2}a(A+B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} + \frac{(A+B-C)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{(A-}{}$$

$$= \frac{(A+B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B+}{d\sqrt{\cos(c+dx}}$$

$$= -\frac{(A-B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B}{ad}$$

Mathematica [C] time = 6.76441, size = 2009, normalized size = 16.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])),x]
```

```
[Out] ((-I/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (
2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c
] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
 E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
```

```
)*(a + a*Sec[c + d*x])) + ((I/2)*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/
2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (((3*I)/2)*C*Cos[c/2 + (
d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*
x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d
*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*S
in[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c
])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x]
)) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*((2*(2*C + A*Cos[c] - B*Cos[c] + C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec
[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] +
C*Sin[(d*x)/2]))/d + (8*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/((A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (2*A*Cos[c/2 +
(d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c
]^2]*(a + a*Sec[c + d*x])) - (2*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (2
*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 4.773, size = 353, normalized size = 2.9

$$-\frac{1}{ad}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-C*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-B+3*C)*sin(1/2*d*x+1/2*c)^
4+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-B+5*C)*sin(1/2*d*
x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)\sec\left(dx+c\right)+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)*sec(d*x + c) + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{A}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{B\sec\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx+\int\frac{C\sec^{2}\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) + In
tegral(B*sec(c + d*x)/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))
), x) + Integral(C*sec(c + d*x)**2/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(
cos(c + d*x))), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

3.1222 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=165

$$\frac{(3A-3B+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A\cdots}{}$$

```
[Out] ((A - 3*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((3*A - 3*B + 5*C)*Elli
pticF[(c + d*x)/2, 2])/(3*a*d) + ((3*A - 3*B + 5*C)*Sin[c + d*x])/(3*a*d*Co
s[c + d*x]^(3/2)) - ((A - 3*B + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.305823, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2748, 2636, 2641, 2639}

$$\frac{(3A-3B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(3A-3B+\cdots}{3ad\,\text{co}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])),x]
```

```
[Out] ((A - 3*B + 3*C)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((3*A - 3*B + 5*C)*Elli
pticF[(c + d*x)/2, 2])/(3*a*d) + ((3*A - 3*B + 5*C)*Sin[c + d*x])/(3*a*d*Co
s[c + d*x]^(3/2)) - ((A - 3*B + 3*C)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A-3B+5C)-\frac{1}{2}a(A-3B+3C)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(A-3B+3C)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(3A-}{}$$

$$= \frac{(3A-3B+5C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(A-3B+3C)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B+C}{d\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{(A-3B+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-3B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(3A-}{3a}$$

Mathematica [C] time = 7.18845, size = 2052, normalized size = 12.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])),x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3
*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2
*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
 + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
```

```
E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])) - (((3*I)/2)*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc
[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (((3*I)/2)*C*Cos[c/2
+ (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d
*x])) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((-2*(-2*B + 2*C + A*Cos[c] - B*Cos[c] + C*Cos[c])*Csc[c/2]*S
ec[c/2]*Sec[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[
(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d)
+ (8*Sec[c]*Sec[c + d*x]*(C*Sin[c] + 3*B*Sin[d*x] - 3*C*Sin[d*x]))/(3*d)))/
((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) -
(2*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2
}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (2*B*Cos[c/2 + (d*x)/2]^2*Cos[c
 + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[
c + d*x])) - (10*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 6.898, size = 494, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*C*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
```

```
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A-B+C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)+(2*B-2*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(
2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

3.1223 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=210

$$-\frac{(3A-5B+5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{3(5A-5B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A}{}$$

```
[Out] (-3*(5*A - 5*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A - 5*B + 5*
C)*EllipticF[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 5*B + 7*C)*Sin[c + d*x])/(5
*a*d*Cos[c + d*x]^(5/2)) - ((3*A - 5*B + 5*C)*Sin[c + d*x])/(3*a*d*Cos[c +
d*x]^(3/2)) + (3*(5*A - 5*B + 7*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.322626, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2748, 2636, 2639, 2641}

$$-\frac{(3A-5B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-5B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A-5B+}{3ad\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])),x]
```

```
[Out] (-3*(5*A - 5*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - ((3*A - 5*B + 5*
C)*EllipticF[(c + d*x)/2, 2])/(3*a*d) + ((5*A - 5*B + 7*C)*Sin[c + d*x])/(5
*a*d*Cos[c + d*x]^(5/2)) - ((3*A - 5*B + 5*C)*Sin[c + d*x])/(3*a*d*Cos[c +
d*x]^(3/2)) + (3*(5*A - 5*B + 7*C)*Sin[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]])
 - ((A - B + C)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-5B+7C)-\frac{1}{2}a(3A-5B+5C)\cos(c+dx}{\cos^{\frac{7}{2}}(c+dx)}}{a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(3A-5B+5C)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} + \cdots$$

$$= \frac{(5A-5B+7C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{(3A-5B+5C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(A-B}{d\cos^{\frac{5}{2}}(c+}$$

$$= -\frac{(3A-5B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-5B+7C)\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{(3A-}{}$$

$$= -\frac{3(5A-5B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{(3A-5B+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + $$

Mathematica [C] time = 7.55618, size = 2111, normalized size = 10.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])),x]
```

```
[Out] (((-3*I)/2)*A*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
```

```
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c
] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(
-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*(a + a*Sec[c + d*x])) + (((3*I)/2)*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x
]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I
)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
+ E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) - (((21*I)/10)*C*C
os[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E
^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d
*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Se
c[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((2*(10*A - 10*B + 16*C + 5*A*Cos[c] - 5*B*Cos[c] + 5*
C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*
(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/d + (8*C*Sec[c]*Sec[c +
 d*x]^3*Sin[d*x])/(5*d) - (8*Sec[c]*Sec[c + d*x]*(-5*B*Sin[c] + 5*C*Sin[c]
- 15*A*Sin[d*x] + 15*B*Sin[d*x] - 24*C*Sin[d*x]))/(15*d) + (8*Sec[c]*Sec[c
+ d*x]^2*(3*C*Sin[c] + 5*B*Sin[d*x] - 5*C*Sin[d*x]))/(15*d)))/((A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])) + (2*A*Cos[c/2
+ (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
ot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot
[c]^2]*(a + a*Sec[c + d*x])) - (10*B*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[
c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
 + (10*C*Cos[c/2 + (d*x)/2]^2*Cos[c + d*x]*Csc[c/2]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [B] time = 8.668, size = 812, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((2*B-2*C)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-A
+B-C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/
2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*A-2*B+2*C)*(-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(
d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec{(dx+c)}^2 + B\sec{(dx+c)} + A}{(a\sec{(dx+c)} + a)\cos{(dx+c)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

3.1224 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=258

$$\frac{5(30A-21B+14C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21a^2d}-\frac{7(11A-8B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(11A-8B+5C)\sin(c+dx)\cos}{3a^2d(\cos(c+dx)+1)}$$

```
[Out] (-7*(11*A - 8*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) + (5*(30*A - 21
*B + 14*C)*EllipticF[(c + d*x)/2, 2])/(21*a^2*d) + (5*(30*A - 21*B + 14*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*a^2*d) - (7*(11*A - 8*B + 5*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d) + ((30*A - 21*B + 14*C)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(7*a^2*d) - ((11*A - 8*B + 5*C)*Cos[c + d*x]^(7/2)*Sin[c
 + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(9/2)*Sin
[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.499248, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2748, 2635, 2639, 2641}

$$\frac{5(30A-21B+14C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^2d}-\frac{7(11A-8B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(11A-8B+5C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx}{3a^2d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```


```
[Out] (-7*(11*A - 8*B + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) + (5*(30*A - 21
*B + 14*C)*EllipticF[(c + d*x)/2, 2])/(21*a^2*d) + (5*(30*A - 21*B + 14*C)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*a^2*d) - (7*(11*A - 8*B + 5*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d) + ((30*A - 21*B + 14*C)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(7*a^2*d) - ((11*A - 8*B + 5*C)*Cos[c + d*x]^(7/2)*Sin[c
 + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(9/2)*Sin
[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}dx$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(-\frac{3}{2}a\right.}{}$$

$$= -\frac{(11A-8B+5C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C}{3d(}$$

$$= -\frac{(11A-8B+5C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C}{3d(}$$

$$= -\frac{7(11A-8B+5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{(30A-21}{}$$

$$= -\frac{7(11A-8B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} + \frac{5(30A-21B+14C}{}$$

$$= -\frac{7(11A-8B+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} + \frac{5(30A-21B+14C}{21a}$$

Mathematica [C] time = 7.28883, size = 2174, normalized size = 8.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] (((-77*I)/5)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*
I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
+ I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1
+ E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I
)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a
*Sec[c + d*x])^2) + (((56*I)/5)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1
/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)
*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((
2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/
2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c
] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - ((7*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/
2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
```

```
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (200*A*Cos[c/2 + (d*x)
/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[
c + d*x])^2) + (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (40*C*Cos[c/2 + (d*x)/2]^4
*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d
*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*((8*(25*A - 20*B + 15*C + 52*A*Cos[c] - 36*B*Cos[c] + 20*C*
Cos[c])*Csc[c])/(5*d) + (4*(107*A - 56*B + 28*C)*Cos[d*x]*Sin[c])/(21*d) -
(8*(2*A - B)*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*A*Cos[3*d*x]*Sin[3*c])/(7*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[
(d*x)/2]))/(3*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(5*A*Sin[(d*x)/2] - 4*B*S
in[(d*x)/2] + 3*C*Sin[(d*x)/2]))/d + (4*(107*A - 56*B + 28*C)*Cos[c]*Sin[d*
x])/(21*d) - (8*(2*A - B)*Cos[2*c]*Sin[2*d*x])/(5*d) + (4*A*Cos[3*c]*Sin[3*
d*x])/(7*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 2.915, size = 513, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/210*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(750*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+1617*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-525*B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1176*B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+350*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+735*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(750*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+1617*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-525*B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1176*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))+350*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+735*C*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+960*A*sin(1/2*d*x+1/2*c)^12+(-2016*A-
672*B)*sin(1/2*d*x+1/2*c)^10+(2608*A+896*B+560*C)*sin(1/2*d*x+1/2*c)^8+(-59
32*A+2296*B-2660*C)*sin(1/2*d*x+1/2*c)^6+(6184*A-3682*B+2940*C)*sin(1/2*d*x
+1/2*c)^4+(-1839*A+1197*B-875*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*
c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x +
 c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*sec
(d*x + c) + a)^2, x)
```

3.1225 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=214

$$-\frac{5(3A-2B+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(56A-35B+20C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A-2B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+d$$

```
[Out] ((56*A - 35*B + 20*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A - 2*B
+ C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A - 2*B + C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(3*a^2*d) + ((56*A - 35*B + 20*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(15*a^2*d) - ((3*A - 2*B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.470087, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5(3A-2B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(56A-35B+20C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A-2B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{(5$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```



```
[Out] ((56*A - 35*B + 20*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A - 2*B
+ C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A - 2*B + C)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(3*a^2*d) + ((56*A - 35*B + 20*C)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(15*a^2*d) - ((3*A - 2*B + C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(\cdots\right.}{\cdots}$$

$$= -\frac{(3A-2B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos\cdots}{3d(a+\cdots}$$

$$= -\frac{(3A-2B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\cos\cdots}{3d(a+\cdots}$$

$$= -\frac{5(3A-2B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{(56A-35B\cdots}{\cdots}$$

$$= \frac{(56A-35B+20C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(3A-2B+C)F\left(\cdots\right.}{3a^2d}$$

Mathematica [C] time = 7.10129, size = 2120, normalized size = 9.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] (((56*I)/5)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((
2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I
)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
 I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 +
 E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)
*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*
Sec[c + d*x])^2) - ((7*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/
4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[
c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*
(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])^2) + ((4*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec
[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/2]^4*C
sc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])
^2) - (40*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-
(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - A
rcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (20*C*Cos[c/2 + (d*x)/2]^4*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2)
 + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((-8*(20*A - 15*B + 10*C + 36*A*Cos[c] - 20*B*Cos[c] + 10*C*Cos[c]
)*Csc[c])/(5*d) - (16*(2*A - B)*Cos[d*x]*Sin[c])/(3*d) + (8*A*Cos[2*d*x]*Si
n[2*c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d
*x)/2] + C*Sin[(d*x)/2]))/(3*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(4*A*Sin[(
d*x)/2] - 3*B*Sin[(d*x)/2] + 2*C*Sin[(d*x)/2]))/d - (16*(2*A - B)*Cos[c]*Si
n[d*x])/(3*d) + (8*A*Cos[2*c]*Sin[2*d*x])/(5*d) + (4*(A - B + C)*Sec[c/2 +
```

```
(d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 2.776, size = 491, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(75*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+168*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+60*C*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(75*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+168*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-50*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-105*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*cos(1/2*d*x+1/2*c)-96*A*sin(1/2*d*x+1/2*c)^10+(128*A+80*B)*sin(1/2*d*x
+1/2*c)^8+(328*A-380*B+120*C)*sin(1/2*d*x+1/2*c)^6+(-526*A+420*B-170*C)*sin
(1/2*d*x+1/2*c)^4+(171*A-125*B+55*C)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+
1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x +
 c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^2, x)
```

3.1226 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=180

$$\frac{(10A-5B+2C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}$$

```
[Out] -(((7*A - 4*B + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((10*A - 5*B + 2*C
)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((10*A - 5*B + 2*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d) - ((7*A - 4*B + C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.450533, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(10A-5B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B+C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(10A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```


```
[Out] -(((7*A - 4*B + C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((10*A - 5*B + 2*C
)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((10*A - 5*B + 2*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d) - ((7*A - 4*B + C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(5A-\right.}{a}$$

$$=-\frac{(7A-4B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B+C)\cos^{\frac{5}{2}}}{3d(a+a\,c}$$

$$=-\frac{(7A-4B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B+C)\cos^{\frac{5}{2}}}{3d(a+a\,c}$$

$$=-\frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(10A-5B+2C)\sqrt{\cos(c-}}{3a^2d}$$

$$=-\frac{(7A-4B+C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(10A-5B+2C)F\left(\frac{1}{2}(c+\right.}{3a^2d}$$

Mathematica [C] time = 6.94298, size = 2064, normalized size = 11.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] ((-7*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[
c + d*x])^2) + ((4*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*
c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
+ E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^2) - (I*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1
/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)
*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((
2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/
2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c
] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (40*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) +
(20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (8*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos
[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((8*(3*A - 2*B + C + 4*A*Cos[c] - 2*B*Cos[c])*Csc[c])/d + (16*A*Cos[d*x]*
Sin[c])/(3*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 2*B*Sin[
(d*x)/2] + C*Sin[(d*x)/2]))/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (16*A*Cos[c]*Sin[d*x])/(3
*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.996, size = 472, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(10*A*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+21*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2
*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(10*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+21*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)+16*A*sin(1/2*d*x+1/2*c)^8+(-76*A+24*B-12*C)*sin(1/2*d*x+1/2*c
)^6+(84*A-34*B+16*C)*sin(1/2*d*x+1/2*c)^4+(-25*A+11*B-5*C)*sin(1/2*d*x+1/2*
c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^2 + B\cos\left(dx+c\right)\sec\left(dx+c\right) + A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2 + 2\,a^2\sec\left(dx+c\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) +
a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^2, x)
```

3.1227 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{(5A-2B-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(5A-2B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B$$

```
[Out] ((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - 2*B - C)*EllipticF[
(c + d*x)/2, 2])/(3*a^2*d) - ((5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.40982, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2977, 2748, 2641, 2639}

$$-\frac{(5A-2B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-2B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B+C)\sin}{3d(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^2,x]
```


```
[Out] ((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - 2*B - C)*EllipticF[
(c + d*x)/2, 2])/(3*a^2*d) - ((5*A - 2*B - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a\right.}{}}{}$$

$$= -\frac{(5A-2B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\,c}{3d(a\,+}$$

$$= -\frac{(5A-2B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\,c}{3d(a\,+}$$

$$= \frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3a^2d}$$

Mathematica [C] time = 6.77093, size = 1628, normalized size = 11.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^2,x]
```

```
[Out] ((4*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((
2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(
```

```
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x)
)*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
 + d*x])^2) - (I*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*
(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2
*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)
*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^(
(2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTa
n[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (8*B*Cos[c/2 +
(d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a
*Sec[c + d*x])^2) - (4*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^
4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-8*(2*A - B
+ 2*A*Cos[c])*Csc[c])/d - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2]
- B*Sin[(d*x)/2]))/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B
*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a +
 a*Sec[c + d*x])^2)
```

Maple [B] time = 2.353, size = 509, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d
*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*
cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4-2*C*cos(1/2*d*x+1/2*c)^4+15*
```

```
A*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2+3*C*cos(1/2*d*x+1/2*c)^2-A+
B-C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx + c\right)^{2} + B \sec \left(dx + c\right) + A\right)\sqrt{\cos \left(dx + c\right)}}{\left(a \sec \left(dx + c\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C \sec \left(dx + c\right)^{2} + B \sec \left(dx + c\right) + A\right)\sqrt{\cos \left(dx + c\right)}}{a^{2} \sec \left(dx + c\right)^{2} + 2\, a^{2} \sec \left(dx + c\right) + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*se
c(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec \left(dx + c\right)^{2} + B \sec \left(dx + c\right) + A\right)\sqrt{\cos \left(dx + c\right)}}{\left(a \sec \left(dx + c\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a)^2, x)
```

3.1228 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{(2A+B+2C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B+C)}{3d(a}$$

```
[Out] -(((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A + B + 2*C)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) + ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^
2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.413859, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2978, 2748, 2641, 2639}

$$\frac{(2A+B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B+C)\sin(c+}{3d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[
c + d*x])^2),x]
```


```
[Out] -(((A - C)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A + B + 2*C)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) + ((A - C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^
2*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B-5C)+\frac{1}{2}a(5A+B-C)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}}{3a^2}$$

$$= \frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} +$$

$$= \frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} -$$

$$= -\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B+2C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(A-C)\sqrt{\cos}}{a^2d(1+}$$

Mathematica [C] time = 6.73531, size = 1620, normalized size = 12.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] ((-I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2
*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
```

```
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))
*Sin[c]))))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
+ d*x])^2) + (I*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*
c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(
-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((
2*I)*d*x))*Sin[c]))))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
+ a*Sec[c + d*x])^2) - (8*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (4*B*Cos[c/2 + (d
*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*S
ec[c + d*x])^2) - (8*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*
Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*(A - C)*Csc[
c])/d + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/d
 - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Si
n[(d*x)/2]))/(3*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/
((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 2.559, size = 509, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3-12*C*cos(1/2*d*x+1/2*c)^6+4*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*C*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*c
os(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+16*C*cos(1/2*d*x+1/2*c)^4+9*A*
cos(1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-3*C*cos(1/2*d*x+1/2*c)^2-A+B-
C)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
```

```
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*s
qrt(cos(d*x + c))), x)
```

3.1229 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{(A+2B-5C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A+2B-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}+\frac{(B-4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(B-4C)}{a^2d\sqrt{\cos}}$$

```
[Out] ((B - 4*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A + 2*B - 5*C)*EllipticF[
(c + d*x)/2, 2])/(3*a^2*d) - ((B - 4*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d
*x]]) + ((A + 2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos
[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^2)
```

Rubi [A] time = 0.439494, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2978, 2748, 2636, 2639, 2641}

$$\frac{(A+2B-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A+2B-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}+\frac{(B-4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(B-4C)\sin(c+}{a^2d\sqrt{\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^2),x]
```

```
[Out] ((B - 4*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A + 2*B - 5*C)*EllipticF[
(c + d*x)/2, 2])/(3*a^2*d) - ((B - 4*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d
*x]]) + ((A + 2*B - 5*C)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos
[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^2)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx=\int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$=-\frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{1}{2}a(A-B+7C)+\frac{3}{2}a(A+B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$=\frac{(A+2B-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}-\frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}+$$

$$=\frac{(A+2B-5C)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))}-\frac{(A-B+C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}+$$

$$=\frac{(A+2B-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(B-4C)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(A+2B-5C)}{3a^2d\sqrt{\cos(c+dx)}(}$$

$$=\frac{(B-4C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A+2B-5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(B-4C)\sin(c}{a^2d\sqrt{\cos(c+}}$$

Mathematica [C] time = 6.89434, size = 1660, normalized size = 9.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] (I*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d
*x])^2) - ((4*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*
(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2
*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)
*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^(
(2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^2) - (4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (8*B*Cos[c/2 + (
d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*
Sec[c + d*x])^2) + (20*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot
[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
+ Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^
4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(2*C - B*C
os[c] + 2*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*
x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (8*Sec[
c/2]*Sec[c/2 + (d*x)/2]*(-(B*Sin[(d*x)/2]) + 2*C*Sin[(d*x)/2]))/d + (16*C*S
ec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/
2])/(3*d)))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c
 + d*x])^2)
```

Maple [B] time = 6.183, size = 559, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(-2*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-5*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*C*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(B-4*C)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(A-10*B+43*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-7*B+37*C)*sin(1/2*d*x+1/2*c)^2)/sin
(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*
x+1/2*c)^2-1)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^2*sec(d*x + c) + a^2*cos(d
*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*c
os(d*x + c)^(3/2)), x)
```

3.1230 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{(2A-5B+10C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{(2A}{}$$

```
[Out] ((A - 4*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A - 5*B + 10*C)*E
llipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B + 10*C)*Sin[c + d*x])/(3*
a^2*d*Cos[c + d*x]^(3/2)) - ((A - 4*B + 7*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[
c + d*x]]) - ((A - 4*B + 7*C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
+ Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.468153, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2978, 2748, 2636, 2641, 2639}

$$\frac{(2A-5B+10C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{(2A-5B+}{3a^2d\,\text{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^2),x]
```


```
[Out] ((A - 4*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A - 5*B + 10*C)*E
llipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B + 10*C)*Sin[c + d*x])/(3*
a^2*d*Cos[c + d*x]^(3/2)) - ((A - 4*B + 7*C)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[
c + d*x]]) - ((A - 4*B + 7*C)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
+ Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a +
a*Cos[c + d*x])^2)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(A-B+3C)+\frac{1}{2}a(A+5B-5C)\cos(c+dx}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}}{3a^2}$$

$$= -\frac{(A-4B+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{(A-4B+7C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{(2A-5B+10C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(A-4B+7C)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(A-4B}{3a^2d\cos^{\frac{3}{2}}(c}$$

$$= \frac{(A-4B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B+10C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(2A}{}$$

Mathematica [C] time = 7.49825, size = 2107, normalized size = 9.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] (I*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d
*x])^2) - ((4*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*
(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2
*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)
*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^(
(2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^2) + ((7*I)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + a*Sec[c + d*x])^2) - (8*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (2
0*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) - (40*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[
c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*((-4*(-2*B + 4*C + A*Cos[c] - 2*B*Cos[c] + 3*C*Cos[c])*Csc[c/2]*Sec[c/2]*S
ec[c])/d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2
```

```
] + C*Sin[(d*x)/2]))/(3*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2]
 - 2*B*Sin[(d*x)/2] + 3*C*Sin[(d*x)/2]))/d + (16*C*Sec[c]*Sec[c + d*x]^2*Si
n[d*x])/(3*d) + (16*Sec[c]*Sec[c + d*x]*(C*Sin[c] + 3*B*Sin[d*x] - 6*C*Sin[
d*x]))/(3*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 9.138, size = 751, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A-
B+C)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2
*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1)
+4*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-2*B+4*C)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(4*B-8*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^3*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^3*sec(d*x + c) + a^2*cos(d
*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*c
os(d*x + c)^(5/2)), x)
```

3.1231 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=250

$$-\frac{5(A-2B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{(20A-35B+56C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A-2B+3C)\sin(c+dx)}{a^2d\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)+1)}$$

```
[Out] -((20*A - 35*B + 56*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(A - 2*B +
 3*C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((20*A - 35*B + 56*C)*Sin[c +
d*x])/(15*a^2*d*Cos[c + d*x]^(5/2)) - (5*(A - 2*B + 3*C)*Sin[c + d*x])/(3*a
^2*d*Cos[c + d*x]^(3/2)) + ((20*A - 35*B + 56*C)*Sin[c + d*x])/(5*a^2*d*Sqr
t[Cos[c + d*x]]) - ((A - 2*B + 3*C)*Sin[c + d*x])/(a^2*d*Cos[c + d*x]^(5/2)
*(1 + Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*(
a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.48978, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2978, 2748, 2636, 2639, 2641}

$$-\frac{5(A-2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(20A-35B+56C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A-2B+3C)\sin(c+dx)}{a^2d\cos^{\frac{5}{2}}(c+dx)(\cos(c+dx)+1)}-\frac{5(A-}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Sec[
c + d*x])^2),x]
```

```
[Out] -((20*A - 35*B + 56*C)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(A - 2*B +
 3*C)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + ((20*A - 35*B + 56*C)*Sin[c +
d*x])/(15*a^2*d*Cos[c + d*x]^(5/2)) - (5*(A - 2*B + 3*C)*Sin[c + d*x])/(3*a
^2*d*Cos[c + d*x]^(3/2)) + ((20*A - 35*B + 56*C)*Sin[c + d*x])/(5*a^2*d*Sqr
t[Cos[c + d*x]]) - ((A - 2*B + 3*C)*Sin[c + d*x])/(a^2*d*Cos[c + d*x]^(5/2)
*(1 + Cos[c + d*x])) - ((A - B + C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*(
a + a*Cos[c + d*x])^2)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(5A-5B+11C)-\frac{1}{2}a(A-7B+7C)\cos(c\cdot}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))}}{3a^2}$$

$$= -\frac{(A-2B+3C)\sin(c+dx)}{a^2d\cos^{\frac{5}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{(A-2B+3C)\sin(c+dx)}{a^2d\cos^{\frac{5}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B+C)\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= \frac{(20A-35B+56C)\sin(c+dx)}{15a^2d\cos^{\frac{5}{2}}(c+dx)} - \frac{5(A-2B+3C)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(A-}{a^2d\cos^{\frac{5}{2}}}$$

$$= -\frac{5(A-2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(20A-35B+56C)\sin(c+dx)}{15a^2d\cos^{\frac{5}{2}}(c+dx)} - 5($$

$$= -\frac{(20A-35B+56C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(A-2B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}$$

Mathematica [C] time = 8.1532, size = 2164, normalized size = 8.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
a*Sec[c + d*x])^2),x]
```

```
[Out] ((-4*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[
c + d*x])^2) + ((7*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*
c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
+ E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])*(a + a*Sec[c + d*x])^2) - (((56*I)/5)*C*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Se
c[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeome
```

```
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/2]^4*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])^2) - (40*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (20*C*Cos[c/2 + (d*x)/2]^4*Csc
[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec
[c/2]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2
) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*((4*(10*A - 20*B + 36*C + 10*A*Cos[c] - 15*B*Cos[c] + 20*C*Cos[c]
)*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin
[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(3*d) + (8*Sec[c/2]*Sec[c/2 +
 (d*x)/2]*(2*A*Sin[(d*x)/2] - 3*B*Sin[(d*x)/2] + 4*C*Sin[(d*x)/2]))/d + (16
*C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) - (16*Sec[c]*Sec[c + d*x]*(-5*B*Si
n[c] + 10*C*Sin[c] - 15*A*Sin[d*x] + 30*B*Sin[d*x] - 54*C*Sin[d*x]))/(15*d)
 + (16*Sec[c]*Sec[c + d*x]^2*(3*C*Sin[c] + 5*B*Sin[d*x] - 10*C*Sin[d*x]))/(
15*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 10.952, size = 1072, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-A
+B-C)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/
2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2-1
)+(4*B-8*C)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*C/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
```

```
)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+(-2*A+4*B-6*C)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)+(4*A-8*B+12*C)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*co
s(d*x + c)^4*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^4*sec(d*x + c) + a^2*cos(d
*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*c
os(d*x + c)^(7/2)), x)
```

3.1232 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=273

$$-\frac{(63A-33B+13C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{7(33A-17B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(63A-33B+13C)\sin(c+dx)}{10d\left(a^3\cos(c+dx)+\right.}$$

```
[Out] (7*(33*A - 17*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((63*A - 33*
B + 13*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((63*A - 33*B + 13*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(6*a^3*d) + (7*(33*A - 17*B + 7*C)*Cos[c + d*x
]^(3/2)*Sin[c + d*x])/(30*a^3*d) - ((A - B + C)*Cos[c + d*x]^(9/2)*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((12*A - 7*B + 2*C)*Cos[c + d*x]^(7/2)
*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((63*A - 33*B + 13*C)*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.659956, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2748, 2635, 2641, 2639}

$$-\frac{(63A-33B+13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(33A-17B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(63A-33B+13C)\sin(c+dx)\cos^{\frac{5}{2}}(c}{10d\left(a^3\cos(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```


```
[Out] (7*(33*A - 17*B + 7*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((63*A - 33*
B + 13*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((63*A - 33*B + 13*C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(6*a^3*d) + (7*(33*A - 17*B + 7*C)*Cos[c + d*x
]^(3/2)*Sin[c + d*x])/(30*a^3*d) - ((A - B + C)*Cos[c + d*x]^(9/2)*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((12*A - 7*B + 2*C)*Cos[c + d*x]^(7/2)
*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((63*A - 33*B + 13*C)*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(-\frac{1}{2}a(\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(12A-7B+2C}{15ad(a}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(12A-7B+2C}{15ad(a}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(12A-7B+2C}{15ad(a}$$

$$= -\frac{(63A-33B+13C)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d} + \frac{7(33A-}{}$$

$$= \frac{7(33A-17B+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(63A-33B+13C)}{6a^3}$$

Mathematica [C] time = 7.4036, size = 2257, normalized size = 8.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (((231*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/
4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[
c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*
(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])^3) - (((119*I)/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*
I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2
*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*
Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(
I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*
d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (((49*I)/
5)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
```

```
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*
c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(
-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((
2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
+ a*Sec[c + d*x])^3) + (84*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (44*B
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (52*C*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec
[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*((-8*(99*A - 59*B + 29*C + 132*A*Cos[c] - 60*B*Cos[c] + 20*C*Cos[c])*Csc
[c])/(5*d) - (32*(3*A - B)*Cos[d*x]*Sin[c])/(3*d) + (16*A*Cos[2*d*x]*Sin[2*
c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/
2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(24*A*Sin[(d
*x)/2] - 19*B*Sin[(d*x)/2] + 14*C*Sin[(d*x)/2]))/(15*d) - (8*Sec[c/2]*Sec[c
/2 + (d*x)/2]*(99*A*Sin[(d*x)/2] - 59*B*Sin[(d*x)/2] + 29*C*Sin[(d*x)/2]))/
(5*d) - (32*(3*A - B)*Cos[c]*Sin[d*x])/(3*d) + (16*A*Cos[2*c]*Sin[2*d*x])/(
5*d) + (8*(24*A - 19*B + 14*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(
A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 3.01, size = 666, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*cos(1/
2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10+160*B*cos(1/2*d*x+1/2*c)^10-228
*A*cos(1/2*d*x+1/2*c)^8-630*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5-1386*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*B*cos(1/2*d*x+1
/2*c)^8+330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+714*B*cos(1/2*
d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*C*cos(1/2*d*x+1/2*c)^8-130*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*C*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
```

```
2*d*x+1/2*c),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6-1058*B*cos(1/2*d*x+1/2*c)
^6+578*C*cos(1/2*d*x+1/2*c)^6-744*A*cos(1/2*d*x+1/2*c)^4+474*B*cos(1/2*d*x+
1/2*c)^4-264*C*cos(1/2*d*x+1/2*c)^4+57*A*cos(1/2*d*x+1/2*c)^2-47*B*cos(1/2*
d*x+1/2*c)^2+37*C*cos(1/2*d*x+1/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^
5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x +
 c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^3, x)
```

3.1233 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=234

$$\frac{(33A-13B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A-49B+9C)\sin(c+dx)\cos}{30d\left(a^3\cos(c+dx)+a^3\right.}$$

```
[Out] -((119*A - 49*B + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*
B + 3*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B + 3*C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[
c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((2*A - B)*Cos[c + d*x]^(5/2)*Sin[
c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((119*A - 49*B + 9*C)*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.631445, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(33A-13B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(119A-49B+9C)\sin(c+dx)\cos^{\frac{3}{2}}(c+}{30d\left(a^3\cos(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```

```
[Out] -((119*A - 49*B + 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*
B + 3*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B + 3*C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(7/2)*Sin[
c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((2*A - B)*Cos[c + d*x]^(5/2)*Sin[
c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - ((119*A - 49*B + 9*C)*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{1}{2}a(\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c}{3ad(a+a\,c}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c}{3ad(a+a\,c}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c}{3ad(a+a\,c}$$

$$= -\frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B+3C}{}$$

$$= -\frac{(119A-49B+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B+3C}{6a}$$

Mathematica [C] time = 7.2029, size = 2206, normalized size = 9.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (((-119*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3
/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x)
)*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*
d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos
[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d
*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(a + a*Sec[c + d*x])^3) + (((49*I)/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*
I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2
*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*
Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(
I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*
d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (((9*I)/5
)*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (
```

```
2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c
] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-
1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2
*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a +
 a*Sec[c + d*x])^3) - (44*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (52*B*
Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sq
rt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c
 + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*((8*(59*A - 29*B + 9*C + 60*A*Cos[c] - 20*B*Cos[c])*Csc[c])/(5*d) + (32*A*
Cos[d*x]*Sin[c])/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] -
 B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(
59*A*Sin[(d*x)/2] - 29*B*Sin[(d*x)/2] + 9*C*Sin[(d*x)/2]))/(5*d) - (8*Sec[c
/2]*Sec[c/2 + (d*x)/2]^3*(19*A*Sin[(d*x)/2] - 14*B*Sin[(d*x)/2] + 9*C*Sin[(
d*x)/2]))/(15*d) + (32*A*Cos[c]*Sin[d*x])/(3*d) - (8*(19*A - 14*B + 9*C)*Se
c[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*T
an[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.836, size = 638, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*cos(1/
2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
48*B*cos(1/2*d*x+1/2*c)^8-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*C*cos(1/2*d*x+
1/2*c)^8+30*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*C*cos(1/2*d
*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^6+578*B*cos
(1/2*d*x+1/2*c)^6-198*C*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-264
```

```
*B*cos(1/2*d*x+1/2*c)^4+114*C*cos(1/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^
2+37*B*cos(1/2*d*x+1/2*c)^2-27*C*cos(1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2
+ 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^3, x)
```

3.1234 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{(13A-3B-C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(49A-9B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B-C)\sin(c+dx)\sqrt{\cos(c+d}}{6d\left(a^3\cos(c+dx)+a^3\right)}$$

```
[Out] ((49*A - 9*B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B - C)
*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B - 2*C)*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B - C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.61, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2977, 2748, 2641, 2639}

$$-\frac{(13A-3B-C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^3,x]
```


```
[Out] ((49*A - 9*B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B - C)
*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B - 2*C)*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B - C)*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-\right.}{(a\cdot}}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\,\mathrm{co}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\,\mathrm{co}}{15ad(a+a}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B-2C)\,\mathrm{co}}{15ad(a+a}$$

$$= \frac{(49A-9B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B-C)F\left(\frac{1}{2}(c+\right.}{6a^3d}$$

Mathematica [C] time = 7.10289, size = 2175, normalized size = 10.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^3,x]
```

```
[Out] (((49*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c
] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(
-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*(a + a*Sec[c + d*x])^3) - (((9*I)/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*
Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*
d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/
E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3
*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(
1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - ((I/5)*C*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[
c + d*x])^3) + (52*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*B*Cos[c/
2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + C
ot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
 + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x
])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-8*
(29*A - 9*B - C + 20*A*Cos[c])*Csc[c])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/
2]*(29*A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(5*d) - (4*Sec[
c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]
))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(14*A*Sin[(d*x)/2] - 9*B*Sin[(d
*x)/2] + 4*C*Sin[(d*x)/2]))/(15*d) + (8*(14*A - 9*B + 4*C)*Sec[c/2 + (d*x)/
2]^2*Tan[c/2])/(15*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)
))/(Sqrt[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a
 + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.704, size = 624, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos(1/2
*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8-30*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*B*cos(1/2*d*x+1/2*c)^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8-10*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+1/2*c)^6+2*C*cos(1/2*d*x+
1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4+24*C*cos(1/2
*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2-17*C*cos(
1/2*d*x+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*se
c(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a)^3, x)
```

3.1235 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{(3A+B+C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(9A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A+B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B\cdots}{}$$

```
[Out] -((9*A + B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B + C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((6*A - B - 4*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((9*A + B - C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.593453, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2978, 2748, 2641, 2639}

$$\frac{(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A+B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B+C)\,\mathrm{s}\cdots}{5d(a\,\mathrm{c}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] -((9*A + B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B + C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((6*A - B - 4*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((9*A + B - C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{1}{2}a(3A-3B-7C)+\frac{1}{2}a(9A+B\cdots\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B-4C)\sqrt{\cos(c+dx)}\sin(c\cdots}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B-4C)\sqrt{\cos(c+dx)}\sin(c\cdots}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B-4C)\sqrt{\cos(c+dx)}\sin(c\cdots}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(9A+B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+B+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A-B+\cdots}{5\cdots}$$

Mathematica [C] time = 6.95539, size = 2167, normalized size = 11.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^3),x]
```

```
[Out] (((-9*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c
] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(
-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*(a + a*Sec[c + d*x])^3) - ((I/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[
c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + ((I/5)*C*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
```

```
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
d*x])^3) - (4*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*B*Cos[c/2 + (d*
x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]
^2]*(a + a*Sec[c + d*x])^3) - (4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*
x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3)
 + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*(9*A +
 B - C)*Csc[c])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2]
- 4*B*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(15*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/
2]*(9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2
]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/
(5*d) - (8*(9*A - 4*B - C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (4*(A -
B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.575, size = 624, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8+10*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-12*C*cos(1/2*d*x+1/2*c)^8+10*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*cos(1/2*d*x+1/2*c)^5-6*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-198*A*cos(1/2*d*x+1/2*c)^6-2*B*cos(1/2*d*x+1/2*c)^6+22*C*cos(1/2*d*x+1/2*
c)^6+114*A*cos(1/2*d*x+1/2*c)^4-24*B*cos(1/2*d*x+1/2*c)^4-6*C*cos(1/2*d*x+1
/2*c)^4-27*A*cos(1/2*d*x+1/2*c)^2+17*B*cos(1/2*d*x+1/2*c)^2-7*C*cos(1/2*d*x
+1/2*c)^2+3*A-3*B+3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)+a^{3}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)*sec(d*x + c)^2 + 3*a^3*cos(d
*x + c)*sec(d*x + c) + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*s
qrt(cos(d*x + c))), x)
```

3.1236 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=191

$$\frac{(A+B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{(A-B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(4A}{}$$

```
[Out] -((A - B - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B + 3*C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A + B - 6*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((A - B - 9*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.59097, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2978, 2748, 2641, 2639}

$$\frac{(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B-9C)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(4A+B-}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] -((A - B - 9*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B + 3*C)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A + B - 6*C)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((A - B - 9*C)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx \\
&= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{-\frac{1}{2}a(A-B-9C)+\frac{1}{2}a(7A+3B-3C)\cos(c+dx}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}}{5a^2} \\
&= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B-6C)\sqrt{\cos(c+dx)}\sin(c}{15ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B-6C)\sqrt{\cos(c+dx)}\sin(c}{15ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B-6C)\sqrt{\cos(c+dx)}\sin(c}{15ad(a+a\cos(c+dx))^2} \\
&= -\frac{(A-B-9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B+}{5a}
\end{aligned}
$$

Mathematica [C] time = 6.93956, size = 2164, normalized size = 11.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^3),x]
```

```
[Out] ((-I/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (
2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c
] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
 E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*(a + a*Sec[c + d*x])^3) + ((I/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]
*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (((9*I)/5)*C*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c +
d*x])^3) - (4*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*B*Cos[c/2 + (
d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[
c]^2]*(a + a*Sec[c + d*x])^3) - (4*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c +
d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3)
 + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((8*(A - B
 - 9*C)*Csc[c])/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*
Sin[(d*x)/2] - 9*C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*
(4*A*Sin[(d*x)/2] + B*Sin[(d*x)/2] - 6*C*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2
]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/
(5*d) + (8*(4*A + B - 6*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A -
B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 2.629, size = 624, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8+10*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-108*C*cos(1/2*d*x+1/2*c)^8+30*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)^5-54*C*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-2*A*cos(1/2*d*x+1/2*c)^6+22*B*cos(1/2*d*x+1/2*c)^6+138*C*cos(1/2*d*x+1/2*
c)^6-24*A*cos(1/2*d*x+1/2*c)^4-6*B*cos(1/2*d*x+1/2*c)^4-24*C*cos(1/2*d*x+1/
2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-7*B*cos(1/2*d*x+1/2*c)^2-3*C*cos(1/2*d*x+1
/2*c)^2-3*A+3*B-3*C)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a^{3}\cos\left(dx+c\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^2*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^2*sec(d*x + c)^2 + 3*a^3*c
os(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + B \sec (dx+c) + A}{(a \sec (dx+c) + a)^3 \cos (dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*c
os(d*x + c)^(3/2)), x)
```

3.1237 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$\frac{(A+3B-13C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(A+9B-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A+9B-49C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(A+3}{6d\sqrt{\cos(c+}}$$

```
[Out] ((A + 9*B - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B - 13*C)
*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A + 9*B - 49*C)*Sin[c + d*x])/(10
*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B - 8*C)*Sin[c + d*x])/(15*a*d*Sqr
t[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) + ((A + 3*B - 13*C)*Sin[c + d*x])/(
6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.633121, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2978, 2748, 2636, 2639, 2641}

$$\frac{(A+3B-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A+9B-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A+9B-49C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(A+3B-13C}{6d\sqrt{\cos(c+dx)}\left(a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] ((A + 9*B - 49*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B - 13*C)
*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((A + 9*B - 49*C)*Sin[c + d*x])/(10
*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B - 8*C)*Sin[c + d*x])/(15*a*d*Sqr
t[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) + ((A + 3*B - 13*C)*Sin[c + d*x])/(
6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A-B+11C)+\frac{5}{2}a(A+B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{(2A+3B-8C)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= \frac{(A+3B-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+9B-49C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A-}{5d\sqrt{\cos(c}}$$

$$= \frac{(A+9B-49C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B-13C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A+9}{10}$$

Mathematica [C] time = 7.2335, size = 2205, normalized size = 9.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^3),x]
```

```
[Out] ((I/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3
*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2
*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
 + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])^3) + (((9*I)/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c
/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) - (((49*I)/5)*C*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
```

```
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[
c + d*x])^3) - (4*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (4*B*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^3) + (52*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
 + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x
])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(
20*C - A*Cos[c] - 9*B*Cos[c] + 29*C*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d)
 - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2] - 29*C
*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] -
B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*
(A*Sin[(d*x)/2] - 6*B*Sin[(d*x)/2] + 11*C*Sin[(d*x)/2]))/(15*d) + (32*C*Sec
[c]*Sec[c + d*x]*Sin[d*x])/d + (8*(A - 6*B + 11*C)*Sec[c/2 + (d*x)/2]^2*Tan
[c/2])/(15*d) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt
[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec
[c + d*x])^3)
```

Maple [B] time = 3.202, size = 789, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+147*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(
```

```
1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-65*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*C*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(A+9*B-49*C)*sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(13*A+147*B-817*C)*sin(1/2*d*x+1/2*c)^6+6*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+43*B-248*C)*sin(1/2*d
*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+69*B-43
9*C)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+a^{3}\cos\left(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^3*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^3*sec(d*x + c)^2 + 3*a^3*c
os(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*c
os(d*x + c)^(5/2)), x)
```

3.1238 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=268

$$\frac{(3A-13B+33C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B+119C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right.}$$

```
[Out] ((9*A - 49*B + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B
+ 33*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((3*A - 13*B + 33*C)*Sin[c +
 d*x])/(6*a^3*d*Cos[c + d*x]^(3/2)) - ((9*A - 49*B + 119*C)*Sin[c + d*x])/(
10*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^3) + ((B - 2*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*
x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((9*A - 49*B + 119*C)*Sin[c + d*x])/(30*
d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.664676, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2978, 2748, 2636, 2641, 2639}

$$\frac{(3A-13B+33C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B+119C)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Sec[
c + d*x])^3),x]
```


```
[Out] ((9*A - 49*B + 119*C)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B
+ 33*C)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((3*A - 13*B + 33*C)*Sin[c +
 d*x])/(6*a^3*d*Cos[c + d*x]^(3/2)) - ((9*A - 49*B + 119*C)*Sin[c + d*x])/(
10*a^3*d*Sqrt[Cos[c + d*x]]) - ((A - B + C)*Sin[c + d*x])/(5*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^3) + ((B - 2*C)*Sin[c + d*x])/(3*a*d*Cos[c + d*
x]^(3/2)*(a + a*Cos[c + d*x])^2) - ((9*A - 49*B + 119*C)*Sin[c + d*x])/(30*
d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(3A-3B+13C)+\frac{1}{2}a(3A+7B-7C)\cos(c+dx}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{(B-2C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{(B-2C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{(B-2C)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= \frac{(3A-13B+33C)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{(9A-49B+119C)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A}{5d\cos^{\frac{3}{2}}(}$$

$$= \frac{(9A-49B+119C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B+33C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} +$$

Mathematica [C] time = 8.02092, size = 2248, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
a*Sec[c + d*x])^3),x]
```

```
[Out] (((9*I)/5)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
-(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
+ (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[
2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-
1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*(a + a*Sec[c + d*x])^3) - (((49*I)/5)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*
Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*
d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/
E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3
*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(
1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3) + (((119*I)/5)
```

```
*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2
*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
 + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1
 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*
I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a +
a*Sec[c + d*x])^3) - (4*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (52*B*Co
s[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(3*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (44*C*Cos[c/2 + (d*x)/2]^6*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*Sec[c + d*x]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c
+ d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((-4*(-20*B + 60*C + 9*A*Cos[c] - 29*B*Cos[c] + 59*C*Cos[c])*Csc[c/2]*Sec[c
/2]*Sec[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Si
n[(d*x)/2] + C*Sin[(d*x)/2]))/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(6*A
*Sin[(d*x)/2] - 11*B*Sin[(d*x)/2] + 16*C*Sin[(d*x)/2]))/(15*d) - (8*Sec[c/2
]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] - 29*B*Sin[(d*x)/2] + 59*C*Sin[(d*x)
/2]))/(5*d) + (32*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (32*Sec[c]*Sec[
c + d*x]*(C*Sin[c] + 3*B*Sin[d*x] - 9*C*Sin[d*x]))/(3*d) - (8*(6*A - 11*B +
 16*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (4*(A - B + C)*Sec[c/2 + (d*
x)/2]^4*Tan[c/2])/(5*d)))/(Sqrt[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] time = 10.748, size = 1040, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/3*(-2
*B+4*C)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+
1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(sin(1/2*d*x+1/2*c)^2
-1)+8*C*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
```

```
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-4*B+12*C)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(A-B+C)*(1/5*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+4/5*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+18/5*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-8/5*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18/5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+(8*B-24*C)*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+a^{3}\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*co
s(d*x + c)^4*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x + c)^2 + 3*a^3*c
os(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*c
os(d*x + c)^(7/2)), x)
```

3.1239 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=278

$$\frac{(339A-108B+17C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{42a^4d}-\frac{(176A-57B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(43A-15B+C)\sin(c+dx)\cos^{\frac{5}{2}}}{42a^4d(\cos(c+dx)+1)^2}$$

```
[Out] -((176*A - 57*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((339*A - 10
8*B + 17*C)*EllipticF[(c + d*x)/2, 2])/(42*a^4*d) + ((339*A - 108*B + 17*C)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(42*a^4*d) - ((43*A - 15*B + C)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(42*a^4*d*(1 + Cos[c + d*x])^2) - ((176*A - 57*B +
8*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*a^4*d*(1 + Cos[c + d*x])) - ((A -
 B + C)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((1
3*A - 6*B - C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x]
)^3)
```

Rubi [A] time = 0.832665, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2748, 2639, 2635, 2641}

$$\frac{(339A-108B+17C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}-\frac{(176A-57B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(43A-15B+C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx}{42a^4d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^4,x]
```


```
[Out] -((176*A - 57*B + 8*C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((339*A - 10
8*B + 17*C)*EllipticF[(c + d*x)/2, 2])/(42*a^4*d) + ((339*A - 108*B + 17*C)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(42*a^4*d) - ((43*A - 15*B + C)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(42*a^4*d*(1 + Cos[c + d*x])^2) - ((176*A - 57*B +
8*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*a^4*d*(1 + Cos[c + d*x])) - ((A -
 B + C)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((1
3*A - 6*B - C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x]
)^3)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
```

```
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}dx$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \int \frac{\cos^{\frac{7}{2}}(c+dx)\left(-\frac{1}{2}a(9A-\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(13A-6B-C)\cos}{35ad(a+a}$$

$$= -\frac{(43A-15B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{42a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos}{7d(a+}$$

$$= -\frac{(43A-15B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{42a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos}{7d(a+}$$

$$= -\frac{(43A-15B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{42a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos}{7d(a+}$$

$$= -\frac{(176A-57B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d} + \frac{(339A-108B+17C}{}$$

$$= -\frac{(176A-57B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d} + \frac{(339A-108B+17C}{42a^4}$$

Mathematica [C] time = 7.59928, size = 2319, normalized size = 8.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^4,x]
```

```
[Out] (((-352*I)/5)*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2,
 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*
x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I
)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*C
os[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2,
3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Co
s[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)
*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] +
 d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])*(a + a*Sec[c + d*x])^4) + (((114*I)/5)*B*Cos[c/2 + (d*x)/2]^8*Csc
[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (((1
```

```
6*I)/5)*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Co
s[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)
*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
- 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*(a + a*Sec[c + d*x])^4) - (904*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c +
d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]
)^4) + (288*B*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) - (136*C*Cos[c/2 +
 (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 +
 Cot[c]^2]*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]^8*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*((16*(96*A - 37*B + 8*C + 80*A*Cos[c] - 20*B*Cos[c]
)*Csc[c])/(5*d) + (64*A*Cos[d*x]*Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x
)/2]^7*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(7*d) + (16*Sec[
c/2]*Sec[c/2 + (d*x)/2]*(96*A*Sin[(d*x)/2] - 37*B*Sin[(d*x)/2] + 8*C*Sin[(d
*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(33*A*Sin[(d*x)/2] - 26*B
*Sin[(d*x)/2] + 19*C*Sin[(d*x)/2]))/(35*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]
^3*(629*A*Sin[(d*x)/2] - 363*B*Sin[(d*x)/2] + 167*C*Sin[(d*x)/2]))/(105*d)
+ (64*A*Cos[c]*Sin[d*x])/(3*d) - (8*(629*A - 363*B + 167*C)*Sec[c/2 + (d*x)
/2]^2*Tan[c/2])/(105*d) + (8*(33*A - 26*B + 19*C)*Sec[c/2 + (d*x)/2]^4*Tan[
c/2])/(35*d) - (4*(A - B + C)*Sec[c/2 + (d*x)/2]^6*Tan[c/2])/(7*d)))/(Cos[c
 + d*x]^(3/2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[
c + d*x])^4)
```

Maple [B] time = 3.224, size = 680, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/840*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-15*A+15*B-
15*C+1902*C*cos(1/2*d*x+1/2*c)^6+1344*C*cos(1/2*d*x+1/2*c)^10+12234*A*cos(1
/2*d*x+1/2*c)^6-5598*B*cos(1/2*d*x+1/2*c)^6-1882*A*cos(1/2*d*x+1/2*c)^4+122
4*B*cos(1/2*d*x+1/2*c)^4-706*C*cos(1/2*d*x+1/2*c)^4+243*A*cos(1/2*d*x+1/2*c
)^2-201*B*cos(1/2*d*x+1/2*c)^2+159*C*cos(1/2*d*x+1/2*c)^2+2240*A*cos(1/2*d*
x+1/2*c)^12+12768*A*cos(1/2*d*x+1/2*c)^10-6216*B*cos(1/2*d*x+1/2*c)^10-2558
8*A*cos(1/2*d*x+1/2*c)^8+10776*B*cos(1/2*d*x+1/2*c)^8-2684*C*cos(1/2*d*x+1/
2*c)^8+6780*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
```

```
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7+14784*A*cos(1/
2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2160*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*c
os(1/2*d*x+1/2*c)^7-4788*B*cos(1/2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3
40*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7+672*C*cos(1/2*d*x+1/2*c
)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/a^4/cos(1/2*d*x+1/2*c)^7/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{4}\sec\left(dx+c\right)^{4}+4\,a^{4}\sec\left(dx+c\right)^{3}+6\,a^{4}\sec\left(dx+c\right)^{2}+4\,a^{4}\sec\left(dx+c\right)+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^4,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(cos(d*x + c))/(a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3
+ 6*a^4*sec(d*x + c)^2 + 4*a^4*sec(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^4,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^4, x)
```

3.1240 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=244

$$-\frac{(108A-17B-4C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{42a^4d}+\frac{(57A-8B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(141A-29B-13C)\sin(c+dx)\cos^{\frac{3}{2}}(\cdots}{210a^4d(\cos(c+dx)+1)^2}$$

```
[Out] ((57*A - 8*B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) - ((108*A - 17*B -
4*C)*EllipticF[(c + d*x)/2, 2])/(42*a^4*d) - ((141*A - 29*B - 13*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) - ((108*A - 17*B
 - 4*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(42*a^4*d*(1 + Cos[c + d*x])) - ((
A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) -
((11*A - 4*B - 3*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c +
 d*x])^3)
```

Rubi [A] time = 0.790196, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2977, 2748, 2641, 2639}

$$-\frac{(108A-17B-4C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}+\frac{(57A-8B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(141A-29B-13C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{210a^4d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^4,x]
```


```
[Out] ((57*A - 8*B - C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) - ((108*A - 17*B -
4*C)*EllipticF[(c + d*x)/2, 2])/(42*a^4*d) - ((141*A - 29*B - 13*C)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) - ((108*A - 17*B
 - 4*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(42*a^4*d*(1 + Cos[c + d*x])) - ((
A - B + C)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) -
((11*A - 4*B - 3*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c +
 d*x])^3)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^4}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \cos^{\frac{5}{2}}(c+dx)\left(-\frac{7}{2}a\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(11A-4B-3C}{35ad(a}$$

$$= -\frac{(141A-29B-13C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B}{}$$

$$= -\frac{(141A-29B-13C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B}{}$$

$$= -\frac{(141A-29B-13C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B}{}$$

$$= \frac{(57A-8B-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d} - \frac{(108A-17B-4C)F\left(\right.}{42a^4d}$$

Mathematica [C] time = 7.39016, size = 2286, normalized size = 9.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^4,x]
```

```
[Out] (((114*I)/5)*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)
*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Co
s[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] +
d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*(a + a*Sec[c + d*x])^4) - (((16*I)/5)*B*Cos[c/2 + (d*x)/2]^8*Csc[c
/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^(
(2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
 (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/
E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-
I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (((2*I
)/5)*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3
*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2
*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
 + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])^4) + (288*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x
]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4
) - (136*B*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) - (32*C*Cos[c/2 + (d
*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((-16*(37*A - 8*B - C + 20*A*Cos[c])*Csc[c])/(5*d) - (
```

```
16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(37*A*Sin[(d*x)/2] - 8*B*Sin[(d*x)/2] - C*Si
n[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(A*Sin[(d*x)/2] - B*S
in[(d*x)/2] + C*Sin[(d*x)/2]))/(7*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(26
*A*Sin[(d*x)/2] - 19*B*Sin[(d*x)/2] + 12*C*Sin[(d*x)/2]))/(35*d) + (8*Sec[c
/2]*Sec[c/2 + (d*x)/2]^3*(363*A*Sin[(d*x)/2] - 167*B*Sin[(d*x)/2] + 41*C*Si
n[(d*x)/2]))/(105*d) + (8*(363*A - 167*B + 41*C)*Sec[c/2 + (d*x)/2]^2*Tan[c
/2])/(105*d) - (8*(26*A - 19*B + 12*C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(35*d
) + (4*(A - B + C)*Sec[c/2 + (d*x)/2]^6*Tan[c/2])/(7*d)))/(Cos[c + d*x]^(3/
2)*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4
)
```

Maple [B] time = 3.206, size = 666, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/840*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6216*A*cos(1
/2*d*x+1/2*c)^10+2160*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7+4788
*A*cos(1/2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1344*B*cos(1/2*d*x+1/2*c)
^10-340*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7-672*B*cos(1/2*d*x+
1/2*c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-168*C*cos(1/2*d*x+1/2*c)^10-80*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7-84*C*cos(1/2*d*x+1/2*c)^7*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-10776*A*cos(1/2*d*x+1/2*c)^8+2684*B*cos(1/2*d*x+1/2*c)^8+8
8*C*cos(1/2*d*x+1/2*c)^8+5598*A*cos(1/2*d*x+1/2*c)^6-1902*B*cos(1/2*d*x+1/2
*c)^6+306*C*cos(1/2*d*x+1/2*c)^6-1224*A*cos(1/2*d*x+1/2*c)^4+706*B*cos(1/2*
d*x+1/2*c)^4-328*C*cos(1/2*d*x+1/2*c)^4+201*A*cos(1/2*d*x+1/2*c)^2-159*B*co
s(1/2*d*x+1/2*c)^2+117*C*cos(1/2*d*x+1/2*c)^2-15*A+15*B-15*C)/a^4/cos(1/2*d
*x+1/2*c)^7/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{4}\sec\left(dx+c\right)^{4}+4\,a^{4}\sec\left(dx+c\right)^{3}+6\,a^{4}\sec\left(dx+c\right)^{2}+4\,a^{4}\sec\left(dx+c\right)+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^4,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^4*se
c(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2 + 4*a^4*sec(d*x
+ c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^4,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a)^4, x)
```

3.1241 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$\frac{(17A+4B+3C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{42a^4d}-\frac{(83A+B-15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{210a^4d(\cos(c+dx)+1)^2}-\frac{(8A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}+$$

```
[Out] -((8*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((17*A + 4*B + 3*C)*Ell
ipticF[(c + d*x)/2, 2])/(42*a^4*d) - ((83*A + B - 15*C)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) + ((8*A + B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(10*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((9*A - 2*B - 5*C)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.774119, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2978, 2748, 2641, 2639}

$$\frac{(17A+4B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}-\frac{(83A+B-15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{210a^4d(\cos(c+dx)+1)^2}-\frac{(8A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}+\frac{(8A+E}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[
c + d*x])^4),x]
```



```
[Out] -((8*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((17*A + 4*B + 3*C)*Ell
ipticF[(c + d*x)/2, 2])/(42*a^4*d) - ((83*A + B - 15*C)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) + ((8*A + B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(10*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((9*A - 2*B - 5*C)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^4}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{1}{2}a(5A-5B-9C)+\frac{1}{2}a(13}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(9A-2B-5C)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{(83A+B-15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{7d(a+a\cos(c+dx}$$

$$= -\frac{(83A+B-15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{7d(a+a\cos(c+dx}$$

$$= -\frac{(83A+B-15C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\cos^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{7d(a+a\cos(c+dx}$$

$$= -\frac{(8A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d} + \frac{(17A+4B+3C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d} - (83A+$$

Mathematica [C] time = 7.207, size = 1862, normalized size = 8.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^4),x]
```

```
[Out] (((-16*I)/5)*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)
*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Co
s[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] +
d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*(a + a*Sec[c + d*x])^4) - (((2*I)/5)*B*Cos[c/2 + (d*x)/2]^8*Csc[c/
2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((
2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
(2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I
)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (136*A*
Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
```

```
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) - (32*B*Cos[c/2 + (d*x)/2]^8*Cs
c[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21
*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a
+ a*Sec[c + d*x])^4) - (8*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) + (C
os[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((16*(8*A + B)*
Csc[c])/(5*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(8*A*Sin[(d*x)/2] + B*Sin[(
d*x)/2]))/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(167*A*Sin[(d*x)/2] - 41
*B*Sin[(d*x)/2] - 15*C*Sin[(d*x)/2]))/(105*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^7*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(7*d) + (8*Sec[c/
2]*Sec[c/2 + (d*x)/2]^5*(19*A*Sin[(d*x)/2] - 12*B*Sin[(d*x)/2] + 5*C*Sin[(d
*x)/2]))/(35*d) - (8*(167*A - 41*B - 15*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(
105*d) + (8*(19*A - 12*B + 5*C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(35*d) - (4*
(A - B + C)*Sec[c/2 + (d*x)/2]^6*Tan[c/2])/(7*d)))/(Cos[c + d*x]^(3/2)*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [B] time = 2.987, size = 595, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/840*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1344*A*cos(
1/2*d*x+1/2*c)^10+340*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7+672*
A*cos(1/2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+168*B*cos(1/2*d*x+1/2*c)^1
0+80*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7+84*B*cos(1/2*d*x+1/2*
c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+60*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1
/2*c)^7-2684*A*cos(1/2*d*x+1/2*c)^8-88*B*cos(1/2*d*x+1/2*c)^8+60*C*cos(1/2*
d*x+1/2*c)^8+1902*A*cos(1/2*d*x+1/2*c)^6-306*B*cos(1/2*d*x+1/2*c)^6-30*C*co
s(1/2*d*x+1/2*c)^6-706*A*cos(1/2*d*x+1/2*c)^4+328*B*cos(1/2*d*x+1/2*c)^4-90
*C*cos(1/2*d*x+1/2*c)^4+159*A*cos(1/2*d*x+1/2*c)^2-117*B*cos(1/2*d*x+1/2*c)
^2+75*C*cos(1/2*d*x+1/2*c)^2-15*A+15*B-15*C)/a^4/cos(1/2*d*x+1/2*c)^7/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^4\cos\left(dx+c\right)\sec\left(dx+c\right)^4+4\,a^4\cos\left(dx+c\right)\sec\left(dx+c\right)^3+6\,a^4\cos\left(dx+c\right)\sec\left(dx+c\right)^2+4\,a^4\cos\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^4,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^4*co
s(d*x + c)*sec(d*x + c)^4 + 4*a^4*cos(d*x + c)*sec(d*x + c)^3 + 6*a^4*cos(d
*x + c)*sec(d*x + c)^2 + 4*a^4*cos(d*x + c)*sec(d*x + c) + a^4*cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^4\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^4,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^4*s
qrt(cos(d*x + c))), x)
```

3.1242 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=229

$$\frac{(4A+3B+4C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{42a^4d}+\frac{(41A+15B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{210a^4d(\cos(c+dx)+1)^2}-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}+\frac{(A-}{}$$

```
[Out] -((A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((4*A + 3*B + 4*C)*Ellipt
icF[(c + d*x)/2, 2])/(42*a^4*d) + ((41*A + 15*B - C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) + ((A - C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(10*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((A - C)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.765289, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2977, 2978, 2748, 2641, 2639}

$$\frac{(4A+3B+4C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}+\frac{(41A+15B-C)\sin(c+dx)\sqrt{\cos(c+dx)}}{210a^4d(\cos(c+dx)+1)^2}-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}+\frac{(A-C)\sin(c}{10a^4d(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^4),x]
```


```
[Out] -((A - C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((4*A + 3*B + 4*C)*Ellipt
icF[(c + d*x)/2, 2])/(42*a^4*d) + ((41*A + 15*B - C)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) + ((A - C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(10*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((A - C)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4}\,dx=\int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx$$

$$=-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{1}{2}a(3A-3B-11C)+\frac{1}{2}a(11A+\right.}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$=-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{(A-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^3}+$$

$$=\frac{(41A+15B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2}-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c}{7d(a+a\cos(c+dx))^4}$$

$$=\frac{(41A+15B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2}-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c}{7d(a+a\cos(c+dx))^4}$$

$$=\frac{(41A+15B-C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2}-\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c}{7d(a+a\cos(c+dx))^4}$$

$$=-\frac{(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}+\frac{(4A+3B+4C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}+\frac{(41A+15B}{210}$$

Mathematica [C] time = 7.06789, size = 1862, normalized size = 8.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^4),x]
```

```
[Out] (((-2*I)/5)*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3
/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x)
)*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*
d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos
[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d
*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*(a + a*Sec[c + d*x])^4) + (((2*I)/5)*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2
]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (32*A*Co
s[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
```

```
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) - (8*B*Cos[c/2 + (d*x)/2]^8*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*
Sec[c + d*x])^4) - (32*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) + (Cos
[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((16*(A - C)*Csc[
c])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(12*A*Sin[(d*x)/2] - 5*B*Sin[(
d*x)/2] - 2*C*Sin[(d*x)/2]))/(35*d) + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Si
n[(d*x)/2] - C*Sin[(d*x)/2]))/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(41*
A*Sin[(d*x)/2] + 15*B*Sin[(d*x)/2] - C*Sin[(d*x)/2]))/(105*d) + (4*Sec[c/2]
*Sec[c/2 + (d*x)/2]^7*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2]))/(
7*d) + (8*(41*A + 15*B - C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(105*d) - (8*(12
*A - 5*B - 2*C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(35*d) + (4*(A - B + C)*Sec[
c/2 + (d*x)/2]^6*Tan[c/2])/(7*d)))/(Cos[c + d*x]^(3/2)*(A + 2*C + 2*B*Cos[c
 + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [B] time = 3.024, size = 595, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/840*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(168*A*cos(1
/2*d*x+1/2*c)^10+80*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7+84*A*c
os(1/2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+60*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*cos(1/2*d*x+1/2*c)^7-168*C*cos(1/2*d*x+1/2*c)^10+80*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*cos(1/2*d*x+1/2*c)^7-84*C*cos(1/2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-88*A*cos(1/2*d*x+1/2*c)^8+60*B*cos(1/2*d*x+1/2*c)^8+248*C*cos(1/2*d*x+
1/2*c)^8-306*A*cos(1/2*d*x+1/2*c)^6-30*B*cos(1/2*d*x+1/2*c)^6-54*C*cos(1/2*
d*x+1/2*c)^6+328*A*cos(1/2*d*x+1/2*c)^4-90*B*cos(1/2*d*x+1/2*c)^4-8*C*cos(1
/2*d*x+1/2*c)^4-117*A*cos(1/2*d*x+1/2*c)^2+75*B*cos(1/2*d*x+1/2*c)^2-33*C*c
os(1/2*d*x+1/2*c)^2+15*A-15*B+15*C)/a^4/cos(1/2*d*x+1/2*c)^7/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{4}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{4}+4\,a^{4}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}+6\,a^{4}\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+4\,a^{4}\cos\left(dx-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^4,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^4*co
s(d*x + c)^2*sec(d*x + c)^4 + 4*a^4*cos(d*x + c)^2*sec(d*x + c)^3 + 6*a^4*c
os(d*x + c)^2*sec(d*x + c)^2 + 4*a^4*cos(d*x + c)^2*sec(d*x + c) + a^4*cos(
d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{4}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^4,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^4*c
os(d*x + c)^(3/2)), x)
```

3.1243 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=234

$$\frac{(3A+4B+17C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{42a^4d}+\frac{(15A-B-83C)\sin(c+dx)\sqrt{\cos(c+dx)}}{210a^4d(\cos(c+dx)+1)^2}+\frac{(B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-$$

```
[Out] ((B + 8*C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((3*A + 4*B + 17*C)*Elli
pticF[(c + d*x)/2, 2])/(42*a^4*d) + ((15*A - B - 83*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) - ((B + 8*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(10*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((5*A + 2*B - 9*C)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.763188, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3041, 2978, 2748, 2641, 2639}

$$\frac{(3A+4B+17C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}+\frac{(15A-B-83C)\sin(c+dx)\sqrt{\cos(c+dx)}}{210a^4d(\cos(c+dx)+1)^2}+\frac{(B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(B+8C}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^4),x]
```


```
[Out] ((B + 8*C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((3*A + 4*B + 17*C)*Elli
pticF[(c + d*x)/2, 2])/(42*a^4*d) + ((15*A - B - 83*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(210*a^4*d*(1 + Cos[c + d*x])^2) - ((B + 8*C)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(10*a^4*d*(1 + Cos[c + d*x])) - ((A - B + C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((5*A + 2*B - 9*C)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^4}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4}\,dx$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{-\frac{1}{2}a(A-B-13C)+\frac{1}{2}a(9A+5B-5C)\cos(c+d}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(5A+2B-9C)\sqrt{\cos(c+dx)}\sin(c}{35ad(a+a\cos(c+dx))^3}$$

$$= \frac{(15A-B-83C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{(15A-B-83C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{(15A-B-83C)\sqrt{\cos(c+dx)}\sin(c+dx)}{210a^4d(1+\cos(c+dx))^2} - \frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{(B+8C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d} + \frac{(3A+4B+17C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d} + \frac{(15A-B-}{210}$$

Mathematica [C] time = 7.09044, size = 1862, normalized size = 7.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^4),x]
```

```
[Out] (((2*I)/5)*B*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/
4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[
c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*
(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])^4) + (((16*I)/5)*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2
]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (8*A*Cos
[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d
*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-
(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - A
rcTan[Cot[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) - (32*B*Cos[c/2 + (d*x)/2]^8*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*
Sec[c + d*x])^4) - (136*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) + (Co
s[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-16*(B + 8*C)*
Csc[c])/(5*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(15*A*Sin[(d*x)/2] - B*Sin
[(d*x)/2] - 83*C*Sin[(d*x)/2]))/(105*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*
(5*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2] - 9*C*Sin[(d*x)/2]))/(35*d) - (4*Sec[c
/2]*Sec[c/2 + (d*x)/2]^7*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Sin[(d*x)/2])
)/(7*d) - (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*(B*Sin[(d*x)/2] + 8*C*Sin[(d*x)/2
]))/(5*d) + (8*(15*A - B - 83*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(105*d) + (
8*(5*A + 2*B - 9*C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(35*d) - (4*(A - B + C)*
Sec[c/2 + (d*x)/2]^6*Tan[c/2])/(7*d)))/(Cos[c + d*x]^(3/2)*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [B] time = 3.019, size = 595, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/840*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(60*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7-168*B*cos(1/2*d*x+1/2*c)^10+80*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^7-84*B*cos(1/2*d*x+1/2*c)^7*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-1344*C*cos(1/2*d*x+1/2*c)^10+340*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*cos(1/2*d*x+1/2*c)^7-672*C*cos(1/2*d*x+1/2*c)^7*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+60*A*cos(1/2*d*x+1/2*c)^8+248*B*cos(1/2*d*x+1/2*c)^8+1684*C*cos(1/2
*d*x+1/2*c)^8-30*A*cos(1/2*d*x+1/2*c)^6-54*B*cos(1/2*d*x+1/2*c)^6-282*C*cos
(1/2*d*x+1/2*c)^6-90*A*cos(1/2*d*x+1/2*c)^4-8*B*cos(1/2*d*x+1/2*c)^4-34*C*c
os(1/2*d*x+1/2*c)^4+75*A*cos(1/2*d*x+1/2*c)^2-33*B*cos(1/2*d*x+1/2*c)^2-9*C
*cos(1/2*d*x+1/2*c)^2-15*A+15*B-15*C)/a^4/cos(1/2*d*x+1/2*c)^7/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{4}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{4}+4\,a^{4}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{3}+6\,a^{4}\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+4\,a^{4}\cos\left(dx-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^4,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^4*co
s(d*x + c)^3*sec(d*x + c)^4 + 4*a^4*cos(d*x + c)^3*sec(d*x + c)^3 + 6*a^4*c
os(d*x + c)^3*sec(d*x + c)^2 + 4*a^4*cos(d*x + c)^3*sec(d*x + c) + a^4*cos(
d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^4\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^4,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^4*c
os(d*x + c)^(5/2)), x)
```

3.1244 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=276

$$\frac{(4A+17B-108C)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{42a^4d}+\frac{(A+8B-57C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(A+8B-57C)\sin(c+dx)}{10a^4d\sqrt{\cos(c+dx)}}+\frac{(4A+}{42a^4d\sqrt{}}$$

```
[Out] ((A + 8*B - 57*C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((4*A + 17*B - 10
8*C)*EllipticF[(c + d*x)/2, 2])/(42*a^4*d) - ((A + 8*B - 57*C)*Sin[c + d*x]
)/(10*a^4*d*Sqrt[Cos[c + d*x]]) + ((13*A + 29*B - 141*C)*Sin[c + d*x])/(210
*a^4*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^2) + ((4*A + 17*B - 108*C)*Sin
[c + d*x])/(42*a^4*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - ((A - B + C)*
Sin[c + d*x])/(7*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B
 - 11*C)*Sin[c + d*x])/(35*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.826389, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3041, 2978, 2748, 2636, 2639, 2641}

$$\frac{(4A+17B-108C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d}+\frac{(A+8B-57C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d}-\frac{(A+8B-57C)\sin(c+dx)}{10a^4d\sqrt{\cos(c+dx)}}+\frac{(4A+17B-}{42a^4d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a + a*Sec[
c + d*x])^4),x]
```


```
[Out] ((A + 8*B - 57*C)*EllipticE[(c + d*x)/2, 2])/(10*a^4*d) + ((4*A + 17*B - 10
8*C)*EllipticF[(c + d*x)/2, 2])/(42*a^4*d) - ((A + 8*B - 57*C)*Sin[c + d*x]
)/(10*a^4*d*Sqrt[Cos[c + d*x]]) + ((13*A + 29*B - 141*C)*Sin[c + d*x])/(210
*a^4*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^2) + ((4*A + 17*B - 108*C)*Sin
[c + d*x])/(42*a^4*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - ((A - B + C)*
Sin[c + d*x])/(7*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B
 - 11*C)*Sin[c + d*x])/(35*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3041

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((a*A - b*B + a*C)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(b*c - a*d)*(2*m + 1)), x
] + Dist[1/(b*(b*c - a*d)*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c +
 d*Sin[e + f*x])^n*Simp[A*(a*c*(m + 1) - b*d*(2*m + n + 2)) + B*(b*c*m + a*
d*(n + 1)) - C*(a*c*m + b*d*(n + 1)) + (d*(a*A - b*B)*(m + n + 2) + C*(b*c*
(2*m + 1) - a*d*(m - n - 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^
```

```
2 - d^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^4}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4}\,dx$$

$$= -\frac{(A-B+C)\sin(c+dx)}{7d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4} + \frac{\int \frac{\frac{1}{2}a(A-B+15C)+\frac{7}{2}a(A+B-C)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{7d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4} + \frac{(3A+4B-11C)\sin(c+dx)}{35ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx)}$$

$$= \frac{(13A+29B-141C)\sin(c+dx)}{210a^4d\sqrt{\cos(c+dx)}(1+\cos(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= \frac{(13A+29B-141C)\sin(c+dx)}{210a^4d\sqrt{\cos(c+dx)}(1+\cos(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= \frac{(13A+29B-141C)\sin(c+dx)}{210a^4d\sqrt{\cos(c+dx)}(1+\cos(c+dx))^2} - \frac{(A-B+C)\sin(c+dx)}{7d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}$$

$$= \frac{(4A+17B-108C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d} - \frac{(A+8B-57C)\sin(c+dx)}{10a^4d\sqrt{\cos(c+dx)}} + \frac{(13}{210a^4d}$$

$$= \frac{(A+8B-57C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^4d} + \frac{(4A+17B-108C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42a^4d} - \frac{(A}{}$$

Mathematica [C] time = 7.44565, size = 2316, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(a +
a*Sec[c + d*x])^4),x]

[Out] (((2*I)/5)*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/
4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d
*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[
c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*
(-1 + E^((2*I)*d*x))*Sin[c])))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*(a + a*Sec[c + d*x])^4) + (((16*I)/5)*B*Cos[c/2 + (d*x)/2]^8*Csc[c/2
]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
```

```
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))))/((A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4) - (((114*I
)/5)*C*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3
*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2
*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
 + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
E^((2*I)*d*x))*Sin[c]))))/((A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + a*Sec[c + d*x])^4) - (32*A*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]
^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4
) - (136*B*Cos[c/2 + (d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^4) + (288*C*Cos[c/2 + (
d*x)/2]^8*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(7*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]^8*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((8*(20*C - A*Cos[c] - 8*B*Cos[c] + 37*C*Cos[c])*Csc[c
/2]*Sec[c/2]*Sec[c])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/
2] + 83*B*Sin[(d*x)/2] - 237*C*Sin[(d*x)/2]))/(105*d) - (16*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(A*Sin[(d*x)/2] + 8*B*Sin[(d*x)/2] - 37*C*Sin[(d*x)/2]))/(5*d)
 + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2] + C*Si
n[(d*x)/2]))/(7*d) + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(2*A*Sin[(d*x)/2] - 9
*B*Sin[(d*x)/2] + 16*C*Sin[(d*x)/2]))/(35*d) + (64*C*Sec[c]*Sec[c + d*x]*Si
n[d*x])/d - (8*(A + 83*B - 237*C)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(105*d) +
(8*(2*A - 9*B + 16*C)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(35*d) + (4*(A - B + C
)*Sec[c/2 + (d*x)/2]^6*Tan[c/2])/(7*d)))/(Cos[c + d*x]^(3/2)*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [B] time = 4.102, size = 1017, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/840*(-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(21*A*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-20*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-85*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
197*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+540*C*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+12*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
```

```
2*d*x+1/2*c)^2)^(1/2)*(21*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-85*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1197*C*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+540*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(21*A*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*A*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+168*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-85*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-1197*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+540*C*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+4*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(21*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-20*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+168*B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-85*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1197*C*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+540*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))*cos(1/2*d*x+1/2*c)-168*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+8*B-57*C)*sin(1/2*d*x+1/2*c)^10+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(148*A+1259*B-9036*C)*sin(1/2*d*x+1/2*c)^8-14*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(53*A+499*B-3621*C)*sin(1/2*d*x
+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(181*A+210
8*B-15597*C)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(59*A+907*B-7053*C)*sin(1/2*d*x+1/2*c)^2)/a^4/cos(1/2*d*x+1/2*c
)^7/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{a^{4}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{4}+4\,a^{4}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+6\,a^{4}\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+4\,a^{4}\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^4,x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a^4*co
s(d*x + c)^4*sec(d*x + c)^4 + 4*a^4*cos(d*x + c)^4*sec(d*x + c)^3 + 6*a^4*c
os(d*x + c)^4*sec(d*x + c)^2 + 4*a^4*cos(d*x + c)^4*sec(d*x + c) + a^4*cos(
d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c
))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(a \sec(dx+c) + a)^4 \cos(dx+c)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))
^4,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^4*c
os(d*x + c)^(7/2)), x)
```

3.1245 $\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=226

$$\frac{2a(16A+18B+21C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+18B+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+18B+2}{315d\sqrt{\cos(c+dx)}\sqrt{a}}$$

```
[Out] (16*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Sec[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(16*A + 18*B + 21*C)*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(A + 9*B)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c
 + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.628749, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4086, 4015, 3805, 3804}

$$\frac{2a(16A+18B+21C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(16A+18B+21C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a(16A+18B+2}{315d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (16*a*(16*A + 18*B + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Sec[c + d*x]]) + (8*a*(16*A + 18*B + 21*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(16*A + 18*B + 21*C)*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(A + 9*B)*Co
s[c + d*x]^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c
 + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$=\frac{2A\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin}{9d}$$

$$=\frac{2a(A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+$$

$$=\frac{2a(16A+18B+21C)\cos^{\frac{3}{2}}(c+dx)\sin}{105d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{8a(16A+18B+21C)\sqrt{\cos(c+dx)}\sin}{315d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{16a(16A+18B+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+$$

Mathematica [A] time = 0.566825, size = 127, normalized size = 0.56

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}((752A+846B+672C)\cos(c+dx)+4(83A+54B+63C)\cos(2(c+dx}{1260d(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(1321*A + 1368*B + 1596*C + (752*A + 846*B + 672*C)*Cos
[c + d*x] + 4*(83*A + 54*B + 63*C)*Cos[2*(c + d*x)] + 80*A*Cos[3*(c + d*x)]
 + 90*B*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])
]*Sin[c + d*x])/(1260*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.401, size = 153, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(35A(\cos(dx+c))^4+40A(\cos(dx+c))^3+45B(\cos(dx+c))^3+48A(\cos(dx+c))^2+54B(c}{315\,d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+40*A*cos(d*x+c)^3+45*B*cos(d*x+
c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+64*A*cos(d*x+c)+
72*B*cos(d*x+c)+84*C*cos(d*x+c)+128*A+144*B+168*C)*(a*(cos(d*x+c)+1)/cos(d*
x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.42619, size = 902, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(1890*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c)))*sin(9/2*d*x + 9/2*c) + 420*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 252*cos(4/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 45*cos(2/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 1890*c
os(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2
*c))) - 420*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) - 252*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) - 45*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*sin(9/2*d*x + 9/2*c) + 45*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) - 18*sqrt(2)*(7*(15*sin(3*
d*x + 3*c) + 5*sin(2*d*x + 2*c) + sin(d*x + c))*cos(7/2*arctan2(sin(d*x + c
), cos(d*x + c))) - (105*cos(3*d*x + 3*c) + 35*cos(2*d*x + 2*c) + 7*cos(d*x
 + c) + 10)*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 7*sin(5/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 35*sin(3/2*arctan2(sin(d*x + c), cos(d*x +
 c))) - 105*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a) - 84*sq
rt(2)*(5*(6*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*arctan2(sin(d*x + c),
cos(d*x + c))) - (30*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 6)*sin(5/2*arctan2
(sin(d*x + c), cos(d*x + c))) - 5*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 30*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.495573, size = 344, normalized size = 1.52

$$\frac{2\left(35A\cos(dx+c)^4+5(8A+9B)\cos(dx+c)^3+3(16A+18B+21C)\cos(dx+c)^2+4(16A+18B+21C)\cos(dx}{315\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*A*cos(d*x + c)^4 + 5*(8*A + 9*B)*cos(d*x + c)^3 + 3*(16*A + 18*B
+ 21*C)*cos(d*x + c)^2 + 4*(16*A + 18*B + 21*C)*cos(d*x + c) + 128*A + 144*
B + 168*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1246 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=178

$$\frac{2a(24A+28B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c}{35d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (4*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]]) + (2*a*(24*A + 28*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(A + 7*B)*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.550503, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4086, 4015, 3805, 3804}

$$\frac{2a(24A+28B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c}{35d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (4*a*(24*A + 28*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]]) + (2*a*(24*A + 28*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(A + 7*B)*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin}{7d}$$

$$= \frac{2a(A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a(24A+28B+35C)\sqrt{\cos(c+dx)}\sin}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a(24A+28B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A] time = 0.379889, size = 105, normalized size = 0.59

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}((141A+28(4B+5C))\cos(c+dx)+6(6A+7B)\cos(2(c+dx))+15A\,\mathrm{c}}{210d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(228*A + 266*B + 280*C + (141*A + 28*(4*B + 5*C))*Cos[c
 + d*x] + 6*(6*A + 7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1
 + Sec[c + d*x])]*Sin[c + d*x])/(210*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.373, size = 120, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(15A(\cos(dx+c))^3+18A(\cos(dx+c))^2+21B(\cos(dx+c))^2+24A\cos(dx+c)+28B\,\mathrm{co}}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+35*C*cos(d*x+c)+48*A+56*B+70*C)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.31795, size = 686, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) - 14*sqrt(2)*(5*(6*sin(2*d*x + 2*c) + sin(d*x + c))*
cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - (30*cos(2*d*x + 2*c) + 5*cos
(d*x + c) + 6)*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 5*sin(3/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 30*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c))))*B*sqrt(a) - 140*(3*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x
+ c)))*sin(d*x + c) - (3*sqrt(2)*cos(d*x + c) + 2*sqrt(2))*sin(3/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 3*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.488651, size = 284, normalized size = 1.6

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{3}+3\left(6\,A+7\,B\right)\cos\left(dx+c\right)^{2}+\left(24\,A+28\,B+35\,C\right)\cos\left(dx+c\right)+48\,A+56\,B+70\,C\right)\sqrt{\frac{a\cos(dx+c)\cdot}{\cos(dx+c)}}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^3 + 3*(6*A + 7*B)*cos(d*x + c)^2 + (24*A + 28*B +
35*C)*cos(d*x + c) + 48*A + 56*B + 70*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
cos(d*x + c)^(7/2), x)
```

3.1247 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx)+C\,\mathrm{s}$

Optimal. Leaf size=129

$$\frac{2a(7A+5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2(A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+}{5d}$$

```
[Out] (2*a*(7*A + 5*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(A + 5*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(5*d)
```

Rubi [A] time = 0.468495, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {4265, 4086, 4013, 3804}

$$\frac{2a(7A+5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2(A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (2*a*(7*A + 5*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(A + 5*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Si
n[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(5*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin}{5d}$$

$$= \frac{2(A+5B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+}}{15d}$$

$$= \frac{2a(7A+5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+$$

Mathematica [A] time = 0.188029, size = 82, normalized size = 0.64

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}\left((4A+5B)\cos(c+dx)+3A\cos^2(c+dx)+8A+10B+15C\right)}{15d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(8*A + 10*B + 15*C + (4*A + 5*B)*Cos[c + d*x] + 3*A*C
os[c + d*x]^2)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(15*d*(1 + Cos[c +
d*x]))
```

Maple [A] time = 0.351, size = 89, normalized size = 0.7

$$-\frac{(-2+2\cos(dx+c))\left(3A(\cos(dx+c))^2+4A\cos(dx+c)+5B\cos(dx+c)+8A+10B+15C\right)}{15d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+8*A
+10*B+15*C)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.23529, size = 433, normalized size = 3.36

$$\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 + 120*sqrt(2)*C*sqrt(a)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 10*
(3*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(d*x + c) - (3*s
qrt(2)*cos(d*x + c) + 2*sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 3*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.480971, size = 225, normalized size = 1.74

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{2}+\left(4\,A+5\,B\right)\cos\left(dx+c\right)+8\,A+10\,B+15\,C\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 8*A + 10*B + 15*C)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*co
s(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
cos(d*x + c)^(5/2), x)
```

3.1248 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\text{s}\right.$

Optimal. Leaf size=140

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.446848, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4086, 4015, 3801, 215}

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2\sqrt{a}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (2*Sqrt[a]*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin}{3d}$$

$$= \frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2}{}$$

$$= \frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2}{}$$

$$= \frac{2\sqrt{a}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{d}$$

Mathematica [A] time = 0.611356, size = 94, normalized size = 0.67

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(A\cos(c+dx)+2A+3B)+3\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\mathrm{s}\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*
C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(2*A + 3*B + A*Cos[c + d*x])*Sin[(c
 + d*x)/2]))/(3*d)
```

Maple [A] time = 0.38, size = 222, normalized size = 1.6

$$-\frac{-1+\cos(dx+c)}{3d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}+4A\sin(dx+c)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/3/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+4*A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)+6*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*C*2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+3*C*2^(1/2)*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))))*cos(d*x+c)^(
1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.27888, size = 513, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*
sqrt(a)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 3*C*sqrt(a)*(log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [A] time = 0.572159, size = 883, normalized size = 6.31

$$\left[\frac{4\,(A\cos(dx+c)+2\,A+3\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3\,(C\cos(dx+c)+C)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\,\sqrt{a}\sqrt{\cdots}}{\cdots}\right)}{6\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(A*cos(d*x + c) + 2*A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(C*cos(d*x + c) + C)*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(A*cos(d*x + c)
 + 2*A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 3*(C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
cos(d*x + c)^(3/2), x)
```

3.1249 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx)+C$

Optimal. Leaf size=139

$$\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{C\sin(c+dx)\sqrt{a\sec}}{d\sqrt{\cos(c+}}$$

```
[Out] (Sqrt[a]*(2*B + C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*(2*A - C)*Sin[c + d*x])/(d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.4504, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4088, 4015, 3801, 215}

$$\frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(2B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{C\sin(c+dx)\sqrt{a\sec}}{d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```

```
[Out] (Sqrt[a]*(2*B + C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*(2*A - C)*Sin[c + d*x])/(d*S
qrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{}$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{C}{}$$

$$= \frac{a(2A-C)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{C}{}$$

$$= \frac{\sqrt{a}(2B+C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{c}}{d}$$

Mathematica [A] time = 0.74716, size = 94, normalized size = 0.68

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2A+C\sec(c+dx))+\sqrt{2}(2B+C)\tanh^{-1}\left(\sqrt{2}\,\mathrm{s}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2
*B + C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(2*A + C*Sec[c + d*x])*Sin[(c
 + d*x)/2]))/(2*d)
```

Maple [B] time = 0.408, size = 304, normalized size = 2.2

$$-\frac{-1+\cos(dx+c)}{2d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(4A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}+2B\sqrt{2}\arctan\left(1/4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/2/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4*A*cos(d*x+c)*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-2*B*2^(1/2)*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c
)+C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))*cos(d*x+c)-C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+2*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c))/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [B] time = 2.32494, size = 1310, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + 2*B*sqrt(a)*(log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)) - (4*sqrt(2)*cos(5/2*d*x
 + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c
) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
+ 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
+ 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arct
an2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x +
5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.693489, size = 986, normalized size = 7.09

$$\left[\frac{4\,(2\,A\cos\left(dx+c\right)+C)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left((2\,B+C)\cos\left(dx+c\right)^{2}+(2\,B+C)\cos\left(dx+c\right)\right)\sqrt{}}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + ((2*B + C)*cos(d*x + c)^2 + (2*B + C)*cos(d*x
 + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(
d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d
*cos(d*x + c)), 1/2*(2*(2*A*cos(d*x + c) + C)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*B + C)*cos(d*x + c)^2 + (2
*B + C)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x
 + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*
sqrt(cos(d*x + c)), x)
```

3.1250 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{a}(8A+4B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+d}{2d}$$

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*(4*B + C)*Sin[c
+ d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*S
ec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.455926, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4088, 4016, 3801, 215}

$$\frac{\sqrt{a}(8A+4B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{C\sin(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(8*A + 4*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*(4*B + C)*Sin[c
+ d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C*Sqrt[a + a*S
ec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
```

```
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\right.}{}$$

$$= \frac{a(4B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(}{2d\cos}$$

$$= \frac{a(4B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(}{2d\cos}$$

$$= \frac{\sqrt{a}(8A+4B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}{4d}$$

Mathematica [A] time = 0.688137, size = 109, normalized size = 0.72

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{2}(8A+4B+3C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(8
*A + 4*B + 3*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*B + 3
*C + 2*C*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] time = 0.356, size = 440, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(8*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-8*A*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+4*B*2^(1
/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
*cos(d*x+c)^2-4*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+3*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-3*C*2^(1/2)*arcta
n(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+
c)^2-8*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-6*C*(-2/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))
*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.59101, size = 2925, normalized size = 19.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2)) - 4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*
d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d
*x + 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))
*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1) - (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x +
```

```
c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)
*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(s
in(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(s
qrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.09178, size = 1068, normalized size = 7.07

$$\left[\frac{4\left((4B+3C)\cos(dx+c)+2C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\left((8A+4B+3C)\cos(dx+c)^3+(8A+\right.}{16\left(d\cos(dx+c)^3+d\,\mathrm{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*((4*B + 3*C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A + 4*B + 3*C)*cos(d*x + c)^3
+ (8*A + 4*B + 3*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*B + 3*C)*cos(d*x
+ c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + ((8*A + 4*B + 3*C)*cos(d*x + c)^3 + (8*A + 4*B + 3*C)*cos(d*x +
```

```
c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(
d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2
)/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
sqrt(cos(d*x + c)), x)
```

3.1251 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{a(8A+6B+5C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a}{12d\cos}$$

```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*(6*B + C)*Sin[c
+ d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*A + 6*B
+ 5*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C
*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.547987, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4088, 4016, 3803, 3801, 215}

$$\frac{a(8A+6B+5C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(8A+6B+5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a}{12d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(8*A + 6*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
+ d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*(6*B + C)*Sin[c
+ d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(8*A + 6*B
+ 5*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (C
*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
```

```
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec$$

$$= \frac{a(6B+C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+}{3d\cos^{\frac{5}{2}}$$

$$= \frac{a(6B+C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(8A+6B+}{8d\cos^{\frac{3}{2}}(c+dx)$$

$$= \frac{a(6B+C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(8A+6B+}{8d\cos^{\frac{3}{2}}(c+dx)$$

$$= \frac{\sqrt{a}(8A+6B+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx}{8d}$$

Mathematica [A] time = 1.36067, size = 140, normalized size = 0.7

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+6B+5C)\cos(2(c+dx))+24A+4(6B+5C)\cos(c+dx)+}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(8*A + 6*B + 5*C)*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*A + 18*B + 31*C + 4*(
6*B + 5*C)*Cos[c + d*x] + 3*(8*A + 6*B + 5*C)*Cos[2*(c + d*x)])*Sin[(c + d*
x)/2]))/(48*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.378, size = 533, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(24*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-24*A*cos(d*x+c)^3*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+18*B
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^
(1/2)*cos(d*x+c)^3-18*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3+15*C*cos(d*x+c)^3*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-15*C*cos(d
*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+36*B*co
s(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+30*C*cos(d*x+c)^2*(-2/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+24*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2)+20*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+16*C*(-2/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2
/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 3.01307, size = 5403, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
```

```
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)
*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2
*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
+ 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos
(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*
cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arct
an2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c
)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*
d*x + 2*c) + 1) + (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c
) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*
sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(
2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*
c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*d*x +
 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(
2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x +
 c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x +
4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
 + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos
(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
```

```
2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) +
 cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4
*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*
c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
 - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + c
os(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d
*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos
(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x
 + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 6
0*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(s
qrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(
2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*c
os(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*cos(6*
d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*cos(6*d*x +
 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.10647, size = 1180, normalized size = 5.93

$$\left[\frac{4\left(3\left(8A+6B+5C\right)\cos\left(dx+c\right)^{2}+2\left(6B+5C\right)\cos\left(dx+c\right)+8C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left((8\right.}{96\left(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(3*(8*A + 6*B + 5*C)*cos(d*x + c)^2 + 2*(6*B + 5*C)*cos(d*x + c) +
 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c) + 3*((8*A + 6*B + 5*C)*cos(d*x + c)^4 + (8*A + 6*B + 5*C)*cos(d*x + c)^3
)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
 c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(
d*x + c)^3), 1/48*(2*(3*(8*A + 6*B + 5*C)*cos(d*x + c)^2 + 2*(6*B + 5*C)*co
s(d*x + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c) + 3*((8*A + 6*B + 5*C)*cos(d*x + c)^4 + (8*A + 6*B + 5*C)*co
s(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
cos(d*x + c)^(3/2), x)
```

3.1252 $\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a(48A+40B+35C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+40B+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+35C)\sqrt{\cos(c+dx)}\sqrt{s}}{64d}$$

```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*(8*B + C)*Si
n[c + d*x])/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A +
 40*B + 35*C)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x
]]) + (a*(48*A + 40*B + 35*C)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a
 + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c
+ d*x]^(7/2))
```

Rubi [A] time = 0.632717, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4088, 4016, 3803, 3801, 215}

$$\frac{a(48A+40B+35C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(48A+40B+35C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}(48A+40B+35C)\sqrt{\cos(c+dx)}\sqrt{s}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```


```
[Out] (Sqrt[a]*(48*A + 40*B + 35*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a*(8*B + C)*Si
n[c + d*x])/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(48*A +
 40*B + 35*C)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x
]]) + (a*(48*A + 40*B + 35*C)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a
 + a*Sec[c + d*x]]) + (C*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c
+ d*x]^(7/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}$$

$$= \frac{C\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{C\sqrt{a+a\sec(c+}}{4d\cos^{\frac{7}{2}}}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+40B}{96d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+40B}{96d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{a(8B+C)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(48A+40B}{96d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{\sqrt{a}(48A+40B+35C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{64d}$$

Mathematica [A] time = 2.16185, size = 178, normalized size = 0.72

$$\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((432A+77(8B+7C))\cos(c+dx)+4(48A+40B+35C)\cos(2$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(48*A + 40*B + 35*C
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (192*A + 160*B + 332*C
 + (432*A + 77*(8*B + 7*C))*Cos[c + d*x] + 4*(48*A + 40*B + 35*C)*Cos[2*(c
+ d*x)] + 144*A*Cos[3*(c + d*x)] + 120*B*Cos[3*(c + d*x)] + 105*C*Cos[3*(c
+ d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 0.381, size = 626, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] -1/384/d*(-1+cos(d*x+c))*(144*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-144*A*cos(d*x+c)^4*2^(1/2
)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+1
20*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))-120*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+105*C*cos(d*x+c)^4*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-105*
C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))*2^(1/2)+288*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)
+240*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+210*C*cos(d*x+c)^3
*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)+160*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+
140*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+128*B*cos(d*x+c)*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+112*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)*sin(d*x+c)+96*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/
cos(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(7/2)
```

Maxima [B] time = 4.13899, size = 8764, normalized size = 35.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(48*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(
7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*
```

sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 8*(60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x

+ 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) + 2) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*
cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 2) - 60*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))
) - 20*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 1
68*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(
sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(
2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 60*(sqrt(2)*co
s(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(3*cos(
4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^
2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*
cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + (420*(sqrt(2)
*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c)
 + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(15/2*arctan2(sin(d*x + c), cos(d*x + c))
) + 140*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*
sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(13/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 1596*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x +
6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 500*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt
(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2
*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) - 500*(sqrt(2)*sin(8*d*x
+ 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2
)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 1596*(sq
rt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt(2)*sin(4*d*x +
 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x +
 c))) - 140*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x + 6*c) + 6*sqrt
(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 420*(sqrt(2)*sin(8*d*x + 8*c) + 4*sqrt(2)*sin(6*d*x
+ 6*c) + 6*sqrt(2)*sin(4*d*x + 4*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 105*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*
d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2
+ 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos
(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d
*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x +
 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*s

in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c
)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin
(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*
sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) + 2) + 105*(2*(4*cos(6*d*x + 6*c) + 6*co
s(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c
)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16
*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos
(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*
(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
+ 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 105*(2*(4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x +
 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36
*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin
(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 +
 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d
*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 105*(2*(4*cos(6*d*x + 6*c
) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)
^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin
(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 420*(sqrt(2)*cos(8*d*
x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(15/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 140*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)
*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(13/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 1596*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*c
os(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(11/2*arctan2(sin(d*x + c), cos(d*x + c))) - 500*(sqrt(2)*cos(
8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*
sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(9/2*arctan2(sin(d*x + c), cos(d*x +
 c))) + 500*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt
(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1596*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)
*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 140*(sqrt(2)*cos
(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqrt(2)*cos(4*d*x + 4*c) + 4
*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 420*(sqrt(2)*cos(8*d*x + 8*c) + 4*sqrt(2)*cos(6*d*x + 6*c) + 6*sqr
t(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))))*C*sqrt(a)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4
*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2

```
 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*co
s(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*
d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*
sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*
c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*si
n(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.65971, size = 1318, normalized size = 5.34

$$\left[\frac{4\left(3\,(48\,A+40\,B+35\,C)\cos\left(dx+c\right)^{3}+2\,(48\,A+40\,B+35\,C)\cos\left(dx+c\right)^{2}+8\,(8\,B+7\,C)\cos\left(dx+c\right)+48\,C\right)\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(3*(48*A + 40*B + 35*C)*cos(d*x + c)^3 + 2*(48*A + 40*B + 35*C)*c
os(d*x + c)^2 + 8*(8*B + 7*C)*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((48*A + 40*B + 35*C)*c
os(d*x + c)^5 + (48*A + 40*B + 35*C)*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x +
c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3
*(48*A + 40*B + 35*C)*cos(d*x + c)^3 + 2*(48*A + 40*B + 35*C)*cos(d*x + c)^
2 + 8*(8*B + 7*C)*cos(d*x + c) + 48*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((48*A + 40*B + 35*C)*cos(d*x + c)^
5 + (48*A + 40*B + 35*C)*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
 + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+a*sec(d*x+c))**(1/2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/
cos(d*x + c)^(5/2), x)
```

3.1253 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=284

$$\frac{2a^2(84A+110B+99C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(336A+374B+429C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{1155d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(336A}{}$$

```
[Out] (16*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Sec[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*C)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(336*A + 374*B +
429*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) +
 (2*a^2*(84*A + 110*B + 99*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*a*(3*A + 11*B)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[
c + d*x]]*Sin[c + d*x])/(99*d) + (2*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.885488, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4017, 4015, 3805, 3804}

$$\frac{2a^2(84A+110B+99C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(336A+374B+429C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{1155d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(336A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2),x]
```


```
[Out] (16*a^2*(336*A + 374*B + 429*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Sec[c + d*x]]) + (8*a^2*(336*A + 374*B + 429*C)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(336*A + 374*B +
429*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) +
 (2*a^2*(84*A + 110*B + 99*C)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*a*(3*A + 11*B)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[
c + d*x]]*Sin[c + d*x])/(99*d) + (2*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(11*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+a\,\mathrm{s}}{}$$

$$=\frac{2A\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,\mathrm{si}}{11d}$$

$$=\frac{2a(3A+11B)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c}}{99d}$$

$$=\frac{2a^2(84A+110B+99C)\cos^{\frac{5}{2}}(c+dx)\,\mathrm{si}}{693d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{2a^2(336A+374B+429C)\cos^{\frac{3}{2}}(c+dx}{1155d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{8a^2(336A+374B+429C)\sqrt{\cos(c+dx}}{3465d\sqrt{a+a\sec(c+dx)}}$$

$$=\frac{16a^2(336A+374B+429C)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 2.10568, size = 158, normalized size = 0.56

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((34734A+44(799B+759C))\cos(c+dx)+8(1743A+1507B+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(55482*A + 59158*B + 65208*C + (34734*A + 44*(799*B +
 759*C))*Cos[c + d*x] + 8*(1743*A + 1507*B + 1287*C)*Cos[2*(c + d*x)] + 493
5*A*Cos[3*(c + d*x)] + 3740*B*Cos[3*(c + d*x)] + 1980*C*Cos[3*(c + d*x)] +
1470*A*Cos[4*(c + d*x)] + 770*B*Cos[4*(c + d*x)] + 315*A*Cos[5*(c + d*x)])*
Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 0.284, size = 187, normalized size = 0.7

$$-\frac{2\,a\,(-1+\cos(dx+c))\left(315\,A\,(\cos(dx+c))^5+735\,A\,(\cos(dx+c))^4+385\,B\,(\cos(dx+c))^4+840\,A\,(\cos(dx+c))^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2
),x)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+735*A*cos(d*x+c)^4+385*B*co
s(d*x+c)^4+840*A*cos(d*x+c)^3+935*B*cos(d*x+c)^3+495*C*cos(d*x+c)^3+1008*A*
cos(d*x+c)^2+1122*B*cos(d*x+c)^2+1287*C*cos(d*x+c)^2+1344*A*cos(d*x+c)+1496
*B*cos(d*x+c)+1716*C*cos(d*x+c)+2688*A+2992*B+3432*C)*cos(d*x+c)^(1/2)*(a*(
cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.49258, size = 1164, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/110880*(21*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(
11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*c
os(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d
*x + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x
+ 11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d
*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c)
, cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/
2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55
```

```
*a*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2
*d*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(
11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) - 44*sqrt(2)*(189*(10*
a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 7*(270*a*cos(4*d*x + 4*c) + 27*a*cos(2*d*x + 2*c) + 5*
a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 135*a*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 189*a*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 1050*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 1890*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*B*sqrt(a) - 132*sqrt(2)*(175*a*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))*sin(2*d*x + 2*c) - 5*(35*a*cos(2*d*x + 2*c) + 6*a)*sin(7/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 126*a*sin(5/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 175*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 1470*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*C*sqrt(a))/d
```

Fricas [A] time = 0.511192, size = 437, normalized size = 1.54

$$\frac{2\left(315\,Aa\cos\left(dx+c\right)^{5}+35\,(21\,A+11\,B)a\cos\left(dx+c\right)^{4}+5\,(168\,A+187\,B+99\,C)a\cos\left(dx+c\right)^{3}+3\,(336\,A+374\,B+\right.}{3465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*A*a*cos(d*x + c)^5 + 35*(21*A + 11*B)*a*cos(d*x + c)^4 + 5*(168
*A + 187*B + 99*C)*a*cos(d*x + c)^3 + 3*(336*A + 374*B + 429*C)*a*cos(d*x +
 c)^2 + 4*(336*A + 374*B + 429*C)*a*cos(d*x + c) + 8*(336*A + 374*B + 429*C
)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(
d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(11/2), x)
```

3.1254 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=232

$$\frac{2a^2(52A+72B+63C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+156B+189C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+15}{315d\sqrt{\cos(c+}$$

```
[Out] (4*a^2*(136*A + 156*B + 189*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Sec[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(52*A + 72*B + 63*C)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(
A + 3*B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) +
 (2*A*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.787625, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4017, 4015, 3805, 3804}

$$\frac{2a^2(52A+72B+63C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(136A+156B+189C)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(136A+15}{315d\sqrt{\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (4*a^2*(136*A + 156*B + 189*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Sec[c + d*x]]) + (2*a^2*(136*A + 156*B + 189*C)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(52*A + 72*B + 63*C)
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(
A + 3*B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) +
 (2*A*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
```

```
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}{9d}$$

$$= \frac{2a(A+3B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(}}{21d}$$

$$= \frac{2a^2(52A+72B+63C)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(136A+156B+189C)\sqrt{\cos(c+}}{315d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{4a^2(136A+156B+189C)\sin(c+dx}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

Mathematica [A] time = 1.42382, size = 123, normalized size = 0.53

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(2(799A+759B+756C)\cos(c+dx)+4(137A+117B+63C)\cos}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(2689*A + 2964*B + 3276*C + 2*(799*A + 759*B + 756*C)
*Cos[c + d*x] + 4*(137*A + 117*B + 63*C)*Cos[2*(c + d*x)] + 170*A*Cos[3*(c
+ d*x)] + 90*B*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c
+ d*x])]*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.352, size = 154, normalized size = 0.7

$$-\frac{2a\left(-1+\cos\left(dx+c\right)\right)\left(35A\left(\cos\left(dx+c\right)\right)^{4}+85A\left(\cos\left(dx+c\right)\right)^{3}+45B\left(\cos\left(dx+c\right)\right)^{3}+102A\left(\cos\left(dx+c\right)\right)^{2}+117B\right.}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*
x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+136*A*cos(d*
x+c)+156*B*cos(d*x+c)+189*C*cos(d*x+c)+272*A+312*B+378*C)*cos(d*x+c)^(1/2)*
(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.43354, size = 949, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) - 6*sqrt(2)*(175*a*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(2*d*x + 2*c) - 5*(35*a*cos(2*d*x + 2*c) + 6*a)*sin(7/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 126*a*sin(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 175*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 1470*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a
) - 504*(10*sqrt(2)*a*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(2*d*x + 2*c) - 5*sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
+ 2*c))) - 10*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (10*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*C*sqrt(a))/d
```

Fricas [A] time = 0.499806, size = 366, normalized size = 1.58

$$\frac{2\left(35\,Aa\cos\left(dx+c\right)^{4}+5\,(17\,A+9\,B)a\cos\left(dx+c\right)^{3}+3\,(34\,A+39\,B+21\,C)a\cos\left(dx+c\right)^{2}+(136\,A+156\,B+189}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^4 + 5*(17*A + 9*B)*a*cos(d*x + c)^3 + 3*(34*A +
39*B + 21*C)*a*cos(d*x + c)^2 + (136*A + 156*B + 189*C)*a*cos(d*x + c) + 2*
(136*A + 156*B + 189*C)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1255 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=181

$$\frac{8a^2(19A+21B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+21B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2(3A+7B)}{}$$

```
[Out] (8*a^2*(19*A + 21*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(19*A + 21*B + 35*C)*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(3*A + 7*B)*Cos[c + d*x]^(3/2)*(
a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a
 + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.600766, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4086, 4013, 3809, 3804}

$$\frac{8a^2(19A+21B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(19A+21B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2(3A+7B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (8*a^2*(19*A + 21*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(19*A + 21*B + 35*C)*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(3*A + 7*B)*Cos[c + d*x]^(3/2)*(
a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a
 + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}{7d}$$

$$= \frac{2(3A+7B)\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c}{35d}$$

$$= \frac{2a(19A+21B+35C)\sqrt{\cos(c+dx)}\sqrt{}}{105d}$$

$$= \frac{8a^2(19A+21B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.98434, size = 100, normalized size = 0.55

$$\frac{a\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((253A+28(9B+5C))\cos(c+dx)+6(13A+7B)\cos(2(c+dx))}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(494*A + 546*B + 700*C + (253*A + 28*(9*B + 5*C))*Cos
[c + d*x] + 6*(13*A + 7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a
*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.319, size = 121, normalized size = 0.7

$$-\frac{2a(-1+\cos(dx+c))\left(15A(\cos(dx+c))^3+39A(\cos(dx+c))^2+21B(\cos(dx+c))^2+52A\cos(dx+c)+63B\cos}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*
x+c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+35*C*cos(d*x+c)+104*A+126*B+175*C)*c
os(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.37214, size = 693, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) - 84*(10*sqrt(2)*a*cos(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 5*sqrt(2)*a*
sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 10*sqrt(2)*a*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (10*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sq
rt(a) + 280*(sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 9*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt
(a))/d
```

Fricas [A] time = 0.493011, size = 302, normalized size = 1.67

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{3}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)^{2}+(52\,A+63\,B+35\,C)a\cos\left(dx+c\right)+(104\,A+126\,B+175\,C)a\right)\sqrt{\cdots}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^3 + 3*(13*A + 7*B)*a*cos(d*x + c)^2 + (52*A + 63
*B + 35*C)*a*cos(d*x + c) + (104*A + 126*B + 175*C)*a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(7/2), x)
```

3.1256 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=192

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(3A+5B)\sin(c+d}{}$$

```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(12*A + 20*B + 15*C)*Sin[c + d
*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(3*A + 5*B)*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*Cos
[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.638865, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4017, 4015, 3801, 215}

$$\frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a(3A+5B)\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (2*a^(3/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(12*A + 20*B + 15*C)*Sin[c + d
*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(3*A + 5*B)*
Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*Cos
[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}{5d}$$

$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec}}{15d}$$

$$= \frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^2(12A+20B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^{3/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c}}{d}$$

Mathematica [A] time = 1.11284, size = 115, normalized size = 0.6

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(2(9A+5B)\cos(c+dx)+3A\cos(2(c+dx))+39\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(15*Sqrt[
2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (39*A + 50*B + 30*C + 2*(9*A + 5*B
)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d)
```

Maple [A] time = 0.401, size = 235, normalized size = 1.2

$$-\frac{a}{30\,d\sin\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(-15\,C\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\left(\cos\left(dx+c\right)+1-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/30/d*a*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-15*C*arcta
n(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(-2/(cos
(d*x+c)+1))^(1/2)*2^(1/2)*sin(d*x+c)+15*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(
1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*si
n(d*x+c)+12*A*cos(d*x+c)^3+24*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+36*A*cos(d*x
+c)+80*B*cos(d*x+c)+60*C*cos(d*x+c)-72*A-100*B-60*C)/sin(d*x+c)
```

Maxima [B] time = 2.412, size = 1022, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x +
 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*
sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))))*A*sqrt(a) + 20*(sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 9*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*B*sqrt(a) + 30*(4*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - a*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) + a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - a*log(2*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*C*s
```

qrt(a))/d

Fricas [A] time = 0.582412, size = 1029, normalized size = 5.36

$$\left[\frac{4\left(3\,Aa\cos\left(dx+c\right)^{2}+\left(9\,A+5\,B\right)a\cos\left(dx+c\right)+\left(18\,A+25\,B+15\,C\right)a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+}{30\left(d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(4*(3*A*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + (18*A + 25*B
+ 15*C)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 15*(C*a*cos(d*x + c) + C*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)))/(d*cos(d*x + c) + d), 1/15*(2*(3*A*a*cos(d*x + c)^2 + (9*A + 5*B)
*a*cos(d*x + c) + (18*A + 25*B + 15*C)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*(C*a*cos(d*x + c) + C*a)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x +
c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(5/2), x)
```

3.1257 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\mathrm{s}\right.$

Optimal. Leaf size=197

$$\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c}{3d\sqrt{}}$$

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(8*A + 6*B - 3*C)*Sin[c
+ d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a*(2*A - 3*C)*
Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt
[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.648714, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4018, 4015, 3801, 215}

$$\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(2B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a(2A-3C)\sin(c}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*(2*B + 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(8*A + 6*B - 3*C)*Sin[c
+ d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a*(2*A - 3*C)*
Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt
[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
```

```
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}{3d}$$

$$=-\frac{a(2A-3C)\sqrt{a+a\sec(c+dx)}\sin(c}{3d\sqrt{\cos(c+dx)}}$$

$$=\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}-$$

$$=\frac{a^2(8A+6B-3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}-$$

$$=\frac{a^{3/2}(2B+3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A] time = 1.34133, size = 122, normalized size = 0.62

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(2(5A+3B)\cos(c+dx)+A\cos(2(c}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2
]*(2*B + 3*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(A + 3*C + 2*(5*A + 3*B
)*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sec[c + d*x]*Sin[(c + d*x)/2]))/(6*d)
```

Maple [B] time = 0.383, size = 366, normalized size = 1.9

$$-\frac{a(-1+\cos(dx+c))}{6d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(4A(\cos(dx+c))^2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}+20A\cos(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/6/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4*A*cos(d*x+c
)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+20*A*cos(d*x+c)*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2)+12*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+6*B
*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c)))*cos(d*x+c)-6*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+9*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-9*C*2^(1/2)*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c
)+6*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)^(1/2)/(
-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.4507, size = 2558, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(20*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*A*sqrt(a) + 6*(2*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 40*sqrt(2)*a*sin(1/2*d*
x + 1/2*c) - 2*sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 20*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2) - 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 5*a*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
```

```
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*B*sqrt(a) - 15*
(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5/2*d*
x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)
*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(3/2*d
*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 -
 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 2*(s
qrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co
s(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/2*
d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/2*d*x + 5/2
*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c))*C*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 0.702067, size = 1119, normalized size = 5.68

$$\left[\frac{4\left(2\,Aa\cos\left(dx+c\right)^{2}+2\,(5\,A+3\,B)a\cos\left(dx+c\right)+3\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left((2\,B+3\,C)a\right.}{12\left(d\cos\left(dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*A*a*cos(d*x + c)^2 + 2*(5*A + 3*B)*a*cos(d*x + c) + 3*C*a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((2
```

```
*B + 3*C)*a*cos(d*x + c)^2 + (2*B + 3*C)*a*cos(d*x + c))*sqrt(a)*log((a*cos
(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x +
c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/6*(2*(2
*A*a*cos(d*x + c)^2 + 2*(5*A + 3*B)*a*cos(d*x + c) + 3*C*a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((2*B + 3*C)*a
*cos(d*x + c)^2 + (2*B + 3*C)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(a\sec\left(dx+c\right) + a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(3/2), x)
```

3.1258 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+\right.$

Optimal. Leaf size=203

$$\frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+12B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+$$

```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(8*A - 4*B -
5*C)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a*(
4*B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]])
+ (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.661237, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4088, 4018, 4015, 3801, 215}

$$\frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(8A+12B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(4B+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(3/2)*(8*A + 12*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(8*A - 4*B -
5*C)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a*(
4*B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]])
+ (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
```

```
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{s}}{}$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{a(4B+3C)\sqrt{a+a\sec(c+dx)}\sin(c+d}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a(}{}$$

$$= \frac{a^2(8A-4B-5C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a(}{}$$

$$= \frac{a^{3/2}(8A+12B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+d}{\sqrt{a+a\sec(c+}}\right.}{4d}$$

Mathematica [A] time = 1.46863, size = 129, normalized size = 0.64

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(2A\cos(2(c+dx))+2A+C)+(4B+7C)\cos(c+dx))+\sqrt{2}(\right.}{8d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(8*A + 12*B + 7*C)*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*((4*B + 7*C)*Cos[c + d
*x] + 2*(2*A + C + 2*A*Cos[2*(c + d*x)]))*Sin[(c + d*x)/2]))/(8*d*Cos[c + d
*x]^(3/2))
```

Maple [B] time = 0.326, size = 472, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/8/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(16*A*cos(d*x+
c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+8*A*2^(1/2)*arctan(1/4*2^(1/2)*(-
2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-8*A*2^(1/2)
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*co
s(d*x+c)^2+12*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-12*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+7*C*2^(1/2)*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c
)^2-7*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))*cos(d*x+c)^2+8*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)+14*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*C*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3
/2)
```

Maxima [B] time = 2.80763, size = 4942, normalized size = 24.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A
*sqrt(a) + 4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
```

d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
 a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3
/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) +
 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*
sqrt(2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*
sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a
*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*s
in(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))

```
- 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
- 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4
*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sq
rt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*C*sqrt(a)/(2*(2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1))/d
```

Fricas [A] time = 1.12052, size = 1157, normalized size = 5.7

$$\left[\frac{4\left(8\,Aa\cos\left(dx+c\right)^{2}+\left(4\,B+7\,C\right)a\cos\left(dx+c\right)+2\,Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\left(8\,A+12\,B+7\,C\right.\right.}{16\left(d\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(8*A*a*cos(d*x + c)^2 + (4*B + 7*C)*a*cos(d*x + c) + 2*C*a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A +
 12*B + 7*C)*a*cos(d*x + c)^3 + (8*A + 12*B + 7*C)*a*cos(d*x + c)^2)*sqrt(a
)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)
^2), 1/8*(2*(8*A*a*cos(d*x + c)^2 + (4*B + 7*C)*a*cos(d*x + c) + 2*C*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*
A + 12*B + 7*C)*a*cos(d*x + c)^3 + (8*A + 12*B + 7*C)*a*cos(d*x + c)^2)*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x
 + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)*sqrt(cos(d*x + c)), x)
```

3.1259 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=201

$$\frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+14B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(2}{}$$

```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 30*
B + 19*C)*Sin[c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (a*(2*B + C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/
2)) + (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.67798, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4088, 4018, 4016, 3801, 215}

$$\frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(24A+14B+11C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (a^(3/2)*(24*A + 14*B + 11*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(24*A + 30*
B + 19*C)*Sin[c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (a*(2*B + C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/
2)) + (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
```

```
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\sqrt{\sec(c+dx)}(a+a\,\mathrm{se}$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(2B+C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{C(a+a\,\mathrm{s}}{}$$

$$= \frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(2B+C)\sqrt{a}}{4}$$

$$= \frac{a^2(24A+30B+19C)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(2B+C)\sqrt{a}}{4}$$

$$= \frac{a^{3/2}(24A+14B+11C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c}}{8d}$$

Mathematica [A] time = 2.29396, size = 141, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(3(8A+14B+11C)\cos(2(c+dx))+24A+4(6B+11C)\cos(c+}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(24*A + 14*B + 11
*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*A + 42*B + 49*C
+ 4*(6*B + 11*C)*Cos[c + d*x] + 3*(8*A + 14*B + 11*C)*Cos[2*(c + d*x)])*Sin
[(c + d*x)/2]))/(48*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.342, size = 534, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] -1/48/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(72*A*cos(d*x
+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*2^(1/2)-72*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+42*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3-42*B*arctan(1/4*2^
(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+
c)^3+33*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))*2^(1/2)-33*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2)+84*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2)+66*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+24*B*cos(
d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+44*C*(-2/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)*sin(d*x+c)+16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/cos(d*x+c)
^(5/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 3.25556, size = 7760, normalized size = 38.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x +
```

```
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d
*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
 + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin
(2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - 6*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*s
qrt(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sq
rt(2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sq
rt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sq
rt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*s
in(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4
*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arc
```

tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt
(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a)/(2*(2*cos(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (132*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a
*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x +
2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*
sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2
*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*
sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2
*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 33*(a*cos(6*d*x
 + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x +
 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) +
a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*c
os(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*a

```
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos
(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4
*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*
c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c)
 + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6
*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9
*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*
a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4
*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x
 + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c
) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)
 + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)
^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*c
os(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*
d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2
*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*c
os(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*
(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*
sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x
 + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*
a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c)
+ 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/(2*(3*cos(4
*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*c
os(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c
) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.11545, size = 1231, normalized size = 6.12

$$\left[\frac{4\left(3\left(8A+14B+11C\right)a\cos\left(dx+c\right)^2+2\left(6B+11C\right)a\cos\left(dx+c\right)+8Ca\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(3*(8*A + 14*B + 11*C)*a*cos(d*x + c)^2 + 2*(6*B + 11*C)*a*cos(d*x
 + c) + 8*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + 3*((24*A + 14*B + 11*C)*a*cos(d*x + c)^4 + (24*A + 14*B + 11*
C)*a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(
d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(3*(8*A + 14*B + 11*C)*a*cos(d*x +
c)^2 + 2*(6*B + 11*C)*a*cos(d*x + c) + 8*C*a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((24*A + 14*B + 11*C)*a*cos(
d*x + c)^4 + (24*A + 14*B + 11*C)*a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x
+ c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx + c\right)^2 + B \sec\left(dx + c\right) + A\right)\left(a \sec\left(dx + c\right) + a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx + c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/sqrt(cos(d*x + c)), x)
```

3.1260 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=253

$$\frac{a^2(112A+88B+75C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+88B+75C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{64d}$$

```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Se
c[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(48*A + 5
6*B + 39*C)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(112*A + 88*B + 75*C)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]) + (a*(8*B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x]
)/(24*d*Cos[c + d*x]^(5/2)) + (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
4*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.790027, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(112A+88B+75C)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^{3/2}(112A+88B+75C)\sqrt{\cos(c+dx)}\sqrt{\sec}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```


```
[Out] (a^(3/2)*(112*A + 88*B + 75*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Se
c[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(48*A + 5
6*B + 39*C)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(112*A + 88*B + 75*C)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]) + (a*(8*B + 3*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x]
)/(24*d*Cos[c + d*x]^(5/2)) + (C*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
4*d*Cos[c + d*x]^(5/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(8B+3C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{C(a+a}{}$$

$$= \frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(8B+3C)\sqrt{a}}{2}$$

$$= \frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(112A+88}{64d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{a^2(48A+56B+39C)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(112A+88}{64d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{a^{3/2}(112A+88B+75C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos}}{64d}$$

Mathematica [A] time = 3.75934, size = 176, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((1008A+1048B+1155C)\cos(c+dx)+4(48A+88B+75C)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(112*A + 88*B + 7
5*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (192*A + 352*B + 49
2*C + (1008*A + 1048*B + 1155*C)*Cos[c + d*x] + 4*(48*A + 88*B + 75*C)*Cos[
2*(c + d*x)] + 336*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(c + d*x)] + 225*C*Cos[
3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 0.382, size = 627, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] 1/384/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(336*A*cos(d*
x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))-336*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1
```

```
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+264*B*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2
^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-264*B*cos(d*x+c
)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))+225*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1-sin(d*x+c)))*2^(1/2)-225*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-672*A*sin(d*x+c)*c
os(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-528*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(co
s(d*x+c)+1))^(1/2)-450*C*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-352*B*cos(d*x+c)^2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-300*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-128*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-240
*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-96*C*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c))/cos(d*x+c)^(7/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 4.49529, size = 10963, normalized size = 43.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] -1/768*(48*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2
4*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*s
in(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*si
n(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*si
n(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*
(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(
2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
```

2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(
2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(2*(2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 8*(132*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*sin(6*d
*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*
x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos
(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*
x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^
2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2
+ 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin

```
(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6
*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x
 + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x +
 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4
*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2
*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*
a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(
4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2
*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4
*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c)
+ a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a
*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*s
in(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x
 + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*
c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 132*(sqrt(2)*a*cos(6*d*x
 + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sqrt(2)
*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*
a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c)
+ 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*
d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt
(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(300*(sqrt(2)*a*sin(
8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c)
+ 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 100*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(13/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1140*(sqrt(2)*a*sin(8*d*x + 8*
c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2
)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(
```

```
2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 228*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(
2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*
x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt
(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d
*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 100*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d
*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c))*co
s(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(sqrt(2)*a*sin(8*d
*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4
*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*
x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*d*
x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*
c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c) +
 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*si
n(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d*x
 + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*
d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(6*
d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x +
4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*c)
 + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c) +
a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*a*
sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4*d
*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2) - 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*cos(
4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*sin(
6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x
+ 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x + 4*
c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c)
+ a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) + 3*
a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin(4
*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2) + 75*(a*cos(8*d*x + 8*c)^2 + 16*a*cos(6*d*x + 6*c)^2 + 36*a*co
s(4*d*x + 4*c)^2 + 16*a*cos(2*d*x + 2*c)^2 + a*sin(8*d*x + 8*c)^2 + 16*a*si
n(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 48*a*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*
x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(8*d*x + 8*c) + 8*(6*a*cos(4*d*x +
4*c) + 4*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 12*(4*a*cos(2*d*x + 2*c
) + a)*cos(4*d*x + 4*c) + 8*a*cos(2*d*x + 2*c) + 4*(2*a*sin(6*d*x + 6*c) +
3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a*sin
(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
 + 2*c))) + 2) - 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x +
6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2
)*a)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*a
*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x +
4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(13/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*
a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x +
 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
228*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*
a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 228*(sqrt(2)*a*cos(8*d*x + 8*c)
+ 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a
*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1140*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c)
+ 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*a*cos(8
*d*x + 8*c) + 4*sqrt(2)*a*cos(6*d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) +
 4*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 300*(sqrt(2)*a*cos(8*d*x + 8*c) + 4*sqrt(2)*a*cos(6*
d*x + 6*c) + 6*sqrt(2)*a*cos(4*d*x + 4*c) + 4*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(a)/
(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8
*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2
*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*
c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1))
/d
```

Fricas [A] time = 1.63185, size = 1361, normalized size = 5.38

$$\left[\frac{4\left(3\,(112\,A+88\,B+75\,C)a\cos{(dx+c)}^{3}+2\,(48\,A+88\,B+75\,C)a\cos{(dx+c)}^{2}+8\,(8\,B+15\,C)a\cos{(dx+c)}+48\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(3*(112*A + 88*B + 75*C)*a*cos(d*x + c)^3 + 2*(48*A + 88*B + 75*C
)*a*cos(d*x + c)^2 + 8*(8*B + 15*C)*a*cos(d*x + c) + 48*C*a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((112*A + 88*
B + 75*C)*a*cos(d*x + c)^5 + (112*A + 88*B + 75*C)*a*cos(d*x + c)^4)*sqrt(a
)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)
^4), 1/384*(2*(3*(112*A + 88*B + 75*C)*a*cos(d*x + c)^3 + 2*(48*A + 88*B +
75*C)*a*cos(d*x + c)^2 + 8*(8*B + 15*C)*a*cos(d*x + c) + 48*C*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((112*A +
 88*B + 75*C)*a*cos(d*x + c)^5 + (112*A + 88*B + 75*C)*a*cos(d*x + c)^4)*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
```

```
*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*
x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/cos(d*x + c)^(3/2), x)
```

3.1261 $\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{a^2(176A+150B+133C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+150B+133C)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A
+ 90*B + 67*C)*Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d
*x]]) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2
)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])/(1
28*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(10*B + 3*C)*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(40*d*Cos[c + d*x]^(7/2)) + (C*(a + a*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.868542, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^2(176A+150B+133C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(176A+150B+133C)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```


```
[Out] (a^(3/2)*(176*A + 150*B + 133*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^2*(80*A
+ 90*B + 67*C)*Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d
*x]]) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2
)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(176*A + 150*B + 133*C)*Sin[c + d*x])/(1
28*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(10*B + 3*C)*Sqrt[a
+ a*Sec[c + d*x]]*Sin[c + d*x])/(40*d*Cos[c + d*x]^(7/2)) + (C*(a + a*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(7/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !LtQ[n, -2^(-1)] && NeQ
```

```
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a$$

$$= \frac{C(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+d\right.}{}$$

$$= \frac{a(10B+3C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{40d\cos^{\frac{7}{2}}(c+dx)} + \frac{C(a}{}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(10B+3}{}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A}{192d\cos}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A}{192d\cos}$$

$$= \frac{a^2(80A+90B+67C)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(176A}{192d\cos}$$

$$= \frac{a^{3/2}(176A+150B+133C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d}$$

Mathematica [A] time = 5.92914, size = 210, normalized size = 0.69

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(880A+1070B+1273C)\cos(c+dx)+4(3280A+3450B-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(176*A + 150*B +
 133*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (10480*A + 11550
*B + 13313*C + 12*(880*A + 1070*B + 1273*C)*Cos[c + d*x] + 4*(3280*A + 3450
*B + 3059*C)*Cos[2*(c + d*x)] + 3520*A*Cos[3*(c + d*x)] + 3000*B*Cos[3*(c +
 d*x)] + 2660*C*Cos[3*(c + d*x)] + 2640*A*Cos[4*(c + d*x)] + 2250*B*Cos[4*(
c + d*x)] + 1995*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d*Cos[c + d*
x]^(9/2))
```

Maple [B] time = 0.382, size = 720, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)
,x)
```

[Out] -1/3840/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2640*A*cos
(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))*2^(1/2)-2640*A*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+2250*B*cos(d*x+c)^5*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-2250*B*cos
(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*
x+c)))*2^(1/2)+1995*C*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-1995*C*cos(d*x+c)^5*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+5280*A*cos
(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4500*B*cos(d*x+c)^4*(-2/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+3990*C*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+3520*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+3000*B*
sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+2660*C*cos(d*x+c)^3*(-2/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+1280*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2)+2400*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2128
*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+960*B*cos(d*x+c)*sin(d
x+c)(-2/(cos(d*x+c)+1))^(1/2)+1824*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
*sin(d*x+c)+768*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/cos(d*x+c)^(9/2)/(-
2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 1.65031, size = 1523, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")

[Out] [1/7680*(4*(15*(176*A + 150*B + 133*C)*a*cos(d*x + c)^4 + 10*(176*A + 150*B
 + 133*C)*a*cos(d*x + c)^3 + 8*(80*A + 150*B + 133*C)*a*cos(d*x + c)^2 + 48
*(10*B + 19*C)*a*cos(d*x + c) + 384*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((176*A + 150*B + 133*C)*a*cos(d
*x + c)^6 + (176*A + 150*B + 133*C)*a*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*
x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c)
- 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), 1/3840*(2*
(15*(176*A + 150*B + 133*C)*a*cos(d*x + c)^4 + 10*(176*A + 150*B + 133*C)*a
*cos(d*x + c)^3 + 8*(80*A + 150*B + 133*C)*a*cos(d*x + c)^2 + 48*(10*B + 19
*C)*a*cos(d*x + c) + 384*C*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 15*((176*A + 150*B + 133*C)*a*cos(d*x + c)^6 +
 (176*A + 150*B + 133*C)*a*cos(d*x + c)^5)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d

```
*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2
)/cos(d*x + c)^(5/2), x)
```

3.1262 $\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=334

$$\frac{2a^2(136A+182B+143C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2522B+2717C)\sin(c+dx)\cos^{\frac{5}{2}}}{9009d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (16*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(8368*A + 9230*B + 10439*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(8368*A
 + 9230*B + 10439*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(2224*A + 2522*B + 2717*C)*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(9009*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1287*d) + (2*a*(
5*A + 13*B)*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(14
3*d) + (2*A*Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(1
3*d)
```

Rubi [A] time = 1.0976, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4017, 4015, 3805, 3804}

$$\frac{2a^2(136A+182B+143C)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{1287d}+\frac{2a^3(2224A+2522B+2717C)\sin(c+dx)\cos^{\frac{5}{2}}}{9009d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2),x]
```


```
[Out] (16*a^3*(8368*A + 9230*B + 10439*C)*Sin[c + d*x])/(45045*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(8368*A + 9230*B + 10439*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(8368*A
 + 9230*B + 10439*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(2224*A + 2522*B + 2717*C)*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(9009*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(136*A + 182*B + 143*C)*
Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1287*d) + (2*a*(
5*A + 13*B)*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(14
3*d) + (2*A*Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(1
3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
```

```
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{13}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{s}}{}$$

$$= \frac{2A\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,\mathrm{s}}{13d}$$

$$= \frac{2a(5A+13B)\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c}{143d}$$

$$= \frac{2a^2(136A+182B+143C)\cos^{\frac{7}{2}}(c+dx}{1287d}$$

$$= \frac{2a^3(2224A+2522B+2717C)\cos^{\frac{5}{2}}(c+}{9009d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{2a^3(8368A+9230B+10439C)\cos^{\frac{3}{2}}(c}{15015d\sqrt{a+a\sec(c+d}}$$

$$= \frac{8a^3(8368A+9230B+10439C)\sqrt{\cos(c}}{45045d\sqrt{a+a\sec(c+d}}$$

$$= \frac{16a^3(8368A+9230B+10439C)\sin(c}{45045d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 2.62763, size = 190, normalized size = 0.57

$$\frac{a^2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(4(453146A+454285B+445588C)\cos(c+dx)+(746519A+676}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(13/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(2798182*A + 2980640*B + 3233516*C + 4*(453146*A +
454285*B + 445588*C)*Cos[c + d*x] + (746519*A + 676000*B + 581152*C)*Cos[2*
(c + d*x)] + 287060*A*Cos[3*(c + d*x)] + 225550*B*Cos[3*(c + d*x)] + 148720
*C*Cos[3*(c + d*x)] + 94010*A*Cos[4*(c + d*x)] + 58240*B*Cos[4*(c + d*x)] +
 20020*C*Cos[4*(c + d*x)] + 23940*A*Cos[5*(c + d*x)] + 8190*B*Cos[5*(c + d*
x)] + 3465*A*Cos[6*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])
/(720720*d)
```

Maple [A] time = 0.288, size = 222, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(3465A(\cos(dx+c))^6+11970A(\cos(dx+c))^5+4095B(\cos(dx+c))^5+18305A(\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2
),x)
```

```
[Out] -2/45045/d*a^2*(-1+cos(d*x+c))*(3465*A*cos(d*x+c)^6+11970*A*cos(d*x+c)^5+40
95*B*cos(d*x+c)^5+18305*A*cos(d*x+c)^4+14560*B*cos(d*x+c)^4+5005*C*cos(d*x+
c)^4+20920*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18590*C*cos(d*x+c)^3+25104*A
*cos(d*x+c)^2+27690*B*cos(d*x+c)^2+31317*C*cos(d*x+c)^2+33472*A*cos(d*x+c)+
36920*B*cos(d*x+c)+41756*C*cos(d*x+c)+66944*A+73840*B+83512*C)*cos(d*x+c)^(
1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.59576, size = 1532, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2882880*(sqrt(2)*(3783780*a^2*cos(12/13*arctan2(sin(13/2*d*x + 13/2*c), c
os(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 1066065*a^2*cos(10/13*arct
an2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c)
 + 459459*a^2*cos(8/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*
c)))*sin(13/2*d*x + 13/2*c) + 193050*a^2*cos(6/13*arctan2(sin(13/2*d*x + 13
/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13/2*c) + 70070*a^2*cos(4/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c)))*sin(13/2*d*x + 13
/2*c) + 20475*a^2*cos(2/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c)))*sin(13/2*d*x + 13/2*c) - 3783780*a^2*cos(13/2*d*x + 13/2*c)*sin(12
/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 1066065*a^2*
cos(13/2*d*x + 13/2*c)*sin(10/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d
*x + 13/2*c))) - 459459*a^2*cos(13/2*d*x + 13/2*c)*sin(8/13*arctan2(sin(13/
2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 193050*a^2*cos(13/2*d*x + 13/2*
c)*sin(6/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) - 7007
0*a^2*cos(13/2*d*x + 13/2*c)*sin(4/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) - 20475*a^2*cos(13/2*d*x + 13/2*c)*sin(2/13*arctan2(sin
(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 6930*a^2*sin(13/2*d*x + 13/
2*c) + 20475*a^2*sin(11/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 1
3/2*c))) + 70070*a^2*sin(9/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x
+ 13/2*c))) + 193050*a^2*sin(7/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*
d*x + 13/2*c))) + 459459*a^2*sin(5/13*arctan2(sin(13/2*d*x + 13/2*c), cos(1
3/2*d*x + 13/2*c))) + 1066065*a^2*sin(3/13*arctan2(sin(13/2*d*x + 13/2*c),
cos(13/2*d*x + 13/2*c))) + 3783780*a^2*sin(1/13*arctan2(sin(13/2*d*x + 13/2
*c), cos(13/2*d*x + 13/2*c))))*A*sqrt(a) + 1144*sqrt(2)*(225*a^2*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 378*a^2*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2100*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4095*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 63*(65*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*cos(9/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(585*a^2*cos(4*d*x + 4*c) +
54*a^2*cos(2*d*x + 2*c) + 5*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*C*sqrt(a) + 130*(770*sqrt(2)*a^2*sin(9/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1287*sqrt(2)*a^2*sin(7/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 6930*sqrt(2)*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8778*sqrt(2)*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 63756*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 33*(266*sqrt(2)*a^2*sin(4*d*x + 4*c) + 39*sqrt(2)*a^2*sin(2*d
*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*(2926*
sqrt(2)*a^2*cos(4*d*x + 4*c) + 429*sqrt(2)*a^2*cos(2*d*x + 2*c) + 42*sqrt(2
)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

Fricas [A] time = 0.516178, size = 544, normalized size = 1.63

$$2\left(3465\,Aa^2\cos\left(dx+c\right)^6+315\,(38\,A+13\,B)a^2\cos\left(dx+c\right)^5+35\,(523\,A+416\,B+143\,C)a^2\cos\left(dx+c\right)^4+5\,(4184\,A -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*A*a^2*cos(d*x + c)^6 + 315*(38*A + 13*B)*a^2*cos(d*x + c)^5 +
 35*(523*A + 416*B + 143*C)*a^2*cos(d*x + c)^4 + 5*(4184*A + 4615*B + 3718*
C)*a^2*cos(d*x + c)^3 + 3*(8368*A + 9230*B + 10439*C)*a^2*cos(d*x + c)^2 +
4*(8368*A + 9230*B + 10439*C)*a^2*cos(d*x + c) + 8*(8368*A + 9230*B + 10439
*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(13/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(
d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(13/2), x)
```

3.1263 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=284

$$\frac{2a^2(32A+44B+33C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{231d}+\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)\cos^{\frac{3}{2}}}{3465d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (4*a^3*(2840*A + 3212*B + 3795*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(1160*A + 13
64*B + 1485*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(231*d) + (2*a*(5*A + 11*B)*Cos[c + d*x]^(7/2)*(a + a*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d) + (2*A*Cos[c + d*x]^(9/2)*(a + a*Se
c[c + d*x])^(5/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 1.01406, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4017, 4015, 3805, 3804}

$$\frac{2a^2(32A+44B+33C)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{231d}+\frac{2a^3(1160A+1364B+1485C)\sin(c+dx)\cos^{\frac{3}{2}}}{3465d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2),x]
```


```
[Out] (4*a^3*(2840*A + 3212*B + 3795*C)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(2840*A + 3212*B + 3795*C)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(1160*A + 13
64*B + 1485*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a^2*(32*A + 44*B + 33*C)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]*Sin[c + d*x])/(231*d) + (2*a*(5*A + 11*B)*Cos[c + d*x]^(7/2)*(a + a*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(99*d) + (2*A*Cos[c + d*x]^(9/2)*(a + a*Se
c[c + d*x])^(5/2)*Sin[c + d*x])/(11*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{s}}{}$$

$$=\frac{2A\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,\mathrm{si}}{11d}$$

$$=\frac{2a(5A+11B)\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c}{99d}$$

$$=\frac{2a^2(32A+44B+33C)\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}{231d}$$

$$=\frac{2a^3(1160A+1364B+1485C)\cos^{\frac{3}{2}}(c+}{3465d\sqrt{a+a\sec(c+dx}}$$

$$=\frac{2a^3(2840A+3212B+3795C)\sqrt{\cos(c+}}{3465d\sqrt{a+a\sec(c+dx}}$$

$$=\frac{4a^3(2840A+3212B+3795C)\sin(c+}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

Mathematica [A] time = 2.32137, size = 157, normalized size = 0.55

$$\frac{a^2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((69890A+68552B+66660C)\cos(c+dx)+16(1625A+1397B}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(114640*A + 124366*B + 137280*C + (69890*A + 68552*
B + 66660*C)*Cos[c + d*x] + 16*(1625*A + 1397*B + 990*C)*Cos[2*(c + d*x)] +
 8675*A*Cos[3*(c + d*x)] + 5720*B*Cos[3*(c + d*x)] + 1980*C*Cos[3*(c + d*x)
] + 2240*A*Cos[4*(c + d*x)] + 770*B*Cos[4*(c + d*x)] + 315*A*Cos[5*(c + d*x
)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(27720*d)
```

Maple [A] time = 0.28, size = 189, normalized size = 0.7

$$-\frac{2a^2(-1+\cos(dx+c))\left(315A(\cos(dx+c))^5+1120A(\cos(dx+c))^4+385B(\cos(dx+c))^4+1775A(\cos(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2
),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B
*cos(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+495*C*cos(d*x+c)^3+21
30*A*cos(d*x+c)^2+2409*B*cos(d*x+c)^2+1980*C*cos(d*x+c)^2+2840*A*cos(d*x+c)
+3212*B*cos(d*x+c)+3795*C*cos(d*x+c)+5680*A+6424*B+7590*C)*cos(d*x+c)^(1/2)
*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.50638, size = 1249, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 34
65*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 318
78*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11
/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1
287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos
```

```
(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2
*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 1
1/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*
x + 11/2*c))))*A*sqrt(a) + 44*sqrt(2)*(225*a^2*sin(7/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 378*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2100*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 4095*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 63*(65*
a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 7*(585*a^2*cos(4*d*x + 4*c) + 54*a^2*cos(2*d*x +
 2*c) + 5*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt
(a) - 660*sqrt(2)*(77*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))*sin(2*d*x + 2*c) - 42*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 77*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 630
*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (77*a^2*cos(2*d
*x + 2*c) + 6*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*
sqrt(a))/d
```

Fricas [A] time = 0.508834, size = 459, normalized size = 1.62

$$\frac{2\left(315\,Aa^2\cos\left(dx+c\right)^5+35\,(32\,A+11\,B)a^2\cos\left(dx+c\right)^4+5\,(355\,A+286\,B+99\,C)a^2\cos\left(dx+c\right)^3+3\,(710\,A+803\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*A*a^2*cos(d*x + c)^5 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^4 + 5*
(355*A + 286*B + 99*C)*a^2*cos(d*x + c)^3 + 3*(710*A + 803*B + 660*C)*a^2*c
os(d*x + c)^2 + (2840*A + 3212*B + 3795*C)*a^2*cos(d*x + c) + 2*(2840*A + 3
212*B + 3795*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(
d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(11/2), x)
```

3.1264 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=231

$$\frac{16a^2(13A+15B+21C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(13A+15B+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(13A+}{}$$

```
[Out] (64*a^3*(13*A + 15*B + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Sec[c + d*x]]) + (16*a^2*(13*A + 15*B + 21*C)*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(13*A + 15*B + 21*C)*Cos[c
 + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(5*A +
9*B)*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (
2*A*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.706126, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4086, 4013, 3809, 3804}

$$\frac{16a^2(13A+15B+21C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{315d}+\frac{64a^3(13A+15B+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(13A+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```



```
[Out] (64*a^3*(13*A + 15*B + 21*C)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Sec[c + d*x]]) + (16*a^2*(13*A + 15*B + 21*C)*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(13*A + 15*B + 21*C)*Cos[c
 + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(5*A +
9*B)*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (
2*A*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}{9d}$$

$$= \frac{2(5A+9B)\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c}{63d}$$

$$= \frac{2a(13A+15B+21C)\cos^{\frac{3}{2}}(c+dx)(a}{105d}$$

$$= \frac{16a^2(13A+15B+21C)\sqrt{\cos(c+dx}}{315d}$$

$$= \frac{64a^3(13A+15B+21C)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 1.5739, size = 124, normalized size = 0.54

$$\frac{a^2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}((3116A+3030B+2352C)\cos(c+dx)+4(254A+180B+63C}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(5653*A + 6240*B + 7476*C + (3116*A + 3030*B + 2352
*C)*Cos[c + d*x] + 4*(254*A + 180*B + 63*C)*Cos[2*(c + d*x)] + 260*A*Cos[3*
(c + d*x)] + 90*B*Cos[3*(c + d*x)] + 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec
[c + d*x])]*Tan[(c + d*x)/2])/(1260*d)
```

Maple [A] time = 0.342, size = 156, normalized size = 0.7

$$-\frac{2\,a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(35\,A\left(\cos\left(dx+c\right)\right)^{4}+130\,A\left(\cos\left(dx+c\right)\right)^{3}+45\,B\left(\cos\left(dx+c\right)\right)^{3}+219\,A\left(\cos\left(dx+c\right)\right)^{2}+18\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+45*B*cos
(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+63*C*cos(d*x+c)^2+292*A*cos
(d*x+c)+345*B*cos(d*x+c)+294*C*cos(d*x+c)+584*A+690*B+903*C)*cos(d*x+c)^(1/
2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.44277, size = 1014, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) - 30*sqrt(2)*(77*a^2*cos(7/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 42*a^2*sin(5/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 77*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 630*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (77*a^2*cos(2*d*x + 2*c) + 6*a^2)*sin(7/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*B*sqrt(a) - 168*(75*sqrt(2)*a^2*cos(5/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 25*sqrt(2)*a^2*sin(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*sqrt(2)*a^2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(25*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*
sqrt(a))/d
```

Fricas [A] time = 0.496033, size = 377, normalized size = 1.63

$$\frac{2\left(35\,Aa^{2}\cos\left(dx+c\right)^{4}+5\,(26\,A+9\,B)a^{2}\cos\left(dx+c\right)^{3}+3\,(73\,A+60\,B+21\,C)a^{2}\cos\left(dx+c\right)^{2}+(292\,A+345\,B+294\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^4 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^3 + 3*(73*
A + 60*B + 21*C)*a^2*cos(d*x + c)^2 + (292*A + 345*B + 294*C)*a^2*cos(d*x +
 c) + (584*A + 690*B + 903*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1265 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=242

$$\frac{2a^3(160A+224B+245C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(40A+56B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a^{5/2}C\sqrt{}}{}$$

```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(160*A + 224*B + 245*C)*Sin[c
+ d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(40*A
+ 56*B + 35*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1
05*d) + (2*a*(5*A + 7*B)*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c
 + d*x])/(7*d)
```

Rubi [A] time = 0.836349, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4017, 4015, 3801, 215}

$$\frac{2a^3(160A+224B+245C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(40A+56B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a^{5/2}C\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (2*a^(5/2)*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(160*A + 224*B + 245*C)*Sin[c
+ d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(40*A
+ 56*B + 35*C)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(1
05*d) + (2*a*(5*A + 7*B)*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c
 + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}{7d}$$

$$= \frac{2a(5A+7B)\cos^{\frac{3}{2}}(c+dx)(a+a\sec(}{35d}$$

$$= \frac{2a^2(40A+56B+35C)\sqrt{\cos(c+dx)}}{105d}$$

$$= \frac{2a^3(160A+224B+245C)\sin(c+d}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= \frac{2a^3(160A+224B+245C)\sin(c+d}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= \frac{2a^{5/2}C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c}}{d}$$

Mathematica [A] time = 1.84082, size = 137, normalized size = 0.57

$$\frac{a^2\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((505A+392B+140C)\cos(c+dx)+6(20A+\right.}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(420*Sq
rt[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(1040*A + 1246*B + 1120*C + (
505*A + 392*B + 140*C)*Cos[c + d*x] + 6*(20*A + 7*B)*Cos[2*(c + d*x)] + 15*
A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(420*d)
```

Maple [A] time = 0.279, size = 270, normalized size = 1.1

$$-\frac{a^2}{210\,d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(60\,A(\cos(dx+c))^4-105\,C\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/210/d*a^2*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(60*A*cos
(d*x+c)^4-105*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)*sin(d*x+c)+105*C*(-2/(cos(d*
x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+s
in(d*x+c)))*2^(1/2)*sin(d*x+c)+180*A*cos(d*x+c)^3+84*B*cos(d*x+c)^3+220*A*c
os(d*x+c)^2+308*B*cos(d*x+c)^2+140*C*cos(d*x+c)^2+460*A*cos(d*x+c)+812*B*co
s(d*x+c)+980*C*cos(d*x+c)-920*A-1204*B-1120*C)/sin(d*x+c)
```

Maxima [B] time = 2.48745, size = 1357, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*
cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^
2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin
(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) - 28*(75*s
qrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 25*sqrt(2)*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 75*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*
(25*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*B*sqrt(a) + 140*(2*sqrt(2)*a^2*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 30*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*
```

```
d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*a^
2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2) + 3*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*a
^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2))*C*sqrt(a))/d
```

Fricas [A] time = 0.597701, size = 1196, normalized size = 4.94

$$\left[\frac{4\left(15\,Aa^2\cos\left(dx+c\right)^3+3\left(20\,A+7\,B\right)a^2\cos\left(dx+c\right)^2+\left(115\,A+98\,B+35\,C\right)a^2\cos\left(dx+c\right)+\left(230\,A+301\,B+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/210*(4*(15*A*a^2*cos(d*x + c)^3 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^2 + (1
15*A + 98*B + 35*C)*a^2*cos(d*x + c) + (230*A + 301*B + 280*C)*a^2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 105*(C*a
^2*cos(d*x + c) + C*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d
*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*
cos(d*x + c) + d), 1/105*(2*(15*A*a^2*cos(d*x + c)^3 + 3*(20*A + 7*B)*a^2*c
os(d*x + c)^2 + (115*A + 98*B + 35*C)*a^2*cos(d*x + c) + (230*A + 301*B + 2
80*C)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 105*(C*a^2*cos(d*x + c) + C*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d
*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(7/2), x)
```

3.1266 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=243

$$\frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A+10B-15C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(2B+5C)\sqrt{\cos(c+}}{}$$

```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(64*A + 70*B + 15*C)*Sin
[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(16*A
+ 10*B - 15*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*
x]]) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(3*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x
])/(5*d)
```

Rubi [A] time = 0.875288, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4086, 4017, 4018, 4015, 3801, 215}

$$\frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(16A+10B-15C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(2B+5C)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(2*B + 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(64*A + 70*B + 15*C)*Sin
[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(16*A
+ 10*B - 15*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*
x]]) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(3*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x
])/(5*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
```

```
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}{5d}$$

$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}(a+a\sec(c+}{3d}$$

$$= -\frac{a^2(16A+10B-15C)\sqrt{a+a\sec(c-}}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(64A+70B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(2B+5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A] time = 1.48285, size = 149, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((181A+160B+60C)\cos(c+dx)+2(14A+5B)\cos(2(c+\right.}{60d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(30*Sqrt[2]*(2*B + 5*C)*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(28*A + 10*B + 30*C + (181
*A + 160*B + 60*C)*Cos[c + d*x] + 2*(14*A + 5*B)*Cos[2*(c + d*x)] + 3*A*Cos
[3*(c + d*x)])*Sin[(c + d*x)/2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.398, size = 410, normalized size = 1.7

$$-\frac{a^2}{60\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-30\,B\sin(dx+c)\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/60/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-30*B*sin(d*x+c)*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+30*B*sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1)
```

```
)^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c)))*cos(d*x+c)-75*C*sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+7
5*C*sin(d*x+c)*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+24*A*cos(d*x+c)^4+
88*A*cos(d*x+c)^3+40*B*cos(d*x+c)^3+232*A*cos(d*x+c)^2+280*B*cos(d*x+c)^2+1
20*C*cos(d*x+c)^2-344*A*cos(d*x+c)-320*B*cos(d*x+c)-60*C*cos(d*x+c)-60*C)/s
in(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 3.64898, size = 11426, normalized size = 47.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/1260*(42*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 210*(2*sqrt(2
)*a^2*sin(3/2*d*x + 3/2*c) + 30*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 3*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*B*sqrt(a) - 5*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*
c) - 1260*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x
 + 2*c)^2*sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*
d*x + 3/2*c) - 60*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3
/2*d*x + 3/2*c) - 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (
25*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x
+ 1/2*c) - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x
 + 1/2*c)^2)*sin(3/2*d*x + 3/2*c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*
cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2
*sin(3/2*d*x + 3/2*c) + 69*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c
) - 198*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) - 198*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2
)*a^2*cos(3/2*d*x + 3/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x
 + 2*c))*sin(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) -
 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x +
 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x
 + 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x +
 2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1
/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2
*c)^2)*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*
d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/
2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*
d*x + 2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*
x + 1/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x
+ 1/2*c)^2)*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(
5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*co
```

s(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*si
n(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/
2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*
d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*c
os(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)
*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2
*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin
(1/2*d*x + 1/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*
a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(
2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*co
s(1/2*d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3
/2*c)*cos(1/2*d*x + 1/2*c) + 189*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) -
21*(60*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*c
os(2*d*x + 2*c) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c
)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x +
2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(
1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(
2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^
2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 +
2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*
sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(
1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^
2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(
1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(
2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x +
5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
qrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqr
t(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 31
5*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x
+ 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*
d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*
cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a
^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*
c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x +
2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 +
2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*co
s(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*
c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a
^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2
*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 315*(a^2*cos(1/2*d*x +
1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(
2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^
2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 +
(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
+ a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d
*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x +
1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2
*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) +

2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 35*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(13/2*d*x + 13/2*c) + 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(11/2*d*x + 11/2*c) + 7*(9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 - (5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 - (5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 4*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(9/2*d*x + 9/2*c) - 390*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(

2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/
2*c))*sin(5/2*d*x + 5/2*c))*sin(7/2*d*x + 7/2*c) - 21*(69*sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c)^2 + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 69*(sqrt(2)*a^
2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 2*(25*sqrt(2)*a^
2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*cos(2*d*x + 2*
c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt
(2)*a^2)*sin(2*d*x + 2*c)^2 + 12*sqrt(2)*a^2 + 138*(sqrt(2)*a^2*cos(2*d*x +
 2*c)*cos(1/2*d*x + 1/2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2
*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (69*sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 - 50*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*
x + 1/2*c) + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 24*sqrt(2)*a^2)*cos(2
*d*x + 2*c) - 10*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) +
 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c)
)*sin(5/2*d*x + 5/2*c) + 105*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 12*sq
rt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*
c))*sin(2*d*x + 2*c) - 252*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*
a^2)*sin(1/2*d*x + 1/2*c) - 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(
2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a
^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5
/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1
/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + s
qrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^
2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a
^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x +
2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x +
2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*
cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)
*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)
^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*
c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(7/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 63*(sqrt(2)*a^2*cos(1/2*d*
x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x
+ 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2
 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*co
s(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt
(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x +
 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1
/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)
*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x
 + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*
a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c
)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/
2*c))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1260*(
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (
sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)
*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*
cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*
c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2
*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt
(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*
d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x
+ 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x
 + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x

```
 + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d
*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(
2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/
2*c))*sin(5/2*d*x + 5/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*C*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*cos(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2*c
os(1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c)*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) +
 2*(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + cos
(1/2*d*x + 1/2*c)^2 + 2*(cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sin(2*d*
x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) +
 sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2))/d
```

Fricas [A] time = 0.718982, size = 1274, normalized size = 5.24

$$\left[\frac{4\left(6\,Aa^2\cos\left(dx+c\right)^3+2\,(14\,A+5\,B)a^2\cos\left(dx+c\right)^2+2\,(43\,A+40\,B+15\,C)a^2\cos\left(dx+c\right)+15\,Ca^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(4*(6*A*a^2*cos(d*x + c)^3 + 2*(14*A + 5*B)*a^2*cos(d*x + c)^2 + 2*(4
3*A + 40*B + 15*C)*a^2*cos(d*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((2*B + 5*C)*a^2*cos(d*x
+ c)^2 + (2*B + 5*C)*a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d
*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
 + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/30*(2*(6*A*a^2*cos(d*x +
c)^3 + 2*(14*A + 5*B)*a^2*cos(d*x + c)^2 + 2*(43*A + 40*B + 15*C)*a^2*cos(d
*x + c) + 15*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + 15*((2*B + 5*C)*a^2*cos(d*x + c)^2 + (2*B + 5*C)*a^2*cos
(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(5/2), x)
```

3.1267 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=253

$$\frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-12B-21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{12d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(8A+20B+19C)\sqrt{\cos(}}{}$$

```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(56*A + 12*B
 - 27*C)*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
 (a^2*(8*A - 12*B - 21*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt
[Cos[c + d*x]]) - (a*(4*A - 3*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
6*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2
)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.880585, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4018, 4015, 3801, 215}

$$\frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{a^2(8A-12B-21C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{12d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(8A+20B+19C)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (a^(5/2)*(8*A + 20*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(56*A + 12*B
 - 27*C)*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
 (a^2*(8*A - 12*B - 21*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt
[Cos[c + d*x]]) - (a*(4*A - 3*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
6*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2
)*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/}}{3d}$$

$$= -\frac{a(4A-3C)(a+a\sec(c+dx))^{3/2}\sin}{6d\sqrt{\cos(c+dx)}}$$

$$= -\frac{a^2(8A-12B-21C)\sqrt{a+a\sec(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(56A+12B-27C)\sin(c+dx)}{12d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(8A+20B+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a+a\sec}}\right.}{4d}$$

Mathematica [A] time = 1.91004, size = 155, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(4\sin\left(\frac{1}{2}(c+dx)\right)(3(2A+4B+11C)\cos(c+dx)+4(8A+3B)\cos(2(c+dx))\right.}{48d\cos^{\frac{3}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(8*A + 20*B + 1
9*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 4*(32*A + 12*B + 6*
C + 3*(2*A + 4*B + 11*C)*Cos[c + d*x] + 4*(8*A + 3*B)*Cos[2*(c + d*x)] + 2*
A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 0.347, size = 536, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(16*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1
))^(1/2)+24*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1+sin(d*x+c)))*cos(d*x+c)^2-24*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+128*A*cos(d*x+c)^2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+60*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-60*B*2^(1/2)*arc
tan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*
x+c)^2+48*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+57*C*2^(1/2)*
arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos
(d*x+c)^2-57*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+24*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2)+66*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+12*C*(-2/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x
+c)^(3/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 21.9635, size = 7309, normalized size = 28.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(4*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*
```

cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*
d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*A*sqrt(a) + 12*(4*sqrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^3 - 4*sqrt(2)*a^2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(2*sqrt(2)
*a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 5*(a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(a^2*cos
(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 5*(a^2*cos(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a
^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 2) - 5*(a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(2*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))

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 + 2) + 4*(2*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + s
qrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(
2)*a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(cos(5/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(5/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2) - 3*(88*sqr
t(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^2*cos(5/2*d*x
 + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 44*sqrt(
2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 76*(a^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(
2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 - 76*(a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
)*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) - 14*sqrt(2)*
a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22*sqrt(2)
*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
```

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1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c))*
cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22*sqrt(2)*a^2*
sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(11*sqrt(2)*a
^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) + 7*sqrt(2)*a^
2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 19*(a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*si
n(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqr
t(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*C*sqrt(a)/(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.14011, size = 1299, normalized size = 5.13

$$\left[\frac{4\left(8\,Aa^2\cos\left(dx+c\right)^3+8\,(8\,A+3\,B)a^2\cos\left(dx+c\right)^2+3\,(4\,B+11\,C)a^2\cos\left(dx+c\right)+6\,Ca^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(4*(8*A*a^2*cos(d*x + c)^3 + 8*(8*A + 3*B)*a^2*cos(d*x + c)^2 + 3*(4*
B + 11*C)*a^2*cos(d*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((8*A + 20*B + 19*C)*a^2*cos(d*x + c
)^3 + (8*A + 20*B + 19*C)*a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3
 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqr
t(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 +
cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/24*(2*(8*A*a^2*c
os(d*x + c)^3 + 8*(8*A + 3*B)*a^2*cos(d*x + c)^2 + 3*(4*B + 11*C)*a^2*cos(d
*x + c) + 6*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
```

```
))*sin(d*x + c) + 3*((8*A + 20*B + 19*C)*a^2*cos(d*x + c)^3 + (8*A + 20*B +
 19*C)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right)(a \sec(dx+c) + a)^{\frac{5}{2}} \cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(3/2), x)
```

3.1268 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+\right.$

Optimal. Leaf size=253

$$\frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+42B+31C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{24d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(40A+38B+25C}{}$$

```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(24*A - 54*
B - 49*C)*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*(24*A + 42*B + 31*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sq
rt[Cos[c + d*x]]) + (a*(6*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(12*d*Sqrt[Cos[c + d*x]]) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3
*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.873365, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4088, 4018, 4015, 3801, 215}

$$\frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2(24A+42B+31C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{24d\sqrt{\cos(c+dx)}}+\frac{a^{5/2}(40A+38B+25C}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```



```
[Out] (a^(5/2)*(40*A + 38*B + 25*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(24*A - 54*
B - 49*C)*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*(24*A + 42*B + 31*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sq
rt[Cos[c + d*x]]) + (a*(6*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(12*d*Sqrt[Cos[c + d*x]]) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3
*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{s}}{}$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{a(6B+5C)(a+a\sec(c+dx))^{3/2}\sin(c+}{12d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^2(24A+42B+31C)\sqrt{a+a\sec(c+dx}}{24d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{a^3(24A-54B-49C)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{a^{5/2}(40A+38B+25C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{a+a\sec}}\right.}{8d}$$

Mathematica [A] time = 2.59182, size = 157, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(18A+6B+17C)\cos(c+dx)+3(8A+22B+25C)\cos(2(c+\right.}{48d\cos^{\frac{5}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(40*A + 38*B +
25*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (24*A + 66*B + 91*
C + 4*(18*A + 6*B + 17*C)*Cos[c + d*x] + 3*(8*A + 22*B + 25*C)*Cos[2*(c + d
*x)] + 24*A*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 0.384, size = 567, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/48/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(120*A*cos(
d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c)))*2^(1/2)-120*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+96*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos
(d*x+c)+1))^(1/2)+114*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3-114*B*arctan(1/4*2^(1/2)*(-2/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3+75*C*cos(
d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c)))*2^(1/2)-75*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2)+132*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+15
0*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+24*B*cos(d*x+c)*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+68*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*s
in(d*x+c)+16*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/cos(d*x+c)^(5/2)/(-2/(
cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.13413, size = 1328, normalized size = 5.25

$$\left[\frac{4\left(48\,Aa^2\cos\left(dx+c\right)^3+3\,(8\,A+22\,B+25\,C)a^2\cos\left(dx+c\right)^2+2\,(6\,B+17\,C)a^2\cos\left(dx+c\right)+8\,Ca^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(48*A*a^2*cos(d*x + c)^3 + 3*(8*A + 22*B + 25*C)*a^2*cos(d*x + c)^
2 + 2*(6*B + 17*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((40*A + 38*B + 25*C)*a^2*c
os(d*x + c)^4 + (40*A + 38*B + 25*C)*a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos
(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x +
c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2
*(48*A*a^2*cos(d*x + c)^3 + 3*(8*A + 22*B + 25*C)*a^2*cos(d*x + c)^2 + 2*(6
*B + 17*C)*a^2*cos(d*x + c) + 8*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((40*A + 38*B + 25*C)*a^2*cos(d*x +
 c)^4 + (40*A + 38*B + 25*C)*a^2*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec(dx+c)^2 + B \sec(dx+c) + A\right)(a \sec(dx+c) + a)^{\frac{5}{2}} \sqrt{\cos(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)*sqrt(cos(d*x + c)), x)
```

3.1269 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+24B+17C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{32d\cos^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(304A+200B+1}{}$$

```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A
+ 392*B + 299*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
 d*x]]) + (a^2*(16*A + 24*B + 17*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(32*d*Cos[c + d*x]^(3/2)) + (a*(8*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(24*d*Cos[c + d*x]^(3/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c +
d*x])/(4*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.891092, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4088, 4018, 4016, 3801, 215}

$$\frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(16A+24B+17C)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{32d\cos^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(304A+200B+1}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (a^(5/2)*(304*A + 200*B + 163*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(432*A
+ 392*B + 299*C)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
 d*x]]) + (a^2*(16*A + 24*B + 17*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(32*d*Cos[c + d*x]^(3/2)) + (a*(8*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(24*d*Cos[c + d*x]^(3/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c +
d*x])/(4*d*Cos[c + d*x]^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\,\mathrm{se}$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(8B+5C)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)} + \frac{C(a+}{}$$

$$= \frac{a^2(16A+24B+17C)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{32d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A+24}{}$$

$$= \frac{a^3(432A+392B+299C)\sin(c+dx)}{192d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(16A+24}{}$$

$$= \frac{a^{5/2}(304A+200B+163C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{c}}{64d}$$

Mathematica [A] time = 4.07389, size = 178, normalized size = 0.7

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((1584A+2056B+2203C)\cos(c+dx)+4(48A+136B+16\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(304*A + 200*B
+ 163*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (192*A + 544*B
+ 844*C + (1584*A + 2056*B + 2203*C)*Cos[c + d*x] + 4*(48*A + 136*B + 163*C
)*Cos[2*(c + d*x)] + 528*A*Cos[3*(c + d*x)] + 600*B*Cos[3*(c + d*x)] + 489*
C*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 0.348, size = 629, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] -1/384/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(912*A*cos
(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))-912*A*cos(d*x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+600*B*cos(d*x+c)^4*2^(1/2)*arctan(1/
4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-600*B*cos(d*
x+c)^4*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))+489*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-489*C*cos(d*x+c)^4*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+1056*A*sin(d*x+
c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)+1200*B*sin(d*x+c)*cos(d*x+c)^3*(-
2/(cos(d*x+c)+1))^(1/2)+978*C*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+192*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+544*B*cos(d*x+
c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+652*C*cos(d*x+c)^2*(-2/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+128*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+368*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*C*(-2/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c))/cos(d*x+c)^(7/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(
1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.64701, size = 1415, normalized size = 5.59

$$\left[\frac{4\left(3\,(176\,A+200\,B+163\,C)a^2\cos(dx+c)^3+2\,(48\,A+136\,B+163\,C)a^2\cos(dx+c)^2+8\,(8\,B+23\,C)a^2\cos(dx+c)\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(3*(176*A + 200*B + 163*C)*a^2*cos(d*x + c)^3 + 2*(48*A + 136*B +
 163*C)*a^2*cos(d*x + c)^2 + 8*(8*B + 23*C)*a^2*cos(d*x + c) + 48*C*a^2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(
(304*A + 200*B + 163*C)*a^2*cos(d*x + c)^5 + (304*A + 200*B + 163*C)*a^2*co
s(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7
*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c
)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(176*A + 200*B + 163*C)*a^2*cos(d*x +
c)^3 + 2*(48*A + 136*B + 163*C)*a^2*cos(d*x + c)^2 + 8*(8*B + 23*C)*a^2*cos
(d*x + c) + 48*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) + 3*((304*A + 200*B + 163*C)*a^2*cos(d*x + c)^5 + (304*A
 + 200*B + 163*C)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/sqrt(cos(d*x + c)), x)
```

3.1270 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=301

$$\frac{a^3(400A+326B+283C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+110B+79C)\sin(c+dx)}{240d\cos^{\frac{5}{2}}(c+dx}$$

```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*
A + 950*B + 787*C)*Sin[c + d*x])/(960*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(400*A + 326*B + 283*C)*Sin[c + d*x])/(128*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(80*A + 110*B + 79*C)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(240*d*Cos[c + d*x]^(5/2)) + (a*(2*B + C)*(a + a*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (C*(a + a*Sec[c
 + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 1.00604, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(400A+326B+283C)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(80A+110B+79C)\sin(c+dx)}{240d\cos^{\frac{5}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```


```
[Out] (a^(5/2)*(400*A + 326*B + 283*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*
Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(1040*
A + 950*B + 787*C)*Sin[c + d*x])/(960*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(400*A + 326*B + 283*C)*Sin[c + d*x])/(128*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(80*A + 110*B + 79*C)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(240*d*Cos[c + d*x]^(5/2)) + (a*(2*B + C)*(a + a*S
ec[c + d*x])^(3/2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (C*(a + a*Sec[c
 + d*x])^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
```

```
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= \frac{a(2B+C)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{C(}{}$$

$$= \frac{a^2(80A+110B+79C)\sqrt{a+a\sec(c+dx)}\sin(c+}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(80A}{}$$

$$= \frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(400A}{128d\,\mathrm{co}}$$

$$= \frac{a^3(1040A+950B+787C)\sin(c+dx)}{960d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(400A}{128d\,\mathrm{co}}$$

$$= \frac{a^{5/2}(400A+326B+283C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d}$$

Mathematica [A] time = 6.32817, size = 212, normalized size = 0.7

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(12(1360A+1950B+2343C)\cos(c+dx)+4(6640A+6730B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(400*A + 326*B
 + 283*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (20560*A + 220
30*B + 24863*C + 12*(1360*A + 1950*B + 2343*C)*Cos[c + d*x] + 4*(6640*A + 6
730*B + 6509*C)*Cos[2*(c + d*x)] + 5440*A*Cos[3*(c + d*x)] + 6520*B*Cos[3*(
c + d*x)] + 5660*C*Cos[3*(c + d*x)] + 6000*A*Cos[4*(c + d*x)] + 4890*B*Cos[
4*(c + d*x)] + 4245*C*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d*Cos[c +
 d*x]^(9/2))
```

Maple [B] time = 0.375, size = 722, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] 1/3840/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(6000*A*co
s(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d
*x+c)))*2^(1/2)-6000*A*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+4890*B*cos(d*x+c)^5*arctan(1/4*2^(
1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-4890*B*co
s(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d
*x+c)))*2^(1/2)+4245*C*cos(d*x+c)^5*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)-4245*C*cos(d*x+c)^5*arctan(1/4*2^(
1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)-12000*A*c
os(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9780*B*cos(d*x+c)^4*(-2/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-8490*C*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)-5440*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-6520*
B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)-5660*C*cos(d*x+c)^3*(-2
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-1280*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2)-3680*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-45
28*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-960*B*cos(d*x+c)*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-2784*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)*sin(d*x+c)-768*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/cos(
d*x+c)^(9/2)/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.67689, size = 1561, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(4*(15*(400*A + 326*B + 283*C)*a^2*cos(d*x + c)^4 + 10*(272*A + 326
*B + 283*C)*a^2*cos(d*x + c)^3 + 8*(80*A + 230*B + 283*C)*a^2*cos(d*x + c)^
2 + 48*(10*B + 29*C)*a^2*cos(d*x + c) + 384*C*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((400*A + 326*B + 283*
C)*a^2*cos(d*x + c)^6 + (400*A + 326*B + 283*C)*a^2*cos(d*x + c)^5)*sqrt(a)
*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^
5), 1/3840*(2*(15*(400*A + 326*B + 283*C)*a^2*cos(d*x + c)^4 + 10*(272*A +
326*B + 283*C)*a^2*cos(d*x + c)^3 + 8*(80*A + 230*B + 283*C)*a^2*cos(d*x +
c)^2 + 48*(10*B + 29*C)*a^2*cos(d*x + c) + 384*C*a^2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((400*A + 326*B + 2
83*C)*a^2*cos(d*x + c)^6 + (400*A + 326*B + 283*C)*a^2*cos(d*x + c)^5)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x
```

```
+ c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")

[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/cos(d*x + c)^(3/2), x)
```

3.1271 $\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{a^3(1304A+1132B+1015C)\sin(c+dx)}{512d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1304A+1132B+1015C)\sin(c+dx)}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(680A+628B+545C)\sin(c+}{960d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)}}$$

```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a +
 a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(68
0*A + 628*B + 545*C)*Sin[c + d*x])/(960*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec
[c + d*x]]) + (a^3*(1304*A + 1132*B + 1015*C)*Sin[c + d*x])/(768*d*Cos[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1304*A + 1132*B + 1015*C)*Sin[
c + d*x])/(512*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(120*A
 + 156*B + 115*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(480*d*Cos[c + d*x
]^(7/2)) + (a*(12*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(60*d*C
os[c + d*x]^(7/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Cos[c
 + d*x]^(7/2))
```

Rubi [A] time = 1.10375, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4088, 4018, 4016, 3803, 3801, 215}

$$\frac{a^3(1304A+1132B+1015C)\sin(c+dx)}{512d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(1304A+1132B+1015C)\sin(c+dx)}{768d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(680A+628B+545C)\sin(c+}{960d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```



```
[Out] (a^(5/2)*(1304*A + 1132*B + 1015*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a +
 a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(512*d) + (a^3*(68
0*A + 628*B + 545*C)*Sin[c + d*x])/(960*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec
[c + d*x]]) + (a^3*(1304*A + 1132*B + 1015*C)*Sin[c + d*x])/(768*d*Cos[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(1304*A + 1132*B + 1015*C)*Sin[
c + d*x])/(512*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(120*A
 + 156*B + 115*C)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(480*d*Cos[c + d*x
]^(7/2)) + (a*(12*B + 5*C)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(60*d*C
os[c + d*x]^(7/2)) + (C*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Cos[c
 + d*x]^(7/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
```

```
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}$$

$$= \frac{C(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{6d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{a(12B+5C)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{60d\cos^{\frac{7}{2}}(c+dx)} + \frac{C(a}{}$$

$$= \frac{a^2(120A+156B+115C)\sqrt{a+a\sec(c+dx)}\sin(c+}{480d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(120A+1}{}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1304A+}{768d\cos^{\frac{5}{2}}}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1304A+}{768d\cos^{\frac{5}{2}}}$$

$$= \frac{a^3(680A+628B+545C)\sin(c+dx)}{960d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(1304A+}{768d\cos^{\frac{5}{2}}}$$

$$= \frac{a^{5/2}(1304A+1132B+1015C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{512d}$$

Mathematica [B] time = 6.58711, size = 947, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (4*Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sqrt[(1 - 2*Sin[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d
*x)/2]^2]*((C*Sin[(c + d*x)/2])/(48*(1 - 2*Sin[(c + d*x)/2]^2)^6) + ((B + 2
*C)*Sin[(c + d*x)/2])/(40*(1 - 2*Sin[(c + d*x)/2]^2)^5) + ((A + 2*B + C)*Si
n[(c + d*x)/2])/(32*(1 - 2*Sin[(c + d*x)/2]^2)^4) + ((2*A + B)*Sin[(c + d*x
)/2])/(24*(1 - 2*Sin[(c + d*x)/2]^2)^3) + (A*Sin[(c + d*x)/2])/(16*(1 - 2*S
in[(c + d*x)/2]^2)^2) + (3*A*(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (
2*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)))/64 + (5*(2*A + B)*((4*Sin[
(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^2 + 3*(Sqrt[2]*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + (2*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2))))/384 + (
7*(A + 2*B + C)*((16*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^3 + 5*((4
*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^2 + 3*(Sqrt[2]*ArcTanh[Sqrt[2
]*Sin[(c + d*x)/2]] + (2*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)))))/3
072 + (3*(B + 2*C)*((96*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^4 + 7*
((16*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^3 + 5*((4*Sin[(c + d*x)/2
])/(1 - 2*Sin[(c + d*x)/2]^2)^2 + 3*(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/
2]] + (2*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2))))))/10240 + (11*C*((
```

```
256*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^5 + 3*((96*Sin[(c + d*x)/2
])/(1 - 2*Sin[(c + d*x)/2]^2)^4 + 7*((16*Sin[(c + d*x)/2])/(1 - 2*Sin[(c +
d*x)/2]^2)^3 + 5*((4*Sin[(c + d*x)/2])/(1 - 2*Sin[(c + d*x)/2]^2)^2 + 3*(Sq
rt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (2*Sin[(c + d*x)/2])/(1 - 2*Sin[(
c + d*x)/2]^2)))))))/122880))/(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*Sec[c + d*x]^(9/2))
```

Maple [B] time = 0.414, size = 815, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)
,x)
```

```
[Out] 1/15360/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(19560*A*
cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))-19560*A*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+16980*B*cos(d*x+c)^6*2^(1/2)*ar
ctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-16980
*B*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))+15225*C*cos(d*x+c)^6*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-15225*C*cos(d*x+c)^6*2^(1/2)
*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-39
120*A*cos(d*x+c)^5*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-33960*B*cos(d*x+c)^
5*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-30450*C*cos(d*x+c)^5*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-26080*A*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-22640*B*cos(d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-20300*C*cos(
d*x+c)^4*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-14720*A*sin(d*x+c)*cos(d*x+c)
^3*(-2/(cos(d*x+c)+1))^(1/2)-18112*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(cos(d*x+c
)+1))^(1/2)-16240*C*cos(d*x+c)^3*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3840*
A*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-11136*B*cos(d*x+c)^2*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-13920*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-3072*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-89
60*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-2560*C*(-2/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c))/cos(d*x+c)^(11/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(
1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.66173, size = 1736, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(4*(15*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^5 + 10*(1304*A
+ 1132*B + 1015*C)*a^2*cos(d*x + c)^4 + 8*(920*A + 1132*B + 1015*C)*a^2*cos
(d*x + c)^3 + 48*(40*A + 116*B + 145*C)*a^2*cos(d*x + c)^2 + 128*(12*B + 35
*C)*a^2*cos(d*x + c) + 1280*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 15*((1304*A + 1132*B + 1015*C)*a^2*cos(d*
x + c)^7 + (1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^6)*sqrt(a)*log((a*co
s(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x +
 c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6), 1/1536
0*(2*(15*(1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^5 + 10*(1304*A + 1132*
B + 1015*C)*a^2*cos(d*x + c)^4 + 8*(920*A + 1132*B + 1015*C)*a^2*cos(d*x +
c)^3 + 48*(40*A + 116*B + 145*C)*a^2*cos(d*x + c)^2 + 128*(12*B + 35*C)*a^2
*cos(d*x + c) + 1280*C*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + 15*((1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^
7 + (1304*A + 1132*B + 1015*C)*a^2*cos(d*x + c)^6)*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^7 + d*cos(d*x +
 c)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2
)/cos(d*x + c)^(5/2), x)
```

3.1272 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=257

$$\frac{2(31A-7B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}}{}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqr
t[a]*d) - (2*(43*A - 91*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.87685, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4022, 4013, 3808, 206}

$$\frac{2(31A-7B+35C)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqr
t[a]*d) - (2*(43*A - 91*B + 35*C)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B + 35*C)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(}{7d\sqrt{a+a\sec(c+}$$

$$= \frac{2(31A-7B+35C)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\cos}{35d\sqrt{a}}$$

$$= -\frac{2(43A-91B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B+35C}{105d\sqrt{a}}$$

$$= -\frac{2(43A-91B+35C)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B+35C}{105d\sqrt{a}}$$

$$= \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{}}{\sqrt{a}d}$$

Mathematica [A] time = 0.912952, size = 178, normalized size = 0.69

$$-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left((43A-91B+35C)\sec^3(c+dx)+(7(B-5C)-31A)\sec^2(c+dx)+3\right.\right.}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] -(Cos[c + d*x]^(5/2)*(105*Sqrt[2]*(A - B + C)*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(7/2) + 2*Sqrt[1 - Sec[c + d*x]
]*(-15*A + 3*(A - 7*B)*Sec[c + d*x] + (-31*A + 7*(B - 5*C))*Sec[c + d*x]^2
+ (43*A - 91*B + 35*C)*Sec[c + d*x]^3))*Sin[c + d*x])/(105*d*Sqrt[1 - Sec[c
 + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.316, size = 286, normalized size = 1.1

$$-\frac{1}{105\,ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(30\,A(\cos(dx+c))^4-36\,A(\cos(dx+c))^3+105\,\arctan\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/105/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(30*A*cos(d*x
+c)^4-36*A*cos(d*x+c)^3+105*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+42*B*cos(d*x+c)^3-105*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)
+105*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2))*sin(d*x+c)+68*A*cos(d*x+c)^2-56*B*cos(d*x+c)^2+70*C*cos(d*x+c)^2-148
*A*cos(d*x+c)+196*B*cos(d*x+c)-140*C*cos(d*x+c)+86*A-182*B+70*C)/a/sin(d*x+
c)
```

Maxima [B] time = 2.57553, size = 1310, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
```

```
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) + 28*(
30*sqrt(2)*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*(10*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 15*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 15*s
qrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 5*sqrt(2)*sin(3/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 30*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*B/sqrt(a) - 140*(3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3
*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)*sin(3/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 6*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))))*C/sqrt(a))/d
```

Fricas [A] time = 0.554093, size = 1141, normalized size = 4.44

$$\left[\frac{4\left(15\,A\cos\left(dx+c\right)^{3}-3\left(A-7\,B\right)\cos\left(dx+c\right)^{2}+\left(31\,A-7\,B+35\,C\right)\cos\left(dx+c\right)-43\,A+91\,B-35\,C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\ldots}}{210\left(ad\cos\left(dx\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(4*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B +
 35*C)*cos(d*x + c) - 43*A + 91*B - 35*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 105*sqrt(2)*((A - B + C)*a*cos(d*x
 + c) + (A - B + C)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x +
c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) +
 a*d), -1/105*(105*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqr
t(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*s
qrt(cos(d*x + c))/sin(d*x + c)) - 2*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(
d*x + c)^2 + (31*A - 7*B + 35*C)*cos(d*x + c) - 43*A + 91*B - 35*C)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(
d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/sqrt(a
*sec(d*x + c) + a), x)
```

3.1273 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=211

$$\frac{2(13A-5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-5}{1}$$

```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(S
qrt[a]*d)) + (2*(13*A - 5*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.66737, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4086, 4022, 4013, 3808, 206}

$$\frac{2(13A-5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-5}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(S
qrt[a]*d)) + (2*(13*A - 5*B + 15*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}\right.}{}$$

$$= -\frac{2(A-5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{5d\sqrt{a+a\sec(}}$$

$$= \frac{2(13A-5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+}}{15d\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{2(13A-5B+15C)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+}}{15d\sqrt{a+a\,\mathrm{se}}}$$

$$= -\frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{\sqrt{a}d}$$

Mathematica [A] time = 0.56791, size = 163, normalized size = 0.77

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(\sqrt{1-\sec(c+dx)}\sec^2(c+dx)(-2(A-5B)\cos(c+dx)+3A\cos(2(c+dx))+29A-10B+\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*((29*A - 10*B + 30*C - 2*(A - 5*B)*Cos[c + d*x] + 3*A*C
os[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^2 + 15*Sqrt[2]*(A - B
+ C)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*
x]^(5/2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [A] time = 0.404, size = 253, normalized size = 1.2

$$\frac{1}{15\,ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(15\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/15/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(15*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c
)-6*A*cos(d*x+c)^3-15*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+15*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+8*A*cos(d*x+c)^2-10*B*co
s(d*x+c)^2-28*A*cos(d*x+c)+20*B*cos(d*x+c)-30*C*cos(d*x+c)+26*A-10*B+30*C)/
a/sin(d*x+c)
```

Maxima [B] time = 2.46897, size = 1045, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) + 10*(3*sqrt(2)*log(cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) - 3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*a
```

```
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)*sin(3/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a) - 30*(sqrt(2)*log(cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1) - sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/sqrt(a))/d
```

Fricas [A] time = 0.55956, size = 1023, normalized size = 4.85

$$\left[\frac{4\left(3A\cos\left(dx+c\right)^2-(A-5B)\cos\left(dx+c\right)+13A-5B+15C\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\frac{15\sqrt{2}((A-\cdots}{\cdots}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(4*(3*A*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 13*A - 5*B + 15*C)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 1
5*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*x + c)^2
 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B + C)*a*cos(
d*x + c) + (A - B + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d
*x + c)^2 - (A - 5*B)*cos(d*x + c) + 13*A - 5*B + 15*C)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a
*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(a
*sec(d*x + c) + a), x)
```

3.1274 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\,\mathrm{s}}{3}$$

```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqr
t[a]*d) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
 d*x]])
```

Rubi [A] time = 0.486442, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4086, 4013, 3808, 206}

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2A\,\mathrm{s}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```


```
[Out] (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqr
t[a]*d) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
 d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sin(c}{3d\sqrt{a+a\sec(c+}}$$

$$= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\sin(c}{3d\sqrt{a+a\sec(c+}}$$

$$= \frac{\sqrt{2}(A-B+C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{}}{\sqrt{a}d}$$

Mathematica [A] time = 0.700131, size = 88, normalized size = 0.54

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(3(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)(A\cos(c+dx)-A+3B)\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(3*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + 2*(-A + 3*B
+ A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Se
c[c + d*x])])
```

Maple [A] time = 0.364, size = 220, normalized size = 1.4

$$-\frac{-2+2\cos(dx+c)}{3ad(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(A\cos(dx+c)\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}-A\sin(dx+c)\sqrt{-2(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(A*cos(d*x+c)*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-A*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+3
*B*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*A*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))-3*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+3*C*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)^(1/2)/a/sin(d*
x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.41577, size = 764, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)
*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a) - 3*(sq
rt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*C/sqrt(a))/d
```

Fricas [A] time = 0.552341, size = 914, normalized size = 5.61

$$\left[\frac{4\,(A\cos(dx+c) - A + 3\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) + \frac{3\,\sqrt{2}((A-B+C)a\cos(dx+c)+(A-B+C)a)\log\left(-\frac{\cos(dx+c)^2 - \frac{2\,\sqrt{}}{}}{}\right.}{\sqrt{a}}}{6\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (
A - B + C)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1
/3*(3*sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
 c))/sin(d*x + c)) - 2*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a
*sec(d*x + c) + a), x)
```

3.1275 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2C\sqrt{}}{}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[
a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.519686, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4086, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2C\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + a
*Sec[c + d*x]],x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B + C)*ArcTanh[(Sqrt[
a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
```

```
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.}{}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.}{}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(2C\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.}{}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}(A}{}$$

Mathematica [A] time = 0.606673, size = 96, normalized size = 0.54

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}C\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-((A - B + C)*ArcTanh[Sin[(c + d*x)/2]]) + Sqrt[2]*C*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d
*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.369, size = 250, normalized size = 1.4

$$-\frac{-1+\cos(dx+c)}{ad(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(2A\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}-C\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)-C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))+C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-2*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2))+2*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-2*C*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)^(1/2)/a/(-2/(cos
(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.41211, size = 1080, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a) + (sqrt(2)*log(cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - sqrt(2)*log(cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 1) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 2) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*s
qrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqr
```

```
t(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*C/sqrt(a))/d
```

Fricas [A] time = 0.623246, size = 1353, normalized size = 7.6

$$\left[\frac{4\,A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(C\cos\left(dx+c\right)+C\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2\left(ad\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 + sqrt(2)*((A - B + C)*a*cos(d*x + c) + (A - B + C)*a)*log(-(cos(d*x + c)^
2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B + C)*a*cos(d*x + c
) + (A - B + C)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (C*cos(d*x + c)
+ C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a
*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^2\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sqrt(cos(c + d*x))/sqrt(a
*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(a
*sec(d*x + c) + a), x)
```

3.1276 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\cdot}{\sqrt{\cdot}}\right.}{\sqrt{a}d}$$

```
[Out] ((2*B - C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d
*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.525722, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4088, 4023, 3808, 206, 3801, 215}

$$\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\cdot}{\sqrt{\cdot}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]),x]
```


```
[Out] ((2*B - C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B + C)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c + d*x])/(d
*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
```

```
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(}}{}}{a}$$

$$= \frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left((2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left((2B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{(2B-C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}(A-B}{}$$

Mathematica [A] time = 0.570939, size = 113, normalized size = 0.62

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B+C)\cos(c+dx)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(2B-C)\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+\right.\right.\right.}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x] + S
qrt[2]*(2*B - C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*C*Sin[(
c + d*x)/2]))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.323, size = 374, normalized size = 2.1

$$-\frac{-1+\cos(dx+c)}{2\,ad\,(\sin(dx+c))^2}\left(-2\,B\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\cos(dx+c)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(-2*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+2*B*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+C*2^(1/2)*
arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos
(d*x+c)-C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*cos(d*x+c)+4*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*cos(d*x+c)-4*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x
+c)+2*C*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*C*arctan(1/2*sin(d*x+c)*(-2/
(cos(d*x+c)+1))^(1/2))*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a/(-
2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.788308, size = 1530, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*C*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - ((2*B - C)*cos(d*x + c)^2 + (2*B - C)*cos(d*x + c))*sqrt(a)*log((a
*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*((A - B + C)*a*cos(d*x + c)^2 +
 (A - B + C)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d
```

```
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(
d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x
+ c)^2 + a*d*cos(d*x + c)), -1/2*(2*sqrt(2)*((A - B + C)*a*cos(d*x + c)^2 +
 (A - B + C)*a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*C*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*B
- C)*cos(d*x + c)^2 + (2*B - C)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x +
c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(sqrt(a*(sec(c + d*x) + 1
))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + B\sec(dx + c) + A}{\sqrt{a\sec(dx + c) + a}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*sqrt(cos(d*x + c))), x)
```

3.1277 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=235

$$-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(8A-4B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{4\sqrt{a}d}$$

```
[Out] ((8*A - 4*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C
)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((4*B - C)*Sin[
c + d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.735191, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4265, 4088, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{(8A-4B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{4\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + a*
Sec[c + d*x]]),x]
```

```
[Out] ((8*A - 4*B + 7*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B + C
)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((4*B - C)*Sin[
c + d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c-\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+}{}}{2a}$$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4B-C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4B-C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - ($$

$$= \frac{C\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4B-C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{(8A-4B+7C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} - \frac{\sqrt{2}(A-}{}$$

Mathematica [A] time = 1.08886, size = 127, normalized size = 0.54

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(8(A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}(8A-4B+7C)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c\right.\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] -(Cos[(c + d*x)/2]*(8*(A - B + C)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(8*A
- 4*B + 7*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(-4*B + C -
 2*C*Sec[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [B] time = 0.349, size = 545, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*A*2^(1/2)*arct
an(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x
+c)^2-8*A*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*cos(d*x+c)^2-4*B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+4*B*2^(1/2)*arctan(1/4*2^
(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+7*C
```

```
*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x
+c)))*cos(d*x+c)^2-7*C*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+16*A*cos(d*x+c)^2*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-16*B*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))-8*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
)+16*C*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+2*C*(-
2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*C*(-2/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c))/a/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.91788, size = 4501, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) + 4*(4*sqrt(2)*cos(3/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(
2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
```

2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2
)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sqrt(a)) - (4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos
(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*sin(4*d*x +
 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*
cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sqrt(2)*sin(4*d*x
 + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*
x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4
*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sq
rt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*cos(4
*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*C/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +

```
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A] time = 1.30974, size = 1667, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*((4*B - C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((8*A - 4*B + 7*C)*cos(d*x + c)^3 +
(8*A - 4*B + 7*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)) + 8*sqrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*x + c)
^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)
^2), 1/8*(8*sqrt(2)*((A - B + C)*a*cos(d*x + c)^3 + (A - B + C)*a*cos(d*x +
 c)^2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*((4*B - C)*cos(d*x + c) + 2*C
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
 ((8*A - 4*B + 7*C)*cos(d*x + c)^3 + (8*A - 4*B + 7*C)*cos(d*x + c)^2)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*
x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{\sqrt{a \sec (dx + c) + a} \cos (dx + c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*cos(d*x + c)^(3/2)), x)
```

3.1278 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=281

$$\frac{(8A-2B+7C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A-1}{}$$

```
[Out] -((8*A - 14*B + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B +
 C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + ((6*B - C)*Si
n[c + d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((8*A - 2*
B + 7*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.926759, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4265, 4088, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(8A-2B+7C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}(A-B+C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{(8A-1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[a + a*
Sec[c + d*x]]),x]
```


```
[Out] -((8*A - 14*B + 9*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a]*d) + (Sqrt[2]*(A - B +
 C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (C*Sin[
c + d*x])/(3*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + ((6*B - C)*Si
n[c + d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((8*A - 2*
B + 7*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4088

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^n*Simp[A*b*(m + n + 1) + b*C*n + (a*C*m + b*B*(
m + n + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m, n
}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)] &&  !LtQ[n, -2^(-1)] && NeQ
[m + n + 1, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+}{}}{3a}$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{C\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(6B-C)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= -\frac{(8A-14B+9C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8\sqrt{a}d} + \frac{\sqrt{2}(}{}$$

Mathematica [A] time = 1.11333, size = 154, normalized size = 0.55

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B+C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{48}\left(2\sin\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3(8A-2B+7C)+2(6B-\right.\right.\right.}{d\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[
a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B + C)*ArcTanh[Sin[(c + d*x)/2]] + (-3*Sqrt[2]*(8
*A - 14*B + 9*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(3*(8*A
 - 2*B + 7*C) + 2*(6*B - C)*Sec[c + d*x] + 8*C*Sec[c + d*x]^2)*Sin[(c + d*x
)/2])/48))/(d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.367, size = 638, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(24*A*cos(d*x+c
)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
```

```
*2^(1/2)-24*A*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))*2^(1/2)-42*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3+42*B*arctan(1/4*2^(1
/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)
^3+27*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))*2^(1/2)-27*C*cos(d*x+c)^3*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)+48*A*cos(d*x+c)^2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2)+96*A*cos(d*x+c)^3*arctan(1/2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))-12*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
-96*B*cos(d*x+c)^3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+42*C*co
s(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+96*C*cos(d*x+c)^3*arctan(1/
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+24*B*cos(d*x+c)*sin(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2)-4*C*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+16*C*
(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/a/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1
/2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 3.41184, size = 7547, normalized size = 26.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(24*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x +
2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - 6*(4*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(
2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sq
rt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*
```

d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*si
n(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(2
*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d
*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)
- 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2
*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(2)*c
os(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
 sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2
*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)) + (84*(sqrt(2)*sin(6*d*x + 6*c) +
3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqr
t(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 312*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*s
in(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 312*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 100*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4
*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 84*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3
*sqrt(2)*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*

c))) + 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
 + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos
(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(s
in(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 +
 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x
+ 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 27*(2*(3
*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x +
6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2
 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) - 27*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*c
os(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c)
 + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x +
4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*co
s(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 48*(sqrt(2)*cos(6*d*x +
 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqr
t(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*co
s(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6
*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4
*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 48*(sqrt(2)*cos(6*d*x +
 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqr
t(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*co
s(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6
*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4
*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 84*(sqrt(2)*cos(6*d*x +
 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 100*(sqrt(2)*cos(6*
d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 312*(sqrt(2)*co
s(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 312*(sqrt(2
)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c

```
) + sqrt(2))*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 100*(sq
rt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 84*
(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C
/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6
*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x +
 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqr
t(a)))/d
```

Fricas [A] time = 1.32253, size = 1789, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(3*(8*A - 2*B + 7*C)*cos(d*x + c)^2 + 2*(6*B - C)*cos(d*x + c) + 8
*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
 + 3*((8*A - 14*B + 9*C)*cos(d*x + c)^4 + (8*A - 14*B + 9*C)*cos(d*x + c)^3
)*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
 c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 48*sqrt(2)*((A - B + C)*a
*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqr
t(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3), -1/48*(48*sqrt(2)*((A - B
+ C)*a*cos(d*x + c)^4 + (A - B + C)*a*cos(d*x + c)^3)*sqrt(-1/a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/
sin(d*x + c)) - 2*(3*(8*A - 2*B + 7*C)*cos(d*x + c)^2 + 2*(6*B - C)*cos(d*x
 + c) + 8*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) + 3*((8*A - 14*B + 9*C)*cos(d*x + c)^4 + (8*A - 14*B + 9*C)*cos(d
*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)
*cos(d*x + c)^(5/2)), x)
```

3.1279 $\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{2}(a-b)(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2bB\sqrt{}}{}$$

```
[Out] (2*b*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(a - b)*(A - B)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*a*A*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.602296, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4265, 4086, 4023, 3808, 206, 3801, 215}

$$-\frac{\sqrt{2}(a-b)(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2bB\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c + d*x]^
2))/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (2*b*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(a - b)*(A - B)*ArcTanh[
(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (2*a*A*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4086

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^(n + 1)*Simp[a*A*m - b*B*n - b*(A*(m + n + 1) + C*n)*Csc[e + f*x],
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && EqQ[a^2 - b^2, 0] &
&  !LtQ[m, -2^(-1)] && (LtQ[n, -2^(-1)] || EqQ[m + n + 1, 0])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
```

```
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(Ab+aB}{\sqrt{\sec(c}}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c}\right.}{}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \left((a-b)(A\right.$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(2(a-b)(\right.}{}$$

$$= \frac{2bB\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{\sqrt{a}d}$$

Mathematica [A] time = 0.416894, size = 137, normalized size = 0.74

$$\frac{\sin(c+dx)\left(\sqrt{2}(a-b)(A-B)\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+2aA\sqrt{1-\sec(c+dx)}-2bB\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{s}\right.\right.}{d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c +
 d*x]^2))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*a*A*Sqrt[1 - Sec[c + d*x]] - 2*b*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[
c + d*x]] + Sqrt[2]*(a - b)*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqr
t[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c +
 d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.389, size = 282, normalized size = 1.5

$$\frac{-1+\cos(dx+c)}{ad(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-2A\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}a-B\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*a+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a*sec(d
*x+c))^(1/2),x)
```

```
[Out] 1/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-2*A*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2)*a-B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*b+B*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*b+2*A*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*a-2*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)*b-2*B*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*a+2*B*arctan(1/2*s
in(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*b)*cos(d*x+c)^(1/2)/a/sin(d*x+c)^2/(-2
/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.69847, size = 1202, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a
*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B*sqrt(a) - (sqrt(2)*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - s
qrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1))*A*b/sqrt(a) + (sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1) - sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1
) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + log(2*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*a
```

```
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - log(2*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + log(2*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 2))*B*b/sqrt(a))/d
```

Fricas [A] time = 0.804709, size = 1439, normalized size = 7.82

$$\left[\frac{4\,Aa\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(Bb\cos\left(dx+c\right)+Bb\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{c}}{\cos(dx+c)^{3}+\cos(}\right.}{2\,(ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a
*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*A*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + (B*b*cos(d*x + c) + B*b)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)) + sqrt(2)*((A - B)*a^2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*c
os(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (2*
A*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
 + sqrt(2)*((A - B)*a^2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*cos(d*x
 + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + (B*b*cos(d*x + c) + B*b)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x
+ c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a
+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(
a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+a
*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(cos(d*
x + c))/sqrt(a*sec(d*x + c) + a), x)
```

3.1280 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{(15A-11B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B+5C)\sin(c+dx)\cos^{\frac{3}{2}}(c+a}{10ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -((15*A - 11*B + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*
Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a
+ a*Sec[c + d*x])^(3/2)) + ((147*A - 95*B + 75*C)*Sin[c + d*x])/(30*a*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B + 15*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A - 5*B + 5
*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.917505, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4084, 4022, 4013, 3808, 206}

$$-\frac{(15A-11B+7C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B+5C)\sin(c+dx)\cos^{\frac{3}{2}}(c+a}{10ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```


```
[Out] -((15*A - 11*B + 7*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*
Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a
+ a*Sec[c + d*x])^(3/2)) + ((147*A - 95*B + 75*C)*Sin[c + d*x])/(30*a*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B + 15*C)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A - 5*B + 5
*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+d}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B+5C)\cos}{10ad\sqrt{a+}}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(39A-35B+15C)}{30ad\sqrt{a}}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B+7}{30ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B+7}{30ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(15A-11B+7C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+d}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 3.06527, size = 135, normalized size = 0.48

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(3(39A+20(C-B))\cos(c+dx)+(10B-6A)\cos(2(c+dx))+3A\cos(3(c+dx))+141A-85B+7\ldots}{30ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-15*(15*A - 11*B + 7*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (141*
A - 85*B + 75*C + 3*(39*A + 20*(-B + C))*Cos[c + d*x] + (-6*A + 10*B)*Cos[2
*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(30*a*d*Sqrt[Cos[c +
d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.29, size = 450, normalized size = 1.6

$$-\frac{-1+\cos(dx+c)}{60\,da^2(\sin(dx+c))^3}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(225\,A\sin(dx+c)\cos(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/60/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(225*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*(-2/(cos(d*x+c)+1))^(1/2)-24*A*cos(d*x+c)^4-165*B*sin(d*x+c)*cos(d*x+c
)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2
)+105*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*(-2/(cos(d*x+c)+1))^(1/2)+225*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)+48*A*cos(d*x+c)^3-165*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(
d*x+c)-40*B*cos(d*x+c)^3+105*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-240*A*cos(d*x+c)^2+160*B*cos(d*x+
c)^2-120*C*cos(d*x+c)^2-78*A*cos(d*x+c)+70*B*cos(d*x+c)-30*C*cos(d*x+c)+294
*A-190*B+150*C)/a^2/sin(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.565991, size = 1365, normalized size = 4.82

$$\left[\frac{15\,\sqrt{2}\left((15\,A-11\,B+7\,C)\cos\left(dx+c\right)^2+2\,(15\,A-11\,B+7\,C)\cos\left(dx+c\right)+15\,A-11\,B+7\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\,\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(2)*((15*A - 11*B + 7*C)*cos(d*x + c)^2 + 2*(15*A - 11*B + 7
*C)*cos(d*x + c) + 15*A - 11*B + 7*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sq
rt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
 + 4*(12*A*cos(d*x + c)^3 - 4*(3*A - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B +
5*C)*cos(d*x + c) + 147*A - 95*B + 75*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d), 1/60*(15*sqrt(2)*((15*A - 11*B + 7*C)*cos(d*x + c)^2 + 2
*(15*A - 11*B + 7*C)*cos(d*x + c) + 15*A - 11*B + 7*C)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*
sin(d*x + c))) + 2*(12*A*cos(d*x + c)^3 - 4*(3*A - 5*B)*cos(d*x + c)^2 + 12
*(9*A - 5*B + 5*C)*cos(d*x + c) + 147*A - 95*B + 75*C)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2
+ 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^(3/2), x)
```

3.1281 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{(11A-7B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B+3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] ((11*A - 7*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sq
rt[2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a +
a*Sec[c + d*x])^(3/2)) - ((19*A - 15*B + 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((7*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.724864, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4084, 4022, 4013, 3808, 206}

$$\frac{(11A-7B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B+3C)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```


```
[Out] ((11*A - 7*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sq
rt[2]*a^(3/2)*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a +
a*Sec[c + d*x])^(3/2)) - ((19*A - 15*B + 3*C)*Sin[c + d*x])/(6*a*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((7*A - 3*B + 3*C)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+d}$$

$$=-\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$=-\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(7A-3B+3C)\sqrt{c}}{6ad\sqrt{a+}}$$

$$=-\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(19A-15B+3C}{6ad\sqrt{\cos(c+dx)}\sqrt{}}$$

$$=-\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(19A-15B+3C}{6ad\sqrt{\cos(c+dx)}\sqrt{}}$$

$$=\frac{(11A-7B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 2.06573, size = 113, normalized size = 0.48

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(-12(A-B)\cos(c+dx)+2A\cos(2(c+dx))-17A+15B-3C)+3(11A-7B+3C)\cos\left(\frac{1}{2}(c+dx)\right)\text{ta}}{6ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*(11*A - 7*B + 3*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (-17*A +
 15*B - 3*C - 12*(A - B)*Cos[c + d*x] + 2*A*Cos[2*(c + d*x)])*Tan[(c + d*x)
/2])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.375, size = 359, normalized size = 1.5

$$-\frac{-1+\cos(dx+c)}{6\,da^{2}(\sin(dx+c))^{3}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(-4\,A\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^{3}+16\,A(\cos(dx+c))^{2}\sqrt{-2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/6/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(-4*A*(-2/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)^3+16*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-12
*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+33*A*sin(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(cos(d*x+c)+1))^(1/2))+7*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-21
*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*B*(-2/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)+9*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))+3*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-19*A*(-2/(cos(d*
x+c)+1))^(1/2)+15*B*(-2/(cos(d*x+c)+1))^(1/2)-3*C*(-2/(cos(d*x+c)+1))^(1/2)
)*cos(d*x+c)^(1/2)/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.553267, size = 1235, normalized size = 5.3

$$\left[\frac{3\sqrt{2}\left((11\,A-7\,B+3\,C)\cos(dx+c)^{2}+2\,(11\,A-7\,B+3\,C)\cos(dx+c)+11\,A-7\,B+3\,C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\cdots}{\cdots}\right.\cdots}{24\,(a\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((11*A - 7*B + 3*C)*cos(d*x + c)^2 + 2*(11*A - 7*B + 3*C)*
cos(d*x + c) + 11*A - 7*B + 3*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*
(4*A*cos(d*x + c)^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B - 3*C)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B
+ 3*C)*cos(d*x + c)^2 + 2*(11*A - 7*B + 3*C)*cos(d*x + c) + 11*A - 7*B + 3*
C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*A*cos(d*x + c)^2 - 12*(A - B)*
cos(d*x + c) - 19*A + 15*B - 3*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^(3/2), x)
```

3.1282 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{(7A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{}{2d}$$

```
[Out] -((7*A - 3*B - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt
[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*
Sec[c + d*x])^(3/2)) + ((5*A - B + C)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.524716, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4084, 4013, 3808, 206}

$$-\frac{(7A-3B-C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B+C)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(3/2),x]
```

```
[Out] -((7*A - 3*B - C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt
[2]*a^(3/2)*d) - ((A - B + C)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*
Sec[c + d*x])^(3/2)) + ((5*A - B + C)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sqrt{\sec(c+dx)}(a+a\sec(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B+C)}{2ad\sqrt{\cos(c+dx)}\sqrt{}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B+C)}{2ad\sqrt{\cos(c+dx)}\sqrt{}}$$

$$= -\frac{(7A-3B-C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 1.60192, size = 96, normalized size = 0.53

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(4A\cos(c+dx)+5A-B+C)-(7A-3B-C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-((7*A - 3*B - C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]) + (5*A - B +
 C + 4*A*Cos[c + d*x])*Tan[(c + d*x)/2])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(
1 + Sec[c + d*x])])
```

Maple [B] time = 0.362, size = 306, normalized size = 1.7

$$\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(4\,A(\cos(dx+c))^2\sqrt{-2\,(\cos(dx+c)+1)^{-1}}+A\cos(dx+c)\sqrt{-2\,(\cos(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 1/2/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4*A*cos(d*x+c)^2
*(-2/(cos(d*x+c)+1))^(1/2)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+7*A*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-B*(-2/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)-3*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2))+C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-C*sin(d*x+c)*arctan(1/2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(
d*x+c)+1))^(1/2)-C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)/a^2/sin(d*x+
c)^3/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.541769, size = 1111, normalized size = 6.14

$$\left[-\frac{\sqrt{2}\left((7A-3B-C)\cos(dx+c)^2+2(7A-3B-C)\cos(dx+c)+7A-3B-C\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(}{\cos}} \cdots}{\cos(\cdots}\right.}{8\left(a^2d\cos(dx+c)^2+2 \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - 3*B - C)*cos(d*x + c)^2 + 2*(7*A - 3*B - C)*cos(d*x
+ c) + 7*A - 3*B - C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*
x + c) + 5*A - B + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d),
1/4*(sqrt(2)*((7*A - 3*B - C)*cos(d*x + c)^2 + 2*(7*A - 3*B - C)*cos(d*x +
c) + 7*A - 3*B - C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c
) + 5*A - B + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a)^(3/2), x)
```

3.1283 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{(3A+B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\,\mathrm{ta}}{\sqrt{a\,\mathrm{sec}}}\right.}{a^{3/2}d}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A + B - 5*C)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.557254, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4084, 4023, 3808, 206, 3801, 215}

$$\frac{(3A+B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\,\mathrm{ta}}{\sqrt{a\,\mathrm{sec}}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((3*A + B - 5*C)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C)*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
```

```
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c\right.}{(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\mathrm{se}}}{}}{2a}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left((3A+B-5C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}}\right.}{4a}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left((3A+B-5C)\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}}\right.}{}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(3A+B-5C)\tanh}{}$$

Mathematica [A] time = 1.72239, size = 118, normalized size = 0.62

$$\frac{-(A-B+C)\tan\left(\frac{1}{2}(c+dx)\right)+(3A+B-5C)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\sqrt{2}C\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}}{2ad\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + B - 5*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + 4*Sqrt[2]*C*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - (A - B + C)*Tan[(c + d*
x)/2])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.313, size = 374, normalized size = 2.

$$-\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\left(-2\,C\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1-\sin(dx+c))\right)\sin(dx+c)\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(-2*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)*2^(1/2)+2*C*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)*2^(1/2)+3*A*sin(d*
x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+A*cos(d*x+c)*(-2/(cos
(d*x+c)+1))^(1/2)+B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))-B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-5*C*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-A*(
-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c)+1))^(1/2)-C*(-2/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/a^2/(-2/(cos(d*
x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [B] time = 2.76856, size = 5785, normalized size = 30.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
 + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin
(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2
*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d
```

*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c
)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
+ 4*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*co
s(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*
sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2
*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a))
 + (4*(sin(3/2*d*x + 3/2*c) - sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 8*(sin(3/2*d*x + 3/2*c) - sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + (2*(2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 4*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*l
og(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (2*(2*cos(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(2/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*log(cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - 4*(cos(3/2*d*x + 3/2*c) - cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 8*(cos(3/2*d*x + 3/2*c) - cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*sin(3/2*d*x + 3/2*c) - 4*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B/((sqrt(2)*a*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*cos(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sqrt(2)*a*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*cos(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(2*sqrt(2)*a*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*cos(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a)*sqrt(a)) + (4*(sin(2*d*x +
2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 +
4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*
sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*

```
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x
 + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2
 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt
(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 4*
(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((sqrt(2)*a*co
s(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*a*cos(2*d*x + 2*c)
 + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)))/d
```

Fricas [A] time = 0.684395, size = 1636, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((3*A + B - 5*C)*cos(d*x + c)^2 + 2*(3*A + B - 5*C)*cos(d*x
+ c) + 3*A + B - 5*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(A - B + C)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) -
4*(C*cos(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(a)*log((a*cos(d*x + c)^3 -
 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4
*(sqrt(2)*((3*A + B - 5*C)*cos(d*x + c)^2 + 2*(3*A + B - 5*C)*cos(d*x + c)
+ 3*A + B - 5*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A - B + C)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(C*cos
(d*x + c)^2 + 2*C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
 a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/
2)*sqrt(cos(d*x + c))), x)
```

3.1284 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{(A-5B+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}}{a^{3/2}d}$$

```
[Out] ((2*B - 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B + 9*C)*ArcTanh[(S
qrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C
)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) + ((A -
 B + 3*C)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.753744, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4265, 4084, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(A-5B+9C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2B-3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] ((2*B - 3*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B + 9*C)*ArcTanh[(S
qrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B + C
)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) + ((A -
 B + 3*C)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
```

```
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2\right.}{(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+3C)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+3C)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+3C)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

$$= \frac{(2B-3C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A-5B+}{}$$

Mathematica [A] time = 1.56391, size = 198, normalized size = 0.82

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2(A-5B+9C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{2(\sin}{}\right.}{d(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+dx))+A+2B\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*(2*(A - 5*B + 9*C)*ArcTanh[Sin[(c + d*x)/2]] - (2*(2*Sqrt[2]*(2*B - 3*C
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + (A - B + 3*C + 2*C
*Sec[c + d*x])*Sin[(c + d*x)/2]))/(-1 + Sin[(c + d*x)/2]^2)))/(d*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.298, size = 551, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*B*sin(d*x+c)
*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1-sin(d*x+c)))+2*B*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(
```

```
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+3*C*sin(d*x+c)*2^(1/2)*cos(
d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))-3*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-5
*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+B
*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+9*C*sin(d*x+c)*cos(d*x+c)*arctan(1/
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1)
)^(1/2)+A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-B*(-2/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)+C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+2*C*(-2/(cos(d*x+c)+1))^(
1/2))/a^2/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.869514, size = 1905, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((A - 5*B + 9*C)*cos(d*x + c)^3 + 2*(A - 5*B + 9*C)*cos(d*x +
 c)^2 + (A - 5*B + 9*C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sq
rt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
 + 4*((A - B + 3*C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*((2*B - 3*C)*cos(d*x + c)^3 + 2*(2
*B - 3*C)*cos(d*x + c)^2 + (2*B - 3*C)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x
 + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x +
c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a
^2*d*cos(d*x + c)), -1/4*(sqrt(2)*((A - 5*B + 9*C)*cos(d*x + c)^3 + 2*(A -
5*B + 9*C)*cos(d*x + c)^2 + (A - 5*B + 9*C)*cos(d*x + c))*sqrt(-a)*arctan(s
qrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/
(a*sin(d*x + c))) - 2*((A - B + 3*C)*cos(d*x + c) + 2*C)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*((2*B - 3*C)*cos(
d*x + c)^3 + 2*(2*B - 3*C)*cos(d*x + c)^2 + (2*B - 3*C)*cos(d*x + c))*sqrt(
-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d
*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(a \sec(dx+c)+a)^{\frac{3}{2}} \cos(dx+c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/
2)*cos(d*x + c)^(3/2)), x)
```

3.1285 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{(5A-9B+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(8A-12B+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}{4a^{3/2}d}$$

```
[Out] ((8*A - 12*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*
C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- ((A - B + C)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(
3/2)) + ((A - B + 2*C)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]) - ((2*A - 6*B + 7*C)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*
Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.985412, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4265, 4084, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{(5A-9B+13C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(8A-12B+19C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}{4a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] ((8*A - 12*B + 19*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*a^(3/2)*d) - ((5*A - 9*B + 13*
C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- ((A - B + C)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(
3/2)) + ((A - B + 2*C)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]) - ((2*A - 6*B + 7*C)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*
Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && LtQ[m, -2^(-1)]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+\right.}{(a+a\sec(c+dx))^{3/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(A-B+2C)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{(8A-12B+19C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{3/2}d} - (5A-$$

Mathematica [A] time = 2.28997, size = 239, normalized size = 0.8

$$-\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(2(5A-9B+13C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sqrt{2}(}{}\right.}{d(a(\sec(c+dx)+1))^{3/2}(A\cos(2(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -((Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*(2*(5*A - 9*B + 13*C)*ArcTanh[Sin[(c + d*x)/2]] + (Sqrt[2]*(8*A - 12*
B + 19*C)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + ((-2*A + 6
*B - 3*C + (8*B - 6*C)*Cos[c + d*x] + (-2*A + 6*B - 7*C)*Cos[2*(c + d*x)])*
Sec[c + d*x]^2*Sin[(c + d*x)/2])/2)/(-1 + Sin[(c + d*x)/2]^2)))/(d*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(3/2)))
```

Maple [B] time = 0.331, size = 731, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*A*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2
^(1/2)*sin(d*x+c)-8*A*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-12*B*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*s
in(d*x+c)+12*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+19*C*arctan(1/4*2^(1/2)*(-2/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+
c)-19*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+4*A*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)^3-20*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)-12*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+36*B*arctan(1/2*sin(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+14*C*(-2/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)^3-52*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)-4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+4*B*(
-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-8*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^
(1/2)+8*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-10*C*cos(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2)+4*C*(-2/(cos(d*x+c)+1))^(1/2))/a^2/sin(d*x+c)^3/(-2/(cos(d*x+
c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.58797, size = 2099, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(2)*((5*A - 9*B + 13*C)*cos(d*x + c)^4 + 2*(5*A - 9*B + 13*C)*
cos(d*x + c)^3 + (5*A - 9*B + 13*C)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) - 4*((2*A - 6*B + 7*C)*cos(d*x + c)^2 - (4*B - 3*C)*cos(d*x
+ c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + ((8*A - 12*B + 19*C)*cos(d*x + c)^4 + 2*(8*A - 12*B + 19*C)*cos(
d*x + c)^3 + (8*A - 12*B + 19*C)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c
)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*
sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d
*cos(d*x + c)^2), 1/8*(2*sqrt(2)*((5*A - 9*B + 13*C)*cos(d*x + c)^4 + 2*(5*
A - 9*B + 13*C)*cos(d*x + c)^3 + (5*A - 9*B + 13*C)*cos(d*x + c)^2)*sqrt(-a
)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))/(a*sin(d*x + c))) - 2*((2*A - 6*B + 7*C)*cos(d*x + c)^2 - (4*B -
```

```
3*C)*cos(d*x + c) - 2*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) + ((8*A - 12*B + 19*C)*cos(d*x + c)^4 + 2*(8*A - 12*B
 + 19*C)*cos(d*x + c)^3 + (8*A - 12*B + 19*C)*cos(d*x + c)^2)*sqrt(-a)*arct
an(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^
4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/
2)*cos(d*x + c)^(5/2)), x)
```

3.1286 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=333

$$\frac{(157A-85B+45C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B+195C)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B}{240a^2d\sqrt{\cos(c+}}$$

```
[Out] -((283*A - 163*B + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/
(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d
*(a + a*Sec[c + d*x])^(5/2)) - ((21*A - 13*B + 5*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((2671*A - 1495*B + 735*C)*
Sin[c + d*x])/(240*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((7
87*A - 475*B + 195*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a +
a*Sec[c + d*x]]) + ((157*A - 85*B + 45*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/
(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 1.15347, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4084, 4020, 4022, 4013, 3808, 206}

$$\frac{(157A-85B+45C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B+195C)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B}{240a^2d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```



```
[Out] -((283*A - 163*B + 75*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/
(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d
*(a + a*Sec[c + d*x])^(5/2)) - ((21*A - 13*B + 5*C)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((2671*A - 1495*B + 735*C)*
Sin[c + d*x])/(240*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((7
87*A - 475*B + 195*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a +
a*Sec[c + d*x]]) + ((157*A - 85*B + 45*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/
(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
```

```
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{}\right.}{}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5}{16ad(a}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5}{16ad(a}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5}{16ad(a}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5}{16ad(a}$$

$$= -\frac{(A-B+C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B+5}{16ad(a}$$

$$= -\frac{(283A-163B+75C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\mathrm{co}}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 4.2826, size = 173, normalized size = 0.52

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)(5(887A-479B+255C)\cos(c+dx)+16(52A-25B+15C)\cos(2(c+dx))-40A\right.}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*(-120*(283*A - 163*B + 75*C)*ArcTanh[Sin[(c + d*x)/2]]*Co
s[(c + d*x)/2]^4 + 4*(3491*A - 1895*B + 975*C + 5*(887*A - 479*B + 255*C)*C
os[c + d*x] + 16*(52*A - 25*B + 15*C)*Cos[2*(c + d*x)] - 40*A*Cos[3*(c + d*
x)] + 40*B*Cos[3*(c + d*x)] + 12*A*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(96
0*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.302, size = 647, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/480/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(4245*A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-192*A*cos(d*x+c)^5-2445*B*sin(d*x+c)*co
s(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)
+1))^(1/2)+1125*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+8490*A*sin(d*x+c)*cos(d*x+c)*arct
an(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+512*
A*cos(d*x+c)^4-4890*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-320*B*cos(d*x+c)^4+2250*C*sin(d
*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(
d*x+c)+1))^(1/2)+4245*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/
(cos(d*x+c)+1))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*arctan(1/2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*B*sin(d*x+c)+19
20*B*cos(d*x+c)^3+1125*C*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/2*sin(d*x+c)*(-
2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-960*C*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+
3430*B*cos(d*x+c)^2-1590*C*cos(d*x+c)^2+3768*A*cos(d*x+c)-2040*B*cos(d*x+c)
+1080*C*cos(d*x+c)+5342*A-2990*B+1470*C)/a^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.584835, size = 1701, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/960*(15*sqrt(2)*((283*A - 163*B + 75*C)*cos(d*x + c)^3 + 3*(283*A - 163*
B + 75*C)*cos(d*x + c)^2 + 3*(283*A - 163*B + 75*C)*cos(d*x + c) + 283*A -
163*B + 75*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x
 + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(96*A*cos(d*x + c)^
4 - 160*(A - B)*cos(d*x + c)^3 + 32*(49*A - 25*B + 15*C)*cos(d*x + c)^2 + 5
*(911*A - 503*B + 255*C)*cos(d*x + c) + 2671*A - 1495*B + 735*C)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/480*(
15*sqrt(2)*((283*A - 163*B + 75*C)*cos(d*x + c)^3 + 3*(283*A - 163*B + 75*C
)*cos(d*x + c)^2 + 3*(283*A - 163*B + 75*C)*cos(d*x + c) + 283*A - 163*B +
75*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^4 - 160*(A
- B)*cos(d*x + c)^3 + 32*(49*A - 25*B + 15*C)*cos(d*x + c)^2 + 5*(911*A - 5
03*B + 255*C)*cos(d*x + c) + 2671*A - 1495*B + 735*C)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
```

```
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec
(d*x + c) + a)^(5/2), x)
```

3.1287 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$\frac{(95A-39B+15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A-147B+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B+19C)\sqrt{\cos(c+d}{}$$

```
[Out] ((163*A - 75*B + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1
6*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(
a + a*Sec[c + d*x])^(5/2)) - ((17*A - 9*B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((299*A - 147*B + 27*C)*Sin[c +
d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((95*A - 39*
B + 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x
]])
```

Rubi [A] time = 0.95592, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4265, 4084, 4020, 4022, 4013, 3808, 206}

$$\frac{(95A-39B+15C)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A-147B+27C)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{(163A-75B+19C)\sqrt{\cos(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```


```
[Out] ((163*A - 75*B + 19*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1
6*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(
a + a*Sec[c + d*x])^(5/2)) - ((17*A - 9*B + C)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((299*A - 147*B + 27*C)*Sin[c +
d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((95*A - 39*
B + 15*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x
]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+d}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}}\right.}{}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B+C)\sqrt{\mathrm{c}}}{16ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B+C)\sqrt{\mathrm{c}}}{16ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B+C)\sqrt{\mathrm{c}}}{16ad(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B+C)\sqrt{\mathrm{c}}}{16ad(a+a\,\mathrm{s}}$$

$$= \frac{(163A-75B+19C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 3.36112, size = 146, normalized size = 0.52

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((-479A+255B-39C)\cos(c+dx)+(48B-80A)\cos(2(c+dx))+8A\cos(3(c+dx)\right.}{192ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*(24*(163*A - 75*B + 19*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(
c + d*x)/2]^4 + 4*(-379*A + 195*B - 27*C + (-479*A + 255*B - 39*C)*Cos[c +
d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)] + 8*A*Cos[3*(c + d*x)])*Sin[(c + d*x
)/2]))/(192*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.305, size = 550, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] -1/48/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(32*A*cos(d*x
+c)^4*(-2/(cos(d*x+c)+1))^(1/2)-192*A*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^
3+96*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-343*A*cos(d*x+c)^2*(-2/(cos(d
*x+c)+1))^(1/2)-489*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))+159*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+225*B*sin(d*
```

```
x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-39*C*cos(d
*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-57*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+204*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2)-489*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-108*B*
(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+225*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2))+12*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-57*C*
sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+299*A*(-2/(cos(
d*x+c)+1))^(1/2)-147*B*(-2/(cos(d*x+c)+1))^(1/2)+27*C*(-2/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)^(1/2)/a^3/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.573833, size = 1569, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(2)*((163*A - 75*B + 19*C)*cos(d*x + c)^3 + 3*(163*A - 75*B +
 19*C)*cos(d*x + c)^2 + 3*(163*A - 75*B + 19*C)*cos(d*x + c) + 163*A - 75*B
 + 19*C)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c)
 - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*A*cos(d*x + c)^3 - 3
2*(5*A - 3*B)*cos(d*x + c)^2 - (503*A - 255*B + 39*C)*cos(d*x + c) - 299*A
+ 147*B - 27*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163*A - 75*B + 19*C)*cos(d*x + c)^3 +
 3*(163*A - 75*B + 19*C)*cos(d*x + c)^2 + 3*(163*A - 75*B + 19*C)*cos(d*x +
 c) + 163*A - 75*B + 19*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(32*A*cos
(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^2 - (503*A - 255*B + 39*C)*cos(d*
x + c) - 299*A + 147*B - 27*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec
(d*x + c) + a)^(5/2), x)
```

3.1288 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=231

$$\frac{(49A-9B+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-$$

```
[Out] -((75*A - 19*B - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16
*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a
 + a*Sec[c + d*x])^(5/2)) - ((13*A - 5*B - 3*C)*Sin[c + d*x])/(16*a*d*Sqrt[
Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((49*A - 9*B + C)*Sin[c + d*x])
/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.739262, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4265, 4084, 4020, 4013, 3808, 206}

$$\frac{(49A-9B+C)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(75A-19B-5C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + a*Sec
[c + d*x])^(5/2),x]
```


```
[Out] -((75*A - 19*B - 5*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16
*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a
 + a*Sec[c + d*x])^(5/2)) - ((13*A - 5*B - 3*C)*Sin[c + d*x])/(16*a*d*Sqrt[
Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((49*A - 9*B + C)*Sin[c + d*x])
/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx)}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B-}{16ad\sqrt{\cos(c+dx)}(}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B-}{16ad\sqrt{\cos(c+dx)}(}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B-}{16ad\sqrt{\cos(c+dx)}(}$$

$$= -\frac{(75A-19B-5C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{16\sqrt{2}a^{5/2}d}$$

Mathematica [A] time = 2.83333, size = 128, normalized size = 0.55

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((85A-13B+5C)\cos(c+dx)+16A\cos(2(c+dx))+65A-9B+C)-8(75A-19B\right.}{64ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*(-8*(75*A - 19*B - 5*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c
+ d*x)/2]^4 + 4*(65*A - 9*B + C + (85*A - 13*B + 5*C)*Cos[c + d*x] + 16*A*C
os[2*(c + d*x)])*Sin[(c + d*x)/2]))/(64*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[
c + d*x]))^(3/2))
```

Maple [B] time = 0.307, size = 500, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] -1/16/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(32*A*(-2/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)^3+53*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)
+75*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)-13*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-19*B*sin(d*x+c)*cos(d*x+c)*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+5*C*cos(d*x+c)^2*(-2/(cos(d*
x+c)+1))^(1/2)-5*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x
+c)+1))^(1/2))-36*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+75*A*sin(d*x+c)*ar
ctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+4*B*(-2/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)-19*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))-4*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*C*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-49*A*(-2/(cos(d*x+c)+1))^(1/2)+9*B*(-2/(co
s(d*x+c)+1))^(1/2)-C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)/a^3/sin(d*
x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.559769, size = 1430, normalized size = 6.19

$$\left[-\frac{\sqrt{2}\left((75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B-5\,C)\cos\left(dx+c\right)^{2}+3\,(75\,A-19\,B-5\,C)\cos\left(dx+c\right)+75\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B - 5*C)*cos(d*x + c)^3 + 3*(75*A - 19*B - 5*C)
*cos(d*x + c)^2 + 3*(75*A - 19*B - 5*C)*cos(d*x + c) + 75*A - 19*B - 5*C)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^2 + (85*A - 13*
B + 5*C)*cos(d*x + c) + 49*A - 9*B + C)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((75*A - 19*B - 5*
C)*cos(d*x + c)^3 + 3*(75*A - 19*B - 5*C)*cos(d*x + c)^2 + 3*(75*A - 19*B -
 5*C)*cos(d*x + c) + 75*A - 19*B - 5*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c)))
+ 2*(32*A*cos(d*x + c)^2 + (85*A - 13*B + 5*C)*cos(d*x + c) + 49*A - 9*B +
C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^
3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec
(d*x + c) + a)^(5/2), x)
```

3.1289 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(19A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-$$

```
[Out] ((19*A + 5*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*S
qrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a +
 a*Sec[c + d*x])^(5/2)) - ((9*A - B - 7*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.559329, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4265, 4084, 4012, 3808, 206}

$$\frac{(19A+5B+3C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + a*Sec[
c + d*x])^(5/2)),x]
```

```
[Out] ((19*A + 5*B + 3*C)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*S
qrt[2]*a^(5/2)*d) - ((A - B + C)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*(a +
 a*Sec[c + d*x])^(5/2)) - ((9*A - B - 7*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
```

```
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c\right.}{(a+a\sec(c+dx))^{5/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\mathrm{se}}}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(9A-B-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(9A-B-7C)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= \frac{(19A+5B+3C)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} -$$

Mathematica [A] time = 2.02348, size = 119, normalized size = 0.65

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(8(19A+5B+3C)\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sin\left(\frac{1}{2}(c+dx)\right)((13A-5B-3C)\cos(c\right.}{64ad\cos^{\frac{3}{2}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*(8*(19*A + 5*B + 3*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c +
d*x)/2]^4 - 4*(9*A - B - 7*C + (13*A - 5*B - 3*C)*Cos[c + d*x])*Sin[(c + d*
x)/2]))/(64*a*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.309, size = 474, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(19*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))+13*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+5*B*
sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-5*B*
(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+3*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-3*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))
^(1/2)+19*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*A
*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+5*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))+4*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*C*sin
(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*C*cos(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2)-9*A*(-2/(cos(d*x+c)+1))^(1/2)+B*(-2/(cos(d*x+c)+1))^
(1/2)+7*C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos
(d*x+c))^(1/2)/a^3/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [B] time = 6.19342, size = 11343, normalized size = 61.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2
+ 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x
+ 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
```

^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
 + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x +
4*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7
/2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10
*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*s
in(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*
sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(
1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
 57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)
+ 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
 - 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*c
os(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2
*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
 + 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
 + 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(
1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x +
3/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*si
n(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a
^2*cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*
cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x
+ 4*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2
*c)^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d
*x + c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos
(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c)
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sq
rt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*
cos(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x +
 3*c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*
d*x + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c
))*sin(3*d*x + 3*c))*sqrt(a)) + (4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arct

an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x

+ 3/2*c))) - 3*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3
*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*co
s(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c)
+ 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c)
- 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3
/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x
+ 3*c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
12*sin(3/2*d*x + 3/2*c))*B/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a
^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt
(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16
*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*
c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2
)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sq
rt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c
) + 6*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + sqrt(2)*a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 12*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x
+ 3*c) + sqrt(2)*a^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sq
rt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)) - (12*(
sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(11*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(3/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sin(4*d
*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(si
n(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
2*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 3*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x
+ 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c)
+ 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) +
6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*

```
x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*
log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) + 3*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*co
s(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 1
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x
+ 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*
sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 6*cos(2*d*x +
2*c) + 4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*(11*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 11*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c
) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*
d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*si
n(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 48*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 48*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*C/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 1
6*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*s
qrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 12*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a
^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*
(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*
a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d*
x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sq
rt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)))/d
```

Fricas [A] time = 0.546091, size = 1354, normalized size = 7.4

$$\left[\frac{\sqrt{2}\left((19\,A+5\,B+3\,C)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B+3\,C)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B+3\,C)\cos\left(dx+c\right)+19\,A+5\,B\right.}{64\left(a^{3}d\,\mathrm{c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 5*B + 3*C)*cos(d*x + c)^3 + 3*(19*A + 5*B + 3*C)*co
s(d*x + c)^2 + 3*(19*A + 5*B + 3*C)*cos(d*x + c) + 19*A + 5*B + 3*C)*sqrt(a
)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((13*A - 5*B - 3*C)*cos(d*x + c) + 9*A
- B - 7*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d), -1/32*(sqrt(2)*((19*A + 5*B + 3*C)*cos(d*x + c)^3 + 3*(19*A +
 5*B + 3*C)*cos(d*x + c)^2 + 3*(19*A + 5*B + 3*C)*cos(d*x + c) + 19*A + 5*B
 + 3*C)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 5*B - 3*C)*cos(d*x
+ c) + 9*A - B - 7*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/
2)*sqrt(cos(d*x + c))), x)
```

3.1290 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=241

$$\frac{(5A+3B-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{\ }}{\sqrt{a}}\right.}{a^{5/2}d}$$

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A + 3*B - 43*C)*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Si
n[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3
*B - 11*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3
/2))
```

Rubi [A] time = 0.749033, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {4265, 4084, 4019, 4023, 3808, 206, 3801, 215}

$$\frac{(5A+3B-43C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{\ }}{\sqrt{a}}\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + a*Sec[
c + d*x])^(5/2)),x]
```

```
[Out] (2*C*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((5*A + 3*B - 43*C)*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt
[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B + C)*Si
n[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3
*B - 11*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3
/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2\right.}{(a+a\sec(c+dx))^{5/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B-11C)\sin(c+d}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B-11C)\sin(c+d}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B-11C)\sin(c+d}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{2C\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(5A+3B-43C)}{}$$

Mathematica [A] time = 3.29063, size = 153, normalized size = 0.63

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((5A+3B-11C)\cos(c+dx)+A+7B-15C)+2(5A+3B-43C)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(\right.\right.}{16a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sqrt{a(\sec(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(5*A + 3*B - 43*C)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqr
t[2]*C*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A + 7*B - 15
*C + (5*A + 3*B - 11*C)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a^2*d*Sqrt[Cos[
c + d*x]]*(1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.3, size = 675, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(16*C*sin(d*x
+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))-16*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(
-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+5*A*cos(d*x+c)^2*(-2/(c
os(d*x+c)+1))^(1/2)-5*A*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
```

```
s(d*x+c)+1))^(1/2))+3*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-3*B*sin(d*x+
c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))+16*C*arctan(
1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)
*2^(1/2)-16*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*sin(d*x+c)*2^(1/2)-11*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)+4
3*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-
4*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-5*A*sin(d*x+c)*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))+4*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*B*
sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*C*cos(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2)+43*C*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*
x+c)+1))^(1/2))-A*(-2/(cos(d*x+c)+1))^(1/2)-7*B*(-2/(cos(d*x+c)+1))^(1/2)+1
5*C*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^(1/2)/a^3/sin(d*x+c)^5/(-2/(cos(d
*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.710573, size = 1998, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*A + 3*B - 43*C)*cos(d*x + c)^3 + 3*(5*A + 3*B - 43*C)*c
os(d*x + c)^2 + 3*(5*A + 3*B - 43*C)*cos(d*x + c) + 5*A + 3*B - 43*C)*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((5*A + 3*B - 11*C)*cos(d*x + c) + A +
 7*B - 15*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) - 32*(C*cos(d*x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) +
C)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x
+ c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A
 + 3*B - 43*C)*cos(d*x + c)^3 + 3*(5*A + 3*B - 43*C)*cos(d*x + c)^2 + 3*(5*
A + 3*B - 43*C)*cos(d*x + c) + 5*A + 3*B - 43*C)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*((5*A + 3*B - 11*C)*cos(d*x + c) + A + 7*B - 15*C)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(C*cos(d*
x + c)^3 + 3*C*cos(d*x + c)^2 + 3*C*cos(d*x + c) + C)*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/
2)*cos(d*x + c)^(3/2)), x)
```

3.1291 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=294

$$\frac{(3A-11B+35C)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(3A-43B+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+$$

```
[Out] ((2*B - 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B + 115*C)*ArcTa
nh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A -
 B + C)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) +
 ((A + 7*B - 15*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^(3/2)) + ((3*A - 11*B + 35*C)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.997417, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4265, 4084, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{(3A-11B+35C)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(3A-43B+115C)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + a*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] ((2*B - 5*C)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B + 115*C)*ArcTa
nh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - ((A -
 B + C)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) +
 ((A + 7*B - 15*C)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^(3/2)) + ((3*A - 11*B + 35*C)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4084

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[((a*A - b*B + a*C)*Cot[e + f*x]*(a + b*Csc[e +
 f*x])^m*(d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*B*n - b*C*n - A*
b*(2*m + n + 1) - (b*B*(m + n + 1) - a*(A*(m + n + 1) - C*(m - n)))*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c\right.}{(a+a\sec(c+dx))^{5/2}}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}}{}}{}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= -\frac{(A-B+C)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(A+7B-15C)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= \frac{(2B-5C)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A-43B+}{}$$

Mathematica [A] time = 3.96492, size = 222, normalized size = 0.76

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left((6A-86B+230C)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right.}{4d\sqrt{\cos(c+dx)}(a(\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((6*A - 86*B +
230*C)*ArcTanh[Sin[(c + d*x)/2]] + 32*Sqrt[2]*(2*B - 5*C)*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]] + ((3*A - 11*B + 67*C + 2*(7*A - 15*B + 55*C)*Cos[c + d*x]
 + (3*A - 11*B + 35*C)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^3*Sec[c + d*x]*Ta
n[(c + d*x)/2])/2))/(4*d*Sqrt[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*(c + d*x)])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.322, size = 972, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)
,x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-16*B*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c
)^2*2^(1/2)*sin(d*x+c)+16*B*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+40*C*arctan(1/4*2^
(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*2^(
1/2)*sin(d*x+c)-40*C*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+3*A*(-2/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^3-3*A*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)-11*B*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+43*B*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-16*B*sin
(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))+16*B*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/
2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+35*C*(-2/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^3-115*C*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)^2*sin(d*x+c)+40*C*sin(d*x+c)*2^(1/2)*cos(d*x+c)*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-40*C*sin(d*x+c
)*2^(1/2)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))+4*A*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-3*A*sin(d*x+c)
*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))-4*B*(-2/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)^2+43*B*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))+20*C*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)-1
15*C*sin(d*x+c)*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))
-7*A*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)+15*B*(-2/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)-39*C*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)-16*C*(-2/(cos(d*x+c)+1))
^(1/2))/a^3/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.963444, size = 2329, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((3*A - 43*B + 115*C)*cos(d*x + c)^4 + 3*(3*A - 43*B + 115*C
)*cos(d*x + c)^3 + 3*(3*A - 43*B + 115*C)*cos(d*x + c)^2 + (3*A - 43*B + 11
5*C)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A - 11*B +
 35*C)*cos(d*x + c)^2 + (7*A - 15*B + 55*C)*cos(d*x + c) + 16*C)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 16*((2*B -
5*C)*cos(d*x + c)^4 + 3*(2*B - 5*C)*cos(d*x + c)^3 + 3*(2*B - 5*C)*cos(d*x
+ c)^2 + (2*B - 5*C)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a
```

```
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c
)^2)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + a^3*d*cos(d*x + c)), -1/32*(sqrt(2)*((3*A - 43*B + 115*C)*cos(d*x + c
)^4 + 3*(3*A - 43*B + 115*C)*cos(d*x + c)^3 + 3*(3*A - 43*B + 115*C)*cos(d*
x + c)^2 + (3*A - 43*B + 115*C)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x +
 c))) - 2*((3*A - 11*B + 35*C)*cos(d*x + c)^2 + (7*A - 15*B + 55*C)*cos(d*x
 + c) + 16*C)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) - 16*((2*B - 5*C)*cos(d*x + c)^4 + 3*(2*B - 5*C)*cos(d*x + c)^3
+ 3*(2*B - 5*C)*cos(d*x + c)^2 + (2*B - 5*C)*cos(d*x + c))*sqrt(-a)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 +
 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + B \sec (dx+c) + A}{(a \sec (dx+c) + a)^{\frac{5}{2}} \cos (dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/
2)*cos(d*x + c)^(5/2)), x)
```

3.1292 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=190

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(5aB+5Ab+7bC)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA+9aC+9bB)}{15d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(7}{45d}$$

```
[Out] (2*(7*a*A + 9*b*B + 9*a*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(5*A*b +
5*a*B + 7*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*A*b + 5*a*B + 7*b*
C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*a*A + 9*b*B + 9*a*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(A*b + a*B)*Cos[c + d*x]^(5/2)*Si
n[c + d*x])/(7*d) + (2*a*A*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.297042, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+7bC)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA+9aC+9bB)}{15d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(7aA+9a}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*(7*a*A + 9*b*B + 9*a*C)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(5*A*b +
5*a*B + 7*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*A*b + 5*a*B + 7*b*
C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*a*A + 9*b*B + 9*a*C)*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(A*b + a*B)*Cos[c + d*x]^(5/2)*Si
n[c + d*x])/(7*d) + (2*a*A*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))\left(C+B\right.$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{3}{2}}($$

$$= \frac{2(Ab+aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a}{}$$

$$= \frac{2(Ab+aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a}{}$$

$$= \frac{2(5Ab+5aB+7bC)\sqrt{\cos(c+dx)}\sin(c+}{21d}$$

$$= \frac{2(7aA+9bB+9aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2(}{}$$

Mathematica [A] time = 1.01293, size = 143, normalized size = 0.75

$$\frac{60\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(5aB+5Ab+7bC)+84E\left(\frac{1}{2}(c+dx)\middle|2\right)(7aA+9aC+9bB)+\sin(c+dx)\sqrt{\cos(c+dx)}(7\cos}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (84*(7*a*A + 9*b*B + 9*a*C)*EllipticE[(c + d*x)/2, 2] + 60*(5*A*b + 5*a*B +
 7*b*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(43*a*A + 36*b*B
+ 36*a*C)*Cos[c + d*x] + 5*(78*A*b + 78*a*B + 84*b*C + 18*(A*b + a*B)*Cos[2
*(c + d*x)] + 7*a*A*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 2.467, size = 565, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*a*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a+720*A*b+720*B*a)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a-1080*A*b-1080*B*a-504*B*b-504*C*
a)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*A*a+840*A*b+840*B*a+504*B*b
+504*C*a+420*C*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a-240*A*b
-240*B*a-126*B*b-126*C*a-210*C*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7
5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+75*B*a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+105*C*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+\left(Ca+Bb\right)\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+Aa\cos\left(dx+c\right)^4+\left(Ba+Ab\right)\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^4*se
c(d*x + c)^2 + A*a*cos(d*x + c)^4 + (B*a + A*b)*cos(d*x + c)^4*sec(d*x + c)
)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\left(b\sec\left(dx+c\right) + a\right)\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(
d*x + c)^(9/2), x)
```

3.1293 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=154

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(5aA+7aC+7bB)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC)}{5d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(5}{21d}$$

```
[Out] (2*(3*A*b + 3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A + 7
*b*B + 7*a*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*a*A + 7*b*B + 7*a*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.269183, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+7aC+7bB)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC)}{5d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(5aA+7a}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*(3*A*b + 3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A + 7
*b*B + 7*a*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*a*A + 7*b*B + 7*a*C
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int \sqrt{\cos(c+dx)}(b+a\cos(c+dx))\left(C+B\right.$$

$$=\frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sqrt{\cos}$$

$$=\frac{2(Ab+aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a}{}$$

$$=\frac{2(Ab+aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a}{}$$

$$=\frac{2(3Ab+3aB+5bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(}{}$$

$$=\frac{2(3Ab+3aB+5bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(}{}$$

Mathematica [A] time = 0.900022, size = 117, normalized size = 0.76

$$\frac{10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(5aA+7aC+7bB)+42E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+5bC)+\sin(c+dx)\sqrt{\cos(c+dx)}(42(a}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (42*(3*A*b + 3*a*B + 5*b*C)*EllipticE[(c + d*x)/2, 2] + 10*(5*a*A + 7*b*B +
 7*a*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(65*a*A + 70*b*B + 7
```

```
0*a*C + 42*(A*b + a*B)*Cos[c + d*x] + 15*a*A*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

Maple [B] time = 2.419, size = 515, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*a*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a-168*A*b-168*B*a)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b+140*C*a)*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b-70*C*a)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*b+35*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a+35*a*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)^{3}+\left(Ba+Ab\right)\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^3*se
c(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A*b)*cos(d*x + c)^3*sec(d*x + c)
)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)\cos(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(
d*x + c)^(7/2), x)
```

3.1294 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=116

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(aB+Ab+3bC)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+5aC+5bB)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c}}{3d}$$

```
[Out] (2*(3*a*A + 5*b*B + 5*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B
 + 3*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.251822, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4112, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab+3bC)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(3aA+5aC+5bB)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*(3*a*A + 5*b*B + 5*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B
 + 3*b*C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))\left(C+B\cos(c+dx)+\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{5bC}{2}+}{}$$

$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a}{}$$

$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a}{}$$

$$= \frac{2(3aA+5bB+5aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(}{}$$

Mathematica [C] time = 6.49148, size = 1569, normalized size = 13.53

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((-4*(3*a*A + 5*b*B + 5*a*C)*Cot[c])/(5*d) + (4*(A*b + a*B)*Cos[d*x]*
Sin[c])/(3*d) + (2*a*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*(A*b + a*B)*Cos[c]*S
in[d*x])/(3*d) + (2*a*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((b + a*Cos[c + d*x])*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*A*b*Cos[c + d*x]^3*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a
 + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(
b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2]) - (4*a*B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*C
```

```
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b*C*Cos[c + d*x]
^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*
(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2]) - (6*a*A*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x])*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (2*b*B*Cos[c + d*x]^3*Csc[c]*(a +
 b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos
[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (2*a*C*Cos[
c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x]))
```

Maple [B] time = 2.481, size = 465, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a+20*A*b+20*B*a)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5
*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+15*C*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)^{2}+\left(Ba+Ab\right)\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2*se
c(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c)^2*sec(d*x + c)
)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(
d*x + c)^(5/2), x)
```

3.1295 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\,\mathrm{se}\right.$

Optimal. Leaf size=106

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(a(A+3C)+3bB)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab-bC)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+$$

```
[Out] (2*(A*b + a*B - b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*b*B + a*(A + 3*C)
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.260655, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4112, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(A+3C)+3bB)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(aB+Ab-bC)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bC\sin(}{d\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (2*(A*b + a*B - b*C)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*b*B + a*(A + 3*C)
)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*C*Sin[c + d*x])/(d*Sqrt[Cos[c + d
*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))\left(C+B\cos(c+dx)+\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2bC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{\frac{1}{2}(-bB-aC)-\frac{1}{2}(A}{}$$

$$= \frac{2bC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sin(c}{3d}$$

$$= \frac{2bC\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA\sqrt{\cos(c+dx)}\sin(c}{3d}$$

$$= \frac{2(Ab+aB-bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3bB}{}$$

Mathematica [C] time = 6.91638, size = 1904, normalized size = 17.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (I*A*b*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
 + 2*d*x])) + (I*a*B*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x])*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Co
```

```
s[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)
*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
- 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) - (I*b*C*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x
])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(7/2)*(a + b*Sec[c
+ d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(A*b + a*B - 2*b*C + A
*b*Cos[2*c] + a*B*Cos[2*c])*Csc[c]*Sec[c])/d + (4*a*A*Cos[d*x]*Sin[c])/(3*d
) + (4*a*A*Cos[c]*Sin[d*x])/(3*d) + (4*b*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d)
)/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
- (4*a*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b*B*Cos[c + d*x]^3*Csc[c]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c +
 d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c +
d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
 - (4*a*C*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 2.518, size = 388, normalized size = 3.7

$$-\frac{2}{3\,d}\left(4\,Aa\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+Aa\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*(4*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+A*a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-2*A*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+3*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+3*a*C
```

```
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b-6*C*b*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)*sec(d*
x + c)^2 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*sec(d*x + c))*sqrt(c
os(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(
d*x + c)^(3/2), x)
```

3.1296 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=112

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(3aB+3Ab+bC)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a(A-C))}{d}+\frac{2(aC+bB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sin(c}{3d\cos^{\frac{3}{2}}(c}$$

```
[Out] (-2*(b*B - a*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A*b + 3*a*B + b*
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*C*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.27805, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {4112, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(3aB+3Ab+bC)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(bB-a(A-C))}{d}+\frac{2(aC+bB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2),x]
```


```
[Out] (-2*(b*B - a*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(3*A*b + 3*a*B + b*
C)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*C*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))\left(C+B\cos(c+d\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2bC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}(bB+aC)-}{}$$

$$= \frac{2bC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2bC\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(bB-a(A-C))E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + $$

Mathematica [C] time = 6.99074, size = 1909, normalized size = 17.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] (I*a*A*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
```

```
 + 2*d*x])) - (I*b*B*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x])*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Co
s[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)
*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
- 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) - (I*a*C*Cos[c + d*x]^3*Csc[c]*(a + b*Sec[c + d*x
])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(7/2)*(a + b*Sec[c
+ d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(a*A - 2*b*B - 2*a*C +
 a*A*Cos[2*c])*Csc[c]*Sec[c])/d + (4*b*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3
*d) + (4*Sec[c]*Sec[c + d*x]*(b*C*Sin[c] + 3*b*B*Sin[d*x] + 3*a*C*Sin[d*x])
)/(3*d)))/((b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])) - (4*A*b*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a*B*Cos[c + d*x]^3*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*S
ec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Co
s[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[
c]^2]) - (4*b*C*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]
]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin
[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 5.604, size = 666, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*b*(sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*b*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(B*b+C*a)*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\sec\left(dx+c\right)^3+\left(Ca+Bb\right)\sec\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt
(cos(d*x + c)), x)
```

3.1297 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(a(3A+C)+bB)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+3bC)}{5d}+\frac{2\sin(c+dx)(5aB+5Ab+3bC)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*(5*A*b + 5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*
(3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*C*Sin[c + d*x])/(5*d*Cos
[c + d*x]^(5/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) +
(2*(5*A*b + 5*a*B + 3*b*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.304602, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(a(3A+C)+bB)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aB+5Ab+3bC)}{5d}+\frac{2\sin(c+dx)(5aB+5Ab+3bC)}{5d\sqrt{\cos(c+dx)}}+\frac{2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```


```
[Out] (-2*(5*A*b + 5*a*B + 3*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(b*B + a*
(3*A + C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*C*Sin[c + d*x])/(5*d*Cos
[c + d*x]^(5/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) +
(2*(5*A*b + 5*a*B + 3*b*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \frac{(b+a\cos(c+dx))\left(C+B\cos(c+dx)+A\cos^2(c+\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2bC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(bB+aC)-\frac{1}{2}(5Ab+5aB+}{\cos}$$

$$= \frac{2bC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}(}{}$$

$$= \frac{2bC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}(-5Ab-$$

$$= \frac{2(bB+a(3A+C))F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2bC\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{2(5Ab+5aB+3bC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(bB+a(3A}{}$$

Mathematica [A] time = 1.55189, size = 136, normalized size = 0.89

$$\frac{10\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(a(3A+C)+bB)+3\sin(2(c+dx))(5aB+5Ab+3bC)-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (-6*(5*A*b + 5*a*B + 3*b*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] +
10*(b*B + a*(3*A + C))*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*(b
*B + a*C)*Sin[c + d*x] + 3*(5*A*b + 5*a*B + 3*b*C)*Sin[2*(c + d*x)] + 6*b*C
*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 7.888, size = 742, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(B*
b+C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C*b/(8*sin(1/2*d*x+1/2*
c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2
*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*(A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\sec\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\sec\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(c + dx))\left(A + B\sec(c + dx) + C\sec^2(c + dx)\right)}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/
2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/sqrt
(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx + c)^2 + B\sec(dx + c) + A\right)(b\sec(dx + c) + a)}{\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt
(cos(d*x + c)), x)
```

3.1298 $\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=190

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(7aB+7Ab+5bC)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+3aC+3bB)}{5d}+\frac{2\sin(c+dx)(7aB+7Ab+5}{21d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(5*a*A + 3*b*B + 3*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A*b +
7*a*B + 5*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*C*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (2*(7*A*b + 7*a*B + 5*b*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (
2*(5*a*A + 3*b*B + 3*a*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.319087, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {4112, 3031, 3021, 2748, 2636, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(7aB+7Ab+5bC)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+3aC+3bB)}{5d}+\frac{2\sin(c+dx)(7aB+7Ab+5bC)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```


```
[Out] (-2*(5*a*A + 3*b*B + 3*a*C)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*A*b +
7*a*B + 5*b*C)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*C*Sin[c + d*x])/(7*
d*Cos[c + d*x]^(7/2)) + (2*(b*B + a*C)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2
)) + (2*(7*A*b + 7*a*B + 5*b*C)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (
2*(5*a*A + 3*b*B + 3*a*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \int \frac{(b+a\cos(c+dx))\left(C+B\cos(c+dx)+A\cos^2(c+\ldots\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2bC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(bB+aC)-\frac{1}{2}(7Ab+7aB+\ldots}{\cos\ldots}$$

$$= \frac{2bC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}(\ldots}{\ldots}$$

$$= \frac{2bC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{5}(-5aA-\ldots$$

$$= \frac{2bC\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(bB+aC)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(7Ab+7\ldots}{21\ldots}$$

$$= -\frac{2(5aA+3bB+3aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(7Ab+7aB\ldots}{\ldots}$$

Mathematica [A] time = 4.27779, size = 173, normalized size = 0.91

$$\frac{10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(7aB+7Ab+5bC)-42E\left(\frac{1}{2}(c+dx)\middle|2\right)(5aA+3aC+3bB)+\frac{\sin(c+dx)(21\cos(c+dx)(15aA+13aC+13\ldots}{\ldots}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] (-42*(5*a*A + 3*b*B + 3*a*C)*EllipticE[(c + d*x)/2, 2] + 10*(7*A*b + 7*a*B
+ 5*b*C)*EllipticF[(c + d*x)/2, 2] + ((70*A*b + 70*a*B + 110*b*C + 21*(15*a
*A + 13*b*B + 13*a*C)*Cos[c + d*x] + 10*(7*A*b + 7*a*B + 5*b*C)*Cos[2*(c +
d*x)] + 105*a*A*Cos[3*(c + d*x)] + 63*b*B*Cos[3*(c + d*x)] + 63*a*C*Cos[3*(
c + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(7/2)))/(105*d)
```

Maple [B] time = 9.324, size = 851, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*b*(-1/56*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*(B*b+C*a)/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^
2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*A*a*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb\sec{(dx+c)}^{3}+(Ca+Bb)\sec{(dx+c)}^{2}+Aa+(Ba+Ab)\sec{(dx+c)}}{\cos{(dx+c)}^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*sec(d*x + c)^3 + (C*a + B*b)*sec(d*x + c)^2 + A*a + (B*a + A*
b)*sec(d*x + c))/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/
2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(C\sec{(dx+c)}^{2}+B\sec{(dx+c)}+A\right)(b\sec{(dx+c)}+a)}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/cos(
d*x + c)^(3/2), x)
```

3.1299 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=250

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^2B+10aAb+14abC+7b^2B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(7A+9C)+18abB+3b^2(3A+5C\right.}{15d}$$

```
[Out] (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2,
2])/(15*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*d) + (2*(4*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(4*A*b + 9*a*B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(63*d) + (2*A*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*
Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.603223, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+14abC+7b^2B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(7A+9C)+18abB+3b^2(3A+5C)\right)}{15d}+\frac{2\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2,
2])/(15*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*(10*a*A*b + 5*a^2*B + 7*b^2*B + 14*a*b*C)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(21*d) + (2*(4*A*b^2 + 18*a*b*B + a^2*(7*A + 9*C))*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(4*A*b + 9*a*B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(63*d) + (2*A*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*
Sin[c + d*x])/(9*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2\left(C\right.$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\,\mathrm{si}}{9d}$$

$$= \frac{2a(4Ab+9aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+d}{63d}$$

$$= \frac{2\left(4Ab^2+18abB+a^2(7A+9C)\right)\cos^{\frac{3}{2}}}{45d}$$

$$= \frac{2\left(4Ab^2+18abB+a^2(7A+9C)\right)\cos^{\frac{3}{2}}}{45d}$$

$$= \frac{2\left(18abB+3b^2(3A+5C)+a^2(7A+9\right.}{15d}$$

$$= \frac{2\left(18abB+3b^2(3A+5C)+a^2(7A+9\right.}{15d}$$

Mathematica [A] time = 1.33587, size = 194, normalized size = 0.78

$$\frac{60\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^2B+2ab(5A+7C)+7b^2B\right)+84E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(7A+9C)+18abB+3b^2(3A+5C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (84*(18*a*b*B + 3*b^2*(3*A + 5*C) + a^2*(7*A + 9*C))*EllipticE[(c + d*x)/2,
 2] + 60*(5*a^2*B + 7*b^2*B + 2*a*b*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2]
+ Sqrt[Cos[c + d*x]]*(7*(36*A*b^2 + 72*a*b*B + a^2*(43*A + 36*C))*Cos[c + d
*x] + 5*(156*a*A*b + 78*a^2*B + 84*b^2*B + 168*a*b*C + 18*a*(2*A*b + a*B)*C
os[2*(c + d*x)] + 7*a^2*A*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 2.587, size = 784, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^2+1440*A*a*b+720*B*a^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^2-2160*A*a*b-504*A*b^2-
1080*B*a^2-1008*B*a*b-504*C*a^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(9
52*A*a^2+1680*A*a*b+504*A*b^2+840*B*a^2+1008*B*a*b+420*B*b^2+504*C*a^2+840*
C*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a^2-480*A*a*b-126*A*
b^2-240*B*a^2-252*B*a*b-210*B*b^2-126*C*a^2-420*C*a*b)*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+150*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
```

```
),2^(1/2))*a^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+75*B*a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+105*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b+210*a*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3
15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^4\sec(dx+c)^3 + Aa^2\cos(dx+c)^4 + \left(Ca^2 + 2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^4*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*co
s(d*x + c)^(9/2), x)
```

3.1300 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=202

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(5A+7C)+14abB+7b^2(A+3C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+6aAb+10abC+5b^2B\right)}{5d}+$$

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d
) + (2*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2
, 2])/(21*d) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(21*d) + (2*a*(4*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(35*d) + (2*A*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(7
*d)
```

Rubi [A] time = 0.569767, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(5A+7C)+14abB+7b^2(A+3C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+6aAb+10abC+5b^2B\right)}{5d}+\frac{2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```



```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*d
) + (2*(14*a*b*B + 7*b^2*(A + 3*C) + a^2*(5*A + 7*C))*EllipticF[(c + d*x)/2
, 2])/(21*d) + (2*(4*A*b^2 + 14*a*b*B + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(21*d) + (2*a*(4*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(35*d) + (2*A*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(7
*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx=\int \frac{(b+a\cos(c+dx))^2\left(C+B\cos(c+\right.}{\sqrt{\cos(c+dx)}}$$

$$=\frac{2A\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2\,\mathrm{si}}{7d}$$

$$=\frac{2a(4Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{35d}$$

$$=\frac{2\left(4Ab^2+14abB+a^2(5A+7C)\right)\sqrt{\mathrm{co}}}{21d}$$

$$=\frac{2\left(4Ab^2+14abB+a^2(5A+7C)\right)\sqrt{\mathrm{co}}}{21d}$$

$$=\frac{2\left(6aAb+3a^2B+5b^2B+10abC\right)E\left(\frac{1}{2}\right.}{5d}$$

Mathematica [C] time = 6.85339, size = 2361, normalized size = 11.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((-4*(6*a*A*b + 3*a^2*B + 5*b^2*B + 10*a*b*C)*Cot[c])/(5*d) + ((23*
a^2*A + 28*A*b^2 + 56*a*b*B + 28*a^2*C)*Cos[d*x]*Sin[c])/(21*d) + (2*a*(2*A
*b + a*B)*Cos[2*d*x]*Sin[2*c])/(5*d) + (a^2*A*Cos[3*d*x]*Sin[3*c])/(7*d) +
((23*a^2*A + 28*A*b^2 + 56*a*b*B + 28*a^2*C)*Cos[c]*Sin[d*x])/(21*d) + (2*a
*(2*A*b + a*B)*Cos[2*c]*Sin[2*d*x])/(5*d) + (a^2*A*Cos[3*c]*Sin[3*d*x])/(7*
d)))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])) - (20*a^2*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*b^2*Cos[c + d*x]
^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*Sqrt[1 + Cot[c]^2]) - (8*a*b*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^2*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^2
*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Arc
Tan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
an[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^2*C*Cos[c + d*x]^4*Csc[c]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c +
 d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c
+ d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]
^2]) - (12*a*A*b*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (6*a^2*B*Cos[c + d*x]^4*Csc[c]*(a + b
*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPF
Q[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
 Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*C
os[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2)
)/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*C
os[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (2*b^2*
B*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
```

```
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])) - (4*a*b*C*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*
(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*x])^2*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 2.666, size = 706, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a
^2+280*B*a*b+140*C*a^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-
84*A*a*b-70*A*b^2-42*B*a^2-140*B*a*b-70*C*a^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-126*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+25*a^2*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1
05*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-210*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b+35*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^4+\left(2\,Cab+Bb^2\right)\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+Aa^2\cos\left(dx+c\right)^3+\left(Ca^2+2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*co
s(d*x + c)^(7/2), x)
```

3.1301 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=186

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)+10abB+5b^2(A-C)\right)}{5d}+$$

```
[Out] (2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a*(2*A*b + a*B - 6*b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*a^2*(A - 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b + a*Cos[c
 + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.551769, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A+3C)+3b^2B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)+10abB+5b^2(A-C)\right)}{5d}+\frac{2a^2(A-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(10*a*b*B + 5*b^2*(A - C) + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(a^2*B + 3*b^2*B + 2*a*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a*(2*A*b + a*B - 6*b*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
(2*a^2*(A - 5*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b + a*Cos[c
 + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^2\left(C+B\cos(c+dx\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int$$

$$= \frac{2a^2(A-5C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{}$$

$$= \frac{2a(2Ab+aB-6bC)\sqrt{\cos(c+dx)}\sin(c+}{3d}$$

$$= \frac{2a(2Ab+aB-6bC)\sqrt{\cos(c+dx)}\sin(c+}{3d}$$

$$= \frac{2\left(10abB+5b^2(A-C)+a^2(3A+5C)\right)E}{5d}$$

Mathematica [C] time = 7.46154, size = 3011, normalized size = 16.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (((3*I)/5)*a^2*A*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (
2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c
] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
 E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) + (I*A*b^2*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d
*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A +
2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((2*I)*a*b*B*Cos[c + d*x]^4
*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b +
 a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (I*
a^2*C*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E
^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d
*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) - (I*b^2*C*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2
, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*
Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2,
 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
+ d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x
])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(3*a^2*A + 5*A*b^2 + 10*a
*b*B + 5*a^2*C - 10*b^2*C + 3*a^2*A*Cos[2*c] + 5*A*b^2*Cos[2*c] + 10*a*b*B*
```

```
Cos[2*c] + 5*a^2*C*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*a*(2*A*b + a*B)*Cos[
d*x]*Sin[c])/(3*d) + (2*a^2*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*a*(2*A*b + a*
B)*Cos[c]*Sin[d*x])/(3*d) + (4*b^2*C*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*a
^2*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Co
s[c + d*x] + A*Cos[2*c + 2*d*x])) - (8*a*A*b*Cos[c + d*x]^4*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*
x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c +
 d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^
2]) - (4*a^2*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^2*B*Cos[c + d*x]^4*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a
 + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(
b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[1 + Cot[c]^2]) - (8*a*b*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^2*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 2.903, size = 932, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/15*(-24*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*a*(6*A*a+10*A*b+5*B*a)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*a^2+10*A*a*b
+5*B*a^2+15*C*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*a*b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-9*A*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2-15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*b^2+5*B*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-30*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+30*a*b*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*C*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
^2+15*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^2\cos(dx+c)^2\sec(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^2\sec(dx+c)^3+Aa^2\cos(dx+c)^2+\left(Ca^2+2\,B\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*co
s(d*x + c)^(5/2), x)
```

3.1302 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2 \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=180

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-C)-b^2B\right)}{d}+\frac{2a^2(A}{}$$

```
[Out] (2*(a^2*B - b^2*B + 2*a*b*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*b
*B + b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b
*(3*b*B + 4*a*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(A - C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*(b + a*Cos[c + d*x])^2*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.545494, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)+6abB+b^2(3A+C)\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2B+2ab(A-C)-b^2B\right)}{d}+\frac{2a^2(A-C)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(a^2*B - b^2*B + 2*a*b*(A - C))*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*b
*B + b^2*(3*A + C) + a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b
*(3*b*B + 4*a*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(A - C)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*(b + a*Cos[c + d*x])^2*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^2\left(C+B\cos(c+dx\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int$$

$$= \frac{2b(3bB+4aC)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2C(b+a\,\text{co}}{3d}$$

$$= \frac{2b(3bB+4aC)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2(A-C)}{}$$

$$= \frac{2b(3bB+4aC)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2(A-C)}{}$$

$$= \frac{2\left(a^2B-b^2B+2ab(A-C)\right)E\left(\frac{1}{2}(c+dx)\right|}{d}$$

Mathematica [C] time = 7.54517, size = 2779, normalized size = 15.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((2*I)*a*A*b*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*
(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2
*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)
*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^(
(2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] +
 A*Cos[2*c + 2*d*x])) + (I*a^2*B*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])
^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (I*b^2*B*Cos[c + d*x]^4*Csc[c]*
(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)
*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^
2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])
/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((
3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*H
ypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*
(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c
 + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((2*I)*a*b*
C*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2
*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))
*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])) + (Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*((-2*(2*a*A*b + a^2*B - 2*b^2*B - 4*a*b*C + 2*a*A*b*Co
s[2*c] + a^2*B*Cos[2*c])*Csc[c]*Sec[c])/d + (4*a^2*A*Cos[d*x]*Sin[c])/(3*d)
 + (4*a^2*A*Cos[c]*Sin[d*x])/(3*d) + (4*b^2*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x
])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(b^2*C*Sin[c] + 3*b^2*B*Sin[d*x] + 6*a*b*
C*Sin[d*x]))/(3*d)))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])) - (4*a^2*A*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
```

```
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^2*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*b^
2*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (8*a*b*B*Cos[c + d*x]^4*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*
x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d
*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]
) - (4*a^2*C*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^2*C*Cos[c + d*x]^4*Csc[
c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b
*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b
+ a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[
1 + Cot[c]^2])
```

Maple [B] time = 6.998, size = 1301, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(-6*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b+6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*A*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4-2*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a^
2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2-8*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+12*B*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2-2*C*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2+
6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2-6*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2-6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*sin(1/2*d
*x+1/2*c)^2-2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
```

```
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2-2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+3*a^2*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+b^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*C*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+24*C*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^2+12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-12*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-12*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*a*b*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)\sec(dx+c)^3 + Aa^2\cos(dx+c) + \left(Ca^2 + 2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x + c
)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (C*a^2 + 2*B*a*b + A*b^2)*cos(d*x +
 c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(d*x + c))*sqrt(cos(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*co
s(d*x + c)^(3/2), x)
```

3.1303 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=201

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^2(A-C)+10abB+b^2(5A+3C)\right)}{5d}+$$

```
[Out] (-2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])
/(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*(3*A + C))*EllipticF[(c + d*x)/2, 2])/
(3*d) + (2*b*(5*b*B + 4*a*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*(
5*A*b^2 + 10*a*b*B + 4*a^2*C + 3*b^2*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*C*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.571035, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2B+2ab(3A+C)+b^2B\right)}{3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^2(A-C)+10abB+b^2(5A+3C)\right)}{5d}+\frac{2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```

```
[Out] (-2*(10*a*b*B - 5*a^2*(A - C) + b^2*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2])
/(5*d) + (2*(3*a^2*B + b^2*B + 2*a*b*(3*A + C))*EllipticF[(c + d*x)/2, 2])/
(3*d) + (2*b*(5*b*B + 4*a*C)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*(
5*A*b^2 + 10*a*b*B + 4*a^2*C + 3*b^2*C)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*C*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^2\left(C+B\cos(c+dx\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int$$

$$= \frac{2b(5bB+4aC)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2C(b+a\,\mathrm{co}}{5d}$$

$$= \frac{2b(5bB+4aC)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5Ab^2+\right.}{}$$

$$= \frac{2b(5bB+4aC)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5Ab^2+\right.}{}$$

$$= -\frac{2\left(10abB-5a^2(A-C)+b^2(5A+3C)\right)}{5d}$$

Mathematica [C] time = 7.64808, size = 3017, normalized size = 15.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (I*a^2*A*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*
I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
+ I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1
+ E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I
)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])) - (I*A*b^2*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[
1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x
))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1
/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x)
)*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*
d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[
c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((2*I)*a*b*B*Cos[c + d*x]^4*Csc[c]*
(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)
*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^
2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])
/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((
3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*H
ypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*
(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c
 + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (I*a^2*C*Co
s[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*
d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin
[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])) - (((3*I)/5)*b^2*C*Cos[c + d*x]^4*Csc[c]*(a + b*Sec[c + d*x])^2*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2
, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*
Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2,
 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
```

```
+ d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x
])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(5*a^2*A - 10*A*b^2 - 20*
a*b*B - 10*a^2*C - 6*b^2*C + 5*a^2*A*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*b^
2*C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]^2*(3*b^2
*C*Sin[c] + 5*b^2*B*Sin[d*x] + 10*a*b*C*Sin[d*x]))/(15*d) + (4*Sec[c]*Sec[c
 + d*x]*(5*b^2*B*Sin[c] + 10*a*b*C*Sin[c] + 15*A*b^2*Sin[d*x] + 30*a*b*B*Si
n[d*x] + 15*a^2*C*Sin[d*x] + 9*b^2*C*Sin[d*x]))/(15*d)))/((b + a*Cos[c + d*
x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (8*a*A*b*Cos[c +
 d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*Sqrt[1 + Cot[c]^2]) - (4*a^2*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^2*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^
2*B*Cos[c + d*x]^4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (8*a*b*C*Cos[c + d*x]^4*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c
+ d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos
[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot
[c]^2])
```

Maple [B] time = 8.06, size = 1000, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*A*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/
5*b^2*C/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2
```

```
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b*(B*b+2*C*a)*(-1/6*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2)))+2*(A*b^2+2*B*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aal\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sq
rt(cos(d*x + c)), x)
```

3.1304 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx)+C\sec^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=249

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+6abC+3b^2B\right)}{5d}$$

```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*
d) + (2*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*b*(7*b*B + 4*a*C)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)
) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sin[c + d*x])/(21*d*Cos[c +
 d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sin[c + d*x])/(5
*d*Sqrt[Cos[c + d*x]]) + (2*C*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos
[c + d*x]^(7/2))
```

Rubi [A] time = 0.60521, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+10aAb+6abC+3b^2B\right)}{5d}+\frac{2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```

```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*EllipticE[(c + d*x)/2, 2])/(5*
d) + (2*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/
2, 2])/(21*d) + (2*b*(7*b*B + 4*a*C)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)
) + (2*(7*A*b^2 + 14*a*b*B + 4*a^2*C + 5*b^2*C)*Sin[c + d*x])/(21*d*Cos[c +
 d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B + 6*a*b*C)*Sin[c + d*x])/(5
*d*Sqrt[Cos[c + d*x]]) + (2*C*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos
[c + d*x]^(7/2))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \frac{(b+a\cos(c+dx))^2\left(C+B\cos(c+dx)+A\cos^2\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(b+a\cos}{}$$

$$= \frac{2b(7bB+4aC)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2C(b+a\cos(c+dx))}{7d\cos^{\frac{7}{2}}(c+}$$

$$= \frac{2b(7bB+4aC)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2+14abB+4\right.}{21d\cos}$$

$$= \frac{2b(7bB+4aC)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7Ab^2+14abB+4\right.}{21d\cos}$$

$$= \frac{2\left(14abB+7a^2(3A+C)+b^2(7A+5C)\right)F\left(\frac{1}{2}(c+\right.}{21d}$$

$$= -\frac{2\left(10aAb+5a^2B+3b^2B+6abC\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Mathematica [A] time = 4.59474, size = 218, normalized size = 0.88

$$\frac{2\left(5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^2(3A+C)+14abB+b^2(7A+5C)\right)-21E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2B+2ab(5A+3C)+3b^2\right.\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-21*(5*a^2*B + 3*b^2*B + 2*a*b*(5*A + 3*C))*EllipticE[(c + d*x)/2, 2] +
 5*(14*a*b*B + 7*a^2*(3*A + C) + b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2]
 + (15*b^2*C*Sin[c + d*x])/Cos[c + d*x]^(7/2) + (21*b*(b*B + 2*a*C)*Sin[c +
 d*x])/Cos[c + d*x]^(5/2) + (5*(7*A*b^2 + 14*a*b*B + 7*a^2*C + 5*b^2*C)*Sin
[c + d*x])/Cos[c + d*x]^(3/2) + (21*(5*a^2*B + 3*b^2*B + 2*a*b*(5*A + 3*C))
*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)
```

Maple [B] time = 10.196, size = 947, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5
*b*(B*b+2*C*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
```

```
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(A*b^2+2*B*a*b+C*a^2)*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*C*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5
/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(2*A*b+B*a)*(-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^2\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\sec(dx+c)^3 + Aa^2 + \left(Ca^2 + 2\,Bab + Ab^2\right)\sec(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\text{se}}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*sec(d*x + c)^3 + A*a^2 +
 (C*a^2 + 2*B*a*b + A*b^2)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*sec(d*x + c))
/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{2}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sq
rt(cos(d*x + c)), x)
```

3.1305 $\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=361

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^3(9A+11C)+165a^2bB+33ab^2(5A+7C)+77b^3B\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(7A+9C)+\right.}{15d}$$

```
[Out] (2*(7*a^3*B + 27*a*b^2*B + 3*b^3*(3*A + 5*C) + 3*a^2*b*(7*A + 9*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(165*a^2*b*B + 77*b^3*B + 33*a*b^2*(5*A + 7
*C) + 5*a^3*(9*A + 11*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (2*(165*a^2*
b*B + 77*b^3*B + 33*a*b^2*(5*A + 7*C) + 5*a^3*(9*A + 11*C))*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(231*d) + (2*(24*A*b^3 + 77*a^3*B + 242*a*b^2*B + 33*a^2*
b*(7*A + 9*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(495*d) + (2*a*(24*A*b^2 +
143*a*b*B + 9*a^2*(9*A + 11*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d) +
(2*(6*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])
/(99*d) + (2*A*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*
d)
```

Rubi [A] time = 0.955049, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3049, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^3(9A+11C)+165a^2bB+33ab^2(5A+7C)+77b^3B\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(7A+9C)+7a^3B+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2),x]
```



```
[Out] (2*(7*a^3*B + 27*a*b^2*B + 3*b^3*(3*A + 5*C) + 3*a^2*b*(7*A + 9*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(165*a^2*b*B + 77*b^3*B + 33*a*b^2*(5*A + 7
*C) + 5*a^3*(9*A + 11*C))*EllipticF[(c + d*x)/2, 2])/(231*d) + (2*(165*a^2*
b*B + 77*b^3*B + 33*a*b^2*(5*A + 7*C) + 5*a^3*(9*A + 11*C))*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(231*d) + (2*(24*A*b^3 + 77*a^3*B + 242*a*b^2*B + 33*a^2*
b*(7*A + 9*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(495*d) + (2*a*(24*A*b^2 +
143*a*b*B + 9*a^2*(9*A + 11*C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d) +
(2*(6*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])
/(99*d) + (2*A*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(11*
d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \sqrt{\cos(c+dx)}(b+a\cos(c+dx))^3\left(C+\right.$$
$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^3\sin(}{11d}$$
$$= \frac{2(6Ab+11aB)\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c}{99d}$$
$$= \frac{2a\left(24Ab^2+143abB+9a^2(9A+11C)\right)c}{693d}$$
$$= \frac{2\left(24Ab^3+77a^3B+242ab^2B+33a^2b(7\right.}{495d}$$
$$= \frac{2\left(24Ab^3+77a^3B+242ab^2B+33a^2b(7\right.}{495d}$$
$$= \frac{2\left(7a^3B+27ab^2B+3b^3(3A+5C)+3a^2\right.}{15d}$$
$$= \frac{2\left(7a^3B+27ab^2B+3b^3(3A+5C)+3a^2\right.}{15d}$$

Mathematica [A] time = 2.02615, size = 286, normalized size = 0.79

$$10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^3(9A+11C)+165a^2bB+33ab^2(5A+7C)+77b^3B\right)+154E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(7A+9C\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2),x]
```

```
[Out] (154*(7*a^3*B + 27*a*b^2*B + 3*b^3*(3*A + 5*C) + 3*a^2*b*(7*A + 9*C))*Ellip
ticE[(c + d*x)/2, 2] + 10*(165*a^2*b*B + 77*b^3*B + 33*a*b^2*(5*A + 7*C) +
5*a^3*(9*A + 11*C))*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(154*(3
6*A*b^3 + 43*a^3*B + 108*a*b^2*B + 3*a^2*b*(43*A + 36*C))*Cos[c + d*x] + 5*
(36*a*(33*A*b^2 + 33*a*b*B + a^2*(16*A + 11*C))*Cos[2*(c + d*x)] + 154*a^2*
(3*A*b + a*B)*Cos[3*(c + d*x)] + 3*(1716*a^2*b*B + 616*b^3*B + 132*a*b^2*(1
3*A + 14*C) + a^3*(531*A + 572*C) + 21*a^3*A*Cos[4*(c + d*x)])))*Sin[c + d*
x])/12)/(1155*d)
```

Maple [B] time = 2.615, size = 1082, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a^3-36960*A*a^2*b-1232
0*B*a^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^3+73920*A*a^2*
b+23760*A*a*b^2+24640*B*a^3+23760*B*a^2*b+7920*C*a^3)*sin(1/2*d*x+1/2*c)^8*
```

```
cos(1/2*d*x+1/2*c)+(-34920*A*a^3-68376*A*a^2*b-35640*A*a*b^2-5544*A*b^3-227
92*B*a^3-35640*B*a^2*b-16632*B*a*b^2-11880*C*a^3-16632*C*a^2*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A*a^3+31416*A*a^2*b+27720*A*a*b^2+5544*
A*b^3+10472*B*a^3+27720*B*a^2*b+16632*B*a*b^2+4620*B*b^3+9240*C*a^3+16632*C
*a^2*b+13860*C*a*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2790*A*a^3-
5544*A*a^2*b-7920*A*a*b^2-1386*A*b^3-1848*B*a^3-7920*B*a^2*b-4158*B*a*b^2-2
310*B*b^3-2640*C*a^3-4158*C*a^2*b-6930*C*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+675*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2475*A*a*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-4851*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-2079*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*b^3+2475*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1155*B*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6237*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a*b^2+825*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3465*C*a*b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-6237*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3465*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos(dx+c)^5\sec(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\cos(dx+c)^5\sec(dx+c)^4 + Aa^3\cos(dx+c)^5 + \left(3\,Ca^2b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^5*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x
 + c)^5*sec(d*x + c)^4 + A*a^3*cos(d*x + c)^5 + (3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*cos(d*x + c)^5*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*cos(d*
x + c)^5*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c)^5*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*co
s(d*x + c)^(11/2), x)
```

3.1306 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3 \left(A + B\sec(c + dx) + C\right.$

Optimal. Leaf size=296

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2b(5A+7C)+5a^3B+21ab^2B+7b^3(A+3C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(7A+9C)+27a\right.}{15d}$$

```
[Out] (2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Elliptic
E[(c + d*x)/2, 2])/(15*d) + (2*(5*a^3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3*
a^2*b*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(8*A*b^3 + 15*a^3
*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63
*d) + (2*a*(24*A*b^2 + 99*a*b*B + 7*a^2*(7*A + 9*C))*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(315*d) + (2*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d
*x])^2*Sin[c + d*x])/(21*d) + (2*A*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^
3*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.910354, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(5A+7C)+5a^3B+21ab^2B+7b^3(A+3C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(7A+9C)+27a^2bB+9a\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(27*a^2*b*B + 15*b^3*B + 9*a*b^2*(3*A + 5*C) + a^3*(7*A + 9*C))*Elliptic
E[(c + d*x)/2, 2])/(15*d) + (2*(5*a^3*B + 21*a*b^2*B + 7*b^3*(A + 3*C) + 3*
a^2*b*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(8*A*b^3 + 15*a^3
*B + 54*a*b^2*B + 9*a^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63
*d) + (2*a*(24*A*b^2 + 99*a*b*B + 7*a^2*(7*A + 9*C))*Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(315*d) + (2*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d
*x])^2*Sin[c + d*x])/(21*d) + (2*A*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^
3*Sin[c + d*x])/(9*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^3 \text{si}}{9d}$$

$$= \frac{2(2Ab+3aB)\sqrt{\cos(c+dx)}(b+a\cos(}{21d}$$

$$= \frac{2a\left(24Ab^2+99abB+7a^2(7A+9C)\right)}{315d}$$

$$= \frac{2\left(8Ab^3+15a^3B+54ab^2B+9a^2b(5A\right.}{63d}$$

$$= \frac{2\left(8Ab^3+15a^3B+54ab^2B+9a^2b(5A\right.}{63d}$$

$$= \frac{2\left(27a^2bB+15b^3B+9ab^2(3A+5C)+\right.}{15d}$$

Mathematica [C] time = 7.27416, size = 3237, normalized size = 10.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((-4*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B + 9*a^3*C + 45*
a*b^2*C)*Cot[c])/(15*d) + ((69*a^2*A*b + 28*A*b^3 + 23*a^3*B + 84*a*b^2*B +
 84*a^2*b*C)*Cos[d*x]*Sin[c])/(21*d) + (a*(19*a^2*A + 54*A*b^2 + 54*a*b*B +
 18*a^2*C)*Cos[2*d*x]*Sin[2*c])/(45*d) + (a^2*(3*A*b + a*B)*Cos[3*d*x]*Sin[
3*c])/(7*d) + (a^3*A*Cos[4*d*x]*Sin[4*c])/(18*d) + ((69*a^2*A*b + 28*A*b^3
+ 23*a^3*B + 84*a*b^2*B + 84*a^2*b*C)*Cos[c]*Sin[d*x])/(21*d) + (a*(19*a^2*
A + 54*A*b^2 + 54*a*b*B + 18*a^2*C)*Cos[2*c]*Sin[2*d*x])/(45*d) + (a^2*(3*A
*b + a*B)*Cos[3*c]*Sin[3*d*x])/(7*d) + (a^3*A*Cos[4*c]*Sin[4*d*x])/(18*d)))
/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
 - (20*a^2*A*b*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, S
in[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(7*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*b^3*Cos[c + d*x]^5*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a
 + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d
*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*S
qrt[1 + Cot[c]^2]) - (20*a^3*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(b + a*Cos[c + d*x])^3*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a*b^
```

2*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^2*b*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^3*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (14*a^3*A*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (18*a*A*b^2*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (18*a^2*b*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (2*b^3*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (6*a^3*C*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (6*a*b^2*C*Cos[c +

```
d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x]))
```

Maple [B] time = 2.827, size = 975, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*a^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*A*a^3+2160*A*a^2*b+720*B*a^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^3-3240*A*a^2*b-1512*A
*a*b^2-1080*B*a^3-1512*B*a^2*b-504*C*a^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+(952*A*a^3+2520*A*a^2*b+1512*A*a*b^2+420*A*b^3+840*B*a^3+1512*B*a^2*
b+1260*B*a*b^2+504*C*a^3+1260*C*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-168*A*a^3-720*A*a^2*b-378*A*a*b^2-210*A*b^3-240*B*a^3-378*B*a^2*b-630
*B*a*b^2-126*C*a^3-630*C*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225
*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+75*B*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+315*B*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2*b-315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+315*a^2*b*C*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+315*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*a^3-945*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos(dx+c)^4\sec(dx+c)^5+\left(3\,Cab^2+Bb^3\right)\cos(dx+c)^4\sec(dx+c)^4+Aa^3\cos(dx+c)^4+\left(3\,Ca^2b+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^4*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x
 + c)^4*sec(d*x + c)^4 + A*a^3*cos(d*x + c)^4 + (3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*cos(d*x + c)^4*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*cos(d*
x + c)^4*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c)^4*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*co
s(d*x + c)^(9/2), x)
```

3.1307 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=277

$$\frac{2\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(3A+5C)+3\right.}{5d}$$

```
[Out] (2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(21*a^2*b*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^
3*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(21*a*b*B + 6*b^2*(
3*A - 7*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*
a^2*(11*A*b + 7*a*B - 35*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*
a*(A - 7*C)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) +
 (2*C*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.905624, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(5A+7C)+21a^2bB+21ab^2(A+3C)+21b^3B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(3A+5C)+3a^3B+1\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(3*a^3*B + 15*a*b^2*B + 5*b^3*(A - C) + 3*a^2*b*(3*A + 5*C))*EllipticE[(
c + d*x)/2, 2])/(5*d) + (2*(21*a^2*b*B + 21*b^3*B + 21*a*b^2*(A + 3*C) + a^
3*(5*A + 7*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(21*a*b*B + 6*b^2*(
3*A - 7*C) + a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*
a^2*(11*A*b + 7*a*B - 35*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*
a*(A - 7*C)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) +
 (2*C*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2$$

$$= \frac{2a(A-7C)\sqrt{\cos(c+dx)}(b+a\cos(c+}{7d}$$

$$= \frac{2a^2(11Ab+7aB-35bC)\cos^{\frac{3}{2}}(c+dx)}{35d}$$

$$= \frac{2a\left(21abB+6b^2(3A-7C)+a^2(5A+\right.}{21d}$$

$$= \frac{2a\left(21abB+6b^2(3A-7C)+a^2(5A+\right.}{21d}$$

$$= \frac{2\left(3a^3B+15ab^2B+5b^3(A-C)+3a^2\right.}{5d}$$

Mathematica [C] time = 8.07596, size = 3915, normalized size = 14.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (((9*I)/5)*a^2*A*b*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[
c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Co
s[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)
*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -
(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
 (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2
*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])) + (I*A*b^3*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c +
 d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (((3*I)/5)*a^3*B*Cos[c +
d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*
```

Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))
/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
 + ((3*I)*a*b^2*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c
 + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] +
(2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*
c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1
+ E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])) + ((3*I)*a^2*b*C*Cos[c + d*x]^5*Csc[c]*(a + b*Se
c[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^
3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (I*b^3*C*Cos[c + d*x
]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((
b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
(Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((-2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B + 15*a^2*b*C - 10
*b^3*C + 9*a^2*A*b*Cos[2*c] + 5*A*b^3*Cos[2*c] + 3*a^3*B*Cos[2*c] + 15*a*b^
2*B*Cos[2*c] + 15*a^2*b*C*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (a*(23*a^2*A + 8
4*A*b^2 + 84*a*b*B + 28*a^2*C)*Cos[d*x]*Sin[c])/(21*d) + (2*a^2*(3*A*b + a*
B)*Cos[2*d*x]*Sin[2*c])/(5*d) + (a^3*A*Cos[3*d*x]*Sin[3*c])/(7*d) + (a*(23*
a^2*A + 84*A*b^2 + 84*a*b*B + 28*a^2*C)*Cos[c]*Sin[d*x])/(21*d) + (4*b^3*C*
Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*a^2*(3*A*b + a*B)*Cos[2*c]*Sin[2*d*x])
/(5*d) + (a^3*A*Cos[3*c]*Sin[3*d*x])/(7*d)))/((b + a*Cos[c + d*x])^3*(A + 2
*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (20*a^3*A*Cos[c + d*x]^5*Csc
[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a +
b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(
b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[1 + Cot[c]^2]) - (4*a*A*b^2*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^2*b*B*
Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)

```
*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[
1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[
Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^3*B*Cos[c + d*x]^5*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x]
)^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x
])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
- (4*a^3*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (12*a*b^2*C*Cos[c + d*x]^5*C
sc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a
+ b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b
 + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt
[1 + Cot[c]^2])
```

Maple [B] time = 3.283, size = 1278, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/105*(240*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^2*(15*A*a+21*A*b+7*B*a)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+28*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(10*A*a^2
+18*A*a*b+15*A*b^2+6*B*a^2+15*B*a*b+5*C*a^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(40*A*a^3+
63*A*a^2*b+105*A*a*b^2+21*B*a^3+105*B*a^2*b+35*C*a^3+105*C*b^3)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-105*A*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+25*A*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+105*A*
a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)-63*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a^3-315*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+105*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+105*B*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-315*C*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
```

```
2*b+105*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*b^3+35*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+315*C*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^3\sec\left(dx+c\right)^4+Aa^3\cos\left(dx+c\right)^3+\left(3\,Ca^2b+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x
 + c)^3*sec(d*x + c)^4 + A*a^3*cos(d*x + c)^3 + (3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*cos(d*x + c)^3*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*cos(d*
x + c)^3*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c)^3*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*co
s(d*x + c)^(7/2), x)
```

3.1308 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=267

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(3a^2b(A+3C)+a^3B+9ab^2B+b^3(3A+C)\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(3A+5C)+15a^2bB+15\right.}{5d}$$

```
[Out] (2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*EllipticE[(c
 + d*x)/2, 2])/(5*d) + (2*(a^3*B + 9*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A +
 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(a^2*B - 6*b^2*B + 3*a*b*(A
- 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*(3*a*A - 15*b*B - 3
5*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(b*B + 2*a*C)*(b + a*Co
s[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*C*(b + a*Cos[c + d*
x])^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.87886, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2b(A+3C)+a^3B+9ab^2B+b^3(3A+C)\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(3A+5C)+15a^2bB+15ab^2(A-\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(15*a^2*b*B - 5*b^3*B + 15*a*b^2*(A - C) + a^3*(3*A + 5*C))*EllipticE[(c
 + d*x)/2, 2])/(5*d) + (2*(a^3*B + 9*a*b^2*B + b^3*(3*A + C) + 3*a^2*b*(A +
 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(a^2*B - 6*b^2*B + 3*a*b*(A
- 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*(3*a*A - 15*b*B - 3
5*a*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(b*B + 2*a*C)*(b + a*Co
s[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*C*(b + a*Cos[c + d*
x])^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^3\left(C+B\cos(c+dx\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int$$

$$= \frac{2(bB+2aC)(b+a\cos(c+dx))^2\sin(c+d}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2(3aA-15bB-35aC)\cos^{\frac{3}{2}}(c+dx)\sin}{15d}$$

$$= \frac{2a\left(a^2B-6b^2B+3ab(A-5C)\right)\sqrt{\cos(c+}}{3d}$$

$$= \frac{2a\left(a^2B-6b^2B+3ab(A-5C)\right)\sqrt{\cos(c+}}{3d}$$

$$= \frac{2\left(15a^2bB-5b^3B+15ab^2(A-C)+a^3(3A\right.}{5d}$$

Mathematica [C] time = 8.29387, size = 3868, normalized size = 14.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (((3*I)/5)*a^3*A*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (
2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c
] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
 E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) + ((3*I)*a*A*b^2*Cos[c + d*x]^5*Csc[c]*(a + b*Sec
[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((3*I)*a^2*b*B*Cos[c
+ d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]
^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
```

x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x)
)*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin
[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])
))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)) - (I*b^3*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4,
-(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
+ (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[
2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d
*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((
2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I
)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
 I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^
((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])) + (I*a^3*C*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x]
)^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometri
c2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I
)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1
/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)
*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((
2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))
Cos[c] + d(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((3*I)*a*b^2*C*Cos[c + d*x]^5*
Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b +
a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos
[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*((-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 10*b^3*B + 5*a^3*C - 30*a*b^
2*C + 3*a^3*A*Cos[2*c] + 15*a*A*b^2*Cos[2*c] + 15*a^2*b*B*Cos[2*c] + 5*a^3*
C*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*a^2*(3*A*b + a*B)*Cos[d*x]*Sin[c])/(3
*d) + (2*a^3*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*a^2*(3*A*b + a*B)*Cos[c]*Sin
[d*x])/(3*d) + (4*b^3*C*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*S
ec[c + d*x]*(b^3*C*Sin[c] + 3*b^3*B*Sin[d*x] + 9*a*b^2*C*Sin[d*x]))/(3*d) +
 (2*a^3*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2
*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*a^2*A*b*Cos[c + d*x]^5*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec
[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Co
s[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]) - (4*A*b^3*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^3*B*Cos[c + d*x]
^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - A

```
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*Sqrt[1 + Cot[c]^2]) - (12*a*b^2*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (12*a
^2*b*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^3*C*Cos[c + d*x]^5*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c
 + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Co
s[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2])
```

Maple [B] time = 8.522, size = 1837, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] 2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(30*A*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-40*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^4-60*B*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*A*a^3*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+10*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2+30*B*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+10*C*b^3*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^8+72*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+40*B*a^3*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4-5*C*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+120*
A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-120*A*a^2*b*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4-180*C*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
4+30*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+90*C*a*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a^3-5*B*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^3-18*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+10*B*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+30*B*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)*b^3*sin(1/2*d*x+1/2*c)^2+10*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1
/2*c)^2-30*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-15*A*a^2*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-45*B*a*b^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-45*a^2*b*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-45*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-90*B*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+90*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin
(1/2*d*x+1/2*c)^2+90*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+
30*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-90*A*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+90*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*
sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^3\cos\left(dx+c\right)^2\sec\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4+Aa^3\cos\left(dx+c\right)^2+\left(3\,Ca^2b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)^2*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x
 + c)^2*sec(d*x + c)^4 + A*a^3*cos(d*x + c)^2 + (3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*cos(d*x + c)^2*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*cos(d*
x + c)^2*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c)^2*sec(d*x + c))*
sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*co
s(d*x + c)^(5/2), x)
```

3.1309 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=274

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(15a^2b(A-C)+5a^3B-1\right.}{5d}$$

```
[Out] (2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*EllipticE[(c
 + d*x)/2, 2])/(5*d) + (2*(9*a^2*b*B + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A +
 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(15*A*b^2 + 35*a*b*B + 24*a^
2*C + 9*b^2*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(5*a*A - 5*
b*B - 9*a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(5*b*B + 6*a*C)*(
b + a*Cos[c + d*x])^2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*C*(b + a
*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.87114, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^3(A+3C)+9a^2bB+3ab^2(3A+C)+b^3B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(15a^2b(A-C)+5a^3B-15ab^2B\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(5*a^3*B - 15*a*b^2*B + 15*a^2*b*(A - C) - b^3*(5*A + 3*C))*EllipticE[(c
 + d*x)/2, 2])/(5*d) + (2*(9*a^2*b*B + b^3*B + 3*a*b^2*(3*A + C) + a^3*(A +
 3*C))*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(15*A*b^2 + 35*a*b*B + 24*a^
2*C + 9*b^2*C)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(5*a*A - 5*
b*B - 9*a*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(5*b*B + 6*a*C)*(
b + a*Cos[c + d*x])^2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*C*(b + a
*Cos[c + d*x])^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^3\left(C+B\cos(c+\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}$$

$$= \frac{2(5bB+6aC)(b+a\cos(c+dx))^2\sin(}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(15Ab^2+35abB+24a^2C+9b^2C\right)}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{2b\left(15Ab^2+35abB+24a^2C+9b^2C\right)}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{2b\left(15Ab^2+35abB+24a^2C+9b^2C\right)}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{2\left(5a^3B-15ab^2B+15a^2b(A-C)-b^3\right.}{5d}$$

Mathematica [C] time = 8.40216, size = 3871, normalized size = 14.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((3*I)*a^2*A*b*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*
d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*
I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
+ I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E
^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])) - (I*A*b^3*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x
])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*
I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-
1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (I*a^3*B*Cos[c + d*x]^5*Csc[c
]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*
I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
```

((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2
*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*
Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(
I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*
d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos
[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((3*I)*a*
b^2*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*
I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*
(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I
*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E
^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d
*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) - ((3*I)*a^2*b*C*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^
3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C +
 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (((3*I)/5)*b^3*C*Cos[c + d*x]^5*
Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b +
a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos
[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*((-2*(15*a^2*A*b - 10*A*b^3 + 5*a^3*B - 30*a*b^2*B - 30*a^2*b*C - 6*b^
3*C + 15*a^2*A*b*Cos[2*c] + 5*a^3*B*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*a^3
*A*Cos[d*x]*Sin[c])/(3*d) + (4*a^3*A*Cos[c]*Sin[d*x])/(3*d) + (4*b^3*C*Sec[
c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]^2*(3*b^3*C*Sin[c
] + 5*b^3*B*Sin[d*x] + 15*a*b^2*C*Sin[d*x]))/(15*d) + (4*Sec[c]*Sec[c + d*x
]*(5*b^3*B*Sin[c] + 15*a*b^2*C*Sin[c] + 15*A*b^3*Sin[d*x] + 45*a*b^2*B*Sin[
d*x] + 45*a^2*b*C*Sin[d*x] + 9*b^3*C*Sin[d*x]))/(15*d)))/((b + a*Cos[c + d*
x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*a^3*A*Cos[c +
 d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*Sqrt[1 + Cot[c]^2]) - (12*a*A*b^2*Cos[c + d*x]^5*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^
3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])
^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) -
(12*a^2*b*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec

```
[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^3*B*Cos[c + d*x]^5*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*
Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b +
 a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
 + Cot[c]^2]) - (4*a^3*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a*b^2*C*Cos[c
 + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 9.362, size = 1419, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*A*a^3*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-4*A*a^3+6*A*a^2*b+2*B*a^3)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*A*a^2*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a*b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2
*B*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+6*B*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+2*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*C*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*s
in(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1
```

```
/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*b^2*(B*b+3*C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b^2+3*B*a
*b+3*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^3\cos(dx+c)\sec(dx+c)^5+\left(3\,Cab^2+Bb^3\right)\cos(dx+c)\sec(dx+c)^4+Aa^3\cos(dx+c)+\left(3\,Ca^2b+3\,Ba\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*cos(d*x + c)*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*cos(d*x +
 c)*sec(d*x + c)^4 + A*a^3*cos(d*x + c) + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c
os(d*x + c)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*cos(d*x + c)*s
ec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c)*sec(d*x + c))*sqrt(cos(d*x
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*co
s(d*x + c)^(3/2), x)
```

3.1310 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=294

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^3(A-C)+15a^2b\right.}{5d}$$

```
[Out] (-2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*EllipticE[
(c + d*x)/2, 2])/(5*d) + (2*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b
^3*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(35*A*b^2 + 63*a*b
*B + 24*a^2*C + 25*b^2*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(98
*a^2*b*B + 21*b^3*B + 24*a^3*C + 21*a*b^2*(5*A + 3*C))*Sin[c + d*x])/(35*d*
Sqrt[Cos[c + d*x]]) + (2*(7*b*B + 6*a*C)*(b + a*Cos[c + d*x])^2*Sin[c + d*x
])/(35*d*Cos[c + d*x]^(5/2)) + (2*C*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(7
*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.895941, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(21a^2b(3A+C)+21a^3B+21ab^2B+b^3(7A+5C)\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-5a^3(A-C)+15a^2bB+3ab^2\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-2*(15*a^2*b*B + 3*b^3*B - 5*a^3*(A - C) + 3*a*b^2*(5*A + 3*C))*EllipticE[
(c + d*x)/2, 2])/(5*d) + (2*(21*a^3*B + 21*a*b^2*B + 21*a^2*b*(3*A + C) + b
^3*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(35*A*b^2 + 63*a*b
*B + 24*a^2*C + 25*b^2*C)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(98
*a^2*b*B + 21*b^3*B + 24*a^3*C + 21*a*b^2*(5*A + 3*C))*Sin[c + d*x])/(35*d*
Sqrt[Cos[c + d*x]]) + (2*(7*b*B + 6*a*C)*(b + a*Cos[c + d*x])^2*Sin[c + d*x
])/(35*d*Cos[c + d*x]^(5/2)) + (2*C*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(7
*d*Cos[c + d*x]^(7/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \int \frac{(b+a\cos(c+dx))^3 \left(C+B\cos(c+dx\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int$$

$$= \frac{2(7bB+6aC)(b+a\cos(c+dx))^2\sin(c+}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(35Ab^2+63abB+24a^2C+25b^2C\right)\mathrm{si}}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(35Ab^2+63abB+24a^2C+25b^2C\right)\mathrm{si}}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\left(35Ab^2+63abB+24a^2C+25b^2C\right)\mathrm{si}}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\left(15a^2bB+3b^3B-5a^3(A-C)+3ab^2\right.}{5d}$$

Mathematica [C] time = 8.49002, size = 3933, normalized size = 13.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (I*a^3*A*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*
I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
+ I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1
+ E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I
)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*C
os[2*c + 2*d*x])) - ((3*I)*a*A*b^2*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x
])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*
I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-
1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I
)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^(
(2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x)
)*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((3*I)*a^2*b*B*Cos[c + d*x]^5
*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
```

```
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b +
 a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (((
3*I)/5)*b^3*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4,
-(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
+ (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[
2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d
*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((
2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I
)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
 I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^
((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])) - (I*a^3*C*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x]
)^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometri
c2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I
)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1
/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)
*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((
2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))
*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^3*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (((9*I)/5)*a*b^2*C*Cos[c + d*x
]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((
b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
(Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((-2*(5*a^3*A - 30*a*A*b^2 - 30*a^2*b*B - 6*b^3*B - 10*a^3*C - 18*
a*b^2*C + 5*a^3*A*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*b^3*C*Sec[c]*Sec[c +
d*x]^4*Sin[d*x])/(7*d) + (4*Sec[c]*Sec[c + d*x]^3*(5*b^3*C*Sin[c] + 7*b^3*B
*Sin[d*x] + 21*a*b^2*C*Sin[d*x]))/(35*d) + (4*Sec[c]*Sec[c + d*x]*(35*A*b^3
*Sin[c] + 105*a*b^2*B*Sin[c] + 105*a^2*b*C*Sin[c] + 25*b^3*C*Sin[c] + 315*a
*A*b^2*Sin[d*x] + 315*a^2*b*B*Sin[d*x] + 63*b^3*B*Sin[d*x] + 105*a^3*C*Sin[
d*x] + 189*a*b^2*C*Sin[d*x]))/(105*d) + (4*Sec[c]*Sec[c + d*x]^2*(21*b^3*B*
Sin[c] + 63*a*b^2*C*Sin[c] + 35*A*b^3*Sin[d*x] + 105*a*b^2*B*Sin[d*x] + 105
*a^2*b*C*Sin[d*x] + 25*b^3*C*Sin[d*x]))/(105*d)))/((b + a*Cos[c + d*x])^3*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (12*a^2*A*b*Cos[c + d*x
]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sqrt[1 + Cot[c]^2]) - (4*A*b^3*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*
Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^3*(A +
 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^3*
```

```
B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a*b^2*B*Cos[c + d*x]^5*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c +
d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c +
 d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^
2]) - (4*a^2*b*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (20*b^3*C*Cos[c + d*x]^5
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(
a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21
*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 10.504, size = 1205, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a^2*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
2/5*b^2*(B*b+3*C*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*C*b^3*(-1/56*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/
2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b^2+3*B*a*b+3
*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
```

```
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a*(3*A*b^2+3*B*a*b+C*a^2
)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb^3\sec\left(dx+c\right)^5+\left(3\,Cab^2+Bb^3\right)\sec\left(dx+c\right)^4+Aa^3+\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)\sec\left(dx+c\right)^3+\left(Ca^3+3\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^3\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sq
rt(cos(d*x + c)), x)
```

3.1311 $\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=357

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(7a^3(3A+C)+21a^2bB+3ab^2(7A+5C)+5b^3B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2b(5A+3C)+15\right.}{15d}$$

```
[Out] (-2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3
*a*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(63*A*b^2 + 99
*a*b*B + 24*a^2*C + 49*b^2*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2
*(54*a^2*b*B + 15*b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*
d*Cos[c + d*x]^(3/2)) + (2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b
^3*(9*A + 7*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(3*b*B + 2*a*C
)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*C*(b
+ a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rubi [A] time = 0.974086, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3031, 3021, 2748, 2636, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(7a^3(3A+C)+21a^2bB+3ab^2(7A+5C)+5b^3B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(9a^2b(5A+3C)+15a^3B+2\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```


```
[Out] (-2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b^3*(9*A + 7*C))*Ellipti
cE[(c + d*x)/2, 2])/(15*d) + (2*(21*a^2*b*B + 5*b^3*B + 7*a^3*(3*A + C) + 3
*a*b^2*(7*A + 5*C))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(63*A*b^2 + 99
*a*b*B + 24*a^2*C + 49*b^2*C)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2
*(54*a^2*b*B + 15*b^3*B + 8*a^3*C + 9*a*b^2*(7*A + 5*C))*Sin[c + d*x])/(63*
d*Cos[c + d*x]^(3/2)) + (2*(15*a^3*B + 27*a*b^2*B + 9*a^2*b*(5*A + 3*C) + b
^3*(9*A + 7*C))*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(3*b*B + 2*a*C
)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*C*(b
+ a*Cos[c + d*x])^3*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \int \frac{(b+a\cos(c+dx))^3\left(C+B\cos(c+dx)+A\cos^2\right.}{\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^3\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(b+a\,\mathrm{co}}{}$$

$$= \frac{2(3bB+2aC)(b+a\cos(c+dx))^2\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2C}{}$$

$$= \frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(63Ab^2+99abB+24a^2C+49b^2C\right)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(21a^2bB+5b^3B+7a^3(3A+C)+3ab^2(7A+5C)\right.}{21d}$$

$$= -\frac{2\left(15a^3B+27ab^2B+9a^2b(5A+3C)+b^3(9A+7\right.}{15d}$$

Mathematica [C] time = 8.2908, size = 3345, normalized size = 9.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((4*(45*a^2*A*b + 9*A*b^3 + 15*a^3*B + 27*a*b^2*B + 27*a^2*b*C + 7
*b^3*C)*Csc[c]*Sec[c])/(15*d) + (4*b^3*C*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(9
*d) + (4*Sec[c]*Sec[c + d*x]^4*(7*b^3*C*Sin[c] + 9*b^3*B*Sin[d*x] + 27*a*b^
2*C*Sin[d*x]))/(63*d) + (4*Sec[c]*Sec[c + d*x]^2*(63*A*b^3*Sin[c] + 189*a*b
^2*B*Sin[c] + 189*a^2*b*C*Sin[c] + 49*b^3*C*Sin[c] + 315*a*A*b^2*Sin[d*x] +
 315*a^2*b*B*Sin[d*x] + 75*b^3*B*Sin[d*x] + 105*a^3*C*Sin[d*x] + 225*a*b^2*
C*Sin[d*x]))/(315*d) + (4*Sec[c]*Sec[c + d*x]^3*(45*b^3*B*Sin[c] + 135*a*b^
2*C*Sin[c] + 63*A*b^3*Sin[d*x] + 189*a*b^2*B*Sin[d*x] + 189*a^2*b*C*Sin[d*x
] + 49*b^3*C*Sin[d*x]))/(315*d) + (4*Sec[c]*Sec[c + d*x]*(105*a*A*b^2*Sin[c
] + 105*a^2*b*B*Sin[c] + 25*b^3*B*Sin[c] + 35*a^3*C*Sin[c] + 75*a*b^2*C*Sin
[c] + 315*a^2*A*b*Sin[d*x] + 63*A*b^3*Sin[d*x] + 105*a^3*B*Sin[d*x] + 189*a
*b^2*B*Sin[d*x] + 189*a^2*b*C*Sin[d*x] + 49*b^3*C*Sin[d*x]))/(105*d)))/((b
+ a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4
*a^3*A*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a*A*b^2*Cos[c + d*x]^5*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec
[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
```

]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Co
s[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]) - (4*a^2*b*B*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*C
os[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (20*b^3*B*Cos[c + d
*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(21*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^3*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A
 + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^3
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2
0*a*b^2*C*Cos[c + d*x]^5*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(7*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (6*a^2*A*b*Cos[c + d*x]^5*Csc
[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*
(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
 (6*A*b^3*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]
+ C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + (2*a^3*B*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c
+ d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*
x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (18*a*b^2*B*Cos[
c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*
x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[

```
2*c + 2*d*x])) + (18*a^2*b*C*Cos[c + d*x]^5*Csc[c]*(a + b*Sec[c + d*x])^3*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^3*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (14*b^3*C*Cos[c + d*x]^5*
Csc[c]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(15*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x]))
```

Maple [B] time = 13.236, size = 1292, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
^2*(B*b+3*C*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))+2*a*(3*A*b^2+3*B*a*b+C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+2*C*b^3*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*
c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))))-2/5*b*(A*b^2+3*B*a*b+3*C*a^2)/(8*sin(1/2*d*x+1/2*c)^6-1
2*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)+2*a^2*(3*A*b+B*a)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Cb^3\sec(dx+c)^5 + \left(3\,Cab^2 + Bb^3\right)\sec(dx+c)^4 + Aa^3 + \left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)\sec(dx+c)^3 + \left(Ca^3 + 3\,Ba\right.}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^3*sec(d*x + c)^5 + (3*C*a*b^2 + B*b^3)*sec(d*x + c)^4 + A*a^3
 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*sec(d*x + c)^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))/sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c) + a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sq
rt(cos(d*x + c)), x)
```

3.1312 $\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=404

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A+3C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\right|}{}$$

```
[Out] (2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*A
 + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(220*a^3*b*B + 308*a*b^3*B
+ 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*EllipticF
[(c + d*x)/2, 2])/(231*d) + (2*(64*A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a
^4*(9*A + 11*C) + 9*a^2*b^2*(101*A + 143*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(693*d) + (2*a*(192*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2*a^2*b*(673*A +
891*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d) + (2*(16*A*b^2 + 55*a*b*B
 + 3*a^2*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c + d*
x])/(231*d) + (2*(8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^3
*Sin[c + d*x])/(99*d) + (2*A*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^4*Sin[
c + d*x])/(11*d)
```

Rubi [A] time = 1.32034, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A+3C)\right)}{231d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2),x]
```


```
[Out] (2*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7*A
 + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(220*a^3*b*B + 308*a*b^3*B
+ 77*b^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*EllipticF
[(c + d*x)/2, 2])/(231*d) + (2*(64*A*b^4 + 660*a^3*b*B + 682*a*b^3*B + 15*a
^4*(9*A + 11*C) + 9*a^2*b^2*(101*A + 143*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(693*d) + (2*a*(192*A*b^3 + 539*a^3*B + 1353*a*b^2*B + 2*a^2*b*(673*A +
891*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3465*d) + (2*(16*A*b^2 + 55*a*b*B
 + 3*a^2*(9*A + 11*C))*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c + d*
x])/(231*d) + (2*(8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^3
*Sin[c + d*x])/(99*d) + (2*A*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^4*Sin[
c + d*x])/(11*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^4\left(C+B\cos(c+dx \ldots\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^4\sin(\ldots}{11d}$$

$$= \frac{2(8Ab+11aB)\sqrt{\cos(c+dx)}(b+a\cos(c \ldots}{99d}$$

$$= \frac{2\left(16Ab^2+55abB+3a^2(9A+11C)\right)\sqrt{c \ldots}}{23 \ldots}$$

$$= \frac{2a\left(192Ab^3+539a^3B+1353ab^2B+2a^2 \ldots\right.}{34 \ldots}$$

$$= \frac{2\left(64Ab^4+660a^3bB+682ab^3B+15a^4(\ldots\right.}{\ldots}$$

$$= \frac{2\left(64Ab^4+660a^3bB+682ab^3B+15a^4(\ldots\right.}{\ldots}$$

$$= \frac{2\left(7a^4B+54a^2b^2B+15b^4B+12ab^3(3A \ldots\right.}{15 \ldots}$$

Mathematica [A] time = 2.51623, size = 320, normalized size = 0.79

$$\frac{10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(66a^2b^2(5A+7C)+5a^4(9A+11C)+220a^3bB+308ab^3B+77b^4(A+3C)\right)+154E\left(\frac{1}{2}(c+d \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2),x]
```

```
[Out] (154*(7*a^4*B + 54*a^2*b^2*B + 15*b^4*B + 12*a*b^3*(3*A + 5*C) + 4*a^3*b*(7
*A + 9*C))*EllipticE[(c + d*x)/2, 2] + 10*(220*a^3*b*B + 308*a*b^3*B + 77*b
^4*(A + 3*C) + 66*a^2*b^2*(5*A + 7*C) + 5*a^4*(9*A + 11*C))*EllipticF[(c +
d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(154*a*(144*A*b^3 + 43*a^3*B + 216*a*b^2*B
 + 4*a^2*b*(43*A + 36*C))*Cos[c + d*x] + 5*(36*a^2*(66*A*b^2 + 44*a*b*B + a
^2*(16*A + 11*C))*Cos[2*(c + d*x)] + 154*a^3*(4*A*b + a*B)*Cos[3*(c + d*x)]
 + 3*(616*A*b^4 + 2288*a^3*b*B + 2464*a*b^3*B + 264*a^2*b^2*(13*A + 14*C) +
 a^4*(531*A + 572*C) + 21*a^4*A*Cos[4*(c + d*x)])))*Sin[c + d*x])/12)/(1155
*d)
```

Maple [B] time = 2.933, size = 1273, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*a^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*A*a^4-49280*A*a^3*b-1232
```

```
0*B*a^4)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^4+98560*A*a^3*
b+47520*A*a^2*b^2+24640*B*a^4+31680*B*a^3*b+7920*C*a^4)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-34920*A*a^4-91168*A*a^3*b-71280*A*a^2*b^2-22176*A*a*
b^3-22792*B*a^4-47520*B*a^3*b-33264*B*a^2*b^2-11880*C*a^4-22176*C*a^3*b)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A*a^4+41888*A*a^3*b+55440*A*a^
2*b^2+22176*A*a*b^3+4620*A*b^4+10472*B*a^4+36960*B*a^3*b+33264*B*a^2*b^2+18
480*B*a*b^3+9240*C*a^4+22176*C*a^3*b+27720*C*a^2*b^2)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(-2790*A*a^4-7392*A*a^3*b-15840*A*a^2*b^2-5544*A*a*b^3-2
310*A*b^4-1848*B*a^4-10560*B*a^3*b-8316*B*a^2*b^2-9240*B*a*b^3-2640*C*a^4-5
544*C*a^3*b-13860*C*a^2*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-6468*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+675*A
*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+4950*A*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1155*
A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1617*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12474*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-3465*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+3300*B
*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+4620*a*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8316*
C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^3*b-13860*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+825
*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+6930*C*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+346
5*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^5\sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^5\sec(dx+c)^5 + Aa^4\cos(dx+c)^5 + \left(6\,Ca^2b^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^5*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x
 + c)^5*sec(d*x + c)^5 + A*a^4*cos(d*x + c)^5 + (6*C*a^2*b^2 + 4*B*a*b^3 +
A*b^4)*cos(d*x + c)^5*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^2*b^2 + 2*A*a*b
^3)*cos(d*x + c)^5*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^5*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c)^5*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+
c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{4}\cos\left(dx+c\right)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)
^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*co
s(d*x + c)^(11/2), x)
```

3.1313 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=377

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(4a^3b(5A+7C)+42a^2b^2B+5a^4B+28ab^3(A+3C)+21b^4B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2\right.}{}$$

```
[Out] (2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4
*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2*
B + 21*b^4*B + 28*a*b^3*(A + 3*C) + 4*a^3*b*(5*A + 7*C))*EllipticF[(c + d*x
)/2, 2])/(21*d) + (2*a*(15*a^3*B + 117*a*b^2*B + 2*b^3*(31*A - 63*C) + 12*a
^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63*d) + (2*a^2*(162*a*b
*B + 3*b^2*(41*A - 105*C) + 7*a^2*(7*A + 9*C))*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(315*d) + (2*a*(5*A*b + 3*a*B - 21*b*C)*Sqrt[Cos[c + d*x]]*(b + a*Cos[
c + d*x])^2*Sin[c + d*x])/(21*d) + (2*a*(A - 9*C)*Sqrt[Cos[c + d*x]]*(b + a
*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.30519, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(5A+7C)+42a^2b^2B+5a^4B+28ab^3(A+3C)+21b^4B\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(3A+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(36*a^3*b*B + 60*a*b^3*B + 15*b^4*(A - C) + 18*a^2*b^2*(3*A + 5*C) + a^4
*(7*A + 9*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(5*a^4*B + 42*a^2*b^2*
B + 21*b^4*B + 28*a*b^3*(A + 3*C) + 4*a^3*b*(5*A + 7*C))*EllipticF[(c + d*x
)/2, 2])/(21*d) + (2*a*(15*a^3*B + 117*a*b^2*B + 2*b^3*(31*A - 63*C) + 12*a
^2*b*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(63*d) + (2*a^2*(162*a*b
*B + 3*b^2*(41*A - 105*C) + 7*a^2*(7*A + 9*C))*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(315*d) + (2*a*(5*A*b + 3*a*B - 21*b*C)*Sqrt[Cos[c + d*x]]*(b + a*Cos[
c + d*x])^2*Sin[c + d*x])/(21*d) + (2*a*(A - 9*C)*Sqrt[Cos[c + d*x]]*(b + a
*Cos[c + d*x])^3*Sin[c + d*x])/(9*d) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2$$

$$= \frac{2a(A-9C)\sqrt{\cos(c+dx)}(b+a\cos(c+}{9d}$$

$$= \frac{2a(5Ab+3aB-21bC)\sqrt{\cos(c+dx)}(b}{21d}$$

$$= \frac{2a^2\left(162abB+3b^2(41A-105C)+7a^2\right.}{315}$$

$$= \frac{2a\left(15a^3B+117ab^2B+2b^3(31A-63C\right.}{}$$

$$= \frac{2a\left(15a^3B+117ab^2B+2b^3(31A-63C\right.}{}$$

$$= \frac{2\left(36a^3bB+60ab^3B+15b^4(A-C)+\right.}{}$$

Mathematica [C] time = 8.41413, size = 4114, normalized size = 10.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((-2*(7*a^4*A + 54*a^2*A*b^2 + 15*A*b^4 + 36*a^3*b*B + 60*a*b^3*B
+ 9*a^4*C + 90*a^2*b^2*C - 30*b^4*C + 7*a^4*A*Cos[2*c] + 54*a^2*A*b^2*Cos[2
*c] + 15*A*b^4*Cos[2*c] + 36*a^3*b*B*Cos[2*c] + 60*a*b^3*B*Cos[2*c] + 9*a^4
*C*Cos[2*c] + 90*a^2*b^2*C*Cos[2*c])*Csc[c]*Sec[c])/(15*d) + (a*(92*a^2*A*b
 + 112*A*b^3 + 23*a^3*B + 168*a*b^2*B + 112*a^2*b*C)*Cos[d*x]*Sin[c])/(21*d
) + (a^2*(19*a^2*A + 108*A*b^2 + 72*a*b*B + 18*a^2*C)*Cos[2*d*x]*Sin[2*c])/
(45*d) + (a^3*(4*A*b + a*B)*Cos[3*d*x]*Sin[3*c])/(7*d) + (a^4*A*Cos[4*d*x]*
Sin[4*c])/(18*d) + (a*(92*a^2*A*b + 112*A*b^3 + 23*a^3*B + 168*a*b^2*B + 11
2*a^2*b*C)*Cos[c]*Sin[d*x])/(21*d) + (4*b^4*C*Sec[c]*Sec[c + d*x]*Sin[d*x])
/d + (a^2*(19*a^2*A + 108*A*b^2 + 72*a*b*B + 18*a^2*C)*Cos[2*c]*Sin[2*d*x])
/(45*d) + (a^3*(4*A*b + a*B)*Cos[3*c]*Sin[3*d*x])/(7*d) + (a^4*A*Cos[4*c]*S
in[4*d*x])/(18*d)))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A
*Cos[2*c + 2*d*x])) - (80*a^3*A*b*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A +
B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(b + a*Cos[c + d*x])^4*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*
a*A*b^3*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
```

ArcTan[Cot[c]]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (20*a^4*B*Cos[c + d*x]^6*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*S
ec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(b +
a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2]) - (8*a^2*b^2*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C
+ 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*b^4*B*Cos
[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*a^3*b*C*Cos[c + d*x]^6*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d
x])^4(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]
) - (16*a*b^3*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c +
d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (14*a^4*A*Cos[c + d*x]^6*
Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(15*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])) - (36*a^2*A*b^2*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^4*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (2*A*b^4*Cos[c + d*x]^6*Csc[c]*(a
+ b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a
*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (24*a
^3*b*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[
Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta

```
n[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) - (8*a*b^3*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c +
 d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*x
])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (6*a^4*C*Cos[c +
d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
+ Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[
1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(
Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(5*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c
+ 2*d*x])) - (12*a^2*b^2*C*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]
*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*x])^4*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (2*b^4*C*Cos[c + d*x]^6*Csc[c
]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b
 + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 3.588, size = 1652, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -2/315*(-1120*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+80*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^3*(28*A*a+36*A*b+9*B*a)*sin(1/2*d*x+1/2*c)^8*cos(1/2*
d*x+1/2*c)-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(259*
A*a^2+540*A*a*b+378*A*b^2+135*B*a^2+252*B*a*b+63*C*a^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+56*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*a*(17*A*a^3+60*A*a^2*b+54*A*a*b^2+30*A*b^3+15*B*a^3+36*B*a^2*b+45*B*a*b^2
+9*C*a^3+30*C*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*A*a^4+160*A*a^3*b+126*A*a^2*b^
2+140*A*a*b^3+40*B*a^4+84*B*a^3*b+210*B*a^2*b^2+21*C*a^4+140*C*a^3*b+105*C*
```

```
b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*A*a^3*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+420*A*a*b^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-
147*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a^4-1134*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2*b^2-315*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+75*B*a^4*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+630*a^2*b^2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+31
5*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)-756*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a^3*b-1260*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+420*a^3*b*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1260*C*a*b^3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-18
9*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^4-1890*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^2+315*C*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^4\sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^4\sec(dx+c)^5 + Aa^4\cos(dx+c)^4 + \left(6\,Ca^2b^2 + 4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x
 + c)^4*sec(d*x + c)^5 + A*a^4*cos(d*x + c)^4 + (6*C*a^2*b^2 + 4*B*a*b^3 +
A*b^4)*cos(d*x + c)^4*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^2*b^2 + 2*A*a*b
^3)*cos(d*x + c)^4*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^4*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c)^4*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{4}\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*co
s(d*x + c)^(9/2), x)
```

3.1314 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=371

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(42a^2b^2(A+3C)+a^4(5A+7C)+28a^3bB+84ab^3B+7b^4(3A+C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3\right.}{}$$

```
[Out] (2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5*
C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*
(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a*(28*a^2*b*B - 42*b^3*B + 3*a*b^2*(13*A - 49*C) + a^3*(5*A
 + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a^2*(54*a*A*b + 21*a^
2*B - 105*b^2*B - 350*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*
a*(a*A - 7*b*B - 21*a*C)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c +
d*x])/(7*d) + (2*(3*b*B + 8*a*C)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c + d*x])/(3*d*Cos[c
+ d*x]^(3/2))
```

Rubi [A] time = 1.29516, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3049, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(A+3C)+a^4(5A+7C)+28a^3bB+84ab^3B+7b^4(3A+C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(3A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(3*a^4*B + 30*a^2*b^2*B - 5*b^4*B + 20*a*b^3*(A - C) + 4*a^3*b*(3*A + 5*
C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(28*a^3*b*B + 84*a*b^3*B + 7*b^4*
(3*A + C) + 42*a^2*b^2*(A + 3*C) + a^4*(5*A + 7*C))*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a*(28*a^2*b*B - 42*b^3*B + 3*a*b^2*(13*A - 49*C) + a^3*(5*A
 + 7*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a^2*(54*a*A*b + 21*a^
2*B - 105*b^2*B - 350*a*b*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*
a*(a*A - 7*b*B - 21*a*C)*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*Sin[c +
d*x])/(7*d) + (2*(3*b*B + 8*a*C)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c + d*x])/(3*d*Cos[c
+ d*x]^(3/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^4\left(C+B\cos(c+dx\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int$$

$$= \frac{2(3bB+8aC)(b+a\cos(c+dx))^3\sin(c+}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2a(7bB-a(A-21C))\sqrt{\cos(c+dx)}(b+}{7d}$$

$$= \frac{2a^2\left(54aAb+21a^2B-105b^2B-350abC\right)}{105d}$$

$$= \frac{2a\left(28a^2bB-42b^3B+3ab^2(13A-49C)+\right.}{21}$$

$$= \frac{2a\left(28a^2bB-42b^3B+3ab^2(13A-49C)+\right.}{21}$$

$$= \frac{2\left(3a^4B+30a^2b^2B-5b^4B+20ab^3(A-C\right.}{5d}$$

Mathematica [C] time = 9.05849, size = 4776, normalized size = 12.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (((12*I)/5)*a^3*A*b*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec
[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4,
 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4,
-(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
+ (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[
2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-
1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c +
 d*x] + A*Cos[2*c + 2*d*x])) + ((4*I)*a*A*b^3*Cos[c + d*x]^6*Csc[c]*(a + b*
Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*H
ypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*
(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^
((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x]
)^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (((3*I)/5)*a^4*B*C
```

os[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2
*d*x])) + ((6*I)*a^2*b^2*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (I*b^4*B*Cos[c + d*x]^6*Csc[c]*(a + b
*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x
])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + ((4*I)*a^3*b*C*Co
s[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*
d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin
[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*
d*x])) - ((4*I)*a*b^3*C*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)
*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Co
s[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] +
d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(13/2)*(a + b*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(12*a^3*A*b + 20*a*A*b^3 +
 3*a^4*B + 30*a^2*b^2*B - 10*b^4*B + 20*a^3*b*C - 40*a*b^3*C + 12*a^3*A*b*C
os[2*c] + 20*a*A*b^3*Cos[2*c] + 3*a^4*B*Cos[2*c] + 30*a^2*b^2*B*Cos[2*c] +
20*a^3*b*C*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (a^2*(23*a^2*A + 168*A*b^2 + 11
2*a*b*B + 28*a^2*C)*Cos[d*x]*Sin[c])/(21*d) + (2*a^3*(4*A*b + a*B)*Cos[2*d*
x]*Sin[2*c])/(5*d) + (a^4*A*Cos[3*d*x]*Sin[3*c])/(7*d) + (a^2*(23*a^2*A + 1
68*A*b^2 + 112*a*b*B + 28*a^2*C)*Cos[c]*Sin[d*x])/(21*d) + (4*b^4*C*Sec[c]*

```
Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(b^4*C*Sin[c] + 3*b
^4*B*Sin[d*x] + 12*a*b^3*C*Sin[d*x]))/(3*d) + (2*a^3*(4*A*b + a*B)*Cos[2*c]
*Sin[2*d*x])/(5*d) + (a^4*A*Cos[3*c]*Sin[3*d*x])/(7*d)))/((b + a*Cos[c + d*
x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (20*a^4*A*Cos[c
 + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(21*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c +
 2*d*x])*Sqrt[1 + Cot[c]^2]) - (8*a^2*A*b^2*Cos[c + d*x]^6*Csc[c]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x
])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*
x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
 - (4*A*b^4*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*a^3*b*B*Cos[c + d*x]^6*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a +
 b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(
b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqr
t[1 + Cot[c]^2]) - (16*a*b^3*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*
C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^4*C*C
os[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
ot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2
*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (24*a^2*b^2*C*Cos[c + d*x]^6*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c
 + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c
]^2]) - (4*b^4*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 10.084, size = 2507, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

[Out] 2/105*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(252*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+420*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-210*A*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-140*B*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-420*a*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+420*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-420*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-630*C*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1344*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2016*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+1680*A*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+1120*B*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1008*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1680*A*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1120*B*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-1680*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+168*A*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+420*A*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+280*B*a^3*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+840*C*a*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-126*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+210*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+70*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+70*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+50*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2+210*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+480*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-960*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-336*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+920*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+504*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+280*C*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-440*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-252*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-420*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-280*C*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+80*A*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+42*B*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+210*B*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+70*C*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+70*C*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+420*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2-504*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2-840*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2+280*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*b*sin(1/2*d*x+1/2*c)^2+840*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c)^2-1260*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2+1260*C*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2-25*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*A*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c

```
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a^4-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-35*a^4*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-35*C*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-840*C*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*b*
sin(1/2*d*x+1/2*c)^2+840*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^3*sin(1/2*d*x+1/2*c
)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^3\sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^3\sec(dx+c)^5 + Aa^4\cos(dx+c)^3 + \left(6\,Ca^2b^2 + 4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^3*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x
 + c)^3*sec(d*x + c)^5 + A*a^4*cos(d*x + c)^3 + (6*C*a^2*b^2 + 4*B*a*b^3 +
A*b^4)*cos(d*x + c)^3*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^2*b^2 + 2*A*a*b
^3)*cos(d*x + c)^3*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^3*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c)^3*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec{(dx+c)}^2 + B\sec{(dx+c)} + A\right)(b\sec{(dx+c)} + a)^4 \cos{(dx+c)}^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*co
s(d*x + c)^(7/2), x)
```

3.1315 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec\right.$

Optimal. Leaf size=388

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(4a^3b(A+3C)+18a^2b^2B+a^4B+4ab^3(3A+C)+b^4B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(30a^2b^2(A-C)+\right.}{}$$

```
[Out] (2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3
*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b^
4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*(5*a^3*B - 105*a*b^2*B + 4*a^2*b*(5*A - 33*C) - 6*b^3*(5*A + 3*C)
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (2*a^2*(50*a*b*B - a^2*(3*A - 5
9*C) + 3*b^2*(5*A + 3*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(5*A
*b^2 + 15*a*b*B + 16*a^2*C + 3*b^2*C)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/
(5*d*Sqrt[Cos[c + d*x]]) + (2*(5*b*B + 8*a*C)*(b + a*Cos[c + d*x])^3*Sin[c
+ d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c + d*x
])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 1.30293, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3033, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(4a^3b(A+3C)+18a^2b^2B+a^4B+4ab^3(3A+C)+b^4B\right)}{3d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(30a^2b^2(A-C)+a^4(3A\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```



```
[Out] (2*(20*a^3*b*B - 20*a*b^3*B + 30*a^2*b^2*(A - C) - b^4*(5*A + 3*C) + a^4*(3
*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(a^4*B + 18*a^2*b^2*B + b^
4*B + 4*a*b^3*(3*A + C) + 4*a^3*b*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*(5*a^3*B - 105*a*b^2*B + 4*a^2*b*(5*A - 33*C) - 6*b^3*(5*A + 3*C)
)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) - (2*a^2*(50*a*b*B - a^2*(3*A - 5
9*C) + 3*b^2*(5*A + 3*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*(5*A
*b^2 + 15*a*b*B + 16*a^2*C + 3*b^2*C)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/
(5*d*Sqrt[Cos[c + d*x]]) + (2*(5*b*B + 8*a*C)*(b + a*Cos[c + d*x])^3*Sin[c
+ d*x])/(15*d*Cos[c + d*x]^(3/2)) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c + d*x
])/(5*d*Cos[c + d*x]^(5/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^4\left(C+B\cos(c+dx\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int$$

$$= \frac{2(5bB+8aC)(b+a\cos(c+dx))^3\sin(c+}{15d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(5Ab^2+15abB+16a^2C+3b^2C\right)(b+a}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2a^2\left(50abB-a^2(3A-59C)+3b^2(5A+\right.}{15d}$$

$$= \frac{2a\left(5a^3B-105ab^2B+4a^2b(5A-33C)-\right.}{15}$$

$$= \frac{2a\left(5a^3B-105ab^2B+4a^2b(5A-33C)-\right.}{15}$$

$$= \frac{2\left(20a^3bB-20ab^3B+30a^2b^2(A-C)-b\right.}{}$$

Mathematica [C] time = 9.20879, size = 4960, normalized size = 12.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (((3*I)/5)*a^4*A*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/
4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[
c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*C
os[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] -
3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (
2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c
] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
 E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) + ((6*I)*a^2*A*b^2*Cos[c + d*x]^6*Csc[c]*(a + b*S
ec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])
^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (I*A*b^4*Cos[c + d*
```

```
x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*
((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
+ I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d
*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*S
in[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Si
n[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Si
n[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(
(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) +
 ((4*I)*a^3*b*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c +
 d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4
, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c
] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3
*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2
*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
 + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 +
E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])) - ((4*I)*a*b^3*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[
c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (I*a^4*C*Cos[c + d*x]^
6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2
*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b
+ a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((
6*I)*a^2*b^2*C*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*
d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*
I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
+ I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E
^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])) - (((3*I)/5)*b^4*C*Cos[c + d*x]^6*Csc[c]*(a + b*Sec
[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
```

```
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4
*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(13/2)*
(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(3*a^4*
A + 30*a^2*A*b^2 - 10*A*b^4 + 20*a^3*b*B - 40*a*b^3*B + 5*a^4*C - 60*a^2*b^
2*C - 6*b^4*C + 3*a^4*A*Cos[2*c] + 30*a^2*A*b^2*Cos[2*c] + 20*a^3*b*B*Cos[2
*c] + 5*a^4*C*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*a^3*(4*A*b + a*B)*Cos[d*x
]*Sin[c])/(3*d) + (2*a^4*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*a^3*(4*A*b + a*B
)*Cos[c]*Sin[d*x])/(3*d) + (4*b^4*C*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) +
 (4*Sec[c]*Sec[c + d*x]^2*(3*b^4*C*Sin[c] + 5*b^4*B*Sin[d*x] + 20*a*b^3*C*S
in[d*x]))/(15*d) + (4*Sec[c]*Sec[c + d*x]*(5*b^4*B*Sin[c] + 20*a*b^3*C*Sin[
c] + 15*A*b^4*Sin[d*x] + 60*a*b^3*B*Sin[d*x] + 90*a^2*b^2*C*Sin[d*x] + 9*b^
4*C*Sin[d*x]))/(15*d) + (2*a^4*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((b + a*Cos[c
 + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (16*a^3*A*b
*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*a*A*b^3*Cos[c + d*x]^6*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c
 + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c
 + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c
]^2]) - (4*a^4*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c
]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[
c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (24*a^2*b^2*B*Cos[c +
d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*
x])*Sqrt[1 + Cot[c]^2]) - (4*b^4*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A +
 B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*
a^3*b*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] +
A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*a*b^3*C*Cos[c + d*x]^6*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*S
ec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b +
a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1
+ Cot[c]^2])
```

Maple [B] time = 11.548, size = 1884, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*A*a^4*(-4*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+1/3*(-12*A*a^4+16*A*a^3*b+4*B*a^4)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+(6*A*a^4-16*A*a^3*b+12*A*a^2*b^2-4*B*a^4+8*B*a^3*b+2*C*a^4)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*A*a^3*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*a^2*b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+8*A*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+2*B*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-8*B*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a^2*b^2*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*a^4*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8
*a^3*b*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+2*b^3*(B*b+4*C*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5
*C*b^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)^2\sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\cos(dx+c)^2\sec(dx+c)^5 + Aa^4\cos(dx+c)^2 + \left(6\,Ca^2b^2 + 4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)^2*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x
 + c)^2*sec(d*x + c)^5 + A*a^4*cos(d*x + c)^2 + (6*C*a^2*b^2 + 4*B*a*b^3 +
A*b^4)*cos(d*x + c)^2*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^2*b^2 + 2*A*a*b
^3)*cos(d*x + c)^2*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d
*x + c)^2*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c)^2*sec(d*x + c))
*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^4\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*co
s(d*x + c)^(5/2), x)
```

3.1316 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4 \left(A + B\sec(c+dx) + C s\right.$

Optimal. Leaf size=384

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)(\cdots}{\cdots}$$

```
[Out] (2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3*
C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^2
*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*b*(413*a^2*b*B + 63*b^3*B + 192*a^3*C + 2*a*b^2*(175*A + 10
1*C))*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) - (2*a^2*(98*a*b*B - a^2*(35
*A - 87*C) + 5*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) +
(2*(35*A*b^2 + 77*a*b*B + 48*a^2*C + 25*b^2*C)*(b + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(7*b*B + 8*a*C)*(b + a*Cos[c + d*x
])^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*C*(b + a*Cos[c + d*x])^4*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 1.31648, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3031, 3023, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(42a^2b^2(3A+C)+7a^4(A+3C)+84a^3bB+28ab^3B+b^4(7A+5C)\right)}{21d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(20a^3b(A\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (2*(5*a^4*B - 30*a^2*b^2*B - 3*b^4*B + 20*a^3*b*(A - C) - 4*a*b^3*(5*A + 3*
C))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(84*a^3*b*B + 28*a*b^3*B + 42*a^2
*b^2*(3*A + C) + 7*a^4*(A + 3*C) + b^4*(7*A + 5*C))*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*b*(413*a^2*b*B + 63*b^3*B + 192*a^3*C + 2*a*b^2*(175*A + 10
1*C))*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]) - (2*a^2*(98*a*b*B - a^2*(35
*A - 87*C) + 5*b^2*(7*A + 5*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d) +
(2*(35*A*b^2 + 77*a*b*B + 48*a^2*C + 25*b^2*C)*(b + a*Cos[c + d*x])^2*Sin[c
 + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*(7*b*B + 8*a*C)*(b + a*Cos[c + d*x
])^3*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*C*(b + a*Cos[c + d*x])^4*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \int \frac{(b+a\cos(c+dx))^4\left(C+B\cos(c+\right.}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}$$

$$= \frac{2(7bB+8aC)(b+a\cos(c+dx))^3\sin(}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\left(35Ab^2+77abB+48a^2C+25b^2C\right)}{105d\cos^{\frac{3}{2}}(c}$$

$$= \frac{2b\left(413a^2bB+63b^3B+192a^3C+2ab^2\right.}{105d\sqrt{\cos(c+}}$$

$$= \frac{2b\left(413a^2bB+63b^3B+192a^3C+2ab^2\right.}{105d\sqrt{\cos(c+}}$$

$$= \frac{2b\left(413a^2bB+63b^3B+192a^3C+2ab^2\right.}{105d\sqrt{\cos(c+}}$$

$$= \frac{2\left(5a^4B-30a^2b^2B-3b^4B+20a^3b(A\right.}{5}$$

Mathematica [C] time = 9.44671, size = 4791, normalized size = 12.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] ((4*I)*a^3*A*b*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c +
d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4,
 -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
 + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos
[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*
d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*
I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c]
+ I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E
^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x]
 + A*Cos[2*c + 2*d*x])) - ((4*I)*a*A*b^3*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c
 + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (I*a^4*B*Cos[c + d*x]^6
*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*
```

```
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b +
 a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((6
*I)*a^2*b^2*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4,
-(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c]
+ (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[
2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d
*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((
2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I
)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
 I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^
((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])) - (((3*I)/5)*b^4*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[
c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - ((4*I)*a^3*b*C*Cos[c +
 d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I
)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x
)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))
*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c]))
)/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
) - (((12*I)/5)*a*b^3*C*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)
*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Co
s[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3
/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos
[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*
Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] +
d*(-1 + E^((2*I)*d*x))*Sin[c])))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^(13/2)*(a + b*Sec[c + d*x]
)^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-2*(20*a^3*A*b - 40*a*A*b^3 +
 5*a^4*B - 60*a^2*b^2*B - 6*b^4*B - 40*a^3*b*C - 24*a*b^3*C + 20*a^3*A*b*Co
s[2*c] + 5*a^4*B*Cos[2*c])*Csc[c]*Sec[c])/(5*d) + (4*a^4*A*Cos[d*x]*Sin[c])
/(3*d) + (4*a^4*A*Cos[c]*Sin[d*x])/(3*d) + (4*b^4*C*Sec[c]*Sec[c + d*x]^4*S
in[d*x])/(7*d) + (4*Sec[c]*Sec[c + d*x]^3*(5*b^4*C*Sin[c] + 7*b^4*B*Sin[d*x
] + 28*a*b^3*C*Sin[d*x]))/(35*d) + (4*Sec[c]*Sec[c + d*x]*(35*A*b^4*Sin[c]
+ 140*a*b^3*B*Sin[c] + 210*a^2*b^2*C*Sin[c] + 25*b^4*C*Sin[c] + 420*a*A*b^3
*Sin[d*x] + 630*a^2*b^2*B*Sin[d*x] + 63*b^4*B*Sin[d*x] + 420*a^3*b*C*Sin[d*
```

```
x] + 252*a*b^3*C*Sin[d*x]))/(105*d) + (4*Sec[c]*Sec[c + d*x]^2*(21*b^4*B*Si
n[c] + 84*a*b^3*C*Sin[c] + 35*A*b^4*Sin[d*x] + 140*a*b^3*B*Sin[d*x] + 210*a
^2*b^2*C*Sin[d*x] + 25*b^4*C*Sin[d*x]))/(105*d)))/((b + a*Cos[c + d*x])^4*(
A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) - (4*a^4*A*Cos[c + d*x]^6
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(
a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*
Sqrt[1 + Cot[c]^2]) - (24*a^2*A*b^2*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A
 + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*A*
b^4*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x
]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*a^3*b*B*Cos[c + d*x]^6*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec
[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Co
s[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]) - (16*a*b^3*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d
*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*
B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^4*C*Cos[c +
 d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*
x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot
[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]
]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d
*x])*Sqrt[1 + Cot[c]^2]) - (8*a^2*b^2*C*Cos[c + d*x]^6*Csc[c]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4
*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^
4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (
20*b^4*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(21*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2])
```

Maple [B] time = 12.151, size = 1624, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3*A*a^4*(2*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-4*A*a^4+8*A*a^3*b+2*B*a^4)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8*A*a^3*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*A*a^2*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-2*B*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+8*B*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+2*a^4*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b^2*(A*b^2+4*B*a*b+6*C*a^2)*(-1/6
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*b^4*(-1/56*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3*(B*b+4*C*a)/(8*sin(1/2*d
*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1
/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\cos(dx+c)\sec(dx+c)^6+\left(4\,Cab^3+Bb^4\right)\cos(dx+c)\sec(dx+c)^5+Aa^4\cos(dx+c)+\left(6\,Ca^2b^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*cos(d*x + c)*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*cos(d*x +
 c)*sec(d*x + c)^5 + A*a^4*cos(d*x + c) + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)
*cos(d*x + c)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^2*b^2 + 2*A*a*b^3)*cos(
d*x + c)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*cos(d*x + c)*se
c(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*cos(d*x + c)*sec(d*x + c))*sqrt(cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c
)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^4\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*co
s(d*x + c)^(3/2), x)
```

3.1317 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^4 \left(A+B\sec(c+dx)+C s\right.$

Optimal. Leaf size=401

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(28a^3b(3A+C)+42a^2b^2B+21a^4B+4ab^3(7A+5C)+5b^4B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(5A\right.}{}$$

```
[Out] (-2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^
4*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(21*a^4*B + 42*a^2*b^
2*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4*a*b^3*(7*A + 5*C))*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(261*a^2*b*B + 75*b^3*B + 64*a^3*C + 2*a*b^2*(147*A
 + 101*C))*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)) + (2*(1098*a^3*b*B + 75
6*a*b^3*B + 192*a^4*C + 21*b^4*(9*A + 7*C) + 7*a^2*b^2*(261*A + 155*C))*Sin
[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]) + (2*(63*A*b^2 + 117*a*b*B + 48*a^2*C
 + 49*b^2*C)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)
) + (2*(9*b*B + 8*a*C)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c + d
*x]^(7/2)) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9
/2))
```

Rubi [A] time = 1.33063, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3031, 3021, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(28a^3b(3A+C)+42a^2b^2B+21a^4B+4ab^3(7A+5C)+5b^4B\right)}{21d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(18a^2b^2(5A+3C)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
 + d*x]^2),x]
```


```
[Out] (-2*(60*a^3*b*B + 36*a*b^3*B - 15*a^4*(A - C) + 18*a^2*b^2*(5*A + 3*C) + b^
4*(9*A + 7*C))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(21*a^4*B + 42*a^2*b^
2*B + 5*b^4*B + 28*a^3*b*(3*A + C) + 4*a*b^3*(7*A + 5*C))*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(261*a^2*b*B + 75*b^3*B + 64*a^3*C + 2*a*b^2*(147*A
 + 101*C))*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)) + (2*(1098*a^3*b*B + 75
6*a*b^3*B + 192*a^4*C + 21*b^4*(9*A + 7*C) + 7*a^2*b^2*(261*A + 155*C))*Sin
[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]) + (2*(63*A*b^2 + 117*a*b*B + 48*a^2*C
 + 49*b^2*C)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)
) + (2*(9*b*B + 8*a*C)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(63*d*Cos[c + d
*x]^(7/2)) + (2*C*(b + a*Cos[c + d*x])^4*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9
/2))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^4 \left(A+B\sec(c+dx)+C\sec^2(c+dx)\right) dx = \int \frac{(b+a\cos(c+dx))^4 \left(C+B\cos(c+dx\right.}{\cos^{\frac{11}{2}}(c+dx)}$$

$$= \frac{2C(b+a\cos(c+dx))^4\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int$$

$$= \frac{2(9bB+8aC)(b+a\cos(c+dx))^3\sin(c+}{63d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2\left(63Ab^2+117abB+48a^2C+49b^2C\right)(b}{315d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2b\left(261a^2bB+75b^3B+64a^3C+2ab^2(14\right.}{315d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2b\left(261a^2bB+75b^3B+64a^3C+2ab^2(14\right.}{315d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{2b\left(261a^2bB+75b^3B+64a^3C+2ab^2(14\right.}{315d\cos^{\frac{3}{2}}(c+dx}$$

$$= -\frac{2\left(60a^3bB+36ab^3B-15a^4(A-C)+1\right.}{}$$

Mathematica [C] time = 9.09068, size = 4150, normalized size = 10.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C
*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(13/2)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +
 d*x]^2)*((-2*(15*a^4*A - 180*a^2*A*b^2 - 18*A*b^4 - 120*a^3*b*B - 72*a*b^3
*B - 30*a^4*C - 108*a^2*b^2*C - 14*b^4*C + 15*a^4*A*Cos[2*c])*Csc[c]*Sec[c]
)/(15*d) + (4*b^4*C*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(9*d) + (4*Sec[c]*Sec[c
 + d*x]^4*(7*b^4*C*Sin[c] + 9*b^4*B*Sin[d*x] + 36*a*b^3*C*Sin[d*x]))/(63*d)
 + (4*Sec[c]*Sec[c + d*x]^2*(63*A*b^4*Sin[c] + 252*a*b^3*B*Sin[c] + 378*a^2
*b^2*C*Sin[c] + 49*b^4*C*Sin[c] + 420*a*A*b^3*Sin[d*x] + 630*a^2*b^2*B*Sin[
d*x] + 75*b^4*B*Sin[d*x] + 420*a^3*b*C*Sin[d*x] + 300*a*b^3*C*Sin[d*x]))/(3
15*d) + (4*Sec[c]*Sec[c + d*x]^3*(45*b^4*B*Sin[c] + 180*a*b^3*C*Sin[c] + 63
*A*b^4*Sin[d*x] + 252*a*b^3*B*Sin[d*x] + 378*a^2*b^2*C*Sin[d*x] + 49*b^4*C*
Sin[d*x]))/(315*d) + (4*Sec[c]*Sec[c + d*x]*(140*a*A*b^3*Sin[c] + 210*a^2*b
^2*B*Sin[c] + 25*b^4*B*Sin[c] + 140*a^3*b*C*Sin[c] + 100*a*b^3*C*Sin[c] + 6
30*a^2*A*b^2*Sin[d*x] + 63*A*b^4*Sin[d*x] + 420*a^3*b*B*Sin[d*x] + 252*a*b^
3*B*Sin[d*x] + 105*a^4*C*Sin[d*x] + 378*a^2*b^2*C*Sin[d*x] + 49*b^4*C*Sin[d
*x]))/(105*d)))/((b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])) - (16*a^3*A*b*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Se
c[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C
 + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (16*a*A*b^3
*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
```

n[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2
)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos
[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*a^4*B*Cos[c + d*x]^6*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d
x])^4(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c +
d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2
]) - (8*a^2*b^2*B*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] +
C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
Sin[d*x - ArcTan[Cot[c]]]])/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (20*b^4*B*Cos[c + d*x]^
6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2
1*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]
)*Sqrt[1 + Cot[c]^2]) - (16*a^3*b*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(b + a*Cos[c + d*x])^4*
(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (80
*a*b^3*C*Cos[c + d*x]^6*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c
+ d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqr
t[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
- ArcTan[Cot[c]]]])/(21*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x
] + A*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (2*a^4*A*Cos[c + d*x]^6*Csc[c
]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b
+ a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (
12*a^2*A*b^2*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*
x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])) + (6*A*b^4*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c +
d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (8*a^3*b*B*Co
s[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c +

```
d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^
2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[
2*c + 2*d*x])) + (24*a*b^3*B*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^4*(A
+ 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (2*a^4*C*Cos[c + d*x]^6*C
sc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
 + (36*a^2*b^2*C*Cos[c + d*x]^6*Csc[c]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c
+ d*x] + C*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b + a*Cos[c + d*x])^4*(A + 2*C + 2*B*
Cos[c + d*x] + A*Cos[2*c + 2*d*x])) + (14*b^4*C*Cos[c + d*x]^6*Csc[c]*(a +
b*Sec[c + d*x])^4*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((HypergeometricP
FQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c
]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[
Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
+ Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*
Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2
))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d*(b + a
*Cos[c + d*x])^4*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x]))
```

Maple [B] time = 15.119, size = 1550, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*A*a^3*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*a^4*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
4*a*b*(2*A*b^2+3*B*a*b+2*C*a^2)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*
b^3*(B*b+4*C*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/
2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*C*b^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/
2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d
*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))))-2/5*b^2*(A*b^2+4*B*a*b+6*C*a^2)/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*a^2*(6*A*b^2+4*B*a*b+C*a^2)*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4\sec(dx+c)^6 + \left(4\,Cab^3 + Bb^4\right)\sec(dx+c)^5 + Aa^4 + \left(6\,Ca^2b^2 + 4\,Bab^3 + Ab^4\right)\sec(dx+c)^4 + 2\left(2\,Ca\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*b^4*sec(d*x + c)^6 + (4*C*a*b^3 + B*b^4)*sec(d*x + c)^5 + A*a^4
 + (6*C*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sec(d*x + c)^4 + 2*(2*C*a^3*b + 3*B*a^
2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + (C*a^4 + 4*B*a^3*b + 6*A*a^2*b^2)*sec(d
*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^4\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*sq
rt(cos(d*x + c)), x)
```

3.1318 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2b(A+3C)+a^3(-B)-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)-5abB+5Ab\right.}{5a^3d}$$

```
[Out] (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^3*
d) - (2*(3*A*b^3 - a^3*B - 3*a*b^2*B + a^2*b*(A + 3*C))*EllipticF[(c + d*x)
/2, 2])/(3*a^4*d) + (2*b^2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(3*a^2*d) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.965389, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b(A+3C)+a^3(-B)-3ab^2B+3Ab^3\right)}{3a^4d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(3A+5C)-5abB+5Ab^2\right)}{5a^3d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```


```
[Out] (2*(5*A*b^2 - 5*a*b*B + a^2*(3*A + 5*C))*EllipticE[(c + d*x)/2, 2])/(5*a^3*
d) - (2*(3*A*b^3 - a^3*B - 3*a*b^2*B + a^2*b*(A + 3*C))*EllipticF[(c + d*x)
/2, 2])/(3*a^4*d) + (2*b^2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(3*a^2*d) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{b+a\cos(c+dx)}\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3Ab}{2}+\frac{1}{2}a(3A+5C)\cos\ldots\right.}{b+a\cos\ldots}}{5a\ldots}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin\ldots}{5ad}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin\ldots}{5ad}$$

$$= \frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2(Ab-a\ldots}{\ldots}$$

$$= \frac{2\left(5Ab^2-5abB+a^2(3A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2\left(3Ab^3\ldots\right.}{\ldots}$$

Mathematica [A] time = 2.39304, size = 274, normalized size = 1.31

$$\frac{6\sin(c+dx)\left(a^2(3A+5C)-5abB+5Ab^2\right)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)}{b\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2*(5*A*b^2 - 5*a*b*B + 3*a^2*(3*A + 5*C))*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2])/(a + b) + 2*a^2*(4*A*b + 5*a*B)*(2*EllipticF[(c + d*x)/2, 2
] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*a^2*Sqrt[C
os[c + d*x]]*(-5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] + (6*(5*A*b
^2 - 5*a*b*B + a^2*(3*A + 5*C))*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]
], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b
^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*S
qrt[Sin[c + d*x]^2]))/(30*a^4*d)
```

Maple [B] time = 6.517, size = 801, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*A/a*(-4*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)-4/3/a^2*(3*A*a+A*b-B*a)*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/a^3
*(3*A*a^2+2*A*a*b+A*b^2-2*B*a^2-B*a*b+C*a^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*(A*a^3+A*a^2*b+A*a*b^2+A*b^3-B*a^3-B*a^2*b-B*a*b^2+C*a^3+C*
a^2*b)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-2*b^2*(A*b^2-B*a*b+C*a^2)/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec
(d*x + c) + a), x)
```

3.1319 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(A+3C)-3abB+3Ab^2\right)}{3a^3d}-\frac{2b\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2(Ab-aB)E}{}$$

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(3*A*b^2 - 3*a*b*B
+ a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*a^3*d) - (2*b*(A*b^2 - a*(b*
B - a*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.668308, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(A+3C)-3abB+3Ab^2\right)}{3a^3d}-\frac{2b\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(3*A*b^2 - 3*a*b*B
+ a^2*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*a^3*d) - (2*b*(A*b^2 - a*(b*
B - a*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{b+a\cos(c+dx)}\,dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{Ab}{2}+\frac{1}{2}a(A+3C)\cos(c+dx)-\frac{3}{2}(Ab-}{\sqrt{\cos(c+dx)}(b+a\cos(c-}}{3a}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{2\int \frac{-\frac{1}{2}aAb-\frac{1}{2}\left(3Ab^2-3abB+a^2(A+3C\right.}{\sqrt{\cos(c+dx)}(b+a\cos(c+a}}{3a^2}$$

$$= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}$$

$$= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\left(3Ab^2-3abB+a^2(A+3C\right.}{3a^3d}$$

Mathematica [A] time = 1.34508, size = 218, normalized size = 1.48

$$\frac{-\frac{6(Ab-aB)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}+\frac{4(A+3C)\left((a+b\right.}{}}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]),x]
```

```
[Out] ((2*(-(A*b) + 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (
4*(A + 3*C)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + 4*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(A*b
 - a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ell
ipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -
ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2])
)/(6*a*d)
```

Maple [B] time = 3.227, size = 945, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*A*a^3-4*A*
a^2*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^3+2*A*a^2*b)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*a^2*b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+3*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^3-3*B*a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3*B*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))*a*b^2+3*a^3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a^2*b)/a^3/(a-b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a), x)
```

3.1320 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{2(Ab-aB)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(a^2*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(a^2*(a + b)*d)
```

Rubi [A] time = 0.391676, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {4112, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x]),x]
```


```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(a^2*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(a^2*(a + b)*d)
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx$$

$$= -\frac{\int \frac{-aC+(Ab-aB)\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a} + \frac{A\int \sqrt{\cos(c+dx)}\,dx}{a}$$

$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^2} + \left(\frac{b(Ab-a}{a^2}\right.$$

$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\left(\frac{b(Ab-}{a^2}\right.}{}$$

Mathematica [F] time = 52.3722, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]),x]
```

```
[Out] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x]), x]
```

Maple [A] time = 2.645, size = 323, normalized size = 3.3

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\,a^2\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(A\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b+C*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))*a^2)/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec
(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sec\left(c+dx\right)+C\sec^{2}\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sqrt(cos(c + d*x))/(a + b
*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec
(d*x + c) + a), x)
```

3.1321 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=118

$$\frac{2A\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}-\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2C\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*A*EllipticF[(c + d*x)/2, 2])/(a
*d) - (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])
/(a*b*(a + b)*d) + (2*C*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.599909, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a+b)}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2C\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Sec[
c + d*x])),x]
```

```
[Out] (-2*C*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*A*EllipticF[(c + d*x)/2, 2])/(a
*d) - (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])
/(a*b*(a + b)*d) + (2*C*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx &= \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}\,dx \\
&= \frac{2C\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}+\frac{2\int \frac{\frac{1}{2}(bB-aC)+\frac{1}{2}b(A-C)\cos(c+dx)-\frac{1}{2}aC\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{b} \\
&= \frac{2C\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}-\frac{2\int \frac{-\frac{1}{2}a(bB-aC)-\frac{1}{2}aAb\cos(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{ab}-\frac{C\int \sqrt{\cos(c+dx)}\,dx}{b} \\
&= -\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2C\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}+\frac{A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a}+\left(-\frac{Ab}{a}+B-\right. \\
&= -\frac{2CE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2\left(\frac{Ab}{a}-B+\frac{aC}{b}\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{(a+b)d}
\end{aligned}$$

Mathematica [A] time = 2.79748, size = 206, normalized size = 1.75

$$\frac{\frac{C\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{b(A-C)\left(2\text{EllipticF}\left(\frac{1}{2}(\right.\right.}{}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Sec[c + d*x])),x]
```

```
[Out] (((2*b*B - 3*a*C)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (b*(
A - C)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a + b)))/a + (2*C*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (C*(2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(b*d)
```

Maple [B] time = 5.244, size = 409, normalized size = 3.5

$$-\frac{1}{d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\text{Ellipti}}{a\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*(-A
*b^2+B*a*b-C*a^2)/b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*C/b*(-(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)\sqrt{\cos\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))/cos(d*x+c)**(1/
2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/((a + b*sec(c + d*x))*sqr
t(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A}{\left(b\sec\left(dx + c\right) + a\right)\sqrt{\cos\left(dx + c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqr
t(cos(d*x + c))), x)
```

3.1322 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=158

$$\frac{2\text{CEllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}+\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2(bB-aC)}{b^2d\sqrt{\cos}}$$

```
[Out] (-2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*C*EllipticF[(c + d*
x)/2, 2])/(3*b*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c
 + d*x)/2, 2])/(b^2*(a + b)*d) + (2*C*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/
2)) + (2*(b*B - a*C)*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.934632, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(Ab^2-a(bB-aC)\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2(bB-aC)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2CF\left(\frac{1}{2}(c+dx\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])),x]
```


```
[Out] (-2*(b*B - a*C)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*C*EllipticF[(c + d*
x)/2, 2])/(3*b*d) + (2*(A*b^2 - a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c
 + d*x)/2, 2])/(b^2*(a + b)*d) + (2*C*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/
2)) + (2*(b*B - a*C)*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))}\,dx$$

$$= \frac{2C\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{3}{2}(bB-aC)+\frac{1}{2}b(3A+C)\cos(c+dx)+\frac{1}{2}aC\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}dx}{3b}$$

$$= \frac{2C\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(b^2(3A+C)-3a(bB-aC)\right)-\frac{1}{4}b\cdots}{\sqrt{\cos(\cdots}}\cdots}{\cdots}$$

$$= \frac{2C\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}a\left(b^2(3A+C)-3a(bB-aC)\right)-\cdots}{\sqrt{\cos(c+dx)}(b+a\cos\cdots}\cdots}{3ab^2}$$

$$= -\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2C\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+d\cdots}{b^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(bB-aC)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2CF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd} + \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\Pi\cdots}{(a+\cdots}$$

Mathematica [A] time = 2.48184, size = 269, normalized size = 1.7

$$-\frac{\frac{6(bB-aC)\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a\sqrt{\sin^2(c+dx)}}+\frac{b\left(8abC-6b\cdots\right.}{\cdots}}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])),x]
```

```
[Out] ((2*b*(6*A*b^2 - 9*a*b*B + 9*a^2*C + 2*b^2*C)*EllipticPi[(2*a)/(a + b), (c
+ d*x)/2, 2])/(a + b) + (b*(-6*b^2*B + 8*a*b*C)*(2*EllipticF[(c + d*x)/2, 2
] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (4*b^2*C*
Sin[c + d*x])/Cos[c + d*x]^(3/2) + (12*b*(b*B - a*C)*Sin[c + d*x])/Sqrt[Cos
[c + d*x]] - (6*(b*B - a*C)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*
EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[
Sin[c + d*x]^2]))/(6*b^3*d)
```

Maple [B] time = 7.112, size = 472, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
```

```
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b^2-B*a*b+C*a^2)/b^2/(a^2-a*b)*a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b)
,2^(1/2))+2*(B*b-C*a)/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c
)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx+c)^2 + B \sec (dx+c) + A}{(b \sec (dx+c) + a) \cos (dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos
(d*x + c)^(3/2)), x)
```

3.1323 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=236

$$\frac{2(bB-aC)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d}+\frac{2\sin(c+dx)\left(5a^2C-5abB+5\right.}{5b^3d\sqrt{\cos(c+dx}}$$

```
[Out] (-2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b
^3*d) + (2*(b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(3*b^2*d) - (2*a*(A*b^2 -
 a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d)
+ (2*C*Sin[c + d*x])/(5*b*d*Cos[c + d*x]^(5/2)) + (2*(b*B - a*C)*Sin[c + d*
x])/(3*b^2*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*
C)*Sin[c + d*x])/(5*b^3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.2782, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d}+\frac{2\sin(c+dx)\left(5a^2C-5abB+5Ab^2+3b^2C\right)}{5b^3d\sqrt{\cos(c+dx)}}-\frac{2a\left(Ab^2-a(bB-aC\right.}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])),x]
```


```
[Out] (-2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*EllipticE[(c + d*x)/2, 2])/(5*b
^3*d) + (2*(b*B - a*C)*EllipticF[(c + d*x)/2, 2])/(3*b^2*d) - (2*a*(A*b^2 -
 a*(b*B - a*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d)
+ (2*C*Sin[c + d*x])/(5*b*d*Cos[c + d*x]^(5/2)) + (2*(b*B - a*C)*Sin[c + d*
x])/(3*b^2*d*Cos[c + d*x]^(3/2)) + (2*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*
C)*Sin[c + d*x])/(5*b^3*d*Sqrt[Cos[c + d*x]])
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b+a\cos(c+dx))}\,dx$$

$$= \frac{2C\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\frac{5}{2}(bB-aC)+\frac{1}{2}b(5A+3C)\cos(c+dx)+\frac{3}{2}aC\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))}dx}{5b}$$

$$= \frac{2C\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{3}{4}(5Ab^2-5abB+5a^2C+3b^2C)+\frac{1}{4}b(\cdots}{\cos^{\frac{3}{2}}(c+\cdots}}{\cdots}$$

$$= \frac{2C\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(5Ab^2-5abB+5a^2C+3b^2\cdots}{5b^3d\sqrt{\cos(c+dx\cdots}}$$

$$= \frac{2C\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(bB-aC)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(5Ab^2-5abB+5a^2C+3b^2\cdots}{5b^3d\sqrt{\cos(c+dx\cdots}}$$

$$= -\frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2C\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + 2(\cdots$$

$$= -\frac{2\left(5Ab^2-5abB+5a^2C+3b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2(bB-aC)F\left(\frac{1}{2}(c+dx\cdots\right.}{3b^2d}$$

Mathematica [A] time = 4.82478, size = 334, normalized size = 1.42

$$-\frac{\frac{2b\left(20a^2C-20abB+15Ab^2+9b^2C\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} + \frac{6\sin(c+dx)\left(5a^2C-5abB+5Ab^2+3b^2C\right)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+\cdots}\right.\right.\right.}{ab\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])),x]
```

```
[Out] ((-2*(-45*a^2*b*B - 10*b^3*B + 45*a^3*C + a*b^2*(45*A + 19*C))*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (2*b*(15*A*b^2 - 20*a*b*B + 20*a^2
*C + 9*b^2*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b),
 (c + d*x)/2, 2])/(a + b)))/a + (6*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*
(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]) + (2*(10*b
*(b*B - a*C)*Sin[c + d*x] + 3*(5*A*b^2 - 5*a*b*B + 5*a^2*C + 3*b^2*C)*Sin[2
*(c + d*x)] + 6*b^2*C*Tan[c + d*x]))/Cos[c + d*x]^(3/2))/(30*b^3*d)
```

Maple [B] time = 10.045, size = 800, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(B*b-C*a)/b^2
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*C/b/(8*sin(1/2*d*x+1/2*c)^6-1
2*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*(A*b^2-B*a*b+C*a^2)*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*(A*b^2-B*a*b+C*
a^2)/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*
c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c
)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos
(d*x + c)^(5/2)), x)
```

3.1324 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=346

$$-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^2(16A-3C)-2a^4(A+3C)+12a^3bB-9ab^3B+15Ab^4\right)}{3a^4d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b(4A\right.}{a^3d}$$

```
[Out] ((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*EllipticE[(c + d*x)/2, 2
])/(a^3*(a^2 - b^2)*d) - ((15*A*b^4 + 12*a^3*b*B - 9*a*b^3*B - a^2*b^2*(16*
A - 3*C) - 2*a^4*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d
) + (b*(5*A*b^4 + 5*a^3*b*B - 3*a*b^3*B - a^2*b^2*(7*A - C) - 3*a^4*C)*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a - b)*(a + b)^2*d) - ((5*A*b^
2 - 3*a*b*B - a^2*(2*A - 3*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2
 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(
a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.21453, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(16A-3C)-2a^4(A+3C)+12a^3bB-9ab^3B+15Ab^4\right)}{3a^4d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b(4A-C)+\right.}{a^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```


```
[Out] ((5*A*b^3 + 2*a^3*B - 3*a*b^2*B - a^2*b*(4*A - C))*EllipticE[(c + d*x)/2, 2
])/(a^3*(a^2 - b^2)*d) - ((15*A*b^4 + 12*a^3*b*B - 9*a*b^3*B - a^2*b^2*(16*
A - 3*C) - 2*a^4*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d
) + (b*(5*A*b^4 + 5*a^3*b*B - 3*a*b^3*B - a^2*b^2*(7*A - C) - 3*a^4*C)*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a - b)*(a + b)^2*d) - ((5*A*b^
2 - 3*a*b*B - a^2*(2*A - 3*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2
 - b^2)*d) + ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(
a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
```

```
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \int \frac{\sqrt{\cos(c+d}}{}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d}$$

$$= -\frac{\left(5Ab^2-3abB-a^2(2A-3C)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} -$$

$$= \frac{\left(5Ab^3+2a^3B-3ab^2B-a^2b(4A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} -$$

Mathematica [A] time = 3.68558, size = 339, normalized size = 0.98

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3b\left(a(aC-bB)+Ab^2\right)}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}+2A\right) - \frac{\frac{8\left(a^2(A+3C)-3abB+2Ab^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{6\sin(c+dx)\left(a^2b\right.}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*A + (3*b*(A*b^2 + a*(-(b*B) + a*C)))/((-a^2 + b^2)
*(b + a*Cos[c + d*x])))*Sin[c + d*x] - ((2*(5*A*b^3 + 6*a^3*B - 3*a*b^2*B -
 a^2*b*(8*A + 3*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8
*(2*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - b
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(5*A*b^3 + 2*a^3*
B - 3*a*b^2*B + a^2*b*(-4*A + C))*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^
2*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/(12*a^2*d)
```

Maple [B] time = 9.059, size = 1123, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*
```

```
A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2+3*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b^2-B*a*b+C*a^2)/a^4*(a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b
/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^3*b*(4*A*b^2-3*B*a*b+2*C*a^2)/(a
^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^2, x)
```

3.1325 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=257

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b(4A+C)+2a^3B-ab^2B+3Ab^3\right)}{a^3d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A-C))-abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)}-\frac{(-a$$

```
[Out] -(((3*A*b^2 - a*b*B - a^2*(2*A - C))*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 -
 b^2)*d)) + ((3*A*b^3 + 2*a^3*B - a*b^2*B - a^2*b*(4*A + C))*EllipticF[(c +
 d*x)/2, 2])/(a^3*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C
- a^2*b^2*(5*A + C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b
)*(a + b)^2*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.800583, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b(4A+C)+2a^3B-ab^2B+3Ab^3\right)}{a^3d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A-C))-abB+3Ab^2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\left(-a^2b^2(5A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^2,x]
```

```
[Out] -(((3*A*b^2 - a*b*B - a^2*(2*A - C))*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 -
 b^2)*d)) + ((3*A*b^3 + 2*a^3*B - a*b^2*B - a^2*b*(4*A + C))*EllipticF[(c +
 d*x)/2, 2])/(a^3*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - a*b^3*B - a^4*C
- a^2*b^2*(5*A + C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b
)*(a + b)^2*d) + ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \int \frac{\frac{1}{2}\left(Ab^2-\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \int \frac{-\frac{1}{2}a\left(Ab^2\right.}{}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2-\right.}{a}$$

$$= -\frac{\left(3Ab^2-abB-a^2(2A-C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(3Ab^3+\right.}{}$$

Mathematica [A] time = 3.28709, size = 301, normalized size = 1.17

$$\frac{\frac{-\frac{2\sin(c+dx)\left(a^2(2A-C)+abB-3Ab^2\right)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}-\frac{8(-aB+Ab+bC)\left((a+b)\text{Ellip}\ldots\right.}{\ldots}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2
)*(b + a*Cos[c + d*x])) + ((2*(-(A*b^2) - a*b*B + a^2*(2*A + C))*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(A*b - a*B + b*C)*((a + b)*El
lipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a
+ b) - (2*(-3*A*b^2 + a*b*B + a^2*(2*A - C))*(2*a*b*EllipticE[ArcSin[Sqrt[C
os[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [B] time = 8.351, size = 856, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2*b*(A*b^2-B*a*b+C*a^2)/a^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b
/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2)))-2/a^2*(3*A*b^2-2*B*a*b+C*a^2)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\sec(c + dx) + C\sec^2(c + dx))\sqrt{\cos(c + dx)}}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sqrt(cos(c + d*x))/(a + b
*sec(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\sec(dx + c)^2 + B\sec(dx + c) + A)\sqrt{\cos(dx + c)}}{(b\sec(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec
(d*x + c) + a)^2, x)
```

3.1326 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=239

$$-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2(-(2A+C))+abB+Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}+\frac{\left(-3a^2b^2(A+C)+a^3b\right.}{}$$

```
[Out] ((A*b^2 - a*(b*B - a*C))*EllipticE[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d) - (
(A*b^2 + a*b*B - a^2*(2*A + C))*EllipticF[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)
*d) + ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*b*(a + b)^2*d) - ((A*b^2 - a*(b
*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b + a*Cos[c +
 d*x]))
```

Rubi [A] time = 0.787957, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2(-(2A+C))+abB+Ab^2\right)}{a^2d\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}+\frac{\left(-3a^2b^2(A+C)+a^3bB+a^4C\right.}{a^2bd(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Sec[
c + d*x])^2),x]
```


```
[Out] ((A*b^2 - a*(b*B - a*C))*EllipticE[(c + d*x)/2, 2])/(a*b*(a^2 - b^2)*d) - (
(A*b^2 + a*b*B - a^2*(2*A + C))*EllipticF[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)
*d) + ((A*b^4 + a^3*b*B + a*b^3*B + a^4*C - 3*a^2*b^2*(A + C))*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*b*(a + b)^2*d) - ((A*b^2 - a*(b
*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b + a*Cos[c +
 d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2}\,dx$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(Ab^2-abB-a^2C+2b^2C\right)+b}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}a\left(Ab^2-abB-a^2C+2b^2C\right)}{\sqrt{\cos(c+}}}{a}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}}{b\left(a^2-b^2\right)d(b+a\cos(c+}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ab\left(a^2-b^2\right)d} - \frac{\left(Ab^2+abB-a^2(2A+C)\right)F\left(\frac{1}{2}(c\right.}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 4.20234, size = 301, normalized size = 1.26

$$-\frac{\frac{\frac{2\sin(c+dx)\left(a(aC-bB)+Ab^2\right)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}+\frac{8b(a(A+C)-bB)\left((a+b)\text{EllipticF}\left(\frac{1}{2}\right.\right.}{a(a+}}{(b-a)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Sec[c + d*x])^2),x]
```

```
[Out] -((4*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^
2)*(b + a*Cos[c + d*x])) + ((2*(-(A*b^2) + a*b*B + 3*a^2*C - 4*b^2*C)*Ellip
ticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(-(b*B) + a*(A + C))*(
(a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2]))/(a*(a + b)) + (2*(A*b^2 + a*(-(b*B) + a*C))*(-2*a*b*EllipticE[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]
], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])
*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/(4*b*d)
```

Maple [B] time = 6.981, size = 809, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a
^2*(A*b^2-B*a*b+C*a^2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+
b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a
*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-2*A*
b+B*a)/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**2/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*s
qrt(cos(d*x + c))), x)
```

3.1327 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=307

$$-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2C-abB+Ab^2-2b^2C\right)}{b^2d\left(a^2-b^2\right)}+\frac{\left(a^2b^2(A+5C)+a^3bB-\right.}{}$$

```
[Out] -(((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2
 - b^2)*d)) - ((A*b^2 - a*(b*B - a*C))*EllipticF[(c + d*x)/2, 2])/(a*b*(a^2
 - b^2)*d) + ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 5*C))*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a*b^2*(a + b)*(a^2 - b^2)*d) + (
(A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[C
os[c + d*x]]) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqr
t[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.1022, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(Ab^2-a(bB-aC)\right)}{abd\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2C-abB+Ab^2-2b^2C\right)}{b^2d\left(a^2-b^2\right)}+\frac{\left(a^2b^2(A+5C)+a^3bB-3a^4C-\right.}{ab^2d(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^2),x]
```


```
[Out] -(((A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2
 - b^2)*d)) - ((A*b^2 - a*(b*B - a*C))*EllipticF[(c + d*x)/2, 2])/(a*b*(a^2
 - b^2)*d) + ((A*b^4 + a^3*b*B - 3*a*b^3*B - 3*a^4*C + a^2*b^2*(A + 5*C))*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a*b^2*(a + b)*(a^2 - b^2)*d) + (
(A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[C
os[c + d*x]]) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqr
t[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2}\,dx$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-Ab^2+abB-3a^2C+2b^2C\right)+b(b\ldots}{\cos\ldots}}{}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+\ldots}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\,c\ldots}$$

$$= \frac{\left(Ab^2-abB+3a^2C-2b^2C\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{\left(Ab^2-a(bB-aC)\right)\sin(c+\ldots}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\,c\ldots}$$

$$= -\frac{\left(Ab^2-abB+3a^2C-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{\left(Ab^2-abB+3a^2C-2b^2C\ldots}{b^2\left(a^2-b^2\right)d\sqrt{\cos(\ldots}}$$

$$= -\frac{\left(Ab^2-abB+3a^2C-2b^2C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{\left(Ab^2-a(bB-aC)\right)F\left(\frac{1}{2}(c\ldots\right.}{ab\left(a^2-b^2\right)d}$$

Mathematica [A] time = 5.02124, size = 340, normalized size = 1.11

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{a\sin(c+dx)\left(a(aC-bB)+Ab^2\right)}{\left(a^2-b^2\right)(a\cos(c+dx)+b)}+2C\tan(c+dx)\right) - \frac{\frac{4b\left(2a^2C-abB+Ab^2-b^2C\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2\sin(c+dx)\left(3a^2C\ldots\right.}{}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])^2),x]
```

```
[Out] (-((((2*(-3*a^2*b*B + 4*b^3*B + 9*a^3*C - a*b^2*(A + 10*C))*EllipticPi[(2*a)
/(a + b), (c + d*x)/2, 2])/(a + b) + (4*b*(A*b^2 - a*b*B + 2*a^2*C - b^2*C)
*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b)))/a + (2*(A*b^2 - a*b*B + 3*a^2*C - 2*b^2*C)*(-2*a*b*EllipticE
[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c
+ d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]
], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*Sq
rt[Cos[c + d*x]]*((a*(A*b^2 + a*(-(b*B) + a*C))*Sin[c + d*x])/((a^2 - b^2)*
(b + a*Cos[c + d*x])) + 2*C*Tan[c + d*x]))/(4*b^2*d)
```

Maple [B] time = 9.099, size = 897, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b^2+B*a*b
-C*a^2)/a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
```

```
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(A*b^2-C*a^2)/b^2
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+2*C/b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2
*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{(b \sec (dx + c) + a)^2 \cos (dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*c
os(d*x + c)^(3/2)), x)
```

3.1328 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=387

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(5a^2C-3abB+3Ab^2-2b^2C\right)}{3b^2d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-5a^3C-ab^2(A-4C)-2b^3B\right)}{b^3d\left(a^2-b^2\right)}-\frac{(}{}$$

```
[Out] -(((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*EllipticE[(c + d*x)/2,
 2])/(b^3*(a^2 - b^2)*d)) + ((3*A*b^2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Ellipt
icF[(c + d*x)/2, 2])/(3*b^2*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - 5*a*b^
3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/((a - b)*b^3*(a + b)^2*d) + ((3*A*b^2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sin
[c + d*x])/(3*b^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((3*a^2*b*B - 2*b^3*B
 - a*b^2*(A - 4*C) - 5*a^3*C)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d
*x]^(3/2)*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.4961, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(5a^2C-3abB+3Ab^2-2b^2C\right)}{3b^2d\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(3a^2bB-5a^3C-ab^2(A-4C)-2b^3B\right)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-a^2b^2(A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])^2),x]
```


```
[Out] -(((3*a^2*b*B - 2*b^3*B - a*b^2*(A - 4*C) - 5*a^3*C)*EllipticE[(c + d*x)/2,
 2])/(b^3*(a^2 - b^2)*d)) + ((3*A*b^2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Ellipt
icF[(c + d*x)/2, 2])/(3*b^2*(a^2 - b^2)*d) - ((3*A*b^4 + 3*a^3*b*B - 5*a*b^
3*B - a^2*b^2*(A - 7*C) - 5*a^4*C)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/((a - b)*b^3*(a + b)^2*d) + ((3*A*b^2 - 3*a*b*B + 5*a^2*C - 2*b^2*C)*Sin
[c + d*x])/(3*b^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((3*a^2*b*B - 2*b^3*B
 - a*b^2*(A - 4*C) - 5*a^3*C)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d
*x]^(3/2)*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))^2}\,dx$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-3Ab^2+3abB-5a^2C+2b^2C \ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C-2b^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(Ab^2-a(bB-aC)\right)\mathrm{s}\ldots}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b\ldots}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C-2b^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(3a^2bB-2b^3B-ab^2(A-4\ldots\right.}{b^3\left(a^2-b^2\right)d\sqrt{\ldots}}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C-2b^2C\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(3a^2bB-2b^3B-ab^2(A-4\ldots\right.}{b^3\left(a^2-b^2\right)d\sqrt{\ldots}}$$

$$= -\frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3Ab^2-3ab\ldots\right.}{3b^2\left(a\ldots\right.}$$

$$= -\frac{\left(3a^2bB-2b^3B-ab^2(A-4C)-5a^3C\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3Ab^2-3ab\ldots\right.}{\ldots}$$

Mathematica [A] time = 7.17638, size = 474, normalized size = 1.22

$$\frac{\frac{\left(-24a^2b^2B+40a^3bC+12aAb^3-28ab^3C+12b^4B\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} + \frac{\sin(c+dx)\cos(2(c+dx))\left(3a^2Ab^2-12a^2b^2C-9a^3bB+15a^4C+\ldots\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + 
b*Sec[c + d*x])^2),x]
```

```
[Out] ((2*(9*a^2*A*b^2 - 12*A*b^4 - 27*a^3*b*B + 30*a*b^3*B + 45*a^4*C - 44*a^2*b
^2*C - 4*b^4*C)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + ((12*a
*A*b^3 - 24*a^2*b^2*B + 12*b^4*B + 40*a^3*b*C - 28*a*b^3*C)*(2*EllipticF[(c
 + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a
 + ((3*a^2*A*b^2 - 9*a^3*b*B + 6*a*b^3*B + 15*a^4*C - 12*a^2*b^2*C)*Cos[2*(
c + d*x)]*(-4*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 4*b*(a + b)*E
llipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*(a^2 - 2*b^2)*EllipticPi[-(a/b
), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[1 - Cos[c + 
d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(12*(a - b)*b^3*(a + b)*d) + (Sqrt[Cos[c
+ d*x]]*((2*Sec[c + d*x]*(b*B*Sin[c + d*x] - 2*a*C*Sin[c + d*x]))/b^3 + (a^
2*A*b^2*Sin[c + d*x] - a^3*b*B*Sin[c + d*x] + a^4*C*Sin[c + d*x])/(b^3*(-a^
2 + b^2)*(b + a*Cos[c + d*x])) + (2*C*Sec[c + d*x]*Tan[c + d*x])/(3*b^2)))/
d
```

Maple [B] time = 12.576, size = 1031, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C/b^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b^2-B*a*b+C*a^2)/b^2*(a^2/b/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)
/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*(B*b-2*C*a)/b^3/(a^2-a*b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+2*(B*b-2*C*a)/b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^2,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*c
os(d*x + c)^(5/2)), x)
```

3.1329 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=538

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^4b^2(128A-15C)-a^2b^4(223A-9C)+8a^6(A+3C)+99a^3b^3B-72a^5bB-45ab^5B+105Ab^6\right)}{12a^5d\left(a^2-b^2\right)^2}$$

```
[Out] -((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B - a^2*b^3*(65*A - 3*C) +
a^4*(24*A*b - 9*b*C))*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) +
((105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B + a^4*b^2*(128*A - 15*
C) - a^2*b^4*(223*A - 9*C) + 8*a^6*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(1
2*a^5*(a^2 - b^2)^2*d) - (b*(35*A*b^6 - 35*a^5*b*B + 38*a^3*b^3*B - 15*a*b^
5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((35*A*b^4
+ 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*
C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]
)^2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^3*B - 5*a^4*C - a^2*b^2*(13*A + C))*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 2.04328, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^4b^2(128A-15C)-a^2b^4(223A-9C)+8a^6(A+3C)+99a^3b^3B-72a^5bB-45ab^5B+105Ab^6\right)}{12a^5d\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```

```
[Out] -((35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*B - a^2*b^3*(65*A - 3*C) +
a^4*(24*A*b - 9*b*C))*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) +
((105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B + a^4*b^2*(128*A - 15*
C) - a^2*b^4*(223*A - 9*C) + 8*a^6*(A + 3*C))*EllipticF[(c + d*x)/2, 2])/(1
2*a^5*(a^2 - b^2)^2*d) - (b*(35*A*b^6 - 35*a^5*b*B + 38*a^3*b^3*B - 15*a*b^
5*B - a^2*b^4*(86*A - 3*C) + 3*a^4*b^2*(21*A - 2*C) + 15*a^6*C)*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((35*A*b^4
+ 33*a^3*b*B - 15*a*b^3*B + a^4*(8*A - 21*C) - a^2*b^2*(61*A - 3*C))*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + ((A*b^2 - a*(b*B - a*
C))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]
)^2) - ((7*A*b^4 + 9*a^3*b*B - 3*a*b^3*B - 5*a^4*C - a^2*b^2*(13*A + C))*Co
s[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\left(7Ab^4+9a^3b\right.}{}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C\right.}{12a^3\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(35Ab^4+33a^3bB-15ab^3B+a^4(8A-21C)-a^2b^2(61A-3C\right.}{12a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(35Ab^5-8a^5B+29a^3b^2B-15ab^4B+3a^4b(8A-3C)-a^2b\right.}{4a^4\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(35Ab^5-8a^5B+29a^3b^2B-15ab^4B+3a^4b(8A-3C)-a^2b\right.}{4a^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.42052, size = 604, normalized size = 1.12

$$\frac{\frac{\left(112a^3Ab^2+16a^5A+24a^2b^3B+24a^3b^2C-96a^4bB+48a^5C-56aAb^4\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}\right)}{a}+\frac{\sin(c+dx)\cos(2(c+dx))\left(195a^2Ab^3-72a^4Ab-\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35*A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a
*b^4*B - 15*a^4*b*C - 3*a^2*b^3*C)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a + b) + ((16*a^5*A + 112*a^3*A*b^2 - 56*a*A*b^4 - 96*a^4*b*B + 24*a^2*
b^3*B + 48*a^5*C + 24*a^3*b^2*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*Ellipt
icPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + ((-72*a^4*A*b + 195*a^2*
A*b^3 - 105*A*b^5 + 24*a^5*B - 87*a^3*b^2*B + 45*a*b^4*B + 27*a^4*b*C - 9*a
^2*b^3*C)*Cos[2*(c + d*x)]*(-4*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1
] + 4*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*(a^2 - 2*b^2)
*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*
Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(48*a^3*(a - b)^2*(a + b
)^2*d) + (Sqrt[Cos[c + d*x]]*((2*A*Sin[c + d*x])/(3*a^3) - (-(A*b^4*Sin[c +
 d*x]) + a*b^3*B*Sin[c + d*x] - a^2*b^2*C*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*
(b + a*Cos[c + d*x])^2) + (-17*a^2*A*b^3*Sin[c + d*x] + 11*A*b^5*Sin[c + d*
x] + 13*a^3*b^2*B*Sin[c + d*x] - 7*a*b^4*B*Sin[c + d*x] - 9*a^4*b*C*Sin[c +
```

```
d*x] + 3*a^2*b^3*C*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])
)))/d
```

Maple [B] time = 14.63, size = 2289, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^5*(4*A*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18
*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*A*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2+3*a^2*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2/a^5*b^2*(5*A*b^2-4*B*a*b+3*C*a^2
)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/a^4*b*(10*A*b^2-6*B*a*b+3*C
*a^2)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^3*(A*b^2-B*a*b+C*a^2)/a^5*(1/2*a^2/b/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
```

```
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)
/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-
a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C \sec\left(dx+c\right)^{2} + B \sec\left(dx+c\right) + A\right) \cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b \sec\left(dx+c\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^3, x)
```

3.1330 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=426

$$-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^3(33A+C)+a^4b(24A+7C)+5a^3b^2B-8a^5B-3ab^4B+15Ab^5\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a\right.}{}$$

```
[Out] ((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*
EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((15*A*b^5 - 8*a^5*B +
 5*a^3*b^2*B - 3*a*b^4*B - a^2*b^3*(33*A + C) + a^4*b*(24*A + 7*C))*Ellipti
cF[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((15*A*b^6 - 15*a^5*b*B + 6*a
^3*b^3*B - 3*a*b^5*B + 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C)
)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d)
+ ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2
)*d*(b + a*Cos[c + d*x])^2) - ((5*A*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a
^2*b^2*(11*A + 3*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*
d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.43768, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^3(33A+C)+a^4b(24A+7C)+5a^3b^2B-8a^5B-3ab^4B+15Ab^5\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(29A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^3,x]
```


```
[Out] ((15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*
EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((15*A*b^5 - 8*a^5*B +
 5*a^3*b^2*B - 3*a*b^4*B - a^2*b^3*(33*A + C) + a^4*b*(24*A + 7*C))*Ellipti
cF[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((15*A*b^6 - 15*a^5*b*B + 6*a
^3*b^3*B - 3*a*b^5*B + 3*a^6*C - a^2*b^4*(38*A + C) + 5*a^4*b^2*(7*A + 2*C)
)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d)
+ ((A*b^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2
)*d*(b + a*Cos[c + d*x])^2) - ((5*A*b^4 + 7*a^3*b*B - a*b^3*B - 3*a^4*C - a
^2*b^2*(11*A + 3*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*
d*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}(}{}}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\left(5Ab^4+7a^3\right.}{}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\left(5Ab^4+7a^3\right.}{}$$

$$= \frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)E}{4a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(15Ab^4+9a^3bB-3ab^3B+a^4(8A-5C)-a^2b^2(29A+C)\right)E}{4a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 6.3533, size = 441, normalized size = 1.04

$$\frac{\frac{16\left(-a^2b(4A+3C)+2a^3B+ab^2B+Ab^3\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{2\sin(c+dx)\left(-a^2b^2(29A+C)+a^4(8A-5C)+9a^3bB-3ab^3B+15Ab^4\right)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(}\right.\right.\right.}{a^2b\sqrt{\sin^2(c+}}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(b*(-5*A*b^4 - 7*a^3*b*B + a*b^3*B + 3*a^4*C + a^2*b
^2*(11*A + 3*C)) + a*(-7*A*b^4 - 9*a^3*b*B + 3*a*b^3*B + 5*a^4*C + a^2*b^2*
(13*A + C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])
^2) + ((2*(5*A*b^4 - 5*a^3*b*B - a*b^3*B + a^4*(8*A + C) + a^2*b^2*(-7*A +
5*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(A*b^3 + 2*a
^3*B + a*b^2*B - a^2*b*(4*A + 3*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - b*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*(15*A*b^4 + 9*a^3*
b*B - 3*a*b^3*B + a^4*(8*A - 5*C) - a^2*b^2*(29*A + C))*(2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]],
-1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16
*a^2*d)
```

Maple [B] time = 13.492, size = 2022, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+
3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a)-2/a^4*b*(4*A*b^2-3*B*a*b+2*C*a^2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2)))-2/a^3*(6*A*b^2-3*B*a*b+C*a^2)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^2
*(A*b^2-B*a*b+C*a^2)/a^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2
+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(
a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right) + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec
(d*x + c) + a)^3, x)
```

3.1331 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=423

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-a^2b^2(5A-3C)+a^4(8A+3C)-7a^3bB+ab^3B+3Ab^4\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(9A+5C\right.}{4a^2bc}$$

```
[Out] -((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*EllipticE[(
c + d*x)/2, 2])/(4*a^2*b*(a^2 - b^2)^2*d) + ((3*A*b^4 - 7*a^3*b*B + a*b^3*B
 - a^2*b^2*(5*A - 3*C) + a^4*(8*A + 3*C))*EllipticF[(c + d*x)/2, 2])/(4*a^3
*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2*
b^4*(2*A - C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*EllipticPi[(2*a)/(a + b), (c
 + d*x)/2, 2])/(4*a^3*(a - b)^2*b*(a + b)^3*d) + ((A*b^2 - a*(b*B - a*C))*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2)
+ ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.35867, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {4112, 3047, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(5A-3C)+a^4(8A+3C)-7a^3bB+ab^3B+3Ab^4\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-a^2b^2(9A+5C)+5a^3\right.}{4a^2bd\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Sec[
c + d*x])^3),x]
```


```
[Out] -((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*EllipticE[(
c + d*x)/2, 2])/(4*a^2*b*(a^2 - b^2)^2*d) + ((3*A*b^4 - 7*a^3*b*B + a*b^3*B
 - a^2*b^2*(5*A - 3*C) + a^4*(8*A + 3*C))*EllipticF[(c + d*x)/2, 2])/(4*a^3
*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B - 10*a^3*b^3*B + a*b^5*B - 3*a^2*
b^4*(2*A - C) - a^6*C + 5*a^4*b^2*(3*A + 2*C))*EllipticPi[(2*a)/(a + b), (c
 + d*x)/2, 2])/(4*a^3*(a - b)^2*b*(a + b)^3*d) + ((A*b^2 - a*(b*B - a*C))*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2)
+ ((3*A*b^4 + 5*a^3*b*B + a*b^3*B - a^4*C - a^2*b^2*(9*A + 5*C))*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(4*a*b*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+dx)}\left(C+B\cos(c+dx)+A\cos^2(c+dx)\right)}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \int \frac{\frac{1}{2}\left(Ab^2-a(bB-aC)\right)-2a(Ab-a\cdots}{\cdots}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\left(3Ab^4+5a^3bB+ab^3B-\cdots\right.}{4ab\left(\cdots\right.}$$

$$= \frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\left(3Ab^4+5a^3bB+ab^3B-\cdots\right.}{4ab\left(\cdots\right.}$$

$$= -\frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2b\left(a^2-b^2\right)^2d} + \frac{\left(Ab\cdots\right.}{\cdots}$$

$$= -\frac{\left(3Ab^4+5a^3bB+ab^3B-a^4C-a^2b^2(9A+5C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2b\left(a^2-b^2\right)^2d} + \frac{\left(3A\cdots\right.}{\cdots}$$

Mathematica [A] time = 5.813, size = 428, normalized size = 1.01

$$\frac{\frac{16b\left(a^2(2A+C)-3abB+b^2(A+2C)\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-\frac{2\sin(c+dx)\left(a^2b^2(9A+5C)-5a^3bB+a^4C-ab^3B-3Ab^4\right)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\cdots\right.\right.}{a^2b\sqrt{\sin^2(c+dx)}}\cdots}{(a-b)^2(a+b)^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*Sqrt[Cos[c + d*x]]*(-(b*(A*b^4 + 3*a^3*b*B + 3*a*b^3*B + a^4*C - 7*a^2
*b^2*(A + C))) + a*(-3*A*b^4 - 5*a^3*b*B - a*b^3*B + a^4*C + a^2*b^2*(9*A +
 5*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) +
 ((2*(-(A*b^4) + a^3*b*B + 5*a*b^3*B + 3*a^4*C - a^2*b^2*(5*A + 9*C))*Ellip
ticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*b*(-3*a*b*B + a^2*(2*A
+ C) + b^2*(A + 2*C))*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*
a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (2*(-3*A*b^4 - 5*a^3*b*B - a*b^3*B
+ a^4*C + a^2*b^2*(9*A + 5*C))*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2
)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*b*d)
```

Maple [B] time = 11.946, size = 1972, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/a
^3*(3*A*b^2-2*B*a*b+C*a^2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2
/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a
^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-
3*A*b+B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b*(A*b^2-B*a*b+C*a^2)/a^3*(1/2*a^
2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/
(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b
)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/
2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**3/cos(d*x+c)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*s
qrt(cos(d*x + c))), x)
```

3.1332 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=409

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(5A+9C)-a^3bB-3a\right.}{4ab^2d\left(a^2-b^2\right)^2}$$

```
[Out] -((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*EllipticE[(
c + d*x)/2, 2])/(4*a*b^2*(a^2 - b^2)^2*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B
 + a^4*C - 7*a^2*b^2*(A + C))*EllipticF[(c + d*x)/2, 2])/(4*a^2*b*(a^2 - b^
2)^2*d) - ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^2*(A - 2*C
) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((A*b^
4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.35396, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-7a^2b^2(A+C)+3a^3bB+a^4C+3ab^3B+Ab^4\right)}{4a^2bd\left(a^2-b^2\right)^2}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(5A+9C)-a^3bB-3a^4C-5a\right.}{4ab^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^3),x]
```


```
[Out] -((A*b^4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*EllipticE[(
c + d*x)/2, 2])/(4*a*b^2*(a^2 - b^2)^2*d) + ((A*b^4 + 3*a^3*b*B + 3*a*b^3*B
 + a^4*C - 7*a^2*b^2*(A + C))*EllipticF[(c + d*x)/2, 2])/(4*a^2*b*(a^2 - b^
2)^2*d) - ((A*b^6 - a^5*b*B + 10*a^3*b^3*B + 3*a*b^5*B - 3*a^4*b^2*(A - 2*C
) - 3*a^6*C - 5*a^2*b^4*(2*A + 3*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(4*a^2*(a - b)^2*b^2*(a + b)^3*d) - ((A*b^2 - a*(b*B - a*C))*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((A*b^
4 - a^3*b*B - 5*a*b^3*B - 3*a^4*C + a^2*b^2*(5*A + 9*C))*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] &&  !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(Ab^2-abB-3a^2C+4b^2C\right)+2b(b}{\sqrt{c}}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\left(Ab^4-a^3bB-5ab^3B-3a\right.}{4b^2\left(a^2\right.}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\left(Ab^4-a^3bB-5ab^3B-3a\right.}{4b^2\left(a^2\right.}$$

$$= -\frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2\left(a^2-b^2\right)^2d} - \frac{\left(Ab^2-\right.}{2}$$

$$= -\frac{\left(Ab^4-a^3bB-5ab^3B-3a^4C+a^2b^2(5A+9C)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2\left(a^2-b^2\right)^2d} + \frac{\left(Ab^4+\right.}{}$$

Mathematica [A] time = 6.26472, size = 444, normalized size = 1.09

$$\frac{\frac{16b\left(a^2bB+a^3C-ab^2(3A+4C)+2b^3B\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)}-\frac{2\sin(c+dx)\left(-a^2b^2(5A+9C)+a^3bB+3a^4C+5ab^3B-Ab^4\right)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a +
b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(b*(3*A*b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^
2*(3*A + 11*C)) - a*(-(A*b^4) + a^3*b*B + 5*a*b^3*B + 3*a^4*C - a^2*b^2*(5*
A + 9*C))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2
) + ((2*(3*a^3*b*B - 9*a*b^3*B + a^2*b^2*(A - 19*C) + 9*a^4*C + b^4*(5*A +
16*C))*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*b*(a^2*b*B
+ 2*b^3*B + a^3*C - a*b^2*(3*A + 4*C))*((a + b)*EllipticF[(c + d*x)/2, 2] -
 b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) - (2*(-(A*b^4) +
 a^3*b*B + 5*a*b^3*B + 3*a^4*C - a^2*b^2*(5*A + 9*C))*(2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1
])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b
^2*d)
```

Maple [B] time = 11.249, size = 1879, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-2*A*b+B*a)/
a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*A/a/(a^2-a*b)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+2*(A*b^2-B*a*b+C*a^2)/a^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a
+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(
a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a
-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*c
os(d*x + c)^(3/2)), x)
```

3.1333 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=496

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+3Ab^4\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(A+29C)+3a^3b\right.}{4b^3d\left(a^2-\right.}$$

```
[Out] ((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*
EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*A*b^4 + a^3*b*B -
7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A + 11*C))*EllipticF[(c + d*x)/2, 2])/(4*a
*b^2*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B + 6*a^3*b^3*B - 15*a*b^5*B +
15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*b*B - 9*a*
b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*Sin[c + d*x])/(4*b
^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d
*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2) + ((3*A*
b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A + 11*C))*Sin[c + d*x])/(
4*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.88118, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {4112, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(3A+11C)+a^3bB-5a^4C-7ab^3B+3Ab^4\right)}{4ab^2d\left(a^2-b^2\right)^2}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\left(a^2b^2(A+29C)+3a^3bB-15a\right.}{4b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])^3),x]
```


```
[Out] ((3*a^3*b*B - 9*a*b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*
EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*A*b^4 + a^3*b*B -
7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A + 11*C))*EllipticF[(c + d*x)/2, 2])/(4*a
*b^2*(a^2 - b^2)^2*d) - ((3*A*b^6 - 3*a^5*b*B + 6*a^3*b^3*B - 15*a*b^5*B +
15*a^6*C + 5*a^2*b^4*(2*A + 7*C) - a^4*b^2*(A + 38*C))*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*b*B - 9*a*
b^3*B + b^4*(5*A - 8*C) - 15*a^4*C + a^2*b^2*(A + 29*C))*Sin[c + d*x])/(4*b
^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - ((A*b^2 - a*(b*B - a*C))*Sin[c + d
*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2) + ((3*A*
b^4 + a^3*b*B - 7*a*b^3*B - 5*a^4*C + a^2*b^2*(3*A + 11*C))*Sin[c + d*x])/(
4*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```


Rule 4112

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x
_)])^(m_.)*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_)] + (C_.)*sec[(e_.) + (f_.)
*(x_)]^2), x_Symbol] :> Dist[d^(m + 2), Int[(b + a*Cos[e + f*x])^m*(d*Cos[e
 + f*x])^(n - m - 2)*(C + B*Cos[e + f*x] + A*Cos[e + f*x]^2), x], x] /; Fre
eQ[{a, b, d, e, f, A, B, C, n}, x] && !IntegerQ[n] && IntegerQ[m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{C+B\cos(c+dx)+A\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}\left(-Ab^2+abB-5a^2C+4b^2\right.}{}}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(b+a\cos(c+dx))^2} + \frac{\left(3Ab^4+a^3bB-7ab^3\right.}{4b^2\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)E\left(\frac{1}{2}(c+dx)\right|}{4b^3\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(3a^3bB-9ab^3B+b^4(5A-8C)-15a^4C+a^2b^2(A+29C)\right)E\left(\frac{1}{2}(c+dx)\right|}{4b^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.30034, size = 594, normalized size = 1.2

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{a^2bB\sin(c+dx)+a^3(-C)\sin(c+dx)-aAb^2\sin(c+dx)}{2b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{-a^3Ab^2\sin(c+dx)+9a^2b^3B\sin(c+dx)-13a^3b^2C\sin(c+dx)-3a^4bB\sin(c+dx)+7a^5}{4b^3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])^3),x]
```

```
[Out] -((2*(-3*a^3*A*b^2 + 9*a*A*b^4 - 9*a^4*b*B + 19*a^2*b^3*B - 16*b^5*B + 45*a
^5*C - 95*a^3*b^2*C + 56*a*b^4*C)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a + b) + ((-8*a^2*A*b^3 - 16*A*b^5 - 8*a^3*b^2*B + 32*a*b^4*B + 40*a^4*b
*C - 80*a^2*b^3*C + 16*b^5*C)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticP
i[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + ((-(a^3*A*b^2) - 5*a*A*b^4
- 3*a^4*b*B + 9*a^2*b^3*B + 15*a^5*C - 29*a^3*b^2*C + 8*a*b^4*C)*Cos[2*(c +
 d*x)]*(-4*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 4*b*(a + b)*Elli
pticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*(a^2 - 2*b^2)*EllipticPi[-(a/b),
-ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[1 - Cos[c + d*x
]^2]*(-1 + 2*Cos[c + d*x]^2)))/(16*(a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Cos[c
 + d*x]]*((-(a*A*b^2*Sin[c + d*x]) + a^2*b*B*Sin[c + d*x] - a^3*C*Sin[c + d
*x])/(2*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (-(a^3*A*b^2*Sin[c + d*x
]) - 5*a*A*b^4*Sin[c + d*x] - 3*a^4*b*B*Sin[c + d*x] + 9*a^2*b^3*B*Sin[c +
d*x] + 7*a^5*C*Sin[c + d*x] - 13*a^3*b^2*C*Sin[c + d*x]))/(4*b^3*(-a^2 + b^2
)^2*(b + a*Cos[c + d*x])) + (2*C*Tan[c + d*x])/b^3))/d
```

Maple [B] time = 14.485, size = 2049, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b^2-C*a^2)
/b^2/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*C/b^3/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+2*(-A*b^2+B*a*b-C*a^2)/a/b*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-
a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a
*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*C/b^3*(-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+
2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(b \sec(dx+c) + a)^3 \cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^3,x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*c
os(d*x + c)^(5/2)), x)
```

3.1334 $\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\big(A+B\sec(c+dx)+C\,\mathrm{s}$

Optimal. Leaf size=457

$$-\frac{2\left(a^2-b^2\right)\left(6a^2b(6A+7C)-75a^3B-24ab^2B+16Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c$$

```
[Out] (-2*(a^2 - b^2)*(16*A*b^3 - 75*a^3*B - 24*a*b^2*B + 6*a^2*b*(6*A + 7*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(31
5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(16*A*b^4 - 57*a^
3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3
15*a^4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(8*A*b^3 + 75*a^3*B - 12*
a*b^2*B + a^2*b*(13*A + 21*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(315*a^3*d) - (2*(6*A*b^2 - 9*a*b*B - 7*a^2*(7*A + 9*C))*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d) + (2*(A*b
 + 9*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*a*d
) + (2*A*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.79839, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(-7a^2(7A+9C)-9abB+6Ab^2\right)\sqrt{a+b\sec(c+dx)}}{315a^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2b(13A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (-2*(a^2 - b^2)*(16*A*b^3 - 75*a^3*B - 24*a*b^2*B + 6*a^2*b*(6*A + 7*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(31
5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(16*A*b^4 - 57*a^
3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*Sqrt[Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3
15*a^4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(8*A*b^3 + 75*a^3*B - 12*
a*b^2*B + a^2*b*(13*A + 21*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(315*a^3*d) - (2*(6*A*b^2 - 9*a*b*B - 7*a^2*(7*A + 9*C))*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d) + (2*(A*b
 + 9*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*a*d
) + (2*A*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec}}{}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{9d}$$

$$= \frac{2(Ab+9aB)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}{63ad}$$

$$= -\frac{2\left(6Ab^2-9abB-7a^2(7A+9C)\right)\cos^{\frac{3}{2}}(c}{315a^2}$$

$$= \frac{2\left(8Ab^3+75a^3B-12ab^2B+a^2b(13A+2\right.}{}$$

$$= \frac{2\left(8Ab^3+75a^3B-12ab^2B+a^2b(13A+2\right.}{}$$

$$= \frac{2\left(8Ab^3+75a^3B-12ab^2B+a^2b(13A+2\right.}{}$$

$$= \frac{2\left(8Ab^3+75a^3B-12ab^2B+a^2b(13A+2\right.}{}$$

$$= -\frac{2\left(a^2-b^2\right)\left(16Ab^3-75a^3B-24ab^2B+6\right.}{315a^4d\sqrt{\cos(c+}}$$

Mathematica [C] time = 24.3367, size = 3595, normalized size = 7.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(((57*a^2*A*b + 32*A*b^3 + 345
*a^3*B - 48*a*b^2*B + 84*a^2*b*C)*Sin[c + d*x])/(630*a^3) + ((133*a^2*A - 1
2*A*b^2 + 18*a*b*B + 126*a^2*C)*Sin[2*(c + d*x)])/(630*a^2) + ((A*b + 9*a*B
)*Sin[3*(c + d*x)])/(126*a) + (A*Sin[4*(c + d*x)])/36))/d - (2*Cos[c + d*x]
^(3/2)*((7*a*A*Sqrt[Cos[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) - (8*A*b^2*Sqrt[Cos[c + d*x]])/(105*a*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (16*A*b^4*Sqrt[Cos[c + d*x]])/(315*a^3*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (19*b*B*Sqrt[Cos[c + d*x]])/(105*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*b^3*B*Sqrt[Cos[c + d*x]])/(105*a^2*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (3*a*C*Sqrt[Cos[c + d*x]])/(5*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*b^2*C*Sqrt[Cos[c + d*x]])
```

```
/(15*a*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (37*A*b*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) - (4*A*b^3*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(315*a^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a*B
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (2*
b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[b + a*Cos[c + d*x]
]) + (7*b*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d
*x]]))*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((-
I)*(a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 2
1*a^4*(7*A + 9*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
 + I*a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C)
+ 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) -
 21*a^4*(7*A + 9*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c
 + d*x)/2]))/(315*a^4*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*(-(Cos[c +
d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a +
b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7
*A + 9*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(
a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3*a^3*
(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] + (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*
(7*A + 9*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/
2]))/(315*a^3*(b + a*Cos[c + d*x])^(3/2)) + (Sqrt[Cos[c + d*x]]*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(-16*A*b^4 + 57*a^3
*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*EllipticE[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-16*A*b^3 + 12
*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*
EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (16*A*b^4 - 57*a
^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*(b + a*Co
s[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a^4*Sqrt[b +
 a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])
^(3/2)*(((16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a
^4*(7*A + 9*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a +
 b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(
7*A + 9*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
 + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c
 + d*x)/2] + I*a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3
*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - a*(16*A*b^4 - 57*a^3*b*B - 24*a*b^
3*B + 6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*(Sec[(c + d*x)/2]^2)^(3/2
)*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*(16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B +
6*a^2*b^2*(4*A + 7*C) - 21*a^4*(7*A + 9*C))*(b + a*Cos[c + d*x])*(Sec[(c +
d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(-16*A*b^4 + 57*a^3
*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*EllipticE[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[
(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)] + ((I/2)*a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b
*(6*A + 3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*
Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
- (a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A + 3*B + 7*C) +
```

```
3*a^3*(49*A + 25*B + 63*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 + ((-a
+ b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*
a*b^3*B - 6*a^2*b^2*(4*A + 7*C) + 21*a^4*(7*A + 9*C))*Sec[(c + d*x)/2]^4*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Ta
n[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(315*a^4*Sqr
t[b + a*Cos[c + d*x]]) - (Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*((-I)*(a + b)*(-16*A*b^4 + 57*a^3*b*B + 24*a*b^3*B - 6*a^2*b^2*(4*A
 + 7*C) + 21*a^4*(7*A + 9*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + I*a*(a + b)*(-16*A*b^3 + 12*a*b^2*(A + 2*B) - 6*a^2*b*(6*A +
3*B + 7*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] + (16*A*b^4 - 57*a^3*b*B - 24*a*b^3*B + 6*a^2*b^2*(4
*A + 7*C) - 21*a^4*(7*A + 9*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(
3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
 Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^4*Sqrt[b + a*Cos[c +
 d*x]])))
```

Maple [B] time = 1.109, size = 4075, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(45*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^5
*(1/(cos(d*x+c)+1))^(3/2)+63*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^5*(1/(cos
(d*x+c)+1))^(3/2)*sin(d*x+c)+63*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^5*(1/(
cos(d*x+c)+1))^(3/2)*sin(d*x+c)+189*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1
/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+147*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(
d*x+c)*a^5*(1/(cos(d*x+c)+1))^(3/2)+35*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)^5*a^5*(1/(cos(d*x+c)+1))^(3/2)+147*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)
*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+75*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*b*(
1/(cos(d*x+c)+1))^(3/2)+57*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(cos
(d*x+c)+1))^(3/2)-12*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c
)+1))^(3/2)+24*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3
/2)+189*C*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+21*
C*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-42*C*((a-
b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-24*A*((a-b)/(a+
b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+8*A*((a-b)/(a+b))^(1/
2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)+49*A*((a-b)/(a+b))^(1/2)*sin(d
*x+c)*cos(d*x+c)^2*a^5*(1/(cos(d*x+c)+1))^(3/2)+49*A*((a-b)/(a+b))^(1/2)*si
n(d*x+c)*cos(d*x+c)^3*a^5*(1/(cos(d*x+c)+1))^(3/2)+45*B*((a-b)/(a+b))^(1/2)
*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^4*a^5+75*B*((a-b)/(a+b))^(1
/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*a^5+75*B*((a-b)/(a+b))
^(1/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*a^5+35*A*((a-b)/(a+b)
)^(1/2)*sin(d*x+c)*cos(d*x+c)^4*a^5*(1/(cos(d*x+c)+1))^(3/2)-16*A*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+147*A*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-147*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^5-75*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
```

(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5+189*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-189*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+84*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+62*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+62*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+2*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+40*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^4*b*(1/(cos(d*x+c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+40*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*a^4*b*(1/(cos(d*x+c)+1))^(3/2)-3*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-8*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)+54*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^4*b*(1/(cos(d*x+c)+1))^(3/2)-11*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+2*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+12*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+54*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)-3*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+132*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+84*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-21*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+13*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-111*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+57*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-6*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+24*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-57*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+57*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-24*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+24*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-147*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-42*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+189*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+42*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-42*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-16*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^5*(1/(cos(d*x+c)+1))^(3/2)-24*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+4*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-16*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+147*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt

```
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
3*b^2-24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+16*A
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4)/a^4/((a-b)/(a+b
))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)^4\sec(dx+c)^2 + B\cos(dx+c)^4\sec(dx+c) + A\cos(dx+c)^4\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4*sec(d*x + c)^2 + B*cos(d*x + c)^4*sec(d*x + c) +
 A*cos(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1335 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=360

$$\frac{2\left(a^2-b^2\right)\left(25a^2A+35a^2C-14abB+8Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(-5a\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^3*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B
 + a^2*b*(19*A + 35*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*Cos[c + d*x]
^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d) + (2*A*Cos[c + d*x]^
(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.29943, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(-5a^2(5A+7C)-7abB+4Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105a^2d}+\frac{2\left(a^2-b^2\right)\left(25a^2A+35a^2C-14abB\right.}{105a^3d\sqrt{\cos(c+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^3*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B
 + a^2*b*(19*A + 35*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]) - (2*(4*A*b^2 - 7*a*b*B - 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*Cos[c + d*x]
^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d) + (2*A*Cos[c + d*x]^
(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

```
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{7d}$$

$$= \frac{2(Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{35ad}$$

$$= -\frac{2\left(4Ab^2-7abB-5a^2(5A+7C)\right)\sqrt{\cos(c}}{105a^2}$$

$$= -\frac{2\left(4Ab^2-7abB-5a^2(5A+7C)\right)\sqrt{\cos(c}}{105a^2}$$

$$= -\frac{2\left(4Ab^2-7abB-5a^2(5A+7C)\right)\sqrt{\cos(c}}{105a^2}$$

$$= -\frac{2\left(4Ab^2-7abB-5a^2(5A+7C)\right)\sqrt{\cos(c}}{105a^2}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A+8Ab^2-14abB+35a\right.}{105a^3d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [C] time = 22.9347, size = 3071, normalized size = 8.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(((115*a^2*A - 16*A*b^2 + 28*a
*b*B + 140*a^2*C)*Sin[c + d*x])/(210*a^2) + ((A*b + 7*a*B)*Sin[2*(c + d*x)]
)/(35*a) + (A*Sin[3*(c + d*x)])/14))/d - (2*Cos[c + d*x]^(3/2)*((19*A*b*Sqr
t[Cos[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*A*b
^3*Sqrt[Cos[c + d*x]])/(105*a^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) + (3*a*B*Sqrt[Cos[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (2*b^2*B*Sqrt[Cos[c + d*x]])/(15*a*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) + (5*a*A*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*C
os[c + d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt
[b + a*Cos[c + d*x]]) + (7*b*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*S
qrt[b + a*Cos[c + d*x]]) + (a*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*S
qrt[b + a*Cos[c + d*x]]))*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sqrt[a +
b*Sec[c + d*x]]*((-I)*(a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*
A + 35*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(
a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B + 35*C))*EllipticF[I
```

```
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (8*A*b^3 + 63*a^3*B - 14*a*
b^2*B + a^2*b*(19*A + 35*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/(105*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*(-(
Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-
I)*(a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*Elliptic
E[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^2 - 2
*a*b*(3*A + 7*B) + a^2*(25*A + 63*B + 35*C))*EllipticF[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] - (8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A
 + 35*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])
)/(105*a^2*(b + a*Cos[c + d*x])^(3/2)) + (Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)
/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(8*A*b^3 + 63*a^3*B -
14*a*b^2*B + a^2*b*(19*A + 35*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63
*B + 35*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
 + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (8*A
*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*(b + a*Cos[c + d*x])*(S
ec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(35*a^3*Sqrt[b + a*Cos[c + d*x]
]) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((8*A*
b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*(b + a*Cos[c + d*x])*(Se
c[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a
^2*b*(19*A + 35*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
]*Tan[(c + d*x)/2] + I*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A +
 63*B + 35*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec
[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan
[(c + d*x)/2] + a*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*(
Sec[(c + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(8*A*b^3 + 63*
a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*
x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(8*A*b^3 + 63*a^3*B -
 14*a*b^2*B + a^2*b*(19*A + 35*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/
(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a +
b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a
+ b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B + 35*C))*EllipticF[I*A
rcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c
 + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] - (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B +
 35*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2
]^2)/(a + b)]) + ((a + b)*(8*A*b^3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A +
35*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c
+ d*x)/2]^2])))/(105*a^3*Sqrt[b + a*Cos[c + d*x]]) - (Cos[c + d*x]^(3/2)*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(8*A*b^3 + 63*a^3*B - 14*
a*b^2*B + a^2*b*(19*A + 35*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] + I*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B
+ 35*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (8*A*b^
3 + 63*a^3*B - 14*a*b^2*B + a^2*b*(19*A + 35*C))*(b + a*Cos[c + d*x])*(Sec[
(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Si
n[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(35*a^3*Sq
rt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 0.837, size = 2829, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(25*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4
*(1/(cos(d*x+c)+1))^(3/2)+19*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^3*b-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
^2+8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-19*A*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+19*A*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-8*A*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^3+8*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^4*(1/(co
s(d*x+c)+1))^(3/2)+25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(c
os(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(
cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/
(cos(d*x+c)+1))^(3/2)+25*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x
+c)+1))^(3/2)+19*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1)
)^(3/2)-4*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+6
3*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+7*B*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-14*B*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+35*C*((a-b)/(a+b))^(1/2)
*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+35*C*((a-b)/(a+b))^(1/2)*a^2*b^2
*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*
sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/
2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+63*B*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+14*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b^2-14*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^3-49*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a^3*b-14*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-35*C
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+35*C*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+35*C*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+18*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+28*B*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+44*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+4*A*cos(d*x+c)*((a-b)/(
```

```
a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+28*B*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-7*B*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+70*C*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+18*A*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-A*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-63
*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-35*C*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^4-25*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^4+8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^
4)/a^3/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2)/sin(d*
x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^(7/2), x)
```

3.1336 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\big(A+B\sec(c+dx)+C$

Optimal. Leaf size=273

$$-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\cdots}{15a^2d\sqrt{\frac{a\cos(c+dx)\cdots}{a+b}}}$$

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]) - (2*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*S
qrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2*A*Cos[c + d*x]^(3/2)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.938143, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sqrt{\cos(c+dx)}\left(-3a^2(3A+5C)-5abB+2Ab^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{a\cdots}{}}}{15a^2d\sqrt{\cos(c+dx)}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]) - (2*(2*A*b^2 - 5*a*b*B - 3*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*S
qrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2*A*Cos[c + d*x]^(3/2)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b}}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin}{5d}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{15ad}$$

$$= -\frac{2\left(a^2-b^2\right)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

Mathematica [C] time = 17.7665, size = 404, normalized size = 1.48

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(\frac{2(5aB+Ab)\sin(c+dx)}{15a}+\frac{1}{5}A\sin(2(c+dx))\right)}{d} - \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+d}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*((2*(A*b + 5*a*B)*Sin[c + d*x]
)/(15*a) + (A*Sin[2*(c + d*x)])/5))/d - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((-I)*(a + b)*(-2*A*b^2
 + 5*a*b*B + 3*a^2*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] + I*a*(a + b)*(9*a*A - 2*A*b + 5*a*(B + 3*C))*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-2*A*b^2 + 5*a*b*B + 3*a^2*(3*
A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])
)/(15*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.552, size = 1966, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1)
^2*(-1+cos(d*x+c))^3*(3*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*(
1/(cos(d*x+c)+1))^(3/2)+9*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*
x+c)+1))^(3/2)+A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3
/2)+5*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+5*B*s
in(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+15*C*((a-b)/(a
+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+3*A*sin(d*x+c)*((a-b)/
(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3/2)+9*A*sin(d*x+c)*((a-b
)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)+5*B*sin(d*x+c)*((a-b
)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3/2)+5*B*sin(d*x+c)*((a
-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)-2*A*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+9*A*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^3-9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^3-5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+15*
C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-15*C*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-2*A*sin(d*x+c)*((a-b)/(a+b))^(1
/2)*b^3*(1/(cos(d*x+c)+1))^(3/2)+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a^2*b-5*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2*b+5*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-15*C*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+15*C*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-7*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^2*b-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a*b^2+9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*b+2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+4*A*sin
(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+4*A
*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-A
*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+1
0*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2
)+15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/
2))/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2)/sin(d
*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
```

```
cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\cos(dx+c)^2\sec(dx+c)^2 + B\cos(dx+c)^2\sec(dx+c) + A\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^(5/2), x)
```

3.1337 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=277

$$-\frac{2\left(Ab^2-a^2(A+3C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{3ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*(A*b^2 - a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]) + (2*b*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b +
 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.02884, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(Ab^2-a^2(A+3C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] (-2*(A*b^2 - a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]) + (2*b*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b +
 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec}}{}$$

$$=\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{3d}$$

$$=\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{3d}$$

$$=\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{3d}$$

$$=\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+}{3d}$$

$$=\frac{2bC\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2}{}$$

$$=-\frac{2\left(\frac{Ab^2}{a}-a(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+a}}$$

Mathematica [C] time = 33.2616, size = 43023, normalized size = 155.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.588, size = 1256, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^3*(A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*
a^2+A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^
2+2*A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*
b+3*B*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^
2+A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b+A*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*b^2+3*B*sin(d*x+c)*(1/(cos(d
*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*b^2-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+A*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*C*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*C*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
-b),I/((a-b)/(a+b))^(1/2))*a*b)*cos(d*x+c)^(1/2)/a/((a-b)/(a+b))^(1/2)/(b+a
*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right) \sqrt{b \sec \left(dx+c\right)+a} \cos \left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
cos(d*x + c)^(3/2), x)
```

3.1338 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+\right.$

Optimal. Leaf size=258

$$\frac{(2aB+bC)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2A-C)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

```
[Out] ((2*a*B + b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B + a
*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A - C)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.9644, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2A-C)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2aB+bC)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(aC+2bB}{d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec
[c + d*x]^2),x]
```


```
[Out] ((2*a*B + b*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B + a
*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A - C)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b}\cdots}{\cdots}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cdots}\right.$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cdots}\right.$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\cdots\right.$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left((-\cdots\right.}{\cdots}$$

$$= \frac{(2bB+aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+d\cdots\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\cdots}$$

$$= \frac{(2aB+bC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\cdots\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 32.7951, size = 64644, normalized size = 250.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] +
 C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.529, size = 1114, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(2*A*cos(d*x+c)^2*sin(d*x+c)*(1/(co
s(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a+2*A*cos(d*x+c)*sin(d*x+c)*(1/(cos(
d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*b+C*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+
c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a+C*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((
a-b)/(a+b))^(1/2)*b+2*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a-2*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b-2*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a+2*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-4*B*
cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*b-2*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+2*
B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*C*cos(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+
C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-C*cos(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b)*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1)
)^(3/2)/sin(d*x+c)^6/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)*(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*
sqrt(cos(d*x + c)), x)
```

3.1339 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=346

$$\frac{(8aA+3aC+4bB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((8*a*A + 4*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
 ((8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) - ((4*b*B + a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^
(3/2)) + ((4*b*B + a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 1.32705, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(a^2(-C)+4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(8aA+3aC+4bB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```


```
[Out] ((8*a*A + 4*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
 ((8*A*b^2 + 4*a*b*B - a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) - ((4*b*B + a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^
(3/2)) + ((4*b*B + a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[
Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
```

```
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+b$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB+aC)\sqrt{a+}}{4bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB+aC)\sqrt{a+}}{4bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB+aC)\sqrt{a+}}{4bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB+aC)\sqrt{a+}}{4bd}$$

$$= \frac{\left(8Ab^2+4abB-a^2C+4b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(\right.}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(8aA+4bB+3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 33.4586, size = 100266, normalized size = 289.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.51, size = 1579, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 1/4/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(-4*B*((a-b)/(a+b))^(1/2)*cos(d*x+
c)^2*sin(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)-C*((a-b)/(a+b))^(1/2)*cos(d*x+
c)^2*a^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-2*C*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-4*B*((a-b)/(a+b))^(1/2)*cos(
```

```
d*x+c)*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(3/2)-3*C*((a-b)/(a+b))^(1/2)*cos(
d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-2*C*sin(d*x+c)*((a-b)/(a+b))
^(1/2)*cos(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(3/2)-2*C*sin(d*x+c)*((a-b)/(a+b))
^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^2+8*A*cos(d*x+c)^2*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b-8*A*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*b^2+16*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-4*B*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b+4*B*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*b^2+8*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-C*cos(d*x+c)^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+C*cos(d*x+c)^2*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*C*cos(d*x+c)^2*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2+2*C*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*C*cos(d*x+c)^2*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2-2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*C*cos(d*x+c)^2*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2)*((b+a*cos(d
*x+c))/cos(d*x+c))^(1/2)/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x
+c)+1))^(3/2)/sin(d*x+c)^6/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*(A + B*sec(c + d*x) + C*sec(c + d*x)**2)/
sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)\sqrt{b\sec(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
sqrt(cos(d*x + c)), x)
```

3.1340 $\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=447

$$\frac{\left(a^2(-C)+18abB+24Ab^2+16b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\left(-3a^2C+6abB+24Ab^2+16\cdots\right.}{24b^2d\sqrt{\cos(c+d\cdots}}$$

```
[Out] ((24*A*b^2 + 18*a*b*B - a^2*C + 16*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) - ((2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
(8*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 + 6*a*b*
B - 3*a^2*C + 16*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + (
(6*b*B + a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*b*d*Cos[c + d*x]^(
3/2)) + ((24*A*b^2 + 6*a*b*B - 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.77156, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\left(-3a^2C+6abB+24Ab^2+16b^2C\right)\sqrt{a+b\sec(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}+\frac{\left(a^2(-C)+18abB+24Ab^2+16b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\cdots}{24bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```


```
[Out] ((24*A*b^2 + 18*a*b*B - a^2*C + 16*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) - ((2*a^2*b*B - 8*b^3*B - a^3*C - 4*a*b^2*(2*A + C))*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
(8*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 + 6*a*b*
B - 3*a^2*C + 16*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + (
(6*b*B + a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*b*d*Cos[c + d*x]^(
3/2)) + ((24*A*b^2 + 6*a*b*B - 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6bB+aC)\sqrt{a+}}{12bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6bB+aC)\sqrt{a+}}{12bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6bB+aC)\sqrt{a+}}{12bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6bB+aC)\sqrt{a+}}{12bd}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6bB+aC)\sqrt{a+}}{12bd}$$

$$= -\frac{\left(2a^2bB-8b^3B-a^3C-4ab^2(2A+C)\right)\sqrt{\frac{b+a\cos(c+}{a+b}}}{8b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

$$= \frac{\left(24Ab^2+18abB-a^2C+16b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{24bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 34.1696, size = 131249, normalized size = 293.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)
)/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.663, size = 2548, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] 1/24/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(-18*B*sin(d*x+c)*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+C*cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-10*C*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-10*C*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-24*A*sin(d*x+c)*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-6*B*sin(d*x
+c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-12*B*si
n(d*x+c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-2*
C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2
)-16*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(3/2)-24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^
3*a*b^2-6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b+6*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2+24*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*cos(d*x+c)^3*a*b^2+2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*cos(d*x+c)^3*a^2*b+4*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*cos(d*x+c)^3*a*b^2-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*cos(d*x+c)^3*a^2*b-16*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*cos(d*x+c)^3*a*b^2+48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(3/2)-16*C*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-12*B*cos(d*x+c)^2*sin(d*x+c)*(
(a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^3-12*B*cos(d*x+c)*sin(d*x+c)*
((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^3-8*C*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-12*B*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+12*B*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+12*B*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*c
os(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+24*A*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^3-8*C*((a-b)
/(a+b))^(1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+48*B*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*b^3-24*B*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+6*C*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a
^3-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^3
+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^3+1
6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^3+3*
C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2))
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/b^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c
))/sin(d*x+c)^6/(1/(cos(d*x+c)+1))^(3/2)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*(a+b*sec(d*x+c))**(1/2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/
cos(d*x + c)^(3/2), x)
```

3.1341 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=455

$$\frac{2\left(a^2-b^2\right)\left(a^2(39Ab+63bC)+75a^3B-18ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c$$

```
[Out] (2*(a^2 - b^2)*(8*A*b^3 + 75*a^3*B - 18*a*b^2*B + a^2*(39*A*b + 63*b*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(31
5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 246*a^
3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/
(315*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b^3 - 75*a^3*B - 9
*a*b^2*B - 2*a^2*b*(44*A + 63*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(315*a^2*d) + (2*(3*A*b^2 + 72*a*b*B + 7*a^2*(7*A + 9*C))*
Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(A
*b + 3*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(21*d
) + (2*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.85682, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(7a^2(7A+9C)+72abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}}{315ad}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(-2a^2b(44A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (2*(a^2 - b^2)*(8*A*b^3 + 75*a^3*B - 18*a*b^2*B + a^2*(39*A*b + 63*b*C))*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(31
5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 246*a^
3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/
(315*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b^3 - 75*a^3*B - 9
*a*b^2*B - 2*a^2*b*(44*A + 63*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(315*a^2*d) + (2*(3*A*b^2 + 72*a*b*B + 7*a^2*(7*A + 9*C))*
Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(A
*b + 3*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(21*d
) + (2*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\mathrm{se}}{}$$

$$= \frac{2A\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\mathrm{si}}{9d}$$

$$= \frac{2(Ab+3aB)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}}{21d}$$

$$= \frac{2\left(3Ab^2+72abB+7a^2(7A+9C)\right)\cos^{\frac{3}{2}}}{315}$$

$$= -\frac{2\left(4Ab^3-75a^3B-9ab^2B-2a^2b(44A\right.}{}$$

$$= -\frac{2\left(4Ab^3-75a^3B-9ab^2B-2a^2b(44A\right.}{}$$

$$= -\frac{2\left(4Ab^3-75a^3B-9ab^2B-2a^2b(44A\right.}{}$$

$$= -\frac{2\left(4Ab^3-75a^3B-9ab^2B-2a^2b(44A\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(8Ab^3+75a^3B-18ab^2B+a\right.}{315a^3d\sqrt{\cos(c}}$$

Mathematica [C] time = 24.4856, size = 3703, normalized size = 8.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*(((402*a^2*A*b - 16*A*b^3 + 345*a^3*B + 36*a*b^2*B + 504*a^2*b*
C)*Sin[c + d*x])/(315*a^2) + ((133*a^2*A + 6*A*b^2 + 144*a*b*B + 126*a^2*C)
*Sin[2*(c + d*x)])/(315*a) + ((10*A*b + 9*a*B)*Sin[3*(c + d*x)])/63 + (a*A*
Sin[4*(c + d*x)])/18))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x]
+ A*Cos[2*c + 2*d*x])) - (4*Cos[c + d*x]^(3/2)*((14*a^2*A*Sqrt[Cos[c + d*x]
])/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (22*A*b^2*Sqrt[Cos[c
+ d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^4*Sqrt
[Cos[c + d*x]])/(315*a^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16
4*a*b*B*Sqrt[Cos[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (4*b^3*B*Sqrt[Cos[c + d*x]])/(35*a*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
```

```
 + d*x]]) + (6*a^2*C*Sqrt[Cos[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]) + (2*b^2*C*Sqrt[Cos[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]*Sq
rt[Sec[c + d*x]]) + (124*a*A*b*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(105*
Sqrt[b + a*Cos[c + d*x]]) + (4*A*b^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
/(315*a*Sqrt[b + a*Cos[c + d*x]]) + (10*a^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (34*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (8*a*b*C*Sqrt[Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]))*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((-I)*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9
*C) + 3*a^2*b^2*(11*A + 21*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 5
7*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2)/(a + b)] - (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A
 + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2
)^(3/2)*Tan[(c + d*x)/2]))/(315*a^3*d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B
*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(7/2)*((-2*Cos[c + d*x]^(3
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(8*A
*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21
*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)
*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A
+ 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Se
c[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] -
(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A
+ 21*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))
/(315*a^2*(b + a*Cos[c + d*x])^(3/2)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(8*A*b^4 + 246*a^3*b*
B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*EllipticE[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 - 6*a*b
^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (8*A*b^4 + 246*a^
3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(b + a*C
os[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a^3*Sqrt[b
+ a*Cos[c + d*x]]) - (4*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(3/2)*(-((8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2
*b^2*(11*A + 21*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*
(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^
2*(11*A + 21*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
ec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*T
an[(c + d*x)/2] + I*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A
+ 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + a*(8*A*b^4 + 246*a^3*b*B - 18
*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(Sec[(c + d*x)/2]^
2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(8*A*b^4 + 246*a^3*b*B - 18*a*b
^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(b + a*Cos[c + d*x])*(
Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(8*A*b^4 +
 246*a^3*b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) +
3*a^2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c +
```

```
d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] - (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^2*b*(13*A + 57*B
+ 21*C) + 3*a^3*(49*A + 25*B + 63*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt
[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(8*A*b^4 + 246*a^3*
b*B - 18*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*Sec[(c + d
*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + (
(-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(
315*a^3*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x
)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B + 21
*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*EllipticE[I*ArcSinh[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^
2*b*(13*A + 57*B + 21*C) + 3*a^3*(49*A + 25*B + 63*C))*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B
+ 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(11*A + 21*C))*(b + a*Cos[c + d*x])*(Sec[(
c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin
[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^3*Sq
rt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 1.086, size = 4075, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(45*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^5
*(1/(cos(d*x+c)+1))^(3/2)+63*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^5*(1/(cos
(d*x+c)+1))^(3/2)*sin(d*x+c)+63*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^5*(1/(
cos(d*x+c)+1))^(3/2)*sin(d*x+c)+189*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1
/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+147*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(
d*x+c)*a^5*(1/(cos(d*x+c)+1))^(3/2)+35*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)^5*a^5*(1/(cos(d*x+c)+1))^(3/2)+147*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)
*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+75*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*b*(
1/(cos(d*x+c)+1))^(3/2)+246*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(co
s(d*x+c)+1))^(3/2)+9*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c
)+1))^(3/2)-18*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3
/2)+189*C*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+126
*C*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+63*C*((a
-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+33*A*((a-b)/(a
+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)-4*A*((a-b)/(a+b))^(1
/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)+49*A*((a-b)/(a+b))^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*a^5*(1/(cos(d*x+c)+1))^(3/2)+49*A*((a-b)/(a+b))^(1/2)*s
in(d*x+c)*cos(d*x+c)^3*a^5*(1/(cos(d*x+c)+1))^(3/2)+45*B*((a-b)/(a+b))^(1/2
)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^4*a^5+75*B*((a-b)/(a+b))^(
1/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*a^5+75*B*((a-b)/(a+b)
)^(1/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*a^5+35*A*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*a^5*(1/(cos(d*x+c)+1))^(3/2)+8*A*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+147*A*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
```

d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-147*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-75*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5+189*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-189*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+189*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+137*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+137*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+53*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+85*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+53*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+85*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+81*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+4*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)+117*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+121*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)-9*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+117*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+81*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+321*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+189*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+189*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+88*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-186*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+246*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-153*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-18*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-246*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+246*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+18*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-18*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-252*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+63*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+189*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-63*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+63*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+8*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^5*(1/(cos(d*x+c)+1))^(3/2)+33*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a

```
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+147*A*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^4*b-33*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3*b^2+33*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b^3-8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4)/a^
3/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^6/(1/(cos(d*x+c)+1))^(3/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(Cb\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{3}+\left(Ca+Bb\right)\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)^{4}+\left(Ba+Ab\right)\cos\left(dx+\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^4*se
c(d*x + c)^2 + A*a*cos(d*x + c)^4 + (B*a + A*b)*cos(d*x + c)^4*sec(d*x + c)
)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1342 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=359

$$\frac{2\left(a^2-b^2\right)\left(25a^2A+35a^2C+21abB-6Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2\right.}{}$$

```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B
 - 2*a^2*b*(41*A + 70*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(
a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]) + (2*(3*A*b^2 + 42*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a*d) + (2*(3*A*b + 7*a*B)*Cos[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]
^(5/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.38295, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2(5A+7C)+42abB+3Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{2\left(a^2-b^2\right)\left(25a^2A+35a^2C+21abB-\right.}{105a^2d\sqrt{\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B + 35*a^2*C)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(6*A*b^3 - 63*a^3*B - 21*a*b^2*B
 - 2*a^2*b*(41*A + 70*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(
a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]) + (2*(3*A*b^2 + 42*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a*d) + (2*(3*A*b + 7*a*B)*Cos[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]
^(5/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
```

b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin}{7d}$$

$$= \frac{2(3Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c}}{35d}$$

$$= \frac{2\left(3Ab^2+42abB+5a^2(5A+7C)\right)\sqrt{\cos}}{105}$$

$$= \frac{2\left(3Ab^2+42abB+5a^2(5A+7C)\right)\sqrt{\cos}}{105}$$

$$= \frac{2\left(3Ab^2+42abB+5a^2(5A+7C)\right)\sqrt{\cos}}{105}$$

$$= \frac{2\left(3Ab^2+42abB+5a^2(5A+7C)\right)\sqrt{\cos}}{105}$$

$$= \frac{2\left(a^2-b^2\right)\left(25a^2A-6Ab^2+21abB+35\right.}{105a^2d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 23.2033, size = 3261, normalized size = 9.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*(((115*a^2*A + 12*A*b^2 + 168*a*b*B + 140*a^2*C)*Sin[c + d*x])/
(105*a) + (2*(8*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a*A*Sin[3*(c + d*x)])/
7))/(d*(b + a*Cos[c + d*x])*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x
])) - (4*Cos[c + d*x]^(3/2)*((164*a*A*b*Sqrt[Cos[c + d*x]])/(105*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*A*b^3*Sqrt[Cos[c + d*x]])/(35*a*Sqr
t[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (6*a^2*B*Sqrt[Cos[c + d*x]])/(5
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]
)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a*b*C*Sqrt[Cos[c + d
*x]])/(3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (10*a^2*A*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (34*A*b^2*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (8*a*b*
B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) + (2*
a^2*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*Sqrt[b + a*Cos[c + d*x]]) +
 (2*b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]])*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*S
ec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b
```

```
^2*B + 2*a^2*b*(41*A + 70*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + I*a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(19*
A + 7*(B + 5*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
- (-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*(b + a*Cos[c +
 d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a^2*d*(b + a*Cos[
c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^
(7/2)*((-2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c
 + d*x]*((-I)*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 7
0*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x
)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b
)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*Ellipt
icF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-6*A*b^3 + 63*a^3*B +
 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]
^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a*(b + a*Cos[c + d*x])^(3/2)) + (2*Sqrt[C
os[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a
+ b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*EllipticE[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-6*A*b^2 + a^2
*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*EllipticF[I*ArcSinh[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a
^2*b*(41*A + 70*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c
+ d*x)/2]))/(35*a^2*Sqrt[b + a*Cos[c + d*x]]) - (4*Cos[c + d*x]^(3/2)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2
*a^2*b*(41*A + 70*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 -
I*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*Ellipt
icE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*a*(
a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] +
a*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*(Sec[(c + d*x)
/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(-6*A*b^3 + 63*a^3*B + 21*a
*b^2*B + 2*a^2*b*(41*A + 70*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(
3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2
*B + 2*a^2*b*(41*A + 70*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)
) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)*(-
6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*EllipticF[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec
[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] - (a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*B + 35*C) + 3*a*b*(
19*A + 7*(B + 5*C)))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 + ((-a + b)*Ta
n[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-6*A*b^3 + 63*a^3*B + 21*a*b^2*B +
2*a^2*b*(41*A + 70*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*
Sqrt[1 + Tan[(c + d*x)/2]^2])))/(105*a^2*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos
[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(-6*A*b
^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A + 70*C))*EllipticE[I*ArcSinh[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-6*A*b^2 + a^2*(25*A + 63*
B + 35*C) + 3*a*b*(19*A + 7*(B + 5*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
```

```
+ d*x)/2]^2)/(a + b)] - (-6*A*b^3 + 63*a^3*B + 21*a*b^2*B + 2*a^2*b*(41*A +
 70*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(
-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[
c + d*x]*Tan[c + d*x]))/(35*a^2*Sqrt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 0.691, size = 2911, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(25*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4
*(1/(cos(d*x+c)+1))^(3/2)+82*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^3*b-51*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b^2-6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-82*A*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+82*A*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+6*A*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^3-6*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^4*(1/(c
os(d*x+c)+1))^(3/2)+25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(
cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/
(cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1
/(cos(d*x+c)+1))^(3/2)+25*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*
x+c)+1))^(3/2)+82*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1
))^(3/2)+3*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+
63*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+42*B*((a
-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+21*B*((a-b)/(a
+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+35*C*((a-b)/(a+b))^(1/
2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+140*C*((a-b)/(a+b))^(1/2)*a^2*
b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/
2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^3*((a-b)/(a+b))^
(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+63*B*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-21*B*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+21*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^3-84*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a^3*b+21*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-1
40*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+140*C*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+140*C*EllipticF((-1+c
```

```
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-105*C*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+39*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*sin
(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+107*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+27*A*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-3*A*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+175*C*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+39*A*cos
(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+27*
A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3
/2)-63*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-35*C*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-25*A*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^4-6*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^4)/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2)/
sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb\cos(dx+c)^3\sec(dx+c)^3 + (Ca+Bb)\cos(dx+c)^3\sec(dx+c)^2 + Aa\cos(dx+c)^3 + (Ba+Ab)\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^3*se
c(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A*b)*cos(d*x + c)^3*sec(d*x + c)
)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(7/2), x)
```

3.1343 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=356

$$-\frac{2\left(-3a^2b(A+5C)-5a^3B+5ab^2B+3Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\left(3a^2(3A+5C\right.}{}$$

```
[Out] (-2*(3*A*b^3 - 5*a^3*B + 5*a*b^2*B - 3*a^2*b*(A + 5*C))*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*C*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) + (2*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(
15*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(3*A*b + 5*a*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(
3/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.39907, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(-3a^2b(A+5C)-5a^3B+5ab^2B+3Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\left(3a^2(3A+5C)+20ab\right.}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (-2*(3*A*b^3 - 5*a^3*B + 5*a*b^2*B - 3*a^2*b*(A + 5*C))*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*C*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) + (2*(3*A*b^2 + 20*a*b*B + 3*a^2*(3*A + 5*C))*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(
15*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(3*A*b + 5*a*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(
3/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
```

```
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$= \frac{2(3Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\,\text{se}}}{15d}$$

$$= \frac{2(3Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\,\text{se}}}{15d}$$

$$= \frac{2(3Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\,\text{se}}}{15d}$$

$$= \frac{2(3Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\,\text{se}}}{15d}$$

$$= \frac{2b^2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2\left(3Ab^3-5a^3B+5ab^2B-3a^2b(A\right.}{15ad\sqrt{\cos(c+dx}}$$

Mathematica [C] time = 35.1835, size = 56321, normalized size = 158.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.52, size = 2220, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1)
^2*(-1+cos(d*x+c))^3*(3*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*(
1/(cos(d*x+c)+1))^(3/2)+9*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*
x+c)+1))^(3/2)+6*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^
(3/2)+5*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+20*
B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+15*C*((a-b)
/(a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+3*A*sin(d*x+c)*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3/2)+9*A*sin(d*x+c)*((
a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)+5*B*sin(d*x+c)*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3/2)+5*B*sin(d*x+c)*
((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)+3*A*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+9*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^3-9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3-5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+
15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-15*C*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+3*A*sin(d*x+c)*((a-b)/(a+b))
^(1/2)*b^3*(1/(cos(d*x+c)+1))^(3/2)+20*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^2*b-20*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2*b+20*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-
30*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+15*C*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-12*A*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+3*A*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a*b^2+9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b-3*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-
15*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+15*C*(1/(a+b
```

```
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-30*C*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+9*A*sin(d*x+c)*((a-b)/
(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+9*A*sin(d*x+c)*((a
-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+9*A*sin(d*x+c)*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+25*B*sin(d*x+c
)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+15*C*((a-b)
/(a+b))^(1/2)*cos(d*x+c)*a^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2))/a/((a-b)/
(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Cb\cos(dx+c)^2\sec(dx+c)^3 + (Ca+Bb)\cos(dx+c)^2\sec(dx+c)^2 + Aa\cos(dx+c)^2 + (Ba+Ab)\cos(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^2*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)^2*se
c(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c)^2*sec(d*x + c)
)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(5/2), x)
```

3.1344 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=340

$$\frac{\left(2a^2(A+3C)+6abB-b^2(2A-3C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\cos(c+dx)}(6aB+8Ab-3bC)\sqrt{a}}{3d\sqrt{\frac{a\cos(c}{a}}}$$

```
[Out] ((6*a*b*B - b^2*(2*A - 3*C) + 2*a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (b*(2*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + ((8*A*b + 6*a*B - 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) - (b*(2*A - 3*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/
2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.38481, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4094, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^2(A+3C)+6abB-b^2(2A-3C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\cos(c+dx)}(6aB+8Ab-3bC)\sqrt{a+b\sec(c}}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] ((6*a*b*B - b^2*(2*A - 3*C) + 2*a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (b*(2*b*B + 3*a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + ((8*A*b + 6*a*B - 3*b*C)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) - (b*(2*A - 3*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/
2)*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
```

```
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\sin}{3d}$$

$$= -\frac{b(2A-3C)\sqrt{a+b\sec(c+dx)}\sin(c+}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(2A-3C)\sqrt{a+b\sec(c+dx)}\sin(c+}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(2A-3C)\sqrt{a+b\sec(c+dx)}\sin(c+}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(2A-3C)\sqrt{a+b\sec(c+dx)}\sin(c+}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(2bB+3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}}$$

$$= \frac{\left(6abB-b^2(2A-3C)+2a^2(A+3C)\right)\sqrt{}}{3d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

Mathematica [C] time = 33.6827, size = 79958, normalized size = 235.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.643, size = 1865, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^3*(3*C*sin(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^2+2*A*sin
(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2-6*A*c
os(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+8*A*cos(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-12*B*cos(d*x+
```

```
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+6*
B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*B*co
s(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-6*B*cos(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-6*C*cos(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*C*cos(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-12*B*cos(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*C*((a-b)/(a+b))^(1/2)*c
os(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+2*A*sin(d*x+c)*(1/(cos(d*
x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-2*A*cos(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+10*A*sin(d*x+c)*cos(d*x+c)^2*
(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b+6*B*sin(d*x+c)*cos(d*x+c)*
(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b+6*B*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-18*C*cos(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+6*C*cos(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*C*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+6*B*sin(d*x+c)*cos(d*x+c)^2
*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a^2+8*A*sin(d*x+c)*cos(d*x+c)
*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*b^2+2*A*sin(d*x+c)*cos(d*x+c)
^3*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a^2+8*A*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*cos(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b)/((a-b)/(a+b))^(1/2)/(b+a*cos(d
*x+c))/(1/(cos(d*x+c)+1))^(3/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec (dx+c)^2 + B \sec (dx+c) + A\right)(b \sec (dx+c) + a)^{\frac{3}{2}} \cos (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\Big(\big(Cb \cos (dx+c) \sec (dx+c)^3 + (Ca+Bb) \cos (dx+c) \sec (dx+c)^2 + Aa \cos (dx+c) + (Ba+Ab) \cos (dx -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)*sec(d*x + c)^3 + (C*a + B*b)*cos(d*x + c)*sec(d*
x + c)^2 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*sec(d*x + c))*sqrt(b
*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \sec \left(dx+c\right)^{2}+B \sec \left(dx+c\right)+A\right)\left(b \sec \left(dx+c\right)+a\right)^{\frac{3}{2}} \cos \left(dx+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*cos(d*x + c)^(3/2), x)
```

3.1345 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+\right.$

Optimal. Leaf size=353

$$\frac{\left(8a^2B+ab(8A+7C)+4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2C+12abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)}{a+b}}}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

```
[Out] ((8*a^2*B + 4*b^2*B + a*b*(8*A + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]) + ((8*A*b^2 + 12*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*a*A - 4*b*B - 5*a*C)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + ((4*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (C*(a + b*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.34075, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(8a^2B+ab(8A+7C)+4b^2B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2C+12abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```

```
[Out] ((8*a^2*B + 4*b^2*B + a*b*(8*A + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]) + ((8*A*b^2 + 12*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*a*A - 4*b*B - 5*a*C)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + ((4*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (C*(a + b*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
```

```
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+\cdots}{\cdots}$$

$$= \frac{C(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \cdots$$

$$= \frac{(4bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(\cdots}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{(4bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(\cdots}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{(4bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(\cdots}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{(4bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(\cdots}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(8Ab^2+12abB+3a^2C+4b^2C\right)\sqrt{\frac{b\cdots}{\cdots}}}{4d\sqrt{\cos(c+dx)}\sqrt{a+\cdots}}$$

$$= \frac{\left(8a^2B+4b^2B+ab(8A+7C)\right)\sqrt{\frac{b+a\cdots}{\cdots}}}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\cdots}}$$

Mathematica [C] time = 34.7861, size = 120732, normalized size = 342.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.599, size = 2099, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)
,x)
```

```
[Out] -1/4/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(2*C*sin(d*x+c)*((a-b)/(a+b))^(1/
2)*(1/(cos(d*x+c)+1))^(3/2)*b^2+8*A*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*b^2-16*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-4*B*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*b^2+5*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*C*cos(d*x+c)^2*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2+4*C*cos(d*x+c)^2*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^2-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*C*cos(d*x+c)^2*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+8*A*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b+5*C*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^2*a^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+8*B*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a*b+4*B*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)+2*C*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^2*a*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+7*C*((a-b)/(a+
b))^(1/2)*cos(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+8*A*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^2-8*A*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2-8*B*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^2-2*C*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+8*A*sin(d*x+c)*cos(d*x
+c)^2*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b+4*B*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(3/2)+2*C*sin(d*x+c)*((a-b
)/(a+b))^(1/2)*cos(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(3/2)-16*A*cos(d*x+c)^2*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b+4*B*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b-24*B*cos(d*x+c)^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-5*C*cos(d*
```

```
x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+8*A*sin(d*x+
c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a^2)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^
(3/2)/sin(d*x+c)^6/(1/(cos(d*x+c)+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2 + B\sec(dx+c) + A\right)(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)*sqrt(cos(d*x + c)), x)
```

3.1346 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=446

$$\frac{\left(a^2(48A+17C)+42abB+8b^2(3A+2C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\left(3a^2C+30abB+2\right.}{24bd\sqrt{c}}$$

```
[Out] ((42*a*b*B + 8*b^2*(3*A + 2*C) + a^2*(48*A + 17*C))*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + ((6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A
+ C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(
a + b)])/(8*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 +
 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + ((2*b*B + a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c
 + d*x]^(3/2)) + ((24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (C*(a + b*Sec[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.81079, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\left(3a^2C+30abB+24Ab^2+16b^2C\right)\sqrt{a+b\sec(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\left(a^2(48A+17C)+42abB+8b^2(3A+2C)\right)\sqrt{\frac{a\cos}{}}}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```


```
[Out] ((42*a*b*B + 8*b^2*(3*A + 2*C) + a^2*(48*A + 17*C))*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + ((6*a^2*b*B + 8*b^3*B - a^3*C + 12*a*b^2*(2*A
+ C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(
a + b)])/(8*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((24*A*b^2 +
 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + ((2*b*B + a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c
 + d*x]^(3/2)) + ((24*A*b^2 + 30*a*b*B + 3*a^2*C + 16*b^2*C)*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (C*(a + b*Sec[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\,\mathrm{se}$$

$$= \frac{C(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx}\right.$$

$$= \frac{(2bB+aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{C(a+b}{}$$

$$= \frac{(2bB+aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(24Ab^2}{}$$

$$= \frac{(2bB+aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(24Ab^2}{}$$

$$= \frac{(2bB+aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(24Ab^2}{}$$

$$= \frac{(2bB+aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(24Ab^2}{}$$

$$= \frac{\left(6a^2bB+8b^3B-a^3C+12ab^2(2A+C)\right)\sqrt{\frac{b+a\cos(c+dx}{a+b}}}{8bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}$$

$$= \frac{\left(42abB+8b^2(3A+2C)+a^2(48A+17C)\right)\sqrt{\frac{b+a\cos(c}{a+b}}}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}$$

Mathematica [C] time = 34.5252, size = 132839, normalized size = 297.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.609, size = 2725, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^3*(-42*B*sin(d*x+c)*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+
```

1))^(3/2)-17*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*
x+c)+1))^(3/2)-22*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(c
os(d*x+c)+1))^(3/2)-22*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1
/(cos(d*x+c)+1))^(3/2)-24*A*sin(d*x+c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b
^2*(1/(cos(d*x+c)+1))^(3/2)-30*B*sin(d*x+c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-12*B*sin(d*x+c)*cos(d*x+c)^3*((a-b)/(a+b))
^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-14*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-16*C*cos(d*x+c)^3*((a-b)/(a+b))^
(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-24*A*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2+48*A*cos(d*x+c)^3*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-48*A*cos(d*x+c)^3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-30*B*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+30*B*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+72*C*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a*b^2+14*C*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b-20*C*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2+3*C*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b-16*C*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2+144*
A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*
x+c)+1))^(3/2)-16*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c)*(1/(cos
(d*x+c)+1))^(3/2)-12*B*cos(d*x+c)^2*sin(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(
d*x+c)+1))^(3/2)*b^3-12*B*cos(d*x+c)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos
(d*x+c)+1))^(3/2)*b^3-8*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c)*(1/
(cos(d*x+c)+1))^(3/2)+36*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+12*B*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+12*B*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^3-8*C*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(3/2)+48*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*b^3-24*B*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a^3+6*C*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^3-3*C*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^3+16*C*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*b^3-3*C*cos(d*x+c)^3*((a-b)/(a+b)
)^(1/2)*a^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2))/b/((a-b)/(a+b))^(1/2)/(b+a

```
*cos(d*x+c))/sin(d*x+c)^6/cos(d*x+c)^(5/2)/(1/(cos(d*x+c)+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/sqrt(cos(d*x + c)), x)
```

3.1347 $\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=551

$$\frac{\left(136a^2bB-3a^3C+12ab^2(28A+19C)+128b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{192bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\left(3a^2C+56a\right.}{9}$$

```
[Out] ((136*a^2*b*B + 128*b^3*B - 3*a^3*C + 12*a*b^2*(28*A + 19*C))*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(192*b*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((8*a^3*b*B - 96*a*b^3*B - 3*a^4*
C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(64*b^2*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) - ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b
^2*(20*A + 13*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(192*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) +
((8*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Cos[c + d*x]^
(5/2)) + ((48*A*b^2 + 56*a*b*B + 3*a^2*C + 36*b^2*C)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(96*b*d*Cos[c + d*x]^(3/2)) + ((24*a^2*b*B + 128*b^3*B - 9
*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(19
2*b^2*d*Sqrt[Cos[c + d*x]]) + (C*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
4*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 2.34832, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\left(3a^2C+56abB+48Ab^2+36b^2C\right)\sqrt{a+b\sec(c+dx)}}{96bd\cos^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(24a^2bB-9a^3C+12ab^2(20A+13C\right.}{192b^2d\sqrt{\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```


```
[Out] ((136*a^2*b*B + 128*b^3*B - 3*a^3*C + 12*a*b^2*(28*A + 19*C))*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(192*b*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((8*a^3*b*B - 96*a*b^3*B - 3*a^4*
C - 24*a^2*b^2*(2*A + C) - 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(64*b^2*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) - ((24*a^2*b*B + 128*b^3*B - 9*a^3*C + 12*a*b
^2*(20*A + 13*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(192*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) +
((8*b*B + 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Cos[c + d*x]^
(5/2)) + ((48*A*b^2 + 56*a*b*B + 3*a^2*C + 36*b^2*C)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(96*b*d*Cos[c + d*x]^(3/2)) + ((24*a^2*b*B + 128*b^3*B - 9
*a^3*C + 12*a*b^2*(20*A + 13*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(19
2*b^2*d*Sqrt[Cos[c + d*x]]) + (C*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(
4*d*Cos[c + d*x]^(5/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] && !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{se}$$

$$= \frac{C(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx}\right.$$

$$= \frac{(8bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{C(a+}{}$$

$$= \frac{(8bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(48Ab}{}$$

$$= \frac{(8bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(48Ab}{}$$

$$= \frac{(8bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(48Ab}{}$$

$$= \frac{(8bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(48Ab}{}$$

$$= \frac{(8bB+3aC)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(48Ab}{}$$

$$= -\frac{\left(8a^3bB-96ab^3B-3a^4C-24a^2b^2(2A+C)-16b^4(\right.}{64b^2d\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{\left(136a^2bB+128b^3B-3a^3C+12ab^2(28A+19C)\right)\sqrt{}}{192bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

Mathematica [C] time = 36.0003, size = 179293, normalized size = 325.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.862, size = 3943, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] -1/192/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c
))^3*(72*C*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*sin(d*
x+c)*b^4+240*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2*(1/(cos(
d*x+c)+1))^(3/2)+176*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^3*(1
/(cos(d*x+c)+1))^(3/2)+136*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^
2*b^2*(1/(cos(d*x+c)+1))^(3/2)+78*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^
2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+120*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^
2*a*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+24*B*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^4*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+112*B*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^4*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+128*B*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+228*C*((a-b)
/(a+b))^(1/2)*cos(d*x+c)^3*a*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-3*C*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+78
*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(
3/2)+6*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1
))^(3/2)+156*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^2*b^2*sin(d*x+c)*(1/(cos(
d*x+c)+1))^(3/2)+72*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a*b^3*sin(d*x+c)*(1/
(cos(d*x+c)+1))^(3/2)+336*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a*b
^3*(1/(cos(d*x+c)+1))^(3/2)+176*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+
c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+120*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^
3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+96*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4
*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+48*B*cos(d*x+c)^4*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b-576*B*cos(d*x+c)^
4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3-4
8*B*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-
112*B*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b^2+160*B*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a*b^3+24*B*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^3*b-24*B*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b^2+128*B*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b^3+48*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*b^4*(1/(cos(d*x+
c)+1))^(3/2)-144*C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a^2*b^2-6*C*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a^3*b-84*C*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-72*C*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+9*C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+156*C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-156*C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-9*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)
^4*a^4*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-96*A*cos(d*x+c)^4*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+240*A*cos(d*x+c)^4*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-240*A*cos(d*x+c)^4*(1/(a+b)
```

```
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+64*B*((a-b)/(a+b))^(1/2)*
cos(d*x+c)*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+96*A*((a-b)/(a+b))^(1/2)
*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^3*sin(d*x+c)*b^4+72*C*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^3*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+96*A*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+128*B*((a-b)/(a
+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)-96*A*cos(d*
x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+64*B*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)-288*A
*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*a^2*b^2-384*A*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*b^4-128*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b^4-18*C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^4-288*C*cos(d*x+c)^4*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4+18*C*cos(d*x+c)^4*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+144*C*cos(d*x+c)^4*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-9*C*cos(d*x+c)^4*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+48*C*((a-b)/(a+b))^(1/2)
*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+192*A*cos(d*x+c)^4*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4)/b^2/((a-b)/(a+b))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)^6/cos(d*x+c)^(7/2)/(1/(cos(d*x+c)+1))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2
)/cos(d*x + c)^(3/2), x)
```

3.1348 $\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=565

$$\frac{2\left(a^2-b^2\right)\left(15a^2b^2(19A+33C)+75a^4(9A+11C)+1254a^3bB-110ab^3B+40Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{3465a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*(a^2 - b^2)*(40*A*b^4 + 1254*a^3*b*B - 110*a*b^3*B + 75*a^4*(9*A + 11*C)
 + 15*a^2*b^2*(19*A + 33*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(3465*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a
^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*a^3*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]) - (2*(20*A*b^4 - 1793*a^3*b*B - 55*a*b^3*B -
75*a^4*(9*A + 11*C) - 5*a^2*b^2*(205*A + 297*C))*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d) + (2*(15*A*b^3 + 539*a^3*B + 8
25*a*b^2*B + 5*a^2*b*(229*A + 297*C))*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(3465*a*d) + (2*(5*A*b^2 + 44*a*b*B + 3*a^2*(9*A + 11*
C))*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(231*d) + (2*
(5*A*b + 11*a*B)*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(99*d) + (2*A*Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(11*d)
```

Rubi [A] time = 2.48463, antiderivative size = 565, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(3a^2(9A+11C)+44abB+5Ab^2\right)\sqrt{a+b\sec(c+dx)}}{231d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(5a^2b(229\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*
Sec[c + d*x]^2),x]
```


```
[Out] (2*(a^2 - b^2)*(40*A*b^4 + 1254*a^3*b*B - 110*a*b^3*B + 75*a^4*(9*A + 11*C)
 + 15*a^2*b^2*(19*A + 33*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(3465*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a
^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*a^3*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]) - (2*(20*A*b^4 - 1793*a^3*b*B - 55*a*b^3*B -
75*a^4*(9*A + 11*C) - 5*a^2*b^2*(205*A + 297*C))*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d) + (2*(15*A*b^3 + 539*a^3*B + 8
25*a*b^2*B + 5*a^2*b*(229*A + 297*C))*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(3465*a*d) + (2*(5*A*b^2 + 44*a*b*B + 3*a^2*(9*A + 11*
C))*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(231*d) + (2*
(5*A*b + 11*a*B)*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(99*d) + (2*A*Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(11*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{s}}{}$$

$$= \frac{2A\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,\mathrm{si}}{11d}$$

$$= \frac{2(5Ab+11aB)\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c}{99d}$$

$$= \frac{2\left(5Ab^2+44abB+3a^2(9A+11C)\right)\cos}{23}$$

$$= \frac{2\left(15Ab^3+539a^3B+825ab^2B+5a^2b(\right.}{}$$

$$= -\frac{2\left(20Ab^4-1793a^3bB-55ab^3B-75a\right.}{}$$

$$= -\frac{2\left(20Ab^4-1793a^3bB-55ab^3B-75a\right.}{}$$

$$= -\frac{2\left(20Ab^4-1793a^3bB-55ab^3B-75a\right.}{}$$

$$= -\frac{2\left(20Ab^4-1793a^3bB-55ab^3B-75a\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)\left(40Ab^4+1254a^3bB-110ab\right.}{34}$$

Mathematica [C] time = 25.7296, size = 4170, normalized size = 7.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
] + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*(((6525*a^4*A + 9330*a^2*A*b^2 - 160*A*b^4 + 16434*a^3*b*B + 44
0*a*b^3*B + 7590*a^4*C + 11880*a^2*b^2*C)*Sin[c + d*x])/(6930*a^2) + ((3095
*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a*b^2*B + 2970*a^2*b*C)*Sin[2*(c +
d*x)])/(3465*a) + ((513*a^2*A + 452*A*b^2 + 836*a*b*B + 396*a^2*C)*Sin[3*(c
 + d*x)])/2772 + (a*(23*A*b + 11*a*B)*Sin[4*(c + d*x)])/198 + (a^2*A*Sin[5*
(c + d*x)])/44))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])) - (4*Cos[c + d*x]^(3/2)*((494*a^2*A*b*Sqrt[Cos[c + d*x]]
)/(231*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (34*A*b^3*Sqrt[Cos[c
+ d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^5*Sqrt
[Cos[c + d*x]])/(693*a^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (14
*a^3*B*Sqrt[Cos[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
 + (62*a*b^2*B*Sqrt[Cos[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]]) - (4*b^4*B*Sqrt[Cos[c + d*x]])/(63*a*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) + (58*a^2*b*C*Sqrt[Cos[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) + (2*b^3*C*Sqrt[Cos[c + d*x]])/(7*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (30*a^3*A*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x
]])/(77*Sqrt[b + a*Cos[c + d*x]]) + (442*a*A*b^2*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]) + (4*A*b^4*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(693*a*Sqrt[b + a*Cos[c + d*x]]) + (58*a^2*b*B*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (62*b^3*B*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(63*Sqrt[b + a*Cos[c + d*x]]) + (10*a^3
*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (
18*a*b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(7*Sqrt[b + a*Cos[c + d*x
]]))*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A
+ B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a + b)*(40*A*b^5 + 1617*a^5*B +
 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A
+ 319*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
 d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a
 + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B + 33*C)
+ 3*a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 209*B + 660*C))*Elliptic
F[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (40*A*b^5 + 1617*a^5*B +
 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A
+ 319*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])
)/(3465*a^3*d*(b + a*Cos[c + d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*
c + 2*d*x])*Sec[c + d*x]^(9/2)*((-2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(40*A*b^5 + 1617*a^5*B + 306
9*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 31
9*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x
)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b
)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B + 33*C) + 3*
a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 209*B + 660*C))*EllipticF[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (40*A*b^5 + 1617*a^5*B + 306
9*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 31
9*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3
465*a^2*(b + a*Cos[c + d*x])^(3/2)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/
2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(40*A*b^5 + 1617*a^5*B
+ 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A
 + 319*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(
a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B + 33*C)
 + 3*a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 209*B + 660*C))*Ellipti
cF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[(
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (40*A*b^5 + 1617*a^5*B
+ 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A
 + 319*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]
))/(1155*a^3*Sqrt[b + a*Cos[c + d*x]]) - (4*Cos[c + d*x]^(3/2)*(Cos[(c + d*
```

```
x)/2]^2*Sec[c + d*x])^(3/2)*(-((40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 11
0*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*(b + a*Cos
[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(40*A*b^5 + 1617*a^5*B
 + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*
A + 319*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
 + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c
 + d*x)/2] + I*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19
*A + 121*B + 33*C) + 3*a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 209*B
 + 660*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c
+ d*x)/2] + a*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^
2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*(Sec[(c + d*x)/2]^2)^(3/2)*
Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B
- 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*(b + a
*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a
 + b)*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(1
7*A + 33*C) + 15*a^4*b*(247*A + 319*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d
*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/
(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*
a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B + 33
*C) + 3*a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 209*B + 660*C))*Elli
pticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-(
(a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] - (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a
^2*b^2*(19*A + 121*B + 33*C) + 3*a^4*(225*A + 539*B + 275*C) + 6*a^3*b*(505
*A + 209*B + 660*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 + ((-a + b)*Ta
n[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(40*A*b^5 + 1617*a^5*B + 3069*a^3*b^
2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C))*Se
c[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sq
rt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]
^2])))/(3465*a^3*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*Sqrt[Cos
[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(40*A*b^5 + 1617*a^5*B + 3069*a
^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C
))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2
]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(
40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B + 33*C) + 3*a^4
*(225*A + 539*B + 275*C) + 6*a^3*b*(505*A + 209*B + 660*C))*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (40*A*b^5 + 1617*a^5*B + 3069*a
^3*b^2*B - 110*a*b^4*B + 15*a^2*b^3*(17*A + 33*C) + 15*a^4*b*(247*A + 319*C
))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos
[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d
*x]*Tan[c + d*x]))/(1155*a^3*Sqrt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 1.532, size = 5307, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2
),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^5\sec(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^5\sec(dx+c)^3+Aa^2\cos(dx+c)^5+\left(Ca^2+2\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^5*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^5*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^5 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^5*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^5*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(
d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec(dx+c)^2+B\sec(dx+c)+A\right)(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*
x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(11/2), x)
```

3.1349 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=452

$$-\frac{2\left(a^2-b^2\right)\left(-6a^2b(19A+28C)-75a^3B-45ab^2B+10Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\,\mathrm{co}}{}$$

```
[Out] (-2*(a^2 - b^2)*(10*A*b^3 - 75*a^3*B - 45*a*b^2*B - 6*a^2*b*(19*A + 28*C))*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(
315*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(10*A*b^4 - 435
*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(5*A*b^3 + 75*a^3*B
 + 135*a*b^2*B + a^2*b*(163*A + 231*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(15*A*b^2 + 90*a*b*B + 7*a^2*(7*A + 9
*C))*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (2
*(5*A*b + 9*a*B)*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(63*d) + (2*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(9*d)
```

Rubi [A] time = 1.91044, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4094, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(7a^2(7A+9C)+90abB+15Ab^2\right)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2b(163A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (-2*(a^2 - b^2)*(10*A*b^3 - 75*a^3*B - 45*a*b^2*B - 6*a^2*b*(19*A + 28*C))*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(
315*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(10*A*b^4 - 435
*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(5*A*b^3 + 75*a^3*B
 + 135*a*b^2*B + a^2*b*(163*A + 231*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(15*A*b^2 + 90*a*b*B + 7*a^2*(7*A + 9
*C))*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (2
*(5*A*b + 9*a*B)*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(63*d) + (2*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(9*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$=\frac{2A\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\sin}{9d}$$

$$=\frac{2(5Ab+9aB)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c-}{63d}$$

$$=\frac{2\left(15Ab^2+90abB+7a^2(7A+9C)\right)\cos}{31}$$

$$=\frac{2\left(5Ab^3+75a^3B+135ab^2B+a^2b(163A\right.}{}$$

$$=\frac{2\left(5Ab^3+75a^3B+135ab^2B+a^2b(163A\right.}{}$$

$$=\frac{2\left(5Ab^3+75a^3B+135ab^2B+a^2b(163A\right.}{}$$

$$=\frac{2\left(5Ab^3+75a^3B+135ab^2B+a^2b(163A\right.}{}$$

$$=-\frac{2\left(a^2-b^2\right)\left(10Ab^3-75a^3B-45ab^2B-\right.}{315a^2d\sqrt{\cos(}}$$

Mathematica [C] time = 24.8359, size = 3785, normalized size = 8.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*(((747*a^2*A*b + 20*A*b^3 + 345*a^3*B + 540*a*b^2*B + 924*a^2*b
*C)*Sin[c + d*x])/(315*a) + ((133*a^2*A + 150*A*b^2 + 270*a*b*B + 126*a^2*C
)*Sin[2*(c + d*x)])/315 + (a*(19*A*b + 9*a*B)*Sin[3*(c + d*x)])/63 + (a^2*A
*Sin[4*(c + d*x)])/18))/(d*(b + a*Cos[c + d*x])^2*(A + 2*C + 2*B*Cos[c + d*
x] + A*Cos[2*c + 2*d*x])) - (4*Cos[c + d*x]^(3/2)*((14*a^3*A*Sqrt[Cos[c + d
*x]])/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (62*a*A*b^2*Sqrt[C
```

os[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*A*b^4*S
qrt[Cos[c + d*x]])/(63*a*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (58
*a^2*b*B*Sqrt[Cos[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) + (2*b^3*B*Sqrt[Cos[c + d*x]])/(7*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (6*a^3*C*Sqrt[Cos[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) + (46*a*b^2*C*Sqrt[Cos[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (58*a^2*A*b*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(35
*Sqrt[b + a*Cos[c + d*x]]) + (62*A*b^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]
])/(63*Sqrt[b + a*Cos[c + d*x]]) + (10*a^3*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c
+ d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (18*a*b^2*B*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(7*Sqrt[b + a*Cos[c + d*x]]) + (34*a^2*b*C*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (2*b^3*C*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/Sqrt[b + a*Cos[c + d*x]])*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[
c + d*x]^2)*((-I)*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7
*A + 9*C) + 3*a^2*b^2*(93*A + 161*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C)
 + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (10*A*b^4 - 435*a^3*b*B - 45
*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*(b + a*Cos[c + d*
x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^2*d*(b + a*Cos[c +
 d*x])^3*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sec[c + d*x]^(9/
2)*((-2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c +
d*x]*((-I)*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*
C) + 3*a^2*b^2*(93*A + 161*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] + I*a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^
3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (10*A*b^4 - 435*a^3*b*B - 45*a*b^3*
B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*(b + a*Cos[c + d*x])*(Se
c[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a*(b + a*Cos[c + d*x])^(3/2
)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c +
d*x]*((-I)*(a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*
C) + 3*a^2*b^2*(93*A + 161*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] + I*a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^
3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (10*A*b^4 - 435*a^3*b*B - 45*a*b^3*
B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*(b + a*Cos[c + d*x])*(Se
c[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a^2*Sqrt[b + a*Cos[c + d*x]
]) - (4*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(((10*A*
b^4 - 435*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161
C))(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(-10*A*
b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A + 161
*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)
/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)
/2] + I*a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A +
 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - a*(10*A*b^4 - 435*a^3*b*B -
 45*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*(Sec[(c + d*x)
/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*(10*A*b^4 - 435*a^3*b*B - 4
5*a*b^3*B - 21*a^4*(7*A + 9*C) - 3*a^2*b^2*(93*A + 161*C))*(b + a*Cos[c + d
x])(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(-10
*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*A +

```
161*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d
*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)*(-10*A*b^3 + 6*a^2*b*(
19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B +
7*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*
x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2)/(a + b)] - (a*(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A +
60*B + 28*C) + 3*a^3*(49*A + 25*B + 63*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*
Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])
/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a
+ b)]) + ((a + b)*(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C
) + 3*a^2*b^2*(93*A + 161*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b
)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(315*a^2*Sqrt[b + a*Cos[c + d*x]]) -
 (2*Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*
(-10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B + 21*a^4*(7*A + 9*C) + 3*a^2*b^2*(93*
A + 161*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
 + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*
(a + b)*(-10*A*b^3 + 6*a^2*b*(19*A + 60*B + 28*C) + 3*a^3*(49*A + 25*B + 63
*C) + 15*a*b^2*(11*A + 3*(B + 7*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] + (10*A*b^4 - 435*a^3*b*B - 45*a*b^3*B - 21*a^4*(7*A + 9
*C) - 3*a^2*b^2*(93*A + 161*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(
3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
 Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^2*Sqrt[b + a*Cos[c +
 d*x]])))
```

Maple [B] time = 1.032, size = 4157, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(45*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^5
*(1/(cos(d*x+c)+1))^(3/2)+63*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^5*(1/(cos
(d*x+c)+1))^(3/2)*sin(d*x+c)+63*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^5*(1/(
cos(d*x+c)+1))^(3/2)*sin(d*x+c)+189*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1
/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+147*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(
d*x+c)*a^5*(1/(cos(d*x+c)+1))^(3/2)+35*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)^5*a^5*(1/(cos(d*x+c)+1))^(3/2)+147*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)
*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+75*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*b*(
1/(cos(d*x+c)+1))^(3/2)+435*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(co
s(d*x+c)+1))^(3/2)+135*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x
+c)+1))^(3/2)+45*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^
(3/2)+189*C*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+2
31*C*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+483*C*
((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+279*A*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+5*A*((a-b)/(a+b)
)^(1/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)+49*A*((a-b)/(a+b))^(1/2)*
sin(d*x+c)*cos(d*x+c)^2*a^5*(1/(cos(d*x+c)+1))^(3/2)+49*A*((a-b)/(a+b))^(1/
2)*sin(d*x+c)*cos(d*x+c)^3*a^5*(1/(cos(d*x+c)+1))^(3/2)+45*B*((a-b)/(a+b))^
```

(1/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^4*a^5+75*B*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*a^5+75*B*((a-b)/(
a+b))^(1/2)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*a^5+35*A*((a-b)/
(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*a^5*(1/(cos(d*x+c)+1))^(3/2)-10*A*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+147*A*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-147*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^5-75*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^5+189*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-1
89*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+294*C*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+212*A*((
a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+212*
A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2
)+170*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(d*x+c)+
1))^(3/2)+80*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3*(1/(cos(
d*x+c)+1))^(3/2)+130*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^4*b*(1
/(cos(d*x+c)+1))^(3/2)+170*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^
3*b^2*(1/(cos(d*x+c)+1))^(3/2)+130*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x
+c)^4*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+270*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*c
os(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-5*A*((a-b)/(a+b))^(1/2)*sin(d*x+
c)*cos(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)+180*B*((a-b)/(a+b))^(1/2)*sin(
d*x+c)*cos(d*x+c)^3*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+442*A*((a-b)/(a+b))^(1/2
)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+80*A*((a-b)/(a+b))
^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+180*B*((a-b)/
(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+180*B*(
(a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+2
70*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(d*x+c)+1))
^(3/2)+510*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)
+1))^(3/2)+294*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3
/2)*sin(d*x+c)+714*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+
1))^(3/2)*sin(d*x+c)+163*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(cos(d
*x+c)+1))^(3/2)-315*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^2*b^3-261*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+435*
B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-405*B*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+45*B*EllipticF((-1+cos(d
x+c))((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-435*B*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^4*b+435*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*b^2-45*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b^3+45*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-3
57*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+483*C*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+189*C*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-483*C*EllipticE((-1+cos(d*x+c))*

```
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+483*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b^3-10*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^5*(1/(cos(d*
x+c)+1))^(3/2)+279*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
3*b^2-155*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-10*
A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+147*A*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-279*A*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+279*A*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^2*b^3+10*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^4)/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)
+1))^(3/2)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2\cos(dx+c)^4\sec(dx+c)^4 + \left(2\,Cab + Bb^2\right)\cos(dx+c)^4\sec(dx+c)^3 + Aa^2\cos(dx+c)^4 + \left(Ca^2 + 2\,Bab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^4*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.1350 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\,\mathrm{s}\right.$

Optimal. Leaf size=441

$$-\frac{2\left(10a^2b^2(A-7C)-5a^4(5A+7C)-56a^3bB+56ab^3B+15Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+a}{}$$

```
[Out] (-2*(15*A*b^4 - 56*a^3*b*B + 56*a*b^3*B + 10*a^2*b^2*(A - 7*C) - 5*a^4*(5*A
 + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)])/(105*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^3*C*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(15*A*b^3 + 63*a^3*B
+ 161*a*b^2*B + 5*a^2*b*(29*A + 49*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(5*A*b + 7*a*B)
*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*
Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.82349, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4094, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2(5A+7C)+56abB+15Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105d}-\frac{2\left(10a^2b^2(A-7C)-5a^4(5A+7C)-\right.}{105ad\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] (-2*(15*A*b^4 - 56*a^3*b*B + 56*a*b^3*B + 10*a^2*b^2*(A - 7*C) - 5*a^4*(5*A
 + 7*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)])/(105*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^3*C*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(15*A*b^3 + 63*a^3*B
+ 161*a*b^2*B + 5*a^2*b*(29*A + 49*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*(15*A*b^2 + 56*a*b*B + 5*a^2*(5*A + 7*C))*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(5*A*b + 7*a*B)
*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*
Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

0] && !GtQ[a + b, 0]

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$= \frac{2(5Ab+7aB)\cos^{\frac{3}{2}}(c+dx)(a+b\sec}{35d}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)}{}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)}{}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)}{}$$

$$= \frac{2\left(15Ab^2+56abB+5a^2(5A+7C)\right)}{}$$

$$= \frac{2b^3C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2\left(15Ab^4-56a^3bB+56ab^3B+10a\right.}{105ad}$$

Mathematica [C] time = 35.285, size = 64878, normalized size = 147.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.75, size = 3164, normalized size = 7.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(25*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4
*(1/(cos(d*x+c)+1))^(3/2)+145*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
```

```
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^3*b-135*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b^2+15*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-145*A*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+145*A*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-15*A*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+15*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^
4*(1/(cos(d*x+c)+1))^(3/2)+25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a
^4*(1/(cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*
a^4*(1/(cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)
*a^4*(1/(cos(d*x+c)+1))^(3/2)+25*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/
(cos(d*x+c)+1))^(3/2)+145*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(
d*x+c)+1))^(3/2)+45*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1
))^(3/2)+63*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)
+77*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+161*B
*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+35*C*((a-b)/
(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+245*C*((a-b)/(a+b))^
(1/2)*a^2*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^4*((a-b)/
(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^3*((a-
b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+63*B*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-161*B*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+161*B*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-119*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^3*b+161*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^2*b^2-245*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*
b+245*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+245*C*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+60*A*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+98*B*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+170*A*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+90*A
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)
+60*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3
/2)+98*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))
^(3/2)+238*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+
c)+1))^(3/2)+280*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(
d*x+c)+1))^(3/2)+60*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/
(cos(d*x+c)+1))^(3/2)+90*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*
b^2*(1/(cos(d*x+c)+1))^(3/2)-63*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^4+63*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4
-35*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-25*A*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
```

```
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+15*A*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-315*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b^2+105*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a*b^3-105*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b
^3-210*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
a*b^3)/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^6/(1/(cos(d*x+c)+1
))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Cb^2\cos(dx+c)^3\sec(dx+c)^4+\left(2\,Cab+Bb^2\right)\cos(dx+c)^3\sec(dx+c)^3+Aa^2\cos(dx+c)^3+\left(Ca^2+2\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^3*sec(d*x + c)^4 + (2*C*a*b + B*b^2)*cos(d*x +
 c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (C*a^2 + 2*B*a*b + A*b^2)*cos
(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3*sec(d*x + c))
*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(7/2), x)
```

3.1351 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=419

$$\frac{\left(4a^2b(4A+15C)+10a^3B+20ab^2B-b^3(16A-15C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\left(6a^2(\right.}{}$$

```
[Out] ((10*a^3*B + 20*a*b^2*B - b^3*(16*A - 15*C) + 4*a^2*b*(4*A + 15*C))*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b^2*(2*b*B + 5*a*C)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((70*a*b*B + b^2*(46*A - 15*C)
+ 6*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (b
*(16*A*b + 10*a*B - 15*b*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[Cos[c + d*x]]) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(3*d) + (2*A*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5
/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 1.828, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4094, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(4a^2b(4A+15C)+10a^3B+20ab^2B-b^3(16A-15C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\left(6a^2(3A+5C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] ((10*a^3*B + 20*a*b^2*B - b^3*(16*A - 15*C) + 4*a^2*b*(4*A + 15*C))*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b^2*(2*b*B + 5*a*C)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((70*a*b*B + b^2*(46*A - 15*C)
+ 6*a^2*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (b
*(16*A*b + 10*a*B - 15*b*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[Cos[c + d*x]]) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(3*d) + (2*A*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5
/2)*Sin[c + d*x])/(5*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\sin}{5d}$$

$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}(a+b\sec(c+d}{3d}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{a+b\sec(c+}}{15d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{a+b\sec(c+}}{15d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{a+b\sec(c+}}{15d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(16Ab+10aB-15bC)\sqrt{a+b\sec(c+}}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(2bB+5aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}}$$

$$= \frac{\left(10a^3B+20ab^2B-b^3(16A-15C)+4a\right.}{15d\sqrt{\cos(c+d}}$$

Mathematica [C] time = 34.8934, size = 86542, normalized size = 206.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.706, size = 2893, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] -1/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c)
)^3*(15*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1)
)^(3/2)+6*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*(1/(cos(d*x+c)+
1))^(3/2)+18*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+
```

```
c)+1))^(3/2)+10*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d
*x+c)+1))^(3/2)+28*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^2*b*(1/(
cos(d*x+c)+1))^(3/2)+68*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2
*(1/(cos(d*x+c)+1))^(3/2)+80*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*
a^2*b*(1/(cos(d*x+c)+1))^(3/2)+70*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+
c)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+30*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b
*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+15*C*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(3/2)+28*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2
*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+18*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x
+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+22*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(
d*x+c)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+10*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*c
os(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+18*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-18*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3+46*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b^3-10*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3+30*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^3-60*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+30*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3-30*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^3-15*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*b^3+46*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*b^3
*(1/(cos(d*x+c)+1))^(3/2)+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a
^3*(1/(cos(d*x+c)+1))^(3/2)+70*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b-90*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^2-70*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b+70*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^2-90*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b+60*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2+30*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b+15*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*b^2-150*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*a*b^2-34*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+46*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b^2+18*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b-46*A*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
```

```
),(-(a+b)/(a-b))^(1/2))*a*b^2+10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x
+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)+30*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*
(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c))/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/s
in(d*x+c)^6/(1/(cos(d*x+c)+1))^(3/2)/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(5/2), x)
```

3.1352 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\right.$

Optimal. Leaf size=427

$$\frac{\left(8a^3(A+3C)+48a^2bB+ab^2(16A+33C)+12b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{12d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\left(24a^2B\right.}{}$$

```
[Out] ((48*a^2*b*B + 12*b^3*B + 8*a^3*(A + 3*C) + a*b^2*(16*A + 33*C))*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(12*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b*(8*A*b^2 + 20*a*b*B + 15*a^2*C
 + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((24*a^2
*B - 12*b^2*B + a*b*(56*A - 27*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(12*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]) - (b*(8*a*A - 12*b*B - 21*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(12*d*Sqrt[Cos[c + d*x]]) - (b*(4*A - 3*C)*(a + b*Sec[c + d*x])^(3/2)*
Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*(a + b*Sec
[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.82817, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4094, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(8a^3(A+3C)+48a^2bB+ab^2(16A+33C)+12b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{12d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\left(24a^2B+ab(56A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] ((48*a^2*b*B + 12*b^3*B + 8*a^3*(A + 3*C) + a*b^2*(16*A + 33*C))*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(12*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b*(8*A*b^2 + 20*a*b*B + 15*a^2*C
 + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((24*a^2
*B - 12*b^2*B + a*b*(56*A - 27*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(12*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]) - (b*(8*a*A - 12*b*B - 21*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(12*d*Sqrt[Cos[c + d*x]]) - (b*(4*A - 3*C)*(a + b*Sec[c + d*x])^(3/2)*
Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*(a + b*Sec
[c + d*x])^(5/2)*Sin[c + d*x])/(3*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4094

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\sin}{3d}$$

$$= -\frac{b(4A-3C)(a+b\sec(c+dx))^{3/2}\sin(c}{6d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{a+b\sec(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{a+b\sec(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{a+b\sec(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(8aA-12bB-21aC)\sqrt{a+b\sec(c+}}{12d\sqrt{\cos(c+dx)}}$$

$$= \frac{b\left(8Ab^2+20abB+15a^2C+4b^2C\right)\sqrt{\frac{b+}{}}}{4d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$= \frac{\left(48a^2bB+12b^3B+8a^3(A+3C)+ab^2(\right.}{12d\sqrt{\cos(c+dx}}$$

Mathematica [C] time = 35.3315, size = 129353, normalized size = 302.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.955, size = 2792, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)
,x)
```

```
[Out] 1/12/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(-56*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^2*b-12*B*sin(d*x+c)*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-27*C*cos(d*x+c)^2*((a-b)/
```

(a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-6*C*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-33*C*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+27*C*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b^2-8*A*sin(d*x+
c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*(1/(cos(d*x+c)+1))^(3/2)-64*A*((a-b
)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-56*A*
((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+
72*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a*b^2
+56*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2*
b-56*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a*b
^2+72*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^
2*b-48*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a
*b^2+120*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*a*b^2-24*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*cos(d*x+c)^2*a^2*b-12*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*cos(d*x+c)^2*a*b^2-18*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^2*a^2*b+6*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^2*a*b^2+90*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a^2*b-27*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^2*b-24*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)
^2*sin(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-12*B*cos(d*x+c)*sin(d*x+c)*((a
-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^3-6*C*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-6*C*((a-b)/(a+b))^(1/2)*b^3*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-8*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^2*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4
*sin(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)-24*B*((a-b)/(a+b))^(1/2)*cos(d*x+c
)^3*sin(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)+8*A*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^3-24*A*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*b^3+48*A*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*b^3-24*B*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^3+24*B*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*a^3+12*B*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*b^3+24*C*EllipticF((-1+cos(d
x+c))((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*a^3-12*C*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*b^3+24*C*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*b^3)*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/

```
2)/sin(d*x+c)^6/(1/(cos(d*x+c)+1))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d
*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x
+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*cos(d*x + c)^(3/2), x)
```

3.1353 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+\right.$

Optimal. Leaf size=453

$$\frac{\left(a^2b(96A+59C)+48a^3B+66ab^2B+8b^3(3A+2C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\left(15a^2C\right.}{}$$

```
[Out] ((48*a^3*B + 66*a*b^2*B + 8*b^3*(3*A + 2*C) + a^2*b*(96*A + 59*C))*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((30*a^2*b*B + 8*b^3*B + 5*a^3*
C + 20*a*b^2*(2*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]) + ((24*A*b^2 + 42*a*b*B + 15*a^2*C + 16*b^2*
C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + ((6*b
*B + 5*a*C)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x
]]) + (C*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.8757, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4096, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\left(15a^2C+42abB+24Ab^2+16b^2C\right)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\left(a^2b(96A+59C)+48a^3B+66ab^2B+8b^3(3A\right.}{24d\sqrt{\cos(c+dx)}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*S
ec[c + d*x]^2),x]
```


```
[Out] ((48*a^3*B + 66*a*b^2*B + 8*b^3*(3*A + 2*C) + a^2*b*(96*A + 59*C))*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((30*a^2*b*B + 8*b^3*B + 5*a^3*
C + 20*a*b^2*(2*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((54*a*b*B - a^2*(48*A - 33*C) + 8*b^2*(3*A + 2*C))*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]) + ((24*A*b^2 + 42*a*b*B + 15*a^2*C + 16*b^2*
C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + ((6*b
*B + 5*a*C)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(12*d*Sqrt[Cos[c + d*x
]]) + (C*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{s}}{}$$

$$= \frac{C(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}$$

$$= \frac{(6bB+5aC)(a+b\sec(c+dx))^{3/2}\sin(c}{12d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{a}}{24d\sqrt{\cos(c+d}}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{a}}{24d\sqrt{\cos(c+d}}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{a}}{24d\sqrt{\cos(c+d}}$$

$$= \frac{\left(24Ab^2+42abB+15a^2C+16b^2C\right)\sqrt{a}}{24d\sqrt{\cos(c+d}}$$

$$= \frac{\left(30a^2bB+8b^3B+5a^3C+20ab^2(2A+\right.}{8d\sqrt{\cos(c+dx)}\sqrt{}}$$

$$= \frac{\left(48a^3B+66ab^2B+8b^3(3A+2C)+a^2\right.}{24d\sqrt{\cos(c+}}$$

Mathematica [C] time = 36.5855, size = 157926, normalized size = 348.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
 + C*Sec[c + d*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 1.01, size = 3162, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)
,x)
```

```
[Out] -1/24/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3*(66*B*sin(d*x+c)*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+59*C*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+34*C*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+34*C*cos(d*x+c)*
```

```
((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+24*A*sin(d*x+
c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)+54*B*sin
(d*x+c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+12*
B*sin(d*x+c)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2
)+26*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(3/2)+16*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*sin(d*x+c)*(1/(cos(d*x+c
)+1))^(3/2)+24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*
x+c)^3*a*b^2-144*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b+48*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*a^2*b*(1/(c
os(d*x+c)+1))^(3/2)+48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*cos(d*x+c)^3*a^2*b-48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*cos(d*x+c)^3*a^3-48*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*cos(d*x+c)^3*a^3+96*A*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*b^2+54*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b-54*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2-120*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a*b^2-26*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b+44*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2-33*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^2*b+16*C*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a*b^2-240*A*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*a*b^2+48*A*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*a^3+24*A*sin(d*x+c)*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(3/2)+16*C*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+12*B*cos(d*x
+c)^2*sin(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^3+12*B*cos(
d*x+c)*sin(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^3+8*C*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-180*B*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a^2*b+36*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a^2*b-12*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a*b^2-24*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+
c)^3*b^3+8*C*((a-b)/(a+b))^(1/2)*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-48
*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*
x+c)^3*b^3+24*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*b^3-30*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*cos(d*x+c)^3*a^3-18*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
```

```
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*cos(d*x+c)^3*a^3+33*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*cos(d*x+c)^3*a^3-16*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*cos(d*x+c)^3*b^3+33*C*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*sin(d*x+c)*(1
/(cos(d*x+c)+1))^(3/2)+48*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a^3
*(1/(cos(d*x+c)+1))^(3/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b)
)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^6/cos(d*x+c)^(5/2)/(1/(cos(d*x+c)+1))^(
3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)*sqrt(cos(d*x + c)), x)
```

3.1354 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=550

$$\frac{\left(a^3(384A+133C)+472a^2bB+4ab^2(132A+89C)+128b^3B\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{192d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\,(5a}{}$$

```
[Out] ((472*a^2*b*B + 128*b^3*B + 4*a*b^2*(132*A + 89*C) + a^3*(384*A + 133*C))*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(1
92*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*b*B + 160*a*b^
3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(64*b*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((264*a^2*b*B + 128*b^3*B + 15*a
^3*C + 4*a*b^2*(108*A + 71*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + ((16*A*b^2 + 24*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(32*d*Cos[c + d*x]^(3/2)) + ((264*a^2*b*B + 128*b^3*B +
15*a^3*C + 4*a*b^2*(108*A + 71*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
192*b*d*Sqrt[Cos[c + d*x]]) + ((8*b*B + 5*a*C)*(a + b*Sec[c + d*x])^(3/2)*S
in[c + d*x])/(24*d*Cos[c + d*x]^(3/2)) + (C*(a + b*Sec[c + d*x])^(5/2)*Sin[
c + d*x])/(4*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 2.38615, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\left(5a^2C+24abB+16Ab^2+12b^2C\right)\sqrt{a+b\sec(c+dx)}}{32d\cos^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(264a^2bB+15a^3C+4ab^2(108A+71C)\right.}{192bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```


```
[Out] ((472*a^2*b*B + 128*b^3*B + 4*a*b^2*(132*A + 89*C) + a^3*(384*A + 133*C))*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(1
92*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*b*B + 160*a*b^
3*B - 5*a^4*C + 120*a^2*b^2*(2*A + C) + 16*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(64*b*d*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((264*a^2*b*B + 128*b^3*B + 15*a
^3*C + 4*a*b^2*(108*A + 71*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + ((16*A*b^2 + 24*a*b*B + 5*a^2*C + 12*b^2*C)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(32*d*Cos[c + d*x]^(3/2)) + ((264*a^2*b*B + 128*b^3*B +
15*a^3*C + 4*a*b^2*(108*A + 71*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
192*b*d*Sqrt[Cos[c + d*x]]) + ((8*b*B + 5*a*C)*(a + b*Sec[c + d*x])^(3/2)*S
in[c + d*x])/(24*d*Cos[c + d*x]^(3/2)) + (C*(a + b*Sec[c + d*x])^(5/2)*Sin[
c + d*x])/(4*d*Cos[c + d*x]^(3/2))
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
```

```
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+$$

$$= \frac{C(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+\right.$$

$$= \frac{(8bB+5aC)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)} + \frac{C}{}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+b\sec(c+}}{32d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+b\sec(c+}}{32d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+b\sec(c+}}{32d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+b\sec(c+}}{32d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(16Ab^2+24abB+5a^2C+12b^2C\right)\sqrt{a+b\sec(c+}}{32d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\left(40a^3bB+160ab^3B-5a^4C+120a^2b^2(2A+C)+\right.}{64bd\sqrt{\cos(c+dx}}$$

$$= \frac{\left(472a^2bB+128b^3B+4ab^2(132A+89C)+a^3(38\right.}{192d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [C] time = 35.9669, size = 180789, normalized size = 328.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.953, size = 4031, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)
,x)
```

[Out] -1/192/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c
))^3*(72*C*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^2*sin(d*
x+c)*b^4-384*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+
c)^4*a^3*b+432*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2*(1/(co
s(d*x+c)+1))^(3/2)+272*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^3*
(1/(cos(d*x+c)+1))^(3/2)+472*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*
a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+254*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2
*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+184*C*((a-b)/(a+b))^(1/2)*cos(d*x+
c)^2*a*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+264*B*((a-b)/(a+b))^(1/2)*co
s(d*x+c)^4*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+208*B*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+128*B*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+356*C*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+13
3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3
/2)+254*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b^2*sin(d*x+c)*(1/(cos(d*x+c
)+1))^(3/2)+118*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*b*sin(d*x+c)*(1/(cos
(d*x+c)+1))^(3/2)+284*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^2*b^2*sin(d*x+c)
*(1/(cos(d*x+c)+1))^(3/2)+72*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a*b^3*sin(d
x+c)(1/(cos(d*x+c)+1))^(3/2)+528*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d
*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+272*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3
*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+184*C*((a-b)/(a+b))^(1/2)*cos(d*
x+c)*a*b^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+96*A*((a-b)/(a+b))^(1/2)*cos
(d*x+c)^4*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)-240*B*cos(d*x+c)^4*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b-960*B*c
os(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*a*b^3-144*B*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^3*b-208*B*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a^2*b^2+352*B*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a*b^3+264*B*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*b-264*B*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^2+128*B*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^3+48*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*b^4
*(1/(cos(d*x+c)+1))^(3/2)-720*C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2-118*C*cos(d*x+c)^4*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b+76*C*cos(d*x+c)^4*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-72*C*cos(d*x+c)^4*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3-15*C*cos(d*x+c)^4*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+284*C*cos(d*x+c)^4*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-284*C*cos(d*x+c)^4*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+15*C*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^4*a^4*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-96*A*cos(d*x+c)
^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+432*A*cos(d*x+

```
c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-432*A*cos(
d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+64*B*((a
-b)/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+96*A*((
a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)^3*sin(d*x+c)*b^4+72*C
*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+9
6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2
)+128*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^
(3/2)+288*A*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a^2*b^2+64*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^4*(1/(cos(d*x+
c)+1))^(3/2)-1440*A*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*a^2*b^2-384*A*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4-128*B*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+30*C*cos(d*x+c)^4*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^4-288*C*cos(d*x+c)
^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4-30
*C*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+144
*C*cos(d*x+c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+15*
C*cos(d*x+c)^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+48*C
*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+192*A*cos(d*x+
c)^4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4)/b/((a-b)/(a+b
))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(7/2)/sin(d*x+c)^6/(1/(cos(d*x+c)+1))^
(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/sqrt(cos(d*x + c)), x)
```

3.1355 $\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=674

$$\frac{\left(4a^2b^2(1180A+809C)+1330a^3bB-15a^4C+3560ab^3B+256b^4(5A+4C)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{1920bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((1330*a^3*b*B + 3560*a*b^3*B - 15*a^4*C + 256*b^4*(5*A + 4*C) + 4*a^2*b^2*
(1180*A + 809*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(1920*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (
(10*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5*C - 40*a^3*b^2*(2*A + C) - 8
0*a*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)])/(128*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) - ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a
^2*b^2*(220*A + 141*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(1920*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]) + ((80*A*b^2 + 110*a*b*B + 15*a^2*C + 64*b^2*C)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(240*d*Cos[c + d*x]^(5/2)) + ((590*a^2*b*B + 360*b^3*B + 1
5*a^3*C + 4*a*b^2*(260*A + 193*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
960*b*d*Cos[c + d*x]^(3/2)) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256
*b^4*(5*A + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(1920*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*b*B + a*C)*(a + b*Sec[c +
d*x])^(3/2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (C*(a + b*Sec[c + d*x]
)^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 2.97369, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4096, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\left(590a^2bB+15a^3C+4ab^2(260A+193C)+360b^3B\right)\sqrt{a+b\sec(c+dx)}}{960bd\cos^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\left(15a^2C+110abB+\right.}{240d\,c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] ((1330*a^3*b*B + 3560*a*b^3*B - 15*a^4*C + 256*b^4*(5*A + 4*C) + 4*a^2*b^2*
(1180*A + 809*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(1920*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (
(10*a^4*b*B - 240*a^2*b^3*B - 96*b^5*B - 3*a^5*C - 40*a^3*b^2*(2*A + C) - 8
0*a*b^4*(4*A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)])/(128*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) - ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256*b^4*(5*A + 4*C) + 12*a
^2*b^2*(220*A + 141*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(1920*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]) + ((80*A*b^2 + 110*a*b*B + 15*a^2*C + 64*b^2*C)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(240*d*Cos[c + d*x]^(5/2)) + ((590*a^2*b*B + 360*b^3*B + 1
5*a^3*C + 4*a*b^2*(260*A + 193*C))*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
960*b*d*Cos[c + d*x]^(3/2)) + ((150*a^3*b*B + 2840*a*b^3*B - 45*a^4*C + 256
*b^4*(5*A + 4*C) + 12*a^2*b^2*(220*A + 141*C))*Sqrt[a + b*Sec[c + d*x]]*Sin
```

```
[c + d*x])/(1920*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*b*B + a*C)*(a + b*Sec[c +
d*x])^(3/2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (C*(a + b*Sec[c + d*x]
)^(5/2)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{se}$$

$$= \frac{C(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(\sqrt{\cos(c+dx}\right.$$

$$= \frac{(2bB+aC)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{C(a+}{}$$

$$= \frac{\left(80Ab^2+110abB+15a^2C+64b^2C\right)\sqrt{a+b\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\left(80Ab^2+110abB+15a^2C+64b^2C\right)\sqrt{a+b\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\left(80Ab^2+110abB+15a^2C+64b^2C\right)\sqrt{a+b\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\left(80Ab^2+110abB+15a^2C+64b^2C\right)\sqrt{a+b\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\left(80Ab^2+110abB+15a^2C+64b^2C\right)\sqrt{a+b\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\left(80Ab^2+110abB+15a^2C+64b^2C\right)\sqrt{a+b\sec(c+}}{240d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{\left(10a^4bB-240a^2b^3B-96b^5B-3a^5C-40a^3b^2(2A\right.}{128b^2d\sqrt{\cos(c+}}$$

$$= \frac{\left(1330a^3bB+3560ab^3B-15a^4C+256b^4(5A+4C)\right.}{1920bd\sqrt{\cos(c+d}}$$

Mathematica [C] time = 37.5182, size = 211844, normalized size = 314.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^
2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 1.381, size = 5292, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+
c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2
)/cos(d*x + c)^(3/2), x)
```

3.1356 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=380

$$\frac{2\left(2a^2b^2(16A+35C)+5a^4(5A+7C)-49a^3bB-56ab^3B+48Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+$$

```
[Out] (2*(48*A*b^4 - 49*a^3*b*B - 56*a*b^3*B + 5*a^4*(5*A + 7*C) + 2*a^2*b^2*(16*
A + 35*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)])/(105*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*
A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^4*d*S
qrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(24*A*b^2 - 28*a*b*B + 5*a^2*(5*A +
 7*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^3*d
) - (2*(6*A*b - 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(35*a^2*d) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(7*a*d)
```

Rubi [A] time = 1.35008, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4265, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(5a^2(5A+7C)-28abB+24Ab^2\right)\sqrt{a+b\sec(c+dx)}}{105a^3d}+\frac{2\left(2a^2b^2(16A+35C)+5a^4(5A+7C\right.}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```


```
[Out] (2*(48*A*b^4 - 49*a^3*b*B - 56*a*b^3*B + 5*a^4*(5*A + 7*C) + 2*a^2*b^2*(16*
A + 35*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)])/(105*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*
A*b^3 - 63*a^3*B - 56*a*b^2*B + a^2*(44*A*b + 70*b*C))*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^4*d*S
qrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(24*A*b^2 - 28*a*b*B + 5*a^2*(5*A +
 7*C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^3*d
) - (2*(6*A*b - 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(35*a^2*d) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(7*a*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
```

```
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7ad} - \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= -\frac{2(6Ab-7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35a^2d} +$$

$$= \frac{2\left(24Ab^2-28abB+5a^2(5A+7C)\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}{105a^3d}$$

$$= \frac{2\left(24Ab^2-28abB+5a^2(5A+7C)\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}{105a^3d}$$

$$= \frac{2\left(24Ab^2-28abB+5a^2(5A+7C)\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}{105a^3d}$$

$$= \frac{2\left(24Ab^2-28abB+5a^2(5A+7C)\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}{105a^3d}$$

$$= \frac{2\left(48Ab^4-49a^3bB-56ab^3B+5a^4(5A+7C)+2a^2b^2(16A+\right.}{105a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

<u>Mathematica</u> [C] time = 19.326, size = 492, normalized size = 1.29

$$\frac{(a\cos(c+dx)+b)\left(\frac{\sin(c+dx)\left(115a^2A+140a^2C-112abB+96Ab^2\right)}{210a^3}+\frac{(7aB-6Ab)\sin(2(c+dx))}{35a^2}+\frac{A\sin(3(c+dx))}{14a}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\left(\cos^2\left(\frac{1}{2}(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*(((115*a^2*A + 96*A*b^2 - 112*a*b*B + 140*a^2*C)*Sin[
c + d*x])/(210*a^3) + ((-6*A*b + 7*a*B)*Sin[2*(c + d*x)])/(35*a^2) + (A*Sin
[3*(c + d*x)])/(14*a)))/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (
2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*
(-48*A*b^3 + 63*a^3*B + 56*a*b^2*B - 2*a^2*b*(22*A + 35*C))*EllipticE[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(-48*A*b^3 + 4*a*b^2*(-3*A
+ 14*B) - 2*a^2*b*(22*A - 7*B + 35*C) + a^3*(25*A + 63*B + 35*C))*EllipticF
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (48*A*b^3 - 63*a^3*B - 56
*a*b^2*B + a^2*(44*A*b + 70*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)
^(3/2)*Tan[(c + d*x)/2]))/(105*a^4*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```

Maple [B] time = 0.771, size = 2829, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1
)^2*(-1+cos(d*x+c))^3*(25*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4
*(1/(cos(d*x+c)+1))^(3/2)-44*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^3*b+12*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*
b^2-48*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+44*A*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-44*A*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+48*A*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-48*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^4*(1
/(cos(d*x+c)+1))^(3/2)+25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(
1/(cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*
(1/(cos(d*x+c)+1))^(3/2)+63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4
*(1/(cos(d*x+c)+1))^(3/2)+25*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos
(d*x+c)+1))^(3/2)-44*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c
)+1))^(3/2)+24*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3
/2)+63*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-28*B
*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+56*B*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+35*C*((a-b)/(a+b))
^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-70*C*((a-b)/(a+b))^(1/2)*a
^2*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^4*((a-b)/(a+b))^
(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+21*B*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+15*A*cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+35*C*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+63*B*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-56*B*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+56*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^3-14*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^3*b+56*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^
2+70*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-70*C*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-70*C*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-3*A*cos(d*x+c)^3*((a-b)/(a+b))^(
1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-7*B*cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-19*A*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+6*A*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-24*A*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)-7*B*cos(d*x+c)
```

```
*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+28*B*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-35*C*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-3*A*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+
6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^
(3/2)-63*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-35*C*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-25*A*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^4-48*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^4)/a^4/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^6/(1/(cos(d
*x+c)+1))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{3}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3*sec(d*x + c)^2 + B*cos(d*x + c)^3*sec(d*x + c) +
 A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/sqrt(b
*sec(d*x + c) + a), x)
```

3.1357 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=291

$$-\frac{2\left(a^2b(7A+15C)-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\left(3a^2(3A+5C)-\right.}{}$$

```
[Out] (-2*(8*A*b^3 - 5*a^3*B - 10*a*b^2*B + a^2*b*(7*A + 15*C))*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A
+ 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b -
 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d
) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.975189, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4265, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\left(a^2b(7A+15C)-5a^3B-10ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\left(3a^2(3A+5C)-10abB+\right.}{15a^3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```


```
[Out] (-2*(8*A*b^3 - 5*a^3*B - 10*a*b^2*B + a^2*b*(7*A + 15*C))*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^2 - 10*a*b*B + 3*a^2*(3*A
+ 5*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b -
 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d
) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} - \frac{\left(2\sqrt{\cos(c+}\right.}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} +$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} +$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} +$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} +$$

$$= -\frac{2\left(8Ab^3-5a^3B-10ab^2B+a^2b(7A+15C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 18.2299, size = 379, normalized size = 1.3

$$\frac{2a\sin(c+dx)(a\cos(c+dx)+b)(3aA\cos(c+dx)+5aB-4Ab)+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia\sec^2\left(\frac{1}{2}(c+dx)\right)\left(a^2(9A+5(B+3C))+\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x
] + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(8*A*b^2 - 10*a*b
*B + 3*a^2*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)] - I*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*(B + 3*C)))*EllipticF
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (8*A*b^2 - 10*a*b*B + 3*a
^2*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*
x)/2]))/Sec[c + d*x]^(3/2))/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]])
```

Maple [B] time = 0.529, size = 1885, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1)^
2*(-1+cos(d*x+c))^3*(-3*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*(
1/(cos(d*x+c)+1))^(3/2)-9*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*
x+c)+1))^(3/2)+4*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^
(3/2)-5*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+10*
B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-15*C*((a-b)
/(a+b))^(1/2)*a^2*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-3*A*sin(d*x+c)*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3/2)-9*A*sin(d*x+c)*((
a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)-5*B*sin(d*x+c)*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3/2)-5*B*sin(d*x+c)*
((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3/2)-8*A*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-9*A*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^3+9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-
15*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+15*C*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-8*A*sin(d*x+c)*((a-b)/(a+b))
^(1/2)*b^3*(1/(cos(d*x+c)+1))^(3/2)+10*B*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^2*b-10*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2*b+10*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-
15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*A*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-8*A*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b+8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^2+A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(cos(d*x+
c)+1))^(3/2)+A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+
c)+1))^(3/2)-4*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(cos(d*
x+c)+1))^(3/2)+5*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(
d*x+c)+1))^(3/2)-15*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*sin(d*x+c)*(1/(cos
(d*x+c)+1))^(3/2))/a^3/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^6/(1
/(cos(d*x+c)+1))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b
*sec(d*x + c) + a), x)
```

3.1358 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=216

$$\frac{2\left(a^2(A+3C)-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*(2*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) - (2*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*a*d)
```

Rubi [A] time = 0.665206, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4265, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2(A+3C)-3abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```

```
[Out] (2*(2*A*b^2 - 3*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) - (2*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*a*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(2\sqrt{\cos}\right.}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left((2Ab-\right.}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\left(\frac{1}{2}b(\right.\right.}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\left(\frac{1}{2}b(\right.\right.}{}$$

$$= \frac{2\left(2Ab^2-3abB+a^2(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 12.4673, size = 359, normalized size = 1.66

$$\frac{4\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(aA\sin(c+dx)(a\cos(c+dx)+b) - \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ia\sec\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(a*A*(b + a*C
os[c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*
(a + b)*(-2*A*b + 3*a*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + I*a*(-2*A*b + a*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2)/(a + b)] + (2*A*b - 3*a*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*
x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*a^2*d*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.57, size = 1012, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))
^3*(A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*
a^2+A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^
2-A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+
3*B*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+
A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b-2*A*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*b^2+3*B*sin(d*x+c)*(1/(cos(d
*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b+2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^2-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*A*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2)*cos(d*
x+c)^(1/2)/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(3/2
)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b
*sec(d*x + c) + a), x)
```

3.1359 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=219

$$-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2C\sqrt{\frac{a\cos}{}}}{d\sqrt{\cos(}}$$

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)])
```

Rubi [A] time = 0.756044, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4265, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2C\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Gamma}{d\sqrt{\cos(c+dx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt[a + b
*Sec[c + d*x]],x]
```


```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
```

```
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{((Ab-a}{}$$

$$= -\frac{\left((Ab-aB)\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(C\sqrt{\frac{b}{}}\right.}{\sqrt{co}}$$

$$= \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left((Ab-aB)\sqrt{\frac{b+a\cos(}{a+b}}\right.}{a\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{\frac{b+a\cos(c+}{a+b}}}{d\sqrt{\cos(c+}}$$

Mathematica [C] time = 33.8512, size = 36160, normalized size = 165.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sqrt
[a + b*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.514, size = 2005, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(A*sin(d*x+c)*cos(d*x+c)^2*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(c
os(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-A*sin
```

```
(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a+A*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(3/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-B*sin(d*x+c)*cos(d*x
+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a+C*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-2*C*sin(d*x+c)*cos(d*x+c)^2*(1/(cos
(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*a+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-2*A*sin(d*x+c)*cos(d*x+c)*(1/(cos(
d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+2*A*
sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^
(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+2*C*sin(d*x+c)*cos(
d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a-4*C*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-A*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a+A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+C*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*
x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-2*C*sin(d*
x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/
((a-b)/(a+b))^(1/2))*a-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a+A*((a-b)/(a+b))
^(1/2)*cos(d*x+c)*a-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b+A*((a-b)/(a+b))^(1/2
)*b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/a/((a-b)/(a+b))^(
1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sec\left(c + dx\right) + C\sec^{2}\left(c + dx\right)\right)\sqrt{\cos\left(c + dx\right)}}{\sqrt{a + b\sec\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)*sqrt(cos(c + d*x))/sqrt(a
 + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx + c\right)^{2} + B\sec\left(dx + c\right) + A\right)\sqrt{\cos\left(dx + c\right)}}{\sqrt{b\sec\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b
*sec(d*x + c) + a), x)
```

3.1360 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=260

$$\frac{(2A+C)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2bB-aC)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{C\sin(c+dx)\sqrt{a}}{bd\sqrt{\cos(}}$$

```
[Out] ((2*A + C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - a*C)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (C*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.97983, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2A+C)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2bB-aC)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{C\sin(c+dx)\sqrt{a+b\sec(c}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]),x]
```


```
[Out] ((2*A + C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - a*C)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (C*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(b*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec\right.}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aC}{2}+A}{}}{b}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{aC}{2}}{\sqrt{\sec(c+}}}{b}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\left(C\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b}}{\sqrt{se}}}{2b}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left((2A+C)\sqrt{b+a\cos(c+dx)}\right)\int \frac{}{\sqrt{b+a}}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

$$= \frac{(2bB-aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{C\sqrt{a+b\sec(c+dx)}\sin(}{bd\sqrt{\cos(c+dx)}}$$

$$= \frac{(2A+C)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2bB-aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2\right.}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{s}}}$$

Mathematica [C] time = 32.5569, size = 52620, normalized size = 202.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.462, size = 866, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] -1/d*(2*A*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*b+4*B*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b+2*C*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-C*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+C*sin(d*x+c)*cos(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*C*sin(d*x+c)*cos(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+C*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-C*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*b-C*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b)*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin
(d*x+c)/(1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{\sqrt{b \sec(dx+c) + a}\sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sec(c + dx) + C\sec^2(c + dx)}{\sqrt{a + b\sec(c + dx)}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x) + C*sec(c + d*x)**2)/(sqrt(a + b*sec(c + d*x))
*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx + c)^2 + B\sec(dx + c) + A}{\sqrt{b\sec(dx + c) + a}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*sqrt(cos(d*x + c))), x)
```

3.1361 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=350

$$\frac{(4bB-aC)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] ((4*b*B - a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b
^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]) - ((4*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d*Cos[c + d*x
]^(3/2)) + ((4*b*B - 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d
*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.28481, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(3a^2C-4abB+8Ab^2+4b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(4bB-3aC)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}-\frac{(4bB-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]),x]
```


```
[Out] ((4*b*B - a*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b
^2 - 4*a*b*B + 3*a^2*C + 4*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]) - ((4*b*B - 3*a*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (C*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d*Cos[c + d*x
]^(3/2)) + ((4*b*B - 3*a*C)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d
*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
```

```
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c\right.}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(}{}}{2}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB-3aC)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB-3aC)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB-3aC)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{C\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4bB-3aC)\sqrt{a+b\sec(c+dx)}\sin(c+d}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(8Ab^2-4abB+3a^2C+4b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{C\sqrt{a+}}{}$$

$$= \frac{(4bB-aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(8Ab^2-4abB+3a^2C+4b^2C\right)}{4b^2d\sqrt{\cos(c+dx}}$$

Mathematica [C] time = 33.4505, size = 98830, normalized size = 282.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[
a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.49, size = 3191, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)
,x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*(cos(d*x+c)+1)*(-8*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a*b-4*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+2*C*cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*a*b-3*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+C*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a*b-2*C*((a-b)/(a+b))^(1/2)*b^2+8*B*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*
x+c)*a*b-4*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+2*C*cos(d*x+c)^2*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(1/2
)-3*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(
d*x+c)*a*b*(1/(cos(d*x+c)+1))^(1/2)+8*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-4*B*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*sin(d*x+c)*a*b+2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-3*C*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
^3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*a*b-8*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+16*A*
cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/2)-8*A*cos(d*x+c)^3*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/
2)+4*B*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin
(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/2)+6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
```

```
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticPi((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^2+8*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^3*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-6*C*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*sin(d*x+c)*a^2-4*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^2+3*C*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d
*x+c)*a^2+16*A*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/2)-8*A*cos(d*x+c)^2
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b^2*(1/(cos
(d*x+c)+1))^(1/2)+4*B*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/2)+6*C*cos(d*x+c)^2*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*
(1/(cos(d*x+c)+1))^(1/2)+8*C*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/2)-6*
C*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*a^2*(1/(cos(d*x+c)+1))^(1/2)-4*C*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b^2*(1/(cos(d*x+c)+1))^(1/2)+3*C*cos(d
*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*
(1/(cos(d*x+c)+1))^(1/2)+4*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-3*C*cos(d
*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+3*C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+2*
C*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+4*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*b^2-4*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/b^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^3/cos(d*x+c)^(3/2
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)
*cos(d*x + c)^(3/2)), x)
```

3.1362 $\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2aB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2bB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rubi [A] time = 0.897641, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {4265, 4072, 4037, 3854, 3858, 2663, 2661, 3859, 2807, 2805, 3856, 2655, 2653}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\,\text{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c + d*x]^
2))/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4072

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.
)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*((c_.) + csc[(e_.) + (f_.)
*(x_)]*(d_.))^(n_.), x_Symbol] :> Dist[1/b^2, Int[(a + b*Csc[e + f*x])^(m +
 1)*(c + d*Csc[e + f*x])^n*(b*B - a*C + b*C*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0]
```

Rule 4037

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
```

```
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(aA+(Ab+aB)\sec(c+dx)+bB\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(Ab+aB)\,\mathrm{s}}{\sqrt{\sec(c+a}}$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}\left(-abB+\right.}{\sqrt{\sec}}}{b^2}$$

$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+}}{\sqrt{\sec(c+dx)}}$$

$$= \left(aB\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx}}{\sqrt{a+b\sec(c+}}$$

$$= \frac{\left(aB\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(bB}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

$$= \frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2bB\sqrt{\frac{b+a\,\mathrm{co}}{a}}}{d\sqrt{\cos(c}}$$

Mathematica [C] time = 6.29192, size = 25325, normalized size = 121.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*A + (A*b + a*B)*Sec[c + d*x] + b*B*Sec[c +
 d*x]^2))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.512, size = 2301, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*a+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d
*x+c))^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(A*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1)
)^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b-A*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+
A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)^2*b-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*b+2*B*sin(d*x+c)*
(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*b+2*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*b-2*B*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)*b+4*B*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*b+A*sin(d*x+c)*(1
/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a-B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*b+2*B*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-A*((a-b)/(a+b))^(1/2)*b
+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a+A*
((a-b)/(a+b))^(1/2)*cos(d*x+c)*b-A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+
c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+A*sin(d*x+c)
*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+B*sin(d*x+c)*cos(d*x+c)^2*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a-2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+2*A*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1
))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*sin(d*x+
c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b+2*B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-A*sin(d*x+c)*(1/(cos(d*x+c)
+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+B*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(cos(d*x+c)+1))^(3/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/(a+b))^
(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\sec(dx+c)+A)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\sec(c+dx))\sqrt{a+b\sec(c+dx)}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a
+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bb\sec(dx+c)^2+Aa+(Ba+Ab)\sec(dx+c)\right)\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*sec(d*x+c)+b*B*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(cos(d*
x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.1363 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=461

$$-\frac{2\left(6a^2b(2A+5C)-5a^3B-40ab^2B+48Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\,(\ldots}{\ldots}$$

```
[Out] (-2*(48*A*b^3 - 5*a^3*B - 40*a*b^2*B + 6*a^2*b*(2*A + 5*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^4*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*A*b^4 + 25*a^3*b*B - 40*a*b
^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^
2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2
*(24*A*b^3 + 5*a^3*B - 20*a*b^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d) - (2*(6*A*b
^2 - 5*a*b*B - a^2*(A - 5*C))*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.5673, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2(-(A-5C))-5abB+6Ab^2\right)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(Ab^2-\ldots\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```


```
[Out] (-2*(48*A*b^3 - 5*a^3*B - 40*a*b^2*B + 6*a^2*b*(2*A + 5*C))*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^4*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(48*A*b^4 + 25*a^3*b*B - 40*a*b
^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^
2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2
*(24*A*b^3 + 5*a^3*B - 20*a*b^2*B - a^2*(9*A*b - 15*b*C))*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d) - (2*(6*A*b
^2 - 5*a*b*B - a^2*(A - 5*C))*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(}\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(6Ab^2\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(24Ab\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(24Ab\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(24Ab\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(24Ab\right.}{}$$

$$= -\frac{2\left(48Ab^3-5a^3B-40ab^2B+6a^2b(2A+5C)\right)\sqrt{\frac{b+a\cos(c}{a+b}}}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}}$$

Mathematica [C] time = 24.839, size = 3870, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((4*(-9*A*b + 5*a*B)*Sin[c + d*x])/(15*a^3) + (4*(A*b^4*Sin[c + d*x
] - a*b^3*B*Sin[c + d*x] + a^2*b^2*C*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(b + a
*Cos[c + d*x])) + (2*A*Sin[2*(c + d*x)])/(5*a^2)))/(d*(A + 2*C + 2*B*Cos[c
+ d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(3/2)) - (4*Cos[c + d*x]^
(3/2)*(b + a*Cos[c + d*x])*((6*a*A*Sqrt[Cos[c + d*x]])/(5*(a^2 - b^2)*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^2*Sqrt[Cos[c + d*x]])/(5*
a*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (32*A*b^4*Sqrt
```

[Cos[c + d*x]])/(5*a^3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (10*b*B*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*b^3*B*Sqrt[Cos[c + d*x]])/(3*a^2*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*C*Sqrt[Cos[c + d*x]])/((a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*b^2*C*Sqrt[Cos[c + d*x]])/(a*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*A*b*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(5*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) - (8*A*b^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(5*a^2*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) + (4*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) - (2*b*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]))*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^4*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((-2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^(3/2)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(5*a^4*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) - (4*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(((48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - a*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(Sec[(c + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*(48*A*b^4 + 25*a^3*b*B - 40*a*b^3*B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(

```
c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B)
 - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x
)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] - (a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B) - 6*a^2*b*(2*A + 5*(B
 + C)) + a^3*(9*A + 5*(B + 3*C)))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 +
 ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-48*A*b^4 - 25*a^3*b*B
 + 40*a*b^3*B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*Sec[(c + d*x)/2]
^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a +
b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(15*a^4
*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(-48*A*b^4 - 25*a^3*b*B + 40*a*b^3*
B + 6*a^2*b^2*(4*A - 5*C) + 3*a^4*(3*A + 5*C))*EllipticE[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 + 4*a*b^2*(9*A + 10*B
) - 6*a^2*b*(2*A + 5*(B + C)) + a^3*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSin
h[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (48*A*b^4 + 25*a^3*b*B - 40*a*b^3*
B - 6*a^2*b^2*(4*A - 5*C) - 3*a^4*(3*A + 5*C))*(b + a*Cos[c + d*x])*(Sec[(c
 + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[
(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(5*a^4*(a^2
- b^2)*Sqrt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 0.613, size = 2418, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)+1)^
5*(-1+cos(d*x+c))^3*(-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(
1/(cos(d*x+c)+1))^(3/2)-12*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^3*b-36*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^
2-48*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^3+24*A*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-48*A*((a-b)/(a+b))^(
1/2)*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)+9*A*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^4+15*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^4-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d
*x+c)+1))^(3/2)-15*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(
d*x+c)+1))^(3/2)-5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(
d*x+c)+1))^(3/2)-9*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1)
)^(3/2)-24*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)-
5*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+20*B*((a-
b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)+40*B*((a-b)/(a+
b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)-15*C*((a-b)/(a+b))^(1/2
```

```
)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-30*C*((a-b)/(a+b))^(1/2)*a^2*b^
2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)-3*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)-25*B*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^3*b+40*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^3+30*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a^3*b+40*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2-30*C
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-30*C*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-3*A*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-5*B*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-3*A*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)-18*A*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-24*A*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)+15*B*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))^(3/2)+20*B*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-
15*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/
2)+3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d*x+c)+1))
^(3/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+
c)+1))^(3/2)+5*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-15
*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-9*A*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-48*A*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^4)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)
/a^4/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x
 + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec
(d*x + c) + a)^(3/2), x)
```

3.1364 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=350

$$\frac{2\left(a^2(A+3C)-6abB+8Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2(-(A-3C))-3a\right.}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] (2*(8*A*b^2 - 6*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) + (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.1145, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2(-(A-3C))-3abB+4Ab^2\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(Ab^2-a(b\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```


```
[Out] (2*(8*A*b^2 - 6*a*b*B + a^2*(A + 3*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) + (2*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B - a^2*(5*A*b - 3*b*C))
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) - (2*(4*A*b^2 - 3*a*b*B - a^2*(A - 3*C))*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
```

```
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+d}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(4Ab^2-3\right.}{}$$

$$= \frac{2\left(8Ab^2-6abB+a^2(A+3C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right.}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 23.1212, size = 3283, normalized size = 9.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*((4*A*Sin[c + d*x])/(3*a^2) - (4*(A*b^3*Sin[c + d*x] - a*b^2*B*Sin[
c + d*x] + a^2*b*C*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/
(d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(
3/2)) + (4*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])*((-10*A*b*Sqrt[Cos[c + d
*x]])/(3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b
^3*Sqrt[Cos[c + d*x]])/(3*a^2*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*a*B*Sqrt[Cos[c + d*x]])/((a^2 - b^2)*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) - (4*b^2*B*Sqrt[Cos[c + d*x]])/(a*(a^2 - b^2)*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*C*Sqrt[Cos[c + d*x]])/((a^2
- b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) + (4*A*b^
2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[b + a*Cos[c
+ d*x]]) - (2*b*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[
b + a*Cos[c + d*x]]) + (2*a*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/((a^2
- b^2)*Sqrt[b + a*Cos[c + d*x]]))*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(
A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*(I*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*
b^2*B + a^2*(-5*A*b + 3*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
```

b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] - I*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (8*A*b^3 + 3*a^3*
B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2
]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d
*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((2
*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*(I
(a + b)(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*EllipticE[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(8*A*b^2 - 6*a*
b*(A + B) + a^2*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)] + (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*(b
+ a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^2*(a^2
 - b^2)*(b + a*Cos[c + d*x])^(3/2)) - (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/
2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*(I*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b
^2*B + a^2*(-5*A*b + 3*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)] - I*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*El
lipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*S
qrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (8*A*b^3 + 3*a^3*B
 - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]
^2)^(3/2)*Tan[(c + d*x)/2]))/(a^3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]]) + (
4*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(((8*A*b^3 + 3
*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d
x)/2]^2)^(5/2))/2 + I(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b
 + 3*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c
+ d*x)/2] - I*a*(a + b)*(8*A*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Ell
ipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - a*
(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*(Sec[(c + d*x)/2]^2)
^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B +
a^2*(-5*A*b + 3*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(
c + d*x)/2]^2)/2 + ((I/2)*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*
A*b + 3*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[
(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - ((I/2)*a*(a + b)*(8*A*b^2 - 6*a*b
(A + B) + a^2(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a
+ b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b))
)/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a*(a + b)*(8*A
*b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*Sec[(c + d*x)/2]^4*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2
]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(8*A*b^3 + 3*
a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/
2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(3*a^3*(a^2 - b^2)*Sqrt[
b + a*Cos[c + d*x]]) + (2*Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(I*(a + b)*(8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3*b*C))*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(8*A*
b^2 - 6*a*b*(A + B) + a^2*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)] + (8*A*b^3 + 3*a^3*B - 6*a*b^2*B + a^2*(-5*A*b + 3
*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-
(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c

```
 + d*x]*Tan[c + d*x]))/(a^3*(a^2 - b^2)*Sqrt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 0.669, size = 1518, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^5*(-1+cos(d*x+c))
^3*(A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(cos(d*x+c)+1))^(3
/2)+A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(cos(d*x+c)+1))^
(3/2)+A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1))^(3
/2)-3*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+c)+1))^
(3/2)-4*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(cos(d*x+c)+1)
)^(3/2)+3*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(cos(d*x+c)+1)
)^(3/2)+3*B*sin(d*x+c)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(cos(d*x+c)+
1))^(3/2)+A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)-4
*A*sin(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-8*A*sin(d*
x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(cos(d*x+c)+1))^(3/2)+3*B*sin(d*x+c)*((a-b)
/(a+b))^(1/2)*a^2*b*(1/(cos(d*x+c)+1))^(3/2)+6*B*sin(d*x+c)*((a-b)/(a+b))^(
1/2)*a*b^2*(1/(cos(d*x+c)+1))^(3/2)-3*C*((a-b)/(a+b))^(1/2)*a^2*b*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(3/2)-A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^3-6*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-8*A
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+5*A*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*A*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*b^3+3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^3+6*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2
*b-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*B*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*C*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b)*cos(d*x+c)^(1/2)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+
1))^(3/2)/a^3/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^
2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^(3/2), x)
```

3.1365 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{2(2Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\left(\right.}{}$$

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(
2*A*b^2 - a*b*B - a^2*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.755281, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4265, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\left(a^2(-(A-C))-abB+2Ab^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(3/2),x]
```


```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(
2*A*b^2 - a*b*B - a^2*(A - C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(a*(a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sqrt{\sec(c+dx)}(a+b\sec(c+}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+}\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\right.}{a^2\sqrt{c}}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left((2Ab-aB)\right.\right.}{a^2\sqrt{\cos}}$$

$$= -\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(2Ab^2-ab\right.}{}$$

Mathematica [C] time = 17.6123, size = 517, normalized size = 2.08

$$\frac{4\sqrt{\cos(c+dx)}(a\cos(c+dx)+b)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)\left(a^2C\sin(c+dx)-abB\sin(c+dx)+Ab^2\sin(c+\right.}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}(A\cos(2c+2dx)+A+2B\cos(c+dx)+2C)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x] + C*Sec[c +
d*x]^2)*(A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(a*
(a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[
c + d*x])^(3/2)) - (4*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a
 + b)*(-2*A*b^2 + a*b*B + a^2*(A - C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-2*A*b + a*(A + B - C))*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*A*b^2 - a*b*B + a^2*(-A + C)
)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a^2*(
a^2 - b^2)*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*Sqrt[Sec[c +
 d*x]]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.546, size = 966, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -2/d*(cos(d*x+c)+1)^5*(-1+cos(d*x+c))^3*(A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3
/2)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2
)*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*
((a-b)/(a+b))^(1/2)*a*b+2*A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b
))^(1/2)*b^2-B*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*b+
C*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2+A*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b-A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2+2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-B*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-B*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2+C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1
/2)*(1/(cos(d*x+c)+1))^(3/2)/a^2/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^2+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec
(d*x + c) + a)^(3/2), x)
```

3.1366 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=311

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\left(Ab^2-a\right.}{ab}$$

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(a*b*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b^2 - a*(b*
B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]])
```

Rubi [A] time = 1.16698, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\left(Ab^2-a(bB-aC)\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{abd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(a*b*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b^2 - a*(b*
B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
```

```
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A+B\sec(c+dx)+C\sec^{\cdot}\right.}{(a+b\sec(c+dx))^{3/2}}$$

$$=-\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$=-\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$=-\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$=-\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(A\sqrt{b+a\cos(c+dx)}\right)}{a\sqrt{\cos(c+dx)}\sqrt{a+}}$$

$$=\frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

$$=\frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+l}\right.}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 35.0312, size = 63246, normalized size = 203.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.456, size = 950, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 2/d*(cos(d*x+c)+1)^5*(-1+cos(d*x+c))^3*(A*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/
2)*((a-b)/(a+b))^(1/2)*b^2-B*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+
b))^(1/2)*a*b+C*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2
+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+A*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b-B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b-2*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-C*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*C*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+2*C*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+C*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)/b/a/(b+a*cos(d*x+c)
)/(a-b)/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(3/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/
2)*sqrt(cos(d*x + c))), x)
```

3.1367 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=393

$$\frac{C\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\left(3a^2C-2abB+2\right.}{b^2d\left(a^2-b^2\right.}$$

```
[Out] (C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 3*a*C)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((2*A*b^2 - 2*a*b*B +
3*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*
x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x
]])
```

Rubi [A] time = 1.49248, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\left(3a^2C-2abB+2Ab^2-b^2C\right)\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^(3/2)),x]
```


```
[Out] (C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 3*a*C)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((2*A*b^2 - 2*a*b*B +
3*a^2*C - b^2*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*
x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b^2 - 2*a*b*B + 3*a^2*C - b^2*C)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x
]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2\right.}{(a+b\sec(c+dx))^{3/2}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c-\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2abB+3a^2C-\right.}{b^2\left(a^2-\right.}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2abB+3a^2C-\right.}{b^2\left(a^2-\right.}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2abB+3a^2C-\right.}{b^2\left(a^2-\right.}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2Ab^2-2abB+3a^2C-\right.}{b^2\left(a^2-\right.}$$

$$= \frac{(2bB-3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(Ab^2-a(bB-a\right.}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+d}$$

$$= \frac{C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2bB-3aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c\right.}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

Mathematica [C] time = 34.7657, size = 111509, normalized size = 283.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + 
b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.488, size = 1503, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)
,x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^3*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^5
*(2*A*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*b^
```

```
2-2*B*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)*((a-b)/(a+b))^(1/2)*a*
b+3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c
)+C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+
C*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b*(1/(cos(d*x+c)+1))^(3/2)+C*sin(d*x+c)*
((a-b)/(a+b))^(1/2)*(1/(cos(d*x+c)+1))^(3/2)*b^2-2*A*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+2*A*cos(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+4*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-2*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b-4*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-4*B*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-6*C*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-4*C*cos(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+6*C*cos(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b)*((a-b)/(a+b))^(1/2)*(1/
(cos(d*x+c)+1))^(3/2)/b^2/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/(a-b)/sin(d*x+c
)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec(dx+c)^2 + B\sec(dx+c) + A}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/
2)*cos(d*x + c)^(3/2)), x)
```

3.1368 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=663

$$\frac{2\left(-4a^2b^3(29A-10C)-a^4b(17A+45C)+80a^3b^2B+5a^5B-80ab^4B+128Ab^5\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right.}{15a^5d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*(128*A*b^5 + 5*a^5*B + 80*a^3*b^2*B - 80*a*b^4*B - 4*a^2*b^3*(29*A - 10*
C) - a^4*b*(17*A + 45*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)])/(15*a^5*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) + (2*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B
+ 5*a^4*b^2*(11*A - 15*C) - 4*a^2*b^4*(53*A - 10*C) + 3*a^6*(3*A + 5*C))*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b
^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (2*(8*A*b^4 + 9*a^3*b*B - 5*a*b^3*B - 2*a^2*b^2*
(6*A - C) - 6*a^4*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*
d*Sqrt[a + b*Sec[c + d*x]]) - (2*(64*A*b^5 - 5*a^5*B + 65*a^3*b^2*B - 40*a*
b^4*B + 2*a^4*b*(7*A - 20*C) - 2*a^2*b^3*(49*A - 10*C))*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d) + (2*(48*A*
b^4 + 50*a^3*b*B - 30*a*b^3*B + a^4*(3*A - 35*C) - a^2*b^2*(71*A - 15*C))*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2
)^2*d)
```

Rubi [A] time = 2.45997, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(-a^2b^2(71A-15C)+a^4(3A-35C)+50a^3bB-30ab^3B+48Ab^4\right)\sqrt{a+b\sec(c+dx)}}{15a^3d\left(a^2-b^2\right)^2}-\frac{2\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```


```
[Out] (2*(128*A*b^5 + 5*a^5*B + 80*a^3*b^2*B - 80*a*b^4*B - 4*a^2*b^3*(29*A - 10*
C) - a^4*b*(17*A + 45*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)])/(15*a^5*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) + (2*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B
+ 5*a^4*b^2*(11*A - 15*C) - 4*a^2*b^4*(53*A - 10*C) + 3*a^6*(3*A + 5*C))*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b
^2 - a*(b*B - a*C))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (2*(8*A*b^4 + 9*a^3*b*B - 5*a*b^3*B - 2*a^2*b^2*
(6*A - C) - 6*a^4*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*
d*Sqrt[a + b*Sec[c + d*x]]) - (2*(64*A*b^5 - 5*a^5*B + 65*a^3*b^2*B - 40*a*
b^4*B + 2*a^4*b*(7*A - 20*C) - 2*a^2*b^3*(49*A - 10*C))*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d) + (2*(48*A*
b^4 + 50*a^3*b*B - 30*a*b^3*B + a^4*(3*A - 35*C) - a^2*b^2*(71*A - 15*C))*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2
)^2*d)
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
```

```
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec^2(c}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+d}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+}\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(8Ab^4+9\right.}{}$$

$$= \frac{2\left(128Ab^5+5a^5B+80a^3b^2B-80ab^4B-4a^2b^3(29A-10C)\right.}{15a^5\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 27.6113, size = 4917, normalized size = 7.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(-14*A*
b + 5*a*B)*Sin[c + d*x])/(15*a^4) - (4*(A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c
+ d*x] + a^2*b^3*C*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2
) - (4*(-15*a^2*A*b^4*Sin[c + d*x] + 11*A*b^6*Sin[c + d*x] + 12*a^3*b^3*B*S
in[c + d*x] - 8*a*b^5*B*Sin[c + d*x] - 9*a^4*b^2*C*Sin[c + d*x] + 5*a^2*b^4
*C*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + (2*A*Sin[2*(
c + d*x)])/(5*a^3)))/(d*Sqrt[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*
Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) - (4*Cos[c + d*x]^(3/2)*(b +
a*Cos[c + d*x])^2*((6*a^2*A*Sqrt[Cos[c + d*x]])/(5*(a^2 - b^2)^2*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (22*A*b^2*Sqrt[Cos[c + d*x]])/(3*(a^2
- b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (424*A*b^4*Sqrt[Cos
[c + d*x]])/(15*a^2*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) + (256*A*b^6*Sqrt[Cos[c + d*x]])/(15*a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) - (16*a*b*B*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (56*b^3*B*Sqrt[Cos[c + d*
x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (32*
b^5*B*Sqrt[Cos[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (2*a^2*C*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (10*b^2*C*Sqrt[Cos[c + d*x]])/((a^2 - b^
2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*b^4*C*Sqrt[Cos[c +
d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(16*a*A*b*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*(a^2 - b^2)^2*Sqrt[b +
 a*Cos[c + d*x]]) - (88*A*b^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*a*
(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (64*A*b^5*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(15*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*B*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c +
d*x]]) + (14*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) - (8*b^4*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
/(3*a^2*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*a*b*C*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*b^3*C*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]))*Sqrt[Sec[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(A + B
*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a + b)*(128*A*b^6 - 40*a^5*b*B + 1
40*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4
*a^2*b^4*(-53*A + 10*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + 2*a^3*b^2*(36*A +
40*B - 15*C) + 4*a^2*b^3*(-29*A + 15*B + 10*C) - a^4*b*(17*A + 45*(B + C))
+ a^5*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] - (128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2
*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*(b + a*Cos[c
 + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^5*(a^2 - b^2)^
2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^
(5/2)*((-2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c
 + d*x]*((-I)*(a + b)*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B
+ 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*
A*b^5 - 16*a*b^4*(6*A + 5*B) + 2*a^3*b^2*(36*A + 40*B - 15*C) + 4*a^2*b^3*(
-29*A + 15*B + 10*C) - a^4*b*(17*A + 45*(B + C)) + a^5*(9*A + 5*(B + 3*C)))
*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^
2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (128*A*b^6 - 40
*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*
A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]
^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/
```

2)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c +
d*x]*((-I)*(a + b)*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B +
5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*Ell
ipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*
b^5 - 16*a*b^4*(6*A + 5*B) + 2*a^3*b^2*(36*A + 40*B - 15*C) + 4*a^2*b^3*(-2
9*A + 15*B + 10*C) - a^4*b*(17*A + 45*(B + C)) + a^5*(9*A + 5*(B + 3*C)))*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (128*A*b^6 - 40*a
^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A
+ 5*C) + 4*a^2*b^4*(-53*A + 10*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2
)^(3/2)*Tan[(c + d*x)/2]))/(5*a^5*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) -
(4*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((128*A*b^
6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a
^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*(b + a*Cos[c + d*x])*(Sec[(c + d
x)/2]^2)^(5/2))/2 - I(a + b)*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80
*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A +
10*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d
*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d
*x)/2] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + 2*a^3*b^2*(36*A +
40*B - 15*C) + 4*a^2*b^3*(-29*A + 15*B + 10*C) - a^4*b*(17*A + 45*(B + C))
+ a^5*(9*A + 5*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]*Tan[(c + d*x)/2] + a*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*
a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A +
10*C))*(Sec[(c + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(128*A
*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) +
3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*(b + a*Cos[c + d*x])*(Sec[(c
+ d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(128*A*b^6 - 40*a
^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A
+ 5*C) + 4*a^2*b^4*(-53*A + 10*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/
(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a +
b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a
+ b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + 2*a^3*b^2*(36*A + 40*B - 15*C) + 4
*a^2*b^3*(-29*A + 15*B + 10*C) - a^4*b*(17*A + 45*(B + C)) + a^5*(9*A + 5*(
B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*
A + 5*B) + 2*a^3*b^2*(36*A + 40*B - 15*C) + 4*a^2*b^3*(-29*A + 15*B + 10*C)
- a^4*b*(17*A + 45*(B + C)) + a^5*(9*A + 5*(B + 3*C)))*Sec[(c + d*x)/2]^4*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c
+ d*x)/2]^2]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(
128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*
C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*Sec[(c + d*x)/2]^4*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(
c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(15*a^5*(a^2 -
b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B -
80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C) + 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*
A + 10*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(
a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + 2*a^3*b^2*(36*A + 40*B - 15*C) +
4*a^2*b^3*(-29*A + 15*B + 10*C) - a^4*b*(17*A + 45*(B + C)) + a^5*(9*A + 5
*(B + 3*C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(
c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (12
8*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B + 5*a^4*b^2*(11*A - 15*C)
+ 3*a^6*(3*A + 5*C) + 4*a^2*b^4*(-53*A + 10*C))*(b + a*Cos[c + d*x])*(Sec[

```
(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Si
n[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(5*a^5*(a^
2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]])))
```

Maple [B] time = 1.135, size = 6912, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+\right.}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2*sec(d*x + c)^2 + B*cos(d*x + c)^2*sec(d*x + c) +
 A*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x
 + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^2 + B\sec\left(dx+c\right) + A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec
(d*x + c) + a)^(5/2), x)
```

3.1369 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=521

$$-\frac{2\left(-2a^2b^2(8A-C)+a^4(-(A+3C))+9a^3bB-8ab^3B+16Ab^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+\cdots}{\cdots}$$

```
[Out] (-2*(16*A*b^4 + 9*a^3*b*B - 8*a*b^3*B - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*C)
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])
/(3*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(16
*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) + a^4*(8
*A*b - 6*b*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a
^3*b*B + 3*a*b^3*B + 4*a^4*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2
- b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B
+ a^4*(A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.8131, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(-a^2b^2(13A-C)+a^4(A-5C)+8a^3bB-4ab^3B+8Ab^4\right)\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2\sin(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```



```
[Out] (-2*(16*A*b^4 + 9*a^3*b*B - 8*a*b^3*B - 2*a^2*b^2*(8*A - C) - a^4*(A + 3*C)
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])
/(3*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(16
*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B - 2*a^2*b^3*(14*A - C) + a^4*(8
*A*b - 6*b*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(10*a^2*A*b^2 - 6*A*b^4 - 7*a
^3*b*B + 3*a*b^3*B + 4*a^4*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2
- b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B
+ a^4*(A - 5*C) - a^2*b^2*(13*A - C))*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos}\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2\right.}{}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(10a^2\right.}{}$$

$$= -\frac{2\left(16Ab^4+9a^3bB-8ab^3B-2a^2b^2(8A-C)-a^4(A+3C\right.}{3a^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

Mathematica [C] time = 25.9744, size = 4327, normalized size = 8.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*A*Sin[c
 + d*x])/(3*a^3) + (4*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x] + a^2*b^2*
C*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (4*(-12*a^2*A
*b^3*Sin[c + d*x] + 8*A*b^5*Sin[c + d*x] + 9*a^3*b^2*B*Sin[c + d*x] - 5*a*b
^4*B*Sin[c + d*x] - 6*a^4*b*C*Sin[c + d*x] + 2*a^2*b^3*C*Sin[c + d*x]))/(3*
a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])))/(d*Sqrt[Cos[c + d*x]]*(A + 2*C +
```

2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) - (4*Cos
[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*((-16*a*A*b*Sqrt[Cos[c + d*x]])/(3*(
a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (56*A*b^3*Sqrt[
Cos[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (32*A*b^5*Sqrt[Cos[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*B*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sq
rt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (10*b^2*B*Sqrt[Cos[c + d*x]])/
((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*b^4*B*Sqr
t[Cos[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) + (4*a*b*C*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) - (4*b^3*C*Sqrt[Cos[c + d*x]])/(3*a*(a^2 - b^2)^2*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*A*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (14*A*b^2*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d
*x]]) - (8*A*b^4*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^
2*Sqrt[b + a*Cos[c + d*x]]) - (4*a*b*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]
])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*b^3*B*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*C*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*
x]]) + (2*b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqr
t[b + a*Cos[c + d*x]]))*Sqrt[Sec[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a + b)*(-16*A*b^5 +
3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^4*(-8*A*b + 6
*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*
x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a +
b)*(-16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*a^3*b*(
-3*A - 3*B + C) + a^4*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] + (16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B + 2*a
^2*b^3*(-14*A + C) + a^4*(8*A*b - 6*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*
x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(A + 2*C + 2*B*Cos
[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)*((-2*Cos[c + d*x
]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*
(-16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^
4*(-8*A*b + 6*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
] + I*a*(a + b)*(-16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C
) + 3*a^3*b*(-3*A - 3*B + C) + a^4*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*
a*b^4*B + 2*a^2*b^3*(-14*A + C) + a^4*(8*A*b - 6*b*C))*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*(b + a*
Cos[c + d*x])^(3/2)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(-16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B +
8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^4*(-8*A*b + 6*b*C))*EllipticE[I*ArcSin
h[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-16*A*b^4 + 4*a*b^3*(
3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*a^3*b*(-3*A - 3*B + C) + a^4*(A
+ 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[
(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (1
6*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B + 2*a^2*b^3*(-14*A + C) + a^4*
(8*A*b - 6*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d
x)/2]))/(a^4(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*Cos[c + d*x]^(3/
2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(((16*A*b^5 - 3*a^5*B + 15*a^3*b
^2*B - 8*a*b^4*B + 2*a^2*b^3*(-14*A + C) + a^4*(8*A*b - 6*b*C))*(b + a*Cos[
c + d*x])*(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(-16*A*b^5 + 3*a^5*B -
15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^4*(-8*A*b + 6*b*C))*Ell
ipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*

```
a*(a + b)*(-16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*
a^3*b*(-3*A - 3*B + C) + a^4*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] - a*(16*A*b^5 - 3*a^5*B + 15*a
^3*b^2*B - 8*a*b^4*B + 2*a^2*b^3*(-14*A + C) + a^4*(8*A*b - 6*b*C))*(Sec[(c
 + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*(16*A*b^5 - 3*a^5*B
+ 15*a^3*b^2*B - 8*a*b^4*B + 2*a^2*b^3*(-14*A + C) + a^4*(8*A*b - 6*b*C))*(
b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/
2)*(a + b)*(-16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^3*(14*
A - C) + a^4*(-8*A*b + 6*b*C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a +
 b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))
/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*a*(a + b)
*(-16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*a^3*b*(-3
*A - 3*B + C) + a^4*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x]
)/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a
+ b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (a*(a + b)
*(-16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*a^3*b*(-3
*A - 3*B + C) + a^4*(A + 3*(B + C)))*Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[
1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-16*A*b^5 + 3*a^5*B
 - 15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^4*(-8*A*b + 6*b*C))*
Sec[(c + d*x)/2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*
Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/
2]^2])))/(3*a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(
3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(-16*A*b^5 + 3*a^5
*B - 15*a^3*b^2*B + 8*a*b^4*B + 2*a^2*b^3*(14*A - C) + a^4*(-8*A*b + 6*b*C)
)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]
^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-
16*A*b^4 + 4*a*b^3*(3*A + 2*B) + 2*a^2*b^2*(8*A - 3*B - C) + 3*a^3*b*(-3*A
- 3*B + C) + a^4*(A + 3*(B + C)))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] + (16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B + 2*a^2*b^
3*(-14*A + C) + a^4*(8*A*b - 6*b*C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]
^2)^(3/2)*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/
2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(a^4*(a^2 - b^2)^2*Sqr
t[b + a*Cos[c + d*x]])))
```

Maple [B] time = 1.163, size = 5097, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)
,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\sec\left(dx+c\right)+A\cos\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)*sec(d*x + c)^2 + B*cos(d*x + c)*sec(d*x + c) + A*c
os(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^
3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec
(d*x + c) + a)^(5/2), x)
```

3.1370 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=401

$$\frac{2\left(-a^2b(9A+C)+3a^3B-2ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(-2a^2b^2(4A+C)\right.}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c}}$$

```
[Out] (2*(8*A*b^3 + 3*a^3*B - 2*a*b^2*B - a^2*b*(9*A + C))*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 6*a^3*b*B - 2*a*
b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*
x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*
(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2*a^2*b^2*(4*A + C))*Sin[c + d*x
])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.22894, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4265, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(-2a^2b^2(4A+C)+5a^3bB-2a^4C-ab^3B+4Ab^4\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a + b*Sec
[c + d*x])^(5/2),x]
```


```
[Out] (2*(8*A*b^3 + 3*a^3*B - 2*a*b^2*B - a^2*b*(9*A + C))*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(8*A*b^4 + 6*a^3*b*B - 2*a*
b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*
x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*
(4*A*b^4 + 5*a^3*b*B - a*b^3*B - 2*a^4*C - 2*a^2*b^2*(4*A + C))*Sin[c + d*x
])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
```

```
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)+C\sec\cdots}{\sqrt{\sec(c+dx)}(a+b\sec(c\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos\cdots\right.}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab\cdots\right.}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab\cdots\right.}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab\cdots\right.}{\cdots}$$

$$= \frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4Ab\cdots\right.}{\cdots}$$

$$= \frac{2\left(8Ab^3+3a^3B-2ab^2B-a^2b(9A+C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\cdots\right.}{3a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d\cdots}}$$

Mathematica [C] time = 24.8534, size = 3834, normalized size = 9.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/(a +
 b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-4*(A*b^3
*Sin[c + d*x] - a*b^2*B*Sin[c + d*x] + a^2*b*C*Sin[c + d*x]))/(3*a^2*(a^2 -
 b^2)*(b + a*Cos[c + d*x])^2) + (4*(9*a^2*A*b^2*Sin[c + d*x] - 5*A*b^4*Sin[
c + d*x] - 6*a^3*b*B*Sin[c + d*x] + 2*a*b^3*B*Sin[c + d*x] + 3*a^4*C*Sin[c
+ d*x] + a^2*b^2*C*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])
)))/(d*Sqrt[Cos[c + d*x]]*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])
*(a + b*Sec[c + d*x])^(5/2)) - (4*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2
*((2*a^2*A*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (10*A*b^2*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*A*b^4*Sqrt[Cos[c + d*x]])/(3*a^2*(a^
2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*a*b*B*Sqrt[Cos
[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (
4*b^3*B*Sqrt[Cos[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (2*a^2*C*Sqrt[Cos[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*b^2*C*Sqrt[Cos[c + d*x]])/(3*(a^2 - b
^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*a*A*b*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (4*A*b
^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[b + a*Cos
[c + d*x]]) + (2*a^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^
2*Sqrt[b + a*Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]
])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a*b*C*Sqrt[Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Sec[c
```

```
 + d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Se
c[c + d*x]^2)*((-I)*(a + b)*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C
) - a^2*b^2*(15*A + C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)] + I*a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A + B) + 3*a^3*(A + B - C) - a^2*
b*(9*A - 3*B + C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
 - (8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*(
b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a*(a^3
 - a*b^2)^2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[
c + d*x])^(5/2)*((-2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(
3/2)*Sin[c + d*x]*((-I)*(a + b)*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A
 - C) - a^2*b^2*(15*A + C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] + I*a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A + B) + 3*a^3*(A + B - C) -
a^2*b*(9*A - 3*B + C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b)] - (8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C
))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*(a
^3 - a*b^2)^2*(b + a*Cos[c + d*x])^(3/2)) + (2*Sqrt[Cos[c + d*x]]*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(8*A*b^4 + 6*a^3*
b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*EllipticE[I*ArcSinh[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A +
 B) + 3*a^3*(A + B - C) - a^2*b*(9*A - 3*B + C))*EllipticF[I*ArcSinh[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2)/(a + b)] - (8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(
A - C) - a^2*b^2*(15*A + C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/(a*(a^3 - a*b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (4*Cos[
c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(-((8*A*b^4 + 6*a^3*
b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*(b + a*Cos[c + d*x])*
(Sec[(c + d*x)/2]^2)^(5/2))/2 - I*(a + b)*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B
+ 3*a^4*(A - C) - a^2*b^2*(15*A + C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A
+ B) + 3*a^3*(A + B - C) - a^2*b*(9*A - 3*B + C))*EllipticF[I*ArcSinh[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + a*(8*A*b^4 + 6*a^3*b*B -
 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*(Sec[(c + d*x)/2]^2)^(3/2)
*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^
4*(A - C) - a^2*b^2*(15*A + C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(
3/2)*Tan[(c + d*x)/2]^2)/2 - ((I/2)*(a + b)*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*
B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d
*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/
(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + ((I/2)*
a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A + B) + 3*a^3*(A + B - C) - a^2*b*(9*A - 3
*B + C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A + B
) + 3*a^3*(A + B - C) - a^2*b*(9*A - 3*B + C))*Sec[(c + d*x)/2]^4*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2
]^2]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(8*A*b^4 +
 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*Sec[(c + d*x)/
2]^4*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a
+ b)*Tan[(c + d*x)/2]^2)/(a + b)])/(2*Sqrt[1 + Tan[(c + d*x)/2]^2])))/(3*a*
(a^3 - a*b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*((-I)*(a + b)*(8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B
```

```
 + 3*a^4*(A - C) - a^2*b^2*(15*A + C))*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 - 2*a*b^2*(3*A + B) + 3*a^3*(A
+ B - C) - a^2*b*(9*A - 3*B + C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] - (8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B + 3*a^4*(A - C) - a^2*b^
2*(15*A + C))*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)
/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^
2*Sec[c + d*x]*Tan[c + d*x]))/(a*(a^3 - a*b^2)^2*Sqrt[b + a*Cos[c + d*x]]))
)
```

Maple [B] time = 1.005, size = 3773, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] 2/3/d*(cos(d*x+c)+1)^5*(-1+cos(d*x+c))^3*(-2*B*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^2*b^3-2*B*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b^2-C*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^3*b^2-C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^4*b-15*A*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)*a^3*b^2+8*A*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)*a*b^4-9*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*cos(d*x+c)*a^4*b+6*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*cos(d*x+c)*a^3*b^2-3*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1
/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+5*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^
2*(1/(cos(d*x+c)+1))^(3/2)-B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos
(d*x+c)+1))^(3/2)-2*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1
))^(3/2)-2*C*((a-b)/(a+b))^(1/2)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+
C*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-C*((a-b)/
(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-11*A*((a-b)/(a+b))
^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+4*A*((a-b)/(a+b))^(1/2)*
sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)-3*A*((a-b)/(a+b))^(1/2)*sin(d*x+c
)*cos(d*x+c)^2*a^5*(1/(cos(d*x+c)+1))^(3/2)+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^5-3*C*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^5+3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*cos(d*x+c)*a^5+3*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)*a^5-3*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*cos(d*x+c)*a^5+8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
```

```
/2))*b^5-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)
*a^4*b+8*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a
^2*b^3+6*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a
^4*b-6*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))
^(3/2)-3*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b*(1/(cos(d*x+c)
+1))^(3/2)+3*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2*(1/(cos(
d*x+c)+1))^(3/2)+3*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3*(1
/(cos(d*x+c)+1))^(3/2)-B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*
(1/(cos(d*x+c)+1))^(3/2)+12*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a*b
^4*(1/(cos(d*x+c)+1))^(3/2)-15*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*
a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+7*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x
+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)-3*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)+6*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1))^(3/2)+C*((a-b)/(a+b))^(1/2)*cos(d*x+c)
*a^4*b*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-3*A*((a-b)/(a+b))^(1/2)*sin(d*x+
c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)-3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^4*b+3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^4*b-3*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-2
*B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+6*B*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-2*B*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+3*C*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a^4*b-C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^3*b^2-3*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-
C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+8*A*((a-b)/(a
+b))^(1/2)*sin(d*x+c)*b^5*(1/(cos(d*x+c)+1))^(3/2)-9*A*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+6*A*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^2*b^3+8*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a*b^4+3*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4
*b-15*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3)*cos(d*
x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(cos(
d*x+c)+1))^(3/2)/a^3/(a+b)/(a-b)^2/(b+a*cos(d*x+c))^2/sin(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*se
c(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec
(d*x + c) + a)^(5/2), x)
```

3.1371 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=378

$$-\frac{2\left(a^2(-(3A+C))+abB+2Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\left(-5a^2b^2(A+C)+2a^3bB+a^4\right.}{3abd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b}}$$

```
[Out] (-2*(2*A*b^2 + a*b*B - a^2*(3*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]) - (2*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*
A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
 b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(A*b^4 + 2*a^3*b*B + 2*a*b^3*B + a
^4*C - 5*a^2*b^2*(A + C))*Sin[c + d*x])/(3*a*b*(a^2 - b^2)^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.22934, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4265, 4098, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(-5a^2b^2(A+C)+2a^3bB+a^4C+2ab^3B+Ab^4\right)}{3abd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a + b*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] (-2*(2*A*b^2 + a*b*B - a^2*(3*A + C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]) - (2*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*
A + 2*C))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
 b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(A*b^4 + 2*a^3*b*B + 2*a*b^3*B + a
^4*C - 5*a^2*b^2*(A + C))*Sin[c + d*x])/(3*a*b*(a^2 - b^2)^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
```

```
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(A + B\sec(c+dx) + C\sec^2(c\right.}{(a+b\sec(c+dx))^{5/2}}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4 + 2a^3bB + 2ab^3\right.}{3ab\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4 + 2a^3bB + 2ab^3\right.}{3ab\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4 + 2a^3bB + 2ab^3\right.}{3ab\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(Ab^2 - a(bB - aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4 + 2a^3bB + 2ab^3\right.}{3ab\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(2Ab^2 + abB - a^2(3A+C)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(2Ab^3 +\right.}{}$$

Mathematica [C] time = 19.3603, size = 673, normalized size = 1.78

$$\frac{(a\cos(c+dx)+b)^3\left(A + B\sec(c+dx) + C\sec^2(c+dx)\right)\left(\frac{4\left(a^2C\sin(c+dx) - abB\sin(c+dx) + Ab^2\sin(c+dx)\right)}{3a\left(a^2-b^2\right)(a\cos(c+dx)+b)^2} + \frac{4\left(-6a^2Ab\sin(c+dx) - 4a^2bC\right.}{}\right.}{d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}(A\cos(2c+2dx) + A + 2B\cos(c+dx) +}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(a +
b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((4*(A*b^2*
Sin[c + d*x] - a*b*B*Sin[c + d*x] + a^2*C*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(
b + a*Cos[c + d*x])^2) + (4*(-6*a^2*A*b*Sin[c + d*x] + 2*A*b^3*Sin[c + d*x]
 + 3*a^3*B*Sin[c + d*x] + a*b^2*B*Sin[c + d*x] - 4*a^2*b*C*Sin[c + d*x]))/(
3*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Sqrt[Cos[c + d*x]]*(A + 2*C +
2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^(5/2)) + (4*Cos
[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]*(Cos[(c + d*x)/2]
^2*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)*((-I)*(a + b
)*(2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*EllipticE[I*ArcSinh[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(-2*A*b^2 + a^2*(3*A - 3*
B + C) + a*b*(3*A - B + 3*C))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
```

```
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] - (2*A*b^3 + 3*a^3*B + a*b^2*B - 2*a^2*b*(3*A + 2*C))*(b + a*Co
s[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^
2*d*(A + 2*C + 2*B*Cos[c + d*x] + A*Cos[2*c + 2*d*x])*(a + b*Sec[c + d*x])^
(5/2))
```

Maple [B] time = 0.785, size = 2767, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)
,x)
```

```
[Out] 2/3/d*(cos(d*x+c)+1)^5*(-1+cos(d*x+c))^3*(3*A*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^3*b-3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a^2*b^2-2*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
*b^3+6*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-2*A*((
a-b)/(a+b))^(1/2)*sin(d*x+c)*b^4*(1/(cos(d*x+c)+1))^(3/2)-C*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)-3*B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*sin(d*x+c)*a^4*(1/(cos(d*x+c)+1))^(3/2)+5*A*((a-b)/(a+b))^(
1/2)*sin(d*x+c)*a^2*b^2*(1/(cos(d*x+c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(
d*x+c)*a*b^3*(1/(cos(d*x+c)+1))^(3/2)-2*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^
3*b*(1/(cos(d*x+c)+1))^(3/2)+B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(c
os(d*x+c)+1))^(3/2)-B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(cos(d*x+c)+1
))^(3/2)-C*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)+4*
C*((a-b)/(a+b))^(1/2)*a^2*b^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(3/2)-3*B*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-B*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+3*B*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^3*b-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^2*b^2+4*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
+C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b-B*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b-B*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^2*b^2-3*C*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b+4*C*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b-3*A*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^3*b-2*A*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^2*b^2+6*A*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b-2*A*(1/(a+b)*(b+a*cos(d*x+c)
```

```
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a*b^3-C*((a-b)/(a+b))^(1/2)*(1/(cos(
d*x+c)+1))^(3/2)*sin(d*x+c)*a^4+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*cos(d*x+c)*a^4-3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*cos(d*x+c)*a^4+C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*cos(d*x+c)*a^4+3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)*a^4+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(cos(d
*x+c)+1))^(3/2)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^2*(1/(cos
(d*x+c)+1))^(3/2)-3*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^3*(1/(c
os(d*x+c)+1))^(3/2)+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b*(1/(c
os(d*x+c)+1))^(3/2)+3*C*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*sin(d*x+c)*(1/
(cos(d*x+c)+1))^(3/2)-2*A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*b^4-3*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2)*cos
(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(c
os(d*x+c)+1))^(3/2)/(a+b)/(a-b)^2/a^2/(b+a*cos(d*x+c))^2/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C\sec\left(dx+c\right)^{2}+B\sec\left(dx+c\right)+A\right)\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)\sec\left(dx+c\right)+a^{3}\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)
^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*s
qrt(cos(d*x + c))/(b^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*s
ec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/(a+b*sec(d*x+c))**(5/2)/cos(d*x+
c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(b \sec(dx+c)+a)^{\frac{5}{2}} \sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*sqrt(cos(d*x + c))), x)
```

3.1372 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=447

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3abd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+\frac{2\sin\cdots}{3\cdots}$$

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)])/(3*a*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*a)/(a + b)])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) - (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*
b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b^2 - a*(b*
B - a*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c
+ d*x])^(3/2)) + (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sin
[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]])
```

Rubi [A] time = 1.64815, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {4265, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+\frac{2\sin(c+dx)\left(a^2b^2(3A+7C)-3a^4C-4ab^3B+Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\left(Ab^2-a(\cdots\right.}{3abd\left(a^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + b*Sec[
c + d*x])^(5/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)])/(3*a*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + (2*C*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (
c + d*x)/2, (2*a)/(a + b)])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) - (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*
b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b^2 - a*(b*
B - a*C))*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c
+ d*x])^(3/2)) + (2*(A*b^4 - 4*a*b^3*B - 3*a^4*C + a^2*b^2*(3*A + 7*C))*Sin
[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]])
```

Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(A+B\sec(c+dx)+C\sec^2\right.}{(a+b\sec(c+dx))^{5/2}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4-4ab^3B-3a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4-4ab^3B-3a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4-4ab^3B-3a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(Ab^4-4ab^3B-3a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2C\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(Ab^2-a(bB-aC)\right)\sin(}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ab\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2C\sqrt{\frac{b+a\cos(c+dx}{a+b}}}{b^2d\sqrt{\cos(c+d}}$$

Mathematica [C] time = 36.9811, size = 119861, normalized size = 268.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(a + 
b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.718, size = 3739, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^3*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)
^5*(-4*B*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
```

a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^2
b^3-B(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b
^2+7*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^3*b
^2+4*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^4*b
+3*A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^3*b^2
+A*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a*b^4-3*A
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^3*b^2-3*C
*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+B*(
(a-b)/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1))^(3/2)+B*((a-b)/(a+
b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+c)+1))^(3/2)-4*B*((a-b)/(a+b))^(1/
2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2)-4*C*((a-b)/(a+b))^(1/2)*a^4*b*
(1/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-C*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(cos(d
*x+c)+1))^(3/2)*sin(d*x+c)+7*C*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(cos(d*x+c)+1
))^(3/2)*sin(d*x+c)+2*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3*(1/(cos(d*x+
c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*a*b^4*(1/(cos(d*x+c)+1))^(3/2
)-3*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*a^5+6*
C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a^5+A*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5-6*C*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^4*b+6*C*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b
^2+6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d
x+c))((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^2*b^3-9*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)*a^3*b^2-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)*a^2*b^3+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)*a^2*b^3+3*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^4-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a
^4*b-6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
a^3*b^2+6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2
))*a^2*b^3+6*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*a*b^4-3*C*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-6
C(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*
x+c)*a^5+A*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*a
^2*b^3+B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c)+1
))^(3/2)+3*A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+
c)+1))^(3/2)-A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(cos(d*
x+c)+1))^(3/2)-3*B*((a-b)/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3*(1/(co

```
s(d*x+c)+1))^(3/2)-C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^4*b*(1/(cos(d*x+c)+1)
)^(3/2)*sin(d*x+c)+6*C*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*b^2*(1/(cos(d*x+c
)+1))^(3/2)*sin(d*x+c)-9*C*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^2*b^3-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-4
*B*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+6*C*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+4*C*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-3*C*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^4*b+7*C*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^2*b^3+A*((a-b)/(a+b))^(1/2)*sin(d*x+c)*b^5*(1/(cos(d*x+c)+1))^(3/2)-
3*A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+A*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+3*A*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3)*cos(d*x+c)^(1/2)*((a-b)/(a+b))^(1/2)
*(1/(cos(d*x+c)+1))^(3/2)/a/(a+b)/(a-b)^2/b^2/(b+a*cos(d*x+c))^2/sin(d*x+c)
^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec (dx + c)^2 + B \sec (dx + c) + A}{(b \sec (dx + c) + a)^{\frac{5}{2}} \cos (dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*cos(d*x + c)^(3/2)), x)
```

3.1373 $\int \frac{A+B\sec(c+dx)+C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=563

$$\frac{\left(5a^2C-2abB+2Ab^2-3b^2C\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4\right.}{3b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}$$

```
[Out] ((2*A*b^2 - 2*a*b*B + 5*a^2*C - 3*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 5*a*C)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^3*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b^4 + 6*a^3*b*B - 14*a*b^3*B - 15*a^4
*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a
)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b*(a^2 -
 b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^
3*b*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Sin[c + d*x])/(3*b^2*(a^2
- b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - ((8*A*b^4 + 6*a^3
*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 2.16349, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules used = {4265, 4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sin(c+dx)\left(a^2b^2(A+9C)+2a^3bB-5a^4C-6ab^3B+3Ab^4\right)}{3b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\sin(c+dx)\left(Ab^2-a(bB-aC)\right)}{3bd\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a + b*Sec[
c + d*x])^(5/2)),x]
```


```
[Out] ((2*A*b^2 - 2*a*b*B + 5*a^2*C - 3*b^2*C)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*b*B - 5*a*C)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^3*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*A*b^4 + 6*a^3*b*B - 14*a*b^3*B - 15*a^4
*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a
)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]) - (2*(A*b^2 - a*(b*B - a*C))*Sin[c + d*x])/(3*b*(a^2 -
 b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)) + (2*(3*A*b^4 + 2*a^
3*b*B - 6*a*b^3*B - 5*a^4*C + a^2*b^2*(A + 9*C))*Sin[c + d*x])/(3*b^2*(a^2
- b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) - ((8*A*b^4 + 6*a^3
*b*B - 14*a*b^3*B - 15*a^4*C + 26*a^2*b^2*C - 3*b^4*C)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```


Rule 4265

```
Int[(cos[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Cos[a
+ b*x])^m*(c*Sec[a + b*x])^m, Int[ActivateTrig[u]/(c*Sec[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{A + B\sec(c+dx) + C\sec^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(A + B\sec(c+dx) + C\sec^2(c+\right.}{(a+b\sec(c+dx))^{5/2}}$$

$$= -\frac{2\left(Ab^2 - a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{2\left(Ab^2 - a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4 + 2a^3bB - 6ab^{\right.}}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2\left(Ab^2 - a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4 + 2a^3bB - 6ab^{\right.}}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2\left(Ab^2 - a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4 + 2a^3bB - 6ab^{\right.}}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2\left(Ab^2 - a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4 + 2a^3bB - 6ab^{\right.}}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2\left(Ab^2 - a(bB-aC)\right)\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2\left(3Ab^4 + 2a^3bB - 6ab^{\right.}}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{(2bB-5aC)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(Ab^2 - a(bB-aC\right.}{3b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\left(2Ab^2 - 2abB + 5a^2C - 3b^2C\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2bB-5aC}{b^3d\sqrt{}}$$

Mathematica [C] time = 38.0549, size = 215866, normalized size = 383.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(a +
b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.839, size = 5561, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2)
,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)**2)/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c
))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \sec(dx+c)^2 + B \sec(dx+c) + A}{(b \sec(dx+c) + a)^{\frac{5}{2}} \cos(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c)+C*sec(d*x+c)^2)/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))
^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*sec(d*x + c)^2 + B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/
2)*cos(d*x + c)^(5/2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                          #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```